PROBLEM 1.1

KNOWN: Heat rate, q, through one-dimensional wall of area A, thickness L, thermal conductivity k and inner temperature, T_1.

FIND: The outer temperature of the wall, T_2.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional conduction in the x-direction, (2) Steady-state conditions, (3) Constant properties.

ANALYSIS: The rate equation for conduction through the wall is given by Fourier's law,

$$q_{cond} = q_x = q''_x \cdot A = -k \frac{dT}{dx} \cdot A = kA \frac{T_1 - T_2}{L}.$$

Solving for T_2 gives

$$T_2 = T_1 - \frac{q_{cond} L}{kA}.$$

Substituting numerical values, find

$$T_2 = 415^\circ C - \frac{3000 W \times 0.025 m}{0.2 W/m \cdot K \times 10 m^2}$$

$$T_2 = 415^\circ C - 37.5^\circ C$$

$$T_2 = 378^\circ C.$$ <

COMMENTS: Note direction of heat flow and fact that T_2 must be less than T_1.

PROBLEM 1.2

KNOWN: Inner surface temperature and thermal conductivity of a concrete wall.

FIND: Heat loss by conduction through the wall as a function of ambient air temperatures ranging from -15 to 38°C.

SCHEMATIC:

$A = 20\ m^2$
$k = 1, 0.75,$ or 1.25 W/m·K
$T_1 = 25\ ^oC$
$T_2 = -15$ to $38\ ^oC$
$T(x)$
$L = 0.30$ m
x

ASSUMPTIONS: (1) One-dimensional conduction in the x-direction, (2) Steady-state conditions, (3) Constant properties, (4) Outside wall temperature is that of the ambient air.

ANALYSIS: From Fourier's law, it is evident that the gradient, $dT/dx = -q''_x/k$, is a constant, and hence the temperature distribution is linear, if q''_x and k are each constant. The heat flux must be constant under one-dimensional, steady-state conditions; and k is approximately constant if it depends only weakly on temperature. The heat flux and heat rate when the outside wall temperature is $T_2 = -15°C$ are

$$q''_x = -k\frac{dT}{dx} = k\frac{T_1 - T_2}{L} = 1\,W/m\cdot K\,\frac{25^\circ C - \left(-15^\circ C\right)}{0.30\,m} = 133.3\,W/m^2\,. \qquad (1)$$

$$q_x = q''_x \times A = 133.3\,W/m^2 \times 20\,m^2 = 2667\,W\,. \qquad (2) <$$

Combining Eqs. (1) and (2), the heat rate q_x can be determined for the range of ambient temperature, $-15 \le T_2 \le 38°C$, with different wall thermal conductivities, k.

Heat loss, qx (W)
3500
2500
1500
500
-500
-1500
-20 -10 0 10 20 30 40
Ambient air temperature, T2 (C)
Wall thermal conductivity, k = 1.25 W/m.K
k = 1 W/m.K, concrete wall
k = 0.75 W/m.K

For the concrete wall, k = 1 W/m·K, the heat loss varies linearily from +2667 W to -867 W and is zero when the inside and ambient temperatures are the same. The magnitude of the heat rate increases with increasing thermal conductivity.

COMMENTS: Without steady-state conditions and constant k, the temperature distribution in a plane wall would not be linear.

PROBLEM 1.3

KNOWN: Dimensions, thermal conductivity and surface temperatures of a concrete slab. Efficiency of gas furnace and cost of natural gas.

FIND: Daily cost of heat loss.

SCHEMATIC:

Furnace, $\eta_f = 0.90$
Natural gas, C_g = $0.01/MJ
Warm air
Concrete, k = 1.4 W/m-K
$T_1 = 17^oC$
q
$T_2 = 10^oC$
L = 11 m
W = 8 m
t = 0.2 m

ASSUMPTIONS: (1) Steady state, (2) One-dimensional conduction, (3) Constant properties.

ANALYSIS: The rate of heat loss by conduction through the slab is

$$q = k\,(LW)\frac{T_1 - T_2}{t} = 1.4\,W/m\cdot K\,(11\,m \times 8\,m)\frac{7°C}{0.20\,m} = 4312\ W$$ <

The daily cost of natural gas that must be combusted to compensate for the heat loss is

$$C_d = \frac{q\,C_g}{\eta_f}(\Delta t) = \frac{4312\,W \times \$0.01/MJ}{0.9 \times 10^6\,J/MJ}(24\,h/d \times 3600\,s/h) = \$4.14/d$$ <

COMMENTS: The loss could be reduced by installing a floor covering with a layer of insulation between it and the concrete.

PROBLEM 1.4

KNOWN: Heat flux and surface temperatures associated with a wood slab of prescribed thickness.

FIND: Thermal conductivity, k, of the wood.

SCHEMATIC:

$T_1 = 40°C$, $q''_x = 40\ W/m^2$, $T(x)$, $T_2 = 20°C$, $L = 0.05\ m$, T, x

ASSUMPTIONS: (1) One-dimensional conduction in the x-direction, (2) Steady-state conditions, (3) Constant properties.

ANALYSIS: Subject to the foregoing assumptions, the thermal conductivity may be determined from Fourier's law, Eq. 1.2. Rearranging,

$$k = q''_x \frac{L}{T_1 - T_2} = 40 \frac{W}{m^2} \frac{0.05m}{(40\text{-}20)^\circ C}$$

$$k = 0.10\ W/m \cdot K.$$ <

COMMENTS: Note that the °C or K temperature units may be used interchangeably when evaluating a temperature difference.

PROBLEM 1.5

KNOWN: Inner and outer surface temperatures of a glass window of prescribed dimensions.

FIND: Heat loss through window.

SCHEMATIC:

T

$T_1 = 15°C$

$A = 1m \times 3m = 3m^2$,
$k = 1.4 W/m \cdot K$

$T_2 = 5°C$

$L = 0.005m$

x

ASSUMPTIONS: (1) One-dimensional conduction in the x-direction, (2) Steady-state conditions, (3) Constant properties.

ANALYSIS: Subject to the foregoing conditions the heat flux may be computed from Fourier's law, Eq. 1.2.

$$q''_x = k \frac{T_1 - T_2}{L}$$

$$q''_x = 1.4 \frac{W}{m \cdot K} \frac{(15\text{-}5)^\circ C}{0.005m}$$

$$q''_x = 2800 \text{ W/m}^2.$$

Since the heat flux is uniform over the surface, the heat loss (rate) is

$$q = q''_x \times A$$

$$q = 2800 \text{ W/m}^2 \times 3m^2$$

$$q = 8400 \text{ W}. \quad <$$

COMMENTS: A linear temperature distribution exists in the glass for the prescribed conditions.

PROBLEM 1.6

KNOWN: Width, height, thickness and thermal conductivity of a single pane window and the air space of a double pane window. Representative winter surface temperatures of single pane and air space.

FIND: Heat loss through single and double pane windows.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional conduction through glass or air, (2) Steady-state conditions, (3) Enclosed air of double pane window is stagnant (negligible buoyancy induced motion).

ANALYSIS: From Fourier's law, the heat losses are

Single Pane: $$q_g = k_g A \frac{T_1 - T_2}{L} = 1.4\ \text{W/m}\cdot\text{K}\left(2\text{m}^2\right)\frac{35\ ^\circ\text{C}}{0.005\text{m}} = 19,600\ \text{W}$$

Double Pane: $$q_a = k_a A \frac{T_1 - T_2}{L} = 0.024\left(2\text{m}^2\right)\frac{25\ ^\circ\text{C}}{0.010\ \text{m}} = 120\ \text{W}$$

COMMENTS: Losses associated with a single pane are unacceptable and would remain excessive, even if the thickness of the glass were doubled to match that of the air space. The principal advantage of the double pane construction resides with the low thermal conductivity of air (~ 60 times smaller than that of glass). For a fixed ambient outside air temperature, use of the double pane construction would also increase the surface temperature of the glass exposed to the room (inside) air.

PROBLEM 1.7

KNOWN: Dimensions of freezer compartment. Inner and outer surface temperatures.

FIND: Thickness of styrofoam insulation needed to maintain heat load below prescribed value.

SCHEMATIC:

ASSUMPTIONS: (1) Perfectly insulated bottom, (2) One-dimensional conduction through 5 walls of area $A = 4m^2$, (3) Steady-state conditions, (4) Constant properties.

ANALYSIS: Using Fourier's law, Eq. 1.2, the heat rate is

$$q = q'' \cdot A = k \frac{\Delta T}{L} A_{total}$$

Solving for L and recognizing that $A_{total} = 5 \times W^2$, find

$$L = \frac{5\, k\, \Delta T\, W^2}{q}$$

$$L = \frac{5 \times 0.03\ W/m \cdot K\ \left[35 - (-10)\right]^{\circ} C\ \left(4m^2\right)}{500\ W}$$

$$L = 0.054m = 54mm.$$ <

COMMENTS: The corners will cause local departures from one-dimensional conduction and a slightly larger heat loss.

PROBLEM 1.8

KNOWN: Dimensions and thermal conductivity of food/beverage container. Inner and outer surface temperatures.

FIND: Heat flux through container wall and total heat load.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible heat transfer through bottom wall, (3) Uniform surface temperatures and one-dimensional conduction through remaining walls.

ANALYSIS: From Fourier's law, Eq. 1.2, the heat flux is

$$q'' = k\frac{T_2 - T_1}{L} = \frac{0.023\ \text{W/m}\cdot\text{K}\ (20-2)^\circ\text{C}}{0.025\ \text{m}} = 16.6\ \text{W/m}^2$$ <

Since the flux is uniform over each of the five walls through which heat is transferred, the heat load is

$$q = q'' \times A_{total} = q''\left[H\left(2W_1 + 2W_2\right) + W_1 \times W_2\right]$$

$$q = 16.6\ \text{W/m}^2\left[0.6\text{m}\left(1.6\text{m} + 1.2\text{m}\right) + \left(0.8\text{m} \times 0.6\text{m}\right)\right] = 35.9\ \text{W}$$ <

COMMENTS: The corners and edges of the container create local departures from one-dimensional conduction, which increase the heat load. However, for H, W_1, W_2 >> L, the effect is negligible.

PROBLEM 1.9

KNOWN: Masonry wall of known thermal conductivity has a heat rate which is 80% of that through a composite wall of prescribed thermal conductivity and thickness.

FIND: Thickness of masonry wall.

SCHEMATIC:

ASSUMPTIONS: (1) Both walls subjected to same surface temperatures, (2) One-dimensional conduction, (3) Steady-state conditions, (4) Constant properties.

ANALYSIS: For steady-state conditions, the conduction heat flux through a one-dimensional wall follows from Fourier's law, Eq. 1.2,

$$q'' = k\frac{\Delta T}{L}$$

where ΔT represents the difference in surface temperatures. Since ΔT is the same for both walls, it follows that

$$L_1 = L_2 \frac{k_1}{k_2} \cdot \frac{q_2''}{q_1''}.$$

With the heat fluxes related as

$$q_1'' = 0.8\, q_2''$$

$$L_1 = 100\text{mm}\,\frac{0.75\ \text{W/m}\cdot\text{K}}{0.25\ \text{W/m}\cdot\text{K}} \times \frac{1}{0.8} = 375\text{mm}.$$ <

COMMENTS: Not knowing the temperature difference across the walls, we cannot find the value of the heat rate.

PROBLEM 1.10

KNOWN: Thickness, diameter and inner surface temperature of bottom of pan used to boil water. Rate of heat transfer to the pan.

FIND: Outer surface temperature of pan for an aluminum and a copper bottom.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional, steady-state conduction through bottom of pan.

ANALYSIS: From Fourier's law, the rate of heat transfer by conduction through the bottom of the pan is

$$q = kA\frac{T_1 - T_2}{L}$$

Hence,

$$T_1 = T_2 + \frac{qL}{kA}$$

where $A = \pi D^2/4 = \pi(0.2\text{m})^2/4 = 0.0314\ \text{m}^2$.

Aluminum: $$T_1 = 110\ {}^\circ\text{C} + \frac{600\text{W}(0.005\ \text{m})}{240\ \text{W/m}\cdot\text{K}\left(0.0314\ \text{m}^2\right)} = 110.40\ {}^\circ\text{C}$$

Copper: $$T_1 = 110\ {}^\circ\text{C} + \frac{600\text{W}(0.005\ \text{m})}{390\ \text{W/m}\cdot\text{K}\left(0.0314\ \text{m}^2\right)} = 110.25\ {}^\circ\text{C}$$

COMMENTS: Although the temperature drop across the bottom is slightly larger for aluminum (due to its smaller thermal conductivity), it is sufficiently small to be negligible for both materials. To a good approximation, the bottom may be considered *isothermal* at T ≈ 110 °C, which is a desirable feature of pots and pans.

PROBLEM 1.11

KNOWN: Dimensions and thermal conductivity of a chip. Power dissipated on one surface.

FIND: Temperature drop across the chip.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Uniform heat dissipation, (4) Negligible heat loss from back and sides, (5) One-dimensional conduction in chip.

ANALYSIS: All of the electrical power dissipated at the back surface of the chip is transferred by conduction through the chip. Hence, from Fourier's law,

$$P = q = kA \frac{\Delta T}{t}$$

or

$$\Delta T = \frac{t \cdot P}{kW^2} = \frac{0.001\ \text{m} \times 4\ \text{W}}{150\ \text{W/m} \cdot \text{K} \left(0.005\ \text{m}\right)^2}$$

$$\Delta T = 1.1^\circ\ \text{C}.$$ <

COMMENTS: For fixed P, the temperature drop across the chip decreases with increasing k and W, as well as with decreasing t.

PROBLEM 1.12

KNOWN: Heat flux gage with thin-film thermocouples on upper and lower surfaces; output voltage, calibration constant, thickness and thermal conductivity of gage.

FIND: (a) Heat flux, (b) Precaution when sandwiching gage between two materials.

SCHEMATIC:

t = 0.25 mm
q″
ΔE = 350 μV
Gage, k = 1.4 W/m·K
5 pair type-K TC junctions, S_{AB} = 40 μV/°C
Gage bonded between laminates

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat conduction in gage, (3) Constant properties.

ANALYSIS: (a) Fourier's law applied to the gage can be written as

$$q'' = k\,\frac{\Delta T}{\Delta x}$$

and the gradient can be expressed as

$$\frac{\Delta T}{\Delta x} = \frac{\Delta E / N}{S_{AB} t}$$

where N is the number of differentially connected thermocouple junctions, S_{AB} is the Seebeck coefficient for type K thermocouples (A-chromel and B-alumel), and $\Delta x = t$ is the gage thickness. Hence,

$$q'' = \frac{k\Delta E}{N S_{AB} t}$$

$$q'' = \frac{1.4\ \mathrm{W/m\cdot K} \times 350 \times 10^{-6}\ \mathrm{V}}{5 \times 40 \times 10^{-6}\ \mathrm{V/^\circ C} \times 0.25 \times 10^{-3}\ \mathrm{m}} = 9800\ \mathrm{W/m^2}.$$ <

(b) The major precaution to be taken with this type of gage is to match its thermal conductivity with that of the material on which it is installed. If the gage is bonded between laminates (see sketch above) and its thermal conductivity is significantly different from that of the laminates, one dimensional heat flow will be disturbed and the gage will read incorrectly.

COMMENTS: If the thermal conductivity of the gage is lower than that of the laminates, will it indicate heat fluxes that are systematically high or low?

PROBLEM 1.13

KNOWN: Hand experiencing convection heat transfer with moving air and water.

FIND: Determine which condition feels colder. Contrast these results with a heat loss of 30 W/m^2 under normal room conditions.

SCHEMATIC:

Water
T_∞ = 10 °C
V = 0.2 m/s
h = 900 W/m^2·K

q''_{cv}
Hand
T_s = 30 °C

Air
T_∞ = -5 °C
V = 35 km/h
h = 40 W/m^2·K

ASSUMPTIONS: (1) Temperature is uniform over the hand's surface, (2) Convection coefficient is uniform over the hand, and (3) Negligible radiation exchange between hand and surroundings in the case of air flow.

ANALYSIS: The hand will feel colder for the condition which results in the larger heat loss. The heat loss can be determined from Newton's law of cooling, Eq. 1.3a, written as

$$q'' = h\left(T_s - T_\infty\right)$$

For the air stream:

$$q''_{air} = 40\,\mathrm{W/m^2 \cdot K}\left[30 - (-5)\right]\mathrm{K} = 1,400\,\mathrm{W/m^2}$$ <

For the water stream:

$$q''_{water} = 900\,\mathrm{W/m^2 \cdot K}\left(30 - 10\right)\mathrm{K} = 18,000\,\mathrm{W/m^2}$$ <

COMMENTS: The heat loss for the hand in the water stream is an order of magnitude larger than when in the air stream for the given temperature and convection coefficient conditions. In contrast, the heat loss in a normal room environment is only 30 W/m^2 which is a factor of 400 times less than the loss in the air stream. In the room environment, the hand would feel comfortable; in the air and water streams, as you probably know from experience, the hand would feel uncomfortably cold since the heat loss is excessively high.

PROBLEM 1.14

KNOWN: Power required to maintain the surface temperature of a long, 25-mm diameter cylinder with an imbedded electrical heater for different air velocities.

FIND: (a) Determine the convection coefficient for each of the air velocity conditions and display the results graphically, and (b) Assuming that the convection coefficient depends upon air velocity as $h = CV^n$, determine the parameters C and n.

SCHEMATIC:

Air; T_s = 300 °C; V, T_∞ = 40 °C; P'_e

V(m/s)	1	2	4	8	12
P'_e (W/m)	450	658	983	1507	1963
h (W/m²·K)	22.0	32.2	48.1	73.8	96.1

ASSUMPTIONS: (1) Temperature is uniform over the cylinder surface, (2) Negligible radiation exchange between the cylinder surface and the surroundings, (3) Steady-state conditions.

ANALYSIS: (a) From an overall energy balance on the cylinder, the power dissipated by the electrical heater is transferred by convection to the air stream. Using Newtons law of cooling on a per unit length basis,

$$P'_e = h(\pi D)(T_s - T_\infty)$$

where P'_e is the electrical power dissipated per unit length of the cylinder. For the V = 1 m/s condition, using the data from the table above, find

$$h = 450\,\mathrm{W/m} \Big/ \pi \times 0.025\,\mathrm{m}\,(300-40)^\circ\mathrm{C} = 22.0\,\mathrm{W/m^2{\cdot}K}$$ <

Repeating the calculations, find the convection coefficients for the remaining conditions which are tabulated above and plotted below. Note that h is not linear with respect to the air velocity.

(b) To determine the (C,n) parameters, we plotted h vs. V on log-log coordinates. Choosing C = 22.12 $W/m^2{\cdot}K(s/m)^n$, assuring a match at V = 1, we can readily find the exponent n from the slope of the h vs. V curve. From the trials with n = 0.8, 0.6 and 0.5, we recognize that n = 0.6 is a reasonable choice. Hence, C = 22.12 and n = 0.6. <

PROBLEM 1.15

KNOWN: Long, 30mm-diameter cylinder with embedded electrical heater; power required to maintain a specified surface temperature for water and air flows.

FIND: Convection coefficients for the water and air flow convection processes, h_w and h_a, respectively.

SCHEMATIC:

ASSUMPTIONS: (1) Flow is cross-wise over cylinder which is very long in the direction normal to flow.

ANALYSIS: The convection heat rate from the cylinder per unit length of the cylinder has the form

$$q' = h(\pi D)\ (T_s - T_\infty)$$

and solving for the heat transfer convection coefficient, find

$$h = \frac{q'}{\pi D\ (T_s - T_\infty)}.$$

Substituting numerical values for the water and air situations:

Water $$h_w = \frac{28 \times 10^3\ \text{W/m}}{\pi \times 0.030\text{m}\ (90\text{-}25)^\circ\ \text{C}} = 4{,}570\ \text{W/m}^2 \cdot \text{K}$$ <

Air $$h_a = \frac{400\ \text{W/m}}{\pi \times 0.030\text{m}\ (90\text{-}25)^\circ\ \text{C}} = 65\ \text{W/m}^2 \cdot \text{K}.$$ <

COMMENTS: Note that the air velocity is 10 times that of the water flow, yet

$$h_w \approx 70 \times h_a.$$

These values for the convection coefficient are typical for forced convection heat transfer with liquids and gases. See Table 1.1.

PROBLEM 1.16

KNOWN: Dimensions of a cartridge heater. Heater power. Convection coefficients in air and water at a prescribed temperature.

FIND: Heater surface temperatures in water and air.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) All of the electrical power is transferred to the fluid by convection, (3) Negligible heat transfer from ends.

ANALYSIS: With P = q_{conv}, Newton's law of cooling yields

$$P = hA\left(T_S - T_\infty\right) = h\pi DL\left(T_S - T_\infty\right)$$

$$T_S = T_\infty + \frac{P}{h\pi DL}.$$

In water,

$$T_s = 20^\circ C + \frac{2000\ W}{5000\ W/m^2 \cdot K \times \pi \times 0.02\ m \times 0.200\ m}$$

$$T_s = 20^\circ C + 31.8^\circ C = 51.8^\circ C.$$ <

In air,

$$T_s = 20^\circ C + \frac{2000\ W}{50\ W/m^2 \cdot K \times \pi \times 0.02\ m \times 0.200\ m}$$

$$T_s = 20^\circ C + 3183^\circ C = 3203^\circ C.$$ <

COMMENTS: (1) Air is much less effective than water as a heat transfer fluid. Hence, the cartridge temperature is much higher in air, so high, in fact, that the cartridge would melt.

(2) In air, the high cartridge temperature would render radiation significant.

PROBLEM 1.17

KNOWN: Length, diameter and calibration of a hot wire anemometer. Temperature of air stream. Current, voltage drop and surface temperature of wire for a particular application.

FIND: Air velocity

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible heat transfer from the wire by natural convection or radiation.

ANALYSIS: If all of the electric energy is transferred by convection to the air, the following equality must be satisfied

$$P_{elec} = EI = hA\left(T_s - T_\infty\right)$$

where $A = \pi DL = \pi\left(0.0005\text{m} \times 0.02\text{m}\right) = 3.14 \times 10^{-5}\text{m}^2$.

Hence,

$$h = \frac{EI}{A\left(T_s - T_\infty\right)} = \frac{5\text{V} \times 0.1\text{A}}{3.14 \times 10^{-5}\text{m}^2\left(50\ {}^\circ\text{C}\right)} = 318\ \text{W/m}^2 \cdot \text{K}$$

$$V = 6.25 \times 10^{-5} h^2 = 6.25 \times 10^{-5}\left(318\ \text{W/m}^2 \cdot \text{K}\right)^2 = 6.3\ \text{m/s}$$ <

COMMENTS: The convection coefficient is sufficiently large to render buoyancy (natural convection) and radiation effects negligible.

PROBLEM 1.18

KNOWN: Chip width and maximum allowable temperature. Coolant conditions.

FIND: Maximum allowable chip power for air and liquid coolants.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible heat transfer from sides and bottom, (3) Chip is at a uniform temperature (isothermal), (4) Negligible heat transfer by radiation in air.

ANALYSIS: All of the electrical power dissipated in the chip is transferred by convection to the coolant. Hence,

$$P = q$$

and from Newton's law of cooling,

$$P = hA(T - T_\infty) = h\,W^2(T - T_\infty).$$

In *air*,

$$P_{max} = 200\ \text{W/m}^2\cdot\text{K}(0.005\ \text{m})^2(85 - 15)\ {}^\circ\text{C} = 0.35\ \text{W}.$$ <

In the *dielectric liquid*

$$P_{max} = 3000\ \text{W/m}^2\cdot\text{K}(0.005\ \text{m})^2(85-15)\ {}^\circ\text{C} = 5.25\ \text{W}.$$ <

COMMENTS: Relative to liquids, air is a poor heat transfer fluid. Hence, in air the chip can dissipate far less energy than in the dielectric liquid.

PROBLEM 1.19

KNOWN: Length, diameter and maximum allowable surface temperature of a power transistor. Temperature and convection coefficient for air cooling.

FIND: Maximum allowable power dissipation.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible heat transfer through base of transistor, (3) Negligible heat transfer by radiation from surface of transistor.

ANALYSIS: Subject to the foregoing assumptions, the power dissipated by the transistor is equivalent to the rate at which heat is transferred by convection to the air. Hence,

$$P_{elec} = q_{conv} = hA\left(T_s - T_\infty\right)$$

where $A = \pi\left(DL + D^2/4\right) = \pi\left[0.012\text{m} \times 0.01\text{m} + \left(0.012\text{m}\right)^2/4\right] = 4.90 \times 10^{-4}\,\text{m}^2$.

For a maximum allowable surface temperature of 85°C, the power is

$$P_{elec} = 100\ \text{W/m}^2 \cdot \text{K}\left(4.90 \times 10^{-4}\text{m}^2\right)\ \left(85 - 25\right)^\circ \text{C} = 2.94\ \text{W}$$ <

COMMENTS: (1) For the prescribed surface temperature and convection coefficient, radiation will be negligible relative to convection. However, conduction through the base could be significant, thereby permitting operation at a larger power.

(2) The *local* convection coefficient varies over the surface, and *hot spots* could exist if there are locations at which the local value of *h* is substantially smaller than the prescribed average value.

PROBLEM 1.20

KNOWN: Air jet impingement is an effective means of cooling logic chips.

FIND: Procedure for measuring convection coefficients associated with a 10 mm × 10 mm chip.

SCHEMATIC:

ASSUMPTIONS: Steady-state conditions.

ANALYSIS: One approach would be to use the actual chip-substrate system, Case (a), to perform the measurements. In this case, the electric power dissipated in the chip would be transferred from the chip by radiation and conduction (to the substrate), as well as by convection to the jet. An energy balance for the chip yields $q_{elec} = q_{conv} + q_{cond} + q_{rad}$. Hence, with $q_{conv} = hA(T_s - T_\infty)$, where A = 100 mm^2 is the surface area of the chip,

$$h = \frac{q_{elec} - q_{cond} - q_{rad}}{A(T_s - T_\infty)} \tag{1}$$

While the electric power (q_{elec}) and the jet (T_∞) and surface (T_s) temperatures may be measured, losses from the chip by conduction and radiation would have to be estimated. Unless the losses are negligible (an unlikely condition), the accuracy of the procedure could be compromised by uncertainties associated with determining the conduction and radiation losses.

A second approach, Case (b), could involve fabrication of a heater assembly for which the conduction and radiation losses are controlled and minimized. A 10 mm × 10 mm copper block (k ~ 400 W/m·K) could be inserted in a poorly conducting substrate (k < 0.1 W/m·K) and a patch heater could be applied to the back of the block and insulated from below. If conduction to both the substrate and insulation could thereby be rendered negligible, heat would be transferred almost exclusively through the block. If radiation were rendered negligible by applying a low emissivity coating (ε < 0.1) to the surface of the copper block, virtually all of the heat would be transferred by convection to the jet. Hence, q_{cond} and q_{rad} may be neglected in equation (1), and the expression may be used to accurately determine h from the known (A) and measured (q_{elec}, T_s, T_∞) quantities.

COMMENTS: Since convection coefficients associated with gas flows are generally small, concurrent heat transfer by radiation and/or conduction must often be considered. However, jet impingement is one of the more effective means of transferring heat by convection and convection coefficients well in excess of 100 W/m^2·K may be achieved.

PROBLEM 1.21

KNOWN: Upper temperature set point, T_{set}, of a bimetallic switch and convection heat transfer coefficient between clothes dryer air and exposed surface of switch.

FIND: Electrical power for heater to maintain T_{set} when air temperature is $T_\infty = 50°C$.

SCHEMATIC:

q_{elec}
← Dryer wall
← Insulation pad
← Electrical heater
← Bimetallic switch
$T_\infty = 50°C$
$h = 25 W/m^2 \cdot K$
q_{conv}
$T_{set} = 70°C$, $A_s = 30 mm^2$

ASSUMPTIONS: (1) Steady-state conditions, (2) Electrical heater is perfectly insulated from dryer wall, (3) Heater and switch are isothermal at T_{set}, (4) Negligible heat transfer from sides of heater or switch, (5) Switch surface, A_s, loses heat only by convection.

ANALYSIS: Define a control volume around the bimetallic switch which experiences heat input from the heater and convection heat transfer to the dryer air. That is,

$$\dot{E}_{in} - \dot{E}_{out} = 0$$
$$q_{elec} - hA_s\left(T_{set} - T_\infty\right) = 0.$$

The electrical power required is,

$$q_{elec} = hA_s\left(T_{set} - T_\infty\right)$$

$$q_{elec} = 25 \text{ W/m}^2 \cdot \text{K} \times 30 \times 10^{-6} \text{ m}^2 \left(70 - 50\right)\text{K} = 15 \text{ mW}.$$ <

COMMENTS: (1) This type of controller can achieve variable operating air temperatures with a single set-point, inexpensive, bimetallic-thermostatic switch by adjusting power levels to the heater.

(2) Will the heater power requirement increase or decrease if the insulation pad is other than perfect?

PROBLEM 1.22

KNOWN: Hot vertical plate suspended in cool, still air. Change in plate temperature with time at the instant when the plate temperature is 225°C.

FIND: Convection heat transfer coefficient for this condition.

SCHEMATIC:

ASSUMPTIONS: (1) Plate is isothermal and of uniform temperature, (2) Negligible radiation exchange with surroundings, (3) Negligible heat lost through suspension wires.

ANALYSIS: As shown in the cooling curve above, the plate temperature decreases with time. The condition of interest is for time t_o. For a control surface about the plate, the conservation of energy requirement is

$$\dot{E}_{in} - \dot{E}_{out} = \dot{E}_{st}$$

$$-2hA_s\left(T_s - T_\infty\right) = Mc_p \frac{dT}{dt}$$

where A_s is the surface area of one side of the plate. Solving for h, find

$$h = \frac{Mc_p}{2A_s\left(T_s - T_\infty\right)} \frac{dT}{dt}$$

$$h = \frac{3.75\ \text{kg} \times 2770\ \text{J/kg} \cdot \text{K}}{2 \times (0.3 \times 0.3)\text{m}^2\ (225 - 25)\text{K}} \times 0.022\ \text{K/s} = 6.4\ \text{W/m}^2 \cdot \text{K}$$ <

COMMENTS: (1) Assuming the plate is very highly polished with emissivity of 0.08, determine whether radiation exchange with the surroundings at 25°C is negligible compared to convection.

(2) We will later consider the criterion for determining whether the isothermal plate assumption is reasonable. If the thermal conductivity of the present plate were high (such as aluminum or copper), the criterion would be satisfied.

PROBLEM 1.23

KNOWN: Width, input power and efficiency of a transmission. Temperature and convection coefficient associated with air flow over the casing.

FIND: Surface temperature of casing.

SCHEMATIC:

$T_\infty = 30°C$
$h_i = 200\ W/m^2\text{-}K$
q
$P_o = \eta P_i$
$P_i = 150$ hp
W = 0.3 m

ASSUMPTIONS: (1) Steady state, (2) Uniform convection coefficient and surface temperature, (3) Negligible radiation.

ANALYSIS: From Newton's law of cooling,

$$q = hA_s\,(T_s - T_\infty) = 6\,hW^2\,(T_s - T_\infty)$$

where the output power is $\eta\ P_i$ and the heat rate is

$$q = P_i - P_o = P_i\,(1-\eta) = 150\,hp \times 746\,W/hp \times 0.07 = 7833\,W$$

Hence,

$$T_s = T_\infty + \frac{q}{6\ hW^2} = 30°C + \frac{7833\ W}{6 \times 200\ W/m^2 \cdot K \times (0.3\,m)^2} = 102.5°C \qquad <$$

COMMENTS: There will, in fact, be considerable variability of the local convection coefficient over the transmission case and the prescribed value represents an average over the surface.

PROBLEM 1.24

KNOWN: Air and wall temperatures of a room. Surface temperature, convection coefficient and emissivity of a person in the room.

FIND: Basis for difference in comfort level between summer and winter.

SCHEMATIC:

Air

q''_{conv}

q''_{rad}

T_{sur} = 27°C (summer)
T_{sur} = -14°C (winter)

T_∞ = 20°C
h = 2 W/m²-K

T_s = 32°C
ε = 0.9

ASSUMPTIONS: (1) Person may be approximated as a small object in a large enclosure.

ANALYSIS: Thermal comfort is linked to heat loss from the human body, and a *chilled* feeling is associated with excessive heat loss. Because the temperature of the room air is fixed, the different summer and winter comfort levels can not be attributed to convection heat transfer from the body. In both cases, the heat flux is

Summer and Winter: $q''_{conv} = h\left(T_s - T_\infty\right) = 2\ \text{W/m}^2\cdot\text{K}\times 12\ ^\circ\text{C} = 24\ \text{W/m}^2$

However, the heat flux due to radiation will differ, with values of

Summer: $q''_{rad} = \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right) = 0.9\times 5.67\times 10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left(305^4 - 300^4\right)\text{K}^4 = 28.3\ \text{W/m}^2$

Winter: $q''_{rad} = \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right) = 0.9\times 5.67\times 10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left(305^4 - 287^4\right)\text{K}^4 = 95.4\ \text{W/m}^2$

There is a significant difference between winter and summer radiation fluxes, and the chilled condition is attributable to the effect of the colder walls on radiation.

COMMENTS: For a representative surface area of A = 1.5 m^2, the heat losses are q_{conv} = 36 W, $q_{rad(summer)}$ = 42.5 W and $q_{rad(winter)}$ = 143.1 W. The winter time radiation loss is significant and if maintained over a 24 h period would amount to 2,950 kcal.

PROBLEM 1.25

KNOWN: Diameter and emissivity of spherical interplanetary probe. Power dissipation within probe.

FIND: Probe surface temperature.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible radiation incident on the probe.

ANALYSIS: Conservation of energy dictates a balance between energy generation within the probe and radiation emission from the probe surface. Hence, at any instant

$$-\dot{E}_{out} + \dot{E}_g = 0$$

$$\varepsilon A_s \sigma T_s^4 = \dot{E}_g$$

$$T_s = \left(\frac{\dot{E}_g}{\varepsilon \pi D^2 \sigma} \right)^{1/4}$$

$$T_s = \left(\frac{150\text{W}}{0.8\pi \, (0.5\text{ m})^2 \, 5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4} \right)^{1/4}$$

$$T_s = 254.7\text{ K.}$$ <

COMMENTS: Incident radiation, as, for example, from the sun, would increase the surface temperature.

PROBLEM 1.26

KNOWN: Spherical shaped instrumentation package with prescribed surface emissivity within a large space-simulation chamber having walls at 77 K.

FIND: Acceptable power dissipation for operating the package surface temperature in the range T_s = 40 to 85°C. Show graphically the effect of emissivity variations for 0.2 and 0.3.

SCHEMATIC:

ASSUMPTIONS: (1) Uniform surface temperature, (2) Chamber walls are large compared to the spherical package, and (3) Steady-state conditions.

ANALYSIS: From an overall energy balance on the package, the internal power dissipation P_e will be transferred by radiation exchange between the package and the chamber walls. From Eq. 1.7,

$$q_{rad} = P_e = \varepsilon A_s \sigma \left(T_s^4 - T_{sur}^4\right)$$

For the condition when T_s = 40°C, with $A_s = \pi D^2$ the power dissipation will be

$$P_e = 0.25(\pi \times 0.10\,\mathrm{m}) \times 5.67 \times 10^{-8}\,\mathrm{W/m^2 \cdot K^4} \times \left[(40+273)^4 - 77^4\right]\mathrm{K}^4 = 4.3\,\mathrm{W}$$ <

Repeating this calculation for the range $40 \le T_s \le 85$°C, we can obtain the power dissipation as a function of surface temperature for the ε = 0.25 condition. Similarly, with 0.2 or 0.3, the family of curves shown below has been obtained.

COMMENTS: (1) As expected, the internal power dissipation increases with increasing emissivity and surface temperature. Because the radiation rate equation is non-linear with respect to temperature, the power dissipation will likewise not be linear with surface temperature.

(2) What is the maximum power dissipation that is possible if the surface temperature is not to exceed 85°C? What kind of a coating should be applied to the instrument package in order to approach this limiting condition?

PROBLEM 1.27

KNOWN: Area, emissivity and temperature of a surface placed in a large, evacuated chamber of prescribed temperature.

FIND: (a) Rate of surface radiation emission, (b) Net rate of radiation exchange between surface and chamber walls.

SCHEMATIC:

ASSUMPTIONS: (1) Area of the enclosed surface is much less than that of chamber walls.

ANALYSIS: (a) From Eq. 1.5, the rate at which radiation is emitted by the surface is

$$q_{emit} = E \cdot A = \varepsilon A \sigma T_s^4$$

$$q_{emit} = 0.8\left(0.5\ m^2\right) 5.67 \times 10^{-8}\ W/m^2 \cdot K^4 \left[(150 + 273)K\right]^4$$

$$q_{emit} = 726\ W.$$ <

(b) From Eq. 1.7, the *net* rate at which radiation is transferred *from* the surface to the chamber walls is

$$q = \varepsilon A \sigma \left(T_s^4 - T_{sur}^4\right)$$

$$q = 0.8\left(0.5\ m^2\right) 5.67 \times 10^{-8}\ W/m^2 \cdot K^4 \left[(423K)^4 - (298K)^4\right]$$

$$q = 547\ W.$$ <

COMMENTS: The foregoing result gives the net heat loss from the surface which occurs at the instant the surface is placed in the chamber. The surface would, of course, cool due to this heat loss and its temperature, as well as the heat loss, would decrease with increasing time. Steady-state conditions would eventually be achieved when the temperature of the surface reached that of the surroundings.

PROBLEM 1.28

KNOWN: Length, diameter, surface temperature and emissivity of steam line. Temperature and convection coefficient associated with ambient air. Efficiency and fuel cost for gas fired furnace.

FIND: (a) Rate of heat loss, (b) Annual cost of heat loss.

SCHEMATIC:

ASSUMPTIONS: (1) Steam line operates continuously throughout year, (2) Net radiation transfer is between small surface (steam line) and large enclosure (plant walls).

ANALYSIS: (a) From Eqs. (1.3a) and (1.7), the heat loss is

$$q = q_{conv} + q_{rad} = A\left[h\left(T_s - T_\infty\right) + \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right)\right]$$

where $A = \pi DL = \pi\left(0.1\text{m} \times 25\text{m}\right) = 7.85\text{m}^2$.

Hence,

$$q = 7.85\text{m}^2\left[10\ \text{W/m}^2 \cdot \text{K}\left(150 - 25\right)\text{K} + 0.8 \times 5.67 \times 10^{-8}\text{W/m}^2 \cdot \text{K}^4\left(423^4 - 298^4\right)\text{K}^4\right]$$

$$q = 7.85\text{m}^2\left(1,250 + 1,095\right)\text{w/m}^2 = \left(9813 + 8592\right)\text{W} = 18,405\ \text{W}$$ <

(b) The annual energy loss is

$$E = qt = 18,405\ \text{W} \times 3600\ \text{s/h} \times 24\text{h/d} \times 365\ \text{d/y} = 5.80 \times 10^{11}\ \text{J}$$

With a furnace energy consumption of $E_f = E/\eta_f = 6.45 \times 10^{11}\ \text{J}$, the annual cost of the loss is

$$C = C_g E_f = 0.01\ \$/\text{MJ} \times 6.45 \times 10^5\text{MJ} = \$6450$$ <

COMMENTS: The heat loss and related costs are unacceptable and should be reduced by insulating the steam line.

PROBLEM 1.29

KNOWN: Exact and approximate expressions for the linearized radiation coefficient, h_r and h_{ra}, respectively.

FIND: (a) Comparison of the coefficients with ε = 0.05 and 0.9 and surface temperatures which may exceed that of the surroundings (T_{sur} = 25°C) by 10 to 100°C; also comparison with a free convection coefficient correlation, (b) Plot of the relative error $(h_r - r_{ra})/h_r$ as a function of the furnace temperature associated with a workpiece at T_s = 25°C having ε = 0.05, 0.2 or 0.9.

ASSUMPTIONS: (1) Furnace walls are large compared to the workpiece and (2) Steady-state conditions.

ANALYSIS: (a) The linearized radiation coefficient, Eq. 1.9, follows from the radiation exchange rate equation,

$$h_r = \varepsilon\sigma\left(T_s + T_{sur}\right)\left(T_s^2 + T_{sur}^2\right)$$

If $T_s \approx T_{sur}$, the coefficient may be approximated by the simpler expression

$$h_{r,a} = 4\varepsilon\sigma\bar{T}^3 \qquad \bar{T} = \left(T_s + T_{sur}\right)/2$$

For the condition of ε = 0.05, $T_s = T_{sur} + 10$ = 35°C = 308 K and T_{sur} = 25°C = 298 K, find that

$$h_r = 0.05\times5.67\times10^{-8}\,\mathrm{W/m^2\cdot K^4}\,(308+298)\left(308^2+298^2\right)\mathrm{K^3} = 0.32\,\mathrm{W/m^2\cdot K}$$ <

$$h_{r,a} = 4\times0.05\times5.67\times10^{-8}\,\mathrm{W/m^2\cdot K^4}\left((308+298)/2\right)^3\mathrm{K^3} = 0.32\,\mathrm{W/m^2\cdot K}$$ <

The free convection coefficient with T_s = 35°C and $T_\infty = T_{sur}$ = 25°C, find that

$$h = 0.98\Delta T^{1/3} = 0.98\left(T_s - T_\infty\right)^{1/3} = 0.98(308-298)^{1/3} = 2.1\,\mathrm{W/m^2\cdot K}$$ <

For the range $T_s - T_{sur}$ = 10 to 100°C with ε = 0.05 and 0.9, the results for the coefficients are tabulated below. For this range of surface and surroundings temperatures, the radiation and free convection coefficients are of comparable magnitude for moderate values of the emissivity, say ε > 0.2. The approximate expression for the linearized radiation coefficient is valid within 2% for these conditions.

(b) The above expressions for the radiation coefficients, h_r and $h_{r,a}$, are used for the workpiece at T_s = 25°C placed inside a furnace with walls which may vary from 100 to 1000°C. The relative error, $(h_r - h_{ra})/h_r$, will be independent of the surface emissivity and is plotted as a function of T_{sur}. For T_{sur} > 150°C, the approximate expression provides estimates which are in error more than 5%. The approximate expression should be used with caution, and only for surface and surrounding temperature differences of 50 to 100°C.

		Coefficients ($W/m^2\cdot K$)		
T_s (°C)	ε	h_r	$h_{r,a}$	h
35	0.05	0.32	0.32	2.1
	0.9	5.7	5.7	
135	0.05	0.51	0.50	4.7
	0.9	9.2	9.0	

PROBLEM 1.30

KNOWN: Chip width, temperature, and heat loss by convection in air. Chip emissivity and temperature of large surroundings.

FIND: Increase in chip power due to radiation.

SCHEMATIC:

q_{conv} = 0.35W, q_{rad}, Air, Chip, T=358K, ε=0.9, T_{sur}=288K, 5mm

ASSUMPTIONS: (1) Steady-state conditions, (2) Radiation exchange between small surface and large enclosure.

ANALYSIS: Heat transfer from the chip due to net radiation exchange with the surroundings is

$$q_{rad} = \varepsilon W^2 \sigma \left(T^4 - T_{sur}^4\right)$$

$$q_{rad} = 0.9(0.005\text{ m})^2 \, 5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4 \left(358^4 - 288^4\right)\text{K}^4$$

$$q_{rad} = 0.0122\text{ W}.$$

The percent increase in chip power is therefore

$$\frac{\Delta P}{P}\times100 = \frac{q_{rad}}{q_{conv}}\times100 = \frac{0.0122\text{ W}}{0.350\text{ W}}\times100 = 3.5\%. \qquad <$$

COMMENTS: For the prescribed conditions, radiation effects are small. Relative to convection, the effect of radiation would increase with increasing chip temperature and decreasing convection coefficient.

PROBLEM 1.31

KNOWN: Width, surface emissivity and maximum allowable temperature of an electronic chip. Temperature of air and surroundings. Convection coefficient.

FIND: (a) Maximum power dissipation for free convection with $h(W/m^2 \cdot K) = 4.2(T - T_\infty)^{1/4}$, (b) Maximum power dissipation for forced convection with $h = 250\ W/m^2 \cdot K$.

SCHEMATIC:

$T_{sur} = 25^oC$
Air
$T_\infty = 25^oC$
$h = 4.2(T_s - T_\infty)^{1/4}$
or
$h = 250\ W/m^2\text{-}K$
q_{rad}
q_{conv}
Substrate
Chip, P_{elec}
$T_s = 85^oC$, $\varepsilon = 0.60$
L = 15 mm

ASSUMPTIONS: (1) Steady-state conditions, (2) Radiation exchange between a small surface and a large enclosure, (3) Negligible heat transfer from sides of chip or from back of chip by conduction through the substrate.

ANALYSIS: Subject to the foregoing assumptions, electric power dissipation by the chip must be balanced by convection and radiation heat transfer from the chip. Hence, from Eq. (1.10),

$$P_{elec} = q_{conv} + q_{rad} = hA\left(T_s - T_\infty\right) + \varepsilon A \sigma \left(T_s^4 - T_{sur}^4\right)$$

where $A = L^2 = (0.015m)^2 = 2.25 \times 10^{-4} m^2$.

(a) If heat transfer is by natural convection,

$$q_{conv} = C\,A\left(T_s - T_\infty\right)^{5/4} = 4.2\ W/m^2 \cdot K^{5/4}\left(2.25 \times 10^{-4} m^2\right)(60K)^{5/4} = 0.158\ W$$

$$q_{rad} = 0.60\left(2.25 \times 10^{-4} m^2\right)5.67 \times 10^{-8}\ W/m^2 \cdot K^4\left(358^4 - 298^4\right)K^4 = 0.065\ W$$

$$P_{elec} = 0.158\ W + 0.065\ W = 0.223\ W$$ <

(b) If heat transfer is by forced convection,

$$q_{conv} = hA\left(T_s - T_\infty\right) = 250\ W/m^2 \cdot K\left(2.25 \times 10^{-4} m^2\right)(60K) = 3.375\ W$$

$$P_{elec} = 3.375\ W + 0.065\ W = 3.44\ W$$ <

COMMENTS: Clearly, radiation and natural convection are inefficient mechanisms for transferring heat from the chip. For $T_s = 85°C$ and $T_\infty = 25°C$, the natural convection coefficient is $11.7\ W/m^2 \cdot K$. Even for forced convection with $h = 250\ W/m^2 \cdot K$, the power dissipation is well below that associated with many of today's processors. To provide acceptable cooling, it is often necessary to attach the chip to a highly conducting substrate and to thereby provide an additional heat transfer mechanism due to conduction from the back surface.

PROBLEM 1.32

KNOWN: Vacuum enclosure maintained at 77 K by liquid nitrogen shroud while baseplate is maintained at 300 K by an electrical heater.

FIND: (a) Electrical power required to maintain baseplate, (b) Liquid nitrogen consumption rate, (c) Effect on consumption rate if aluminum foil (ε_p = 0.09) is bonded to baseplate surface.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) No heat losses from backside of heater or sides of plate, (3) Vacuum enclosure large compared to baseplate, (4) Enclosure is evacuated with negligible convection, (5) Liquid nitrogen (LN2) is heated only by heat transfer to the shroud, and (6) Foil is intimately bonded to baseplate.

PROPERTIES: Heat of vaporization of liquid nitrogen (given): 125 kJ/kg.

ANALYSIS: (a) From an energy balance on the baseplate,

$$\dot{E}_{in} - \dot{E}_{out} = 0 \qquad q_{elec} - q_{rad} = 0$$

and using Eq. 1.7 for radiative exchange between the baseplate and shroud,

$$q_{elec} = \varepsilon_p A_p \sigma \left(T_p^4 - T_{sh}^4\right).$$

Substituting numerical values, with $A_p = \left(\pi D_p^2 / 4\right)$, find

$$q_{elec} = 0.25\left(\pi (0.3\text{ m})^2 / 4\right) 5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4\left(300^4 - 77^4\right)\text{K}^4 = 8.1\text{ W}.$$ <

(b) From an energy balance on the enclosure, radiative transfer heats the liquid nitrogen stream causing evaporation,

$$\dot{E}_{in} - \dot{E}_{out} = 0 \qquad q_{rad} - \dot{m}_{LN2} h_{fg} = 0$$

where $\dot{m}_{LN2}$ is the liquid nitrogen consumption rate. Hence,

$$\dot{m}_{LN2} = q_{rad} / h_{fg} = 8.1\text{ W} / 125\text{ kJ/kg} = 6.48\times10^{-5}\text{ kg/s} = 0.23\text{ kg/h}.$$ <

(c) If aluminum foil (ε_p = 0.09) were bonded to the upper surface of the baseplate,

$$q_{rad,foil} = q_{rad}\left(\varepsilon_f / \varepsilon_p\right) = 8.1\text{ W}\left(0.09/0.25\right) = 2.9\text{ W}$$

and the liquid nitrogen consumption rate would be reduced by

(0.25 - 0.09)/0.25 = 64% to 0.083 kg/h. <

PROBLEM 1.33

KNOWN: Width, input power and efficiency of a transmission. Temperature and convection coefficient for air flow over the casing. Emissivity of casing and temperature of surroundings.

FIND: Surface temperature of casing.

SCHEMATIC:

$T_\infty = 30^oC$
$h_i = 200\ W/m^2\text{-}K$
q
$P_o = \eta P_i$
$P_i = 150$ hp
W = 0.3 m

ASSUMPTIONS: (1) Steady state, (2) Uniform convection coefficient and surface temperature, (3) Radiation exchange with large surroundings.

ANALYSIS: Heat transfer from the case must balance heat dissipation in the transmission, which may be expressed as $q = P_i - P_o = P_i\,(1 - \eta) = 150\text{ hp} \times 746\text{ W/hp} \times 0.07 = 7833\text{ W}$. Heat transfer from the case is by convection and radiation, in which case

$$q = A_s\left[h\left(T_s - T_\infty\right) + \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right)\right]$$

where $A_s = 6\ W^2$. Hence,

$$7833\,\text{W} = 6\left(0.30\,\text{m}\right)^2\left[200\,\text{W/m}^2\cdot\text{K}\left(T_s - 303\text{K}\right) + 0.8\times5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left(T_s^4 - 303^4\right)\text{K}^4\right]$$

A trial-and-error solution yields

$$T_s \approx 373\,\text{K} = 100°\text{C}$$ <

COMMENTS: (1) For $T_s \approx 373$ K, $q_{conv} \approx 7{,}560$ W and $q_{rad} \approx 270$ W, in which case heat transfer is dominated by convection, (2) If radiation is neglected, the corresponding surface temperature is T_s = 102.5°C.

PROBLEM 1.34

KNOWN: Resistor connected to a battery operating at a prescribed temperature in air.

FIND: (a) Considering the resistor as the system, determine corresponding values for $\dot{E}_{in}(W)$, $\dot{E}_g(W)$, $\dot{E}_{out}(W)$ and $\dot{E}_{st}(W)$. If a control surface is placed about the entire system, determine the values for $\dot{E}_{in}$, $\dot{E}_g$, $\dot{E}_{out}$, and $\dot{E}_{st}$. (b) Determine the volumetric heat generation rate within the resistor, $\dot{q}$ (W/m^3), (c) Neglecting radiation from the resistor, determine the convection coefficient.

SCHEMATIC:

ASSUMPTIONS: (1) Electrical power is dissipated uniformly within the resistor, (2) Temperature of the resistor is uniform, (3) Negligible electrical power dissipated in the lead wires, (4) Negligible radiation exchange between the resistor and the surroundings, (5) No heat transfer occurs from the battery, (5) Steady-state conditions.

ANALYSIS: (a) Referring to Section 1.3.1, the conservation of energy requirement for a control volume at an instant of time, Eq 1.11a, is

$$\dot{E}_{in} + \dot{E}_g - \dot{E}_{out} = \dot{E}_{st}$$

where $\dot{E}_{in}$, $\dot{E}_{out}$ correspond to *surface* inflow and outflow processes, respectively. The energy generation term $\dot{E}_g$ is associated with conversion of some other energy form (chemical, electrical, electromagnetic or nuclear) to thermal energy. The energy storage term $\dot{E}_{st}$ is associated with changes in the internal, kinetic and/or potential energies of the matter in the control volume. $\dot{E}_g$, $\dot{E}_{st}$ are *volumetric* phenomena. The electrical power delivered by the battery is P = VI = 24V×6A = 144 W.

Control volume: Resistor.

$$\dot{E}_{in} = 0 \qquad \dot{E}_{out} = 144\,W$$

<

$$\dot{E}_g = 144\,W \qquad \dot{E}_{st} = 0$$

The $\dot{E}_g$ term is due to conversion of electrical energy to thermal energy. The term $\dot{E}_{out}$ is due to convection from the resistor surface to the air.

Continued...

PROBLEM 1.34 (Cont.)

Control volume: Battery-Resistor System.

$$\dot{E}_{in} = 0 \qquad \dot{E}_{out} = 144\,W \qquad \dot{E}_g = 0 \qquad \dot{E}_{st} = -144\,W$$ <

The $\dot{E}_{st}$ term represents the decrease in the chemical energy within the battery. The conversion of chemical energy to electrical energy and its subsequent conversion to thermal energy are processes internal to the system which are not associated with $\dot{E}_{st}$ or $\dot{E}_g$. The $\dot{E}_{out}$ term is due to convection from the resistor surface to the air.

(b) From the energy balance on the resistor with volume, $\forall = (\pi D^2/4)L$,

$$\dot{E}_g = \dot{q}\forall \qquad 144\,W = \dot{q}\left(\pi\,(0.06\,m)^2/4\right)\times 0.25\,m \qquad \dot{q} = 2.04\times 10^5\ W/m^3$$ <

(c) From the energy balance on the resistor and Newton's law of cooling with $A_s = \pi DL + 2(\pi D^2/4)$,

$$\dot{E}_{out} = q_{cv} = hA_s\left(T_s - T_\infty\right)$$

$$144\,W = h\left[\pi\times 0.06\,m\times 0.25\,m + 2\left(\pi\times 0.06^2\ m^2/4\right)\right](95-25)^\circ C$$

$$144\,W = h\left[0.0471 + 0.0057\right]m^2\,(95-25)^\circ C$$

$$h = 39.0\,W/m^2\cdot K$$ <

COMMENTS: (1) In using the conservation of energy requirement, Eq. 1.11a, it is important to recognize that $\dot{E}_{in}$ and $\dot{E}_{out}$ will always represent *surface* processes and $\dot{E}_g$ and $\dot{E}_{st}$, *volumetric* processes. The generation term $\dot{E}_g$ is associated with a *conversion* process from some form of energy to *thermal energy*. The storage term $\dot{E}_{st}$ represents the rate of change of *internal energy*.

(2) From Table 1.1 and the magnitude of the convection coefficient determined from part (c), we conclude that the resistor is experiencing forced, rather than free, convection.

PROBLEM 1.35

KNOWN: Thickness and initial temperature of an aluminum plate whose thermal environment is changed.

FIND: (a) Initial rate of temperature change, (b) Steady-state temperature of plate, (c) Effect of emissivity and absorptivity on steady-state temperature.

SCHEMATIC:

ASSUMPTIONS: (1) Negligible end effects, (2) Uniform plate temperature at any instant, (3) Constant properties, (4) Adiabatic bottom surface, (5) Negligible radiation from surroundings, (6) No internal heat generation.

ANALYSIS: (a) Applying an energy balance, Eq. 1.11a, at an instant of time to a control volume about the plate, $\dot{E}_{in} - \dot{E}_{out} = \dot{E}_{st}$, it follows for a unit surface area.

$$\alpha_S G_S \left(1\text{m}^2\right) - E\left(1\text{m}^2\right) - q''_{conv}\left(1\text{m}^2\right) = (d/dt)(McT) = \rho\left(1\text{m}^2 \times L\right)c\left(dT/dt\right).$$

Rearranging and substituting from Eqs. 1.3 and 1.5, we obtain

$$dT/dt = (1/\rho Lc)\left[\alpha_S G_S - \varepsilon\sigma T_i^4 - h\left(T_i - T_\infty\right)\right].$$

$$dT/dt = \left(2700\,\text{kg/m}^3 \times 0.004\,\text{m} \times 900\,\text{J/kg}\cdot\text{K}\right)^{-1} \times$$

$$\left[0.8 \times 900\,\text{W/m}^2 - 0.25 \times 5.67 \times 10^{-8}\,\text{W/m}^2\cdot\text{K}^4\,(298\,\text{K})^4 - 20\,\text{W/m}^2\cdot\text{K}\,(25-20)^\circ\text{C}\right]$$

$$dT/dt = 0.052^\circ\text{C/s}.$$ <

(b) Under steady-state conditions, $\dot{E}_{st} = 0$, and the energy balance reduces to

$$\alpha_S G_S = \varepsilon\sigma T^4 + h\left(T - T_\infty\right) \qquad (2)$$

$$0.8 \times 900\,\text{W/m}^2 = 0.25 \times 5.67 \times 10^{-8}\,\text{W/m}^2\cdot\text{K}^4 \times T^4 + 20\,\text{W/m}^2\cdot\text{K}\left(T - 293\,\text{K}\right)$$

The solution yields T = 321.4 K = 48.4°C. <

(c) Using the IHT *First Law Model* for an *Isothermal Plane Wall*, parametric calculations yield the following results.

COMMENTS: The surface radiative properties have a significant effect on the plate temperature, which decreases with increasing ε and decreasing α_S. If a low temperature is desired, the plate coating should be characterized by a large value of ε/α_S. The temperature also decreases with increasing h.

PROBLEM 1.36

KNOWN: Surface area of electronic package and power dissipation by the electronics. Surface emissivity and absorptivity to solar radiation. Solar flux.

FIND: Surface temperature without and with incident solar radiation.

SCHEMATIC:

ASSUMPTIONS: Steady-state conditions.

ANALYSIS: Applying conservation of energy to a control surface about the compartment, at any instant

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = 0.$$

It follows that, with the solar input,

$$\alpha_S A_S q_S'' - A_S E + P = 0$$

$$\alpha_S A_S q_S'' - A_S \varepsilon \sigma T_S^4 + P = 0$$

$$T_S = \left(\frac{\alpha_S A_S q_S'' + P}{A_S \varepsilon \sigma} \right)^{1/4}.$$

In the shade $\left(q_S'' = 0\right)$,

$$T_S = \left(\frac{1000 \text{ W}}{1 \text{ m}^2 \times 1 \times 5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4} \right)^{1/4} = 364 \text{ K.}$$ <

In the sun,

$$T_S = \left(\frac{0.25 \times 1 \text{ m}^2 \times 750 \text{ W/m}^2 + 1000 \text{ W}}{1 \text{ m}^2 \times 1 \times 5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4} \right)^{1/4} = 380 \text{ K.}$$ <

COMMENTS: In orbit, the space station would be continuously cycling between shade and sunshine, and a steady-state condition would not exist.

PROBLEM 1.37

KNOWN: Daily hot water consumption for a family of four and temperatures associated with ground water and water storage tank. Unit cost of electric power. Heat pump COP.

FIND: Annual heating requirement and costs associated with using electric resistance heating or a heat pump.

SCHEMATIC:

Water, T_i = 15 °C
T_f = 55 °C, V = 100 gal
ΔU_t
Q
Electric heater or heat pump, COP = 3

ASSUMPTIONS: (1) Process may be modelled as one involving heat addition in a closed system, (2) Properties of water are constant.

PROPERTIES: Table A-6, Water (T_{ave} = 308 K): $\rho = v_f^{-1}$ = 993 kg/m^3, $c_{p,f}$ = 4.178 kJ/kg·K.

ANALYSIS: From Eq. 1.11c, the daily heating requirement is $Q_{daily} = \Delta U_t = Mc\Delta T$ $= \rho Vc\left(T_f - T_i\right)$. With V = 100 gal/264.17 gal/m^3 = 0.379 m^3,

$$Q_{daily} = 993\text{kg}/\text{m}^3\left(0.379\,\text{m}^3\right)4.178\text{kJ/kg}\cdot\text{K}\left(40^\circ\text{C}\right) = 62,900\,\text{kJ}$$

The annual heating requirement is then, $Q_{annual} = 365\,\text{days}\left(62,900\,\text{kJ/day}\right) = 2.30\times10^7$ kJ , or, with 1 kWh = 1 kJ/s (3600 s) = 3600 kJ,

$$Q_{annual} = 6380\,\text{kWh}$$ <

With electric resistance heating, $Q_{annual} = Q_{elec}$ and the associated cost, C, is

$$C = 6380\,\text{kWh}\left(\$0.08/\text{kWh}\right) = \$510$$ <

If a heat pump is used, $Q_{annual} = \text{COP}\left(W_{elec}\right)$. Hence,

$$W_{elec} = Q_{annual}/\left(\text{COP}\right) = 6380\text{kWh}/\left(3\right) = 2130\,\text{kWh}$$

The corresponding cost is

$$C = 2130\,\text{kWh}\left(\$0.08/\text{kWh}\right) = \$170$$ <

COMMENTS: Although annual operating costs are significantly lower for a heat pump, corresponding capital costs are much higher. The feasibility of this approach depends on other factors such as geography and seasonal variations in COP, as well as the time value of money.

PROBLEM 1.38

KNOWN: Initial temperature of water and tank volume. Power dissipation, emissivity, length and diameter of submerged heaters. Expressions for convection coefficient associated with natural convection in water and air.

FIND: (a) Time to raise temperature of water to prescribed value, (b) Heater temperature shortly after activation and at conclusion of process, (c) Heater temperature if activated in air.

SCHEMATIC:

Water
T_i = 295 K
T_f = 335 K
Heater
L = 250 mm
D = 25 mm
q_1 = 500 W
Tank
$\forall$ = 10 gal

ASSUMPTIONS: (1) Negligible heat loss from tank to surroundings, (2) Water is *well-mixed* (at a uniform, but time varying temperature) during heating, (3) Negligible changes in thermal energy storage for heaters, (4) Constant properties, (5) Surroundings afforded by tank wall are large relative to heaters.

ANALYSIS: (a) Application of conservation of energy to a closed system (the water) at an instant, Eq. (1.11d), yields

$$\frac{dU}{dt} = Mc\frac{dT}{dt} = \rho\forall c\frac{dT}{dt} = q = 3q_1$$

Hence,

$$\int_0^t dt = \left(\rho\forall c/3q_1\right)\int_{T_i}^{T_f} dT$$

$$t = \frac{990\ \text{kg/m}^3 \times 10\text{gal}\left(3.79\times10^{-3}\text{m}^3/\text{gal}\right)4180\text{J/kg}\cdot\text{K}}{3\times500\ \text{W}}\left(335-295\right)\text{K} = 4180\ \text{s}$$ <

(b) From Eq. (1.3a), the heat rate by convection from each heater is

$$q_1 = Aq_1'' = Ah\left(T_s - T\right) = \left(\pi DL\right)370\left(T_s - T\right)^{4/3}$$

Hence,

$$T_s = T + \left(\frac{q_1}{370\pi DL}\right)^{3/4} = T + \left(\frac{500\ \text{W}}{370\ \text{W/m}^2\cdot\text{K}^{4/3}\times\pi\times0.025\ \text{m}\times0.250\ \text{m}}\right)^{3/4} = \left(T+24\right)\text{K}$$

With water temperatures of $T_i \approx 295$ K and $T_f = 335$ K shortly after the start of heating and at the end of heating, respectively,

$T_{s,i} = 319$ K $\qquad T_{s,f} = 359$ K <

Continued …..

PROBLEM 1.38 (Continued)

(c) From Eq. (1.10), the heat rate in air is

$$q_1 = \pi DL\left[0.70\left(T_s - T_\infty\right)^{4/3} + \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right)\right]$$

Substituting the prescribed values of q_1, D, L, $T_\infty = T_{sur}$ and ε, an iterative solution yields

$T_s = 830$ K <

COMMENTS: In part (c) it is presumed that the heater can be operated at $T_s = 830$ K without experiencing burnout. The much larger value of T_s for air is due to the smaller convection coefficient. However, with q_{conv} and q_{rad} equal to 59 W and 441 W, respectively, a significant portion of the heat dissipation is effected by radiation.

PROBLEM 1.39

KNOWN: Power consumption, diameter, and inlet and discharge temperatures of a hair dryer.

FIND: (a) Volumetric flow rate and discharge velocity of heated air, (b) Heat loss from case.

SCHEMATIC:

$T_\infty = 20^oC$
$h = 4\ W/m^2\text{-}K$
Air
$T_{sur} = 20^oC$
q_{rad}
q_{conv}
$T_s = 40^oC$, $\varepsilon = 0.8$
$D = 70$ mm
V_o, $T_o = 45^oC$
Air $\dot{\forall}$, $T_i = 20^oC$
$\rho = 1.10\ kg/m^3$
$c_p = 1007$ J/kg-K
L = 150 mm
$P_{elec} = 500$ W

ASSUMPTIONS: (1) Steady-state, (2) Constant air properties, (3) Negligible potential and kinetic energy changes of air flow, (4) Negligible work done by fan, (5) Negligible heat transfer from casing of dryer to ambient air (Part (a)), (6) Radiation exchange between a small surface and a large enclosure (Part (b)).

ANALYSIS: (a) For a control surface about the air flow passage through the dryer, conservation of energy for an open system reduces to

$$\dot{m}(u+pv)_i - \dot{m}(u+pv)_o + q = 0$$

where u + pv = i and q = P_{elec}. Hence, with $\dot{m}(i_i - i_o) = \dot{m}c_p(T_i - T_o)$,

$$\dot{m}c_p(T_o - T_i) = P_{elec}$$

$$\dot{m} = \frac{P_{elec}}{c_p(T_o - T_i)} = \frac{500\ \text{W}}{1007\ \text{J/kg}\cdot\text{K}\left(25^\circ\text{C}\right)} = 0.0199\ \text{kg/s}$$

$$\dot{\forall} = \frac{\dot{m}}{\rho} = \frac{0.0199\ \text{kg/s}}{1.10\ \text{kg/m}^3} = 0.0181\ \text{m}^3/\text{s}$$ <

$$V_o = \frac{\dot{\forall}}{A_c} = \frac{4\dot{\forall}}{\pi D^2} = \frac{4\times 0.0181\ \text{m}^3/\text{s}}{\pi(0.07\ \text{m})^2} = 4.7\ \text{m/s}$$ <

(b) Heat transfer from the casing is by convection and radiation, and from Eq. (1.10)

$$q = hA_s(T_s - T_\infty) + \varepsilon A_s\sigma\left(T_s^4 - T_{sur}^4\right)$$

Continued …..

where $A_s = \pi DL = \pi(0.07\ m \times 0.15\ m) = 0.033\ m^2$. Hence,

$$q = 4 W/m^2 \cdot K\left(0.033\ m^2\right)\left(20^\circ C\right) + 0.8 \times 0.033\ m^2 \times 5.67 \times 10^{-8}\ W/m^2 \cdot K^4\left(313^4 - 293^4\right)K^4$$

$$q = 2.64\ W + 3.33\ W = 5.97\ W$$ <

The heat loss is much less than the electrical power, and the assumption of negligible heat loss is justified.

COMMENTS: Although the mass flow rate is invariant, the volumetric flow rate increases as the air is heated in its passage through the dryer, causing a reduction in the density. However, for the prescribed temperature rise, the change in ρ, and hence the effect on $\dot{\forall}$, is small.

PROBLEM 1.40

KNOWN: Speed, width, thickness and initial and final temperatures of 304 stainless steel in an annealing process. Dimensions of annealing oven and temperature, emissivity and convection coefficient of surfaces exposed to ambient air and large surroundings of equivalent temperatures. Thickness of pad on which oven rests and pad surface temperatures.

FIND: Oven operating power.

SCHEMATIC:

$T_\infty = 300$ K
$h = 10$ W/m^2·K
Oven
P_{elec}
$L_o = 25$ m
$T_s = 350$ K
$\varepsilon_s = 0.8$
St. St. 304
$H_o = 2$ m
$T_i = 300$ K
$T_o = 1250$ K
$t_s = 0.008$ m
$V_s = 0.01$ m/s
$W_s = 2$ m
$T_s = 350$ K
$T_{sur} = 300$ K
$T_b = 300$ K
$t_c = 0.5$ m
Concrete

ASSUMPTIONS: (1) steady-state, (2) Constant properties, (3) Negligible changes in kinetic and potential energy.

PROPERTIES: Table A.1, St.St.304 $\left(\overline{T} = (T_i + T_o)/2 = 775\,K\right)$: $\rho = 7900$ kg/m^3, $c_p = 578$ J/kg·K; Table A.3, Concrete, T = 300 K: $k_c = 1.4$ W/m·K.

ANALYSIS: The rate of energy addition to the oven must balance the rate of energy transfer to the steel sheet and the rate of heat loss from the oven. With $\dot{E}_{in} - \dot{E}_{out} = 0$, it follows that

$$P_{elec} + \dot{m}(u_i - u_o) - q = 0$$

where heat is transferred from the oven. With $\dot{m} = \rho V_s (W_s t_s)$, $(u_i - u_o) = c_p (T_i - T_o)$, and

$$q = (2H_oL_o + 2H_oW_o + W_oL_o) \times \left[h(T_s - T_\infty) + \varepsilon_s \sigma \left(T_s^4 - T_{sur}^4\right)\right] + k_c (W_oL_o)(T_s - T_b)/t_c,$$

it follows that

$$P_{elec} = \rho V_s (W_s t_s) c_p (T_o - T_i) + (2H_oL_o + 2H_oW_o + W_oL_o) \times \left[h(T_s - T_o) + \varepsilon_s \sigma \left(T_s^4 - T_{sur}^4\right)\right] + k_c (W_oL_o)(T_s - T_b)/t_c$$

$$P_{elec} = 7900\,\text{kg/m}^3 \times 0.01\,\text{m/s}\,(2\,\text{m} \times 0.008\,\text{m})\,578\,\text{J/kg}\cdot\text{K}\,(1250 - 300)\,\text{K}$$
$$+ (2 \times 2\text{m} \times 25\text{m} + 2 \times 2\text{m} \times 2.4\text{m} + 2.4\text{m} \times 25\text{m})[10\,\text{W/m}^2 \cdot \text{K}\,(350 - 300)\,\text{K}$$
$$+ 0.8 \times 5.67 \times 10^{-8}\,\text{W/m}^2 \cdot \text{K}^4 \left(350^4 - 300^4\right)\text{K}^4] + 1.4\,\text{W/m}\cdot\text{K}\,(2.4\text{m} \times 25\text{m})(350 - 300)\,\text{K}/0.5\text{m}$$

Continued.….

PROBLEM 1.40 (Cont.)

$$P_{elec} = 694,000\,W + 169.6\,m^2 \left(500 + 313\right) W/m^2 + 8400\,W$$

$$= \left(694,000 + 84,800 + 53,100 + 8400\right) W = 840\,kW$$ <

COMMENTS: Of the total energy input, 83% is transferred to the steel while approximately 10%, 6% and 1% are lost by convection, radiation and conduction from the oven. The convection and radiation losses can both be reduced by adding insulation to the side and top surfaces, which would reduce the corresponding value of T_s.

PROBLEM 1.41

KNOWN: Hot plate-type wafer thermal processing tool based upon heat transfer modes by conduction through gas within the gap and by radiation exchange across gap.

FIND: (a) Radiative and conduction heat fluxes across gap for specified hot plate and wafer temperatures and gap separation; initial time rate of change in wafer temperature for each mode, and (b) heat fluxes and initial temperature-time change for gap separations of 0.2, 0.5 and 1.0 mm for hot plate temperatures 300 < T_h < 1300°C. Comment on the relative importance of the modes and the influence of the gap distance. Under what conditions could a wafer be heated to 900°C in less than 10 seconds?

SCHEMATIC:

d = 0.78 mm
D >> L
L = 0.2 mm
Hot plate $T_h = 600^oC$
Stagnant gas, k
Wafer $T_{w,i} = 20^oC$
q''_{rad}, q''_{cond}
Wafer, $\dot{E}_{st}$

ASSUMPTIONS: (1) Steady-state conditions for flux calculations, (2) Diameter of hot plate and wafer much larger than gap spacing, approximating plane, infinite planes, (3) One-dimensional conduction through gas, (4) Hot plate and wafer are blackbodies, (5) Negligible heat losses from wafer backside, and (6) Wafer temperature is uniform at the onset of heating.

PROPERTIES: Wafer: ρ = 2700 kg/m^3, c = 875 J/kg·K; Gas in gap: k = 0.0436 W/m·K.

ANALYSIS: (a) The radiative heat flux between the hot plate and wafer for T_h = 600°C and T_w = 20° C follows from the rate equation,

$$q''_{rad} = \sigma\left(T_h^4 - T_w^4\right) = 5.67\times10^{-8}\,W/m^2\cdot K^4\left((600+273)^4 - (20+273)^4\right)K^4 = 32.5\,kW/m^2 \quad <$$

The conduction heat flux through the gas in the gap with L = 0.2 mm follows from Fourier's law,

$$q''_{cond} = k\frac{T_h - T_w}{L} = 0.0436\,W/m\cdot K\,\frac{(600-20)K}{0.0002\ m} = 126\ kW/m^2 \quad <$$

The initial time rate of change of the wafer can be determined from an energy balance on the wafer at the instant of time the heating process begins,

$$\dot{E}''_{in} - \dot{E}''_{out} = \dot{E}''_{st} \qquad\qquad \dot{E}''_{st} = \rho c\, d\left(\frac{dT_w}{dt}\right)_i$$

where $\dot{E}''_{out} = 0$ and $\dot{E}''_{in} = q''_{rad}$ or q''_{cond}. Substituting numerical values, find

$$\left.\frac{dT_w}{dt}\right)_{i,rad} = \frac{q''_{rad}}{\rho c d} = \frac{32.5\times10^3\,W/m^2}{2700\,kg/m^3\times875\ J/kg\cdot K\times0.00078\ m} = 17.6\ K/s \quad <$$

$$\left.\frac{dT_w}{dt}\right)_{i,cond} = \frac{q''_{cond}}{\rho c d} = 68.4\ K/s \quad <$$

Continued …..

PROBLEM 1.41 (Cont.)

(b) Using the foregoing equations, the heat fluxes and initial rate of temperature change for each mode can be calculated for selected gap separations L and range of hot plate temperatures T_h with T_w = 20°C.

In the left-hand graph, the conduction heat flux increases linearly with T_h and inversely with L as expected. The radiative heat flux is independent of L and highly non-linear with T_h, but does not approach that for the highest conduction heat rate until T_h approaches 1200°C.

The general trends for the initial temperature-time change, $(dT_w/dt)_i$, follow those for the heat fluxes. To reach 900°C in 10 s requires an average temperature-time change rate of 90 K/s. Recognizing that (dT_w/dt) will decrease with increasing T_w, this rate could be met only with a very high T_h and the smallest L.

PROBLEM 1.42

KNOWN: Silicon wafer, radiantly heated by lamps, experiencing an annealing process with known backside temperature.

FIND: Whether temperature difference across the wafer thickness is less than 2°C in order to avoid damaging the wafer.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in wafer, (3) Radiation exchange between upper surface of wafer and surroundings is between a small object and a large enclosure, and (4) Vacuum condition in chamber, no convection.

PROPERTIES: Wafer: k = 30 W/m·K, $\varepsilon = \alpha_\ell = 0.65$.

ANALYSIS: Perform a surface energy balance on the upper surface of the wafer to determine $T_{w,u}$. The processes include the absorbed radiant flux from the lamps, radiation exchange with the chamber walls, and conduction through the wafer.

$$\dot{E}''_{in} - \dot{E}''_{out} = 0$$

$$\alpha_\ell q''_s - q''_{rad} - q''_{cd} = 0$$

$$\alpha_\ell q''_s - \varepsilon\sigma\left(T_{w,u}^4 - T_{sur}^4\right) - k\frac{T_{w,u} - T_{w,\ell}}{L} = 0$$

$$0.65\times3.0\times10^5\,\mathrm{W/m^2} - 0.65\times5.67\times10^{-8}\,\mathrm{W/m^2\cdot K^4}\left(T_{w,u}^4 - (27+273)^4\right)\mathrm{K}^4$$

$$-30\,\mathrm{W/m\cdot K}\left[T_{w,u} - (997+273)\right]\mathrm{K}/0.00078\ \mathrm{m} = 0$$

$$T_{w,u} = 1273\,\mathrm{K} = 1000°\mathrm{C}$$ <

COMMENTS: (1) The temperature difference for this steady-state operating condition, $T_{w,u} - T_{w,l}$, is larger than 2°C. Warping of the wafer and inducing slip planes in the crystal structure could occur.

(2) The radiation exchange rate equation requires that temperature must be expressed in kelvin units. Why is it permissible to use kelvin or Celsius temperature units in the conduction rate equation?

(3) Note how the surface energy balance, Eq. 1.12, is represented schematically. It is essential to show the control surfaces, and then identify the rate processes associated with the surfaces. Make sure the directions (in or out) of the process are consistent with the energy balance equation.

PROBLEM 1.43

KNOWN: Silicon wafer positioned in furnace with top and bottom surfaces exposed to hot and cool zones, respectively.

FIND: (a) Initial rate of change of the wafer temperature corresponding to the wafer temperature $T_{w,i} = 300\,K$, and (b) Steady-state temperature reached if the wafer remains in this position. How significant is convection for this situation? Sketch how you'd expect the wafer temperature to vary as a function of vertical distance.

SCHEMATIC:

PROBLEM 1.43 (Cont.)

$$0.65\sigma\left(1500^4 - T_{w,ss}^4\right)K^4 + 0.65\sigma\left(330^4 - T_{w,ss}^4\right)K^4$$

$$-8\,W/m^2 \cdot K\left(T_{w,ss} - 700\right)K - 4\,W/m^2 \cdot K\left(T_{w,ss} - 700\right)K = 0$$

$$T_{w,ss} = 1251\ K$$ <

To determine the relative importance of the convection processes, re-solve the energy balance above ignoring those processes to find $\left(d\,T_w / dt\right)_i = 101\ K/s$ and $T_{w,ss} = 1262\ K$. We conclude that the radiation exchange processes control the initial time rate of temperature change and the steady-state temperature.

If the wafer were elevated above the present operating position, its temperature would increase, since the lower surface would begin to experience radiant exchange with progressively more of the hot zone chamber. Conversely, by lowering the wafer, the upper surface would experience less radiant exchange with the hot zone chamber, and its temperature would decrease. The temperature-distance trend might appear as shown in the sketch.

PROBLEM 1.44

KNOWN: Radial distribution of heat dissipation in a cylindrical container of radioactive wastes. Surface convection conditions.

FIND: Total energy generation rate and surface temperature.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible temperature drop across thin container wall.

ANALYSIS: The rate of energy generation is

$$\dot{E}_g = \int \dot{q} dV = \dot{q}_o \int_0^{r_o} \left[1-(r/r_o)^2\right] 2\pi r L dr$$

$$\dot{E}_g = 2\pi L \dot{q}_o \left(r_o^2/2 - r_o^2/4\right)$$

or per unit length,

$$\dot{E}'_g = \frac{\pi \dot{q}_o r_o^2}{2}. \qquad \text{<}$$

Performing an energy balance for a control surface about the container yields, at an instant,

$$\dot{E}'_g - \dot{E}'_{out} = 0$$

and substituting for the convection heat rate per unit length,

$$\frac{\pi \dot{q}_o r_o^2}{2} = h(2\pi r_o)(T_s - T_\infty)$$

$$T_s = T_\infty + \frac{\dot{q}_o r_o}{4h}. \qquad \text{<}$$

COMMENTS: The temperature within the radioactive wastes increases with decreasing r from T_s at r_o to a maximum value at the centerline.

PROBLEM 1.45

KNOWN: Rod of prescribed diameter experiencing electrical dissipation from passage of electrical current and convection under different air velocity conditions. See Example 1.3.

FIND: Rod temperature as a function of the electrical current for $0 \le I \le 10$ A with convection coefficients of 50, 100 and 250 $W/m^2 \cdot K$. Will variations in the surface emissivity have a significant effect on the rod temperature?

SCHEMATIC:

$T_\infty = 300$ K
$h = 100$ W/m^2-K
$T_{sur} = 300$ K
q'_{conv}
q'_{rad}
T, D = 1 mm
$\varepsilon = 0.8$, $R'_e = 0.4$ Ω/m
I = 5.2 A
$\dot{E}'_{gen}$

ASSUMPTIONS: (1) Steady-state conditions, (2) Uniform rod temperature, (3) Radiation exchange between the outer surface of the rod and the surroundings is between a small surface and large enclosure.

ANALYSIS: The energy balance on the rod for steady-state conditions has the form,

$$q'_{conv} + q'_{rad} = \dot{E}'_{gen}$$

$$\pi Dh\left(T - T_\infty\right) + \pi D\varepsilon\sigma\left(T^4 - T_{sur}^4\right) = I^2 R'_e$$

Using this equation in the Workspace of IHT, the rod temperature is calculated and plotted as a function of current for selected convection coefficients.

Rod temperature, T (C)
Current, I (amperes)
h = 50 W/m^2.K
h = 100 W/m^2.K
h = 250 W/m^2.K

COMMENTS: (1) For forced convection over the cylinder, the convection heat transfer coefficient is dependent upon air velocity approximately as $h \sim V^{0.6}$. Hence, to achieve a 5-fold change in the convection coefficient (from 50 to 250 $W/m^2 \cdot K$), the air velocity must be changed by a factor of nearly 15.

Continued …..

(2) For the condition of I = 4 A with h = 50 W/m^2·K with T = 63.5°C, the convection and radiation exchange rates per unit length are, respectively, $q'_{cv} = 5.7\ W/m$ and $q'_{rad} = 0.67\ W/m$. We conclude that convection is the dominate heat transfer mode and that changes in surface emissivity could have only a minor effect. Will this also be the case if h = 100 or 250 W/m^2·K?

(3) What would happen to the rod temperature if there was a "loss of coolant" condition where the air flow would cease?

(4) The Workspace for the IHT program to calculate the heat losses and perform the parametric analysis to generate the graph is shown below. It is good practice to provide commentary with the code making your solution logic clear, and to summarize the results. It is also good practice to show plots in *customary* units, that is, the units used to prescribe the problem. As such the graph of the rod temperature is shown above with Celsius units, even though the calculations require temperatures in kelvins.

```
// Energy balance; from Ex. 1.3, Comment 1
-q'cv - q'rad + Edot'g = 0
q'cv = pi*D*h*(T - Tinf)
q'rad = pi*D*eps*sigma*(T^4 - Tsur^4)
sigma = 5.67e-8

// The generation term has the form
Edot'g = I^2*R'e
qdot = I^2*R'e / (pi*D^2/4)

// Input parameters
D = 0.001
Tsur = 300
T_C = T – 273          // Representing temperature in Celsius units using _C subscript
eps = 0.8
Tinf = 300
h = 100
//h = 50               // Values of coefficient for parameter study
//h = 250
I = 5.2                // For graph, sweep over range from 0 to 10 A
//I = 4                // For evaluation of heat rates with h = 50 W/m^2.K
R'e = 0.4

/*  Base case results: I = 5.2 A with h = 100 W/m^2.K, find T = 60 C (Comment 2 case).
Edot'g     T       T_C     q'cv    q'rad   qdot                D       I       R'e
           Tinf    Tsur    eps     h       sigma
10.82      332.6   59.55   10.23   0.5886  1.377E7             0.001   5.2     0.4
           300     300     0.8     100     5.67E-8   */

/* Results: I = 4 A with h = 50 W/m^2.K, find q'cv =  5.7 W/m and q'rad = 0.67 W/m
Edot'g     T       T_C     q'cv    q'rad   qdot      D       I       R'e
     Tinf  Tsur    eps     h       sigma
6.4        336.5   63.47   5.728   0.6721  8.149E6 0.001     4       0.4
     300   300     0.8     50      5.67E-8    */
```

PROBLEM 1.46

KNOWN: Long bus bar of prescribed diameter and ambient air and surroundings temperatures. Relations for the electrical resistivity and free convection coefficient as a function of temperature.

FIND: (a) Current carrying capacity of the bus bar if its surface temperature is not to exceed 65°C; compare relative importance of convection and radiation exchange heat rates, and (b) Show graphically the operating temperature of the bus bar as a function of current for the range $100 \le I \le 5000$ A for bus-bar diameters of 10, 20 and 40 mm. Plot the ratio of the heat transfer by convection to the total heat transfer for these conditions.

SCHEMATIC:

Conduit, $T_{sur} = 30^oC$
Bus-bar
Air
T_∞, h(T,D)
$h(T,D) = CD^{-0.25}(T - T_\infty)^{0.25}$, $C = 1.21\ W\text{-}m^{-1.75}K^{-1.25}$
q'_{cv} q'_{rad}
I
$T = 65^oC$
$\varepsilon = 0.85$
D = 20mm
$\dot{E}'_{gen}$, $\rho_e(T)$

ASSUMPTIONS: (1) Steady-state conditions, (2) Bus bar and conduit are very long in direction normal to page, (3) Uniform bus-bar temperature, (4) Radiation exchange between the outer surface of the bus bar and the conduit is between a small surface and a large enclosure.

PROPERTIES: Bus-bar material, $\rho_e = \rho_{e,o}\left[1+\alpha\left(T-T_o\right)\right]$, $\rho_{e,o} = 0.0171\mu\Omega\cdot m$, $T_o = 25°C$, $\alpha = 0.00396\,K^{-1}$.

ANALYSIS: An energy balance on the bus-bar for a unit length as shown in the schematic above has the form

$$\dot{E}'_{in} - \dot{E}'_{out} + \dot{E}'_{gen} = 0$$

$$-q'_{rad} - q'_{conv} + I^2R'_e = 0$$

$$-\varepsilon\pi D\sigma\left(T^4 - T_{sur}^4\right) - h\pi D\left(T - T_\infty\right) + I^2\rho_e / A_c = 0$$

where $R'_e = \rho_e / A_c$ and $A_c = \pi D^2/4$. Using the relations for $\rho_e(T)$ and $h(T,D)$, and substituting numerical values with T = 65°C, find

$$q'_{rad} = 0.85\pi(0.020m)\times 5.67\times 10^{-8} W/m^2\cdot K^4\left([65+273]^4 - [30+273]^4\right)K^4 = 223\,W/m$$ <

$$q'_{conv} = 7.83\,W/m^2\cdot K\ \pi(0.020m)(65-30)K = 17.2\,W/m$$ <

where $h = 1.21\,W\cdot m^{-1.75}\cdot K^{-1.25}(0.020m)^{-0.25}(65-30)^{0.25} = 7.83\,W/m^2\cdot K$

$$I^2R'_e = I^2\left(198.2\times 10^{-6}\Omega\cdot m\right)/\pi(0.020)^2\,m^2/4 = 6.31\times 10^{-5} I^2\ W/m$$

where $\rho_e = 0.0171\times 10^{-6}\Omega\cdot m\left[1 + 0.00396\,K^{-1}(65-25)K\right] = 198.2\,\mu\Omega\cdot m$

The maximum allowable current capacity and the ratio of the convection to total heat transfer rate are

$$I = 1950\,A \qquad q'_{cv}/\left(q'_{cv} + q'_{rad}\right) = q'_{cv}/q'_{tot} = 0.072$$ <

For this operating condition, convection heat transfer is only 7.2% of the total heat transfer.

(b) Using these equations in the Workspace of IHT, the bus-bar operating temperature is calculated and plotted as a function of the current for the range $100 \le I \le 5000$ A for diameters of 10, 20 and 40 mm. Also shown below is the corresponding graph of the ratio (expressed in percentage units) of the heat transfer by convection to the total heat transfer, q'_{cv}/q'_{tot}.

Continued …..

PROBLEM 1.46 (Cont.)

Bar temperature, Ts (C) vs. Current, I (A)

D = 10 mm
D = 20 mm
D = 40 mm

Ratio q'cv / q'tot, (%) vs. Bus bar temperature, T (C)

D = 10 mm
D = 20 mm
D = 40 mm

COMMENTS: (1) The trade-off between current-carrying capacity, operating temperature and bar diameter is shown in the first graph. If the surface temperature is not to exceed 65°C, the maximum current capacities for the 10, 20 and 40-mm diameter bus bars are 960, 1950, and 4000 A, respectively.

(2) From the second graph with q'_{cv} / q'_{tot} vs. T, note that the convection heat transfer rate is always a small fraction of the total heat transfer. That is, radiation is the dominant mode of heat transfer. Note also that the convection contribution increases with increasing diameter.

(3) The Workspace for the IHT program to perform the parametric analysis and generate the graphs is shown below. It is good practice to provide commentary with the code making your solution logic clear, and to summarize the results.

```
/* Results: base-case conditions, Part (a)
I        R'e        cvovertot  hbar    q'cv    q'rad   rhoe       D      Tinf_C   Ts_C
         Tsur_C     eps
1950     6.309E-5   7.171      7.826   17.21   222.8   1.982E-8   0.02   30       65
         30         0.85 */

// Energy balance, on a per unit length basis; steady-state conditions
// Edot'in - Edot'out + Edot'gen = 0
-q'cv - q'rad + Edot'gen = 0
q'cv = hbar * P * (Ts - Tinf)
P = pi * D
q'rad = eps * sigma * (Ts^4 - Tsur^4)
sigma = 5.67e-8
Edot'gen = I^2 * R'e
R'e = rhoe / Ac
rhoe = rhoeo * (1 + alpha * (Ts - To) )
To = 25 + 273
Ac = pi * D^2 / 4

// Convection coefficient
hbar = 1.21 * (D^-0.25) * (Ts - Tinf)^0.25          // Compact convection coeff. correlation
// Convection vs. total heat rates
cvovertot = q'cv / (q'cv + q'rad) * 100

// Input parameters
D = 0.020
// D = 0.010                // Values of diameter for parameter study
// D = 0.040
// I = 1950                 // Base case condition unknown
rhoeo = 0.01711e-6
alpha = 0.00396
Tinf_C  = 30
Tinf = Tinf_C + 273
Ts_C = 65                   // Base case condition to determine current
Ts = Ts_C + 273
Tsur_C = 30
Tsur = Tsur_C + 273
eps = 0.85
```

PROBLEM 1.47

KNOWN: Elapsed times corresponding to a temperature change from 15 to 14°C for a reference sphere and test sphere of unknown composition suddenly immersed in a stirred water-ice mixture. Mass and specific heat of reference sphere.

FIND: Specific heat of the test sphere of known mass.

SCHEMATIC:

Water - ice mixture
T_∞, h
q_{conv}
T(t), T(0) - T(t) = (15 -14)°C
Δt_r = 6.35 s, Δt_t = 4.59 s
Reference (r) and test (t) spheres
M_r = 0.515 kg M_t = 1.263 kg
c_r = 447 J/kg-K c_t = ?

ASSUMPTIONS: (1) Spheres are of equal diameter, (2) Spheres experience temperature change from 15 to 14°C, (3) Spheres experience same convection heat transfer rate when the time rates of surface temperature are observed, (4) At any time, the temperatures of the spheres are uniform, (5) Negligible heat loss through the thermocouple wires.

PROPERTIES: Reference-grade sphere material: c_r = 447 J/kg K.

ANALYSIS: Apply the conservation of energy requirement at an instant of time, Eq. 1.11a, after a sphere has been immersed in the ice-water mixture at T_∞.

$$\dot{E}_{in} - \dot{E}_{out} = \dot{E}_{st}$$

$$-q_{conv} = Mc\frac{dT}{dt}$$

where $q_{conv} = h\,A_s\left(T - T_\infty\right)$. Since the temperatures of the spheres are uniform, the change in energy storage term can be represented with the time rate of temperature change, dT/dt. The convection heat rates are equal at this instant of time, and hence the change in energy storage terms for the reference (r) and test (t) spheres must be equal.

$$M_r\ c_r\ \frac{dT}{dt}\bigg)_r = M_t\ c_t\ \frac{dT}{dt}\bigg)_t$$

Approximating the instantaneous differential change, dT/dt, by the difference change over a short period of time, ΔT/Δt, the specific heat of the test sphere can be calculated.

$$0.515\ \text{kg} \times 447\,\text{J/kg}\cdot\text{K}\ \frac{(15-14)\text{K}}{6.35\,\text{s}} = 1.263\,\text{kg} \times c_t \times \frac{(15-14)\text{K}}{4.59\,\text{s}}$$

$$c_t = 132\ \text{J/kg}\cdot\text{K}$$ <

COMMENTS: Why was it important to perform the experiments with the reference and test spheres over the same temperature range (from 15 to 14°C)? Why does the analysis require that the spheres have uniform temperatures at all times?

PROBLEM 1.48

KNOWN: Inner surface heating and new environmental conditions associated with a spherical shell of prescribed dimensions and material.

FIND: (a) Governing equation for variation of wall temperature with time. Initial rate of temperature change, (b) Steady-state wall temperature, (c) Effect of convection coefficient on canister temperature.

SCHEMATIC:

ASSUMPTIONS: (1) Negligible temperature gradients in wall, (2) Constant properties, (3) Uniform, time-independent heat flux at inner surface.

PROPERTIES: *Table A.1*, Stainless Steel, AISI 302: ρ = 8055 kg/m^3, c_p = 510 J/kg·K.

ANALYSIS: (a) Performing an energy balance on the shell at an instant of time, $\dot{E}_{in} - \dot{E}_{out} = \dot{E}_{st}$. Identifying relevant processes and solving for dT/dt,

$$q_i''\left(4\pi r_i^2\right) - h\left(4\pi r_o^2\right)\left(T - T_\infty\right) = \rho \frac{4}{3}\pi\left(r_o^3 - r_i^3\right)c_p \frac{dT}{dt}$$

$$\frac{dT}{dt} = \frac{3}{\rho c_p\left(r_o^3 - r_i^3\right)}\left[q_i'' r_i^2 - h r_o^2\left(T - T_\infty\right)\right].$$

Substituting numerical values for the initial condition, find

$$\left.\frac{dT}{dt}\right)_i = \frac{3\left[10^5 \frac{W}{m^2}(0.5m)^2 - 500\frac{W}{m^2 \cdot K}(0.6m)^2(500-300)K\right]}{8055\frac{kg}{m^3}510\frac{J}{kg \cdot K}\left[(0.6)^3 - (0.5)^3\right]m^3}$$

$$\left.\frac{dT}{dt}\right)_i = -0.089\,K/s.$$ <

(b) Under steady-state conditions with $\dot{E}_{st}$ = 0, it follows that

$$q_i''\left(4\pi r_i^2\right) = h\left(4\pi r_o^2\right)\left(T - T_\infty\right)$$

Continued …..

PROBLEM 1.48 (Cont.)

$$T = T_\infty + \frac{q_i''}{h}\left(\frac{r_i}{r_o}\right)^2 = 300\text{K} + \frac{10^5\,\text{W/m}^2}{500\,\text{W/m}^2\cdot\text{K}}\left(\frac{0.5\text{m}}{0.6\text{m}}\right)^2 = 439\text{K}$$ <

(c) Parametric calculations were performed using the IHT *First Law Model* for an *Isothermal Hollow Sphere*. As shown below, there is a sharp increase in temperature with decreasing values of $h < 1000$ $W/m^2 \cdot K$. For $T > 380$ K, boiling will occur at the canister surface, and for $T > 410$ K a condition known as film boiling (Chapter 10) will occur. The condition corresponds to a precipitous reduction in h and increase in T.

Temperature, T(K)

Convection coefficient, h(W/m^2.K)

Although the canister remains well below the melting point of stainless steel for $h = 100$ $W/m^2 \cdot K$, boiling should be avoided, in which case the convection coefficient should be maintained at $h > 1000$ $W/m^2 \cdot K$.

COMMENTS: The governing equation of part (a) is a first order, nonhomogenous differential equation with constant coefficients. Its solution is $\theta = (S/R)\left(1 - e^{-Rt}\right) + \theta_i e^{-Rt}$, where $\theta \equiv T - T_\infty$,

$S \equiv 3q_i'' r_i^2 / \rho c_p \left(r_o^3 - r_i^3\right)$, $R = 3hr_o^2/\rho c_p \left(r_o^3 - r_i^3\right)$. Note results for $t \to \infty$ and for $S = 0$.

PROBLEM 1.49

KNOWN: Boiling point and latent heat of liquid oxygen. Diameter and emissivity of container. Free convection coefficient and temperature of surrounding air and walls.

FIND: Mass evaporation rate.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Temperature of container outer surface equals boiling point of oxygen.

ANALYSIS: (a) Applying an energy balance to a control surface about the container, it follows that, at any instant,

$$\dot{E}_{in} - \dot{E}_{out} = 0 \qquad \text{or} \qquad q_{conv} + q_{rad} - q_{evap} = 0 .$$

The evaporative heat loss is equal to the product of the mass rate of vapor production and the heat of vaporization. Hence,

$$\left[h\left(T_\infty - T_s\right) + \varepsilon\sigma\left(T_{sur}^4 - T_s^4\right)\right]A_s - \dot{m}_{evap}h_{fg} = 0 \qquad (1)$$

$$\dot{m}_{evap} = \frac{\left[h\left(T_\infty - T_s\right) + \varepsilon\sigma\left(T_{sur}^4 - T_s^4\right)\right]\pi D^2}{h_{fg}}$$

$$\dot{m}_{evap} = \frac{\left[10\,W/m^2 \cdot K\left(298 - 263\right)K + 0.2 \times 5.67 \times 10^{-8}\,W/m^2 \cdot K^4\left(298^4 - 263^4\right)K^4\right]\pi\left(0.5\,m\right)^2}{214\,kJ/kg}$$

$$\dot{m}_{evap} = \frac{\left(350 + 35.2\right)W/m^2\left(0.785\ m^2\right)}{214\,kJ/kg} = 1.41 \times 10^{-3}\,kg/s .$$ <

(b) Using the energy balance, Eq. (1), the mass rate of vapor production can be determined for the range of emissivity 0.2 to 0.94. The effect of increasing emissivity is to increase the heat rate into the container and, hence, increase the vapor production rate.

COMMENTS: To reduce the loss of oxygen due to vapor production, insulation should be applied to the outer surface of the container, in order to reduce q_{conv} and q_{rad}. Note from the calculations in part (a), that heat transfer by convection is greater than by radiation exchange.

PROBLEM 1.50

KNOWN: Frost formation of 2-mm thickness on a freezer compartment. Surface exposed to convection process with ambient air.

FIND: Time required for the frost to melt, t_m.

SCHEMATIC:

Freezer compartment wall
Ambient air
$T_\infty = 20^\circ C$
$h = 2\ W/m^2\text{-}K$
Frost layer
Adiabatic surface
Frost, $T_f = 0^\circ C$
ΔE_{st}
E_{in}
dx
x
0 $\quad x_o = 2$ mm

ASSUMPTIONS: (1) Frost is isothermal at the fusion temperature, T_f, (2) The water melt falls away from the exposed surface, (3) Negligible radiation exchange at the exposed surface, and (4) Backside surface of frost formation is adiabatic.

PROPERTIES: Frost, $\rho_f = 770\ kg/m^3$, $h_{sf} = 334\ kJ/kg$.

ANALYSIS: The time t_m required to melt a 2-mm thick frost layer may be determined by applying an energy balance, Eq 1.11b, over the differential time interval dt and to a differential control volume extending inward from the surface of the layer dx. From the schematic above, the energy *in* is the convection heat flux over the time period dt and the change in energy storage is the latent energy change within the control volume, $A_s \cdot dx$.

$$E_{in} - E_{out} = E_{st}$$

$$q''_{conv} A_s dt = dU_{\ell at}$$

$$h A_s \left(T_\infty - T_f\right) dt = -\rho_f A_s h_{sf} dx$$

Integrating both sides of the equation and defining appropriate limits, find

$$h\left(T_\infty - T_f\right)\int_0^{t_m} dt = -\rho_f h_{sf} \int_{x_o}^{0} dx$$

$$t_m = \frac{\rho_f h_{sf} x_o}{h\left(T_\infty - T_f\right)}$$

$$t_m = \frac{700\ kg/m^3 \times 334 \times 10^3\ J/kg \times 0.002m}{2\ W/m^2 \cdot K\left(20-0\right)K} = 11{,}690\ s = 3.2\ hour$$ <

COMMENTS: (1) The energy balance could be formulated intuitively by recognizing that the total heat *in* by convection during the time interval $t_m \left(q''_{cv} \cdot t_m\right)$ must be equal to the total latent energy for melting the frost layer $\left(\rho\, x_o h_{sf}\right)$. This equality is directly comparable to the derived expression above for t_m.

(2) Explain why the energy storage term in the analysis has a negative sign, and the limits of integration are as shown. *Hint*: Recall from the formulation of Eq. 1.11b, that the storage term represents the change between the final and initial states.

PROBLEM 1.51

KNOWN: Vertical slab of Woods metal initially at its fusion temperature, T_f, joined to a substrate. Exposed surface is irradiated with laser source, $G_\ell \left(W/m^2\right)$.

FIND: Instantaneous rate of melting per unit area, $\dot{m}''_m$ (kg/s·m^2), and the material removed in a period of 2 s, (a) Neglecting heat transfer from the irradiated surface by convection and radiation exchange, and (b) Allowing for convection and radiation exchange.

SCHEMATIC:

ASSUMPTIONS: (1) Woods metal slab is isothermal at the fusion temperature, T_f, and (2) The melt runs off the irradiated surface.

ANALYSIS: (a) The instantaneous rate of melting per unit area may be determined by applying an energy balance, Eq 1.11a, on the metal slab at an instant of time neglecting convection and radiation exchange from the irradiated surface.

$$\dot{E}''_{in} - \dot{E}''_{out} = \dot{E}''_{st} \qquad \alpha_\ell G_\ell = \frac{d}{dt}\left(-M'' h_{sf}\right) = -h_{sf}\frac{dM''}{dt}$$

where $dM''/dt = \dot{m}''_m$ is the time rate of change of mass in the control volume. Substituting values,

$$0.4 \times 5000\,W/m^2 = -33{,}000\,J/kg \times \dot{m}''_m \qquad \dot{m}''_m = -60.6 \times 10^{-3}\,kg/s\cdot m^2$$ $<$

The material removed in a 2s period per unit area is

$$M''_{2s} = \dot{m}''_m \cdot \Delta t = 121\,g/m^2$$ $<$

(b) The energy balance considering convection and radiation exchange with the surroundings yields

$$\alpha_\ell G_\ell - q''_{cv} - q''_{rad} = -h_{sf}\,\dot{m}''_m$$

$$q''_{cv} = h\left(T_f - T_\infty\right) = 15\,W/m^2\cdot K\left(72-20\right)K = 780\,W/m^2$$

$$q''_{rad} = \varepsilon\sigma\left(T_f^4 - T_\infty^4\right) = 0.4 \times 5.67 \times 10^{-8}\,W/m^2\cdot K\left([72+273]^4 - [20+273]^4\right)K^4 = 154\,W/m^2$$

$$\dot{m}''_m = -32.3 \times 10^{-3}\,kg/s\cdot m^2 \qquad M_{2s} = 64\,g/m^2$$ $<$

COMMENTS: (1) The effects of heat transfer by convection and radiation reduce the estimate for the material removal rate by a factor of two. The heat transfer by convection is nearly 5 times larger than by radiation exchange.

(2) Suppose the work piece were horizontal, rather than vertical, and the melt puddled on the surface rather than ran off. How would this affect the analysis?

(3) Lasers are common heating sources for metals processing, including the present application of melting (heat transfer with phase change), as well as for heating work pieces during milling and turning (laser-assisted machining).

PROBLEM 1.52

KNOWN: Hot formed paper egg carton of prescribed mass, surface area and water content exposed to infrared heater providing known radiant flux.

FIND: Whether water content can be reduced from 75% to 65% by weight during the 18s period carton is on conveyor.

SCHEMATIC:

ASSUMPTIONS: (1) All the radiant flux from the heater bank is absorbed by the carton, (2) Negligible heat loss from carton by convection and radiation, (3) Negligible mass loss occurs from bottom side.

PROPERTIES: Water (given): h_{fg} = 2400 kJ/kg.

ANALYSIS: Define a control surface about the carton, and write the conservation of energy requirement for an interval of time, Δt,

$$E_{in} - E_{out} = \Delta E_{st} = 0$$

where E_{in} is due to the absorbed radiant flux, q_h'', from the heater and E_{out} is the energy leaving due to evaporation of water from the carton. Hence.

$$q_h'' \cdot A_s \cdot \Delta t = \Delta M \cdot h_{fg}.$$

For the prescribed radiant flux q_h'',

$$\Delta M = \frac{q_h'' A_s \Delta t}{h_{fg}} = \frac{5000\ W/m^2 \times 0.0625\ m^2 \times 18s}{2400\ kJ/kg} = 0.00234\ kg.$$

The chief engineer's requirement was to remove 10% of the water content, or

$$\Delta M_{req} = M \times 0.10 = 0.220\ kg \times 0.10 = 0.022\ kg$$

which is nearly an order of magnitude larger than the evaporative loss. Considering heat losses by convection and radiation, the actual water removal from the carton will be less than ΔM. Hence, the purchase should not be recommended, since the desired water removal cannot be achieved. $<$

PROBLEM 1.53

KNOWN: Average heat sink temperature when total dissipation is 20 W with prescribed air and surroundings temperature, sink surface area and emissivity.

FIND: Sink temperature when dissipation is 30 W.

SCHEMATIC:

Power devices, P_e=20 or 30W
Sink, T_s=42°C with P_e=20W, A_s=0.045m², ε = 0.8
q_{rad}
q_{conv}
Air
T_{sur}=27°C
T_∞=27°C

ASSUMPTIONS: (1) Steady-state conditions, (2) All dissipated power in devices is transferred to the sink, (3) Sink is isothermal, (4) Surroundings and air temperature remain the same for both power levels, (5) Convection coefficient is the same for both power levels, (6) Heat sink is a small surface within a large enclosure, the surroundings.

ANALYSIS: Define a control volume around the heat sink. Power dissipated within the devices is transferred into the sink, while the sink loses heat to the ambient air and surroundings by convection and radiation exchange, respectively.

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$P_e - hA_s\left(T_s - T_\infty\right) - A_s\varepsilon\sigma\left(T_s^4 - T_{sur}^4\right) = 0. \qquad (1)$$

Consider the situation when P_e = 20 W for which T_s = 42°C; find the value of h.

$$h = \left[P_e / A_s - \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right)\right] / \left(T_s - T_\infty\right)$$

$$h = \left[20\ \text{W}/0.045\ \text{m}^2 - 0.8 \times 5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4 \left(315^4 - 300^4\right)\text{K}^4\right] / \left(315 - 300\right)\text{K}$$

$$h = 24.4\ \text{W/m}^2 \cdot \text{K}.$$

For the situation when P_e = 30 W, using this value for h with Eq. (1), obtain

$$30\ \text{W} - 24.4\ \text{W/m}^2 \cdot \text{K} \times 0.045\ \text{m}^2 \left(T_s - 300\right)\text{K}$$
$$-0.045\ \text{m}^2 \times 0.8 \times 5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4 \left(T_s^4 - 300^4\right)\text{K}^4 = 0$$

$$30 = 1.098\left(T_s - 300\right) + 2.041 \times 10^{-9}\left(T_s^4 - 300^4\right).$$

By trial-and-error, find

$$T_s \approx 322\ \text{K} = 49^\circ\text{C}.$$ <

COMMENTS: (1) It is good practice to express all temperatures in kelvin units when using energy balances involving radiation exchange.

(2) Note that we have assumed A_s is the same for the convection and radiation processes. Since not all portions of the fins are completely exposed to the surroundings, $A_{s,rad}$ is less than $A_{s,conv} = A_s$.

(3) Is the assumption that the heat sink is isothermal reasonable?

PROBLEM 1.54

KNOWN: Number and power dissipation of PCBs in a computer console. Convection coefficient associated with heat transfer from individual components in a board. Inlet temperature of cooling air and fan power requirement. Maximum allowable temperature rise of air. Heat flux from component most susceptible to thermal failure.

FIND: (a) Minimum allowable volumetric flow rate of air, (b) Preferred location and corresponding surface temperature of most thermally sensitive component.

SCHEMATIC:

$(T_o - T_i)_{max} = 15^{o}C$

P_b

$h = 200\ W/m^2\text{-}K$

$P_b = 20\ W$

$P_f = 25\ W$

$q'' = 1\ W/cm^2$

$\rho = 1.161\ kg/m^3$

$c_p = 1007\ J/kg$

Air

$\dot{\forall}, T_i = 20^{o}C$

ASSUMPTIONS: (1) Steady-state, (2) Constant air properties, (3) Negligible potential and kinetic energy changes of air flow, (4) Negligible heat transfer from console to ambient air, (5) Uniform convection coefficient for all components.

ANALYSIS: (a) For a control surface about the air space in the console, conservation of energy for an open system, Eq. (1.11e), reduces to

$$\dot{m}(u+pv)_i - \dot{m}(u+pv)_o + q - \dot{W} = 0$$

where $u + pv = i$, $q = 5P_b$, and $\dot{W} = -P_f$. Hence, with $\dot{m}(i_i - i_o) = \dot{m}c_p(T_i - T_o)$,

$$\dot{m}c_p(T_o - T_i) = 5\ P_b + P_f$$

For a maximum allowable temperature rise of 15°C, the required mass flow rate is

$$\dot{m} = \frac{5\ P_b + P_f}{c_p(T_o - T_i)} = \frac{5\times 20\ W + 25\ W}{1007\ J/kg\cdot K\left(15\ ^{\circ}C\right)} = 8.28\times 10^{-3}\,kg/s$$

The corresponding volumetric flow rate is

$$\dot{\forall} = \frac{\dot{m}}{\rho} = \frac{8.28\times 10^{-3}\,kg/s}{1.161\ kg/m^3} = 7.13\times 10^{-3}\,m^3/s$$ <

(b) The component which is most susceptible to thermal failure should be mounted at the bottom of one of the PCBs, where the air is coolest. From the corresponding form of Newton's law of cooling, $q'' = h(T_s - T_i)$, the surface temperature is

$$T_s = T_i + \frac{q''}{h} = 20^{\circ}C + \frac{1\times 10^4\ W/m^2}{200\ W/m^2\cdot K} = 70^{\circ}C$$ <

COMMENTS: (1) Although the mass flow rate is invariant, the volumetric flow rate increases as the air is heated in its passage through the console, causing a reduction in the density. However, for the prescribed temperature rise, the change in ρ, and hence the effect on $\dot{\forall}$, is small. (2) If the thermally sensitive component were located at the top of a PCB, it would be exposed to warmer air (T_o = 35°C) and the surface temperature would be T_s = 85°C.

PROBLEM 1.55

KNOWN: Top surface of car roof absorbs solar flux, $q''_{S,abs}$, and experiences for case (a): convection with air at T_∞ and for case (b): the same convection process and radiation emission from the roof.

FIND: Temperature of the plate, T_s, for the two cases. Effect of airflow on roof temperature.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible heat transfer to auto interior, (3) Negligible radiation from atmosphere.

ANALYSIS: (a) Apply an energy balance to the control surfaces shown on the schematic. For an instant of time, $\dot{E}_{in} - \dot{E}_{out} = 0$. Neglecting radiation emission, the relevant processes are convection between the plate and the air, q''_{conv}, and the absorbed solar flux, $q''_{S,abs}$. Considering the roof to have an area A_s,

$$q''_{S,abs} \cdot A_s - hA_s\left(T_s - T_\infty\right) = 0$$

$$T_s = T_\infty + q''_{S,abs}/h$$

$$T_s = 20^\circ C + \frac{800 W/m^2}{12 W/m^2 \cdot K} = 20^\circ C + 66.7^\circ C = 86.7^\circ C \qquad <$$

(b) With radiation emission from the surface, the energy balance has the form

$$q''_{S,abs} \cdot A_s - q_{conv} - E \cdot A_s = 0$$

$$q''_{S,abs} A_s - hA_s\left(T_s - T_\infty\right) - \varepsilon A_s \sigma T_s^4 = 0.$$

Substituting numerical values, with temperature in absolute units (K),

$$800\frac{W}{m^2} - 12\frac{W}{m^2 \cdot K}\left(T_s - 293K\right) - 0.8 \times 5.67 \times 10^{-8}\frac{W}{m^2 \cdot K^4} T_s^4 = 0$$

$$12T_s + 4.536 \times 10^{-8} T_s^4 = 4316$$

It follows that T_s = 320 K = 47°C. $<$

Continued.….

PROBLEM 1.55 (Cont.)

(c) Parametric calculations were performed using the IHT *First Law Model* for an *Isothermal Plane Wall*. As shown below, the roof temperature depends strongly on the velocity of the auto relative to the ambient air. For a convection coefficient of h = 40 W/m^2·K, which would be typical for a velocity of 55 mph, the roof temperature would exceed the ambient temperature by less than 10°C.

COMMENTS: By considering radiation emission, T_s decreases, as expected. Note the manner in which q''_{conv} is formulated using Newton's law of cooling; since q''_{conv} is shown leaving the control surface, the rate equation must be $h\left(T_S - T_\infty\right)$ and not $h\left(T_\infty - T_S\right)$.

PROBLEM 1.56

KNOWN: Detector and heater attached to cold finger immersed in liquid nitrogen. Detector surface of $\varepsilon = 0.9$ is exposed to large vacuum enclosure maintained at 300 K.

FIND: (a) Temperature of detector when no power is supplied to heater, (b) Heater power (W) required to maintain detector at 195 K, (c) Effect of finger thermal conductivity on heater power.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction through cold finger, (3) Detector and heater are very thin and isothermal at T_s, (4) Detector surface is small compared to enclosure surface.

PROPERTIES: Cold finger (given): k = 10 W/m·K.

ANALYSIS: Define a control volume about detector and heater and apply conservation of energy requirement on a rate basis, Eq. 1.11a,

$$\dot{E}_{in} - \dot{E}_{out} = 0 \quad (1)$$

where

$$\dot{E}_{in} = q_{rad} + q_{elec}; \qquad \dot{E}_{out} = q_{cond} \quad (2,3)$$

Combining Eqs. (2,3) with (1), and using the appropriate rate equations,

$$\varepsilon A_s \sigma \left(T_{sur}^4 - T_s^4\right) + q_{elec} = kA_s\left(T_s - T_L\right)/L. \quad (4)$$

(a) Where $q_{elec} = 0$, substituting numerical values

$$0.9 \times 5.67 \times 10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left(300^4 - T_s^4\right)\text{K}^4 = 10\,\text{W/m}\cdot\text{K}\left(T_s - 77\right)\text{K}/0.050\,\text{m}$$

$$5.103 \times 10^{-8}\left(300^4 - T_s^4\right) = 200\left(T_s - 77\right)$$

$$T_s = 79.1\,\text{K}$$ <

Continued…..

PROBLEM 1.56 (Cont.)

(b) When T_s = 195 K, Eq. (4) yields

$$0.9\times[\pi\,(0.005\,\text{m})^2/4]\times 5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left(300^4-195^4\right)\text{K}^4+q_{elec}$$

$$=10\,\text{W/m}\cdot\text{K}\times[\pi\,(0.005\,\text{m})^2/4]\times(195-77)\,\text{K}/0.050\,\text{m}$$

$$q_{elec}=0.457\,\text{W}=457\,\text{mW}$$ <

(c) Calculations were performed using the *First Law Model* for a *Nonisothermal Plane Wall*. With net radiative transfer to the detector fixed by the prescribed values of T_s and T_{sur}, Eq. (4) indicates that q_{elec} increases linearly with increasing k.

Heat transfer by conduction through the finger material increases with its thermal conductivity. Note that, for k = 0.1 W/m·K, q_{elec} = -2 mW, where the minus sign implies the need for a heat *sink*, rather than a heat source, to maintain the detector at 195 K. In this case q_{rad} exceeds q_{cond}, and a heat sink would be needed to dispose of the difference. A conductivity of k = 0.114 W/m·K yields a precise balance between q_{rad} and q_{cond}. Hence to circumvent heaving to use a heat sink, while minimizing the heater power requirement, k should exceed, but remain as close as possible to the value of 0.114 W/m·K. Using a graphite fiber composite, with the fibers oriented normal to the direction of conduction, Table A.2 indicates a value of k ≈ 0.54 W/m·K at an average finger temperature of $\overline{T}$ = 136 K. For this value, q_{elec} = 18 mW

COMMENTS: The heater power requirement could be further reduced by decreasing ε.

PROBLEM 1.57

KNOWN: Conditions at opposite sides of a furnace wall of prescribed thickness, thermal conductivity and surface emissivity.

FIND: Effect of wall thickness and outer convection coefficient on surface temperatures. Recommended values of L and h_2.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Negligible radiation exchange at surface 1, (4) Surface 2 is exposed to large surroundings.

ANALYSIS: The unknown temperatures may be obtained by simultaneously solving energy balance equations for the two surface. At surface 1,

$$q''_{conv,1} = q''_{cond}$$

$$h_1\left(T_{\infty,1} - T_1\right) = k\left(T_1 - T_2\right)/L \qquad (1)$$

At surface 2,

$$q''_{cond} = q''_{conv} + q''_{rad}$$

$$k\left(T_1 - T_2\right)/L = h_2\left(T_2 - T_{\infty,2}\right) + \varepsilon\sigma\left(T_2^4 - T_{sur}^4\right) \qquad (2)$$

Using the IHT *First Law Model* for a *Nonisothermal Plane Wall*, we obtain

Continued …..

PROBLEM 1.57 (Cont.)

Both q''_{cond} and T_2 decrease with increasing wall thickness, and for the prescribed value of h_2 = 10 W/m^2·K, a value of L ≥ 0.275 m is needed to maintain T_2 ≤ 373 K = 100 °C. Note that inner surface temperature T_1, and hence the temperature difference, $T_1 - T_2$, increases with increasing L.

Performing the calculations for the prescribed range of h_2, we obtain

For the prescribed value of L = 0.15 m, a value of h_2 ≥ 24 W/m^2·K is needed to maintain T_2 ≤ 373 K. The variation has a negligible effect on T_1, causing it to decrease slightly with increasing h_2, but does have a strong influence on T_2.

COMMENTS: If one wishes to avoid use of active (forced convection) cooling on side 2, reliance will have to be placed on free convection, for which h_2 ≈ 5 W/m^2·K. The minimum wall thickness would then be L = 0.40 m.

PROBLEM 1.58

KNOWN: Furnace wall with inner surface temperature T_1 = 352°C and prescribed thermal conductivity experiencing convection and radiation exchange on outer surface. See Example 1.5.

FIND: (a) Outer surface temperature T_2 resulting from decreasing the wall thermal conductivity k or increasing the convection coefficient h by a factor of two; benefit of applying a low emissivity coating ($\varepsilon < 0.8$); comment on the effectiveness of these strategies to reduce risk of burn injury when $T_2 \leq 65°C$; and (b) Calculate and plot T_2 as a function of h for the range $20 \leq h \leq 100\ W/m^2 \cdot K$ for three materials with k = 0.3, 0.6, and 1.2 W/m·K; what conditions will provide for safe outer surface temperatures.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in wall, (3) Radiation exchange is between small surface and large enclosure, (4) Inner surface temperature remains constant for all conditions.

ANALYSIS: (a) The surface (x = L) energy balance is

$$k\frac{T_1 - T_2}{L} = h\left(T_2 - T_\infty\right) + \varepsilon\sigma\left(T_2^4 - T_{sur}^4\right)$$

With T_1 = 352°C, the effects of parameters h, k and ε on the outer surface temperature are calculated and tabulated below.

Conditions	$k\left(W/m\cdot K\right)$	$h\left(W/m^2\cdot K\right)$	ε	$T_2\left(°C\right)$
Example 1.5	1.2	20	0.8	100
Decrease k by ½	0.6	20	0.8	69
Increase h by 2	1.2	40	0.8	73
Change k and h	0.6	40	0.8	51
Decrease ε	1.2	20	0.1	115

(b) Using the energy balance relation in the Workspace of IHT, the outer surface temperature can be calculated and plotted as a function of the convection coefficient for selected values of the wall thermal conductivity.

Continued …..

PROBLEM 1.58 (Cont.)

Outer surface temperature, T2 (C)

100
80
60
40
20

20 40 60 80 100

Convection coefficient, h (W/m^2.K)

k = 1.2 W/m.K
k = 0.6 W/m.K
k = 0.3 W/m.K

COMMENTS: (1) From the parameter study of part (a), note that decreasing the thermal conductivity is more effective in reducing T_2 than is increasing the convection coefficient. Only if both changes are made will T_2 be in the safe range.

(2) From part (a), note that applying a low emissivity coating is not beneficial. Did you suspect that before you did the analysis? Give a physical explanation for this result.

(3) From the parameter study graph we conclude that safe wall conditions ($T_2 \leq 65°C$) can be maintained for these conditions: with k = 1.2 W/m·K when h > 55 $W/m^2 \cdot K$; with k = 0.6 W/m·K when h > 25 $W/m^2 \cdot K$; and with k = 0.3 W/m·K when h > 20 W/m·K.

PROBLEM 1.59

KNOWN: Inner surface temperature, thickness and thermal conductivity of insulation exposed at its outer surface to air of prescribed temperature and convection coefficient.

FIND: Outer surface temperature.

SCHEMATIC:

Insulation, $k = 0.1$ W/m·K

Air

$T_\infty = 35°C$

$h = 500$ W/m²·K

$T_1 = 400°C$

q''_{cond}

q''_{conv}

T_2

$L = 0.025m$

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in the insulation, (3) Negligible radiation exchange between outer surface and surroundings.

ANALYSIS: From an energy balance at the outer surface at an instant of time,

$$q''_{cond} = q''_{conv}.$$

Using the appropriate rate equations,

$$k\frac{(T_1 - T_2)}{L} = h(T_2 - T_\infty).$$

Solving for T_2, find

$$T_2 = \frac{\frac{k}{L}T_1 + h\,T_\infty}{h + \frac{k}{L}} = \frac{\frac{0.1\text{ W/m}\cdot\text{K}}{0.025\text{m}}\left(400^\circ\text{C}\right) + 500\frac{\text{W}}{\text{m}^2\cdot\text{K}}\left(35^\circ\text{C}\right)}{500\frac{\text{W}}{\text{m}^2\cdot\text{K}} + \frac{0.1\text{ W/m}\cdot\text{K}}{0.025\text{m}}}$$

$$T_2 = 37.9^\circ\text{C}.$$ <

COMMENTS: If the temperature of the surroundings is approximately that of the air, radiation exchange between the outer surface and the surroundings will be negligible, since T_2 is small. In this case convection makes the dominant contribution to heat transfer from the outer surface, and assumption (3) is excellent.

PROBLEM 1.60

KNOWN: Thickness and thermal conductivity, k, of an oven wall. Temperature and emissivity, ε, of front surface. Temperature and convection coefficient, h, of air. Temperature of large surroundings.

FIND: (a) Temperature of back surface, (b) Effect of variations in k, h and ε.

SCHEMATIC:

$k = 0.7$ W/m·K; q''_{rad}; q''_{conv}; q''_{cond}; $T_\infty = 300$ K; $h = 20$ W/m²·K; T_1; $T_2 = 400$ K; ε = 0.8; $L = 0.05$ m; $T_{sur} = 300$ K

ASSUMPTIONS: (1) Steady-state, (2) One-dimensional conduction, (3) Radiation exchange with large surroundings.

ANALYSIS: (a) Applying an energy balance, Eq. 1.13, at an instant of time to the front surface and substituting the appropriate rate equations, Eqs. 1.2, 1.3a and 1.7, find

$$k\frac{T_1 - T_2}{L} = h\left(T_2 - T_\infty\right) + \varepsilon\sigma\left(T_2^4 - T_{sur}^4\right).$$

Substituting numerical values, find

$$T_1 - T_2 = \frac{0.05\,\text{m}}{0.7\,\text{W/m}\cdot\text{K}}\left[20\frac{\text{W}}{\text{m}^2\cdot\text{K}}100\,\text{K} + 0.8\times5.67\times10^{-8}\frac{\text{W}}{\text{m}^2\cdot\text{K}^4}\left[(400\,\text{K})^4 - (300\,\text{K})^4\right]\right] = 200\,\text{K}\,.$$

Since T_2 = 400 K, it follows that T_1 = 600 K. <

(b) Parametric effects may be evaluated by using the IHT *First Law* Model for a *Nonisothermal Plane* Wall. Changes in k strongly influence conditions for k < 20 W/m·K, but have a negligible effect for larger values, as T_2 approaches T_1 and the heat fluxes approach the corresponding limiting values

Temperature, T2(K) vs. Thermal conductivity, k(W/m.K)

Heat flux, q''(W/m^2) vs. Thermal conductivity, k(W/m.K)

Conduction heat flux, q''cond(W/m^2)
Convection heat flux, q''conv(W/m^2)
Radiation heat flux, q''rad(W/m^2)

PROBLEM 1.60 (Cont.)

The implication is that, for k > 20 W/m·K, heat transfer by conduction in the wall is extremely efficient relative to heat transfer by convection and radiation, which become the *limiting* heat transfer processes. Larger fluxes could be obtained by increasing ε and h and/or by decreasing T_∞ and T_{sur}.

With increasing h, the front surface is cooled more effectively (T_2 decreases), and although q''_{rad} decreases, the reduction is exceeded by the increase in q''_{conv}. With a reduction in T_2 and fixed values of k and L, q''_{cond} must also increase.

Temperature, T2(K)
Convection coefficient, h(W/m^2.K)

Heat flux, q''(W/m^2)
Convection coefficient, h(W/m^2.K)
Conduction heat flux, q''cond(W/m^2)
Convection heat flux, q''conv(W/m^2)
Radiation heat flux, q''rad(W/m^2)

The surface temperature also decreases with increasing ε, and the increase in q''_{rad} exceeds the reduction in q''_{conv}, allowing q''_{cond} to increase with ε.

Temperature, T2(K)
Emissivity

Heat flux, q''(W/m^2)
Emissivity
Conduction heat flux, q''cond(W/m^2)
Convection heat flux, q''conv(W/m^2)
Radiation heat flux, q''rad(W/m^2)

COMMENTS: Conservation of energy, of course, dictates that, irrespective of the prescribed conditions, $q''_{cond} = q''_{conv} + q''_{rad}$.

PROBLEM 1.61

KNOWN: Temperatures at 10 mm and 20 mm from the surface and in the adjoining airflow for a thick steel casting.

FIND: Surface convection coefficient, h.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state, (2) One-dimensional conduction in the x-direction, (3) Constant properties, (4) Negligible generation.

ANALYSIS: From a surface energy balance, it follows that

$$q''_{cond} = q''_{conv}$$

where the convection rate equation has the form

$$q''_{conv} = h\left(T_\infty - T_0\right),$$

and q''_{cond} can be evaluated from the temperatures prescribed at surfaces 1 and 2. That is, from Fourier's law,

$$q''_{cond} = k\frac{T_1 - T_2}{x_2 - x_1}$$

$$q''_{cond} = 15\frac{W}{m \cdot K}\frac{(50-40)^\circ C}{(20\text{-}10)\times 10^{-3} m} = 15,000\ W/m^2.$$

Since the temperature gradient in the solid must be linear for the prescribed conditions, it follows that

$$T_0 = 60^\circ C.$$

Hence, the convection coefficient is

$$h = \frac{q''_{cond}}{T_\infty - T_0}$$

$$h = \frac{15,000\ W/m^2}{40^\circ C} = 375\ W/m^2 \cdot K.$$ <

COMMENTS: The accuracy of this procedure for measuring h depends strongly on the validity of the assumed conditions.

PROBLEM 1.62

KNOWN: Duct wall of prescribed thickness and thermal conductivity experiences prescribed heat flux q''_o at outer surface and convection at inner surface with known heat transfer coefficient.

FIND: (a) Heat flux at outer surface required to maintain inner surface of duct at T_i = 85°C, (b) Temperature of outer surface, T_o, (c) Effect of h on T_o and q''_o.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in wall, (3) Constant properties, (4) Backside of heater perfectly insulated, (5) Negligible radiation.

ANALYSIS: (a) By performing an energy balance on the wall, recognize that $q''_o = q''_{cond}$. From an energy balance on the top surface, it follows that $q''_{cond} = q''_{conv} = q''_o$. Hence, using the convection rate equation,

$$q''_o = q''_{conv} = h\left(T_i - T_\infty\right) = 100\,\text{W/m}^2\cdot\text{K}\left(85-30\right)^\circ\text{C} = 5500\,\text{W/m}^2. \quad <$$

(b) Considering the duct wall and applying Fourier's Law,

$$q''_o = k\frac{\Delta T}{\Delta X} = k\frac{T_o - T_i}{L}$$

$$T_o = T_i + \frac{q''_o L}{k} = 85^\circ\text{C} + \frac{5500\,\text{W/m}^2 \times 0.010\,\text{m}}{20\,\text{W/m}\cdot\text{K}} = \left(85+2.8\right)^\circ\text{C} = 87.8^\circ\text{C}. \quad <$$

(c) For T_i = 85°C, the desired results may be obtained by simultaneously solving the energy balance equations

$$q''_o = k\frac{T_o - T_i}{L} \qquad \text{and} \qquad k\frac{T_o - T_i}{L} = h\left(T_i - T_\infty\right)$$

Using the IHT *First Law* Model for a *Nonisothermal Plane Wall*, the following results are obtained.

Since q''_{conv} increases linearly with increasing h, the applied heat flux q''_o must be balanced by an increase in q''_{cond}, which, with fixed k, T_i and L, necessitates an increase in T_o.

COMMENTS: The temperature difference across the wall is small, amounting to a maximum value of $\left(T_o - T_i\right)$ = 5.5°C for h = 200 $W/m^2\cdot K$. If the wall were thinner (L < 10 mm) or made from a material with higher conductivity (k > 20 W/m·K), this difference would be reduced.

PROBLEM 1.63

KNOWN: Dimensions, average surface temperature and emissivity of heating duct. Duct air inlet temperature and velocity. Temperature of ambient air and surroundings. Convection coefficient.

FIND: (a) Heat loss from duct, (b) Air outlet temperature.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state, (2) Constant air properties, (3) Negligible potential and kinetic energy changes of air flow, (4) Radiation exchange between a small surface and a large enclosure.

ANALYSIS: (a) Heat transfer from the surface of the duct to the ambient air and the surroundings is given by Eq. (1.10)

$$q = hA_s\left(T_s - T_\infty\right) + \varepsilon A_s \sigma\left(T_s^4 - T_{sur}^4\right)$$

where A_s = L (2W + 2H) = 15 m (0.7 m + 0.5 m) = 16.5 m^2. Hence,

$$q = 4\ \text{W/m}^2\cdot\text{K}\times 16.5\ \text{m}^2\left(45^\circ\text{C}\right) + 0.5\times 16.5\ \text{m}^2\times 5.67\times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4\left(323^4 - 278^4\right)\text{K}^4$$

$$q = q_{conv} + q_{rad} = 2970\ \text{W} + 2298\ \text{W} = 5268\ \text{W} \qquad <$$

(b) With i = u + pv, $\dot{W}$ = 0 and the third assumption, Eq. (1.11e) yields,

$$\dot{m}\left(i_i - i_o\right) = \dot{m}c_p\left(T_i - T_o\right) = q$$

where the sign on q has been reversed to reflect the fact that heat transfer is *from* the system.

With $\dot{m} = \rho V A_c = 1.10\ \text{kg/m}^3 \times 4\ \text{m/s}\left(0.35\text{m}\times 0.20\text{m}\right) = 0.308\ \text{kg/s}$, the outlet temperature is

$$T_o = T_i - \frac{q}{\dot{m}c_p} = 58^\circ\text{C} - \frac{5268\ \text{W}}{0.308\ \text{kg/s}\times 1008\ \text{J/kg}\cdot\text{K}} = 41^\circ\text{C} \qquad <$$

COMMENTS: The temperature drop of the air is large and unacceptable, unless the intent is to use the duct to heat the basement. If not, the duct should be insulated to insure maximum delivery of thermal energy to the intended space(s).

PROBLEM 1.64

KNOWN: Uninsulated pipe of prescribed diameter, emissivity, and surface temperature in a room with fixed wall and air temperatures. See Example 1.2.

FIND: (a) Which option to reduce heat loss to the room is more effective: reduce by a factor of two the convection coefficient (from 15 to 7.5 $W/m^2 \cdot K$) or the emissivity (from 0.8 to 0.4) and (b) Show graphically the heat loss as a function of the convection coefficient for the range $5 \le h \le 20$ $W/m^2 \cdot K$ for emissivities of 0.2, 0.4 and 0.8. Comment on the relative efficacy of reducing heat losses associated with the convection and radiation processes.

SCHEMATIC:

$T_\infty = 25^oC$
$h = 15\ W/m^2\text{-}K$
q'_{conv}
q'_{rad}
$T_{sur} = 25^oC$
$T_s = 200^oC$
D = 70 mm
$\varepsilon = 0.8$

ASSUMPTIONS: (1) Steady-state conditions, (2) Radiation exchange between pipe and the room is between a small surface in a much larger enclosure, (3) The surface emissivity and absorptivity are equal, and (4) Restriction of the air flow does not alter the radiation exchange process between the pipe and the room.

ANALYSIS: (a) The heat rate from the pipe to the room per unit length is

$$q' = q'/L = q'_{conv} + q'_{rad} = h(\pi D)(T_s - T_\infty) + \varepsilon(\pi D)\sigma\left(T_s^4 - T_{sur}^4\right)$$

Substituting numerical values for the two options, the resulting heat rates are calculated and compared with those for the conditions of Example 1.2. We conclude that both options are comparably effective.

Conditions	$h\left(W/m^2 \cdot K\right)$	ε	$q'(W/m)$
Base case, Example 1.2	15	0.8	998
Reducing h by factor of 2	7.5	0.8	788
Reducing ε by factor of 2	15	0.4	709

(b) Using IHT, the heat loss can be calculated as a function of the convection coefficient for selected values of the surface emissivity.

Heat loss, q' (/m)
1200
800
400
0
5
10
15
20
Convection coefficient, h (W/m^2.K)
eps = 0.8, bare pipe
eps = 0.4, coated pipe
eps = 0.2, coated pipe

Continued …..

COMMENTS: (1) In Example 1.2, Comment 3, we read that the heat rates by convection and radiation exchange were comparable for the base case conditions (577 vs. 421 W/m). It follows that reducing the key transport parameter (h or ε) by a factor of two yields comparable reductions in the heat loss. Coating the pipe to reduce the emissivity might to be the more practical option as it may be difficult to control air movement.

(2) For this pipe size and thermal conditions (T_s and T_∞), the minimum possible convection coefficient is approximately 7.5 $W/m^2 \cdot K$, corresponding to free convection heat transfer to quiescent ambient air. Larger values of h are a consequence of forced air flow conditions.

(3) The Workspace for the IHT program to calculate the heat loss and generate the graph for the heat loss as a function of the convection coefficient for selected emissivities is shown below. It is good practice to provide commentary with the code making your solution logic clear, and to summarize the results.

```
// Heat loss per unit pipe length; rate equation from Ex. 1.2
q' = q'cv + q'rad
q'cv = pi*D*h*(Ts - Tinf)
q'rad = pi*D*eps*sigma*(Ts^4 - Tsur^4)
sigma = 5.67e-8

// Input parameters
D = 0.07
Ts_C = 200        // Representing temperatures in Celsius units using _C subscripting
Ts = Ts_C +273
Tinf_C = 25
Tinf = Tinf_C + 273
h = 15            // For graph, sweep over range from 5 to 20
Tsur_C = 25
Tsur = Tsur_C + 273
eps = 0.8
//eps = 0.4       // Values of emissivity for parameter study
//eps = 0.2

/* Base case results
Tinf   Ts     Tsur   q'      q'cv    q'rad   D      Tinf_C  Ts_C   Tsur_C
       eps    h      sigma
298    473    298    997.9   577.3   420.6   0.07   25      200    25
       0.8    15     5.67E-8    */
```

PROBLEM 1.65

KNOWN: Conditions associated with surface cooling of plate glass which is initially at 600°C. Maximum allowable temperature gradient in the glass.

FIND: Lowest allowable air temperature, T_∞

SCHEMATIC:

ASSUMPTIONS: (1) Surface of glass exchanges radiation with large surroundings at $T_{sur} = T_\infty$, (2) One-dimensional conduction in the x-direction.

ANALYSIS: The maximum temperature gradient will exist at the surface of the glass and at the instant that cooling is initiated. From the surface energy balance, Eq. 1.12, and the rate equations, Eqs. 1.1, 1.3a and 1.7, it follows that

$$-k\frac{dT}{dx} - h\left(T_s - T_\infty\right) - \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right) = 0$$

or, with $(dT/dx)_{max} = -15°C/mm = -15{,}000°C/m$ and $T_{sur} = T_\infty$,

$$-1.4\frac{W}{m\cdot K}\left[-15{,}000\frac{°C}{m}\right] = 5\frac{W}{m^2\cdot K}\left(873 - T_\infty\right)K$$

$$+0.8\times 5.67\times 10^{-8}\frac{W}{m^2\cdot K^4}\left[873^4 - T_\infty^4\right]K^4.$$

T_∞ may be obtained from a trial-and-error solution, from which it follows that, for $T_\infty = 618K$,

$$21{,}000\frac{W}{m^2} \approx 1275\frac{W}{m^2} + 19{,}730\frac{W}{m^2}.$$

Hence the lowest allowable air temperature is

$$T_\infty \approx 618K = 345^\circ C.$$ <

COMMENTS: (1) Initially, cooling is determined primarily by radiation effects.

(2) For fixed T_∞, the surface *temperature gradient* would *decrease* with *increasing* time into the cooling process. Accordingly, T_∞ could be decreasing with increasing time and still keep within the maximum allowable temperature gradient.

PROBLEM 1.66

KNOWN: Hot-wall oven, in lieu of infrared lamps, with temperature T_{sur} = 200°C for heating a coated plate to the cure temperature. See Example 1.6.

FIND: (a) The plate temperature T_s for prescribed convection conditions and coating emissivity, and (b) Calculate and plot T_s as a function of T_{sur} for the range $150 \le T_{sur} \le 250$°C for ambient air temperatures of 20, 40 and 60°C; identify conditions for which acceptable curing temperatures between 100 and 110°C may be maintained.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible heat loss from back surface of plate, (3) Plate is small object in large isothermal surroundings (hot oven walls).

ANALYSIS: (a) The temperature of the plate can be determined from an energy balance on the plate, considering radiation exchange with the hot oven walls and convection with the ambient air.

$$\dot{E}''_{in} - \dot{E}''_{out} = 0 \qquad \text{or} \qquad q''_{rad} - q''_{conv} = 0$$

$$\varepsilon\sigma\left(T_{sur}^4 - T_s^4\right) - h\left(T_s - T_\infty\right) = 0$$

$$0.5 \times 5.67 \times 10^{-8}\ W/m^2 \cdot K^4\left([200+273]^4 - T_s^4\right)K^4 - 15\ W/m^2 \cdot K\left(T_s - [20+273]\right)K = 0$$

$$T_s = 357\ K = 84°C$$ <

(b) Using the energy balance relation in the Workspace of IHT, the plate temperature can be calculated and plotted as a function of oven wall temperature for selected ambient air temperatures.

COMMENTS: From the graph, acceptable cure temperatures between 100 and 110°C can be maintained for these conditions: with T_∞ = 20°C when $225 \le T_{sur} \le 240$°C; with T_∞ = 40°C when $205 \le T_{sur} \le 220$°C; and with T_∞ = 60°C when $175 \le T_{sur} \le 195$°C.

PROBLEM 1.67

KNOWN: Operating conditions for an electrical-substitution radiometer having the same receiver temperature, T_s, in electrical and optical modes.

FIND: Optical power of a laser beam and corresponding receiver temperature when the indicated electrical power is 20.64 mW.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Conduction losses from backside of receiver negligible in optical mode, (3) Chamber walls form large isothermal surroundings; negligible effects due to aperture, (4) Radiation exchange between the receiver surface and the chamber walls is between small surface and large enclosure, (5) Negligible convection effects.

PROPERTIES: Receiver surface: $\varepsilon = 0.95$, $\alpha_{opt} = 0.98$.

ANALYSIS: The schematic represents the operating conditions for the *electrical mode* with the optical beam blocked. The temperature of the receiver surface can be found from an energy balance on the receiver, considering the electrical power input, conduction loss from the backside of the receiver, and the radiation exchange between the receiver and the chamber.

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$P_{elec} - q_{loss} - q_{rad} = 0$$

$$P_{elec} - 0.05\,P_{elec} - \varepsilon\, A_s \sigma \left(T_s^4 - T_{sur}^4\right) = 0$$

$$20.64\times10^{-3}\,\mathrm{W}\,(1-0.05) - 0.95\left(\pi 0.015^2/4\right)\mathrm{m}^2 \times 5.67\times10^{-8}\,\mathrm{W/m^2\cdot K^4}\left(T_s^4 - 77^4\right)\mathrm{K}^4 = 0$$

$$T_s = 213.9\ \mathrm{K}$$ <

For the *optical mode* of operation, the optical beam is incident on the receiver surface, there is no electrical power input, and the receiver temperature is the same as for the electrical mode. The optical power of the beam can be found from an energy balance on the receiver considering the absorbed beam power and radiation exchange between the receiver and the chamber.

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$\alpha_{opt}\, P_{opt} - q_{rad} = 0.98\,P_{opt} - 19.60\,\mathrm{mW} = 0$$

$$P_{opt} = 19.99\,\mathrm{mW}$$ <

where q_{rad} follows from the previous energy balance using T_s = 213.9K.

COMMENTS: Recognizing that the receiver temperature, and hence the radiation exchange, is the same for both modes, an energy balance could be directly written in terms of the absorbed optical power and equivalent electrical power, $\alpha_{opt}\, P_{opt} = P_{elec} - q_{loss}$.

PROBLEM 1.68

KNOWN: Surface temperature, diameter and emissivity of a hot plate. Temperature of surroundings and ambient air. Expression for convection coefficient.

FIND: (a) Operating power for prescribed surface temperature, (b) Effect of surface temperature on power requirement and on the relative contributions of radiation and convection to heat transfer from the surface.

SCHEMATIC:

ASSUMPTIONS: (1) Plate is of uniform surface temperature, (2) Walls of room are large relative to plate, (3) Negligible heat loss from bottom or sides of plate.

ANALYSIS: (a) From an energy balance on the hot plate, $P_{elec} = q_{conv} + q_{rad} = A_p \left(q''_{conv} + q''_{rad}\right)$.

Substituting for the area of the plate and from Eqs. (1.3a) and (1.7), with h = 0.70 $(T_s - T_\infty)^{1/3}$, it follows that

$$P_{elec} = \left(\pi D^2/4\right)\left[0.70\left(T_s - T_\infty\right)^{4/3} + \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right)\right]$$

$$P_{elec} = \pi\left(0.3\text{m}\right)^2/4\left[0.70\left(175\right)^{4/3} + 0.8\times5.67\times10^{-8}\left(473^4 - 298^4\right)\right]\ \text{W/m}^2$$

$$P_{elec} = 0.0707\ \text{m}^2\left[685\ \text{W/m}^2 + 1913\ \text{W/m}^2\right] = 48.4\ \text{W} + 135.2\ \text{W} = 190.6\ \text{W} \qquad <$$

(b) As shown graphically, both the radiation and convection heat rates, and hence the requisite electric power, increase with increasing surface temperature.

However, because of its dependence on the fourth power of the surface temperature, the increase in radiation is more pronounced. The significant relative effect of radiation is due to the small convection coefficients characteristic of natural convection, with $3.37 \le h \le 5.2\ \text{W/m}^2\cdot\text{K}$ for $100 \le T_s < 300°\text{C}$.

COMMENTS: Radiation losses could be reduced by applying a low emissivity coating to the surface, which would have to maintain its integrity over the range of operating temperatures.

PROBLEM 1.69

KNOWN: Long bus bar of rectangular cross-section and ambient air and surroundings temperatures. Relation for the electrical resistivity as a function of temperature.

FIND: (a) Temperature of the bar with a current of 60,000 A, and (b) Compute and plot the operating temperature of the bus bar as a function of the convection coefficient for the range $10 \le h \le 100$ $W/m^2 \cdot K$. Minimum convection coefficient required to maintain a safe-operating temperature below 120°C. Will increasing the emissivity significantly affect this result?

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Bus bar is long, (3) Uniform bus-bar temperature, (3) Radiation exchange between the outer surface of the bus bar and its surroundings is between a small surface and a large enclosure.

PROPERTIES: Bus-bar material, $\rho_e = \rho_{e,o}\left[1+\alpha\left(T-T_o\right)\right]$, $\rho_{e,o} = 0.0828\,\mu\Omega \cdot m$, $T_o = 25°C$, $\alpha = 0.0040\,K^{-1}$.

ANALYSIS: (a) An energy balance on the bus-bar for a unit length as shown in the schematic above has the form

$$\dot{E}'_{in} - \dot{E}'_{out} + \dot{E}'_{gen} = 0 \qquad -q'_{rad} - q'_{conv} + I^2 R'_e = 0$$

$$-\varepsilon P\sigma\left(T^4 - T^4_{sur}\right) - hP\left(T - T_\infty\right) + I^2\rho_e / A_c = 0$$

where $P = 2(H+W)$, $R'_e = \rho_e / A_c$ and $A_c = H \times W$. Substituting numerical values,

$$-0.8\times 2(0.600+0.200)m \times 5.67\times10^{-8} W/m^2\cdot K^4\left(T^4 - [30+273]^4\right)K^4$$

$$-10\,W/m^2\cdot K \times 2(0.600+0.200)m\left(T-[30+273]\right)K$$

$$+(60,000\,A)^2\left\{0.0828\times10^{-6}\Omega\cdot m\left[1+0.0040\,K^{-1}\left(T-[25+273]\right)K\right]\right\}/(0.600\times0.200)m^2 = 0$$

Solving for the bus-bar temperature, find $T = 426\ K = 153°C.$ <

(b) Using the energy balance relation in the Workspace of IHT, the bus-bar operating temperature is calculated as a function of the convection coefficient for the range $10 \le h \le 100$ $W/m^2 \cdot K$. From this graph we can determine that to maintain a safe operating temperature below 120°C, the minimum convection coefficient required is

$$h_{min} = 16\ W/m^2 \cdot K.$$ <

Continued …..

Using the same equations, we can calculate and plot the heat transfer rates by convection and radiation as a function of the bus-bar temperature.

Note that convection is the dominant mode for low bus-bar temperatures; that is, for low current flow. As the bus-bar temperature increases toward the safe-operating limit (120°C), convection and radiation exchange heat transfer rates become comparable. Notice that the relative importance of the radiation exchange rate increases with increasing bus-bar temperature.

COMMENTS: (1) It follows from the second graph that increasing the surface emissivity will be only significant at higher temperatures, especially beyond the safe-operating limit.

(2) The Workspace for the IHT program to perform the parametric analysis and generate the graphs is shown below. It is good practice to provide commentary with the code making your solution logic clear, and to summarize the results.

```
/* Results for base case conditions:
Ts_C   q'cv    q'rad    rhoe       H      I      Tinf_C   Tsur_C   W      alpha
       eps     h
153.3  1973    1786     1.253E-7   0.6    6E4    30       30       0.2    0.004
       0.8     10   */

// Surface energy balance on a per unit length basis
-q'cv - q'rad + Edot'gen = 0
q'cv = h * P * (Ts - Tinf)
P = 2 * (W + H)            // perimeter of the bar experiencing surface heat transfer
q'rad = eps * sigma * (Ts^4 - Tsur^4) * P
sigma = 5.67e-8
Edot'gen = I^2 * Re'
Re' = rhoe / Ac
rhoe = rhoeo * ( 1 + alpha * (Ts - Teo))
Ac = W * H

// Input parameters
I = 60000
alpha = 0.0040             // temperature coefficient, K^-1; typical value for cast aluminum
rhoeo = 0.0828e-6          // electrical resistivity at the reference temperature, Teo; microohm-m
Teo = 25 + 273             // reference temperature, K
W = 0.200
H = 0.600
Tinf_C = 30
Tinf = Tinf_C + 273
h = 10
eps = 0.8
Tsur_C = 30
Tsur = Tsur_C + 273
Ts_C = Ts - 273
```

PROBLEM 1.70

KNOWN: Solar collector designed to heat water operating under prescribed solar irradiation and loss conditions.

FIND: (a) Useful heat collected per unit area of the collector, q''_u, (b) Temperature rise of the water flow, $T_o - T_i$, and (c) Collector efficiency.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) No heat losses out sides or back of collector, (3) Collector area is small compared to sky surroundings.

PROPERTIES: *Table A.6,* Water (300K): c_p = 4179 J/kg·K.

ANALYSIS: (a) Defining the collector as the control volume and writing the conservation of energy requirement on a per unit area basis, find that

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = \dot{E}_{st}.$$

Identifying processes as per above right sketch,

$$q''_{solar} - q''_{rad} - q''_{conv} - q''_u = 0$$

where $q''_{solar} = 0.9\ q''_S$; that is, 90% of the solar flux is absorbed in the collector (Eq. 1.6). Using the appropriate rate equations, the useful heat rate per unit area is

$$q''_u = 0.9\ q''_S - \varepsilon\sigma\left(T_{cp}^4 - T_{sky}^4\right) - h\left(T_s - T_\infty\right)$$

$$q''_u = 0.9 \times 700 \frac{W}{m^2} - 0.94 \times 5.67 \times 10^{-8} \frac{W}{m^2 \cdot K^4}\left(303^4 - 263^4\right)K^4 - 10 \frac{W}{m^2 \cdot K}\left(30 - 25\right)^\circ C$$

$$q''_u = 630\ W/m^2 - 194\ W/m^2 - 50\ W/m^2 = 386\ W/m^2.$$ <

(b) The total useful heat collected is $q''_u \cdot A$. Defining a control volume about the water tubing, the useful heat causes an enthalpy change of the flowing water. That is,

$$q''_u \cdot A = \dot{m} c_p \left(T_i - T_o\right) \qquad \text{or}$$

$$\left(T_i - T_o\right) = 386\ W/m^2 \times 3m^2 / 0.01 kg/s \times 4179 J/kg \cdot K = 27.7^\circ C.$$ <

(c) The efficiency is $\eta = q''_u / q''_S = \left(386\ W/m^2\right) / \left(700\ W/m^2\right) = 0.55$ or 55%. <

COMMENTS: Note how the sky has been treated as large surroundings at a uniform temperature T_{sky}.

PROBLEM 1.71

KNOWN: Surface-mount transistor with prescribed dissipation and convection cooling conditions.

FIND: (a) Case temperature for mounting arrangement with air-gap and conductive paste between case and circuit board, (b) Consider options for increasing $\dot{E}_g$, subject to the constraint that T_c = 40°C.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Transistor case is isothermal, (3) Upper surface experiences convection; negligible losses from edges, (4) Leads provide conduction path between case and board, (5) Negligible radiation, (6) Negligible energy generation in leads due to current flow, (7) Negligible convection from surface of leads.

PROPERTIES: (Given): Air, $k_{g,a}$ = 0.0263 W/m·K; Paste, $k_{g,p}$ = 0.12 W/m·K; Metal leads, k_ℓ = 25 W/m·K.

ANALYSIS: (a) Define the transistor as the system and identify modes of heat transfer.

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = \Delta\dot{E}_{st} = 0$$

$$-q_{conv} - q_{cond,gap} - 3q_{lead} + \dot{E}_g = 0$$

$$-hA_s(T_c - T_\infty) - k_g A_s \frac{T_c - T_b}{t} - 3k_\ell A_c \frac{T_c - T_b}{L} + \dot{E}_g = 0$$

where $A_s = L_1 \times L_2$ = 4 × 8 mm^2 = 32 × 10^{-6} m^2 and A_c = t × w = 0.25 × 1 mm^2 = 25 × 10^{-8} m^2.

Rearranging and solving for T_c,

$$T_c = \left\{hA_sT_\infty + \left[k_gA_s/t + 3\left(k_\ell A_c/L\right)\right]T_b + \dot{E}_g\right\} / \left[hA_s + k_gA_s/t + 3\left(k_\ell A_c/L\right)\right]$$

Substituting numerical values, with the *air-gap condition* ($k_{g,a}$ = 0.0263 W/m·K)

$$T_c = \left\{50\,\text{W/m}^2\cdot\text{K} \times 32\times10^{-6}\,\text{m}^2 \times 20^\circ\text{C} + \left[\left(0.0263\,\text{W/m}\cdot\text{K} \times 32\times10^{-6}\,\text{m}^2/0.2\times10^{-3}\,\text{m}\right)\right.\right.$$

$$\left.\left. +3\left(25\,\text{W/m}\cdot\text{K}\times25\times10^{-8}\,\text{m}^2/4\times10^{-3}\,\text{m}\right)\right]35^\circ\text{C}\right\} / \left[1.600\times10^{-3} + 4.208\times10^{-3} + 4.688\times10^{-3}\right]\text{W/K}$$

$$T_c = 47.0^\circ\text{C}.$$ <

Continued.….

PROBLEM 1.71 (Cont.)

With the *paste condition* ($k_{g,p}$ = 0.12 W/m·K), T_c = 39.9°C. As expected, the effect of the conductive paste is to improve the coupling between the circuit board and the case. Hence, T_c decreases.

(b) Using the keyboard to enter model equations into the workspace, IHT has been used to perform the desired calculations. For values of k_ℓ = 200 and 400 W/m·K and convection coefficients in the range from 50 to 250 W/m^2·K, the energy balance equation may be used to compute the power dissipation for a maximum allowable case temperature of 40°C.

Power dissipation, Edotg(W)
Convection coefficient, h(W/m^2.K)
kl = 400 W/m.K
kl = 200 W/m.K

As indicated by the energy balance, the power dissipation increases linearly with increasing h, as well as with increasing k_ℓ. For h = 250 W/m^2·K (enhanced air cooling) and k_ℓ = 400 W/m·K (copper leads), the transistor may dissipate up to 0.63 W.

COMMENTS: Additional benefits may be derived by increasing heat transfer across the gap separating the case from the board, perhaps by inserting a highly conductive material in the gap.

PROBLEM 1.72(a)

KNOWN: Solar radiation is incident on an asphalt paving.

FIND: Relevant heat transfer processes.

SCHEMATIC:

The relevant processes shown on the schematic include:

q''_S Incident solar radiation, a large portion of which $q''_{S,abs}$, is absorbed by the asphalt surface,

q''_{rad} Radiation emitted by the surface to the air,

q''_{conv} Convection heat transfer from the surface to the air, and

q''_{cond} Conduction heat transfer from the surface into the asphalt.

Applying the surface energy balance, Eq. 1.12,

$$q''_{S,abs} - q''_{rad} - q''_{conv} = q''_{cond}.$$

COMMENTS: (1) q''_{cond} and q''_{conv} could be evaluated from Eqs. 1.1 and 1.3, respectively.

(2) It has been assumed that the pavement surface temperature is higher than that of the underlying pavement and the air, in which case heat transfer by conduction and convection are from the surface.

(3) For simplicity, radiation incident on the pavement due to atmospheric emission has been ignored (see Section 12.8 for a discussion). Eq. 1.6 may then be used for the absorbed solar irradiation and Eq. 1.5 may be used to obtain the emitted radiation q''_{rad}.

(4) With the rate equations, the energy balance becomes

$$q''_{S,abs} - \varepsilon\,\sigma\,T_s^4 - h\left(T_s - T_\infty\right) = -k\left.\frac{dT}{dx}\right]_s.$$

PROBLEM 1.72(b)

KNOWN: Physical mechanism for microwave heating.

FIND: Comparison of (a) cooking in a microwave oven with a conventional radiant or convection oven and (b) a microwave clothes dryer with a conventional dryer.

(a) Microwave cooking occurs as a result of volumetric thermal energy generation *throughout* the food, without heating of the food container or the oven wall. Conventional cooking relies on radiant heat transfer from the oven walls and/or convection heat transfer from the air space to the surface of the food and subsequent heat transfer by conduction to the core of the food. Microwave cooking is more efficient and is achieved in less time.

(b) In a microwave dryer, the microwave radiation would heat the water, but not the fabric, directly (the fabric would be heated indirectly by energy transfer from the water). By heating the water, energy would go directly into evaporation, unlike a conventional dryer where the walls and air are first heated electrically or by a gas heater, and thermal energy is subsequently transferred to the wet clothes. The microwave dryer would still require a rotating drum and air flow to remove the water vapor, but is able to operate more efficiently and at lower temperatures. For a more detailed description of microwave drying, see *Mechanical Engineering*, March 1993, page 120.

PROBLEM 1.72(c)

KNOWN: Surface temperature of exposed arm exceeds that of the room air and walls.

FIND: Relevant heat transfer processes.

SCHEMATIC:

Neglecting evaporation from the surface of the skin, the only relevant heat transfer processes are:

q_{conv} Convection heat transfer from the skin to the room air, and

q_{rad} Net radiation exchange between the surface of the skin and the surroundings (walls of the room).

You are not imagining things. Even though the room air is maintained at a fixed temperature ($T_\infty = 15°C$), the inner surface temperature of the outside walls, T_{sur}, will decrease with decreasing outside air temperature. Upon exposure to these walls, body heat loss will be larger due to increased q_{rad}.

COMMENTS: The foregoing reasoning assumes that the thermostat measures the true room air temperature and is shielded from radiation exchange with the outside walls.

PROBLEM 1.72(d)

KNOWN: Tungsten filament is heated to 2900 K in an air-filled glass bulb.

FIND: Relevant heat transfer processes.

SCHEMATIC:

The relevant processes associated with the filament and bulb include:

$q_{rad,f}$ Radiation emitted by the tungsten filament, a portion of which is transmitted through the glass,

$q_{conv,f}$ Free convection from filament to air of temperature $T_{a,i} < T_f$,

$q_{rad,g,i}$ Radiation emitted by inner surface of glass, a small portion of which is intercepted by the filament,

$q_{conv,g,i}$ Free convection from air to inner glass surface of temperature $T_{g,i} < T_{a,i}$,

$q_{cond,g}$ Conduction through glass wall,

$q_{conv,g,o}$ Free convection from outer glass surface to room air of temperature $T_{a,o} < T_{g,o}$, and

$q_{rad,g\text{-}sur}$ Net radiation heat transfer between outer glass surface and surroundings, such as the walls of a room, of temperature $T_{sur} < T_{g,o}$.

COMMENTS: If the glass bulb is evacuated, no convection is present within the bulb; that is, $q_{conv,f} = q_{conv,g,i} = 0$.

PROBLEM 1.72(e)

KNOWN: Geometry of a composite insulation consisting of a honeycomb core.

FIND: Relevant heat transfer processes.

SCHEMATIC:

The above schematic represents the cross section of a single honeycomb cell and surface slabs. Assumed direction of gravity field is downward. Assuming that the bottom (inner) surface temperature exceeds the top (outer) surface temperature $\left(T_{s,i} > T_{s,o}\right)$, heat transfer is in the direction shown.

Heat may be transferred to the inner surface by convection and radiation, whereupon it is transferred through the composite by

$q_{cond,i}$	Conduction through the inner solid slab,
$q_{conv,hc}$	Free convection through the cellular airspace,
$q_{cond,hc}$	Conduction through the honeycomb wall,
$q_{rad,hc}$	Radiation between the honeycomb surfaces, and
$q_{cond,o}$	Conduction through the outer solid slab.

Heat may then be transferred from the outer surface by convection and radiation. Note that for a single cell under steady state conditions,

$$q_{rad,i} + q_{conv,i} = q_{cond,i} = q_{conv,hc} + q_{cond,hc}$$

$$+ q_{rad,hc} = q_{cond,o} = q_{rad,o} + q_{conv,o}.$$

COMMENTS: Performance would be enhanced by using materials of low thermal conductivity, k, and emissivity, ε. Evacuating the airspace would enhance performance by eliminating heat transfer due to free convection.

PROBLEM 1.72(f)

KNOWN: A thermocouple junction is used, with or without a radiation shield, to measure the temperature of a gas flowing through a channel. The wall of the channel is at a temperature much less than that of the gas.

FIND: (a) Relevant heat transfer processes, (b) Temperature of junction relative to that of gas, (c) Effect of radiation shield.

SCHEMATIC:

ASSUMPTIONS: (1) Junction is small relative to channel walls, (2) Steady-state conditions, (3) Negligible heat transfer by conduction through the thermocouple leads.

ANALYSIS: (a) The relevant heat transfer processes are:

q_{rad} Net radiation transfer from the junction to the walls, and

q_{conv} Convection transfer from the gas to the junction.

(b) From a surface energy balance on the junction,

$$q_{conv} = q_{rad}$$

or from Eqs. 1.3a and 1.7,

$$h\,A\left(T_j - T_g\right) = \varepsilon\,A\,\sigma\left(T_j^4 - T_s^4\right).$$

To satisfy this equality, it follows that

$$T_s < T_j < T_g.$$

That is, the junction assumes a temperature between that of the channel wall and the gas, thereby sensing a temperature which is less than that of the gas.

(c) The measurement error $\left(T_g - T_j\right)$ is reduced by using a radiation shield as shown in the schematic. The junction now exchanges radiation with the shield, whose temperature must exceed that of the channel wall. The radiation loss from the junction is therefore reduced, and its temperature more closely approaches that of the gas.

PROBLEM 1.72(g)

KNOWN: Fireplace cavity is separated from room air by two glass plates, open at both ends.

FIND: Relevant heat transfer processes.

SCHEMATIC:

The relevant heat transfer processes associated with the double-glazed, glass fire screen are:

$q_{rad,1}$ Radiation from flames and cavity wall, portions of which are absorbed and transmitted by the two panes,

$q_{rad,2}$ Emission from inner surface of inner pane to cavity,

$q_{rad,3}$ Net radiation exchange between outer surface of inner pane and inner surface of outer pane,

$q_{rad,4}$ Net radiation exchange between outer surface of outer pane and walls of room,

$q_{conv,1}$ Convection between cavity gases and inner pane,

q_{conv2} Convection across air space between panes,

$q_{conv,3}$ Convection from outer surface to room air,

$q_{cond,1}$ Conduction across inner pane, and

$q_{cond,2}$ Conduction across outer pane.

COMMENTS: (1) Much of the luminous portion of the flame radiation is transmitted to the room interior.

(2) All convection processes are buoyancy driven (free convection).

PROBLEM 1.73(a)

KNOWN: Room air is separated from ambient air by one or two glass panes.

FIND: Relevant heat transfer processes.

SCHEMATIC:

Single pane Double pane

The relevant processes associated with single (above left schematic) and double (above right schematic) glass panes include.

$q_{conv,1}$ Convection from room air to inner surface of first pane,

$q_{rad,1}$ Net radiation exchange between room walls and inner surface of first pane,

$q_{cond,1}$ Conduction through first pane,

$q_{conv,s}$ Convection across airspace between panes,

$q_{rad,s}$ Net radiation exchange between outer surface of first pane and inner surface of second pane (across airspace),

$q_{cond,2}$ Conduction through a second pane,

$q_{conv,2}$ Convection from outer surface of single (or second) pane to ambient air,

$q_{rad,2}$ Net radiation exchange between outer surface of single (or second) pane and surroundings such as the ground, and

q_S Incident solar radiation during day; fraction transmitted to room is smaller for double pane.

COMMENTS: Heat loss from the room is significantly reduced by the double pane construction.

PROBLEM 1.73(b)

KNOWN: Configuration of a flat plate solar collector.

FIND: Relevant heat transfer processes with and without a cover plate.

SCHEMATIC:

The relevant processes without (above left schematic) and with (above right schematic) include:

q_S	Incident solar radiation, a large portion of which is absorbed by the absorber plate. Reduced with use of cover plate (primarily due to reflection off cover plate).
$q_{rad,\infty}$	Net radiation exchange between absorber plate or cover plate and surroundings,
$q_{conv,\infty}$	Convection from absorber plate or cover plate to ambient air,
$q_{rad,a-c}$	Net radiation exchange between absorber and cover plates,
$q_{conv,a-c}$	Convection heat transfer across airspace between absorber and cover plates,
q_{cond}	Conduction through insulation, and
q_{conv}	Convection to working fluid.

COMMENTS: The cover plate acts to significantly reduce heat losses by convection and radiation from the absorber plate to the surroundings.

PROBLEM 1.73(c)

KNOWN: Configuration of a solar collector used to heat air for agricultural applications.

FIND: Relevant heat transfer processes.

SCHEMATIC:

Assume the temperature of the absorber plates exceeds the ambient air temperature. At the *cover plates*, the relevant processes are:

$q_{conv,a-i}$	Convection from inside air to inner surface,
$q_{rad,p-i}$	Net radiation transfer from absorber plates to inner surface,
$q_{conv,i-o}$	Convection across airspace between covers,
$q_{rad,i-o}$	Net radiation transfer from inner to outer cover,
$q_{conv,o-\infty}$	Convection from outer cover to ambient air,
$q_{rad,o}$	Net radiation transfer from outer cover to surroundings, and
q_S	Incident solar radiation.

Additional processes relevant to the *absorber plates* and *airspace* are:

$q_{S,t}$	Solar radiation transmitted by cover plates,
$q_{conv,p-a}$	Convection from absorber plates to inside air, and
q_{cond}	Conduction through insulation.

PROBLEM 1.73(d)

KNOWN: Features of an evacuated tube solar collector.

FIND: Relevant heat transfer processes for one of the tubes.

SCHEMATIC:

The relevant heat transfer processes for one of the evacuated tube solar collectors includes:

q_S Incident solar radiation including contribution due to reflection off panel (most is transmitted),

$q_{conv,o}$ Convection heat transfer from outer surface to ambient air,

$q_{rad,o\text{-}sur}$ Net rate of radiation heat exchange between outer surface of outer tube and the surroundings, including the panel,

$q_{S,t}$ Solar radiation transmitted through outer tube and incident on inner tube (most is absorbed),

$q_{rad,i\text{-}o}$ Net rate of radiation heat exchange between outer surface of inner tube and inner surface of outer tube, and

$q_{conv,i}$ Convection heat transfer to working fluid.

There is also conduction heat transfer through the inner and outer tube walls. If the walls are thin, the temperature drop across the walls will be small.

PROBLEM 2.1

KNOWN: Steady-state, one-dimensional heat conduction through an axisymmetric shape.

FIND: Sketch temperature distribution and explain shape of curve.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state, one-dimensional conduction, (2) Constant properties, (3) No internal heat generation.

ANALYSIS: Performing an energy balance on the object according to Eq. 1.11a, $\dot{E}_{in} - \dot{E}_{out} = 0$, it follows that

$$\dot{E}_{in} - \dot{E}_{out} = q_x$$

and that $q_x \neq q_x(x)$. That is, the heat rate within the object is everywhere constant. From Fourier's law,

$$q_x = -kA_x \frac{dT}{dx},$$

and since q_x and k are both constants, it follows that

$$A_x \frac{dT}{dx} = \text{Constant.}$$

That is, the product of the cross-sectional area normal to the heat rate and temperature gradient remains a constant and independent of distance x. It follows that since A_x increases with x, then dT/dx must decrease with increasing x. Hence, the temperature distribution appears as shown above.

COMMENTS: (1) Be sure to recognize that dT/dx is the slope of the temperature distribution. (2) What would the distribution be when $T_2 > T_1$? (3) How does the heat flux, q''_x, vary with distance?

PROBLEM 2.2

KNOWN: Hot water pipe covered with thick layer of insulation.

FIND: Sketch temperature distribution and give brief explanation to justify shape.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional (radial) conduction, (3) No internal heat generation, (4) Insulation has uniform properties independent of temperature and position.

ANALYSIS: Fourier's law, Eq. 2.1, for this one-dimensional (cylindrical) radial system has the form

$$q_r = -kA_r \frac{dT}{dr} = -k(2\pi r\ell)\frac{dT}{dr}$$

where $A_r = 2\pi r\ell$ and ℓ is the axial length of the pipe-insulation system. Recognize that for steady-state conditions with no internal heat generation, an energy balance on the system requires $\dot{E}_{in} = \dot{E}_{out}$ since $\dot{E}_g = \dot{E}_{st} = 0.$ Hence

$$q_r = \text{Constant.}$$

That is, q_r is independent of radius (r). Since the thermal conductivity is also constant, it follows that

$$r\left[\frac{dT}{dr}\right] = \text{Constant.}$$

This relation requires that the product of the radial temperature gradient, dT/dr, and the radius, r, remains constant throughout the insulation. For our situation, the temperature distribution must appear as shown in the sketch.

COMMENTS: (1) Note that, while q_r is a constant and independent of r, q''_r is not a constant. How does $q''_r(r)$ vary with r? (2) Recognize that the radial temperature gradient, dT/dr, decreases with increasing radius.

PROBLEM 2.3

KNOWN: A spherical shell with prescribed geometry and surface temperatures.

FIND: Sketch temperature distribution and explain shape of the curve.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in radial (spherical coordinates) direction, (3) No internal generation, (4) Constant properties.

ANALYSIS: Fourier's law, Eq. 2.1, for this one-dimensional, radial (spherical coordinate) system has the form

$$q_r = -k\, A_r \frac{dT}{dr} = -k \left(4\pi r^2\right)\frac{dT}{dr}$$

where A_r is the surface area of a sphere. For steady-state conditions, an energy balance on the system yields $\dot{E}_{in} = \dot{E}_{out}$, since $\dot{E}_g = \dot{E}_{st} = 0$. Hence,

$$q_{in} = q_{out} = q_r \neq q_r(r).$$

That is, q_r is a constant, independent of the radial coordinate. Since the thermal conductivity is constant, it follows that

$$r^2\left[\frac{dT}{dr}\right] = \text{Constant.}$$

This relation requires that the product of the radial temperature gradient, dT/dr, and the radius squared, r^2, remains constant throughout the shell. Hence, the temperature distribution appears as shown in the sketch.

COMMENTS: Note that, for the above conditions, $q_r \neq q_r(r)$; that is, q_r is everywhere constant. How does q_r'' vary as a function of radius?

PROBLEM 2.4

KNOWN: Symmetric shape with prescribed variation in cross-sectional area, temperature distribution and heat rate.

FIND: Expression for the thermal conductivity, k.

SCHEMATIC:

$q = 6000W$

$A(x) = (1-x)$
$T(x) = 300(1-2x-x^3)$

Units
T-K
x-m
A-m²

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in x-direction, (3) No internal heat generation.

ANALYSIS: Applying the energy balance, Eq. 1.11a, to the system, it follows that, since $\dot{E}_{in} = \dot{E}_{out}$,

$$q_x = \text{Constant} \neq f(x).$$

Using Fourier's law, Eq. 2.1, with appropriate expressions for A_x and T, yields

$$q_x = -k\, A_x \frac{dT}{dx}$$
$$6000\text{W} = -k \cdot (1\text{-}x)\text{m}^2 \cdot \frac{d}{dx}\left[300\left(1-2x\text{-}x^3\right)\right]\frac{\text{K}}{\text{m}}.$$

Solving for k and recognizing its units are W/m·K,

$$k = \frac{-6000}{(1\text{-}x)\left[300\left(-2-3x^2\right)\right]} = \frac{20}{(1-x)\left(2+3x^2\right)}.$$ <

COMMENTS: (1) At x = 0, k = 10W/m·K and k → ∞ as x → 1. (2) Recognize that the 1-D assumption is an approximation which becomes more inappropriate as the area change with x, and hence two-dimensional effects, become more pronounced.

PROBLEM 2.5

KNOWN: End-face temperatures and temperature dependence of k for a truncated cone.

FIND: Variation with axial distance along the cone of q_x, q''_x, k, and dT/dx.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional conduction in x (negligible temperature gradients along y), (2) Steady-state conditions, (3) Adiabatic sides, (4) No internal heat generation.

ANALYSIS: For the prescribed conditions, it follows from conservation of energy, Eq. 1.11a, that for a differential control volume, $\dot{E}_{in} = \dot{E}_{out}$ or $q_x = q_{x+dx}$. Hence

q_x is independent of x.

Since A(x) *increases* with *increasing* x, it follows that $q''_x = q_x / A(x)$ *decreases* with *increasing* x. Since T *decreases* with *increasing* x, k *increases* with *increasing* x. Hence, from Fourier's law, Eq. 2.2,

$$q''_x = -k\,\frac{dT}{dx},$$

it follows that | dT/dx | *decreases* with increasing x.

PROBLEM 2.6

KNOWN: Temperature dependence of the thermal conductivity, k(T), for heat transfer through a plane wall.

FIND: Effect of k(T) on temperature distribution, T(x).

ASSUMPTIONS: (1) One-dimensional conduction, (2) Steady-state conditions, (3) No internal heat generation.

ANALYSIS: From Fourier's law and the form of k(T),

$$q_x'' = -k\ \frac{dT}{dx} = -\left(k_o + aT\right)\frac{dT}{dx}. \qquad (1)$$

The shape of the temperature distribution may be inferred from knowledge of $d^2T/dx^2 = d(dT/dx)/dx$. Since q_x'' is independent of x for the prescribed conditions,

$$\frac{dq_x''}{dx} = -\frac{d}{dx}\left[\left(k_o + aT\right)\frac{dT}{dx}\right] = 0$$

$$-\left(k_o + aT\right)\frac{d^2T}{dx^2} - a\left[\frac{dT}{dx}\right]^2 = 0.$$

Hence,

$$\frac{d^2T}{dx^2} = \frac{-a}{k_o + aT}\left[\frac{dT}{dx}\right]^2 \qquad \text{where} \begin{cases} k_o + aT = k > 0 \\ \left[\frac{dT}{dx}\right]^2 > 0 \end{cases}$$

from which it follows that for

$$a > 0:\ d^2T/dx^2 < 0$$

$$a = 0:\ d^2T/dx^2 = 0$$

$$a < 0:\ d^2T/dx^2 > 0.$$

COMMENTS: The shape of the distribution could also be inferred from Eq. (1). Since T decreases with increasing x,

a > 0: k decreases with increasing x = > | dT/dx | increases with increasing x

a = 0: $k = k_o$ = > dT/dx is constant

a < 0: k increases with increasing x = > | dT/dx | decreases with increasing x.

PROBLEM 2.7

KNOWN: Thermal conductivity and thickness of a one-dimensional system with no internal heat generation and steady-state conditions.

FIND: Unknown surface temperatures, temperature gradient or heat flux.

SCHEMATIC:

T(x)
L = 0.5m
T_1
q''_x
$\frac{dT}{dx}$, Temperature gradient
T_2
k = 25 W/m·K
x

ASSUMPTIONS: (1) One-dimensional heat flow, (2) No internal heat generation, (3) Steady-state conditions, (4) Constant properties.

ANALYSIS: The rate equation and temperature gradient for this system are

$$q''_x = -k\frac{dT}{dx} \qquad \text{and} \qquad \frac{dT}{dx} = \frac{T_1 - T_2}{L}. \qquad (1,2)$$

Using Eqs. (1) and (2), the unknown quantities can be determined.

(a) $\frac{dT}{dx} = \frac{(400-300)\text{K}}{0.5\text{m}} = 200\ \text{K/m}$

$$q''_x = -25\frac{\text{W}}{\text{m}\cdot\text{K}} \times 200\frac{\text{K}}{\text{m}} = -5000\ \text{W/m}^2.$$ <

400K, 300K, q''_x, x

(b) $$q''_x = -25\frac{\text{W}}{\text{m}\cdot\text{K}} \times \left[-250\frac{\text{K}}{\text{m}}\right] = 6250\ \text{W/m}^2$$

$$T_2 = T_1 - L\left[\frac{dT}{dx}\right] = 1000^\circ\text{C-0.5m}\left[\text{-250}\frac{\text{K}}{\text{m}}\right]$$

$$T_2 = 225^\circ\text{C}.$$ <

q''_x, T_2, $\frac{dT}{dx}$ = -250, 100°C, x

(c) $$q''_x = -25\frac{\text{W}}{\text{m}\cdot\text{K}} \times 200\frac{\text{K}}{\text{m}} = -5000\ \text{W/m}^2$$

$$T_2 = 80^\circ\text{C-0.5m}\left[200\frac{\text{K}}{\text{m}}\right] = -20^\circ\text{C}.$$ <

80°C, $\frac{dT}{dx}$ = +200 $\frac{K}{m}$, T_2, q''_x, x

(d) $$\frac{dT}{dx} = -\frac{q''_x}{k} = -\frac{4000\ \text{W/m}^2}{25\ \text{W/m}\cdot\text{K}} = -160\frac{\text{K}}{\text{m}}$$

$$T_1 = L\left[\frac{dT}{dx}\right] + T_2 = 0.5\text{m}\left[\text{-160}\frac{\text{K}}{\text{m}}\right] + \left(-5^\circ\text{C}\right)\sqrt{a^2 + b^2}$$

$$T_1 = -85^\circ\text{C}.$$ <

q_x = 4000 $\frac{W}{m^2}$, -5°C, T_1, x

(e) $$\frac{dT}{dx} = -\frac{q''_x}{k} = -\frac{\left(-3000\ \text{W/m}^2\right)}{25\ \text{W/m}\cdot\text{K}} = 120\frac{\text{K}}{\text{m}}$$

$$T_2 = 30^\circ\text{C-0.5m}\left[120\frac{\text{K}}{\text{m}}\right] = -30^\circ\text{C}.$$

30°C, T_2, q''_x = -3000 $\frac{W}{m^2}$

PROBLEM 2.8

KNOWN: One-dimensional system with prescribed thermal conductivity and thickness.

FIND: Unknowns for various temperature conditions and sketch distribution.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) No internal heat generation, (4) Constant properties.

ANALYSIS: The rate equation and temperature gradient for this system are

$$q''_x = -k\frac{dT}{dx} \qquad \text{and} \qquad \frac{dT}{dx} = \frac{T_2 - T_1}{L}. \qquad (1,2)$$

Using Eqs. (1) and (2), the unknown quantities for each case can be determined.

(a) $$\frac{dT}{dx} = \frac{(-20-50)\,\text{K}}{0.25\text{m}} = -280\ \text{K/m}$$

$$q''_x = -50\frac{\text{W}}{\text{m}\cdot\text{K}} \times \left[-280\frac{\text{K}}{\text{m}}\right] = 14.0\ \text{kW/m}^2.$$ <

(b) $$\frac{dT}{dx} = \frac{(-10-(-30))\,\text{K}}{0.25\text{m}} = 80\ \text{K/m}$$

$$q''_x = -50\frac{\text{W}}{\text{m}\cdot\text{K}} \times \left[80\frac{\text{K}}{\text{m}}\right] = -4.0\ \text{kW/m}^2.$$ <

(c) $$q''_x = -50\frac{\text{W}}{\text{m}\cdot\text{K}} \times \left[160\frac{\text{K}}{\text{m}}\right] = -8.0\ \text{kW/m}^2$$

$$T_2 = L\cdot\frac{dT}{dx} + T_1 = 0.25\text{m} \times \left[160\frac{\text{K}}{\text{m}}\right] + 70^\circ\text{C}.$$

$$T_2 = 110^\circ\text{C}.$$ <

(d) $$q''_x = -50\frac{\text{W}}{\text{m}\cdot\text{K}} \times \left[-80\frac{\text{K}}{\text{m}}\right] = 4.0\ \text{kW/m}^2$$

$$T_1 = T_2 - L\cdot\frac{dT}{dx} = 40^\circ\text{C} - 0.25\text{m}\left[-80\frac{\text{K}}{\text{m}}\right].\in$$

$$T_1 = 60^\circ\text{C}.$$ <

(e) $$q''_x = -50\frac{\text{W}}{\text{m}\cdot\text{K}} \times \left[200\frac{\text{K}}{\text{m}}\right] = -10.0\ \text{kW/m}^2$$

$$T_1 = T_2 - L\cdot\frac{dT}{dx} = 30^\circ\text{C} - 0.25\text{m}\left[200\frac{\text{K}}{\text{m}}\right] = -20^\circ\text{C}.$$ <

PROBLEM 2.9

KNOWN: Plane wall with prescribed thermal conductivity, thickness, and surface temperatures.

FIND: Heat flux, q''_x, and temperature gradient, dT/dx, for the three different coordinate systems shown.

SCHEMATIC:

$T_1 = 400K$
$T_2 = 600K$
$k = 100 W/m \cdot K$
$L = 100 mm$

(a) (b) (c)

ASSUMPTIONS: (1) One-dimensional heat flow, (2) Steady-state conditions, (3) No internal generation, (4) Constant properties.

ANALYSIS: The rate equation for conduction heat transfer is

$$q''_x = -k\frac{dT}{dx}, \qquad (1)$$

where the temperature gradient is constant throughout the wall and of the form

$$\frac{dT}{dx} = \frac{T(L) - T(0)}{L}. \qquad (2)$$

Substituting numerical values, find the temperature gradients,

(a) $$\frac{dT}{dx} = \frac{T_2 - T_1}{L} = \frac{(600-400)K}{0.100m} = 2000\ K/m$$ <

(b) $$\frac{dT}{dx} = \frac{T_1 - T_2}{L} = \frac{(400-600)K}{0.100m} = -2000\ K/m$$ <

(c) $$\frac{dT}{dx} = \frac{T_2 - T_1}{L} = \frac{(600-400)K}{0.100m} = 2000\ K/m.$$ <

The heat rates, using Eq. (1) with k = 100 W/m·K, are

(a) $$q''_x = -100\frac{W}{m \cdot K} \times 2000\ K/m = -200\ kW/m^2$$ <

(b) $$q''_x = -100\frac{W}{m \cdot K}(-2000\ K/m) = +200\ kW/m^2$$ <

(c) $$q''_x = -100\frac{W}{m \cdot K} \times 2000\ K/m = -200\ kW/m^2$$ <

PROBLEM 2.10

KNOWN: Temperature distribution in solid cylinder and convection coefficient at cylinder surface.

FIND: Expressions for heat rate at cylinder surface and fluid temperature.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional, radial conduction, (2) Steady-state conditions, (3) Constant properties.

ANALYSIS: The heat rate from Fourier's law for the radial (cylindrical) system has the form

$$q_r = -kA_r \frac{dT}{dr}.$$

Substituting for the temperature distribution, $T(r) = a + br^2$,

$$q_r = -k(2\pi rL)\, 2br \;=\; -4\pi kbLr^2.$$

At the outer surface ($r = r_o$), the conduction heat rate is

$$q_{r=r_o} = -4\pi kbLr_o^2. \qquad <$$

From a surface energy balance at $r = r_o$,

$$q_{r=r_o} = q_{conv} = h(2\pi r_o L)\left[T(r_o) - T_\infty\right],$$

Substituting for $q_{r=r_o}$ and solving for T_∞,

$$T_\infty \;=\; T(r_o) + \frac{2kbr_o}{h}$$

$$T_\infty \;=\; a + br_o^2 + \frac{2kbr_o}{h}$$

$$T_\infty \;=\; a + br_o\left[r_o + \frac{2k}{h}\right]. \qquad <$$

PROBLEM 2.11

KNOWN: Two-dimensional body with specified thermal conductivity and two isothermal surfaces of prescribed temperatures; one surface, A, has a prescribed temperature gradient.

FIND: Temperature gradients, $\partial T/\partial x$ and $\partial T/\partial y$, at the surface B.

SCHEMATIC:

ASSUMPTIONS: (1) Two-dimensional conduction, (2) Steady-state conditions, (3) No heat generation, (4) Constant properties.

ANALYSIS: At the surface A, the temperature gradient in the x-direction must be zero. That is, $(\partial T/\partial x)_A = 0$. This follows from the requirement that the heat flux vector must be normal to an isothermal surface. The heat rate at the surface A is given by Fourier's law written as

$$q'_{y,A} = -k \cdot w_A \left.\frac{\partial T}{\partial y}\right]_A = -10\frac{W}{m \cdot K} \times 2m \times 30\frac{K}{m} = -600 W/m.$$

On the surface B, it follows that

$$(\partial T/\partial y)_B = 0$$ <

in order to satisfy the requirement that the heat flux vector be normal to the isothermal surface B. Using the conservation of energy requirement, Eq. 1.11a, on the body, find

$$q'_{y,A} - q'_{x,B} = 0 \qquad \text{or} \qquad q'_{x,B} = q'_{y,A}.$$

Note that,

$$q'_{x,B} = -k \cdot w_B \left.\frac{\partial T}{\partial x}\right]_B$$

and hence

$$(\partial T/\partial x)_B = \frac{-q'_{y,A}}{k \cdot w_B} = \frac{-(-600 \text{ W/m})}{10 \text{ W/m}\cdot\text{K} \times 1\text{m}} = 60 \text{ K/m}.$$ <

COMMENTS: Note that, in using the conservation requirement, $q'_{in} = +q'_{y,A}$ and $q'_{out} = +q'_{x,B}$.

PROBLEM 2.12

KNOWN: Length and thermal conductivity of a shaft. Temperature distribution along shaft.

FIND: Temperature and heat rates at ends of shaft.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in x, (3) Constant properties.

ANALYSIS: Temperatures at the top and bottom of the shaft are, respectively,

$$T(0) = 100°C \qquad\qquad T(L) = -40°C.$$ <

Applying Fourier's law, Eq. 2.1,

$$q_x = -kA\frac{dT}{dx} = -25\ W/m\cdot K\left(0.005\ m^2\right)\left(-150 + 20x\right)^\circ C/m$$

$$q_x = 0.125\left(150\ -\ 20x\right)W.$$

Hence,

$$q_x(0) = 18.75\ W \qquad\qquad q_x(L) = 16.25\ W.$$ <

The difference in heat rates, $q_x(0) > q_x(L)$, is due to heat losses q_ℓ from the side of the shaft.

COMMENTS: Heat loss from the side requires the existence of temperature gradients over the shaft cross-section. Hence, specification of T as a function of only x is an approximation.

PROBLEM 2.13

KNOWN: A rod of constant thermal conductivity k and variable cross-sectional area $A_x(x) = A_o e^{ax}$ where A_o and a are constants.

FIND: (a) Expression for the conduction heat rate, $q_x(x)$; use this expression to determine the temperature distribution, T(x); and sketch of the temperature distribution, (b) Considering the presence of volumetric heat generation rate, $\dot{q} = \dot{q}_o \exp(-ax)$, obtain an expression for $q_x(x)$ when the left face, x = 0, is well insulated.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional conduction in the rod, (2) Constant properties, (3) Steady-state conditions.

ANALYSIS: Perform an energy balance on the control volume, A(x)·dx,

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = 0$$

$$q_x - q_{x+dx} + \dot{q} \cdot A(x) \cdot dx = 0$$

The conduction heat rate terms can be expressed as a Taylor series and substituting expressions for $\dot{q}$ and A(x),

$$-\frac{d}{dx}(q_x) + \dot{q}_o \exp(-ax) \cdot A_o \exp(ax) = 0 \quad (1)$$

$$q_x = -k \cdot A(x) \frac{dT}{dx} \quad (2)$$

(a) With no internal generation, $\dot{q}_o = 0$, and from Eq. (1) find

$$-\frac{d}{dx}(q_x) = 0$$ <

indicating that the heat rate is constant with x. By combining Eqs. (1) and (2)

$$-\frac{d}{dx}\left(-k \cdot A(x) \frac{dT}{dx}\right) = 0 \qquad \text{or} \qquad A(x) \cdot \frac{dT}{dx} = C_1 \quad (3)$$ <

Continued...

That is, the product of the cross-sectional area and the temperature gradient is a constant, independent of x. Hence, with T(0) > T(L), the temperature distribution is exponential, and as shown in the sketch above. Separating variables and integrating Eq. (3), the general form for the temperature distribution can be determined,

$$A_o \exp(ax) \cdot \frac{dT}{dx} = C_1$$

$$dT = C_1 A_o^{-1} \exp(-ax) dx$$

$$T(x) = -C_1 A_o a \exp(-ax) + C_2$$ <

We could use the two temperature boundary conditions, $T_o = T(0)$ and $T_L = T(L)$, to evaluate C_1 and C_2 and, hence, obtain the temperature distribution in terms of T_o and T_L.

(b) With the internal generation, from Eq. (1),

$$-\frac{d}{dx}(q_x) + \dot{q}_o A_o = 0 \qquad \text{or} \qquad q_x = \dot{q}_o A_o x$$ <

That is, the heat rate increases linearly with x.

COMMENTS: In part (b), you could determine the temperature distribution using Fourier’s law and knowledge of the heat rate dependence upon the x-coordinate. Give it a try!

PROBLEM 2.14

KNOWN: Dimensions and end temperatures of a cylindrical rod which is insulated on its side.

FIND: Rate of heat transfer associated with different rod materials.

SCHEMATIC:

D=25mm
T_1=100°C
T_2 = 0°C
L = 0.1m
x

ASSUMPTIONS: (1) One-dimensional conduction along cylinder axis, (2) Steady-state conditions, (3) Constant properties.

PROPERTIES: The properties may be evaluated from *Tables A-1* to *A-3* at a mean temperature of 50°C = 323K and are summarized below.

ANALYSIS: The heat transfer rate may be obtained from Fourier's law. Since the axial temperature gradient is linear, this expression reduces to

$$q = kA\frac{T_1 - T_2}{L} = k\frac{\pi(0.025\text{m})^2}{4}\frac{(100-0)°\text{C}}{0.1\text{m}} = 0.491(\text{m}\cdot°\text{C})\cdot k$$

	Cu (pure)	Al (2024)	St.St. (302)	SiN	Oak	Magnesia (85%)	Pyrex	
k(W/m·K)	401	177	16.3	14.9	0.19	0.052	1.4	
q(W)	197	87	8.0	7.3	0.093	0.026	0.69	<

COMMENTS: The k values of Cu and Al were obtained by linear interpolation; the k value of St.St. was obtained by linear extrapolation, as was the value for SiN; the value for magnesia was obtained by linear interpolation; and the values for oak and pyrex are for 300 K.

PROBLEM 2.15

KNOWN: One-dimensional system with prescribed surface temperatures and thickness.

FIND: Heat flux through system constructed of these materials: (a) pure aluminum, (b) plain carbon steel, (c) AISI 316, stainless steel, (d) pyroceram, (e) teflon and (f) concrete.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) No heat generation, (4) Constant thermal properties.

PROPERTIES: The thermal conductivity is evaluated at the average temperature of the system, T = $(T_1+T_2)/2 = (325+275)K/2 = 300K$. Property values and table identification are shown below.

ANALYSIS: For this system, Fourier's law can be written as

$$q_x'' = -k\frac{dT}{dx} = -k\frac{T_2 - T_1}{L}.$$

Substituting numerical values, the heat flux is

$$q_x'' = -k\frac{(275-325)K}{20\times10^{-3}m} = +2500\frac{K}{m}\cdot k$$

where q_x'' will have units W/m^2 if k has units W/m·K. The heat fluxes for each system follow.

Material	Table	Thermal conductivity k(W/m·K)	Heat flux $q_x''\left(kW/m^2\right)$
(a) Pure Aluminum	A-1	237	593 <
(b) Plain carbon steel	A-1	60.5	151
(c) AISI 316, S.S.	A-1	13.4	33.5
(d) Pyroceram	A-2	3.98	9.95
(e) Teflon	A-3	0.35	0.88
(f) Concrete	A-3	1.4	3.5

COMMENTS: Recognize that the thermal conductivity of these solid materials varies by more than two orders of magnitude.

PROBLEM 2.16

KNOWN: Different thicknesses of three materials: rock, 18 ft; wood, 15 in; and fiberglass insulation, 6 in.

FIND: The insulating quality of the materials as measured by the R-value.

PROPERTIES: *Table A-3* (300K):

Material	Thermal conductivity, W/m·K
Limestone	2.15
Softwood	0.12
Blanket (glass, fiber 10 kg/m^3)	0.048

ANALYSIS: The R-value, a quantity commonly used in the construction industry and building technology, is defined as

$$R \equiv \frac{L(\text{in})}{k\left(\text{Btu}\cdot\text{in}/\text{h}\cdot\text{ft}^2\cdot{}^\circ\text{F}\right)}.$$

The R-value can be interpreted as the thermal resistance of a 1 ft^2 cross section of the material. Using the conversion factor for thermal conductivity between the SI and English systems, the R-values are:

Rock, Limestone, 18 ft:

$$R=\frac{18\ \text{ft}\times 12\,\frac{\text{in}}{\text{ft}}}{2.15\,\frac{\text{W}}{\text{m}\cdot\text{K}}\times 0.5778\,\frac{\text{Btu/h}\cdot\text{ft}\cdot{}^\circ\text{F}}{\text{W/m}\cdot\text{K}}\times 12\,\frac{\text{in}}{\text{ft}}}=14.5\ \left(\text{Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F}\right)^{-1}$$

Wood, Softwood, 15 in:

$$R=\frac{15\ \text{in}}{0.12\,\frac{\text{W}}{\text{m}\cdot\text{K}}\times 0.5778\,\frac{\text{Btu}/\text{h}\cdot\text{ft}\cdot{}^\circ\text{F}}{\text{W}/\text{m}\cdot\text{K}}\times 12\,\frac{\text{in}}{\text{ft}}}=18\ \left(\text{Btu}/\text{h}\cdot\text{ft}^2\cdot{}^\circ\text{F}\right)^{-1}$$

Insulation, Blanket, 6 in:

$$R=\frac{6\ \text{in}}{0.048\,\frac{\text{W}}{\text{m}\cdot\text{K}}\times 0.5778\,\frac{\text{Btu}/\text{h}\cdot\text{ft}\cdot{}^\circ\text{F}}{\text{W}/\text{m}\cdot\text{K}}\times 12\,\frac{\text{in}}{\text{ft}}}=18\ \left(\text{Btu}/\text{h}\cdot\text{ft}^2\cdot{}^\circ\text{F}\right)^{-1}$$

COMMENTS: The R-value of 19 given in the advertisement is reasonable.

PROBLEM 2.17

KNOWN: Electrical heater sandwiched between two identical cylindrical (30 mm dia. × 60 mm length) samples whose opposite ends contact plates maintained at T_o.

FIND: (a) Thermal conductivity of SS316 samples for the prescribed conditions (A) and their average temperature, (b) Thermal conductivity of Armco iron sample for the prescribed conditions (B), (c) Comment on advantages of experimental arrangement, lateral heat losses, and conditions for which $\Delta T_1 \neq \Delta T_2$.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional heat transfer in samples, (2) Steady-state conditions, (3) Negligible contact resistance between materials.

PROPERTIES: *Table A.2*, Stainless steel 316 $\left(\overline{T} = 400\text{ K}\right)$: $k_{ss} = 15.2\text{ W/m}\cdot\text{K}$; Armco iron $\left(\overline{T} = 380\text{ K}\right)$: $k_{iron} = 71.6\text{ W/m}\cdot\text{K}$.

ANALYSIS: (a) For Case A recognize that half the heater power will pass through each of the samples which are presumed identical. Apply Fourier's law to a sample

$$q = kA_c \frac{\Delta T}{\Delta x}$$

$$k = \frac{q\Delta x}{A_c \Delta T} = \frac{0.5(100\text{V} \times 0.353\text{A}) \times 0.015\text{ m}}{\pi(0.030\text{ m})^2 / 4 \times 25.0^\circ\text{C}} = 15.0\text{ W/m}\cdot\text{K}.$$ <

The total temperature drop across the length of the sample is $\Delta T_1(L/\Delta x)$ = 25°C (60 mm/15 mm) = 100°C. Hence, the heater temperature is T_h = 177°C. Thus the average temperature of the sample is

$$\overline{T} = \left(T_o + T_h\right)/2 = 127^\circ\text{C} = 400\text{ K}$$ <

.

We compare the calculated value of k with the tabulated value (see above) at 400 K and note the good agreement.

(b) For Case B, we assume that the thermal conductivity of the SS316 sample is the same as that found in Part (a). The heat rate through the Armco iron sample is

Continued …..

$$q_{iron} = q_{heater} - q_{ss} = 100V \times 0.601A - 15.0\ W/m \cdot K \times \frac{\pi(0.030\ m)^2}{4} \times \frac{15.0°C}{0.015\ m}$$

$$q_{iron} = (60.1 - 10.6)W = 49.5\ W$$

where

$$q_{ss} = k_{ss} A_c \Delta T_2 / \Delta x_2 .$$

Applying Fourier's law to the iron sample,

$$k_{iron} = \frac{q_{iron}\Delta x_2}{A_c \Delta T_2} = \frac{49.5\ W \times 0.015\ m}{\pi(0.030\ m)^2 / 4 \times 15.0°C} = 70.0\ W/m \cdot K. \qquad <$$

The total drop across the iron sample is 15°C(60/15) = 60°C; the heater temperature is (77 + 60)°C = 137°C. Hence the average temperature of the iron sample is

$$\overline{T} = (137 + 77)°C / 2 = 107°C = 380\ K. \qquad <$$

We compare the computed value of k with the tabulated value (see above) at 380 K and note the good agreement.

(c) The principal advantage of having two identical samples is the assurance that all the electrical power dissipated in the heater will appear as equivalent heat flows through the samples. With only one sample, heat can flow from the backside of the heater even though insulated.

Heat leakage out the lateral surfaces of the cylindrically shaped samples will become significant when the sample thermal conductivity is comparable to that of the insulating material. Hence, the method is suitable for metallics, but must be used with caution on nonmetallic materials.

For any combination of materials in the upper and lower position, we expect $\Delta T_1 = \Delta T_2$. However, if the insulation were improperly applied along the lateral surfaces, it is possible that heat leakage will occur, causing $\Delta T_1 \neq \Delta T_2$.

PROBLEM 2.18

KNOWN: Comparative method for measuring thermal conductivity involving two identical samples stacked with a reference material.

FIND: (a) Thermal conductivity of test material and associated temperature, (b) Conditions for which $\Delta T_{t,1} \neq \Delta T_{t,2}$

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat transfer through samples and reference material, (3) Negligible thermal contact resistance between materials.

PROPERTIES: *Table A.2*, Armco iron $\left(\overline{T} = 350\ K\right)$: $k_r = 69.2\ W/m\cdot K$.

ANALYSIS: (a) Recognizing that the heat rate through the samples and reference material, all of the same diameter, is the same, it follows from Fourier's law that

$$k_t \frac{\Delta T_{t,1}}{\Delta x} = k_r \frac{\Delta T_r}{\Delta x} = k_t \frac{\Delta T_{t,2}}{\Delta x}$$

$$k_t = k_r \frac{\Delta T_r}{\Delta T_t} = 69.2\ W/m\cdot K \frac{2.49^\circ C}{3.32^\circ C} = 51.9\ W/m\cdot K. \qquad <$$

We should assign this value a temperature of 350 K. <

(b) If the test samples are identical in every respect, $\Delta T_{t,1} \neq \Delta T_{t,2}$ if the thermal conductivity is highly dependent upon temperature. Also, if there is heat leakage out the lateral surface, we can expect $\Delta T_{t,2} < \Delta T_{t,1}$. Leakage could be influential, if the thermal conductivity of the test material were less than an order of magnitude larger than that of the insulating material.

PROBLEM 2.19

KNOWN: Identical samples of prescribed diameter, length and density initially at a uniform temperature T_i, sandwich an electric heater which provides a uniform heat flux q''_o for a period of time Δt_o. Conditions shortly after energizing and a long time after de-energizing heater are prescribed.

FIND: Specific heat and thermal conductivity of the test sample material. From these properties, identify type of material using Table A.1 or A.2.

SCHEMATIC:

ASSUMPTIONS: (1) One dimensional heat transfer in samples, (2) Constant properties, (3) Negligible heat loss through insulation, (4) Negligible heater mass.

ANALYSIS: Consider a control volume about the samples and heater, and apply conservation of energy over the time interval from t = 0 to ∞

$$E_{in} - E_{out} = \Delta E = E_f - E_i$$

$$P\Delta t_o - 0 = Mc_p\left[T(\infty) - T_i\right]$$

where energy inflow is prescribed by the Case A power condition and the final temperature T_f by Case B. Solving for c_p,

$$c_p = \frac{P\Delta t_o}{M\left[T(\infty) - T_i\right]} = \frac{15\ \text{W} \times 120\ \text{s}}{2 \times 3965\ \text{kg/m}^3\left(\pi \times 0.060^2/4\right)\text{m}^2 \times 0.010\ \text{m}\left[33.50 - 23.00\right]^\circ\text{C}}$$

$$c_p = 765\ \text{J/kg}\cdot\text{K}$$ <

where $M = \rho V = 2\rho(\pi D^2/4)L$ is the mass of both samples. For Case A, the transient thermal response of the heater is given by

Continued …..

PROBLEM 2.19 (Cont.)

$$T_o(t) - T_i = 2q_o''\left[\frac{t}{\pi\rho c_p k}\right]^{1/2}$$

$$k = \frac{t}{\pi\rho c_p}\left[\frac{2q_o''}{T_o(t) - T_i}\right]^2$$

$$k = \frac{30\text{ s}}{\pi \times 3965\text{ kg/m}^3 \times 765\text{ J/kg}\cdot\text{K}}\left[\frac{2 \times 2653\text{ W/m}^2}{(24.57\ -\ 23.00)^\circ\text{C}}\right]^2 = 36.0\text{ W/m}\cdot\text{K}$$ <

where

$$q_o'' = \frac{P}{2A_s} = \frac{P}{2(\pi D^2/4)} = \frac{15\text{ W}}{2(\pi \times 0.060^2/4)\text{m}^2} = 2653\text{ W/m}^2.$$

With the following properties now known,

$\rho = 3965$ kg/m^3 $\qquad$ $c_p = 765$ J/kg·K $\qquad$ $k = 36$ W/m·K

entries in Table A.1 are scanned to determine whether these values are typical of a metallic material. Consider the following,

- metallics with low ρ generally have higher thermal conductivities,
- specific heats of both types of materials are of similar magnitude,
- the low k value of the sample is typical of poor metallic conductors which generally have much higher specific heats,
- more than likely, the material is nonmetallic.

From Table A.2, the second entry, polycrystalline aluminum oxide, has properties at 300 K corresponding to those found for the samples. <

PROBLEM 2.20

KNOWN: Temperature distribution, T(x,y,z), within an infinite, homogeneous body at a given instant of time.

FIND: Regions where the temperature changes with time.

SCHEMATIC:

$T(x,y,z) = x^2 - 2y^2 + z^2 - xy + 2yz$

Infinite medium

ASSUMPTIONS: (1) Constant properties of infinite medium and (2) No internal heat generation.

ANALYSIS: The temperature distribution throughout the medium, at any instant of time, must satisfy the heat equation. For the three-dimensional cartesian coordinate system, with constant properties and no internal heat generation, the heat equation, Eq. 2.15, has the form

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}. \qquad (1)$$

If T(x,y,z) satisfies this relation, conservation of energy is satisfied at every point in the medium. Substituting T(x,y,z) into the Eq. (1), first find the gradients, $\partial T/\partial x$, $\partial T/\partial y$, and $\partial T/\partial z$.

$$\frac{\partial}{\partial x}(2x - y) + \frac{\partial}{\partial y}(-4y - x + 2z) + \frac{\partial}{\partial z}(2z + 2y) = \frac{1}{\alpha}\frac{\partial T}{\partial t}.$$

Performing the differentiations,

$$2 - 4 + 2 = \frac{1}{\alpha}\frac{\partial T}{\partial t}.$$

Hence,

$$\frac{\partial T}{\partial t} = 0$$

which implies that, at the prescribed instant, the temperature is everywhere independent of time.

COMMENTS: Since we do not know the initial and boundary conditions, we cannot determine the temperature distribution, T(x,y,z), at any future time. We can only determine that, for this special instant of time, the temperature will not change.

PROBLEM 2.21

KNOWN: Diameter D, thickness L and initial temperature T_i of pan. Heat rate from stove to bottom of pan. Convection coefficient h and variation of water temperature $T_\infty(t)$ during Stage 1. Temperature T_L of pan surface in contact with water during Stage 2.

FIND: Form of heat equation and boundary conditions associated with the two stages.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional conduction in pan bottom, (2) Heat transfer from stove is uniformly distributed over surface of pan in contact with the stove, (3) Constant properties.

ANALYSIS:

Stage 1

Heat Equation: $$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

Boundary Conditions: $$-k\left.\frac{\partial T}{\partial x}\right|_{x=0} = q_o'' = \frac{q_o}{\left(\pi D^2/4\right)}$$

$$-k\left.\frac{\partial T}{\partial x}\right|_{x=L} = h\left[T(L,t) - T_\infty(t)\right]$$

Initial Condition: $$T(x,0) = T_i$$

Stage 2

Heat Equation: $$\frac{d^2 T}{dx^2} = 0$$

Boundary Conditions: $$-k\left.\frac{dT}{dx}\right|_{x=0} = q_o''$$

$$T(L) = T_L$$

COMMENTS: Stage 1 is a transient process for which $T_\infty(t)$ must be determined separately. As a first approximation, it could be estimated by neglecting changes in thermal energy storage by the pan bottom and assuming that all of the heat transferred from the stove acted to increase thermal energy storage within the water. Hence, with $q \approx Mc_p\, dT_\infty/dt$, where M and c_p are the mass and specific heat of the water in the pan, $T_\infty(t) \approx (q/Mc_p)\, t$.

PROBLEM 2.22

KNOWN: Steady-state temperature distribution in a cylindrical rod having uniform heat generation of $\dot{q}_1 = 5\times10^7\ \text{W/m}^3$.

FIND: (a) Steady-state centerline and surface heat transfer rates per unit length, q_r'. (b) Initial time rate of change of the centerline and surface temperatures in response to a change in the generation rate from $\dot{q}_1$ to $\dot{q}_2 = 10^8\ \text{W/m}^3$.

SCHEMATIC:

$r_o = 0.025\text{m}$

$T(r) = 800 - 4.167\cdot10^5 r^2$

$\dot{q} = \dot{q}_1 = 5\cdot10^7\ \text{W/m}^3$

$k = 30\ \text{W/m}\cdot\text{K}$

$\rho = 1100\ \text{kg/m}^3,\ c_p = 800\ \text{J/kg}\cdot\text{K}$

ASSUMPTIONS: (1) One-dimensional conduction in the r direction, (2) Uniform generation, and (3) Steady-state for $\dot{q}_1 = 5\times10^7\ \text{W/m}^3$.

ANALYSIS: (a) From the rate equations for cylindrical coordinates,

$$q_r'' = -k\frac{\partial T}{\partial r} \qquad q = -kA_r\frac{\partial T}{\partial r}.$$

Hence,

$$q_r = -k(2\pi rL)\frac{\partial T}{\partial r}$$

or

$$q_r' = -2\pi kr\frac{\partial T}{\partial r}$$

where ∂T/∂r may be evaluated from the prescribed temperature distribution, T(r).

At r = 0, the gradient is (∂T/∂r) = 0. Hence, from Eq. (1) the heat rate is

$$q_r'(0) = 0.$$ <

At $r = r_o$, the temperature gradient is

$$\left.\frac{\partial T}{\partial r}\right]_{r=r_o} = -2\left[4.167\times10^5\frac{\text{K}}{\text{m}^2}\right](r_o) = -2\left(4.167\times10^5\right)(0.025\text{m})$$

$$\left.\frac{\partial T}{\partial r}\right]_{r=r_o} = -0.208\times10^5\ \text{K/m}.$$

Continued …..

PROBLEM 2.22(Cont.)

Hence, the heat rate at the outer surface ($r = r_o$) per unit length is

$$q'_r(r_o) = -2\pi[30 \text{ W/m}\cdot\text{K}](0.025\text{m})\left[-0.208\times10^5 \text{ K/m}\right]$$

$$q'_r(r_o) = 0.980\times10^5 \text{ W/m}.$$ <

(b) Transient (time-dependent) conditions will exist when the generation is changed, and for the prescribed assumptions, the temperature is determined by the following form of the heat equation, Eq. 2.20

$$\frac{1}{r}\frac{\partial}{\partial r}\left[kr\frac{\partial T}{\partial r}\right] + \dot{q}_2 = \rho c_p \frac{\partial T}{\partial t}$$

Hence

$$\frac{\partial T}{\partial t} = \frac{1}{\rho c_p}\left[\frac{1}{r}\frac{\partial}{\partial r}\left[kr\frac{\partial T}{\partial r}\right] + \dot{q}_2\right].$$

However, initially (at t = 0), the temperature distribution is given by the prescribed form, $T(r) = 800 - 4.167\times10^5 r^2$, and

$$\frac{1}{r}\frac{\partial}{\partial r}\left[kr\frac{\partial T}{\partial r}\right] = \frac{k}{r}\frac{\partial}{\partial r}\left[r\left(-8.334\times10^5\cdot r\right)\right]$$

$$= \frac{k}{r}\left(-16.668\times10^5\cdot r\right)$$

$$= 30 \text{ W/m}\cdot\text{K}\left[-16.668\times10^5 \text{ K/m}^2\right]$$

$$= -5\times10^7 \text{ W/m}^3 \text{ (the original } \dot{q} = \dot{q}_1).$$

Hence, everywhere in the wall,

$$\frac{\partial T}{\partial t} = \frac{1}{1100 \text{ kg/m}^3 \times 800 \text{ J/kg}\cdot\text{K}}\left[-5\times10^7 + 10^8\right] \text{W/m}^3$$

or

$$\frac{\partial T}{\partial t} = 56.82 \text{ K/s}.$$ <

COMMENTS: (1) The value of $(\partial T/\partial t)$ will decrease with increasing time, until a new steady-state condition is reached and once again $(\partial T/\partial t) = 0$.

(2) By applying the energy conservation requirement, Eq. 1.11a, to a unit length of the rod for the steady-state condition, $\dot{E}'_{in} - E'_{out} + \dot{E}'_{gen} = 0$. Hence $q'_r(0) - q'_r(r_o) = -\dot{q}_1\left(\pi r_o^2\right)$.

PROBLEM 2.23

KNOWN: Temperature distribution in a one-dimensional wall with prescribed thickness and thermal conductivity.

FIND: (a) The heat generation rate, $\dot{q}$, in the wall, (b) Heat fluxes at the wall faces and relation to $\dot{q}$.

SCHEMATIC:

k = 50 W/m·K, $\dot{q}$
$T(x) = a + bx^2$
x
L = 50 mm
T(°C)
a = 200°C
b = -2000°C/m²
L
$q''_x(0)$
$\dot{E}_g$
$q''_x(L)$

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat flow, (3) Constant properties.

ANALYSIS: (a) The appropriate form of the heat equation for steady-state, one-dimensional conditions with constant properties is Eq. 2.15 re-written as

$$\dot{q} = -k\frac{d}{dx}\left[\frac{dT}{dx}\right]$$

Substituting the prescribed temperature distribution,

$$\dot{q} = -k\frac{d}{dx}\left[\frac{d}{dx}\left(a + bx^2\right)\right] = -k\frac{d}{dx}[2bx] = -2bk$$

$$\dot{q} = -2\left(-2000^\circ C/m^2\right)\times 50\ W/m\cdot K = 2.0\times 10^5\ W/m^3. \qquad <$$

(b) The heat fluxes at the wall faces can be evaluated from Fourier's law,

$$q''_x(x) = -k\left.\frac{dT}{dx}\right]_x.$$

Using the temperature distribution T(x) to evaluate the gradient, find

$$q''_x(x) = -k\frac{d}{dx}\left[a + bx^2\right] = -2kbx.$$

The fluxes at x = 0 and x = L are then

$$q''_x(0) = 0 \qquad <$$

$$q''_x(L) = -2kbL = -2\times 50W/m\cdot K\left(-2000^\circ C/m^2\right)\times 0.050m$$

$$q''_x(L) = 10{,}000\ W/m^2. \qquad <$$

COMMENTS: From an overall energy balance on the wall, it follows that, for a unit area,

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = 0 \qquad q''_x(0) - q''_x(L) + \dot{q}L = 0$$

$$\dot{q} = \frac{q''_x(L) - q''_x(0)}{L} = \frac{10{,}000\ W/m^2 - 0}{0.050m} = 2.0\times 10^5 W/m^3.$$

PROBLEM 2.24

KNOWN: Wall thickness, thermal conductivity, temperature distribution, and fluid temperature.

FIND: (a) Surface heat rates and rate of change of wall energy storage per unit area, and (b) Convection coefficient.

SCHEMATIC:

$k = 1\ W/m\cdot K$
$T(x) = 200 - 200x + 30x^2$
200°C
$\dot{E}_{ST}$
142.7°C
q''_{in}
q''_{out}
Fluid
$T_\infty = 100°C, h$
x
L = 0.3m

ASSUMPTIONS: (1) One-dimensional conduction in x, (2) Constant k.

ANALYSIS: (a) From Fourier's law,

$$q''_x = -k\frac{\partial T}{\partial x} = (200 - 60x)\cdot k$$

$$q''_{in} = q''_{x=0} = 200\frac{°C}{m}\times 1\frac{W}{m\cdot K} = 200\ W/m^2$$ <

$$q''_{out} = q''_{x=L} = (200 - 60\times 0.3)°C/m\ \times\ 1\ W/m\cdot K = 182\ W/m^2.$$ <

Applying an energy balance to a control volume about the wall, Eq. 1.11a,

$$\dot{E}''_{in} - \dot{E}''_{out} = \dot{E}''_{st}$$

$$\dot{E}''_{st} = q''_{in} - q''_{out} = 18\ W/m^2.$$ <

(b) Applying a surface energy balance at x = L,

$$q''_{out} = h\left[T(L) - T_\infty\right]$$

$$h = \frac{q''_{out}}{T(L) - T_\infty} = \frac{182\ W/m^2}{(142.7 - 100)°C}$$

$$h = 4.3\ W/m^2\cdot K.$$ <

COMMENTS: (1) From the heat equation,

$$(\partial T/\partial t) = (k/\rho c_p)\ \partial^2 T/\partial x^2 = 60(k/\rho c_p),$$

it follows that the temperature is increasing with time at every point in the wall.

(2) The value of h is small and is typical of free convection in a gas.

PROBLEM 2.25

KNOWN: Analytical expression for the steady-state temperature distribution of a plane wall experiencing uniform volumetric heat generation $\dot{q}$ while convection occurs at both of its surfaces.

FIND: (a) Sketch the temperature distribution, T(x), and identify significant physical features, (b) Determine $\dot{q}$, (c) Determine the surface heat fluxes, $q''_x(-L)$ and $q''_x(+L)$; how are these fluxes related to the generation rate; (d) Calculate the convection coefficients at the surfaces x = L and x = +L, (e) Obtain an expression for the heat flux distribution, $q''_x(x)$; explain significant features of the distribution; (f) If the source of heat generation is suddenly deactivated ($\dot{q}$ = 0), what is the rate of change of energy stored at this instant; (g) Determine the temperature that the wall will reach eventually with $\dot{q} = 0$; determine the energy that must be removed by the fluid per unit area of the wall to reach this state.

SCHEMATIC:

Fluid
$T_\infty = 20^oC$, h_l

$T = a + bx + cx^2$
$\dot{q}$, k = 5 W/m-K
$\rho = 2600\ kg/m^3$
$c_p = 800\ J/kg\text{-}K$

a = 82.0°C, x(m)
b = -210°C/m
c = - 2x10⁴ °C/m²

$q''_x(-L)$ $q''_x(+L)$
T(-L) T(+L)
- L x +L = 20 mm

Fluid
$T_\infty = 20^oC$, h_r

ASSUMPTIONS: (1) Steady-state conditions, (2) Uniform volumetric heat generation, (3) Constant properties.

ANALYSIS: (a) Using the analytical expression in the Workspace of IHT, the temperature distribution appears as shown below. The significant features include (1) parabolic shape, (2) maximum does not occur at the mid-plane, T(-5.25 mm) = 83.3°C, (3) the gradient at the x = +L surface is greater than at x = -L. Find also that T(-L) = 78.2°C and T(+L) = 69.8°C for use in part (d).

Temperature distribution

Temperature, T(x) (C)
90
85
80
75
70
-20 -10 0 10 20
x-coordinate, x (mm)

(b) Substituting the temperature distribution expression into the appropriate form of the heat diffusion equation, Eq. 2.15, the rate of volumetric heat generation can be determined.

$$\frac{d}{dx}\left(\frac{dT}{dx}\right)+\frac{\dot{q}}{k}=0 \qquad \text{where} \qquad T(x)=a+bx+cx^2$$

$$\frac{d}{dx}(0+b+2cx)+\frac{\dot{q}}{k}=(0+2c)+\frac{\dot{q}}{k}=0$$

Continued …..

PROBLEM 2.25 (Cont.)

$$\dot{q} = -2ck = -2\left(-2\times10^{4}\,{}^\circ C/m^2\right)5\,W/m\cdot K = 2\times10^{5}\,W/m^3$$ <

(c) The heat fluxes at the two boundaries can be determined using Fourier's law and the temperature distribution expression.

$$q''_x(x) = -k\frac{dT}{dx} \qquad \text{where} \qquad T(x) = a + bx + cx^2$$

$$q''_x(-L) = -k\left[0 + b + 2cx\right]_{x=-L} = -\left[b - 2cL\right]k$$

$$q''_x(-L) = -\left[-210^\circ C/m - 2\left(-2\times10^{4}\,{}^\circ C/m^2\right)0.020m\right]\times 5\,W/m\cdot K = -2950\,W/m^2$$ <

$$q''_x(+L) = -\left(b + 2cL\right)k = +5050\,W/m^2$$ <

From an overall energy balance on the wall as shown in the sketch below, $\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = 0$,

$$+q''_x(-L) - q''_x(+L) + 2\dot{q}L \overset{?}{=} 0 \quad \text{or} \quad -2950\,W/m^2 - 5050\,W/m^2 + 8000\,W/m^2 = 0$$

where $2\dot{q}L = 2\times2\times10^{5}\,W/m^3 \times 0.020\,m = 8000\,W/m^2$, so the equality is satisfied

Part (c) Overall energy balance

Part (d) Surface energy balances

(d) The convection coefficients, h_l and h_r, for the left- and right-hand boundaries (x = -L and x= +L, respectively), can be determined from the convection heat fluxes that are equal to the conduction fluxes at the boundaries. See the surface energy balances in the sketch above. See also part (a) result for T(-L) and T(+L).

$$q''_{cv,\ell} = q''_x(-L)$$

$$h_l\left[T_\infty - T(-L)\right] = h_l\left[20 - 78.2\right]K = -2950\,W/m^2 \qquad h_l = 51\,W/m^2\cdot K$$ <

$$q''_{cv,r} = q''_x(+L)$$

$$h_r\left[T(+L) - T_\infty\right] = h_r\left[69.8 - 20\right]K = +5050\,W/m^2 \qquad h_r = 101\,W/m^2\cdot K$$ <

(e) The expression for the heat flux distribution can be obtained from Fourier's law with the temperature distribution

$$q''_x(x) = -k\frac{dT}{dx} = -k\left[0 + b + 2cx\right]$$

$$q''_x(x) = -5\,W/m\cdot K\left[-210^\circ C/m + 2\left(-2\times10^{4}\,{}^\circ C/m^2\right)\right]x = 1050 + 2\times10^{5}x$$ <

Continued …..

The distribution is linear with the x-coordinate. The maximum temperature will occur at the location where $q''_x(x_{max}) = 0$,

$$x_{max} = -\frac{1050\,W/m^2}{2\times10^5\ W/m^3} = -5.25\times10^{-3}m = -5.25\,mm$$ <

(f) If the source of the heat generation is suddenly deactivated so that $\dot{q} = 0$, the appropriate form of the heat diffusion equation for the ensuing transient conduction is

$$k\frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}\right) = \rho\, c_p \frac{\partial T}{\partial t}$$

At the instant this occurs, the temperature distribution is still T(x) = a + bx + cx^2. The right-hand term represents the rate of energy storage per unit volume,

$$\dot{E}''_{st} = k\frac{\partial}{\partial x}[0+b+2cx] = k[0+2c] = 5\,W/m\cdot K\times2\left(-2\times10^4{}^\circ C/m^2\right) = -2\times10^5\ W/m^3$$ <

(g) With no heat generation, the wall will eventually ($t \to \infty$) come to equilibrium with the fluid, $T(x,\infty) = T_\infty = 20°C$. To determine the energy that must be removed from the wall to reach this state, apply the conservation of energy requirement over an interval basis, Eq. 1.11b. The "initial" state is that corresponding to the steady-state temperature distribution, T_i, and the "final" state has $T_f = 20°C$. We've used T_∞ as the reference condition for the energy terms.

$$E''_{in} - E''_{out} = \Delta E''_{st} = E''_f - E''_i \qquad \text{with} \quad E''_{in} = 0.$$

$$-E''_{out} = \rho\, c_p 2L\left(T_f - T_\infty\right) - \rho\, c_p \int_{-L}^{+L}\left(T_i - T_\infty\right)dx$$

$$E''_{out} = \rho\, c_p \int_{-L}^{+L}\left[a+bx+cx^2 - T_\infty\right]dx = \rho\, c_p\left[ax + bx^2/2 + cx^3/3 - T_\infty x\right]_{-L}^{+L}$$

$$E''_{out} = \rho\, c_p\left[2aL + 0 + 2cx^3/3 - 2T_\infty L\right]$$

$$E''_{out} = 2600\,kg/m^3\times800\,J/kg\cdot K\Big[2\times82°C\times0.020m + 2\left(-2\times10^4{}^\circ C/m^2\right)$$
$$(0.020m)^3/3 - 2(20°C)0.020m\Big]$$

$$E''_{out} = 4.94\times10^6\ J/m^2$$ <

COMMENTS: (1) In part (a), note that the temperature gradient is larger at x = + L than at x = - L. This is consistent with the results of part (c) in which the conduction heat fluxes are evaluated.

Continued …..

PROBLEM 2.25 (Cont.)

(2) In evaluating the conduction heat fluxes, $q''_x(x)$, it is important to recognize that this flux is in the positive x-direction. See how this convention is used in formulating the energy balance in part (c).

(3) It is good practice to represent energy balances with a schematic, clearly defining the system or surface, showing the CV or CS with dashed lines, and labeling the processes. Review again the features in the schematics for the energy balances of parts (c & d).

(4) Re-writing the heat diffusion equation introduced in part (b) as

$$-\frac{d}{dx}\left(-k\frac{dT}{dx}\right)+\dot{q}=0$$

recognize that the term in parenthesis is the heat flux. From the differential equation, note that if the differential of this term is a constant $(\dot{q}/k)$, then the term must be a linear function of the x-coordinate. This agrees with the analysis of part (e).

(5) In part (f), we evaluated $\dot{E}_{st}$, the rate of energy change stored in the wall at the instant the volumetric heat generation was deactivated. Did you notice that $\dot{E}_{st} = -2\times10^5\ W/m^3$ is the same value of the deactivated $\dot{q}$? How do you explain this?

PROBLEM 2.26

KNOWN: Steady-state conduction with uniform internal energy generation in a plane wall; temperature distribution has quadratic form. Surface at x=0 is prescribed and boundary at x = L is insulated.

FIND: (a) Calculate the internal energy generation rate, $\dot{q}$, by applying an overall energy balance to the wall, (b) Determine the coefficients a, b, and c, by applying the boundary conditions to the prescribed form of the temperature distribution; plot the temperature distribution and label as Case 1, (c) Determine new values for a, b, and c for conditions when the convection coefficient is halved, and the generation rate remains unchanged; plot the temperature distribution and label as Case 2; (d) Determine new values for a, b, and c for conditions when the generation rate is doubled, and the convection coefficient remains unchanged ($h = 500\ W/m^2 \cdot K$); plot the temperature distribution and label as Case 3.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction with constant properties and uniform internal generation, and (3) Boundary at x = L is adiabatic.

ANALYSIS: (a) The internal energy generation rate can be calculated from an overall energy balance on the wall as shown in the schematic below.

$$\dot{E}''_{in} - \dot{E}''_{out} + \dot{E}''_{gen} = 0 \qquad \text{where} \qquad \dot{E}''_{in} = q''_{conv}$$

$$h\left(T_\infty - T_o\right) + \dot{q}\,L = 0 \qquad (1)$$

$$\dot{q} = -h\left(T_\infty - T_o\right)/L = -500\,W/m^2 \cdot K\left(20-120\right)°C/0.050\ m = 1.0\times10^6\ W/m^3 \quad <$$

(a) Overall energy balance

(b) Surface energy balances

(b) The coefficients of the temperature distribution, $T(x) = a + bx + cx^2$, can be evaluated by applying the boundary conditions at x = 0 and x = L. See Table 2.1 for representation of the boundary conditions, and the schematic above for the relevant surface energy balances.

Boundary condition at x = 0, convection surface condition

$$\dot{E}''_{in} - \dot{E}''_{out} = q''_{conv} - q''_x(0) = 0 \qquad \text{where} \qquad q''_x(0) = -k\left.\frac{dT}{dx}\right)_{x=0}$$

$$h\left(T_\infty - T_o\right) - \left[-k\left(0 + b + 2cx\right)_{x=0}\right] = 0$$

Continued …..

$$b = -h\left(T_\infty - T_o\right)/k = -500\,W/m^2 \cdot K\left(20 - 120\right)°C/5\,W/m \cdot K = 1.0\times10^4\,K/m \quad <$$

Boundary condition at x = L, adiabatic or insulated surface

$$\dot{E}_{in} - \dot{E}_{out} = -q''_x\left(L\right) = 0 \qquad \text{where} \qquad q''_x\left(L\right) = -k\left.\frac{dT}{dx}\right)_{x=L}$$

$$k\left[0 + b + 2cx\right]_{x=L} = 0 \tag{3}$$

$$c = -b/2L = -1.0\times10^4\,K/m/\left(2\times0.050m\right) = -1.0\times10^5\,K/m^2 \quad <$$

Since the surface temperature at x = 0 is known, T(0) = T_o = 120°C, find

$$T\left(0\right) = 120°C = a + b\cdot0 + c\cdot0 \qquad \text{or} \qquad a = 120°C \tag{4} \quad <$$

Using the foregoing coefficients with the expression for T(x) in the Workspace of IHT, the temperature distribution can be determined and is plotted as Case 1 in the graph below.

(c) Consider Case 2 when the convection coefficient is halved, h_2 = h/2 = 250 W/m^2·K, $\dot{q} = 1\times10^6$ W/m^3 and other parameters remain unchanged except that $T_o \neq 120°C$. We can determine a, b, and c for the temperature distribution expression by repeating the analyses of parts (a) and (b).

Overall energy balance on the wall, see Eqs. (1,4)

$$a = T_o = \dot{q}L/h + T_\infty = 1\times10^6\,W/m^3\times0.050m/250\,W/m^2\cdot K + 20°C = 220°C \quad <$$

Surface energy balance at x = 0, see Eq. (2)

$$b = -h\left(T_\infty - T_o\right)/k = -250\,W/m^2\cdot K\left(20 - 220\right)°C/5\,W/m\cdot K = 1.0\times10^4\,K/m \quad <$$

Surface energy balance at x = L, see Eq. (3)

$$c = -b/2L = -1.0\times10^4\,K/m/\left(2\times0.050m\right) = -1.0\times10^5\,K/m^2 \quad <$$

The new temperature distribution, T_2 (x), is plotted as Case 2 below.

(d) Consider Case 3 when the internal energy volumetric generation rate is doubled, $\dot{q}_3 = 2\dot{q} = 2\times10^6\,W/m^3$, h = 500 W/m^2·K, and other parameters remain unchanged except that $T_o \neq 120°C$. Following the same analysis as part (c), the coefficients for the new temperature distribution, T (x), are

$$a = 220°C \qquad b = 2\times10^4\,K/m \qquad c = -2\times10^5\,K/m^2 \quad <$$

and the distribution is plotted as Case 3 below.

Continued …..

Temperature, T (C)

800
700
600
500
400
300
200
100

0 5 10 15 20 25 30 35 40 45 50

Wall position, x (mm)

1. h = 500 W/m^2.K, qdot = 1e6 W/m^3
2. h = 250 W/m^2.K, qdot = 1e6 W/m^3
3. h = 500 W/m^2.K, qdot = 2e6 W/m^3

COMMENTS: Note the following features in the family of temperature distributions plotted above. The temperature gradients at x = L are zero since the boundary is insulated (adiabatic) for all cases. The shapes of the distributions are all quadratic, with the maximum temperatures at the insulated boundary.

By halving the convection coefficient for Case 2, we expect the surface temperature T_o to increase relative to the Case 1 value, since the same heat flux is removed from the wall $(\dot{q}L)$ but the convection resistance has increased.

By doubling the generation rate for Case 3, we expect the surface temperature T_o to increase relative to the Case 1 value, since double the amount of heat flux is removed from the wall $(2\dot{q}L)$.

Can you explain why T_o is the same for Cases 2 and 3, yet the insulated boundary temperatures are quite different? Can you explain the relative magnitudes of T(L) for the three cases?

PROBLEM 2.27

KNOWN: Temperature distribution and distribution of heat generation in central layer of a solar pond.

FIND: (a) Heat fluxes at lower and upper surfaces of the central layer, (b) Whether conditions are steady or transient, (c) Rate of thermal energy generation for the entire central layer.

SCHEMATIC:

ASSUMPTIONS: (1) Central layer is stagnant, (2) One-dimensional conduction, (3) Constant properties

ANALYSIS: (a) The desired fluxes correspond to conduction fluxes in the central layer at the lower and upper surfaces. A general form for the conduction flux is

$$q''_{cond} = -k\frac{\partial T}{\partial x} = -k\left[\frac{A}{ka}e^{-ax} + B\right].$$

Hence,

$$q''_l = q''_{cond(x=L)} = -k\left[\frac{A}{ka}e^{-aL} + B\right] \quad q''_u = q''_{cond(x=0)} = -k\left[\frac{A}{ka} + B\right].$$ <

(b) Conditions are steady if $\partial T/\partial t = 0$. Applying the heat equation,

$$\frac{\partial^2 T}{\partial x^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad -\frac{A}{k}e^{-ax} + \frac{A}{k}e^{-ax} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

Hence conditions are *steady* since

$$\partial T/\partial t = 0 \qquad \text{(for all } 0 \le \times \le L).$$ <

(c) For the central layer, the energy generation is

$$\dot{E}''_g = \int_0^L \dot{q}\,dx = A\int_0^L e^{-ax}\,dx$$

$$\dot{E}_g = -\frac{A}{a}e^{-ax}\Big|_0^L = -\frac{A}{a}\left(e^{-aL} - 1\right) = \frac{A}{a}\left(1 - e^{-aL}\right).$$ <

Alternatively, from an overall energy balance,

$$q''_2 - q''_1 + \dot{E}''_g = 0 \qquad \dot{E}''_g = q''_1 - q''_2 = \left(-q''_{cond(x=0)}\right) - \left(-q''_{cond(x=L)}\right)$$

$$\dot{E}_g = k\left[\frac{A}{ka} + B\right] - k\left[\frac{A}{ka}e^{-aL} + B\right] = \frac{A}{a}\left(1 - e^{-aL}\right).$$

COMMENTS: Conduction is in the negative x-direction, necessitating use of minus signs in the above energy balance.

PROBLEM 2.28

KNOWN: Temperature distribution in a semi-transparent medium subjected to radiative flux.

FIND: (a) Expressions for the heat flux at the front and rear surfaces, (b) Heat generation rate $\dot{q}(x)$, (c) Expression for absorbed radiation per unit surface area in terms of A, a, B, C, L, and k.

SCHEMATIC:

$q''_x(0)$, $\dot{E}_g$, $q''_x(L)$

Laser irradiation

Semitransparent medium, $T(x) = -\frac{A}{ka^2} e^{-ax} + Bx + C$

x, L

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in medium, (3) Constant properties, (4) All laser irradiation is absorbed and can be characterized by an internal volumetric heat generation term $\dot{q}(x)$.

ANALYSIS: (a) Knowing the temperature distribution, the surface heat fluxes are found using Fourier's law,

$$q''_x = -k\left[\frac{dT}{dx}\right] = -k\left[-\frac{A}{ka^2}(-a)e^{-ax} + B\right]$$

Front Surface, x=0: $$q''_x(0) = -k\left[+\frac{A}{ka}\cdot 1 + B\right] = -\left[\frac{A}{a} + kB\right]$$ <

Rear Surface, x=L: $$q''_x(L) = -k\left[+\frac{A}{ka}e^{-aL} + B\right] = -\left[\frac{A}{a}e^{-aL} + kB\right].$$ <

(b) The heat diffusion equation for the medium is

$$\frac{d}{dx}\left(\frac{dT}{dx}\right) + \frac{\dot{q}}{k} = 0 \quad \text{or} \quad \dot{q} = -k\frac{d}{dx}\left(\frac{dT}{dx}\right)$$

$$\dot{q}(x) = -k\frac{d}{dx}\left[+\frac{A}{ka}e^{-ax} + B\right] = Ae^{-ax}.$$ <

(c) Performing an energy balance on the medium,

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = 0$$

recognize that $\dot{E}_g$ represents the absorbed irradiation. On a unit area basis

$$\dot{E}''_g = -\dot{E}''_{in} + \dot{E}''_{out} = -q''_x(0) + q''_x(L) = +\frac{A}{a}\left(1 - e^{-aL}\right).$$ <

Alternatively, evaluate $\dot{E}''_g$ by integration over the volume of the medium,

$$\dot{E}''_g = \int_0^L \dot{q}(x)dx = \int_0^L Ae^{-ax}dx = -\frac{A}{a}\left[e^{-ax}\right]_0^L = \frac{A}{a}\left(1 - e^{-aL}\right).$$

PROBLEM 2.29

KNOWN: Steady-state temperature distribution in a one-dimensional wall of thermal conductivity, $T(x) = Ax^3 + Bx^2 + Cx + D$.

FIND: Expressions for the heat generation rate in the wall and the heat fluxes at the two wall faces (x = 0,L).

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat flow, (3) Homogeneous medium.

ANALYSIS: The appropriate form of the heat diffusion equation for these conditions is

$$\frac{d^2T}{dx^2} + \frac{\dot{q}}{k} = 0 \quad \text{or} \quad \dot{q} = -k\frac{d^2T}{dx^2}.$$

Hence, the generation rate is

$$\dot{q} = -k\frac{d}{dx}\left[\frac{dT}{dx}\right] = -k\frac{d}{dx}\left[3Ax^2 + 2Bx + C + 0\right]$$

$$\dot{q} = -k\left[6Ax + 2B\right] \qquad <$$

which is linear with the coordinate x. The heat fluxes at the wall faces can be evaluated from Fourier's law,

$$q_x'' = -k\frac{dT}{dx} = -k\left[3Ax^2 + 2Bx + C\right]$$

using the expression for the temperature gradient derived above. Hence, the heat fluxes are:

Surface x=0:

$$q_x''(0) = -kC \qquad <$$

Surface x=L:

$$q_x''(L) = -k\left[3AL^2 + 2BL + C\right]. \qquad <$$

COMMENTS: (1) From an overall energy balance on the wall, find

$$\dot{E}_{in}'' - \dot{E}_{out}'' + \dot{E}_g'' = 0$$

$$q_x''(0) - q_x''(L) + \dot{E}_g'' = (-kC) - (-k)\left[3AL^2 + 2BL + C\right] + \dot{E}_g'' = 0$$

$$\dot{E}_g'' = -3AkL^2 - 2BkL.$$

From integration of the volumetric heat rate, we can also find $\dot{E}_g''$ as

$$\dot{E}_g'' = \int_0^L \dot{q}(x)dx = \int_0^L -k\left[6Ax + 2B\right]dx = -k\left[3Ax^2 + 2Bx\right]_0^L$$

$$\dot{E}_g'' = -3AkL^2 - 2BkL.$$

PROBLEM 2.30

KNOWN: Plane wall with no internal energy generation.

FIND: Determine whether the prescribed temperature distribution is possible; explain your reasoning. With the temperatures T(0) = 0°C and T_∞ = 20°C fixed, compute and plot the temperature T(L) as a function of the convection coefficient for the range $10 \le h \le 100\ \text{W/m}^2\cdot\text{K}$.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional conduction, (2) No internal energy generation, (3) Constant properties, (4) No radiation exchange at the surface x = L, and (5) Steady-state conditions.

ANALYSIS: (a) Is the prescribed temperature distribution possible? If so, the energy balance at the surface x = L as shown above in the Schematic, must be satisfied.

$$\dot{E}_{in} - \dot{E}_{out} \;?\!=\!?\; 0 \qquad q''_x(L) - q''_{cv} \;?\!=\!?\; 0 \qquad (1,2)$$

where the conduction and convection heat fluxes are, respectively,

$$q''_x(L) = -k\left.\frac{dT}{dx}\right)_{x=L} = -k\frac{T(L)-T(0)}{L} = -4.5\,\text{W/m}\cdot\text{K}\times(120-0)^\circ\text{C}\big/0.18\,\text{m} = -3000\,\text{W/m}^2$$

$$q''_{cv} = h\left[T(L)-T_\infty\right] = 30\,\text{W/m}^2\cdot\text{K}\times(120-20)^\circ\text{C} = 3000\,\text{W/m}^2$$

Substituting the heat flux values into Eq. (2), find (-3000) - (3000) ≠ 0 and therefore, the temperature distribution is not possible.

(b) With T(0) = 0°C and T_∞ = 20°C, the temperature at the surface x = L, T(L), can be determined from an overall energy balance on the wall as shown above in the Schematic,

$$\dot{E}_{in} - \dot{E}_{out} = 0 \qquad q''_x(0) - q''_{cv} = 0 \qquad -k\frac{T(L)-T(0)}{L} - h\left[T(L)-T_\infty\right] = 0$$

$$-4.5\,\text{W/m}\cdot\text{K}\left[T(L)-0^\circ\text{C}\right]\big/0.18\,\text{m} - 30\,\text{W/m}^2\cdot\text{K}\left[T(L)-20^\circ\text{C}\right] = 0$$

T(L) = 10.9°C <

Using this same analysis, T(L) as a function of the convection coefficient can be determined and plotted. We don't expect T(L) to be linearly dependent upon h. Note that as h increases to larger values, T(L) approaches T_∞. To what value will T(L) approach as h decreases?

PROBLEM 2.31

KNOWN: Coal pile of prescribed depth experiencing uniform volumetric generation with convection, absorbed irradiation and emission on its upper surface.

FIND: (a) The appropriate form of the heat diffusion equation (HDE) and whether the prescribed temperature distribution satisfies this HDE; conditions at the bottom of the pile, x = 0; sketch of the temperature distribution with labeling of key features; (b) Expression for the conduction heat rate at the location x = L; expression for the surface temperature T_s based upon a surface energy balance at x = L; evaluate T_S and T(0) for the prescribed conditions; (c) Based upon typical daily averages for G_S and h, compute and plot T_S and T(0) for (1) h = 5 $W/m^2 \cdot K$ with $50 \le G_S \le 500$ W/m^2, (2) G_S = 400 W/m^2 with $5 \le h \le 50$ $W/m^2 \cdot K$.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional conduction, (2) Uniform volumetric heat generation, (3) Constant properties, (4) Negligible irradiation from the surroundings, and (5) Steady-state conditions.

PROPERTIES: *Table A.3*, Coal (300K): k = 0.26 W/m.K

ANALYSIS: (a) For one-dimensional, steady-state conduction with uniform volumetric heat generation and constant properties the heat diffusion equation (HDE) follows from Eq. 2.16,

$$\frac{d}{dx}\left(\frac{dT}{dx}\right)+\frac{\dot{q}}{k}=0 \qquad (1)$$ <

Substituting the temperature distribution into the HDE, Eq. (1),

$$T(x)=T_S+\frac{\dot{q}L^2}{2k}\left(1-\frac{x^2}{L^2}\right) \qquad \frac{d}{dx}\left[0+\frac{\dot{q}L^2}{2k}\left(0-\frac{2x}{L^2}\right)\right]+\frac{\dot{q}}{k}\,?=?\,0 \qquad (2,3)$$

we find that it does indeed satisfy the HDE for all values of x. <

From Eq. (2), note that the temperature distribution must be quadratic, with maximum value at x = 0. At x = 0, the heat flux is

$$q''_x(0)=-k\left.\frac{dT}{dx}\right)_{x=0}=-k\left[0+\frac{\dot{q}L^2}{2k}\left(0-\frac{2x}{L^2}\right)\right]_{x=0}=0$$

so that the gradient at x = 0 is zero. Hence, the bottom is insulated.

(b) From an overall energy balance on the pile, the conduction heat flux at the surface must be

$$q''_x(L)=\dot{E}''_g=\dot{q}L$$ <

Continued...

PROBLEM 2.31 (Cont.)

From a surface energy balance per unit area shown in the Schematic above,

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = 0 \qquad q''_x(L) - q''_{cv} + G_{S,abs} - E = 0$$

$$\dot{q}L - h\left(T_s - T_\infty\right) + 0.95G_S - \varepsilon\sigma T_s^4 = 0 \qquad (4)$$

$$20\ \mathrm{W/m^3} \times 1\ \mathrm{m} - 5\ \mathrm{W/m^2 \cdot K}\left(T_s - 298\ \mathrm{K}\right) + 0.95 \times 400\ \mathrm{W/m^2} - 0.95 \times 5.67 \times 10^{-8}\ \mathrm{W/m^2 \cdot K^4}\, T_s^4 = 0$$

$$T_s = 295.7\ \mathrm{K} = 22.7°\mathrm{C}$$ <

From Eq. (2) with x = 0, find

$$T(0) = T_s + \frac{\dot{q}L^2}{2k} = 22.7°\mathrm{C} + \frac{30\ \mathrm{W/m^2} \times (1\ \mathrm{m})^2}{2 \times 0.26\ \mathrm{W/m \cdot K}} = 61.1°\mathrm{C} \qquad (5)$$ <

where the thermal conductivity for coal was obtained from Table A.3.

(c) Two plots are generated using Eq. (4) and (5) for T_s and T(0), respectively; (1) with h = 5 W/m^2·K for $50 \le G_S \le 500$ W/m^2 and (2) with G_S = 400 W/m^2 for $5 \le h \le 50$ W/m^2·K.

From the T vs. h plot with G_S = 400 W/m^2, note that the convection coefficient does not have a major influence on the surface or bottom coal pile temperatures. From the T vs. G_S plot with h = 5 W/m^2·K, note that the solar irradiation has a very significant effect on the temperatures. The fact that T_s is less than the ambient air temperature, T_∞, and, in the case of very low values of G_S, below freezing, is a consequence of the large magnitude of the emissive power E.

COMMENTS: In our analysis we ignored irradiation from the sky, an environmental radiation effect you'll consider in Chapter 12. Treated as large isothermal surroundings, $G_{sky} = \sigma T_{sky}^4$ where T_{sky} = -30°C for very clear conditions and nearly air temperature for cloudy conditions. For low G_S conditions we should consider G_{sky}, the effect of which will be to predict higher values for T_s and T(0).

PROBLEM 2.32

KNOWN: Cylindrical system with negligible temperature variation in the r,z directions.

FIND: (a) Heat equation beginning with a properly defined control volume, (b) Temperature distribution $T(\phi)$ for steady-state conditions with no internal heat generation and constant properties, (c) Heat rate for Part (b) conditions.

SCHEMATIC:

ASSUMPTIONS: (1) T is independent of r,z, (2) $\Delta r = (r_o - r_i) << r_i$.

ANALYSIS: (a) Define the control volume as $V = r_i d\phi \cdot \Delta r \cdot L$ where L is length normal to page. Apply the conservation of energy requirement, Eq. 1.11a,

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = \dot{E}_{st} \qquad q_\phi - q_{\phi+d\phi} + \dot{q}V = \rho V c \frac{\partial T}{\partial t} \qquad (1,2)$$

where

$$q_\phi = -k(\Delta r \cdot L)\frac{\partial T}{r_i \partial \phi} \qquad q_{\phi+d\phi} = q_\phi + \frac{\partial}{\partial \phi}(q_\phi)d\phi. \qquad (3,4)$$

Eqs. (3) and (4) follow from Fourier's law, Eq. 2.1, and from Eq. 2.7, respectively. Combining Eqs. (3) and (4) with Eq. (2) and canceling like terms, find

$$\frac{1}{r_i^2}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right) + \dot{q} = \rho c \frac{\partial T}{\partial t}. \qquad (5) <$$

Since temperature is independent of r and z, this form agrees with Eq. 2.20.

(b) For steady-state conditions with $\dot{q} = 0$, the heat equation, (5), becomes

$$\frac{d}{d\phi}\left[k\frac{dT}{d\phi}\right] = 0. \qquad (6)$$

With constant properties, it follows that $dT/d\phi$ is constant which implies $T(\phi)$ is linear in ϕ. That is,

$$\frac{dT}{d\phi} = \frac{T_2 - T_1}{\phi_2 - \phi_1} = +\frac{1}{\pi}(T_2 - T_1) \quad \text{or} \quad T(\phi) = T_1 + \frac{1}{\pi}(T_2 - T_1)\phi. \qquad (7,8) <$$

(c) The heat rate for the conditions of Part (b) follows from Fourier's law, Eq. (3), using the temperature gradient of Eq. (7). That is,

$$q_\phi = -k(\Delta r \cdot L)\frac{1}{r_i}\left[+\frac{1}{\pi}(T_2 - T_1)\right] = -k\left[\frac{r_o - r_i}{\pi r_i}\right]L(T_2 - T_1). \qquad (9) <$$

COMMENTS: Note the expression for the temperature gradient in Fourier's law, Eq. (3), is $\partial T / r_i \partial \phi$ not $\partial T / \partial \phi$. For the conditions of Parts (b) and (c), note that q_ϕ is independent of ϕ; this is first indicated by Eq. (6) and confirmed by Eq. (9).

PROBLEM 2.33

KNOWN: Heat diffusion with internal heat generation for one-dimensional cylindrical, radial coordinate system.

FIND: Heat diffusion equation.

SCHEMATIC:

ASSUMPTIONS: (1) Homogeneous medium.

ANALYSIS: Control volume has volume, $V = A_r \cdot dr = 2\pi r \cdot dr \cdot 1$, with unit thickness normal to page. Using the conservation of energy requirement, Eq. 1.11a,

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = \dot{E}_{st}$$

$$q_r - q_{r+dr} + \dot{q}V = \rho V c_p \frac{\partial T}{\partial t}.$$

Fourier's law, Eq. 2.1, for this one-dimensional coordinate system is

$$q_r = -kA_r \frac{\partial T}{\partial r} = -k \times 2\pi r \cdot 1 \times \frac{\partial T}{\partial r}.$$

At the outer surface, r+dr, the conduction rate is

$$q_{r+dr} = q_r + \frac{\partial}{\partial r}(q_r)dr = q_r + \frac{\partial}{\partial r}\left[-k \cdot 2\pi r \cdot \frac{\partial T}{\partial r}\right]dr.$$

Hence, the energy balance becomes

$$q_r - \left[q_r + \frac{\partial}{\partial r}\left[-k2\pi r \frac{\partial T}{\partial r}\right]dr\right] + \dot{q} \cdot 2\pi r dr = \rho \cdot 2\pi r dr \cdot c_p \frac{\partial T}{\partial t}$$

Dividing by the factor $2\pi r\, dr$, we obtain

$$\frac{1}{r}\frac{\partial}{\partial r}\left[kr\frac{\partial T}{\partial r}\right] + \dot{q} = \rho c_p \frac{\partial T}{\partial t}.$$

<

COMMENTS: (1) Note how the result compares with Eq. 2.20 when the terms for the ϕ,z coordinates are eliminated. (2) Recognize that we did not require $\dot{q}$ and k to be independent of r.

PROBLEM 2.34

KNOWN: Heat diffusion with internal heat generation for one-dimensional spherical, radial coordinate system.

FIND: Heat diffusion equation.

SCHEMATIC:

ASSUMPTIONS: (1) Homogeneous medium.

ANALYSIS: Control volume has the volume, $V = A_r \cdot dr = 4\pi r^2 dr$. Using the conservation of energy requirement, Eq. 1.11a,

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = \dot{E}_{st}$$

$$q_r - q_{r+dr} + \dot{q}V = \rho V c_p \frac{\partial T}{\partial t}.$$

Fourier's law, Eq. 2.1, for this coordinate system has the form

$$q_r = -kA_r \frac{\partial T}{\partial r} = -k \cdot 4\pi r^2 \cdot \frac{\partial T}{\partial r}.$$

At the outer surface, r+dr, the conduction rate is

$$q_{r+dr} = q_r + \frac{\partial}{\partial r}(q_r)dr = q_r + \frac{\partial}{\partial r}\left[-k \cdot 4\pi r^2 \cdot \frac{\partial T}{\partial r}\right]dr.$$

Hence, the energy balance becomes

$$q_r - \left[q_r + \frac{\partial}{\partial r}\left[-k \cdot 4\pi r^2 \cdot \frac{\partial T}{\partial r}\right]dr\right] + \dot{q} \cdot 4\pi r^2 dr = \rho \cdot 4\pi r^2 dr \cdot c_p \frac{\partial T}{\partial t}.$$

Dividing by the factor $4\pi r^2 dr$, we obtain

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left[kr^2 \frac{\partial T}{\partial r}\right] + \dot{q} = \rho c_p \frac{\partial T}{\partial t}.$$

<

COMMENTS: (1) Note how the result compares with Eq. 2.23 when the terms for the θ,ϕ directions are eliminated.

(2) Recognize that we did not require $\dot{q}$ and k to be independent of the coordinate r.

PROBLEM 2.35

KNOWN: Three-dimensional system – described by cylindrical coordinates (r,ϕ,z) – experiences transient conduction and internal heat generation.

FIND: Heat diffusion equation.

SCHEMATIC: See also Fig. 2.9.

ASSUMPTIONS: (1) Homogeneous medium.

ANALYSIS: Consider the differential control volume identified above having a volume given as V = dr·rdϕ·dz. From the conservation of energy requirement,

$$q_r - q_{r+dr} + q_\phi - q_{\phi+d\phi} + q_z - q_{z+dz} + \dot{E}_g = \dot{E}_{st}. \tag{1}$$

The generation and storage terms, both representing volumetric phenomena, are

$$\dot{E}_g = \dot{q}V = \dot{q}(dr \cdot rd\phi \cdot dz) \qquad \dot{E}_g = \rho V c \partial T/\partial t = \rho(dr \cdot rd\phi \cdot dz)c\,\partial T/\partial t. \tag{2,3}$$

Using a Taylor series expansion, we can write

$$q_{r+dr} = q_r + \frac{\partial}{\partial r}(q_r)dr, \quad q_{\phi+d\phi} = q_\phi + \frac{\partial}{\partial \phi}(q_\phi)d\phi, \quad q_{z+dz} = q_z + \frac{\partial}{\partial z}(q_z)dz. \tag{4,5,6}$$

Using Fourier's law, the expressions for the conduction heat rates are

$$q_r = -kA_r\partial T/\partial r = -k(rd\phi \cdot dz)\partial T/\partial r \tag{7}$$

$$q_\phi = -kA_\phi\partial T/r\partial\phi = -k(dr \cdot dz)\partial T/r\partial\phi \tag{8}$$

$$q_z = -kA_z\partial T/\partial z = -k(dr \cdot rd\phi)\partial T/\partial z. \tag{9}$$

Note from the above, right schematic that the gradient in the ϕ-direction is ∂T/r∂ϕ and not ∂T/∂ϕ. Substituting Eqs. (2), (3) and (4), (5), (6) into Eq. (1),

$$-\frac{\partial}{\partial r}(q_r)dr - \frac{\partial}{\partial\phi}(q_\phi)d\phi - \frac{\partial}{\partial z}(q_z)dz + \dot{q}\,dr \cdot rd\phi \cdot dz = \rho(dr \cdot rd\phi \cdot dz)c\frac{\partial T}{\partial t}. \tag{10}$$

Substituting Eqs. (7), (8) and (9) for the conduction rates, find

$$-\frac{\partial}{\partial r}\left[-k(rd\phi \cdot dz)\frac{\partial T}{\partial r}\right]dr - \frac{\partial}{\partial\phi}\left[-k(drdz)\frac{\partial T}{r\partial\phi}\right]d\phi - \frac{\partial}{\partial z}\left[-k(dr \cdot rd\phi)\frac{\partial T}{\partial z}\right]dz$$

$$+\dot{q}\,dr \cdot rd\phi \cdot dz = \rho(dr \cdot rd\phi \cdot dz)c\frac{\partial T}{\partial t}. \tag{11}$$

Dividing Eq. (11) by the volume of the CV, Eq. 2.20 is obtained.

$$\frac{1}{r}\frac{\partial}{\partial r}\left[kr\frac{\partial T}{\partial r}\right] + \frac{1}{r^2}\frac{\partial}{\partial\phi}\left[k\frac{\partial T}{\partial\phi}\right] + \frac{\partial}{\partial z}\left[k\frac{\partial T}{\partial z}\right] + \dot{q} = \rho c\frac{\partial T}{\partial t}$$

<

PROBLEM 2.36

KNOWN: Three-dimensional system – described by cylindrical coordinates (r,ϕ,θ) – experiences transient conduction and internal heat generation.

FIND: Heat diffusion equation.

SCHEMATIC: See Figure 2.10.

ASSUMPTIONS: (1) Homogeneous medium.

ANALYSIS: The differential control volume is V = dr·rsinθdϕ·rdθ, and the conduction terms are identified in Figure 2.10. Conservation of energy requires

$$q_r - q_{r+dr} + q_\phi - q_{\phi+d\phi} + q_\theta - q_{\theta+d\theta} + \dot{E}_g = \dot{E}_{st}. \qquad (1)$$

The generation and storage terms, both representing volumetric phenomena, are

$$\dot{E}_g = \dot{q}V = \dot{q}[dr \cdot r\sin\theta d\phi \cdot rd\theta] \qquad \dot{E}_{st} = \rho Vc\frac{\partial T}{\partial t} = \rho[dr \cdot r\sin\theta d\phi \cdot rd\theta]c\frac{\partial T}{\partial t}. \qquad (2,3)$$

Using a Taylor series expansion, we can write

$$q_{r+dr} = q_r + \frac{\partial}{\partial r}(q_r)dr, \quad q_{\phi+d\phi} = q_\phi + \frac{\partial}{\partial\phi}(q_\phi)d\phi, \quad q_{\theta+d\theta} = q_\theta + \frac{\partial}{\partial\theta}(q_\theta)d\theta. \qquad (4,5,6)$$

From Fourier's law, the conduction heat rates have the following forms.

$$q_r = -kA_r\partial T/\partial r = -k[r\sin\theta d\phi \cdot rd\theta]\partial T/\partial r \qquad (7)$$

$$q_\phi = -kA_\phi\partial T/r\sin\theta\partial\phi = -k[dr \cdot rd\theta]\partial T/r\sin\theta\partial\phi \qquad (8)$$

$$q_\theta = -kA_\theta\partial T/r\partial\theta = -k[dr \cdot r\sin\theta d\phi]\partial T/r\partial\theta. \qquad (9)$$

Substituting Eqs. (2), (3) and (4), (5), (6) into Eq. (1), the energy balance becomes

$$-\frac{\partial}{\partial r}(q_r)dr - \frac{\partial}{\partial_\phi}(q_\phi)d\phi - \frac{\partial}{\partial\theta}(q_\theta)d\theta + \dot{q}[dr \cdot r\sin\theta d\phi \cdot rd\theta] = \rho[dr \cdot r\sin\theta d\phi \cdot rd\theta]c\frac{\partial T}{\partial t} \qquad (10)$$

Substituting Eqs. (7), (8) and (9) for the conduction rates, find

$$-\frac{\partial}{\partial\theta}\left[-k[r\sin\theta d\phi \cdot rd\theta]\frac{\partial T}{\partial r}\right]dr - \frac{\partial}{\partial\phi}\left[-k[dr \cdot rd\theta]\frac{\partial T}{r\sin\theta\partial\phi}\right]d\phi$$

$$-\frac{\partial}{\partial\theta}\left[-k[dr \cdot r\sin\theta d\phi]\frac{\partial T}{r\partial\theta}\right]d\theta + \dot{q}[dr \cdot r\sin\theta d\phi \cdot rd\theta] = \rho[dr \cdot r\sin\theta d\phi \cdot rd\theta]c\frac{\partial T}{\partial t} \qquad (11)$$

Dividing Eq. (11) by the volume of the control volume, V, Eq. 2.23 is obtained.

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left[kr^2\frac{\partial T}{\partial r}\right] + \frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial\phi}\left[k\frac{\partial T}{\partial \phi}\right] + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left[k\sin\theta\frac{\partial T}{\partial\theta}\right] + \dot{q} = \rho c\frac{\partial T}{\partial t}. \qquad <$$

COMMENTS: Note how the temperature gradients in Eqs. (7) - (9) are formulated. The numerator is always ∂T while the denominator is the dimension of the control volume in the specified coordinate direction.

PROBLEM 2.37

KNOWN: Temperature distribution in steam pipe insulation.

FIND: Whether conditions are steady-state or transient. Manner in which heat flux and heat rate vary with radius.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional conduction in r, (2) Constant properties.

ANALYSIS: From Equation 2.20, the heat equation reduces to

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)=\frac{1}{\alpha}\frac{\partial T}{\partial t}.$$

Substituting for T(r),

$$\frac{1}{\alpha}\frac{\partial T}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{C_1}{r}\right)=0.$$

Hence, steady-state conditions exist. <

From Equation 2.19, the radial component of the heat flux is

$$q_r''=-k\frac{\partial T}{\partial r}=-k\frac{C_1}{r}.$$

Hence, q_r'' decreases with increasing $r\left(q_r''\alpha 1/r\right)$. <

At any radial location, the heat rate is

$$q_r=2\pi rLq_r''=-2\pi kC_1L$$

Hence, q_r is independent of r. <

COMMENTS: The requirement that q_r is invariant with r is consistent with the energy conservation requirement. If q_r is constant, the flux must vary inversely with the area perpendicular to the direction of heat flow. Hence, q_r'' varies inversely with r.

PROBLEM 2.38

KNOWN: Inner and outer radii and surface temperatures of a long circular tube with internal energy generation.

FIND: Conditions for which a linear radial temperature distribution may be maintained.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional, steady-state conduction, (2) Constant properties.

ANALYSIS: For the assumed conditions, Eq. 2.20 reduces to

$$\frac{k}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)+\dot{q}=0$$

If $\dot{q}$ = 0 or $\dot{q}$ = constant, it is clearly impossible to have a linear radial temperature distribution. However, we may use the heat equation to infer a special form of $\dot{q}$ (r) for which dT/dr is a constant (call it C_1). It follows that

$$\frac{k}{r}\frac{d}{dr}(r\,C_1)+\dot{q}=0$$

$$\dot{q}=-\frac{C_1 k}{r}$$

<

where $C_1 = (T_2 - T_1)/(r_2 - r_1)$. Hence, if the generation rate varies inversely with radial location, the radial temperature distribution is linear.

COMMENTS: Conditions for which $\dot{q} \propto (1/r)$ would be unusual.

PROBLEM 2.39

KNOWN: Radii and thermal conductivity of conducting rod and cladding material. Volumetric rate of thermal energy generation in the rod. Convection conditions at outer surface.

FIND: Heat equations and boundary conditions for rod and cladding.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in r, (3) Constant properties.

ANALYSIS: From Equation 2.20, the appropriate forms of the heat equation are

Conducting Rod:

$$\frac{k_r}{r}\frac{d}{dr}\left(r\frac{dT_r}{dt}\right)+\dot{q}=0$$ <

Cladding:

$$\frac{d}{dr}\left(r\frac{dT_c}{dr}\right)=0.$$ <

Appropriate boundary conditions are:

(a) $dT_r / dr|_{r=0} = 0$ <

(b) $T_r(r_i) = T_c(r_i)$ <

(c) $k_r \frac{dT_r}{dr}|_{r_i} = k_c \frac{dT_c}{dr}|_{r_i}$ <

(d) $k_c \frac{dT_c}{dr}|_{r_o} = h\left[T_c(r_o) - T_\infty\right]$ <

COMMENTS: Condition (a) corresponds to symmetry at the centerline, while the interface conditions at $r = r_i$ (b,c) correspond to requirements of thermal equilibrium and conservation of energy. Condition (d) results from conservation of energy at the outer surface.

PROBLEM 2.40

KNOWN: Steady-state temperature distribution for hollow cylindrical solid with volumetric heat generation.

FIND: (a) Determine the inner radius of the cylinder, r_i, (b) Obtain an expression for the volumetric rate of heat generation, $\dot{q}$, (c) Determine the axial distribution of the heat flux at the outer surface, $q''_r(r_o, z)$, and the heat rate at this outer surface; is the heat rate *in* or *out* of the cylinder; (d) Determine the radial distribution of the heat flux at the end faces of the cylinder, $q''_z(r, +z_o)$ and $q''_z(r, -z_o)$, and the corresponding heat rates; are the heat rates *in* or *out* of the cylinder; (e) Determine the relationship of the surface heat rates to the heat generation rate; is an overall energy balance satisfied?

SCHEMATIC:

+z_o = 2.5 m
Insulated boundary
$T(r,z) = a + br^2 + c\ln(r) + dz^2$ r(m), z(m)
$a = 20°C$ $c = -12°C$
$b = 150°C/m^2$ $d = -300°C/m^2$
k = 16 W/m-K
z
-z_o = 2.5 m
0 r_i r_o = 1 m r

ASSUMPTIONS: (1) Steady-state conditions, (2) Two-dimensional conduction with constant properties and volumetric heat generation.

ANALYSIS: (a) Since the inner boundary, r = r_i, is adiabatic, then $q''_r(r_i, z) = 0$. Hence the temperature gradient in the r-direction must be zero.

$$\left.\frac{\partial T}{\partial r}\right)_{r_i} = 0 + 2br_i + c/r_i + 0 = 0$$

$$r_i = +\left(-\frac{c}{2b}\right)^{1/2} = \left(-\frac{-12°C}{2\times150°C/m^2}\right)^{1/2} = 0.2\ m$$ <

(b) To determine $\dot{q}$, substitute the temperature distribution into the heat diffusion equation, Eq. 2.20, for two-dimensional (r,z), steady-state conduction

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\partial}{\partial z}\left(\frac{\partial T}{\partial z}\right) + \frac{\dot{q}}{k} = 0$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r[0 + 2br + c/r + 0]\right) + \frac{\partial}{\partial z}(0 + 0 + 0 + 2dz) + \frac{\dot{q}}{k} = 0$$

$$\frac{1}{r}[4br + 0] + 2d + \frac{\dot{q}}{k} = 0$$

$$\dot{q} = -k[4b - 2d] = -16\ W/m\cdot K\left[4\times150°C/m^2 - 2\left(-300°C/m^2\right)\right]$$

$$\dot{q} = 0\ W/m^3$$ <

(c) The heat flux and the heat rate at the outer surface, r = r_o, may be calculated using Fourier's law. Note that the sign of the heat flux in the positive r-direction is negative, and hence the heat flow is *into* the cylinder.

$$q''_r(r_o, z) = -k\left.\frac{\partial T}{\partial r}\right)_{r_o} = -k[0 + 2br_o + c/r_o + 0]$$

Continued …..

$$q''_r\left(r_{o,}z\right) = -16\ \text{W/m}\cdot\text{K}\left[2\times150°\text{C/m}^2\times1\ \text{m} - 12°\text{C}/1\ \text{m}\right] = -4608\ \text{W/m}^2$$ <

$$q_r\left(r_o\right) = A_r\, q''_r\left(r_{o,}z\right) \qquad \text{where} \qquad A_r = 2\pi r_o\left(2z_o\right)$$

$$q_r\left(r_o\right) = -4\pi\times1\ \text{m}\times2.5\ \text{m}\times4608\ \text{W/m}^2 = -144,765\ \text{W}$$ <

(d) The heat fluxes and the heat rates at end faces, z = + z_o and – z_o, may be calculated using Fourier's law. The direction of the heat rate *in* or *out* of the end face is determined by the sign of the heat flux in the positive z-direction.

At the upper end face, z = + z_o: heat rate is *out* of the cylinder <

$$q''_z\left(r,+z_o\right) = -k\left.\frac{\partial T}{\partial z}\right)_{z_o} = -k\left[0+0+0+2dz_o\right]$$

$$q''_z\left(r,+z_o\right) = -16\ \text{W/m}\cdot\text{K}\times2\left(-300°\text{C/m}^2\right)2.5\ \text{m} = +24,000\ \text{W/m}^2$$ <

$$q_z\left(+z_o\right) = A_z\, q''_z\left(r,+z_o\right) \qquad \text{where} \qquad A_z = \pi\left(r_o^2 - r_i^2\right)$$

$$q_z\left(+z_o\right) = \pi\left(1^2 - 0.2^2\right)\text{m}^2\times24,000\ \text{W/m}^2 = +72,382\ \text{W}$$ <

At the lower end face, z = - z_o: heat rate is *out* of the cylinder <

$$q''_z\left(r,-z_o\right) = -k\left.\frac{\partial T}{\partial z}\right)_{-z_o} = -k\left[0+0+0+2dz_o\right]$$

$$q''_z\left(r,-z_o\right) = -16\ \text{W/m}^2\cdot\text{K}\times2\left(-300°\text{C/m}\right)\left(-2.5\ \text{m}\right) = -24,000\ \text{W/m}^2$$ <

$$q_z\left(-z_o\right) = -72,382\ \text{W}$$ <

(e) The heat rates from the surfaces and the volumetric heat generation can be related through an overall energy balance on the cylinder as shown in the sketch.

$q''_z(r,+z_o)$ = +24,000 W/m^2
$q_z(r,+z_o)$ = +72,382 W
$\dot{E}_{gen} = \dot{q}\,\forall$
z
$q''_r(r_o,z)$ = -4,608 W/m^2
$q_r(r_o,z)$ = -144,765 W
r
$q''_z(r,-z_o)$ = -24,000 W/m^2
$q_z(r,-z_o)$ = -72,382 W

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = 0 \qquad \text{where} \qquad \dot{E}_{gen} = \dot{q}\forall = 0$$

$$\dot{E}_{in} = -q_r\left(r_o\right) = -\left(-144,765\ \text{W}\right) = +144,765\ \text{W}$$ <

$$\dot{E}_{out} = +q_z\left(z_o\right) - q_z\left(-z_o\right) = \left[72,382 - \left(-72,382\right)\right]\text{W} = +144,764\ \text{W}$$ <

The overall energy balance is satisfied.

COMMENTS: When using Fourier's law, the heat flux q''_z denotes the heat flux in the positive z-direction. At a boundary, the sign of the numerical value will determine whether heat is flowing into or out of the boundary.

PROBLEM 2.41

KNOWN: An electric cable with an insulating sleeve experiences convection with adjoining air and radiation exchange with large surroundings.

FIND: (a) Verify that prescribed temperature distributions for the cable and insulating sleeve satisfy their appropriate heat diffusion equations; sketch temperature distributions labeling key features; (b) Applying Fourier's law, verify the conduction heat rate expression for the sleeve, q'_r, in terms of $T_{s,1}$ and $T_{s,2}$; apply a surface energy balance to the cable to obtain an alternative expression for q'_r in terms of $\dot{q}$ and r_1; (c) Apply surface energy balance around the outer surface of the sleeve to obtain an expression for which $T_{s,2}$ can be evaluated; (d) Determine $T_{s,1}$, $T_{s,2}$, and T_o for the specified geometry and operating conditions; and (e) Plot $T_{s,1}$, $T_{s,2}$, and T_o as a function of the outer radius for the range $15.5 \le r_2 \le 20$ mm.

SCHEMATIC:

$T_{sur} = 35\,^oC$

$T_\infty = 25\,^oC$
$h = 25\ W/m^2 \cdot K$

Electrical cable, k_c, $\dot{q}$
Insulation sleeve, $k_s = 0.15\ W/m \cdot K$
$T_{s,1}$
$r_1 = 15$ mm
$r_2 = 15.5$ mm
$T_{s,2}$

ASSUMPTIONS: (1) One-dimensional, radial conduction, (2) Uniform volumetric heat generation in cable, (3) Negligible thermal contact resistance between the cable and sleeve, (4) Constant properties in cable and sleeve, (5) Surroundings large compared to the sleeve, and (6) Steady-state conditions.

ANALYSIS: (a) The appropriate forms of the heat diffusion equation (HDE) for the insulation and cable are identified. The temperature distributions are valid if they satisfy the relevant HDE.

Insulation: The temperature distribution is given as

$$T(r) = T_{s,2} + \left(T_{s,1} - T_{s,2}\right)\frac{\ln(r/r_2)}{\ln(r_1/r_2)} \tag{1}$$

and the appropriate HDE (radial coordinates, SS, $\dot{q} = 0$), Eq. 2.20,

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$$

$$\frac{d}{dr}\left(r\left[0 + \left(T_{s,1} - T_{s,2}\right)\frac{1/r}{\ln(r_1/r_2)}\right]\right) = \frac{d}{dr}\left(\frac{T_{s,1} - T_{s,2}}{\ln(r_1/r_2)}\right) \overset{?}{=} 0$$

Hence, the temperature distribution satisfies the HDE. $<$

Cable: The temperature distribution is given as

$$T(r) = T_{s,1} + \frac{\dot{q}r_1^2}{4k_c}\left(1 - \frac{r^2}{r_1^2}\right) \tag{2}$$

and the appropriate HDE (radial coordinates, SS, $\dot{q}$ uniform), Eq. 2.20,

Continued...

PROBLEM 2.41 (Cont.)

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)+\frac{\dot{q}}{k_c}=0$$

$$\frac{1}{r}\frac{d}{dr}\left(r\left[0+\frac{\dot{q}r_1^2}{4k_c}\left(0-\frac{2r}{r_1^2}\right)\right]\right)+\frac{\dot{q}}{k_c}?=?0$$

$$\frac{1}{r}\frac{d}{dr}\left(-\frac{\dot{q}r_1^2}{4k_c}\frac{2r^2}{r_1^2}\right)+\frac{\dot{q}}{k_c}?=?0$$

$$\frac{1}{r}\left(-\frac{\dot{q}r_1^2}{4k_c}\frac{4r}{r_1^2}\right)+\frac{\dot{q}}{k_c}?=?0$$

Hence the temperature distribution satisfies the HDE. <

The temperature distributions in the cable, $0 \le r \le r_1$, and sleeve, $r_1 \le r \le r_2$, and their key features are as follows:

(1) Zero gradient, symmetry condition,

(2) Increasing gradient with increasing radius, r, because of $\dot{q}$,

(3) Discontinuous T(r) across cable-sleeve interface because of different thermal conductivities,

(4) Decreasing gradient with increasing radius, r, since heat rate is constant.

(b) Using Fourier's law for the radial-cylindrical coordinate, the heat rate through the *insulation* (sleeve) per unit length is

$$q_r' = -kA_r'\frac{dT}{dr} = -k2\pi r\frac{dT}{dr}$$ <

and substituting for the temperature distribution, Eq. (1),

$$q_r' = -k_s 2\pi r\left[0+\left(T_{s,1}-T_{s,2}\right)\frac{1/r}{\ln\left(r_1/r_2\right)}\right] = 2\pi k_s\frac{\left(T_{s,1}-T_{s,2}\right)}{\ln\left(r_2/r_1\right)} \quad (3)$$ <

Applying an energy balance to a control surface placed around the cable,

$$\dot{E}_{in}-\dot{E}_{out}=0$$

$$\dot{q}\forall_c' - q_r' = 0$$

where $\dot{q}\forall_c$ represents the dissipated electrical power in the cable

Continued...

PROBLEM 2.41 (Cont.)

$$\dot{q}\left(\pi r_1^2\right) - q'_r = 0 \qquad \text{or} \qquad q'_r = \pi \dot{q} r_1^2 \qquad (4)$$ <

(c) Applying an energy balance to a control surface placed around the outer surface of the sleeve,

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$q'_r - q'_{cv} - q'_{rad} = 0$$

$$\pi \dot{q} r_1^2 - h\left(2\pi r_2\right)\left(T_{s,2} - T_\infty\right) - \varepsilon\left(2\pi r_2\right)\sigma\left(T_{s,2}^4 - T_{sur}^4\right) = 0 \qquad (5)$$ <

This relation can be used to determine $T_{s,2}$ in terms of the variables $\dot{q}$, r_1, r_2, h, T_∞, ε and T_{sur}.

(d) Consider a cable-sleeve system with the following prescribed conditions:

r_1 = 15 mm	k_c = 200 W/m·K	h = 25 W/m^2·K	ε = 0.9
r_2 = 15.5 mm	k_s = 0.15 W/m·K	T_∞ = 25°C	T_{sur} = 35°C

For 250 A with R'_e = 0.005 Ω/m, the volumetric heat generation rate is

$$\dot{q} = I^2 R'_e / \forall'_c = I^2 R'_e \Big/ \left(\pi r_1^2\right)$$

$$\dot{q} = \left(250\,A\right)^2 \times 0.005\,\Omega/m \Big/ \left(\pi \times 0.015^2\,m^2\right) = 4.42 \times 10^5\ W/m^3$$

Substituting numerical values in appropriate equations, we can evaluate $T_{s,1}$, $T_{s,2}$ and T_o.

Sleeve outer surface temperature, $T_{s,2}$: Using Eq. (5),

$$\pi \times 4.42 \times 10^5\,W/m^3 \times \left(0.015m\right)^2 - 25\,W/m^2 \cdot K \times \left(2\pi \times 0.0155m\right)\left(T_{s,2} - 298K\right)$$
$$-0.9 \times \left(2\pi \times 0.0155m\right) \times 5.67 \times 10^{-8}\,W/m^2 \cdot K^4 \left(T_{s,2}^4 - 308^4\right)K^4 = 0$$

$$T_{s,2} = 395\,K = 122^\circ C$$ <

Sleeve-cable interface temperature, $T_{s,1}$: Using Eqs. (3) and (4), with $T_{s,2}$ = 395 K,

$$\pi \dot{q} r_1^2 = 2\pi k_s \frac{\left(T_{s,1} - T_{s,2}\right)}{\ln\left(r_2/r_1\right)}$$

$$\pi \times 4.42 \times 10^5\,W/m^3 \times \left(0.015\,m\right)^2 = 2\pi \times 0.15\,W/m \cdot K \frac{\left(T_{s,1} - 395\,K\right)}{\ln\left(15.5/15.0\right)}$$

$$T_{s,1} = 406\,K = 133^\circ C$$ <

Continued...

PROBLEM 2.41 (Cont.)

Cable centerline temperature, T_o: Using Eq. (2) with $T_{s,1}$ = 133°C,

$$T_o = T(0) = T_{s,1} + \frac{\dot{q} r_1^2}{4k_c}$$

$$T_o = 133^\circ C + 4.42 \times 10^5 \, W/m^3 \times (0.015\,m)^2 \Big/ (4 \times 200\,W/m \cdot K) = 133.1^\circ C$$ <

(e) With all other conditions remaining the same, the relations of part (d) can be used to calculate T_o, $T_{s,1}$ and $T_{s,2}$ as a function of the sleeve outer radius r_2 for the range $15.5 \le r_2 \le 20$ mm.

Temperature, Ts1 or Ts2 (C)
200
180
160
140
120
100
15 16 17 18 19 20
Sleeve outer radius, r2 (mm)
Inner sleeve, r1
Outer sleeve, r2

On the plot above T_o would show the same behavior as $T_{s,1}$ since the temperature rise between cable center and its surface is 0.12°C. With increasing r_2, we expect $T_{s,2}$ to decrease since the heat flux decreases with increasing r_2. We expect $T_{s,1}$ to increase with increasing r_2 since the thermal resistance of the sleeve increases.

PROBLEM 2.42

KNOWN: Temperature distribution in a spherical shell.

FIND: Whether conditions are steady-state or transient. Manner in which heat flux and heat rate vary with radius.

SCHEMATIC:

Spherical shell, $T(r) = \frac{C_1}{r} + C_2$

ASSUMPTIONS: (1) One-dimensional conduction in r, (2) Constant properties.

ANALYSIS: From Equation 2.23, the heat equation reduces to

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right) = \frac{1}{\alpha}\frac{\partial T}{\partial t}.$$

Substituting for T(r),

$$\frac{1}{\alpha}\frac{\partial T}{\partial t} = -\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{C_1}{r^2}\right) = 0.$$

Hence, steady-state conditions exist. <

From Equation 2.22, the radial component of the heat flux is

$$q''_r = -k\frac{\partial T}{\partial r} = -k\frac{C_1}{r^2}.$$

Hence, q''_r decreases with increasing $r^2 \left(q''_r \alpha 1/r^2\right)$. <

At any radial location, the heat rate is

$$q_r = 4\pi r^2 q''_r = 4\pi k C_1.$$

Hence, q_r is independent of r. <

COMMENTS: The fact that q_r is independent of r is consistent with the energy conservation requirement. If q_r is constant, the flux must vary inversely with the area perpendicular to the direction of heat flow. Hence, q''_r varies inversely with r^2.

PROBLEM 2.43

KNOWN: Spherical container with an exothermic reaction enclosed by an insulating material whose outer surface experiences convection with adjoining air and radiation exchange with large surroundings.

FIND: (a) Verify that the prescribed temperature distribution for the insulation satisfies the appropriate form of the heat diffusion equation; sketch the temperature distribution and label key features; (b) Applying Fourier's law, verify the conduction heat rate expression for the insulation layer, q_r, in terms of $T_{s,1}$ and $T_{s,2}$; apply a surface energy balance to the container and obtain an alternative expression for q_r in terms of $\dot{q}$ and r_1; (c) Apply a surface energy balance around the outer surface of the insulation to obtain an expression to evaluate $T_{s,2}$; (d) Determine $T_{s,2}$ for the specified geometry and operating conditions; (e) Compute and plot the variation of $T_{s,2}$ as a function of the outer radius for the range $201 \le r_2 \le 210$ mm; explore approaches for reducing $T_{s,2} \le$ 45°C to eliminate potential risk for burn injuries to personnel.

SCHEMATIC:

Reaction, T_O, $\dot{q} = \dot{q}_O \exp(-A/T_O)$
Insulation, k
$T_{s,1}$
$T_{s,2}$
T_{sur}
Air
T_∞, h
r_1
r_2

ASSUMPTIONS: (1) One-dimensional, radial spherical conduction, (2) Isothermal reaction in container so that $T_o = T_{s,1}$, (2) Negligible thermal contact resistance between the container and insulation, (3) Constant properties in the insulation, (4) Surroundings large compared to the insulated vessel, and (5) Steady-state conditions.

ANALYSIS: The appropriate form of the heat diffusion equation (HDE) for the insulation follows from Eq. 2.23,

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = 0 \qquad (1) \quad <$$

The temperature distribution is given as

$$T(r) = T_{s,1} - \left(T_{s,1} - T_{s,2}\right)\left[\frac{1-(r_1/r)}{1-(r_1/r_2)}\right] \qquad (2)$$

Substitute T(r) into the HDE to see if it is satisfied:

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\left[0 - \left(T_{s,1} - T_{s,2}\right)\frac{0+\left(r_1/r^2\right)}{1-(r_1/r_2)}\right]\right) ?=? 0$$

$$\frac{1}{r^2}\frac{d}{dr}\left(+\left(T_{s,1} - T_{s,2}\right)\frac{r_1}{1-(r_1/r_2)}\right) ?=? 0 \qquad <$$

and since the expression in parenthesis is independent of r, T(r) does indeed satisfy the HDE. The temperature distribution in the insulation and its key features are as follows:

Continued...

PROBLEM 2.43 (Cont.)

(1) $T_{s,1} > T_{s,2}$

(2) Decreasing gradient with increasing radius, r, since the heat rate is constant through the insulation.

(b) Using Fourier's law for the radial-spherical coordinate, the heat rate through the insulation is

$$q_r = -kA_r \frac{dT}{dr} = -k\left(4\pi r^2\right)\frac{dT}{dr} \qquad <$$

and substituting for the temperature distribution, Eq. (2),

$$q_r = -k\pi r^2 \left[0 - \left(T_{s,1} - T_{s,2}\right) \frac{0 + \left(r_1 / r^2\right)}{1 - \left(r_1 / r_2\right)} \right]$$

$$q_r = \frac{4\pi k\left(T_{s,1} - T_{s,2}\right)}{\left(1/r_1\right) - \left(1/r_2\right)} \qquad (3) \quad <$$

Applying an energy balance to a control surface about the container at $r = r_1$,

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$\dot{q}\forall - q_r = 0$$

where $\dot{q}\forall$ represents the generated heat in the container,

$$q_r = \left(4/3\right)\pi r_1^3 \dot{q} \qquad (4) \quad <$$

(c) Applying an energy balance to a control surface placed around the outer surface of the insulation,

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$q_r - q_{cv} - q_{rad} = 0$$

$$q_r - hA_s\left(T_{s,2} - T_\infty\right) - \varepsilon A_s \sigma\left(T_{s,2}^4 - T_{sur}^4\right) = 0 \qquad (5) \quad <$$

Continued...

where

$$A_s = 4\pi r_2^2 \tag{6}$$

These relations can be used to determine $T_{s,2}$ in terms of the variables $\dot{q}$, r_1, r_2, h, T_∞, ε and T_{sur}.

(d) Consider the reactor system operating under the following conditions:

r_1 = 200 mm	h = 5 W/m^2·K	ε = 0.9
r_2 = 208 mm	T_∞ = 25°C	T_{sur} = 35°C
k = 0.05 W/m·K		

The heat generated by the exothermic reaction provides for a volumetric heat generation rate,

$$\dot{q} = \dot{q}_o \exp(-A/T_o) \qquad q_o = 5000\,W/m^3 \qquad A = 75\,K \tag{7}$$

where the temperature of the reaction is that of the inner surface of the insulation, $T_o = T_{s,1}$. The following system of equations will determine the operating conditions for the reactor.

Conduction rate equation, insulation, Eq. (3),

$$q_r = \frac{4\pi \times 0.05\,W/m\cdot K\left(T_{s,1} - T_{s,2}\right)}{\left(1/0.200\,m - 1/0.208\,m\right)} \tag{8}$$

Heat generated in the reactor, Eqs. (4) and (7),

$$q_r = 4/3\pi\,(0.200\,m)^3\,\dot{q} \tag{9}$$

$$\dot{q} = 5000\,W/m^3 \exp\left(-75\,K/T_{s,1}\right) \tag{10}$$

Surface energy balance, insulation, Eqs. (5) and (6),

$$q_r - 5\,W/m^2\cdot K\,A_s\left(T_{s,2} - 298\,K\right) - 0.9A_s\,5.67\times10^{-8}\,W/m^2{\cdot}K^4\left(T_{s,2}^4 - (308\,K)^4\right) = 0 \tag{11}$$

$$A_s = 4\pi\,(0.208\,m)^2 \tag{12}$$

Solving these equations simultaneously, find that

$$T_{s,1} = 94.3^\circ C \qquad T_{s,2} = 52.5^\circ C$$ <

That is, the reactor will be operating at $T_o = T_{s,1}$ = 94.3°C, very close to the desired 95°C operating condition.

(e) From the above analysis, we found the outer surface temperature $T_{s,2}$ = 52.5°C represents a potential burn risk to plant personnel. Using the above system of equations, Eqs. (8)-(12), we have explored the effects of changes in the convection coefficient, h, and the insulation thermal conductivity, k, as a function of insulation thickness, $t = r_2 - r_1$.

Continued...

PROBLEM 2.43 (Cont.)

Outer surface temperature, Ts2 (C)

Insulation thickness, (r2 - r1) (mm)

k = 0.05 W/m.K, h = 5 W/m^2.K
k = 0.01 W/m.K, h = 5 W/m^2.K
k = 0.05 W/m.K, h = 15 W/m^2.K

Reaction temperature, To (C)

Insulation thickness, (r2-r1) (mm)

k = 0.05 W/m.K, h = 5 W/m^2.K
k = 0.01 W/m.K, h = 5 W/m^2.K
k = 0.05 W/m.K, h = 15 W/m^2.K

In the $T_{s,2}$ vs. $(r_2 - r_1)$ plot, note that decreasing the thermal conductivity from 0.05 to 0.01 W/m·K slightly increases $T_{s,2}$ while increasing the convection coefficient from 5 to 15 $W/m^2 \cdot K$ markedly decreases $T_{s,2}$. Insulation thickness only has a minor effect on $T_{s,2}$ for either option. In the T_o vs. $(r_2 - r_1)$ plot, note that, for all the options, the effect of increased insulation is to increase the reaction temperature. With k = 0.01 W/m·K, the reaction temperature increases beyond 95°C with less than 2 mm insulation. For the case with h = 15 $W/m^2 \cdot K$, the reaction temperature begins to approach 95°C with insulation thickness around 10 mm. We conclude that by selecting the proper insulation thickness and controlling the convection coefficient, the reaction could be operated around 95°C such that the outer surface temperature would not exceed 45°C.

PROBLEM 2.44

KNOWN: One-dimensional system, initially at a uniform temperature T_i, is suddenly exposed to a uniform heat flux at one boundary, while the other boundary is insulated.

FIND: (a) Proper form of heat equation and boundary and initial conditions, (b) Temperature distributions for following conditions: initial condition ($t \le 0$), and several times after heater is energized; will a steady-state condition be reached; (c) Heat flux at x = 0, L/2, L as a function of time; (d) Expression for uniform temperature, T_f, reached after heater has been switched off following an elapsed time, t_e, with the heater on.

SCHEMATIC:

Insulation
System, mass M, c_p
Electrical heater, area A_s

ASSUMPTIONS: (1) One-dimensional conduction, (2) No internal heat generation, (3) Constant properties.

ANALYSIS: (a) The appropriate form of the heat equation follows from Eq. 2.15. Also, the appropriate boundary and initial conditions are:

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

Initial condition: $T(x,0) = T_i$ Uniform temperature

Boundary conditions: $x = 0$ $\quad q''_o = -k\partial T/\partial x)_0$

$x = L$ $\quad \partial T/\partial x)_L = 0$

(b) The temperature distributions are as follows:

Slope at this boundary is always zero.
T(x,t)
Slope at this boundary is always constant: $\frac{dT}{dx}\Big)_0 = -q''_o/k$
$T_i = T(x,0)$

<

No steady-state condition will be reached since $\dot{E}_{in} = \dot{E}_{st}$ and $\dot{E}_{in}$ is constant.

(c)The heat flux as a function of time for positions x = 0, L/2 and L is as follows:

$q''(x,t)$
$q''_x(0,t)$
$q''_x(L/2,t)$
$q''_x(L,t)$

<

(d) If the heater is energized until $t = t_e$ and then switched off, the system will eventually reach a uniform temperature, T_f. Perform an energy balance on the system, Eq. 1.11b, for an interval of time $\Delta t = t_e$,

$$E_{in} = E_{st} \qquad E_{in} = Q_{in} = \int_0^{t_e} q''_o A_s dt = q''_o A_s t_e \qquad E_{st} = Mc(T_f - T_i)$$

It follows that $\quad q''_o A_s t_e = Mc(T_f - T_i) \quad$ or $\quad T_f = T_i + \dfrac{q''_o A_s t_e}{Mc}$.

<

PROBLEM 2.45

KNOWN: Plate of thickness 2L, initially at a uniform temperature of $T_i = 200°C$, is suddenly quenched in a liquid bath of $T_\infty = 20°C$ with a convection coefficient of 100 $W/m^2 \cdot K$.

FIND: (a) On T-x coordinates, sketch the temperature distributions for the initial condition ($t \le 0$), the steady-state condition ($t \to \infty$), and two intermediate times; (b) On $q''_x - t$ coordinates, sketch the variation with time of the heat flux at x = L, (c) Determine the heat flux at x = L and for t = 0; what is the temperature gradient for this condition; (d) By performing an energy balance on the plate, determine the amount of energy per unit surface area of the plate (J/m^2) that is transferred to the bath over the time required to reach steady-state conditions; and (e) Determine the energy transferred to the bath during the quenching process using the exponential-decay relation for the surface heat flux.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional conduction, (2) Constant properties, and (3) No internal heat generation.

ANALYSIS: (a) The temperature distributions are shown in the sketch below.

(b) The heat flux at the surface x = L, $q''_x(L,t)$, is initially a maximum value, and decreases with increasing time as shown in the sketch above.

(c) The heat flux at the surface x = L at time t = 0, $q''_x(L,0)$, is equal to the convection heat flux with the surface temperature as $T(L,0) = T_i$.

$$q''_x(L,0) = q''_{conv}(t=0) = h(T_i - T_\infty) = 100\,W/m^2 \cdot K(200-20)°C = 18.0\,kW/m^2 \qquad <$$

From a surface energy balance as shown in the sketch considering the conduction and convection fluxes at the surface, the temperature gradient can be calculated.

Continued …..

PROBLEM 2.45 (Cont.)

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$q''_x(L,0) - q''_{conv}(t=0) = 0 \qquad \text{with} \qquad q''_x(L,0) = -k\left.\frac{\partial T}{\partial x}\right)_{x=L}$$

$$\left.\frac{\partial T}{\partial x}\right)_{L,0} = -q''_{conv}(t=0)/k = -18\times10^3\ \text{W/m}^2/50\ \text{W/m}\cdot\text{K} = -360\ \text{K/m}$$ <

$T(L,0) = T_i$
$q''_x(L,0)$
$q''_{conv}(t=0)$
T_∞, h

(d) The energy transferred from the plate to the bath over the time required to reach steady-state conditions can be determined from an energy balance on a time interval basis, Eq. 1.11b. For the initial state, the plate has a uniform temperature T_i; for the final state, the plate is at the temperature of the bath, T_∞.

$$E''_{in} - E''_{out} = \Delta E''_{st} = E''_f - E''_i \qquad \text{with} \qquad E''_{in} = 0,$$

$$-E''_{out} = \rho c_p (2L)[T_\infty - T_i]$$

$$E''_{out} = -2770\ \text{kg/m}^3 \times 875\ \text{J/kg}\cdot\text{K}(2\times0.010\ \text{m})[20-200]\text{K} = +8.73\times10^6\ \text{J/m}^2$$ <

(e) The energy transfer from the plate to the bath during the quenching process can be evaluated from knowledge of the surface heat flux as a function of time. The area under the curve in the $q''_x(L,t)$ vs. time plot (see schematic above) represents the energy transferred during the quench process.

$$E''_{out} = 2\int_{t=0}^{\infty} q''_x(L,t)dt = 2\int_{t=0}^{\infty} Ae^{-Bt}dt$$

$$E''_{out} = 2A\left[-\frac{1}{B}e^{-Bt}\right]_0^\infty = 2A\left[-\frac{1}{B}(0-1)\right] = 2A/B$$

$$E''_{out} = 2\times1.80\times10^4\ \text{W/m}^2/4.126\times10^{-3}\text{s}^{-1} = 8.73\times10^6\ \text{J/m}^2$$ <

COMMENTS: (1) Can you identify and explain the important features in the temperature distributions of part (a)?

(2) The maximum heat flux from the plate occurs at the instant the quench process begins and is equal to the convection heat flux. At this instant, the gradient in the plate at the surface is a maximum. If the gradient is too large, excessive thermal stresses could be induced and cracking could occur.

(3) In this thermodynamic analysis, we were able to determine the energy transferred during the quenching process. We cannot determine the rate at which cooling of the plate occurs without solving the heat diffusion equation.

PROBLEM 2.46

KNOWN: Plane wall, initially at a uniform temperature, is suddenly exposed to convective heating.

FIND: (a) Differential equation and initial and boundary conditions which may be used to find the temperature distribution, T(x,t); (b) Sketch T(x,t) for these conditions: initial ($t \le 0$), steady-state, $t \to \infty$, and two intermediate times; (c) Sketch heat fluxes as a function of time for surface locations; (d) Expression for total energy transferred to wall per unit volume (J/m^3).

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional conduction, (2) Constant properties, (3) No internal heat generation.

ANALYSIS: (a) For one-dimensional conduction with constant properties, the heat equation has the form,

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

and the conditions are:

$$\begin{cases} \text{Initial, } t \le 0: & T(x,0) = T_i & \text{uniform} \\ \text{Boundaries: } x = 0 & \partial T/\partial x)_0 = 0 & \text{adiabatic} \\ \qquad x = L & -k\partial T/\partial x)_L = h\left[T(L,t) - T_\infty\right] & \text{convection} \end{cases}$$

(b) The temperature distributions are shown on the sketch.

Note that the gradient at x = 0 is always zero, since this boundary is adiabatic. Note also that the gradient at x = L decreases with time.

(c) The heat flux, $q''_x(x,t)$, as a function of time, is shown on the sketch for the surfaces x = 0 and x = L.

Continued …..

For the surface at $x = 0$, $q''_x(0,t) = 0$ since it is adiabatic. At x = L and t = 0, $q''_x(L,0)$ is a maximum

$$q''_x(L,0) = h\left[T(L,0) - T_\infty\right]$$

where T(L,0) = T_i. The gradient, and hence the flux, decrease with time.

(d) The total energy transferred to the wall may be expressed as

$$E_{in} = \int_0^\infty q''_{conv} A_s dt$$

$$E_{in} = hA_s \int_0^\infty \left(T_\infty - T(L,t)\right) dt$$

Dividing both sides by A_sL, the energy transferred per unit volume is

$$\frac{E_{in}}{V} = \frac{h}{L}\int_0^\infty \left[T_\infty - T(L,t)\right] dt \qquad \left[J/m^3\right]$$

COMMENTS: Note that the heat flux at x = L is into the wall and is hence in the negative x direction.

PROBLEM 2.47

KNOWN: Plane wall, initially at a uniform temperature T_i, is suddenly exposed to convection with a fluid at T_∞ at one surface, while the other surface is exposed to a constant heat flux q_o''.

FIND: (a) Temperature distributions, T(x,t), for initial, steady-state and two intermediate times, (b) Corresponding heat fluxes on $q_x'' - x$ coordinates, (c) Heat flux at locations x = 0 and x = L as a function of time, (d) Expression for the steady-state temperature of the heater, $T(0,\infty)$, in terms of q_o'', T_∞, k, h and L.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional conduction, (2) No heat generation, (3) Constant properties.

ANALYSIS: (a) For $T_i < T_\infty$, the temperature distributions are

Note the constant gradient at x = 0 since $q_x''(0) = q_o''$.

(b) The heat flux distribution, $q_x''(x,t)$, is determined from knowledge of the temperature gradients, evident from Part (a), and Fourier's law.

(c) On $q_x''(x,t) - t$ coordinates, the heat fluxes at the boundaries are shown above.

(d) Perform a surface energy balance at x = L and an energy balance on the wall:

$$q''_{cond} = q''_{conv} = h\left[T(L,\infty) - T_\infty\right] \quad (1), \qquad q''_{cond} = q_o''. \quad (2)$$

For the wall, under steady-state conditions, Fourier's law gives

$$q_o'' = -k\frac{dT}{dx} = k\frac{T(0,\infty) - T(L,\infty)}{L}. \qquad (3)$$

Combine Eqs. (1), (2), (3) to find:

$$T(0,\infty) = T_\infty + \frac{q_o''}{1/h + L/k}.$$

PROBLEM 2.48

KNOWN: Plane wall, initially at a uniform temperature T_o, has one surface (x = L) suddenly exposed to a convection process ($T_\infty > T_o$,h), while the other surface (x = 0) is maintained at T_o. Also, wall experiences uniform volumetric heating $\dot{q}$ such that the maximum steady-state temperature will exceed T_∞.

FIND: (a) Sketch temperature distribution (T vs. X) for following conditions: initial ($t \le 0$), steady-state ($t \to \infty$), and two intermediate times; also show distribution when there is no heat flow at the x = L boundary, (b) Sketch the heat flux $\left(q''_x \text{ vs. } t\right)$ at the boundaries x = 0 and L.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional conduction, (2) Constant properties, (3) Uniform volumetric generation, (4) $T_o < T_\infty$ and $\dot{q}$ large enough that $T(x,\infty) > T_\infty$.

ANALYSIS: (a) The initial and boundary conditions for the wall can be written as

Initial ($t \le 0$):	$T(x,0) = T_o$	Uniform temperature
Boundary:	$x = 0 \quad T(0,t) = T_o$	Constant temperature
	$x = L \quad -k\left.\frac{\partial T}{\partial x}\right)_{x=L} = h\left[T(L,t) - T_\infty\right]$	Convection process.

The temperature distributions are shown on the T-x coordinates below. Note the special condition when the heat flux at (x = L) is zero.

(b) The heat flux as a function of time at the boundaries, $q''_x(0,t)$ and $q''_x(L,t)$, can be inferred from the temperature distributions using Fourier's law.

COMMENTS: Since $T(x,\infty) > T_\infty$ and $T_\infty > T_o$, heat transfer at both boundaries must be out of the wall. Hence, it follows from an overall energy balance on the wall that $+q''_x(0,\infty) - q''_x(L,\infty) + \dot{q}L = 0$.

PROBLEM 2.49

KNOWN: Plane wall, initially at a uniform temperature T_o, has one surface (x = L) suddenly exposed to a convection process ($T_\infty < T_o$, h), while the other surface (x = 0) is maintained at T_o. Also, wall experiences uniform volumetric heating $\dot{q}$ such that the maximum steady-state temperature will exceed T_∞.

FIND: (a) Sketch temperature distribution (T vs. x) for following conditions: initial ($t \le 0$), steady-state ($t \to \infty$), and two intermediate times; identify key features of the distributions, (b) Sketch the heat flux $\left(q''_x \text{ vs. } t\right)$ at the boundaries x = 0 and L; identify key features of the distributions.

SCHEMATIC:

$\dot{q}(t \ge 0)$ T(L,t) $T_\infty < T_o$, h $T(0,t) = T_o$ x L = 0.1m

ASSUMPTIONS: (1) One-dimensional conduction, (2) Constant properties, (3) Uniform volumetric generation, (4) $T_\infty < T_o$ and $\dot{q}$ large enough that $T(x,\infty) > T_o$.

ANALYSIS: (a) The initial and boundary conditions for the wall can be written as

Initial ($t \le 0$):	$T(x,0) = T_o$	Uniform temperature
Boundary:	$x = 0 \quad T(0,t) = T_o$	Constant temperature
	$x = L \quad -k\left.\dfrac{\partial T}{\partial x}\right)_{x=L} = h\left[T(L,t) - T_\infty\right]$	Convection process.

The temperature distributions are shown on the T-x coordinates below. Note that the maximum temperature occurs under steady-state conditions not at the midplane, but to the right toward the surface experiencing convection. The temperature gradients at x = L increase for t > 0 since the convection heat rate from the surface increases as the surface temperature increases.

(b) The heat flux as a function of time at the boundaries, $q''_x(0,t)$ and $q''_x(L,t)$, can be inferred from the temperature distributions using Fourier's law. At the surface x = L, the convection heat flux at t = 0 is $q''_x(L,0) = h\left(T_o - T_\infty\right)$. Because the surface temperature dips slightly at early times, the convection heat flux decreases slightly, and then increases until the steady-state condition is reached. For the steady-state condition, heat transfer at both boundaries must be out of the wall. It follows from an overall energy balance on the wall that $+q''_x(0,\infty) - q''_x(L,\infty) + \dot{q}L = 0$.

T(x,t) Steady-state $T(x,\infty)$ t T_o T_∞ Initial, T(x,0) 0 L x

+ $q''_x(L,0) = h[T(L,0) - T_\infty]$ $q''_x(L,t)$ $q''_x(x,t)$ $\dot{q}L$ 0 t - $q''_x(0,t)$

PROBLEM 2.50

KNOWN: Interfacial heat flux and outer surface temperature of adjoining, equivalent plane walls.

FIND: (a) Form of temperature distribution at representative times during the heating process, (b) Variation of heat flux with time at the interface and outer surface.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional conduction, (2) Constant properties.

ANALYSIS: (a) With symmetry about the interface, consideration of the temperature distribution may be restricted to $0 \le x \le L$. During early stages of the process, heat transfer is *into* the material from the outer surface, as well as from the interface. During later stages and the eventual steady state, heat is transferred *from* the material at the outer surface. At steady-state, dT/dx = $-\left(q_o''/2\right)/k = \text{const}$. and T(0,t) = T_o + $\left(q_o''/2\right)L/k$.

(b) At the outer surface, the heat flux is initially negative, but increases with time, approaching q_o''/2. It is zero when $dT/dx|_{x=L} = 0$.

PROBLEM 2.51

KNOWN: Temperature distribution in a plane wall of thickness L experiencing uniform volumetric heating $\dot{q}$ having one surface (x = 0) insulated and the other exposed to a convection process characterized by T_∞ and h. Suddenly the volumetric heat generation is deactivated while convection continues to occur.

FIND: (a) Determine the magnitude of the volumetric energy generation rate associated with the initial condition, (b) On T-x coordinates, sketch the temperature distributions for the initial condition ($T \le 0$), the steady-state condition ($t \to \infty$), and two intermediate times; (c) On q''_x - t coordinates, sketch the variation with time of the heat flux at the boundary exposed to the convection process, $q''_x(L,t)$; calculate the corresponding value of the heat flux at t = 0; and (d) Determine the amount of energy removed from the wall per unit area (J/m^2) by the fluid stream as the wall cools from its initial to steady-state condition.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional conduction, (2) Constant properties, and (3) Uniform internal volumetric heat generation for t < 0.

ANALYSIS: (a) The volumetric heating rate can be determined by substituting the temperature distribution for the initial condition into the appropriate form of the heat diffusion equation.

$$\frac{d}{dx}\left(\frac{dT}{dx}\right)+\frac{\dot{q}}{k}=0 \qquad \text{where} \qquad T(x,0)=a+bx^2$$

$$\frac{d}{dx}(0+2bx)+\frac{\dot{q}}{k}=0+2b+\frac{\dot{q}}{k}=0$$

$$\dot{q}=-2kb=-2\times 90\,W/m\cdot K\left(-1.0\times 10^{4}\,{}^\circ C/m^2\right)=1.8\times 10^{6}\,W/m^3$$ <

(b) The temperature distributions are shown in the sketch below.

Continued …..

(c) The heat flux at the exposed surface x = L, $q''_x(L,0)$, is initially a maximum value and decreases with increasing time as shown in the sketch above. The heat flux at t = 0 is equal to the convection heat flux with the surface temperature T(L,0). See the surface energy balance represented in the schematic.

$$q''_x(L,0) = q''_{conv}(t=0) = h\left(T(L,0) - T_\infty\right) = 1000\,\mathrm{W/m^2 \cdot K}\,(200-20)\,°\mathrm{C} = 1.80\times10^5\,\mathrm{W/m^2}$$ <

where $T(L,0) = a + bL^2 = 300°\mathrm{C} - 1.0\times10^4 °\mathrm{C/m^2}\,(0.1\mathrm{m})^2 = 200°\mathrm{C}.$

T(L,0) = a + bx²
q"conv(t=0)
q"x(L,0)
T∞,h

(d) The energy removed from the wall to the fluid as it cools from its initial to steady-state condition can be determined from an energy balance on a time interval basis, Eq. 1.11b. For the initial state, the wall has the temperature distribution $T(x,0) = a + bx^2$; for the final state, the wall is at the temperature of the fluid, $T_f = T_\infty$. We have used T_∞ as the reference condition for the energy terms.

$$E''_{in} - E''_{out} = \Delta E''_{st} = E''_f - E''_i \qquad \text{with} \qquad E''_{in} = 0$$

$$-E''_{out} = \rho c_p L\left[T_f - T_\infty\right] - \rho c_p \int_{x=0}^{x=L}\left[T(x,0) - T_\infty\right]dx$$

$$E''_{out} = \rho c_p \int_{x=0}^{x=L}\left[a + bx^2 - T_\infty\right]dx = \rho c_p\left[ax + bx^3/3 - T_\infty x\right]_0^L$$

$$E''_{out} = 7000\,\mathrm{kg/m^3} \times 450\,\mathrm{J/kg\cdot K}\left[300\times0.1 - 1.0\times10^4\,(0.1)^3/3 - 20\times0.1\right]\mathrm{K\cdot m}$$

$$E''_{out} = 7.77\times10^7\,\mathrm{J/m^2}$$ <

COMMENTS: (1) In the temperature distributions of part (a), note these features: initial condition has quadratic form with zero gradient at the adiabatic boundary; for the steady-state condition, the wall has reached the temperature of the fluid; for all distributions, the gradient at the adiabatic boundary is zero; and, the gradient at the exposed boundary decreases with increasing time.

(2) In this thermodynamic analysis, we were able to determine the energy transferred during the cooling process. However, we cannot determine the rate at which cooling of the wall occurs without solving the heat diffusion equation.

PROBLEM 2.52

KNOWN: Temperature as a function of position and time in a plane wall suddenly subjected to a change in surface temperature, while the other surface is insulated.

FIND: (a) Validate the temperature distribution, (b) Heat fluxes at x = 0 and x = L, (c) Sketch of temperature distribution at selected times and surface heat flux variation with time, (d) Effect of thermal diffusivity on system response.

SCHEMATIC:

$\alpha, T(x,0) = T_i$

$T(L,t) = T_s$

$$\frac{T(x,t)-T_s}{T_i-T_s} = C_1 \exp\left(-\frac{\pi^2}{4}\frac{\alpha t}{L^2}\right)\cos\left(\frac{\pi}{2}\frac{x}{L}\right)$$

x L

ASSUMPTIONS: (1) One-dimensional conduction in x, (2) Constant properties.

ANALYSIS: (a) To be valid, the temperature distribution must satisfy the appropriate forms of the heat equation and boundary conditions. Substituting the distribution into Equation 2.15, it follows that

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$$-C_1(T_i - T_s)\exp\left(-\frac{\pi^2}{4}\frac{\alpha t}{L^2}\right)\left(\frac{\pi}{2L}\right)^2 \cos\left(\frac{\pi}{2}\frac{x}{L}\right)$$

$$= -\frac{C_1}{\alpha}(T_i - T_s)\left(\frac{\pi^2}{4}\frac{\alpha}{L^2}\right)\exp\left(-\frac{\pi^2}{4}\frac{\alpha t}{L^2}\right)\cos\left(\frac{\pi}{2}\frac{x}{L}\right).$$ <

Hence, the heat equation is satisfied. Applying boundary conditions at x = 0 and x = L, it follows that

$$\frac{\partial T}{\partial x}\Big|_{x=0} = -\frac{C_1\pi}{2L}(T_i - T_s)\exp\left(-\frac{\pi^2}{4}\frac{\alpha t}{L^2}\right)\sin\left(\frac{\pi}{2}\frac{x}{L}\right)\Big|_{x=0} = 0$$ <

and

$$T(L,t) = T_s + C_1(T_i - T_s)\exp\left(-\frac{\pi^2}{4}\frac{\alpha t}{L^2}\right)\cos\left(\frac{\pi}{2}\frac{x}{L}\right)\Big|_{x=L} = T_s.$$ <

Hence, the boundary conditions are also satisfied.

(b) The heat flux has the form

$$q''_x = -k\frac{\partial T}{\partial x} = +\frac{kC_1\pi}{2L}(T_i - T_s)\exp\left(-\frac{\pi^2}{4}\frac{\alpha t}{L^2}\right)\sin\left(\frac{\pi}{2}\frac{x}{L}\right).$$

Continued …..

Hence, $q''_x(0) = 0,$ <

$$q''_x(L) = +\frac{kC_1\pi}{2L}(T_i - T_s)\exp\left(-\frac{\pi^2}{4}\frac{\alpha t}{L^2}\right).$$ <

(c) The temperature distribution and surface heat flux variations are:

(d) For materials A and B of different α,

$$\frac{[T(x,t) - T_s]_A}{[T(x,t) - T_s]_B} = \exp\left[-\frac{\pi^2}{4L^2}(\alpha_A - \alpha_B)t\right]$$

Hence, if $\alpha_A > \alpha_B$, $T(x,t) \to T_s$ more rapidly for Material A. If $\alpha_A < \alpha_B$, $T(x,t) \to T_s$ more rapidly for Material B. <

COMMENTS: Note that the prescribed function for T(x,t) does not reduce to T_i for t $\to$ 0. For times at or close to zero, the function is not a valid solution of the problem. At such times, the solution for T(x,t) must include additional terms. The solution is consideed in Section 5.5.1 of the text.

PROBLEM 2.53

KNOWN: Thin electrical heater dissipating 4000 W/m^2 sandwiched between two 25-mm thick plates whose surfaces experience convection.

FIND: (a) On T-x coordinates, sketch the steady-state temperature distribution for $-L \le x \le +L$; calculate values for the surfaces x = L and the mid-point, x = 0; label this distribution as Case 1 and explain key features; (b) Case 2: sudden loss of coolant causing existence of adiabatic condition on the x = +L surface; sketch temperature distribution on same T-x coordinates as part (a) and calculate values for x = 0, ± L; explain key features; (c) Case 3: further loss of coolant and existence of adiabatic condition on the x = - L surface; situation goes undetected for 15 minutes at which time power to the heater is deactivated; determine the eventual ($t \rightarrow \infty$) uniform, steady-state temperature distribution; sketch temperature distribution on same T-x coordinates as parts (a,b); and (d) On T-t coordinates, sketch the temperature-time history at the plate locations x = 0, ± L during the transient period between the steady-state distributions for Case 2 and Case 3; at what location and when will the temperature in the system achieve a maximum value?

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional conduction, (2) Constant properties, (3) No internal volumetric generation in plates, and (3) Negligible thermal resistance between the heater surfaces and the plates.

ANALYSIS: (a) Since the system is symmetrical, the heater power results in equal conduction fluxes through the plates. By applying a surface energy balance on the surface x = +L as shown in the schematic, determine the temperatures at the mid-point, x = 0, and the exposed surface, x + L.

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$q''_x(+L) - q''_{conv} = 0 \qquad \text{where} \qquad q''_x(+L) = q''_o/2$$

$$q''_o/2 - h\left[T(+L) - T_\infty\right] = 0$$

$$T_1(+L) = q''_o/2h + T_\infty = 4000\,W/m^2 / \left(2 \times 400\,W/m^2 \cdot K\right) + 20°C = 25°C$$ <

From Fourier's law for the conduction flux through the plate, find T(0).

$$q''_x = q''_o/2 = k\left[T(0) - T(+L)\right]/L$$

$$T_1(0) = T_1(+L) + q''_o L/2k = 25°C + 4000\,W/m^2 \cdot K \times 0.025m / \left(2 \times 5\,W/m \cdot K\right) = 35°C$$ <

The temperature distribution is shown on the T-x coordinates below and labeled Case 1. The key features of the distribution are its symmetry about the heater plane and its linear dependence with distance.

Continued …..

PROBLEM 2.53 (Cont.)

(b) Case 2: sudden loss of coolant with the existence of an adiabatic condition on surface x = +L. For this situation, all the heater power will be conducted to the coolant through the left-hand plate. From a surface energy balance and application of Fourier's law as done for part (a), find

$$T_2(-L) = q_o''/h + T_\infty = 4000\,\text{W/m}^2/400\,\text{W/m}^2\cdot\text{K} + 20°\text{C} = 30°\text{C}$$ <

$$T_2(0) = T_2(-L) + q_o''L/k = 30°\text{C} + 4000\,\text{W/m}^2 \times 0.025\ \text{m}/5\,\text{W/m}\cdot\text{K} = 50°\text{C}$$ <

The temperature distribution is shown on the T-x coordinates above and labeled Case 2. The distribution is linear in the left-hand plate, with the maximum value at the mid-point. Since no heat flows through the right-hand plate, the gradient must zero and this plate is at the maximum temperature as well. The maximum temperature is higher than for Case 1 because the heat flux through the left-hand plate has increased two-fold.

(c) Case 3: sudden loss of coolant occurs at the x = -L surface also. For this situation, there is no heat transfer out of either plate, so that for a 15-minute period, Δt_o, the heater dissipates 4000 W/m^2 and then is deactivated. To determine the eventual, uniform steady-state temperature distribution, apply the conservation of energy requirement on a time-interval basis, Eq. 1.11b. The initial condition corresponds to the temperature distribution of Case 2, and the final condition will be a uniform, elevated temperature $T_f = T_3$ representing Case 3. We have used T_∞ as the reference condition for the energy terms.

$$E_{in}'' - E_{out}'' + E_{gen}'' = \Delta E_{st}'' = E_f'' - E_i'' \quad (1)$$

Note that $E_{in}'' - E_{out}'' = 0$, and the dissipated electrical energy is

$$E_{gen}'' = q_o''\Delta t_o = 4000\,\text{W/m}^2\,(15\times 60)\,\text{s} = 3.600\times 10^6\ \text{J/m}^2 \quad (2)$$

For the final condition,

$$E_f'' = \rho c(2L)[T_f - T_\infty] = 2500\,\text{kg/m}^3 \times 700\,\text{J/kg}\cdot\text{K}\,(2\times 0.025\text{m})[T_f - 20]°\text{C}$$
$$E_f'' = 8.75\times 10^4\,[T_f - 20]\,\text{J/m}^2 \quad (3)$$

where $T_f = T_3$, the final uniform temperature, Case 3. For the initial condition,

$$E_i'' = \rho c\int_{-L}^{+L}[T_2(x) - T_\infty]dx = \rho c\left\{\int_{-L}^{0}[T_2(x) - T_\infty]dx + \int_{0}^{+L}[T_2(0) - T_\infty]dx\right\} \quad (4)$$

where $T_2(x)$ is linear for $-L \le x \le 0$ and constant at $T_2(0)$ for $0 \le x \le +L$.

$$T_2(x) = T_2(0) + [T_2(0) - T_2(L)]x/L \qquad -L \le x \le 0$$

$$T_2(x) = 50°\text{C} + [50 - 30]°\text{C}x/0.025\text{m}$$

$$T_2(x) = 50°\text{C} + 800x \quad (5)$$

Substituting for $T_2(x)$, Eq. (5), into Eq. (4)

Continued …..

PROBLEM 2.53 (Cont.)

$$E_i'' = \rho c \left\{ \int_{-L}^{0} \left[50 + 800x - T_\infty \right] dx + \left[T_2(0) - T_\infty \right] L \right\}$$

$$E_i'' = \rho c \left\{ \left[50x + 400x^2 - T_\infty x \right]_{-L}^{0} + \left[T_2(0) - T_\infty \right] L \right\}$$

$$E_i'' = \rho c \left\{ -\left[-50L + 400L^2 + T_\infty L \right] + \left[T_2(0) - T_\infty \right] L \right\}$$

$$E_i'' = \rho c L \left\{ +50 - 400L - T_\infty + T_2(0) - T_\infty \right\}$$

$$E_i'' = 2500\,\text{kg/m}^3 \times 700\,\text{J/kg}\cdot\text{K} \times 0.025\ \text{m} \left\{ +50 - 400 \times 0.025 - 20 + 50 - 20 \right\} \text{K}$$

$$E_i'' = 2.188 \times 10^6\ \text{J/m}^2 \qquad (6)$$

Returning to the energy balance, Eq. (1), and substituting Eqs. (2), (3) and (6), find $T_f = T_3$.

$$3.600 \times 10^6\ \text{J/m}^2 = 8.75 \times 10^4 \left[T_3 - 20 \right] - 2.188 \times 10^6\ \text{J/m}^2$$

$$T_3 = (66.1 + 20)\,°\text{C} = 86.1\,°\text{C}$$ <

The temperature distribution is shown on the T-x coordinates above and labeled Case 3. The distribution is uniform, and considerably higher than the maximum value for Case 2.

(d) The temperature-time history at the plate locations x = 0, ± L during the transient period between the distributions for Case 2 and Case 3 are shown on the T-t coordinates below.

Note the temperatures for the locations at time t = 0 corresponding to the instant when the surface x = - L becomes adiabatic. These temperatures correspond to the distribution for Case 2. The heater remains energized for yet another 15 minutes and then is deactivated. The midpoint temperature, T(0,t), is always the hottest location and the maximum value slightly exceeds the final temperature T_3.

PROBLEM 2.54

KNOWN: Radius and length of coiled wire in hair dryer. Electric power dissipation in the wire, and temperature and convection coefficient associated with air flow over the wire.

FIND: (a) Form of heat equation and conditions governing transient, thermal behavior of wire during start-up, (b) Volumetric rate of thermal energy generation in the wire, (c) Sketch of temperature distribution at selected times during start-up, (d) Variation with time of heat flux at r = 0 and r = r_o.

SCHEMATIC:

L = 0.5 m
$\dot{E}_{gen}$
$q''(r_o)$
r_o = 1 mm

ASSUMPTIONS: (1) One-dimensional, radial conduction, (2) Constant properties, (3) Uniform volumetric heating, (4) Negligible radiation from surface of wire.

ANALYSIS: (a) The general form of the heat equation for cylindrical coordinates is given by Eq. 2.20. For one-dimensional, radial conduction and constant properties, the equation reduces to

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\dot{q}=\frac{\rho c_p}{k}\frac{\partial T}{\partial t}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$$ <

The initial condition is $\quad T(r,0)=T_i$ <

The boundary conditions are: $\quad \partial T/\partial r|_{r=0}=0$ <

$$-k\left.\frac{\partial T}{\partial r}\right|_{r=r_o}=h\left[T(r_o,t)-T_\infty\right]$$ <

(b) The volumetric rate of thermal energy generation is

$$\dot{q}=\frac{\dot{E}_g}{\forall}=\frac{P_{elec}}{\pi r_o^2 L}=\frac{500\ \text{W}}{\pi(0.001\text{m})^2(0.5\text{m})}=3.18\times10^8\ \text{W/m}^3$$ <

Under steady-state conditions, all of the thermal energy generated within the wire is transferred to the air by convection. Performing an energy balance for a control surface about the wire, $-\dot{E}_{out}+\dot{E}_g=0$, it follows that $-2\pi r_o L\ q''(r_o, t\to\infty)+P_{elec}=0$. Hence,

$$q''(r_o,t\to\infty)=\frac{P_{elec}}{2\pi r_o L}=\frac{500\ \text{W}}{2\pi(0.001\text{m})0.5\text{m}}=1.59\times10^5\ \text{W/m}^2$$ <

T(r,t)
Steady-state, T(r,∞)
t
T_i
Initial, T(r,0)
0 r_o r

q''_r
$q''(r_o,\infty)$
$q''(r_o,t)$
$q''(0,t)$
0 t

COMMENTS: The symmetry condition at r = 0 imposes the requirement that $\partial T/\partial r|_{r=0}=0$, and hence $q''(0,t)=0$ throughout the process. The temperature at r_o, and hence the convection heat flux, increases steadily during the start-up, and since conduction to the surface must be balanced by convection from the surface at all times, $\left|\partial T/\partial r\right|_{r=r_o}$ also increases during the start-up.

PROBLEM 3.1

KNOWN: One-dimensional, plane wall separating hot and cold fluids at $T_{\infty,1}$ and $T_{\infty,2}$, respectively.

FIND: Temperature distribution, T(x), and heat flux, q''_x, in terms of $T_{\infty,1}$, $T_{\infty,2}$, h_1, h_2, k and L.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional conduction, (2) Steady-state conditions, (3) Constant properties, (4) Negligible radiation, (5) No generation.

ANALYSIS: For the foregoing conditions, the general solution to the heat diffusion equation is of the form, Equation 3.2,

$$T(x) = C_1 x + C_2. \qquad (1)$$

The constants of integration, C_1 and C_2, are determined by using surface energy balance conditions at x = 0 and x = L, Equation 2.23, and as illustrated above,

$$-k\frac{dT}{dt}\bigg]_{x=0} = h_1\left[T_{\infty,1} - T(0)\right] \qquad -k\frac{dT}{dx}\bigg]_{x=L} = h_2\left[T(L) - T_{\infty,2}\right]. \qquad (2,3)$$

For the BC at x = 0, Equation (2), use Equation (1) to find

$$-k(C_1 + 0) = h_1\left[T_{\infty,1} - (C_1 \cdot 0 + C_2)\right] \qquad (4)$$

and for the BC at x = L to find

$$-k(C_1 + 0) = h_2\left[(C_1 L + C_2) - T_{\infty,2}\right]. \qquad (5)$$

Multiply Eq. (4) by h_2 and Eq. (5) by h_1, and add the equations to obtain C_1. Then substitute C_1 into Eq. (4) to obtain C_2. The results are

$$C_1 = -\frac{\left(T_{\infty,1} - T_{\infty,2}\right)}{k\left[\frac{1}{h_1} + \frac{1}{h_2} + \frac{L}{k}\right]} \qquad C_2 = -\frac{\left(T_{\infty,1} - T_{\infty,2}\right)}{h_1\left[\frac{1}{h_1} + \frac{1}{h_2} + \frac{L}{k}\right]} + T_{\infty,1}$$

$$T(x) = -\frac{\left(T_{\infty,1} - T_{\infty,2}\right)}{\left[\frac{1}{h_1} + \frac{1}{h_2} + \frac{L}{k}\right]}\left[\frac{x}{k} + \frac{1}{h_1}\right] + T_{\infty,1}. \qquad <$$

From Fourier's law, the heat flux is a constant and of the form

$$q''_x = -k\frac{dT}{dx} = -k\,C_1 = +\frac{\left(T_{\infty,1} - T_{\infty,2}\right)}{\left[\frac{1}{h_1} + \frac{1}{h_2} + \frac{L}{k}\right]}. \qquad <$$

PROBLEM 3.2

KNOWN: Temperatures and convection coefficients associated with air at the inner and outer surfaces of a rear window.

FIND: (a) Inner and outer window surface temperatures, $T_{s,i}$ and $T_{s,o}$, and (b) $T_{s,i}$ and $T_{s,o}$ as a function of the outside air temperature $T_{\infty,o}$ and for selected values of outer convection coefficient, h_o.

SCHEMATIC:

Glass
$T_{\infty,o} = -10\ ^oC$
$h_o = 65\ W/m^2 \cdot K$
$T_{s,o}$
q''
$T_{s,i}$
$T_{\infty,i} = 40\ ^oC$
$h_i = 30\ W/m^2 \cdot K$
$L = 0.004m$

$T_{\infty,o}$ $T_{s,o}$ $T_{s,i}$ $T_{\infty,i}$
$1/h_o$ L/k $1/h_i$ q''

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Negligible radiation effects, (4) Constant properties.

PROPERTIES: *Table A-3*, Glass (300 K): k = 1.4 W/m·K.

ANALYSIS: (a) The heat flux may be obtained from Eqs. 3.11 and 3.12,

$$q'' = \frac{T_{\infty,i} - T_{\infty,o}}{\frac{1}{h_o} + \frac{L}{k} + \frac{1}{h_i}} = \frac{40^\circ C - \left(-10^\circ C\right)}{\frac{1}{65\, W/m^2 \cdot K} + \frac{0.004\, m}{1.4\, W/m \cdot K} + \frac{1}{30\, W/m^2 \cdot K}}$$

$$q'' = \frac{50^\circ C}{\left(0.0154 + 0.0029 + 0.0333\right) m^2 \cdot K/W} = 968\, W/m^2 .$$

Hence, with $q'' = h_i \left(T_{\infty,i} - T_{\infty,o}\right)$, the inner surface temperature is

$$T_{s,i} = T_{\infty,i} - \frac{q''}{h_i} = 40^\circ C - \frac{968\, W/m^2}{30\, W/m^2 \cdot K} = 7.7^\circ C$$ <

Similarly for the outer surface temperature with $q'' = h_o \left(T_{s,o} - T_{\infty,o}\right)$ find

$$T_{s,o} = T_{\infty,o} - \frac{q''}{h_o} = -10^\circ C - \frac{968\, W/m^2}{65\, W/m^2 \cdot K} = 4.9^\circ C$$ <

(b) Using the same analysis, $T_{s,i}$ and $T_{s,o}$ have been computed and plotted as a function of the outside air temperature, $T_{\infty,o}$, for outer convection coefficients of h_o = 2, 65, and 100 $W/m^2 \cdot K$. As expected, $T_{s,i}$ and $T_{s,o}$ are linear with changes in the outside air temperature. The difference between $T_{s,i}$ and $T_{s,o}$ increases with increasing convection coefficient, since the heat flux through the window likewise increases. This difference is larger at lower outside air temperatures for the same reason. Note that with h_o = 2 $W/m^2 \cdot K$, $T_{s,i}$ - $T_{s,o}$, is too small to show on the plot.

Continued …..

COMMENTS: (1) The largest resistance is that associated with convection at the inner surface. The values of $T_{s,i}$ and $T_{s,o}$ could be increased by increasing the value of h_i.

(2) The *IHT Thermal Resistance Network Model* was used to create a model of the window and generate the above plot. The Workspace is shown below.

```
// Thermal Resistance Network Model:
// The Network:

// Heat rates into node j,qij, through thermal resistance Rij
q21 = (T2 - T1) / R21
q32 = (T3 - T2) / R32
q43 = (T4 - T3) / R43

// Nodal energy balances
q1 + q21 = 0
q2 - q21 + q32 = 0
q3 - q32 + q43 = 0
q4 - q43 = 0

/* Assigned variables list: deselect the qi, Rij and Ti which are unknowns; set qi = 0 for embedded nodal points
at which there is no external source of heat. */
T1 =  Tinfo          // Outside air temperature, C
//q1 =               // Heat rate, W
T2 =  Tso            // Outer surface temperature, C
q2 =  0              // Heat rate, W; node 2, no external heat source
T3 =  Tsi            // Inner surface temperature, C
q3 =  0              // Heat rate, W; node 2, no external heat source
T4 = Tinfi           // Inside air temperature, C
//q4 =               // Heat rate, W

// Thermal Resistances:
R21 =  1 / ( ho * As )      // Convection thermal resistance, K/W; outer surface
R32 =  L / ( k * As )       // Conduction thermal resistance, K/W; glass
R43 =  1 / ( hi * As )      // Convection thermal resistance, K/W; inner surface

// Other Assigned Variables:
Tinfo = -10          // Outside air temperature, C
ho = 65              // Convection coefficient, W/m^2.K; outer surface
L = 0.004            // Thickness, m; glass
k = 1.4              // Thermal conductivity, W/m.K; glass
Tinfi = 40           // Inside air temperature, C
hi = 30              // Convection coefficient, W/m^2.K; inner surface
As = 1               // Cross-sectional area, m^2; unit area
```

PROBLEM 3.3

KNOWN: Desired inner surface temperature of rear window with prescribed inside and outside air conditions.

FIND: (a) Heater power per unit area required to maintain the desired temperature, and (b) Compute and plot the electrical power requirement as a function of $T_{\infty,o}$ for the range -30 ≤ $T_{\infty,o}$ ≤ 0°C with h_o of 2, 20, 65 and 100 $W/m^2 \cdot K$. Comment on heater operation needs for low h_o. If h ~ V^n, where V is the vehicle speed and n is a positive exponent, how does the vehicle speed affect the need for heater operation?

SCHEMATIC:

$T_{s,i}$ = 15 °C
Film-type heater, q''_h
L = 0.004 m
Window glass
Inside air
Ambient air
$T_{\infty,i}$ = 25 °C
h_i = 65 $W/m^2 \cdot K$
$T_{\infty,o}$ = -10 °C, h_o = 65 $W/m^2 \cdot K$

$T_{\infty,i}$ — $1/h_i A$ — $T_{s,i}$ — L/kA — $1/h_o A$ — $T_{\infty,o}$ → q; q''_h

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat transfer, (3) Uniform heater flux, q''_h, (4) Constant properties, (5) Negligible radiation effects, (6) Negligible film resistance.

PROPERTIES: *Table A-3*, Glass (300 K): k = 1.4 W/m·K.

ANALYSIS: (a) From an energy balance at the inner surface and the thermal circuit, it follows that for a unit surface area,

$$\frac{T_{\infty,i} - T_{s,i}}{1/h_i} + q''_h = \frac{T_{s,i} - T_{\infty,o}}{L/k + 1/h_o}$$

$$q''_h = \frac{T_{s,i} - T_{\infty,o}}{L/k + 1/h_o} - \frac{T_{\infty,i} - T_{s,i}}{1/h_i} = \frac{15^\circ C - \left(-10^\circ C\right)}{\frac{0.004\,m}{1.4\,W/m \cdot K} + \frac{1}{65\,W/m^2 \cdot K}} - \frac{25^\circ C - 15^\circ C}{\frac{1}{10\,W/m^2 \cdot K}}$$

$$q''_h = (1370 - 100)\,W/m^2 = 1270\,W/m^2$$ <

(b) The heater electrical power requirement as a function of the exterior air temperature for different exterior convection coefficients is shown in the plot. When h_o = 2 $W/m^2 \cdot K$, the heater is unecessary, since the glass is maintained at 15°C by the interior air. If h ~ V^n, we conclude that, with higher vehicle speeds, the exterior convection will increase, requiring increased heat power to maintain the 15°C condition.

Heater power (W/m^2)
3500
3000
2500
2000
1500
1000
500
0
-30
-20
-10
0
Exterior air temperature, Tinfo (C)
h = 20 W/m^2.K
h = 65 W/m^2.K
h = 100 W/m^2.K

COMMENTS: With q''_h = 0, the inner surface temperature with $T_{\infty,o}$ = -10°C would be given by

$$\frac{T_{\infty,i} - T_{s,i}}{T_{\infty,i} - T_{\infty,o}} = \frac{1/h_i}{1/h_i + L/k + 1/h_o} = \frac{0.10}{0.118} = 0.846, \qquad \text{or} \qquad T_{s,i} = 25^\circ C - 0.846\left(35^\circ C\right) = -4.6^\circ C.$$

PROBLEM 3.4

KNOWN: Curing of a transparent film by radiant heating with substrate and film surface subjected to known thermal conditions.

FIND: (a) Thermal circuit for this situation, (b) Radiant heat flux, q''_o (W/m^2), to maintain bond at curing temperature, T_o, (c) Compute and plot q''_o as a function of the film thickness for $0 \le L_f \le 1$ mm, and (d) If the film is not transparent, determine q''_o required to achieve bonding; plot results as a function of L_f.

SCHEMATIC:

T_∞ = 20 °C
h = 50 W/m^2 • K
q''_o
k_f = 0.025 W/m•K
L_f = 0.25 mm
Film
Bond, T_o = 60 °C
L_s = 1.0 mm
Substrate
k_s = 0.05 W/m•K
T_1 = 30 °C

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat flow, (3) All the radiant heat flux q''_o is absorbed at the bond, (4) Negligible contact resistance.

ANALYSIS: (a) The thermal circuit for this situation is shown at the right. Note that terms are written on a per unit area basis.

q''_2 ← T_∞ — R''_{cv} — T_s — R''_f — T_o (q''_o) — R''_s — T_1 → q''_1

(b) Using this circuit and performing an energy balance on the film-substrate interface,

$$q''_o = q''_1 + q''_2 \qquad\qquad q''_o = \frac{T_o - T_\infty}{R''_{cv} + R''_f} + \frac{T_o - T_1}{R''_s}$$

where the thermal resistances are

$$R''_{cv} = 1/h = 1\big/50\,\mathrm{W/m^2\cdot K} = 0.020\,\mathrm{m^2\cdot K/W}$$

$$R''_f = L_f/k_f = 0.00025\,\mathrm{m}/0.025\,\mathrm{W/m\cdot K} = 0.010\,\mathrm{m^2\cdot K/W}$$

$$R''_s = L_s/k_s = 0.001\,\mathrm{m}/0.05\,\mathrm{W/m\cdot K} = 0.020\,\mathrm{m^2\cdot K/W}$$

$$q''_o = \frac{(60-20)^\circ\mathrm{C}}{[0.020+0.010]\,\mathrm{m^2\cdot K/W}} + \frac{(60-30)^\circ\mathrm{C}}{0.020\,\mathrm{m^2\cdot K/W}} = (133+1500)\,\mathrm{W/m^2} = 2833\,\mathrm{W/m^2} \qquad <$$

(c) For the transparent film, the radiant flux required to achieve bonding as a function of film thickness L_f is shown in the plot below.

(d) If the film is opaque (not transparent), the thermal circuit is shown below. In order to find q''_o, it is necessary to write two energy balances, one around the T_s node and the second about the T_o node.

q_2'' ← T_∞ — R''_{cv} — T_s (q_o'') — R''_f — T_o — R''_s — T_1 → q_1''

.

The results of the analyses are plotted below.

Continued...

COMMENTS: (1) When the film is transparent, the radiant flux is absorbed on the bond. The flux required decreases with increasing film thickness. Physically, how do you explain this? Why is the relationship not linear?

(2) When the film is opaque, the radiant flux is absorbed on the surface, and the flux required increases with increasing thickness of the film. Physically, how do you explain this? Why is the relationship linear?

(3) The IHT Thermal Resistance Network Model was used to create a model of the film-substrate system and generate the above plot. The Workspace is shown below.

// Thermal Resistance Network Model:
// The Network:


```
// Heat rates into node j,qij, through thermal resistance Rij
q21 = (T2 - T1) / R21
q32 = (T3 - T2) / R32
q43 = (T4 - T3) / R43

// Nodal energy balances
q1 + q21 = 0
q2 - q21 + q32 = 0
q3 - q32 + q43 = 0
q4 - q43 = 0

/* Assigned variables list: deselect the qi, Rij and Ti which are unknowns; set qi = 0 for embedded nodal points
at which there is no external source of heat. */
T1 = Tinf          // Ambient air temperature, C
//q1 =             // Heat rate, W; film side
T2 = Ts            // Film surface temperature, C
q2 = 0             // Radiant flux, W/m^2; zero for part (a)
T3 = To            // Bond temperature, C
q3 = qo            // Radiant flux, W/m^2; part (a)
T4 = Tsub          // Substrate temperature, C
//q4 =             // Heat rate, W; substrate side

// Thermal Resistances:
R21 =  1 / ( h * As )      // Convection resistance, K/W
R32 =  Lf / (kf * As)      // Conduction resistance, K/W; film
R43 =  Ls / (ks * As)      // Conduction resistance, K/W; substrate

// Other Assigned Variables:
Tinf = 20          // Ambient air temperature, C
h = 50             // Convection coefficient, W/m^2.K
Lf = 0.00025       // Thickness, m; film
kf = 0.025         // Thermal conductivity, W/m.K; film
To = 60            // Cure temperature, C
Ls = 0.001         // Thickness, m; substrate
ks = 0.05          // Thermal conductivity, W/m.K; substrate
Tsub = 30          // Substrate temperature, C
As = 1             // Cross-sectional area, m^2; unit area
```

PROBLEM 3.5

KNOWN: Thicknesses and thermal conductivities of refrigerator wall materials. Inner and outer air temperatures and convection coefficients.

FIND: Heat gain per surface area.

SCHEMATIC:

Refrigerated air
$L_i = 0.050$ m
$L_p = 0.003$ m
Ambient air
$T_{\infty,i} = 4^oC$
$h_i = 5\ W/m^2\text{-}K$
$T_{\infty,o} = 25^oC$
$h_o = 5\ W/m^2\text{-}K$
Insulation
$k_i = 0.046$ W/m-K
Panel (2)
$k_p = 60$ W/m-K
$T_{\infty,i}$ $T_{\infty,o}$
q”
$1/h_i$ L_p/k_p L_i/k_i L_p/k_p $1/h_o$

ASSUMPTIONS: (1) One-dimensional heat transfer, (2) Steady-state conditions, (3) Negligible contact resistance, (4) Negligible radiation, (5) Constant properties.

ANALYSIS: From the thermal circuit, the heat gain per unit surface area is

$$q'' = \frac{T_{\infty,o} - T_{\infty,i}}{(1/h_i) + (L_p/k_p) + (L_i/k_i) + (L_p/k_p) + (1/h_o)}$$

$$q'' = \frac{(25-4)^\circ C}{2\left(1/5\,W/m^2\cdot K\right) + 2\left(0.003m/60\,W/m\cdot K\right) + \left(0.050m/0.046\,W/m\cdot K\right)}$$

$$q'' = \frac{21^\circ C}{(0.4+0.0001+1.087)m^2\cdot K/W} = 14.1\ W/m^2 \qquad <$$

COMMENTS: Although the contribution of the panels to the total thermal resistance is negligible, that due to convection is not inconsequential and is comparable to the thermal resistance of the insulation.

PROBLEM 3.6

KNOWN: Design and operating conditions of a heat flux gage.

FIND: (a) Convection coefficient for water flow (T_s = 27°C) and error associated with neglecting conduction in the insulation, (b) Convection coefficient for air flow (T_s = 125°C) and error associated with neglecting conduction and radiation, (c) Effect of convection coefficient on error associated with neglecting conduction for T_s = 27°C.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state, (2) One-dimensional conduction, (3) Constant k.

ANALYSIS: (a) The electric power dissipation is balanced by convection to the water and conduction through the insulation. An energy balance applied to a control surface about the foil therefore yields

$$P''_{elec} = q''_{conv} + q''_{cond} = h\left(T_s - T_\infty\right) + k\left(T_s - T_b\right)/L$$

Hence,

$$h = \frac{P''_{elec} - k\left(T_s - T_b\right)/L}{T_s - T_\infty} = \frac{2000\,W/m^2 - 0.04\,W/m\cdot K\left(2\,K\right)/0.01\,m}{2\,K}$$

$$h = \frac{\left(2000 - 8\right)W/m^2}{2\,K} = 996\,W/m^2\cdot K$$ <

If conduction is neglected, a value of h = 1000 W/m^2·K is obtained, with an attendant error of (1000 - 996)/996 = 0.40%

(b) In air, energy may also be transferred from the foil surface by radiation, and the energy balance yields

$$P''_{elec} = q''_{conv} + q''_{rad} + q''_{cond} = h\left(T_s - T_\infty\right) + \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right) + k\left(T_s - T_b\right)/L$$

Hence,

$$h = \frac{P''_{elec} - \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right) - k\left(T_s - T_\infty\right)/L}{T_s - T_\infty}$$

$$= \frac{2000\,W/m^2 - 0.15\times5.67\times10^{-8}\,W/m^2\cdot K^4\left(398^4 - 298^4\right)K^4 - 0.04\,W/m\cdot K\,(100\,K)/0.01\,m}{100\,K}$$

$$= \frac{\left(2000 - 146 - 400\right)W/m^2}{100\,K} = 14.5\,W/m^2\cdot K$$ <

Continued...

PROBLEM 3.6 (Cont.)

If conduction, radiation, or conduction and radiation are neglected, the corresponding values of h and the percentage errors are 18.5 $W/m^2 \cdot K$ (27.6%), 16 $W/m^2 \cdot K$ (10.3%), and 20 $W/m^2 \cdot K$ (37.9%).

(c) For a fixed value of T_s = 27°C, the conduction loss remains at q''_{cond} = 8 W/m^2, which is also the fixed difference between P''_{elec} and q''_{conv}. Although this difference is not clearly shown in the plot for $10 \le h \le 1000$ $W/m^2 \cdot K$, it is revealed in the subplot for $10 \le 100$ $W/m^2 \cdot K$.

Errors associated with neglecting conduction decrease with increasing h from values which are significant for small h (h < 100 $W/m^2 \cdot K$) to values which are negligible for large h.

COMMENTS: In liquids (large h), it is an excellent approximation to neglect conduction and assume that all of the dissipated power is transferred to the fluid.

PROBLEM 3.7

KNOWN: A layer of fatty tissue with fixed inside temperature can experience different outside convection conditions.

FIND: (a) Ratio of heat loss for different convection conditions, (b) Outer surface temperature for different convection conditions, and (c) Temperature of still air which achieves same cooling as moving air (*wind chill* effect).

SCHEMATIC:

$L = 0.003\text{m}$; $T_{s,1} = 36°C$; $T_{s,2}$; Fatty tissue; q''_{cond}; q''_{conv}; $T_\infty = -15°C$, $h = 25\ W/m^2\cdot°C$, or $h = 65\ W/m^2\cdot°C$

ASSUMPTIONS: (1) One-dimensional conduction through a plane wall, (2) Steady-state conditions, (3) Homogeneous medium with constant properties, (4) No internal heat generation (metabolic effects are negligible), (5) Negligible radiation effects.

PROPERTIES: *Table A-3*, Tissue, fat layer: k = 0.2 W/m·K.

ANALYSIS: The thermal circuit for this situation is

$T_{s,1}$ — L/kA — $T_{s,2}$ — $1/hA$ — T_∞ → q

Hence, the heat rate is

$$q = \frac{T_{s,1} - T_\infty}{R_{tot}} = \frac{T_{s,1} - T_\infty}{L/kA + 1/hA}.$$

Therefore,

$$\frac{q''_{calm}}{q''_{windy}} = \frac{\left[\frac{L}{k} + \frac{1}{h}\right]_{windy}}{\left[\frac{L}{k} + \frac{1}{h}\right]_{calm}}.$$

Applying a surface energy balance to the outer surface, it also follows that

$$q''_{cond} = q''_{conv}.$$

Continued …..

PROBLEM 3.7 (Cont.)

Hence,

$$\frac{k}{L}\left(T_{s,1}-T_{s,2}\right)=h\left(T_{s,2}-T_{\infty}\right)$$

$$T_{s,2}=\frac{T_{\infty}+\frac{k}{hL}T_{s,1}}{1+\frac{k}{hL}}.$$

To determine the wind chill effect, we must determine the heat loss for the windy day and use it to evaluate the hypothetical ambient air temperature, T'_{∞}, which would provide the same heat loss on a calm day, Hence,

$$q''=\frac{T_{s,1}-T_{\infty}}{\left[\frac{L}{k}+\frac{1}{h}\right]_{windy}}=\frac{T_{s,1}-T'_{\infty}}{\left[\frac{L}{k}+\frac{1}{h}\right]_{calm}}$$

From these relations, we can now find the results sought:

(a) $$\frac{q''_{calm}}{q''_{windy}}=\frac{\frac{0.003\text{ m}}{0.2\text{ W/m}\cdot\text{K}}+\frac{1}{65\text{ W/m}^2\cdot\text{K}}}{\frac{0.003\text{ m}}{0.2\text{ W/m}\cdot\text{K}}+\frac{1}{25\text{ W/m}^2\cdot\text{K}}}=\frac{0.015+0.0154}{0.015+0.04}$$

$$\frac{q''_{calm}}{q''_{windy}}=0.553$$ <

(b) $$T_{s,2}\Big]_{calm}=\frac{-15^{\circ}\text{C}+\frac{0.2\text{ W/m}\cdot\text{K}}{\left(25\text{ W/m}^2\cdot\text{K}\right)\left(0.003\text{ m}\right)}36^{\circ}\text{C}}{1+\frac{0.2\text{ W/m}\cdot\text{K}}{\left(25\text{ W/m}^2\cdot\text{K}\right)\left(0.003\text{ m}\right)}}=22.1^{\circ}\text{C}$$ <

$$T_{s,2}\Big]_{windy}=\frac{-15^{\circ}\text{C}+\frac{0.2\text{ W/m}\cdot\text{K}}{\left(65\text{ W/m}^2\cdot\text{K}\right)\left(0.003\text{m}\right)}36^{\circ}\text{C}}{1+\frac{0.2\text{ W/m}\cdot\text{K}}{\left(65\text{ W/m}^2\cdot\text{K}\right)\left(0.003\text{m}\right)}}=10.8^{\circ}\text{C}$$ <

(c) $$T'_{\infty}=36^{\circ}\text{C}-\left(36+15\right)^{\circ}\text{C}\frac{\left(0.003/0.2+1/25\right)}{\left(0.003/0.2+1/65\right)}=-56.3^{\circ}\text{C}$$ <

COMMENTS: The wind chill effect is equivalent to a decrease of $T_{s,2}$ by 11.3°C and increase in the heat loss by a factor of $(0.553)^{-1}$ = 1.81.

PROBLEM 3.8

KNOWN: Dimensions of a thermopane window. Room and ambient air conditions.

FIND: (a) Heat loss through window, (b) Effect of variation in outside convection coefficient for double and triple pane construction.

SCHEMATIC (Double Pane):

Glass; Window, 0.8 m x 0.5 m; $L = 0.007$ m

Air: $T_{\infty,i} = 20$ °C, $h_i = 10$ W/m²·K; Air; Air: $T_{\infty,o} = -10$ °C, $h_o = 80$ W/m²·K

$T_{\infty,i}$ — $\frac{1}{h_i A}$ — $\frac{L}{k_g A}$ — $\frac{L}{k_a A}$ — $\frac{L}{k_g A}$ — $\frac{1}{h_o A}$ — $T_{\infty,o}$ → q

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat transfer, (3) Constant properties, (4) Negligible radiation effects, (5) Air between glass is stagnant.

PROPERTIES: *Table A-3*, Glass (300 K): $k_g = 1.4$ W/m·K; *Table A-4*, Air (T = 278 K): $k_a = 0.0245$ W/m·K.

ANALYSIS: (a) From the thermal circuit, the heat loss is

$$q = \frac{T_{\infty,i} - T_{\infty,o}}{\frac{1}{A}\left(\frac{1}{h_i} + \frac{L}{k_g} + \frac{L}{k_a} + \frac{L}{k_g} + \frac{1}{h_o}\right)}$$

$$q = \frac{20^\circ\text{C} - \left(-10^\circ\text{C}\right)}{\left(\frac{1}{0.4\,\text{m}^2}\right)\left(\frac{1}{10\,\text{W/m}^2\cdot\text{K}} + \frac{0.007\,\text{m}}{1.4\,\text{W/m}\cdot\text{K}} + \frac{0.007\,\text{m}}{0.0245\,\text{W/m}\cdot\text{K}} + \frac{0.007\,\text{m}}{1.4\,\text{W/m}\cdot\text{K}} + \frac{1}{80\,\text{W/m}^2\cdot\text{K}}\right)}$$

$$q = \frac{30^\circ\text{C}}{(0.25 + 0.0125 + 0.715 + 0.0125 + 0.03125)\,\text{K/W}} = \frac{30^\circ\text{C}}{1.021\,\text{K/W}} = 29.4\ \text{W}$$ <

(b) For the triple pane window, the additional pane and airspace increase the total resistance from 1.021 K/W to 1.749 K/W, thereby reducing the heat loss from 29.4 to 17.2 W. The effect of h_o on the heat loss is plotted as follows.

Heat loss, q(W): 15, 18, 21, 24, 27, 30

Outside convection coefficient, ho(W/m^2.K): 10, 28, 46, 64, 82, 100

Double pane
Triple pane

Continued...

PROBLEM 3.8 (Cont.)

Changes in h_o influence the heat loss at small values of h_o, for which the outside convection resistance is not negligible relative to the total resistance. However, the resistance becomes negligible with increasing h_o, particularly for the triple pane window, and changes in h_o have little effect on the heat loss.

COMMENTS: The largest contribution to the thermal resistance is due to conduction across the enclosed air. Note that this air could be in motion due to free convection currents. If the corresponding convection coefficient exceeded 3.5 $W/m^2 \cdot K$, the thermal resistance would be less than that predicted by assuming conduction across stagnant air.

PROBLEM 3.9

KNOWN: Thicknesses of three materials which form a composite wall and thermal conductivities of two of the materials. Inner and outer surface temperatures of the composite; also, temperature and convection coefficient associated with adjoining gas.

FIND: Value of unknown thermal conductivity, k_B.

SCHEMATIC:

$T_{s,i} = 600°C$, $T_{s,o} = 20°C$, $T_\infty = 800°C$, $h = 25 W/m^2 \cdot K$

$L_A = 0.3 m$, $L_B = L_C = 0.15 m$, $k_A = 20 W/m \cdot K$, $k_C = 50 W/m \cdot K$

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible contact resistance, (5) Negligible radiation effects.

ANALYSIS: Referring to the thermal circuit, the heat flux may be expressed as

$$q'' = \frac{T_{s,i} - T_{s,o}}{\frac{L_A}{k_A} + \frac{L_B}{k_B} + \frac{L_C}{k_C}} = \frac{(600-20)^\circ C}{\frac{0.3 m}{20 W/m \cdot K} + \frac{0.15 m}{k_B} + \frac{0.15 m}{50 W/m \cdot K}}$$

$$q'' = \frac{580}{0.018 + 0.15/k_B} W/m^2. \qquad (1)$$

The heat flux may be obtained from

$$q'' = h\left(T_\infty - T_{s,i}\right) = 25 W/m^2 \cdot K (800\text{-}600)^\circ C \qquad (2)$$

$$q'' = 5000 W/m^2.$$

Substituting for the heat flux from Eq. (2) into Eq. (1), find

$$\frac{0.15}{k_B} = \frac{580}{q''} - 0.018 = \frac{580}{5000} - 0.018 = 0.098$$

$$k_B = 1.53 W/m \cdot K. \qquad <$$

COMMENTS: Radiation effects are likely to have a significant influence on the net heat flux at the inner surface of the oven.

PROBLEM 3.10

KNOWN: Properties and dimensions of a composite oven window providing an outer surface safe-to-touch temperature $T_{s,o}$ = 43°C with outer convection coefficient h_o = 30 W/m^2·K and ε = 0.9 when the oven wall air temperatures are $T_w = T_a$ = 400°C. See Example 3.1.

FIND: Values of the outer convection coefficient h_o required to maintain the safe-to-touch condition when the oven wall-air temperature is raised to 500°C or 600°C.

SCHEMATIC:

L_A = 41.8 mm
L_B = 20.9 mm
$T_{s,i}$, ε = 0.9
$T_{s,o}$ = 43°C, ε = 0.9
$T_{w,i}$ = 500 or 600°C
$T_{w,o}$ = 25°C
$T_a = T_{w,i}$
h_i = 25 W/m^2-K
T_∞ = 25°C
h_o = ?
A, k_A = 0.15 W/m-K
B, k_B = 0.08 W/m-K

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in window with no contact resistance and constant properties, (3) Negligible absorption in window material, (4) Radiation exchange processes are between small surface and large isothermal surroundings.

ANALYSIS: From the analysis in the Ex. 3.1 Comment 2, the surface energy balances at the inner and outer surfaces are used to determine the required value of h_o when $T_{s,o}$ = 43°C and $T_{w,i} = T_a$ = 500 or 600°C.

$$\varepsilon\sigma\left(T_{w,i}^4 - T_{s,i}^4\right) + h_i\left(T_a - T_{s,i}\right) = \frac{T_{s,i} - T_{s,o}}{\left(L_A / k_A\right) + \left(L_B / k_B\right)}$$

$$\frac{T_{s,i} - T_{s,o}}{\left(L_A / k_A\right) + \left(L_B / k_B\right)} = \varepsilon\sigma\left(T_{s,o}^4 - T_{w,o}^4\right) + h_o\left(T_{s,o} - T_\infty\right)$$

Using these relations in IHT, the following results were calculated:

$T_{w,i}$, T_s(°C)	$T_{s,i}$(°C)	h_o(W/m^2·K)
400	392	30
500	493	40.4
600	594	50.7

COMMENTS: Note that the window inner surface temperature is closer to the oven air-wall temperature as the outer convection coefficient increases. Why is this so?

PROBLEM 3.11

KNOWN: Drying oven wall having material with known thermal conductivity sandwiched between thin metal sheets. Radiation and convection conditions prescribed on inner surface; convection conditions on outer surface.

FIND: (a) Thermal circuit representing wall and processes and (b) Insulation thickness required to maintain outer wall surface at T_o = 40°C.

SCHEMATIC:

q''_{rad} = 100 W/m^2
Oven air
$T_{\infty,i}$ = 300°C
h_i = 30 W/m^2-K
T_i
L
Insulation, k = 0.05 W/m-K
T_o = 40°C
Ambient air
$T_{\infty,o}$ = 25°C
h_o = 10 W/m^2-K
$T_{\infty,i}$ T_i T_o $T_{\infty,o}$
q''_i $R''_{cv,i}$ R''_{cd} $R''_{cv,o}$ q''_o
q''_{rad}

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in wall, (3) Thermal resistance of metal sheets negligible.

ANALYSIS: (a) The thermal circuit is shown above. Note labels for the temperatures, thermal resistances and the relevant heat fluxes.

(b) Perform energy balances on the i- and o- nodes finding

$$\frac{T_{\infty,i} - T_i}{R''_{cv,i}} + \frac{T_o - T_i}{R''_{cd}} + q''_{rad} = 0 \quad (1)$$

$$\frac{T_i - T_o}{R''_{cd}} + \frac{T_{\infty,o} - T_o}{R''_{cv,o}} = 0 \quad (2)$$

where the thermal resistances are

$$R''_{cv,i} = 1/h_i = 0.0333\ m^2 \cdot K/W \quad (3)$$

$$R''_{cd} = L/k = L/0.05\ m^2 \cdot K/W \quad (4)$$

$$R''_{cv,o} = 1/h_o = 0.0100\ m^2 \cdot K/W \quad (5)$$

Substituting numerical values, and solving Eqs. (1) and (2) simultaneously, find

$$L = 86\ mm$$ <

COMMENTS: (1) The temperature at the inner surface can be found from an energy balance on the i-node using the value found for L.

$$\frac{T_{\infty,i} - T_i}{R''_{cv,o}} + \frac{T_{\infty,o} - T_i}{R''_{cd} + R''_{cv,i}} + q''_{rad} = 0 \qquad T_i = 298.3°C$$

It follows that T_i is close to $T_{\infty,i}$ since the wall represents the dominant resistance of the system.

(2) Verify that $q''_i = 50\ W/m^2$ and $q''_o = 150\ W/m^2$. Is the overall energy balance on the system satisfied?

PROBLEM 3.12

KNOWN: Configurations of exterior wall. Inner and outer surface conditions.

FIND: Heating load for each of the three cases.

SCHEMATIC:

Case (a): L_p 10 mm, L_f 50 mm, L_w 10 mm; Plaster board (p), Urethane foam (f), Wood (w)

Case (b): L_g 3 mm; Glass (g)

Case (b): L_g 3 mm, L_a 5 mm, L_g 3 mm; Glass (g), Air(a), Glass (g)

$h_i = 5\ \text{W/m}^2\cdot\text{K}$, $T_{\infty,i} = 20\ ^\circ\text{C}$ $\quad$ $h_o = 15\ \text{W/m}^2\cdot\text{K}$, $T_{\infty,o} = -15\ ^\circ\text{C}$

ASSUMPTIONS: (1) Steady-state, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible radiation effects.

PROPERTIES: (T = 300 K): *Table A.3*: plaster board, k_p = 0.17 W/m·K; urethane, k_f = 0.026 W/m·K; wood, k_w = 0.12 W/m·K; glass, k_g = 1.4 W/m·K. *Table A.4*: air, k_a = 0.0263 W/m·K.

ANALYSIS: (a) The heat loss may be obtained by dividing the overall temperature difference by the total thermal resistance. For the composite wall of unit surface area, A = 1 m^2,

$$q = \frac{T_{\infty,i} - T_{\infty,o}}{\left[\left(1/h_i\right) + \left(L_p/k_p\right) + \left(L_f/k_f\right) + \left(L_w/k_w\right) + \left(1/h_o\right)\right]/A}$$

$$q = \frac{20^\circ\text{C} - \left(-15^\circ\text{C}\right)}{\left[\left(0.2 + 0.059 + 1.92 + 0.083 + 0.067\right)\text{m}^2\cdot\text{K/W}\right]/1\,\text{m}^2}$$

$$q = \frac{35^\circ\text{C}}{2.33\,\text{K/W}} = 15.0\,\text{W}$$ <

(b) For the single pane of glass,

$$q = \frac{T_{\infty,i} - T_{\infty,o}}{\left[\left(1/h_i\right) + \left(L_g/k_g\right) + \left(1/h_o\right)\right]/A}$$

$$q = \frac{35^\circ\text{C}}{\left[\left(0.2 + 0.002 + 0.067\right)\text{m}^2\cdot\text{K/W}\right]/1\,\text{m}^2} = \frac{35^\circ\text{C}}{0.269\,\text{K/W}} = 130.3\,\text{W}$$ <

(c) For the double pane window,

$$q = \frac{T_{\infty,i} - T_{\infty,o}}{\left[\left(1/h_i\right) + 2\left(L_g/k_g\right) + \left(L_a/k_a\right) + \left(1/h_o\right)\right]/A}$$

$$q = \frac{35^\circ\text{C}}{\left[\left(0.2 + 0.004 + 0.190 + 0.067\right)\text{m}^2\cdot\text{K/W}\right]/1\,\text{m}^2} = \frac{35^\circ\text{C}}{0.461\,\text{K/W}} = 75.9\,\text{W}$$ <

COMMENTS: The composite wall is clearly superior from the standpoint of reducing heat loss, and the dominant contribution to its total thermal resistance (82%) is associated with the foam insulation. Even with double pane construction, heat loss through the window is significantly larger than that for the composite wall.

PROBLEM 3.13

KNOWN: Composite wall of a house with prescribed convection processes at inner and outer surfaces.

FIND: (a) Expression for thermal resistance of house wall, R_{tot}; (b) Total heat loss, q(W); (c) Effect on heat loss due to increase in outside heat transfer convection coefficient, h_o; and (d) Controlling resistance for heat loss from house.

SCHEMATIC:

Plaster board, k_p
Fiberglass blanket (28 kg/m³), k_b
Plywood siding, k_s
$A = 350\ m^2$
$h_i = 30\ W/m^2 \cdot K$
$T_i = 20°C$
$h_o = 60\ W/m^2 \cdot K$, $T_o = -15°C$
$L_p = 10\ mm$
$L_b = 100\ mm$
$L_s = 20\ mm$

ASSUMPTIONS: (1) One-dimensional conduction, (2) Steady-state conditions, (3) Negligible contact resistance.

PROPERTIES: *Table A-3*, $\left(\bar{T} = (T_i + T_o)/2 = (20-15)^\circ C/2 = 2.5^\circ C \approx 300K\right)$: Fiberglass blanket, 28 kg/m^3, k_b = 0.038 W/m·K; Plywood siding, k_s = 0.12 W/m·K; Plasterboard, k_p = 0.17 W/m·K.

ANALYSIS: (a) The expression for the total thermal resistance of the house wall follows from Eq. 3.18.

$$R_{tot} = \frac{1}{h_i A} + \frac{L_p}{k_p A} + \frac{L_b}{k_b A} + \frac{L_s}{k_s A} + \frac{1}{h_o A}.$$ <

(b) The total heat loss through the house wall is

$$q = \Delta T / R_{tot} = (T_i - T_o)/R_{tot}.$$

Substituting numerical values, find

$$R_{tot} = \frac{1}{30 W/m^2 \cdot K \times 350 m^2} + \frac{0.01 m}{0.17 W/m \cdot K \times 350 m^2} + \frac{0.10 m}{0.038 W/m \cdot K \times 350 m^2}$$
$$+ \frac{0.02 m}{0.12 W/m \cdot K \times 350 m^2} + \frac{1}{60 W/m^2 \cdot K \times 350 m^2}$$
$$R_{tot} = [9.52 + 16.8 + 752 + 47.6 + 4.76] \times 10^{-5}\ ^\circ C/W = 831 \times 10^{-5}\ ^\circ C/W$$

The heat loss is then,

$$q = [20-(-15)]^\circ C / 831 \times 10^{-5}\ ^\circ C/W = 4.21\ kW.$$ <

(c) If h_o changes from 60 to 300 W/m^2·K, $R_o = 1/h_oA$ changes from 4.76 × 10^{-5} °C/W to 0.95 × 10^{-5} °C/W. This reduces R_{tot} to 826 × 10^{-5} °C/W, which is a 0.5% decrease and hence a 0.5% increase in q.

(d) From the expression for R_{tot} in part (b), note that the insulation resistance, L_b/k_bA, is 752/830 ≈ 90% of the total resistance. Hence, this material layer controls the resistance of the wall. From part (c) note that a 5-fold decrease in the outer convection resistance due to an increase in the wind velocity has a negligible effect on the heat loss.

PROBLEM 3.14

KNOWN: Composite wall of a house with prescribed convection processes at inner and outer surfaces.

FIND: Daily heat loss for prescribed diurnal variation in ambient air temperature.

SCHEMATIC:

Plaster board, k_p
Fiberglass blanket (28kg/m³), k_b
Plywood siding, k_s
$A = 200 m^2$
Inside air
$h_i = 30 W/m^2 \cdot K$
$T_{\infty,i} = 20°C$
$h_o = 60 W/m^2 \cdot K$
$T_{\infty,o} = 273 + 5 \sin\left(\frac{2\pi}{24} t\right) \quad 0 \le t \le 12h$
$T_{\infty,o} = 273 + 11 \sin\left(\frac{2\pi}{24} t\right) \quad 12 \le t \le 24h$
$L_p = 10mm$, $L_b = 100mm$, $L_s = 20mm$

ASSUMPTIONS: (1) One-dimensional, steady-state conduction (negligible change in wall thermal energy storage over 24h period), (2) Negligible contact resistance.

PROPERTIES: *Table A-3*, T ≈ 300 K: Fiberglass blanket (28 kg/m^3), k_b = 0.038 W/m·K; Plywood, k_s = 0.12 W/m·K; Plasterboard, k_p = 0.17 W/m·K.

ANALYSIS: The heat loss may be approximated as $Q = \int_0^{24h} \frac{T_{\infty,i} - T_{\infty,o}}{R_{tot}} dt$ where

$$R_{tot} = \frac{1}{A}\left[\frac{1}{h_i} + \frac{L_p}{k_p} + \frac{L_b}{k_b} + \frac{L_s}{k_s} + \frac{1}{h_o}\right]$$

$$R_{tot} = \frac{1}{200 m^2}\left[\frac{1}{30\ W/m^2 \cdot K} + \frac{0.01m}{0.17\ W/m \cdot K} + \frac{0.1m}{0.038\ W/m \cdot K} + \frac{0.02m}{0.12\ W/m \cdot K} + \frac{1}{60\ W/m^2 \cdot K}\right]$$

$$R_{tot} = 0.01454\ K/W.$$

Hence the heat rate is

$$Q = \frac{1}{R_{tot}}\left\{\int_0^{12h}\left[293 - \left[273 + 5\sin\frac{2\pi}{24}t\right]\right]dt + \int_{12}^{24h}\left[293 - \left[273 + 11\sin\frac{2\pi}{24}t\right]\right]dt\right\}$$

$$Q = 68.8\frac{W}{K}\left\{\left[20t + 5\left[\frac{24}{2\pi}\right]\cos\frac{2\pi t}{24}\right]\Bigg|_0^{12} + \left[20t + 11\left[\frac{24}{2\pi}\right]\cos\frac{2\pi t}{24}\right]\Bigg|_{12}^{24}\right\} K \cdot h$$

$$Q = 68.8\left\{\left[240 + \frac{60}{\pi}(-1-1)\right] + \left[480 - 240 + \frac{132}{\pi}(1+1)\right]\right\} W \cdot h$$

$$Q = 68.8\ \{480\text{-}38.2\text{+}84.03\}\ W \cdot h$$

$$Q = 36.18\ kW \cdot h = 1.302 \times 10^8 J.$$ <

COMMENTS: From knowledge of the fuel cost, the total daily heating bill could be determined. For example, at a cost of 0.10\$/kW·h, the heating bill would be \$3.62/day.

PROBLEM 3.15

KNOWN: Dimensions and materials associated with a composite wall (2.5m × 6.5m, 10 studs each 2.5m high).

FIND: Wall thermal resistance.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Temperature of composite depends only on x (surfaces normal to x are isothermal), (3) Constant properties, (4) Negligible contact resistance.

PROPERTIES: *Table A-3* (T ≈ 300K): Hardwood siding, k_A = 0.094 W/m·K; Hardwood, k_B = 0.16 W/m·K; Gypsum, k_C = 0.17 W/m·K; Insulation (glass fiber paper faced, 28 kg/m^3), k_D = 0.038 W/m·K.

ANALYSIS: Using the isothermal surface assumption, the thermal circuit associated with a single unit (enclosed by dashed lines) of the wall is

$$\left(L_A / k_A A_A\right) = \frac{0.008\text{m}}{0.094\ \text{W/m}\cdot\text{K}\ \left(0.65\text{m}\times 2.5\text{m}\right)} = 0.0524\ \text{K/W}$$

$$\left(L_B / k_B A_B\right) = \frac{0.13\text{m}}{0.16\ \text{W/m}\cdot\text{K}\ \left(0.04\text{m}\times 2.5\text{m}\right)} = 8.125\ \text{K/W}$$

$$\left(L_D / k_D A_D\right) = \frac{0.13\text{m}}{0.038\ \text{W/m}\cdot\text{K}\left(0.61\text{m}\times 2.5\text{m}\right)} = 2.243\ \text{K/W}$$

$$\left(L_C / k_C A_C\right) = \frac{0.012\text{m}}{0.17\ \text{W/m}\cdot\text{K}\left(0.65\text{m}\times 2.5\text{m}\right)} = 0.0434\ \text{K/W.}$$

The equivalent resistance of the core is

$$R_{eq} = \left(1/R_B + 1/R_D\right)^{-1} = \left(1/8.125 + 1/2.243\right)^{-1} = 1.758\ \text{K/W}$$

and the total unit resistance is

$$R_{tot,1} = R_A + R_{eq} + R_C = 1.854\ \text{K/W.}$$

With 10 such units in parallel, the total wall resistance is

$$R_{tot} = \left(10\times 1/R_{tot,1}\right)^{-1} = 0.1854\ \text{K/W.}$$ <

COMMENTS: If surfaces parallel to the heat flow direction are assumed adiabatic, the thermal circuit and the value of R_{tot} will differ.

PROBLEM 3.16

KNOWN: Conditions associated with maintaining heated and cooled conditions within a refrigerator compartment.

FIND: Coefficient of performance (COP).

SCHEMATIC:

$T_\infty = 20\ ^oC$
$h = 50\ W/m^2 \cdot K$

Case (a): $T_{\infty,i} = 90\ ^oC$, q_{out}, Electric heater, $q_{elec} = 20$ W, Unplugged

$T_{\infty,o} = 25\ ^oC$

Case (b): $T_{\infty,i} = 5\ ^oC$, q_{out}, Cooling coil, q_{out}, q_{in}, Plugged, $W_{in} = 125{,}000$ J, $\Delta t = 12$ h

ASSUMPTIONS: (1) Steady-state operating conditions, (2) Negligible radiation, (3) Compartment completely sealed from ambient air.

ANALYSIS: The Case (a) experiment is performed to determine the overall thermal resistance to heat transfer between the interior of the refrigerator and the ambient air. Applying an energy balance to a control surface about the refrigerator, it follows from Eq. 1.11a that, at any instant,

$$\dot{E}_g - \dot{E}_{out} = 0$$

Hence,

$$q_{elec} - q_{out} = 0$$

where $q_{out} = \left(T_{\infty,i} - T_{\infty,o}\right)/R_t$. It follows that

$$R_t = \frac{T_{\infty,i} - T_{\infty,o}}{q_{elec}} = \frac{(90-25)^\circ C}{20\,W} = 3.25^\circ C/W$$

For Case (b), heat transfer from the ambient air to the compartment (the heat load) is balanced by heat transfer to the refrigerant ($q_{in} = q_{out}$). Hence, the thermal energy transferred from the refrigerator over the 12 hour period is

$$Q_{out} = q_{out}\Delta t = q_{in}\Delta t = \frac{T_{\infty,i} - T_{\infty,o}}{R_t}\Delta t$$

$$Q_{out} = \frac{(25-5)^\circ C}{3.25^\circ\ C/W}\left(12\,h \times 3600\,s/h\right) = 266{,}000\,J$$

The coefficient of performance (COP) is therefore

$$COP = \frac{Q_{out}}{W_{in}} = \frac{266{,}000}{125{,}000} = 2.13$$ <

COMMENTS: The ideal (Carnot) COP is

$$COP)_{ideal} = \frac{T_c}{T_h - T_c} = \frac{278\,K}{(298-278)\,K} = 13.9$$

and the system is operating well below its peak possible performance.

PROBLEM 3.17

KNOWN: Total floor space and vertical distance between floors for a square, flat roof building.

FIND: (a) Expression for width of building which minimizes heat loss, (b) Width and number of floors which minimize heat loss for a prescribed floor space and distance between floors. Corresponding heat loss, percent heat loss reduction from 2 floors.

SCHEMATIC:

ASSUMPTIONS: Negligible heat loss to ground.

ANALYSIS: (a) To minimize the heat loss q, the exterior surface area, A_s, must be minimized. From Fig. (a)

$$A_s = W^2 + 4WH = W^2 + 4WN_f H_f$$

where

$$N_f = A_f / W^2$$

Hence,

$$A_s = W^2 + 4WA_f H_f / W^2 = W^2 + 4A_f H_f / W$$

The optimum value of W corresponds to

$$\frac{dA_s}{dW} = 2W - \frac{4A_f H_f}{W^2} = 0$$

or

$$W_{op} = (2A_f H_f)^{1/3}$$ <

The competing effects of W on the areas of the roof and sidewalls, and hence the basis for an optimum, is shown schematically in Fig. (b).

(b) For A_f = 32,768 m^2 and H_f = 4 m,

$$W_{op} = \left(2 \times 32,768\,\text{m}^2 \times 4\,\text{m}\right)^{1/3} = 64\,\text{m}$$ <

Continued …..

PROBLEM 3.17 (Cont.)

Hence,

$$N_f = \frac{A_f}{W^2} = \frac{32,768\,m^2}{(64\,m)^2} = 8$$ <

and

$$q = UA_s\Delta T = 1\,W/m^2 \cdot K\left[(64\,m)^2 + \frac{4\times 32,768\,m^2\times 4\,m}{64\,m}\right]25^\circ C = 307,200\,W$$ <

For $N_f = 2$,

$$W = (A_f/N_f)^{1/2} = (32,768\ m^2/2)^{1/2} = 128\ m$$

$$q = 1\,W/m^2 \cdot K\left[(128\,m)^2 + \frac{4\times 32,768\,m^2\times 4\,m}{128\,m}\right]25^\circ C = 512,000\,W$$

% reduction in q = (512,000 - 307,200)/512,000 = 40% <

COMMENTS: Even the minimum heat loss is excessive and could be reduced by reducing U.

PROBLEM 3.18

KNOWN: Concrete wall of 150 mm thickness experiences a flash-over fire with prescribed radiant flux and hot-gas convection on the fire-side of the wall. Exterior surface condition is 300°C, typical ignition temperature for most household and office materials.

FIND: (a) Thermal circuit representing wall and processes and (b) Temperature at the fire-side of the wall; comment on whether wall is likely to experience structural collapse for these conditions.

SCHEMATIC:

q''_{rad} = 25 kW/m^2

Hot gases

T_∞ = 400°C

h = 200 W/m^2-K

Concrete, k = 1.4 W/m-K

T_o

T_L = 300°C

x

L = 150 mm

q''_{cv}, T_∞, R''_{cv}, q''_{rad}, T_o, R''_{cd}, T_L, q''_L

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in wall, (3) Constant properties.

PROPERTIES: *Table A-3*, Concrete (stone mix, 300 K): k = 1.4 W/m·K.

ANALYSIS: (a) The thermal cirucit is shown above. Note labels for the temperatures, thermal resistances and the relevant heat fluxes.

(b) To determine the fire-side wall surface temperatures, perform an energy balance on the o-node.

$$\frac{T_\infty - T_o}{R''_{cv}} + q''_{rad} = \frac{T_L - T_o}{R''_{cd}}$$

where the thermal resistances are

$$R''_{cv} = 1/h_i = 1/200\ \mathrm{W/m^2 \cdot K} = 0.00500\ \mathrm{m^2 \cdot K/W}$$

$$R''_{cd} = L/k = 0.150\ \mathrm{m}/1.4\,\mathrm{W/m \cdot K} = 0.107\ \mathrm{m^2 \cdot K/W}$$

Substituting numerical values,

$$\frac{(400 - T_o)\mathrm{K}}{0.005\ \mathrm{m^2 \cdot K/W}} + 25{,}000\ \mathrm{W/m^2}\ \frac{(300 - T_o)\mathrm{K}}{0.107\ \mathrm{m^2 \cdot K/W}} = 0$$

$$T_o = 515°\mathrm{C}$$ <

COMMENTS: (1) The fire-side wall surface temperature is within the 350 to 600°C range for which explosive spalling could occur. It is likely the wall will experience structural collapse for these conditions.

(2) This steady-state condition is an extreme condition, as the wall may fail before near steady-state conditions can be met.

PROBLEM 3.19

KNOWN: Representative dimensions and thermal conductivities for the layers of fire-fighter's protective clothing, a turnout coat.

FIND: (a) Thermal circuit representing the turnout coat; tabulate thermal resistances of the layers and processes; and (b) For a prescribed radiant heat flux on the fire-side surface and temperature of T_i =.60°C at the inner surface, calculate the fire-side surface temperature, T_o.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction through the layers, (3) Heat is transferred by conduction and radiation exchange across the stagnant air gaps, (3) Constant properties.

PROPERTIES: *Table A-4*, Air (470 K, 1 atm): $k_{ab} = k_{cd} = 0.0387$ W/m·K.

ANALYSIS: (a) The thermal circuit is shown with labels for the temperatures and thermal resistances.

The conduction thermal resistances have the form $R''_{cd} = L/k$ while the radiation thermal resistances across the air gaps have the form

$$R''_{rad} = \frac{1}{h_{rad}} = \frac{1}{4\sigma T_{avg}^3}$$

The linearized radiation coefficient follows from Eqs. 1.8 and 1.9 with ε = 1 where T_{avg} represents the average temperature of the surfaces comprising the gap

$$h_{rad} = \sigma\left(T_1 + T_2\right)\left(T_1^2 + T_2^2\right) \approx 4\sigma T_{avg}^3$$

For the radiation thermal resistances tabulated below, we used T_{avg} = 470 K.

Continued …..

PROBLEM 3.19 (Cont.)

	Shell (s)	Air gap (a-b)	Barrier (mb)	Air gap (c-d)	Liner (tl)	*Total* (tot)
$R''_{cd}\left(m^2 \cdot K/W\right)$	0.01702	0.0259	0.04583	0.0259	0.00921	--
$R''_{rad}\left(m^2 \cdot K/W\right)$	--	0.04264	--	0.04264	--	--
$R''_{gap}\left(m^2 \cdot K/W\right)$	--	0.01611	--	0.01611	--	--
R''_{total}	--	--	--	--	--	0.1043

From the thermal circuit, the resistance across the gap for the conduction and radiation processes is

$$\frac{1}{R''_{gap}} = \frac{1}{R''_{cd}} + \frac{1}{R''_{rad}}$$

and the total thermal resistance of the turn coat is

$$R''_{tot} = R''_{cd,s} + R''_{gap,a-b} + R''_{cd,mb} + R''_{gap,c-d} + R''_{cd,tl}$$

(b) If the heat flux through the coat is 0.25 W/cm^2, the fire-side surface temperature T_o can be calculated from the rate equation written in terms of the overall thermal resistance.

$$q'' = \left(T_o - T_i\right)/R''_{tot}$$

$$T_o = 66°C + 0.25\ W/cm^2 \times \left(10^2 cm/m\right)^2 \times 0.1043\ m^2 \cdot K/W$$

$$T_o = 327°C$$

COMMENTS: (1) From the tabulated results, note that the thermal resistance of the moisture barrier (mb) is nearly 3 times larger than that for the shell or air gap layers, and 4.5 times larger than the thermal liner layer.

(2) The air gap conduction and radiation resistances were calculated based upon the average temperature of 470 K. This value was determined by setting $T_{avg} = (T_o + T_i)/2$ and solving the equation set using *IHT* with $k_{air} = k_{air}(T_{avg})$.

PROBLEM 3.20

KNOWN: Materials and dimensions of a composite wall separating a combustion gas from a liquid coolant.

FIND: (a) Heat loss per unit area, and (b) Temperature distribution.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional heat transfer, (2) Steady-state conditions, (3) Constant properties, (4) Negligible radiation effects.

PROPERTIES: *Table A-1*, St. St. (304) $\left(\bar{T} \approx 1000\text{K}\right)$: k = 25.4 W/m·K; *Table A-2*, Beryllium Oxide (T ≈ 1500K): k = 21.5 W/m·K.

ANALYSIS: (a) The desired heat flux may be expressed as

$$q'' = \frac{T_{\infty,1} - T_{\infty,2}}{\frac{1}{h_1} + \frac{L_A}{k_A} + R_{t,c} + \frac{L_B}{k_B} + \frac{1}{h_2}} = \frac{(2600 - 100)^\circ \text{C}}{\left[\frac{1}{50} + \frac{0.01}{21.5} + 0.05 + \frac{0.02}{25.4} + \frac{1}{1000}\right]\frac{\text{m}^2.\text{K}}{\text{W}}}$$

$$q'' = 34{,}600 \text{ W/m}^2.$$ <

(b) The composite surface temperatures may be obtained by applying appropriate rate equations. From the fact that $q'' = h_1\left(T_{\infty,1} - T_{s,1}\right)$, it follows that

$$T_{s,1} = T_{\infty,1} - \frac{q''}{h_1} = 2600^\circ \text{C} - \frac{34{,}600 \text{ W/m}^2}{50 \text{ W/m}^2 \cdot \text{K}} 1908^\circ \text{C}.$$

With $q'' = \left(k_A / L_A\right)\left(T_{s,1} - T_{c,1}\right)$, it also follows that

$$T_{c,1} = T_{s,1} - \frac{L_A q''}{k_A} = 1908^\circ \text{C} - \frac{0.01\text{m} \times 34{,}600 \text{ W/m}^2}{21.5 \text{ W/m} \cdot \text{K}} = 1892^\circ \text{C}.$$

Similarly, with $q'' = \left(T_{c,1} - T_{c,2}\right) / R_{t,c}$

$$T_{c,2} = T_{c,1} - R_{t,c} q'' = 1892^\circ\text{C} - 0.05 \frac{\text{m}^2 \cdot \text{K}}{\text{W}} \times 34{,}600 \frac{\text{W}}{\text{m}^2} = 162^\circ \text{C}$$

Continued …..

PROBLEM 3.20 (Cont.)

and with $q'' = (k_B / L_B)(T_{c,2} - T_{s,2})$,

$$T_{s,2} = T_{c,2} - \frac{L_B q''}{k_B} = 162^\circ C - \frac{0.02\text{m} \times 34{,}600\ \text{W/m}^2}{25.4\ \text{W/m}\cdot\text{K}} = 134.6^\circ C.$$

The temperature distribution is therefore of the following form:

$T_{\infty,1} = 2600°C$
$T_{s,1} = 1908°C$
$T_{c,1} = 1892°C$
$T_{c,2} = 162°C$
$T_{s,2} = 134.6°C$
$T_{\infty,2} = 100°C$

<

COMMENTS: (1) The calculations may be checked by recomputing q'' from

$$q'' = h_2 (T_{s,2} - T_{\infty,2}) = 1000\text{W/m}^2 \cdot \text{K}(134.6\text{-}100)^\circ \text{C} = 34{,}600\text{W/m}^2$$

(2) The initial *estimates* of the mean material temperatures are in error, particularly for the stainless steel. For improved accuracy the calculations should be repeated using k values corresponding to T ≈ 1900°C for the oxide and T ≈ 115°C for the steel.

(3) The major contributions to the total resistance are made by the combustion gas boundary layer and the contact, where the temperature drops are largest.

PROBLEM 3.21

KNOWN: Thickness, overall temperature difference, and pressure for two stainless steel plates.

FIND: (a) Heat flux and (b) Contact plane temperature drop.

SCHEMATIC:

$T_{s,1}$ — $\frac{L}{k}$ — $R''_{t,c}$ — $\frac{L}{k}$ — $T_{s,2}$ → q''

0.01m, 0.01m; $T_{s,1}$; $T_{s,2}$; Contact pressure 1 bar; $T_{s,1} - T_{s,2} = 100°C$; Stainless steel

ASSUMPTIONS: (1) One-dimensional heat transfer, (2) Steady-state conditions, (3) Constant properties.

PROPERTIES: *Table A-1*, Stainless Steel (T ≈ 400K): k = 16.6 W/m·K.

ANALYSIS: (a) With $R''_{t,c} \approx 15\times10^{-4}\ m^2\cdot K/W$ from Table 3.1 and

$$\frac{L}{k} = \frac{0.01m}{16.6\ W/m\cdot K} = 6.02\times10^{-4} m^2\cdot K/W,$$

it follows that

$$R''_{tot} = 2(L/k) + R''_{t,c} \approx 27\times10^{-4} m^2\cdot K/W;$$

hence

$$q'' = \frac{\Delta T}{R''_{tot}} = \frac{100^\circ C}{27\times10^{-4} m^2\cdot K/W} = 3.70\times10^4 W/m^2. \qquad <$$

(b) From the thermal circuit,

$$\frac{\Delta T_c}{T_{s,1} - T_{s,2}} = \frac{R''_{t,c}}{R''_{tot}} = \frac{15\times10^{-4} m^2\cdot K/W}{27\times10^{-4} m^2\cdot K/W} = 0.556.$$

Hence,

$$\Delta T_c = 0.556\left(T_{s,1} - T_{s,2}\right) = 0.556\left(100^\circ C\right) = 55.6^\circ C. \qquad <$$

COMMENTS: The contact resistance is significant relative to the conduction resistances. The value of $R''_{t,c}$ would diminish, however, with increasing pressure.

PROBLEM 3.22

KNOWN: Temperatures and convection coefficients associated with fluids at inner and outer surfaces of a composite wall. Contact resistance, dimensions, and thermal conductivities associated with wall materials.

FIND: (a) Rate of heat transfer through the wall, (b) Temperature distribution.

SCHEMATIC:

$T_{\infty,1} = 200°C$, $h_1 = 10\ W/m^2 \cdot K$, $k_A = 0.1\ W/m\cdot K$, $H=2m$, $W=2.5m$, $A=5m^2$, $L_A=0.01m$, $L_B=0.02m$, $k_B = 0.04\ W/m\cdot K$, $T_{\infty,2} = 40°C$, $h_2 = 20\ W/m^2\cdot K$, $R''_{t,c} = 0.30\ m^2\cdot K/W$

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat transfer, (3) Negligible radiation, (4) Constant properties.

ANALYSIS: (a) Calculate the total resistance to find the heat rate,

$$R_{tot} = \frac{1}{h_1 A} + \frac{L_A}{k_A A} + R_{t,c} + \frac{L_B}{k_B A} + \frac{1}{h_2 A}$$

$$R_{tot} = \left[\frac{1}{10\times5} + \frac{0.01}{0.1\times5} + \frac{0.3}{5} + \frac{0.02}{0.04\times5} + \frac{1}{20\times5}\right]\frac{K}{W}$$

$$R_{tot} = [0.02 + 0.02 + 0.06 + 0.10 + 0.01]\frac{K}{W} = 0.21\frac{K}{W}$$

$$q = \frac{T_{\infty,1} - T_{\infty,2}}{R_{tot}} = \frac{(200-40)^\circ C}{0.21\ K/W} = 762\ W.$$ <

(b) It follows that

$$T_{s,1} = T_{\infty,1} - \frac{q}{h_1 A} = 200^\circ C - \frac{762\ W}{50\ W/K} = 184.8^\circ C$$

$$T_A = T_{s,1} - \frac{qL_A}{k_A A} = 184.8^\circ C - \frac{762 W \times 0.01 m}{0.1 \frac{W}{m\cdot K} \times 5 m^2} = 169.6^\circ C$$

$$T_B = T_A - qR_{t,c} = 169.6^\circ C - 762 W \times 0.06 \frac{K}{W} = 123.8^\circ C$$

$$T_{s,2} = T_B - \frac{qL_B}{k_B A} = 123.8^\circ C - \frac{762 W \times 0.02 m}{0.04 \frac{W}{m\cdot K} \times 5 m^2} = 47.6^\circ C$$

$$T_{\infty,2} = T_{s,2} - \frac{q}{h_2 A} = 47.6^\circ C - \frac{762 W}{100 W/K} = 40^\circ C$$

PROBLEM 3.23

KNOWN: Outer and inner surface convection conditions associated with zirconia-coated, Inconel turbine blade. Thicknesses, thermal conductivities, and interfacial resistance of the blade materials. Maximum allowable temperature of Inconel.

FIND: Whether blade operates below maximum temperature. Temperature distribution in blade, with and without the TBC.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional, steady-state conduction in a composite plane wall, (2) Constant properties, (3) Negligible radiation.

ANALYSIS: For a unit area, the total thermal resistance with the TBC is

$$R''_{tot,w} = h_o^{-1} + (L/k)_{Zr} + R''_{t,c} + (L/k)_{In} + h_i^{-1}$$

$$R''_{tot,w} = \left(10^{-3} + 3.85\times10^{-4} + 10^{-4} + 2\times10^{-4} + 2\times10^{-3}\right) m^2 \cdot K/W = 3.69\times10^{-3} m^2 \cdot K/W$$

With a heat flux of

$$q''_w = \frac{T_{\infty,o} - T_{\infty,i}}{R''_{tot,w}} = \frac{1300\,K}{3.69\times10^{-3}\,m^2 \cdot K/W} = 3.52\times10^5\,W/m^2$$

the inner and outer surface temperatures of the Inconel are

$$T_{s,i(w)} = T_{\infty,i} + (q''_w/h_i) = 400\,K + \left(3.52\times10^5\,W/m^2 / 500\,W/m^2 \cdot K\right) = 1104\,K$$

$$T_{s,o(w)} = T_{\infty,i} + \left[(1/h_i) + (L/k)_{In}\right] q''_w = 400\,K + \left(2\times10^{-3} + 2\times10^{-4}\right) m^2 \cdot K/W \left(3.52\times10^5\,W/m^2\right) = 1174\,K$$

Without the TBC, $R''_{tot,wo} = h_o^{-1} + (L/k)_{In} + h_i^{-1} = 3.20\times10^{-3} m^2 \cdot K/W$, and $q''_{wo} = \left(T_{\infty,o} - T_{\infty,i}\right)/R''_{tot,wo}$ = (1300 K)/3.20×10^{-3} m^2·K/W = 4.06×10^5 W/m^2. The inner and outer surface temperatures of the Inconel are then

$$T_{s,i(wo)} = T_{\infty,i} + (q''_{wo}/h_i) = 400\,K + \left(4.06\times10^5\,W/m^2 / 500\,W/m^2 \cdot K\right) = 1212\,K$$

$$T_{s,o(wo)} = T_{\infty,i} + \left[(1/h_i) + (L/k)_{In}\right] q''_{wo} = 400\,K + \left(2\times10^{-3} + 2\times10^{-4}\right) m^2 \cdot K/W \left(4.06\times10^5\,W/m^2\right) = 1293\,K$$

Continued...

PROBLEM 3.23 (Cont.)

Use of the TBC facilitates operation of the Inconel below T_{max} = 1250 K.

COMMENTS: Since the durability of the TBC decreases with increasing temperature, which increases with increasing thickness, limits to the thickness are associated with reliability considerations.

PROBLEM 3.24

KNOWN: Size and surface temperatures of a cubical freezer. Materials, thicknesses and interface resistances of freezer wall.

FIND: Cooling load.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state, (2) One-dimensional conduction, (3) Constant properties.

PROPERTIES: *Table A-1*, Aluminum 2024 (~267K): k_{al} = 173 W/m·K. *Table A-1*, Carbon steel AISI 1010 (~295K): k_{st} = 64 W/m·K. *Table A-3* (~300K): k_{ins} = 0.039 W/m·K.

ANALYSIS: For a unit wall surface area, the total thermal resistance of the composite wall is

$$R''_{tot} = \frac{L_{al}}{k_{al}} + R''_{t,c} + \frac{L_{ins}}{k_{ins}} + R''_{t,c} + \frac{L_{st}}{k_{st}}$$

$$R''_{tot} = \frac{0.00635\text{m}}{173\text{ W/m}\cdot\text{K}} + 2.5\times10^{-4}\,\frac{\text{m}^2\cdot\text{K}}{\text{W}} + \frac{0.100\text{m}}{0.039\text{ W/m}\cdot\text{K}} + 2.5\times10^{-4}\,\frac{\text{m}^2\cdot\text{K}}{\text{W}} + \frac{0.00635\text{m}}{64\text{ W/m}\cdot\text{K}}$$

$$R''_{tot} = \left(3.7\times10^{-5} + 2.5\times10^{-4} + 2.56 + 2.5\times10^{-4} + 9.9\times10^{-5}\right)\text{m}^2\cdot\text{K/W}$$

Hence, the heat flux is

$$q'' = \frac{T_{s,o} - T_{s,i}}{R''_{tot}} = \frac{\left[22 - (-6)\right]^\circ\text{C}}{2.56\text{ m}^2\cdot\text{K/W}} = 10.9\,\frac{\text{W}}{\text{m}^2}$$

and the cooling load is

$$q = A_s\,q'' = 6\,W^2\,q'' = 54\text{m}^2 \times 10.9\text{ W/m}^2 = 590\,\text{W}$$ <

COMMENTS: Thermal resistances associated with the cladding and the adhesive joints are negligible compared to that of the insulation.

PROBLEM 3.25

KNOWN: Thicknesses and thermal conductivity of window glass and insulation. Contact resistance. Environmental temperatures and convection coefficients. Furnace efficiency and fuel cost.

FIND: (a) Reduction in heat loss associated with the insulation, (b) Heat losses for prescribed conditions, (c) Savings in fuel costs for 12 hour period.

SCHEMATIC:

L_w = 0.006 m
L_{ins} = 0.025 m
A_s = 12 m^2
Ambient air
$T_{\infty,o}$ = -12°C
h_o = 20 W/m^2-K
Window
k_w = 1.4 W/m-K
Room air
$T_{\infty,i}$ = 20°C
h_i = 2 or 5 W/m^2-K
Insulation, k_{ins} = 0.027 W/m-K
$R''_{t,c}$ = 0.002 m^2 -K/W
q''
$T_{\infty,o}$ $R''_{cnv,o}$ $R''_{cnd,w}$ $R''_{t,c}$ $R''_{cnd,ins}$ $R''_{cnv,i}$ $T_{\infty,i}$

ASSUMPTIONS: (1) Steady-state, (2) One-dimensional heat transfer, (3) Constant properties.

ANALYSIS: (a) The percentage reduction in heat loss is

$$R_q = \frac{q''_{wo} - q''_{with}}{q''_{wo}} \times 100\% = \left(1 - \frac{q''_{with}}{q''_{wo}}\right) \times 100\% = \left(1 - \frac{R''_{tot,wo}}{R''_{tot,with}}\right) \times 100\%$$

where the total thermal resistances without and with the insulation, respectively, are

$$R''_{tot,wo} = R''_{cnv,o} + R''_{cnd,w} + R''_{cnv,i} = \frac{1}{h_o} + \frac{L_w}{k_w} + \frac{1}{h_i}$$

$$R''_{tot,wo} = (0.050 + 0.004 + 0.200)\,m^2 \cdot K/W = 0.254\ m^2 \cdot K/W$$

$$R''_{tot,with} = R''_{cnv,o} + R''_{cnd,w} + R''_{t,c} + R''_{cnd,ins} + R''_{cnv,i} = \frac{1}{h_o} + \frac{L_w}{k_w} + R''_{t,c} + \frac{L_{ins}}{k_{ins}} + \frac{1}{h_i}$$

$$R''_{tot,with} = (0.050 + 0.004 + 0.002 + 0.926 + 0.500)\,m^2 \cdot K/W = 1.482\ m^2 \cdot K/W$$

$$R_q = (1 - 0.254/1.482) \times 100\% = 82.9\% \qquad <$$

(b) With $A_s = 12\ m^2$, the heat losses without and with the insulation are

$$q_{wo} = A_s\left(T_{\infty,i} - T_{\infty,o}\right)/R''_{tot,wo} = 12\,m^2 \times 32°C/0.254\,m^2 \cdot K/W = 1512\,W \qquad <$$

$$q_{with} = A_s\left(T_{\infty,i} - T_{\infty,o}\right)/R''_{tot,with} = 12\,m^2 \times 32°C/1.482\,m^2 \cdot K/W = 259\ W \qquad <$$

(c) With the windows covered for 12 hours per day, the daily savings are

$$S = \frac{(q_{wo} - q_{with})}{\eta_f}\Delta t\ C_g \times 10^{-6}\ MJ/J = \frac{(1512 - 259)\,W}{0.8} 12h \times 3600\,s/h \times \$0.01/MJ \times 10^{-6}\,MJ/J = \$0.677$$

COMMENTS: (1) The savings may be insufficient to justify the cost of the insulation, as well as the daily tedium of applying and removing the insulation. However, the losses are significant and unacceptable. The owner of the building should install double pane windows. (2) The dominant contributions to the total thermal resistance are made by the insulation and convection at the inner surface.

PROBLEM 3.26

KNOWN: Surface area and maximum temperature of a chip. Thickness of aluminum cover and chip/cover contact resistance. Fluid convection conditions.

FIND: Maximum chip power.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat transfer, (3) Negligible heat loss from sides and bottom, (4) Chip is isothermal.

PROPERTIES: *Table A.1*, Aluminum (T ≈ 325 K): k = 238 W/m·K.

ANALYSIS: For a control surface about the chip, conservation of energy yields

$$\dot{E}_g - \dot{E}_{out} = 0$$

or

$$P_c - \frac{(T_c - T_\infty)A}{\left[(L/k) + R''_{t,c} + (1/h)\right]} = 0$$

$$P_{c,max} = \frac{(85-25)^\circ C\left(10^{-4} m^2\right)}{\left[(0.002/238) + 0.5\times10^{-4} + (1/1000)\right] m^2 \cdot K/W}$$

$$P_{c,max} = \frac{60\times10^{-4}\ {}^\circ C \cdot m^2}{\left(8.4\times10^{-6} + 0.5\times10^{-4} + 10^{-3}\right) m^2 \cdot K/W}$$

$$P_{c,max} = 5.7\ W.$$ <

COMMENTS: The dominant resistance is that due to convection $\left(R_{conv} > R_{t,c} >> R_{cond}\right)$.

PROBLEM 3.27

KNOWN: Operating conditions for a board mounted chip.

FIND: (a) Equivalent thermal circuit, (b) Chip temperature, (c) Maximum allowable heat dissipation for dielectric liquid (h_o = 1000 W/m^2·K) and air (h_o = 100 W/m^2·K). Effect of changes in circuit board temperature and contact resistance.

SCHEMATIC:

q''_o $\quad h_o$, $T_\infty = 20\ ^oC$ $\quad q''_c, T_c$ $\quad L_b = 0.005$ m $\quad q''_i$ $\quad R''_{t,c}$ $\quad k_b$ $\quad h_i = 40$ W/m^2•K $\quad T_{\infty,i} = 20\ ^oC$

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Negligible chip thermal resistance, (4) Negligible radiation, (5) Constant properties.

PROPERTIES: *Table A-3*, Aluminum oxide (polycrystalline, 358 K): k_b = 32.4 W/m·K.

ANALYSIS: (a)

$T_{\infty,i}$ $\quad T_c$ $\quad T_{\infty,o}$ $\quad q''_i$ $\quad 1/h_i$ $\quad (L/k)_b$ $\quad R''_{t,c}$ $\quad 1/h_o$ $\quad q''_o$ $\quad q''_c$

(b) Applying conservation of energy to a control surface about the chip $\left(\dot{E}_{in} - \dot{E}_{out} = 0\right)$,

$$q''_c - q''_i - q''_o = 0$$

$$q''_c = \frac{T_c - T_{\infty,i}}{1/h_i + (L/k)_b + R''_{t,c}} + \frac{T_c - T_{\infty,o}}{1/h_o}$$

With $q''_c = 3\times10^4\ W/m^2$, h_o = 1000 W/m^2·K, k_b = 1 W/m·K and $R''_{t,c} = 10^{-4} m^2\cdot K/W$,

$$3\times10^4\ W/m^2 = \frac{T_c - 20^\circ C}{\left(1/40 + 0.005/1 + 10^{-4}\right)m^2\cdot K/W} + \frac{T_c - 20^\circ C}{(1/1000)m^2\cdot K/W}$$

$$3\times10^4\ W/m^2 = \left(33.2T_c - 664 + 1000T_c - 20,000\right)W/m^2\cdot K$$

$$1003T_c = 50,664$$

$$T_c = 49^\circ C. \qquad <$$

(c) For T_c = 85°C and h_o = 1000 W/m^2·K, the foregoing energy balance yields

$$q''_c = 67,160\ W/m^2 \qquad <$$

with q''_o = 65,000 W/m^2 and q''_i = 2160 W/m^2. Replacing the dielectric with air (h_o = 100 W/m^2·K), the following results are obtained for different combinations of k_b and $R''_{t,c}$.

Continued...

PROBLEM 3.27 (Cont.)

k_b (W/m·K)	$R''_{t,c}$ (m^2·K/W)	q''_i (W/m^2)	q''_o (W/m^2)	q''_c (W/m^2)
1	10^{-4}	2159	6500	8659
32.4	10^{-4}	2574	6500	9074
1	10^{-5}	2166	6500	8666
32.4	10^{-5}	2583	6500	9083

<

COMMENTS: 1. For the conditions of part (b), the total internal resistance is 0.0301 m^2·K/W, while the outer resistance is 0.001 m^2·K/W. Hence

$$\frac{q''_o}{q''_i} = \frac{\left(T_c - T_{\infty,o}\right)/R''_o}{\left(T_c - T_{\infty,i}\right)/R''_i} = \frac{0.0301}{0.001} = 30\,.$$

and only approximately 3% of the heat is dissipated through the board.

2. With h_o = 100 W/m^2·K, the outer resistance increases to 0.01 m^2·K/W, in which case $q''_o/q''_i = R''_i/R''_o$ = 0.0301/0.01 = 3.1 and now almost 25% of the heat is dissipated through the board. Hence, although measures to reduce R''_i would have a negligible effect on q''_c for the liquid coolant, some improvement may be gained for air-cooled conditions. As shown in the table of part (b), use of an aluminum oxide board increase q''_i by 19% (from 2159 to 2574 W/m^2) by reducing R''_i from 0.0301 to 0.0253 m^2·K/W. Because the initial contact resistance ($R''_{t,c} = 10^{-4}\,m^2 \cdot K/W$) is already much less than R''_i, any reduction in its value would have a negligible effect on q''_i. The largest gain would be realized by increasing h_i, since the inside convection resistance makes the dominant contribution to the total internal resistance.

PROBLEM 3.28

KNOWN: Dimensions, thermal conductivity and emissivity of base plate. Temperature and convection coefficient of adjoining air. Temperature of surroundings. Maximum allowable temperature of transistor case. Case-plate interface conditions.

FIND: (a) Maximum allowable power dissipation for an air-filled interface, (b) Effect of convection coefficient on maximum allowable power dissipation.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state, (2) Negligible heat transfer from the enclosure, to the surroundings. (3) One-dimensional conduction in the base plate, (4) Radiation exchange at surface of base plate is with large surroundings, (5) Constant thermal conductivity.

PROPERTIES: Aluminum-aluminum interface, air-filled, 10 μm roughness, 10^5 N/m^2 contact pressure (Table 3.1): $R''_{t,c} = 2.75\times10^{-4}\ \text{m}^2\cdot\text{K/W}$.

ANALYSIS: (a) With all of the heat dissipation transferred through the base plate,

$$P_{elec} = q = \frac{T_{s,c} - T_\infty}{R_{tot}} \tag{1}$$

where $R_{tot} = R_{t,c} + R_{cnd} + \left[(1/R_{cnv}) + (1/R_{rad})\right]^{-1}$

$$R_{tot} = \frac{R''_{t,c}}{A_c} + \frac{L}{kW^2} + \frac{1}{W^2}\left(\frac{1}{h + h_r}\right) \tag{2}$$

and
$$h_r = \varepsilon\sigma\left(T_{s,p} + T_{sur}\right)\left(T_{s,p}^2 + T_{sur}^2\right) \tag{3}$$

To obtain $T_{s,p}$, the following energy balance must be performed on the plate surface,

$$q = \frac{T_{s,c} - T_{s,p}}{R_{t,c} + R_{cnd}} = q_{cnv} + q_{rad} = hW^2\left(T_{s,p} - T_\infty\right) + h_r W^2\left(T_{s,p} - T_{sur}\right) \tag{4}$$

With $R_{t,c} = 2.75 \times 10^{-4}\ \text{m}^2\cdot\text{K/W}/2\times10^{-4}\ \text{m}^2 = 1.375$ K/W, $R_{cnd} = 0.006\ \text{m}/(240\ \text{W/m}\cdot\text{K} \times 4 \times 10^{-4}\ \text{m}^2)$ = 0.0625 K/W, and the prescribed values of h, W, $T_\infty = T_{sur}$ and ε, Eq. (4) yields a surface temperature of $T_{s,p}$ = 357.6 K = 84.6°C and a power dissipation of

Continued …..

PROBLEM 3.28 (Cont.)

$$P_{elec} = q = 0.268\ \text{W}$$ <

The convection and radiation resistances are R_{cnv} = 625 m·K/W and R_{rad} = 345 m·K/W, where h_r = 7.25 W/m^2·K.

(b) With the major contribution to the total resistance made by convection, significant benefit may be derived by increasing the value of h.

For h = 200 W/m^2·K, R_{cnv} = 12.5 m·K/W and $T_{s,p}$ = 351.6 K, yielding R_{rad} = 355 m·K/W. The effect of radiation is then negligible.

COMMENTS: (1) The plate conduction resistance is negligible, and even for h = 200 W/m^2·K, the contact resistance is small relative to the convection resistance. However, $R_{t,c}$ could be rendered negligible by using indium foil, instead of an air gap, at the interface. From Table 3.1, $R''_{t,c} = 0.07 \times 10^{-4}\ \text{m}^2 \cdot \text{K/W}$, in which case $R_{t,c}$ = 0.035 m·K/W.

(2) Because $A_c < W^2$, heat transfer by conduction in the plate is actually two-dimensional, rendering the conduction resistance even smaller.

PROBLEM 3.29

KNOWN: Conduction in a conical section with prescribed diameter, D, as a function of x in the form $D = ax^{1/2}$.

FIND: (a) Temperature distribution, T(x), (b) Heat transfer rate, q_x.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in x-direction, (3) No internal heat generation, (4) Constant properties.

PROPERTIES: *Table A-2*, Pure Aluminum (500K): k= 236 W/m·K.

ANALYSIS: (a) Based upon the assumptions, and following the same methodology of Example 3.3, q_x is a constant independent of x. Accordingly,

$$q_x = -kA\frac{dT}{dx} = -k\left[\pi\left(ax^{1/2}\right)^2/4\right]\frac{dT}{dx} \qquad (1)$$

using $A = \pi D^2/4$ where $D = ax^{1/2}$. Separating variables and identifying limits,

$$\frac{4q_x}{\pi\, a^2 k}\int_{x_1}^{x}\frac{dx}{x} = -\int_{T_1}^{T} dT. \qquad (2)$$

Integrating and solving for T(x) and then for T_2,

$$T(x) = T_1 - \frac{4q_x}{\pi\, a^2 k}\ln\frac{x}{x_1} \qquad T_2 = T_1 - \frac{4q_x}{\pi\, a^2 k}\ln\frac{x_2}{x_1}. \qquad (3,4)$$

Solving Eq. (4) for q_x and then substituting into Eq. (3) gives the results,

$$q_x = -\frac{\pi}{4}a^2 k\left(T_1 - T_2\right)/\ln\left(x_1/x_2\right) \qquad (5)$$

$$T(x) = T_1 + \left(T_1 - T_2\right)\frac{\ln\left(x/x_1\right)}{\ln\left(x_1/x_2\right)}.$$ <

From Eq. (1) note that $(dT/dx)\cdot x$ = Constant. It follows that T(x) has the distribution shown above.

(b) The heat rate follows from Eq. (5),

$$q_x = \frac{\pi}{4}\times 0.5^2\,\text{m}\times 236\frac{\text{W}}{\text{m}\cdot\text{K}}\left(600-400\right)\text{K}/\ln\frac{25}{125} = 5.76\,\text{kW}.$$ <

PROBLEM 3.30

KNOWN: Geometry and surface conditions of a truncated solid cone.

FIND: (a) Temperature distribution, (b) Rate of heat transfer across the cone.

SCHEMATIC:

$T_1 = 100°C$, Aluminum, $D = ax^{3/2}$, $T_2 = 20°C$, $x_1 = 0.075m$, $x_2 = 0.225m$, $a = 1m^{-1/2}$, q_x, x

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in x, (3) Constant properties.

PROPERTIES: *Table A-1*, Aluminum (333K): k = 238 W/m·K.

ANALYSIS: (a) From Fourier's law, Eq. (2.1), with $A = \pi D^2/4 = \left(\pi a^2/4\right)x^3$, it follows that

$$\frac{4q_x dx}{\pi a^2 x^3} = -k dT.$$

Hence, since q_x is independent of x,

$$\frac{4q_x}{\pi a^2}\int_{x_1}^{x}\frac{dx}{x^3} = -k\int_{T_1}^{T} dT$$

or

$$\frac{4q_x}{\pi a^2}\left[-\frac{1}{2x^2}\right]\Bigg|_{x_1}^{x} = -k\left(T - T_1\right).$$

Hence

$$T = T_1 + \frac{2q_x}{\pi a^2 k}\left[\frac{1}{x^2} - \frac{1}{x_1^2}\right]. \qquad <$$

(b) From the foregoing expression, it also follows that

$$q_x = \frac{\pi a^2 k}{2}\frac{T_2 - T_1}{\left[1/x_2^2 - 1/x_1^2\right]}$$

$$q_x = \frac{\pi\left(1m^{-1}\right)238\ W/m\cdot K}{2}\times\frac{(20-100)^\circ C}{\left[(0.225)^{-2} - (0.075)^{-2}\right]m^{-2}}$$

$$q_x = 189\ W. \qquad <$$

COMMENTS: The foregoing results are approximate due to use of a one-dimensional model in treating what is inherently a two-dimensional problem.

PROBLEM 3.31

KNOWN: Temperature dependence of the thermal conductivity, k.

FIND: Heat flux and form of temperature distribution for a plane wall.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional conduction through a plane wall, (2) Steady-state conditions, (3) No internal heat generation.

ANALYSIS: For the assumed conditions, q_x and A(x) are constant and Eq. 3.21 gives

$$q''_x \int_0^L dx = -\int_{T_o}^{T_1} (k_o + aT) dT$$

$$q''_x = \frac{1}{L}\left[k_o (T_o - T_1) + \frac{a}{2}\left(T_o^2 - T_1^2\right)\right].$$

From Fourier's law,

$$q''_x = -(k_o + aT)\ dT/dx.$$

Hence, since the product of (k_o+aT) and dT/dx) is constant, decreasing T with increasing x implies,

a > 0: decreasing (k_o+aT) and increasing |dT/dx| with increasing x

a = 0: k = k_o => constant (dT/dx)

a < 0: increasing (k_o+aT) and decreasing |dT/dx| with increasing x.

The temperature distributions appear as shown in the above sketch.

PROBLEM 3.32

KNOWN: Temperature dependence of tube wall thermal conductivity.

FIND: Expressions for heat transfer per unit length and tube wall thermal (conduction) resistance.

SCHEMATIC:

r_i, T_i

$k = k_o(1 + aT)$

r_o, T_o

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional radial conduction, (3) No internal heat generation.

ANALYSIS: From Eq. 3.24, the appropriate form of Fourier's law is

$$q_r = -kA_r \frac{dT}{dr} = -k(2\pi\, rL)\frac{dT}{dr}$$

$$q'_r = -2\pi\, kr\frac{dT}{dr}$$

$$q'_r = -2\pi\, rk_o(1+aT)\frac{dT}{dr}.$$

Separating variables,

$$-\frac{q'_r}{2\pi}\frac{dr}{r} = k_o(1+aT)dT$$

and integrating across the wall, find

$$-\frac{q'_r}{2\pi}\int_{r_i}^{r_o}\frac{dr}{r} = k_o\int_{T_i}^{T_o}(1+aT)dT$$

$$-\frac{q'_r}{2\pi}\ln\frac{r_o}{r_i} = k_o\left[T + \frac{aT^2}{2}\right]\Bigg|_{T_i}^{T_o}$$

$$-\frac{q'_r}{2\pi}\ln\frac{r_o}{r_i} = k_o\left[(T_o - T_i) + \frac{a}{2}\left(T_o^2 - T_i^2\right)\right]$$

$$q'_r = -2\pi k_o\left[1 + \frac{a}{2}(T_o + T_i)\right]\frac{(T_o - T_i)}{\ln(r_o/r_i)}. \qquad <$$

It follows that the overall thermal resistance per unit length is

$$R'_t = \frac{\Delta T}{q'_r} = \frac{\ln(r_o/r_i)}{2\pi k_o\left[1 + \frac{a}{2}(T_o + T_i)\right]}. \qquad <$$

COMMENTS: Note the necessity of the stated assumptions to treating q'_r as independent of r.

PROBLEM 3.33

KNOWN: Steady-state temperature distribution of convex shape for material with $k = k_o(1 + \alpha T)$ where α is a constant and the mid-point temperature is ΔT_o higher than expected for a linear temperature distribution.

FIND: Relationship to evaluate α in terms of ΔT_o and T_1, T_2 (the temperatures at the boundaries).

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) No internal heat generation, (4) α is positive and constant.

ANALYSIS: At any location in the wall, Fourier's law has the form

$$q''_x = -k_o(1+\alpha T)\frac{dT}{dx}. \qquad (1)$$

Since q''_x is a constant, we can separate Eq. (1), identify appropriate integration limits, and integrate to obtain

$$\int_0^L q''_x dx = -\int_{T_1}^{T_2} k_o(1+\alpha T)dT \qquad (2)$$

$$q''_x = -\frac{k_o}{L}\left[\left(T_2 + \frac{\alpha T_2^2}{2}\right) - \left(T_1 + \frac{\alpha T_1^2}{2}\right)\right]. \qquad (3)$$

We could perform the same integration, but with the upper limits at x = L/2, to obtain

$$q''_x = -\frac{2k_o}{L}\left[\left(T_{L/2} + \frac{\alpha T_{L/2}^2}{2}\right) - \left(T_1 + \frac{\alpha T_1^2}{2}\right)\right] \qquad (4)$$

where

$$T_{L/2} = T(L/2) = \frac{T_1 + T_2}{2} + \Delta T_o. \qquad (5)$$

Setting Eq. (3) equal to Eq. (4), substituting from Eq. (5) for $T_{L/2}$, and solving for α, it follows that

$$\alpha = \frac{2\Delta T_o}{\left(T_2^2 + T_1^2\right)/2 - \left[(T_1 + T_2)/2 + \Delta T_o\right]^2}.$$ <

PROBLEM 3.34

KNOWN: Hollow cylinder of thermal conductivity k, inner and outer radii, r_i and r_o, respectively, and length L.

FIND: Thermal resistance using the alternative conduction analysis method.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional radial conduction, (3) No internal volumetric generation, (4) Constant properties.

ANALYSIS: For the differential control volume, energy conservation requires that $q_r = q_{r+dr}$ for steady-state, one-dimensional conditions with no heat generation. With Fourier's law,

$$q_r = -kA\frac{dT}{dr} = -k\left(2\pi\ rL\right)\frac{dT}{dr} \qquad (1)$$

where $A = 2\pi rL$ is the area normal to the direction of heat transfer. Since q_r is constant, Eq. (1) may be separated and expressed in integral form,

$$\frac{q_r}{2\pi\ L}\int_{r_i}^{r_o}\frac{dr}{r} = -\int_{T_i}^{T_o} k\left(T\right)dT.$$

Assuming k is constant, the heat rate is

$$q_r = \frac{2\pi\ Lk\left(T_i - T_o\right)}{\ln\left(r_o / r_i\right)}.$$

Remembering that the thermal resistance is defined as

$$R_t \equiv \Delta T/q$$

it follows that for the hollow cylinder,

$$R_t = \frac{\ln\left(r_o / r_i\right)}{2\pi\ LK}.$$ <

COMMENTS: Compare the *alternative* method used in this analysis with the *standard* method employed in Section 3.3.1 to obtain the same result.

PROBLEM 3.35

KNOWN: Thickness and inner surface temperature of calcium silicate insulation on a steam pipe. Convection and radiation conditions at outer surface.

FIND: (a) Heat loss per unit pipe length for prescribed insulation thickness and outer surface temperature. (b) Heat loss and radial temperature distribution as a function of insulation thickness.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties.

PROPERTIES: *Table A-3*, Calcium Silicate (T = 645 K): k = 0.089 W/m·K.

ANALYSIS: (a) From Eq. 3.27 with $T_{s,2}$ = 490 K, the heat rate per unit length is

$$q' = q_r/L = \frac{2\pi k\left(T_{s,1} - T_{s,2}\right)}{\ln\left(r_2/r_1\right)}$$

$$q' = \frac{2\pi\left(0.089\,W/m\cdot K\right)\left(800-490\right)K}{\ln\left(0.08\,m/0.06\,m\right)}$$

$$q' = 603\,W/m\,.$$ <

(b) Performing an energy for a control surface around the outer surface of the insulation, it follows that

$$q'_{cond} = q'_{conv} + q'_{rad}$$

$$\frac{T_{s,1} - T_{s,2}}{\ln\left(r_2/r_1\right)/2\pi k} = \frac{T_{s,2} - T_\infty}{1/\left(2\pi r_2 h\right)} + \frac{T_{s,2} - T_{sur}}{1/\left(2\pi r_2 h_r\right)}$$

where $h_r = \varepsilon\sigma\left(T_{s,2} + T_{sur}\right)\left(T_{s,2}^2 + T_{sur}^2\right)$. Solving this equation for $T_{s,2}$, the heat rate may be determined from

$$q' = 2\pi r_2\left[h\left(T_{s,2} - T_\infty\right) + h_r\left(T_{s,2} - T_{sur}\right)\right]$$

Continued...

and from Eq. 3.26 the temperature distribution is

$$T(r) = \frac{T_{s,1} - T_{s,2}}{\ln(r_1/r_2)} \ln\left(\frac{r}{r_2}\right) + T_{s,2}$$

As shown below, the outer surface temperature of the insulation $T_{s,2}$ and the heat loss q' decay precipitously with increasing insulation thickness from values of $T_{s,2} = T_{s,1} = 800$ K and $q' = 11,600$ W/m, respectively, at $r_2 = r_1$ (no insulation).

When plotted as a function of a dimensionless radius, $(r - r_1)/(r_2 - r_1)$, the temperature decay becomes more pronounced with increasing r_2.

Note that $T(r_2) = T_{s,2}$ increases with decreasing r_2 and a linear temperature distribution is approached as r_2 approaches r_1.

COMMENTS: An insulation layer thickness of 20 mm is sufficient to maintain the outer surface temperature and heat rate below 350 K and 1000 W/m, respectively.

PROBLEM 3.36

KNOWN: Temperature and volume of hot water heater. Nature of heater insulating material. Ambient air temperature and convection coefficient. Unit cost of electric power.

FIND: Heater dimensions and insulation thickness for which annual cost of heat loss is less than \$50.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional, steady-state conduction through side and end walls, (2) Conduction resistance dominated by insulation, (3) Inner surface temperature is approximately that of the water ($T_{s,1}$ = 55°C), (4) Constant properties, (5) Negligible radiation.

PROPERTIES: *Table A.3*, Urethane Foam (T = 300 K): k = 0.026 W/m·K.

ANALYSIS: To minimize heat loss, tank dimensions which minimize the total surface area, $A_{s,t}$, should be selected. With $L = 4\forall/\pi D^2$, $A_{s,t} = \pi DL + 2\left(\pi D^2/4\right) = 4\forall/D + \pi D^2/2$, and the tank diameter for which $A_{s,t}$ is an extremum is determined from the requirement

$$dA_{s,t}/dD = -4\forall/D^2 + \pi D = 0$$

It follows that

$$D = \left(4\forall/\pi\right)^{1/3} \qquad \text{and} \qquad L = \left(4\forall/\pi\right)^{1/3}$$

With $d^2A_{s,t}/dD^2 = 8\forall/D^3 + \pi > 0$, the foregoing conditions yield the desired minimum in $A_{s,t}$. Hence, for $\forall$ = 100 gal × 0.00379 m³/gal = 0.379 m³,

$$D_{op} = L_{op} = 0.784\,\text{m}$$ <

The total heat loss through the side and end walls is

$$q = \frac{T_{s,1} - T_\infty}{\dfrac{\ln\left(r_2/r_1\right)}{2\pi k L_{op}} + \dfrac{1}{h 2\pi r_2 L_{op}}} + \frac{2\left(T_{s,1} - T_\infty\right)}{\dfrac{\delta}{k\left(\pi D_{op}^2/4\right)} + \dfrac{1}{h\left(\pi D_{op}^2/4\right)}}$$

We begin by estimating the heat loss associated with a 25 mm thick layer of insulation. With $r_1 = D_{op}/2$ = 0.392 m and $r_2 = r_1 + \delta$ = 0.417 m, it follows that

Continued...

$$q = \frac{(55-20)^\circ C}{\dfrac{\ln(0.417/0.392)}{2\pi(0.026\,W/m\cdot K)0.784\,m} + \dfrac{1}{\left(2\,W/m^2\cdot K\right)2\pi(0.417\,m)0.784\,m}}$$

$$+\frac{2(55-20)^\circ C}{\dfrac{0.025\,m}{(0.026\,W/m\cdot K)\pi/4(0.784\,m)^2} + \dfrac{1}{\left(2\,W/m^2\cdot K\right)\pi/4(0.784\,m)^2}}$$

$$q = \frac{35^\circ C}{(0.483+0.243)\,K/W} + \frac{2\left(35^\circ C\right)}{(1.992+1.036)\,K/W} = (48.2+23.1)\,W = 71.3\,W$$

The annual energy loss is therefore

$$Q_{annual} = 71.3\,W\,(365\,days)(24\ h/day)\left(10^{-3}\ kW/W\right) = 625\,kWh$$

With a unit electric power cost of \$0.08/kWh, the annual cost of the heat loss is

C = (\$0.08/kWh)625 kWh = \$50.00

Hence, an insulation thickness of

δ = 25 mm <

will satisfy the prescribed cost requirement.

COMMENTS: Cylindrical containers of aspect ratio L/D = 1 are seldom used because of floor space constraints. Choosing L/D = 2, $\forall = \pi D^3/2$ and $D = (2\forall/\pi)^{1/3}$ = 0.623 m. Hence, L = 1.245 m, r_1 = 0.312m and r_2 = 0.337 m. It follows that q = 76.1 W and C = \$53.37. The 6.7% increase in the annual cost of the heat loss is small, providing little justification for using the optimal heater dimensions.

PROBLEM 3.37

KNOWN: Inner and outer radii of a tube wall which is heated electrically at its outer surface and is exposed to a fluid of prescribed h and T_∞. Thermal contact resistance between heater and tube wall and wall inner surface temperature.

FIND: Heater power per unit length required to maintain a heater temperature of 25°C.

SCHEMATIC:

$r_o = 75\,mm$, $r_i = 25\,mm$, $T_o = 25°C$, $T_i = 5°C$, Electrical heater (q'), Fluid, $h = 100\,W/m^2\cdot K$, $T_\infty = -10°C$, $k = 10\,W/m\cdot K$, $R'_{t,c} = 0.01\,m\cdot K/W$

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible temperature drop across heater.

ANALYSIS: The thermal circuit has the form

T_i, q'_a, $\frac{\ln(r_o/r_i)}{2\pi k}$, $R'_{t,c}$, T_o, q', $(1/h\pi D_o)$, T_∞, q'_b

Applying an energy balance to a control surface about the heater,

$$q' = q'_a + q'_b$$

$$q' = \frac{T_o - T_i}{\frac{\ln(r_o/r_i)}{2\pi k} + R'_{t,c}} + \frac{T_o - T_\infty}{(1/h\pi D_o)}$$

$$q' = \frac{(25\text{-}5)^\circ C}{\frac{\ln(75mm/25mm)}{2\pi \times 10\ W/m\cdot K} + 0.01\frac{m\cdot K}{W}} + \frac{\left[25-(-10)\right]^\circ C}{\left[1/\left(100\ W/m^2\cdot K \times \pi \times 0.15m\right)\right]}$$

$$q' = (728 + 1649)\ W/m$$

$$q' = 2377\ W/m.$$ <

COMMENTS: The conduction, contact and convection resistances are 0.0175, 0.01 and 0.021 m ·K/W, respectively,

PROBLEM 3.38

KNOWN: Inner and outer radii of a tube wall which is heated electrically at its outer surface. Inner and outer wall temperatures. Temperature of fluid adjoining outer wall.

FIND: Effect of wall thermal conductivity, thermal contact resistance, and convection coefficient on total heater power and heat rates to outer fluid and inner surface.

SCHEMATIC:

T_o = 25 °C; Electrical heater, q'; Fluid; r_o = 75 mm; r_i = 25 mm; T_i = 5 °C; $10 \le h \le 1000$ W/m²·K; T_∞ = -10 °C; $1 \le k \le 200$ W/m·K; $0 \le R'_{t,c} \le 0.1$ m·K/W

Thermal circuit: q_i' ← T_i — $\frac{ln(r_o/r_i)}{2\pi k}$ — $R'_{t,c}$ — T_o (with q' entering) — $(1/2\pi r_o h)$ — T_∞ → q_o'

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible temperature drop across heater, (5) Negligible radiation.

ANALYSIS: Applying an energy balance to a control surface about the heater,

$$q' = q_i' + q_o'$$

$$q' = \frac{T_o - T_i}{\frac{\ln(r_o/r_i)}{2\pi k} + R'_{t,c}} + \frac{T_o - T_\infty}{(1/2\pi r_o h)}$$

Selecting nominal values of k = 10 W/m·K, $R'_{t,c}$ = 0.01 m·K/W and h = 100 W/m²·K, the following parametric variations are obtained

Heat rate (W/m) vs. Thermal conductivity, k(W/m.K) — legend: qi, q, qo

Heat rate(W/m) vs. Contact resistance, Rtc(m.K/W) — legend: qi, q, qo

Continued...

PROBLEM 3.38 (Cont.)

Heat rate(W/m)

20000
16000
12000
8000
4000
0

0 200 400 600 800 1000

Convection coefficient, h(W/m^2.K)

qi
q
qo

For a prescribed value of h, q'_o is fixed, while q'_i, and hence q', increase and decrease, respectively, with increasing k and $R'_{t,c}$. These trends are attributable to the effects of k and $R'_{t,c}$ on the total (conduction plus contact) resistance separating the heater from the inner surface. For fixed k and $R'_{t,c}$, q'_i is fixed, while q'_o, and hence q', increase with increasing h due to a reduction in the convection resistance.

COMMENTS: For the prescribed nominal values of k, $R'_{t,c}$ and h, the electric power requirement is q' = 2377 W/m. To maintain the prescribed heater temperature, q' would increase with any changes which reduce the conduction, contact and/or convection resistances.

PROBLEM 3.39

KNOWN: Wall thickness and diameter of stainless steel tube. Inner and outer fluid temperatures and convection coefficients.

FIND: (a) Heat gain per unit length of tube, (b) Effect of adding a 10 mm thick layer of insulation to outer surface of tube.

SCHEMATIC:

r_3 = 30 mm
r_2 = 20 mm
r_1 = 18 mm
Pharmaceutical
$T_{\infty,i}$ = 6°C
h_i = 400 W/m²-K
Ambient air
$T_{\infty,o}$ = 23°C
h_o = 6 W/m²-K
Insulation
k_{ins} = 0.05 W/m-K
$T_{\infty,i}$, $R'_{cond,ss}$, $R'_{cond,ins}$, $T_{\infty,o}$, q', $R'_{conv,i}$, $R'_{conv,o}$

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional radial conduction, (3) Constant properties, (4) Negligible contact resistance between tube and insulation, (5) Negligible effect of radiation.

PROPERTIES: *Table A-1*, St. St. 304 (~280K): k_{st} = 14.4 W/m·K.

ANALYSIS: (a) Without the insulation, the total thermal resistance per unit length is

$$R'_{tot} = R'_{conv,i} + R'_{cond,st} + R'_{conv,o} = \frac{1}{2\pi r_i h_i} + \frac{\ln(r_2/r_i)}{2\pi k_{st}} + \frac{1}{2\pi r_2 h_o}$$

$$R'_{tot} = \frac{1}{2\pi(0.018\text{m})400\ \text{W/m}^2\cdot\text{K}} + \frac{\ln(20/18)}{2\pi(14.4\ \text{W/m}\cdot\text{K})} + \frac{1}{2\pi(0.020\text{m})6\ \text{W/m}^2\cdot\text{K}}$$

$$R'_{tot} = \left(0.0221 + 1.16\times10^{-3} + 1.33\right)\text{m}\cdot\text{K/W} = 1.35\ \text{m}\cdot\text{K/W}$$

The heat gain per unit length is then

$$q' = \frac{T_{\infty,o} - T_{\infty,i}}{R'_{tot}} = \frac{(23-6)°\text{C}}{1.35\,\text{m}\cdot\text{K/W}} = 12.6\ \text{W/m}$$ <

(b) With the insulation, the total resistance per unit length is now $R'_{tot} = R'_{conv,i} + R'_{cond,st} + R'_{cond,ins} + R'_{conv,o}$, where $R'_{conv,i}$ and $R'_{cond,st}$ remain the same. The thermal resistance of the insulation is

$$R'_{cond,ins} = \frac{\ln(r_3/r_2)}{2\pi k_{ins}} = \frac{\ln(30/20)}{2\pi(0.05\ \text{W/m}\cdot\text{K})} = 1.29\ \text{m}\cdot\text{K/W}$$

and the outer convection resistance is now

$$R'_{conv,o} = \frac{1}{2\pi r_3 h_o} = \frac{1}{2\pi(0.03\text{m})6\ \text{W/m}^2\cdot\text{K}} = 0.88\ \text{m}\cdot\text{K/W}$$

The total resistance is now

$$R'_{tot} = \left(0.0221 + 1.16\times10^{-3} + 1.29 + 0.88\right)\text{m}\cdot\text{K/W} = 2.20\,\text{m}\cdot\text{K/W}$$

Continued …..

and the heat gain per unit length is

$$q' = \frac{T_{\infty,o} - T_{\infty,i}}{R'_{tot}} = \frac{17°C}{2.20\ m \cdot K/W} = 7.7\ W/m$$

COMMENTS: (1) The validity of assuming negligible radiation may be assessed for the worst case condition corresponding to the bare tube. Assuming a tube outer surface temperature of $T_s = T_{\infty,i}$ = 279K, large surroundings at $T_{sur} = T_{\infty,o}$ = 296K, and an emissivity of ε = 0.7, the heat gain due to net radiation exchange with the surroundings is $q'_{rad} = \varepsilon\sigma(2\pi r_2)\left(T_{sur}^4 - T_s^4\right) = 7.7\ W/m$. Hence, the net rate of heat transfer by radiation to the tube surface is comparable to that by convection, and the assumption of negligible radiation is inappropriate.

(2) If heat transfer from the air is by natural convection, the value of h_o with the insulation would actually be less than the value for the bare tube, thereby further reducing the heat gain. Use of the insulation would also increase the outer surface temperature, thereby reducing net radiation transfer from the surroundings.

(3) The critical radius is $r_{cr} = k_{ins}/h \approx 8$ mm $< r_2$. Hence, as indicated by the calculations, heat transfer is reduced by the insulation.

PROBLEM 3.40

KNOWN: Diameter, wall thickness and thermal conductivity of steel tubes. Temperature of steam flowing through the tubes. Thermal conductivity of insulation and emissivity of aluminum sheath. Temperature of ambient air and surroundings. Convection coefficient at outer surface and maximum allowable surface temperature.

FIND: (a) Minimum required insulation thickness (r3 – r2) and corresponding heat loss per unit length, (b) Effect of insulation thickness on outer surface temperature and heat loss.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state, (2) One-dimensional radial conduction, (3) Negligible contact resistances at the material interfaces, (4) Negligible steam side convection resistance ($T_{\infty,i} = T_{s,i}$), (5) Negligible conduction resistance for aluminum sheath, (6) Constant properties, (7) Large surroundings.

ANALYSIS: (a) To determine the insulation thickness, an energy balance must be performed at the outer surface, where $q' = q'_{conv,o} + q'_{rad}$. With $q'_{conv,o} = 2\pi r_3 h_o \left(T_{s,o} - T_{\infty,o}\right)$, $q'_{rad} = 2\pi r_3 \ \varepsilon\sigma \left(T_{s,o}^4 - T_{sur}^4\right)$, $q' = \left(T_{s,i} - T_{s,o}\right)/\left(R'_{cond,st} + R'_{cond,ins}\right)$, $R'_{cond,st} = \ell n\left(r_2/r_1\right)/2\pi k_{st}$, and $R'_{cond,ins} = \ell n\left(r_3/r_2\right)/2\pi k_{ins}$, it follows that

$$\frac{2\pi\left(T_{s,i} - T_{s,o}\right)}{\dfrac{\ell n\left(r_2/r_1\right)}{k_{st}} + \dfrac{\ell n\left(r_3/r_2\right)}{k_{ins}}} = 2\pi r_3\left[h_o\left(T_{s,o} - T_{\infty,o}\right) + \varepsilon\sigma\left(T_{s,o}^4 - T_{sur}^4\right)\right]$$

$$\frac{2\pi\left(848 - 323\right)\text{K}}{\dfrac{\ell n\left(0.18/0.15\right)}{35\ \text{W/m}\cdot\text{K}} + \dfrac{\ell n\left(r_3/0.18\right)}{0.10\ \text{W/m}\cdot\text{K}}} = 2\pi r_3\left[6\ \text{W/m}^2\cdot\text{K}\left(323 - 300\right)\text{K} + 0.20\times 5.67\times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4\left(323^4 - 300^4\right)\text{K}^4\right]$$

A trial-and-error solution yields r_3 = 0.394 m = 394 mm, in which case the insulation thickness is

$$t_{ins} = r_3 - r_2 = 214\,\text{mm}$$ <

The heat rate is then

$$q' = \frac{2\pi\left(848 - 323\right)\text{K}}{\dfrac{\ell n\left(0.18/0.15\right)}{35\,\text{W/m}\cdot\text{K}} + \dfrac{\ell n\left(0.394/0.18\right)}{0.10\,\text{W/m}\cdot\text{K}}} = 420\,\text{W/m}$$ <

(b) The effects of r_3 on $T_{s,o}$ and q' have been computed and are shown below.

Conditioned …..

Beyond $r_3 \approx 0.40$m, there are rapidly diminishing benefits associated with increasing the insulation thickness.

COMMENTS: Note that the thermal resistance of the insulation is much larger than that for the tube wall. For the conditions of Part (a), the radiation coefficient is $h_r = 1.37$ W/m, and the heat loss by radiation is less than 25% of that due to natural convection $\left(q'_{rad} = 78\,\text{W/m},\ q'_{conv,o} = 342\ \text{W/m}\right)$.

PROBLEM 3.41

KNOWN: Thin electrical heater fitted between two concentric cylinders, the outer surface of which experiences convection.

FIND: (a) Electrical power required to maintain outer surface at a specified temperature, (b) Temperature at the center

SCHEMATIC:

$A, k_A = 0.15\ \frac{W}{m \cdot K}$ — $r_1 = 20\ mm$ — $r_2 = 40\ mm$

Thin electrical heater $(r = r_1)$

$B, k_B = 1.5\ \frac{W}{m \cdot K}$ — $T_\infty = -15°C$ — $h = 50\ W/m^2 \cdot K$

ASSUMPTIONS: (1) One-dimensional, radial conduction, (2) Steady-state conditions, (3) Heater element has negligible thickness, (4) Negligible contact resistance between cylinders and heater, (5) Constant properties, (6) No generation.

ANALYSIS: (a) Perform an energy balance on the composite system to determine the power required to maintain $T(r_2) = T_s = 5°C$.

T_∞, h — q'_{conv} — T_s — $\dot{E}'_{in} = q'_{elec}$

$$\dot{E}'_{in} - \dot{E}'_{out} + \dot{E}_{gen} = \dot{E}_{st}$$

$$+q'_{elec} - q'_{conv} = 0.$$

Using Newton's law of cooling,

$$q'_{elec} = q'_{conv} = h \cdot 2\pi\ r_2 \left(T_s - T_\infty\right)$$

$$q'_{elec} = 50\frac{W}{m^2 \cdot K} \times 2\pi\left(0.040 m\right)\ \left[5-\left(-15\right)\right]^\circ C = 251\ W/m. \qquad <$$

(b) From a control volume about Cylinder A, we recognize that the cylinder must be isothermal, that is,

$$T(0) = T(r_1).$$

Represent Cylinder B by a thermal circuit:

$T(r_1)$ — R'_B — T_s — q'

$$q' = \frac{T\left(r_1\right) - T_s}{R'_B}$$

For the cylinder, from Eq. 3.28,

$$R'_B = \ln r_2 / r_1 / 2\pi\ k_B$$

giving

$$T\left(r_1\right) = T_s + q'R'_B = 5^\circ C + 253.1\frac{W}{m}\frac{\ln 40/20}{2\pi \times 1.5\ W/m \cdot K} = 23.5^\circ C$$

Hence, $T(0) = T(r_1) = 23.5°C$. $\qquad <$

Note that k_A has no influence on the temperature $T(0)$.

PROBLEM 3.42

KNOWN: Electric current and resistance of wire. Wire diameter and emissivity. Thickness, emissivity and thermal conductivity of coating. Temperature of ambient air and surroundings. Expression for heat transfer coefficient at surface of the wire or coating.

FIND: (a) Heat generation per unit length and volume of wire, (b) Temperature of uninsulated wire, (c) Inner and outer surface temperatures of insulation.

SCHEMATIC:

Insulation
$k_i = 0.25$ W/m-K
$\varepsilon_i = 0.90$
Air
$T_\infty = 20^oC$
$T_{sur} = 20^oC$
$T_{s,2}$, $r_2 = 3$ mm
$T_{s,1}$, $r_1 = 1$ mm
$h = 1.25[(T - T_\infty)/D]^{1/4}$
I = 20 A
Wire T, $R'_{elec} = 0.01\ \Omega$/m, $\varepsilon_w = 0.3$

ASSUMPTIONS: (1) Steady-state, (2) One-dimensional radial conduction through insulation, (3) Constant properties, (4) Negligible contact resistance between insulation and wire, (5) Negligible radial temperature gradients in wire, (6) Large surroundings.

ANALYSIS: (a) The rates of energy generation per unit length and volume are, respectively,

$$\dot{E}'_g = I^2 R'_{elec} = (20\,A)^2 (0.01\Omega/m) = 4\ W/m$$
<

$$\dot{q} = \dot{E}'_g / A_c = 4\dot{E}'_g / \pi D^2 = 16\,W/m / \pi(0.002m)^2 = 1.27\times10^6\ W/m^3$$
<

(b) Without the insulation, an energy balance at the surface of the wire yields

$$\dot{E}'_g = q' = q'_{conv} + q'_{rad} = \pi D h (T - T_\infty) + \pi D \varepsilon_w \sigma \left(T^4 - T_{sur}^4\right)$$

where $h = 1.25[(T - T_\infty)/D]^{1/4}$. Substituting,

$$4\,W/m = 1.25\pi(0.002m)^{3/4}(T-293)^{5/4} + \pi(0.002m)0.3\times5.67\times10^{-8}\,W/m^2\cdot K^4\left(T^4 - 293^4\right)K^4$$

and a trial-and-error solution yields

$$T = 331K = 58°C$$
<

(c) Performing an energy balance at the outer surface,

$$\dot{E}'_g = q' = q'_{conv} + q'_{rad} = \pi D h \left(T_{s,2} - T_\infty\right) + \pi D \varepsilon_i \sigma \left(T_{s,2}^4 - T_{sur}^4\right)$$

$$4\,W/m = 1.25\pi(0.006m)^{3/4}\left(T_{s,2} - 293\right)^{5/4} + \pi(0.006m)0.9\times5.67\times10^{-8}\,W/m^2\cdot K^4\left(T_{s,2}^4 - 293^4\right)K^4$$

and an iterative solution yields the following value of the surface temperature

$$T_{s,2} = 307.8\,K = 34.8°C$$
<

The inner surface temperature may then be obtained from the following expression for heat transfer by conduction in the insulation.

Continued …..

PROBLEM 3.42 (Cont.)

$$q' = \frac{T_{s,i} - T_2}{R'_{cond}} = \frac{T_{s,i} - T_{s,2}}{\ell n(r_2/r_1)/2\pi k_i}$$

$$4\,W = \frac{2\pi(0.25\,W/m\cdot K)(T_{s,i} - 307.8\,K)}{\ell n\,3}$$

$$T_{s,i} = 310.6\,K = 37.6°C$$ <

As shown below, the effect of increasing the insulation thickness is to *reduce*, not increase, the surface temperatures.

Surface temperatures, C
Insulation thickness, mm
Inner surface temperature, C
Outer surface temperature, C

This behavior is due to a reduction in the total resistance to heat transfer with increasing r_2. Although the convection, h, and radiation, $h_r = \varepsilon\sigma(T_{s,2} + T_{sur})(T_{s,2}^2 + T_{sur}^2)$, coefficients decrease with increasing r_2, the corresponding increase in the surface area is more than sufficient to provide for a reduction in the total resistance. Even for an insulation thickness of t = 4 mm, h = h + h_r = (7.1 + 5.4) $W/m^2\cdot K$ = 12.5 $W/m^2\cdot K$, and r_{cr} = k/h = 0.25 W/m·K/12.5 $W/m^2\cdot K$ = 0.020m = 20 mm > r_2 = 5 mm. The outer radius of the insulation is therefore well below the critical radius.

PROBLEM 3.43

KNOWN: Diameter of electrical wire. Thickness and thermal conductivity of rubberized sheath. Contact resistance between sheath and wire. Convection coefficient and ambient air temperature. Maximum allowable sheath temperature.

FIND: Maximum allowable power dissipation per unit length of wire. Critical radius of insulation.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state, (2) One-dimensional radial conduction through insulation, (3) Constant properties, (4) Negligible radiation exchange with surroundings.

ANALYSIS: The maximum insulation temperature corresponds to its inner surface and is independent of the contact resistance. From the thermal circuit, we may write

$$\dot{E}'_g = q' = \frac{T_{in,i} - T_\infty}{R'_{cond} + R'_{conv}} = \frac{T_{in,i} - T_\infty}{\left[\ell n\left(r_{in,o}/r_{in,i}\right)/2\pi k\right] + \left(1/2\pi r_{in,o} h\right)}$$

where $r_{in,i} = D/2 = 0.001\text{m}$, $r_{in,o} = r_{in,i} + t = 0.003\text{m}$, and $T_{in,i} = T_{max} = 50°\text{C}$ yields the maximum allowable power dissipation. Hence,

$$\dot{E}'_{g,max} = \frac{(50-20)°\text{C}}{\dfrac{\ell n\, 3}{2\pi \times 0.13\,\text{W/m}\cdot\text{K}} + \dfrac{1}{2\pi\,(0.003\text{m})\,10\,\text{W/m}^2\cdot\text{K}}} = \frac{30°\text{C}}{(1.35 + 5.31)\,\text{m}\cdot\text{K/W}} = 4.51\,\text{W/m} \quad <$$

The critical insulation radius is also unaffected by the contact resistance and is given by

$$r_{cr} = \frac{k}{h} = \frac{0.13\,\text{W/m}\cdot\text{K}}{10\,\text{W/m}^2\cdot\text{K}} = 0.013\text{m} = 13\,\text{mm} \quad <$$

Hence, $r_{in,o} < r_{cr}$ and $\dot{E}'_{g,max}$ could be increased by increasing $r_{in,o}$ up to a value of 13 mm (t = 12 mm).

COMMENTS: The contact resistance affects the temperature of the wire, and for $q' = \dot{E}'_{g,max} = 4.51\,\text{W/m}$, the outer surface temperature of the wire is $T_{w,o} = T_{in,i} + q' R'_{t,c} = 50°\text{C} + (4.51\,\text{W/m})\left(3\times10^{-4}\,\text{m}^2\cdot\text{K/W}\right)/\pi\,(0.002\text{m}) = 50.2°\text{C}$. Hence, the temperature change across the contact resistance is negligible.

PROBLEM 3.44

KNOWN: Long rod experiencing uniform volumetric generation of thermal energy, $\dot{q}$, concentric with a hollow ceramic cylinder creating an enclosure filled with air. Thermal resistance per unit length due to radiation exchange between enclosure surfaces is R'_{rad}. The free convection coefficient for the enclosure surfaces is h = 20 W/m^2·K.

FIND: (a) Thermal circuit of the system that can be used to calculate the surface temperature of the rod, T_r; label all temperatures, heat rates and thermal resistances; evaluate the thermal resistances; and (b) Calculate the surface temperature of the rod.

SCHEMATIC:

Rod
$\dot{q} = 2\times10^6$ W/m^3
Air space
h = 20 W/m^2-K
R'_{rad} = 0.30 m-K/W
T_r
T_i
D_r D_i D_o D
D_r = 20 mm
D_i = 40 mm
D_o = 120 mm
T_o = 25°C
Ceramic, k = 1.75 W/m-K

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional, radial conduction through the hollow cylinder, (3) The enclosure surfaces experience free convection and radiation exchange.

ANALYSIS: (a) The thermal circuit is shown below. Note labels for the temperatures, thermal resistances and the relevant heat fluxes.

Enclosure, radiation exchange (given):

$$R'_{rad} = 0.30\ \text{m}\cdot\text{K/W}$$

Enclosure, free convection:

$$R'_{cv,rod} = \frac{1}{h\pi D_r} = \frac{1}{20\ \text{W/m}^2\cdot\text{K}\times\pi\times0.020\text{m}} = 0.80\ \text{m}\cdot\text{K/W}$$

$$R'_{cv,cer} = \frac{1}{h\pi D_i} = \frac{1}{20\ \text{W/m}^2\cdot\text{K}\times\pi\times0.040\text{m}} = 0.40\ \text{m}\cdot\text{K/W}$$

Ceramic cylinder, conduction:

$$R'_{cd} = \frac{\ell n(D_o/D_i)}{2\pi k} = \frac{\ell n(0.120/0.040)}{2\pi\times1.75\ \text{W/m}\cdot\text{K}} = 0.10\ \text{m}\cdot\text{K/W}$$

The thermal resistance between the enclosure surfaces (r-i) due to convection and radiation exchange is

$$\frac{1}{R'_{enc}} = \frac{1}{R'_{rad}} + \frac{1}{R'_{cv,rod} + R'_{cv,cer}}$$

$$R'_{enc} = \left[\frac{1}{0.30} + \frac{1}{0.80+0.40}\right]^{-1} \text{m}\cdot\text{K/W} = 0.24\ \text{m}\cdot\text{K/W}$$

The total resistance between the rod surface (r) and the outer surface of the cylinder (o) is

$$R'_{tot} = R'_{enc} + R'_{cd} = (0.24+0.1)\text{m}\cdot\text{K/W} = 0.34\ \text{m}\cdot\text{K/W}$$

Continued …..

PROBLEM 3.44 (Cont.)

q' T_r R'_{rad} T_i R'_{cd} $T_o = 25^oC$

$R'_{cv,rod}$ $R'_{cv,cer}$

T_∞

(a) Thermal circuit

CV $\dot{E}'_g$ q'

(b) Overall energy balance on rod

(b) From an energy balance on the rod (see schematic) find T_r.

$$\dot{E}'_{in} - \dot{E}'_{out} + \dot{E}'_{gen} = 0$$

$$-q + \dot{q}\forall = 0$$

$$-(T_r - T_i)/R'_{tot} + \dot{q}\left(\pi D_r^2/4\right) = 0$$

$$-(T_r - 25)K/0.34\ m\cdot K/W + 2\times10^6 W/m^3\left(\pi\times0.020m^2/4\right) = 0$$

$$T_r = 239°C$$ <

COMMENTS: In evaluating the convection resistance of the air space, it was necessary to define an average air temperature (T_∞) and consider the convection coefficients for each of the space surfaces. As you'll learn later in Chapter 9, correlations are available for directly estimating the convection coefficient (h_{enc}) for the enclosure so that $q_{cv} = h_{enc}\ (T_r - T_1)$.

PROBLEM 3.45

KNOWN: Tube diameter and refrigerant temperature for evaporator of a refrigerant system. Convection coefficient and temperature of outside air.

FIND: (a) Rate of heat extraction without frost formation, (b) Effect of frost formation on heat rate, (c) Time required for a 2 mm thick frost layer to melt in ambient air for which h = 2 W/m^2·K and T_∞ = 20°C.

SCHEMATIC:

Frost
k = 0.4 W/m K
ρ = 700 kg/m^3
h_{sf} = 3.34 x 10^5 J/kg
Ambient air
$T_{\infty,o}$ = -3 °C
h = 100 W/m^2•K
Refrigerant
$T_{\infty,i}$ = -18 °C
Part(b)
$T_{s,2}$
r_1 = 5 mm
$T_{s,1}$
$r_2 = r_1 + \delta$
$(0 \le \delta \le 4$ mm)
Part(c)
r
q_{conv}
dr

ASSUMPTIONS: (1) One-dimensional, steady-state conditions, (2) Negligible convection resistance for refrigerant flow $\left(T_{\infty,i} = T_{s,1}\right)$, (3) Negligible tube wall conduction resistance, (4) Negligible radiation exchange at outer surface.

ANALYSIS: (a) The cooling capacity in the defrosted condition (δ = 0) corresponds to the rate of heat extraction from the airflow. Hence,

$$q' = h2\pi r_1\left(T_{\infty,o} - T_{s,1}\right) = 100\,W/m^2 \cdot K\left(2\pi \times 0.005\,m\right)\left(-3+18\right)^\circ C$$

$$q' = 47.1\,W/m$$ <

(b) With the frost layer, there is an additional (conduction) resistance to heat transfer, and the extraction rate is

$$q' = \frac{T_{\infty,o} - T_{s,1}}{R'_{conv} + R'_{cond}} = \frac{T_{\infty,o} - T_{s,1}}{1/\left(h2\pi r_2\right) + \ln\left(r_2/r_1\right)/2\pi k}$$

For 5 ≤ r_2 ≤ 9 mm and k = 0.4 W/m·K, this expression yields

Continued...

PROBLEM 3.45 (Cont.)

The heat extraction, and hence the performance of the evaporator coil, decreases with increasing frost layer thickness due to an increase in the total resistance to heat transfer. Although the convection resistance decreases with increasing δ, the reduction is exceeded by the increase in the conduction resistance.

(c) The time t_m required to melt a 2 mm thick frost layer may be determined by applying an energy balance, Eq. 1.11b, over the differential time interval dt and to a differential control volume extending inward from the surface of the layer.

$$\dot{E}_{in} dt = dE_{st} = dU_{lat}$$

$$h(2\pi rL)(T_{\infty,o} - T_f)dt = -h_{sf}\,\rho d\forall = -h_{sf}\,\rho(2\pi rL)dr$$

$$h(T_{\infty,o} - T_f)\int_0^{t_m} dt = -\rho h_{sf}\int_{r_2}^{r_1} dr$$

$$t_m = \frac{\rho h_{sf}(r_2 - r_1)}{h(T_{\infty,o} - T_f)} = \frac{700\,kg/m^3\left(3.34\times10^5\,J/kg\right)(0.002\,m)}{2\,W/m^2\cdot K(20-0)^\circ C}$$

$$t_m = 11,690\,s = 3.25\,h$$ <

COMMENTS: The tube radius r_1 exceeds the critical radius r_{cr} = k/h = 0.4 W/m·K/100 W/m^2·K = 0.004 m, in which case any frost formation will reduce the performance of the coil.

PROBLEM 3.46

KNOWN: Conditions associated with a composite wall and a thin electric heater.

FIND: (a) Equivalent thermal circuit, (b) Expression for heater temperature, (c) Ratio of outer and inner heat flows and conditions for which ratio is minimized.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional, steady-state conduction, (2) Constant properties, (3) Isothermal heater, (4) Negligible contact resistance(s).

ANALYSIS: (a) On the basis of a unit axial length, the circuit, thermal resistances, and heat rates are as shown in the schematic.

(b) Performing an energy balance for the heater, $\dot{E}_{in} = \dot{E}_{out}$, it follows that

$$q''_h\left(2\pi r_2\right) = q'_i + q'_o = \frac{T_h - T_{\infty,i}}{\left(h_i 2\pi r_1\right)^{-1} + \frac{\ln\left(r_2/r_1\right)}{2\pi k_B}} + \frac{T_h - T_{\infty,o}}{\left(h_o 2\pi r_3\right)^{-1} + \frac{\ln\left(r_3/r_2\right)}{2\pi k_A}}$$ <

(c) From the circuit,

$$\frac{q'_o}{q'_i} = \frac{\left(T_h - T_{\infty,o}\right)}{\left(T_h - T_{\infty,i}\right)} \times \frac{\left(h_i 2\pi r_1\right)^{-1} + \frac{\ln\left(r_2/r_1\right)}{2\pi k_B}}{\left(h_o 2\pi r_3\right)^{-1} + \frac{\ln\left(r_3/r_2\right)}{2\pi k_A}}$$ <

To reduce q'_o/q'_i, one could increase k_B, h_i, and r_3/r_2, while reducing k_A, h_o and r_2/r_1.

COMMENTS: Contact resistances between the heater and materials A and B could be important.

PROBLEM 3.47

KNOWN: Electric current flow, resistance, diameter and environmental conditions associated with a cable.

FIND: (a) Surface temperature of bare cable, (b) Cable surface and insulation temperatures for a thin coating of insulation, (c) Insulation thickness which provides the lowest value of the maximum insulation temperature. Corresponding value of this temperature.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in r, (3) Constant properties.

ANALYSIS: (a) The rate at which heat is transferred to the surroundings is fixed by the rate of heat generation in the cable. Performing an energy balance for a control surface about the cable, it follows that $\dot{E}_g = q$ or, for the bare cable, $I^2 R'_e L = h(\pi D_i L)(T_s - T_\infty)$. With $q' = I^2 R'_e = (700\text{A})^2 \left(6\times10^{-4}\Omega/\text{m}\right) = 294\ \text{W/m}$, it follows that

$$T_s = T_\infty + \frac{q'}{h\pi D_i} = 30^\circ\text{C} + \frac{294\ \text{W/m}}{\left(25\ \text{W/m}^2\cdot\text{K}\right)\pi(0.005\text{m})}$$

$$T_s = 778.7^\circ\text{C}.$$ <

(b) With a thin coating of insulation, there exist contact and convection resistances to heat transfer from the cable. The heat transfer rate is determined by heating within the cable, however, and therefore remains the same.

$$q = \frac{T_s - T_\infty}{R_{t,c} + \frac{1}{h\pi D_i L}} = \frac{T_s - T_\infty}{\frac{R''_{t,c}}{\pi D_i L} + \frac{1}{h\pi D_i L}}$$

$$q' = \frac{\pi D_i (T_s - T_\infty)}{R''_{t,c} + 1/h}$$

and solving for the surface temperature, find

$$T_s = \frac{q'}{\pi D_i}\left[R''_{t,c} + \frac{1}{h}\right] + T_\infty = \frac{294\ \text{W/m}}{\pi(0.005\text{m})}\left[0.02\frac{\text{m}^2\cdot\text{K}}{\text{W}} + 0.04\frac{\text{m}^2\cdot\text{K}}{\text{W}}\right] + 30^\circ\text{C}$$

$$T_s = 1153^\circ\text{C}.$$ <

Continued …..

PROBLEM 3.47 (Cont.)

The insulation temperature is then obtained from

$$q = \frac{T_s - T_i}{R_{t,c}}$$

or

$$T_i = T_s - qR_{t,c} = 1153^\circ C - q\frac{R''_{t,c}}{\pi D_i L} = 1153^\circ C - \frac{294\frac{W}{m} \times 0.02\frac{m^2 \cdot K}{W}}{\pi(0.005m)}$$

$$T_i = 778.7^\circ C.$$ <

(c) The maximum insulation temperature could be reduced by reducing the resistance to heat transfer from the outer surface of the insulation. Such a reduction is possible if $D_i < D_{cr}$. From Example 3.4,

$$r_{cr} = \frac{k}{h} = \frac{0.5\ W/m \cdot K}{25\ W/m^2 \cdot K} = 0.02m.$$

Hence, $D_{cr} = 0.04m > D_i = 0.005m$. To minimize the maximum temperature, which exists at the inner surface of the insulation, add insulation in the amount

$$t = \frac{D_o - D_i}{2} = \frac{D_{cr} - D_i}{2} = \frac{(0.04 - 0.005)m}{2}$$

$$t = 0.0175m.$$ <

The cable surface temperature may then be obtained from

$$q' = \frac{T_s - T_\infty}{\frac{R''_{t,c}}{\pi D_i} + \frac{\ln(D_{cr}/D_i)}{2\pi k} + \frac{1}{h\pi D_{cr}}} = \frac{T_s - 30^\circ C}{\frac{0.02\ m^2 \cdot K/W}{\pi(0.005m)} + \frac{\ln(0.04/0.005)}{2\pi(0.5\ W/m \cdot K)} + \frac{1}{25\frac{W}{m^2 \cdot K}\pi(0.04m)}}$$

Hence,

$$294\frac{W}{m} = \frac{T_s - 30^\circ C}{(1.27+0.66+0.32)m \cdot K/W} = \frac{T_s - 30^\circ C}{2.25\ m \cdot K/W}$$

$$T_s = 692.5^\circ C$$

Recognizing that $q = (T_s - T_i)/R_{t,c}$, find

$$T_i = T_s - qR_{t,c} = T_s - q\frac{R''_{t,c}}{\pi D_i L} = 692.5^\circ C - \frac{294\frac{W}{m} \times 0.02\frac{m^2 \cdot K}{W}}{\pi(0.005m)}$$

$$T_i = 318.2^\circ C.$$ <

COMMENTS: Use of the critical insulation thickness in lieu of a thin coating has the effect of reducing the maximum insulation temperature from 778.7°C to 318.2°C. Use of the critical insulation thickness also reduces the cable surface temperature to 692.5°C from 778.7°C with no insulation or from 1153°C with a thin coating.

PROBLEM 3.48

KNOWN: Saturated steam conditions in a pipe with prescribed surroundings.

FIND: (a) Heat loss per unit length from bare pipe and from insulated pipe, (b) Pay back period for insulation.

SCHEMATIC:

Steam Costs:
 \$4 for 10^9 J
Insulation Cost:
 \$100 per meter
Operation time:
 7500 h/yr

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat transfer, (3) Constant properties, (4) Negligible pipe wall resistance, (5) Negligible steam side convection resistance (pipe inner surface temperature is equal to steam temperature), (6) Negligible contact resistance, (7) $T_{sur} = T_\infty$.

PROPERTIES: *Table A-6*, Saturated water (p = 20 bar): $T_{sat} = T_s = 486K$; *Table A-3*, Magnesia, 85% ($T \approx 392K$): k = 0.058 W/m·K.

ANALYSIS: (a) Without the insulation, the heat loss may be expressed in terms of radiation and convection rates,

$$q' = \varepsilon\pi D\sigma\left(T_s^4 - T_{sur}^4\right) + h(\pi D)(T_s - T_\infty)$$

$$q' = 0.8\pi(0.2\text{m})5.67\times10^{-8}\ \frac{W}{m^2\cdot K^4}\left(486^4 - 298^4\right)K^4$$

$$+20\frac{W}{m^2\cdot K}(\pi\times0.2\text{m})\ (486\text{-}298)K$$

$$q' = (1365+2362)\,W/m = 3727\ W/m.$$ <

With the insulation, the thermal circuit is of the form

Continued …..

From an energy balance at the outer surface of the insulation,

$$q'_{cond} = q'_{conv} + q'_{rad}$$

$$\frac{T_{s,i} - T_{s,o}}{\ln(D_o / D_i)/2\pi\ k} = h\pi\ D_o\left(T_{s,o} - T_\infty\right) + \varepsilon\sigma\pi\ D_o\left(T_{s,o}^4 - T_{sur}^4\right)$$

$$\frac{(486 - T_{s,o})K}{\dfrac{\ln(0.3m/0.2m)}{2\pi(0.058\ W/m\cdot K)}} = 20\frac{W}{m^2\cdot K}\pi(0.3m)(T_{s,o} - 298K)$$

$$+0.8\times5.67\times10^{-8}\frac{W}{m^2\cdot K^4}\pi(0.3m)\left(T_{s,o}^4 - 298^4\right)K^4.$$

By trial and error, we obtain

$$T_{s,o} \approx 305K$$

in which case

$$q' = \frac{(486\text{-}305)K}{\dfrac{\ln(0.3m/0.2m)}{2\pi(0.055\ W/m\cdot K)}} = 163\ W/m.$$ <

(b) The yearly energy savings per unit length of pipe due to use of the insulation is

$$\frac{Savings}{Yr\cdot m} = \frac{Energy\ Savings}{Yr.}\times\frac{Cost}{Energy}$$

$$\frac{Savings}{Yr\cdot m} = (3727 - 163)\frac{J}{s\cdot m}\times3600\frac{s}{h}\times7500\frac{h}{Yr}\times\frac{\$4}{10^9 J}$$

$$\frac{Savings}{Yr\cdot m} = \$385/Yr\cdot m.$$

The pay back period is then

$$\text{Pay Back Period} = \frac{\text{Insulation Costs}}{\text{Savings/Yr.}\cdot m} = \frac{\$100/m}{\$385/Yr\cdot m}$$

Pay Back Period = 0.26 Yr = 3.1 mo. <

COMMENTS: Such a low pay back period is more than sufficient to justify investing in the insulation.

PROBLEM 3.49

KNOWN: Temperature and convection coefficient associated with steam flow through a pipe of prescribed inner and outer diameters. Outer surface emissivity and convection coefficient. Temperature of ambient air and surroundings.

FIND: Heat loss per unit length.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat transfer, (3) Constant properties, (4) Surroundings form a large enclosure about pipe.

PROPERTIES: *Table A-1*, Steel, AISI 1010 (T ≈ 450 K): k = 56.5 W/m·K.

ANALYSIS: Referring to the thermal circuit, it follows from an energy balance on the outer surface that

$$\frac{T_{\infty,i} - T_{s,o}}{R_{conv,i} + R_{cond}} = \frac{T_{s,o} - T_{\infty,o}}{R_{conv,o}} + \frac{T_{s,o} - T_{sur}}{R_{rad}}$$

or from Eqs. 3.9, 3.28 and 1.7,

$$\frac{T_{\infty,i} - T_{s,o}}{(1/\pi\ D_i h_i) + \ln(D_o/D_i)/2\pi k} = \frac{T_{s,o} - T_{\infty,o}}{(1/\pi\ D_o h_o)} + \varepsilon\pi\ D_o\sigma\left(T_{s,o}^4 - T_{sur}^4\right)$$

$$\frac{523\text{K} - T_{s,o}}{\left(\pi\times 0.6\text{m}\times 500\ \text{W/m}^2\cdot\text{K}\right)^{-1} + \dfrac{\ln\ (75/60)}{2\pi\times 56.5\ \text{W/m}\cdot\text{K}}} = \frac{T_{s,o} - 293\text{K}}{\left(\pi\times 0.075\text{m}\times 25\ \text{W/m}^2\cdot\text{K}\right)^{-1}}$$

$$+0.8\pi\times(0.075\text{m})\times 5.67\times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4\left[T_{s,o}^4 - 293^4\right]\text{K}^4$$

$$\frac{523 - T_{s,o}}{0.0106+0.0006} = \frac{T_{s,o} - 293}{0.170} + 1.07\times 10^{-8}\left[T_{s,o}^4 - 293^4\right].$$

From a trial-and-error solution, $T_{s,o} \approx 502\text{K}$. Hence the heat loss is

$$q' = \pi\ D_o h_o\left(T_{s,o} - T_{\infty,o}\right) + \varepsilon\pi\ D_o\sigma\left(T_{s,o}^4 - T_{sur}^4\right)$$

$$q' = \pi(0.075\text{m})\,25\ \text{W/m}^2\cdot\text{K}\,(502\text{-}293) + 0.8\ \pi(0.075\text{m})\,5.67\times 10^{-8}\ \frac{\text{W}}{\text{m}^2\cdot\text{K}^4}\left[502^4 - 243^4\right]\text{K}^4$$

$$q' = 1231\ \text{W/m} + 600\ \text{W/m} = 1831\ \text{W/m}.$$ <

COMMENTS: The thermal resistance between the outer surface and the surroundings is much larger than that between the outer surface and the steam.

PROBLEM 3.50

KNOWN: Temperature and convection coefficient associated with steam flow through a pipe of prescribed inner and outer radii. Emissivity of outer surface magnesia insulation, and convection coefficient. Temperature of ambient air and surroundings.

FIND: Heat loss per unit length q' and outer surface temperature $T_{s,o}$ as a function of insulation thickness. Recommended insulation thickness. Corresponding annual savings and temperature distribution.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat transfer, (3) Constant properties, (4) Surroundings form a large enclosure about pipe.

PROPERTIES: *Table A-1*, Steel, AISI 1010 (T ≈ 450 K): k_s = 56.5 W/m·K. *Table A-3*, Magnesia, 85% (T ≈ 365 K): k_m = 0.055 W/m·K.

ANALYSIS: Referring to the thermal circuit, it follows from an energy balance on the outer surface that

$$\frac{T_{\infty,i} - T_{s,o}}{R'_{conv,i} + R'_{cond,s} + R'_{cond,m}} = \frac{T_{s,o} - T_{\infty,o}}{R'_{conv,o}} + \frac{T_{s,o} - T_{sur}}{R'_{rad}}$$

or from Eqs. 3.9, 3.28 and 1.7,

$$\frac{T_{\infty,i} - T_{s,o}}{\left(1/2\pi r_1 h_i\right) + \ln\left(r_2/r_1\right)/2\pi k_s + \ln\left(r_3/r_2\right)/2\pi k_m} = \frac{T_{s,o} - T_{\infty,o}}{\left(1/2\pi r_3 h_o\right)} + \frac{T_{s,o} - T_{sur}}{\left[\left(2\pi r_3\right)\varepsilon\sigma\left(T_{s,o} + T_{sur}\right)\left(T_{s,o}^2 + T_{sur}^2\right)\right]^{-1}}$$

This expression may be solved for $T_{s,o}$ as a function of r_3, and the heat loss may then be determined by evaluating either the left-or right-hand side of the energy balance equation. The results are plotted as follows.

Continued...

PROBLEM 3.50 (Cont.)

The rapid decay in q' with increasing r_3 is attributable to the dominant contribution which the insulation begins to make to the total thermal resistance. The inside convection and tube wall conduction resistances are fixed at 0.0106 m·K/W and 6.29×10^{-4} m·K/W, respectively, while the resistance of the insulation increases to approximately 2 m·K/W at $r_3 = 0.075$ m.

The heat loss may be reduced by almost 91% from a value of approximately 1830 W/m at $r_3 = r_2 = 0.0375$ m (no insulation) to 172 W/m at $r_3 = 0.0575$ m and by only an additional 3% if the insulation thickness is increased to $r_3 = 0.0775$ m. Hence, an insulation thickness of $(r_3 - r_2) = 0.020$ m is recommended, for which $q' = 172$ W/m. The corresponding annual savings (AS) in energy costs is therefore

$$AS = \left[(1830 - 172)\,W/m\right]\frac{\$4}{10^9\,J}\times 7000\,\frac{h}{y}\times 3600\,\frac{s}{h} = \$167/m$$ <

The corresponding temperature distribution is

The temperature in the insulation decreases from $T(r) = T_2 = 521$ K at $r = r_2 = 0.0375$ m to $T(r) = T_3 = 309$ K at $r = r_3 = 0.0575$ m.

Continued...

PROBLEM 3.50 (Cont.)

COMMENTS: 1. The annual energy and costs savings associated with insulating the steam line are substantial, as is the reduction in the outer surface temperature (from $T_{s,o} \approx 502$ K for $r_3 = r_2$, to 309 K for $r_3 = 0.0575$ m).

2. The increase in R'_{rad} to a maximum value of 0.63 m·K/W at $r_3 = 0.0455$ m and the subsequent decay is due to the competing effects of h_{rad} and $A'_3 = (1/2\pi r_3)$. Because the initial decay in $T_3 = T_{s,o}$ with increasing r_3, and hence, the reduction in h_{rad}, is more pronounced than the increase in A'_3, R'_{rad} increases with r_3. However, as the decay in $T_{s,o}$, and hence h_{rad}, becomes less pronounced, the increase in A'_3 becomes more pronounced and R'_{rad} decreases with increasing r_3.

PROBLEM 3.51

KNOWN: Pipe wall temperature and convection conditions associated with water flow through the pipe and ice layer formation on the inner surface.

FIND: Ice layer thickness δ.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional, steady-state conduction, (2) Negligible pipe wall thermal resistance, (3) negligible ice/wall contact resistance, (4) Constant k.

PROPERTIES: *Table A.3*, Ice (T = 265 K): k ≈ 1.94 W/m·K.

ANALYSIS: Performing an energy balance for a control surface about the ice/water interface, it follows that, for a unit length of pipe,

$$q'_{conv} = q'_{cond}$$

$$h_i\,(2\pi r_1)\left(T_{\infty,i} - T_{s,i}\right) = \frac{T_{s,i} - T_{s,o}}{\ln\left(r_2/r_1\right)/2\pi k}$$

Dividing both sides of the equation by r_2,

$$\frac{\ln\left(r_2/r_1\right)}{\left(r_2/r_1\right)} = \frac{k}{h_i r_2} \times \frac{T_{s,i} - T_{s,o}}{T_{\infty,i} - T_{s,i}} = \frac{1.94\,\text{W/m}\cdot\text{K}}{\left(2000\,\text{W/m}^2\cdot\text{K}\right)\left(0.05\,\text{m}\right)} \times \frac{15^\circ\text{C}}{3^\circ\text{C}} = 0.097$$

The equation is satisfied by $r_2/r_1 = 1.114$, in which case $r_1 = 0.050\text{ m}/1.114 = 0.045\text{ m}$, and the ice layer thickness is

$$\delta = r_2 - r_1 = 0.005\,\text{m} = 5\,\text{mm}$$ <

COMMENTS: With no flow, $h_i \to 0$, in which case $r_1 \to 0$ and complete blockage could occur. The pipe should be insulated.

PROBLEM 3.52

KNOWN: Inner surface temperature of insulation blanket comprised of two semi-cylindrical shells of different materials. Ambient air conditions.

FIND: (a) Equivalent thermal circuit, (b) Total heat loss and material outer surface temperatures.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional, radial conduction, (3) Infinite contact resistance between materials, (4) Constant properties.

ANALYSIS: (a) The thermal circuit is,

$$R'_{conv,A} = R'_{conv,B} = 1/\pi\, r_2 h$$

$$R'_{cond(A)} = \frac{\ln(r_2/r_1)}{\pi\, k_A}$$

$$R'_{cond(\mathbf{B})} = \frac{\ln(r_2/r_i)}{\pi\, k_B}$$

<

The conduction resistances follow from Section 3.3.1 and Eq. 3.28. Each resistance is larger by a factor of 2 than the result of Eq. 3.28 due to the reduced area.

(b) Evaluating the thermal resistances and the heat rate $(q' = q'_A + q'_B)$,

$$R'_{conv} = \left(\pi \times 0.1\text{m} \times 25\ \text{W/m}^2\cdot\text{K}\right)^{-1} = 0.1273\ \text{m}\cdot\text{K/W}$$

$$R'_{cond(A)} = \frac{\ln(0.1\text{m}/0.05\text{m})}{\pi \times 2\ \text{W/m}\cdot\text{K}} = 0.1103\ \text{m}\cdot\text{K/W} \quad R'_{cond(B)} = 8\ R'_{cond(A)} = 0.8825\ \text{m}\cdot\text{K/W}$$

$$q' = \frac{T_{s,1} - T_\infty}{R'_{cond(A)} + R'_{conv}} + \frac{T_{s,1} - T_\infty}{R'_{cond(B)} + R'_{conv}}$$

$$q' = \frac{(500-300)\text{K}}{(0.1103+0.1273)\text{m}\cdot\text{K/W}} + \frac{(500-300)\text{K}}{(0.8825+0.1273)\text{m}\cdot\text{K/W}} = (842+198)\text{W/m} = 1040\ \text{W/m}.$$ <

Hence, the temperatures are

$$T_{s,2(A)} = T_{s,1} - q'_A R'_{cond(A)} = 500\text{K} - 842\frac{\text{W}}{\text{m}} \times 0.1103\frac{\text{m}\cdot\text{K}}{\text{W}} = 407\text{K}$$ <

$$T_{s,2(B)} = T_{s,1} - q'_B R'_{cond(B)} = 500\text{K} - 198\frac{\text{W}}{\text{m}} \times 0.8825\frac{\text{m}\cdot\text{K}}{\text{W}} = 325\text{K}.$$ <

COMMENTS: The total heat loss can also be computed from $q' = (T_{s,1} - T_\infty)/R_{equiv}$,

where $R_{equiv} = \left[\left(R'_{cond(A)} + R'_{conv,A}\right)^{-1} + \left(R'_{cond(B)} + R'_{conv,B}\right)^{-1}\right]^{-1} = 0.1923\ \text{m}\cdot\text{K/W}.$

Hence $q' = (500-300)\text{K}/0.1923\ \text{m}\cdot\text{K/W} = 1040\ \text{W/m}.$

PROBLEM 3.53

KNOWN: Surface temperature of a circular rod coated with bakelite and adjoining fluid conditions.

FIND: (a) Critical insulation radius, (b) Heat transfer per unit length for bare rod and for insulation at critical radius, (c) Insulation thickness needed for 25% heat rate reduction.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in r, (3) Constant properties, (4) Negligible radiation and contact resistance.

PROPERTIES: *Table A-3*, Bakelite (300K): k = 1.4 W/m·K.

ANALYSIS: (a) From Example 3.4, the critical radius is

$$r_{cr} = \frac{k}{h} = \frac{1.4\ \text{W/m}\cdot\text{K}}{140\ \text{W/m}^2\cdot\text{K}} = 0.01\text{m}.$$ <

(b) For the bare rod,

$$q' = h\left(\pi\ D_i\right)\ \left(T_i - T_\infty\right)$$

$$q' = 140\frac{\text{W}}{\text{m}^2\cdot\text{K}}\left(\pi\times 0.01\text{m}\right)\ \left(200-25\right)^\circ\text{C} = 770\ \text{W/m}$$ <

For the critical insulation thickness,

$$q' = \frac{T_i - T_\infty}{\frac{1}{2\pi\ r_{cr}h} + \frac{\ln\left(r_{cr}/r_i\right)}{2\pi\ k}} = \frac{\left(200-25\right)^\circ\text{C}}{\frac{1}{2\pi\times\left(0.01\text{m}\right)\times 140\ \text{W/m}^2\cdot\text{K}} + \frac{\ln\left(0.01\text{m}/0.005\text{m}\right)}{2\pi\times 1.4\ \text{W/m}\cdot\text{K}}}$$

$$q' = \frac{175^\circ\text{C}}{\left(0.1137+0.0788\right)\ \text{m}\cdot\text{K/W}} = 909\ \text{W/m}$$ <

(c) The insulation thickness needed to reduce the heat rate to 577 W/m is obtained from

$$q' = \frac{T_i - T_\infty}{\frac{1}{2\pi\ rh} + \frac{\ln\left(r/r_i\right)}{2\pi\ k}} = \frac{\left(200-25\right)^\circ\text{C}}{\frac{1}{2\pi\left(r\right)140\ \text{W/m}^2\cdot\text{K}} + \frac{\ln\left(r/0.005\text{m}\right)}{2\pi\times 1.4\ \text{W/m}\cdot\text{K}}} = 577\frac{\text{W}}{\text{m}}$$

From a trial-and-error solution, find

$$r \approx 0.06\ \text{m}.$$

The desired insulation thickness is then

$$\delta = \left(r - r_i\right) \approx \left(0.06 - 0.005\right)\text{m} = 55\ \text{mm}.$$ <

PROBLEM 3.54

KNOWN: Geometry of an oil storage tank. Temperature of stored oil and environmental conditions.

FIND: Heater power required to maintain a prescribed inner surface temperature.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in radial direction, (3) Constant properties, (4) Negligible radiation.

PROPERTIES: *Table A-3*, Pyrex (300K): k = 1.4 W/m·K.

ANALYSIS: The rate at which heat must be supplied is equal to the loss through the cylindrical and hemispherical sections. Hence,

$$q = q_{cyl} + 2q_{hemi} = q_{cyl} + q_{spher}$$

or, from Eqs. 3.28 and 3.36,

$$q = \frac{T_{s,i} - T_\infty}{\dfrac{\ln\left(D_o / D_i\right)}{2\pi\ Lk} + \dfrac{1}{\pi\ D_o Lh}} + \frac{T_{s,i} - T_\infty}{\dfrac{1}{2\pi\ k}\left[\dfrac{1}{D_i} - \dfrac{1}{D_o}\right] + \dfrac{1}{\pi D_o^2 h}}$$

$$q = \frac{(400-300)\mathrm{K}}{\dfrac{\ln 1.04}{2\pi(2\mathrm{m})1.4\ \mathrm{W/m\cdot K}} + \dfrac{1}{\pi(1.04\mathrm{m})2\mathrm{m}\left(10\ \mathrm{W/m^2\cdot K}\right)}}$$

$$+\frac{(400-300)\mathrm{K}}{\dfrac{1}{2\pi(1.4\ \mathrm{W/m\cdot K})}(1-0.962)\mathrm{m}^{-1} + \dfrac{1}{\pi(1.04\mathrm{m})^2 10\ \mathrm{W/m^2\cdot K}}}$$

$$q = \frac{100\mathrm{K}}{2.23\times10^{-3}\ \mathrm{K/W} + 15.30\times10^{-3}\ \mathrm{K/W}} + \frac{100\mathrm{K}}{4.32\times10^{-3}\ \mathrm{K/W} + 29.43\times10^{-3}}$$

q = 5705W + 2963W = 8668W. <

PROBLEM 3.55

KNOWN: Diameter of a spherical container used to store liquid oxygen and properties of insulating material. Environmental conditions.

FIND: (a) Reduction in evaporative oxygen loss associated with a prescribed insulation thickness, (b) Effect of insulation thickness on evaporation rate.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state, one-dimensional conduction, (2) Negligible conduction resistance of container wall and contact resistance between wall and insulation, (3) Container wall at boiling point of liquid oxygen.

ANALYSIS: (a) Applying an energy balance to a control surface about the insulation, $\dot{E}_{in} - \dot{E}_{out} = 0$, it follows that $q_{conv} + q_{rad} = q_{cond} = q$. Hence,

$$\frac{T_\infty - T_{s,2}}{R_{t,conv}} + \frac{T_{sur} - T_{s,2}}{R_{t,rad}} = \frac{T_{s,2} - T_{s,1}}{R_{t,cond}} = q \qquad (1)$$

where $R_{t,conv} = \left(4\pi r_2^2 h\right)^{-1}$, $R_{t,rad} = \left(4\pi r_2^2 h_r\right)^{-1}$, $R_{t,cond} = (1/4\pi k)\left[(1/r_1) - (1/r_2)\right]$, and, from Eq. 1.9, the radiation coefficient is $h_r = \varepsilon\sigma\left(T_{s,2} + T_{sur}\right)\left(T_{s,2}^2 + T_{sur}^2\right)$. With t = 10 mm ($r_2$ = 260 mm), ε = 0.2 and $T_\infty = T_{sur}$ = 298 K, an iterative solution of the energy balance equation yields $T_{s,2} \approx 297.7$ K, where $R_{t,conv}$ = 0.118 K/W, $R_{t,rad}$ = 0.982 K/W and $R_{t,cond}$ = 76.5 K/W. With the insulation, it follows that the heat gain is

$$q_w \approx 2.72 \text{ W}$$

Without the insulation, the heat gain is

$$q_{wo} = \frac{T_\infty - T_{s,1}}{R_{t,conv}} + \frac{T_{sur} - T_{s,1}}{R_{t,rad}}$$

where, with $r_2 = r_1$, $T_{s,1}$ = 90 K, $R_{t,conv}$ = 0.127 K/W and $R_{t,rad}$ = 3.14 K/W. Hence,

$$q_{wo} = 1702 \text{ W}$$

With the oxygen mass evaporation rate given by $\dot{m} = q/h_{fg}$, the percent reduction in evaporated oxygen is

$$\%\ \text{Reduction} = \frac{\dot{m}_{wo} - \dot{m}_w}{\dot{m}_{wo}} \times 100\% = \frac{q_{wo} - q_w}{q_{wo}} \times 100\%$$

Hence,

$$\%\ \text{Reduction} = \frac{(1702 - 2.7)\text{ W}}{1702 \text{ W}} \times 100\% = 99.8\% \qquad <$$

Continued...

PROBLEM 3.55 (Cont.)

(b) Using Equation (1) to compute $T_{s,2}$ and q as a function of r_2, the corresponding evaporation rate, $\dot{m} = q/h_{fg}$, may be determined. Variations of q and $\dot{m}$ with r_2 are plotted as follows.

Because of its extremely low thermal conductivity, significant benefits are associated with using even a thin layer of insulation. Nearly three-order magnitude reductions in q and $\dot{m}$ are achieved with $r_2 = 0.26$ m. With increasing r_2, q and $\dot{m}$ decrease from values of 1702 W and 8×10^{-3} kg/s at $r_2 = 0.25$ m to 0.627 W and 2.9×10^{-6} kg/s at $r_2 = 0.30$ m.

COMMENTS: Laminated metallic-foil/glass-mat insulations are extremely effective and corresponding conduction resistances are typically much larger than those normally associated with surface convection and radiation.

PROBLEM 3.56

KNOWN: Sphere of radius r_i, covered with insulation whose outer surface is exposed to a convection process.

FIND: Critical insulation radius, r_{cr}.

SCHEMATIC:

T_∞, h ; T_i ; r ; r_i ; k ; q ; T_∞ — $R_{t,conv}$ — $R_{t,cond}$ — T_i

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional radial (spherical) conduction, (3) Constant properties, (4) Negligible radiation at surface.

ANALYSIS: The heat rate follows from the thermal circuit shown in the schematic,

$$q = \left(T_i - T_\infty\right)/R_{tot}$$

where $R_{tot} = R_{t,conv} + R_{t,cond}$ and

$$R_{t,conv} = \frac{1}{hA_s} = \frac{1}{4\pi\ hr^2} \qquad (3.9)$$

$$R_{t,cond} = \frac{1}{4\pi\ k}\left[\frac{1}{r_i} - \frac{1}{r}\right] \qquad (3.36)$$

If q is a maximum or minimum, we need to find the condition for which

$$\frac{d\,R_{tot}}{dr} = 0.$$

It follows that

$$\frac{d}{dr}\left[\frac{1}{4\pi k}\left[\frac{1}{r_i} - \frac{1}{r}\right] + \frac{1}{4\pi\ hr^2}\right] = \left[+\frac{1}{4\pi\ k}\frac{1}{r^2} - \frac{1}{2\pi\ h}\frac{1}{r^3}\right] = 0$$

giving

$$r_{cr} = 2\frac{k}{h}$$

The second derivative, evaluated at $r = r_{cr}$, is

$$\frac{d}{dr}\left[\frac{dR_{tot}}{dr}\right] = -\frac{1}{2\pi\ k}\ \frac{1}{r^3} + \frac{3}{2\pi\ h}\ \frac{1}{r^4}\Bigg]_{r=r_{cr}}$$

$$= \frac{1}{(2k/h)^3}\left\{-\frac{1}{2\pi\ k} + \frac{3}{2\pi\ h}\frac{1}{2k/h}\right\} = \frac{1}{(2k/h)^3}\frac{1}{2\pi\ k}\left\{-1 + \frac{3}{2}\right\} > 0$$

Hence, it follows no optimum R_{tot} exists. We refer to this condition as the critical insulation radius. See Example 3.4 which considers this situation for a cylindrical system.

PROBLEM 3.57

KNOWN: Thickness of hollow aluminum sphere and insulation layer. Heat rate and inner surface temperature. Ambient air temperature and convection coefficient.

FIND: Thermal conductivity of insulation.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional radial conduction, (3) Constant properties, (4) Negligible contact resistance, (5) Negligible radiation exchange at outer surface.

PROPERTIES: *Table A-1*, Aluminum (523K): $k \approx 230$ W/m·K.

ANALYSIS: From the thermal circuit,

$$q = \frac{T_1 - T_\infty}{R_{tot}} = \frac{T_1 - T_\infty}{\dfrac{1/r_1 - 1/r_2}{4\pi\, k_{A1}} + \dfrac{1/r_2 - 1/r_3}{4\pi\, k_I} + \dfrac{1}{h 4\pi\, r_3^2}}$$

$$q = \frac{(250 - 20)^\circ\ \mathrm{C}}{\left[\dfrac{1/0.15 - 1/0.18}{4\pi(230)} + \dfrac{1/0.18 - 1/0.30}{4\pi\, k_I} + \dfrac{1}{30(4\pi)(0.3)^2}\right]\dfrac{\mathrm{K}}{\mathrm{W}}} = 80\ \mathrm{W}$$

or

$$3.84\times10^{-4} + \frac{0.177}{k_I} + 0.029 = \frac{230}{80} = 2.875.$$

Solving for the unknown thermal conductivity, find

$$k_I = 0.062\ \mathrm{W/m\cdot K}.$$ <

COMMENTS: The dominant contribution to the total thermal resistance is made by the insulation. Hence uncertainties in knowledge of h or k_{A1} have a negligible effect on the accuracy of the k_I measurement.

PROBLEM 3.58

KNOWN: Dimensions of spherical, stainless steel liquid oxygen (LOX) storage container. Boiling point and latent heat of fusion of LOX. Environmental temperature.

FIND: Thermal isolation system which maintains boil-off below 1 kg/day.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional, steady-state conditions, (2) Negligible thermal resistances associated with internal and external convection, conduction in the container wall, and contact between wall and insulation, (3) Negligible radiation at exterior surface, (4) Constant insulation thermal conductivity.

PROPERTIES: *Table A.1*, 304 Stainless steel (T = 100 K): k_s = 9.2 W/m·K; *Table A.3*, Reflective, aluminum foil-glass paper insulation (T = 150 K): k_i = 0.000017 W/m·K.

ANALYSIS: The heat gain associated with a loss of 1 kg/day is

$$q = \dot{m} h_{fg} = \frac{1\,\text{kg/day}}{86,400\,\text{s/day}}\left(2.13\times10^5\ \text{J/kg}\right) = 2.47\,\text{W}$$

With an overall temperature difference of $\left(T_\infty - T_{bp}\right)$ = 150 K, the corresponding total thermal resistance is

$$R_{tot} = \frac{\Delta T}{q} = \frac{150\,\text{K}}{2.47\,\text{W}} = 60.7\,\text{K/W}$$

Since the conduction resistance of the steel wall is

$$R_{t,cond,s} = \frac{1}{4\pi k_s}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = \frac{1}{4\pi\left(9.2\,\text{W/m}\cdot\text{K}\right)}\left(\frac{1}{0.35\,\text{m}} - \frac{1}{0.40\,\text{m}}\right) = 2.4\times10^{-3}\,\text{K/W}$$

it is clear that exclusive reliance must be placed on the insulation and that a special insulation of very low thermal conductivity should be selected. The best choice is a highly reflective foil/glass matted insulation which was developed for cryogenic applications. It follows that

$$R_{t,cond,i} = 60.7\,\text{K/W} = \frac{1}{4\pi k_i}\left(\frac{1}{r_2} - \frac{1}{r_3}\right) = \frac{1}{4\pi\left(0.000017\,\text{W/m}\cdot\text{K}\right)}\left(\frac{1}{0.40\,\text{m}} - \frac{1}{r_3}\right)$$

which yields r_3 = 0.4021 m. The minimum insulation thickness is therefore $\delta = (r_3 - r_2)$ = 2.1 mm.

COMMENTS: The heat loss could be reduced well below the maximum allowable by adding more insulation. Also, in view of weight restrictions associated with launching space vehicles, consideration should be given to fabricating the LOX container from a lighter material.

PROBLEM 3.59

KNOWN: Diameter and surface temperature of a spherical cryoprobe. Temperature of surrounding tissue and effective convection coefficient at interface between frozen and normal tissue.

FIND: Thickness of frozen tissue layer.

SCHEMATIC:

r_1 = 0.0015 m
r_2
Probe
$T_{s,1}$ = -30 °C
$T_{s,2}$ = 0 °C
Normal tissue
T_∞ = 37 °C
h = 50 W/m²·K
q_{cond}
Frozen tissue, k = 1.5 W/m·K
q_{conv}

ASSUMPTIONS: (1) One-dimensional, steady-state conditions, (2) Negligible contact resistance between probe and frozen tissue, (3) Constant properties.

ANALYSIS: Performing an energy balance for a control surface about the phase front, it follows that

$$q_{conv} - q_{cond} = 0$$

Hence,

$$h\left(4\pi r_2^2\right)\left(T_\infty - T_{s,2}\right) = \frac{T_{s,2} - T_{s,1}}{\left[(1/r_1) - (1/r_2)\right]/4\pi k}$$

$$r_2^2\left[(1/r_1) - (1/r_2)\right] = \frac{k}{h}\frac{\left(T_{s,2} - T_{s,1}\right)}{\left(T_\infty - T_{s,2}\right)}$$

$$\left(\frac{r_2}{r_1}\right)\left[\left(\frac{r_2}{r_1}\right) - 1\right] = \frac{k}{hr_1}\frac{\left(T_{s,2} - T_{s,1}\right)}{\left(T_\infty - T_{s,2}\right)} = \frac{1.5\,\mathrm{W/m\cdot K}}{\left(50\,\mathrm{W/m^2\cdot K}\right)(0.0015\,\mathrm{m})}\left(\frac{30}{37}\right)$$

$$\left(\frac{r_2}{r_1}\right)\left[\left(\frac{r_2}{r_1}\right) - 1\right] = 16.2$$

$$\left(r_2/r_1\right) = 4.56$$

It follows that r_2 = 6.84 mm and the thickness of the frozen tissue is

$$\delta = r_2 - r_1 = 5.34\,\mathrm{mm}$$ <

PROBLEM 3.60

KNOWN: Inner diameter, wall thickness and thermal conductivity of spherical vessel containing heat generating medium. Inner surface temperature without insulation. Thickness and thermal conductivity of insulation. Ambient air temperature and convection coefficient.

FIND: (a) Thermal energy generated within vessel, (b) Inner surface temperature of vessel with insulation.

SCHEMATIC:

Insulation $k_i = 0.04$ W/m-K

St steel wall $k_w = 17$ W/m-K

Pharmaceuticals, $\dot{E}_{gen}$

$T_{s,1}$, q

$r_1 = 0.50$ m

$r_2 = 0.51$ m

$r_3 = 0.53$ m

Air

$T_\infty = 25°C$

$h = 6$ W/m^2-K

q, $T_{s,1}$, $\frac{1}{4\pi k_w}\left[\frac{1}{r_1}-\frac{1}{r_2}\right]$, $\frac{1}{4\pi k_i}\left[\frac{1}{r_2}-\frac{1}{r_3}\right]$, $\frac{1}{h4\pi r_3^2}$, T_∞

ASSUMPTIONS: (1) Steady-state, (2) One-dimensional, radial conduction, (3) Constant properties, (4) Negligible contact resistance, (5) Negligible radiation.

ANALYSIS: (a) From an energy balance performed at an instant for a control surface about the pharmaceuticals, $\dot{E}_g = q$, in which case, without the insulation

$$\dot{E}_g = q = \frac{T_{s,1} - T_\infty}{\frac{1}{4\pi k_w}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) + \frac{1}{4\pi r_2^2 h}} = \frac{(50-25)°C}{\frac{1}{4\pi(17\,W/m\cdot K)}\left(\frac{1}{0.50m} - \frac{1}{0.51m}\right) + \frac{1}{4\pi(0.51m)^2\,6\,W/m^2\cdot K}}$$

$$\dot{E}_g = q = \frac{25°C}{\left(1.84\times10^{-4} + 5.10\times10^{-2}\right)K/W} = 489\,W$$ <

(b) With the insulation,

$$T_{s,1} = T_\infty + q\left[\frac{1}{4\pi k_w}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) + \frac{1}{4\pi k_i}\left(\frac{1}{r_2} - \frac{1}{r_3}\right) + \frac{1}{4\pi r_3^2 h}\right]$$

$$T_{s,1} = 25°C + 489\,W\left[1.84\times10^{-4} + \frac{1}{4\pi(0.04)}\left(\frac{1}{0.51} - \frac{1}{0.53}\right) + \frac{1}{4\pi(0.53)^2\,6}\right]\frac{K}{W}$$

$$T_{s,1} = 25°C + 489\,W\left[1.84\times10^{-4} + 0.147 + 0.047\right]\frac{K}{W} = 120°C$$ <

COMMENTS: The thermal resistance associated with the vessel wall is negligible, and without the insulation the dominant resistance is due to convection. The thermal resistance of the insulation is approximately three times that due to convection.

PROBLEM 3.61

KNOWN: Spherical tank of 1-m diameter containing an exothermic reaction and is at 200°C when the ambient air is at 25°C. Convection coefficient on outer surface is 20 W/m^2·K.

FIND: Determine the thickness of urethane foam required to reduce the exterior temperature to 40°C. Determine the percentage reduction in the heat rate achieved using the insulation.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional, radial (spherical) conduction through the insulation, (3) Convection coefficient is the same for bare and insulated exterior surface, and (3) Negligible radiation exchange between the insulation outer surface and the ambient surroundings.

PROPERTIES: *Table A-3,* urethane, rigid foam (300 K): k = 0.026 W/m·K.

ANALYSIS: (a) The heat transfer situation for the heat rate from the tank can be represented by the thermal circuit shown above. The heat rate from the tank is

$$q = \frac{T_t - T_\infty}{R_{cd} + R_{cv}}$$

where the thermal resistances associated with conduction within the insulation (Eq. 3.35) and convection for the exterior surface, respectively, are

$$R_{cd} = \frac{\left(1/r_t - 1/r_o\right)}{4\pi k} = \frac{\left(1/0.5 - 1/r_o\right)}{4\pi \times 0.026\ \mathrm{W/m\cdot K}} = \frac{\left(1/0.5 - 1/r_o\right)}{0.3267}\mathrm{K/W}$$

$$R_{cv} = \frac{1}{hA_s} = \frac{1}{4\pi h r_o^2} = \frac{1}{4\pi \times 20\ \mathrm{W/m^2\cdot K} \times r_o^2} = 3.979\times10^{-3} r_o^{-2}\mathrm{K/W}$$

To determine the required insulation thickness so that T_o = 40°C, perform an energy balance on the o-node.

$$\frac{T_t - T_o}{R_{cd}} + \frac{T_\infty - T_o}{R_{cv}} = 0$$

$$\frac{(200-40)\mathrm{K}}{\left(1/0.5 - 1/r_o\right)/0.3267\ \mathrm{K/W}} + \frac{(25-40)\mathrm{K}}{3.979\times10^{-3} r_o^2\ \mathrm{K/W}} = 0$$

$$r_o = 0.5135\ \mathrm{m} \qquad t = r_o - r_i = (0.5135 - 0.5000)\ \mathrm{m} = 13.5\ \mathrm{mm}$$ <

From the rate equation, for the bare and insulated surfaces, respectively,

$$q_o = \frac{T_t - T_\infty}{1/4\pi h r_t^2} = \frac{(200-25)\mathrm{K}}{0.01592\ \mathrm{K/W}} = 10.99\ \mathrm{kW}$$

$$q_{ins} = \frac{T_t - T_\infty}{R_{cd} + R_{cv}} = \frac{(200-25)}{(0.161 + 0.01592)\mathrm{K/W}} = 0.994\ \mathrm{kW}$$

Hence, the percentage reduction in heat loss achieved with the insulation is,

$$\frac{q_{ins} - q_o}{q_o}\times100 = -\frac{0.994 - 10.99}{10.99}\times100 = 91\%$$ <

PROBLEM 3.62

KNOWN: Dimensions and materials used for composite spherical shell. Heat generation associated with stored material.

FIND: Inner surface temperature, T_1, of lead (proposal is flawed if this temperature exceeds the melting point).

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional conduction, (2) Steady-state conditions, (3) Constant properties at 300K, (4) Negligible contact resistance.

PROPERTIES: *Table A-1*, Lead: k = 35.3 W/m·K, MP = 601K; St.St.: 15.1 W/m·K.

ANALYSIS: From the thermal circuit, it follows that

$$q=\frac{T_1-T_\infty}{R_{tot}}=\dot{q}\left[\frac{4}{3}\pi\ r_1^3\right]$$

Evaluate the thermal resistances,

$$R_{Pb}=\left[1/\left(4\pi\times 35.3\ \mathrm{W/m\cdot K}\right)\right]\ \left[\frac{1}{0.25\mathrm{m}}-\frac{1}{0.30\mathrm{m}}\right]=0.00150\ \mathrm{K/W}$$

$$R_{St.St.}=\left[1/\left(4\pi\times 15.1\ \mathrm{W/m\cdot K}\right)\right]\ \left[\frac{1}{0.30\mathrm{m}}-\frac{1}{0.31\mathrm{m}}\right]=0.000567\ \mathrm{K/W}$$

$$R_{conv}=\left[1/\left(4\pi\times 0.31^2\ \mathrm{m}^2\times 500\ \mathrm{W/m^2\cdot K}\right)\right]=0.00166\ \mathrm{K/W}$$

$$R_{tot}=0.00372\ \mathrm{K/W}.$$

The heat rate is $q=5\times10^5\ \mathrm{W/m^3}\left(4\pi/3\right)\left(0.25\mathrm{m}\right)^3=32,725\ \mathrm{W}$. The inner surface temperature is

$$T_1=T_\infty+R_{tot}\ q=283\mathrm{K}+0.00372\mathrm{K/W}\left(32{,}725\ \mathrm{W}\right)$$

$$T_1=405\ \mathrm{K}<\mathrm{MP}=601\mathrm{K}.$$ <

Hence, from the thermal standpoint, the proposal is adequate.

COMMENTS: In fabrication, attention should be given to maintaining a good thermal contact. A protective outer coating should be applied to prevent long term corrosion of the stainless steel.

PROBLEM 3.63

KNOWN: Dimensions and materials of composite (lead and stainless steel) spherical shell used to store radioactive wastes with constant heat generation. Range of convection coefficients h available for cooling.

FIND: (a) Variation of maximum lead temperature with h. Minimum allowable value of h to maintain maximum lead temperature at or below 500 K. (b) Effect of outer radius of stainless steel shell on maximum lead temperature for h = 300, 500 and 1000 $W/m^2 \cdot K$.

SCHEMATIC:

Lead
Stainless steel
$r_1 = 0.25$ m
$r_2 = 0.30$ m
$r_3 \geq 0.30$ m
Fluid
$100 \leq h \leq 1000$ $W/m^2 \cdot K$
$T_\infty = 10\ ^oC$
Radioactive wastes
$\dot{q} = 5 \times 10^5$ W/m^3

T_1 — Pb — T_2 — St.St. — T_3 — T_∞ → q

$$\frac{1}{4\pi k_{Pb}}\left(\frac{1}{r_1}-\frac{1}{r_2}\right) \quad \frac{1}{4\pi k_{St.St.}}\left(\frac{1}{r_2}-\frac{1}{r_3}\right) \quad \frac{1}{4\pi r_3^2 h}$$

ASSUMPTIONS: (1) One-dimensional conduction, (2) Steady-state conditions, (3) Constant properties at 300 K, (4) Negligible contact resistance.

PROPERTIES: *Table A-1*, Lead: k = 35.3 W/m·K, St. St.: 15.1 W/m·K.

ANALYSIS: (a) From the schematic, the maximum lead temperature T_1 corresponds to $r = r_1$, and from the thermal circuit, it may be expressed as

$$T_1 = T_\infty + R_{tot} q$$

where q = $\dot{q}(4/3)\pi r_1^3 = 5\times10^5\ W/m^3\ (4\pi/3)(0.25\,m)^3 = 32,725\,W$. The total thermal resistance is

$$R_{tot} = R_{cond,Pb} + R_{cond,St.St} + R_{conv}$$

where expressions for the component resistances are provided in the schematic. Using the *Resistance Network* model and *Thermal Resistance* tool pad of IHT, the following result is obtained for the variation of T_1 with h.

Maximum Pb Temperature, T(r1) (K)
700
600
500
400
300
100 200 300 400 500 600 700 800 900 1000
Convection coefficient, h(W/m^2.K)
T_1

Continued...

PROBLEM 3.63 (Cont.)

To maintain T_1 below 500 K, the convection coefficient must be maintained at

$$h \geq 181 \text{ W/m}^2\cdot\text{K}$$ <

(b) The effect of varying the outer shell radius over the range $0.3 \leq r_3 \leq 0.5$ m is shown below.

Maximum Pb temperature, T(r1) (K)

600
550
500
450
400
350

0.3 0.35 0.4 0.45 0.5

Outer radius of steel shell, r3(m)

h = 300 W/m^2.K
h = 500 W/m^2.K
h = 1000 W/m^2.k

For h = 300, 500 and 1000 $\text{W/m}^2\cdot\text{K}$, the maximum allowable values of the outer radius are r_3 = 0.365, 0.391 and 0.408 m, respectively.

COMMENTS: For a maximum allowable value of T_1 = 500 K, the maximum allowable value of the total thermal resistance is $R_{tot} = (T_1 - T_\infty)/q$, or R_{tot} = (500 - 283)K/32,725 W = 0.00663 K/W. Hence, any increase in $R_{cond,St.St}$ due to increasing r_3 must be accompanied by an equivalent reduction in R_{conv}.

PROBLEM 3.64

KNOWN: Representation of the eye with a contact lens as a composite spherical system subjected to convection processes at the boundaries.

FIND: (a) Thermal circuits with and without contact lens in place, (b) Heat loss from anterior chamber for both cases, and (c) Implications of the heat loss calculations.

SCHEMATIC:

$T_{\infty,i}, h_i$ — Anterior chamber — k_1, Cornea — k_2, Contact lens — $T_{\infty,o}, h_o$

r_1=10.2mm	k_1=0.35 W/m·K
r_2=12.7mm	k_2=0.80 W/m·K
r_3=16.5mm	
$T_{\infty,i}$=37°C	h_i=12 W/m^2·K
$T_{\infty,o}$=21°C	h_o=6 W/m^2·K

ASSUMPTIONS: (1) Steady-state conditions, (2) Eye is represented as 1/3 sphere, (3) Convection coefficient, h_o, unchanged with or without lens present, (4) Negligible contact resistance.

ANALYSIS: (a) Using Eqs. 3.9 and 3.36 to express the resistance terms, the thermal circuits are:

Without lens: q_{wo} → $T_{\infty,i}$ — $3/h_i 4\pi r_1^2$ — $\frac{3}{4\pi k_1}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$ — $3/h_o 4\pi r_2^2$ — $T_{\infty,o}$ <

With lens: q_w → $T_{\infty,i}$ — $3/h_i 4\pi r_1^2$ — $\frac{3}{4\pi k_1}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$ — $\frac{3}{4\pi k_2}\left(\frac{1}{r_2}-\frac{1}{r_3}\right)$ — $3/h_o 4\pi r_3^2$ — $T_{\infty,o}$ <

(b) The heat losses for both cases can be determined as $q = (T_{\infty,i} - T_{\infty,o})/R_t$, where R_t is the thermal resistance from the above circuits.

Without lens:

$$R_{t,wo} = \frac{3}{12\,\text{W/m}^2\cdot\text{K}\,4\pi\left(10.2\times10^{-3}\text{m}\right)^2} + \frac{3}{4\pi\times0.35\text{ W/m}\cdot\text{K}}\left[\frac{1}{10.2}-\frac{1}{12.7}\right]\frac{1}{10^{-3}}\text{m}$$

$$+\frac{3}{6\text{ W/m}^2\cdot\text{K}\,4\pi\left(12.7\times10^{-3}\text{m}\right)^2} = 191.2\text{ K/W}+13.2\text{ K/W}+246.7\text{ K/W}=451.1\text{ K/W}$$

With lens:

$$R_{t,w} = 191.2\text{ K/W}+13.2\text{ K/W}+\frac{3}{4\pi\times0.80\text{ W/m}\cdot\text{K}}\left[\frac{1}{12.7}-\frac{1}{16.5}\right]\frac{1}{10^{-3}}\text{m}$$

$$+\frac{3}{6\text{W/m}^2\cdot\text{K}\,4\pi\left(16.5\times10^{-3}\text{m}\right)^2} = 191.2\text{ K/W}+13.2\text{ K/W}+5.41\text{ K/W}+146.2\text{ K/W}=356.0\text{ K/W}$$

Hence the heat loss rates from the anterior chamber are

Without lens: $q_{wo} = (37-21)^\circ\text{C}/451.1\text{ K/W}=35.5\text{mW}$ <

With lens: $q_w = (37-21)^\circ\text{C}/356.0\text{ K/W}=44.9\text{mW}$ <

(c) The heat loss from the anterior chamber increases by approximately 20% when the contact lens is in place, implying that the outer radius, r_3, is less than the critical radius.

PROBLEM 3.65

KNOWN: Thermal conductivity and inner and outer radii of a hollow sphere subjected to a uniform heat flux at its outer surface and maintained at a uniform temperature on the inner surface.

FIND: (a) Expression for radial temperature distribution, (b) Heat flux required to maintain prescribed surface temperatures.

SCHEMATIC:

$r_1 = 50\text{ mm}$, $T_{s,1} = 20^\circ C$
$r_2 = 100\text{ mm}$, $T_{s,2} = 50^\circ C$
$k = 10\text{ W/m}\cdot\text{K}$
q_2''

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional radial conduction, (3) No generation, (4) Constant properties.

ANALYSIS: (a) For the assumptions, the temperature distribution may be obtained by integrating Fourier's law, Eq. 3.33. That is,

$$\frac{q_r}{4\pi}\int_{r_1}^{r}\frac{dr}{r^2} = -k\int_{T_{s,1}}^{T} dT \quad \text{or} \quad -\frac{q_r}{4\pi}\frac{1}{r}\Bigg|_{r_1}^{r} = -k\left(T - T_{s,1}\right).$$

Hence,

$$T(r) = T_{s,1} + \frac{q_r}{4\pi\, k}\left[\frac{1}{r} - \frac{1}{r_1}\right]$$

or, with $q_2'' \equiv q_r / 4\pi\, r_2^2$,

$$T(r) = T_{s,1} + \frac{q_2'' r_2^2}{k}\left[\frac{1}{r} - \frac{1}{r_1}\right]$$ <

(b) Applying the above result at r_2,

$$q_2'' = \frac{k\left(T_{s,2} - T_{s,1}\right)}{r_2^2\left[\frac{1}{r_2} - \frac{1}{r_1}\right]} = \frac{10\text{ W/m}\cdot\text{K}\ (50-20)^\circ\text{C}}{(0.1\text{m})^2\left[\frac{1}{0.1} - \frac{1}{0.05}\right]\frac{1}{\text{m}}} = -3000\text{ W/m}^2.$$ <

COMMENTS: (1) The desired temperature distribution could also be obtained by solving the appropriate form of the heat equation,

$$\frac{d}{dr}\left[r^2\frac{dT}{dr}\right] = 0$$

and applying the boundary conditions $T(r_1) = T_{s,1}$ and $-k\frac{dT}{dr}\Big]_{r_2} = q_2''$.

(2) The negative sign on q_2'' implies heat transfer in the negative r direction.

PROBLEM 3.66

KNOWN: Volumetric heat generation occurring within the cavity of a spherical shell of prescribed dimensions. Convection conditions at outer surface.

FIND: Expression for steady-state temperature distribution in shell.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional radial conduction, (2) Steady-state conditions, (3) Constant properties, (4) Uniform generation within the shell cavity, (5) Negligible radiation.

ANALYSIS: For the prescribed conditions, the appropriate form of the heat equation is

$$\frac{d}{dr}\left[r^2\frac{dT}{dr}\right]=0$$

Integrate twice to obtain,

$$r^2\frac{dT}{dr}=C_1 \qquad \text{and} \qquad T=-\frac{C_1}{r}+C_2. \qquad (1,2)$$

The boundary conditions may be obtained from energy balances at the inner and outer surfaces. At the inner surface (r_i),

$$\dot{E}_g=\dot{q}\left(4/3\pi\ r_i^3\right)=q_{cond,i}=-k\left(4\pi\ r_i^2\right)dT/dr)_{r_i} \qquad dT/dr)_{r_i}=-\dot{q}r_i/3k. \qquad (3)$$

At the outer surface (r_o),

$$q_{cond,o}=-k4\pi\ r_o^2\ dT/dr)_{r_o}=q_{conv}=h4\pi\ r_o^2\left[T\left(r_o\right)-T_\infty\right]$$

$$dT/dr)_{r_o}=-\left(h/k\right)\left[T\left(r_o\right)-T_\infty\right]. \qquad (4)$$

From Eqs. (1) and (3), $C_1=-\dot{q}r_i^3/3k$. From Eqs. (1), (2) and (4)

$$-\frac{\dot{q}r_i^3}{3kr_o^2}=-\left[\frac{h}{k}\right]\left[\frac{\dot{q}r_i^3}{3r_o k}+C_2-T_\infty\right]$$

$$C_2=\frac{\dot{q}r_i^3}{3hr_o^2}-\frac{\dot{q}r_i^3}{3r_o k}+T_\infty.$$

Hence, the temperature distribution is

$$T=\frac{\dot{q}r_i^3}{3k}\left[\frac{1}{r}-\frac{1}{r_o}\right]+\frac{\dot{q}r_i^3}{3hr_o^2}+T_\infty.$$ <

COMMENTS: Note that $\dot{E}_g=q_{cond,i}=q_{cond,o}=q_{conv}$.

PROBLEM 3.67

KNOWN: Spherical tank of 3-m diameter containing LP gas at -60°C with 250 mm thickness of insulation having thermal conductivity of 0.06 W/m·K. Ambient air temperature and convection coefficient on the outer surface are 20°C and 6 $W/m^2 \cdot K$, respectively.

FIND: (a) Determine the radial position in the insulation at which the temperature is 0°C and (b) If the insulation is pervious to moisture, what conclusions can be reached about ice formation? What effect will ice formation have on the heat gain? How can this situation be avoided?

SCHEMATIC:

Tank, T_t = -60°C, r_i = 1.5 m

$T_{oo}(r_{oo})$ = 0°C

Ambient air

T_∞ = 20°C
h = 6 W/m²-K

Insulation, k = 0.06 W/m-K

t_{ins} = 250 mm

r_i, r_o, r, q, T_t, T_o, T_∞, R_{ins}, R_{cv}

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional, radial (spherical) conduction through the insulation, and (3) Negligible radiation exchange between the insulation outer surface and the ambient surroundings.

ANALYSIS: (a) The heat transfer situation can be represented by the thermal circuit shown above. The heat gain to the tank is

$$q = \frac{T_\infty - T_t}{R_{ins} + R_{cv}} = \frac{[20 - (-60)]\,K}{\left(0.1263 + 4.33 \times 10^{-3}\right) K/W} = 612.4\ W$$

where the thermal resistances for the insulation (see Table 3.3) and the convection process on the outer surface are, respectively,

$$R_{ins} = \frac{1/r_i - 1/r_o}{4\pi k} = \frac{(1/1.50 - 1/1.75)\,m^{-1}}{4\pi \times 0.06\ W/m \cdot K} = 0.1263\ K/W$$

$$R_{cv} = \frac{1}{hA_s} = \frac{1}{h4\pi r_o^2} = \frac{1}{6\ W/m^2 \cdot K \times 4\pi (1.75\ m)^2} = 4.33 \times 10^{-3} K/W$$

To determine the location within the insulation where T_{oo} (r_{oo}) = 0°C, use the conduction rate equation, Eq. 3.35,

$$q = \frac{4\pi k (T_{oo} - T_t)}{(1/r_i - 1/r_{oo})} \qquad r_{oo} = \left[\frac{1}{r_i} - \frac{4\pi k (T_{oo} - T_t)}{q}\right]^{-1}$$

and substituting numerical values, find

$$r_{oo} = \left[\frac{1}{1.5\ m} - \frac{4\pi \times 0.06\ W/m \cdot K (0 - (-60))K}{612.4\ W}\right]^{-1} = 1.687\ m$$ <

(b) With r_{oo} = 1.687 m, we'd expect the region of the insulation $r_i \le r \le r_{oo}$ to be filled with ice formations if the insulation is pervious to water vapor. The effect of the ice formation is to substantially increase the heat gain since k_{ice} is nearly twice that of k_{ins}, and the ice region is of thickness (1.687 – 1.50)m = 187 mm. To avoid ice formation, a vapor barrier should be installed at a radius larger than r_{oo}.

PROBLEM 3.68

KNOWN: Radius and heat dissipation of a hemispherical source embedded in a substrate of prescribed thermal conductivity. Source and substrate boundary conditions.

FIND: Substrate temperature distribution and surface temperature of heat source.

SCHEMATIC:

ASSUMPTIONS: (1) Top surface is adiabatic. Hence, hemispherical source in semi-infinite medium is equivalent to spherical source in infinite medium (with q = 8 W) and heat transfer is one-dimensional in the radial direction, (2) Steady-state conditions, (3) Constant properties, (4) No generation.

ANALYSIS: Heat equation reduces to

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right)=0 \qquad r^2 dT/dr=C_1$$

$$T(r)=-C_1/r+C_2.$$

Boundary conditions:

$$T(\infty)=T_\infty \qquad T(r_o)=T_s$$

Hence, $C_2 = T_\infty$ and

$$T_s=-C_1/r_o+T_\infty \quad \text{and} \quad C_1=r_o(T_\infty-T_s).$$

The temperature distribution has the form

$$T(r)=T_\infty+(T_s-T_\infty)r_o/r$$ <

and the heat rate is

$$q=-kAdT/dr=-k2\pi\, r^2\left[-(T_s-T_\infty)r_o/r^2\right]=k2\pi\, r_o(T_s-T_\infty)$$

It follows that

$$T_s-T_\infty=\frac{q}{k2\pi\, r_o}=\frac{4\ \text{W}}{125\ \text{W/m}\cdot\text{K}\ 2\pi\left(10^{-4}\ \text{m}\right)}=50.9^\circ\text{C}$$

$$T_s=77.9^\circ\text{C}.$$ <

COMMENTS: For the semi-infinite (or infinite) medium approximation to be valid, the substrate dimensions must be much larger than those of the transistor.

PROBLEM 3.69

KNOWN: Critical and normal tissue temperatures. Radius of spherical heat source and radius of tissue to be maintained above the critical temperature. Tissue thermal conductivity.

FIND: General expression for radial temperature distribution in tissue. Heat rate required to maintain prescribed thermal conditions.

SCHEMATIC:

Tissue
$k = 0.5$ W/m·K
$T_b = 37\ ^oC$
r
q
$r_o = 0.5$ mm
T_o
$r_c = 5$ mm
$T_c = 42\ ^oC$

ASSUMPTIONS: (1) One-dimensional, steady-state conduction, (2) Constant k.

ANALYSIS: The appropriate form of the heat equation is

$$\frac{1}{r^2}\frac{d}{dr}\left(r\frac{dT}{dr}\right)=0$$

Integrating twice,

$$\frac{dT}{dr}=\frac{C_1}{r^2}$$

$$T(r)=-\frac{C_1}{r}+C_2$$

Since $T \to T_b$ as $r \to \infty$, $C_2 = T_b$. At $r = r_o$, $q = -k\left(4\pi r_o^2\right)dT/dr|_{r_o} = -4\pi k r_o^2\, C_1 / r_o^2 = -4\pi k C_1$.

Hence, $C_1 = -q/4\pi k$ and the temperature distribution is

$$T(r)=\frac{q}{4\pi kr}+T_b$$ <

It follows that

$$q=4\pi kr\left[T(r)-T_b\right]$$

Applying this result at $r = r_c$,

$$q=4\pi\left(0.5\,\text{W/m}\cdot\text{K}\right)\left(0.005\,\text{m}\right)\left(42-37\right)^\circ\text{C}=0.157\,\text{W}$$ <

COMMENTS: At $r_o = 0.0005$ m, $T(r_o) = \left[q/\left(4\pi k r_o\right)\right]+T_b = 92°C$. Proximity of this temperature to the boiling point of water suggests the need to operate at a lower power dissipation level.

PROBLEM 3.70

KNOWN: Cylindrical and spherical shells with uniform heat generation and surface temperatures.

FIND: Radial distributions of temperature, heat flux and heat rate.

SCHEMATIC:

Cylindrical shell

Spherical shell

ASSUMPTIONS: (1) One-dimensional, steady-state conduction, (2) Uniform heat generation, (3) Constant k.

ANALYSIS: (a) For the *cylindrical shell*, the appropriate form of the heat equation is

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)+\frac{\dot{q}}{k}=0$$

The general solution is

$$T(r)=-\frac{\dot{q}}{4k}r^2+C_1\ln r+C_2$$

Applying the boundary conditions, it follows that

$$T(r_1)=T_{s,1}=-\frac{\dot{q}}{4k}r_1^2+C_1\ln r_1+C_2$$

$$T(r_2)=T_{s,2}=-\frac{\dot{q}}{4k}r_2^2+C_1\ln r_2+C_2$$

which may be solved for

$$C_1=\left[(\dot{q}/4k)\left(r_2^2-r_1^2\right)+\left(T_{s,2}-T_{s,1}\right)\right]\Big/\ln(r_2/r_1)$$

$$C_2=T_{s,2}+(\dot{q}/4k)r_2^2-C_1\ln r_2$$

Hence,

$$T(r)=T_{s,2}+(\dot{q}/4k)\left(r_2^2-r^2\right)+\left[(\dot{q}/4k)\left(r_2^2-r_1^2\right)+\left(T_{s,2}-T_{s,1}\right)\right]\frac{\ln(r/r_2)}{\ln(r_2/r_1)}$$ <

With $q''=-k\,dT/dr$, the heat flux distribution is

$$q''(r)=\frac{\dot{q}}{2}r-\frac{k\left[(\dot{q}/4k)\left(r_2^2-r_1^2\right)+\left(T_{s,2}-T_{s,1}\right)\right]}{r\ln(r_2/r_1)}$$ <

Continued...

PROBLEM 3.70 (Cont.)

Similarly, with q = q'' A(r) = q'' (2πrL), the heat rate distribution is

$$q(r) = \pi L \dot{q} r^2 - \frac{2\pi Lk\left[(\dot{q}/4k)\left(r_2^2 - r_1^2\right) + \left(T_{s,2} - T_{s,1}\right)\right]}{\ln(r_2/r_1)}$$ <

(b) For the *spherical shell*, the heat equation and general solution are

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{dT}{dr}\right) + \frac{\dot{q}}{k} = 0$$

$$T(r) = -(\dot{q}/6k) r^2 - C_1/r + C_2$$

Applying the boundary conditions, it follows that

$$T(r_1) = T_{s,1} = -(\dot{q}/6k) r_1^2 - C_1/r_1 + C_2$$

$$T(r_2) = T_{s,2} = -(\dot{q}/6k) r_2^2 - C_1/r_2 + C_2$$

Hence,

$$C_1 = \left[(\dot{q}/6k)\left(r_2^2 - r_1^2\right) + \left(T_{s,2} - T_{s,1}\right)\right] \Big/ \left[(1/r_1) - (1/r_2)\right]$$

$$C_2 = T_{s,2} + (\dot{q}/6k) r_2^2 + C_1/r_2$$

and

$$T(r) = T_{s,2} + (\dot{q}/6k)\left(r_2^2 - r^2\right) - \left[(\dot{q}/6k)\left(r_2^2 - r_1^2\right) + \left(T_{s,2} - T_{s,1}\right)\right]\frac{(1/r) - (1/r_2)}{(1/r_1) - (1/r_2)}$$ <

With q''(r) = - k dT/dr, the heat flux distribution is

$$q''(r) = \frac{\dot{q}}{3} r - \frac{\left[(\dot{q}/6)\left(r_2^2 - r_1^2\right) + k\left(T_{s,2} - T_{s,1}\right)\right]}{(1/r_1) - (1/r_2)} \frac{1}{r^2}$$ <

and, with q = $q''\left(4\pi r^2\right)$, the heat rate distribution is

$$q(r) = \frac{4\pi\dot{q}}{3} r^3 - \frac{4\pi\left[(\dot{q}/6)\left(r_2^2 - r_1^2\right) + k\left(T_{s,2} - T_{s,1}\right)\right]}{(1/r_1) - (1/r_2)}$$ <

PROBLEM 3.71

KNOWN: Temperature distribution in a composite wall.

FIND: (a) Relative magnitudes of interfacial heat fluxes, (b) Relative magnitudes of thermal conductivities, and (c) Heat flux as a function of distance x.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties.

ANALYSIS: (a) For the prescribed conditions (one-dimensional, steady-state, constant k), the parabolic temperature distribution in C implies the existence of heat generation. Hence, since dT/dx *increases* with *decreasing* x, the heat flux in C *increases* with *decreasing* x. Hence,

$$q''_3 > q''_4$$

However, the linear temperature distributions in A and B indicate no generation, in which case

$$q''_2 = q''_3$$

(b) Since conservation of energy requires that $q''_{3,B} = q''_{3,C}$ and $dT/dx)_B < dT/dx)_C$, it follows from Fourier's law that

$$k_B > k_C.$$

Similarly, since $q''_{2,A} = q''_{2,B}$ and $dT/dx)_A > dT/dx)_B$, it follows that

$$k_A < k_B.$$

(c) It follows that the flux distribution appears as shown below.

COMMENTS: Note that, with $dT/dx)_{4,C} = 0$, the interface at 4 is adiabatic.

PROBLEM 3.72

KNOWN: Plane wall with internal heat generation which is insulated at the inner surface and subjected to a convection process at the outer surface.

FIND: Maximum temperature in the wall.

SCHEMATIC:

$k = 25\ W/m\cdot K$
$\dot{q} = 0.3\times10^6\ W/m^3$
T_o
T_s
Insulation
$T_\infty = 92°C$
$h = 500\ W/m^2\cdot K$
x
$L = 0.1m$

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction with uniform volumetric heat generation, (3) Inner surface is adiabatic.

ANALYSIS: From Eq. 3.42, the temperature at the inner surface is given by Eq. 3.43 and is the maximum temperature within the wall,

$$T_o = \dot{q}L^2/2k + T_s.$$

The outer surface temperature follows from Eq. 3.46,

$$T_s = T_\infty + \dot{q}L/h$$

$$T_s = 92°C + 0.3\times10^6\ \frac{W}{m^3}\times0.1m/500W/m^2\cdot K = 92°C + 60°C = 152°C.$$

It follows that

$$T_o = 0.3\times10^6 W/m^3\times(0.1m)^2/2\times25W/m\cdot K + 152°C$$

$$T_o = 60°C + 152°C = 212°C.$$ <

COMMENTS: The heat flux leaving the wall can be determined from knowledge of h, T_s and T_∞ using Newton's law of cooling.

$$q''_{conv} = h(T_s - T_\infty) = 500W/m^2\cdot K(152-92)°C = 30kW/m^2.$$

This same result can be determined from an energy balance on the entire wall, which has the form

$$\dot{E}_g - \dot{E}_{out} = 0$$

where

$$\dot{E}_g = \dot{q}AL \qquad \text{and} \qquad \dot{E}_{out} = q''_{conv}\cdot A.$$

Hence,

$$q''_{conv} = \dot{q}L = 0.3\times10^6 W/m^3\times0.1m = 30kW/m^2.$$

PROBLEM 3.73

KNOWN: Composite wall with outer surfaces exposed to convection process.

FIND: (a) Volumetric heat generation and thermal conductivity for material B required for special conditions, (b) Plot of temperature distribution, (c) T_1 and T_2, as well as temperature distributions corresponding to loss of coolant condition where h = 0 on surface A.

SCHEMATIC:

$T_1 = 261\ ^oC$, $T_2 = 211\ ^oC$; T_∞, h; $T_\infty = 25\ ^oC$, $h = 1000\ W/m^2{\cdot}K$; A, B ($\dot{q}$), C; L_A, $2L_B$, L_C

L_A = 30 mm
L_B = 30 mm
L_C = 20 mm
k_A = 25 W/m·K
k_C = 50 W/m·K

ASSUMPTIONS: (1) Steady-state, one-dimensional heat transfer, (2) Negligible contact resistance at interfaces, (3) Uniform generation in B; zero in A and C.

ANALYSIS: (a) From an energy balance on wall B,

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = \dot{E}_{st}$$

$$-q_1'' - q_2'' + 2\dot{q}L_B = 0$$

$$\dot{q}_B = \left(q_1'' + q_2''\right)/2L_B \,.$$

(q_1'', q_2'', $\dot{E}_g$, B, $2L_B = 60$ mm)

To determine the heat fluxes, q_1'' and q_2'', construct thermal circuits for A and C:

$T_\infty = 25\ ^oC$ — $R''_{conv} = 1/h$ — $R''_A = L_A/k_A$ — $T_1 = 261\ ^oC$ ← q_1''

q_2'' → $T_2 = 211\ ^oC$ — $R''_C = L_C/k_C$ — $R''_{conv} = 1/h$ — $T_\infty = 25\ ^oC$

$$q_1'' = \left(T_1 - T_\infty\right)/\left(1/h + L_A/k_A\right) \qquad q_2'' = \left(T_2 - T_\infty\right)/\left(L_C/k_C + 1/h\right)$$

$$q_1'' = (261-25)^\circ C \Big/ \left(\frac{1}{1000\,W/m^2\cdot K} + \frac{0.030\,m}{25\,W/m\cdot K}\right) \qquad q_2'' = (211-25)^\circ C \Big/ \left(\frac{0.020\,m}{50\,W/m\cdot K} + \frac{1}{1000\,W/m^2\cdot K}\right)$$

$$q_1'' = 236^\circ C/(0.001 + 0.0012)\,m^2\cdot K/W \qquad q_2'' = 186^\circ C/(0.0004 + 0.001)\,m^2\cdot K/W$$

$$q_1'' = 107,273\,W/m^2 \qquad q_2'' = 132,857\,W/m^2$$

Using the values for q_1'' and q_2'' in Eq. (1), find

$$\dot{q}_B = \left(106,818 + 132,143\,W/m^2\right)/2\times 0.030\,m = 4.00\times 10^6\,W/m^3\,.$$ <

To determine k_B, use the general form of the temperature and heat flux distributions in wall B,

$$T(x) = -\frac{\dot{q}_B}{2k_B}x^2 + C_1 x + C_2 \qquad q_x''(x) = -k_B\left[-\frac{\dot{q}}{k_B}x + C_1\right] \qquad (1,2)$$

there are 3 unknowns, C_1, C_2 and k_B, which can be evaluated using three conditions,

Continued...

PROBLEM 3.73 (Cont.)

$$T(-L_B) = T_1 = -\frac{\dot{q}_B}{2k_B}(-L_B)^2 - C_1 L_B + C_2 \qquad \text{where } T_1 = 261°C \qquad (3)$$

$$T(+L_B) = T_2 = -\frac{\dot{q}_B}{2k_B}(+L_B)^2 + C_1 L_B + C_2 \qquad \text{where } T_2 = 211°C \qquad (4)$$

$$q''_x(-L_B) = -q''_1 = -k_B\left[-\frac{\dot{q}_B}{k_B}(-L_B) + C_1\right] \qquad \text{where } q''_1 = 107{,}273 \text{ W/m}^2 \qquad (5)$$

Using IHT to solve Eqs. (3), (4) and (5) simultaneously with $\dot{q}_B = 4.00 \times 10^6$ W/m^3, find

$k_B = 15.3 \text{ W/m} \cdot \text{K}$ <

(b) Following the method of analysis in the *IHT Example 3.6, User-Defined Functions*, the temperature distribution is shown in the plot below. The important features are (1) Distribution is quadratic in B, but non-symmetrical; linear in A and C; (2) Because thermal conductivities of the materials are different, discontinuities exist at each interface; (3) By comparison of gradients at x = $-L_B$ and $+L_B$, find $q''_2 > q''_1$.

(c) Using the same method of analysis as for Part (c), the temperature distribution is shown in the plot below when h = 0 on the surface of A. Since the left boundary is adiabatic, material A will be isothermal at T_1. Find

$T_1 = 835°C$ $\qquad T_2 = 360°C$ <

PROBLEM 3.74

KNOWN: Composite wall exposed to convection process; inside wall experiences a uniform heat generation.

FIND: (a) Neglecting interfacial thermal resistances, determine T_1 and T_2, as well as the heat fluxes through walls A and C, and (b) Determine the same parameters, but consider the interfacial contact resistances. Plot temperature distributions.

SCHEMATIC:

$T_\infty = 25\ ^oC$
$h = 1000\ W/m^2 \cdot K$

$k_A = 25\, W/m\cdot K \qquad L_A = 30\, mm$

$k_B = 15\, W/m\cdot K \qquad L_B = 30\, mm$

$k_C = 50\, W/m\cdot K \qquad L_C = 20\, mm$

$\dot{q}_B = 4\times 10^6\, W/m^3$

ASSUMPTIONS: (1) One-dimensional, steady-state heat flow, (2) Negligible contact resistance between walls, part (a), (3) Uniform heat generation in B, zero in A and C, (4) Uniform properties, (5) Negligible radiation at outer surfaces.

ANALYSIS: (a) The temperature distribution in wall B follows from Eq. 3.41,

$$T(x) = \frac{\dot{q}_B L_B^2}{2k_B}\left(1 - \frac{x^2}{L_B^2}\right) + \frac{T_2 - T_1}{2}\frac{x}{L_B} + \frac{T_1 - T_2}{2}. \qquad (1)$$

The heat fluxes to the neighboring walls are found using Fourier's law,

$$q''_x = -k\frac{dT}{dx}.$$

$$\text{At } x = -L_B: \quad q''_x(-L_B) - k_B\left[+\frac{\dot{q}_B}{k_B}(L_B) + \frac{T_2 - T_1}{2L_B}\right] = q''_1 \; (2)$$

$$\text{At } x = +L_B: \quad q''_x(L_B) - k_B\left[-\frac{\dot{q}_B}{k_B}(L_B) + \frac{T_2 - T_1}{2L_B}\right] = q''_2 \; (3)$$

The heat fluxes, q''_1 and q''_2, can be evaluated by thermal circuits.

Substituting numerical values, find

$$q''_1 = (T_\infty - T_1)^\circ C \Big/ (1/h + L_A/k_A) = (25 - T_1)^\circ C \Big/ \left(1/1000\, W/m^2\cdot K + 0.03\, m/25\, W/m\cdot K\right)$$

$$q''_1 = (25 - T_1)^\circ C \Big/ (0.001 + 0.0012)\, K/W = 454.6(25 - T_1) \qquad (4)$$

$$q''_2 = (T_2 - T_\infty)^\circ C \Big/ (1/h + L_C/k_C) = (T_2 - 25)^\circ C \Big/ \left(1/1000\, W/m^2\cdot K + 0.02\, m/50\, W/m\cdot K\right)$$

$$q''_2 = (T_2 - 25)^\circ C \Big/ (0.001 + 0.0004)\, K/W = 714.3(T_2 - 25). \qquad (5)$$

Continued...

Substituting the expressions for the heat fluxes, Eqs. (4) and (5), into Eqs. (2) and (3), a system of two equations with two unknowns is obtained.

Eq. (2): $-4\times10^6\,\mathrm{W/m^3}\times0.03\,\mathrm{m}+15\,\mathrm{W/m\cdot K}\,\dfrac{T_2-T_1}{2\times0.03\,\mathrm{m}}=q_1''$

$$-1.2\times10^5\,\mathrm{W/m^2}-2.5\times10^2\left(T_2-T_1\right)\mathrm{W/m^2}=454.6\left(25-T_1\right)$$

$$704.6\,T_1-250\,T_2=131{,}365 \qquad (6)$$

Eq. (3): $+4\times10^6\,\mathrm{W/m^3}\times0.03\,\mathrm{m}-15\,\mathrm{W/m\cdot K}\,\dfrac{T_2-T_1}{2\times0.03\,\mathrm{m}}=q_2''$

$$+1.2\times10^5\,\mathrm{W/m^2}-2.5\times10^2\left(T_2-T_1\right)\mathrm{W/m^2}=714.3\left(T_2-25\right)$$

$$250\,T_1-964\,T_2=-137{,}857 \qquad (7)$$

Solving Eqs. (6) and (7) simultaneously, find

T_1 = 260.9°C $\qquad$ T_2 = 210.0°C $\qquad$ <

From Eqs. (4) and (5), the heat fluxes at the interfaces and through walls A and C are, respectively,

$$q_1''=454.6\left(25-260.9\right)=-107{,}240\,\mathrm{W/m^2} \qquad <$$

$$q_2''=714.3\left(210-25\right)=+132{,}146\,\mathrm{W/m^2}. \qquad <$$

Note directions of the heat fluxes.

(b) Considering interfacial contact resistances, we will use a different approach. The general solution for the temperature and heat flux distributions in each of the materials is

$$T_A(x)=C_1x+C_2 \qquad q_x''=-k_AC_1 \qquad -\left(L_A+L_B\right)\le x\le -L_B \qquad (1,2)$$

$$T_B(x)=-\frac{\dot q_B}{2k_B}x^2+C_3x+C_4 \qquad q_x''=-\frac{\dot q_B}{k_B}x+C_3 \qquad -L_B\le x\le L_B \qquad (3,4)$$

$$T_C(x)=C_5x+C_6 \qquad q_x''=-k_CC_5 \qquad +L_B\le x\le\left(L_B+L_C\right) \qquad (5,6)$$

To determine C_1 ... C_6 and the distributions, we need to identify boundary conditions using surface energy balances.

At x = -(L_A + L_B):

$$-q_x''\left(-L_A-L_B\right)+q_{cv}''=0 \qquad (7)$$

$$-\left(-k_AC_1\right)+h\left[T_\infty-T_A\left(-L_A-L_B\right)\right] \qquad (8)$$

At x = -L_B: The heat flux must be continuous, but the temperature will be discontinuous across the contact resistance.

$$q_{x,A}''\left(-L_B\right)=q_{x,B}''\left(-L_B\right) \qquad (9)$$

$$q_{x,A}''\left(-L_B\right)=\left[T_{1A}\left(-L_B\right)-T_{1B}\left(-L_B\right)\right]/R_{tc,AB}'' \qquad (10)$$

Continued...

PROBLEM 3.74 (Cont.)

At x = + L_B: The same conditions apply as for x = -L_B,

$$q''_{x,B}(+L_B) = q''_{x,C}(+L_B) \quad (11)$$

$$q''_{x,B}(+L_B) = \left[T_{2B}(+L_B) - T_{2C}(+L_B)\right] / R''_{tc,BC} \quad (12)$$

At x = +(L_B + L_C):

$$-q_{x,C}(L_B + L_C) - q''_{cv} = 0 \quad (13)$$

$$-(-k_C C_5) - h\left[T_C(L_B + L_C) - T_\infty\right] = 0 \quad (14)$$

Following the method of analysis in IHT Example 3.6, User-Defined Functions, we solve the system of equations above for the constants C_1 ... C_6 for conditions with negligible and prescribed values for the interfacial constant resistances. The results are tabulated and plotted below; q''_1 and q''_2 represent heat fluxes leaving surfaces A and C, respectively.

Conditions	T_{1A} (°C)	T_{1B} (°C)	T_{2B} (°C)	T_{2C} (°C)	q''_1 (kW/m^2)	q''_2 (kW/m^2)
$R''_{tc} = 0$	260	260	210	210	106.8	132.0
$R''_{tc} \neq 0$	233	470	371	227	94.6	144.2

COMMENTS: (1) The results for part (a) can be checked using an energy balance on wall B,

$$\dot{E}_{in} - \dot{E}_{out} = -\dot{E}_g$$

$$q''_1 - q''_2 = -\dot{q}_B \times 2L_B$$

where

$$q''_1 - q''_2 = -107,240 - 132,146 = 239,386\ \text{W/m}^2$$

$$-\dot{q}_B L_B = -4 \times 10^6\ \text{W/m}^3 \times 2(0.03\ \text{m}) = -240,000\ \text{W/m}^2 .$$

Hence, we have confirmed proper solution of Eqs. (6) and (7).

(2) Note that the effect of the interfacial contact resistance is to increase the temperature at all locations. The total heat flux leaving the composite wall ($q_1 + q_2$) will of course be the same for both cases.

PROBLEM 3.75

KNOWN: Composite wall of materials A and B. Wall of material A has uniform generation, while wall B has no generation. The inner wall of material A is insulated, while the outer surface of material B experiences convection cooling. Thermal contact resistance between the materials is $R''_{t,c} = 10^{-4}\,m^2 \cdot K/W$. See Ex. 3.6 that considers the case without contact resistance.

FIND: Compute and plot the temperature distribution in the composite wall.

SCHEMATIC:

$R''_{t,c} = 10^{-4}$ m²-K/W

T_0, T_2, Insulation, A, B, k_A, k_B, T_{1A}, T_{1B}

$T_\infty = 30°C$, $h = 1000\ W/m^2\text{-}K$

$\dot{q}_A = 1.5\times10^6\ W/m^3$, $k_A = 75\ W/m\text{-}K$

$\dot{q}_B = 0$, $k_B = 150\ W/m\text{-}K$

$L_A = 50$ mm, $L_B = 20$ mm, x

$q''_x(L_A) = \dot{q}_A L_A$ → T_{1A} — $R''_{t,c}$ — T_{1B} — $R''_{cond,B}$ — T_2 — R''_{conv} — T_∞

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction with constant properties, and (3) Inner surface of material A is adiabatic.

ANALYSIS: From the analysis of Ex. 3.6, we know the temperature distribution in material A is parabolic with zero slope at the inner boundary, and that the distribution in material B is linear. At the interface between the two materials, $x = L_A$, the temperature distribution will show a discontinuity.

$$T_A(x) = \frac{\dot{q} L_A^2}{2k_A}\left(1 - \frac{x^2}{L_A^2}\right) + T_{1A} \qquad 0 \le x \le L_A$$

$$T_B(x) = T_{1B} - (T_{1B} - T_2)\frac{x - L_A}{L_B} \qquad L_A \le x \le L_A + L_B$$

Considering the thermal circuit above (see also Ex. 3.6) including the thermal contact resistance,

$$q'' = \dot{q} L_A = \frac{T_{1A} - T_\infty}{R''_{tot}} = \frac{T_{1B} - T_\infty}{R''_{cond,B} + R''_{conv}} = \frac{T_2 - T_\infty}{R''_{conv}}$$

find $T_A(0) = 147.5°C$, $T_{1A} = 122.5°C$, $T_{1B} = 115°C$, and $T_2 = 105°C$. Using the foregoing equations in IHT, the temperature distributions for each of the materials can be calculated and are plotted on the graph below.

Effect of thermal contact resistance on temperature distribution

COMMENTS: (1) The effect of the thermal contact resistance between the materials is to increase the maximum temperature of the system.

(2) Can you explain why the temperature distribution in the material B is not affected by the presence of the thermal contact resistance at the materials' interface?

PROBLEM 3.76

KNOWN: Plane wall of thickness 2L, thermal conductivity k with uniform energy generation $\dot{q}$. For case 1, boundary at x = -L is perfectly insulated, while boundary at x = +L is maintained at T_o = 50°C. For case 2, the boundary conditions are the same, but a thin dielectric strip with thermal resistance $R''_t = 0.0005\ m^2 \cdot K/W$ is inserted at the mid-plane.

FIND: (a) Sketch the temperature distribution for case 1 on T-x coordinates and describe key features; identify and calculate the maximum temperature in the wall, (b) Sketch the temperature distribution for case 2 on the same T-x coordinates and describe the key features; (c) What is the temperature difference between the two walls at x = 0 for case 2? And (d) What is the location of the maximum temperature of the composite wall in case 2; calculate this temperature.

SCHEMATIC:

$\dot{q}$ = 5x10^6 W/m^3
k = 50 W/m-K
L = 20 mm

Case 1: $\dot{q}$,k; T_o; -L, x, +L

Case 2: R''_t = 0.0005 m^2-K/W; T_o = 50°C; A, B; $\dot{q}$,k; -L, x, +L

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in the plane and composite walls, and (3) Constant properties.

ANALYSIS: (a) For case 1, the temperature distribution, $T_1(x)$ vs. x, is parabolic as shown in the schematic below and the gradient is zero at the insulated boundary, x = -L. From Eq. 3.43,

$$T_1(-L) - T_1(+L) = \frac{\dot{q}(2L)^2}{2k} = \frac{5\times10^6\ W/m^3\,(2\times0.020\ m)^2}{2\times50\ W/m\cdot K} = 80°C$$

and since $T_1(+L) = T_o$ = 50°C, the maximum temperature occurs at x = -L,

$$T_1(-L) = T_1(+L) + 80°C = 130°C$$

(b) For case 2, the temperature distribution, $T_2(x)$ vs. x, is piece-wise parabolic, with zero gradient at x = -L and a drop across the dielectric strip, ΔT_{AB}. The temperature gradients at either side of the dielectric strip are equal.

Parts (a,b) Temperature distributions

T(x); $T_2(0)$; $T_2(x)$; $T_1(0)$; $T_1(x)$; ΔT_{AB}; T_o; -L; 0; +L; x

Part (d) Surface energy balance

ΔT_{AB}; A; B; $q''_x(0)$; k; k; x = 0

(c) For case 2, the temperature drop across the thin dielectric strip follows from the surface energy balance shown above.

$$q''_x(0) = \Delta T_{AB}/R''_t \qquad\qquad q''_x(0) = \dot{q}L$$

$$\Delta T_{AB} = R''_t\,\dot{q}L = 0.0005\ m^2\cdot K/W \times 5\times10^6\ W/m^3 \times 0.020\ m = 50°C.$$

(d) For case 2, the maximum temperature in the composite wall occurs at x = -L, with the value,

$$T_2(-L) = T_1(-L) + \Delta T_{AB} = 130°C + 50°C = 180°C$$

<

PROBLEM 3.77

KNOWN: Geometry and boundary conditions of a nuclear fuel element.

FIND: (a) Expression for the temperature distribution in the fuel, (b) Form of temperature distribution for the entire system.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional heat transfer, (2) Steady-state conditions, (3) Uniform generation, (4) Constant properties, (5) Negligible contact resistance between fuel and cladding.

ANALYSIS: (a) The general solution to the heat equation, Eq. 3.39,

$$\frac{d^2T}{dx^2}+\frac{\dot{q}}{k_f}=0 \qquad (-L \le x \le +L)$$

is $\quad T=-\frac{\dot{q}}{2k_f}x^2+C_1x+C_2.$

The insulated wall at x = - (L+b) dictates that the heat flux at x = - L is zero (for an energy balance applied to a control volume about the wall, $\dot{E}_{in}=\dot{E}_{out}=0$). Hence

$$\left.\frac{dT}{dx}\right]_{x=-L}=-\frac{\dot{q}}{k_f}(-L)+C_1=0 \qquad \text{or} \qquad C_1=-\frac{\dot{q}L}{k_f}$$

$$T=-\frac{\dot{q}}{2k_f}x^2-\frac{\dot{q}L}{k_f}x+C_2.$$

The value of $T_{s,1}$ may be determined from the energy conservation requirement that $\dot{E}_g=q_{cond}=q_{conv}$, or on a unit area basis.

$$\dot{q}(2L)=\frac{k_s}{b}\left(T_{s,1}-T_{s,2}\right)=h\left(T_{s,2}-T_\infty\right).$$

Hence,

$$T_{s,1}=\frac{\dot{q}(2\,Lb)}{k_s}+T_{s,2} \qquad \text{where} \qquad T_{s,2}=\frac{\dot{q}(2L)}{h}+T_\infty$$

$$T_{s,1}=\frac{\dot{q}(2\,Lb)}{k_s}+\frac{\dot{q}(2L)}{h}+T_\infty.$$

Continued …..

Hence from Eq. (1),

$$T(L) = T_{s,1} = \frac{\dot{q}(2\,Lb)}{k_s} + \frac{\dot{q}(2\,L)}{h} + T_\infty = -\frac{3}{2}\ \frac{\dot{q}\left(L^2\right)}{k_f} + C_2$$

which yields

$$C_2 = T_\infty + \dot{q}L\left[\frac{2b}{k_s} + \frac{2}{h} + \frac{3}{2}\ \frac{L}{k_f}\right]$$

Hence, the temperature distribution for $(-L \le x \le +L)$ is

$$T = -\frac{\dot{q}}{2k_f}x^2 - \frac{\dot{q}L}{k_f}x + \dot{q}L\left[\frac{2b}{k_s} + \frac{2}{h} + \frac{3}{2}\ \frac{L}{k_f}\right] + T_\infty$$ <

(b) For the temperature distribution shown below,

$(-L - b) \le x \le -L$: $dT/dx=0,\ T=T_{max}$
$-L \le x \le +L$: $|dT/dx| \uparrow$ with $\uparrow$ x
$+L \le x \le L+b$: (dT/dx) is const.

PROBLEM 3.78

KNOWN: Thermal conductivity, heat generation and thickness of fuel element. Thickness and thermal conductivity of cladding. Surface convection conditions.

FIND: (a) Temperature distribution in fuel element with one surface insulated and the other cooled by convection. Largest and smallest temperatures and corresponding locations. (b) Same as part (a) but with equivalent convection conditions at both surfaces, (c) Plot of temperature distributions.

SCHEMATIC:

L = 0.15 m
b = 0.003 m
Insulated surface (part a)
Cladding k_s = 15 W/m-K
T_∞ = 200°C
h = 10,000 W/m²-K
Fuel, $\dot{q} = 2\times10^7$ W/m³, k_f = 60 W/m-K

ASSUMPTIONS: (1) One-dimensional heat transfer, (2) Steady-state, (3) Uniform generation, (4) Constant properties, (5) Negligible contact resistance.

ANALYSIS: (a) From Eq. C.1,

$$T(x) = \frac{\dot{q}L^2}{2k_f}\left(1 - \frac{x^2}{L^2}\right) + \frac{T_{s,2} - T_{s,1}}{2}\frac{x}{L} + \frac{T_{s,1} + T_{s,2}}{2} \tag{1}$$

With an insulated surface at x = -L, Eq. C.10 yields

$$T_{s,1} - T_{s,2} = \frac{2\dot{q}L^2}{k_f} \tag{2}$$

and with convection at x = L + b, Eq. C.13 yields

$$U\left(T_{s,2} - T_\infty\right) = \dot{q}L - \frac{k_f}{2L}\left(T_{s,2} - T_{s,1}\right)$$

$$T_{s,1} - T_{s,2} = \frac{2LU}{k_f}\left(T_{s,2} - T_\infty\right) - \frac{2\dot{q}L^2}{k_f} \tag{3}$$

Substracting Eq. (2) from Eq. (3),

$$0 = \frac{2LU}{k_f}\left(T_{s,2} - T_\infty\right) - \frac{4\dot{q}L^2}{k_f}$$

$$T_{s,2} = T_\infty + \frac{2\dot{q}L}{U} \tag{4}$$

Continued …..

PROBLEM 3.78 (Cont.)

and substituting into Eq. (2)

$$T_{s,1} = T_\infty + 2\dot{q}L\left(\frac{L}{k_f} + \frac{1}{U}\right) \qquad (5)$$

Substituting Eqs. (4) and (5) into Eq. (1),

$$T(x) = -\frac{\dot{q}}{2k_f}x^2 - \frac{\dot{q}L}{k_f}x + \dot{q}L\left(\frac{2}{U} + \frac{3}{2}\frac{L}{k_f}\right) + T_\infty$$

or, with $U^{-1} = h^{-1} + b/k_s$,

$$T(x) = -\frac{\dot{q}}{2k_f}x^2 - \frac{\dot{q}L}{k_f}x + \dot{q}L\left(\frac{2b}{k_s} + \frac{2}{h} + \frac{3}{2}\frac{L}{k_f}\right) + T_\infty \qquad (6)$$ <

The maximum temperature occurs at x = - L and is

$$T(-L) = 2\dot{q}L\left(\frac{b}{k_s} + \frac{1}{h} + \frac{L}{k_f}\right) + T_\infty$$

$$T(-L) = 2\times 2\times 10^7\ \text{W/m}^3 \times 0.015\,\text{m}\left(\frac{0.003\text{m}}{15\ \text{W/m}\cdot\text{K}} + \frac{1}{10,000\ \text{W/m}^2\cdot\text{K}} + \frac{0.015\,\text{m}}{60\ \text{W/m}\cdot\text{K}}\right) + 200°\text{C} = 530°\text{C}$$ <

The lowest temperature is at x = + L and is

$$T(+L) = -\frac{3}{2}\frac{\dot{q}L^2}{k_f} + \dot{q}L\left(\frac{2b}{k_s} + \frac{2}{h} + \frac{3}{2}\frac{L}{k_f}\right) + T_\infty = 380°\text{C}$$ <

(b) If a convection condition is maintained at x = - L, Eq. C.12 reduces to

$$U\left(T_\infty - T_{s,1}\right) = -\dot{q}L - \frac{k_f}{2L}\left(T_{s,2} - T_{s,1}\right)$$

$$T_{s,1} - T_{s,2} = \frac{2LU}{k_f}\left(T_{s,1} - T_\infty\right) - \frac{2\dot{q}L^2}{k_f} \qquad (7)$$

Subtracting Eq. (7) from Eq. (3),

$$0 = \frac{2LU}{k_f}\left(T_{s,2} - T_\infty - T_{s,1} + T_\infty\right) \qquad \text{or} \qquad T_{s,1} = T_{s,2}$$

Hence, from Eq. (7)

Continued …..

PROBLEM 3.78 (Cont.)

$$T_{s,1} = T_{s,2} = \frac{\dot{q}L}{U} + T_\infty = \dot{q}L\left(\frac{1}{h} + \frac{b}{k_s}\right) + T_\infty \tag{8}$$

Substituting into Eq. (1), the temperature distribution is

$$T(x) = \frac{\dot{q}L^2}{2k_f}\left(1 - \frac{x^2}{L^2}\right) + \dot{q}L\left(\frac{1}{h} + \frac{b}{k_s}\right) + T_\infty \tag{9}$$ <

The maximum temperature is at x = 0 and is

$$T(0) = \frac{2\times10^7\ \mathrm{W/m^3}\,(0.015\,\mathrm{m})^2}{2\times60\ \mathrm{W/m\cdot K}} + 2\times10^7\ \mathrm{W/m^3}\times0.015\,\mathrm{m}\left(\frac{1}{10,000\ \mathrm{W/m^2\cdot K}} + \frac{0.003\,\mathrm{m}}{15\ \mathrm{W/m\cdot K}}\right) + 200°\mathrm{C}$$

$$T(0) = 37.5°\mathrm{C} + 90°\mathrm{C} + 200°\mathrm{C} = 327.5°\mathrm{C}$$ <

The minimum temperature at x = ± L is

$$T_{s,1} = T_{s,2} = 2\times10^7\ \mathrm{W/m^3}\,(0.015\,\mathrm{m})\left(\frac{1}{10,000\ \mathrm{W/m^2\cdot K}} + \frac{0.003\,\mathrm{m}}{15\ \mathrm{W/m\cdot K}}\right) + 200°\mathrm{C} = 290°\mathrm{C}$$ <

(c) The temperature distributions are as shown.

The amount of heat generation is the same for both cases, but the ability to transfer heat from both surfaces for case (b) results in lower temperatures throughout the fuel element.

COMMENTS: Note that for case (a), the temperature in the insulated cladding is constant and equivalent to $T_{s,1}$ = 530°C.

PROBLEM 3.79

KNOWN: Wall of thermal conductivity k and thickness L with uniform generation $\dot{q}$; strip heater with uniform heat flux q_o''; prescribed inside and outside air conditions (h_i, $T_{\infty,i}$, h_o, $T_{\infty,o}$).

FIND: (a) Sketch temperature distribution in wall if none of the heat generated within the wall is lost to the outside air, (b) Temperatures at the wall boundaries T(0) and T(L) for the prescribed condition, (c) Value of q_o'' required to maintain this condition, (d) Temperature of the outer surface, T(L), if $\dot{q}=0$ but q_o'' corresponds to the value calculated in (c).

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Uniform volumetric generation, (4) Constant properties.

ANALYSIS: (a) If none of the heat generated within the wall is lost to the *outside* of the chamber, the gradient at x = 0 must be zero. Since $\dot{q}$ is uniform, the temperature distribution is parabolic, with $T(L) > T_{\infty,i}$.

(b) To find temperatures at the boundaries of wall, begin with the general solution to the appropriate form of the heat equation (Eq.3.40).

$$T(x) = -\frac{\dot{q}}{2k}x^2 + C_1x + C_2 \qquad (1)$$

From the first boundary condition,

$$\left.\frac{dT}{dx}\right|_{x=o} = 0 \quad \rightarrow \quad C_1 = 0. \qquad (2)$$

Two approaches are possible using different forms for the second boundary condition.

Approach No. 1: With boundary condition $\rightarrow T(0) = T_1$

$$T(x) = -\frac{\dot{q}}{2k}x^2 + T_1 \qquad (3)$$

To find T_1, perform an overall energy balance on the wall

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = 0$$

$$-h\left[T(L) - T_{\infty,i}\right] + \dot{q}L = 0 \qquad T(L) = T_2 = T_{\infty,i} + \frac{\dot{q}L}{h} \qquad (4)$$

Continued …..

PROBLEM 3.79 (Cont.)

and from Eq. (3) with x = L and T(L) = T_2,

$$T(L) = -\frac{\dot{q}}{2k}L^2 + T_1 \quad \text{or} \quad T_1 = T_2 + \frac{\dot{q}}{2k}L^2 = T_{\infty,i} + \frac{\dot{q}L}{h} + \frac{\dot{q}L^2}{2k} \qquad (5,6)$$

Substituting numerical values into Eqs. (4) and (6), find

$$T_2 = 50^\circ\text{C} + 1000\ \text{W/m}^3 \times 0.200\ \text{m}/20\ \text{W/m}^2\cdot\text{K} = 50^\circ\text{C} + 10^\circ\text{C} = 60^\circ\text{C}$$ <

$$T_1 = 60^\circ\text{C} + 1000\ \text{W/m}^3 \times (0.200\ \text{m})^2 / 2\times 4\ \text{W/m}\cdot\text{K} = 65^\circ\text{C}.$$ <

Approach No. 2: Using the boundary condition

$$-k\frac{dT}{dx}\Big|_{x=L} = h\left[T(L) - T_{\infty,i}\right]$$

yields the following temperature distribution which can be evaluated at x = 0,L for the required temperatures,

$$T(x) = -\frac{\dot{q}}{2k}\left(x^2 - L^2\right) + \frac{\dot{q}L}{h} + T_{\infty,i}.$$

(c) The value of q''_o when T(0) = T_1 = 65°C follows from the circuit

$$q''_o = \frac{T_1 - T_{\infty,o}}{1/h_o}$$

$$q''_o = 5\ \text{W/m}^2\cdot\text{K}\,(65\text{-}25)^\circ\text{C} = 200\ \text{W/m}^2.$$ <

(d) With $\dot{q} = 0$, the situation is represented by the thermal circuit shown. Hence,

$$q''_o = q''_a + q''_b$$

$$q''_o = \frac{T_1 - T_{\infty,o}}{1/h_o} + \frac{T_1 - T_{\infty,i}}{L/k + 1/h_i}$$

which yields

$$T_1 = 55^\circ\text{C}.$$ <

PROBLEM 3.80

KNOWN: Wall of thermal conductivity k and thickness L with uniform generation and strip heater with uniform heat flux q''_o; prescribed inside and outside air conditions ($T_{\infty,i}$, h_i, $T_{\infty,o}$, h_o). Strip heater acts to guard against heat losses from the wall to the outside.

FIND: Compute and plot q''_o and T(0) as a function of $\dot{q}$ for $200 \le \dot{q} \le 2000$ W/m^3 and $T_{\infty,i}$ = 30, 50 and 70°C.

SCHEMATIC:

Strip heater, q''_o; Wall, $\dot{q}$, $k = 4$ W/m·K; Outside chamber; Inside chamber; x; L = 200 mm

$T_{\infty,o} = 25$ °C, $h_o = 5$ W/m^2·K; $T_{\infty,i} = 50$ °C, $h_i = 20$ W/m^2·K

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Uniform volumetric generation, (4) Constant properties.

ANALYSIS: If no heat generated within the wall will be lost to the outside of the chamber, the gradient at the position x = 0 must be zero. Since $\dot{q}$ is uniform, the temperature distribution must be parabolic as shown in the sketch.

T (0); Gradient is zero; T(L); $T_{\infty,i}$; $T_{\infty,o}$; 0; L; x

To determine the required heater flux q''_o as a function of the operation conditions $\dot{q}$ and $T_{\infty,i}$, the analysis begins by considering the temperature distribution in the wall and then surface energy balances at the two wall surfaces. The analysis is organized for easy treatment with equation-solving software.

Temperature distribution in the wall, T(x): The general solution for the temperature distribution in the wall is, Eq. 3.40,

$$T(x) = -\frac{\dot{q}}{2k}x^2 + C_1 x + C_2$$

and the guard condition at the outer wall, x = 0, requires that the conduction heat flux be zero. Using Fourier's law,

$$q''_x(0) = -k\left.\frac{dT}{dx}\right)_{x=0} = -kC_1 = 0 \qquad (C_1 = 0) \qquad (1)$$

At the outer wall, x = 0,

$$T(0) = C_2 \qquad (2)$$

Surface energy balance, x = 0:

q''_o; $q''_{cv,o}$; $q''_x(0)$; T (0); x = 0

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$q''_o - q''_{cv,o} - q''_x(0) = 0 \qquad (3)$$

$$q''_{cv,o} = h\left(T(0) - T_{\infty,o}\right), \; q''_x(0) = 0 \qquad (4a,b)$$

Continued...

PROBLEM 3.80 (Cont.)

Surface energy balance, x = L:

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$q''_x(L) - q''_{cv,i} = 0 \qquad (5)$$

$$q''_x(L) = -k\left.\frac{dT}{dx}\right)_{x=L} = +\dot{q}L \qquad (6)$$

$$q''_{cv,i} = h\left[T(L) - T_{\infty,i}\right]$$

$$q''_{cv,i} = h\left[-\frac{\dot{q}}{2k}L^2 + T(0) - T_{\infty,i}\right] \qquad (7)$$

$q''_x(L)$ → $q''_{cv,i}$, $T(L)$, $x = L$

Solving Eqs. (1) through (7) simultaneously with appropriate numerical values and performing the parametric analysis, the results are plotted below.

Heater flux, q"o (W/m^2)
400
300
200
100
0
0 500 1000 1500 2000
Volumetric generation rate, qdot (W/m^3)
Tinfi = 30 C
Tinfi = 50 C
Tinfi = 70 C

Wall temperature, T(0) (C)
120
100
80
60
40
20
0 500 1000 1500 2000
Volumetric generation rate, qdot (W/m^3)
Tinfi = 30 C
Tinfi = 50 C
Tinfi = 70 C

From the first plot, the heater flux q''_o is a linear function of the volumetric generation rate $\dot{q}$. As expected, the higher $\dot{q}$ and $T_{\infty,i}$, the higher the heat flux required to maintain the guard condition ($q''_x(0) = 0$). Notice that for any $\dot{q}$ condition, equal changes in $T_{\infty,i}$ result in equal changes in the required q''_o. The outer wall temperature T(0) is also linearly dependent upon $\dot{q}$. From our knowledge of the temperature distribution, it follows that for any $\dot{q}$ condition, the outer wall temperature T(0) will track changes in $T_{\infty,i}$.

PROBLEM 3.81

KNOWN: Plane wall with prescribed nonuniform volumetric generation having one boundary insulated and the other isothermal.

FIND: Temperature distribution, T(x), in terms of x, L, k, $\dot{q}_o$ and T_o.

SCHEMATIC:

$\dot{q}(x) = \dot{q}_o(1-\frac{x}{L})$ — Insulation — T_o — x — L

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in x-direction, (3) Constant properties.

ANALYSIS: The appropriate form the heat diffusion equation is

$$\frac{d}{dx}\left[\frac{dT}{dx}\right]+\frac{\dot{q}}{k}=0.$$

Noting that $\dot{q}=\dot{q}(x)=\dot{q}_o\ (1-x/L)$, substitute for $\dot{q}(x)$ into the above equation, separate variables and then integrate,

$$d\left[\frac{dT}{dx}\right]=-\frac{\dot{q}_o}{k}\left[1-\frac{x}{L}\right]dx \qquad \frac{dT}{dx}=-\frac{\dot{q}_o}{k}\left[x-\frac{x^2}{2L}\right]+C_1.$$

Separate variables and integrate again to obtain the general form of the temperature distribution in the wall,

$$dT=-\frac{\dot{q}_o}{k}\left[x-\frac{x^2}{2L}\right]dx+C_1dx \qquad T(x)=-\frac{\dot{q}_o}{k}\left[\frac{x^2}{2}-\frac{x^3}{6L}\right]+C_1x+C_2.$$

Identify the boundary conditions at x = 0 and x = L to evaluate C_1 and C_2. At x = 0,

$$T(0)=T_o=-\frac{\dot{q}_o}{k}(0-0)+C_1\cdot 0+C_2 \qquad \text{hence, } C_2=T_o$$

At x = L,

$$\left.\frac{dT}{dx}\right]_{x=L}=0=-\frac{\dot{q}_o}{k}\left[L-\frac{L^2}{2L}\right]+C_1 \qquad \text{hence, } C_1=\frac{\dot{q}_oL}{2k}$$

The temperature distribution is

$$T(x)=-\frac{\dot{q}_o}{k}\left[\frac{x^2}{2}-\frac{x^3}{6L}\right]+\frac{\dot{q}_oL}{2k}x+T_o.$$ <

COMMENTS: It is good practice to test the final result for satisfying BCs. The heat flux at x = 0 can be found using Fourier's law or from an overall energy balance

$$\dot{E}_{out}=\dot{E}_g=\int_0^L \dot{q}dV \qquad \text{to obtain} \qquad q''_{out}=\dot{q}_oL/2.$$

PROBLEM 3.82

KNOWN: Distribution of volumetric heating and surface conditions associated with a quartz window.

FIND: Temperature distribution in the quartz.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Negligible radiation emission and convection at inner surface (x = 0) and negligible emission from outer surface, (4) Constant properties.

ANALYSIS: The appropriate form of the heat equation for the quartz is obtained by substituting the prescribed form of $\dot{q}$ into Eq. 3.39.

$$\frac{d^2T}{dx^2} + \frac{\alpha(1-\beta)q_o''}{k} e^{-\alpha x} = 0$$

Integrating,

$$\frac{dT}{dx} = +\frac{(1-\beta)q_o''}{k} e^{-\alpha x} + C_1 \qquad T = -\frac{(1-\beta)}{k\alpha} q_o'' e^{-\alpha x} + C_1 x + C_2$$

Boundary Conditions:

$$-k\, dT/dx)_{x=o} = \beta q_o''$$
$$-k\, dT/dx)_{x=L} = h\left[T(L) - T_\infty\right]$$

Hence, at x = 0:

$$-k\left[\frac{(1-\beta)}{k} q_o'' + C_1\right] = \beta q_o''$$
$$C_1 = -q_o''/k$$

At x = L:

$$-k\left[\frac{(1-\beta)}{k} q_o'' e^{-\alpha L} + C_1\right] = h\left[-\frac{(1-\beta)}{k\alpha} q_o'' e^{-\alpha L} + C_1 L + C_2 - T_\infty\right]$$

Substituting for C_1 and solving for C_2,

$$C_2 = \frac{q_o''}{h}\left[1-(1-\beta)e^{-\alpha L}\right] + \frac{q_o''}{k} + \frac{q_o''(1-\beta)}{k\alpha} e^{-\alpha L} + T_\infty.$$

Hence,

$$T(x) = \frac{(1-\beta)q_o''}{k\alpha}\left[e^{-\alpha L} - e^{-\alpha x}\right] + \frac{q_o''}{k}(L-x) + \frac{q_o''}{h}\left[1-(1-\beta)e^{-\alpha L}\right] + T_\infty. \quad <$$

COMMENTS: The temperature distribution depends strongly on the radiative coefficients, α and β. For $\alpha \to \infty$ or $\beta = 1$, the heating occurs entirely at x = 0 (no volumetric heating).

PROBLEM 3.83

KNOWN: Radial distribution of heat dissipation in a cylindrical container of radioactive wastes. Surface convection conditions.

FIND: Radial temperature distribution.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible temperature drop across container wall.

ANALYSIS: The appropriate form of the heat equation is

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)=-\frac{\dot{q}}{k}=-\frac{\dot{q}_o}{k}\left(1-\frac{r^2}{r_o^2}\right)$$

$$r\frac{dT}{dr}=-\frac{\dot{q}_o r^2}{2k}+\frac{\dot{q}r^4}{4kr_o^2}+C_1 \qquad T=-\frac{\dot{q}_o r^2}{4k}+\frac{\dot{q}_o r^4}{16kr_o^2}+C_1\ln r+C_2.$$

From the boundary conditions,

$$\frac{dT}{dr}\bigg|_{r=0}=0\rightarrow C_1=0 \qquad -k\frac{dT}{dr}\bigg|_{r=r_o}=h\left[T\left(r_o\right)-T_\infty\right]$$

$$+\frac{\dot{q}_o r_o}{2}-\frac{\dot{q}_o r_o}{4}=h\left[-\frac{\dot{q}_o r_o^2}{4k}+\frac{\dot{q}_o r_o^2}{16k}+C_2-T_\infty\right]$$

$$C_2=\frac{\dot{q}_o r_o}{4h}+\frac{3\dot{q}_o r_o^2}{16k}+T_\infty.$$

Hence

$$T(r)=T_\infty+\frac{\dot{q}_o r_o}{4h}+\frac{\dot{q}_o r_o^2}{k}\left[\frac{3}{16}-\frac{1}{4}\left(\frac{r}{r_o}\right)^2+\frac{1}{16}\left(\frac{r}{r_o}\right)^4\right].$$

<

COMMENTS: Applying the above result at r_o yields

$$T_s=T\left(r_o\right)=T_\infty+\left(\dot{q}_o r_o\right)/4h$$

The same result may be obtained by applying an energy balance to a control surface about the container, where $\dot{E}_g=q_{conv}$. The maximum temperature exists at r = 0.

PROBLEM 3.84

KNOWN: Cylindrical shell with uniform volumetric generation is insulated at inner surface and exposed to convection on the outer surface.

FIND: (a) Temperature distribution in the shell in terms of r_i, r_o, $\dot{q}$, h, T_∞ and k, (b) Expression for the heat rate per unit length at the outer radius, $q'(r_o)$.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional radial (cylindrical) conduction in shell, (3) Uniform generation, (4) Constant properties.

ANALYSIS: (a) The general form of the temperature distribution and boundary conditions are

$$T(r) = -\frac{\dot{q}}{4k}r^2 + C_1 \ln r + C_2$$

at $r = r_i$: $$\left.\frac{dT}{dr}\right)_{r_i} = 0 = -\frac{\dot{q}}{2k}r_i + C_1\frac{1}{r_i} + 0 \qquad C_1 = \frac{\dot{q}}{2k}r_i^2$$

at $r = r_o$: $$-k\left.\frac{dT}{dr}\right)_{r_o} = h\left[T(r_o) - T_\infty\right] \qquad \text{surface energy balance}$$

$$k\left[-\frac{\dot{q}}{2k}r_o + \left(\frac{\dot{q}}{2k}r_i^2 \cdot \frac{1}{r_o}\right)\right] = h\left[-\frac{\dot{q}}{4k}r_o^2 + \left(\frac{\dot{q}}{2k}r_i^2\right)\ln r_o + C_2 - T_\infty\right]$$

$$C_2 = -\frac{\dot{q}r_o}{2h}\left[1 + \left(\frac{r_i}{r_o}\right)^2\right] + \frac{\dot{q}r_o^2}{2k}\left[\frac{1}{2} - \left(\frac{r_i}{r_o}\right)^2 \ln r_o\right] + T_\infty$$

Hence,

$$T(r) = \frac{\dot{q}}{4k}\left(r_o^2 - r^2\right) + \frac{\dot{q}r_i^2}{2k}\ln\left(\frac{r}{r_o}\right) - \frac{\dot{q}r_o}{2h}\left[1 + \left(\frac{r_i}{r_o}\right)^2\right] + T_\infty. \qquad <$$

(b) From an overall energy balance on the shell,

$$q_r'(r_o) = \dot{E}_g' = \dot{q}\pi\left(r_o^2 - r_i^2\right). \qquad <$$

Alternatively, the heat rate may be found using Fourier's law and the temperature distribution,

$$q_r'(r) = -k(2\pi\, r_o)\left.\frac{dT}{dr}\right)_{r_o} = -2\pi\, kr_o\left[-\frac{\dot{q}}{2k}r_o + \frac{\dot{q}r_i^2}{2k}\,\frac{1}{r_o} + 0 + 0\right] = \dot{q}\pi\left(r_o^2 - r_i^2\right)$$

PROBLEM 3.85

KNOWN: The solid tube of Example 3.7 with inner and outer radii, 50 and 100 mm, and a thermal conductivity of 5 W/m·K. The inner surface is cooled by a fluid at 30°C with a convection coefficient of 1000 $W/m^2 \cdot K$.

FIND: Calculate and plot the temperature distributions for volumetric generation rates of 1×10^5, 5×10^5, and 1×10^6 W/m^3. Use Eq. (7) with Eq. (10) of the Example 3.7 in the *IHT Workspace*.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional radial conduction, (3) Constant properties and (4) Uniform volumetric generation.

ANALYSIS: From Example 3.7, the temperature distribution in the tube is given by Eq. (7),

$$T(r) = T_{s,2} + \frac{\dot{q}}{4k}\left(r_2^2 - r^2\right) - \frac{\dot{q}}{2k} r_2^2 \ell n\left(\frac{r_2}{r}\right) \qquad r_1 \le r \le r_2 \tag{1}$$

The temperature at the inner boundary, $T_{s,1}$, follows from the surface energy balance, Eq. (10),

$$\pi \dot{q}\left(r_2^2 - r_1^2\right) = h 2\pi r_1 \left(T_{s,1} - T_\infty\right) \tag{2}$$

For the conditions prescribed in the schematic with $\dot{q} = 1 \times 10^5 W/m^3$, Eqs. (1) and (2), with r = r_1 and T(r) = $T_{s,1}$, are solved simultaneously to find $T_{s,2}$ = 69.3°C. Eq. (1), with $T_{s,2}$ now a known parameter, can be used to determine the temperature distribution, T(r). The results for different values of the generation rate are shown in the graph.

COMMENTS: (1) The temperature distributions are parabolic with a zero gradient at the insulated outer boundary, r = r_2. The effect of increasing $\dot{q}$ is to increase the maximum temperature in the tube, which always occurs at the outer boundary.

(2) The equations used to generate the graphical result in the *IHT Workspace* are shown below.

```
// The temperature distribution, from Eq. 7, Example 3.7
T_r = Ts2 + qdot/(4*k) * (r2^2 – r^2) – qgot / (2*k) * r2^2*ln (r2/r)
// The temperature at the inner surface, from Eq. 7
Ts1 = Ts2 + qdot / (4*k) * (r2^2 – r1^2) – qdot / (2*k) * r2^2 * ln (r2/r1)
// The energy balance on the surface, from Eq. 10
pi * qdot * (r2^2 – r1^2) = h * 2 * pi * r1 * (Ts1 – Tinf)
```

PROBLEM 3.86

KNOWN: Diameter, resistivity, thermal conductivity, emissivity, voltage, and maximum temperature of heater wire. Convection coefficient and air exit temperature. Temperature of surroundings.

FIND: Maximum operating current, heater length and power rating.

SCHEMATIC:

$T_{sur} = 50^oC$

Nichrome wire
D = 1 mm, L
$\rho_e = 10^{-6}\ \Omega$-m
k = 25 W/m-K
$\varepsilon = 0.20$
$T_{max} = 1200^oC$

V = 110 V

I

Air
$T_\infty = 50^oC$
$h = 250\ W/m^2$-K

$T_s \approx 1200^oC$
$T_o = T_{max}$

ASSUMPTIONS: (1) Steady-state, (2) Uniform wire temperature, (3) Constant properties, (4) Radiation exchange with large surroundings.

ANALYSIS: Assuming a uniform wire temperature, $T_{max} = T(r = 0) \equiv T_o \approx T_s$, the maximum volumetric heat generation may be obtained from Eq. (3.55), but with the total heat transfer coefficient, $h_t = h + h_r$, used in lieu of the convection coefficient h. With

$$h_r = \varepsilon\sigma\left(T_s + T_{sur}\right)\left(T_s^2 + T_{sur}^2\right) = 0.20\times 5.67\times 10^{-8}\,W/m^2\cdot K^4\,(1473+323)\,K\left(1473^2+323\right)^2 K^2 = 46.3\,W/m^2\cdot K$$

$$h_t = (250+46.3)\,W/m^2\cdot K = 296.3\,W/m^2\cdot K$$

$$\dot{q}_{max} = \frac{2\,h_t}{r_o}\left(T_s - T_\infty\right) = \frac{2\left(296.3\,W/m^2\cdot K\right)}{0.0005m}(1150^\circ C) = 1.36\times 10^9\ W/m^3$$

Hence, with $\dot{q} = \dfrac{I^2 R_e}{\forall} = \dfrac{I^2\left(\rho_e L/A_c\right)}{LA_c} = \dfrac{I^2\rho_e}{A_c^2} = \dfrac{I^2\rho_e}{\left(\pi D^2/4\right)^2}$

$$I_{max} = \left(\frac{\dot{q}_{max}}{\rho_e}\right)^{1/2}\frac{\pi D^2}{4} = \left(\frac{1.36\times 10^9\,W/m^3}{10^{-6}\,\Omega\cdot m}\right)^{1/2}\frac{\pi\,(0.001m)^2}{4} = 29.0\,A$$ <

Also, with $\Delta E = I\,R_e = I\,(\rho_e L/A_c)$,

$$L = \frac{\Delta E\cdot A_c}{I_{max}\,\rho_e} = \frac{110\,V\left[\pi\,(0.001m)^2/4\right]}{29.0\,A\left(10^{-6}\,\Omega\cdot m\right)} = 2.98m$$ <

and the power rating is

$$P_{elec} = \Delta E\cdot I_{max} = 110\,V\,(29\,A) = 3190\,W = 3.19\,kW$$ <

COMMENTS: To assess the validity of assuming a uniform wire temperature, Eq. (3.53) may be used to compute the centerline temperature corresponding to $\dot{q}_{max}$ and a surface temperature of 1200°C. It follows that $T_o = \dfrac{\dot{q}\,r_o^2}{4\,k} + T_s = \dfrac{1.36\times 10^9\ W/m^3\,(0.0005m)^2}{4\,(25\,W/m\cdot K)} + 1200^\circ C = 1203^\circ C$. With only a

3°C temperature difference between the centerline and surface of the wire, the assumption is *excellent*.

PROBLEM 3.87

KNOWN: Energy generation in an aluminum-clad, thorium fuel rod under specified operating conditions.

FIND: (a) Whether prescribed operating conditions are acceptable, (b) Effect of $\dot{q}$ and h on acceptable operating conditions.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional conduction in r-direction, (2) Steady-state conditions, (3) Constant properties, (4) Negligible temperature gradients in aluminum and contact resistance between aluminum and thorium.

PROPERTIES: *Table A-1*, Aluminum, pure: M.P. = 933 K; *Table A-1*, Thorium: M.P. = 2023 K, k ≈ 60 W/m·K.

ANALYSIS: (a) System failure would occur if the melting point of either the thorium or the aluminum were exceeded. From Eq. 3.53, the maximum thorium temperature, which exists at r = 0, is

$$T(0) = \frac{\dot{q} r_o^2}{4k} + T_s = T_{Th,max}$$

where, from the energy balance equation, Eq. 3.55, the surface temperature, which is also the aluminum temperature, is

$$T_s = T_\infty + \frac{\dot{q} r_o}{2h} = T_{Al}$$

Hence,

$$T_{Al} = T_s = 95^\circ C + \frac{7\times10^8\ W/m^3 \times 0.0125\ m}{14,000\ W/m^2\cdot K} = 720^\circ C = 993\ K$$

$$T_{Th,max} = \frac{7\times10^8\ W/m^3\ (0.0125m)^2}{4\times60\ W/m\cdot K} + 993\ K = 1449\ K$$ <

Although $T_{Th,max} < M.P._{Th}$ and the thorium would not melt, $T_{al} > M.P._{Al}$ and the cladding would melt under the proposed operating conditions. The problem could be eliminated by *decreasing* $\dot{q}$, *increasing* h or using a cladding material with a higher melting point.

(b) Using the one-dimensional, steady-state conduction model (solid cylinder) of the IHT software, the following radial temperature distributions were obtained for parametric variations in $\dot{q}$ and h.

Continued...

For h = 10,000 W/m$^2\cdot$K, which represents a reasonable upper limit with water cooling, the temperature of the aluminum would be well below its melting point for $\dot{q} = 7 \times 10^8$ W/m^3, but would be close to the melting point for $\dot{q} = 8 \times 10^8$ W/m^3 and would exceed it for $\dot{q} = 9 \times 10^8$ W/m^3. Hence, under the best of conditions, $\dot{q} \approx 7 \times 10^8$ W/m^3 corresponds to the maximum allowable energy generation. However, if coolant flow conditions are constrained to provide values of h < 10,000 W/m$^2\cdot$K, volumetric heating would have to be reduced. Even for $\dot{q}$ as low as 2×10^8 W/m^3, operation could not be sustained for h = 2000 W/m$^2\cdot$K.

The effects of $\dot{q}$ and h on the centerline and surface temperatures are shown below.

For h = 2000 and 5000 W/m$^2\cdot$K, the melting point of thorium would be approached for $\dot{q} \approx 4.4 \times 10^8$ and 8.5×10^8 W/m^3, respectively. For h = 2000, 5000 and 10,000 W/m$^2\cdot$K, the melting point of aluminum would be approached for $\dot{q} \approx 1.6 \times 10^8$, 4.3×10^8 and 8.7×10^8 W/m^3. Hence, the envelope of acceptable operating conditions must call for a reduction in $\dot{q}$ with decreasing h, from a maximum of $\dot{q} \approx 7 \times 10^8$ W/m^3 for h = 10,000 W/m$^2\cdot$K.

COMMENTS: Note the problem which would arise in the event of a *loss of coolant*, for which case h would *decrease* drastically.

PROBLEM 3.88

KNOWN: Radii and thermal conductivities of reactor fuel element and cladding. Fuel heat generation rate. Temperature and convection coefficient of coolant.

FIND: (a) Expressions for temperature distributions in fuel and cladding, (b) Maximum fuel element temperature for prescribed conditions, (c) Effect of h on temperature distribution.

SCHEMATIC:

Coolant
$r_2 = 9$ mm
$r_1 = 6$ mm
Fuel element ($k_f = 2$ W/m·K, $\dot{q} = 2\times10^8$ W/m^3)
Cladding ($k_c = 25$ W/m·K)
q''_{cond} (f) q''_{cond} (c) q''_{conv}
r_1 r_2

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Negligible contact resistance, (4) Constant properties.

ANALYSIS: (a) From Eqs. 3.49 and 3.23, the heat equations for the fuel (f) and cladding (c) are

$$\frac{1}{r}\frac{d}{dr}\left(\frac{dT_f}{dr}\right) = -\frac{\dot{q}}{k_f} \qquad (0 \le r \le r_1) \qquad\qquad \frac{1}{r}\frac{d}{dr}\left(r\frac{dT_c}{dr}\right) = 0 \qquad (r_1 \le r \le r_2)$$

Hence, integrating both equations twice,

$$\frac{dT_f}{dr} = -\frac{\dot{q}r}{2k_f} + \frac{C_1}{k_f r} \qquad T_f = -\frac{\dot{q}r^2}{4k_f} + \frac{C_1}{k_f}\ln r + C_2 \tag{1,2}$$

$$\frac{dT_c}{dr} = \frac{C_3}{k_c r} \qquad T_c = \frac{C_3}{k_c}\ln r + C_4 \tag{3,4}$$

The corresponding boundary conditions are:

$$dT_f/dr\big)_{r=0} = 0 \qquad T_f(r_1) = T_c(r_1) \tag{5,6}$$

$$-k_f\frac{dT_f}{dr}\bigg)_{r=r_1} = -k_c\frac{dT_c}{dr}\bigg)_{r=r_1} \qquad -k_c\frac{dT_c}{dr}\bigg)_{r=r_2} = h\left[T_c(r_2) - T_\infty\right] \tag{7,8}$$

Note that Eqs. (7) and (8) are obtained from surface energy balances at r_1 and r_2, respectively. Applying Eq. (5) to Eq. (1), it follows that $C_1 = 0$. Hence,

$$T_f = -\frac{\dot{q}r^2}{4k_f} + C_2 \tag{9}$$

From Eq. (6), it follows that

$$-\frac{\dot{q}r_1^2}{4k_f} + C_2 = \frac{C_3 \ln r_1}{k_c} + C_4 \tag{10}$$

Continued...

PROBLEM 3.88 (Cont.)

Also, from Eq. (7),

$$\frac{\dot{q}r_1}{2} = -\frac{C_3}{r_1} \qquad \text{or} \qquad C_3 = -\frac{\dot{q}r_1^2}{2} \tag{11}$$

Finally, from Eq. (8), $-\frac{C_3}{r_2} = h\left[\frac{C_3}{k_c}\ln r_2 + C_4 - T_\infty\right]$ or, substituting for C_3 and solving for C_4

$$C_4 = \frac{\dot{q}r_1^2}{2r_2h} + \frac{\dot{q}r_1^2}{2k_c}\ln r_2 + T_\infty \tag{12}$$

Substituting Eqs. (11) and (12) into (10), it follows that

$$C_2 = \frac{\dot{q}r_1^2}{4k_f} - \frac{\dot{q}r_1^2 \ln r_1}{2k_c} + \frac{\dot{q}r_1^2}{2r_2h} + \frac{\dot{q}r_1^2}{2k_c}\ln r_2 + T_\infty$$

$$C_2 = \frac{\dot{q}r_1^2}{4k_f} + \frac{\dot{q}r_1^2}{2k_c}\ln\frac{r_2}{r_1} + \frac{\dot{q}r_1^2}{2r_2h} T_\infty \tag{13}$$

Substituting Eq. (13) into (9),

$$T_f = \frac{\dot{q}}{4k_f}\left(r_1^2 - r^2\right) + \frac{\dot{q}r_1^2}{2k_c}\ln\frac{r_2}{r_1} + \frac{\dot{q}r_1^2}{2r_2h} + T_\infty \tag{14}$$ <

Substituting Eqs. (11) and (12) into (4),

$$T_c = \frac{\dot{q}r_1^2}{2k_c}\ln\frac{r_2}{r} + \frac{\dot{q}r_1^2}{2r_2h} + T_\infty \,. \tag{15}$$ <

(b) Applying Eq. (14) at r = 0, the maximum fuel temperature for h = 2000 $W/m^2 \cdot K$ is

$$T_f(0) = \frac{2\times10^8\ W/m^3 \times (0.006\,m)^2}{4\times 2\ W/m\cdot K} + \frac{2\times10^8\ W/m^3 \times (0.006\,m)^2}{2\times 25\ W/m\cdot K}\ln\frac{0.009\,m}{0.006\,m}$$

$$+ \frac{2\times10^8\ W/m^3\,(0.006\,m)^2}{2\times(0.09\,m)\,2000\ W/m^2\cdot K} + 300\,K$$

$$T_f(0) = (900 + 58.4 + 200 + 300)\,K = 1458\,K\,.$$ <

(c) Temperature distributions for the prescribed values of h are as follows:

Temperature, Tf(K)
Radius in fuel element, r(m)
h = 2000 W/m^2.K
h = 5000 W/m^2.K
h = 10000 W/m^2.K

Temperature, Tc(K)
Radius in cladding, r(m)
h = 2000 W/m^2.K
h = 5000 W/m^2.K
h = 10000 W/m^2.K

Continued...

Clearly, the ability to control the maximum fuel temperature by increasing h is limited, and even for $h \rightarrow \infty$, $T_f(0)$ exceeds 1000 K. The overall temperature drop, $T_f(0) - T_\infty$, is influenced principally by the low thermal conductivity of the fuel material.

COMMENTS: For the prescribed conditions, Eq. (14) yields, $T_f(0) - T_f(r_1) = \dot{q}r_1^2/4k_f = (2\times10^8\ W/m^3)(0.006\ m)^3/8\ W/m\cdot K = 900\ K$, in which case, with no cladding and $h \rightarrow \infty$, $T_f(0) = 1200$ K. To reduce $T_f(0)$ below 1000 K for the prescribed material, it is necessary to reduce $\dot{q}$.

PROBLEM 3.89

KNOWN: Dimensions and properties of tubular heater and external insulation. Internal and external convection conditions. Maximum allowable tube temperature.

FIND: (a) Maximum allowable heater current for adiabatic outer surface, (3) Effect of internal convection coefficient on heater temperature distribution, (c) Extent of heat loss at outer surface.

SCHEMATIC:

Stainless steel
$k = 15$ W/m·K
$\rho_e = 0.7 \times 10^{-6}\ \Omega\cdot$m
$T_{max} = 1400$ K
Refractory
$k_i = 1.0$ W/m·K
$\delta = 25, 50$ mm
Air
$T_{\infty,2} = 300$ K
$h_2 = 25$ W/m$^2\cdot$K
$\dot{q}$
Air
$T_{\infty,1} = 400$ K
$100 \le h_1 \le 1000$ W/m^2
$r_3 = r_2 + \delta,\ T_{s,3}$
$r_2 = 35$ mm, $T_{s,2}$
$r_1 = 25$ mm, $T_{s,1}$

ASSUMPTIONS: (1) One-dimensional, steady-state conditions, (2) Constant properties, (3) Uniform heat generation, (4) Negligible radiation at outer surface, (5) Negligible contact resistance.

ANALYSIS: (a) From Eqs. 7 and 10, respectively, of Example 3.7, we know that

$$T_{s,2} - T_{s,1} = \frac{\dot{q}}{2k} r_2^2 \ln\frac{r_2}{r_1} - \frac{\dot{q}}{4k}\left(r_2^2 - r_1^2\right) \tag{1}$$

and

$$T_{s,1} = T_{\infty,1} + \frac{\dot{q}\left(r_2^2 - r_1^2\right)}{2h_1 r_1} \tag{2}$$

Hence, eliminating $T_{s,1}$, we obtain

$$T_{s,2} - T_{\infty,1} = \frac{\dot{q} r_2^2}{2k}\left[\ln\frac{r_2}{r_1} - \frac{1}{2}\left(1 - r_1^2/r_2^2\right) + \frac{k}{h_1 r_1}\left(1 - r_1^2/r_2^2\right)\right]$$

Substituting the prescribed conditions ($h_1 = 100$ W/m$^2\cdot$K),

$$T_{s,2} - T_{\infty,1} = 1.237 \times 10^{-4}\left(\text{m}^3\cdot\text{K/W}\right)\dot{q}\left(\text{W/m}^3\right)$$

Hence, with T_{max} corresponding to $T_{s,2}$, the maximum allowable value of $\dot{q}$ is

$$\dot{q}_{max} = \frac{1400 - 400}{1.237\times10^{-4}} = 8.084\times10^6\ \text{W/m}^3$$

with

$$\dot{q} = \frac{I^2 R_e}{\forall} = \frac{I^2 \rho_e L/A_c}{LA_c} = \frac{\rho_e I^2}{\left[\pi\left(r_2^2 - r_1\right)\right]^2}$$

$$I_{max} = \pi\left(r_2^2 - r_1^2\right)\left(\frac{\dot{q}}{\rho_e}\right)^{1/2} = \pi\left(0.035^2 - 0.025^2\right)\text{m}^2\left(\frac{8.084\times10^6\ \text{W/m}^3}{0.7\times10^{-6}\,\Omega\cdot\text{m}}\right)^{1/2} = 6406\ \text{A} \quad <$$

Continued …..

PROBLEM 3.89 (Cont.)

(b) Using the one-dimensional, steady-state conduction model of IHT (hollow cylinder; convection at inner surface and adiabatic outer surface), the following temperature distributions were obtained.

The results are consistent with key implications of Eqs. (1) and (2), namely that the value of h_1 has no effect on the temperature drop across the tube ($T_{s,2} - T_{s,1} = 30$ K, irrespective of h_1), while $T_{s,1}$ decreases with increasing h_1. For h_1 = 100, 500 and 1000 $W/m^2 \cdot K$, respectively, the ratio of the temperature drop between the inner surface and the air to the temperature drop across the tube, $(T_{s,1} - T_{\infty,1})/(T_{s,2} - T_{s,1})$, decreases from 970/30 = 32.3 to 194/30 = 6.5 and 97/30 = 3.2. Because the outer surface is insulated, the heat rate to the airflow is fixed by the value of $\dot{q}$ and, irrespective of h_1,

$$q'(r_1) = \pi\left(r_2^2 - r_1^2\right)\dot{q} = -15,240\,\text{W}$$ <

(c) Heat loss from the outer surface of the tube to the surroundings depends on the total thermal resistance

$$R_{tot} = \frac{\ln(r_3/r_2)}{2\pi L k_i} + \frac{1}{2\pi r_3 L h_2}$$

or, for a unit area on surface 2,

$$R''_{tot,2} = (2\pi r_2 L) R_{tot} = \frac{r_2 \ln(r_3/r_2)}{k_i} + \frac{r_2}{r_3 h_2}$$

Again using the capabilities of IHT (hollow cylinder; convection at inner surface and heat transfer from outer surface through $R''_{tot,2}$), the following temperature distributions were determined for the tube and insulation.

Continued...

PROBLEM 3.89 (Cont.)

Heat losses through the insulation, $q'(r_2)$, are 4250 and 3890 W/m for δ = 25 and 50 mm, respectively, with corresponding values of $q'(r_1)$ equal to -10,990 and -11,350 W/m. Comparing the tube temperature distributions with those predicted for an adiabatic outer surface, it is evident that the losses reduce tube wall temperatures predicted for the adiabatic surface and also shift the maximum temperature from r = 0.035 m to r ≈ 0.033 m. Although the tube outer and insulation inner surface temperatures, $T_{s,2} = T(r_2)$, increase with increasing insulation thickness, Fig. (c), the insulation outer surface temperature decreases.

COMMENTS: If the intent is to maximize heat transfer to the airflow, heat losses to the ambient should be reduced by selecting an insulation material with a significantly smaller thermal conductivity.

PROBLEM 3.90

KNOWN: Electric current I is passed through a pipe of resistance R'_e to melt ice under steady-state conditions.

FIND: (a) Temperature distribution in the pipe wall, (b) Time to completely melt the ice.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional radial conduction, (3) Constant properties, (4) Uniform heat generation in the pipe wall, (5) Outer surface of the pipe is adiabatic, (6) Inner surface is at a constant temperature, T_m.

PROPERTIES: *Table A-3,* Ice (273K): $\rho = 920$ kg/m^3; *Handbook Chem. & Physics,* Ice: Latent heat of fusion, $h_{sf} = 3.34\times10^5$ J/kg.

ANALYSIS: (a) The appropriate form of the heat equation is Eq. 3.49, and the general solution, Eq. 3.51 is

$$T(r) = -\frac{\dot{q}}{4k}r^2 + C_1 \ln r + C_2$$

where

$$\dot{q} = \frac{I^2 R'_e}{\pi\left(r_2^2 - r_1^2\right)}.$$

Applying the boundary condition $(dT/dr)_{r_2} = 0$, it follows that

$$0 = \frac{\dot{q}r_2}{2k} + \frac{C_1}{r_2}$$

Hence $$C_1 = \frac{\dot{q}r_2^2}{2k}$$

and $$T(r) = -\frac{\dot{q}}{4k}r^2 + \frac{\dot{q}r_2^2}{2k}\ln r + C_2.$$

Continued …..

Applying the second boundary condition, $T(r_1) = T_m$, it follows that

$$T_m = -\frac{\dot{q}}{4k} r_1^2 + \frac{\dot{q} r_2^2}{2k} \ln r_1 + C_2.$$

Solving for C_2 and substituting into the expression for T(r), find

$$T(r) = T_m + \frac{\dot{q} r_2^2}{2k} \ln \frac{r}{r_1} - \frac{\dot{q}}{4k}\left(r^2 - r_1^2\right). \qquad <$$

(b) Conservation of energy dictates that the energy required to completely melt the ice, E_m, must equal the energy which reaches the inner surface of the pipe by conduction through the wall during the melt period. Hence from Eq. 1.11b

$$\Delta E_{st} = E_{in} - E_{out} + E_{gen}$$

$$\Delta E_{st} = E_m = t_m \cdot q_{cond,r_1}$$

or, for a unit length of pipe,

$$\rho\left(\pi\, r_1^2\right) h_{sf} = t_m \left[-k\left(2\pi\, r_1\right)\left[\frac{dT}{dr}\right]_{r_1}\right]$$

$$\rho\left(\pi\, r_1^2\right) h_{sf} = -2\pi r_1 k t_m \left[\frac{\dot{q} r_2^2}{2k r_1} - \frac{\dot{q} r_1}{2k}\right]$$

$$\rho\left(\pi\, r_1^2\right) h_{sf} = -t_m \dot{q} \pi \left(r_2^2 - r_1^2\right).$$

Dropping the minus sign, which simply results from the fact that conduction is in the negative r direction, it follows that

$$t_m = \frac{\rho h_{sf} r_1^2}{\dot{q}\left(r_2^2 - r_1^2\right)} = \frac{\rho h_{sf} \pi\, r_1^2}{I^2 R_e'}.$$

With r_1 = 0.05m, I = 100 A and $R_e' = 0.30\ \Omega/\text{m}$, it follows that

$$t_m = \frac{920\text{kg/m}^3 \times 3.34\times10^5 \text{J/kg} \times \pi \times (0.05\text{m})^2}{(100\text{A})^2 \times 0.30\Omega/\text{m}}$$

or $\quad t_m = 804\text{s}. \qquad <$

COMMENTS: The foregoing expression for t_m could also be obtained by recognizing that all of the energy which is generated by electrical heating in the pipe wall must be transferred to the ice. Hence,

$$I^2 R_e' t_m = \rho h_{sf} \pi\, r_1^2.$$

PROBLEM 3.91

KNOWN: Materials, dimensions, properties and operating conditions of a gas-cooled nuclear reactor.

FIND: (a) Inner and outer surface temperatures of fuel element, (b) Temperature distributions for different heat generation rates and maximum allowable generation rate.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible contact resistance, (5) Negligible radiation.

PROPERTIES: *Table A.1*, Thoriun: $T_{mp} \approx 2000$ K; *Table A.2*, Graphite: $T_{mp} \approx 2300$ K.

ANALYSIS: (a) The outer surface temperature of the fuel, T_2, may be determined from the rate equation

$$q' = \frac{T_2 - T_\infty}{R'_{tot}}$$

where

$$R'_{tot} = \frac{\ln(r_3/r_2)}{2\pi\, k_g} + \frac{1}{2\pi r_3 h} = \frac{\ln(14/11)}{2\pi\,(3\,\text{W/m}\cdot\text{K})} + \frac{1}{2\pi\,(0.014\,\text{m})\left(2000\,\text{W/m}^2\cdot\text{K}\right)} = 0.0185\,\text{m}\cdot\text{K/W}$$

and the heat rate per unit length may be determined by applying an energy balance to a control surface about the fuel element. Since the interior surface of the element is essentially adiabatic, it follows that

$$q' = \dot{q}\pi\left(r_2^2 - r_1^2\right) = 10^8\,\text{W/m}^3 \times \pi\left(0.011^2 - 0.008^2\right)\text{m}^2 = 17,907\,\text{W/m}$$

Hence,

$$T_2 = q'R'_{tot} + T_\infty = 17,907\,\text{W/m}\,(0.0185\,\text{m}\cdot\text{K/W}) + 600\,\text{K} = 931\,\text{K}$$ <

With zero heat flux at the inner surface of the fuel element, Eq. C.14 yields

$$T_1 = T_2 + \frac{\dot{q}r_2^2}{4k_t}\left(1 - \frac{r_1^2}{r_2^2}\right) - \frac{\dot{q}r_1^2}{2k_t}\ln\left(\frac{r_2}{r_1}\right)$$

$$T_1 = 931\,\text{K} + \frac{10^8\,\text{W/m}^3\,(0.011\,\text{m})^2}{4\times 57\,\text{W/m}\cdot\text{K}}\left[1 - \left(\frac{0.008}{0.011}\right)^2\right] - \frac{10^8\,\text{W/m}^3\,(0.008\,\text{m})^2}{2\times 57\,\text{W/m}\cdot\text{K}}\ln\left(\frac{0.011}{0.008}\right)$$

Continued...

PROBLEM 3.91 (Cont.)

$$T_1 = 931\,K + 25\,K - 18\,K = 938\,K$$ <

(b) The temperature distributions may be obtained by using the IHT model for one-dimensional, steady-state conduction in a hollow tube. For the fuel element ($\dot{q} > 0$), an adiabatic surface condition is prescribed at r_1, while heat transfer from the outer surface at r_2 to the coolant is governed by the thermal resistance $R''_{tot,2} = 2\pi r_2 R'_{tot} = 2\pi(0.011\ m)0.0185\ m\cdot K/W = 0.00128\ m^2\cdot K/W$. For the graphite ($\dot{q}$ = 0), the value of T_2 obtained from the foregoing solution is prescribed as an inner boundary condition at r_2, while a convection condition is prescribed at the outer surface (r_3). For $1 \times 10^8 \le \dot{q} \le 5 \times 10^8\ W/m^3$, the following distributions are obtained.

Temperature, T(K)
2500
2100
1700
1300
900
500
0.008 0.009 0.01 0.011
Radial location in fuel, r(m)
qdot = 5E8
qdot = 3E8
qdot = 1E8

Temperature, T(K)
2500
2100
1700
1300
900
500
0.011 0.012 0.013 0.014
Radial location in graphite, r(m)
qdot = 5E8
qdot = 3E8
qdot = 1E8

The comparatively large value of k_t yields small temperature variations across the fuel element, while the small value of k_g results in large temperature variations across the graphite. Operation at $\dot{q} = 5 \times 10^8\ W/m^3$ is clearly unacceptable, since the melting points of thorium and graphite are exceeded and approached, respectively. To prevent softening of the materials, which would occur below their melting points, the reactor should not be operated much above $\dot{q} = 3 \times 10^8\ W/m^3$.

COMMENTS: A contact resistance at the thorium/graphite interface would increase temperatures in the fuel element, thereby reducing the maximum allowable value of $\dot{q}$.

PROBLEM 3.92

KNOWN: Long rod experiencing uniform volumetric generation encapsulated by a circular sleeve exposed to convection.

FIND: (a) Temperature at the interface between rod and sleeve and on the outer surface, (b) Temperature at center of rod.

SCHEMATIC:

Rod, $k_r = 0.5\ W/m\cdot K$, $\dot{q} = 24{,}000\ W/m^3$; Sleeve, $k_s = 4\ W/m\cdot K$; T_1, T_2, r_1, r_2; $T_\infty = 27°C$, $h = 25\ W/m^2\cdot K$

ASSUMPTIONS: (1) One-dimensional radial conduction in rod and sleeve, (2) Steady-state conditions, (3) Uniform volumetric generation in rod, (4) Negligible contact resistance between rod and sleeve.

ANALYSIS: (a) Construct a thermal circuit for the sleeve,

q' → T_1 — R_s — T_2 — R_{conv} — T_∞

where

$$q' = \dot{E}'_{gen} = \dot{q}\pi\ D_1^2/4 = 24{,}000\ W/m^3 \times \pi \times (0.20\ m)^2/4 = 754.0\ W/m$$

$$R'_s = \frac{\ln(r_2/r_1)}{2\pi\ k_s} = \frac{\ln(400/200)}{2\pi \times 4\ W/m\cdot K} = 2.758\times10^{-2}\ m\cdot\ K/W$$

$$R_{conv} = \frac{1}{h\pi\ D_2} = \frac{1}{25\ W/m^2\cdot K\times\pi\times0.400\ m} = 3.183\times10^{-2}\ m\cdot\ K/W$$

The rate equation can be written as

$$q' = \frac{T_1 - T_\infty}{R'_s + R'_{conv}} = \frac{T_2 - T_\infty}{R'_{conv}}$$

$$T_1 = T_\infty + q'(R'_s + R'_{conv}) = 27°C + 754\ W/m\left(2.758\times10^{-2} + 3.183\times10^{-2}\right)K/W\cdot m = 71.8°C \quad <$$

$$T_2 = T_\infty + q'R'_{conv} = 27°C + 754\ W/m \times 3.183\times10^{-2}\ m\cdot\ K/W = 51.0°C. \quad <$$

(b) The temperature at the center of the rod is

$$T(0) = T_o = \frac{\dot{q}r_1^2}{4k_r} + T_1 = \frac{24{,}000\ W/m^3(0.100\ m)^2}{4\times0.5\ W/m\cdot K} + 71.8°C = 192°C. \quad <$$

COMMENTS: The thermal resistances due to conduction in the sleeve and convection are comparable. Will increasing the sleeve outer diameter cause the surface temperature T_2 to increase or decrease?

PROBLEM 3.93

KNOWN: Radius, thermal conductivity, heat generation and convection conditions associated with a solid sphere.

FIND: Temperature distribution.

SCHEMATIC:

r_o ; $\dot{q}$; h, T_∞

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional radial conduction, (3) Constant properties, (4) Uniform heat generation.

ANALYSIS: Integrating the appropriate form of the heat diffusion equation,

$$\frac{1}{r^2}\frac{d}{dr}\left[kr^2\frac{dT}{dr}\right]+\dot{q}=0 \qquad \text{or} \qquad \frac{d}{dr}\left[r^2\frac{dT}{dr}\right]=-\frac{\dot{q}r^2}{k}$$

$$r^2\frac{dT}{dr}=-\frac{\dot{q}r^3}{3k}+C_1 \qquad \frac{dT}{dr}=-\frac{\dot{q}r}{3k}+\frac{C_1}{r^2}$$

$$T(r)=-\frac{\dot{q}r^2}{6k}-\frac{C_1}{r}+C_2.$$

The boundary conditions are: $\left.\frac{dT}{dr}\right]_{r=0}=0$ hence $C_1=0,$ and

$$-k\left.\frac{dT}{dr}\right]_{r_o}=h\left[T(r_o)-T_\infty\right].$$

Substituting into the second boundary condition ($r = r_o$), find

$$\frac{\dot{q}r_o}{3}=h\left[-\frac{\dot{q}r_o^2}{6k}+C_2-T_\infty\right] \qquad C_2=\frac{\dot{q}r_o}{3h}+\frac{\dot{q}r_o^2}{6k}+T_\infty.$$

The temperature distribution has the form

$$T(r)=\frac{\dot{q}}{6k}\left(r_o^2-r^2\right)+\frac{\dot{q}r_o}{3h}+T_\infty.$$ <

COMMENTS: To verify the above result, obtain $T(r_o) = T_s$,

$$T_s=\frac{\dot{q}r_o}{3h}+T_\infty$$

Applying energy balance to the control volume about the sphere,

$$\dot{q}\left[\frac{4}{3}\pi\, r_o^3\right]=h4\pi\, r_o^2\left(T_s-T_\infty\right) \qquad \text{find} \qquad T_s=\frac{\dot{q}r_o}{3h}+T_\infty.$$

PROBLEM 3.94

KNOWN: Radial distribution of heat dissipation of a spherical container of radioactive wastes. Surface convection conditions.

FIND: Radial temperature distribution.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible temperature drop across container wall.

ANALYSIS: The appropriate form of the heat equation is

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right)=-\frac{\dot{q}}{k}=-\frac{\dot{q}_o}{k}\left[1-\left(\frac{r}{r_o}\right)^2\right].$$

Hence $$r^2\frac{dT}{dr}=-\frac{\dot{q}_o}{k}\left(\frac{r^3}{3}-\frac{r^5}{5r_o^2}\right)+C_1$$

$$T=-\frac{\dot{q}_o}{k}\left(\frac{r^2}{6}-\frac{r^4}{20r_o^2}\right)-\frac{C_1}{r}+C_2.$$

From the boundary conditions,

$$dT/dr\,|_{r=0}=0 \quad \text{and} \quad -k\,dT/dr\,|_{r=r_o}=h\left[T(r_o)-T_\infty\right]$$

it follows that $C_1 = 0$ and

$$\dot{q}_o\left(\frac{r_o}{3}-\frac{r_o}{5}\right)=h\left[-\frac{\dot{q}_o}{k}\left(\frac{r_o^2}{6}-\frac{r_o^2}{20}\right)+C_2-T_\infty\right]$$

$$C_2=\frac{2r_o\dot{q}_o}{15h}+\frac{7\dot{q}_o r_o^2}{60k}+T_\infty.$$

Hence $$T(r)=T_\infty+\frac{2r_o\dot{q}_o}{15h}+\frac{\dot{q}r_o^2}{k}\left[\frac{7}{60}-\frac{1}{6}\left(\frac{r}{r_o}\right)^2+\frac{1}{20}\left(\frac{r}{r_o}\right)^4\right].$$ <

COMMENTS: Applying the above result at r_o yields

$$T_s=T(r_o)=T_\infty+\left(2r_o\dot{q}_o/15h\right).$$

The same result may be obtained by applying an energy balance to a control surface about the container, where $\dot{E}_g=q_{conv}$. The maximum temperature exists at r = 0.

PROBLEM 3.95

KNOWN: Dimensions and thermal conductivity of a spherical container. Thermal conductivity and volumetric energy generation within the container. Outer convection conditions.

FIND: (a) Outer surface temperature, (b) Container inner surface temperature, (c) Temperature distribution within and center temperature of the wastes, (d) Feasibility of operating at twice the energy generation rate.

SCHEMATIC:

Stainless steel
k_{ss} = 15 W/m·K
q_{conv}
r_i = 0.5 m
r_o = 0.6 m
$T_{s,o}$
$T_{s,i}$
T_∞ = 25 °C
h = 1000 W/m²·K
Radioactive wastes
k_{rw} = 20 W/m·K, $\dot{q}$ = 10^5 W/m³

ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) One-dimensional radial conduction.

ANALYSIS: (a) For a control volume which includes the container, conservation of energy yields $\dot{E}_g - \dot{E}_{out} = 0$, or $\dot{q}V - q_{conv} = 0$. Hence

$$\dot{q}(4/3)\left(\pi r_i^3\right) = h4\pi r_o^2\left(T_{s,o} - T_\infty\right)$$

and with $\dot{q} = 10^5$ W/m³,

$$T_{s,o} = T_\infty + \frac{\dot{q}r_i^3}{3hr_o^2} = 25^\circ C + \frac{10^5\,W/m^2\,(0.5\,m)^3}{3000\,W/m^2\cdot K\,(0.6\,m)^2} = 36.6^\circ C.$$ <

(b) Performing a surface energy balance at the outer surface, $\dot{E}_{in} - \dot{E}_{out} = 0$ or $q_{cond} - q_{conv} = 0$. Hence

$$\frac{4\pi k_{ss}\left(T_{s,i} - T_{s,o}\right)}{(1/r_i) - (1/r_o)} = h4\pi r_o^2\left(T_{s,o} - T_\infty\right)$$

$$T_{s,i} = T_{s,o} + \frac{h}{k_{ss}}\left(\frac{r_o}{r_i} - 1\right)r_o\left(T_{s,o} - T_\infty\right) = 36.6^\circ C + \frac{1000\,W/m^2\cdot K}{15\,W/m\cdot K}(0.2)0.6\,m\left(11.6^\circ C\right) = 129.4^\circ C.$$ <

(c) The heat equation in spherical coordinates is

$$k_{rw}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) + \dot{q}r^2 = 0.$$

Solving,

$$r^2\frac{dT}{dr} = -\frac{\dot{q}r^3}{3k_{rw}} + C_1 \qquad \text{and} \qquad T(r) = -\frac{\dot{q}r^2}{6k_{rw}} - \frac{C_1}{r} + C_2$$

Applying the boundary conditions,

$$\left.\frac{dT}{dr}\right|_{r=0} = 0 \qquad \text{and} \qquad T(r_i) = T_{s,i}$$

$$C_1 = 0 \qquad \text{and} \qquad C_2 = T_{s,i} + \dot{q}r_i^2/6k_{rw}.$$

Continued...

PROBLEM 3.95 (Cont.)

Hence

$$T(r) = T_{s,i} + \frac{\dot{q}}{6k_{rw}}\left(r_i^2 - r^2\right)$$ <

At r = 0,

$$T(0) = T_{s,i} + \frac{\dot{q}r_i^2}{6k_{rw}} = 129.4^\circ C + \frac{10^5\ W/m^3\,(0.5\,m)^2}{6\left(20\,W/m\cdot K\right)} = 337.7^\circ C$$ <

(d) The feasibility assessment may be performed by using the IHT model for one-dimensional, steady-state conduction in a solid sphere, with the surface boundary condition prescribed in terms of the total thermal resistance

$$R''_{tot,i} = \left(4\pi r_i^2\right)R_{tot} = R''_{cnd,i} + R''_{cnv,i} = \frac{r_i^2\left[(1/r_i) - (1/r_o)\right]}{k_{ss}} + \frac{1}{h}\left(\frac{r_i}{r_o}\right)^2$$

where, for r_o = 0.6 m and h = 1000 $W/m^2\cdot K$, $R''_{cnd,i} = 5.56 \times 10^{-3}\ m^2\cdot K/W$, $R''_{cnv,i} = 6.94 \times 10^{-4}\ m^2\cdot K/W$, and $R''_{tot,i} = 6.25 \times 10^{-3}\ m^2\cdot K/W$. Results for the center temperature are shown below.

Clearly, even with $r_o = 0.54$ m = $r_{o,min}$ and h = 10,000 $W/m^2\cdot K$ (a practical upper limit), T(0) > 475°C and the desired condition can not be met. The corresponding resistances are $R''_{cnd,i} = 2.47 \times 10^{-3}\ m^2\cdot K/W$, $R''_{cnv,i} = 8.57 \times 10^{-5}\ m^2\cdot K/W$, and $R''_{tot,i} = 2.56 \times 10^{-3}\ m^2\cdot K/W$. The conduction resistance remains dominant, and the effect of reducing $R''_{cnv,i}$ by increasing h is small. *The proposed extension is not feasible.*

COMMENTS: A value of $\dot{q} = 1.79 \times 10^5\ W/m^3$ would allow for operation at T(0) = 475°C with r_o = 0.54 m and h = 10,000 $W/m^2\cdot K$.

PROBLEM 3.96

KNOWN: Carton of apples, modeled as 80-mm diameter spheres, ventilated with air at 5°C and experiencing internal volumetric heat generation at a rate of 4000 J/kg·day.

FIND: (a) The apple center and surface temperatures when the convection coefficient is 7.5 $W/m^2 \cdot K$, and (b) Compute and plot the apple temperatures as a function of air velocity, V, for the range $0.1 \le V \le 1$ m/s, when the convection coefficient has the form $h = C_1 V^{0.425}$, where $C_1 = 10.1\ W/m^2 \cdot K \cdot (m/s)^{0.425}$.

SCHEMATIC:

Apple, D = 80 mm
$\dot{q}$ = 4000 J/kg•day
ρ = 840 kg/m^3, k = 0.5 W/m•K
$\dot{q}$, k
T_o
T_s
Air
T_∞ = 5 °C V = 0.5 m/s
h = 7.5 W/m^2•K
q_{cv}

ASSUMPTIONS: (1) Apples can be modeled as spheres, (2) Each apple experiences flow of ventilation air at $T_\infty = 5°C$, (3) One-dimensional radial conduction, (4) Constant properties and (5) Uniform heat generation.

ANALYSIS: (a) From Eq. C.24, the temperature distribution in a solid sphere (apple) with uniform generation is

$$T(r) = \frac{\dot{q} r_o^2}{6k}\left(1 - \frac{r^2}{r_o^2}\right) + T_s \qquad (1)$$

To determine T_s, perform an energy balance on the apple as shown in the sketch above, with volume $V = 4/3 \pi r_o^3$,

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = 0 \qquad\qquad -q_{cv} + \dot{q}\forall = 0$$

$$-h\left(4\pi r_o^2\right)\left(T_s - T_\infty\right) + \dot{q}\left(4/3\pi r_o^3\right) = 0 \qquad (2)$$

$$-7.5\,W/m^2 \cdot K\left(4\pi \times 0.040^2\,m^2\right)\left(T_s - 5^\circ C\right) + 38.9\,W/m^3\left(4/3\pi \times 0.040^3\,m^3\right) = 0$$

where the volumetric generation rate is

$$\dot{q} = 4000\,J/kg \cdot day$$

$$\dot{q} = 4000\,J/kg \cdot day \times 840\,kg/m^3 \times (1\,day/24\,hr) \times (1\,hr/3600\,s)$$

$$\dot{q} = 38.9\,W/m^3$$

and solving for T_s, find

$$T_s = 5.14^\circ C$$ <

From Eq. (1), at r = 0, with T_s, find

$$T(0) = \frac{38.9\,W/m^3 \times 0.040^2\,m^2}{6 \times 0.5\,W/m \cdot K} + 5.14^\circ C = 0.12^\circ C + 5.14^\circ C = 5.26^\circ C$$ <

Continued...

PROBLEM 3.96 (Cont.)

(b) With the convection coefficient depending upon velocity,

$$h = C_1 V^{0.425}$$

with C_1 = 10.1 W/m^2·K·(m/s)$^{0.425}$, and using the energy balance of Eq. (2), calculate and plot T_s as a function of ventilation air velocity V. With very low velocities, the center temperature is nearly 0.5°C higher than the air. From our earlier calculation we know that $T(0) - T_s$ = 0.12°C and is independent of V.

COMMENTS: (1) While the temperature within the apple is nearly isothermal, the center temperature will track the ventilation air temperature which will increase as it passes through stacks of cartons.

(2) The IHT Workspace used to determine T_s for the base condition and generate the above plot is shown below.

```
// The temperature distribution, Eq (1),
T_r = qdot * ro^2 / (4 * k) * ( 1- r^2/ro^2 )  +  Ts

// Energy balance on the apple, Eq (2)
- qcv + qdot * Vol = 0
Vol = 4 / 3 * pi *  ro ^3

// Convection rate equation:
qcv = h* As * ( Ts - Tinf )
As = 4 * pi * ro^2

// Generation rate:
qdot = qdotm * (1/24) * (1/3600) * rho          // Generation rate, W/m^3; Conversions: days/h and h/sec

// Assigned variables:
ro = 0.080              // Radius of apple, m
k = 0.5                 // Thermal conductivity, W/m.K
qdotm = 4000            // Generation rate, J/kg.K
rho = 840               // Specific heat, J/kg.K
r = 0                   // Center, m; location for T(0)
h =  7.5                // Convection coefficient, W/m^2.K; base case, V = 0.5 m/s
//h = C1 * V^0.425      // Correlation
//C1 = 10.1
//V = 0.5               // Air velocity, m/s; range 0.1 to 1 m/s
Tinf = 5                // Air temperature, C
```

PROBLEM 3.97

KNOWN: Plane wall, long cylinder and sphere, each with characteristic length a, thermal conductivity k and uniform volumetric energy generation rate $\dot{q}$.

FIND: (a) On the same graph, plot the dimensionless temperature, $[T(x \text{ or } r) - T(a)]/[\dot{q}a^2/2k]$, *vs.* the dimensionless characteristic length, x/a or r/a, for each shape; (b) Which shape has the smallest temperature difference between the center and the surface? Explain this behavior by comparing the ratio of the volume-to-surface area; and (c) Which shape would be preferred for use as a nuclear fuel element? Explain why?

SCHEMATIC:

Plane wall — $\dot{q}$,k — $T(a) = T_s$ — x — a

Long cylinder — $\dot{q}$,k — $T(a) = T_s$ — r — a

Sphere — $\dot{q}$,k — $T(a) = T_s$ — r = a

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties and (4) Uniform volumetric generation.

ANALYSIS: (a) For each of the shapes, with $T(a) = T_s$, the dimensionless temperature distributions can be written by inspection from results in Appendix C.3.

Plane wall, Eq. C.22
$$\frac{T(x) - T_s}{\dot{q}a^2/2k} = 1 - \left(\frac{x}{a}\right)^2$$

Long cylinder, Eq. C.23
$$\frac{T(r) - T_s}{\dot{q}a^2/2k} = \frac{1}{2}\left[1 - \left(\frac{r}{a}\right)^2\right]$$

Sphere, Eq. C.24
$$\frac{T(r) - T_s}{\dot{q}a^2/2k} = \frac{1}{3}\left[1 - \left(\frac{r}{a}\right)^2\right]$$

The dimensionless temperature distributions using the foregoing expressions are shown in the graph below.

Dimensionless temperature distribution

(T_x,r-Ts) / (qdot*a^2/2*k)

1
0.8
0.6
0.4
0.2
0

0 0.2 0.4 0.6 0.8 1

Dimensionless length, x/a or r/a

Plane wall, 2a
Long cylinder, a
Sphere, a

Continued …..

(b) The sphere shape has the smallest temperature difference between the center and surface, T(0) – T(a). The ratio of volume-to-surface-area, $\forall/A_s$, for each of the shapes is

Plane wall $$\frac{\forall}{A_s} = \frac{a(1\times 1)}{(1\times 1)} = a$$

Long cylinder $$\frac{\forall}{A_s} = \frac{\pi a^2 \times 1}{2\pi a \times 1} = \frac{a}{2}$$

Sphere $$\frac{\forall}{A_s} = \frac{4\pi a^3/3}{4\pi a^2} = \frac{a}{3}$$

The smaller the $\forall/A_s$ ratio, the smaller the temperature difference, T(0) – T(a).

(c) The sphere would be the preferred element shape since, for a given $\forall/A_s$ ratio, which controls the generation and transfer rates, the sphere will operate at the lowest temperature.

PROBLEM 3.98

KNOWN: Radius, thickness, and incident flux for a radiation heat gauge.

FIND: Expression relating incident flux to temperature difference between center and edge of gauge.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in r (negligible temperature drop across foil thickness), (3) Constant properties, (4) Uniform incident flux, (5) Negligible heat loss from foil due to radiation exchange with enclosure wall, (6) Negligible contact resistance between foil and heat sink.

ANALYSIS: Applying energy conservation to a circular ring extending from r to r + dr,

$$q_r + q_i''(2\pi\ rdr) = q_{r+dr}, \qquad q_r = -k(2\pi\ rt)\frac{dT}{dr}, \qquad q_{r+dr} = q_r + \frac{dq_r}{dr}dr.$$

Rearranging, find that

$$q_i''(2\pi\ rdr) = \frac{d}{dr}\left[(-k2\pi\ rt)\frac{dT}{dr}\right]dr$$

$$\frac{d}{dr}\left[r\frac{dT}{dr}\right] = -\frac{q_i''}{kt}r.$$

Integrating,

$$r\frac{dT}{dr} = -\frac{q_i''r^2}{2kt} + C_1 \qquad \text{and} \qquad T(r) = -\frac{q_i''r^2}{4kt} + C_1 \ln r + C_2.$$

With $dT/dr|_{r=0} = 0$, $C_1 = 0$ and with $T(r = R) = T(R)$,

$$T(R) = -\frac{q_i''R^2}{4kt} + C_2 \qquad \text{or} \qquad C_2 = T(R) + \frac{q_i''R^2}{4kt}.$$

Hence, the temperature distribution is

$$T(r) = \frac{q_i''}{4kt}\left(R^2 - r^2\right) + T(R).$$

Applying this result at r = 0, it follows that

$$q_i'' = \frac{4kt}{R^2}\left[T(0) - T(R)\right] = \frac{4kt}{R^2}\Delta T.$$

<

COMMENTS: This technique allows for determination of a radiation flux from measurement of a temperature difference. It becomes inaccurate if emission from the foil becomes significant.

PROBLEM 3.98

KNOWN: Radius, thickness, and incident flux for a radiation heat gauge.

FIND: Expression relating incident flux to temperature difference between center and edge of gauge.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in r (negligible temperature drop across foil thickness), (3) Constant properties, (4) Uniform incident flux, (5) Negligible heat loss from foil due to radiation exchange with enclosure wall, (6) Negligible contact resistance between foil and heat sink.

ANALYSIS: Applying energy conservation to a circular ring extending from r to r + dr,

$$q_r + q_i''(2\pi\ rdr) = q_{r+dr}, \qquad q_r = -k(2\pi\ rt)\frac{dT}{dr}, \qquad q_{r+dr} = q_r + \frac{dq_r}{dr}dr.$$

Rearranging, find that

$$q_i''(2\pi\ rdr) = \frac{d}{dr}\left[(-k2\pi\ rt)\frac{dT}{dr}\right]dr$$

$$\frac{d}{dr}\left[r\frac{dT}{dr}\right] = -\frac{q_i''}{kt}r.$$

Integrating,

$$r\frac{dT}{dr} = -\frac{q_i''r^2}{2kt} + C_1 \qquad \text{and} \qquad T(r) = -\frac{q_i''r^2}{4kt} + C_1 \ln r + C_2.$$

With $dT/dr|_{r=0} = 0$, $C_1 = 0$ and with $T(r = R) = T(R)$,

$$T(R) = -\frac{q_i''R^2}{4kt} + C_2 \qquad \text{or} \qquad C_2 = T(R) + \frac{q_i''R^2}{4kt}.$$

Hence, the temperature distribution is

$$T(r) = \frac{q_i''}{4kt}\left(R^2 - r^2\right) + T(R).$$

Applying this result at r = 0, it follows that

$$q_i'' = \frac{4kt}{R^2}\left[T(0) - T(R)\right] = \frac{4kt}{R^2}\Delta T.$$ <

COMMENTS: This technique allows for determination of a radiation flux from measurement of a temperature difference. It becomes inaccurate if emission from the foil becomes significant.

PROBLEM 3.99

KNOWN: Net radiative flux to absorber plate.

FIND: (a) Maximum absorber plate temperature, (b) Rate of energy collected per tube.

SCHEMATIC:

t = 0.006m, q''_{rad} = 800 W/m², Al alloy, q'_x, q'_{x+dx}, dx, Line of symmetry (dT/dx = 0), T_w = 60°C, x = L/2 = 0.1 m

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional (x) conduction along absorber plate, (3) Uniform radiation absorption at plate surface, (4) Negligible losses by conduction through insulation, (5) Negligible losses by convection at absorber plate surface, (6) Temperature of absorber plate at x = 0 is approximately that of the water.

PROPERTIES: *Table A-1,* Aluminum alloy (2024-T6): k ≈ 180 W/m·K.

ANALYSIS: The absorber plate acts as an extended surface (a conduction-radiation system), and a differential equation which governs its temperature distribution may be obtained by applying Eq.1.11a to a differential control volume. For a unit length of tube

$$q'_x + q''_{rad}(dx) - q'_{x+dx} = 0.$$

With
$$q'_{x+dx} = q'_x + \frac{dq'_x}{dx}dx$$

and
$$q'_x = -kt\frac{dT}{dx}$$

it follows that,

$$q''_{rad} - \frac{d}{dx}\left[-kt\frac{dT}{dx}\right] = 0$$

$$\frac{d^2T}{dx^2} + \frac{q''_{rad}}{kt} = 0$$

Integrating twice it follows that, the general solution for the temperature distribution has the form,

$$T(x) = -\frac{q''_{rad}}{2kt}x^2 + C_1x + C_2.$$

Continued …..

The boundary conditions are:

$$T(0) = T_w \qquad C_2 = T_w$$

$$\left.\frac{dT}{dx}\right]_{x=L/2} = 0 \qquad C_1 = \frac{q''_{rad} L}{2kt}$$

Hence,

$$T(x) = \frac{q''_{rad}}{2kt}\ x(L - x) + T_w.$$

The maximum absorber plate temperature, which is at x = L/2, is therefore

$$T_{max} = T(L/2) = \frac{q''_{rad} L^2}{8kt} + T_w.$$

The rate of energy collection per tube may be obtained by applying Fourier's law at x = 0. That is, energy is transferred to the tubes via conduction through the absorber plate. Hence,

$$q' = 2\left[-k\,t\left.\frac{dT}{dx}\right]_{x=0}\right]$$

where the factor of two arises due to heat transfer from both sides of the tube. Hence,

$$q' = -Lq''_{rad}.$$

Hence $$T_{max} = \frac{800\dfrac{W}{m^2}(0.2m)^2}{8\left[180\dfrac{W}{m\cdot K}\right](0.006m)} + 60^\circ C$$

or $$T_{max} = 63.7^\circ C$$ <

and $$q' = -0.2m \times 800\ W/m^2$$

or $$q' = -160\ W/m.$$ <

COMMENTS: Convection losses in the typical flat plate collector, which is not evacuated, would reduce the value of q'.

PROBLEM 3.100

KNOWN: Surface conditions and thickness of a solar collector absorber plate. Temperature of working fluid.

FIND: (a) Differential equation which governs plate temperature distribution, (b) Form of the temperature distribution.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Adiabatic bottom surface, (4) Uniform radiation flux and convection coefficient at top, (5) Temperature of absorber plate at x = 0 corresponds to that of working fluid.

ANALYSIS: (a) Performing an energy balance on the differential control volume,

$$q'_x + dq'_{rad} = q'_{x+dx} + dq'_{conv}$$

where

$$q'_{x+dx} = q'_x + \left(dq'_x / dx\right)dx$$
$$dq'_{rad} = q''_{rad} \cdot dx$$
$$dq'_{conv} = h\left(T - T_\infty\right) \cdot dx$$

Hence,

$$q''_{rad} dx = \left(dq'_x / dx\right)dx + h\left(T - T_\infty\right)dx.$$

From Fourier's law, the conduction heat rate per unit width is

$$q'_x = -k\, t\, dT/dx \qquad \frac{d^2T}{dx^2} - \frac{h}{kT}\left(T - T_\infty\right) + \frac{q''_{rad}}{kt} = 0.$$ <

(b) Defining $\theta = T - T_\infty, d^2T/dx^2 = d^2\theta / dx^2$ and the differential equation becomes,

$$\frac{d^2\theta}{dx^2} - \frac{h}{kt}\theta + \frac{q''_{rad}}{kt} = 0.$$

It is a second-order, differential equation with constant coefficients and a source term, and its general solution is of the form

$$\theta = C_1 e^{+\lambda x} + C_2 e^{-\lambda x} + S/\lambda^2$$

where $\lambda = \left(h/kt\right)^{1/2}$, $S = q''_{rad} / kt$.

Appropriate boundary conditions are:

$$\theta\left(0\right) = T_o - T_\infty \equiv \theta_0, \qquad d\theta/dx)_{x=L} = 0.$$

Hence,

$$\theta_o = C_1 + C_2 + S/\lambda^2$$

$$d\theta/dx)_{x=L} = C_1\ \lambda e^{+\lambda L} - C_2\ \lambda e^{-\lambda L} = 0 \qquad C_2 = C_1\ e^{2\lambda L}$$

Hence,

$$C_1 = \left(\theta_0 - S/\lambda^2\right) / \left(1 + e^{2\lambda L}\right) \qquad C_2 = \left(\theta_0 - S/\lambda^2\right) / \left(1 + e^{-2\lambda L}\right)$$

$$\theta = \left(\theta_0 - S/\lambda^2\right) \left[\frac{e^{\lambda x}}{1+e^{2\lambda L}} + \frac{e^{-\lambda x}}{1+e^{-2\lambda L}}\right] + S/\lambda^2.$$ <

PROBLEM 3.101

KNOWN: Dimensions of a plate insulated on its bottom and thermally joined to heat sinks at its ends. Net heat flux at top surface.

FIND: (a) Differential equation which determines temperature distribution in plate, (b) Temperature distribution and heat loss to heat sinks.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state, (2) One-dimensional conduction in x (W,L>>t), (3) Constant properties, (4) Uniform surface heat flux, (5) Adiabatic bottom, (6) Negligible contact resistance.

ANALYSIS: (a) Applying conservation of energy to the differential control volume, $q_x + dq = q_{x+dx}$, where $q_{x+dx} = q_x + (dq_x/dx)\,dx$ and $dq = q_o''\,(W \cdot dx)$. Hence, $(dq_x/dx) - q_o''\,W = 0$. From Fourier's law, $q_x = -k(t \cdot W)\,dT/dx$. Hence, the differential equation for the temperature distribution is

$$-\frac{d}{dx}\left[ktW\,\frac{dT}{dx}\right] - q_o''\,W = 0 \qquad \frac{d^2T}{dx^2} + \frac{q_o''}{kt} = 0.$$ <

(b) Integrating twice, the general solution is,

$$T(x) = -\frac{q_o''}{2kt}x^2 + C_1\,x + C_2$$

and appropriate boundary conditions are $T(0) = T_o$, and $T(L) = T_o$. Hence, $T_o = C_2$, and

$$T_o = -\frac{q_o''}{2kt}L^2 + C_1 L + C_2 \qquad \text{and} \qquad C_1 = \frac{q_o'' L}{2kt}.$$

Hence, the temperature distribution is

$$T(x) = -\frac{q_o'' L}{2kt}\left(x^2 - Lx\right) + T_o.$$ <

Applying Fourier's law at x = 0, and at x = L,

$$q(0) = -k(Wt)\,dT/dx)_{x=0} = -kWt\left[-\frac{q_o''}{kt}\right]\left[x - \frac{L}{2}\right]\Bigg|_{x=0} = -\frac{q_o'' WL}{2}$$

$$q(L) = -k(Wt)dT/dx)_{x=L} = -kWt\left[-\frac{q_o''}{kt}\right]\left[x - \frac{L}{2}\right]\Bigg|_{x=L} = +\frac{q_o'' WL}{2}$$

Hence the heat loss from the plates is $q = 2\left(q_o'' WL/2\right) = q_o'' WL$. <

COMMENTS: (1) Note signs associated with q(0) and q(L). (2) Note symmetry about x = L/2. Alternative boundary conditions are $T(0) = T_o$ and $dT/dx)_{x=L/2} = 0$.

PROBLEM 3.102

KNOWN: Dimensions and surface conditions of a plate thermally joined at its ends to heat sinks at different temperatures.

FIND: (a) Differential equation which determines temperature distribution in plate, (b) Temperature distribution and an expression for the heat rate from the plate to the sinks, and (c) Compute and plot temperature distribution and heat rates corresponding to changes in different parameters.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in x (W,L >> t), (3) Constant properties, (4) Uniform surface heat flux and convection coefficient, (5) Negligible contact resistance.

ANALYSIS: (a) Applying conservation of energy to the differential control volume

$$q_x + dq_o = q_{x+dx} + dq_{conv}$$

where

$$q_{x+dx} = q_x + \left(dq_x/dx\right)dx \qquad\qquad dq_{conv} = h\left(T - T_\infty\right)\left(W \cdot dx\right)$$

Hence,

$$q_x + q_o''\left(W \cdot dx\right) = q_x + \left(dq_x/dx\right)dx + h\left(T - T_\infty\right)\left(W \cdot dx\right) \qquad\qquad \frac{dq_x}{dx} + hW\left(T - T_\infty\right) = q_o''W\,.$$

Using Fourier's law, $q_x = -k\left(t \cdot W\right)dT/dx$,

$$-ktW\frac{d^2T}{dx^2} + hW\left(T - T_\infty\right) = q_o'' \qquad\qquad \frac{d^2T}{dx^2} - \frac{h}{kt}\left(T - T_\infty\right) + \frac{q_o''}{kt} = 0\,.$$ <

(b) Introducing $\theta \equiv T - T_\infty$, the differential equation becomes

$$\frac{d^2\theta}{dx^2} - \frac{h}{kt}\theta + \frac{q_o''}{kt} = 0\,.$$

This differential equation is of second order with constant coefficients and a source term. With $\lambda^2 \equiv h/kt$ and $S \equiv q_o''/kt$, it follows that the general solution is of the form

$$\theta = C_1 e^{+\lambda x} + C_2 e^{-\lambda x} + S/\lambda^2\,. \qquad (1)$$

Appropriate boundary conditions are: $\theta(0) = T_o - T_\infty \equiv \theta_o \qquad \theta(L) = T_L - T_\infty \equiv \theta_L$ (2,3)

Substituting the boundary conditions, Eqs. (2,3) into the general solution, Eq. (1),

$$\theta_o = C_1 e^0 + C_2 e^0 + S/\lambda^2 \qquad\qquad \theta_L = C_1 e^{+\lambda L} + C_2 e^{-\lambda L} + S/\lambda^2 \qquad (4,5)$$

To solve for C_2, multiply Eq. (4) by $-e^{+\lambda L}$ and add the result to Eq. (5),

$$-\theta_o e^{+\lambda L} + \theta_L = C_2\left(-e^{+\lambda L} + e^{-\lambda L}\right) + S/\lambda^2\left(-e^{+\lambda L} + 1\right)$$

$$C_2 = \left[\left(\theta_L - \theta_o e^{+\lambda L}\right) - S/\lambda^2\left(-e^{+\lambda L} + 1\right)\right] \Big/ \left(-e^{+\lambda L} + e^{-\lambda L}\right) \qquad (6)$$

Continued...

PROBLEM 3.102 (Cont.)

Substituting for C_2 from Eq. (6) into Eq. (4), find

$$C_1 = \theta_o - \left\{ \left[\left(\theta_L - \theta_o e^{+\lambda L} \right) - S/\lambda^2 \left(-e^{+\lambda L} + 1 \right) \right] \Big/ \left(-e^{+\lambda L} + e^{-\lambda L} \right) \right\} - S/\lambda^2 \qquad (7)$$

Using C_1 and C_2 from Eqs. (6,7) and Eq. (1), the temperature distribution can be expressed as

$$\theta(x) = \left[e^{+\lambda x} - \frac{\sinh(\lambda x)}{\sinh(\lambda L)} e^{+\lambda L} \right] \theta_o + \frac{\sinh(\lambda x)}{\sinh(\lambda L)} \theta_L + \left[-\left(1 - e^{+\lambda L}\right) \frac{\sinh(\lambda x)}{\sinh(\lambda L)} + \left(1 - e^{+\lambda L}\right) \right] \frac{S}{\lambda^2} \qquad (8)$$

<

The heat rate from the plate is $q_p = -q_x(0) + q_x(L)$ and using Fourier's law, the conduction heat rates, with $A_c = W \cdot t$, are

$$q_x(0) = -kA_c \left.\frac{d\theta}{dx}\right)_{x=0} = -kA_c \left\{ \left[\lambda e^0 - \frac{e^{\lambda L}}{\sinh(\lambda L)} \lambda \right] \theta_o + \frac{\lambda}{\sinh(\lambda L)} \theta_L + \left[-\frac{1 - e^{+\lambda L}}{\sinh(\lambda L)} \lambda - \lambda \right] \frac{S}{\lambda^2} \right\}$$

<

$$q_x(L) = -kA_c \left.\frac{d\theta}{dx}\right)_{x=L} = -kA_c \left\{ \left[\lambda e^{\lambda L} - \frac{e^{\lambda L}}{\sinh(\lambda L)} \lambda \cosh(\lambda L) \right] \theta_o + \frac{\lambda \cosh(\lambda L)}{\sinh(\lambda L)} \theta_L + \left[-\frac{1 - e^{+\lambda L}}{\sinh(\lambda L)} \lambda \cosh(\lambda L) - \lambda e^{+\lambda L} \right] \frac{S}{\lambda^2} \right\}$$

<

(c) For the prescribed base-case conditions listed below, the temperature distribution (solid line) is shown in the accompanying plot. As expected, the maximum temperature does not occur at the midpoint, but slightly toward the x-origin. The sink heat rates are

$$q''_x(0) = -17.22\,\text{W} \qquad\qquad q''_x(L) = 23.62\,\text{W}$$

<

Temperature, T(x) (C)

Distance, x (mm)

q"o = 20,000 W/m^2; h = 50 W/m^2.K
q"o = 30,000 W/m^2; h = 50 W/m^2.K
q"o = 20,000 W/m^2; h = 200 W/m^2.K
q"o = 4927 W/m^2 with q"x(0) = 0; h = 200 W/m^2.K

The additional temperature distributions on the plot correspond to changes in the following parameters, with all the remaining parameters unchanged: (i) q''_o = 30,000 W/m^2, (ii) h = 200 W/m^2·K, (iii) the value of q''_o for which $q''_x(0)$ = 0 with h = 200 W/m^2·K. The condition for the last curve is q''_o = 4927 W/m^2 for which the temperature gradient at x = 0 is zero.

Base case conditions are: q''_o = 20,000 W/m^2, T_o = 100°C, T_L = 35°C, T_∞ = 25°C, k = 25 W/m·K, h = 50 W/m^2·K, L = 100 mm, t = 5 mm, W = 30 mm.

PROBLEM 3.103

KNOWN: Thin plastic film being bonded to a metal strip by laser heating method; strip dimensions and thermophysical properties are prescribed as are laser heating flux and convection conditions.

FIND: (a) Expression for temperature distribution for the region with the plastic strip, $-w_1/2 \le x \le w_1/2$, (b) Temperature at the center (x = 0) and the edge of the plastic strip ($x = \pm w_1/2$) when the laser flux is 10,000 W/m^2; (c) Plot the temperature distribution for the strip and point out special features.

SCHEMATIC:

q''_o = 10,000 W/m^2
T_∞ = 25 °C
h = 10 W/m^2·K
Metal strip, k = 60W/m·K
T_∞ = 25 °C
$w_1/2$ = 20mm
d = 1.25 mm
$w_2/2 \to \infty$

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in x-direction only, (3) Plastic film has negligible thermal resistance, (4) Upper and lower surfaces have uniform convection coefficients, (5) Edges of metal strip are at air temperature (T_∞), that is, strip behaves as infinite fin so that $w_2 \to \infty$, (6) All the incident laser heating flux q''_o is absorbed by the film.

PROPERTIES: Metal strip (given): ρ = 7850 kg/m^3, c_p = 435 J/kg·m^3, k = 60 W/m·K.

ANALYSIS: (a) The strip-plastic film arrangement can be modeled as an infinite fin of uniform cross section a portion of which is exposed to the laser heat flux on the upper surface. The general solutions for the two regions of the strip, in terms of $\theta \equiv T(x) - T_\infty$, are

$$0 \le x \le w_1/2 \qquad \theta_1(x) = C_1 e^{+mx} + C_2 e^{-mx} + M/m^2 \tag{1}$$

$$M = q''_o P/2kA_c = q''_o/kd \qquad m = (2h/kd)^{1/2} \tag{2,3}$$

$$w_1/2 \le x \le \infty \qquad \theta_2(x) = C_3 e^{+mx} + C_4 e^{-mx}. \tag{4}$$

Four boundary conditions can be identified to evaluate the constants:

At x = 0:
$$\frac{d\theta_1}{dx}(0) = 0 = C_1 m e^0 - C_2 m e^{-0} + 0 \quad \rightarrow \quad C_1 = C_2 \tag{5}$$

At x = w_1/2:
$$\theta(w_1/2) = \theta_2(w_1/2)$$
$$C_1 e^{+mw_1/2} + C_2 e^{-mw_1/2} + M/m^2 = C_3 e^{+mw_1/2} + C_4 e^{-mw_1/2} \tag{6}$$

At x = w_1/2:
$$d\theta_1(w_1/2)/dx = d\theta_2(w_1/2)/dx$$
$$mC_1 e^{+mw_1/2} - mC_2 e^{-mw_1/2} + 0 = mC_3 e^{+mw_1/2} - mC_4 e^{-mw_1/2} \tag{7}$$

At x → ∞:
$$\theta_2(\infty) = 0 = C_3 e^{\infty} + C_4 e^{-\infty} \quad \rightarrow \quad C_3 = 0 \tag{8}$$

With $C_3 = 0$ and $C_1 = C_2$, combine Eqs. (6 and 7) to eliminate C_4 to find

$$C_1 = C_2 = -\frac{M/m^2}{2e^{mw_1/2}}. \tag{9}$$

and using Eq. (6) with Eq. (9) find

$$C_4 = M/m^2 \sinh(mw_1/2) e^{-mx_1/2} \tag{10}$$

Continued...

PROBLEM 3.103 (Cont.)

Hence, the temperature distribution in the region (1) under the plastic film, $0 \le x \le w_1/2$, is

$$\theta_1(x) = -\frac{M/m^2}{2e^{m w_1/w}}\left(e^{+mx} + e^{-mx}\right) + \frac{M}{m^2} = \frac{M}{m^2}\left(1 - e^{-m w_1/2}\cosh mx\right) \qquad (11)$$

<

and for the region (2), $x \ge w_1/2$,

$$\theta_2(x) = \frac{M}{m^2}\sinh\left(m w_1/2\right)e^{-mx} \qquad (12)$$

(b) Substituting numerical values into the temperature distribution expression above, $\theta_1(0)$ and $\theta_1(w_1/2)$ can be determined. First evaluate the following parameters:

$$M = 10,000\,\text{W/m}^2 / 60\,\text{W/m}\cdot\text{K} \times 0.00125\,\text{m} = 133,333\,\text{K/m}^2$$

$$m = \left(2 \times 10\,\text{W/m}^2\cdot\text{K} / 60\,\text{W/m}\cdot\text{K} \times 0.00125\,\text{m}\right)^{1/2} = 16.33\,\text{m}^{-1}$$

Hence, for the midpoint x = 0,

$$\theta_1(0) = \frac{133,333\,\text{K/m}^2}{\left(16.33\,\text{m}^{-1}\right)^2}\left[1 - \exp\left(-16.33\,\text{m}^{-1} \times 0.020\,\text{m}\right) \times \cosh(0)\right] = 139.3\,\text{K}$$

$$T_1(0) = \theta_1(0) + T_\infty = 139.3\,\text{K} + 25^\circ\text{C} = 164.3^\circ\text{C}.$$

<

For the position $x = w_1/2 = 0.020$ m,

$$\theta_1(w_1/2) = 500.0\left[1 - 0.721\cosh\left(16.33\,\text{m}^{-1} \times 0.020\,\text{m}\right)\right] = 120.1\,\text{K}$$

$$T_1(w_1/2) = 120.1\,\text{K} + 25^\circ\text{C} = 145.1^\circ\text{C}.$$

<

(c) The temperature distributions, $\theta_1(x)$ and $\theta_2(x)$, are shown in the plot below. Using IHT, Eqs. (11) and (12) were entered into the workspace and a graph created. The special features are noted:

(1) No gradient at midpoint, x = 0; symmetrical distribution.

(2) No discontinuity of gradient at $w_1/2$ (20 mm).

(3) Temperature excess and gradient approach zero with increasing value of x.

COMMENTS: How wide must the strip be in order to satisfy the infinite fin approximation such that $\theta_2(x \to \infty) = 0$? For x = 200 mm, find θ_2(200 mm) = 6.3°C; this would be a poor approximation. When x = 300 mm, θ_2(300 mm) = 1.2°C; hence when $w_2/2$ = 300 mm, the strip is a reasonable approximation to an infinite fin.

PROBLEM 3.104

KNOWN: Thermal conductivity, diameter and length of a wire which is annealed by passing an electrical current through the wire.

FIND: Steady-state temperature distribution along wire.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction along the wire, (3) Constant properties, (4) Negligible radiation, (5) Uniform convection coefficient h.

ANALYSIS: Applying conservation of energy to a differential control volume,

$$q_x + \dot{E}_g - dq_{conv} - q_{x+dx} = 0$$

$$q_{x+dx} = q_x + \frac{dq_x}{dx}dx \qquad q_x = -k\left(\pi\ D^2/4\right)dT/dx$$

$$dq_{conv} = h(\pi\ D\ dx)\ (T - T_\infty) \qquad \dot{E}_g = \dot{q}\left(\pi\ D^2/4\right)dx.$$

Hence,

$$k\left(\pi\ D^2/4\right)\ \frac{d^2T}{dx^2}dx + \dot{q}\left(\pi\ D^2/4\right)dx - h(\pi\ Ddx)\ (T - T_\infty) = 0$$

or, with $\theta \equiv T - T_\infty$,
$$\frac{d^2\theta}{dx^2} - \frac{4h}{kD}\theta + \frac{\dot{q}}{k} = 0$$

The solution (general and particular) to this nonhomogeneous equation is of the form

$$\theta = C_1\ e^{mx} + C_2\ e^{-mx} + \frac{\dot{q}}{km^2}$$

where $m^2 = (4h/kD)$. The boundary conditions are:

$$\left.\frac{d\theta}{dx}\right]_{x=0} = 0 = m\ C_1\ e^0 - mC_2\ e^0 \rightarrow \qquad C_1 = C_2$$

$$\theta(L) = 0 = C_1\left(e^{mL} + e^{-mL}\right) + \frac{\dot{q}}{km^2} \rightarrow \qquad C_1 = \frac{-\dot{q}/km^2}{e^{mL} + e^{-mL}} = C_2$$

The temperature distribution has the form

$$T = T_\infty - \frac{\dot{q}}{km^2}\left[\frac{e^{mx} + e^{-mx}}{e^{mL} + e^{-mL}} - 1\right] = T_\infty - \frac{\dot{q}}{km^2}\left[\frac{\cosh mx}{\cosh mL} - 1\right].$$ <

COMMENTS: This process is commonly used to anneal wire and spring products. To check the result, note that $T(L) = T(-L) = T_\infty$.

PROBLEM 3.105

KNOWN: Electric power input and mechanical power output of a motor. Dimensions of housing, mounting pad and connecting shaft needed for heat transfer calculations. Temperature of ambient air, tip of shaft, and base of pad.

FIND: Housing temperature.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in pad and shaft, (3) Constant properties, (4) Negligible radiation.

ANALYSIS: Conservation of energy yields

$$P_{elec} - P_{mech} - q_h - q_p - q_s = 0$$

$$q_h = h_h A_h \left(T_h - T_\infty\right), \qquad q_p = k_p W^2 \frac{\left(T_h - T_\infty\right)}{t}, \qquad q_s = M \frac{\cosh mL - \theta_L / \theta_b}{\sinh mL}$$

$$\theta_L = 0, \qquad mL = \left(4 h_s L^2 / k_s D\right)^{1/2}, \qquad M = \left(\frac{\pi^2}{4} D^3 h_s k_s\right)^{1/2} \left(T_h - T_\infty\right).$$

Hence

$$q_s = \frac{\left(\left[\pi^2 / 4\right] D^3 h_s k_s\right)^{1/2} \left(T_h - T_\infty\right)}{\tanh\left(4 h_s L^2 / k_s D\right)^{1/2}}$$

Substituting, and solving for $(T_h - T_\infty)$,

$$T_h - T_\infty = \frac{P_{elec} - P_{mech}}{h_h A_h + k_p W^2 / t + \left(\left(\pi^2 / 4\right) D^3 h_s k_s\right)^{1/2} / \tanh\left(4 h_s L^2 / k_s D\right)^{1/2}}$$

$$\left(\left(\pi^2 / 4\right) D^3 h_s k_s\right)^{1/2} = 6.08 \text{ W/K}, \qquad \left(4 h_s L^2 / k_s D\right)^{1/2} = 3.87, \qquad \tanh mL = 0.999$$

$$T_h - T_\infty = \frac{(25 - 15) \times 10^3 \text{ W}}{\left[10 \times 2 + 0.5(0.7)^2 / 0.05 + 6.08 / 0.999\right] \text{W/K}} = \frac{10^4 \text{ W}}{(20 + 4.90 + 6.15) \text{W/K}}$$

$$T_h - T_\infty = 322.1 \text{K} \qquad T_h = 347.1^\circ \text{C}$$ <

COMMENTS: (1) T_h is large enough to provide significant heat loss by radiation from the housing. Assuming an emissivity of 0.8 and surroundings at 25°C, $q_{rad} = \varepsilon A_h \left(T_h^4 - T_{sur}^4\right) = 4347$ W, which compares with $q_{conv} = h A_h \left(T_h - T_\infty\right) = 5390$ W. Radiation has the effect of decreasing T_h. (2) The infinite fin approximation, $q_s = M$, is excellent.

PROBLEM 3.106

KNOWN: Dimensions and thermal conductivity of pipe and flange. Inner surface temperature of pipe. Ambient temperature and convection coefficient.

FIND: Heat loss through flange.

SCHEMATIC:

$T_\infty = 20°C$, $h = 10\ W/m^2\text{-}K$, $t = 15$ mm, $T_{s,i} = 300°C$, $D_i = 120$ mm, $D_o = 150$ mm, $k = 40$ W/m-K, $D_f = 250$ mm; thermal circuit: q, T_∞, $R_{t,f}$, $R_{t,w}$, $T_{s,i}$

ASSUMPTIONS: (1) Steady-state, (2) One-dimensional radial conduction in pipe and flange, (3) Constant thermal conductivity, (4) Negligible radiation exchange with surroundings.

ANALYSIS: From the thermal circuit, the heat loss through the flanges is

$$q = \frac{T_{s,i} - T_\infty}{R_{t,w} + R_{t,f}} = \frac{T_{s,i} - T_\infty}{\left[\ell n\left(D_o / D_i\right)/4\pi tk\right] + \left(1/hA_f\eta_f\right)}$$

Since convection heat transfer only occurs from one surface of a flange, the connected flanges may be modeled as a single annular fin of thickness $t' = 2t = 30\,\text{mm}$. Hence, $r_{2c} = \left(D_f / 2\right) + t'/2 = 0.140\,\text{m}$, $A_f = 2\pi\left(r_{2c}^2 - r_1^2\right) = 2\pi\left(r_{2c}^2 - D_o/2\right) = 2\pi\left(0.140^2 - 0.06^2\right)\text{m}^2 = 0.101\,\text{m}^2$, $L_c = L + t'/2 = \left(D_f - D_o\right)/2 + t = 0.065\,\text{m}$, $A_p = L_c\,t' = 0.00195\,\text{m}^2$, $L_c^{2/2}\left(h/kA_p\right)^{1/2} = 0.188$. With $r_{2c}/r_1 = r_{2c}/(D_o/2) = 1.87$, Fig. 3.19 yields $\eta_f = 0.94$. Hence,

$$q = \frac{300°\text{C} - 20°\text{C}}{\left[\ell n(1.25)/4\pi \times 0.03\,\text{m} \times 40\,\text{W/m}\cdot\text{K}\right] + \left(1/10\,\text{W/m}^2\cdot\text{K} \times 0.101\,\text{m}^2 \times 0.94\right)}$$

$$q = \frac{280°\text{C}}{(0.0148 + 1.053)\,\text{K/W}} = 262\,\text{W}$$ <

COMMENTS: Without the flange, heat transfer from a section of pipe of width $t' = 2t$ is $q = \left(T_{s,i} - T_\infty\right)/\left(R_{t,w} + R_{t,cnv}\right)$, where $R_{t,cnv} = \left(h \times \pi D_o t'\right)^{-1} = 7.07\,\text{K/W}$. Hence, $q = 39.5\,\text{W}$, and there is significant heat transfer enhancement associated with the extended surfaces afforded by the flanges.

PROBLEM 3.107

KNOWN: TC wire leads attached to the upper and lower surfaces of a cylindrically shaped solder bead. Base of bead attached to cylinder head operating at 350°C. Constriction resistance at base and TC wire convection conditions specified.

FIND: (a) Thermal circuit that can be used to determine the temperature difference between the two intermediate metal TC junctions, $(T_1 - T_2)$; label temperatures, thermal resistances and heat rates; and (b) Evaluate $(T_1 - T_2)$ for the prescribed conditions. Comment on assumptions made in building the model.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in solder bead; no losses from lateral and top surfaces; (3) TC wires behave as infinite fins, (4) Negligible thermal contact resistance between TC wire terminals and bead.

ANALYSIS: (a) The thermal circuit is shown above. Note labels for the temperatures, thermal resistances and the relevant heat fluxes. The thermal resistances are as follows:

Constriction (con) resistance, see Table 4.1, case 10

$$R_{con} = 1/\left(2k_{bead}D_{sol}\right) = 1/\left(2\times 40\ \mathrm{W/m\cdot K}\times 0.006\ \mathrm{m}\right) = 2.08\ \mathrm{K/W}$$

TC (tc) wires, infinitely long fins; Eq. 3.80

$$R_{tc,1} = R_{tc,2} = R_{fin} = \left(hPk_w A_c\right)^{-0.5} \qquad P = \pi D_w,\ A_c = \pi D_w^2/4$$

$$R_{tc} = \left(100\ \mathrm{W/m^2\cdot K}\times\pi^2\times(0.003\ \mathrm{m})^3\times 70\ \mathrm{W/m\cdot K}/4\right)^{-0.5} = 46.31\ \mathrm{K/W}$$

Solder bead (sol), cylinder D_{sol} *and* L_{sol}

$$R_{sol} = L_{sol}/\left(k_{sol}A_{sol}\right) \qquad A_{sol} = \pi D_{sol}^2/4$$

$$R_{sol} = 0.010\ \mathrm{m}/\left(10\ \mathrm{W/m\cdot K}\times\pi\,(0.006\ \mathrm{m})^2/4\right) = 35.37\ \mathrm{K/W}$$

(b) Perform energy balances on the 1- and 2-nodes, solve the equations simultaneously to find T_1 and T_2, from which $(T_1 - T_2)$ can be determined.

Continued …..

PROBLEM 3.107 (Cont.)

Node 1 $$\frac{T_2 - T_1}{R_{sol}} + \frac{T_{head} - T_1}{R_{con}} + \frac{T_\infty - T_1}{R_{tc,1}} = 0$$

Node 2 $$\frac{T_\infty - T_2}{R_{tc,2}} + \frac{T_1 - T_2}{R_{sol}} = 0$$

Substituting numerical values with the equations in the *IHT* Workspace, find

$$T_1 = 359°C \qquad T_2 = 199.2°C \qquad T_1 - T_2 = 160°C$$

COMMENTS: (1) With this arrangement, the TC indicates a systematically low reading of the cylinder head. The size of the solder bead (L_{sol}) needs to be reduced substantially.

(2) The model neglects heat losses from the top and lateral sides of the solder bead, the effect of which would be to increase our estimate for ($T_1 - T_2$). Constriction resistance is important; note that $T_{head} - T_1 = 26°C$.

PROBLEM 3.108

KNOWN: Rod (D, k, 2L) that is perfectly insulated over the portion of its length $-L \le x \le 0$ and experiences convection (T_∞, h) over the portion $0 \le x \le +L$. One end is maintained at T_1 and the other is separated from a heat sink at T_3 with an interfacial thermal contact resistance R''_{tc}.

FIND: (a) Sketch the temperature distribution T vs. x and identify key features; assume $T_1 > T_3 > T_2$; (b) Derive an expression for the mid-point temperature T_2 in terms of thermal and geometric parameters of the system, (c) Using, numerical values, calculate T_2 and plot the temperature distribution. Describe key features and compare to your sketch of part (a).

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in rod for $-L \le x \le 0$, (3) Rod behaves as one-dimensional extended surface for $0 \le x \le +L$, (4) Constant properties.

ANALYSIS: (a) The sketch for the temperature distribution is shown below. Over the insulated portion of the rod, the temperature distribution is linear. A temperature drop occurs across the thermal contact resistance at x = +L. The distribution over the exposed portion of the rod is non-linear. The minimum temperature of the system could occur in this portion of the rod.

(b) To derive an expression for T_2, begin with the general solution from the conduction analysis for a fin of uniform cross-sectional area, Eq. 3.66.

$$\theta(x) = C_1 e^{mx} + C_2 e^{-mx} \qquad 0 \le x \le +L \tag{1}$$

where $m = (hP/kA_c)^{1/2}$ and $\theta = T(x) - T_\infty$. The arbitrary constants are determined from the boundary conditions.

At x = 0, thermal resistance of rod

$$q_x(0) = -kA_c \left.\frac{d\theta}{dx}\right)_{x=0} = kA_c \frac{\theta_1 - \theta(0)}{L} \qquad \theta_1 = T_1 - T_\infty$$

$$mC_1e^0 - mC_2e^0 = \frac{1}{L}\left[\theta_1 - \left(C_1e^0 + C_2e^0\right)\right] \tag{2}$$

Continued …..

PROBLEM 3.108 (Cont.)

At x=L, thermal contact resistance

$$q_x(+L) = -kA_c \left.\frac{d\theta}{dx}\right)_{x=L} = \frac{\theta(L)-\theta_3}{R''_{tc}/A_c} \qquad \theta_3 = T_3 - T_\infty$$

$$-k\left[mC_1e^{mL} - mC_2e^{-mL}\right] = \frac{1}{R''_{tc}}\left[C_1e^{mL} + C_2e^{-mL} - \theta_3\right] \tag{3}$$

Eqs. (2) and (3) cannot be rearranged easily to find explicit forms for C_1 and C_2. The constraints will be evaluated numerically in part (c). Knowing C_1 and C_2, Eq. (1) gives

$$\theta_2 = \theta(0) = T_2 - T_\infty = C_1e^0 + C_2e^0 \tag{4}$$

(c) With Eqs. (1-4) in the *IHT Workspace* using numerical values shown in the schematic, find T_2 = 62.1°C. The temperature distribution is shown in the graph below.

Temperature distribution in rod

COMMENTS: (1) The purpose of asking you to sketch the temperature distribution in part (a) was to give you the opportunity to identify the relevant thermal processes and come to an understanding of the system behavior.

(2) Sketch the temperature distributions for the following conditions and explain their key features: (a) $R''_{tc} = 0$, (b) $R''_{tc} \to \infty$, and (c) the exposed portion of the rod behaves as an infinitely long fin; that is, k is very large.

PROBLEM 3.109

KNOWN: Long rod in oven with air temperature at 400°C has one end firmly pressed against surface of a billet; thermocouples imbedded in rod at locations 25 and 120 mm from the billet indicate 325 and 375°C, respectively.

FIND: The temperature of the billet, T_b.

SCHEMATIC:

Rod
Oven wall
Air $T_\infty = 400°C$
x(mm)
120
$T_2 = 375°C$
T_b
25
$T_1 = 325°C$
Billet

ASSUMPTIONS: (1) Steady-state conditions, (2) Rod is infinitely long with uniform cross-sectional area, (3) Uniform convection coefficient along rod.

ANALYSIS: For an infinitely long rod of uniform cross-sectional area, the temperature distribution is

$$\theta(x) = \theta_b e^{-mx} \qquad (1)$$

where

$$\theta(x) = T(x) - T_\infty \qquad \theta_b = T(0) - T_\infty = T_b - T_\infty.$$

Substituting values for T_1 and T_2 at their respective distances, x_1 and x_2, into Eq. (1), it is possible to evaluate m,

$$\frac{\theta(x_1)}{\theta(x_2)} = \frac{\theta_b e^{-mx_1}}{\theta_b e^{-mx_2}} = e^{-m(x_1 - x_2)}$$

$$\frac{(325-400)^\circ C}{(375\text{-}400)^\circ C} = e^{-m(0.025-0.120)m} \qquad m=11.56.$$

Using the value for m with Eq. (1) at location x_1, it is now possible to determine the rod base or billet temperature,

$$\theta(x_1) = T_1 - T_\infty = (T_b - T_\infty)e^{-mx}$$

$$(325-400)^\circ C = (T_b - 400)^\circ C\ e^{-11.56 \times 0.025}$$

$$T_b = 300^\circ C. \qquad <$$

COMMENTS: Using the criteria mL ≥ 2.65 (see Example 3.8) for the infinite fin approximation, the insertion length should be ≥ 229 mm to justify the approximation,

PROBLEM 3.110

KNOWN: Temperature sensing probe of thermal conductivity k, length L and diameter D is mounted on a duct wall; portion of probe L_i is exposed to water stream at $T_{\infty,i}$ while other end is exposed to ambient air at $T_{\infty,o}$; convection coefficients h_i and h_o are prescribed.

FIND: (a) Expression for the measurement error, $\Delta T_{err} = T_{tip} - T_{\infty,i}$, (b) For prescribed $T_{\infty,i}$ and $T_{\infty,o}$, calculate ΔT_{err} for immersion to total length ratios of 0.225, 0.425, and 0.625, (c) Compute and plot the effects of probe thermal conductivity and water velocity (h_i) on ΔT_{err}.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in probe, (3) Probe is thermally isolated from the duct, (4) Convection coefficients are uniform over their respective regions.

PROPERTIES: Probe material (given): k = 177 W/m·K.

ANALYSIS: (a) To derive an expression for $\Delta T_{err} = T_{tip} - T_{\infty,i}$, we need to determine the temperature distribution in the immersed length of the probe $T_i(x)$. Consider the probe to consist of two regions: $0 \le x_i \le L_i$, the immersed portion, and $0 \le x_o \le (L - L_i)$, the ambient-air portion where the origin corresponds to the location of the duct wall. Use the results for the temperature distribution and fin heat rate of Case A, Table 3.4:

Temperature distribution in region i:

$$\frac{\theta_i}{\theta_{b,i}} = \frac{T_i(x_i) - T_{\infty,i}}{T_o - T_{\infty,i}} = \frac{\cosh(m_i(L_i - x_i)) + (h_i/m_i k)\sinh(L_i - x_i)}{\cosh(m_i L_i) + (h_i/m_i k)\sinh(m_i L_i)} \quad (1)$$

and the tip temperature, $T_{tip} = T_i(L_i)$ at $x_i = L_i$, is

$$\frac{T_{tip} - T_{\infty,i}}{T_o - T_{\infty,i}} = A = \frac{\cosh(0) + (h_i/m_i k)\sinh(0)}{\cosh(m_i L_i) + (h_i/m_i k)\sinh(m_i L_i)} \quad (2)$$

and hence

$$\Delta T_{err} = T_{tip} - T_{\infty,i} = A\left(T_o - T_{\infty,i}\right) \quad (3)$$ <

where T_o is the temperature at $x_i = x_o = 0$ which at present is unknown, but can be found by setting the fin heat rates equal, that is,

$$q_{f,o} = -q_{f,i} \quad (4)$$

Continued...

PROBLEM 3.110 (Cont.)

$$\left(h_o P k A_c\right)^{1/2} \theta_{b,o} \cdot B = -\left(h_i P k A_c\right)^{1/2} \theta_{b,i} \cdot C$$

Solving for T_o, find

$$\frac{\theta_{b,o}}{\theta_{b,i}} = \frac{T_o - T_{\infty,o}}{T_o - T_{\infty,i}} = -\left(h_i P k A_c\right)^{1/2} \theta_{b,i} \cdot C$$

$$T_o = \left[T_{\infty,o} + \left(\frac{h_i}{h_o}\right)^{1/2} \frac{C}{B} T_{\infty,i} \right] \Bigg/ \left[1 + \left(\frac{h_i}{h_o}\right)^{1/2} \frac{C}{B} \right] \quad (5)$$

where the constants B and C are,

$$B = \frac{\sinh(m_o L_o) + (h_o/m_o k)\cosh(m_o L_o)}{\cosh(m_o L_o) + (h_o/m_o k)\sinh(m_o L_o)} \quad (6)$$

$$C = \frac{\sinh(m_i L_i) + (h_i/m_i k)\cosh(m_i L_i)}{\cosh(m_i L_i) + (h_i/m_i k)\sinh(m_i L_i)} \quad (7)$$

(b) To calculate the immersion error for prescribed immersion lengths, L_i/L = 0.225, 0.425 and 0.625, we use Eq. (3) as well as Eqs. (2, 6, 7 and 5) for A, B, C, and T_o, respectively. Results of these calculations are summarized below.

L_i/L	L_o (mm)	L_i (mm)	A	B	C	T_o (°C)	ΔT_{err} (°C)	
0.225	155	45	0.2328	0.5865	0.9731	76.7	-0.76	<
0.425	115	85	0.0396	0.4639	0.992	77.5	-0.10	<
0.625	75	125	0.0067	0.3205	0.9999	78.2	-0.01	<

(c) The probe behaves as a fin having ends exposed to the cool ambient air and the hot ambient water whose temperature is to be measured. If the thermal conductivity is *decreased*, heat transfer along the probe length is likewise decreased, the tip temperature will be closer to the water temperature. If the velocity of the water *decreases*, the convection coefficient will decrease, and the difference between the tip and water temperatures will increase.

Temperature error, Tinfo - Ttip (C)

Immersion length ratio, Li/L

Base case: k = 177 W/m.K; ho = 1100 W/m^2.K
Low velocity flow: k = 177 W/m.K; ho = 500 W/m^2.K
Low conductivity probe: k = 50 W/m.K; ho = 1100 W/m^2.K

PROBLEM 3.111

KNOWN: Rod protruding normally from a furnace wall covered with insulation of thickness L_{ins} with the length L_o exposed to convection with ambient air.

FIND: (a) An expression for the exposed surface temperature T_o as a function of the prescribed thermal and geometrical parameters. (b) Will a rod of L_o = 100 mm meet the specified operating limit, $T_0 \le 100°C$? If not, what design parameters would you change?

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in rod, (3) Negligible thermal contact resistance between the rod and hot furnace wall, (4) Insulated section of rod, L_{ins}, experiences no lateral heat losses, (5) Convection coefficient uniform over the exposed portion of the rod, L_o, (6) Adiabatic tip condition for the rod and (7) Negligible radiation exchange between rod and its surroundings.

ANALYSIS: (a) The rod can be modeled as a thermal network comprised of two resistances in series: the portion of the rod, L_{ins}, covered by insulation, R_{ins}, and the portion of the rod, L_o, experiencing convection, and behaving as a fin with an adiabatic tip condition, R_{fin}. For the insulated section:

$$R_{ins} = L_{ins}/kA_c \qquad (1)$$

For the fin, Table 3.4, Case B, Eq. 3.76,

$$R_{fin} = \theta_b/q_f = \frac{1}{(hPkA_c)^{1/2}\tanh(mL_o)} \qquad (2)$$

$$m = (hP/kA_c)^{1/2} \qquad A_c = \pi D^2/4 \qquad P = \pi D \qquad (3,4,5)$$

From the thermal network, by inspection,

$$\frac{T_o - T_\infty}{R_{fin}} = \frac{T_w - T_\infty}{R_{ins} + R_{fin}} \qquad T_o = T_\infty + \frac{R_{fin}}{R_{ins} + R_{fin}}(T_w - T_\infty) \qquad (6) <$$

(b) Substituting numerical values into Eqs. (1) - (6) with L_o = 200 mm,

$$T_o = 25°C + \frac{6.298}{6.790 + 6.298}(200 - 25)°C = 109°C \qquad <$$

$$R_{ins} = \frac{0.200\,m}{60\,W/m\cdot K \times 4.909\times10^{-4}\,m^2} = 6.790\,K/W \qquad A_c = \pi(0.025\,m)^2/4 = 4.909\times10^{-4}\,m^2$$

$$R_{fin} = 1\Big/\left(0.0347\,W^2/K^2\right)^{1/2}\tanh(6.324\times0.200) = 6.298\,K/W$$

$$(hPkA_c) = \left(15\,W/m^2\cdot K\times\pi(0.025\,m)\times60\,W/m\cdot K\times4.909\times10^{-4}\,m^2\right) = 0.0347\,W^2/K^2$$

Continued...

$$m = \left(hP/kA_c\right)^{1/2} = \left(15\,W/m^2 \cdot K \times \pi\,(0.025\,m)\Big/60\,W/m \cdot K \times 4.909 \times 10^{-4}\,m^2\right)^{1/2} = 6.324\,m^{-1}$$

Consider the following design changes aimed at reducing $T_o \leq 100°C$. (1) Increasing length of the fin portions: with L_o = 200 mm, the fin already behaves as an infinitely long fin. Hence, increasing L_o will not result in reducing T_o. (2) Decreasing the thermal conductivity: backsolving the above equation set with T_0 = 100°C, find the required thermal conductivity is k = 14 W/m·K. Hence, we could select a stainless steel alloy; see Table A.1. (3) Increasing the insulation thickness: find that for T_o = 100°C, the required insulation thickness would be L_{ins} = 211 mm. This design solution might be physically and economically unattractive. (4) A very practical solution would be to introduce thermal contact resistance between the rod base and the furnace wall by "tack welding" (rather than a continuous bead around the rod circumference) the rod in two or three places. (5) A less practical solution would be to increase the convection coefficient, since to do so, would require an air handling unit.

COMMENTS: (1) Would replacing the rod by a thick-walled tube provide a practical solution?

(2) The *IHT Thermal Resistance Network Model* and the *Thermal Resistance Tool* for a *fin* with an *adiabatic tip* were used to create a model of the rod. The Workspace is shown below.

```
// Thermal Resistance Network Model:
// The Network:
```

q1 → 1 —R21— 2 ← q2 —R32— 3 ← q3

```
// Heat rates into node j,qij, through thermal resistance Rij
q21 = (T2 - T1) / R21
q32 = (T3 - T2) / R32

// Nodal energy balances
q1 + q21 = 0
q2 - q21 + q32 = 0
q3 - q32 = 0

/* Assigned variables list: deselect the qi, Rij and Ti which are unknowns; set qi = 0 for embedded nodal
points at which there is no external source of heat. */
T1 = Tw              // Furnace wall temperature, C
//q1 =               // Heat rate, W
T2 = To              // To, beginning of rod exposed length
q2 =  0              // Heat rate, W; node 2; no external heat source
T3 =  Tinf           // Ambient air temperature, C
//q3 =               // Heat rate, W

// Thermal Resistances:
// Rod - conduction resistance
R21 =  Lins / (k * Ac)        // Conduction resistance, K/W
Ac = pi * D^2 / 4             // Cross sectional area of rod, m^2

// Thermal Resistance Tools - Fin with Adiabatic Tip:
R32 =  Rfin                   //  Resistance of fin, K/W
/* Thermal resistance of a fin of uniform cross sectional area Ac, perimeter P, length L, and thermal
conductivity k with an adiabatic tip condition experiencing convection with a fluid at Tinf and coefficient h, */
Rfin = 1/ ( tanh (m*Lo) * (h * P * k * Ac ) ^ (1/2) )        // Case B, Table 3.4
m = sqrt(h*P / (k*Ac))
P = pi * D                    // Perimeter, m

// Other Assigned Variables:
Tw = 200             // Furnace wall temperature, C
k = 60               // Rod thermal conductivity, W/m.K
Lins = 0.200         // Insulated length, m
D = 0.025            // Rod diameter, m
h = 15               // Convection coefficient, W/m^2.K
Tinf = 25            // Ambient air temperature,C
Lo = 0.200           // Exposed length, m
```

PROBLEM 3.112

KNOWN: Rod (D, k, 2L) inserted into a perfectly insulating wall, exposing one-half of its length to an airstream (T_∞, h). An electromagnetic field induces a uniform volumetric energy generation $(\dot{q})$ in the imbedded portion.

FIND: (a) Derive an expression for T_b at the base of the exposed half of the rod; the exposed region may be approximated as a very long fin; (b) Derive an expression for T_o at the end of the imbedded half of the rod, and (c) Using numerical values, plot the temperature distribution in the rod and describe its key features. Does the rod behave as a very long fin?

SCHEMATIC:

T_o $\dot{q}$ T_b $T_\infty = 20°C$ $h = 100\ W/m^2\text{-}K$ Rod -L x +L

D = 5 mm
L = 50 mm
k = 25 W/m-K
$\dot{q} = 1\text{x}10^6\ W/m^3$

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in imbedded portion of rod, (3) Imbedded portion of rod is perfectly insulated, (4) Exposed portion of rod behaves as an infinitely long fin, and (5) Constant properties.

ANALYSIS: (a) Since the exposed portion of the rod ($0 \le x \le +L$) behaves as an infinite fin, the fin heat rate using Eq. 3.80 is

$$q_x(0) = q_f = M = (hPkA_c)^{1/2}(T_b - T_\infty) \qquad (1)$$

From an energy balance on the imbedded portion of the rod,

$$q_f = \dot{q}\,A_c L \qquad (2)$$

Combining Eqs. (1) and (2), with $P = \pi D$ and $A_c = \pi D^2/4$, find

$$T_b = T_\infty + q_f\,(hPkA_c)^{-1/2} = T_\infty + \dot{q}A_c^{1/2}L\,(hPk)^{-1/2} \qquad (3)$$ <

(b) The imbedded portion of the rod ($-L \le x \le 0$) experiences one-dimensional heat transfer with uniform $\dot{q}$. From Eq. 3.43,

$$T_o = \frac{\dot{q}L^2}{2k} + T_b$$ <

(c) The temperature distribution T(x) for the rod is piecewise parabolic and exponential,

$$T(x) - T_b = \frac{\dot{q}L^2}{2k}\left(\frac{x}{L}\right)^2 \qquad -L \le x \le 0$$

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = \exp(-mx) \qquad 0 \le x \le +L$$

Continued …..

The gradient at x = 0 will be continuous since we used this condition in evaluating T_b. The distribution is shown below with T_o = 105.4°C and T_b = 55.4°C.

T(x) over embedded and exposed portions of rod

PROBLEM 3.113

KNOWN: Very long rod (D, k) subjected to induction heating experiences uniform volumetric generation $(\dot{q})$ over the center, 30-mm long portion. The unheated portions experience convection (T_∞, h).

FIND: Calculate the temperature of the rod at the mid-point of the heated portion within the coil, T_o, and at the edge of the heated portion, T_b.

SCHEMATIC:

$\dot{q}$ = 7.5x10^6 W/m^3
T_∞ = 20°C
h = 10 W/m^2-K
Very long rod, 5 mm dia.
k = 25 W/m-K
L = 15 mm
x
$q''_x(L) = q_f$
$\dot{q}$
x = 0
x = L
Fin

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction with uniform $\dot{q}$ in portion of rod within the coil; no convection from lateral surface of rod, (3) Exposed portions of rod behave as infinitely long fins, and (4) Constant properties.

ANALYSIS: The portion of the rod within the coil, $0 \le x \le + L$, experiences one-dimensional conduction with uniform generation. From Eq. 3.43,

$$T_o = \frac{\dot{q}L^2}{2k} + T_b \qquad (1)$$

The portion of the rod beyond the coil, $L \le x \le \infty$, behaves as an infinitely long fin for which the heat rate from Eq. 3.80 is

$$q_f = q_x(L) = (hPkA_c)^{1/2}(T_b - T_\infty) \qquad (2)$$

where $P = \pi D$ and $A_c = \pi D^2/4$. From an overall energy balance on the imbedded portion of the rod as illustrated in the schematic above, find the heat rate as

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = 0$$

$$-q_f + \dot{q}A_c L = 0$$

$$q_f = \dot{q}A_c L \qquad (3)$$

Combining Eqs. (1-3),

$$T_b = T_\infty + \dot{q}A_c^{1/2}L(hPk)^{-1/2} \qquad (4)$$

$$T_o = T_\infty + \frac{\dot{q}L^2}{2k} + \dot{q}A_c^{1/2}L(hPk)^{-1/2} \qquad (5)$$

and substituting numerical values find

$$T_o = 305°C \qquad T_b = 272°C$$

<

PROBLEM 3.114

KNOWN: Dimensions, end temperatures and volumetric heating of wire leads. Convection coefficient and ambient temperature.

FIND: (a) Equation governing temperature distribution in the leads, (b) Form of the temperature distribution.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state, (2) One-dimensional conduction in x, (3) Uniform volumetric heating, (4) Uniform h (both sides), (5) Negligible radiation.

ANALYSIS: (a) Performing an energy balance for the differential control volume,

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = 0 \qquad\qquad q_x - q_{x+dx} - dq_{conv} + \dot{q}dV = 0$$

$$-kA_c \frac{dT}{dx} - \left[-kA_c \frac{dT}{dx} - \frac{d}{dx}\left(kA_c \frac{dT}{dx}\right)dx\right] - hPdx\left(T - T_\infty\right) + \dot{q}A_c dx = 0$$

$$\frac{d^2T}{dx^2} - \frac{hP}{kA_c}\left(T - T_\infty\right) + \frac{\dot{q}}{k} = 0$$

<

(b) With a *reduced temperature* defined as $\Theta \equiv T - T_\infty - \left(\dot{q}A_c/hP\right)$ and $m^2 \equiv hP/kA_c$, the differential equation may be rendered homogeneous, with a general solution and boundary conditionsas shown

$$\frac{d^2\Theta}{dx^2} - m^2\Theta = 0 \qquad\qquad \Theta\left(x\right) = C_1e^{mx} + C_2e^{-mx}$$

$$\Theta_b = C_1 + C_2 \qquad\qquad \Theta_c = C_1e^{mL} + C_2e^{-mL}$$

it follows that

$$C_1 = \frac{\Theta_b e^{-mL} - \Theta_c}{e^{-mL} - e^{mL}} \qquad\qquad C_2 = \frac{\Theta_c - \Theta_b e^{mL}}{e^{-mL} - e^{mL}}$$

$$\Theta\left(x\right) = \frac{\left(\Theta_b e^{-mL} - \Theta_c\right)e^{mx} + \left(\Theta_c - \Theta_b e^{mL}\right)e^{-mx}}{e^{-mL} - e^{mL}}$$

<

COMMENTS: If $\dot{q}$ is large and h is small, temperatures within the lead may readily exceed the prescribed boundary temperatures.

PROBLEM 3.115

KNOWN: Disk-shaped electronic device (D, L_d, k_d) dissipates electrical power (P_e) at one of its surfaces. Device is bonded to a cooled base (T_o) using a thermal pad (L_p, k_A). Long fin (D, k_f) is bonded to the heat-generating surface using an identical thermal pad. Fin is cooled by convection (T_∞, h).

FIND: (a) Construct a thermal circuit of the system, (b) Derive an expression for the temperature of the heat-generating device, T_d, in terms of circuit thermal resistance, T_o and T_∞; write expressions for the thermal resistances; and (c) Calculate T_d for the prescribed conditions.

SCHEMATIC:

L_p, L_d
Heat generating surface, P_e = 15 W, T_d
Air
$T_o = 20^oC$
Long fin, D, k_f
Device, k_d
Thermal pads, k_p
$T_\infty = 15^oC$
h = 250 W/m²-K

Device
L_d = 3 mm
k_d = 25 W/m-K
Pads
L_p = 1.5 mm
k_p = 50 W/m-K
Fins
D = 6 mm
k_f = 230 W/m-K

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction through thermal pads and device; no losses from lateral surfaces; (3) Fin is infinitely long, (4) Negligible contact resistance between components of the system, and (5) Constant properties.

ANALYSIS: (a) The thermal circuit is shown below with thermal resistances associated with conduction (pads, R_p; device, R_d) and for the long fin, R_f.

q_a → T_o — R_p — R_d — T_d (P_e) — R_p — R_f — T_∞ ← q_b

(b) To obtain an expression for T_d, perform an energy balance about the d-node

$$\dot{E}_{in} - \dot{E}_{out} = q_a + q_b + P_e = 0 \quad (1)$$

Using the conduction rate equation with the circuit

$$q_a = \frac{T_o - T_d}{R_f + R_d} \qquad q_b = \frac{T_\infty - T_d}{R_p + R_f} \quad (2,3)$$

Combine with Eq. (1), and solve for T_d,

$$T_d = \frac{P_e + T_o/\left(R_p + R_d\right) + T_\infty/\left(R_p + R_f\right)}{1/\left(R_p + R_d\right) + 1/\left(R_p + R_f\right)} \quad (4)$$

where the thermal resistances with P = πD and $A_c = \pi D^2/4$ are

$$R_p = L_p / k_p A_c \qquad R_d = L_d / k_d A_c \qquad R_f = \left(hPk_f A_c\right)^{-1/2} \quad (5,6,7)$$

(c) Substituting numerical values with the foregoing relations, find

$$R_p = 1.061 \text{ K/W} \qquad R_d = 4.244 \text{ K/W} \qquad R_f = 5.712 \text{ K/W}$$

and the device temperature as

$$T_d = 62.4°C$$ <

COMMENTS: What fraction of the power dissipated in the device is removed by the fin? Answer: q_b/P_e = 47%.

PROBLEM 3.116

KNOWN: Dimensions and thermal conductivity of a gas turbine blade. Temperature and convection coefficient of gas stream. Temperature of blade base and maximum allowable blade temperature.

FIND: (a) Whether blade operating conditions are acceptable, (b) Heat transfer to blade coolant.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional, steady-state conduction in blade, (2) Constant k, (3) Adiabatic blade tip, (4) Negligible radiation.

ANALYSIS: Conditions in the blade are determined by Case B of Table 3.4.

(a) With the maximum temperature existing at x = L, Eq. 3.75 yields

$$\frac{T(L)-T_\infty}{T_b-T_\infty}=\frac{1}{\cosh mL}$$

$$m=\left(hP/kA_c\right)^{1/2}=\left(250\,W/m^2\cdot K\times 0.11\,m/20\,W/m\cdot K\times 6\times 10^{-4}\,m^2\right)^{1/2}$$

$m = 47.87\ m^{-1}$ and $mL = 47.87\ m^{-1} \times 0.05\ m = 2.39$

From Table B.1, cosh mL = 5.51. Hence,

$$T(L)=1200^\circ C+(300-1200)^\circ C/5.51=1037^\circ C$$ <

and the operating conditions are acceptable.

(b) With $M=\left(hPkA_c\right)^{1/2}\Theta_b=\left(250\,W/m^2\cdot K\times 0.11\,m\times 20\,W/m\cdot K\times 6\times 10^{-4}\,m^2\right)^{1/2}\left(-900^\circ C\right)=-517\,W$,

Eq. 3.76 and Table B.1 yield

$$q_f=M\tanh mL=-517\,W\left(0.983\right)=-508\,W$$

Hence, $q_b=-q_f=508\,W$ <

COMMENTS: Radiation losses from the blade surface and convection from the tip will contribute to reducing the blade temperatures.

PROBLEM 3.117

KNOWN: Dimensions of disc/shaft assembly. Applied angular velocity, force, and torque. Thermal conductivity and inner temperature of disc.

FIND: (a) Expression for the friction coefficient μ, (b) Radial temperature distribution in disc, (c) Value of μ for prescribed conditions.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional radial conduction, (3) Constant k, (4) Uniform disc contact pressure p, (5) All frictional heat dissipation is transferred to shaft from base of disc.

ANALYSIS: (a) The normal force acting on a differential ring extending from r to r+dr on the contact surface of the disc may be expressed as $dF_n = p2\pi r dr$. Hence, the tangential force is $dF_t = \mu p 2\pi r dr$, in which case the torque may be expressed as

$$d\tau = 2\pi\mu p r^2 dr$$

For the entire disc, it follows that

$$\tau = 2\pi\mu p \int_o^{r_2} r^2 dr = \frac{2\pi}{3}\mu p r_2^3$$

where $p = F/\pi r_2^2$. Hence,

$$\mu = \frac{3}{2}\frac{\tau}{F r_2}$$ <

(b) Performing an energy balance on a differential control volume in the disc, it follows that

$$q_{cond,r} + dq_{fric} - q_{cond,r+dr} = 0$$

With $dq_{fric} = \omega d\tau = 2\mu F\omega\left(r^2/r_2^2\right)dr$, $q_{cond,r+dr} = q_{cond,r} + \left(dq_{cond,r}/dr\right)dr$, and

$q_{cond,r} = -k(2\pi r t)dT/dr$, it follows that

$$2\mu F\omega\left(r^2/r_2^2\right)dr + 2\pi k t\frac{d(r dT/dr)}{dr}dr = 0$$

or

$$\frac{d(r dT/dr)}{dr} = -\frac{\mu F\omega}{\pi k t r_2^2}r^2$$

Integrating twice,

Continued...

PROBLEM 3.117 (Cont.)

$$\frac{dT}{dr} = -\frac{\mu F\omega}{3\pi k t r_2^2} r^2 + \frac{C_1}{r}$$

$$T = -\frac{\mu F\omega}{9\pi k t r_2^2} r^3 + C_1 \ell n r + C_2$$

Since the disc is well insulated at $r = r_2$, $dT/dr|_{r_2} = 0$ and

$$C_1 = \frac{\mu F\omega r_2}{3\pi k t}$$

With $T(r_1) = T_1$, it also follows that

$$C_2 = T_1 + \frac{\mu F\omega}{9\pi k t r_2^2} r_1^3 - C_1 \ell n r_1$$

Hence,

$$T(r) = T_1 - \frac{\mu F\omega}{9\pi k t r_2^2}\left(r^3 - r_1^3\right) + \frac{\mu F\omega r_2}{3\pi k t} \ell n \frac{r}{r_1}$$ <

(c) For the prescribed conditions,

$$\mu = \frac{3}{2} \frac{8 N \cdot m}{200 N (0.18 m)} = 0.333$$ <

Since the maximum temperature occurs at r = r_2,

$$T_{max} = T(r_2) = T_1 - \frac{\mu F\omega r_2}{9\pi k t}\left[1 - \left(\frac{r_1}{r_2}\right)^3\right] + \frac{\mu F\omega r_2}{3\pi k t} \ell n\left(\frac{r_2}{r_1}\right)$$

With $(\mu F\omega r_2 / 3\pi k t) = (0.333 \times 200 N \times 40 rad/s \times 0.18 m / 3\pi \times 15 W/m \cdot K \times 0.012 m) = 282.7^\circ C$,

$$T_{max} = 80^\circ C - \frac{282.7^\circ C}{3}\left[1 - \left(\frac{0.02}{0.18}\right)^3\right] + 282.7^\circ C \ell n\left(\frac{0.18}{0.02}\right)$$

$$T_{max} = 80^\circ C - 94.1^\circ C + 621.1^\circ C = 607^\circ C$$ <

COMMENTS: The maximum temperature is excessive, and the disks should be actively cooled (by convection) at their outer surfaces.

PROBLEM 3.118

KNOWN: Extended surface of rectangular cross-section with heat flow in the longitudinal direction.

FIND: Determine the conditions for which the transverse (y-direction) temperature gradient is negligible compared to the longitudinal gradient, such that the 1-D analysis of Section 3.6.1 is valid by finding: (a) An expression for the conduction heat flux at the surface, $q''_y(t)$, in terms of T_s and T_o, assuming the transverse temperature distribution is parabolic, (b) An expression for the convection heat flux at the surface for the x-location; equate the two expressions, and identify the parameter that determines the ratio $(T_o - T_s)/(T_s - T_\infty)$; and (c) Developing a criterion for establishing the validity of the 1-D assumption used to model an extended surface.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Uniform convection coefficient and (3) Constant properties.

ANALYSIS: (a) Referring to the schematics above, the conduction heat flux at the surface y = t at any x-location follows from Fourier's law using the parabolic transverse temperature distribution.

$$q''_y(t) = -k\left.\frac{\partial T}{\partial y}\right)_{y=t} = -k\left(\left[T_s(x) - T_o(x)\right]\frac{2y}{t^2}\right)_{y=t} = -\frac{2k}{t}\left[T_s(x) - T_o(x)\right] \qquad (1)$$

(b) The convection heat flux at the surface of any x-location follows from the rate equation

$$q''_{cv} = h\left[T_s(x) - T_\infty\right] \qquad (2)$$

Performing a surface energy balance as represented schematically above, equating Eqs. (1) and (2) provides

$$q''_y(t) = q''_{cv}$$

$$-\frac{2k}{t}\left[T_s(x) - T_o(x)\right] = h\left[T_s(x) - T_\infty\right]$$

$$\frac{T_s(x) - T_o(x)}{T_s(x) - T_\infty(x)} = -0.5\frac{ht}{k} = -0.5\ \mathrm{Bi} \qquad (3)$$

where Bi = ht/k, the Biot number, represents the ratio of the convection to the conduction thermal resistances,

$$\mathrm{Bi} = \frac{R''_{cd}}{R''_{cv}} = \frac{t/k}{1/h} \qquad (4)$$

(c) The transverse gradient (heat flow) will be negligible compared to the longitudinal gradient when Bi << 1, say, 0.1, an order of magnitude smaller. This is the criterion to validate the one-dimensional assumption used to model extended surfaces.

COMMENTS: The coefficient 0.5 in Eq. (3) is a consequence of the parabolic distribution assumption. This distribution represents the simplest polynomial expression that could approximate the real distribution.

PROBLEM 3.119

KNOWN: Long, aluminum cylinder acts as an extended surface.

FIND: (a) Increase in heat transfer if diameter is tripled and (b) Increase in heat transfer if copper is used in place of aluminum.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, (4) Uniform convection coefficient, (5) Rod is infinitely long.

PROPERTIES: *Table A-1,* Aluminum (pure): k = 240 W/m·K; *Table A-1,* Copper (pure): k = 400 W/m·K.

ANALYSIS: (a) For an infinitely long fin, the fin heat rate from Table 3.4 is

$$q_f = M = \left(hPkA_c\right)^{1/2}\theta_b$$

$$q_f = \left(h\,\pi\,D\,k\,\pi\,D^2/4\right)^{1/2}\theta_b = \frac{\pi}{2}\left(hk\right)^{1/2}D^{3/2}\;\theta_b.$$

where $P = \pi D$ and $A_c = \pi D^2/4$ for the circular cross-section. Note that $q_f \propto D^{3/2}$. Hence, if the diameter is tripled,

$$\frac{q_{f(3D)}}{q_{f(D)}} = 3^{3/2} = 5.2$$

and there is a 420% increase in heat transfer. <

(b) In changing from aluminum to copper, since $q_f \propto k^{1/2}$, it follows that

$$\frac{q_f\,(Cu)}{q_f\,(A1)} = \left[\frac{k_{Cu}}{k_{A1}}\right]^{1/2} = \left[\frac{400}{240}\right]^{1/2} = 1.29$$

and there is a 29% increase in the heat transfer rate. <

COMMENTS: (1) Because fin effectiveness is enhanced by maximizing $P/A_c = 4/D$, the use of a larger number of small diameter fins is preferred to a single large diameter fin.

(2) From the standpoint of cost and weight, aluminum is preferred over copper.

PROBLEM 3.120

KNOWN: Length, diameter, base temperature and environmental conditions associated with a brass rod.

FIND: Temperature at specified distances along the rod.

SCHEMATIC:

Air $h = 30 W/m^2 \cdot K$, $T_\infty = 20°C$

$T_b = 200°C$, x_1, x_2, x, L, D

$L = 0.1m$
$D = 0.005m$
$x_1 = 0.025m$
$x_2 = 0.05m$

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible radiation, (5) Uniform convection coefficient h.

PROPERTIES: *Table A-1*, Brass $\left(\bar{T} = 110°C\right)$: $k = 133\ W/m \cdot K$.

ANALYSIS: Evaluate first the fin parameter

$$m = \left[\frac{hP}{kA_c}\right]^{1/2} = \left[\frac{h\pi D}{k\pi D^2/4}\right]^{1/2} = \left[\frac{4h}{kD}\right]^{1/2} = \left[\frac{4 \times 30\ W/m^2 \cdot K}{133\ W/m \cdot K \times 0.005m}\right]^{1/2}$$

$$m = 13.43\ m^{-1}.$$

Hence, m L = (13.43)×0.1 = 1.34 and from the results of Example 3.8, it is advisable not to make the infinite rod approximation. Thus from Table 3.4, the temperature distribution has the form

$$\theta = \frac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL}\theta_b$$

Evaluating the hyperbolic functions, cosh mL = 2.04 and sinh mL = 1.78, and the parameter

$$\frac{h}{mk} = \frac{30\ W/m^2 \cdot K}{13.43 m^{-1}\ (133\ W/m \cdot K)} = 0.0168,$$

with θ_b = 180°C the temperature distribution has the form

$$\theta = \frac{\cosh m(L-x) + 0.0168 \sinh m(L-x)}{2.07}(180°C).$$

The temperatures at the prescribed location are tabulated below.

x(m)	cosh m(L-x)	sinh m(L-x)	θ	T(°C)	
x_1 = 0.025	1.55	1.19	136.5	156.5	<
x_2 = 0.05	1.24	0.725	108.9	128.9	<
L = 0.10	1.00	0.00	87.0	107.0	<

COMMENTS: If the rod were approximated as infinitely long: $T(x_1)$ = 148.7°C, $T(x_2)$ = 112.0°C, and T(L) = 67.0°C. The assumption would therefore result in significant underestimates of the rod temperature.

PROBLEM 3.121

KNOWN: Thickness, length, thermal conductivity, and base temperature of a rectangular fin. Fluid temperature and convection coefficient.

FIND: (a) Heat rate per unit width, efficiency, effectiveness, thermal resistance, and tip temperature for different tip conditions, (b) Effect of convection coefficient and thermal conductivity on the heat rate.

SCHEMATIC:

T_∞ = 25°C
h = 100 W/m²-K
Air
Aluminum alloy
k = 180 W/m-K
T_b = 100°C
L = 10 mm
t = 1 mm

ASSUMPTIONS: (1) Steady-state, (2) One-dimensional conduction along fin, (3) Constant properties, (4) Negligible radiation, (5) Uniform convection coefficient, (6) Fin width is much longer than thickness (w >> t).

ANALYSIS: (a) The fin heat transfer rate for Cases A, B and D are given by Eqs. (3.72), (3.76) and (3.80), where $M \approx (2\ hw^2tk)^{1/2}\ (T_b - T_\infty) = (2 \times 100\ W/m^2{\cdot}K \times 0.001m \times 180\ W/m{\cdot}K)^{1/2}\ (75°C)\ w = 450\ w\ W$, $m \approx (2h/kt)^{1/2} = (200\ W/m^2{\cdot}K/180\ W/m{\cdot}K \times 0.001m)^{1/2} = 33.3m^{-1}$, $mL \approx 33.3m^{-1} \times 0.010m = 0.333$, and $(h/mk) \approx (100\ W/m^2{\cdot}K/33.3m^{-1} \times 180\ W/m{\cdot}K) = 0.0167$. From Table B-1, it follows that sinh mL ≈ 0.340, cosh mL ≈ 1.057, and tanh mL ≈ 0.321. From knowledge of q_f, Eqs. (3.86), (3.81) and (3.83) yield

$$\eta_f \approx \frac{q'_f}{h(2L+t)\theta_b}, \quad \varepsilon_f \approx \frac{q'_f}{ht\,\theta_b}, \quad R'_{t,f} = \frac{\theta_b}{q'_f}$$

Case A: From Eq. (3.72), (3.86), (3.81), (3.83) and (3.70),

$$q'_f = \frac{M}{w}\frac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL} = 450\ W/m\ \frac{0.340 + 0.0167 \times 1.057}{1.057 + 0.0167 \times 0.340} = 151\ W/m \qquad <$$

$$\eta_f = \frac{151\ W/m}{100\ W/m^2 \cdot K(0.021m)75°C} = 0.96 \qquad <$$

$$\varepsilon_f = \frac{151\ W/m}{100\ W/m^2 \cdot K(0.001m)75°C} = 20.1, \quad R'_{t,f} = \frac{75°C}{151\ W/m} = 0.50\ m \cdot K/W \qquad <$$

$$T(L) = T_\infty + \frac{\theta_b}{\cosh mL + (h/mk)\sinh mL} = 25°C + \frac{75°C}{1.057 + (0.0167)0.340} = 95.6°C \qquad <$$

Case B: From Eqs. (3.76), (3.86), (3.81), (3.83) and (3.75)

$$q'_f = \frac{M}{w}\tanh mL = 450\ W/m(0.321) = 144\ W/m \qquad <$$

$$\eta_f = 0.92, \ \varepsilon_f = 19.2, \ R'_{t,f} = 0.52\ m \cdot K/W \qquad <$$

$$T(L) = T_\infty + \frac{\theta_b}{\cosh mL} = 25°C + \frac{75°C}{1.057} = 96.0°C \qquad <$$

Continued …..

Case D (L → ∞): From Eqs. (3.80), (3.86), (3.81), (3.83) and (3.79)

$$q_f' = \frac{M}{w} = 450\,\text{W/m} \qquad <$$

$$\eta_f = 0,\ \varepsilon_f = 60.0,\ R_{t,f}' = 0.167\,\text{m}\cdot\text{K/W},\ T(L) = T_\infty = 25°\text{C} \qquad <$$

(b) The effect of h on the heat rate is shown below for the aluminum and stainless steel fins.

Variation of qf' with h (k=180 W/m.K)

Heat rate, qf'(W/m) vs. Convection coefficient, h(W/m^2.K)

qfA'
qfB'
qfD'

Variation of qf' with h (k=15W/m.K)

Heat rate, qf'(W/m) vs. Convection coefficient, h(W/m^2.K)

qfA'
qfB'
qfD'

For both materials, there is little difference between the Case A and B results over the entire range of h. The difference (percentage) increases with decreasing h and increasing k, but even for the worst case condition (h = 10 W/m^2·K, k = 180 W/m·K), the heat rate for Case A (15.7 W/m) is only slightly larger than that for Case B (14.9 W/m). For aluminum, the heat rate is significantly over-predicted by the infinite fin approximation over the entire range of h. For stainless steel, it is over-predicted for small values of h, but results for all three cases are within 1% for h > 500 W/m^2·K.

COMMENTS: From the results of Part (a), we see there is a slight reduction in performance (smaller values of q_f', η_f and ε_f, as well as a larger value of $R_{t,f}'$) associated with insulating the tip.

Although $\eta_f = 0$ for the infinite fin, q_f' and ε_f are substantially larger than results for L = 10 mm, indicating that performance may be significantly improved by increasing L.

PROBLEM 3.122

KNOWN: Thickness, length, thermal conductivity, and base temperature of a rectangular fin. Fluid temperature and convection coefficient.

FIND: (a) Heat rate per unit width, efficiency, effectiveness, thermal resistance, and tip temperature for different tip conditions, (b) Effect of fin length and thermal conductivity on the heat rate.

SCHEMATIC:

$T_\infty = 25^oC$
$h = 100\ W/m^2\text{-}K$
Air
Aluminum alloy
$k = 180\ W/m\text{-}K$
$T_b = 100^oC$
L = 10 mm
t = 1 mm

ASSUMPTIONS: (1) Steady-state, (2) One-dimensional conduction along fin, (3) Constant properties, (4) Negligible radiation, (5) Uniform convection coefficient, (6) Fin width is much longer than thickness (w >> t).

ANALYSIS: (a) The fin heat transfer rate for Cases A, B and D are given by Eqs. (3.72), (3.76) and (3.80), where $M \approx (2\ hw^2tk)^{1/2}\ (T_b - T_\infty) = (2 \times 100\ W/m^2{\cdot}K \times 0.001m \times 180\ W/m{\cdot}K)^{1/2}\ (75°C)\ w = 450\ w\ W$, $m \approx (2h/kt)^{1/2} = (200\ W/m^2{\cdot}K/180\ W/m{\cdot}K \times 0.001m)^{1/2} = 33.3m^{-1}$, $mL \approx 33.3m^{-1} \times 0.010m = 0.333$, and $(h/mk) \approx (100\ W/m^2{\cdot}K/33.3m^{-1} \times 180\ W/m{\cdot}K) = 0.0167$. From Table B-1, it follows that sinh mL ≈ 0.340, cosh mL ≈ 1.057, and tanh mL ≈ 0.321. From knowledge of q_f, Eqs. (3.86), (3.81) and (3.83) yield

$$\eta_f \approx \frac{q_f'}{h(2L+t)\theta_b},\quad \varepsilon_f \approx \frac{q_f'}{ht\,\theta_b},\quad R_{t,f}' = \frac{\theta_b}{q_f'}$$

Case A: From Eq. (3.72), (3.86), (3.81), (3.83) and (3.70),

$$q_f' = \frac{M}{w}\frac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL} = 450\ W/m\,\frac{0.340 + 0.0167 \times 1.057}{1.057 + 0.0167 \times 0.340} = 151\ W/m$$ <

$$\eta_f = \frac{151\ W/m}{100\ W/m^2 \cdot K\,(0.021m)\,75°C} = 0.96$$ <

$$\varepsilon_f = \frac{151\ W/m}{100\ W/m^2 \cdot K\,(0.001m)\,75°C} = 20.1,\quad R_{t,f}' = \frac{75°C}{151\ W/m} = 0.50\ m \cdot K/W$$ <

$$T(L) = T_\infty + \frac{\theta_b}{\cosh mL + (h/mk)\sinh mL} = 25°C + \frac{75°C}{1.057 + (0.0167)0.340} = 95.6°C$$ <

Case B: From Eqs. (3.76), (3.86), (3.81), (3.83) and (3.75)

$$q_f' = \frac{M}{w}\tanh mL = 450\ W/m\,(0.321) = 144\ W/m$$ <

$$\eta_f = 0.92,\ \varepsilon_f = 19.2,\ R_{t,f}' = 0.52\ m \cdot K/W$$ <

$$T(L) = T_\infty + \frac{\theta_b}{\cosh mL} = 25°C + \frac{75°C}{1.057} = 96.0°C$$ <

Continued …..

PROBLEM 3.122 (Cont.)

Case D ($L \rightarrow \infty$): From Eqs. (3.80), (3.86), (3.81), (3.83) and (3.79)

$$q_f' = \frac{M}{w} = 450\,\text{W/m}$$ <

$$\eta_f = 0,\ \varepsilon_f = 60.0,\ R_{t,f}' = 0.167\,\text{m}\cdot\text{K/W},\ T(L) = T_\infty = 25°\text{C}$$ <

(b) The effect of L on the heat rate is shown below for the aluminum and stainless steel fins.

Variation of qf' with L (k=180 W/m .K)

Heat rate, qf'(W/m)

500
400
300
200
100
0

0 0.01 0.02 0.03 0.04 0.05

Fin length, L(m)

qfA'
qfB'
qfD'

Variation of qf' with L (k=15 W/m .K)

Heat rate, qf'(W/m)

150
120
90
60
30
0

0 0.01 0.02 0.03 0.04 0.05

Fin length, L(m)

qfA'
qfB'
qfD'

For both materials, differences between the Case A and B results diminish with increasing L and are within 1% of each other at $L \approx 27$ mm and $L \approx 13$ mm for the aluminum and steel, respectively. At $L = 3$ mm, results differ by 14% and 13% for the aluminum and steel, respectively. The Case A and B results approach those of the infinite fin approximation more quickly for stainless steel due to the larger temperature gradients, $|dT/dx|$, for the smaller value of k.

COMMENTS: From the results of Part (a), we see there is a slight reduction in performance (smaller values of q_f', η_f and ε_f, as well as a larger value of $R_{t,f}'$) associated with insulating the tip.

Although $\eta_f = 0$ for the infinite fin, q_f' and ε_f are substantially larger than results for $L = 10$ mm, indicating that performance may be significantly improved by increasing L.

PROBLEM 3.123

KNOWN: Length, thickness and temperature of straight fins of rectangular, triangular and parabolic profiles. Ambient air temperature and convection coefficient.

FIND: Heat rate per unit width, efficiency and volume of each fin.

SCHEMATIC:

Rectangular
$L_c = L + t/2 = 16.5$ mm
t = 3 mm
L = 15 mm
$T_b = 100°C$
Air
Triangular
$T_\infty = 20°C$
$h = 50\ W/m^2\text{-}K$
Parabolic
Aluminum alloy
k = 185 W/m-K

ASSUMPTIONS: (1) Steady-state, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible radiation, (5) Uniform convection coefficient.

ANALYSIS: For each fin,

$$q'_f = q'_{max} = \eta_f\, hA'_f\, \theta_b, \qquad V' = A_p$$

where η_f depends on the value of $m = (2h/kt)^{1/2} = (100\ W/m^2{\cdot}K/185\ W/m{\cdot}K \times 0.003m)^{1/2} = 13.4m^{-1}$ and the product $mL = 13.4m^{-1} \times 0.015m = 0.201$ or $mL_c = 0.222$. Expressions for η_f, A'_f and A_p are obtained from Table 3-5.

Rectangular Fin:

$$\eta_f = \frac{\tanh\ mL_c}{mL_c} = \frac{0.218}{0.222} = 0.982,\ \ A'_f = 2L_c = 0.033m \qquad <$$

$$q' = 0.982\left(50\ W/m^2 \cdot K\right)0.033m\left(80°C\right) = 129.6\ W/m,\ \ V' = tL = 4.5\times10^{-5}\ m^2 \qquad <$$

Triangular Fin:

$$\eta_f = \frac{1}{mL}\frac{I_1(2mL)}{I_0(2mL)} = \frac{0.205}{(0.201)1.042} = 0.978,\ \ A'_f = 2\left[L^2 + (t/2)^2\right]^{1/2} = 0.030m \qquad <$$

$$q' = 0.978\left(50\ W/m^2 \cdot K\right)0.030m\left(80°C\right) = 117.3\ W/m,\ \ V' = (t/2)L = 2.25\times10^{-5} m^2 \qquad <$$

Parabolic Fin:

$$\eta_f = \frac{2}{\left[4(mL)^2 + 1\right]^{1/2} + 1} = 0.963,\ \ A'_f = \left[C_1 L + \left(L^2/t\right)\ln\left(t/L + C_1\right)\right] = 0.030m \qquad <$$

$$q'_f = 0.963\left(50\ W/m^2 \cdot K\right)0.030m\left(80°C\right) = 115.6\ W/m,\ \ V' = (t/3)L = 1.5\times10^{-5} m^2 \qquad <$$

COMMENTS: Although the heat rate is slightly larger (~10%) for the rectangular fin than for the triangular or parabolic fins, the heat rate per unit volume (or mass) is larger and largest for the triangular and parabolic fins, respectively.

PROBLEM 3.124

KNOWN: Melting point of solder used to join two long copper rods.

FIND: Minimum power needed to solder the rods.

SCHEMATIC:

x ← | → x
Copper
→ ∞
∞ ←
D = 1cm
Junction
$T_b = 650°C$
Air
$T_\infty = 25°C$
$h = 10 W/m^2 \cdot K$

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction along the rods, (3) Constant properties, (4) No internal heat generation, (5) Negligible radiation exchange with surroundings, (6) Uniform h, and (7) Infinitely long rods.

PROPERTIES: *Table A-1:* Copper $\bar{T} = (650 + 25)^\circ C \approx 600K$: $k = 379$ W/m·K.

ANALYSIS: The junction must be maintained at 650°C while energy is transferred by conduction from the junction (along both rods). The minimum power is twice the fin heat rate for an infinitely long fin,

$$q_{min} = 2q_f = 2\left(hPkA_c\right)^{1/2}\left(T_b - T_\infty\right).$$

Substituting numerical values,

$$q_{min} = 2\left[10\frac{W}{m^2 \cdot K}(\pi \times 0.01m)\left[379\frac{W}{m \cdot K}\right]\frac{\pi}{4}(0.01m)^2\right]^{1/2}(650 - 25)^\circ C.$$

Therefore,

$$q_{min} = 120.9\ W.$$ <

COMMENTS: Radiation losses from the rods may be significant, particularly near the junction, thereby requiring a larger power input to maintain the junction at 650°C.

PROBLEM 3.125

KNOWN: Dimensions and end temperatures of pin fins.

FIND: (a) Heat transfer by convection from a single fin and (b) Total heat transfer from a 1 m^2 surface with fins mounted on 4mm centers.

SCHEMATIC:

$T(0)=100°C$, $T_\infty = 0°C$, $h = 100 W/m^2\cdot K$, Air, $T(L) = 0°C$, $q_{cond,i}$, $q_{cond,o}$, $D = 1mm$, q_{conv}, $L = 25mm$, x

ASSUMPTIONS: (1) Steady-state, (2) One-dimensional conduction along rod, (3) Constant properties, (4) No internal heat generation, (5) Negligible radiation.

PROPERTIES: *Table A-1*, Copper, pure (323K): k ≈ 400 W/m·K.

ANALYSIS: (a) By applying conservation of energy to the fin, it follows that

$$q_{conv} = q_{cond,i} - q_{cond,o}$$

where the conduction rates may be evaluated from knowledge of the temperature distribution. The general solution for the temperature distribution is

$$\theta(x) = C_1 e^{mx} + C_2 e^{-mx} \qquad \theta \equiv T - T_\infty .$$

The boundary conditions are $\theta(0) \equiv \theta_o = 100°C$ and $\theta(L) = 0$. Hence

$$\theta_o = C_1 + C_2$$

$$0 = C_1 e^{mL} + C_2 e^{-mL}$$

Therefore, $\quad C_2 = C_1 e^{2mL}$

$$C_1 = \frac{\theta_o}{1-e^{2mL}}, \qquad C_2 = -\frac{\theta_o e^{2mL}}{1-e^{2mL}}$$

and the temperature distribution has the form

$$\theta = \frac{\theta_o}{1-e^{2mL}}\left[e^{mx} - e^{2mL-mx}\right].$$

The conduction heat rate can be evaluated by Fourier's law,

$$q_{cond} = -kA_c \frac{d\theta}{dx} = -\frac{kA_c\theta_o}{1-e^{2mL}} m\left[e^{mx} + e^{2mL-mx}\right]$$

or, with $m = \left(hP/kA_c\right)^{1/2}$,

$$q_{cond} = -\frac{\theta_o \left(hPkA_c\right)^{1/2}}{1-e^{2mL}}\left[e^{mx} + e^{2mL-mx}\right].$$

Continued …..

Hence at x = 0,

$$q_{cond,i} = -\frac{\theta_o \left(hPkA_c\right)^{1/2}}{1-e^{2mL}}\left(1+e^{2mL}\right)$$

at x = L

$$q_{cond,o} = -\frac{\theta_o \left(hPkA_c\right)^{1/2}}{1-e^{2mL}}\left(2e^{mL}\right)$$

Evaluating the fin parameters:

$$m = \left[\frac{hP}{kA_c}\right]^{1/2} = \left[\frac{4h}{kD}\right]^{1/2} = \left[\frac{4\times100\ W/m^2\cdot K}{400\ W/m\cdot K\times0.001m}\right]^{1/2} = 31.62\ m^{-1}$$

$$\left(hPkA_c\right)^{1/2} = \left[\frac{\pi^2}{4}D^3hk\right]^{1/2} = \left[\frac{\pi^2}{4}\times(0.001m)^3\times100\frac{W}{m^2\cdot K}\times400\frac{W}{m\cdot K}\right]^{1/2} = 9.93\times10^{-3}\frac{W}{K}$$

$$mL = 31.62\ m^{-1}\times0.025m = 0.791, \qquad e^{mL} = 2.204, \qquad e^{2mL} = 4.865$$

The conduction heat rates are

$$q_{cond,i} = \frac{-100K\left(9.93\times10^{-3}\ W/K\right)}{-3.865}\times5.865 = 1.507\ W$$

$$q_{cond,o} = \frac{-100K\left(9.93\times10^{-3}\ W/K\right)}{-3.865}\times4.408 = 1.133\ W$$

and from the conservation relation,

$$q_{conv} = 1.507\ W - 1.133\ W = 0.374\ W.$$ <

(b) The total heat transfer rate is the heat transfer from N = 250×250 = 62,500 rods and the heat transfer from the remaining (bare) surface ($A = 1m^2 - NA_c$). Hence,

$$q = N\,q_{cond,i} + hA\theta_o = 62,500\ (1.507\ W) + 100W/m^2\cdot K\left(0.951\ m^2\right)\ 100K$$

$$q = 9.42\times10^4\ W + 0.95\times10^4\ W = 1.037\times10^5\ W.$$

COMMENTS: (1) The fins, which cover only 5% of the surface area, provide for more than 90% of the heat transfer from the surface.

(2) The fin effectiveness, $\varepsilon \equiv q_{cond,i}/hA_c\theta_o$, is ε = 192, and the fin efficiency, $\eta \equiv \left(q_{conv}/h\pi\ DL\theta_o\right)$, is $\eta = 0.48$.

(3) The temperature distribution, $\theta(x)/\theta_o$, and the conduction term, $q_{cond,i}$, could have been obtained directly from Eqs. 3.77and 3.78, respectively.

(4) Heat transfer by convection from a single fin could also have been obtained from Eq. 3.73.

PROBLEM 3.126

KNOWN: Pin fin of thermal conductivity k, length L and diameter D connecting two devices (L_g,k_g) experiencing volumetric generation of thermal energy $(\dot{q})$. Convection conditions are prescribed (T_∞, h).

FIND: Expression for the device surface temperature T_b in terms of device, convection and fin parameters.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Pin fin is of uniform cross-section with constant h, (3) Exposed surface of device is at a uniform temperature T_b, (4) Backside of device is insulated, (5) Device experiences 1-D heat conduction with uniform volumetric generation, (6) Constant properties, and (7) No contact resistance between fin and devices.

ANALYSIS: Recognizing symmetry, the pin fin is modeled as a fin of length L/2 with insulated tip. Perform a surface energy balance,

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$q_d - q_s - q_f = 0 \qquad (1)$$

The heat rate q_d can be found from an energy balance on the entire device to find

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = 0$$

$$-q_d + \dot{q}V = 0$$

$$q_d = \dot{q}A_g L_g \qquad (2)$$

The fin heat rate, q_f, follows from Case B, Table 3.4

$$q_f = M \tanh mL/2 = \left(hPkA_c\right)^{1/2}\left(T_b - T_\infty\right)\tanh\left(mL/2\right), \qquad m = \left(hP/kA_c\right)^{1/2} \qquad (3,4)$$

$$P/A_c = \pi D/\left(\pi D^2/4\right) = 4/D \quad \text{and} \quad PA_c = \pi^2 D^3/4. \qquad (5,6)$$

Hence, the heat rate expression can be written as

$$\dot{q}A_g L_g = h\left(A_g - A_c\right)\left(T_b - T_\infty\right) + \left(hk\left(\pi^2 D^3/4\right)\right)^{1/2}\tanh\left(\left(\frac{4h}{kD}\right)^{1/2}\cdot\frac{L}{2}\right)\left(T_b - T_\infty\right) \qquad (7)$$

Solve now for T_b,

$$T_b = T_\infty + \dot{q}A_g L_g \Big/ \left[h\left(A_g - A_c\right) + \left(hk\left(\pi^2 D^3/4\right)\right)^{1/2}\tanh\left(\left(\frac{4h}{kD}\right)^{1/2}\cdot\frac{L}{2}\right)\right] \qquad (8)$$ <

PROBLEM 3.127

KNOWN: Positions of equal temperature on two long rods of the same diameter, but different thermal conductivity, which are exposed to the same base temperature and ambient air conditions.

FIND: Thermal conductivity of rod B, k_B.

SCHEMATIC:

$x_A = 0.15m$ Rod A, $k_A = 70 W/m \cdot K$

$T_o = 100°C$ $T_A = T_B$ $T_\infty = 20°C$

$x_B = 0.075m$ Rod B, k_B

ASSUMPTIONS: (1) Steady-state conditions, (2) Rods are infinitely long fins of uniform cross-sectional area, (3) Uniform heat transfer coefficient, (4) Constant properties.

ANALYSIS: The temperature distribution for the infinite fin has the form

$$\frac{\theta}{\theta_b} = \frac{T(x) - T_\infty}{T_o - T_\infty} = e^{-mx} \qquad m = \left[\frac{hP}{kA_c}\right]^{1/2}. \qquad (1,2)$$

For the two positions prescribed, x_A and x_B, it was observed that

$$T_A(x_A) = T_B(x_B) \qquad \text{or} \qquad \theta_A(x_A) = \theta_B(x_B). \qquad (3)$$

Since θ_b is identical for both rods, Eq. (1) with the equality of Eq. (3) requires that

$$m_A x_A = m_B x_B$$

Substituting for m from Eq. (2) gives

$$\left[\frac{hP}{k_A A_c}\right]^{1/2} x_A = \left[\frac{hP}{k_B A_c}\right]^{1/2} x_B.$$

Recognizing that h, P and A_c are identical for each rod and rearranging,

$$k_B = \left[\frac{x_B}{x_A}\right]^2 k_A$$

$$k_B = \left[\frac{0.075m}{0.15m}\right]^2 \times 70 \text{ W/m} \cdot \text{K} = 17.5 \text{ W/m} \cdot \text{K}.$$

<

COMMENTS: This approach has been used as a method for determining the thermal conductivity. It has the attractive feature of not requiring power or temperature measurements, assuming of course, a reference material of known thermal conductivity is available.

PROBLEM 3.128

KNOWN: Slender rod of length L with ends maintained at T_o while exposed to convection cooling ($T_\infty < T_o$, h).

FIND: Temperature distribution for three cases, when rod has thermal conductivity (a) k_A, (b) $k_B < k_A$, and (c) k_A for $0 \le x \le L/2$ and k_B for $L/2 \le x \le L$.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, and (4) Negligible thermal resistance between the two materials (A, B) at the mid-span for case (c).

ANALYSIS: (a, b) The effect of thermal conductivity on the temperature distribution when all other conditions (T_o, h, L) remain the same is to reduce the minimum temperature with decreasing thermal conductivity. Hence, as shown in the sketch, the mid-span temperatures are $T_B(0.5L) < T_A(0.5L)$ for $k_B < k_A$. The temperature distribution is, of course, symmetrical about the mid-span.

(c) For the composite rod, the temperature distribution can be reasoned by considering the boundary condition at the mid-span.

$$q''_{x,A}(0.5L) = q''_{x,B}(0.5L)$$

$$-k_A \left.\frac{dT}{dx}\right)_{A,x=0.5L} = -k_B \left.\frac{dT}{dx}\right)_{B,x=0.5L}$$

Since $k_A > k_B$, it follows that

$$\left(\frac{dT}{dx}\right)_{A,x=0.5L} < \left(\frac{dT}{dx}\right)_{B,x=0.5L}.$$

It follows that the minimum temperature in the rod must be in the k_B region, $x > 0.5L$, and the temperature distribution is not symmetrical about the mid-span.

COMMENTS: (1) Recognize that the area under the curve on the T-x coordinates is proportional to the fin heat rate. What conclusions can you draw regarding the relative magnitudes of q_{fin} for cases (a), (b) and (c)?

(2) If L is increased substantially, how would the temperature distribution be affected?

PROBLEM 3.129

KNOWN: Base temperature, ambient fluid conditions, and temperatures at a prescribed distance from the base for two long rods, with one of known thermal conductivity.

FIND: Thermal conductivity of other rod.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state, (2) One-dimensional conduction along rods, (3) Constant properties, (4) Negligible radiation, (5) Negligible contact resistance at base, (6) Infinitely long rods, (7) Rods are identical except for their thermal conductivity.

ANALYSIS: With the assumption of infinitely long rods, the temperature distribution is

$$\frac{\theta}{\theta_b} = \frac{T - T_\infty}{T_b - T_\infty} = e^{-mx}$$

or

$$\ln \frac{T - T_\infty}{T_b - T_\infty} = -mx = \left[\frac{hP}{kA}\right]^{1/2} x$$

Hence, for the two rods,

$$\frac{\ln\left[\dfrac{T_A - T_\infty}{T_b - T_\infty}\right]}{\ln\left[\dfrac{T_B - T_\infty}{T_b - T_\infty}\right]} = \left[\frac{k_B}{k_A}\right]^{1/2}$$

$$k_B^{1/2} = k_A^{1/2} \frac{\ln\left[\dfrac{T_A - T_\infty}{T_b - T_\infty}\right]}{\ln\left[\dfrac{T_B - T_\infty}{T_b - T_\infty}\right]} = (200)^{1/2} \frac{\ln \dfrac{75-25}{100-25}}{\ln \dfrac{60-25}{100-25}} = 7.524$$

$$k_B = 56.6 \text{ W/m} \cdot \text{K}.$$ <

COMMENTS: Providing conditions for the two rods may be maintained nearly identical, the above method provides a convenient means of measuring the thermal conductivity of solids.

PROBLEM 3.130

KNOWN: Arrangement of fins between parallel plates. Temperature and convection coefficient of air flow in finned passages. Maximum allowable plate temperatures.

FIND: (a) Expressions relating fin heat transfer rates to end temperatures, (b) Maximum power dissipation for each plate.

SCHEMATIC:

Plate dimensions:
$D=100\,mm \times W=200\,mm \times t=1\,mm$; $N_f=50$

$T_o=400K$, $L=12\,mm$, $q_{f,o}$, $q_{f,L}$, x, $t=1\,mm$, $T_\infty=300K$, $h=150\,W/m^2\cdot K$, $T_L=350K$

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in fins, (3) Constant properties, (4) Negligible radiation, (5) All of the heat is dissipated to the air, (6) Uniform h, (7) Negligible variation in T_∞, (8) Negligible contact resistance.

PROPERTIES: *Table A.1,* Aluminum (pure), 375 K: k = 240 W/m·K.

ANALYSIS: (a) The general solution for the temperature distribution in fin is

$$\theta(x) \equiv T(x) - T_\infty = C_1 e^{mx} + C_2 e^{-mx}$$

Boundary conditions: $\theta(0) = \theta_o = T_o - T_\infty$, $\theta(L) = \theta_L = T_L - T_\infty$.

Hence $\theta_o = C_1 + C_2$ $\theta_L = C_1 e^{mL} + C_2 e^{-mL}$

$$\theta_L = C_1 e^{mL} + (\theta_o - C_1) e^{-mL}$$

$$C_1 = \frac{\theta_L - \theta_o e^{-mL}}{e^{mL} - e^{-mL}} \qquad C_2 = \theta_o - \frac{\theta_L - \theta_o e^{-mL}}{e^{mL} - e^{-mL}} = \frac{\theta_o e^{mL} - \theta_L}{e^{mL} - e^{-mL}}.$$

Hence
$$\theta(x) = \frac{\theta_L e^{mx} - \theta_o e^{m(x-L)} + \theta_o e^{m(L-x)} - \theta_L e^{-mx}}{e^{mL} - e^{-mL}}$$

$$\theta(x) = \frac{\theta_o\left[e^{m(L-x)} - e^{-m(L-x)}\right] + \theta_L\left(e^{mx} - e^{-mx}\right)}{e^{mL} - e^{-mL}}$$

$$\theta(x) = \frac{\theta_o \sinh m(L-x) + \theta_L \sinh mx}{\sinh mL}.$$

The fin heat transfer rate is then

$$q_f = -kA_c\frac{dT}{dx} = -kDt\left[-\frac{\theta_o m}{\sinh mL}\cosh m(L-x) + \frac{\theta_L m}{\sinh mL}\cosh mx\right].$$

Hence
$$q_{f,o} = kDt\left(\frac{\theta_o m}{\tanh mL} - \frac{\theta_L m}{\sinh mL}\right)$$ <

$$q_{f,L} = kDt\left(\frac{\theta_o m}{\sinh mL} - \frac{\theta_L m}{\tanh mL}\right).$$ <

Continued …..

(b) $$m = \left(\frac{hP}{kA_c}\right)^{1/2} = \left(\frac{50\ W/m^2 \cdot K\left(2\times0.1\ m + 2\times0.001\ m\right)}{240\ W/m\cdot K\times0.1\ m\times0.001\ m}\right)^{1/2} = 35.5\ m^{-1}$$

$$mL = 35.5\ m^{-1}\times0.012\ m = 0.43$$

$$\sinh\ mL = 0.439 \qquad \tanh\ mL = 0.401 \qquad \theta_o = 100\ K \qquad \theta_L = 50\ K$$

$$q_{f,o} = 240\ W/m\cdot K\times0.1\ m\times0.001\ m\left(\frac{100\ K\times35.5\ m^{-1}}{0.401} - \frac{50\ K\times35.5\ m^{-1}}{0.439}\right)$$

$$q_{f,o} = 115.4\ W \qquad (\textit{from}\ \text{the top plate})$$

$$q_{f,L} = 240\ W/m\cdot K\times0.1\ m\times0.001\ m\left(\frac{100\ K\times35.5\ m^{-1}}{0.439} - \frac{50\ K\times35.5\ m^{-1}}{0.401}\right)$$

$$q_{f,L} = 87.8\ W. \qquad (\textit{into}\ \text{the bottom plate})$$

Maximum power dissipations are therefore

$$q_{o,max} = N_f q_{f,o} + \left(W - N_f t\right)Dh\theta_o$$

$$q_{o,max} = 50\times115.4\ W + \left(0.200 - 50\times0.001\right)m\times0.1\ m\times150\ W/m^2\cdot K\times100\ K$$

$$q_{o,max} = 5770\ W + 225\ W = 5995\ W$$ <

$$q_{L,max} = -N_f q_{f,L} + \left(W - N_f t\right)Dh\theta_o$$

$$q_{L,max} = -50\times87.8W + \left(0.200 - 50\times0.001\right)m\times0.1\ m\times150\ W/m^2\cdot K\times50\ K$$

$$q_{L,max} = -4390\ W + 112W = -4278\ W.$$ <

COMMENTS: (1) It is of interest to determine the air velocity needed to prevent excessive heating of the air as it passes between the plates. If the air temperature change is restricted to $\Delta T_\infty = 5$ K, its flowrate must be

$$\dot{m}_{air} = \frac{q_{tot}}{c_p\Delta T_\infty} = \frac{1717\ W}{1007\ J/kg\cdot K\times5\ K} = 0.34\ kg/s.$$

Its mean velocity is then

$$V_{air} = \frac{\dot{m}_{air}}{\rho_{air}A_c} = \frac{0.34\ kg/s}{1.16\ kg/m^3\times0.012\ m\left(0.2 - 50\times0.001\right)m} = 163\ m/s.$$

Such a velocity would be impossible to maintain. To reduce it to a reasonable value, e.g. 10 m/s, A_c would have to be increased substantially by increasing W (and hence the space between fins) and by increasing L. The present configuration is impractical from the standpoint that 1717 W could not be transferred to air in such a small volume.

(2) A negative value of $q_{L,max}$ implies that heat must be transferred from the bottom plate to the air to maintain the plate at 350 K.

PROBLEM 3.131

KNOWN: Conditions associated with an array of straight rectangular fins.

FIND: Thermal resistance of the array.

SCHEMATIC:

ASSUMPTIONS: (1) Constant properties, (2) Uniform convection coefficient, (3) Symmetry about midplane.

ANALYSIS: (a) Considering a one-half section of the array, the corresponding resistance is

$$R_{t,o} = (\eta_o h A_t)^{-1}$$

where $A_t = NA_f + A_b$. With S = 4 mm and t = 1 mm, it follows that N = W_1/S = 250, A_f = $2(L/2)W_2 = 0.008\ m^2$, $A_b = W_2(W_1 - Nt) = 0.75\ m^2$, and $A_t = 2.75\ m^2$.
The overall surface efficiency is

$$\eta_o = 1 - \frac{NA_f}{A_t}(1 - \eta_f)$$

where the fin efficiency is

$$\eta_f = \frac{\tanh m(L/2)}{m(L/2)} \qquad \text{and} \qquad m = \left(\frac{hP}{kA_c}\right)^{1/2} = \left[\frac{h(2t + 2W_2)}{ktW_2}\right]^{1/2} \approx \left(\frac{2h}{kt}\right)^{1/2} = 38.7\text{m}^{-1}$$

With m(L/2) = 0.155, it follows that η_f = 0.992 and η_o = 0.994. Hence

$$R_{t,o} = \left(0.994 \times 150\text{W/m}^2 \cdot \text{K} \times 2.75\text{m}^2\right)^{-1} = 2.44 \times 10^{-3}\text{K/W}$$ <

(b) The requirements that t ≥ 0.5 m and (S - t) > 2 mm are based on manufacturing and flow passage restriction constraints. Repeating the foregoing calculations for representative values of t and (S - t), we obtain

S (mm)	N	t (mm)	$R_{t,o}$ (K/W)
2.5	400	0.5	0.00169
3	333	0.5	0.00193
3	333	1	0.00202
4	250	0.5	0.00234
4	250	2	0.00268
5	200	0.5	0.00264
5	200	3	0.00334

COMMENTS: Clearly, the thermal performance of the fin array improves ($R_{t,o}$ decreases) with increasing N. Because $\eta_f \approx 1$ for the entire range of conditions, there is a slight degradation in performance ($R_{t,o}$ increases) with increasing t and fixed N. The reduced performance is associated with the reduction in surface area of the exposed base. Note that the overall thermal resistance for the entire fin array (top and bottom) is $R_{t,o}/2 = 1.22 \times 10^{-2}$ K/W.

PROBLEM 3.132

KNOWN: Width and maximum allowable temperature of an electronic chip. Thermal contact resistance between chip and heat sink. Dimensions and thermal conductivity of heat sink. Temperature and convection coefficient associated with air flow through the heat sink.

FIND: (a) Maximum allowable chip power for heat sink with prescribed number of fins, fin thickness, and fin pitch, and (b) Effect of fin thickness/number and convection coefficient on performance.

SCHEMATIC:

$T_c = 85^oC$

$R''_{t,c} = 2\times10^{-6}$ m^2-K/W

k = 180 W/m-K

W = 20 mm

L_b = 3 mm

L_f = 15 mm

t

S

δ = 1.8 mm

Air

$T_\infty = 20^oC$

h = 100 W/m^2-K

q_c T_c $R_{t,c}$ $R_{t,b}$ $R_{t,o}$ T_∞

ASSUMPTIONS: (1) Steady-state, (2) One-dimensional heat transfer, (3) Isothermal chip, (4) Negligible heat transfer from top surface of chip, (5) Negligible temperature rise for air flow, (6) Uniform convection coefficient associated with air flow through channels and over outer surfaces of heat sink, (7) Negligible radiation.

ANALYSIS: (a) From the thermal circuit,

$$q_c = \frac{T_c - T_\infty}{R_{tot}} = \frac{T_c - T_\infty}{R_{t,c} + R_{t,b} + R_{t,o}}$$

where $R_{t,c} = R''_{t,c} / W^2 = 2\times10^{-6} m^2 \cdot K/W / (0.02m)^2 = 0.005\ K/W$ and $R_{t,b} = L_b / k\left(W^2\right)$

$= 0.003m / 180\ W/m\cdot K\,(0.02m)^2 = 0.042\ K/W$. From Eqs. (3.103), (3.102), and (3.99)

$$R_{t,o} = \frac{1}{\eta_o h A_t}, \qquad \eta_o = 1 - \frac{N A_f}{A_t}(1 - \eta_f), \qquad A_t = N A_f + A_b$$

where $A_f = 2WL_f = 2 \times 0.02m \times 0.015m = 6 \times 10^{-4}\ m^2$ and $A_b = W^2 - N(tW) = (0.02m)^2 - 11(0.182 \times 10^{-3}\ m \times 0.02m) = 3.6 \times 10^{-4}\ m^2$. With $mL_f = (2h/kt)^{1/2} L_f = (200\ W/m^2\cdot K/180\ W/m\cdot K \times 0.182 \times 10^{-3} m)^{1/2}\ (0.015m) = 1.17$, $\tanh mL_f = 0.824$ and Eq. (3.87) yields

$$\eta_f = \frac{\tanh\ mL_f}{mL_f} = \frac{0.824}{1.17} = 0.704$$

It follows that $A_t = 6.96 \times 10^{-3}\ m^2$, $\eta_o = 0.719$, $R_{t,o} = 2.00$ K/W, and

$$q_c = \frac{(85-20)°C}{(0.005 + 0.042 + 2.00)K/W} = 31.8\ W \qquad <$$

(b) The following results are obtained from parametric calculations performed to explore the effect of decreasing the number of fins and increasing the fin thickness.

Continued …..

PROBLEM 3.132 (Cont.)

N	t(mm)	η_f	$R_{t,o}$ (K/W)	q_c (W)	A_t (m^2)
6	1.833	0.957	2.76	23.2	0.00378
7	1.314	0.941	2.40	26.6	0.00442
8	0.925	0.919	2.15	29.7	0.00505
9	0.622	0.885	1.97	32.2	0.00569
10	0.380	0.826	1.89	33.5	0.00632
11	0.182	0.704	2.00	31.8	0.00696

Although η_f (and η_o) increases with decreasing N (increasing t), there is a reduction in A_t which yields a minimum in $R_{t,o}$, and hence a maximum value of q_c, for N = 10. For N = 11, the effect of h on the performance of the heat sink is shown below.

With increasing h from 100 to 1000 $W/m^2 \cdot K$, $R_{t,o}$ decreases from 2.00 to 0.47 K/W, despite a decrease in η_f (and η_o) from 0.704 (0.719) to 0.269 (0.309). The corresponding increase in q_c is significant.

COMMENTS: The heat sink significantly increases the allowable heat dissipation. If it were not used and heat was simply transferred by convection from the surface of the chip with h = 100 $W/m^2 \cdot K$, R_{tot} = 2.05 K/W from Part (a) would be replaced by $R_{cnv} = 1/hW^2$ = 25 K/W, yielding q_c = 2.60 W.

PROBLEM 3.133

KNOWN: Number and maximum allowable temperature of power transistors. Contact resistance between transistors and heat sink. Dimensions and thermal conductivity of heat sink. Temperature and convection coefficient associated with air flow through and along the sides of the heat sink.

FIND: (a) Maximum allowable power dissipation per transistor, (b) Effect of the convection coefficient and fin length on the transistor power.

SCHEMATIC:

$T_t = 100^oC$, $R_{t,c} = 0.045$ K/W, $N_t = 9$, $L_b = 6$ mm, $L_f = 30$ mm, t = 3 mm, k = 180 W/m-K, $N_f = 25$, W = 150 mm, Air, $T_\infty = 27^oC$, h = 100 W/m²-K

Thermal circuit: T_c — $(R_{t,c})_{equiv}$ — $R_{t,b}$ — $R_{t,o}$ — T_∞, $N_t q_t$

ASSUMPTIONS: (1) Steady-state, (2) One-dimensional heat transfer, (3) Isothermal transistors, (4) Negligible heat transfer from top surface of heat sink (all heat transfer is through the heat sink), (5) Negligible temperature rise for the air flow, (6) Uniform convection coefficient, (7) Negligible radiation.

ANALYSIS: (a) From the thermal circuit,

$$N_t\, q_t = \frac{T_t - T_\infty}{\left(R_{t,c}\right)_{equiv} + R_{t,b} + R_{t,o}}$$

For the array of transistors, the corresponding contact resistance is the equivalent resistance associated with the component resistances, in which case,

$$\left(R_{t,c}\right)_{equiv} = \left[N_t\left(1/R_{t,c}\right)\right]^{-1} = \left(9/0.045\,K/W\right)^{-1} = 5\times10^{-3}\ K/W$$

The thermal resistance associated with the base of the heat sink is

$$R_{t,b} = \frac{L_b}{k(W)^2} = \frac{0.006m}{180\ W/m\cdot K\left(0.150m\right)^2} = 1.48\times10^{-3}\ K/W$$

From Eqs. (3.103), (3.102) and (3.99), the thermal resistance associated with the fin array and the corresponding overall efficiency and total surface area are

$$R_{t,o} = \frac{1}{\eta_o\, h\, A_t}, \qquad \eta_o = 1 - \frac{N_f A_f}{A_t}\left(1-\eta_f\right), \qquad A_t = N_f A_f + A_b$$

Each fin has a surface area of $A_f \approx 2\ W\ L_f = 2 \times 0.15m \times 0.03m = 9 \times 10^{-3}\ m^2$, and the area of the exposed base is $A_b = W^2 - N_f\,(tW) = (0.15m)^2 - 25\,(0.003m \times 0.15m) = 1.13 \times 10^{-2}\ m^2$. With $mL_f = (2h/kt)^{1/2}\ L_f = (200\ W/m^2{\cdot}K/180\ W/m{\cdot}K \times 0.003m)^{1/2}\ (0.03m) = 0.577$, $\tanh mL_f = 0.520$ and Eq. (3.87) yields

$$\eta_f = \frac{\tanh\ mL_f}{mL_f} = \frac{0.520}{0.577} = 0.902$$

Hence, with $A_t = [25\ (9 \times 10^{-3}) + 1.13 \times 10^{-2}]m^2 = 0.236m^2$,

Continued …..

$$\eta_o = 1 - \frac{25\left(0.009\text{m}^2\right)}{0.236\text{m}^2}\left(1-0.901\right) = 0.907$$

$$R_{t,o} = \left(0.907 \times 100\ \text{W/m}^2 \cdot \text{K} \times 0.236\text{m}^2\right)^{-1} = 0.0467\ \text{K/W}$$

The heat rate per transistor is then

$$q_t = \frac{1}{9}\,\frac{\left(100-27\right)^\circ\text{C}}{\left(0.0050+0.0015+0.0467\right)\text{K/W}} = 152\ \text{W}$$ <

(b) As shown below, the transistor power dissipation may be enhanced by increasing h and/or L_f.

However, in each case, the effect of the increase diminishes due to an attendant reduction in η_f. For example, as h increases from 100 to 1000 $\text{W/m}^2\cdot\text{K}$ for L_f = 30 mm, η_f decreases from 0.902 to 0.498.

COMMENTS: The heat sink significantly increases the allowable transistor power. If it were not used and heat was simply transferred from a surface of area W^2 = 0.0225 m^2 with h = 100 $\text{W/m}^2\cdot\text{K}$, the corresponding thermal resistance would be $R_{t,cnv} = (hW^2)^{-1}$ K/W = 0.44 and the transistor power would be $q_t = (T_t - T_\infty)/N_t\, R_{t,cnv}$ = 18.4 W.

PROBLEM 3.134

KNOWN: Geometry and cooling arrangement for a chip-circuit board arrangement. Maximum chip temperature.

FIND: (a) Equivalent thermal circuit, (b) Maximum chip heat rate.

SCHEMATIC:

Pins (16), D_p=1.5mm, L_p=15mm
$T_{\infty,o}$=20°C
h_o=1000W/m²·K
A_b
T_c=75°C, q_c, A_c=(0.0127m)²
A_b=A_c−16$A_{c,f}$
L_b=0.005m
k_b=1W/m·K
$T_{\infty,i}$=20°C
h_i=40W/m²·K
$R''_{t,c}$=10⁻⁴m²·K/W

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat transfer in chip-board assembly, (3) Negligible pin-chip contact resistance, (4) Constant properties, (5) Negligible chip thermal resistance, (6) Uniform chip temperature.

PROPERTIES: *Table A.1,* Copper (300 K): k ≈ 400 W/m·K.

ANALYSIS: (a) The thermal circuit is

$(h_oA_b)^{-1}$
q_i $T_{\infty,i}$ $1/h_iA_c$ L_b/k_bA_c $R''_{t,c}/A_c$ T_c q_c q_b q_f R_f $T_{\infty,o}$

$$R_f = \frac{\theta_b}{16q_f} = \frac{\cosh mL + (h_o/mk)\sinh mL}{16\left(h_oPkA_{c,f}\right)^{1/2}\left[\sinh mL + (h_o/mk)\cosh mL\right]}$$

(b) The maximum chip heat rate is

$$q_c = 16q_f + q_b + q_i.$$

Evaluate these parameters

$$m = \left(\frac{h_oP}{kA_{c,f}}\right)^{1/2} = \left(\frac{4h_o}{kD_p}\right)^{1/2} = \left(\frac{4\times1000\ \text{W/m}^2\cdot\text{K}}{400\ \text{W/m}\cdot\text{K}\times0.0015\ \text{m}}\right)^{1/2} = 81.7\ \text{m}^{-1}$$

$$mL = \left(81.7\ \text{m}^{-1}\times0.015\ \text{m}\right) = 1.23,\quad \sinh mL = 1.57,\quad \cosh mL = 1.86$$

$$(h/mk) = \frac{1000\ \text{W/m}^2\cdot\text{K}}{81.7\ \text{m}^{-1}\times400\ \text{W/m}\cdot\text{K}} = 0.0306$$

$$M = \left(h_o\pi\ D_pk\pi\ D_p^2/4\right)^{1/2}\theta_b$$

$$M = \left[1000\ \text{W/m}^2\cdot\text{K}\left(\pi^2/4\right)(0.0015\ \text{m})^3\ 400\ \text{W/m}\cdot\text{K}\right]^{1/2}\left(55^\circ\text{C}\right) = 3.17\ \text{W}.$$

Continued …..

The fin heat rate is

$$q_f = M\frac{\sinh mL+(h/mk)\cosh mL}{\cosh mL+(h/mk)\sinh mL} = 3.17\ \text{W}\,\frac{1.57+0.0306\times1.86}{1.86+0.0306\times1.57}$$

$$q_f = 2.703\ \text{W}.$$

The heat rate from the board by convection is

$$q_b = h_o A_b \theta_b = 1000\ \text{W/m}^2\cdot\text{K}\left[(0.0127\ \text{m})^2 - (16\pi/4)(0.0015\ \text{m})^2\right]55^\circ\text{C}$$

$$q_b = 7.32\ \text{W}.$$

The convection heat rate is

$$q_i = \frac{T_c - T_{\infty,i}}{\left(1/h_i + R''_{t,c} + L_b/k_b\right)(1/A_c)} = \frac{(0.0127\ \text{m})^2(55^\circ\text{C})}{\left(1/40+10^{-4}+0.005/1\right)\text{m}^2\cdot\text{K/W}}$$

$$q_i = 0.29\ \text{W}.$$

Hence, the maximum chip heat rate is

$$q_c = [16(2.703)+7.32+0.29]\,\text{W} = [43.25+7.32+0.29]\,\text{W}$$

$$q_c = 50.9\ \text{W}.$$ <

COMMENTS: (1) The fins are extremely effective in enhancing heat transfer from the chip (assuming negligible contact resistance). Their effectiveness is $\varepsilon = q_f/\left(\pi D_p^2/4\right)h_o\theta_b = 2.703$ W/0.097 W = 27.8

(2) Without the fins, q_c = 1000 W/m²·K(0.0127 m)²55°C + 0.29 W = 9.16 W. Hence the fins provide for a (50.9 W/9.16 W) × 100% = 555% enhancement of heat transfer.

(3) With the fins, the chip heat flux is 50.9 W/(0.0127 m)^2 or $q''_c = 3.16\times10^5\ \text{W/m}^2 = 31.6$ W/cm^2.

(4) If the infinite fin approximation is made, q_f = M = 3.17 W, and the actual fin heat transfer is overestimated by 17%.

PROBLEM 3.135

KNOWN: Geometry of pin fin array used as heat sink for a computer chip. Array convection and chip substrate conditions.

FIND: Effect of pin diameter, spacing and length on maximum allowable chip power dissipation.

SCHEMATIC:

Physical System:

Copper (k = 400 W/m·K)
Pins (N), D_p, L_p
$T_{\infty,o}$ = 20 °C
h_o = 250 W/m²·K
A_b
T_c = 75 °C, q_c, A_c = (0.0127 m)²
$A_b = A_c - NA_{c,b}$
$R''_{t,c} = 10^{-4}$ m²·K/W
L_b = 0.005 m
$T_{\infty,i}$ = 20 °C
k_b = 1 W/m·K
h_i = 40 W/m²·K

Thermal Circuit:

$T_{\infty,i}$, T_c, $T_{\infty,o}$
q_i, $1/h_iA_c$, L_b/k_bA_c, $R''_{t,c}/A_c$, $R_{t,o}$, q_t
q_c

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat transfer in chip-board assembly, (3) Negligible pin-chip contact resistance, (4) Constant properties, (5) Negligible chip thermal resistance, (6) Uniform chip temperature.

ANALYSIS: The total power dissipation is $q_c = q_i + q_t$, where

$$q_i = \frac{T_c - T_{\infty,i}}{\left(1/h_i + R''_{t,c} + L_b/k_b\right)/A_c} = 0.3\text{W}$$

and

$$q_t = \frac{T_c - T_{\infty,o}}{R_{t,o}}$$

The resistance of the pin array is

$$R_{t,o} = \left(\eta_o h_o A_t\right)^{-1}$$

where

$$\eta_o = 1 - \frac{NA_f}{A_t}\left(1 - \eta_f\right)$$

$$A_t = NA_f + A_b$$

$$A_f = \pi D_p L_c = \pi D_p \left(L_p + D_p/4\right)$$

Subject to the constraint that $N^{1/2}D_p \le 9$ mm, the foregoing expressions may be used to compute q_t as a function of D_p for L_p = 15 mm and values of N = 16, 25 and 36. Using the IHT *Performance Calculation, Extended Surface Model* for the *Pin Fin Array*, we obtain

Continued...

Clearly, it is desirable to maximize the number of pins and the pin diameter, so long as flow passages are not constricted to the point of requiring an excessive pressure drop to maintain the prescribed convection coefficient. The maximum heat rate for the fin array (q_t = 33.1 W) corresponds to N = 36 and D_p = 1.5 mm. Further improvement could be obtained by using N = 49 pins of diameter D_p = 1.286 mm, which yield q_t = 37.7 W.

Exploring the effect of L_p for N = 36 and D_p = 1.5 mm, we obtain

Clearly, there are benefits to increasing L_p, although the effect diminishes due to an attendant reduction in η_f (from η_f = 0.887 for L_p = 15 mm to η_f = 0.471 for L_p = 50 mm). Although a heat dissipation rate of q_t = 56.7 W is obtained for L_p = 50 mm, package volume constraints could preclude such a large fin length.

COMMENTS: By increasing N, D_p and/or L_p, the total surface area of the array, A_t, is increased, thereby reducing the array thermal resistance, $R_{t,o}$. The effects of D_p and N are shown for L_p = 15 mm.

PROBLEM 3.136

KNOWN: Copper heat sink dimensions and convection conditions.

FIND: (a) Maximum allowable heat dissipation for a prescribed chip temperature and interfacial chip/heat-sink contact resistance, (b) Effect of fin length and width on heat dissipation.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat transfer in chip-heat sink assembly, (3) Constant k, (4) Negligible chip thermal resistance, (5) Negligible heat transfer from back of chip, (6) Uniform chip temperature.

ANALYSIS: (a) For the prescribed system, the chip power dissipation may be expressed as

$$q_c = \frac{T_c - T_\infty}{R_{t,c} + R_{cond,b} + R_{t,o}}$$

where $$R_{t,c} = \frac{R''_{t,c}}{W_c^2} = \frac{5\times10^{-6}\,m^2\cdot K/W}{(0.016m)^2} = 0.0195\,K/W$$

$$R_{cond,b} = \frac{L_b}{kW_c^2} = \frac{0.003m}{400\,W/m\cdot K\,(0.016m)^2} = 0.0293\,K/W$$

The thermal resistance of the fin array is

$$R_{t,o} = (\eta_o h A_t)^{-1}$$

where $$\eta_o = 1 - \frac{NA_f}{A_t}(1-\eta_f)$$

and $$A_t = NA_f + A_b = N(4wL_c) + \left(W_c^2 - Nw^2\right)$$

Continued...

With w = 0.25 mm, S = 0.50 mm, L_f = 6 mm, N = 1024, and $L_c \approx L_f + w/4 = 6.063\times10^{-3}$ m, it follows that $A_f = 6.06\times10^{-6}\,m^2$ and $A_t = 6.40\times10^{-3}\,m^2$. The fin efficiency is

$$\eta_f = \frac{\tanh mL_c}{mL_c}$$

where $m = \left(hP/kA_c\right)^{1/2} = \left(4h/kw\right)^{1/2}$ = 245 m^{-1} and mL_c = 1.49. It follows that η_f = 0.608 and η_o = 0.619, in which case

$$R_{t,o} = \left(0.619\times1500\,W/m^2\cdot K\times6.40\times10^{-3}\,m^2\right) = 0.168\,K/W$$

and the maximum allowable heat dissipation is

$$q_c = \frac{(85-25)^\circ C}{(0.0195+0.0293+0.168)K/W} = 276W$$ <

(b) The IHT *Performance Calculation, Extended Surface Model* for the *Pin Fin Array* has been used to determine q_c as a function of L_f for four different cases, each of which is characterized by the closest allowable fin spacing of (S - w) = 0.25 mm.

Case	w (mm)	S (mm)	N
A	0.25	0.50	1024
B	0.35	0.60	711
C	0.45	0.70	522
D	0.55	0.80	400

Maximum heat rate, qc(W)

Fin length, Lf(mm)

w = 0.25 mm, S = 0.50 mm, N =1024
w = 0.35 mm, S = 0.60 mm, N = 711
w = 0.45 mm, S = 0.70 mm, N = 522
w = 0.55 mm, S = 0.80 mm, N = 400

With increasing w and hence decreasing N, there is a reduction in the total area A_t associated with heat transfer from the fin array. However, for Cases A through C, the reduction in A_t is more than balanced by an increase in η_f (and η_o), causing a reduction in $R_{t,o}$ and hence an increase in q_c. As the fin efficiency approaches its limiting value of η_f = 1, reductions in A_t due to increasing w are no longer balanced by increases in η_f, and q_c begins to decrease. Hence there is an optimum value of w, which depends on L_f. For the conditions of this problem, L_f = 10 mm and w = 0.55 mm provide the largest heat dissipation.

Problem 3.137

KNOWN: Two finned heat sinks, Designs A and B, prescribed by the number of fins in the array, N, fin dimensions of square cross-section, w, and length, L, with different convection coefficients, h.

FIND: Determine which fin arrangement is superior. Calculate the heat rate, q_f, efficiency, η_f, and effectiveness, ε_f, of a single fin, as well as, the total heat rate, q_t, and overall efficiency, η_o, of the array. Also, compare the total heat rates per unit volume.

SCHEMATIC:

53 mm
57 mm
L = 30mm
54 pins, 9x6 array (Design A)
T_b = 75 °C

Design	Fin dimensions Cross section w x w (mm)	Length L (mm)	Number of fins	Convection coefficient (W/m^2·K)
A	1 x 1	30	6 x 9	125
B	3 x 3	7	14 x 17	375

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in fins, (3) Convection coefficient is uniform over fin and prime surfaces, (4) Fin tips experience convection, and (5) Constant properties.

ANALYSIS: Following the treatment of Section 3.6.5, the overall efficiency of the array, Eq. (3.98), is

$$\eta_o = \frac{q_t}{q_{max}} = \frac{q_t}{hA_t\theta_b} \tag{1}$$

where A_t is the total surface area, the sum of the exposed portion of the base (prime area) plus the fin surfaces, Eq. 3.99,

$$A_t = N \cdot A_f + A_b \tag{2}$$

where the surface area of a single fin and the prime area are

$$A_f = 4(L \times W) + w^2 \tag{3}$$

$$A_b = b1 \times b2 - N \cdot A_c \tag{4}$$

Combining Eqs. (1) and (2), the total heat rate for the array is

$$q_t = N\eta_f hA_f\theta_b + hA_b\theta_b \tag{5}$$

where η_f is the efficiency of a single fin. From Table 4.3, Case A, for the tip condition with convection, the single fin efficiency based upon Eq. 3.86,

$$\eta_f = \frac{q_f}{hA_f\theta_b} \tag{6}$$

Continued...

where

$$q_f = M\frac{\sinh(mL) + (h/mk)\cosh(mL)}{\cosh(mL) + (h/mk)\sinh(mL)} \tag{7}$$

$$M = (hPkA_c)^{1/2}\theta_b \qquad m = (hP/kA_c)^{1/2} \qquad P = 4w \qquad A_c = w^2 \qquad (8,9,10)$$

The single fin effectiveness, from Eq. 3.81,

$$\varepsilon_f = \frac{q_f}{hA_c\theta_b} \tag{11}$$

Additionally, we want to compare the performance of the designs with respect to the array volume, vol

$$q_f''' = q_f/\forall = q_f/(b1 \cdot b2 \cdot L) \tag{12}$$

The above analysis was organized for easy treatment with equation-solving software. Solving Eqs. (1) through (11) simultaneously with appropriate numerical values, the results are tabulated below.

Design	q_t (W)	q_f (W)	η_o	η_f	ε_f	q_f''' (W/m^3)
A	113	1.80	0.804	0.779	31.9	1.25×10^6
B	165	0.475	0.909	0.873	25.3	7.81×10^6

COMMENTS: (1) Both designs have good efficiencies and effectiveness. Clearly, Design B is superior because the heat rate is nearly 50% larger than Design A for the same board footprint. Further, the space requirement for Design B is four times less ($\forall = 2.12\times10^{-5}$ vs. 9.06×10^{-5} m^3) and the heat rate per unit volume is 6 times greater.

(2) Design A features 54 fins compared to 238 fins for Design B. Also very significant to the performance comparison is the magnitude of the convection coefficient which is 3 times larger for Design B. Estimating convection coefficients for fin arrays (and tube banks) is discussed in Chapter 7.6. Of concern is how the fins alter the flow past the fins and whether the convection coefficient is uniform over the array.

(3) The *IHT Extended Surfaces Model*, for a *Rectangular Pin Fin Array* could have been used to solve this problem.

PROBLEM 3.138

KNOWN: Geometrical characteristics of a plate with pin fin array on both surfaces. Inner and outer convection conditions.

FIND: (a) Heat transfer rate with and without pin fin arrays, (b) Effect of using silver solder to join the pins and the plate.

SCHEMATIC:

$T_{\infty,o} = 20\,^oC$
$h_o = 100\ W/m^2\cdot K$
$D_p = 4$ mm
$N = 400$ copper pins on 160 mm x 160 mm copper plate (both sides)
$L_p = 20$ mm
$L_w = 5$ mm
$R''_{t,c} = 0$ (integral machining) or $R''_{t,c} = 5 \times 10^{-6}\ m^2\cdot K/W$ (silver solder)
$T_{\infty,o}$, $R_{t,o(c)o}$, R_w, $R_{t,o(c)i}$, $T_{\infty,i}$, q
$T_{\infty,i} = 65\,^oC$
$h_i = 5\ W/m^2\cdot K$

ASSUMPTIONS: (1) Steady-state conditions, (2) Constant k, (3) Negligible radiation.

PROPERTIES: Table A-1: Copper, $\overline{T} \approx 315$ K, k = 400 W/m·K.

ANALYSIS: (a) The heat rate may be expressed as

$$q = \frac{T_{\infty,i} - T_{\infty,o}}{R_{t,o(c),i} + R_w + R_{t,o(c),o}}$$

where

$$R_{t,o(c)} = \left(\eta_{o(c)} h A_t\right)^{-1},$$

$$\eta_{o(c)} = 1 - \frac{NA_f}{A_t}\left(1 - \frac{\eta_f}{C_1}\right),$$

$$A_t = NA_f + A_b,$$

$$A_f = \pi D_p L_c \approx \pi D_p \left(L + D/4\right),$$

$$A_b = W^2 - NA_{c,b} = W^2 - N\left(\pi D_p^2/4\right),$$

$$\eta_f = \frac{\tanh mL_c}{mL_c}, \qquad m = \left(4h/kD_p\right)^{1/2},$$

Continued...

$$C_1 = 1 + \eta_f h A_f \left(R''_{t,c} / A_{c,b} \right),$$

and

$$R_w = \frac{L_w}{W^2 k}.$$

Calculations may be expedited by using the IHT *Performance Calculation, Extended Surface* Model for the *Pin Fin* Array. For $R''_{t,c}$ = 0, C_1 = 1, and with W = 0.160 m, R_w = 0.005 m/(0.160 m)^2 400 W/m·K = 4.88×10^{-4} K/W. For the prescribed array geometry, we also obtain $A_{c,b} = 1.26 \times 10^{-5}$ m^2, $A_f = 2.64 \times 10^{-4}$ m^2, $A_b = 2.06 \times 10^{-2}$ m^2, and A_t = 0.126 m^2.

On the outer surface, where h_o = 100 W/m^2·K, m = 15.8 m^{-1}, η_f = 0.965, η_o = 0.970 and $R_{t,o}$ = 0.0817 K/W. On the inner surface, where h_i = 5 W/m^2·K, m = 3.54 m^{-1}, η_f = 0.998, η_o = 0.999 and $R_{t,o}$ = 1.588 K/W.

Hence, the heat rate is

$$q = \frac{(65-20)^\circ \mathrm{C}}{\left(1.588 + 4.88\times10^{-4} + 0.0817\right)\mathrm{K/W}} = 26.94\mathrm{W} \qquad <$$

Without the fins,

$$q = \frac{T_{\infty,i} - T_{\infty,o}}{(1/h_i A_w) + R_w + (1/h_o A_w)} = \frac{(65-20)^\circ \mathrm{C}}{\left(7.81 + 4.88\times10^{-4} + 0.39\right)} = 5.49\mathrm{W} \qquad <$$

Hence, the fin arrays provide nearly a five-fold increase in heat rate.

(b) With use of the silver solder, $\eta_{o(c),o}$ = 0.962 and $R_{t,o(c),o}$ = 0.0824 K/W. Also, $\eta_{o(c),i}$ = 0.998 and $R_{t,o(c),i}$ = 1.589 K/W. Hence

$$q = \frac{(65-20)^\circ \mathrm{C}}{\left(1.589 + 4.88\times10^{-4} + 0.0824\right)\mathrm{K/W}} = 26.92\mathrm{W} \qquad <$$

Hence, the effect of the contact resistance is negligible.

COMMENTS: The dominant contribution to the total thermal resistance is associated with internal conditions. If the heat rate must be increased, it should be done by increasing h_i.

PROBLEM 3.139

KNOWN: Long rod with internal volumetric generation covered by an electrically insulating sleeve and supported with a ribbed spider.

FIND: Combination of convection coefficient, spider design, and sleeve thermal conductivity which enhances volumetric heating subject to a maximum centerline temperature of 100°C.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional radial heat transfer in rod, sleeve and hub, (3) Negligible interfacial contact resistances, (4) Constant properties, (5) Adiabatic outer surface.

ANALYSIS: The system heat rate per unit length may be expressed as

$$q' = \dot{q}\left(\pi r_o^2\right) = \frac{T_1 - T_\infty}{R'_{sleeve} + R'_{hub} + R'_{t,o}}$$

where

$$R'_{sleeve} = \frac{\ln\left(r_1/r_o\right)}{2\pi k_s}, \quad R'_{hub} = \frac{\ln\left(r_2/r_1\right)}{2\pi k_r} = 3.168\times10^{-4}\,\mathrm{m\cdot K/W}, \quad R'_{t,o} = \frac{1}{\eta_o h A'_t},$$

$$\eta_o = 1 - \frac{NA'_f}{A'_t}\left(1-\eta_f\right), \qquad A'_f = 2\left(r_3 - r_2\right), \qquad A'_t = NA'_f + \left(2\pi r_3 - Nt\right),$$

$$\eta_f = \frac{\tanh m\left(r_3 - r_2\right)}{m\left(r_3 - r_2\right)}, \qquad m = \left(2h/k_r t\right)^{1/2}.$$

The rod centerline temperature is related to T_1 through

$$T_o = T(0) = T_1 + \frac{\dot{q} r_o^2}{4k}$$

Calculations may be expedited by using the IHT *Performance Calculation, Extended Surface Model* for the *Straight Fin Array*. For base case conditions of k_s = 0.5 W/m·K, h = 20 W/m^2·K, t = 4 mm and N = 12, R'_{sleeve} = 0.0580 m·K/W, $R'_{t,o}$ = 0.0826 m·K/W, η_f = 0.990, q' = 387 W/m, and $\dot{q}$ = 1.23 × 10^6 W/m^3. As shown below, $\dot{q}$ may be increased by increasing h, where h = 250 W/m^2·K represents a reasonable upper limit for airflow. However, a more than 10-fold increase in h yields only a 63% increase in $\dot{q}$.

Continued...

PROBLEM 3.139 (Cont.)

The difficulty is that, by significantly increasing h, the thermal resistance of the fin array is reduced to 0.00727 m·K/W, rendering the sleeve the dominant contributor to the total resistance.

Similar results are obtained when N and t are varied. For values of t = 2, 3 and 4 mm, variations of N in the respective ranges $12 \le N \le 26$, $12 \le N \le 21$ and $12 \le N \le 17$ were considered. The upper limit on N was fixed by requiring that $(S - t) \ge 2$ mm to avoid an excessive resistance to airflow between the ribs. As shown below, the effect of increasing N is small, and there is little difference between results for the three values of t.

In contrast, significant improvement is associated with changing the sleeve material, and it is only necessary to have $k_s \approx 25$ W/m·K (e.g. a boron sleeve) to approach an upper limit to the influence of k_s.

For h = 250 W/m^2·K and k_s = 25 W/m·K, only a slight improvement is obtained by increasing N. Hence, the recommended conditions are:

$$h = 250\,W/m^2 \cdot K, \qquad k_s = 25\,W/m \cdot K, \qquad N = 12, \qquad t = 4mm$$ <

COMMENTS: The upper limit to $\dot{q}$ is reached as the total thermal resistance approaches zero, in which case $T_1 \rightarrow T_\infty$. Hence $\dot{q}_{max} = 4k\left(T_o - T_\infty\right)/r_o^2 = 4.5\times10^6\ W/m^3$.

PROBLEM 3.140

KNOWN: Geometrical and convection conditions of internally finned, concentric tube air heater.

FIND: (a) Thermal circuit, (b) Heat rate per unit tube length, (c) Effect of changes in fin array.

SCHEMATIC:

Air
$T_{\infty,o}$ = 25 °C
h_o = 200 W/m² K
t
r_1 = 13 mm
r_2 = 16 mm
r_3 = 40 mm
Water
$T_{\infty,i}$ = 90 °C
h_i = 5000 W/m²·K
Steel
(k = 20 W/m·K)
q' → $T_{\infty,i}$ — $R'_{conv,i}$ — R'_{cond} — $R'_{t,o}$ — $T_{\infty,o}$

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat transfer in radial direction, (3) Constant k, (4) Adiabatic outer surface.

ANALYSIS: (a) For the thermal circuit shown schematically,

$$R'_{conv,i} = \left(h_i 2\pi r_1\right)^{-1}, \qquad R'_{cond} = \ln\left(r_2/r_1\right)/2\pi k \text{, and} \qquad R'_{t,o} = \left(\eta_o h_o A'_t\right)^{-1},$$

where

$$\eta_o = 1 - \frac{NA'_f}{A'_t}\left(1-\eta_f\right), \quad A'_f = 2L = 2\left(r_3 - r_2\right), \quad A'_t = NA'_f + \left(2\pi r_2 - Nt\right), \text{ and } \quad \eta_f = \frac{\tanh mL}{mL}.$$

(b)
$$q' = \frac{\left(T_{\infty,i} - T_{\infty,o}\right)}{R'_{conv,i} + R'_{cond} + R'_{t,o}}$$

Substituting the known conditions, it follows that

$$R'_{conv,i} = \left(5000\,\text{W/m}^2\cdot\text{K} \times 2\pi \times 0.013\text{m}\right)^{-1} = 2.45\times10^{-3}\,\text{m}\cdot\text{K/W}$$

$$R'_{cond} = \ln\left(0.016\text{m}/0.013\text{m}\right)/2\pi\left(20\,\text{W/m}\cdot\text{K}\right) = 1.65\times10^{-3}\,\text{m}\cdot\text{K/W}$$

$$R'_{t,o} = \left(0.575\times200\,\text{W/m}^2\cdot\text{K}\times0.461\text{m}\right)^{-1} = 18.86\times10^{-3}\,\text{m}\cdot\text{K/W}$$

where η_f = 0.490. Hence,

$$q' = \frac{\left(90-25\right)^\circ\text{C}}{\left(2.45+1.65+18.86\right)\times10^{-3}\,\text{m}\cdot\text{K/W}} = 2831\,\text{W/m}$$ <

(c) The small value of η_f suggests that some benefit may be gained by increasing t, as well as by increasing N. With the requirement that Nt ≤ 50 mm, we use the IHT *Performance Calculation, Extended Surface Model* for the *Straight Fin Array* to consider the following range of conditions: t = 2 mm, 12 ≤ N ≤ 25; t = 3 mm, 8 ≤ N ≤ 16; t = 4 mm, 6 ≤ N ≤ 12; t = 5 mm, 5 ≤ N ≤ 10. Calculations based on the foregoing model are plotted as follows.

Continued...

PROBLEM 3.140 (Cont.)

5000
4000
3000
2000
Heat rate, q'(W/m)
5 10 15 20 25
Number of fins, N
t = 2 mm
t = 3 mm
t = 4 mm
t = 5 mm

By increasing t from 2 to 5 mm, η_f increases from 0.410 to 0.598. Hence, for fixed N, q' increases with increasing t. However, from the standpoint of maximizing q'_t, it is clearly preferable to use the larger number of thinner fins. Hence, subject to the prescribed constraint, we would choose t = 2 mm and N = 25, for which q' = 4880 W/m.

COMMENTS: (1) The air side resistance makes the dominant contribution to the total resistance, and efforts to increase q' by reducing $R'_{t,o}$ are well directed. (2) A fin thickness any smaller than 2 mm would be difficult to manufacture.

PROBLEM 3.141

KNOWN: Dimensions and number of rectangular aluminum fins. Convection coefficient with and without fins.

FIND: Percentage increase in heat transfer resulting from use of fins.

SCHEMATIC:

L = 50 mm
t = 0.5mm
Aluminum

$N = 250\ m^{-1}$
w = width
$h_w = 30 W/m^2 \cdot K$ (with fins)
$h_{wo} = 40 W/m^2 \cdot K$ (without fins)

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible radiation, (5) Negligible fin contact resistance, (6) Uniform convection coefficient.

PROPERTIES: *Table A-1*, Aluminum, pure: $k \approx 240$ W/m·K.

ANALYSIS: Evaluate the fin parameters

$$L_c = L + t/2 = 0.05025 m$$

$$A_p = L_c t = 0.05025 m \times 0.5 \times 10^{-3} m = 25.13 \times 10^{-6}\ m^2$$

$$L_c^{3/2}\left(h_w / kA_p\right)^{1/2} = (0.05025 m)^{3/2}\left[\frac{30\ W/m^2 \cdot K}{240\ W/m \cdot K \times 25.13 \times 10^{-6} m^2}\right]^{1/2}$$

$$L_c^{3/2}\left(h_w / kA_p\right)^{1/2} = 0.794$$

It follows from Fig. 3.18 that $\eta_f \approx 0.72$. Hence,

$$q_f = \eta_f q_{max} = 0.72\ h_w 2wL\ \theta_b$$

$$q_f = 0.72 \times 30\ W/m^2 \cdot K \times 2 \times 0.05 m \times (w\ \theta_b) = 2.16\ W/m \cdot K\,(w\ \theta_b)$$

With the fins, the heat transfer from the walls is

$$q_w = N\ q_f + (1 - Nt)\,w\,h_w\ \theta_b$$

$$q_w = 250 \times 2.16 \frac{W}{m \cdot K}(w\ \theta_b) + \left(1m - 250 \times 5 \times 10^{-4} m\right) \times 30\ W/m^2 \cdot K\ (w\ \theta_b)$$

$$q_w = (540 + 26.3)\frac{W}{m \cdot K}(w\ \theta_b) = 566\ w\ \theta_b.$$

Without the fins, $q_{wo} = h_{wo}\ 1m \times w\ \theta_b = 40\ w\ \theta_b$. Hence the percentage increase in heat transfer is

$$\frac{q_w - q_{wo}}{q_{wo}} = \frac{(566 - 40)\,w\ \theta_b}{40\ w\ \theta_b} = 13.15 = 1315\%$$ <

COMMENTS: If the infinite fin approximation is made, it follows that $q_f = (hPkA_c)^{1/2}\ \theta_b = [h_w 2wkwt]^{1/2}\ \theta_b = (30 \times 2 \times 240 \times 5 \times 10^{-4})^{1/2}\ w\ \theta_b = 2.68\ w\ \theta_b$. Hence, q_f is overestimated.

PROBLEM 3.142

KNOWN: Dimensions, base temperature and environmental conditions associated with rectangular and triangular stainless steel fins.

FIND: Efficiency, heat loss per unit width and effectiveness associated with each fin.

SCHEMATIC:

T_b = 100°C
L = 20mm
t = 6mm
t
L

T_b = 100°C
T_∞ = 20 °C
h = 75 W/m²·K
L

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible radiation, (5) Uniform convection coefficient.

PROPERTIES: *Table A-1*, Stainless Steel 304 (T = 333 K): k = 15.3 W/m·K.

ANALYSIS: For the rectangular fin, with $L_c = L + t/2$, evaluate the parameter

$$L_c^{3/2}\left(h/kA_p\right)^{1/2} = (0.023\,\text{m})^{3/2}\left[\frac{75\,\text{W/m}^2\cdot\text{K}}{15.3\,\text{W/m}\cdot\text{K}\,(0.023\,\text{m})(0.006\,\text{m})}\right]^{1/2} = 0.66\,.$$

Hence, from Fig. 3.18, the fin efficiency is

$$\eta_f \approx 0.79$$ <

From Eq. 3.86, the fin heat rate is $q_f = \eta_f hA_f\theta_b = \eta_f hPL_c\theta_b = \eta_f h2wL_c\theta_b$ or, per unit width,

$$q_f' = \frac{q_f}{w} = 0.79\left(75\,\text{W/m}^2\cdot\text{K}\right)2(0.023\,\text{m})80^\circ\text{C} = 218\,\text{W/m}\,.$$ <

From Eq. 3.81, the fin effectiveness is

$$\varepsilon_f = \frac{q_f}{hA_{c,b}\theta_b} = \frac{q_f'\times w}{h(t\times w)\theta_b} = \frac{218\,\text{W/m}}{75\,\text{W/m}^2\cdot\text{K}\,(0.006\,\text{m})80^\circ\text{C}} = 6.06\,.$$ <

For the triangular fin with

$$L_c^{3/2}\left(h/kA_p\right)^{1/2} = (0.02\,\text{m})^{3/2}\left[\frac{75\,\text{W/m}^2\cdot\text{K}}{(15.3\,\text{W/m}\cdot\text{K})(0.020\,\text{m})(0.003\,\text{m})}\right]^{1/2} = 0.81\,,$$

find from Figure 3.18,

$$\eta_f \approx 0.78\,,$$ <

From Eq. 3.86 and Table 3.5 find

$$q_f' = \eta_f hA_f'\theta_b = \eta_f h2\left[L^2 + (t/2)^2\right]^{1/2}\theta_b$$

$$q_f' = 0.78\times 75\,\text{W/m}^2\cdot\text{K}\times 2\left[(0.02)^2 + (0.006/2)^2\right]^{1/2}\text{m}\left(80^\circ\text{C}\right) = 187\,\text{W/m}\,.$$ <

and from Eq. 3.81, the fin effectiveness is

$$\varepsilon_f = \frac{q_f'\times w}{h(t\times w)\theta_b} = \frac{187\,\text{W/m}}{75\,\text{W/m}^2\cdot\text{K}\,(0.006\,\text{m})80^\circ\text{C}} = 5.19$$ <

COMMENTS: Although it is 14% less effective, the triangular fin offers a 50% weight savings.

PROBLEM 3.143

KNOWN: Dimensions, base temperature and environmental conditions associated with a triangular, aluminum fin.

FIND: (a) Fin efficiency and effectiveness, (b) Heat dissipation per unit width.

SCHEMATIC:

t = 2mm
L = 6mm
$T_b = 250\ ^oC$
w = Fin width
Air
$T_\infty = 20\ ^oC$
$h = 40\ W/m^2{\cdot}K$

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible radiation and base contact resistance, (5) Uniform convection coefficient.

PROPERTIES: *Table A-1*, Aluminum, pure (T ≈ 400 K): k = 240 W/m·K.

ANALYSIS: (a) With $L_c = L = 0.006$ m, find

$$A_p = Lt/2 = (0.006\,m)(0.002\,m)/2 = 6\times10^{-6}\,m^2,$$

$$L_c^{3/2}\left(h/kA_p\right)^{1/2} = (0.006\,m)^{3/2}\left(\frac{40\,W/m^2\cdot K}{240\,W/m\cdot K\times6\times10^{-6}\,m^2}\right)^{1/2} = 0.077$$

and from Fig. 3.18, the fin efficiency is

$$\eta_f \approx 0.99\,.$$ <

From Eq. 3.86 and Table 3.5, the fin heat rate is

$$q_f = \eta_f\,q_{max} = \eta_f\,hA_{f\,(tri)}\theta_b = 2\eta_f\,hw\left[L^2 + (t/2)^2\right]^{1/2}\theta_b\,.$$

From Eq. 3.81, the fin effectiveness is

$$\varepsilon_f = \frac{q_f}{hA_{c,b}\theta_b} = \frac{2\eta_f\,hw\left[L^2 + (t/2)^2\right]^{1/2}\theta_b}{g(w\cdot t)\theta_b} = \frac{2\eta_f\left[L^2 + (t/2)^2\right]^{1/2}}{t}$$

$$\varepsilon_f = \frac{2\times0.99\left[(0.006)^2 + (0.002/2)^2\right]^{1/2}\,m}{0.002\,m} = 6.02$$ <

(b) The heat dissipation per unit width is

$$q_f' = (q_f/w) = 2\eta_f\,h\left[L^2 + (t/2)^2\right]^{1/2}\theta_b$$

$$q_f' = 2\times0.99\times40\,W/m^2\cdot K\left[(0.006)^2 + (0.002/2)^2\right]^{1/2}m\times(250-20)^\circ C = 110.8\,W/m\,.$$ <

COMMENTS: The triangular profile is known to provide the maximum heat dissipation per unit fin mass.

PROBLEM 3.144

KNOWN: Dimensions and base temperature of an annular, aluminum fin of rectangular profile. Ambient air conditions.

FIND: (a) Fin heat loss, (b) Heat loss per unit length of tube with 200 fins spaced at 5 mm increments.

SCHEMATIC:

$T_b = 250^oC$ — Aluminum — $t = 1mm$

$N' = 200$ fins/m — L = 10 mm — $T_\infty = 25\ ^oC$, $h = 25\ W/m^2{\cdot}K$

$r_1 = 12.5mm$

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible radiation and contact resistance, (5) Uniform convection coefficient.

PROPERTIES: *Table A-1*, Aluminum, pure (T ≈ 400 K): k = 240 W/m·K.

ANALYSIS: (a) The fin parameters for use with Figure 3.19 are

$$r_{2c} = r_2 + t/2 = (12.5\,mm + 10\,mm) + 0.5\,mm = 23\,mm = 0.023\,m$$

$$r_{2c}/r_1 = 1.84 \qquad\qquad L_c = L + t/2 = 10.5\,mm = 0.0105\,m$$

$$A_p = L_c t = 0.0105\,m \times 0.001\,m = 1.05\times10^{-5}\,m^2$$

$$L_c^{3/2}\left(h/kA_p\right)^{1/2} = (0.0105\,m)^{3/2}\left(\frac{25\,W/m^2\cdot K}{240\,W/m\cdot K\times1.05\times10^{-5}m^2}\right)^{1/2} = 0.15\,.$$

Hence, the fin effectiveness is $\eta_f \approx 0.97$, and from Eq. 3.86 and Fig. 3.5, the fin heat rate is

$$q_f = \eta_f q_{max} = \eta_f hA_{f\,(ann)}\theta_b = 2\pi\eta_f h\left(r_{2,c}^2 - r_1^2\right)\theta_b$$

$$q_f = 2\pi\times0.97\times25\,W/m^2\cdot K\times\left[(0.023\,m)^2 - (0.0125\,m)^2\right]225^\circ C = 12.8\,W\,.$$ <

(b) Recognizing that there are N = 200 fins per meter length of the tube, the total heat rate considering contributions due to the fin and base (unfinned surfaces is

$$q' = N'q_f + h(1 - N't)2\pi r_1\theta_b$$

$$q' = 200\,m^{-1}\times12.8\,W + 25\,W/m^2\cdot K\left(1 - 200\,m^{-1}\times0.001\,m\right)\times2\pi\times(0.0125\,m)225^\circ C$$

$$q' = (2560\,W + 353\,W)/m = 2.91\,kW/m\,.$$ <

COMMENTS: Note that, while covering only 20% of the tube surface area, the tubes account for more than 85% of the total heat dissipation.

PROBLEM 3.145

KNOWN: Dimensions and base temperature of aluminum fins of rectangular profile. Ambient air conditions.

FIND: (a) Fin efficiency and effectiveness, (b) Rate of heat transfer per unit length of tube.

SCHEMATIC:

$T_b = 200^oC$
L = 15 mm
t = 4mm
N' = 125m^{-1}
Aluminum fins
$T_\infty = 20\ ^oC$
$h = 40\ W/m^2 \cdot K$
Air
r_1 = 25mm

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional radial conduction in fins, (3) Constant properties, (4) Negligible radiation, (5) Negligible base contact resistance, (6) Uniform convection coefficient.

PROPERTIES: *Table A-1*, Aluminum, pure (T ≈ 400 K): k = 240 W/m·K.

ANALYSIS: (a) The fin parameters for use with Figure 3.19 are

$$r_{2c} = r_2 + t/2 = 40\,\text{mm} + 2\,\text{mm} = 0.042\,\text{m} \qquad L_c = L + t/2 = 15\,\text{mm} + 2\,\text{mm} = 0.017\,\text{m}$$

$$r_{2c}/r_1 = 0.042\,\text{m}/0.025\,\text{m} = 1.68 \qquad A_p = L_c t = 0.017\,\text{m} \times 0.004\,\text{m} = 6.8\times10^{-5}\,\text{m}^2$$

$$L_c^{3/2}\left(h/kA_p\right)^{1/2} = \left(0.017\,\text{m}\right)^{3/2}\left[40\,\text{W/m}^2\cdot\text{K}\Big/240\,\text{W/m}\cdot\text{K}\times6.8\times10^{-5}\,\text{m}^2\right]^{1/2} = 0.11$$

The fin efficiency is $\eta_f \approx 0.97$. From Eq. 3.86 and Fig. 3.5,

$$q_f = \eta_f q_{max} = \eta_f hA_{f\,(ann)}\theta_b = 2\pi\eta_f h\left[r_{2c}^2 - r_1^2\right]\theta_b$$

$$q_f = 2\pi\times0.97\times40\,\text{W/m}^2\cdot\text{K}\left[(0.042)^2 - (0.025)^2\right]\text{m}^2\times180^\circ\text{C} = 50\,\text{W}$$ <

From Eq. 3.81, the fin effectiveness is

$$\varepsilon_f = \frac{q_f}{hA_{c,b}\theta_b} = \frac{50\,\text{W}}{40\,\text{W/m}^2\cdot\text{K}\ 2\pi(0.025\,\text{m})(0.004\,\text{m})180^\circ\text{C}} = 11.05$$ <

(b) The rate of heat transfer per unit length is

$$q' = N'q_f + h\left(1 - N't\right)\left(2\pi r_1\right)\theta_b$$

$$q' = 125\times50\,\text{W/m} + 40\,\text{W/m}^2\cdot\text{K}\left(1 - 125\times0.004\right)\left(2\pi\times0.025\,\text{m}\right)\times180^\circ\text{C}$$

$$q' = \left(6250 + 565\right)\text{W/m} = 6.82\,\text{kW/m}$$ <

COMMENTS: Note the dominant contribution made by the fins to the total heat transfer.

PROBLEM 3.146

KNOWN: Dimensions, base temperature, and contact resistance for an annular, aluminum fin. Ambient fluid conditions.

FIND: Fin heat transfer with and without base contact resistance.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible radiation, (5) Uniform convection coefficient.

PROPERTIES: *Table A-1*, Aluminum, pure (T ≈ 350 K): k ≈ 240 W/m·K.

ANALYSIS: With the contact resistance, the fin heat loss is $q_f = \dfrac{T_w - T_\infty}{R_{t,c} + R_f}$ where

$$R_{t,c} = R''_{t,c} / A_b = 2\times10^{-4}\,\mathrm{m^2\cdot K/W} \big/ 2\pi\,(0.015\,\mathrm{m})(0.002\,\mathrm{m}) = 1.06\,\mathrm{K/W}\,.$$

From Eqs. 3.83 and 3.86, the fin resistance is

$$R_f = \frac{\theta_b}{q_f} = \frac{\theta_b}{\eta_f\, q_{max}} = \frac{\theta_b}{\eta_f\, hA_f\theta_b} = \frac{1}{2\pi h\eta_f\left(r_{2,c}^2 - r_1^2\right)}\,.$$

Evaluating parameters,

$$r_{2,c} = r_2 + t/2 = 30\,\mathrm{mm} + 1\,\mathrm{mm} = 0.031\,\mathrm{m} \qquad L_c = L + t/2 = 0.016\,\mathrm{m}$$

$$r_{2c}/r_1 = 0.031/0.015 = 2.07\,\mathbf{Z} \qquad A_p = L_c t = 3.2\times10^{-5}\,\mathrm{m^2}$$

$$L_c^{3/2}\left(h/kA_p\right)^{1/2} = (0.016\,\mathrm{m})^{3/2}\left[75\,\mathrm{W/m^2\cdot K}\big/240\,\mathrm{W/m\cdot K}\times3.2\times10^{-5}\,\mathrm{m^2}\right]^{1/2} = 0.20$$

find the fin efficiency from Figure 3.19 as $\eta_f = 0.94$. Hence,

$$R_f = \frac{1}{2\pi\left(75\,\mathrm{W/m^2\cdot K}\right)0.94\left[(0.031\,\mathrm{m})^2 - (0.015\,\mathrm{m})^2\right]} = 3.07\,\mathrm{K/W}$$

$$q_f = \frac{(100-25)^\circ\mathrm{C}}{(1.06+3.07)\,\mathrm{K/W}} = 18.2\,\mathrm{W}\,.$$ <

Without the contact resistance, $T_w = T_b$ and

$$q_f = \frac{\theta_b}{R_f} = \frac{75^\circ\mathrm{C}}{3.07\,\mathrm{K/W}} = 24.4\,\mathrm{W}\,.$$ <

COMMENTS: To maximize fin performance, every effort should be made to minimize contact resistance.

PROBLEM 3.147

KNOWN: Dimensions and materials of a finned (annular) cylinder wall. Heat flux and ambient air conditions. Contact resistance.

FIND: Surface and interface temperatures (a) without and (b) with an interface contact resistance.

SCHEMATIC:

T_i T_1 $k = 50\ W/m\cdot K$
$r_i = 60mm$
$r_1 = 66mm$
$r_b = 70mm$
$r_o = 95mm$
T_b
$t = 2mm$
$\delta = 2mm$
Aluminum, $k = 240\ W/m\cdot K$
$T_\infty = 320K$
$h = 100 W/m^2\cdot K$
$q_i'' = 10^5 W/m^2$
T_i R_c $T_{1,i}$ $R_{t,c}$ $T_{1,o}$ R_w T_b R_b R_f T_∞

ASSUMPTIONS: (1) One-dimensional, steady-state conditions, (2) Constant properties, (3) Uniform h over surfaces, (4) Negligible radiation.

ANALYSIS: The analysis may be performed per unit length of cylinder or for a 4 mm long section. The following calculations are based on a unit length. The inner surface temperature may be obtained from

$$q' = \frac{T_i - T_\infty}{R'_{tot}} = q_i''(2\pi\ r_i) = 10^5\ W/m^2 \times 2\pi \times 0.06\ m = 37,700\ W/m$$

where $R'_{tot} = R'_c + R'_{t,c} + R'_w + R'_{equiv}$; $R'_{equiv} = (1/R'_f + 1/R'_b)^{-1}$.

R'_c, *Conduction resistance of cylinder wall:*

$$R'_c = \frac{\ln(r_1/r_i)}{2\pi\ k} = \frac{\ln(66/60)}{2\pi(50\ W/m\cdot K)} = 3.034\times10^{-4}\ m\cdot K/W$$

$R'_{t,c}$, *Contact resistance:*

$$R'_{t,c} = R''_{t,c}/2\pi\ r_1 = 10^{-4}\ m^2\cdot K/W/2\pi\times0.066\ m = 2.411\times10^{-4}\ m\cdot K/W$$

R'_w, *Conduction resistance of aluminum base:*

$$R'_w = \frac{\ln(r_b/r_1)}{2\pi\ k} = \frac{\ln(70/66)}{2\pi\times240\ W/m\cdot K} = 3.902\times10^{-5}\ m\cdot K/W$$

R'_b, *Resistance of prime or unfinned surface:*

$$R'_b = \frac{1}{hA'_b} = \frac{1}{100\ W/m^2\cdot K\times0.5\times2\pi(0.07\ m)} = 454.7\times10^{-4}\ m\cdot K/W$$

R'_f, *Resistance of fins:* The fin resistance may be determined from

$$R'_f = \frac{T_b - T_\infty}{q'_f} = \frac{1}{\eta_f hA'_f}$$

The fin efficiency may be obtained from Fig. 3.19,

$$r_{2c} = r_o + t/2 = 0.096\ m \qquad L_c = L + t/2 = 0.026\ m$$

Continued …..

PROBLEM 3.147 (Cont.)

$$A_p = L_c t = 5.2\times10^{-5}\ m^2 \qquad r_{2c}/r_1 = 1.45 \qquad L_c^{3/2}\left(h/kA_p\right)^{1/2} = 0.375$$

Fig. 3.19 $\rightarrow \eta_f \approx 0.88$.

The total fin surface area per meter length

$$A'_f = 250\left[\pi\left(r_o^2 - r_b^2\right)\times2\right] = 250\ m^{-1}\left[2\pi\left(0.096^2 - 0.07^2\right)\right]m^2 = 6.78\ m.$$

Hence $$R'_f = \left[0.88\times100\ W/m^2\cdot K\times6.78\ m\right]^{-1} = 16.8\times10^{-4}\ m\cdot K/W$$

$$1/R'_{equiv} = \left(1/16.8\times10^{-4} + 1/454.7\times10^{-4}\right)W/m\cdot K = 617.2\ W/m\cdot K$$

$$R'_{equiv} = 16.2\times10^{-4}\ m\cdot K/W.$$

Neglecting the *contact resistance,*

$$R'_{tot} = (3.034 + 0.390 + 16.2)10^{-4}\ m\cdot K/W = 19.6\times10^{-4}\ m\cdot K/W$$

$$T_i = q'R'_{tot} + T_\infty = 37,700\ W/m\times19.6\times10^{-4}\ m\cdot K/W + 320\ K = 393.9\ K$$ <

$$T_1 = T_i - q'R'_w = 393.9\ K - 37,700\ W/m\times3.034\times10^{-4}\ m\cdot K/W = 382.5\ K$$ <

$$T_b = T_1 - q'R'_b = 382.5\ K - 37,700\ W/m\times3.902\times10^{-5}\ m\cdot K/W = 381.0\ K.$$ <

Including the *contact resistance,*

$$R'_{tot} = \left(19.6\times10^{-4} + 2.411\times10^{-4}\right)m\cdot K/W = 22.0\times10^{-4}\ m\cdot K/W$$

$$T_i = 37,700\ W/m\times22.0\times10^{-4}\ m\cdot K/W + 320\ K = 402.9\ K$$ <

$$T_{1,i} = 402.9\ K - 37,700\ W/m\times3.034\times10^{-4}\ m\cdot K/W = 391.5\ K$$ <

$$T_{1,o} = 391.5\ K - 37,700\ W/m\times2.411\times10^{-4}\ m\cdot K/W = 382.4\ K$$ <

$$T_b = 382.4\ K - 37,700\ W/m\times3.902\times10^{-5}\ m\cdot K/W = 380.9\ K.$$ <

COMMENTS: (1) The effect of the contact resistance is small.

(2) The effect of including the aluminum fins may be determined by computing T_i without the fins. In this case $R'_{tot} = R'_c + R'_{conv}$, where

$$R'_{conv} = \frac{1}{h2\pi\, r_1} = \frac{1}{100\ W/m^2\cdot K\ 2\pi(0.066\ m)} = 241.1\times10^{-4}\ m\cdot K/W.$$

Hence, $R_{tot} = 244.1\times10^{-4}\ m\cdot K/W$, and

$$T_i = q'R'_{tot} + T_\infty = 37,700\ W/m\times244.1\times10^{-4}\ m\cdot K/W + 320\ K = 1240\ K.$$

Hence, the fins have a significant effect on reducing the cylinder temperature.

(3) The overall surface efficiency is

$$\eta_o = 1 - \left(A'_f / A'_t\right)(1 - \eta_f) = 1 - 6.78\ m/7.00\ m(1 - 0.88) = 0.884.$$

It follows that $q' = \eta_o h_o A'_t \theta_b = 37,700\ W/m$, which agrees with the prescribed value.

PROBLEM 3.148

KNOWN: Dimensions and materials of a finned (annular) cylinder wall. Combustion gas and ambient air conditions. Contact resistance.

FIND: (a) Heat rate per unit length and surface and interface temperatures, (b) Effect of increasing the fin thickness.

SCHEMATIC:

k_w = 50 W/m·K
T_i T_1 T_b
T_g = 1100 K
h_g = 150 W/m²·K
r_i = 60 mm
r_1 = 66 mm
r_b = 70 mm
r_o = 95 mm
t = 2 mm
δ = 2 mm
T_∞= 320 K
h = 100 W/m²·K
Aluminum
k = 240 W/m·K

T_g T_i $T_{1,i}$ $T_{1,o}$ T_b T_∞ → q'
R'_g R'_w $R'_{t,c}$ R'_b $R'_{t,o}$

ASSUMPTIONS: (1) One-dimensional, steady-state conditions, (2) Constant properties, (3) Uniform h over surfaces, (4) Negligible radiation.

ANALYSIS: (a) The heat rate per unit length is

$$q' = \frac{T_g - T_\infty}{R'_{tot}}$$

where $R'_{tot} = R'_g + R'_w + R'_{t,c} + R'_b + R'_{t,o}$, and

$$R'_g = \left(h_g 2\pi r_i\right)^{-1} = \left(150\,\text{W/m}^2\cdot\text{K}\times 2\pi\times 0.06\text{m}\right)^{-1} = 0.0177\,\text{m}\cdot\text{K/W},$$

$$R'_w = \frac{\ln(r_1/r_i)}{2\pi k_w} = \frac{\ln(66/60)}{2\pi(50\,\text{W/m}\cdot\text{K})} = 3.03\times10^{-4}\,\text{m}\cdot\text{K/W},$$

$$R'_{t,c} = \left(R''_{t,c}/2\pi r_1\right) = 10^{-4}\,\text{m}^4\cdot\text{K/W}\Big/2\pi\times 0.066\text{m} = 2.41\times10^{-4}\,\text{m}\cdot\text{K/W}$$

$$R'_b = \frac{\ln(r_b/r_1)}{2\pi k} = \frac{\ln(70/66)}{2\pi\times 240\,\text{W/m}\cdot\text{K}} = 3.90\times10^{-5}\,\text{m}\cdot\text{K/W},$$

$$R_{t,o} = \left(\eta_o h A'_t\right)^{-1},$$

$$\eta_o = 1 - \frac{N'A_f}{A'_t}\left(1-\eta_f\right),$$

$$A_f = 2\pi\left(r_{oc}^2 - r_b^2\right)$$

$$A'_t = N'A_f + \left(1 - N't\right)2\pi r_b$$

$$\eta_f = \frac{(2r_b/m)}{\left(r_{oc}^2 - r_b^2\right)}\frac{K_1(mr_b)I_1(mr_{oc}) - I_1(mr_b)K_1(mr_{oc})}{I_0(mr_1)K_1(mr_{oc}) + K_0(mr_b)I_1(mr_{oc})}$$

$$r_{oc} = r_o + (t/2), \qquad m = (2h/kt)^{1/2}$$

Continued...

PROBLEM 3.148 (Cont.)

Once the heat rate is determined from the foregoing expressions, the desired interface temperatures may be obtained from

$$T_i = T_g - q'R'_g$$

$$T_{1,i} = T_g - q'\left(R'_g + R'_w\right)$$

$$T_{1,o} = T_g - q'\left(R'_g + R'_w + R'_{t,c}\right)$$

$$T_b = T_g - q'\left(R'_g + R'_w + R'_{t,c} + R'_b\right)$$

For the specified conditions we obtain A'_t = 7.00 m, η_f = 0.902, η_o = 0.906 and $R'_{t,o}$ = 0.00158 m·K/W. It follows that

$$q' = 39,300\,\mathrm{W/m}$$ <

$$T_i = 405\mathrm{K}, \quad T_{1,i} = 393\mathrm{K}, \quad T_{1,o} = 384\mathrm{K}, \quad T_b = 382\mathrm{K}$$ <

(b) The *Performance Calculation, Extended Surface* Model for the *Circular Fin* Array may be used to assess the effects of fin thickness and spacing. Increasing the fin thickness to t = 3 mm, with δ = 2 mm, reduces the number of fins per unit length to 200. Hence, although the fin efficiency increases (η_f = 0.930), the reduction in the total surface area (A'_t = 5.72 m) yields an increase in the resistance of the fin array ($R'_{t,o}$ = 0.00188 m·K/W), and hence a reduction in the heat rate (q' = 38,700 W/m) and an increase in the interface temperatures (T_i = 415 K, $T_{1,i}$ = 404 K, $T_{1,o}$ = 394 K, and T_b = 393 K).

COMMENTS: Because the gas convection resistance exceeds all other resistances by at least an order of magnitude, incremental changes in $R_{t,o}$ will not have a significant effect on q' or the interface temperatures.

PROBLEM 3.149

KNOWN: Dimensions of finned aluminum sleeve inserted over transistor. Contact resistance and convection conditions.

FIND: Measures for increasing heat dissipation.

SCHEMATIC: See Example 3.10.

ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible heat transfer from top and bottom of transistor, (3) One-dimensional radial heat transfer, (4) Constant properties, (5) Negligible radiation.

ANALYSIS: With $2\pi r_2$ = 0.0188 m and Nt = 0.0084 m, the existing gap between fins is extremely small (0.87 mm). Hence, by increasing N and/or t, it would become even more difficult to maintain satisfactory airflow between the fins, and this option is not particularly attractive.

Because the fin efficiency for the prescribed conditions is close to unity (η_f = 0.998), there is little advantage to replacing the aluminum with a material of higher thermal conductivity (e.g. Cu with k ~ 400 W/m·K). However, the large value of η_f suggests that significant benefit could be gained by increasing the fin length, $L = r_3 - r_2$.

It is also evident that the thermal contact resistance is large, and from Table 3.2, it's clear that a significant reduction could be effected by using indium foil or a conducting grease in the contact zone. Specifically, a reduction of $R''_{t,c}$ from 10^{-3} to 10^{-4} or even 10^{-5} m^2·K/W is certainly feasible.

Table 1.1 suggests that, by increasing the velocity of air flowing over the fins, a larger convection coefficient may be achieved. A value of h = 100 W/m^2·K would not be unreasonable.

As options for enhancing heat transfer, we therefore use the IHT *Performance Calculation, Extended Surface Model* for the *Straight Fin Array* to explore the effect of parameter variations over the ranges $10 \le L \le 20$ mm, $10^{-5} \le R''_{t,c} \le 10^{-3}$ m^2·K/W and $25 \le h \le 100$ W/m^2·K. As shown below, there is a significant enhancement in heat transfer associated with reducing $R''_{t,c}$ from 10^{-3} to 10^{-4} m^2·K/W, for which $R_{t,c}$ decreases from 13.26 to 1.326 K/W. At this value of $R''_{t,c}$, the reduction in $R_{t,o}$ from 23.45 to 12.57 K/W which accompanies an increase in L from 10 to 20 mm becomes significant, yielding a heat rate of q_t = 4.30 W for $R''_{t,c} = 10^{-4}$ m^2·K/W and L = 20 mm. However, since $R_{t,o} >> R_{t,c}$, little benefit is gained by further reducing $R''_{t,c}$ to 10^{-5} m^2·K/W.

Heat rate, qt(W)
Fin length, L(m)
h = 25 W/m^2.K, R"t,c = E-3 m^2.K/W
h = 25 W/m^2.K, R"t,c = E-4 m^2.K/W
h = 25 W/m^2.K, R"t,c = E-5m^2.K/W

Continued...

PROBLEM 3.149 (Cont.)

To derive benefit from a reduction in $R''_{t,c}$ to 10^{-5} $m^2 \cdot K/W$, an additional reduction in $R_{t,o}$ must be made. This can be achieved by increasing h, and for L = 20 mm and h = 100 $W/m^2 \cdot K$, $R_{t,o}$ = 3.56 K/W. With $R''_{t,c}$ = 10^{-5} $m^2 \cdot K/W$, a value of q_t = 16.04 W may be achieved.

Heat rate, qt(W)

20
16
12
8
4
0

0.01 0.012 0.014 0.016 0.018 0.02

Fin length, L(m)

h = 25 W/m^2.K, R"t,c = E-5 m^2.K/W
h = 50 W/m^2.K, R"t,c = E-5 m^2.K/W
h = 100 W/m^2.K, R"t,c = E-5 m^2.K/W

COMMENTS: In assessing options for enhancing heat transfer, the limiting (largest) resistance(s) should be identified and efforts directed at their reduction.

PROBLEM 3.150

KNOWN: Diameter and internal fin configuration of copper tubes submerged in water. Tube wall temperature and temperature and convection coefficient of gas flow through the tube.

FIND: Rate of heat transfer per tube length.

SCHEMATIC:

T_g = 750 K
h = 30 W/m²·K
D = 50 mm
T_s = 350 K
h, T_g
t = 5 mm
Copper fin (k = 400 W/m·K)
L = 25 mm
$W = \pi D$
T_s

ASSUMPTIONS: (1) Steady-state, (2) One-dimensional fin conduction, (3) Constant properties, (4) Negligible radiation, (5) Uniform convection coefficient, (6) Tube wall may be unfolded and represented as a plane wall with four straight, rectangular fins, each with an adiabatic tip.

ANALYSIS: The rate of heat transfer per unit tube length is:

$$q_t' = \eta_o h A_t' \left(T_g - T_s\right)$$

$$\eta_o = 1 - \frac{NA_f'}{A_t'}\left(1 - \eta_f\right)$$

$$NA_f' = 4 \times 2L = 8\left(0.025\text{m}\right) = 0.20\text{m}$$

$$A_t' = NA_f' + A_b' = 0.20\text{m} + \left(\pi D - 4t\right) = 0.20\text{m} + \left(\pi \times 0.05\text{m} - 4 \times 0.005\text{m}\right) = 0.337\text{m}$$

For an adiabatic fin tip,

$$\eta_f = \frac{q_f}{q_{max}} = \frac{M \tanh mL}{h\left(2L \cdot 1\right)\left(T_g - T_s\right)}$$

$$M = \left[h2\left(1\text{m} + t\right)k\left(1\text{m} \times t\right)\right]^{1/2}\left(T_g - T_s\right) \approx \left[30\,\text{W/m}^2 \cdot \text{K}\left(2\text{m}\right)400\,\text{W/m}\cdot\text{K}\left(0.005\text{m}^2\right)\right]^{1/2}\left(400\text{K}\right) = 4382\text{W}$$

$$mL = \left\{\left[h2\left(1\text{m} + t\right)\right]/\left[k\left(1\text{m} \times t\right)\right]\right\}^{1/2} L \approx \left[\frac{30\,\text{W/m}^2 \cdot \text{K}\left(2\text{m}\right)}{400\,\text{W/m}\cdot\text{K}\left(0.005\text{m}^2\right)}\right]^{1/2} 0.025\text{m} = 0.137$$

Hence, tanh mL = 0.136, and

$$\eta_f = \frac{4382\text{W}\left(0.136\right)}{30\,\text{W/m}^2 \cdot \text{K}\left(0.05\text{m}^2\right)\left(400\text{K}\right)} = \frac{595\text{W}}{600\text{W}} = 0.992$$

$$\eta_o = 1 - \frac{0.20}{0.337}\left(1 - 0.992\right) = 0.995$$

$$q_t' = 0.995\left(30\,\text{W/m}^2 \cdot \text{K}\right)0.337\text{m}\left(400\text{K}\right) = 4025\,\text{W/m}$$

COMMENTS: Alternatively, $q_t' = 4q_f' + h\left(A_t' - A_f'\right)\left(T_g - T_s\right)$. Hence, q' = 4(595 W/m) + 30 W/m²·K (0.137 m)(400 K) = (2380 + 1644) W/m = 4024 W/m.

PROBLEM 3.151

KNOWN: Internal and external convection conditions for an internally finned tube. Fin/tube dimensions and contact resistance.

FIND: Heat rate per unit tube length and corresponding effects of the contact resistance, number of fins, and fin/tube material.

SCHEMATIC:

N fins (t = 5 mm)
$h_g = 30\ W/m^2\cdot K$
$T_g = 750\ K$
$R''_{t,c} = 10^{-4}\ m^2\cdot K/W$
$r_1 = 25$ mm
$r_2 = 30$ mm
$h_w = 2000\ W/m^2\cdot K$
$T_w = 350\ K$

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat transfer, (3) Constant properties, (4) Negligible radiation, (5) Uniform convection coefficient on finned surfaces, (6) Tube wall may be unfolded and approximated as a plane surface with N straight rectangular fins.

PROPERTIES: Copper: k = 400 W/m·K; St.St.: k = 20 W/m·K.

ANALYSIS: The heat rate per unit length may be expressed as

$$q' = \frac{T_g - T_w}{R'_{t,o(c)} + R'_{cond} + R'_{conv,o}}$$

where

$$R_{t,o(c)} = \left(\eta_{o(c)} h_g A'_t\right), \quad \eta_{o(c)} = 1 - \frac{NA'_f}{A'_t}\left(1 - \frac{\eta_f}{C_1}\right), \quad C_1 = 1 + \eta_f h_g A'_f \left(R''_{t,c}/A'_{c,b}\right),$$

$$A'_t = NA'_f + \left(2\pi r_1 - Nt\right), \quad A'_f = 2r_1, \quad \eta_f = \tanh mr_1/mr_1, \quad m = \left(2h_g/kt\right)^{1/2} \quad A'_{c,b} = t,$$

$$R'_{cond} = \frac{\ln\left(r_2/r_1\right)}{2\pi k}, \quad \text{and} \quad R'_{conv,o} = \left(2\pi r_2 h_w\right)^{-1}.$$

Using the IHT *Performance Calculation, Extended Surface Model* for the *Straight Fin Array*, the following results were obtained. For the *base case*, q' = 3857 W/m, where $R'_{t,o(c)}$ = 0.101 m·K/W, R'_{cond} = 7.25×10^{-5} m·K/W and $R'_{conv,o}$ = 0.00265 m·K/W. If the contact resistance is eliminated ($R''_{t,c}$ = 0), q' = 3922 W/m, where $R'_{t,o}$ = 0.0993 m·K/W. If the number of fins is increased to N = 8, q' = 5799 W/m, with $R'_{t,o(c)}$ = 0.063 m·K/W. If the material is changed to stainless steel, q' = 3591 W/m, with $R'_{t,o(c)}$ = 0.107 m·K/W and R'_{cond} = 0.00145 m·K/W.

COMMENTS: The small reduction in q' associated with use of stainless steel is perhaps surprising, in view of the large reduction in k. However, because h_g is small, the reduction in k does not significantly reduce the fin efficiency (η_f changes from 0.994 to 0.891). Hence, the heat rate remains large. The influence of k would become more pronounced with increasing h_g.

PROBLEM 3.152

KNOWN: Design and operating conditions of a tubular, air/water heater.

FIND: (a) Expressions for heat rate per unit length at inner and outer surfaces, (b) Expressions for inner and outer surface temperatures, (c) Surface heat rates and temperatures as a function of volumetric heating $\dot{q}$ for prescribed conditions. Upper limit to $\dot{q}$.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state, (2) Constant properties, (3) One-dimensional heat transfer.

PROPERTIES: Table A-1: Aluminum, T = 300 K, k_a = 237 W/m·K.

ANALYSIS: (a) Applying Equation C.8 to the inner and outer surfaces, it follows that

$$q'(r_i) = \dot{q}\pi r_i^2 - \frac{2\pi k_s}{\ln(r_o/r_i)}\left[\frac{\dot{q}r_o^2}{4k_s}\left(1-\frac{r_i^2}{r_o^2}\right)+\left(T_{s,o}-T_{s,i}\right)\right]$$ <

$$q'(r_o) = \dot{q}\pi r_o^2 - \frac{2\pi k_s}{\ln(r_o/r_i)}\left[\frac{\dot{q}r_o^2}{4k_s}\left(1-\frac{r_i^2}{r_o^2}\right)+\left(T_{s,o}-T_{s,i}\right)\right]$$ <

(b) From Equations C.16 and C.17, energy balances at the inner and outer surfaces are of the form

$$h_i\left(T_{\infty,i}-T_{s,i}\right) = \frac{\dot{q}r_i}{2} - \frac{k_s\left[\frac{\dot{q}r_o^2}{4k_s}\left(1-\frac{r_i^2}{r_o^2}\right)+\left(T_{s,o}-T_{s,i}\right)\right]}{r_i\ln(r_o/r_i)}$$ <

$$U_o\left(T_{s,o}-T_{\infty,o}\right) = \frac{\dot{q}r_o}{2} - \frac{k_s\left[\frac{\dot{q}r_o^2}{4k_s}\left(1-\frac{r_i^2}{r_o^2}\right)+\left(T_{s,o}-T_{s,i}\right)\right]}{r_o\ln(r_o/r_i)}$$ <

Accounting for the fin array and the contact resistance, Equation 3.104 may be used to cast the overall heat transfer coefficient U_o in the form

$$U_o = \frac{q'(r_o)}{A'_w\left(T_{s,o}-T_{\infty,o}\right)} = \frac{1}{A'_w R'_{t,o(c)}} = \frac{A'_t}{A'_w}\eta_{o(c)}h_o$$

where $\eta_{o(c)}$ is determined from Equations 3.105a,b and $A'_w = 2\pi r_o$.

Continued...

PROBLEM 3.152 (Cont.)

(c) For the prescribed conditions and a representative range of $10^7 \le \dot{q} \le 10^8$ W/m^3, use of the relations of part (b) with the capabilities of the IHT *Performance Calculation Extended Surface Model* for a *Circular Fin Array* yields the following graphical results.

It is in this range that the upper limit of $T_{s,i}$ = 373 K is exceeded for $\dot{q} = 4.9 \times 10^7$ W/m^3, while the corresponding value of $T_{s,o}$ = 379 K is well below the prescribed upper limit. The expressions of part (a) yield the following results for the surface heat rates, where heat transfer in the negative r direction corresponds to $q'(r_i) < 0$.

For $\dot{q} = 4.9 \times 10^7$ W/m^3, $q'(r_i) = -2.30 \times 10^4$ W/m and $q'(r_o) = 1.93 \times 10^4$ W/m.

COMMENTS: The foregoing design provides for comparable heat transfer to the air and water streams. This result is a consequence of the nearly equivalent thermal resistances associated with heat transfer from the inner and outer surfaces. Specifically, $R'_{conv,i} = (h_i 2\pi r_i)^{-1}$ = 0.00318 m·K/W is slightly smaller than $R'_{t,o(c)}$ = 0.00411 m·K/W, in which case $|q'(r_i)|$ is slightly larger than $q'(r_o)$, while $T_{s,i}$ is slightly smaller than $T_{s,o}$. Note that the solution must satisfy the energy conservation requirement, $\pi\left(r_o^2 - r_i^2\right)\dot{q} = |q'(r_i)| + q'(r_o)$.

PROBLEM 4.1

KNOWN: Method of separation of variables (Section 4.2) for two-dimensional, steady-state conduction.

FIND: Show that negative or zero values of λ^2, the separation constant, result in solutions which cannot satisfy the boundary conditions.

SCHEMATIC:

ASSUMPTIONS: (1) Two-dimensional, steady-state conduction, (2) Constant properties.

ANALYSIS: From Section 4.2, identification of the separation constant λ^2 leads to the two ordinary differential equations, 4.6 and 4.7, having the forms

$$\frac{d^2X}{dx^2} + l^2 X = 0 \qquad\qquad \frac{d^2Y}{dy^2} - l^2 Y = 0 \qquad (1,2)$$

and the temperature distribution is

$$q(x,y) = X(x) \cdot Y(y). \qquad (3)$$

Consider now the situation when $\lambda^2 = 0$. From Eqs. (1), (2), and (3), find that

$$X = C_1 + C_2 x, \qquad Y = C_3 + C_4 y \quad \text{and} \quad q(x,y) = (C_1 + C_2 x)\ (C_3 + C_4 y). \qquad (4)$$

Evaluate the constants - C_1, C_2, C_3 and C_4 - by substitution of the boundary conditions:

$$x = 0: \quad q(0,y) = (C_1 + C_2 \cdot 0)(C_3 + C_4 y) = 0 \qquad C_1 = 0$$

$$y = 0: \quad q(x,0) = (0 + C_2 X)(C_3 + C_4 \cdot 0) = 0 \qquad C_3 = 0$$

$$x = L: \quad q(L,0) = (0 + C_2 L)(0 + C_4 y) = 0 \qquad C_2 = 0$$

$$y = W: \quad q(x,W) = (0 + 0 \cdot x)(0 + C_4 W) = 1 \qquad 0 \neq 1$$

The last boundary condition leads to an impossibility ($0 \neq 1$). We therefore conclude that a λ^2 value of zero will not result in a form of the temperature distribution which will satisfy the boundary conditions. Consider now the situation when $\lambda^2 < 0$. The solutions to Eqs. (1) and (2) will be

$$X = C_5 e^{-lx} + C_6 e^{+lx}, \qquad Y = C_7 \cos ly + C_8 \sin ly \qquad (5,6)$$

and

$$q(x,y) = \left[C_5 e^{-lx} + C_6 e^{+lx}\right]\ \left[C_7 \cos ly + C_8 \sin ly\right]. \qquad (7)$$

Evaluate the constants for the boundary conditions.

$$y = 0: \quad q(x,0) = \left[C_5 e^{-lx} + C_6 e^{-lx}\right]\ \left[C_7 \cos 0 + C_8 \sin 0\right] = 0 \qquad C_7 = 0$$

$$x = 0: \quad q(0,y) = \left[C_5 e^0 + C_6 e^0\right]\ \left[0 + C_8 \sin ly\right] = 0 \qquad C_8 = 0$$

If $C_8 = 0$, a trivial solution results or $C_5 = -C_6$.

$$x = L: \quad q(L,y) = C_5\left[e^{-xL} - e^{+xL}\right] C_8 \sin ly = 0.$$

From the last boundary condition, we require C_5 or C_8 is zero; either case leads to a trivial solution with either no x or y dependence.

PROBLEM 4.2

KNOWN: Two-dimensional rectangular plate subjected to prescribed uniform temperature boundary conditions.

FIND: Temperature at the mid-point using the exact solution considering the first five non-zero terms; assess error resulting from using only first three terms. Plot the temperature distributions T(x,0.5) and T(1,y).

SCHEMATIC:

y(m), W=1, T_2 = 150 °C, T_1 = 50 °C, T_1 = 50 °C, (1,0.5), 0, L = 2, x(m), T_1 = 50 °C

ASSUMPTIONS: (1) Two-dimensional, steady-state conduction, (2) Constant properties.

ANALYSIS: From Section 4.2, the temperature distribution is

$$\theta(x,y) \equiv \frac{T-T_1}{T_2-T_1} = \frac{2}{\pi}\sum_{n=1}^{\theta} \frac{(-1)^{n+1}+1}{n} \sin\left(\frac{n\pi x}{L}\right) \cdot \frac{\sinh(n\pi y/L)}{\sinh(n\pi W/L)}. \qquad (1,4.19)$$

Considering now the point (x,y) = (1.0,0.5) and recognizing x/L = 1/2, y/L = 1/4 and W/L = 1/2,

$$\theta(1,0.5) \equiv \frac{T-T_1}{T_2-T_1} = \frac{2}{\pi}\sum_{n=1}^{\theta} \frac{(-1)^{n+1}+1}{n} \sin\left(\frac{n\pi}{2}\right) \cdot \frac{\sinh(n\pi/4)}{\sinh(n\pi/2)}.$$

When n is even (2, 4, 6 ...), the corresponding term is zero; hence we need only consider n = 1, 3, 5, 7 and 9 as the first five non-zero terms.

$$\theta(1,0.5) = \frac{2}{\pi}\left\{2\sin\left(\frac{\pi}{2}\right)\frac{\sinh(\pi/4)}{\sinh(\pi/2)} + \frac{2}{3}\sin\left(\frac{3\pi}{2}\right)\frac{\sinh(3\pi/4)}{\sinh(3\pi/2)} + \right.$$

$$\left.\frac{2}{5}\sin\left(\frac{5\pi}{2}\right)\frac{\sinh(5\pi/4)}{\sinh(5\pi/2)} + \frac{2}{7}\sin\left(\frac{7\pi}{2}\right)\frac{\sinh(7\pi/4)}{\sinh(7\pi/2)} + \frac{2}{9}\sin\left(\frac{9\pi}{2}\right)\frac{\sinh(9\pi/4)}{\sinh(9\pi/2)}\right\}$$

$$\theta(1,0.5) = \frac{2}{\pi}[0.755 - 0.063 + 0.008 - 0.001 + 0.000] = 0.445 \qquad (2)$$

$$T(1,0.5) = \theta(1,0.5)(T_2 - T_1) + T_1 = 0.445(150-50) + 50 = 94.5^\circ C.$$ <

If only the first three terms of the series, Eq. (2), are considered, the result will be θ(1,0.5) = 0.46; that is, there is less than a 0.2% effect.

Using Eq. (1), and writing out the first five terms of the series, expressions for θ(x,0.5) or T(x,0.5) and θ(1,y) or T(1,y) were keyboarded into the IHT workspace and evaluated for sweeps over the x or y variable. Note that for T(1,y), that as y → 1, the upper boundary, T(1,1) is greater than 150°C. Upon examination of the magnitudes of terms, it becomes evident that more than 5 terms are required to provide an accurate solution.

T(x,0.5) or T(1,y), C
150
130
110
90
70
50
0 0.2 0.4 0.6 0.8 1
x or y coordinate (m)
T(1,y)
T(x,0.5)

PROBLEM 4.3

KNOWN: Temperature distribution in the two-dimensional rectangular plate of Problem 4.2.

FIND: Expression for the heat rate per unit thickness from the lower surface ($0 \le x \le 2$, 0) and result based on first five non-zero terms of the infinite series.

SCHEMATIC:

ASSUMPTIONS: (1) Two-dimensional, steady-state conduction, (2) Constant properties.

ANALYSIS: The heat rate per unit thickness *from the plate* along the lower surface is

$$q'_{out} = -\int_{x=0}^{x=2} dq'_y(x,0) = -\int_{x=0}^{x=2} -k\left.\frac{\partial T}{\partial y}\right|_{y=0} dx = k(T_2 - T_1)\int_{x=0}^{x=2}\left.\frac{\partial \theta}{\partial y}\right|_{y=0} dx \qquad (1)$$

where from the solution to Problem 4.2,

$$\theta \equiv \frac{T - T_1}{T_2 - T_1} = \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}+1}{n}\sin\left(\frac{n\pi x}{L}\right)\frac{\sinh(n\pi y/L)}{\sinh(n\pi W/L)}. \qquad (2)$$

Evaluate the gradient of θ from Eq. (2) and substitute into Eq. (1) to obtain

$$q'_{out} = k(T_2 - T_1)\int_{x=0}^{x=2}\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}+1}{n}\sin\left(\frac{n\pi x}{L}\right)\left.\frac{(n\pi/L)\cosh(n\pi y/L)}{\sinh(n\pi W/L)}\right|_{y=0} dx$$

$$q'_{out} = k(T_2 - T_1)\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}+1}{n}\frac{1}{\sinh(n\pi W/L)}\left[-\cos\left(\frac{n\pi x}{L}\right)\Bigg|_{x=0}^{2}\right]$$

$$q'_{out} = k(T_2 - T_1)\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}+1}{n}\frac{1}{\sinh(n\pi/L)}\left[1-\cos(n\pi)\right]$$ <

To evaluate the first five, non-zero terms, recognize that since $\cos(n\pi) = 1$ for n = 2, 4, 6 ..., only the n-odd terms will be non-zero. Hence,

Continued …..

$$q'_{out} = 50\,W/m\cdot K(150-50)^\circ C\frac{2}{\pi}\left\{\frac{(-1)^2+1}{1}\cdot\frac{1}{\sinh(\pi/2)}(2)+\frac{(-1)^4+1}{3}\cdot\frac{1}{\sinh(3\pi/2)}\cdot(2)\right.$$

$$\left.+\frac{(-1)^6+1}{5}\cdot\frac{1}{\sinh(5\pi/2)}(2)+\frac{(-1)^8+1}{7}\cdot\frac{1}{\sinh(7\pi/2)}(2)+\frac{(-1)^{10}+1}{9}\cdot\frac{1}{\sinh(9\pi/2)}(2)\right\}$$

$$q'_{out} = 3.183\,kW/m[1.738+0.024+0.00062+(...)] = 5.611\,kW/m$$ <

COMMENTS: If the foregoing procedure were used to evaluate the heat rate into the upper surface, $q'_{in} = -\int_{x=0}^{x=2} dq'_y(x,W)$, it would follow that

$$q'_{in} = k(T_2-T_1)\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}+1}{n}\coth(n\pi/2)[1-\cos(n\pi)]$$

However, with coth(nπ/2) ≥ 1, irrespective of the value of n, and with $\sum_{n=1}^{\infty}\left[(-1)^{n+1}+1\right]/n$ being a divergent series, the complete series does not converge and $q'_{in}\to\infty$. This physically untenable condition results from the temperature discontinuities imposed at the upper left and right corners.

PROBLEM 4.4

KNOWN: Rectangular plate subjected to prescribed boundary conditions.

FIND: Steady-state temperature distribution.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state, 2-D conduction, (2) Constant properties.

ANALYSIS: The solution follows the method of Section 4.2. The product solution is

$$T(x,y) = X(x)\cdot Y(y) = (C_1 \cos \lambda x + C_2 \sin \lambda x)\left(C_3 e^{-\lambda y} + C_4 e^{+\lambda y}\right)$$

and the boundary conditions are: T(0,y) = 0, T(a,y) = 0, T(x,0) = 0, T(x.b) = Ax. Applying BC#1, T(0,y) = 0, find $C_1 = 0$. Applying BC#2, T(a,y) = 0, find that $\lambda = n\pi/a$ with n = 1,2,…. Applying BC#3, T(x,0) = 0, find that $C_3 = -C_4$. Hence, the product solution is

$$T(x,y) = X(x)\cdot Y(y) = C_2 C_4 \sin\left[\frac{n\pi}{a}x\right]\left(e^{+\lambda y} - e^{-\lambda y}\right).$$

Combining constants and using superposition, find

$$T(x,y) = \sum_{n=1}^{\infty} C_n \sin\left[\frac{n\pi x}{a}\right] \sinh\left[\frac{n\pi y}{a}\right].$$

To evaluate C_n, use orthogonal functions with Eq. 4.16 to find

$$C_n = \int_0^a Ax\cdot\sin\left[\frac{n\pi x}{a}\right]\cdot dx/\sinh\left[\frac{n\pi b}{a}\right]\int_0^a \sin^2\left[\frac{n\pi x}{a}\right]dx,$$

noting that y = b. The numerator, denominator and C_n, respectively, are:

$$A\int_0^a x\cdot\sin\frac{n\pi x}{a}\cdot dx = A\left[\left[\frac{a}{n\pi}\right]^2 \sin\left[\frac{n\pi x}{a}\right] - \frac{ax}{n\pi}\cos\left[\frac{n\pi x}{a}\right]\right]_0^a = \frac{Aa^2}{n\pi}\left[-\cos(n\pi)\right] = \frac{Aa^2}{n\pi}(-1)^{n+1},$$

$$\sinh\left[\frac{n\pi b}{a}\right]\int_0^a \sin^2\frac{n\pi x}{a}\cdot dx = \sinh\left[\frac{n\pi b}{a}\right]\left[\frac{1}{2}x - \frac{1}{4n\pi}\sin\left[\frac{2n\pi x}{a}\right]\right]_0^a = \frac{a}{2}\cdot\sinh\left[\frac{n\pi b}{a}\right],$$

$$C_n = \frac{Aa^2}{n\pi}(-1)^{n+1}/\frac{a}{2}\sinh\left[\frac{n\pi b}{a}\right] = 2Aa\,(-1)^{n+1}/n\pi\,\sinh\left[\frac{n\pi b}{a}\right].$$

Hence, the temperature distribution is

$$T(x,y) = \frac{2\,Aa}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\cdot\sin\left[\frac{n\pi x}{a}\right]\frac{\sinh\left[\frac{n\pi y}{a}\right]}{\sinh\left[\frac{n\pi b}{a}\right]}.$$

<

PROBLEM 4.5

KNOWN: Long furnace of refractory brick with prescribed surface temperatures and material thermal conductivity.

FIND: Shape factor and heat transfer rate per unit length using the flux plot method

SCHEMATIC:

ASSUMPTIONS: (1) Furnace length normal to page, ℓ, >> cross-sectional dimensions, (2) Two-dimensional, steady-state conduction, (3) Constant properties.

ANALYSIS: Considering the cross-section, the cross-hatched area represents a symmetrical element. Hence, the heat rte for the entire furnace per unit length is

$$q' = \frac{q}{\ell} = 4\frac{S}{\ell}k(T_1 - T_2) \tag{1}$$

where S is the shape factor for the symmetrical section. Selecting three temperature increments (N = 3), construct the flux plot shown below.

From Eq. 4.26, $S = \frac{M\ell}{N}$ or $\frac{S}{\ell} = \frac{M}{N} = \frac{8.5}{3} = 2.83$ <

and from Eq. (1), $q' = 4 \times 2.83 \times 1.2 \frac{W}{m \cdot K}(600 - 60)^\circ C = 7.34 \text{ kW/m}.$ <

COMMENTS: The shape factor can also be estimated from the relations of Table 4.1. The symmetrical section consists of two plane walls (horizontal and vertical) with an adjoining edge. Using the appropriate relations, the numerical values are, in the same order,

$$S = \frac{0.75m}{0.5m}\ell + 0.54\ell + \frac{0.5m}{0.5m}\ell = 3.04\ell$$

Note that this result compares favorably with the flux plot result of 2.83ℓ.

PROBLEM 4.6

KNOWN: Hot pipe embedded eccentrically in a circular system having a prescribed thermal conductivity.

FIND: The shape factor and heat transfer per unit length for the prescribed surface temperatures.

SCHEMATIC:

$T_1=150°C$
$D=20mm$
$k=0.5 W/m\cdot K$
$T_2=35°C$
$R=40mm$
Cross-hatched region is a symmetrical section; length measured normal to page is ℓ.

ASSUMPTIONS: (1) Two-dimensional conduction, (2) Steady-state conditions, (3) Length ℓ >> diametrical dimensions.

ANALYSIS: Considering the cross-sectional view of the pipe system, the symmetrical section shown above is readily identified. Selecting four temperature increments (N = 4), construct the flux plot shown below.

N= 1 2 3 4
1 2 3 4 5 6 7 8
$T_1=150°C$
$T_2=35°C$
Estimate as 1/2 heat flow lane, M=8.5

For the symmetrical section:
$S_o = M\ell/N = \frac{8.5}{4}\ell = 2.13\ell$

For the pipe system:
$S = 2S_o = 4.26\ell$

For the pipe system, the heat rate per unit length is

$$q' = \frac{q}{\ell} = kS(T_1 - T_2) = 0.5\frac{W}{m \cdot K} \times 4.26(150-35)^{\circ} C = 245 \text{ W/m.}$$ <

COMMENTS: Note that in the lower, right-hand quadrant of the flux plot, the curvilinear squares are irregular. Further work is required to obtain an improved plot and, hence, obtain a more accurate estimate of the shape factor.

PROBLEM 4.7

KNOWN: Structural member with known thermal conductivity subjected to a temperature difference.

FIND: (a) Temperature at a prescribed point P, (b) Heat transfer per unit length of the strut, (c) Sketch the 25, 50 and 75°C isotherms, and (d) Same analysis on the shape but with adiabatic-isothermal boundary conditions reversed.

SCHEMATIC:

ASSUMPTIONS: (1) Two-dimensional conduction, (2) Steady-state conditions, (3) Constant properties.

ANALYSIS: (a) Using the methodology of Section 4.3.1, construct a flux plot. Note the line of symmetry which passes through the point P is an isotherm as shown above. It follows that

$$T(P) = (T_1 + T_2)/2 = (100 + 0)^\circ C/2 = 50^\circ C.$$ <

(b) The flux plot on the symmetrical section is now constructed to obtain the shape factor from which the heat rate is determined. That is, from Eq. 4.25 and 4.26,

$$q = kS(T_1 - T_2) \quad \text{and} \quad S = M\ell/N. \tag{1,2}$$

From the plot of the symmetrical section,

$$S_o = 4.2\ell/4 = 1.05\ell.$$

For the full section of the strut,

$$M = M_o = 4.2$$

but N = $2N_o$ = 8. Hence,

$$S = S_o/2 = 0.53\ell$$

and with $q' = q/\ell$, giving

$$q'/\ell = 75\,W/m\cdot K \times 0.53(100 - 0)^\circ C = 3975\,W/m.$$ <

(c) The isotherms for T = 50, 75 and 100°C are shown on the flux plot. The T = 25°C isotherm is symmetric with the T = 75°C isotherm.

(d) By reversing the adiabatic and isothermal boundary conditions, the two-dimensional shape appears as shown in the sketch below. The symmetrical element to be flux plotted is the same as for the strut, except the symmetry line is now an adiabat.

Continued...

PROBLEM 4.7 (Cont.)

From the flux plot, find $M_o = 3.4$ and $N_o = 4$, and from Eq. (2)

$$S_o = M_o \ell / N_o = 3.4\ell/4 = 0.85\ell \qquad S = 2S_o = 1.70\ell$$

and the heat rate per unit length from Eq. (1) is

$$q' = 75\,\text{W/m}\cdot\text{K} \times 1.70\,(100-0)^\circ\text{C} = 12{,}750\,\text{W/m}$$ <

From the flux plot, estimate that

$$T(P) \approx 40^\circ\text{C}.$$ <

COMMENTS: (1) By inspection of the shapes for parts (a) and (b), it is obvious that the heat rate for the latter will be greater. The calculations show the heat rate is greater by more than a factor of three.

(2) By comparing the flux plots for the two configurations, and corresponding roles of the adiabats and isotherms, would you expect the shape factor for parts (a) to be the reciprocal of part (b)?

PROBLEM 4.8

KNOWN: Relative dimensions and surface thermal conditions of a V-grooved channel.

FIND: Flux plot and shape factor.

SCHEMATIC:

W/4
T_1
$T_1 > T_2$
W/2
Cross section of solid
T_2
W

ASSUMPTIONS: (1) Two-dimensional conduction, (2) Steady-state conditions, (3) Constant properties.

ANALYSIS: With symmetry about the midplane, only one-half of the object need be considered as shown below.

Choosing 6 temperature increments (N = 6), it follows from the plot that M ≈ 7. Hence from Eq. 4.26, the shape factor for the half section is

$$S = \frac{M}{N}\ell = \frac{7}{6}\ell = 1.17\ell.$$

For the complete system, the shape factor is then

$$S = 2.34\ell.$$ <

T_1
N
1
2
3
4
5
6
M= 7 6 5 4 3 2 1
T_2
Symmetry adiabat

PROBLEM 4.9

KNOWN: Long conduit of inner circular cross section and outer surfaces of square cross section.

FIND: Shape factor and heat rate for the two applications when outer surfaces are insulated or maintained at a uniform temperature.

SCHEMATIC:

ASSUMPTIONS: (1) Two-dimensional, steady-state conduction, (2) Constant properties and (3) Conduit is very long.

ANALYSIS: The adiabatic symmetry lines for each of the applications is shown above. Using the flux plot methodology and selecting four temperature increments (N = 4), the flux plots are as shown below.

For the symmetrical sections, $S = 2S_o$, where $S_o = M\ell/N$ and the heat rate for each application is $q = 2(S_o/\ell)k(T_1 - T_2)$.

Application	M	N	S_o/ℓ	q' (W/m)	
A	10.3	4	2.58	11,588	<
B	6.2	4	1.55	6,975	<

COMMENTS: (1) For application A, most of the heat lanes leave the inner surface (T_1) on the upper portion.

(2) For application B, most of the heat flow lanes leave the inner surface on the upper portion (that is, lanes 1-4). Because the lower, right-hand corner is insulated, the entire section experiences small heat flows (lane 6 + 0.2). Note the shapes of the isotherms near the right-hand, insulated boundary and that they intersect the boundary normally.

PROBLEM 4.10

KNOWN: Shape and surface conditions of a support column.

FIND: (a) Heat transfer rate per unit length. (b) Height of a rectangular bar of equivalent thermal resistance.

SCHEMATIC:

0.3m, $T_1 = 100°C$, 0.3m, 0.3m, H, 0.6m, $T_2 = 0°C$

ASSUMPTIONS: (1)Steady-state conditions, (2) Negligible three-dimensional conduction effects, (3) Constant properties, (4) Adiabatic sides.

PROPERTIES: *Table A-1*, Steel, AISI 1010 (323K): k = 62.7 W/m·K.

ANALYSIS: (a) From the flux plot for the half section, M ≈ 5 and N ≈ 8. Hence for the full section

M= 1 2 3 4 5

N= 1 2 3 4 5 6 7 8

$$S = 2\frac{M\ell}{N} \approx 1.25\ell$$

$$q = Sk(T_1 - T_2)$$

$$q' \approx 1.25 \times 62.7 \frac{W}{m \cdot K}(100 - 0)^\circ C$$

$$q' \approx 7.8 \text{ kW/m}. \qquad <$$

(b) The rectangular bar provides for one-dimensional heat transfer. Hence,

$$q = k\,A\frac{(T_1 - T_2)}{H} = k(0.3\ell)\frac{(T_1 - T_2)}{H}$$

Hence,

$$H = \frac{0.3k(T_1 - T_2)}{q'} = \frac{0.3m(62.7\text{ W/m}\cdot\text{K})(100^\circ C)}{7800\text{ W/m}} = 0.24m. \qquad <$$

COMMENTS: The fact that H < 0.3m is consistent with the requirement that the thermal resistance of the trapezoidal column must be less than that of a rectangular bar of the same height and top width (because the width of the trapezoidal column increases with increasing distance, x, from the top). Hence, if the rectangular bar is to be of equivalent resistance, it must be of smaller height.

PROBLEM 4.11

KNOWN: Hollow prismatic bars fabricated from plain carbon steel, 1m in length with prescribed temperature difference.

FIND: Shape factors and heat rate per unit length.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Two-dimensional conduction, (3) Constant properties.

PROPERTIES: *Table A-1*, Steel, Plain Carbon (400K), k = 57 W/m·K.

ANALYSIS: Construct a flux plot on the symmetrical sections (shaded-regions) of each of the bars.

The shape factors for the symmetrical sections are,

$$S_{o,A} = \frac{M\ell}{N} = \frac{4}{4}\ell = 1\ell \qquad S_{o,B} = \frac{M\ell}{N} = \frac{3.5}{4}\ell = 0.88\ell.$$

Since each of these sections is ¼ of the bar cross-section, it follows that

$$S_A = 4 \times 1\ell = 4\ell \qquad S_B = 4 \times 0.88\ell = 3.5\ell.$$ <

The heat rate per unit length is $q' = q/\ell = k(S/\ell)(T_1 - T_2)$,

$$q'_A = 57\frac{W}{m \cdot K} \times 4(500 - 300)K = 45.6 \text{ kW/m}$$ <

$$q'_B = 57\frac{W}{m \cdot K} \times 3.5(500 - 300)K = 39.9 \text{ kW/m}.$$ <

PROBLEM 4.12

KNOWN: Two-dimensional, square shapes, 1 m to a side, maintained at uniform temperatures as prescribed, perfectly insulated elsewhere.

FIND: Using the flux plot method, estimate the heat rate per unit length normal to the page if the thermal conductivity is 50 W/m·K

ASSUMPTIONS: (1) Steady-state, two-dimensional conduction, (2) Constant properties.

ANALYSIS: Use the methodology of Section 4.3.1 to construct the flux plots to obtain the shape factors from which the heat rates can be calculated. With Figure (a), begin at the lower-left side making the isotherms almost equally spaced, since the heat flow will only slightly spread toward the right. Start sketching the adiabats in the vicinity of the T_2 surface. The dashed line represents the adiabat which separates the shape into two segments. Having recognized this feature, it was convenient to identify partial heat lanes. Figure (b) is less difficult to analyze since the isotherm intervals are nearly regular in the lower left-hand corner.

(a) (b)

The shape factors are calculated from Eq. 4.26 and the heat rate from Eq. 4.25.

$$S' = \frac{M}{N} = \frac{0.5+3+0.5+0.5+0.2}{6}$$

$$S' = 0.70$$

$$q' = kS'(T_1 - T_2)$$

$$q' = 10\,\text{W/m}\cdot\text{K} \times 0.70(100-0)\text{K} = 700\,\text{W/m}$$

$$S' = \frac{M}{N} = \frac{4.5}{5} = 0.90$$

$$q' = kS'(T_1 - T_2)$$

$$q' = 10\,\text{W/m}\cdot\text{K} \times 0.90(100-0)\text{K} = 900\,\text{W/m}$$ <

COMMENTS: Using a finite-element package with a fine mesh, we determined heat rates of 956 and 915 W/m, respectively, for Figures (a) and (b). The estimate for the less difficult Figure (b) is within 2% of the numerical method result. For Figure (a), our flux plot result was 27% low.

PROBLEM 4.13

KNOWN: Uniform media of prescribed geometry.

FIND: (a) Shape factor expressions from thermal resistance relations for the plane wall, cylindrical shell and spherical shell, (b) Shape factor expression for the isothermal sphere of diameter D buried in an infinite medium.

ASSUMPTIONS: (1) Steady-state conditions, (2) Uniform properties.

ANALYSIS: (a) The relationship between the shape factor and thermal resistance of a shape follows from their definitions in terms of heat rates and overall temperature differences.

$$q = kS\Delta T \qquad (4.25), \qquad q = \frac{\Delta T}{R_t} \qquad (3.19), \qquad S = 1/kR_t \qquad (4.27)$$

Using the thermal resistance relations developed in Chapter 3, their corresponding shape factors are:

Plane wall: $$R_t = \frac{L}{kA} \qquad S = \frac{A}{L}. \qquad <$$

Cylindrical shell: $$R_t = \frac{\ln(r_2/r_1)}{2\pi Lk} \qquad S = \frac{2\pi L}{\ln r_2/r_1}. \qquad <$$

(*L into the page*)

Spherical shell: $$R_t = \frac{1}{4\pi k}\left[\frac{1}{r_1} - \frac{1}{r_2}\right] \qquad S = \frac{4\pi}{1/r_1 - 1/r_2}.$$

<

(b) The shape factor for the sphere of diameter D in an infinite medium can be derived using the alternative conduction analysis of Section 3.1. For this situation, q_r is a constant and Fourier's law has the form

$$q_r = -k\left(4\pi r^2\right)\frac{dT}{dr}.$$

Separate variables, identify limits and integrate.

$$-\frac{q_r}{4\pi k}\int_{D/2}^{\infty}\frac{dr}{r^2} = \int_{T_1}^{T_2} dT \qquad -\frac{q_r}{4\pi k}\left[-\frac{1}{r}\right]_{D/2}^{\infty} = -\frac{q_r}{4\pi k}\left[0 - \frac{2}{D}\right] = \left(T_2 - T_1\right)$$

$$q_r = 4\pi k\left[\frac{D}{2}\right]\left(T_1 - T_2\right) \qquad \text{or} \qquad S = 2\pi D. \qquad <$$

COMMENTS: Note that the result for the buried sphere, $S = 2\pi D$, can be obtained from the expression for the spherical shell with $r_2 = \infty$. Also, the shape factor expression for the "isothermal sphere buried in a semi-infinite medium" presented in Table 4.1 provides the same result with $z \to \infty$.

PROBLEM 4.14

KNOWN: Heat generation in a buried spherical container.

FIND: (a) Outer surface temperature of the container, (b) Representative isotherms and heat flow lines.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Soil is a homogeneous medium with constant properties.

PROPERTIES: *Table A-3*, Soil (300K): k = 0.52 W/m·K.

ANALYSIS: (a) From an energy balance on the container, $q = \dot{E}_g$ and from the first entry in Table 4.1,

$$q = \frac{2\pi D}{1 - D/4z} k \left(T_1 - T_2\right).$$

Hence,

$$T_1 = T_2 + \frac{q}{k} \frac{1 - D/4z}{2\pi D} = 20^\circ C + \frac{500W}{0.52 \frac{W}{m \cdot K}} \frac{1 - 2m/40m}{2\pi \left(2m\right)} = 92.7^\circ C$$ <

(b) The isotherms may be viewed as spherical surfaces whose center moves downward with increasing radius. The surface of the soil is an isotherm for which the center is at $z = \infty$.

PROBLEM 4.15

KNOWN: Temperature, diameter and burial depth of an insulated pipe.

FIND: Heat loss per unit length of pipe.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction through insulation, two-dimensional through soil, (3) Constant properties, (4) Negligible oil convection and pipe wall conduction resistances.

PROPERTIES: *Table A-3*, Soil (300K): k = 0.52 W/m·K; *Table A-3*, Cellular glass (365K): k = 0.069 W/m·K.

ANALYSIS: The heat rate can be expressed as

$$q = \frac{T_1 - T_2}{R_{tot}}$$

where the thermal resistance is $R_{tot} = R_{ins} + R_{soil}$. From Eq. 3.28,

$$R_{ins} = \frac{\ell n\left(D_2/D_1\right)}{2\pi\ Lk_{ins}} = \frac{\ell n\left(0.7\text{m}/0.5\text{m}\right)}{2\pi\ L \times 0.069\ \text{W/m}\cdot\text{K}} = \frac{0.776\text{m}\cdot\text{K/W}}{L}.$$

From Eq. 4.27 and Table 4.1,

$$R_{soil} = \frac{1}{SK_{soil}} = \frac{\cosh^{-1}\left(2z/D_2\right)}{2\pi\ Lk_{soil}} = \frac{\cosh^{-1}\left(3/0.7\right)}{2\pi \times \left(0.52\ \text{W/m}\cdot\text{K}\right)L} = \frac{0.653}{L}\text{m}\cdot\text{K/W}.$$

Hence,

$$q = \frac{(120-0)^\circ\ \text{C}}{\frac{1}{L}\left(0.776+0.653\right)\frac{\text{m}\cdot\text{K}}{\text{W}}} = 84\frac{\text{W}}{\text{m}} \times L$$

$$q' = q/L = 84\ \text{W/m}.$$ <

COMMENTS: (1) Contributions of the soil and insulation to the total resistance are approximately the same. The heat loss may be reduced by burying the pipe deeper or adding more insulation.

(2) The convection resistance associated with the oil flow through the pipe may be significant, in which case the foregoing result would overestimate the heat loss. A calculation of this resistance may be based on results presented in Chapter 8.

(3) Since z > 3D/2, the shape factor for the soil can also be evaluated from $S = 2\pi L/\ell n\,(4z/D)$ of Table 4.1, and an equivalent result is obtained.

PROBLEM 4.16

KNOWN: Operating conditions of a buried superconducting cable.

FIND: Required cooling load.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Two-dimensional conduction in soil, (4) One-dimensional conduction in insulation.

ANALYSIS: The heat rate per unit length is

$$q' = \frac{T_g - T_n}{R'_g + R'_I}$$

$$q' = \frac{T_g - T_n}{\left[k_g\left(2\pi/\ln\left(4z/D_o\right)\right)\right]^{-1} + \ln\left(D_o/D_i\right)/2\pi\ k_i}$$

where Tables 3.3 and 4.1 have been used to evaluate the insulation and ground resistances, respectively. Hence,

$$q' = \frac{(300-77)\text{K}}{\left[(1.2\text{ W/m}\cdot\text{K})\left(2\pi\ /\ln(8/0.2)\right)\right]^{-1} + \ln(2)/\ 2\pi\times 0.005\text{ W/m}\cdot\text{K}}$$

$$q' = \frac{223\text{ K}}{(0.489+22.064)\text{m}\cdot\text{K/W}}$$

$$q' = 9.9\text{ W/m.}$$ <

COMMENTS: The heat gain is small and the dominant contribution to the thermal resistance is made by the insulation.

PROBLEM 4.17

KNOWN: Electrical heater of cylindrical shape inserted into a hole drilled normal to the surface of a large block of material with prescribed thermal conductivity.

FIND: Temperature reached when heater dissipates 50 W with the block at 25°C.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Block approximates semi-infinite medium with constant properties, (3) Negligible heat loss to surroundings above block surface, (4) Heater can be approximated as isothermal at T_1.

ANALYSIS: The temperature of the heater surface follows from the rate equation written as

$$T_1 = T_2 + q/kS$$

where S can be estimated from the conduction shape factor given in Table 4.1 for a "vertical cylinder in a semi-infinite medium,"

$$S = 2\pi L/\ell n\left(4L/D\right).$$

Substituting numerical values, find

$$S = 2\pi \times 0.1\text{m}/\ell n\left[\frac{4\times 0.1\text{m}}{0.005\text{m}}\right] = 0.143\text{m}.$$

The temperature of the heater is then

$$T_1 = 25°\text{C} + 50\text{ W}/(5\text{ W/m}\cdot\text{K} \times 0.143\text{m}) = 94.9°\text{C}.$$ <

COMMENTS: (1) Note that the heater has L >> D, which is a requirement of the shape factor expression.

(2) Our calculation presumes there is negligible thermal contact resistance between the heater and the medium. In practice, this would not be the case unless a conducting paste were used.

(3) Since L >> D, assumption (3) is reasonable.

(4) This configuration has been used to determine the thermal conductivity of materials from measurement of q and T_1.

PROBLEM 4.18

KNOWN: Surface temperatures of two parallel pipe lines buried in soil.

FIND: Heat transfer per unit length between the pipe lines.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Two-dimensional conduction, (3) Constant properties, (4) Pipe lines are buried very deeply, approximating burial in an infinite medium, (5) Pipe length >> D_1 or D_2 and w > D_1 or D_2.

ANALYSIS: The heat transfer rate per unit length from the hot pipe to the cool pipe is

$$q' = \frac{q}{L} = \frac{S}{L} k \left(T_1 - T_2\right).$$

The shape factor S for this configuration is given in Table 4.1 as

$$S = \frac{2\pi L}{\cosh^{-1}\left[\frac{4w^2 - D_1^2 - D_2^2}{2D_1 D_2}\right]}.$$

Substituting numerical values,

$$\frac{S}{L} = 2\pi / \cosh^{-1}\left[\frac{4\times(0.5\text{m})^2 - (0.1\text{m})^2 - (0.075\text{m})^2}{2\times 0.1\text{m}\times 0.075\text{m}}\right] = 2\pi / \cosh^{-1}(65.63)$$

$$\frac{S}{L} = 2\pi / 4.88 = 1.29.$$

Hence, the heat rate per unit length is

$$q' = 1.29 \times 0.5\,\text{W/m}\cdot\text{K}\,(175-5)^\circ\text{C} = 110\ \text{W/m}.$$ <

COMMENTS: The heat gain to the cooler pipe line will be larger than 110 W/m if the soil temperature is greater than 5°C. How would you estimate the heat gain if the soil were at 25°C?

PROBLEM 4.19

KNOWN: Tube embedded in the center plane of a concrete slab.

FIND: (a) The shape factor and heat transfer rate per unit length using the appropriate tabulated relation, (b) Shape factor using flux plot method.

SCHEMATIC:

ASSUMPTIONS: (1) Two-dimensional conduction, (2) Steady-state conditions, (3) Constant properties, (4) Concrete slab infinitely long in horizontal plane, L >> z.

PROPERTIES: *Table A-3*, Concrete, stone mix (300K): k = 1.4 W/m·K.

ANALYSIS: (a) If we relax the restriction that z >> D/2, the embedded tube-slab system corresponds to the fifth case of Table 4.1. Hence,

$$S = \frac{2\pi L}{\ln(8z/\pi D)}$$

where L is the length of the system normal to the page, z is the half-thickness of the slab and D is the diameter of the tube. Substituting numerical values, find

$$S = 2\pi L/\ln(8\times 50\text{mm}/\pi 50\text{mm}) = 6.72\text{L}.$$

Hence, the heat rate per unit length is

$$q' = \frac{q}{L} = \frac{S}{L}k(T_1 - T_2) = 6.72\times 1.4\frac{\text{W}}{\text{m}\cdot\text{K}}(85-20)^\circ\text{C} = 612\text{ W}.$$

(b) To find the shape factor using the flux plot method, first identify the symmetrical section bounded by the symmetry adiabats formed by the horizontal and vertical center lines. Selecting four temperature increments (N = 4), the flux plot can then be constructed.

From Eq. 4.26, the shape factor of the symmetrical section is

$$S_o = ML/N = 6L/4 = 1.5L.$$

For the tube-slab system, it follows that $S = 4S_o = 6.0L$, which compares favorably with the result obtained from the shape factor relation.

PROBLEM 4.20

KNOWN: Dimensions and boundary temperatures of a steam pipe embedded in a concrete casing.

FIND: Heat loss per unit length.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible steam side convection resistance, pipe wall resistance and contact resistance (T_1 = 450K), (3) Constant properties.

PROPERTIES: *Table A-3*, Concrete (300K): k = 1.4 W/m·K.

ANALYSIS: The heat rate can be expressed as

$$q = Sk\Delta T_{1-2} = Sk\left(T_1 - T_2\right)$$

From Table 4.1, the shape factor is

$$S = \frac{2\pi L}{\ell n\left[\frac{1.08\,w}{D}\right]}.$$

Hence,

$$q' = \left[\frac{q}{L}\right] = \frac{2\pi k\left(T_1 - T_2\right)}{\ell n\left[\frac{1.08\,w}{D}\right]}$$

$$q' = \frac{2\pi \times 1.4\,\text{W/m}\cdot\text{K} \times (450 - 300)\,\text{K}}{\ell n\left[\frac{1.08 \times 1.5\text{m}}{0.5\text{m}}\right]} = 1122\ \text{W/m}.$$ <

COMMENTS: Having neglected the steam side convection resistance, the pipe wall resistance, and the contact resistance, the foregoing result overestimates the actual heat loss.

PROBLEM 4.21

KNOWN: Thin-walled copper tube enclosed by an eccentric cylindrical shell; intervening space filled with insulation.

FIND: Heat loss per unit length of tube; compare result with that of a concentric tube-shell arrangement.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Thermal resistances of copper tube wall and outer shell wall are negligible, (4) Two-dimensional conduction in insulation.

ANALYSIS: The heat loss per unit length written in terms of the shape factor S is $q' = k(S/\ell)(T_1 - T_2)$ and from Table 4.1 for this geometry,

$$\frac{S}{\ell} = 2\pi/\cosh^{-1}\left[\frac{D^2 + d^2 - 4z^2}{2Dd}\right].$$

Substituting numerical values, all dimensions in mm,

$$\frac{S}{\ell} = 2\pi/\cosh^{-1}\left[\frac{120^2 + 30^2 - 4(20)^2}{2\times120\times30}\right] = 2\pi/\cosh^{-1}(1.903) = 4.991.$$

Hence, the heat loss is

$$q' = 0.05\text{W/m}\cdot\text{K}\times4.991(85-35)^\circ\text{C} = 12.5\text{ W/m}.$$

<

If the copper tube were concentric with the shell, but all other conditions were the same, the heat loss would be

$$q'_c = \frac{2\pi k(T_1 - T_2)}{\ell n(D_2/D_1)}$$

using Eq. 3.27. Substituting numerical values,

$$q'_c = 2\pi\times0.05\frac{\text{W}}{\text{m}\cdot\text{K}}(85-35)^\circ\text{C}/\ell n\left(^{120}/_{30}\right)$$

$$q'_c = 11.3\text{ W/m}.$$

COMMENTS: As expected, the heat loss with the eccentric arrangement is larger than that for the concentric arrangement. The effect of the eccentricity is to increase the heat loss by (12.5 - 11.3)/11.3 ≈ 11%.

PROBLEM 4.22

KNOWN: Cubical furnace, 350 mm external dimensions, with 50 mm thick walls.

FIND: The heat loss, q(W).

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Two-dimensional conduction, (3) Constant properties.

PROPERTIES: *Table A-3*, Fireclay brick $\left(\overline{T}=\left(T_1+T_2\right)/2=610\text{K}\right)$: $k\approx 1.1\ \text{W/m}\cdot\text{K}$.

ANALYSIS: Using relations for the shape factor from Table 4.1,

Plane Walls (6) $\qquad S_W=\dfrac{A}{L}=\dfrac{0.25\times 0.25\text{m}^2}{0.05\text{m}}=1.25\text{m}$

Edges (12) $\qquad S_E=0.54D=0.54\times 0.25\text{m}=0.14\text{m}$

Corners (8) $\qquad S_C=0.15L=0.15\times 0.05\text{m}=0.008\text{m}.$

The heat rate in terms of the shape factors is

$$q=kS\left(T_1-T_2\right)=k\left(6S_W+12S_E+8S_C\right)\ \left(T_1-T_2\right)$$

$$q=1.1\frac{\text{W}}{\text{m}\cdot\text{K}}\left(6\times 1.25\text{m}+12\times 0.14\text{m}+0.15\times 0.008\text{m}\right)\ \left(600-75\right)^\circ\text{C}$$

$$q=5.30\ \text{kW}.$$ <

COMMENTS: Note that the restrictions for S_E and S_C have been met.

PROBLEM 4.23

KNOWN: Dimensions, thermal conductivity and inner surface temperature of furnace wall. Ambient conditions.

FIND: Heat loss.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state, (2) Uniform convection coefficient over entire outer surface of container.

ANALYSIS: From the thermal circuit, the heat loss is

$$q = \frac{T_{s,i} - T_\infty}{R_{cond(2D)} + R_{conv}}$$

where $R_{conv} = 1/hA_{s,o} = 1/6(hW^2) = 1/6[5\ W/m^2 \cdot K(5\ m)^2] = 0.00133$ K/W. From Eq. (4.27), the two-dimensional conduction resistance is

$$R_{cond(2D)} = \frac{1}{Sk}$$

where the shape factor S must include the effects of conduction through the 8 corners, 12 edges and 6 plane walls. Hence, using the relations for Cases 8 and 9 of Table 4.1,

$$S = 8(0.15L) + 12 \times 0.54(W - 2L) + 6\,A_{s,i}/L$$

where $A_{s,i} = (W - 2L)^2$. Hence,

$$S = [8(0.15 \times 0.35) + 12 \times 0.54(4.30) + 6(52.83)]\,m$$

$$S = (0.42 + 27.86 + 316.98)\,m = 345.26\,m$$

and $R_{cond(2D)} = 1/(345.26\ m \times 1.4\ W/m \cdot K) = 0.00207$ K/W. Hence

$$q = \frac{(1100 - 25)^\circ\,C}{(0.00207 + 0.00133)\,K/W} = 316\,kW$$ <

COMMENTS: The heat loss is extremely large and measures should be taken to insulate the furnace.

PROBLEM 4.24

KNOWN: Platen heated by passage of hot fluid in poor thermal contact with cover plates exposed to cooler ambient air.

FIND: (a) Heat rate per unit thickness from each channel, q'_i, (b) Surface temperature of cover plate, T_s, (c) q'_i and T_s if lower surface is perfectly insulated, (d) Effect of changing centerline spacing on q'_i and T_s

SCHEMATIC:

Cover plate, B; Plate, A; Contact resistance; Cover plate, B; Air T_∞, h_o; T_s; L_B; L_A; D; L_o

D=15 mm $\quad L_o$=60 mm

L_A=30 mm $\quad L_B$=7.5 mm

T_i=150°C $\quad h_i$=1000 W/m^2·K

T_∞=25°C $\quad h_o$=200 W/m^2·K

k_A=20 W/m·K $\quad k_B$=75 W/m·K

$R''_{t,c} = 2.0\times10^{-4}\ m^2\cdot K/W$

ASSUMPTIONS: (1) Steady-state conditions, (2) Two-dimensional conduction in platen, but one-dimensional in coverplate, (3) Temperature of interfaces between A and B is uniform, (4) Constant properties.

ANALYSIS: (a) The heat rate per unit thickness from each channel can be determined from the following thermal circuit representing the quarter section shown.

$q'_i/4 \rightarrow T_i$ — $\frac{1}{h_i(\pi D/4)}$ — $\frac{1}{k_A S'}$ — $\frac{R''_{t,c}}{L_o/2}$ — $\frac{L_B}{k_B(L_o/2)}$ — T_s — $\frac{1}{h_o(L_o/2)}$ — T_∞

The value for the shape factor is $S' = 1.06$ as determined from the flux plot shown on the next page. Hence, the heat rate is

$$q'_i = 4(T_i - T_\infty)/R'_{tot} \tag{1}$$

$$R'_{tot} = [1/1000\ W/m^2\cdot K(p0.015m/4) + 1/20\ W/m\cdot K\times1.06$$
$$+2.0\times10^{-4} m^2\cdot K/W(0.060m/2) + 0.0075m/75\ W/m\cdot K(0.060m/2)$$
$$+1/200\ W/m^2\cdot K(0.060m/2)]$$

$$R'_{tot} = [0.085 + 0.047 + 0.0067 + 0.0033 + 0.1667]\ m\cdot K/W$$
$$R'_{tot} = 0.309\ m\cdot K/W$$

$$q'_i = 4(150-25)K/0.309\ m\cdot K/W = 1.62\ kW/m.$$ <

(b) The surface temperature of the cover plate also follows from the thermal circuit as

$$q'_i/4 = \frac{T_s - T_\infty}{1/h_o(L_o/2)} \tag{2}$$

Continued …..

PROBLEM 4.24 (Cont.)

$$T_s = T_\infty + \frac{q_i'}{4}\frac{1}{h_o(L_o/2)} = 25^\circ C + \frac{1.62\ kW}{4} \times 0.167\ m\cdot K/W$$

$$T_s = 25^\circ C + 67.6^\circ C \approx 93^\circ C.$$ <

(c,d) The effect of the centerline spacing on q_i' and T_s can be understood by examining the relative magnitudes of the thermal resistances. The dominant resistance is that due to the ambient air convection process which is inversely related to the spacing L_o. Hence, from Eq. (1), the heat rate will increase nearly linearly with an increase in L_o,

$$q_i' \sim \frac{1}{R_{tot}'} \approx \frac{1}{1/h_o(L_o/2)} \sim L_o.$$

From Eq. (2), find

$$\Delta T = T_s - T_\infty = \frac{q_i'}{4}\frac{1}{h_o(L_o/2)} \sim q_i' \cdot L_o^{-1} \sim L_o \cdot L_o^{-1} \approx 1.$$

Hence we conclude that ΔT will not increase with a change in L_o. Does this seem reasonable? What effect does L_o have on Assumptions (2) and (3)?

If the lower surface were insulated, the heat rate would be decreased nearly by half. This follows again from the fact that the overall resistance is dominated by the surface convection process. The temperature difference, T_s - T_∞, would only increase slightly.

$S' = S/l = M/N = 4.25/4 = 1.06$

PROBLEM 4.25

KNOWN: Long constantan wire butt-welded to a large copper block forming a thermocouple junction on the surface of the block.

FIND: (a) The measurement error (T_j - T_o) for the thermocouple for prescribed conditions, and (b) Compute and plot (T_j - T_o) for h = 5, 10 and 25 W/m^2·K for block thermal conductivity 15 ≤ k ≤ 400 W/m·K. When is it advantageous to use smaller diameter wire?

SCHEMATIC:

T_∞ = 25 °C
h = 10 W/m^2·K
Thermocouple wire, D = 1 mm, k_t
T_j
copper block
T_o = 125 °C

T_∞ — R_{fin} — T_j — R_{block} — T_o ← q_f

ASSUMPTIONS: (1) Steady-state conditions, (2) Thermocouple wire behaves as a fin with constant heat transfer coefficient, (3) Copper block has uniform temperature, except in the vicinity of the junction.

PROPERTIES: *Table A-1*, Copper (pure, 400 K), k_b = 393 W/m·K; Constantan (350 K), k_t ≈ 25 W/m·K.

ANALYSIS: The thermocouple wire behaves as a long fin permitting heat to flow from the surface thereby depressing the sensing junction temperature below that of the block T_o. In the block, heat flows into the circular region of the wire-block interface; the thermal resistance to heat flow within the block is approximated as a disk of diameter D on a semi-infinite medium (k_b, T_o). The thermocouple-block combination can be represented by a thermal circuit as shown above. The thermal resistance of the fin follows from the heat rate expression for an infinite fin, $R_{fin} = (hPk_tA_c)^{-1/2}$.

From Table 4.1, the shape factor for the disk-on-a-semi-infinite medium is given as S = 2D and hence $R_{block} = 1/k_bS = 1/2k_bD$. From the thermal circuit,

$$T_o - T_j = \frac{R_{block}}{R_{fin} + R_{block}}(T_o - T_\infty) = \frac{1.27}{1273 + 1.27}(125-25)^\circ C \approx 0.001(125-25)^\circ C = 0.1^\circ C. \quad <$$

with $P = \pi D$ and $A_c = \pi D^2/4$ and the thermal resistances as

$$R_{fin} = \left(10\,W/m^2\cdot K\,(\pi/4)\,25\,W/m\cdot K\times\left(1\times10^{-3}\,m\right)^3\right)^{-1/2} = 1273\,K/W$$

$$R_{block} = (1/2)\times 393\,W/m\cdot K\times 10^{-3}\,m = 1.27\,K/W.$$

(b) We keyed the above equations into the IHT workspace, performed a sweep on k_b for selected values of h and created the plot shown. When the block thermal conductivity is low, the error (T_o - T_j) is larger, increasing with increasing convection coefficient. A smaller diameter wire will be advantageous for low values of k_b and higher values of h.

Error, To-Tj (C)
0 1 2 3 4 5
0 100 200 300 400
Block thermal conductivity, kb (W/m.K)
h = 25 W/m^2.K; D = 1 mm
h = 10 W/m^2.K; D = 1mm
h = 5 W/m^2.K; D = 1mm
h = 25 W/m^2.K; D = 5 mm

PROBLEM 4.26

KNOWN: Dimensions, shape factor, and thermal conductivity of square rod with drilled interior hole. Interior and exterior convection conditions.

FIND: Heat rate and surface temperatures.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state, two-dimensional conduction, (2) Constant properties, (3) Uniform convection coefficients at inner and outer surfaces.

ANALYSIS: The heat loss can be expressed as

$$q = \frac{T_{\infty,1} - T_{\infty,2}}{R_{conv,1} + R_{cond(2D)} + R_{conv,2}}$$

where

$$R_{conv,1} = (h_1 \pi D_1 L)^{-1} = \left(50\,\text{W/m}^2 \cdot \text{K} \times \pi \times 0.25\,\text{m} \times 2\,\text{m}\right)^{-1} = 0.01273\,\text{K/W}$$

$$R_{cond(2D)} = (Sk)^{-1} = (8.59\,\text{m} \times 150\,\text{W/m} \cdot \text{K})^{-1} = 0.00078\,\text{K/W}$$

$$R_{conv,2} = (h_2 \times 4wL)^{-1} = \left(4\,\text{W/m}^2 \cdot \text{K} \times 4\,\text{m} \times 1\text{m}\right)^{-1} = 0.0625\,\text{K/W}$$

Hence,

$$q = \frac{(300 - 25)^\circ\,\text{C}}{0.076\,\text{K/W}} = 3.62\,\text{kW} \qquad <$$

$$T_1 = T_{\infty,1} - qR_{conv,1} = 300^\circ\text{C} - 46^\circ\text{C} = 254^\circ\text{C} \qquad <$$

$$T_2 = T_{\infty,2} + qR_{conv,2} = 25^\circ\text{C} + 226^\circ\text{C} = 251^\circ\text{C} \qquad <$$

COMMENTS: The largest resistance is associated with convection at the outer surface, and the conduction resistance is much smaller than both convection resistances. Hence, $(T_2 - T_{\infty,2}) > (T_{\infty,1} - T_1) >> (T_1 - T_2)$.

PROBLEM 4.27

KNOWN: Long fin of aluminum alloy with prescribed convection coefficient attached to different base materials (aluminum alloy or stainless steel) with and without thermal contact resistance $R''_{t,j}$ at the junction.

FIND: (a) Heat rate q_f and junction temperature T_j for base materials of aluminum and stainless steel, (b) Repeat calculations considering thermal contact resistance, $R''_{t,j}$, and (c) Plot as a function of h for the range $10 \le h \le 1000$ W/m^2·K for each base material.

SCHEMATIC:

Base material, T_b = 100 °C
D = mm
$R''_{t,j} = 3 \times 10^{-5}$ m^2·K/W, Part (b)
T_∞ = 25 °C
h = 50 W/m^2·K

(a) T_b — R_b — T_j — R_f — T_∞ → q_f

(b) T_b — R_b — T_j — $R_{t,j}$ — R_f — T_∞ → q_f

ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Infinite fin.

PROPERTIES: (Given) Aluminum alloy, k = 240 W/m·K, Stainless steel, k = 15 W/m·K.

ANALYSIS: (a,b) From the thermal circuits, the heat rate and junction temperature are

$$q_f = \frac{T_b - T_\infty}{R_{tot}} = \frac{T_b - T_\infty}{R_b + R_{t,j} + R_f} \qquad (1)$$

$$T_j = T_\infty + q_f R_f \qquad (2)$$

and, with $P = \pi D$ and $A_c = \pi D^2/4$, from Tables 4.1 and 3.4 find

$$R_b = 1/Sk_b = 1/(2Dk_b) = (2 \times 0.005\,m \times k_b)^{-1}$$

$$R_{t,j} = R''_{t,j}/A_c = 3\times10^{-5} m^2 \cdot K/W \big/ \pi (0.005\,m)^2/4 = 1.528\,K/W$$

$$R_f = (hPkA_c)^{-1/2} = \left[50\,W/m^2 \cdot K \pi^2 (0.005\,m)^2\, 240\,W/m \cdot K/4\right]^{-1/2} = 16.4\,K/W$$

		Without $R''_{t,j}$		With $R''_{t,j}$	
Base	R_b (K/W)	q_f (W)	T_j (°C)	q_f (W)	T_j (°C)
Al alloy	0.417	4.46	98.2	4.09	92.1
St. steel	6.667	3.26	78.4	3.05	75.1

(c) We used the *IHT Model for Extended Surfaces, Performance Calculations, Rectangular Pin Fin* to calculate q_f for $10 \le h \le 100$ W/m^2·K by replacing R''_{tc} (thermal resistance at fin base) by the sum of the contact and spreading resistances, $R''_{t,j} + R''_b$.

Continued...

PROBLEM 4.27 (Cont.)

Fin heat rate, qf (W)

Convection coefficient, h (W/m^2.K)

Base material - aluminum alloy
Base material - stainless steel

COMMENTS: (1) From part (a), the aluminum alloy base material has negligible effect on the fin heat rate and depresses the base temperature by only 2°C. The effect of the stainless steel base material is substantial, reducing the heat rate by 27% and depressing the junction temperature by 25°C.

(2) The contact resistance reduces the heat rate and increases the temperature depression relatively more with the aluminum alloy base.

(3) From the plot of q_f vs. h, note that at low values of h, the heat rates are nearly the same for both materials since the fin is the dominant resistance. As h increases, the effect of R''_b becomes more important.

PROBLEM 4.28

KNOWN: Igloo constructed in hemispheric shape sits on ice cap; igloo wall thickness and inside/outside convection coefficients (h_i, h_o) are prescribed.

FIND: (a) Inside air temperature $T_{\infty,i}$ when outside air temperature is $T_{\infty,o}$ = -40°C assuming occupants provide 320 W within igloo, (b) Perform parameter sensitivity analysis to determine which variables have significant effect on T_i.

SCHEMATIC:

T_∞ = -40 °C, h_o = 15 W/m^2; r_i = 1.8 m; r_o = 2.3 m; $T_{\infty,i}$, h_i = 6 W/m^2·K; $\dot{E}_g$ = 320 W; Ice cap, T_{ic} = -20 °C

Thermal circuit: q, $T_{\infty,i}$ (4) — $R_{cv,c}$ — (3) — R_{wall} — (2) — $R_{cv,o}$ — (1) $T_{\infty,o}$; (4) — $R_{cv,f}$ — (5) — R_{cap} — (6) T_{ic}

ASSUMPTIONS: (1) Steady-state conditions, (2) Convection coefficient is the same on floor and ceiling of igloo, (3) Floor and ceiling are at uniform temperature, (4) Floor-ice cap resembles disk on semi-infinite medium, (5) One-dimensional conduction through igloo walls.

PROPERTIES: Ice and compacted snow (given): k = 0.15 W/m·K.

ANALYSIS: (a) The thermal circuit representing the heat loss from the igloo to the outside air and through the floor to the ice cap is shown above. The heat loss is

$$q = \frac{T_{\infty,i} - T_{\infty,o}}{R_{cv,c} + R_{wall} + R_{cv,o}} + \frac{T_{\infty,i} - T_{ic}}{R_{cv,f} + R_{cap}}.$$

Convection, ceiling: $$R_{cv,c} = \frac{2}{h_i\left(4\pi r_i^2\right)} = \frac{2}{6\,\text{W/m}^2\cdot\text{K}\times 4\pi\,(1.8\,\text{m})^2} = 0.00819\,\text{K/W}$$

Convection, outside: $$R_{cv,o} = \frac{2}{h_o\left(4\pi r_o^2\right)} = \frac{2}{15\,\text{W/m}^2\cdot\text{K}\times 4\pi\,(2.3\,\text{m})^2} = 0.00201\,\text{K/W}$$

Convection, floor: $$R_{cv,f} = \frac{1}{h_i\left(\pi r_i^2\right)} = \frac{1}{6\,\text{W/m}^2\cdot\text{K}\times \pi\,(1.8\,\text{m})^2} = 0.01637\,\text{K/W}$$

Conduction, wall: $$R_{wall} = 2\left[\frac{1}{4\pi k}\left(\frac{1}{r_i} - \frac{1}{r_o}\right)\right] = 2\left[\frac{1}{4\pi\times 0.15\,\text{W/m}\cdot\text{K}}\left(\frac{1}{1.8} - \frac{1}{2.3}\right)\text{m}\right] = 0.1281\,\text{K/W}$$

Conduction, ice cap: $$R_{cap} = \frac{1}{kS} = \frac{1}{4kr_i} = \frac{1}{4\times 0.15\,\text{W/m}\cdot\text{K}\times 1.8\,\text{m}} = 0.9259\,\text{K/W}$$

where S was determined from the shape factor of Table 4.1. Hence,

$$q = 320\,\text{W} = \frac{T_{\infty,i} - (-40)^\circ\text{C}}{(0.00818 + 0.1281 + 0.0020)\,\text{K/W}} + \frac{T_{\infty,i} - (-20)^\circ\text{C}}{(0.01637 + 0.9259)\,\text{K/W}}$$

$$320\text{ W} = 7.232(T_{\infty,i} + 40) + 1.06(T_{\infty,i} + 20) \qquad T_{\infty,i} = 1.1^\circ\text{C}.$$ <

Continued...

(b) Begin the parameter sensitivity analysis to determine important variables which have a significant influence on the inside air temperature by examining the thermal resistances associated with the processes present in the system and represented by the network.

Process	Symbols		Value (K/W)
Convection, outside	$R_{cv,o}$	R21	0.0020
Conduction, wall	R_{wall}	R32	0.1281
Convection, ceiling	$R_{cv,c}$	R43	0.0082
Convection, floor	$R_{cv,f}$	R54	0.0164
Conduction, ice cap	R_{cap}	R65	0.9259

It follows that the convection resistances are negligible relative to the conduction resistance across the igloo wall. As such, only changes to the wall thickness will have an appreciable effect on the inside air temperature relative to the outside ambient air conditions. We don't want to make the igloo walls thinner and thereby allow the air temperature to dip below freezing for the prescribed environmental conditions.

Using the *IHT Thermal Resistance Network Model*, we used the circuit builder to construct the network and perform the energy balances to obtain the inside air temperature as a function of the outside convection coefficient for selected increased thicknesses of the wall.

Air temperature, Tinfi (C)

Outside coefficient, ho (W/m^2.K)

Wall thickness, (ro-ri) = 0.5 m
(ro-ri) = 0.75 m
(ro-ri) = 1.0 m

COMMENTS: (1) From the plot, we can see that the influence of the outside air velocity which controls the outside convection coefficient h_o is negligible.

(2) The thickness of the igloo wall is the dominant thermal resistance controlling the inside air temperature.

PROBLEM 4.29

KNOWN: Diameter and maximum allowable temperature of an electronic component. Contact resistance between component and large aluminum heat sink. Temperature of heat sink and convection conditions at exposed component surface.

FIND: (a) Thermal circuit, (b) Maximum operating power of component.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Negligible heat loss from sides of chip.

ANALYSIS: (a) The thermal circuit is:

where $R_{2D,cond}$ is evaluated from the shape factor S = 2D of Table 4.1.

(b) Performing an energy balance for a control surface about the component,

$$P = q_{air} + q_{sink} = h\left(\pi\ D^2/4\right)\left(T_c - T_\infty\right) + \frac{T_c - T_b}{R''_{t,c}/\left(\pi\ D^2/4\right) + 1/2Dk}$$

$$P = 25\ \text{W/m}^2\cdot\text{K}\left(\pi/4\right)\left(0.01\ \text{m}\right)^2 75^\circ\text{C} + \frac{75^\circ\text{C}}{\left\{\left[0.5\times10^{-4}/\left(\pi/4\right)\left(0.01\right)^2\right] + \left(0.02\times237\right)^{-1}\right\}\text{K/W}}$$

$$P = 0.15\ \text{W} + \frac{75^\circ\text{C}}{\left(0.64+0.21\right)\text{K/W}} = 0.15\ \text{W} + 88.49\ \text{W} = 88.6\ \text{W}. \qquad <$$

COMMENTS: The convection resistance is much larger than the cumulative contact and conduction resistance. Hence, virtually all of the heat dissipated in the component is transferred through the block. The two-dimensional conduction resistance is significantly underestimated by use of the shape factor S = 2D. Hence, the maximum allowable power is less than 88.6 W.

PROBLEM 4.30

KNOWN: Disc-shaped electronic devices dissipating 100 W mounted to aluminum alloy block with prescribed contact resistance.

FIND: (a) Temperature device will reach when block is at 27°C assuming all the power generated by the device is transferred by conduction to the block and (b) For the operating temperature found in part (a), the permissible operating power with a 30-pin fin heat sink.

SCHEMATIC:

ASSUMPTIONS: (1) Two-dimensional, steady-state conduction, (2) Device is at uniform temperature, T_1, (3) Block behaves as semi-infinite medium.

PROPERTIES: *Table A.1*, Aluminum alloy 2024 (300 K): k = 177 W/m·K.

ANALYSIS: (a) The thermal circuit for the conduction heat flow between the device and the block shown in the above Schematic where R_e is the thermal contact resistance due to the epoxy-filled interface,

$$R_e = R''_{t,c}/A_c = R''_{t,c}\Big/\left(\pi D^2/4\right)$$

$$R_e = 5\times10^{-5}\ \mathrm{K\cdot m^2/W}\Big/\left(\pi\left(0.020\,\mathrm{m}\right)^2\right)\Big/4 = 0.159\,\mathrm{K/W}$$

The thermal resistance between the device and the block is given in terms of the conduction shape factor, Table 4.1, as

$$R_b = 1/Sk = 1/(2Dk)$$

$$R_b = 1/\left(2\times0.020\,\mathrm{m}\times177\,\mathrm{W/m\cdot K}\right) = 0.141\,\mathrm{K/W}$$

From the thermal circuit,

$$T_1 = T_2 + q_d\left(R_b + R_e\right)$$

$$T_1 = 27^\circ\mathrm{C} + 100\,\mathrm{W}\left(0.141 + 0.159\right)\mathrm{K/W}$$

$$T_1 = 27^\circ\mathrm{C} + 30^\circ\mathrm{C} = 57^\circ\mathrm{C}$$ <

(b) The schematic below shows the device with the 30-pin fin heat sink with fins and base material of copper (k = 400 W/m·K). The airstream temperature is 27°C and the convection coefficient is 1000 W/m^2·K.

Continued...

PROBLEM 4.30 (Cont.)

T_∞, h

Pin fins (30), D =1.5 mm
L = 15 mm

Device

Copper, 5 mm thickness

Epoxy

(b)

T_2, T_1, T_∞

q_{cd}, R_b, R_e, R_e, R_c, R_{fin}, q_{cv}

q_d

The thermal circuit for this system has two paths for the device power: to the block by conduction, q_{cd}, and to the ambient air by conduction to the fin array, q_{cv},

$$q_d = \frac{T_1 - T_2}{R_b + R_e} + \frac{T_1 - T_\infty}{R_e + R_c + R_{fin}} \tag{3}$$

where the thermal resistance of the fin base material is

$$R_c = \frac{L_c}{k_c A_c} = \frac{0.005\,\text{m}}{400\,\text{W/m}\cdot\text{K}\left(\pi 0.02^2/4\right)\text{m}^2} = 0.03979\,\text{K/W} \tag{4}$$

and R_{fin} represents the thermal resistance of the fin array (see Section 3.6.5),

$$R_{fin} = R_{t,o} = \frac{1}{\eta_o h A_t} \tag{5, 3.103}$$

$$\eta_o = 1 - \frac{NA_f}{A_t}\left(1 - \eta_f\right) \tag{6, 3.102}$$

where the fin and prime surface area is

$$A_t = NA_f + A_b \tag{3.99}$$

$$A_t = N\left(\pi D_f L\right) + \left[\pi D_d^2/4 - N\left(\pi D_f^2/4\right)\right]$$

where A_f is the fin surface area, D_d is the device diameter and D_f is the fin diameter.

$$A_t = 30\left(\pi \times 0.0015\,\text{m} \times 0.015\,\text{m}\right) + \left[\pi\left(0.020\,\text{m}\right)^2/4 - 30\left(\pi\left(0.0015\,\text{m}\right)^2/4\right)\right]$$

A_t = 0.06362 m^2 + 0.0002611 m^2 = 0.06388 m^2

Using the *IHT Model*, *Extended Surfaces*, *Performance Calculations*, *Rectangular Pin Fin*, find the fin efficiency as

$$\eta_f = 0.8546 \tag{7}$$

Continued...

Substituting numerical values into Eq. (6), find

$$\eta_o = 1 - \frac{30 \times \pi \times 0.0015\,\text{m} \times 0.015\,\text{m}}{0.06388\,\text{m}^2}(1 - 0.8546)$$

$$\eta_o = 0.8552$$

and the fin array thermal resistance is

$$R_{fin} = \frac{1}{0.8552 \times 1000\,\text{W/m}^2 \cdot \text{K} \times 0.06388\,\text{m}^2} = 0.01831\,\text{K/W}$$

Returning to Eq. (3), with $T_1 = 57°C$ from part (a), the permissible heat rate is

$$q_d = \frac{(57-27)^\circ\text{C}}{(0.141 + 0.159)\,\text{K/W}} + \frac{(57-27)^\circ\text{C}}{(0.159 + 0.03979 + 0.01831)\,\text{K/W}}$$

$$q_d = 100\,\text{W} + 138.2\,\text{W} = 238\,\text{W}$$ <

COMMENTS: (1) Recognize in the part (b) analysis, that thermal resistances of the fin base and array are much smaller than the resistance due to the epoxy contact interfaces.

(2) In calculating the fin efficiency, η_f, using the IHT Model it is not necessary to know the base temperature as η_f depends only upon geometric parameters, thermal conductivity and the convection coefficient.

PROBLEM 4.31

KNOWN: Dimensions and surface temperatures of a square channel. Number of chips mounted on outer surface and chip thermal contact resistance.

FIND: Heat dissipation per chip and chip temperature.

SCHEMATIC:

ASSUMPTIONS: (1) Steady state, (2) Approximately uniform channel inner and outer surface temperatures, (3) Two-dimensional conduction through channel wall (negligible end-wall effects), (4) Constant thermal conductivity.

ANALYSIS: The total heat rate is determined by the two-dimensional conduction resistance of the channel wall, $q = (T_2 - T_1)/R_{t,cond(2D)}$, with the resistance determined by using Eq. 4.27 with Case 11 of Table 4.1. For W/w = 1.6 > 1.4

$$R_{t,cond(2D)} = \frac{0.930 \ln(W/w) - 0.050}{2\pi\ L\ k} = \frac{0.387}{2\pi(0.160\text{m})240\ \text{W/m}\cdot\text{K}} = 0.00160\ \text{K/W}$$

The heat rate per chip is then

$$q_c = \frac{T_2 - T_1}{N\ R_{t,cond(2D)}} = \frac{(50-20)^\circ\text{C}}{120(0.0016\ \text{K/W})} = 156.3\ \text{W}$$ <

and, with $q_c = (T_c - T_2)/R_{t,c}$, the chip temperature is

$$T_c = T_2 + R_{t,c}\ q_c = 50^\circ\text{C} + (0.2\ \text{K/W})156.3\ \text{W} = 81.3^\circ\text{C}$$ <

COMMENTS: (1) By acting to *spread* heat flow lines away from a chip, the channel wall provides an excellent *heat sink* for dissipating heat generated by the chip. However, recognize that, in practice, there will be temperature variations on the inner and outer surfaces of the channel, and if the prescribed values of T_1 and T_2 represent minimum and maximum inner and outer surface temperatures, respectively, the rate is overestimated by the foregoing analysis. (2) The shape factor may also be determined by combining the expression for a plane wall with the result of Case 8 (Table 4.1). With $S = [4(wL)/(W-w)/2] + 4(0.54\ L) = 2.479$ m, $R_{t,cond(2D)} = 1/(Sk) = 0.00168$ K/W.

PROBLEM 4.32

KNOWN: Dimensions and thermal conductivity of concrete duct. Convection conditions of ambient air. Inlet temperature of water flow through the duct.

FIND: (a) Heat loss per duct length near inlet, (b) Minimum allowable flow rate corresponding to maximum allowable temperature rise of water.

SCHEMATIC:

ASSUMPTIONS: (1) Steady state, (2) Negligible water-side convection resistance, pipe wall conduction resistance, and pipe/concrete contact resistance (temperature at inner surface of concrete corresponds to that of water), (3) Constant properties, (4) Negligible flow work and kinetic and potential energy changes.

ANALYSIS: (a) From the thermal circuit, the heat loss per unit length near the entrance is

$$q' = \frac{T_i - T_\infty}{R'_{cond(2D)} + R'_{conv}} = \frac{T_i - T_\infty}{\dfrac{\ln(1.08\,w/D)}{2\pi k} + \dfrac{1}{h(4w)}}$$

where $R'_{cond(2D)}$ is obtained by using the shape factor of Case 6 from Table 4.1 with Eq. (4.27). Hence,

$$q' = \frac{(90-0)°C}{\dfrac{\ln(1.08 \times 0.3\text{m}/0.15\text{m})}{2\pi(1.4\,\text{W/m}\cdot\text{K})} + \dfrac{1}{25\,\text{W/m}^2\cdot\text{K}(1.2\text{m})}} = \frac{90°C}{(0.0876+0.0333)\,\text{K}\cdot\text{m/W}} = 745\ \text{W/m}$$ <

(b) From Eq. (1.11e), with $q = q'L$ and $(T_i - T_o) = 5°C$,

$$\dot{m} = \frac{q'L}{u_i - u_o} = \frac{q'L}{c(T_i - T_o)} = \frac{745\,\text{W/m}(100\text{m})}{4207\,\text{J/kg}\cdot\text{K}(5°C)} = 3.54\ \text{kg/s}$$ <

COMMENTS: The small reduction in the temperature of the water as it flows from inlet to outlet induces a slight departure from two-dimensional conditions and a small reduction in the heat rate per unit length. A slightly conservative value (upper estimate) of $\dot{m}$ is therefore obtained in part (b).

PROBLEM 4.33

KNOWN: Dimensions and thermal conductivities of a heater and a finned sleeve. Convection conditions on the sleeve surface.

FIND: (a) Heat rate per unit length, (b) Generation rate and centerline temperature of heater, (c) Effect of fin parameters on heat rate.

SCHEMATIC:

$T_\infty = 50°C$
$h = 500\ W/m^2\text{-}K$
Air
$T_s = 300°C$
w = 40 mm
N fins
$k_s = 240\ W/m\text{-}K$
t
Heater
D = 20 mm
$k_h = 400\ W/m\text{-}K$
L

ASSUMPTIONS: (1) Steady state, (2) Constant properties, (3) Negligible contact resistance between heater and sleeve, (4) Uniform convection coefficient at outer surfaces of sleeve, (5) Uniform heat generation, (6) Negligible radiation.

ANALYSIS: (a) From the thermal circuit, the desired heat rate is

$$q' = \frac{T_s - T_\infty}{R'_{cond(2D)} + R'_{t,o}} = \frac{T_s - T_\infty}{R'_{tot}}$$

The two-dimensional conduction resistance, may be estimated from Eq. (4.27) and Case 6 of Table 4.2

$$R'_{cond(2D)} = \frac{1}{S'k_s} = \frac{\ln(1.08w/D)}{2\pi k_s} = \frac{\ln(2.16)}{2\pi(240\,W/m\cdot K)} = 5.11\times10^{-4}\ m\cdot K/W$$

The thermal resistance of the fin array is given by Eq. (3.103), where η_o and A_t are given by Eqs. (3.102) and (3.99) and η_f is given by Eq. (3.89). With $L_c = L + t/2 = 0.022$ m, $m = (2h/k_s t)^{1/2} = 32.3\ m^{-1}$ and $mL_c = 0.710$,

$$\eta_f = \frac{\tanh\ mL_c}{mL_c} = \frac{0.61}{0.71} = 0.86$$

$$A'_t = NA'_f + A'_b = N(2L+t) + (4w - Nt) = 0.704m + 0.096m = 0.800m$$

$$\eta_o = 1 - \frac{NA'_f}{A'_t}(1-\eta_f) = 1 - \frac{0.704m}{0.800m}(0.14) = 0.88$$

$$R'_{t,o} = (\eta_o h A'_t)^{-1} = \left(0.88\times500\,W/m^2\cdot K\times0.80m\right)^{-1} = 2.84\times10^{-3} m\cdot K/W$$

$$q' = \frac{(300-50)°C}{\left(5.11\times10^{-4} + 2.84\times10^{-3}\right)m\cdot K/W} = 74,600\,W/m$$ <

(b) Eq. (3.55) may be used to determine $\dot{q}$, if h is replaced by an overall coefficient based on the surface area of the heater. From Eq. (3.32),

$$U_s A'_s = U_s \pi D = \left(R'_{tot}\right)^{-1} = \left(3.35\,\text{m}\cdot\text{K/W}\right)^{-1} = 298\,\text{W/m}\cdot\text{K}$$

$$U_s = 298\,\text{W/m}\cdot\text{K}/\left(\pi\times 0.02\text{m}\right) = 4750\,\text{W/m}^2\cdot\text{K}$$

$$\dot{q} = 4U_s\left(T_s - T_\infty\right)/D = 4\left(4750\,\text{W/m}^2\cdot\text{K}\right)\left(250°\text{C}\right)/0.02\text{m} = 2.38\times 10^8\,\text{W/m}^3 \quad <$$

From Eq. (3.53) the centerline temperature is

$$T(0) = \frac{\dot{q}\left(D/2\right)^2}{4k_h} + T_s = \frac{2.38\times 10^8\,\text{W/m}^3\left(0.01\text{m}\right)^2}{4\left(400\,\text{W/m}\cdot\text{K}\right)} + 300°\text{C} = 315°\text{C} \quad <$$

(c) Subject to the prescribed constraints, the following results have been obtained for parameter variations corresponding to $16 \le N \le 40$, $2 \le t \le 8$ mm and $20 \le L \le 40$ mm.

N	t(mm)	L(mm)	η_f	$q'(\text{W/m})$
16	4	20	0.86	74,400
16	8	20	0.91	77,000
28	4	20	0.86	107,900
32	3	20	0.83	115,200
40	2	20	0.78	127,800
40	2	40	0.51	151,300

Clearly there is little benefit to simply increasing t, since there is no change in A'_t and only a marginal increase in η_f. However, due to an attendant increase in A'_t, there is significant benefit to increasing N for fixed t (no change in η_f) and additional benefit in concurrently increasing N while decreasing t. In this case the effect of increasing A'_t exceeds that of decreasing η_f. The same is true for increasing L, although there is an upper limit at which diminishing returns would be reached. The upper limit to L could also be influenced by manufacturing constraints.

COMMENTS: Without the sleeve, the heat rate would be $q' = \pi Dh\left(T_s - T_\infty\right) = 7850\,\text{W/m}$, which is well below that achieved by using the increased surface area afforded by the sleeve.

PROBLEM 4.34

KNOWN: Dimensions of chip array. Conductivity of substrate. Convection conditions. Contact resistance. Expression for resistance of spreader plate. Maximum chip temperature.

FIND: Maximum chip heat rate.

SCHEMATIC:

Air; $T_\infty = 15^oC$; $h = 100\ W/m^2\text{-}K$; $T_h = 85^oC$; $A_r = L_h^2/S_h^2 = 0.25$; $L_h = 5$ mm; $k_{sub} = 80$ W/m-K; $S_h = 10$ mm; $R''_{t,c} = 0.5\times10^{-4}\ m^2\text{-}K/W$

T_∞, T_h, T_∞; q_h, $R_{h,cnv}$, $R_{t(sp)}$, $R_{t,c}$, $R_{sp,cnv}$, q_{sp}

ASSUMPTIONS: (1) Steady-state, (2) Constant thermal conductivity, (3) Negligible radiation, (4) All heat transfer is by convection from the chip and the substrate surface (negligible heat transfer from bottom or sides of substrate).

ANALYSIS: From the thermal circuit,

$$q = q_h + q_{sp} = \frac{T_h - T_\infty}{R_{h,cnv}} + \frac{T_h - T_\infty}{R_{t(sp)} + R_{t,c} + R_{sp,cnv}}$$

$$R_{h,cnv} = \left(h\,A_{s,n}\right)^{-1} = \left(hL_h^2\right)^{-1} = \left[100\,W/m^2\cdot K\left(0.005m\right)^2\right]^{-1} = 400\,K/W$$

$$R_{t(sp)} = \frac{1-1.410\,A_r + 0.344\,A_r^3 + 0.043\,A_r^5 + 0.034\,A_r^7}{4\,k_{sub}\,L_h} = \frac{1-0.353+0.005+0+0}{4\left(80\,W/m\cdot K\right)\left(0.005m\right)} = 0.408\,K/W$$

$$R_{t,c} = \frac{R''_{t,c}}{L_h^2} = \frac{0.5\times10^{-4}m^2\cdot K/W}{\left(0.005m\right)^2} = 2.000\,K/W$$

$$R_{sp,cnv} = \left[h\left(A_{sub} - A_{s,h}\right)\right]^{-1} = \left[100\,W/m^2\cdot K\left(0.010m^2 - 0.005m^2\right)\right]^{-1} = 133.3\,K/W$$

$$q = \frac{70°C}{400\,K/W} + \frac{70°C}{\left(0.408+2+133.3\right)K/W} = 0.18\,W + 0.52\,W = 0.70\,W \qquad <$$

COMMENTS: (1) The thermal resistances of the substrate and the chip/substrate interface are much less than the substrate convection resistance. Hence, the heat rate is increased almost in proportion to the additional surface area afforded by the substrate. An increase in the spacing between chips (S_h) would increase q correspondingly.

(2) In the limit $A_r \to 0$, $R_{t(sp)}$ reduces to $2\pi^{1/2}k_{sub}D_h$ for a circular heat source and $4k_{sub}L_h$ for a square source.

PROBLEM 4.35

KNOWN: Internal corner of a two-dimensional system with prescribed convection boundary conditions.

FIND: Finite-difference equations for these situations: (a) Horizontal boundary is perfectly insulated and vertical boundary is subjected to a convection process (T_∞,h), (b) Both boundaries are perfectly insulated; compare result with Eq. 4.45.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Two-dimensional conduction, (3) Constant properties, (4) No internal generation.

ANALYSIS: Consider the nodal network shown above and also as Case 2, Table 4.2. Having defined the control volume – the shaded area of unit thickness normal to the page – next identify the heat transfer processes. Finally, perform an energy balance wherein the processes are expressed using appropriate rate equations.

(a) With the horizontal boundary insulated and the vertical boundary subjected to a convection process, the energy balance results in the following finite-difference equation:

$$\dot{E}_{in} - \dot{E}_{out} = 0 \qquad q_1 + q_2 + q_3 + q_4 + q_5 + q_6 = 0$$

$$k(\Delta y \cdot 1)\frac{T_{m-1,n} - T_{m,n}}{\Delta x} + k\left[\frac{\Delta x}{2}\cdot 1\right]\frac{T_{m,n-1} - T_{m,n}}{\Delta y} + h\left[\frac{\Delta y}{2}\cdot 1\right]\left(T_\infty - T_{m,n}\right)$$

$$+0 + k\left[\frac{\Delta y}{2}\cdot 1\right]\frac{T_{m+1,n} - T_{m,n}}{\Delta x} + k(\Delta x \cdot 1)\frac{T_{m,n+1} - T_{m,n}}{\Delta y} = 0.$$

Letting $\Delta x = \Delta y$, and regrouping, find

$$2\left(T_{m-1,n} + T_{m,n+1}\right) + \left(T_{m+1,n} + T_{m,n-1}\right) + \frac{h\Delta x}{k}T_\infty - \left[6 + \frac{h\Delta x}{k}\right]T_{m,n} = 0. \qquad <$$

(b) With both boundaries insulated, the energy balance would have $q_3 = q_4 = 0$. The same result would be obtained by letting h = 0 in the previous result. Hence,

$$2\left(T_{m-1,n} + T_{m,n+1}\right) + \left(T_{m+1,n} + T_{m,n-1}\right) - 6\, T_{m,n} = 0. \qquad <$$

Note that this expression compares exactly with Eq. 4.45 when h = 0, which corresponds to insulated boundaries.

PROBLEM 4.36

KNOWN: Plane surface of two-dimensional system.

FIND: The finite-difference equation for nodal point on this boundary when (a) insulated; compare result with Eq. 4.46, and when (b) subjected to a constant heat flux.

SCHEMATIC:

ASSUMPTIONS: (1) Two-dimensional, steady-state conduction with no generation, (2) Constant properties, (3) Boundary is adiabatic.

ANALYSIS: (a) Performing an energy balance on the control volume, $(\Delta x/2)\cdot\Delta y$, and using the conduction rate equation, it follows that

$$\dot{E}_{in} - \dot{E}_{out} = 0 \qquad q_1 + q_2 + q_3 = 0 \qquad (1,2)$$

$$k(\Delta y \cdot 1)\frac{T_{m-1,n} - T_{m,n}}{\Delta x} + k\left[\frac{\Delta x}{2}\cdot 1\right]\frac{T_{m,n-1} - T_{m,n}}{\Delta y} + k\left[\frac{\Delta x}{2}\cdot 1\right]\frac{T_{m,n+1} - T_{m,n}}{\Delta y} = 0. \quad (3)$$

Note that there is no heat rate across the control volume surface at the insulated boundary. Recognizing that $\Delta x = \Delta y$, the above expression reduces to the form

$$2T_{m-1,n} + T_{m,n-1} + T_{m,n+1} - 4T_{m,n} = 0. \qquad (4)$$ <

The Eq. 4.46 of Table 4.3 considers the same configuration but with the boundary subjected to a convection process. That is,

$$\left(2T_{m-1,n} + T_{m,n-1} + T_{m,n+1}\right) + \frac{2h\Delta x}{k}T_\infty - 2\left[\frac{h\Delta x}{k} + 2\right]T_{m,n} = 0. \qquad (5)$$

Note that, if the boundary is insulated, $h = 0$ and Eq. 4.46 reduces to Eq. (4).

(b) If the surface is exposed to a constant heat flux, q''_o, the energy balance has the form $q_1 + q_2 + q_3 + q''_o \cdot \Delta y = 0$ and the finite difference equation becomes

$$2T_{m-1,n} + T_{m,n-1} + T_{m,n+1} - 4T_{m,n} = -\frac{q''_o \Delta x}{k}.$$ <

COMMENTS: Equation (4) can be obtained by using the “interior node” finite-difference equation, Eq. 4.33, where the insulated boundary is treated as a symmetry plane as shown below.

PROBLEM 4.37

KNOWN: External corner of a two-dimensional system whose boundaries are subjected to prescribed conditions.

FIND: Finite-difference equations for these situations: (a) Upper boundary is perfectly insulated and side boundary is subjected to a convection process, (b) Both boundaries are perfectly insulated; compare result with Eq. 4.47.

SCHEMATIC:

$T_{m-1,n}$, T_{m-n}, Insulated boundary, $\Delta y = \Delta x$, Δx, T_{∞}, h, Material, k, $T_{m,n-1}$, Boundary exposed to convection process, $\Delta x/2$, q_4, $T_{m,n}$, $\Delta y/2$, q_1, q_3, q_2

ASSUMPTIONS: (1) Steady-state conditions, (2) Two-dimensional conduction, (3) Constant properties, (4) No internal generation.

ANALYSIS: Consider the nodal point configuration shown in the schematic and also as Case 4, Table 4.2. The control volume about the node – shaded area above of unit thickness normal to the page – has dimensions, $(\Delta x/2)(\Delta y/2)\cdot 1$. The heat transfer processes at the surface of the CV are identified as $q_1, q_2 \cdots$. Perform an energy balance wherein the processes are expressed using the appropriate rate equations.

(a) With the upper boundary insulated and the side boundary subjected to a convection process, the energy balance has the form

$$\dot{E}_{in} - \dot{E}_{out} = 0 \qquad q_1 + q_2 + q_3 + q_4 = 0 \qquad (1,2)$$

$$k\left[\frac{\Delta y}{2}\cdot 1\right]\frac{T_{m-1,n} - T_{m,n}}{\Delta x} + k\left[\frac{\Delta x}{2}\cdot 1\right]\frac{T_{m,n-1} - T_{m,n}}{\Delta y} + h\left[\frac{\Delta y}{2}\cdot 1\right]\left(T_{\infty} - T_{m,n}\right) + 0 = 0.$$

Letting $\Delta x = \Delta y$, and regrouping, find

$$T_{m,n-1} + T_{m-1,n} + \frac{h\Delta x}{k}T_{\infty} - 2\left[\frac{1}{2}\frac{h\Delta x}{k} + 1\right]T_{m,n} = 0. \qquad (3) <$$

(b) With both boundaries insulated, the energy balance of Eq. (2) would have $q_3 = q_4 = 0$. The same result would be obtained by letting h = 0 in the finite-difference equation, Eq. (3). The result is

$$T_{m,n-1} + T_{m-1,n} - 2T_{m,n} = 0. \qquad <$$

Note that this expression is identical to Eq. 4.47 when h = 0, in which case both boundaries are insulated.

COMMENTS: Note the convenience resulting from formulating the energy balance by *assuming* that all the heat flow is *into the node*.

PROBLEM 4.38

KNOWN: Conduction in a one-dimensional (radial) *cylindrical* coordinate system with volumetric generation.

FIND: Finite-difference equation for (a) Interior node, m, and (b) Surface node, n, with convection.

SCHEMATIC:

Note: control volumes are cylindrical shells

(a) Interior node, m (b) Surface node with convection, n

ASSUMPTIONS: (1) Steady-state, one-dimensional (radial) conduction in *cylindrical* coordinates, (2) Constant properties.

ANALYSIS: (a) The network has nodes spaced at equal Δr increments with m = 0 at the center; hence, r = mΔr (or nΔr). The control volume is $V = 2\pi\, r \cdot \Delta r \cdot \ell = 2\pi\,(m\Delta r)\,\Delta r \cdot \ell$. The energy balance is $\dot{E}_{in} + \dot{E}_g = q_a + q_b + \dot{q}V = 0$

$$k\left[2\pi\left[r - \frac{\Delta r}{2}\right]\ell\right]\frac{T_{m-1} - T_m}{\Delta r} + k\left[2\pi\left[r + \frac{\Delta r}{2}\right]\ell\right]\frac{T_{m+1} - T_m}{\Delta r} + \dot{q}\left[2\pi\,(m\Delta r)\,\Delta r\ell\right] = 0.$$

Recognizing that r = mΔr, canceling like terms, and regrouping find

$$\left[m - \frac{1}{2}\right]T_{m-1} + \left[m + \frac{1}{2}\right]T_{m+1} - 2mT_m + \frac{\dot{q}m\Delta r^2}{k} = 0.$$ <

(b) The control volume for the surface node is $V = 2\pi\, r \cdot (\Delta r/2) \cdot \ell$. The energy balance is $\dot{E}_{in} + \dot{E}_g = q_d + q_{conv} + \dot{q}V = 0$. Use Fourier's law to express q_d and Newton's law of cooling for q_{conv} to obtain

$$k\left[2\pi\left[r - \frac{\Delta r}{2}\right]\ell\right]\frac{T_{n-1} - T_n}{\Delta r} + h\left[2\pi\, r\ell\right](T_\infty - T_n) + \dot{q}\left[2\pi\,(n\Delta r)\frac{\Delta r}{2}\ell\right] = 0.$$

Let r = nΔr, cancel like terms and regroup to find

$$\left[n - \frac{1}{2}\right]T_{n-1} - \left[\left[n - \frac{1}{2}\right] + \frac{hn\Delta r}{k}\right]T_n + \frac{\dot{q}n\Delta r^2}{2k} + \frac{hn\Delta r}{k}T_\infty = 0.$$

<

COMMENTS: (1) Note that when m or n becomes very large compared to ½, the finite-difference equation becomes independent of m or n. Then the cylindrical system approximates a rectangular one.

(2) The finite-difference equation for the center node (m = 0) needs to be treated as a special case. The control volume is

$V = \pi\,(\Delta r/2)^2\,\ell$ and the energy balance is

$$\dot{E}_{in} + \dot{E}_g = q_a + \dot{q}V = k\left[2\pi\left[\frac{\Delta r}{2}\right]\ell\right]\frac{T_1 - T_0}{\Delta r} + \dot{q}\left[\pi\left[\frac{\Delta r}{2}\right]^2\ell\right] = 0.$$

Regrouping, the finite-difference equation is $-T_o + T_1 + \frac{\dot{q}\Delta r^2}{4k} = 0.$

PROBLEM 4.39

KNOWN: Two-dimensional cylindrical configuration with prescribed radial (Δr) and angular ($\Delta\phi$) spacings of nodes.

FIND: Finite-difference equations for nodes 2, 3 and 1.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Two-dimensional conduction in cylindrical coordinates (r,ϕ), (3) Constant properties.

ANALYSIS: The method of solution is to define the appropriate control volume for each node, to identify relevant processes and then to perform an energy balance.

(a) Node 2. This is an *interior* node with control volume as shown above. The energy balance is $\dot{E}_{in} = q'_a + q'_b + q'_c + q'_d = 0$. Using Fourier's law for each process, find

$$k\left[\left[r_i + \frac{3}{2}\Delta r\right]\Delta f\right]\frac{(T_5 - T_2)}{\Delta r} + k(\Delta r)\frac{(T_3 - T_2)}{(r_i + \Delta r)\Delta f} +$$

$$+ k\left[\left[r_i + \frac{1}{2}\Delta r\right]\Delta f\right]\frac{(T_i - T_2)}{\Delta r} + k(\Delta r)\frac{(T_1 - T_2)}{(r_i + \Delta r)\Delta f} = 0.$$

Canceling terms and regrouping yields,

$$-2\left[(r_i + \Delta r) + \frac{(\Delta r)^2}{(\Delta f)^2}\frac{1}{(r_i + \Delta r)}\right]T_2 + \left[r_i + \frac{3}{2}\Delta r\right]T_5 + \frac{(\Delta r)^2}{(r_i + \Delta r)(\Delta f)^2}(T_3 + T_1) + \left[r_i + \frac{1}{2}\Delta r\right]T_i = 0.$$

(b) Node 3. The adiabatic surface behaves as a symmetry surface. We can utilize the result of Part (a) to write the finite-difference equation by inspection as

$$-2\left[(r_i + \Delta r) + \frac{(\Delta r)^2}{(\Delta f)^2}\frac{1}{(r_i + \Delta r)}\right]T_3 + \left[r_i + \frac{3}{2}\Delta r\right]T_6 + \frac{2(\Delta r)^2}{(r_i + \Delta r)(\Delta f)^2}\cdot T_2 + \left[r_i + \frac{1}{2}\Delta r\right]T_i = 0.$$

(c) Node 1. The energy balance is $q'_a + q'_b + q'_c + q'_d = 0$. Substituting,

$$k\left[\left[r_i + \frac{3}{2}\Delta r\right]\frac{\Delta f}{2}\right]\frac{(T_4 - T_1)}{\Delta r} + k(\Delta r)\frac{(T_2 - T_1)}{(r_i + \Delta r)\Delta f} +$$

$$+ k\left[\left[r_i + \frac{1}{2}\Delta r\right]\frac{\Delta f}{2}\right]\frac{(T_i - T_1)}{\Delta r} + h(\Delta r)(T_\infty - T_1) = 0$$

<

This expression could now be rearranged.

PROBLEM 4.40

KNOWN: Heat generation and thermal boundary conditions of bus bar. Finite-difference grid.

FIND: Finite-difference equations for selected nodes.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Two-dimensional conduction, (3) Constant properties.

ANALYSIS: (a) Performing an energy balance on the control volume, $(\Delta x/2)(\Delta y/2)\cdot 1$, find the FDE for node 1,

$$\frac{T_o - T_1}{R''_{t,c}/(\Delta y/2)1} + h_u\left(\frac{\Delta x}{2}\cdot 1\right)(T_\infty - T_1) + \frac{k(\Delta y/2\cdot 1)}{\Delta x}(T_2 - T_1)$$

$$+\frac{k(\Delta x/2\cdot 1)}{\Delta y}(T_6 - T_1) + \dot{q}\left[(\Delta x/2)(\Delta y/2)1\right] = 0$$

$$\left(\Delta x/kR''_{t,c}\right)T_o + \left(h_u \Delta x/k\right)T_\infty + T_2 + T_6$$

$$+\dot{q}(\Delta x)^2/2k - \left[\left(\Delta x/kR''_{t,c}\right) + \left(h_u \Delta x/k\right) + 2\right]T_1 = 0. \qquad <$$

(b) Performing an energy balance on the control volume, $(\Delta x)(\Delta y/2)\cdot 1$, find the FDE for node 13,

$$h_l(\Delta x \cdot 1)(T_\infty - T_{13}) + (k/\Delta x)(\Delta y/2\cdot 1)(T_{12} - T_{13})$$
$$+(k/\Delta y)(\Delta x\cdot 1)(T_8 - T_{13}) + (k/\Delta x)(\Delta y/2\cdot 1)(T_{14} - T_{13}) + \dot{q}(\Delta x\cdot \Delta y/2\cdot 1) = 0$$

$$\left(h_l \Delta x/k\right)T_\infty + 1/2\left(T_{12} + 2T_8 + T_{14}\right) + \dot{q}(\Delta x)^2/2k - \left(h_l \Delta x/k + 2\right)T_{13} = 0. \qquad <$$

COMMENTS: For fixed T_o and T_∞, the relative amounts of heat transfer to the air and heat sink are determined by the values of h and $R''_{t,c}$.

PROBLEM 4.41

KNOWN: Nodal point configurations corresponding to a diagonal surface boundary subjected to a convection process and to the tip of a machine tool subjected to constant heat flux and convection cooling.

FIND: Finite-difference equations for the node m,n in the two situations shown.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state, 2-D conduction, (2) Constant properties.

ANALYSIS: (a) The control volume about node m,n has triangular shape with sides Δx and Δy while the diagonal (surface) length is $\sqrt{2}\ \Delta x$. The heat rates associated with the control volume are due to conduction, q_1 and q_2, and to convection, q_c. Performing an energy balance, find

$$\dot{E}_{in} - \dot{E}_{out} = 0 \qquad q_1 + q_2 + q_c = 0$$

$$k(\Delta x \cdot 1)\frac{T_{m,n-1} - T_{m,n}}{\Delta y} + k(\Delta y \cdot 1)\frac{T_{m+1,n} - T_{m,n}}{\Delta x} + h\left(\sqrt{2}\ \Delta x \cdot 1\right)\left(T_\infty - T_{m,n}\right) = 0.$$

Note that we have considered the tool to have unit depth normal to the page. Recognizing that $\Delta x = \Delta y$, dividing each term by k and regrouping, find

$$T_{m,n-1} + T_{m+1,n} + \sqrt{2}\ \cdot \frac{h\Delta x}{k} T_\infty - \left[2 + \sqrt{2}\ \cdot \frac{h\Delta x}{k}\right] T_{m,n} = 0. \quad <$$

(b) The control volume about node m,n has triangular shape with sides $\Delta x/2$ and $\Delta y/2$ while the lower diagonal surface length is $\sqrt{2}\,(\Delta x/2)$. The heat rates associated with the control volume are due to the constant heat flux, q_a, to conduction, q_b, and to the convection process, q_c. Perform an energy balance,

$$\dot{E}_{in} - \dot{E}_{out} = 0 \qquad q_a + q_b + q_c = 0$$

$$q_o'' \cdot \left[\frac{\Delta x}{2} \cdot 1\right] + k \cdot \left[\frac{\Delta y}{2} \cdot 1\right] \frac{T_{m+1,n} - T_{m,n}}{\Delta x} + h \cdot \left[\sqrt{2} \cdot \frac{\Delta x}{2}\right]\left(T_\infty - T_{m,n}\right) = 0.$$

Recognizing that $\Delta x = \Delta y$, dividing each term by k/2 and regrouping, find

$$T_{m+1,n} + \sqrt{2} \cdot \frac{h\Delta x}{k} \cdot T_\infty + q_o'' \cdot \frac{\Delta x}{k} - \left(1 + \sqrt{2} \cdot \frac{h\Delta x}{k}\right) T_{m,n} = 0. \quad <$$

COMMENTS: Note the appearance of the term $h\Delta x/k$ in both results, which is a dimensionless parameter (the *Biot number*) characterizing the relative effects of convection and conduction.

PROBLEM 4.42

KNOWN: Nodal point on boundary between two materials.

FIND: Finite-difference equation for steady-state conditions.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Two-dimensional conduction, (3) Constant properties, (4) No internal heat generation, (5) Negligible thermal contact resistance at interface.

ANALYSIS: The control volume is defined about nodal point 0 as shown above. The conservation of energy requirement has the form

$$\sum_{i=1}^{6} q_i = q_1 + q_2 + q_3 + q_4 + q_5 + q_6 = 0$$

since all heat rates are shown as *into* the CV. Each heat rate can be written using Fourier's law,

$$k_A \cdot \frac{\Delta y}{2} \cdot \frac{T_1 - T_0}{\Delta x} + k_A \cdot \Delta x \cdot \frac{T_2 - T_0}{\Delta y} + k_A \cdot \frac{\Delta y}{2} \cdot \frac{T_3 - T_0}{\Delta x}$$

$$+ k_B \cdot \frac{\Delta y}{2} \cdot \frac{T_3 - T_0}{\Delta x} + k_B \cdot \Delta x \cdot \frac{T_4 - T_0}{\Delta y} + k_B \cdot \frac{\Delta y}{2} \cdot \frac{T_1 - T_0}{\Delta x} = 0.$$

Recognizing that $\Delta x = \Delta y$ and regrouping gives the relation,

$$-T_0 + \frac{1}{4} T_1 + \frac{k_A}{2(k_A + k_B)} T_2 + \frac{1}{4} T_3 + \frac{k_B}{2(k_A + k_B)} T_4 = 0.$$

<

COMMENTS: Note that when $k_A = k_B$, the result agrees with Eq. 4.33 which is appropriate for an interior node in a medium of fixed thermal conductivity.

PROBLEM 4.43

KNOWN: Two-dimensional grid for a system with no internal volumetric generation.

FIND: Expression for heat rate per unit length normal to page crossing the isothermal boundary.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Two-dimensional heat transfer, (3) Constant properties.

ANALYSIS: Identify the surface nodes (T_s) and draw control volumes about these nodes. Since there is no heat transfer in the direction parallel to the isothermal surfaces, the heat rate out of the constant temperature surface boundary is

$$q' = q'_a + q'_b + q'_c + q'_d + q'_e + q'_f$$

For each q'_i, use Fourier's law and pay particular attention to the manner in which the cross-sectional area and gradients are specified.

$$q' = k(\Delta y/2)\frac{T_1 - T_s}{\Delta x} + k(\Delta y)\frac{T_2 - T_s}{\Delta x} + k(\Delta y)\frac{T_3 - T_s}{\Delta x}$$
$$+ k(\Delta x)\frac{T_5 - T_s}{\Delta y} + k(\Delta x)\frac{T_6 - T_s}{\Delta y} + k(\Delta x/2)\frac{T_7 - T_s}{\Delta y}$$

Regrouping with $\Delta x = \Delta y$, find

$$q' = k\left[0.5T_1 + T_2 + T_3 + T_5 + T_6 + 0.5T_7 - 5T_s\right].$$ <

COMMENTS: Looking at the corner node, it is important to recognize the areas associated with q'_c and q'_d (Δy and Δx, respectively).

PROBLEM 4.44

KNOWN: One-dimensional fin of uniform cross section insulated at one end with prescribed base temperature, convection process on surface, and thermal conductivity.

FIND: Finite-difference equation for these nodes: (a) Interior node, m and (b) Node at end of fin, n, where x = L.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction.

ANALYSIS: (a) The control volume about node m is shown in the schematic; the node spacing and control volume length in the x direction are both Δx. The uniform cross-sectional area and fin perimeter are A_c and P, respectively. The heat transfer process on the control surfaces, q_1 and q_2, represent conduction while q_c is the convection heat transfer rate between the fin and ambient fluid. Performing an energy balance, find

$$\dot{E}_{in} - \dot{E}_{out} = 0 \qquad q_1 + q_2 + q_c = 0$$

$$kA_c \frac{T_{m-1} - T_m}{\Delta x} + kA_c \frac{T_{m+1} - T_m}{\Delta x} + hP\Delta x\left(T_\infty - T_m\right) = 0.$$

Multiply the expression by $\Delta x/kA_c$ and regroup to obtain

$$T_{m-1} + T_{m+1} + \frac{hP}{kA_c} \cdot \Delta x^2 T_\infty - \left[2 + \frac{hP}{kA_c}\Delta x^2\right] T_m = 0 \qquad 1<m<n$$ <

Considering now the special node m = 1, then the m-1 node is T_b, the base temperature. The finite-difference equation would be

$$T_b + T_2 + \frac{hP}{kA_c}\Delta x^2 T_\infty - \left[2 + \frac{hP}{kA_c}\Delta x^2\right] T_1 = 0 \qquad m=1$$ <

(b) The control volume of length $\Delta x/2$ about node n is shown in the schematic. Performing an energy balance,

$$\dot{E}_{in} - \dot{E}_{out} = 0 \qquad q_3 + q_4 + q_c = 0$$

$$kA_c \frac{T_{n-1} - T_n}{\Delta x} + 0 + hP\frac{\Delta x}{2}\left(T_\infty - T_n\right) = 0.$$

Note that $q_4 = 0$ since the end (x = L) is insulated. Multiplying by $\Delta x/kA_c$ and regrouping,

$$T_{n-1} + \frac{hP}{kA_c} \cdot \frac{\Delta x^2}{2} T_\infty - \left[\frac{hP}{kA_c} \cdot \frac{\Delta x^2}{2} + 1\right] T_n = 0.$$ <

COMMENTS: The value of Δx will be determined by the selection of n; that is, $\Delta x = L/n$. Note that the grouping, hP/kA_c, appears in the finite-difference and differential forms of the energy balance.

PROBLEM 4.45

KNOWN: Two-dimensional network with prescribed nodal temperatures and thermal conductivity of the material.

FIND: Heat rate per unit length normal to page, q'.

SCHEMATIC:

Node	T_i(°C)
1	120.55
2	120.64
3	121.29
4	123.89
5	134.57
6	150.49
7	147.14

ASSUMPTIONS: (1) Steady-state conditions, (2) Two-dimensional heat transfer, (3) No internal volumetric generation, (4) Constant properties.

ANALYSIS: Construct control volumes around the nodes on the surface maintained at the uniform temperature T_s and indicate the heat rates. The heat rate per unit length is $q' = q'_a + q'_b + q'_c + q'_d + q'_e$ or in terms of conduction terms between nodes,

$$q' = q'_1 + q'_2 + q'_3 + q'_4 + q'_5 + q'_7.$$

Each of these rates can be written in terms of nodal temperatures and control volume dimensions using Fourier's law,

$$q' = k \cdot \frac{\Delta x}{2} \cdot \frac{T_1 - T_s}{\Delta y} + k \cdot \Delta x \cdot \frac{T_2 - T_s}{\Delta y} + k \cdot \Delta x \frac{T_3 - T_s}{\Delta y} + k \cdot \Delta x \frac{T_4 - T_s}{\Delta y}$$
$$+ k \cdot \Delta x \frac{T_5 - T_s}{\Delta y} + k \cdot \frac{\Delta y}{2} \cdot \frac{T_7 - T_s}{\Delta x}.$$

and since $\Delta x = \Delta y$,

$$q' = k[(1/2)(T_1 - T_s) + (T_2 - T_s) + (T_3 - T_s)$$
$$+ (T_4 - T_s) + (T_5 - T_s) + (1/2)(T_7 - T_s)].$$

Substituting numerical values, find

$$q' = 50\text{ W/m}\cdot\text{K}[(1/2)(120.55 - 100) + (120.64 - 100) + (121.29 - 100)$$
$$+ (123.89 - 100) + (134.57 - 100) + (1/2)(147.14 - 100)]$$

$$q' = 6711\text{ W/m}.$$ <

COMMENTS: For nodes a through d, there is no heat transfer into the control volumes in the x-direction. Look carefully at the energy balance for node e, $q'_e = q'_5 + q'_7$, and how q'_5 and q'_7 are evaluated.

PROBLEM 4.46

KNOWN: Nodal temperatures from a steady-state, finite-difference analysis for a one-eighth symmetrical section of a square channel.

FIND: (a) Beginning with properly defined control volumes, derive the finite-difference equations for nodes 2, 4 and 7, and determine T_2, T_4 and T_7, and (b) Heat transfer loss per unit length from the channel, q'.

SCHEMATIC:

Node	T(°C)
1	430
3	394
6	492
8,9	600

ASSUMPTIONS: (1) Steady-state conditions, (2) Two-dimensional conduction, (3) No internal volumetric generation, (4) Constant properties.

ANALYSIS: (a) Define control volumes about the nodes 2, 4, and 7, taking advantage of symmetry where appropriate and performing energy balances, $\dot{E}_{in} - \dot{E}_{out} = 0$, with $\Delta x = \Delta y$,

Node 2: $q'_a + q'_b + q'_c + q'_d = 0$

$$h\Delta x\left(T_\infty - T_2\right) + k\left(\Delta y/2\right)\frac{T_3 - T_2}{\Delta x} + k\Delta x\frac{T_6 - T_2}{\Delta y} + k\left(\Delta y/2\right)\frac{T_1 - T_2}{\Delta x} = 0$$

$$T_2 = \left[0.5T_1 + 0.5T_3 + T_6 + \left(h\Delta x/k\right)T_\infty\right]/\left[2 + \left(h\Delta x/k\right)\right]$$

$$T_2 = \left[0.5\times 430 + 0.5\times 394 + 492 + \left(50\,\text{W/m}^2\cdot\text{K}\times 0.01\,\text{m}/1\,\text{W/m}\cdot\text{K}\right)300\right]\text{K}\Big/\left[2 + 0.50\right]$$

$$T_2 = 422\,\text{K}$$ <

Node 4: $q'_a + q'_b + q'_c = 0$

$$h\left(\Delta x/2\right)\left(T_\infty - T_4\right) + 0 + k\left(\Delta y/2\right)\frac{T_3 - T_4}{\Delta x} = 0$$

$$T_4 = \left[T_3 + \left(h\Delta x/k\right)T_\infty\right]/\left[1 + \left(h\Delta x/k\right)\right]$$

$$T_4 = \left[394 + 0.5\times 300\right]\text{K}/\left[1 + 0.5\right] = 363\,\text{K}$$ <

Continued...

PROBLEM 4.46 (Cont.)

Node 7: From the first schematic, recognizing that the diagonal is a symmetry adiabat, we can treat node 7 as an interior node, hence

$$T_7 = 0.25\left(T_3 + T_3 + T_6 + T_6\right) = 0.25\left(394 + 394 + 492 + 492\right)K = 443\,K$$ <

(b) The heat transfer loss from the upper surface can be expressed as the sum of the convection rates from each node as illustrated in the first schematic,

$$q'_{cv} = q'_1 + q'_2 + q'_3 + q'_4$$

$$q'_{cv} = h\left(\Delta x/2\right)\left(T_1 - T_\infty\right) + h\Delta x\left(T_2 - T_\infty\right) + h\Delta x\left(T_3 - T_\infty\right) + h\left(\Delta x/2\right)\left(T_4 - T_\infty\right)$$

$$q'_{cv} = 50\,W/m^2 \cdot K \times 0.1\,m\left[\left(430 - 300\right)/2 + \left(422 - 300\right) + \left(394 - 300\right) + \left(363 - 300\right)/2\right]K$$

$$q'_{cv} = 156\,W/m$$ <

COMMENTS: (1) Always look for symmetry conditions which can greatly simplify the writing of the nodal equation as was the case for Node 7.

(2) Consider using the *IHT Tool*, *Finite-Difference Equations*, for *Steady-State*, *Two-Dimensional* heat transfer to determine the nodal temperatures T_1 - T_7 when only the boundary conditions T_8, T_9 and (T_∞,h) are specified.

PROBLEM 4.47

KNOWN: Steady-state temperatures (K) at three nodes of a long rectangular bar.

FIND: (a) Temperatures at remaining nodes and (b) heat transfer per unit length from the bar using nodal temperatures; compare with result calculated using knowledge of $\dot{q}$.

SCHEMATIC:

$\dot{q} = 5\times10^7 W/m^3$
$k = 20 W/m\cdot K$
$\Delta x = \Delta y = 5 mm$
$T_A = 398.0K$
$T_B = 374.6K$
$T_c = 348.5K$

ASSUMPTIONS: (1) Steady-state, 2-D conduction, (2) Constant properties.

ANALYSIS: (a) The finite-difference equations for the nodes (1,2,3,A,B,C) can be written by inspection using Eq. 4.39 and recognizing that the adiabatic boundary can be represented by a symmetry plane.

$$\sum T_{neighbors} - 4T_i + \dot{q}\Delta x^2/k = 0 \quad \text{and} \quad \frac{\dot{q}\Delta x^2}{k} = \frac{5\times10^7 \text{ W/m}^3 (0.005\text{m})^2}{20 \text{ W/m}\cdot\text{K}} = 62.5\text{K}.$$

Node A (to find T_2*):*

$$2T_2 + 2T_B - 4T_A + \dot{q}\Delta x^2/k = 0$$

$$T_2 = \frac{1}{2}(-2\times374.6 + 4\times398.0 - 62.5)\text{K} = 390.2\text{K} \quad <$$

Node 3 (to find T_3*):*

$$T_c + T_2 + T_B + 300\text{K} - 4T_3 + \dot{q}\Delta x^2/k = 0$$

$$T_3 = \frac{1}{4}(348.5 + 390.2 + 374.6 + 300 + 62.5)\text{K} = 369.0\text{K} \quad <$$

Node 1 (to find T_1*):*

$$300 + 2T_C + T_2 - 4T_1 + \dot{q}\Delta x^2/k = 0$$

$$T_1 = \frac{1}{4}(300 + 2\times348.5 + 390.2 + 62.5) = 362.4\text{K} \quad <$$

(b) The heat rate out of the bar is determined by calculating the heat rate out of each control volume around the 300K nodes. Consider the node in the upper left-hand corner; from an energy balance

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = 0 \quad \text{or} \quad q'_a = q'_{a,in} + \dot{E}_g \quad \text{where} \quad \dot{E}_g = \dot{q}V.$$

Hence, for the entire bar $\quad q'_{bar} = q'_a + q'_b + q'_c + q'_d + q'_e + q'_f$, or

$$q'_{bar} = \left[k\frac{\Delta y}{2}\frac{T_1 - 300}{\Delta x} + \dot{q}\left[\frac{\Delta x}{2}\cdot\frac{\Delta y}{2}\right]\right]_a + \left[k\Delta y\frac{T_C - 300}{\Delta x} + \dot{q}\left[\frac{\Delta x}{2}\cdot\Delta y\right]\right]_b + \left[\dot{q}\left[\frac{\Delta x}{2}\cdot\frac{\Delta y}{2}\right]\right]_c +$$
$$\left[k\Delta x\frac{T_C - 300}{\Delta y} + \dot{q}\left[\Delta x\cdot\frac{\Delta y}{2}\right]\right]_d + \left[k\Delta x\frac{T_3 - 300}{\Delta y} + \dot{q}\left[\Delta x\cdot\frac{\Delta y}{2}\right]\right]_e + \left[k\frac{\Delta x}{2}\frac{T_B - 300}{\Delta y} + \dot{q}\left[\frac{\Delta x}{2}\cdot\frac{\Delta y}{2}\right]\right]_f.$$

Substituting numerical values, find $q'_{bar} = 7{,}502.5$ W/m. From an overall energy balance on the bar,

$$q'_{bar} = \dot{E}'_g = \dot{q}V/\ell = \dot{q}(3\Delta x\cdot2\Delta y) = 5\times10^7 \text{ W/m}^3\times6(0.005\text{m})^2 = 7{,}500 \text{ W/m}. \quad <$$

As expected, the results of the two methods agree. Why must that be?

PROBLEM 4.48

KNOWN: Steady-state temperatures at selected nodal points of the symmetrical section of a flow channel with uniform internal volumetric generation of heat. Inner and outer surfaces of channel experience convection.

FIND: (a) Temperatures at nodes 1, 4, 7, and 9, (b) Heat rate per unit length (W/m) from the outer surface A to the adjacent fluid, (c) Heat rate per unit length (W/m) from the inner fluid to surface B, and (d) Verify that results are consistent with an overall energy balance.

SCHEMATIC:

$\Delta y = \Delta x = 25$ mm; $T_{\infty,i} = 50°C$, $h_i = 500\ W/m^2\text{-}K$; $\dot{q} = 10^6\ W/m^3$, $k = 10$ W/m-K; $T_{\infty,o} = 25°C$, $h_o = 250\ W/m^2\text{-}K$. Insulation, Surface A, Surface B, Symmetry plane.

$T_2 = 95.47°C$
$T_3 = 117.3°C$
$T_5 = 79.79°C$
$T_6 = 77.29°C$
$T_8 = 87.28°C$
$T_{10} = 77.65°C$

ASSUMPTIONS: (1) Steady-state, two-dimensional conduction, (2) Constant properties.

ANALYSIS: (a) The nodal finite-difference equations are obtained from energy balances on control volumes about the nodes shown in the schematics below.

Node 1

$$q'_a + q'_b + q'_c + q'_d + \dot{E}'_g = 0$$

$$0 + k(\Delta y/2)\frac{T_2 - T_1}{\Delta x} + k(\Delta x/2)\frac{T_3 - T_1}{\Delta y} + 0 + \dot{q}(\Delta x \cdot \Delta y/4) = 0$$

$$T_1 = (T_2 + T_3)/2 + \dot{q}\Delta x^2/2k$$

$$T_1 = (95.47 + 117.3)°C/2 + 10^6\ W/m^3 (25\times 25)\times 10^{-6}\ m^2/(2\times 10\ W/m\cdot K) = 122.0°C$$

Node 1: T_1, T_2, T_3, q'_a, q'_b, q'_c, q'_d, $\dot{E}'_g$. Node 4: T_2, T_3, T_4, T_5, T_8, $T_{\infty,i}$, h_i, q'_a, q'_b, q'_c, q'_d, q'_e, q'_f, $\dot{E}'_g$.

Node 4

$$q'_a + q'_b + q'_c + q'_d + q'_e + q'_f + \dot{E}'_g = 0$$

$$k(\Delta x/2)\frac{T_2 - T_4}{\Delta y} + h_i(\Delta y/2)(T_{\infty,i} - T_4) + h_i(\Delta x/2)(T_\infty - T_4) +$$

Continued …..

PROBLEM 4.48 (Cont.)

$$k(\Delta y/2)\frac{T_5 - T_4}{\Delta x} + k(\Delta x)\frac{T_8 - T_4}{\Delta y} + k(\Delta y)\frac{T_3 - T_4}{\Delta x} + \dot{q}(3\Delta x \cdot \Delta y/4) = 0$$

$$T_4 = \left[T_2 + 2T_3 + T_5 + 2T_8 + 2(h_i \Delta x/k)T_{\infty,i} + \left(3\dot{q}\Delta x^2/2k\right)\right] \Big/ \left[6 + 2(h_i \Delta x/k)\right]$$

$$T_4 = 94.50°C \qquad <$$

Node 7

$$q'_a + q'_b + q'_c + q'_d + \dot{E}'_g = 0$$

$$k(\Delta x/2)\frac{T_3 - T_7}{\Delta y} + k(\Delta y/2)\frac{T_8 - T_7}{\Delta x} + h_o(\Delta x/2)\left(T_{\infty,o} - T_7\right) + 0 + \dot{q}(\Delta x \cdot \Delta y/4) = 0$$

$$T_7 = \left[T_3 + T_8 + (h_o \Delta x/k)T_{\infty,o} + \dot{q}\Delta x^2/2k\right] \Big/ (2 + h_o \Delta x/k)$$

$$T_7 = 95.80°C \qquad <$$

Node 7

Node 9

Node 9

$$q'_a + q'_b + q'_c + q'_d + \dot{E}'_g = 0$$

$$k(\Delta x)\frac{T_5 - T_9}{\Delta y} + k(\Delta y/2)\frac{T_{10} - T_9}{\Delta y} + h_o(\Delta x)\left(T_{\infty,o} - T_9\right)$$

$$+ k(\Delta y/2)\frac{T_8 - T_9}{\Delta x} + \dot{q}(\Delta x \cdot \Delta y/2) = 0$$

$$T_9 = \left[T_5 + 0.5T_8 + 0.5T_{10} + (h_o \Delta x/k)T_{\infty,o} + \dot{q}\Delta x^2/2k\right] / (2 + h_o \Delta x/k)$$

$$T_9 = 79.67°C \qquad <$$

(b) The heat rate per unit length from the outer surface A to the adjacent fluid, q'_A, is the sum of the convection heat rates from the outer surfaces of nodes 7, 8, 9 and 10.

$$q'_A = h_o\left[(\Delta x/2)\left(T_7 - T_{\infty,o}\right) + \Delta x\left(T_8 - T_{\infty,o}\right) + \Delta x\left(T_9 - T_{\infty,o}\right) + (\Delta x/2)\left(T_{10} - T_{\infty,o}\right)\right]$$

$$q'_A = 250\ \text{W/m}^2 \cdot \text{K}\left[(25/2)(95.80 - 25) + 25(87.28 - 25)\right.$$

$$\left. + 25(79.67 - 25) + (25/2)(77.65 - 25)\right] \times 10^{-3}\,\text{m} \cdot \text{K}$$

Continued …..

$$q'_A = 1117\ \text{W/m} \qquad <$$

(c) The heat rate per unit length from the inner fluid to the surface B, q'_B, is the sum of the convection heat rates from the inner surfaces of nodes 2, 4, 5 and 6.

$$q'_B = h_i\left[(\Delta y/2)\left(T_{\infty,i} - T_2\right) + (\Delta y/2 + \Delta x/2)\left(T_{\infty,i} - T_4\right) + \Delta x\left(T_{\infty,i} - T_5\right) + (\Delta x/2)\left(T_{\infty,i} - T_6\right)\right]$$

$$q'_B = 500\ \text{W/m}^2\cdot\text{K}\left[(25/2)(50-95.47) + (25/2+25/2)(50-94.50)\right.$$

$$\left. +25(50-79.79) + (25/2)(50-77.29)\right]\times 10^{-3}\ \text{m}\cdot\text{K}$$

$$q'_B = -1383\ \text{W/m} \qquad <$$

(d) From an overall energy balance on the section, we see that our results are consistent since the conservation of energy requirement is satisfied.

$$\dot{E}'_{in} - \dot{E}'_{out} + \dot{E}'_{gen} = -q'_A + q'_B + \dot{E}'_{gen} = (-1117 - 1383 + 2500)\text{W/m} = 0$$

where $\dot{E}'_{gen} = \dot{q}\forall' = 10^6\ \text{W/m}^3\left[25\times 50 + 25\times 50\right]\times 10^{-6}\ \text{m}^2 = 2500\ \text{W/m}$

COMMENTS: The nodal finite-difference equations for the four nodes can be obtained by using IHT Tool *Finite-Difference Equations* | *Two-Dimensional* | *Steady-state*. Options are provided to build the FDEs for interior, corner and surface nodal arrangements including convection and internal generation. The IHT code lines for the FDEs are shown below.

```
/* Node 1: interior node; e, w, n, s labeled 2, 2, 3, 3. */
0.0   = fd_2d_int(T1,T2,T2,T3,T3,k,qdot,deltax,deltay)

/* Node 4: internal corner node, e-n orientation; e, w, n, s labeled 5, 3, 2, 8. */
0.0   = fd_2d_ic_en(T4,T5,T3,T2,T8,k,qdot,deltax,deltay,Tinfi,hi,q• a4
q• a4 = 0                  // Applied heat flux, W/m^2; zero flux shown

/* Node 7: plane surface node, s-orientation; e, w, n labeled 8, 8, 3. */
0.0   = fd_2d_psur_s(T7,T8,T8,T3,k,qdot,deltax,deltay,Tinfo,ho,q• a7
q• a7=0                    // Applied heat flux, W/m^2; zero flux shown

/* Node 9: plane surface node, s-orientation; e, w, n labeled 10, 8, 5. */
0.0 = fd_2d_psur_s(T9, T10, T8, T5,k,qdot,deltax,deltay,Tinfo,ho,q• a9
q• a9 = 0                  // Applied heat flux, W/m^2; zero flux shown
```

PROBLEM 4.49

KNOWN: Outer surface temperature, inner convection conditions, dimensions and thermal conductivity of a heat sink.

FIND: Nodal temperatures and heat rate per unit length.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state, (2) Two-dimensional conduction, (3) Uniform outer surface temperature, (4) Constant thermal conductivity.

ANALYSIS: (a) To determine the heat rate, the nodal temperatures must first be computed from the corresponding finite-difference equations. From an energy balance for node 1,

$$h(\Delta x/2\cdot 1)(T_\infty - T_1) + k(\Delta y/2\cdot 1)\frac{T_2 - T_1}{\Delta x} + k(\Delta x\cdot 1)\frac{T_5 - T_1}{\Delta y} = 0$$

$$-\left(3 + \frac{h\Delta x}{k}\right)T_1 + T_2 + 2T_5 + \frac{h\Delta x}{k}T_\infty = 0 \qquad (1)$$

With nodes 2 and 3 corresponding to Case 3 of Table 4.2,

$$T_1 - 2\left(\frac{h\Delta x}{k} + 2\right)T_2 + T_3 + 2T_6 + \frac{2h\Delta x}{k}T_\infty = 0 \qquad (2)$$

$$T_2 - \left(\frac{h\Delta x}{k} + 2\right)T_3 + T_7 + \frac{h\Delta x}{k}T_\infty = 0 \qquad (3)$$

where the symmetry condition is invoked for node 3. Applying an energy balance to node 4, we obtain

$$-2T_4 + T_5 + T_s = 0 \qquad (4)$$

The interior nodes 5, 6 and 7 correspond to Case 1 of Table 4.2. Hence,

$$T_1 + T_4 - 4T_5 + T_6 + T_s = 0 \qquad (5)$$

$$T_2 + T_5 - 4T_6 + T_7 + T_s = 0 \qquad (6)$$

$$T_3 + 2T_6 - 4T_7 + T_s = 0 \qquad (7)$$

where the symmetry condition is invoked for node 7. With $T_s = 50°C$, $T_\infty = 20°C$, and $h\Delta x/k = 5000\ W/m^2\cdot K\,(0.005m)/240\ W/m\cdot K = 0.1042$, the solution to Eqs. (1) – (7) yields

$$T_1 = 46.61°C,\ T_2 = 45.67°C,\ T_3 = 45.44°C,\ T_4 = 49.23°C$$

$$T_5 = 48.46°C,\ T_6 = 48.00°C,\ T_7 = 47.86°C \qquad <$$

Continued …..

The heat rate per unit length of channel may be evaluated by computing convection heat transfer from the inner surface. That is,

$$q' = 8h\left[\Delta x/2\left(T_1 - T_\infty\right) + \Delta x\left(T_2 - T_\infty\right) + \Delta x/2\left(T_3 - T_\infty\right)\right]$$

$$q' = 8\times 5000\,\mathrm{W/m^2\cdot K}\left[0.0025\mathrm{m}\left(46.61-20\right)^\circ\mathrm{C} + 0.005\mathrm{m}\left(45.67-20\right)^\circ\mathrm{C}\right.$$

$$\left.+0.0025\mathrm{m}\left(45.44-20\right)^\circ\mathrm{C}\right] = 10,340\ \mathrm{W/m}$$ <

(b) Since $h = 5000\ \mathrm{W/m^2\cdot K}$ is at the high end of what can be achieved through forced convection, we consider the effect of reducing h. Representative results are as follows

$h\left(\mathrm{W/m^2\cdot K}\right)$	$T_1\,(^\circ\mathrm{C})$	$T_2\,(^\circ\mathrm{C})$	$T_3\,(^\circ\mathrm{C})$	$T_4\,(^\circ\mathrm{C})$	$T_5\,(^\circ\mathrm{C})$	$T_6\,(^\circ\mathrm{C})$	$T_7\,(^\circ\mathrm{C})$	$q'\,(\mathrm{W/m})$
200	49.84	49.80	49.79	49.96	49.93	49.91	49.90	477
1000	49.24	49.02	48.97	49.83	49.65	49.55	49.52	2325
2000	48.53	48.11	48.00	49.66	49.33	49.13	49.06	4510
5000	46.61	45.67	45.44	49.23	48.46	48.00	47.86	10,340

There are two resistances to heat transfer between the outer surface of the heat sink and the fluid, that due to conduction in the heat sink, $R_{cond(2D)}$, and that due to convection from its inner surface to the fluid, R_{conv}. With decreasing h, the corresponding increase in R_{conv} reduces heat flow and increases the uniformity of the temperature field in the heat sink. The nearly 5-fold reduction in q' corresponding to the 5-fold reduction in h from 1000 to 200 $\mathrm{W/m^2\cdot K}$ indicates that the convection resistance is dominant $\left(R_{conv} \gg R_{cond(2D)}\right)$.

COMMENTS: To check our finite-difference solution, we could assess its consistency with conservation of energy requirements. For example, an energy balance performed at the inner surface requires a balance between convection from the surface and conduction to the surface, which may be expressed as

$$q' = k\left(\Delta x\cdot 1\right)\frac{\left(T_5 - T_1\right)}{\Delta y} + k\left(\Delta x\cdot 1\right)\frac{T_6 - T_2}{\Delta y} + k\left(\Delta x/2\cdot 1\right)\frac{T_7 - T_3}{\Delta y}$$

Substituting the temperatures corresponding to $h = 5000\ \mathrm{W/m^2\cdot K}$, the expression yields $q' = 10,340\,\mathrm{W/m}$, and, as it must be, conservation of energy is precisely satisfied. Results of the analysis may also be checked by using the expression $q' = \left(T_s - T_\infty\right)/\left(R'_{cond(2D)} + R'_{conv}\right)$, where, for $h = 5000\ \mathrm{W/m^2\cdot K}$, $R'_{conv} = \left(1/4hw\right) = 2.5\times 10^{-3}\,\mathrm{m\cdot K/W}$, and from Eq. (4.27) and Case 11 of Table 4.1, $R'_{cond} = \left[0.930\ \ln\left(W/w\right) - 0.05\right]/2\pi k = 3.94\times 10^{-4}\,\mathrm{m\cdot K/W}$. Hence, $q' = \left(50-20\right)^\circ\mathrm{C}/\left(2.5\times 10^{-3} + 3.94\times 10^{-4}\right)\mathrm{m\cdot K/W} = 10,370\ \mathrm{W/m}$, and the agreement with the finite-difference solution is excellent. Note that, even for $h = 5000\ \mathrm{W/m^2\cdot K}$, $R'_{conv} \gg R'_{cond(2D)}$.

PROBLEM 4.50

KNOWN: Steady-state temperatures (°C) associated with selected nodal points in a two-dimensional system.

FIND: (a) Temperatures at nodes 1, 2 and 3, (b) Heat transfer rate per unit thickness from the system surface to the fluid.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state, two-dimensional conduction, (2) Constant properties.

ANALYSIS: (a) Using the finite-difference equations for Nodes 1, 2 and 3:

Node 1, Interior node, Eq. 4.33: $T_1 = \frac{1}{4} \cdot \sum T_{neighbors}$

$$T_1 = \frac{1}{4}(172.9 + 137.0 + 132.8 + 200.0)^{\circ}C = 160.7^{\circ}C$$ <

Node 2, Insulated boundary, Eq. 4.46 with h = 0, $T_{m,n} = T_2$

$$T_2 = \frac{1}{4}\left(T_{m-1,n} + T_{m+1,n} + 2T_{m,n-1}\right)$$

$$T_2 = \frac{1}{4}(129.4 + 45.8 + 2 \times 103.5)^{\circ} C = 95.6^{\circ}C$$ <

Node 3, Plane surface with convection, Eq. 4.46, $T_{m,n} = T_3$

$$2\left[\frac{h\Delta x}{k} + 2\right]T_3 = \left(2T_{m-1,n} + T_{m,n+1} + T_{m,n-1}\right) + \frac{2h\Delta x}{k}T_\infty$$

$$h\Delta x/k = 50 W/m^2 \cdot K \times 0.1 m/1.5 W/m \cdot K = 3.33$$

$$2(3.33 + 2)T_3 = (2 \times 103.5 + 45.8 + 67.0)^{\circ}C + 2 \times 3.33 \times 30^{\circ}C$$

$$T_3 = \frac{1}{10.66}(319.80 + 199.80)^{\circ}C = 48.7^{\circ}C$$ <

(b) The heat rate per unit thickness from the surface to the fluid is determined from the sum of the convection rates from each control volume surface.

$$q'_{conv} = q'_a + q'_b + q'_c + q'_d$$

$$q_i = h\Delta y_i\,(T_i - T_\infty)$$

$$q'_{conv} = 50\frac{W}{m^2 \cdot K}\left[\frac{0.1}{2}m\,(45.8 - 30.0)^{\circ}C + 0.1m\,(48.7 - 30.0)^{\circ}C + 0.1m\,(67.0 - 30.0)^{\circ}C + \frac{0.1m}{2}(200.0 - 30.0)^{\circ}C\right]$$

$$q'_{conv} = (39.5 + 93.5 + 185.0 + 425)\ W/m = 743\ W/m.$$ <

PROBLEM 4.51

KNOWN: Nodal temperatures from a steady-state finite-difference analysis for a cylindrical fin of prescribed diameter, thermal conductivity and convection conditions (T_∞, h).

FIND: (a) The fin heat rate, q_f, and (b) Temperature at node 3, T_3.

SCHEMATIC:

$T_0 = 100.0°C$
$T_1 = 93.4°C$
$T_2 = 89.5°C$

$D = 12$ mm, $k = 15$ W/m·K

$T_\infty = 25$ °C, $h = 25$ W/m^2·K

ASSUMPTIONS: (a) The fin heat rate, q_f, is that of conduction at the base plane, x = 0, and can be found from an energy balance on the control volume about node 0, $\dot{E}_{in} - \dot{E}_{out} = 0$,

$$q_f + q_1 + q_{conv} = 0 \qquad \text{or} \qquad q_f = -q_1 - q_{conv}.$$

Writing the appropriate rate equation for q_1 and q_{conv}, with $A_c = \pi D^2/4$ and $P = \pi D$,

$$q_f = -kA_c \frac{T_1 - T_0}{\Delta x} - hP(\Delta x/2)(T_\infty - T_0) = -\frac{\pi k D^2}{4\Delta x}(T_1 - T_0) - (\pi/2) D h \Delta x (T_\infty - T_0)$$

Substituting numerical values, with $\Delta x = 0.010$ m, find

$$q_f = -\frac{\pi \times 15\,\text{W/m}\cdot\text{K}(0.012\,\text{m})^2}{4 \times 0.010\,\text{m}}(93.4 - 100)^\circ\text{C}$$
$$-\frac{\pi}{2} \times 0.012\,\text{m} \times 25\,\text{W/m}^2\cdot\text{K} \times 0.010\,\text{m}(25 - 100)^\circ\text{C}$$

$$q_f = (1.120 + 0.353)\,\text{W} = 1.473\,\text{W}.$$ <

(b) To determine T_3, derive the finite-difference equation for node 2, perform an energy balance on the control volume shown above, $\dot{E}_{in} - \dot{E}_{out} = 0$,

$$q_{cv} + q_3 + q_1 = 0$$

$$hP\Delta x(T_\infty - T_2) + kA_c\frac{T_3 - T_2}{\Delta x} + kA_c\frac{T_1 - T_2}{\Delta x} = 0$$

$$T_3 = -T_1 + 2T_2 - \frac{hP\Delta x^2}{kA_c}\Delta x^2[T_\infty - T_2]$$

Substituting numerical values, find

$$T_2 = 89.2^\circ\text{C}$$ <

COMMENTS: Note that in part (a), the convection heat rate from the outer surface of the control volume is significant (25%). It would have been poor approximation to ignore this term.

PROBLEM 4.52

KNOWN: Long rectangular bar having one boundary exposed to a convection process (T_∞, h) while the other boundaries are maintained at a constant temperature (T_s).

FIND: (a) Using a grid spacing of 30 mm and the Gauss-Seidel method, determine the nodal temperatures and the heat rate per unit length into the bar from the fluid, (b) Effect of grid spacing and convection coefficient on the temperature field.

SCHEMATIC:

Fluid
T_∞ = 100 °C
h = 100 W/m²·K
T_s = 50 °C
Δy = 30 mm
Δx = 30 mm
(a)

T_∞, h
Δx = 15 mm
Δy = 15 m
T_s
(b)

ASSUMPTIONS: (1) Steady-state, two-dimensional conduction, (2) Constant properties.

ANALYSIS: (a) With the grid spacing Δx = Δy = 30 mm, three nodes are created. Using the finite-difference equations as shown in Table 4.2, but written in the form required of the Gauss-Seidel method (see Section 4.5.2), and with Bi = hΔx/k = 100 W/m²·K × 0.030 m/1 W/m·K = 3, we obtain:

Node 1: $$T_1 = \frac{1}{(Bi+2)}\left(T_2 + T_s + BiT_\infty\right) = \frac{1}{5}\left(T_2 + 50 + 3\times 100\right) = \frac{1}{5}\left(T_2 + 350\right) \quad (1)$$

Node 2: $$T_2 = \frac{1}{4}\left(T_1 + 2T_s + T_3\right) = \frac{1}{4}\left(T_1 + T_3 + 2\times 50\right) = \frac{1}{4}\left(T_1 + T_3 + 100\right) \quad (2)$$

Node 3: $$T_3 = \frac{1}{4}\left(T_2 + 3T_s\right) = \frac{1}{4}\left(T_2 + 3\times 50\right) = \frac{1}{4}\left(T_2 + 150\right) \quad (3)$$

Denoting each nodal temperature with a superscript to indicate iteration step, e.g. T_1^k, calculate values as shown below.

k	T_1	T_2	T_3 (°C)	
0	85	60	55	← initial guess
1	82.00	59.25	52.31	
2	81.85	58.54	52.14	
3	81.71	58.46	52.12	
4	81.69	58.45	52.11	

By the 4th iteration, changes are of order 0.02°C, suggesting that further calculations may not be necessary.

Continued...

PROBLEM 4.52 (Cont.)

In finite-difference form, the heat rate from the fluid to the bar is

$$q'_{conv} = h(\Delta x/2)(T_\infty - T_s) + h\Delta x(T_\infty - T_1) + h(\Delta x/2)(T_\infty - T_s)$$

$$q'_{conv} = h\Delta x(T_\infty - T_s) + h\Delta x(T_\infty - T_1) = h\Delta x\left[(T_\infty - T_s) + (T_\infty - T_1)\right]$$

$$q'_{conv} = 100\,\text{W/m}^2\cdot\text{K} \times 0.030\,\text{m}\left[(100-50)+(100-81.7)\right]^\circ\text{C} = 205\,\text{W/m}\,.$$ <

(b) Using the *Finite-Difference Equations* option from the *Tools* portion of the IHT menu, the following two-dimensional temperature field was computed for the grid shown in schematic (b), where x and y are in mm and the temperatures are in °C.

y\x	0	15	30	45	60
0	50	80.33	85.16	80.33	50
15	50	63.58	67.73	63.58	50
30	50	56.27	58.58	56.27	50
45	50	52.91	54.07	52.91	50
60	50	51.32	51.86	51.32	50
75	50	50.51	50.72	50.51	50
90	50	50	50	50	50

The improved prediction of the temperature field has a significant influence on the heat rate, where, accounting for the symmetrical conditions,

$$q' = 2h(\Delta x/2)(T_\infty - T_s) + 2h(\Delta x)(T_\infty - T_1) + h(\Delta x)(T_\infty - T_2)$$

$$q' = h(\Delta x)\left[(T_\infty - T_s) + 2(T_\infty - T_1) + (T_\infty - T_2)\right]$$

$$q' = 100\,\text{W/m}^2\cdot\text{K}(0.015\,\text{m})\left[50 + 2(19.67) + 14.84\right]^\circ\text{C} = 156.3\,\text{W/m}$$ <

Additional improvements in accuracy could be obtained by reducing the grid spacing to 5 mm, although the requisite number of finite-difference equations would increase from 12 to 108, significantly increasing problem *set-up* time.

An increase in h would increase temperatures everywhere within the bar, particularly at the heated surface, as well as the rate of heat transfer by convection to the surface.

COMMENTS: (1) Using the matrix-inversion method, the exact solution to the system of equations (1, 2, 3) of part (a) is T_1 = 81.70°C, T_2 = 58.44°C, and T_3 = 52.12°C. The fact that only 4 iterations were required to obtain agreement within 0.01°C is due to the close initial guesses.

(2) Note that the rate of heat transfer by convection to the top surface of the rod must balance the rate of heat transfer by conduction to the sides and bottom of the rod.

NOTE TO INSTRUCTOR: Although the problem statement calls for calculations with $\Delta x = \Delta y = 5$ mm and for plotting associated isotherms, the instructional value and benefit-to-effort ratio are small. Hence, it is recommended that this portion of the problem not be assigned.

PROBLEM 4.53

KNOWN: Square shape subjected to uniform surface temperature conditions.

FIND: (a) Temperature at the four specified nodes; estimate the midpoint temperature T_o, (b) Reducing the mesh size by a factor of 2, determine the corresponding nodal temperatures and compare results, and (c) For the finer grid, plot the 75, 150, and 250°C isotherms.

SCHEMATIC:

(a) Coarse grid

(b) Finer grid

ASSUMPTIONS: (1) Steady-state, two-dimensional conduction, (2) Constant properties.

ANALYSIS: (a) The finite-difference equation for each node follows from Eq. 4.33 for an interior point written in the form, $T_i = 1/4\Sigma T_{neighbors}$. Using the Gauss-Seidel iteration method, Section 4.5.2, the finite-difference equations for the four nodes are:

$$T_1^k = 0.25\left(100 + T_2^{k-1} + T_3^{k-1} + 50\right) = 0.25T_2^{k-1} + 0.25T_3^{k-1} + 37.5$$

$$T_2^k = 0.25\left(100 + 200 + T_4^{k-1} + T_1^{k-1}\right) = 0.25T_1^{k-1} + 0.25T_4^{k-1} + 75.0$$

$$T_3^k = 0.25\left(T_1^{k-1} + T_4^{k-1} + 300 + 50\right) = 0.25T_1^{k-1} + 0.25T_4^{k-1} + 87.5$$

$$T_4^k = 0.25\left(T_2^{k-1} + 200 + 300 + T_3^{k-1}\right) = 0.25T_2^{k-1} + 0.25T_3^{k-1} + 125.0$$

The iteration procedure using a hand calculator is implemented in the table below. Initial estimates are entered on the k = 0 row.

k	T_1 (°C)	T_2 (°C)	T_3 (°C)	T_4 (°C)
0	100	150	150	250
1	112.50	165.63	178.13	210.94
2	123.44	158.60	171.10	207.43
3	119.93	156.40	169.34	206.55
4	119.05	156.40	168.90	206.33
5	118.83	156.29	168.79	206.27
6	118.77	156.26	168.76	206.26
7	118.76	156.25	168.76	206.25

<

Continued...

PROBLEM 4.53 (Cont.)

By the seventh iteration, the convergence is approximately 0.01°C. The midpoint temperature can be estimated as

$$T_o = (T_1 + T_2 + T_3 + T_4)/2 = (118.76 + 156.25 + 168.76 + 206.25)^\circ C/4 = 162.5^\circ C$$

(b) Because all the nodes are interior ones, the nodal equations can be written by inspection directly into the IHT workspace and the set of equations solved for the nodal temperatures (°C).

Mesh	T_o	T_1	T_2	T_3	T_4
Coarse	162.5	118.76	156.25	168.76	206.25
Fine	162.5	117.4	156.1	168.9	207.6

The maximum difference for the interior points is 1.5°C (node 4), but the estimate at the center, T_o, is the same, independently of the mesh size. In terms of the boundary surface temperatures,

$$T_o = (50 + 100 + 200 + 300)^\circ C/4 = 162.5^\circ C$$

Why must this be so?

(c) To generate the isotherms, it would be necessary to employ a contour-drawing routine using the tabulated temperature distribution (°C) obtained from the finite-difference solution. Using these values as a guide, try sketching a few isotherms.

-	100	100	100	100	100	-
50	86.0	105.6	119	131.7	151.6	200
50	88.2	117.4	138.7	156.1	174.6	200
50	99.6	137.1	162.5	179.2	190.8	200
50	123.0	168.9	194.9	207.6	209.4	200
50	173.4	220.7	240.6	246.8	239.0	200
-	300	300	300	300	300	-

COMMENTS: Recognize that this finite-difference solution is only an approximation to the temperature distribution, since the heat conduction equation has been solved for only four (or 25) discrete points rather than for all points if an analytical solution had been obtained.

PROBLEM 4.54

KNOWN: Long bar of square cross section, three sides of which are maintained at a constant temperature while the fourth side is subjected to a convection process.

FIND: (a) The mid-point temperature and heat transfer rate between the bar and fluid; a numerical technique with grid spacing of 0.2 m is suggested, and (b) Reducing the grid spacing by a factor of 2, find the midpoint temperature and the heat transfer rate. Also, plot temperature distribution across the surface exposed to the fluid.

SCHEMATIC:

T_s

Bar, 0.8 x 0.8 m
k = 2 W/m·K

T_s = 300 °C

T_∞ = 100 °C
h = 10 W/m^2•K

Symmetry line

T_s = 300 °C

T_2 T_1, T_4 T_3, T_6 T_5, T_8 T_7

$\Delta x = \Delta y$
Δy = 0.2 mm

y, x

q'_a q'_b q'_c

ASSUMPTIONS: (1) Steady-state, two-dimensional conduction, (2) Constant properties.

ANALYSIS: (a) Considering symmetry, the nodal network is shown above. The matrix inversion method of solution will be employed. The finite-difference equations are:

Nodes 1, 3, 5 - Interior nodes, Eq. 4.33; written by inspection.
Nodes 2, 4, 6 - Also can be treated as interior points, considering symmetry.
Nodes 7, 8 - On a plane with convection, Eq. 4.46; noting that $h\Delta x/k$ =
$10\ \text{W/m}^2\cdot\text{K} \times 0.2\ \text{m}/2\text{W/m}\cdot\text{K} = 1$, find

Node 7: $(2T_5 + 300 + T_8) + 2\times1\cdot100 - 2(1+2)T_7 = 0$
Node 8: $(2T_6 + T_7 + T_7) + 2\times1\cdot100 - 2(1+2)T_8 = 0$

The solution matrix [T] can be found using a stock matrix program using the [A] and [C] matrices shown below to obtain the solution matrix [T] (Eq. 4.52). Alternatively, the set of equations could be entered into the IHT workspace and solved for the nodal temperatures.

$$A = \begin{bmatrix} -4 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 2 & -4 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & -4 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & -4 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -4 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 2 & -4 & 0 & 1 \\ 0 & 0 & 0 & 0 & 2 & 0 & -6 & 1 \\ 0 & 0 & 0 & 0 & 0 & 2 & 2 & -6 \end{bmatrix} \quad C = \begin{bmatrix} -600 \\ -300 \\ -300 \\ 0 \\ -300 \\ 0 \\ -500 \\ -200 \end{bmatrix} \quad T = \begin{bmatrix} 292.2 \\ 289.2 \\ 279.7 \\ 272.2 \\ 254.5 \\ 240.1 \\ 198.1 \\ 179.4 \end{bmatrix}$$

From the solution matrix, [T], find the mid-point temperature as

$T_4 = 272.2°\text{C}$ <

Continued...

PROBLEM 4.54 (Cont.)

The heat rate by convection between the bar and fluid is given as,

$$q'_{conv} = 2\left(q'_a + q'_b + q'_c\right)$$

$$q'_{conv} = 2x\left[h\left(\Delta x/2\right)\left(T_8 - T_\infty\right) + h\left(\Delta x\right)\left(T_7 - T_\infty\right) + h\left(\Delta x/2\right)\left(300 - T_\infty\right)\right]$$

$$q'_{conv} = 2x\left[10\,W/m^2 \cdot K \times \left(0.2\,m/2\right)\left[\left(179.4 - 100\right) + 2\left(198.1 - 100\right) + \left(300 - 100\right)\right]K\right]$$

$$q'_{conv} = 952\,W/m\,.$$ <

(b) Reducing the grid spacing by a factor of 2, the nodal arrangement will appear as shown. The finite-difference equation for the interior and centerline nodes were written by inspection and entered into the IHT workspace. The *IHT Finite-Difference Equations Tool* for 2-D, SS conditions, was used to obtain the FDE for the nodes on the exposed surface.

The midpoint temperature T_{13} and heat rate for the finer mesh are

$T_{13} = 271.0°C$ $\qquad q' = 834\ W/m$ <

COMMENTS: The midpoint temperatures for the coarse and finer meshes agree closely, $T_4 = 272°C$ vs. $T_{13} = 271.0°C$, respectively. However, the estimate for the heat rate is substantially influenced by the mesh size; $q' = 952$ vs. 834 W/m for the coarse and finer meshes, respectively.

PROBLEM 4.55

KNOWN: Volumetric heat generation in a rectangular rod of uniform surface temperature.

FIND: (a) Temperature distribution in the rod, and (b) With boundary conditions unchanged, heat generation rate causing the midpoint temperature to reach 600 K.

SCHEMATIC:

$T_s = 300$ K
$\dot{q} = 5 \times 10^7$ W/m³, $k = 20$ W/m·K
$\Delta y = 5$ mm
$\Delta x = 5$ mm

ASSUMPTIONS: (1) Steady-state, two-dimensional conduction, (2) Constant properties, (3) Uniform volumetric heat generation.

ANALYSIS: (a) From symmetry it follows that six unknown temperatures must be determined. Since all nodes are interior ones, the finite-difference equations may be obtained from Eq. 4.39 written in the form

$$T_i = 1/2 \sum T_{neighbors} + 1/4\left(\dot{q}\left(\Delta x \Delta y 1\right)/k\right).$$

With $\dot{q}\left(\Delta x \Delta y\right)/4k$ = 62.5 K, the system of finite-difference equations is

$$T_1 = 0.25\left(T_s + T_2 + T_4 + T_s\right) + 15.625 \qquad (1)$$

$$T_2 = 0.25\left(T_s + T_3 + T_5 + T_1\right) + 15.625 \qquad (2)$$

$$T_3 = 0.25\left(T_s + T_2 + T_6 + T_2\right) + 15.625 \qquad (3)$$

$$T_4 = 0.25\left(T_1 + T_5 + T_1 + T_s\right) + 15.625 \qquad (4)$$

$$T_5 = 0.25\left(T_2 + T_6 + T_2 + T_4\right) + 15.625 \qquad (5)$$

$$T_6 = 0.25\left(T_3 + T_5 + T_3 + T_5\right) + 15.625 \qquad (6)$$

With T_s = 300 K, the set of equations was written directly into the IHT workspace and solved for the nodal temperatures,

T_1	T_2	T_3	T_4	T_5	T_6 (K)
348.6	368.9	374.6	362.4	390.2	398.0

<

(b) With the boundary conditions unchanged, the $\dot{q}$ required for T_6 = 600 K can be found using the same set of equations in the IHT workspace, but with these changes: (1) replace the last term on the RHS (15.625) of Eqs. (1-6) by $\dot{q}$ (ΔxΔy)/4k = (0.005 m)^2 $\dot{q}$ /4×20 W/m·K = 3.125 × 10^{-7} $\dot{q}$ and (2) set T_6 = 600 K. The set of equations has 6 unknown, five nodal temperatures plus $\dot{q}$. Solving find

$$\dot{q} = 1.53 \times 10^8 \text{ W/m}^3$$

<

PROBLEM 4.56

KNOWN: Flue of square cross section with prescribed geometry, thermal conductivity and inner and outer surface temperatures.

FIND: Heat loss per unit length from the flue, q'.

SCHEMATIC:

150mm
Symmetry line
$k = 0.85 W/m \cdot K$
$T_i = 350°C$
$T_o = 25°C$
Air
Flue gas
300mm
25 25 25 25 25
25
1 2 3 4
2′ 3′
350 350 350
Symmetry line
$\Delta x = \Delta y = 75mm$

ASSUMPTIONS: (1) Steady-state, two-dimensional conduction, (2) Constant properties, (3) No internal generation.

ANALYSIS: Taking advantage of symmetry, the nodal network using the suggested 75mm grid spacing is shown above. To obtain the heat rate, we first need to determine the unknown temperatures T_1, T_2, T_3 and T_4. Recognizing that these nodes may be treated as interior nodes, the nodal equations from Eq. 4.33 are

$$(T_2 + 25 + T_2 + 350) - 4T_1 = 0$$
$$(T_1 + 25 + T_3 + 350) - 4T_2 = 0$$
$$(T_2 + 25 + T_4 + 350) - 4T_3 = 0$$
$$(T_3 + 25 + 25 + T_3) - 4T_4 = 0.$$

The Gauss-Seidel iteration method is convenient for this system of equations and following the procedures of Section 4.5.2, they are rewritten as,

$$T_1^k = 0.50\, T_2^{k-1} + 93.75$$
$$T_2^k = 0.25\, T_1^k + 0.25\, T_3^{k-1} + 93.75$$
$$T_3^k = 0.25\, T_2^k + 0.25\, T_4^{k-1} + 93.75$$
$$T_4^k = 0.50\, T_3^k + 12.5.$$

The iteration procedure is implemented in the table on the following page, one row for each iteration k. The initial estimates, for k = 0, are all chosen as (350 + 25)/2 ≈ 185°C. Iteration is continued until the maximum temperature difference is less than 0.2°C, i.e., $\varepsilon < 0.2°C$.

Note that if the system of equations were organized in matrix form, Eq. 4.52, diagonal dominance would exist. Hence there is no need to reorder the equations since the magnitude of the diagonal element is greater than that of other elements in the same row.

Continued …..

PROBLEM 4.56 (Cont.)

k	T_1(°C)	T_2(°C)	T_3(°C)	T_4(°C)	
0	185	185	185	185	← initial estimate
1	186.3	186.6	186.6	105.8	
2	187.1	187.2	167.0	96.0	
3	187.4	182.3	163.3	94.2	
4	184.9	180.8	162.5	93.8	
5	184.2	180.4	162.3	93.7	
6	184.0	180.3	162.3	93.6	
7	183.9	180.3	162.2	93.6	← ε <0.2°C

From knowledge of the temperature distribution, the heat rate may be obtained by summing the heat rates across the nodal control volume surfaces, as shown in the sketch.

The heat rate leaving the outer surface of this flue section is,

$$q' = q'_a + q'_b + q'_c + q'_d + q'_e$$

$$q' = k\frac{\Delta x}{\Delta y}\left[\frac{1}{2}(T_1 - 25) + (T_2 - 25) + (T_3 - 25) + (T_4 - 25) + 0\right]$$

$$q' = 0.85\frac{\text{W}}{\text{m}\cdot\text{K}}\left[\frac{1}{2}(183.9 - 25) + (180.3 - 25) + (162.2 - 26) + (93.6 - 25)\right]$$

$$q' = 374.5 \text{ W/m.}$$

Since this flue section is 1/8 the total cross section, the total heat loss from the flue is

$$q' = 8 \times 374.5 \text{ W/m} = 3.00 \text{ kW/m.}$$ <

COMMENTS: The heat rate could have been calculated at the inner surface, and from the above sketch has the form

$$q' = k\frac{\Delta x}{\Delta y}\left[\frac{1}{2}(350 - T_1) + (350 - T_2) + (350 - T_3)\right] = 374.5 \text{ W/m.}$$

This result should compare very closely with that found for the outer surface since the conservation of energy requirement must be satisfied in obtaining the nodal temperatures.

PROBLEM 4.57

KNOWN: Flue of square cross section with prescribed geometry, thermal conductivity and inner and outer surface convective conditions.

FIND: (a) Heat loss per unit length, q', by convection to the air, (b) Effect of grid spacing and convection coefficients on temperature field; show isotherms.

SCHEMATIC:

$k = 0.85$ W/m·K; Air: $T_{\infty,o} = 25\,^oC$, $h_o = 5$ W/m^2·K; Flue gas: $T_{\infty,i} = 350\,^oC$, $h_i = 100$ W/m^2·K; 150 mm; 300 mm; $\Delta y = \Delta x$, $\Delta x = 75$ mm

ASSUMPTIONS: (1) Steady-state, two-dimensional conduction, (2) Constant properties.

ANALYSIS: (a) Taking advantage of symmetry, the nodal network for a 75 mm grid spacing is shown in schematic (a). To obtain the heat rate, we need first to determine the temperatures T_i. Recognize that there are four types of nodes: interior (4-7), plane surface with convection (1, 2, 8-11), internal corner with convection (3), and external corner with convection (12). Using the appropriate relations from Table 4.2, the finite-difference equations are

Node		Equation
1	$\left(2T_4 + T_2 + T_2\right) + \frac{2h_i\Delta x}{k}T_{\infty,i} - \left(\frac{h_i\Delta x}{k} + 2\right)T_1 = 0$	4.46
2	$\left(2T_5 + T_3 + T_1\right) + \frac{2h_i\Delta x}{k}T_{\infty,i} - 2\left(\frac{h_i\Delta x}{k} + 2\right)T_2 = 0$	4.46
3	$2\left(T_6 + T_6\right) + \left(T_2 + T_2\right) + \frac{2h_i\Delta x}{k}T_{\infty,i} - 2\left(3 + \frac{h_i\Delta x}{k}\right)T_3 = 0$	4.45
4	$\left(T_8 + T_5 + T_1 + T_5\right) - 4T_4 = 0$	4.33
5	$\left(T_9 + T_6 + T_2 + T_4\right) - 4T_5 = 0$	4.33
6	$\left(T_{10} + T_7 + T_3 + T_5\right) - 4T_6 = 0$	4.33
7	$\left(T_{11} + T_{11} + T_6 + T_6\right) - 4T_7 = 0$	4.33
8	$\left(2T_4 + T_9 + T_9\right) + \frac{2h_o\Delta x}{k}T_{\infty,o} - 2\left(\frac{h_o\Delta x}{k} + 2\right)T_8 = 0$	4.46
9	$\left(2T_5 + T_{10} + T_8\right) + \frac{2h_o\Delta x}{k}T_{\infty,o} - 2\left(\frac{h_o\Delta x}{k} + 2\right)T_9 = 0$	4.46
10	$\left(2T_6 + T_{11} + T_9\right) + \frac{2h_o\Delta x}{k}T_{\infty,o} - 2\left(\frac{h_o\Delta x}{k} + 2\right)T_{10} = 0$	4.46
11	$\left(2T_7 + T_{12} + T_{10}\right) + \frac{2h_o\Delta x}{k}T_{\infty,o} - 2\left(\frac{h_o\Delta x}{k} + 2\right)T_{11} = 0$	4.46
12	$\left(T_{11} + T_{11}\right) + \frac{2h_o\Delta x}{k}T_{\infty,o} - 2\left(\frac{h_o\Delta x}{k} + 1\right)T_{12} = 0$	4.47

Continued...

PROBLEM 4.57 (Cont.)

The Gauss-Seidel iteration is convenient for this system of equations. Following procedures of Section 4.5.2, the system of equations is rewritten in the proper form. Note that diagonal dominance is present; hence, no re-ordering is necessary.

$$T_1^k = 0.09239T_2^{k-1} + 0.09239T_4^{k-1} + 285.3$$
$$T_2^k = 0.04620T_1^k + 0.04620T_3^{k-1} + 0.09239T_5^{k-1} + 285.3$$
$$T_3^k = 0.08457T_2^k + 0.1692T_6^{k-1} + 261.2$$
$$T_4^k = 0.25T_1^k + 0.50T_5^{k-1} + 0.25T_8^{k-1}$$
$$T_5^k = 0.25T_2^k + 0.25T_4^k + 0.25T_6^{k-1} + 0.25T_9^{k-1}$$
$$T_6^k = 0.25T_3^k + 0.25T_5^k + 0.25T_7^{k-1} + 0.25T_9^{k-1}$$
$$T_7^k = 0.50T_6^k + 0.50T_{11}^{k-1}$$
$$T_8^k = 0.4096T_4^k + 0.4096T_9^{k-1} + 4.52$$
$$T_9^k = 0.4096T_5^k + 0.2048T_8^k + 0.2048T_{10}^{k-1} + 4.52$$
$$T_{10}^k = 0.4096T_6^k + 0.2048T_9^k + 0.2048T_{11}^{k-1} + 4.52$$
$$T_{11}^k = 0.4096T_7^k + 0.2048T_{10}^k + 0.2048T_{12}^{k-1} + 4.52$$
$$T_{12}^k = 0.6939T_{11}^k + 7.65$$

The initial estimates (k = 0) are carefully chosen to minimize calculation labor; let $\varepsilon < 1.0$.

k	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	T_{10}	T_{11}	T_{12}
0	340	330	315	250	225	205	195	160	150	140	125	110
1	338.9	336.3	324.3	237.2	232.1	225.4	175.2	163.1	161.7	155.6	130.7	98.3
2	338.3	337.4	328.0	241.4	241.5	226.6	178.6	169.6	170.0	158.9	130.4	98.1
3	338.8	338.4	328.2	247.7	245.7	230.6	180.5	175.6	173.7	161.2	131.6	98.9
4	339.4	338.8	328.9	251.6	248.7	232.9	182.3	178.7	176.0	162.9	132.8	99.8
5	339.8	339.2	329.3	254.0	250.5	234.5	183.7	180.6	177.5	164.1	133.8	100.5
6	340.1	339.4	329.7	255.4	251.7	235.7	184.7	181.8	178.5	164.7	134.5	101.0
7	340.3	339.5	329.9	256.4	252.5	236.4	185.5	182.7	179.1	165.6	135.1	101.4

The heat loss to the outside air for the upper surface (Nodes 8 through 12) is of the form

$$q' = h_o \Delta x \left[\frac{1}{2}\left(T_8 - T_{\infty,o}\right) + \left(T_9 - T_{\infty,o}\right) + \left(T_{10} - T_{\infty,o}\right) + \left(T_{11} - T_{\infty,o}\right) + \frac{1}{2}\left(T_{12} - T_{\infty,o}\right) \right]$$

$$q' = 5\,W/m^2 \cdot K \times 0.075\,m \left[\frac{1}{2}(182.7 - 25) + (179.1 - 25) + (165.6 - 25) + (135.1 - 25) + \frac{1}{2}(101.4 - 25) \right] {}^\circ C = 195\,W/m$$

Hence, for the entire flue cross-section, considering symmetry,

$$q'_{tot} = 8 \times q' = 8 \times 195\,W/m = 1.57\,kW/m$$ <

The convection heat rate at the inner surface is

$$q'_{tot} = 8 \times h_i \Delta x \left[\frac{1}{2}\left(T_{\infty,i} - T_1\right) + \left(T_{\infty,i} - T_2\right) + \frac{1}{2}\left(T_{\infty,i} - T_3\right) \right] = 8 \times 190.5\,W/m = 1.52\,kW/m$$

which is within 2.5% of the foregoing result. The calculation would be identical if $\varepsilon = 0$.

Continued...

PROBLEM 4.57 (Cont.)

(b) Using the *Finite-Difference Equations* option from the *Tools* portion of the IHT menu, the following two-dimensional temperature field was computed for the grid shown in the schematic below, where x and y are in mm and the temperatures are in °C.

(0,0) (300,0) x (mm)

y (mm)

$\Delta x = \Delta y = 25$ mm

(0,150) (150,150)

y\x	0	25	50	75	100	125	150	175	200	225	250	275	300
0	180.7	180.2	178.4	175.4	171.1	165.3	158.1	149.6	140.1	129.9	119.4	108.7	98.0
25	204.2	203.6	201.6	198.2	193.3	186.7	178.3	168.4	157.4	145.6	133.4	121.0	
50	228.9	228.3	226.2	222.6	217.2	209.7	200.1	188.4	175.4	161.6	147.5		
75	255.0	254.4	252.4	248.7	243.1	235.0	223.9	209.8	194.1	177.8			
100	282.4	281.8	280.1	276.9	271.6	263.3	250.5	232.8	213.5				
125	310.9	310.5	309.3	307.1	303.2	296.0	282.2	257.5					
150	340.0	340.0	339.6	339.1	337.9	335.3	324.7						

Agreement between the temperature fields for the (a) and (b) grids is good, with the largest differences occurring at the interior and exterior corners. Ten isotherms generated using *FEHT* are shown on the symmetric section below. Note how the heat flow is nearly normal to the flue wall around the mid-section. In the corner regions, the isotherms are curved and we'd expect that grid size might influence the accuracy of the results. Convection heat transfer to the inner surface is

132 C

Isotherms
Minimum = 132.3 C
Maximum = 339.0 C

316 C

$$q' = 8h_i\Delta x\left[\left(T_{\infty,i} - T_1\right)/2 + \left(T_{\infty,i} - T_2\right) + \left(T_{\infty,i} - T_3\right) + \left(T_{\infty,i} - T_4\right) + \left(T_{\infty,i} - T_5\right) + \left(T_{\infty,i} - T_6\right) + \left(T_{\infty,i} - T_7\right)/2\right] = 1.52\,\text{kW/m}$$

and the agreement with results of the coarse grid is excellent.

The heat rate increases with increasing h_i and h_o, while temperatures in the wall increase and decrease, respectively, with increasing h_i and h_o.

PROBLEM 4.58

KNOWN: Rectangular air ducts having surfaces at 80°C in a concrete slab with an insulated bottom and upper surface maintained at 30°C.

FIND: Heat rate from each duct per unit length of duct, q'.

SCHEMATIC:

Air duct, Concrete, L=150mm, T_2=30°C, T_1=80°C, Concrete, see section below, 1.5L, L, L, L

ASSUMPTIONS: (1) Steady-state conditions, (2) Two-dimensional conduction, (3) No internal volumetric generation, (4) Constant properties.

PROPERTIES: Concrete (given): k = 1.4 W/m·K.

ANALYSIS: Taking advantage of symmetry, the nodal network, using the suggested grid spacing

$$\Delta x = 2\Delta y = 37.50 \text{ mm}$$
$$\Delta y = 0.125L = 18.75 \text{ mm}$$

where L = 150 mm, is shown in the sketch. To evaluate the heat rate, we need the temperatures T_1, T_2, T_3, T_4, and T_5. All the nodes may be treated as interior nodes (considering symmetry for those nodes on insulated boundaries), Eq. 4.33. Use matrix notation, Eq. 4.52, [A][T] = [C], and perform the inversion.

T_2=30°C, T_1 =80°C, $\frac{\Delta x}{\Delta y}=2$, Δy, 1.5L, 0.25L, Δx, L, nodes 1–33

The heat rate per unit length from the prescribed section of the duct follows from an energy balance on the nodes at the top isothermal surface.

q', T_s=30°C, q_1', q_2', q_3', q_4', q_5', T_1=41.70, T_2=44.26, T_3=53.92, T_4=54.89, T_5=54.98°C, $\Delta x = 2\Delta y$

$$q' = q_1' + q_2' + q_3' + q_4' + q_5'$$
$$q' = k(\Delta x/2)\frac{T_1 - T_s}{\Delta y} + k\cdot\Delta x\frac{T_2 - T_s}{\Delta y} + k\cdot\Delta x\frac{T_3 - T_s}{\Delta y} + k\cdot\Delta x\frac{T_4 - T_s}{\Delta y} + k(\Delta x/2)\frac{T_5 - T_s}{\Delta y}$$
$$q' = k\left[(T_1 - T_s) + 2(T_2 - T_s) + 2(T_3 - T_s) + 2(T_4 - T_s) + (T_5 - T_s)\right]$$
$$q' = 1.4 \text{ W/m}\cdot\text{K}\left[(41.70-30) + 2(44.26-30) + 2(53.92-30) + 2(54.89-30) + (54.98-30)\right]$$
$$q' = 228 \text{ W/m.}$$

Since the section analyzed represents one-half of the region about an air duct, the heat loss per unit length for each duct is,

$$q'_{duct} = 2xq' = 456 \text{ W/m.}$$ <

Continued …..

PROBLEM 4.58 (Cont.)

Coefficient matrix [A]

-1.0	.2	0	0	0	.4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
.1	-1.0	.1	0	0	0	.4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	.1	-1.0	.1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	.1	-1.0	.1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	.2	-1.0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
.4	0	0	0	0	-1.0	.2	.4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	.4	0	0	0	.1	-1.0	0	.4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	.4	0	-1.0	.2	.4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	.4	.1	-1.0	0	.4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	.4	0	-1.0	.2	.4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	.4	.1	-1.0	0	.4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	.4	0	-1.0	.2	.4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	.4	.1	-1.0	0	.4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	.4	0	-1.0	.2	.4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	.4	.1	-1.0	0	.4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	.4	0	-1.0	.2	.4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	.4	.1	-1.0	0	.4	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	.4	0	-1.0	.2	.4	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	.4	.1	-1.0	0	.4	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	.4	0	-1.0	.2	.4	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	.4	.1	-1.0	0	.4	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	.4	0	-1.0	.2	.4	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	.4	.1	-1.0	0	.4	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	.4	0	-1.0	.2	0	0	0	.4	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	.4	.1	-1.0	.1	0	0	0	.4	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	.1	-1.0	.1	0	0	0	.4	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	.1	-1.0	.1	0	0	0	.4	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	.2	-1.0	0	0	0	0	.4
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	.8	0	0	0	0	-1.0	.2	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	.8	0	0	0	.1	-1.0	.1	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	.8	0	0	0	.1	-1.0	.1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	.8	0	0	0	.1	-1.0	.1
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	.8	0	0	0	.2	-1.0

RHS Vector	*Solution Vector*
-12.0	41.70
-12	44.26
-44.0	53.92
-44.0	54.89
44	54.98
0	52.13
-80.0	56.75
0	60.24
-80.0	64.58
0	66.19
-80.0	69.64
0	70.41
-80.0	72.98
0	73.35
-80.0	75.20
0	75.37
-80.0	76.68
0	76.73
-80.0	77.66
0	77.62
-80.0	78.30
0	78.16
-80.0	78.68
0	78.45
0	78.85
-32.0	79.75
-32.0	79.94
-32.0	79.97
0	78.54
0	78.91
0	79.68
0	79.92
0	79.96

PROBLEM 4.59

KNOWN: Dimensions and operating conditions for a gas turbine blade with embedded channels.

FIND: Effect of applying a zirconia, thermal barrier coating.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state, two-dimensional conduction, (2) Constant properties, (3) Negligible radiation.

ANALYSIS: Preserving the nodal network of Example 4.4 and adding surface nodes for the TBC, finite-difference equations previously developed for nodes 7 through 21 are still appropriate, while new equations must be developed for nodes 1c-6c, 1o-6o, and 1i-6i. Considering node 3c as an example, an energy balance yields

$$h_o \Delta x \left(T_{\infty,o} - T_{3c}\right) + \frac{k_c \left(\Delta y_c/2\right)}{\Delta x}\left(T_{2c} - T_{3c}\right) + \frac{k_c \left(\Delta y_c/2\right)}{\Delta x}\left(T_{4c} - T_{3c}\right) + \frac{k_c \Delta x}{\Delta y_c}\left(T_{3o} - T_{3c}\right) = 0$$

or, with Δx = 1 mm and Δy_c = 0.5 mm,

$$0.25\left(T_{2c} + T_{4c}\right) + 2T_{3o} - \left(2.5 + \frac{h_o \Delta x}{k_c}\right)T_{3c} = -\frac{h_o \Delta x}{k_c} T_{\infty,o}$$

Similar expressions may be obtained for the other 5 nodal points on the outer surface of the TBC.

Applying an energy balance to node 3o at the inner surface of the TBC, we obtain

$$\frac{k_c \Delta x}{\Delta y_c}\left(T_{3c} - T_{3o}\right) + \frac{k_c \left(\Delta y_c/2\right)}{\Delta x}\left(T_{2o} - T_{3o}\right) + \frac{k_c \left(\Delta y_c/2\right)}{\Delta x}\left(T_{4o} - T_{3o}\right) + \frac{\Delta x}{R''_{t,c}}\left(T_{3i} - T_{3o}\right) = 0$$

or,

$$2T_{3c} + 0.25\left(T_{2o} + T_{4o}\right) + \frac{\Delta x}{k_c R''_{t,c}} T_{3i} - \left(2.5 + \frac{\Delta x}{k_c R''_{t,c}}\right)T_{3o} = 0$$

Similar expressions may be obtained for the remaining nodal points on the inner surface of the TBC (outer region of the contact resistance).

Continued...

PROBLEM 4.59 (Cont.)

Applying an energy balance to node 3i at the outer surface of the turbine blade, we obtain

$$\frac{\Delta x}{R''_{t,c}}\left(T_{3o}-T_{3i}\right)+\frac{k\left(\Delta y/2\right)}{\Delta x}\left(T_{2i}-T_{3i}\right)+\frac{k\left(\Delta y/2\right)}{\Delta x}\left(T_{4i}-T_{3i}\right)+\frac{k\Delta x}{\Delta y}\left(T_9-T_{3i}\right)=0$$

or,

$$\frac{\Delta x}{kR''_{t,c}}T_{3o}+0.5\left(T_{2,i}+T_{4,i}\right)+T_9-\left(2+\frac{\Delta x}{kR''_{t,c}}\right)T_{3i}=0$$

Similar expressions may be obtained for the remaining nodal points on the inner region of the contact resistance.

The 33 finite-difference equations were entered into the workspace of IHT from the keyboard (model equations are appended), and for h_o = 1000 W/m^2·K, $T_{\infty,o}$ = 1700 K, h_i = 200 W/m^2·K and $T_{\infty,i}$ = 400 K, the following temperature field was obtained, where coordinate (x,y) locations are in mm and temperatures are in °C.

y\x	0	1	2	3	4	5
0	1536	1535	1534	1533	1533	1532
0.5	1473	1472	1471	1469	1468	1468
0.5	1456	1456	1454	1452	1451	1451
1.5	1450	1450	1447	1446	1444	1444
2.5	1446	1445	1441	1438	1437	1436
3.5	1445	1443	1438	0	0	0

Note the significant reduction in the turbine blade temperature, as, for example, from a surface temperature of T_1 = 1526 K without the TBC to T_{1i} = 1456 K with the coating. Hence, the coating is serving its intended purpose.

COMMENTS: (1) Significant additional benefits may still be realized by increasing h_i. (2) The foregoing solution may be used to determine the temperature field without the TBC by setting $k_c \to \infty$ and $R''_{t,c} \to 0$.

PROBLEM 4.60

KNOWN: Bar of rectangular cross-section subjected to prescribed boundary conditions.

FIND: Using a numerical technique with a grid spacing of 0.1m, determine the temperature distribution and the heat transfer rate from the bar to the fluid.

SCHEMATIC:

T=200°C
k=1.5W/m·K
Note symmetry
0.4m·0.6m
T=200°C
T_∞=30°C
h=50W/m²K
200 200 200 200 200°C
T_∞, h
Symmetry adiabat
Δx=Δy=0.1m

ASSUMPTIONS: (1) Steady-state conditions, (2) Two-dimensional conduction, (3) Constant properties.

ANALYSIS: The nodal network has Δx = Δy = 0.1m. Note the adiabat corresponding to system symmetry. The finite-difference equations for each node can be written using either Eq. 4.33, for interior nodes, or Eq. 4.46, for a plane surface with convection. In the case of adiabatic surfaces, Eq. 4.46 is used with h = 0. Note that

$$\frac{h\Delta x}{k} = \frac{50\text{W/m}^2\cdot\text{K}\times 0.1\text{m}}{1.5\text{ W/m}\cdot\text{K}} = 3.333.$$

Node	*Finite-Difference Equations*
1	$-4T_1 + 2T_2 + 2T_4 = 0$
2	$-4T_2 + T_1 + T_3 + 2T_5 = 0$
3	$-4T_3 + 200 + 2T_6 + T_2 = 0$
4	$-4T_4 + T_1 + 2T_5 + T_7 = 0$
5	$-4T_5 + T_2 + T_6 + T_8 + T_4 = 0$
6	$-4T_6 + T_5 + T_3 + 200 + T_9 = 0$
7	$-4T_7 + T_4 + 2T_8 + T_{10} = 0$
8	$-4T_8 + T_7 + T_5 + T_9 + T_{11} = 0$
9	$-4T_9 + T_8 + T_6 + 200 + T_{12} = 0$
10	$-4T_{10} + T_7 + 2T_{11} + T_{13} = 0$
11	$-4T_{11} + T_{10} + T_8 + T_{12} + T_{14} = 0$
12	$-4T_{12} + T_{11} + T_9 + 200 + T_{15} = 0$
13	$2T_{10} + T_{14} + 6.666\times30 - 10.666\ T_{13} = 0$
14	$2T_{11} + T_{13} + T_{15} + 6.666\times30 - 2(3.333+2)T_{14} = 0$
15	$2T_{12} + T_{14} + 200 + 6.666\times30 - 2(3.333+2)\ T_{15} = 0$

Using the matrix inversion method, Section 4.5.2, the above equations can be written in the form [A] [T] = [C] where [A] and [C] are shown on the next page. Using a stock matrix inversion routine, the temperatures [T] are determined.

Continued …..

PROBLEM 4.60 (Cont.)

$$[A]=\begin{bmatrix} -4 & 2 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -4 & 1 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -4 & 0 & 0 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -4 & 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & -4 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & -4 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -4 & 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & -4 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & -4 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -4 & 2 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & -4 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & -4 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & -10.66 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 1 & -10.66 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 1 & -10.66 \end{bmatrix}$$

$$[C]=\begin{bmatrix} 0 \\ 0 \\ -200 \\ 0 \\ 0 \\ -200 \\ 0 \\ 0 \\ -200 \\ 0 \\ 0 \\ -200 \\ -200 \\ -200 \\ -400 \end{bmatrix} \qquad [T]=\begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \\ T_7 \\ T_8 \\ T_9 \\ T_{10} \\ T_{11} \\ T_{12} \\ T_{13} \\ T_{14} \\ T_{15} \end{bmatrix} = \begin{bmatrix} 153.9 \\ 159.7 \\ 176.4 \\ 148.0 \\ 154.4 \\ 172.9 \\ 129.4 \\ 137.0 \\ 160.7 \\ 95.6 \\ 103.5 \\ 132.8 \\ 45.8 \\ 48.7 \\ 67.0 \end{bmatrix} \left({}^\circ C\right)$$

Considering symmetry, the heat transfer rate to the fluid is twice the convection rate from the surfaces of the control volumes exposed to the fluid. Using Newton's law of cooling, considering a unit thickness of the bar, find

$$q_{conv} = 2\left[h\cdot\frac{\Delta y}{2}\cdot\left(T_{13}-T_\infty\right)+h\cdot\Delta y\cdot\left(T_{14}-T_\infty\right)+h\cdot\Delta y\left(T_{15}-T_\infty\right)+h\cdot\frac{\Delta y}{2}\left(200-T_\infty\right)\right]$$

$$q_{conv} = 2h\cdot\Delta y\left[\frac{1}{2}\left(T_{13}-T_\infty\right)+\left(T_{14}-T_\infty\right)+\left(T_{15}-T_\infty\right)+\frac{1}{2}\left(200-T_\infty\right)\right]$$

$$q_{conv} = 2\times 50\frac{W}{m^2\cdot K}\times 0.1m\left[\frac{1}{2}\left(45.8-30\right)+\left(48.7-30\right)+\left(67.0-30\right)+\frac{1}{2}\left(200-30\right)\right]$$

$$q_{conv} = 1487\ W/m. \qquad <$$

PROBLEM 4.61

KNOWN: Upper surface and grooves of a plate are maintained at a uniform temperature T_1, while the lower surface is maintained at T_2 or is exposed to a fluid at T_∞.

FIND: (a) Heat rate per width of groove spacing (w) for isothermal top and bottom surfaces using a finite-difference method with Δx = 40 mm, (b) Effect of grid spacing and convection at bottom surface.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state, two-dimensional conduction, (2) Constant properties.

ANALYSIS: (a) Using a space increment of Δx = 40 mm, the symmetrical section shown in schematic (a) corresponds to one-half the groove spacing. There exist only two interior nodes for which finite-difference equations must be written.

Node a:

$$4T_a - (T_1 + T_b + T_2 + T_1) = 0$$

$$4T_a - (200 + T_b + 20 + 200) = 0 \qquad \text{or} \qquad 4T_a - T_b = 420 \qquad (1)$$

Node b:

$$4T_b - (T_1 + T_a + T_2 + T_a) = 0$$

$$4T_b - (200 + 2T_a + 20) = 0 \qquad \text{or} \qquad -2T_a + 4T_b = 220 \qquad (2)$$

Multiply Eq. (2) by 2 and subtract from Eq. (1) to obtain

$7T_b = 860$ or $T_b = 122.9°C$

From Eq. (1),

$4T_a - 122.9 = 420$ or $T_a = (420 + 122.9)/4 = 135.7°C$.

The heat transfer through the symmetrical section is equal to the sum of heat flows through control volumes adjacent to the lower surface. From the schematic,

$$q' = q'_1 + q'_2 + q'_3 = k\left(\frac{\Delta x}{2}\right)\frac{T_1 - T_2}{\Delta y} + k(\Delta x)\frac{T_a - T_2}{\Delta y} + k\left(\frac{\Delta x}{2}\right)\frac{T_b - T_2}{\Delta y}.$$

Continued...

Noting that $\Delta x = \Delta y$, regrouping and substituting numerical values, find

$$q' = k\left[\frac{1}{2}(T_1 - T_2) + (T_a - T_2) + \frac{1}{2}(T_b - T_2)\right]$$

$$q' = 15\,W/m \cdot K\left[\frac{1}{2}(200-20) + (135.7-20) + \frac{1}{2}(122.9-20)\right] = 3.86\,kW/m\,.$$

For the full groove spacing, q'_{total} = 2 × 3.86 kW/m = 7.72 kW/m. <

(b) Using the *Finite-Difference Equations* option from the *Tools* portion of the IHT menu, the following two-dimensional temperature field was computed for the grid shown in schematic (b), where x and y are in mm and the nodal temperatures are in °C. Nodes 2-54 are interior nodes, with those along the symmetry adiabats characterized by $T_{m-1,n} = T_{m+1,n}$, while nodes 55-63 lie on a plane surface.

y\x	0	10	20	30	40	50	60	70	80
0					200	200	200	200	200
10				200	191	186.6	184.3	183.1	182.8
20			200	186.7	177.2	171.2	167.5	165.5	164.8
30		200	182.4	169.5	160.1	153.4	149.0	146.4	145.5
40	200	175.4	160.3	148.9	140.1	133.5	128.7	125.7	124.4
50	141.4	134.3	125.7	118.0	111.6	106.7	103.1	100.9	100.1
60	97.09	94.62	90.27	85.73	81.73	78.51	76.17	74.73	74.24
70	57.69	56.83	55.01	52.95	51.04	49.46	48.31	47.60	47.36
80	20	20	20	20	20	20	20	20	20

The foregoing results were computed for $h = 10^7\ W/m^2\cdot K$ ($h \to \infty$) and T_∞ = 20°C, which is tantamount to prescribing an isothermal bottom surface at 20°C. Agreement between corresponding results for the coarse and fine grids is surprisingly good (T_a = 135.7°C ↔ T_{23} = 140.1°C; T_b = 122.9°C ↔ T_{27} = 124.4°C). The heat rate is

$$q' = 2\times k\left[(T_{46} - T_{55})/2 + (T_{47} - T_{56}) + (T_{48} - T_{57}) + (T_{49} - T_{58}) + (T_{50} - T_{59}) + (T_{51} - T_{60}) + (T_{52} - T_{61}) + (T_{53} - T_{62}) + (T_{54} - T_{63})/2\right]$$

$$q' = 2\times 15\,W/m\cdot K\,[18.84 + 36.82 + 35.00 + 32.95 + 31.04 + 29.46 + 28.31 + 27.6 + 13.68]^\circ C = 7.61\,kW/m$$ <

The agreement with q' = 7.72 kW/m from the coarse grid of part (a) is excellent and a fortuitous consequence of compensating errors. With reductions in the convection coefficient from $h \to \infty$ to h = 1000, 200 and 5 $W/m^2\cdot K$, the corresponding increase in the thermal resistance reduces the heat rate to values of 6.03, 3.28 and 0.14 kW/m, respectively. With decreasing h, there is an overall increase in nodal temperatures, as, for example, from 191°C to 199.8°C for T_2 and from 20°C to 196.9°C for T_{55}.

NOTE TO INSTRUCTOR: To reduce computational effort, while achieving the same educational objectives, the problem statement has been changed to allow for convection at the bottom, rather than the top, surface.

PROBLEM 4.62

KNOWN: Rectangular plate subjected to uniform temperature boundaries.

FIND: Temperature at the midpoint using a finite-difference method with space increment of 0.25m

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Two-dimensional conduction, (3) Constant properties.

ANALYSIS: For the nodal network above, 12 finite-difference equations must be written. It follows that node 8 represents the midpoint of the rectangle. Since all nodes are interior nodes, Eq. 4.33 is appropriate and is written in the form

$$4T_m - \sum T_{neighbors} = 0.$$

For nodes on the symmetry adiabat, the neighboring nodes include two symmetrical nodes. Hence, for Node 4, the neighbors are T_b, T_8 and $2T_3$. Because of the simplicity of the finite-difference equations, we may proceed directly to the matrices [A] and [C] – see Eq. 4.52 – and matrix inversion can be used to find the nodal temperatures T_m.

$$A = \begin{bmatrix} -4 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -4 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -4 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & -4 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -4 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & -4 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & -4 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 2 & -4 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -4 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & -4 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & -4 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 2 & -4 \end{bmatrix} \quad C = \begin{bmatrix} -200 \\ -150 \\ -150 \\ -150 \\ -50 \\ 0 \\ 0 \\ 0 \\ -100 \\ -50 \\ -50 \\ -50 \end{bmatrix} \quad T = \begin{bmatrix} 96.5 \\ 112.9 \\ 118.9 \\ 120.4 \\ 73.2 \\ 86.2 \\ 92.3 \\ 94.0 \\ 59.9 \\ 65.5 \\ 69.9 \\ 71.0 \end{bmatrix}$$

The temperature at the midpoint (Node 8) is

$$T(1,0.5) = T_8 = 94.0^\circ C.$$ <

COMMENTS: Using the exact analytical, solution – see Eq. 4.19 and Problem 4.2 – the midpoint temperature is found to be 94.5°C. To improve the accuracy of the finite-difference method, it would be necessary to decrease the nodal mesh size.

PROBLEM 4.63

KNOWN: Long bar with trapezoidal shape, uniform temperatures on two surfaces, and two insulated surfaces.

FIND: Heat transfer rate per unit length using finite-difference method with space increment of 10mm.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Two-dimensional conduction, (3) Constant properties.

ANALYSIS: The heat rate can be found after the temperature distribution has been determined. Using the nodal network shown above with Δx = 10mm, nine finite-difference equations must be written. Nodes 1-4 and 6-8 are interior nodes and their finite-difference equations can be written directly from Eq. 4.33. For these nodes

$$T_{m,n+1} + T_{m,n-1} + T_{m+1,n} + T_{m-1,n} - 4T_{m,n} = 0 \qquad m = 1-4,\ 6-8. \tag{1}$$

For nodes 5 and 9 located on the diagonal, insulated boundary, the appropriate finite-difference equation follows from an energy balance on the control volume shown above (upper-right corner of schematic),

$$\dot{E}_{in} - \dot{E}_{out} = q_a + q_b = 0$$

$$k(\Delta y \cdot 1)\frac{T_{m-1,n} - T_{m,n}}{\Delta x} + k(\Delta x \cdot 1)\frac{T_{m,n-1} - T_{m,n}}{\Delta y} = 0.$$

Since $\Delta x = \Delta y$, the finite-difference equation for nodes 5 and 9 is of the form

$$T_{m-1,n} + T_{m,n-1} - 2T_{m,n} = 0 \qquad m = 5,9. \tag{2}$$

The system of 9 finite-difference equations is first written in the form of Eqs. (1) or (2) and then written in explicit form for use with the Gauss-Seidel iteration method of solution; see Section 4.5.2.

Node	*Finite-difference equation*	*Gauss-Seidel form*
1	$T_2+T_2+T_6+100-4T_1 = 0$	$T_1 = 0.5T_2+0.25T_6+25$
2	$T_3+T_1+T_7+100-4T_2 = 0$	$T_2 = 0.25(T_1+T_3+T_7)+25$
3	$T_4+T_2+T_8+100-4T_3 = 0$	$T_3 = 0.25(T_2+T_4+T_8)+25$
4	$T_5+T_3+T_9+100-4T_4 = 0$	$T_4 = 0.25(T_3+T_5+T_9)+25$
5	$100+T_4-2T_5 = 0$	$T_5 = 0.5T_4+50$
6	$T_7+T_7+25+T_1-4T_6 = 0$	$T_6 = 0.25T_1+0.5T_7+6.25$
7	$T_8+T_6+25+T_2-4T_7 = 0$	$T_7 = 0.25(T_2+T_6+T_8)+6.25$
8	$T_9+T_7+25+T_3-4T_8 = 0$	$T_8 = 0.25(T_3+T_7+T_9)+6.25$
9	$T_4+T_8-2T_9 = 0$	$T_9 = 0.5(T_4+T_8)$

Continued …..

The iteration process begins after an initial guess (k = 0) is made. The calculations are shown in the table below.

k	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9(°C)
0	75	75	80	85	90	50	50	60	75
1	75.0	76.3	80.0	86.3	92.5	50.0	52.5	57.5	72.5
2	75.7	76.9	80.0	86.3	93.2	51.3	52.2	57.5	71.9
3	76.3	77.0	80.2	86.3	93.2	51.3	52.7	57.3	71.9
4	76.3	77.3	80.2	86.3	93.2	51.7	52.7	57.5	71.8
5	76.6	77.3	80.3	86.3	93.2	51.7	52.9	57.4	71.9
6	76.6	77.5	80.3	86.4	93.2	51.9	52.9	57.5	71.9

Note that by the sixth iteration the change is less than 0.3°C; hence, we assume the temperature distribution is approximated by the last row of the table.

The heat rate per unit length can be determined by evaluating the heat rates in the x-direction for the control volumes about nodes 6, 7, and 8. From the schematic, find that

$$q' = q_1' + q_2' + q_3'$$

$$q' = k\Delta y \frac{T_8 - 25}{\Delta x} + k\Delta y \frac{T_7 - 25}{\Delta x} + k\frac{\Delta y}{2}\,\frac{T_6 - 25}{\Delta x}$$

Recognizing that $\Delta x = \Delta y$ and substituting numerical values, find

$$q' = 20\frac{W}{m\cdot K}\left[(57.5-25)+(52.9-25)+\frac{1}{2}(51.9-25)\right]K$$

$$q' = 1477 \text{ W/m.}$$ <

COMMENTS: (1) Recognize that, while the temperature distribution may have been determined to a reasonable approximation, the uncertainty in the heat rate could be substantial. This follows since the heat rate is based upon a gradient and hence on temperature differences.

(2) Note that the initial guesses (k = 0) for the iteration are within 5°C of the final distribution. The geometry is simple enough that the guess can be very close. In some instances, a flux plot may be helpful and save labor in the calculation.

(3) In writing the FDEs, the iteration index (superscript k) was not included to simplify expression of the equations. However, the most recent value of $T_{m,n}$ is always used in the computations. Note that this system of FDEs is diagonally dominant and no rearrangement is required.

PROBLEM 4.64

KNOWN: Edge of adjoining walls (k = 1 W/m·K) represented by symmetrical element bounded by the diagonal symmetry adiabat and a section of the wall thickness over which the temperature distribution is assumed to be linear.

FIND: (a) Temperature distribution, heat rate and shape factor for the edge using the nodal network with = $\Delta x = \Delta y = 10$ mm; compare shape factor result with that from Table 4.1; (b) Assess the validity of assuming linear temperature distributions across sections at various distances from the edge.

SCHEMATIC:

Linear temperature distribution

T_2, T_1, T_2, L

q'_a q'_b q'_c q'_d $T_2 = 0\ ^oC$

$T_2 = 0\ ^oC$

$T_c = 25$

$T_b = 50$

$T_a = 75$

$T_1 = 100\ ^oC$

T_a, T_2

$\Delta x = \Delta y$

$\Delta x = 10$ mm

ASSUMPTIONS: (1) Two-dimensional, steady-state conduction, (2) Constant properties, and (3) Linear temperature distribution at specified locations across the section.

ANALYSIS: (a) Taking advantage of symmetry along the adiabat diagonal, all the nodes may be treated as interior nodes. Across the left-hand boundary, the temperature distribution is specified as linear. The finite-difference equations required to determine the temperature distribution, and hence the heat rate, can be written by inspection.

$$T_3 = 0.25(T_2 + T_4 + T_6 + T_c)$$

$$T_4 = 0.25(T_2 + T_5 + T_7 + T_3)$$

$$T_5 = 0.25(T_2 + T_2 + T_4 + T_4)$$

$$T_6 = 0.25(T_3 + T_7 + T_8 + T_b)$$

$$T_7 = 0.25(T_4 + T_4 + T_6 + T_6)$$

$$T_8 = 0.25(T_6 + T_6 + T_a + T_a)$$

The heat rate for both surfaces of the edge is

$$q'_{tot} = 2\left[q'_a + q'_b + q'_c + q'_d\right]$$

$$q'_{tot} = 2\left[k(\Delta x/2)(T_c - T_2)/\Delta y + k\Delta x(T_3 - T_2)/\Delta y + k\Delta x(T_4 - T_2)/\Delta y + k\Delta x(T_5 - T_2)/\Delta x\right]$$

The shape factor for the full edge is defined as

$$q'_{tot} = kS'(T_1 - T_2)$$

Solving the above equation set in IHT, the temperature (°C) distribution is

Continued...

PROBLEM 4.64 (Cont.)

0	0	0	0	0
25	18.75	12.5	6.25	
50	37.5	25.0		
75	56.25			
00				

<

and the heat rate and shape factor are

$$q'_{tot} = 100\,W/m \qquad S = 1$$ <

From Table 4.1, the edge shape factor is 0.54, considerably below our estimate from this coarse grid analysis.

(b) The effect of the linear temperature distribution on the shape factor estimate can be explored using a more extensive grid as shown below. The FDE analysis was performed with the linear distribution imposed as the different sections a, b, c, d, e. Following the same approach as above, find

Location of linear distribution	(a)	(b)	(c)	(d)	(e)
Shape factor, S	0.797	0.799	0.809	0.857	1.00

The shape factor estimate decreases as the imposed linear temperature distribution section is located further from the edge. We conclude that assuming the temperature distribution across the section directly at the edge is a poor-one.

COMMENTS: The grid spacing for this analysis is quite coarse making the estimates in poor agreement with the Table 4.1 result. However, the analysis does show the effect of positioning the linear temperature distribution condition.

PROBLEM 4.65

KNOWN: Long triangular bar insulated on the diagonal while sides are maintained at uniform temperatures T_a and T_b.

FIND: (a) Using a nodal network with five nodes to the side, and beginning with properly defined control volumes, derive the finite-difference equations for the interior and diagonal nodes and obtain the temperature distribution; sketch the 25, 50 and 75°C isotherms and (b) Recognizing that the insulated diagonal surface can be treated as a symmetry line, show that the diagonal nodes can be treated as interior nodes, and write the finite-difference equations by inspection.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Two-dimensional heat transfer, and (3) Constant properties.

ANALYSIS: (a) For the nodal network shown above, nodes 2, 4, 5, 7, 8 and 9 are interior nodes and, since Δx = Δy, the corresponding finite-difference equations are of the form, Eq. 4.33,

$$T_j = 1/4 \sum T_{neighbors} \qquad (1)$$

For a node on the adiabatic, diagonal surface, an energy balance, $\dot{E}_{in} - \dot{E}_{out} = 0$, yields

$$q'_a + q'_b + q'_c = 0$$

$$0 + k\Delta x \frac{T_5 - T_3}{\Delta y} + k\Delta y \frac{T_2 - T_3}{\Delta x} = 0$$

$$T_3 = 1/2\left(T_2 + T_5\right) \qquad (2)$$

That is, for the diagonal nodes, m,

$$T_m = 1/2 \sum T_{neighbors} \qquad (3)$$

To obtain the temperature distributions, enter Eqs. (1, 2, 3) into the IHT workspace and solve for the nodal temperatures (°C), tabulated according to the nodal arrangement:

Continued...

–				
00	85.71			
00	71.43	50.00		
00	50.00	28.57	14.29	
–	0	0	0	–

The 25, 50 and 75°C isotherms are sketched below, using an interpolation scheme to scale the isotherms on the triangular bar.

100 °C

75 °C

50 °C

25 °C

0 °C

(b) If we consider the insulated surface as a symmetry plane, the nodal network appears as shown. As such, the diagonal nodes can be treated as interior nodes, as Eq. (1) above applies. Recognize the form is the same as that of Eq. (2) or (3).

T_a T_a T_a T_a

1 2 2 3 5 5 6

COMMENTS: Always look for symmetry conditions which can greatly simplify the writing of nodal equations. In this situation, the adiabatic surface can be treated as a symmetry plane such that the nodes can be treated as interior nodes, and the finite-difference equations can be written by inspection.

PROBLEM 4.66

KNOWN: Straight fin of uniform cross section with prescribed thermal conditions and geometry; tip condition allows for convection.

FIND: (a) Calculate the fin heat rate, q'_f, and tip temperature, T_L, assuming one-dimensional heat transfer in the fin; calculate the Biot number to determine whether the one-dimensional assumption is valid, (b) Using the finite-element software FEHT, perform a two-dimensional analysis to determine the fin heat rate and the tip temperature; display the isotherms; describe the temperature field and the heat flow pattern inferred from the display, and (c) Validate your FEHT code against the 1-D analytical solution for a fin using a thermal conductivity of 50 and 500 W/m·K.

SCHEMATIC:

$T_b = 200^oC$
T_∞, h
$T_\infty = 100^oC$
$h = 500\ W/m^2\text{-}K$
q'_f
$k = 5\ W/m\text{-}K$
T_L
T_∞, h
T_∞, h
x
w = 20 mm
L = 200 mm

ASSUMPTIONS: (1) Steady-state conduction with constant properties, (2) Negligible radiation exchange, (3) Uniform convection coefficient.

ANALYSIS: (a) Assuming one-dimensional conduction, q'_L and T_L can be determined using Eqs. 3.72 and 3.70, respectively, from Table 3.4, Case A. Alternatively, use the IHT *Model | Extended Surfaces | Temperature Distribution and Heat Rate | Straight Fin | Rectangular*. These results are tabulated below and labeled as "1-D." The Biot number for the fin is

$$Bi = \frac{h(t/2)}{k} = \frac{500\ W/m^2 \cdot K\ (0.020\ m/2)}{5\ W/m \cdot K} = 1$$

(b, c) The fin can be drawn as a two-dimensional outline in FEHT with convection boundary conditions on the exposed surfaces, and with a uniform temperature on the base. Using a fine mesh (at least 1280 elements), solve for the temperature distribution and use the *View | Temperature Contours* command to view the isotherms and the *Heat Flow* command to determine the heat rate into the fin base. The results of the analysis are summarized in the table below.

k (W/m·K)	Bi	Tip temperature, T_L (°C)		Fin heat rate, q'_f (W/m)		Difference* (%)
		1-D	2-D	1-D	2-D	
5	1	100	100	1010	805	20
50	0.1	100.3	100	3194	2990	6.4
500	0.01	123.8	124	9812	9563	2.5

* Difference = $(q'_{f,1D} - q'_{f,2D}) \times 100 / q'_{f,1D}$

COMMENTS: (1) From part (a), since $Bi = 1 > 0.1$, the internal conduction resistance is not negligible. Therefore significant transverse temperature gradients exist, and the one-dimensional conduction assumption in the fin is a poor one.

Continued …..

PROBLEM 4.66 (Cont.)

(2) From the table, with k = 5 W/m·K (Bi = 1), the 2-D fin heat rate obtained from the FEA analysis is 20% lower than that for the 1-D analytical analysis. This is as expected since the 2-D model accounts for transverse thermal resistance to heat flow. Note, however, that analyses predict the same tip temperature, a consequence of the fin approximating an infinitely long fin (mL = 20.2 >> 2.56; see Ex. 3.8 Comments).

(3) For the k = 5 W/m·K case, the FEHT isotherms show considerable curvature in the region near the fin base. For example, at x = 10 and 20 mm, the difference between the centerline and surface temperatures are 15 and 7°C.

(4) From the table, with increasing thermal conductivity, note that Bi decreases, and the one-dimensional heat transfer assumption becomes more appropriate. The difference for the case when k = 500 W/m·K is mostly due to the approximate manner in which the heat rate is calculated in the FEA software.

PROBLEM 4.67

KNOWN: Long rectangular bar having one boundary exposed to a convection process (T_∞, h) while the other boundaries are maintained at constant temperature T_s.

FIND: Using the finite-element method of FEHT, (a) Determine the temperature distribution, plot the isotherms, and identify significant features of the distribution, (b) Calculate the heat rate per unit length (W/m) into the bar from the air stream, and (c) Explore the effect on the heat rate of increasing the convection coefficient by factors of two and three; explain why the change in the heat rate is not proportional to the change in the convection coefficient.

SCHEMATIC:

Fluid
$T_\infty = 100°C$
$h = 100\ W/m^2\text{-}K$
0.5q'
Symmetry plane
k = 1 W/m-K
90 mm
$T_s = 50°C$
30 mm

ASSUMPTIONS: (1) Steady-state, two dimensional conduction, (2) Constant properties.

ANALYSIS: (a) The symmetrical section shown in the schematic is drawn in FEHT with the specified boundary conditions and material property. The *View* | *Temperature Contours* command is used to represent ten isotherms (isopotentials) that have minimum and maximum values of 53.9°C and 85.9°C, respectively.

Tinf = 100 C, h = 100 W/m^2-K
FEHT Analysis Results
View/Temperature Contours
Isotherms (10)
Minimum T = 53.9 C
Maximum T = 85.9 C
Ts = 50 C
Symmetry plane
Ts = 50 C

Because of the symmetry boundary condition, the isotherms are normal to the center-plane indicating an adiabatic surface. Note that the temperature change along the upper surface of the bar is substantial (≈ 40°C), whereas the lower half of the bar has less than a 3°C change. That is, the lower half of the bar is largely unaffected by the heat transfer conditions at the upper surface.

(b, c) Using the *View* | *Heat Flows* command considering the upper surface boundary with selected convection coefficients, the heat rates into the bar from the air stream were calculated.

$h\left(W/m^2 \cdot K\right)$	100	200	300
$q'(W/m)$	128	175	206

Increasing the convection coefficient by factors of 2 and 3, increases the heat rate by 37% and 61%, respectively. The heat rate from the bar to the air stream is controlled by the thermal resistances of the bar (conduction) and the convection process. Since the conduction resistance is significant, we should not expect the heat rate to change proportionally to the change in convection resistance.

PROBLEM 4.68

KNOWN: Log rod of rectangular cross-section of Problem 4.55 that experiences uniform heat generation while its surfaces are maintained at a fixed temperature. Use the finite-element software FEHT as your analysis tool.

FIND: (a) Represent the temperature distribution with representative isotherms; identify significant features; and (b) Determine what heat generation rate will cause the midpoint to reach 600 K with unchanged boundary conditions.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, and (2) Two-dimensional conduction with constant properties.

ANALYSIS: (a) Using *FEHT*, do the following: in *Setup*, enter an appropriate scale; *Draw* the outline of the symmetrical section shown in the above schematic; *Specify* the *Boundary Conditions* (zero heat flux or adiabatic along the symmetrical lines, and isothermal on the edges). Also *Specify* the *Material Properties* and *Generation* rate. *Draw* three *Element Lines* as shown on the annotated version of the *FEHT* screen below. To reduce the mesh, hit *Draw/Reduce Mesh* until the desired fineness is achieved (256 elements is a good choice).

Continued …

PROBLEM 4.68 (Cont.)

After hitting *Run*, C*heck* and then *Calculate*, use the *View/Temperature Contours* and select the 10-isopotential option to display the isotherms as shown in an annotated copy of the *FEHT* screen below.

y (mm)
T(0,10) = 401.3 K
10 isotherms
Max = 401.3 K
Min = 310.1 K
(0,0)
x (mm)

The isotherms are normal to the symmetrical lines as expected since those surfaces are adiabatic. The isotherms, especially near the center, have an elliptical shape. Along the x = 0 axis and the y = 10 mm axis, the temperature gradient is nearly linear. The hottest point is of course the center for which the temperature is

(T(0, 10 mm) = 401.3 K. <

The temperature of this point can be read using the *View/Temperatures* or *View*|*Tabular Output* command.

(b) To determine the required generation rate so that T(0, 10 mm) = 600 K, it is necessary to re-run the model with several guessed values of $\dot{q}$. After a few trials, find

$$\dot{q} = 1.48 \times 10^8 \text{ W/m}^3$$ <

PROBLEM 4.69

KNOWN: Symmetrical section of a flow channel with prescribed values of $\dot{q}$ and k, as well as the surface convection conditions. See Problem 4.5(S).

FIND: Using the finite-element method of FEHT, (a) Determine the temperature distribution and plot the isotherms; identify the coolest and hottest regions, and the region with steepest gradients; describe the heat flow field, (b) Calculate the heat rate per unit length (W/m) from the outer surface A to the adjacent fluid, (c) Calculate the heat rate per unit length (W/m) to surface B from the inner fluid, and (d) Verify that the results are consistent with an overall energy balance on the section.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state, two-dimensional conduction, (2) Constant properties.

ANALYSIS: (a) The symmetrical section shown in the schematic is drawn in FEHT with the specified boundary conditions, material property and generation. The *View | Temperature Contours* command is used to represent ten isotherms (isopotentials) that have minimum and maximum values of 82.1°C and 125.2°C.

The hottest region of the section is the upper vertical leg (left-hand corner). The coolest region is in the lower horizontal leg at the far right-hand boundary. The maximum and minimum section temperatures (125°C and 77°C), respectively, are higher than either adjoining fluid. Remembering that heat flow lines are normal to the isotherms, heat flows from the hottest corner directly to the inner fluid and downward into the lower leg and then flows out surface A and the lower portion of surface B.

Continued …..

(b, c) Using the *View | Heat Flows* command considering the boundaries for surfaces A and B, the heat rates are:

$$q'_s = 1135\ \text{W/m} \qquad\qquad q'_B = -1365\ \text{W/m}.$$ <

From an energy balance on the section, we note that the results are consistent since conservation of energy is satisfied.

$$\dot{E}'_{in} - \dot{E}'_{out} + \dot{E}_g = 0$$

$$-q'_A + q'_B + \dot{q}\forall' = 0$$

$$-1135\ \text{W/m} + (-1365\ \text{W/m}) + 2500\ \text{W/m} = 0$$ <

where $\dot{q}\forall' = 1\times10^6\ \text{W/m}^3 \times [25\times50 + 25\times50]\times10^{-6}\text{m}^2 = 2500\ \text{W/m}.$

COMMENTS: (1) For background on setting up this problem in FEHT, see the tutorial example of the User's Manual. While the boundary conditions are different, and the internal generation term is to be included, the procedure for performing the analysis is the same.

(2) The heat flow distribution can be visualized using the *View | Temperature Gradients* command. The direction and magnitude of the heat flow is represented by the directions and lengths of arrows. Compare the heat flow distribution to the isotherms shown above.

FEHT Analysis Results
View/Temperature Gradients

PROBLEM 4.70

KNOWN: Hot-film flux gage for determining the convection coefficient of an adjoining fluid stream by measuring the dissipated electric power, P_e, and the average surface temperature, $T_{s,f}$.

FIND: Using the finite-element method of *FEHT*, determine the fraction of the power dissipation that is conducted into the quartz substrate considering three cases corresponding to convection coefficients of 500, 1000 and 2000 $W/m^2 \cdot K$.

SCHEMATIC:

Fluid stream
$T_\infty = 25°C$
h = 500, 1000 or 2000 W/m^2-K
Hot-thin film, $T_{s,f}$
$P_e'' = 5000\ W/m^2$, w = 4mm
Quartz substrate
k = 1.4 W/m-K
2 mm
W = 40 mm
Insulation

P_e''
T_∞,h
Substrate
w/2
W/2

ASSUMPTIONS: (1) Steady-state, two-dimensional conduction, (2) Constant substrate properties, (3) Uniform convection coefficient over the hot-film and substrate surfaces, (4) Uniform power dissipation over hot film.

ANALYSIS: The symmetrical section shown in the schematic above (right) is drawn into *FEHT* specifying the substrate material property. On the upper surface, a convection boundary condition (T_∞,h) is specified over the full width W/2. Additionally, an applied uniform flux $\left(P_e'',\ W/m^2\right)$ boundary condition is specified for the hot-film region (w/2). The remaining surfaces of the two-dimensional system are specified as adiabatic. In the schematic below, the electrical power dissipation P_e' (W/m) in the hot film is transferred by convection from the film surface, $q'_{cv,f}$, and from the adjacent substrate surface, $q'_{cv,s}$.

P_e'
$q'_{cv,f}$
$q'_{cv,s}$
T_∞,h
Substrate

The analysis evaluates the fraction, F, of the dissipated electrical power that is conducted into the substrate and convected to the fluid stream,

$$F = q'_{cv,s} / P_e' = 1 - q'_{cv,f} / P_e'$$

where $\quad P_e' = P_e''(w/2) = 5000\ W/m^2 \times (0.002\ m) = 10\ W/m.$

After solving for the temperature distribution, the *View|Heat Flow* command is used to evaluate $q'_{cv,f}$ for the three values of the convection coefficient.

Continued …..

PROBLEM 4.70 (Cont.)

Case	$h(W/m^2 \cdot K)$	$q'_{cv,f}\ (W/m)$	F(%)	$T_{s,f}$ (°C)
1	500	5.64	43.6	30.9
2	1000	6.74	32.6	28.6
3	2000	7.70	23.3	27.0

COMMENTS: (1) For the ideal hot-film flux gage, there is negligible heat transfer to the substrate, and the convection coefficient of the air stream is calculated from the measured electrical power, P''_e, the average film temperature (by a thin-film thermocouple), $T_{s,f}$, and the fluid stream temperature, T_∞, as $h = P''_e / (T_{s,f} - T_\infty)$. The purpose in performing the present analysis is to estimate a correction factor to account for heat transfer to the substrate.

(2) As anticipated, the fraction of the dissipated electrical power conducted into the substrate, F, decreases with increasing convection coefficient. For the case of the largest convection coefficient, F amounts to 25%, making it necessary to develop a reliable, accurate heat transfer model to estimate the applied correction. Further, this condition limits the usefulness of this gage design to flows with high convection coefficients.

(3) A reduction in F, and hence the effect of an applied correction, could be achieved with a substrate material having a lower thermal conductivity than quartz. However, quartz is a common substrate material for fabrication of thin-film heat-flux gages and thermocouples. By what other means could you reduce F?

(4) In addition to the tutorial example in the *FEHT* User's Manual, the solved models for Examples 4.3 and 4.4 are useful for developing skills helpful in solving this problem.

PROBLEM 4.71

KNOWN: Hot-plate tool for micro-lithography processing of 300-mm silicon wafer consisting of an aluminum alloy equalizing block (EB) heated by ring-shaped main and trim electrical heaters (MH and TH) providing two-zone control.

FIND: The assignment is to size the heaters, MH and TH, by specifying their applied heat fluxes, q''_{mh} and q''_{th}, and their radial extents, Δr_{mh} and Δr_{th}, to maintain an operating temperature of 140°C with a uniformity of 0.1°C. Consider these steps in the analysis: (a) Perform an energy balance on the EB to obtain an initial estimate for the heater fluxes with $q''_{mh} = q''_{th}$ extending over the full radial limits; using *FEHT*, determine the upper surface temperature distribution and comment on whether the desired uniformity has been achieved; (b) Re-run your *FEHT* code with different values of the heater fluxes to obtain the best uniformity possible and plot the surface temperature distribution; (c) Re-run your *FEHT* code for the best arrangement found in part (b) using the representative distribution of the convection coefficient (see schematic for h(r) for downward flowing gas across the upper surface of the EB; adjust the heat flux of TH to obtain improved uniformity; and (d) Suggest changes to the design for improving temperature uniformity.

SCHEMATIC:

<

Gas $T_\infty = 25°C$, $h = 10\ W/m^2\text{-}K$ or h(r)

Equilizing block (EB), k = 75 W/m-K

MH, q''_{mh} TH, q''_{th} T_∞,h   w

r_1 r_2 r_3 r_4 r_o r

$h(r) = h_o[1+a(r/r_o)^n]$
$h_o = 5.4\ W/m^2\text{-}K$
a = n = 1.5

Dimensions (mm)
r_1 = 30
r_2 = 90
r_3 = 120
r_4 = 150
r_o = 170
w = 30

ASSUMPTIONS: (1) Steady-state conditions, (2) Two-dimensional conduction with uniform and constant properties in EB, (3) Lower surface of EB perfectly insulated, (4) Uniform convection coefficient over upper EB surface, unless otherwise specified and (5) negligible radiation exchange between the EB surfaces and the surroundings.

ANALYSIS: (a) To obtain initial estimates for the MH and TH fluxes, perform an overall energy balance on the EB as illustrated in the schematic below.

Gas T_∞,h q_{conv} $T_s = 140°C$

EB T_∞,h   w

q''_{mh} q''_{th}

r_1 r_2 r_3 r_4 r_o r

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$q''_{mh}\pi\left(r_2^2 - r_1^2\right) + q''_{th}\pi\left(r_4^2 - r_3^2\right) - h\left[\pi r_o^2 + 2\pi r_o w\right]\left(T_s - T_\infty\right) = 0$$

Continued …..

PROBLEM 4.71 (Cont.)

Substituting numerical values and letting $q''_{mh} = q''_{th}$, find

$$q''_{mh} = q''_{th} = 2939 \text{ W/m}^2$$ <

Using *FEHT*, the analysis is performed on an axisymmetric section of the EB with the nodal arrangement as shown below.

T1 Tinf, h T14
EB, k
Scale: 1 cm = 10 mm
Tinf, h
Insulated MH, q''mh Insulated TH, q''th Insulated

The *Temperature Contour* view command is used to create the temperature distribution shown below. The temperatures at the center (T_1) and the outer edge of the wafer (r = 150 mm, T_{14}) are read from the *Tabular Output* page. The *Temperature Gradients* view command is used to obtain the heat flow distribution when the line length is proportional to the magnitude of the heat rate.

T1 = 141.1 C Tinf, h T14 = 139.3 C
10 isothermal lines
Min 138.9 C, max 141.9 C
Tinf, h
MH TH

Heat flow lines

From the analysis results, for this base case design $(q''_{mh} = q''_{th})$, the temperature difference across the radius of the wafer is 1.7°C, much larger than the design goal of 0.1°C. The upper surface temperature distribution is shown in the graph below.

Continued …..

EB surface temperature distribution

(Chart: T(r,z), (C) vs. Radial position, r (mm))

(b) From examination of the results above, we conclude that if q''_{mh} is reduced and q''_{th} increased, the EB surface temperature uniformity could improve. The results of three trials compared to the base case are tabulated below.

Trial	q''_{mh} $\left(W/m^2\right)$	q''_{th} $\left(W/m^2\right)$	T_1 (°C)	T_{14} (°C)	$T_1 - T_{14}$ (°C)
Base	2939	2939	141.1	139.3	1.8
1	2880 (-2%)	2997 (+2%)	141.1	139.4	1.7
2	2880 (-2%)	3027 (+3%)	141.7	140.0	1.7
3	2910 (-1%)	2997 (+2%)	141.7	139.9	1.8
Part (c)	2939	2939	141.7	139.1	2.6
Part (d) k=150 W/m·K	2939	2939	140.4	139.5	0.9
Part (d) k=300 W/m·K	2939	2939	140.0	139.6	0.4

The strategy of changing the heater fluxes (trials 1-3) has not resulted in significant improvements in the EB surface temperature uniformity.

Continued …..

PROBLEM 4.71 (Cont.)

(c) Using the same *FEHT* code as with part (b), base case, the boundary conditions on the upper surface of the EB were specified by the function h(r) shown in the schematic. The value of h(r) ranged from 5.4 to 13.5 $W/m^2 \cdot K$ between the centerline and EB edge. The result of the analysis is tabulated above, labeled as part (c). Note that the temperature uniformity has become significantly poorer.

(d) There are at least two options that should be considered in the re-design to improve temperature uniformity. *Higher thermal conductivity material for the EB.* Aluminum alloy is the material most widely used in practice for reasons of low cost, ease of machining, and durability of the heated surface. The results of analyses for thermal conductivity values of 150 and 300 W/m·K are tabulated above, labeled as part (d). Using pure or oxygen-free copper could improve the temperature uniformity to better than 0.5°C.

Distributed heater elements. The initial option might be to determine whether temperature uniformity could be improved using two elements, but located differently. Another option is a single element heater spirally embedded in the lower portion of the EB. By appropriately positioning the element as a function of the EB radius, improved uniformity can be achieved. This practice is widely used where precise and uniform temperature control is needed.

PROBLEM 4.72

KNOWN: Straight fin of uniform cross section with insulated end.

FIND: (a) Temperature distribution using finite-difference method and validity of assuming one-dimensional heat transfer, (b) Fin heat transfer rate and comparison with analytical solution, Eq. 3.76, (c) Effect of convection coefficient on fin temperature distribution and heat rate.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in fin, (3) Constant properties, (4) Uniform film coefficient.

ANALYSIS: (a) From the analysis of Problem 4.45, the finite-difference equations for the nodal arrangement can be directly written. For the nodal spacing $\Delta x = 4$ mm, there will be 12 nodes. With $\ell \gg w$ representing the distance normal to the page,

$$\frac{hP}{kA_c}\cdot\Delta x^2 \approx \frac{h\cdot 2\ell}{k\cdot\ell\cdot w}\Delta x^2 = \frac{h\cdot 2}{kw}\Delta x^2 = \frac{500\,\text{W/m}^2\cdot\text{K}\times 2}{50\,\text{W/m}\cdot\text{K}\times 6\times 10^{-3}\,\text{m}}\left(4\times 10^{-3}\text{mm}\right) = 0.0533$$

Node 1: $100 + T_2 + 0.0533\times 30 - (2 + 0.0533)T_1 = 0$ or $-2.053T_1 + T_2 = -101.6$

Node n: $T_{n+1} + T_{n-1} + 1.60 - 2.0533T_n = 0$ or $T_{n-1} - 2.053T_n + T_{n-1} = -1.60$

Node 12: $T_{11} + (0.0533/2)30 - (0.0533/2 + 1)T_{12} = 0$ or $T_{11} - 1.0267T_{12} = -0.800$

Using matrix notation, Eq. 4.52, where [A] [T] = [C], the A-matrix is tridiagonal and only the non-zero terms are shown below. A matrix inversion routine was used to obtain [T].

Tridiagonal Matrix A						*Column Matrices*		
Nonzero Terms			Values			Node	C	T
	$a_{1,1}$	$a_{1,2}$		-2.053	1	1	-101.6	85.8
$a_{2,1}$	$a_{2,2}$	$a_{2,3}$	1	-2.053	1	2	-1.6	74.5
$a_{3,2}$	$a_{3,3}$	$a_{3,4}$	1	-2.053	1	3	-1.6	65.6
$a_{4,3}$	$a_{4,4}$	$a_{4,5}$	1	-2.053	1	4	-1.6	58.6
$a_{5,4}$	$a_{5,5}$	$a_{5,6}$	1	-2.053	1	5	-1.6	53.1
$a_{6,5}$	$a_{6,6}$	$a_{6,7}$	1	-2.053	1	6	-1.6	48.8
$a_{7,6}$	$a_{7,7}$	$a_{7,8}$	1	-2.053	1	7	-1.6	45.5
$a_{8,7}$	$a_{8,8}$	$a_{8,9}$	1	-2.053	1	8	-1.6	43.0
$a_{9,8}$	$a_{9,9}$	$a_{9,10}$	1	-2.053	1	9	-1.6	41.2
$a_{10,9}$	$a_{10,10}$	$a_{10,11}$	1	-2.053	1	10	-1.6	39.9
$a_{11,10}$	$a_{11,11}$	$a_{11,12}$	1	-2.053	1	11	-1.6	39.2
$a_{12,11}$	$a_{12,12}$	$a_{12,13}$	1	-1.027	1	12	-0.8	38.9

The assumption of one-dimensional heat conduction is justified when $Bi \equiv h(w/2)/k < 0.1$. Hence, with $Bi = 500\ \text{W/m}^2\cdot\text{K}(3\times 10^{-3}\ \text{m})/50\ \text{W/m}\cdot\text{K} = 0.03$, the assumption is reasonable.

Continued...

PROBLEM 4.72 (Cont.)

(b) The fin heat rate can be most easily found from an energy balance on the control volume about Node 0,

$$q'_f = q'_1 + q'_{conv} = k \cdot w \frac{T_0 - T_1}{\Delta x} + h\left(2\frac{\Delta x}{2}\right)(T_0 - T_\infty)$$

$$q'_f = 50\,W/m\cdot K\left(6\times10^{-3}m\right)\frac{(100-85.8)^\circ C}{4\times10^{-3}m} + 500\,W/m^2\cdot K\left(2\cdot\frac{4\times10^{-3}m}{2}\right)(100-30)^\circ C$$

$$q'_f = (1065+140)\,W/m = 1205\,W/m\,.$$ <

From Eq. 3.76, the fin heat rate is

$$q = \left(hPkA_c\right)^{1/2}\cdot\theta_b\cdot\tanh mL\,.$$

Substituting numerical values with $P = 2(w + \ell) \approx 2\ell$ and $A_c = w\cdot\ell$, $m = (hP/kA_c)^{1/2} = 57.74\ m^{-1}$ and $M = (hPkA_c)^{1/2} = 17.32\,\ell$ W/K. Hence, with $\theta_b = 70°C$,

$$q' = 17.32\,W/K\times70\,K\times\tanh(57.44\times0.048) = 1203\,W/m$$

and the finite-difference result agrees very well with the exact (analytical) solution.

(c) Using the IHT *Finite-Difference Equations Tool Pad* for *1D, SS* conditions, the fin temperature distribution and heat rate were computed for h = 10, 100, 500 and 1000 $W/m^2\cdot K$. Results are plotted as follows.

The temperature distributions were obtained by first creating a *Lookup Table* consisting of 4 rows of nodal temperatures corresponding to the 4 values of h and then using the *LOOKUPVAL2* interpolating function with the *Explore* feature of the IHT menu. Specifically, the function T_EVAL = LOOKUPVAL2(t0467, h, x) was entered into the workspace, where t0467 is the file name given to the Lookup Table. For each value of h, *Explore* was used to compute T(x), thereby generating 4 data sets which were placed in the *Browser* and used to generate the plots. The variation of q' with h was simply generated by using the *Explore* feature to solve the finite-difference model equations for values of h incremented by 10 from 10 to 1000 $W/m^2\cdot K$.

Although q'_f increases with increasing h, the effect of changes in h becomes less pronounced. This trend is a consequence of the reduction in fin temperatures, and hence the fin efficiency, with increasing h. For $10 \le h \le 1000\ W/m^2\cdot K$, $0.95 \ge \eta_f \ge 0.24$. Note the nearly isothermal fin for h = 10 $W/m^2\cdot K$ and the pronounced temperature decay for h = 1000 $W/m^2\cdot K$.

PROBLEM 4.73

KNOWN: Pin fin of 10 mm diameter and length 250 mm with base temperature of 100°C experiencing radiation exchange with the surroundings and free convection with ambient air.

FIND: Temperature distribution using finite-difference method with five nodes. Fin heat rate and relative contributions by convection and radiation.

SCHEMATIC:

Quiescent air, $T_\infty = 25\ ^oC$; $T_{sur} = 25\ ^oC$; $T_b = 100\ ^oC$; Stainless steel rod $k = 14$ W/m·K, $\varepsilon = 0.2$; nodes 1, 2, 3, 4, 5; Δx; $D = 10$ mm; $L = 250$ mm; x

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in fin, (3) Constant properties, (4) Fin approximates small object in large enclosure, (5) Fin tip experiences convection and radiation, (6) $h_{fc} = 2.89[0.6 + 0.624(T - T_\infty)^{1/6}]^2$.

ANALYSIS: To apply the finite-difference method, define the 5-node system shown above where $\Delta x = L/5$. Perform energy balances on the nodes to obtain the finite-difference equations for the nodal temperatures.

Interior node, n = 1, 2, 3 or 4

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$q_a + q_b + q_c + q_d = 0 \qquad (1)$$

q_a, q_b, q_c, q_d; $n-1$, n, $n+1$

$$h_{r,n}P\Delta x(T_{sur} - T_n) + kA_c\frac{T_{n+1} - T_n}{\Delta x} + h_{fc,n}P\Delta x(T_\infty - T_n) + kA_c\frac{T_{n-1} - T_n}{\Delta x} = 0 \qquad (2)$$

where the free convection coefficient is

$$h_{fc,n} = 2.89\left[0.6 + 0.624(T_n - T_\infty)^{1/6}\right]^2 \qquad (3)$$

and the linearized radiation coefficient is

$$h_{r,n} = \varepsilon\sigma(T_n + T_{sur})\left(T_n^2 + T_{sur}^2\right) \qquad (4)$$

with $P = \pi D$ and $A_c = \pi D^2/4$. (5,6)

Tip node, n = 5

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$q_a + q_b + q_c + q_d + q_e = 0$$

q_a, q_b, q_c, q_d, q_e; T_4, T_5

$$h_{r,5}(P\Delta x/2)(T_{sur} - T_5) + h_{r,5}A_c(T_{sur} - T_5) + h_{fc,5}A_c(T_\infty - T_5)$$
$$+ h_{fc,5}(P\Delta x/2)(T_\infty - T_5) + kA_c\frac{T_4 - T_5}{\Delta x} = 0 \qquad (7)$$

Continued...

Knowing the nodal temperatures, the heat rates are evaluated as:

Fin Heat Rate: Perform an energy balance around Node b.

q_a q_b T_b T_1 q_{fin} q_c $\Delta x/2$

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$q_a + q_b + q_c + q_{fin} = 0$$

$$h_{r,b}\left(P\Delta x/2\right)\left(T_{sur} - T_b\right) + h_{fc,b}\left(P\Delta x/2\right)\left(T_\infty - T_b\right) + kA_c\frac{\left(T_1 - T_b\right)}{\Delta x} + q_{fin} = 0 \qquad (8)$$

where $h_{r,b}$ and $h_{fc,b}$ are evaluated at T_b.

Convection Heat Rate: To determine the portion of the heat rate by convection from the fin surface, we need to sum contributions from each node. Using the convection heat rate terms from the foregoing energy balances, for, respectively, node b, nodes 1, 2, 3, 4 and node 5.

$$q_{cv} = -q_b)_b - \sum q_c)_{1-4} - \left(q_c + q_d\right)_5 \qquad (9)$$

Radiation Heat Rate: In the same manner,

$$q_{rad} = -q_a)_b - \sum q_b)_{1-4} - \left(q_a + q_b\right)_5$$

The above equations were entered into the IHT workspace and the set of equations solved for the nodal temperatures and the heat rates. Summary of key results including the temperature distribution and heat rates is shown below.

Node	b	1	2	3	4	5	Fin
T_j (°C)	100	58.5	40.9	33.1	29.8	28.8	-
q_{cv} (W)	0.603	0.451	0.183	0.081	0.043	0.015	1.375
q_{fin} (W)	-	-	-	-	-	-	1.604
q_{rad} (W)	-	-	-	-	-	-	0.229
h_{cv} ($W/m^2\cdot K$)	10.1	8.6	7.3	6.4	5.7	5.5	-
h_{rod} ($W/m^2\cdot K$)	1.5	1.4	1.3	1.3	1.2	1.2	-

<

COMMENTS: From the tabulated results, it is evident that free convection is the dominant node. Note that the free convection coefficient varies almost by a factor of two over the length of the fin.

PROBLEM 4.74

KNOWN: Thin metallic foil of thickness, t, whose edges are thermally coupled to a sink at temperature T_{sink} is exposed on the top surface to an ion beam heat flux, q''_s, and experiences radiation exchange with the vacuum enclosure walls at T_{sur}.

FIND: Temperature distribution across the foil.

SCHEMATIC:

Vacuum enclosure, T_{sur}; Ion beam, q''_s; Grid hole pattern; Grid; Water-cooled electrode sink, T_{sink}; x

ASSUMPTIONS: (1) One-dimensional, steady-state conduction in the foil, (2) Constant properties, (3) Upper and lower surfaces of foil experience radiation exchange, (4) Foil is of unit length normal to the page.

ANALYSIS: The 10-node network representing the foil is shown below.

T_{sur}; q''_s; 2, 1, 2, 3, 9, 10; T_{sink}; Δx; x; L; T_{sur}; t; q'_a, q'_b, q'_c, q'_d, q'_e; T_{n-1}, T_n, T_{n+1}

From an energy balance on node n, $\dot{E}_{in} - \dot{E}_{out} = 0$, for a unit depth,

$$q'_a + q'_b + q'_c + q'_d + q'_e = 0$$

$$q''_s \Delta x + h_{r,n}\Delta x\left(T_{sur} - T_n\right) + k(t)\left(T_{n+1} - T_n\right)/\Delta x + h_{r,n}\Delta x\left(T_{sur} - T_n\right) + k(t)\left(T_{n-1} - T_n\right)/\Delta x = 0 \quad (1)$$

where the linearized radiation coefficient for node n is

$$h_{r,n} = \varepsilon\sigma\left(T_{sur} + T_n\right)\left(T_{sur}^2 + T_n^2\right) \quad (2)$$

Solving Eq. (1) for T_n find,

$$T_n = \left[\left(T_{n+1} + T_{n-1}\right) + \left(2h_{r,n}\Delta x^2/kt\right)T_{sur} + \left(\Delta x^2/kt\right)q''_s\right]\Big/\left[\left(h_{r,n}\Delta x^2/kt\right) + 2\right] \quad (3)$$

which, considering symmetry, applies also to node 1. Using IHT for Eqs. (3) and (2), the set of finite-difference equations was solved for the temperature distribution (K):

T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	T_{10}
374.1	374.0	373.5	372.5	370.9	368.2	363.7	356.6	345.3	327.4

Continued...

COMMENTS: (1) If the temperature gradients were excessive across the foil, it would wrinkle; most likely since its edges are constrained, the foil will bow.

(2) The IHT workspace for the finite-difference analysis follows:

```
// The nodal equations:
T1 = ( (T2 + T2) + A1 * Tsur +  B *q''s ) / ( A1 + 2)
A1= 2 * hr1 * deltax^2 / (k * t)
hr1 = eps * sigma * (Tsur + T1) * (Tsur^2 + T1^2)
sigma = 5.67e-8
B = deltax^2 / (k * t)

T2 = ( (T1 + T3) + A2 * Tsur + B *q''s ) / ( A2 + 2)
A2= 2 * hr2 * deltax^2 / (k * t)
hr2 = eps * sigma * (Tsur + T2) * (Tsur^2 + T2^2)

T3 = ( (T2 + T4) + A3 * Tsur + B *q''s ) / ( A3 + 2)
A3= 2 * hr3 * deltax^2 / (k * t)
hr3 = eps * sigma * (Tsur + T3) * (Tsur^2 + T3^2)

T4 = ( (T3 + T5) + A4 * Tsur + B *q''s ) / ( A4 + 2)
A4= 2 * hr4 * deltax^2 / (k * t)
hr4 = eps * sigma * (Tsur + T4) * (Tsur^2 + T4^2)

T5 = ( (T4 + T6) + A5 * Tsur + B *q''s ) / ( A5 + 2)
A5= 2 * hr5 * deltax^2 / (k * t)
hr5 = eps * sigma * (Tsur + T5) * (Tsur^2 + T5^2)

T6 = ( (T5 + T7) + A6 * Tsur + B *q''s ) / ( A6 + 2)
A6= 2 * hr6 * deltax^2 / (k * t)
hr6 = eps * sigma * (Tsur + T6) * (Tsur^2 + T6^2)

T7 = ( (T6 + T8) + A7 * Tsur + B *q''s ) / ( A7 + 2)
A7= 2 * hr7 * deltax^2 / (k * t)
hr7 = eps * sigma * (Tsur + T7) * (Tsur^2 + T7^2)

T8 = ( (T7 + T9) + A8 * Tsur + B *q''s ) / ( A8 + 2)
A8= 2 * hr8 * deltax^2 / (k * t)
hr8 = eps * sigma * (Tsur + T8) * (Tsur^2 + T8^2)

T9 = ( (T8 + T10) + A9 * Tsur + B *q''s ) / ( A9 + 2)
A9= 2 * hr9 * deltax^2 / (k * t)
hr9 = eps * sigma * (Tsur + T9) * (Tsur^2 + T9^2)

T10 = ( (T9 + Tsink) + A10 * Tsur + B *q''s ) / ( A10 + 2)
A10= 2 * hr10 * deltax^2 / (k * t)
hr10 = eps * sigma * (Tsur + T10) * (Tsur^2 + T10^2)

// Assigned variables
deltax = L / 10          // Spatial increment, m
L = 0.150                // Foil length, m
t = 0.00025              // Foil thickness, m
eps = 0.45               // Emissivity
Tsur = 300               // Surroundings temperature, K
k = 40                   // Foil thermal conductivity, W/m.K
Tsink = 300              // Sink temperature, K
q''s = 600               // Ion beam heat flux, W/m^2

/* Data Browser results: Temperature distribution (K)  and linearized radiation cofficients
(W/m^2.K):

T1      T2      T3      T4      T5      T6      T7      T8      T9      T10
374.1   374     373.5   372.5   370.9   368.2   363.7   356.6   345.3   327.4

hr1     hr2     hr3     hr4     hr5     hr6     hr7     hr8     hr9     hr10
3.956   3.953   3.943   3.926   3.895   3.845   3.765   3.639   3.444   3.157 */
```

PROBLEM 4.75

KNOWN: Electrical heating elements with known dissipation rate embedded in a ceramic plate of known thermal conductivity; lower surface is insulated, while upper surface is exposed to a convection process.

FIND: (a) Temperature distribution within the plate using prescribed grid spacing, (b) Sketch isotherms to illustrate temperature distribution, (c) Heat loss by convection from exposed surface (compare with element dissipation rate), (d) Advantage, if any, in not setting $\Delta x = \Delta y$, (e) Effect of grid size and convection coefficient on the temperature field.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state, two-dimensional conduction in ceramic plate, (2) Constant properties, (3) No internal generation, except for Node 7 (or Node 15 for part (e)), (4) Heating element approximates a line source of negligible wire diameter.

ANALYSIS: (a) The prescribed grid for the symmetry element shown above consists of 12 nodal points. Nodes 1-3 are points on a surface experiencing convection; nodes 4-6 and 8-12 are interior nodes. Node 7 is a special case of the interior node having a generation term; because of symmetry, q'_{ht} = 25 W/m. The finite-difference equations are derived as follows:

Continued...

PROBLEM 4.75 (Cont.)

Surface Node 2. From an energy balance on the prescribed control volume with $\Delta x/\Delta y = 3$,

$\dot{E}_{in} - \dot{E}_{out} = q'_a + q'_b + q'_c + q'_d = 0;$

$$k\frac{\Delta y}{2}\frac{T_1 - T_2}{\Delta x} + h\Delta x\left(T_\infty - T_2\right) + k\frac{\Delta y}{2}\frac{T_3 - T_2}{\Delta x} + k\Delta x\frac{T_5 - T_2}{\Delta y} = 0 .$$

Regrouping, find

$$T_2\left(1 + 2N\frac{\Delta x}{\Delta y} + 1 + 2\left(\frac{\Delta x}{\Delta y}\right)^2\right) = T_1 + T_3 + 2\left(\frac{\Delta x}{\Delta y}\right)^2 T_5 + 2N\frac{\Delta x}{\Delta y}T_\infty$$

where $N = h\Delta x/k = 100\ W/m^2\cdot K \times 0.006\ m/2\ W/m\cdot K = 0.30\ K$. Hence, with $T_\infty = 30°C$,

$$T_2 = 0.04587T_1 + 0.04587T_3 + 0.82569T_5 + 2.4771 \qquad (1)$$

From this FDE, the forms for nodes 1 and 3 can also be deduced.

Interior Node 7. From an energy balance on the prescribed control volume, with $\Delta x/\Delta y = 3$, $\dot{E}'_{in} - \dot{E}'_g = 0$, where $\dot{E}'_g = 2\,q'_{ht}$ and E'_{in} represents the conduction terms. Hence,

$q'_a + q'_b + q'_c + q'_d + 2q'_{ht} = 0$, or

$$k\Delta y\frac{T_8 - T_7}{\Delta x} + k\Delta x\frac{T_4 - T_7}{\Delta y} + k\Delta y\frac{T_8 - T_7}{\Delta x} + k\Delta x\frac{T_{10} - T_7}{\Delta y} + 2q'_{ht} = 0$$

Regrouping,

$$T_7\left[1 + \left(\frac{\Delta x}{\Delta y}\right)^2 + 1 + \left(\frac{\Delta x}{\Delta y}\right)^2\right] = T_8 + \left(\frac{\Delta x}{\Delta y}\right)^2 T_4 + T_8 + \left(\frac{\Delta x}{\Delta y}\right)^2 T_{10} + \frac{2q'_{ht}}{k}\left(\frac{\Delta x}{\Delta y}\right)$$

Recognizing that $\Delta x/\Delta y = 3$, $q'_{ht} = 25\ W/m$ and $k = 2\ W/m\cdot K$, the FDE is

$$T_7 = 0.0500T_8 + 0.4500T_4 + 0.0500T_8 + 0.4500T_{10} + 3.7500 \qquad (2)$$

The FDEs for the remaining nodes may be deduced from this form. Following the procedure described in Section 4.5.2 for the Gauss-Seidel method, the system of FDEs has the form:

$$T_1^k = 0.09174T_2^{k-1} + 0.8257T_4^{k-1} + 2.4771$$

$$T_2^k = 0.04587T_1^k + 0.04587T_3^{k-1} + 0.8257T_5^{k-1} + 2.4771$$

$$T_3^k = 0.09174T_2^k + 0.8257T_6^{k-1} + 2.4771$$

$$T_4^k = 0.4500T_1^k + 0.1000T_5^{k-1} + 0.4500T_7^{k-1}$$

$$T_5^k = 0.4500T_2^k + 0.0500T_4^k + 0.0500T_6^{k-1} + 0.4500T_8^{k-1}$$

$$T_6^k = 0.4500T_3^k + 0.1000T_5^k + 0.4500T_9^{k-1}$$

$$T_7^k = 0.4500T_4^k + 0.1000T_8^{k-1} + 0.4500T_{10}^{k-1} + 3.7500$$

$$T_8^k = 0.4500T_5^k + 0.0500T_7^k + 0.0500T_9^{k-1} + 0.4500T_{11}^{k-1}$$

$$T_9^k = 0.4500T_6^k + 0.1000T_8^k + 0.4500T_{12}^{k-1}$$

$$T_{10}^k = 0.9000T_7^k + 0.1000T_{11}^{k-1}$$

$$T_{11}^k = 0.9000T_8^k + 0.0500T_{10}^{k-1} + 0.0500T_{12}^{k-1}$$

$$T_{12}^k = 0.9000T_9^k + 0.1000T_{11}^k$$

Continued …..

PROBLEM 4.75 (Cont.)

Note the use of the superscript k to denote the level of iteration. Begin the iteration procedure with rational estimates for T_i (k = 0) and prescribe the convergence criterion as $\varepsilon \le 0.1$.

k/T_i	1	2	3	4	5	6	7	8	9	10	11	12
0	55.0	50.0	45.0	61.0	54.0	47.0	65.0	56.0	49.0	60.0	55.0	50.0
1	57.4	51.7	46.0	60.4	53.8	48.1	63.5	54.6	49.6	62.7	54.8	50.1
2	57.1	51.6	46.9	59.7	53.2	48.7	64.3	54.3	49.9	63.4	54.5	50.4
∞	55.80	49.93	47.67	59.03	51.72	49.19	63.89	52.98	50.14	62.84	53.35	50.46

The last row with k = ∞ corresponds to the solution obtained by matrix inversion. It appears that at least 20 iterations would be required to satisfy the convergence criterion using the Gauss-Seidel method.

(b) Selected isotherms are shown in the sketch of the nodal network.

1 2 3
53.4 °C
4 6
49.2 °C
55.8 °C
7 9
49.9 °C
59.0 °C
10 12
11

Note that the isotherms are normal to the adiabatic surfaces.

(c) The heat loss by convection can be expressed as

$$q'_{conv} = h\left[\frac{1}{2}\Delta x\left(T_1 - T_\infty\right) + \Delta x\left(T_2 - T_\infty\right) + \frac{1}{2}\Delta x\left(T_3 - T_\infty\right)\right]$$

$$q'_{conv} = 100\,\mathrm{W/m^2 \cdot K} \times 0.006\,\mathrm{m}\left[\frac{1}{2}(55.80-30) + (49.93-30) + \frac{1}{2}(47.67-30)\right] = 25.00\,\mathrm{W/m}. \quad <$$

As expected, the heat loss by convection is equal to the heater element dissipation. This follows from the conservation of energy requirement.

(d) For this situation, choosing $\Delta x = 3\Delta y$ was advantageous from the standpoint of precision and effort. If we had chosen $\Delta x = \Delta y = 2$ mm, there would have been 28 nodes, doubling the amount of work, but with improved precision.

(e) Examining the effect of grid size by using the *Finite-Difference Equations* option from the *Tools* portion of the IHT Menu, the following temperature field was computed for $\Delta x = \Delta y = 2$ mm, where x and y are in mm and the temperatures are in °C.

y\x	0	2	4	6	8	10	12
0	55.04	53.88	52.03	50.32	49.02	48.24	47.97
2	58.71	56.61	54.17	52.14	50.67	49.80	49.51
4	66.56	59.70	55.90	53.39	51.73	50.77	50.46
6	63.14	59.71	56.33	53.80	52.09	51.11	50.78

Continued …..

PROBLEM 4.75 (Cont.)

Agreement with the results of part (a) is excellent, except in proximity to the heating element, where T_{15} = 66.6°C for the fine grid exceeds T_7 = 63.9°C for the coarse grid by 2.7°C.

For h = 10 W/m^2·K, the maximum temperature in the ceramic corresponds to T_{15} = 254°C, and the heater could still be operated at the prescribed power. With h = 10 W/m^2·K, the critical temperature of T_{15} = 400°C would be reached with a heater power of approximately 82 W/m.

COMMENTS: (1) The method used to obtain the rational estimates for T_i (k = 0) in part (a) is as follows. Assume 25 W/m is transferred by convection uniformly over the surface; find $\overline{T}_{surf} \approx 50°C$. Set T_2 = 50°C and recognize that T_1 and T_3 will be higher and lower, respectively. Assume 25 W/m is conducted uniformly to the outer nodes; find $T_5 - T_2 \approx 4°C$. For the remaining nodes, use intuition to guess reasonable values. (2) In selecting grid size (and whether $\Delta x = \Delta y$), one should consider the region of largest temperature gradients.

NOTE TO INSTRUCTOR: Although the problem statement calls for calculations with $\Delta x = \Delta y = 1$ mm, the instructional value and benefit-to-effort ratio are small. Hence, consideration of this grid size is not recommended.

PROBLEM 4.76

KNOWN: Silicon chip mounted in a dielectric substrate. One surface of system is convectively cooled while the remaining surfaces are well insulated.

FIND: Whether maximum temperature in chip will exceed 85°C.

SCHEMATIC:

Coolant
$T_\infty = 20°C$
$h = 500 W/m^2 \cdot K$
$H = 12 mm$
$H/4$
$L/3$
Chip, $k_c = 50 W/m \cdot K$, $\dot{q} = 10^7 W/m^3$
Substrate, $k_s = 5 W/m \cdot K$
$L = 27 mm$

ASSUMPTIONS: (1) Steady-state conditions, (2) Two-dimensional conduction, (3) Negligible contact resistance between chip and substrate, (4) Upper surface experiences uniform convection coefficient, (5) Other surfaces are perfectly insulated.

ANALYSIS: Performing an energy balance on the chip assuming it is *perfectly insulated from* the substrate, the maximum temperature occurring at the interface with the dielectric substrate will be, according to Eqs. 3.43 and 3.46,

$$T_{max} = \frac{\dot{q}(H/4)^2}{2k_c} + \frac{\dot{q}(H/4)}{h} + T_\infty = \frac{10^7 \ W/m^3 (0.003 \ m)^2}{2 \times 50 \ W/m \cdot K} + \frac{10^7 \ W/m^3 (0.003 \ m)}{500 \ W/m^2 \cdot K} + 20^\circ C = 80.9^\circ C.$$

Since $T_{max} < 85°C$ for the assumed situation, for the actual two-dimensional situation with the conducting dielectric substrate, the maximum temperature should be less than 80°C.

Using the suggested grid spacing of 3 mm, construct the nodal network and write the finite-difference equation for each of the nodes taking advantage of symmetry of the system. Note that we have chosen to *not* locate nodes on the system surfaces for two reasons: (1) fewer total number of nodes, 20 vs. 25, and (2) Node 5 corresponds to center of chip which is likely the point of maximum temperature. Using these numerical values,

y, T_∞, h, $\Delta x = 3mm$, $\Delta y = 3mm$, x
1 2 3 4 5
6 7 8 9 10
11 12 13 14 15
16 17 18 19 20

$$\frac{h\Delta x}{k_s} = \frac{500 \ W/m^2 \cdot K \times 0.003 \ m}{5 \ W/m \cdot K} = 0.30 \qquad a = \frac{2}{(k_s/k_c)+1} = \frac{2}{5/50+1} = 1.818$$

$$\frac{h\Delta x}{k_c} = \frac{500 \ W/m^2 \cdot K \times 0.003 \ m}{5 \ W/m \cdot K} = 0.030 \qquad b = \frac{2}{(k_c/k_s)+1} = \frac{2}{50/5+1} = 0.182$$

$$\frac{\dot{q}\Delta x \Delta y}{k_c} = 1.800 \qquad g = \frac{1}{k_c/k_s+1} = 0.0910$$

find the nodal equations:

Node 1 $$k_s \Delta x \frac{T_6 - T_1}{\Delta y} + k_s \Delta y \frac{T_2 - T_1}{\Delta x} + h\Delta x (T_\infty - T_1) = 0$$

Continued …..

$$-\left(2+\frac{h\Delta x}{k_s}\right)T_1 + T_2 + T_6 = -\frac{h\Delta x}{k_s}T_\infty \qquad -2.30T_1 + T_2 + T_6 = -6.00 \qquad (1)$$

Node 2 $\quad T_1 - 3.3T_2 + T_3 + T_7 = -6.00$

(2)

Node 3

$$k_s\Delta y\frac{T_2 - T_3}{\Delta x} + \frac{T_4 - T_3}{(\Delta x/2)/k_c\Delta y + (\Delta x/2)/k_s\Delta y} + k_s\Delta x\frac{T_8 - T_3}{\Delta y} + h\Delta x(T_\infty - T_3) = 0$$

$$T_2 - (2 + a + (h\Delta x/k_s)T_3) + aT_4 + T_8 = -(h\Delta x/k)T_\infty$$

$$T_2 - 4.12T_3 + 1.82T_4 + T_8 = -6.00 \qquad (3)$$

Node 4

$$\frac{T_3 - T_4}{(\Delta x/2)/k_s\Delta y + (\Delta x/2)/k_c\Delta y} + k_c\Delta y\frac{T_5 - T_4}{\Delta x} + \frac{T_9 - T_4}{(\Delta y/2)/k_s\Delta x + (\Delta y/2)k_c\Delta x}$$

$$+\dot{q}(\Delta x\Delta y) + h\Delta x(T_\infty - T_4) = 0$$

$$bT_3 - (1 + 2b + [h\Delta x/k_c])T_4 + T_5 + bT_9 = -(h\Delta x/k_c)T_\infty - \dot{q}\Delta x\Delta y/k_c$$

$$0.182T_3 - 1.39T_4 + T_5 + 0.182T_9 = -2.40 \qquad (4)$$

Node 5

$$k_c\Delta y\frac{T_4 - T_5}{\Delta x} + \frac{T_{10} - T_5}{(\Delta y/2)/k_s(\Delta x/2) + (\Delta y/2)/k_c(\Delta x/2)} + h(\Delta x/2)(T_\infty - T_5) + \dot{q}\Delta y(\Delta x/2) = 0$$

$$2T_4 - 2.21T_5 + 0.182T_{10} = -2.40 \qquad (5)$$

Nodes 6 and 11

$$k_s\Delta x(T_1 - T_6)/\Delta y + k_s\Delta y(T_7 - T_6)/\Delta x + k_s\Delta x(T_{11} - T_6)/\Delta y = 0$$

$$T_1 - 3T_6 + T_7 + T_{11} = 0 \qquad T_6 - 3T_{11} + T_{12} + T_{16} = 0 \qquad (6,11)$$

Nodes 7, 8, 12, 13, 14 Treat as interior points,

$$T_2 + T_6 - 4T_7 + T_8 + T_{12} = 0 \qquad T_3 + T_7 - 4T_8 + T_9 + T_{13} = 0 \qquad (7,8)$$

$$T_7 + T_{11} - 4T_{12} + T_{13} + T_{17} = 0 \qquad T_8 + T_{12} - 4T_{13} + T_{14} + T_{18} = 0 \qquad (12,13)$$

$$T_9 + T_{13} - 4T_{14} + T_{15} + T_{19} = 0 \qquad (14)$$

Node 9

$$k_s\Delta y\frac{T_8 - T_9}{\Delta x} + \frac{T_4 - T_9}{(\Delta y/2)/k_c\Delta x + (\Delta y/2)/k_s\Delta x} + k_s\Delta y\frac{T_{10} - T_9}{\Delta x} + k_s\Delta x\frac{T_{14} - T_9}{\Delta y} = 0$$

$$1.82T_4 + T_8 - 4.82T_9 + T_{10} + T_{14} = 0 \qquad (9)$$

Node 10 Using the result of Node 9 and considering symmetry,

$$1.82T_5 + 2T_9 - 4.82T_{10} + T_{15} = 0 \qquad (10)$$

Node 15 Interior point considering symmetry $\quad T_{10} + 2T_{14} - 4T_{15} + T_{20} = 0 \qquad (15)$

Node 16 By inspection, $\quad T_{11} - 2T_{16} + T_{17} = 0 \qquad (16)$

Continued …..

Nodes 17, 18, 19, 20

$$T_{12} + T_{16} - 3T_{17} + T_{18} = 0 \qquad T_{13} + T_{17} - 3T_{18} + T_{19} = 0 \qquad (17,18)$$

$$T_{14} + T_{18} - 3T_{19} + T_{20} = 0 \qquad T_{15} + 2T_{19} - 3T_{20} = 0 \qquad (19,20)$$

Using the matrix inversion method, the above system of finite-difference equations is written in matrix notation, Eq. 4.52, [A][T] = [C] where

$$[A] = \begin{matrix}
-2.3 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & -3.3 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & -4.12 & 1.82 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & .182 & -1.39 & 1 & 0 & 0 & 0 & .182 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 2 & -2.21 & 0 & 0 & 0 & 0 & .182 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & -3 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 1 & -4 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & -4 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1.82 & 0 & 0 & 0 & 1 & -4.82 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1.82 & 0 & 0 & 0 & 2 & -4.82 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -3 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & -4 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & -4 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & -4 & 1 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 2 & -4 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -2 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & -3 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & -3 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & -3 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 2 & -3
\end{matrix} \qquad [C] = \begin{matrix} -6 \\ -6 \\ -6 \\ -2.4 \\ -2.4 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{matrix}$$

and the temperature distribution (°C), in geometrical representation, is

34.46	36.13	40.41	45.88	46.23
37.13	38.37	40.85	43.80	44.51
38.56	39.38	40.81	42.72	42.78
39.16	39.77	40.76	41.70	42.06

The maximum temperature is T_5 = 46.23°C which is indeed less than 85°C. <

COMMENTS: (1) The convection process for the energy balances of Nodes 1 through 5 were simplified by assuming the node temperature is also that of the surface. Considering Node 2, the energy balance processes for q_a, q_b and q_c are identical (see Eq. (2)); however,

$$q_{conv} = \frac{T_\infty - T_2}{1/h + \Delta y/2k} \approx h\left(T_\infty - T_2\right)$$

where $h\Delta y/2k = 5\ \text{W/m}^2\cdot\text{K} \times 0.003\ \text{m}/2 \times 50\ \text{W/m}\cdot\text{K} = 1.5 \times 10^{-4} \ll 1$. Hence, for this situation, the simplification is justified.

PROBLEM 4.77

KNOWN: Electronic device cooled by conduction to a heat sink.

FIND: (a) Beginning with a symmetrical element, find the thermal resistance per unit depth between the device and lower surface of the sink, $R'_{t,d-s}$ (m·K/W) using the flux plot method; compare result with thermal resistance based upon assumption of one-dimensional conduction in rectangular domains of (i) width w_d and length L and (ii) width w_s and length L; (b) Using a coarse (5x5) nodal network, determine $R'_{t,d-s}$; (c) Using nodal networks with finer grid spacings, determine the effect of grid size on the precision of the thermal resistance calculation; (d) Using a fine nodal network, determine the effect of device width on $R'_{t,d-s}$ with w_d/w_s = 0.175, 0.275, 0.375 and 0.475 keeping w_s and L fixed.

SCHEMATIC:

w_s = 48 mm
w_d = 18 mm
Device, T_d = 85 °C
L = 24 mm
Sink material
k = 300 W/m·K
Cool surface, T_s = 25 °C

ASSUMPTIONS: (1) Steady-state, two-dimensional conduction, (2) Constant properties, and (3) No internal generation, (4) Top surface not covered by device is insulated.

ANALYSIS: (a) Begin the flux plot for the symmetrical element noting that the temperature drop along the left-hand symmetry line will be almost linear. Choosing to sketch five isotherms and drawing the adiabats, find

M, T_d, Symmetry line, 1, 2, 0.75 x
N, 1, 2, 3, 4, 5
T_s
Symmetry line

$$N = 5 \qquad M = 2.75$$

so that the shape factor for the device to the sink considering two symmetrical elements per unit depth is

$$S' = 2S'_o = 2\frac{M}{N} = 1.10$$

and the thermal resistance per unit depth is

$$R'_{t,d-s,fp} = 1/kS' = 1/300\,W/m\cdot K \times 1.10 = 3.03\times 10^{-3}\,m\cdot K/W$$ <

The thermal resistances for the two rectangular domains are represented schematically below.

w_d
L
w_s w_s
(i) (ii)

Continued...

$$R'_{t,d-s,i} = \frac{L}{kw_d} = \frac{0.024\,m}{300\,W/m\cdot K\times 0.018\,m} = 4.44\times 10^{-3}\ m\cdot K/W$$ <

$$R'_{t,d-s,ii} = \frac{L}{kw_s} = \frac{0.024\,m}{300\,W/m\cdot K\times 0.048\,m} = 1.67\times 10^{-3}\ m\cdot K/W$$ <

We expect the flux plot result to be bounded by the results for the rectangular domains. The *spreading* effect can be seen by comparing $R'_{f,d-s,fp}$ with $R'_{t,d-s,i}$.

(b) The coarse 5x5 nodal network is shown in the sketch including the nodes adjacent to the symmetry lines and the adiabatic surface. As such, all the finite-difference equations are interior nodes and can be written by inspection directly onto the IHT workspace. Alternatively, one could use the *IHT Finite-Difference Equations Tool*. The temperature distribution (°C) is tabulated in the same arrangement as the nodal network.

85.00	85.00	62.31	53.26	50.73
65.76	63.85	55.49	50.00	48.20
50.32	49.17	45.80	43.06	42.07
37.18	36.70	35.47	34.37	33.95
25.00	25.00	25.00	25.00	25.00

The thermal resistance between the device and sink per unit depth is

$$R'_{t,s-d} = \frac{T_d - T_s}{2q'_{tot}}$$

Performing an energy balance on the device nodes, find

$$q'_{tot} = q'_a + q'_b + q'_c$$

$$q'_{tot} = k(\Delta y/2)\frac{T_d - T_1}{\Delta x} + k\Delta x\frac{T_d - T_5}{\Delta y} + k(\Delta x/2)\frac{T_d - T_4}{\Delta y}$$

$$q'_{tot} = 300\,W/m\cdot K\left[(85-62.31)/2 + (85-63.85) + (85-65.76)/2\right]K = 1.263\times 10^4\ W/m$$

$$R'_{t,s-d} = \frac{(85-25)K}{2\times 1.263\times 10^4\ W/m} = 2.38\times 10^{-3} m\cdot K/W$$ <

(c) The effect of grid size on the precision of the thermal resistance estimate should be tested by systematically reducing the nodal spacing Δx and Δy. This is a considerable amount of work even with IHT since the equations need to be individually entered. A more generalized, powerful code would be

Continued...

PROBLEM 4.77 (Cont.)

required which allows for automatically selecting the grid size. Using FEHT, a finite-element package, with eight elements across the device, representing a much finer mesh, we found

$$R'_{t,s-d} = 3.64 \times 10^{-3}\ \text{m} \cdot \text{K/W}$$

(d) Using the same tool, with the finest mesh, the thermal resistance was found as a function of w_d/w_s with fixed w_s and L.

As expected, as w_d increases, $R'_{t,d-s}$ decreases, and eventually will approach the value for the rectangular domain (ii). The spreading effect is shown for the base case, $w_d/w_s = 0.375$, where the thermal resistance of the sink is less than that for the rectangular domain (i).

COMMENTS: It is useful to compare the results for estimating the thermal resistance in terms of precision requirements and level of effort,

	$R'_{t,d-s} \times 10^3$ (m·K/W)
Rectangular domain (i)	4.44
Flux plot	3.03
Rectangular domain (ii)	1.67
FDE, 5x5 network	2.38
FEA, fine mesh	3.64

PROBLEM 4.78

KNOWN: Nodal network and boundary conditions for a water-cooled cold plate.

FIND: (a) Steady-state temperature distribution for prescribed conditions, (b) Means by which operation may be extended to larger heat fluxes.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Two-dimensional conduction, (3) Constant properties.

ANALYSIS: Finite-difference equations must be obtained for each of the 28 nodes. Applying the energy balance method to regions 1 and 5, which are similar, it follows that

Node 1: $\quad (\Delta y/\Delta x)T_2 + (\Delta x/\Delta y)T_6 - [(\Delta y/\Delta x) + (\Delta x/\Delta y)]T_1 = 0$

Node 5: $\quad (\Delta y/\Delta x)T_4 + (\Delta x/\Delta y)T_{10} - [(\Delta y/\Delta x) + (\Delta x/\Delta y)]T_5 = 0$

Nodal regions 2, 3 and 4 are similar, and the energy balance method yields a finite-difference equation of the form

Nodes 2,3,4:

$$(\Delta y/\Delta x)(T_{m-1,n} + T_{m+1,n}) + 2(\Delta x/\Delta y)T_{m,n-1} - 2[(\Delta y/\Delta x) + (\Delta x/\Delta y)]T_{m,n} = 0$$

Energy balances applied to the remaining combinations of similar nodes yield the following finite-difference equations.

Continued...

PROBLEM 4.78 (Cont.)

Nodes 6, 14: $(\Delta x/\Delta y)T_1 + (\Delta y/\Delta x)T_7 - [(\Delta x/\Delta y) + (\Delta y/\Delta x) + (h\Delta x/k)]T_6 = -(h\Delta x/k)T_\infty$

$$(\Delta x/\Delta y)T_{19} + (\Delta y/\Delta x)T_{15} - [(\Delta x/\Delta y) + (\Delta y/\Delta x) + (h\Delta x/k)]T_{14} = -(h\Delta x/k)T_\infty$$

Nodes 7, 15: $(\Delta y/\Delta x)(T_6 + T_8) + 2(\Delta x/\Delta y)T_2 - 2[(\Delta y/\Delta x) + (\Delta x/\Delta y) + (h\Delta x/k)]T_7 = -(2h\Delta x/k)T_\infty$

$$(\Delta y/\Delta x)(T_{14} + T_{16}) + 2(\Delta x/\Delta y)T_{20} - 2[(\Delta y/\Delta x) + (\Delta x/\Delta y) + (h\Delta x/k)]T_{15} = -(2h\Delta x/k)T_\infty$$

Nodes 8, 16: $(\Delta y/\Delta x)T_7 + 2(\Delta y/\Delta x)T_9 + (\Delta x/\Delta y)T_{11} + 2(\Delta x/\Delta y)T_3 - [3(\Delta y/\Delta x) + 3(\Delta x/\Delta y)$

$$+ (h/k)(\Delta x + \Delta y)]T_8 = -(h/k)(\Delta x + \Delta y)T_\infty$$

$$(\Delta y/\Delta x)T_{15} + 2(\Delta y/\Delta x)T_{17} + (\Delta x/\Delta y)T_{11} + 2(\Delta x/\Delta y)T_{21} - [3(\Delta y/\Delta x) + 3(\Delta x/\Delta y)$$

$$+ (h/k)(\Delta x + \Delta y)]T_{16} = -(h/k)(\Delta x + \Delta y)T_\infty$$

Node 11: $(\Delta x/\Delta y)T_8 + (\Delta x/\Delta y)T_{16} + 2(\Delta y/\Delta x)T_{12} - 2[(\Delta x/\Delta y) + (\Delta y/\Delta x) + (h\Delta y/k)]T_{11} = -(2h\Delta y/k)T_\infty$

Nodes 9, 12, 17, 20, 21, 22:

$$(\Delta y/\Delta x)T_{m-1,n} + (\Delta y/\Delta x)T_{m+1,n} + (\Delta x/\Delta y)T_{m,n+1} + (\Delta x/\Delta y)T_{m,n-1} - 2[(\Delta x/\Delta y) + (\Delta y/\Delta x)]T_{m,n} = 0$$

Nodes 10, 13, 18, 23:

$$(\Delta x/\Delta y)T_{n+1,m} + (\Delta x/\Delta y)T_{n-1,m} + 2(\Delta y/\Delta x)T_{m-1,n} - 2[(\Delta x/\Delta y) + (\Delta y/\Delta x)]T_{m,n} = 0$$

Node 19: $(\Delta x/\Delta y)T_{14} + (\Delta x/\Delta y)T_{24} + 2(\Delta y/\Delta x)T_{20} - 2[(\Delta x/\Delta y) + (\Delta y/\Delta x)]T_{19} = 0$

Nodes 24, 28: $(\Delta x/\Delta y)T_{19} + (\Delta y/\Delta x)T_{25} - [(\Delta x/\Delta y) + (\Delta y/\Delta x)]T_{24} = -(q_o''\Delta x/k)$

$$(\Delta x/\Delta y)T_{23} + (\Delta y/\Delta x)T_{27} - [(\Delta x/\Delta y) + (\Delta y/\Delta x)]T_{28} = -(q_o''\Delta x/k)$$

Nodes 25, 26, 27:

$$(\Delta y/\Delta x)T_{m-1,n} + (\Delta y/\Delta x)T_{m+1,n} + 2(\Delta x/\Delta y)T_{m,n+1} - 2[(\Delta x/\Delta y) + (\Delta y/\Delta x)]T_{m,n} = -(2q_o''\Delta x/k)$$

Evaluating the coefficients and solving the equations simultaneously, the steady-state temperature distribution (°C), tabulated according to the node locations, is:

23.77	23.91	24.27	24.61	24.74
23.41	23.62	24.31	24.89	25.07
		25.70	26.18	26.33
28.90	28.76	28.26	28.32	28.35
30.72	30.67	30.57	30.53	30.52
32.77	32.74	32.69	32.66	32.65

Alternatively, the foregoing results may readily be obtained by accessing the IHT *Tools* pat and using the *2-D, SS, Finite-Difference Equations* options (model equations are appended). Maximum and minimum cold plate temperatures are at the bottom (T_{24}) and top center (T_1) locations respectively.

(b) For the prescribed conditions, the maximum allowable temperature (T_{24} = 40°C) is reached when q_o'' = 1.407×10^5 W/m² (14.07 W/cm²). Options for extending this limit could include use of a copper cold plate (k ≈ 400 W/m·K) and/or increasing the convection coefficient associated with the coolant. With k = 400 W/m·K, a value of q_o'' = 17.37 W/cm² may be maintained. With k = 400 W/m·K and h = 10,000 W/m²·K (a practical upper limit), q_o'' = 28.65 W/cm². Additional, albeit small, improvements may be realized by relocating the coolant channels closer to the base of the cold plate.

COMMENTS: The accuracy of the solution may be confirmed by verifying that the results satisfy the overall energy balance

$$q_o''(4\Delta x) = h[(\Delta x/2)(T_6 - T_\infty) + \Delta x(T_7 - T_\infty) + (\Delta x + \Delta y)(T_8 - T_\infty)/2$$

$$+ \Delta y(T_{11} - T_\infty) + (\Delta x + \Delta y)(T_{16} - T_\infty)/2 + \Delta x(T_{15} - T_\infty) + (\Delta x/2)(T_{14} - T_\infty)].$$

PROBLEM 4.79

KNOWN: Heat sink for cooling computer chips fabricated from copper with microchannels passing fluid with prescribed temperature and convection coefficient.

FIND: (a) Using a square nodal network with 100 μm spatial increment, determine the temperature distribution and the heat rate to the coolant per unit channel length for maximum allowable chip temperature $T_{c,max}$ = 75°C; estimate the thermal resistance betweeen the chip surface and the fluid, $R'_{t,c-f}$ (m·K/W); maximum allowable heat dissipation for a chip that measures 10 x 10 mm on a side; (b) The effect of grid spacing by considering spatial increments of 50 and 25 μm; and (c) Consistent with the requirement that a + b = 400 μm, explore altering the sink dimensions to decrease the thermal resistance.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state, two-dimensional conduction, (2) Constant properties, and (3) Convection coefficient is uniform over the microchannel surface and independent of the channel dimensions and shape.

ANALYSIS: (a) The square nodal network with Δx = Δy = 100 μm is shown below. Considering symmetry, the nodes 1, 2, 3, 4, 7, and 9 can be treated as interior nodes and their finite-difference equations representing nodal energy balances can be written by inspection. Using the, *IHT Finite-Difference Equations Tool*, appropriate FDEs for the nodes experiencing surface convection can be obtained. The IHT code including results is included in the Comments. Having the temperature distribution, the heat rate to the coolant per unit channel length for two symmetrical elements can be obtained by applying Newton's law of cooling to the surface nodes,

$$q'_{cv} = 2\left[h(\Delta y/2 + \Delta x/2)(T_5 - T_\infty) + h(\Delta x/2)(T_6 - T_\infty) + h(\Delta y)(T_8 - T_\infty)h(\Delta y/2)(T_{10} - T_\infty)\right]$$

$$q'_{cv} = 2\times 30,000\,\mathrm{W/m^2\cdot K}\times 100\times 10^{-6}\,\mathrm{m}\left[(74.02-25)+(74.09-25)/2+(73.60-25)+(73.37-25)/2\right]\mathrm{K}$$

$$q'_{cv} = 878\,\mathrm{W/m} \qquad <$$

The thermal resistance between the chip and fluid per unit length for each microchannel is

$$R'_{t,c-f} = \frac{T_c - T_\infty}{q'_{cv}} = \frac{(75-25)^\circ\mathrm{C}}{878\,\mathrm{W/m}} = 5.69\times 10^{-2}\,\mathrm{m\cdot K/W} \qquad <$$

The maximum allowable heat dissipation for a 10 mm × 10 mm chip is

$$P_{chip,max} = q''_c \times A_{chip} = 2.20\times 10^6\,\mathrm{W/m^2}\times(0.01\times 0.01)\,\mathrm{m^2} = 220\,\mathrm{W} \qquad <$$

where A_{chip} = 10 mm × 10 mm and the heat flux on the chip surface ($w_f + w_s$) is

$$q''_c = q'_{cv}/(w_f + w_s) = 878\,\mathrm{W/m}/(200+200)\times 10^{-6}\,\mathrm{m} = 2.20\times 10^6\,\mathrm{W/m^2}$$

Continued...

(b) To investigate the effect of grid spacing, the analysis was repreated with a spatial increment of 50 μm (32 nodes as shown above) with the following results

$$q'_{cv} = 881\,W/m \qquad R'_{t,c-f} = 5.67 \times 10^{-2}\,m \cdot K/W$$

<

Using a finite-element package with a mesh around 25 μm, we found $R'_{t,c-f} = 5.70 \times 10^{-2}\,m \cdot K/W$ which suggests the grid spacing effect is not very significant.

(c) Requring that the overall dimensions of the symmetrical element remain unchanged, we explored what effect changes in the microchannel cross-section would have on the overall thermal resistance, $R'_{t,c-f}$. It is important to recognize that the sink conduction path represents the dominant resistance, since for the convection process

$$R'_{t,cv} = 1/A'_s = 1\Big/\left(30,000\,W/m^2 \cdot K \times 600 \times 10^{-6}\,m\right) = 5.55 \times 10^{-2}\,m \cdot K/W$$

where $A'_s = (w_f + 2b) = 600$ μm.

Using a finite-element package, the thermal resistances per unit length for three additional channel cross-sections were determined and results summarized below.

Case	Microchannel (μm) Height	Half-width	$R'_{t,c-s} \times 10^2$ (m·K/W)
A	200	100	5.70
B	133	150	6.12
C	300	100	4.29
D	250	150	4.25

Continued...

COMMENTS: (1) The IHT Workspace for the 5x5 coarse node analysis with results follows.

```
// Finite-difference equations - energy balances
// First row - treating as interior nodes considering symmetry
T1 = 0.25 * ( Tc + T2 + T4 + T2 )
T2 = 0.25 * ( Tc + T3 + T5 + T1 )
T3 = 0.25 * ( Tc + T2 + T6 + T2 )

/* Second row - Node 4 treat as interior node; for others, use Tools: Finite-Difference Equations,
Two-Dimensional, Steady-State; be sure to delimit replicated q''a = 0 equations.  */
T4 = 0.25 * ( T1 + T5+ T7 + T5 )
/* Node 5: internal corner node, e-s orientation; e, w, n, s labeled 6, 4, 2, 8. */
0.0 = fd_2d_ic_es(T5,T6,T4,T2,T8,k,qdot,deltax,deltay,Tinf,h,q''a)
q''a = 0                        // Applied heat flux, W/m^2; zero flux shown
/* Node 6: plane surface node, s-orientation; e, w, n labeled 5, 5, 3 . */
0.0 = fd_2d_psur_s(T6,T5,T5,T3,k,qdot,deltax,deltay,Tinf,h,q''a)
//q''a = 0                      // Applied heat flux, W/m^2; zero flux shown

/* Third row - Node 7 treat as interior node; for others, use Tools: Finite-Difference Equations,
Two-Dimensional, Steady-State; be sure to delimit replicated q''a = 0 equations.  */
T7 = 0.25 * (T4 + T8 + T9 + T8)
/* Node 8: plane surface node, e-orientation; w, n, s labeled 7, 5, 10. */
0.0 = fd_2d_psur_e(T8,T7,T5,T10,k,qdot,deltax,deltay,Tinf,h,q''a)
//q''a = 0                      // Applied heat flux, W/m^2; zero flux shown

/* Fourth row - Node 9 treat as interior node; for others, use Tools: Finite-Difference Equations,
Two-Dimensional, Steady-State; be sure to delimit replicated q''a = 0 equations.  */
T9 = 0.25 * (T7 + T10 +T7 + T10)
/* Node 10: plane surface node, e-orientation; w, n, s labeled 9, 8, 8. */
0.0 = fd_2d_psur_e(T10,T9,T8,T8,k,qdot,deltax,deltay,Tinf,h,q''a)
//q''a = 0                      // Applied heat flux, W/m^2; zero flux shown

// Assigned variables
// For the FDE functions,
qdot = 0                        // Volumetric generation, W/m^3
deltax = deltay                 // Spatial increments
deltay = 100e-6                 // Spatial increment, m
Tinf = 25                       // Microchannel fluid temperature, C
h = 30000                            // Convection coefficient, W/m^2.K
// Sink and chip parameters
k = 400                         // Sink thermal conductivity, W/m.K
Tc = 75                         // Maximum chip operating temperature, C
wf = 200e-6                     // Channel width, m
ws = 200e-6                     // Sink width, m

/* Heat rate per unit length, for two symmetrical elements about one microchannel, */
q'cv= 2 * (q'5 + q'6 + q'8 + q'10)
q'5 = h* (deltax / 2 + deltay / 2) * (T5 - Tinf)
q'6 = h * deltax / 2 * (T6 - Tinf)
q'8 = h * deltax  * (T8 - Tinf)
q'10 = h * deltax / 2 * (T10 - Tinf)

/* Thermal resistance between chip and fluid, per unit channel length, */
R'tcf = (Tc - Tinf) / q'cv      // Thermal resistance, m.K/W

// Total power for a chip of 10mm x 10mm, Pchip (W),
q''c = q'cv / (wf + ws)         // Heat flux on chip surface, W/m^2
Pchip = Achip * q''c            // Power, W
Achip = 0.01 * 0.01             // Chip area, m^2

/* Data Browser results: chip power, thermal resistance, heat rates and temperature distribution
Pchip    R'tcf     q''c      q'cv
219.5    0.05694   2.195E6   878.1

T1       T2        T3        T4        T5        T6        T7        T8        T9        T10
74.53    74.52     74.53     74.07     74.02     74.09     73.7      73.6      73.53     73.37 */
```

PROBLEM 4.80

KNOWN: Longitudinal rib (k = 10 W/m·K) with rectangular cross-section with length L= 8 mm and width w = 4 mm. Base temperature T_b and convection conditions, T_∞ and h, are prescribed.

FIND: (a) Temperature distribution and fin base heat rate using a finite-difference method with $\Delta x = \Delta y$ = 2 mm for a total of $5 \times 3 = 15$ nodal points and regions; compare results with those obtained assuming one-dimensional heat transfer in rib; and (b) The effect of grid spacing by reducing nodal spacing to $\Delta x = \Delta y$ = 1 mm for a total of $9 \times 3 = 27$ nodal points and regions considering symmetry of the centerline; and (c) A criterion for which the one-dimensional approximation is reasonable; compare the heat rate for the range $1.5 \le L/w \le 10$, keeping L constant, as predicted by the two-dimensional, finite-difference method and the one-dimensional fin analysis.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, and (3) Convection coefficient uniform over rib surfaces, including tip.

ANALYSIS: (a) The rib is represented by a 5×3 nodal grid as shown above where the symmetry plane is an adiabatic surface. The *IHT Tool, Finite-Difference Equations*, for *Two-Dimensional, Steady-State* conditions is used to formulate the nodal equations (see Comment 2 below) which yields the following nodal temperatures (° C)

45	39.3	35.7	33.5	32.2
45	40.0	36.4	34.0	32.6
45	39.3	35.7	33.5	32.2

Note that the fin tip temperature is

$$T_{tip} = T_{12} = 32.6^\circ C$$ <

The fin heat rate per unit width normal to the page, q'_{fin}, can be determined from energy balances on the three base nodes as shown in the schematic below.

$$q'_{fin} = q'_a + q'_b + q'_c + q'_d + q'_e$$
$$q'_a = h(\Delta x/2)(T_b - T_\infty)$$
$$q'_b = k(\Delta y/2)(T_b - T_1)/\Delta x$$
$$q'_c = k(\Delta y)(T_b - T_5)/\Delta x$$
$$q'_d = k(\Delta y/2)(T_b - T_9)\Delta x$$
$$q'_3 = h(\Delta x/2)(T_b - T_\infty)$$

Continued...

PROBLEM 4.80 (Cont.)

Substituting numerical values, find

$$q'_{fin} = (12.0 + 28.4 + 50.0 + 28.4 + 12.0)\,W/m = 130.8\,W/m \quad <$$

Using the *IHT Model*, *Extended Surfaces*, *Heat Rate* and *Temperature Distributions* for *Rectangular*, *Straight Fins*, with convection tip condition, the one-dimensional fin analysis yields

$$q'_f = 131\,W/m \qquad T_{tip} = 32.2^\circ C \quad <$$

(b) With $\Delta x = L/8 = 1$ mm and $\Delta x = 1$ mm, for a total of $9 \times 3 = 27$ nodal points and regions, the grid appears as shown below. Note the rib centerline is a plane of symmetry.

Using the same IHT FDE Tool as above with an appropriate expression for the fin heat rate, Eq. (1), the fin heat rate and tip temperature were determined.

	1-D analysis	2-D analysis (nodes) (5×3)	(9×3)	
T_{tip} (°C)	32.2	32.6	32.6	<
q'_{fin} (W/m)	131	131	129	<

(c) To determine when the one-dimensional approximation is reasonable, consider a rib of constant length, L = 8 mm, and vary the thickness w for the range $1.5 \le L/w \le 10$. Using the above IHT model for the 27 node grid, the fin heat rates for 1-D, q'_{1d}, and 2-D, q'_{2d}, analysis were determined as a function of w with the error in the approximation evaluated as

$$\text{Error}(\%) = (q'_{2d} - q'_{1d}) \times 100 / q'_{1d}$$

Note that for small L/w, a thick rib, the 1-D approximation is poor. For large L/w, a thin rib which approximates a fin, we would expect the 1-D approximation to become increasingly more satisfactory. The discrepancy at large L/w must be due to discretization error; that is, the grid is too coarse to accurately represent the slender rib.

PROBLEM 4.81

KNOWN: Bottom half of an I-beam exposed to hot furnace gases.

FIND: (a) The heat transfer rate per unit length into the beam using a coarse nodal network (5×4) considering the temperature distribution across the web is uniform and (b) Assess the reasonableness of the uniform web-flange interface temperature assumption.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state, two-dimensional conduction, and (2) Constant properties.

ANALYSIS: (a) The symmetrical section of the I-beam is shown in the Schematic above indicating the web-flange interface temperature is uniform, $T_w = 100°C$. The nodal arrangement to represent this system is shown below. The nodes on the line of symmetry have been shown for convenience in deriving the nodal finite-difference equations.

Using the *IHT Finite-Difference Equations Tool*, the set of nodal equations can be readily formulated. The temperature distribution (°C) is tabulated in the same arrangement as the nodal network.

100.00	100.00	215.8	262.9	284.8
166.6	177.1	222.4	255.0	272.0
211.7	219.5	241.9	262.7	274.4
241.4	247.2	262.9	279.3	292.9

The heat rate to the beam can be determined from energy balances about the web-flange interface nodes as shown in the sketch below.

Continued...

$$q'_w = q'_a + q'_b + q'_c$$

$$q'_w = k(\Delta y/2)\frac{T_1 - T_w}{\Delta x} + k(\Delta x)\frac{T_5 - T_w}{\Delta y} + k(\Delta x/2)\frac{T_4 - T_w}{\Delta y}$$

$$q'_w = 10\,\text{W/m}\cdot\text{K}\left[(215.8-100)/2 + (177.1-100) + (166.6-100)/2\right]\text{K} = 1683\,\text{W/m}$$ <

(b) The schematic below poses the question concerning the reasonableness of the uniform temperature assumption at the web-flange interface. From the analysis above, note that T_1 = 215.8°C vs. T_w = 100°C indicating that this assumption is a poor one. This L-shaped section has strong two-dimensional behavior. To illustrate the effect, we performed an analysis with T_w = 100°C located nearly 2 × times further up the web than it is wide. For this situation, the temperature difference at the web-flange interface across the width of the web was nearly 40°C. The steel beam with its low thermal conductivity has substantial internal thermal resistance and given the L-shape, the uniform temperature assumption (T_w) across the web-flange interface is inappropriate.

PROBLEM 4.82

KNOWN: Plane composite wall with exposed surfaces maintained at fixed temperatures. Material A has temperature-dependent thermal conductivity.

FIND: Heat flux through the wall (a) assuming a uniform thermal conductivity in material A evaluated at the average temperature of the section, and considering the temperature-dependent thermal conductivity of material A using (b) a finite-difference method of solution in IHT with a space increment of 1 mm and (c) the finite-element method of FEHT.

SCHEMATIC:

$k_o = 4.4$ W/m-K
$\alpha = 0.008$ K^{-1}
$T_o = 300$ K
$k_a = k_o [1 + \alpha(T-T_o)]$
$k_b = 1$ W/m-K
T_{AB}
$T_1 = 600$ K
$T_2 = 300$ K
A B
x (mm) 20 25

q''_x → T_1 — R''_A — T_{AB} — R''_B — T_2

ASSUMPTIONS: (1) Steady-state, one-dimensional conduction, (2) No thermal contact resistance between the materials, and (3) No internal generation.

ANALYSIS: (a) From the thermal circuit in the above schematic, the heat flux is

$$q''_x = \frac{T_1 - T_2}{R''_A + R''_B} = \frac{T_{AB} - T_2}{R''_B} \qquad (1, 2)$$

and the thermal resistances of the two sections are

$$R''_A = L_A / k_A \qquad R''_B = L_B / k_B \qquad (3, 4)$$

The thermal conductivity of material A is evaluated at the average temperature of the section

$$k_A = k_o \left\{1 + \alpha\left[\left(T_1 + T_{AB}\right)/2 - T_o\right]\right\} \qquad (5)$$

Substituting numerical values and solving the system of equations simultaneously in IHT, find

$$T_{AB} = 563.2 \text{ K} \qquad q''_x = 52.64 \text{ kW/m}^2$$ <

(b) The nodal arrangement for the finite-difference method of solution is shown in the schematic below. FDEs must be written for the internal nodes (02 – 10, 12 – 15) and the A-B interface node (11) considering in section A, the temperature-dependent thermal conductivity.

01 02 03 08 09 10 11 12 15 16
$T_{01} = 600$ K $k_a(T)$ k_b $T_{16} = 300$ K
$\Delta x = 1$ mm
x

Interior Nodes, Section A (m = 02, 03 … 10)
Referring to the schematic below, the energy balance on node *m* is written in terms of the heat fluxes at the control surfaces using Fourier's law with the thermal conductivity based upon the average temperature of adjacent nodes. The heat fluxes into node *m* are

Continued …..

PROBLEM 4.82 (Cont.)

$$q''_c = k_a(m, m+1)\frac{T_{m+1} - T_m}{\Delta x} \quad (1)$$

$$q''_d = k_a(m-1, m)\frac{T_{m-1} - T_m}{\Delta x} \quad (2)$$

and the FDEs are obtained from the energy balance written as

$$q''_c + q''_d = 0 \quad (3)$$

$$k_a(m, m+1)\frac{T_{m+1} - T_m}{\Delta x} + k_a(m-1, m)\frac{T_{m-1} - T_m}{\Delta x} = 0 \quad (4)$$

where the thermal conductivities averaged over the path between the nodes are expressed as

$$k_a(m-1, m) = k_o\left\{1 + \alpha\left[\left(T_{m-1} + T_m\right)/2 - T_o\right]\right\} \quad (5)$$

$$k_a(m, m+1) = k_o\left\{1 + \alpha\left[\left(T_m + T_{m+1}\right)/2 - T_o\right]\right\} \quad (6)$$

T_{m-1} T_m T_{m+1} q''_d q''_c $k_a = (m-1,m)$ $k_a = (m,m+1)$

Interior nodes, Section A

T_{10} T_{11} T_{12} q''_d q''_c $k_a = (T_{10},T_{11})$ k_b

A-B interface node

A-B Interface Node 11

Referring to the above schematic, the energy balance on the interface node, $q''_c + q''_d = 0$, has the form

$$k_b\frac{T_{12} - T_{11}}{\Delta x} + k_a(10,11)\frac{T_{10} - T_{11}}{\Delta x} = 0 \quad (7)$$

where the thermal conductivity in the section A path is

$$k(10,11) = k_o\left\{1 + \left[\left(T_{10} + T_{11}\right)/2 - T_o\right]\right\} \quad (8)$$

Interior Nodes, Section B (n = 12 …15)

Since the thermal conductivity in Section B is uniform, the FDEs have the form

$$T_n = \left(T_{n-1} + T_{n+1}\right)/2 \quad (9)$$

And the heat flux in the x-direction is

$$q''_x = k_b\frac{T_n - T_{n+1}}{\Delta x} \quad (10)$$

Finite-Difference Method of Solution

The foregoing FDE equations for section A nodes (m = 02 to 10), the AB interface node and their respective expressions for the thermal conductivity, k (m, m +1), and for section B nodes are entered into the IHT workspace and solved for the temperature distribution. The heat flux can be evaluated using Eq. (2) or (10). A portion of the IHT code is contained in the Comments, and the results of the analysis are tabulated below.

$$T_{11} = T_{AB} = 563.2\ \text{K} \qquad q''_x = 52.64\ \text{kW/m}^2$$ <

Continued …..

(c) The finite-element method of FEHT can be used readily to obtain the heat flux considering the temperature-dependent thermal conductivity of section A. Draw the composite wall outline with properly scaled section thicknesses in the x-direction with an arbitrary y-direction dimension. In the *Specify* | *Materials Properties* box for the thermal conductivity, specify k_a as 4.4*[1 + 0.008*(T – 300)] having earlier selected *Set* | *Temperatures in K*. The results of the analysis are

$$T_{AB} = 563\ K \qquad q''_x = 5.26\ kW/m^2$$ <

COMMENTS: (1) The results from the three methods of analysis compare very well. Because the thermal conductivity in section A is linear, and moderately dependent on temperature, the simplest method of using an overall section average, part (a), is recommended. This same method is recommended when using tabular data for temperature-dependent properties.

(2) For the finite-difference method of solution, part (b), the heat flux was evaluated at several nodes within section A and in section B with identical results. This is a consequence of the technique for averaging k_a over the path between nodes in computing the heat flux into a node.

(3) To illustrate the use of IHT in solving the finite-difference method of solution, lines of code for representative nodes are shown below.

```
// FDEs – Section A
k01_02 * (T01-T02)/deltax + k02_03 * (T03-T02)/deltax = 0
k01_02 = ko * (1+ alpha * ((T01 + T02)/2 – To))
k02_03 = ko * (1 + alpha * ((T02 + T03)/2 – To))

k02_03 * (T02 – T03)/deltax + k03_04 * (T04 – T03)/deltax = 0
k03_04 = ko * (1 + alpha * ((T03 + T04)/2 – To))

// Interface, node 11
k11 * (T10 –T11)/deltax + kb * (T12 –T11)/deltax =0
k11 = ko * (1 + alpha * ((T10 + T11)/2 – To))

// Section B (using Tools/FDE/One-dimensional/Steady-state)
/* Node 12: interior node; */
0.0 = fd_1d_int(T12, T13, T11, kb, qdot, deltax)
```

(4) The solved models for Text Examples 4.3 and 4.4, plus the tutorial of the User’s Manual, provide background for developing skills in using FEHT.

PROBLEM 4.83

KNOWN: Upper surface of a platen heated by hot fluid through the flow channels is used to heat a process fluid.

FIND: (a) The maximum allowable spacing, W, between channel centerlines that will provide a uniform temperature requirement of 5°C on the upper surface of the platen, and (b) Heat rate per unit length from the flow channel for this condition.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state, two-dimensional conduction with constant properties, and (2) Lower surface of platen is adiabatic.

ANALYSIS: As shown in the schematic above for a symmetrical section of the platen-flow channel arrangement, the temperature uniformity requirement will be met when $T_1 - T_2 = 5°C$. The maximum temperature, T_1, will occur directly over the flow channel centerline, while the minimum surface temperature, T_2, will occur at the mid-span between channel centerlines.

We chose to use FEHT to obtain the temperature distribution and heat rate for guessed values of the channel centerline spacing, W. The following method of solution was used: (1) Make an initial guess value for W; try W = 100 mm, (2) Draw an outline of the symmetrical section, and assign properties and boundary conditions, (3) Make a copy of this file so that in your second trial, you can use the *Draw | Move Node* option to modify the section width, W/2, larger or smaller, (4) Draw element lines within the outline to create triangular elements, (5) Use the *Draw | Reduce Mesh* command to generate a suitably fine mesh, then solve for the temperature distribution, (6) Use the *View | Temperatures* command to determine the temperatures T_1 and T_2, (7) If, $T_1 - T_2 \approx 5°C$, use the *View | Heat Flows* command to find the heat rate, otherwise, change the width of the section outline and repeat the analysis. The results of our three trials are tabulated below.

Trial	W (mm)	T_1 (°C)	T_2 (°C)	$T_1 - T_2$ (°C)	q' (W/m)
1	100	108	98	10	--
2	60	119	118	1	--
3	80	113	108	5	1706

COMMENTS: (1) In addition to the tutorial example in the FEHT User's Manual, the solved models for Examples 4.3 and 4.4 of the Text are useful for developing skills in using this problem-solving tool.

(2) An alternative numerical method of solution would be to create a nodal network, generate the finite-difference equations and solve for the temperature distribution and the heat rate. The FDEs should allow for a non-square grid, $\Delta x \neq \Delta y$, so that different values for W/2 can be accommodated by changing the value of Δx. Even using the IHT tool for building FDEs (*Tools | Finite-Difference Equations | Steady-State*) this method of solution is very labor intensive because of the large number of nodes required for obtaining good estimates.

PROBLEM 4.84

KNOWN: Silicon chip mounted in a dielectric substrate. One surface of system is convectively cooled, while the remaining surfaces are well insulated. See Problem 4.77. Use the finite-element software *FEHT* as your analysis tool.

FIND: (a) The temperature distribution in the substrate-chip system; does the maximum temperature exceed 85°C?; (b) Volumetric heating rate that will result in a maximum temperature of 85°C; and (c) Effect of reducing thickness of substrate from 12 to 6 mm, keeping all other dimensions unchanged with $\dot{q} = 1\times10^7$ W/m^3; maximum temperature in the system for these conditions, and fraction of the power generated within the chip removed by convection directly from the chip surface.

SCHEMATIC:

$T_\infty = 20^\circ$C
$h = 500$ W/m^2-K
y
Chip
$k_c = 50$ W/m-K
$\dot{q} = 1\times10^7$ W/m^3
H = 12 mm
L/3 = 9 mm
H/4 = 3 mm
Substrate
$k_s = 5$ W/m-K
x
L = 27 mm

ASSUMPTIONS: (1) Steady-state conditions, (2) Two-dimensional conduction in system, and (3) Uniform convection coefficient over upper surface.

ANALYSIS: Using *FEHT*, the symmetrical section is represented in the workspace as two connected regions, chip and substrate. *Draw* first the chip outline; *Specify* the material and generation parameters. Now, *Draw* the outline of the substrate, connecting the nodes of the interfacing surfaces; *Specify* the material parameters for this region. Finally, *Assign* the *Boundary Conditions*: zero heat flux for the symmetry and insulated surfaces, and convection for the upper surface. Draw *Element Lines*, making the triangular elements near the chip and surface smaller than near the lower insulated boundary as shown in a copy of the *FEHT* screen on the next page. Use the *Draw*|*Reduce M*esh command and *Run* the model.

(a) Use the *View*|*Temperature* command to see the nodal temperatures through out the system. As expected, the hottest location is on the centerline of the chip at the bottom surface. At this location, the temperature is

$$T(0,\ 9\ \text{mm}) = 46.7^\circ\text{C}$$ <

(b) Run the model again, with different values of the generation rate until the temperature at this location is T(0, 9 mm) = 85°C, finding

$$\dot{q} = 2.43\times10^7\ \text{W/m}^3$$ <

Continued …..

PROBLEM 4.84 (Cont.)

(c) Returning to the model code with the conditions of part (a), reposition the nodes on the lower boundary, as well as the intermediate ones, to represent a substrate that is of 6-mm, rather than 12-mm thickness. Find the maximum temperature as

$$T(0,3\text{ mm}) = 47.5°\text{C}$$ <

Using the *View|Heat Flow* command, click on the adjacent line segments forming the chip surface exposed to the convection process. The heat rate per unit width (normal to the page) is

$$q'_{chip,cv} = 60.26\text{ W/m}$$

The total heat generated within the chip is

$$q'_{tot} = \dot{q}(L/6 \times H/4) = 1\times10^{7}\text{ W/m}^3 \times (0.0045\times0.003)\text{m}^2 = 135\text{ W/m}$$

so that the fraction of the power dissipated by the chip that is convected directly to the coolant stream is

$$F = q'_{chip,cv}/q'_{tot} = 60.26/135 = 45\%$$ <

COMMENTS: (1) Comparing the maximum temperatures for the system with the 12-mm and 6-mm thickness substrates, note that the effect of halving the substrate thickness is to raise the maximum temperature by less than 1°C. The thicker substrate does not provide significantly improved heat removal capability.

(2) Without running the code for part (b), estimate the magnitude of $\dot{q}$ that would make T(0, 9 mm) = 85°C. Did you get $\dot{q} = 2.43\times10^{7}$ W/m^3? Why?

PROBLEM 5.1

KNOWN: Electrical heater attached to backside of plate while front surface is exposed to convection process (T_∞,h); initially plate is at a uniform temperature of the ambient air and suddenly heater power is switched on providing a constant q''_o.

FIND: (a) Sketch temperature distribution, T(x,t), (b) Sketch the heat flux at the outer surface, $q''_x(L,t)$ as a function of time.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional conduction, (2) Constant properties, (3) Negligible heat loss from heater through insulation.

ANALYSIS: (a) The temperature distributions for four time conditions including the initial distribution, T(x,0), and the steady-state distribution, T(x,∞), are as shown above.

Note that the temperature gradient at x = 0, $-dT/dx)_{x=0}$, for t > 0 will be a constant since the flux, $q''_x(0)$, is a constant. Noting that $T_o = T(0,\infty)$, the steady-state temperature distribution will be linear such that

$$q''_o = k\frac{T_o - T(L,\infty)}{L} = h\left[T(L,\infty) - T_\infty\right].$$

(b) The heat flux at the front surface, x = L, is given by $q''_x(L,t) = -k(dT/dx)_{x=L}$. From the temperature distribution, we can construct the heat flux-time plot.

COMMENTS: At early times, the temperature and heat flux at x = L will not change from their initial values. Hence, we show a zero slope for $q''_x(L,t)$ at early times. Eventually, the value of $q''_x(L,t)$ will reach the steady-state value which is q''_o.

PROBLEM 5.2

KNOWN: Plane wall whose inner surface is insulated and outer surface is exposed to an airstream at T_∞. Initially, the wall is at a uniform temperature equal to that of the airstream. Suddenly, a radiant source is switched on applying a uniform flux, q''_o, to the outer surface.

FIND: (a) Sketch temperature distribution on T-x coordinates for initial, steady-state, and two intermediate times, (b) Sketch heat flux at the outer surface, $q''_x(L,t)$, as a function of time.

SCHEMATIC:

q''o, when t > 0
Air
T∞,h
x
L
T(x,t)
Steady state, T(x,∞)
At intermediate times, T(x,t)
Initial condition, T(x,0)=T∞
T∞
0
L
x

ASSUMPTIONS: (1) One-dimensional conduction, (2) Constant properties, (3) No internal generation, $\dot{E}_g = 0$, (4) Surface at x = 0 is perfectly insulated, (5) All incident radiant power is absorbed and negligible radiation exchange with surroundings.

ANALYSIS: (a) The temperature distributions are shown on the T-x coordinates and labeled accordingly. Note these special features: (1) Gradient at x = 0 is always zero, (2) gradient is more steep at early times and (3) for steady-state conditions, the radiant flux is equal to the convective heat flux (this follows from an energy balance on the CS at x = L),
$q''_o = q''_{conv} = h\left[T(L,\infty) - T_\infty\right]$.

q''o
q''conv
(a)
x = L
(b)
q''x(L,t)
+
0
-q''o
t

(b) The heat flux at the outer surface, $q''_x(L,t)$, as a function of time appears as shown above.

COMMENTS: The sketches must reflect the initial and boundary conditions:

$T(x,0) = T_\infty$	uniform initial temperature.
$-k\frac{\partial T}{\partial x}\Big\vert_{x=0} = 0$	insulated at x = 0.
$-k\frac{\partial T}{\partial x}\Big\vert_{x=L} = h\left[T(L,t) - T_\infty\right] - q''_o$	surface energy balance at x = L.

PROBLEM 5.3

KNOWN: Microwave and radiant heating conditions for a slab of beef.

FIND: Sketch temperature distributions at specific times during heating and cooling.

SCHEMATIC:

Slab of beef of thickness 2L with microwave (uniform internal) heating or radiant (uniform surface) heating.

+L
x
-L

ASSUMPTIONS: (1) One-dimensional conduction in x, (2) Uniform internal heat generation for microwave, (3) Uniform surface heating for radiant oven, (4) Heat loss from surface of meat to surroundings is negligible during the heating process, (5) Symmetry about midplane.

ANALYSIS:

Microwave
T(°C)
100
50
t_0
t_1
t_2
t_3
-L
0
x
+L

Radiant
T(°C)
100
50
t_0
t_1
t_2
t_3
-L
0
x
+L

COMMENTS: (1) With uniform generation and negligible surface heat loss, the temperature distribution remains nearly uniform during *microwave heating*. During the subsequent surface cooling, the maximum temperature is at the midplane.

(2) The interior of the meat is heated by conduction from the hotter surfaces during *radiant heating*, and the lowest temperature is at the midplane. The situation is reversed shortly after cooling begins, and the maximum temperature is at the midplane.

PROBLEM 5.4

KNOWN: Plate initially at a uniform temperature T_i is suddenly subjected to convection process (T_∞,h) on both surfaces. After elapsed time t_o, plate is insulated on both surfaces.

FIND: (a) Assuming Bi >> 1, sketch on T - x coordinates: initial and steady-state ($t \rightarrow \infty$) temperature distributions, $T(x,t_o)$ and distributions for two intermediate times $t_o < t < \infty$, (b) Sketch on T - t coordinates midplane and surface temperature histories, (c) Repeat parts (a) and (b) assuming Bi << 1, and (d) Obtain expression for $T(x,\infty) = T_f$ in terms of plate parameters (M,c_p), thermal conditions (T_i, T_∞, h), surface temperature T(L,t) and heating time t_o.

SCHEMATIC:

Time, t	Process
t = 0	Uniform T_i
0 - t_o	Heating, (T_∞,h)
t > t_o	Insulated

T_∞,h

Surface area, A_s (both faces)
Mass M
$T(x,0)=T_i$
-L x L
T_∞,h

ASSUMPTIONS: (1) One-dimensional conduction, (2) Constant properties, (3) No internal generation, (4) Plate is perfectly insulated for $t > t_o$, (5) $T(0, t < t_o) < T_\infty$.

ANALYSIS: (a,b) With Bi >> 1, appreciable temperature gradients exist in the plate following exposure to the heating process.

T(x,t)
T_∞
$T(x,t_o)$ ③
⑤
T_f
④
$T(x,\infty)$ ②
$t>t_o$
T_i
T(x,0) ①
0 L x

T(x,t)
T(L,t)
T(0,t)
0 t_o t

On *T-x* coordinates: (1) initial, uniform temperature, (2) steady-state conditions when $t \rightarrow \infty$, (3) distribution at t_o just before plate is covered with insulation, (4) gradients are always zero (symmetry), and (5) when $t > t_o$ (dashed lines) gradients approach zero everywhere.

(c) If Bi << 1, plate is space-wise isothermal (no gradients). On *T-x* coordinates, the temperature distributions are flat; on T-t coordinates, T(L,t) = T(0,t).

(d) The conservation of energy requirement for the interval of time $\Delta t = t_o$ is

$$E_{in} - E_{out} = \Delta E = E_{final} - E_{initial} \qquad 2\int_0^{t_o} hA_s\left[T_\infty - T(L,t)\right]dt - 0 = Mc_p\left(T_f - T_i\right)$$

where E_{in} is due to convection heating over the period of time $t = 0 \rightarrow t_o$. With knowledge of T(L,t), this expression can be integrated and a value for T_f determined.

PROBLEM 5.5

KNOWN: Diameter and initial temperature of steel balls cooling in air.

FIND: Time required to cool to a prescribed temperature.

SCHEMATIC:

D = 0.012 m

Steel, T_i = 1150K
k = 40 W/m·K
ρ = 7800 kg/m³
c = 600 J/kg·K

Air
T_∞ = 325K
h = 20 W/m²·K

ASSUMPTIONS: (1) Negligible radiation effects, (2) Constant properties.

ANALYSIS: Applying Eq. 5.10 to a sphere ($L_c = r_o/3$),

$$Bi = \frac{hL_c}{k} = \frac{h(r_o/3)}{k} = \frac{20\ W/m^2 \cdot K\ (0.002m)}{40\ W/m \cdot K} = 0.001.$$

Hence, the temperature of the steel remains approximately uniform during the cooling process, and the lumped capacitance method may be used. From Eqs. 5.4 and 5.5,

$$t = \frac{r V c_p}{hA_s} \ln \frac{T_i - T_\infty}{T - T_\infty} = \frac{r\left(p D^3/6\right)c_p}{hp\ D^2} \ln \frac{T_i - T_\infty}{T - T_\infty}$$

$$t = \frac{7800 kg/m^3 (0.012m) 600 J/kg \cdot K}{6 \times 20\ W/m^2 \cdot K} \ln \frac{1150 - 325}{400 - 325}$$

$$t = 1122\ s\ = 0.312h$$ <

COMMENTS: Due to the large value of T_i, radiation effects are likely to be significant during the early portion of the transient. The effect is to shorten the cooling time.

PROBLEM 5.6

KNOWN: The temperature-time history of a pure copper sphere in an air stream.

FIND: The heat transfer coefficient between the sphere and the air stream.

SCHEMATIC:

$T_\infty = 27°C$

$T(0) = 66°C$

$T(69s) = 55°C$

$D = 12.7mm$

ASSUMPTIONS: (1) Temperature of sphere is spatially uniform, (2) Negligible radiation exchange, (3) Constant properties.

PROPERTIES: *Table A-1*, Pure copper (333K): $\rho = 8933$ kg/m^3, $c_p = 389$ J/kg·K, k = 398 W/m·K.

ANALYSIS: The time-temperature history is given by Eq. 5.6 with Eq. 5.7.

$$\frac{\theta(t)}{\theta_i} = \exp\left(-\frac{t}{R_t C_t}\right) \qquad \text{where} \qquad R_t = \frac{1}{hA_s} \qquad A_s = \pi D^2$$

$$C_t = \rho V c_p \qquad V = \frac{\pi D^3}{6}$$

$$\theta = T - T_\infty.$$

Recognize that when t = 69s,

$$\frac{\theta(t)}{\theta_i} = \frac{(55-27)°C}{(66-27)°C} = 0.718 = \exp\left(-\frac{t}{\tau_t}\right) = \exp\left(-\frac{69s}{\tau_t}\right)$$

and noting that $\tau_t = R_t C_t$ find

$$\tau_t = 208s.$$

Hence,

$$h = \frac{\rho V c_p}{A_s \tau_t} = \frac{8933 \text{ kg/m}^3 \left(\pi\, 0.0127^3 \text{ m}^3/6\right) 389\text{J/kg}\cdot\text{K}}{\pi\, 0.0127^2 \text{m}^2 \times 208\text{s}}$$

$$h = 35.3 \text{ W/m}^2\cdot\text{K}. \qquad <$$

COMMENTS: Note that with $L_c = D_o/6$,

$$Bi = \frac{hL_c}{k} = 35.3 \text{ W/m}^2\cdot\text{K} \times \frac{0.0127}{6}\text{m}/398 \text{ W/m}\cdot\text{K} = 1.88\times10^{-4}.$$

Hence, Bi < 0.1 and the spatially isothermal assumption is reasonable.

PROBLEM 5.7

KNOWN: Solid steel sphere (AISI 1010), coated with dielectric layer of prescribed thickness and thermal conductivity. Coated sphere, initially at uniform temperature, is suddenly quenched in an oil bath.

FIND: Time required for sphere to reach 140°C.

SCHEMATIC:

Sphere, D = 300mm
AISI 1010 steel,
$T_i = T(0) = 500°C$
Oil bath
$T_\infty = 100°C$
$h = 3300\ W/m^2\cdot K$
Dielectric layer
$k = 0.04 W/m\cdot K$
$\ell = 2mm$

PROPERTIES: *Table A-1*, AISI 1010 Steel $\left(\bar{T} = [500 + 140]^\circ C/2 = 320^\circ C \approx 600K\right)$:

$r = 7832\ kg/m^3,\ c = 559\ J/kg\cdot K,\ k = 48.8\ W/m\cdot K.$

ASSUMPTIONS: (1) Steel sphere is space-wise isothermal, (2) Dielectric layer has negligible thermal capacitance compared to steel sphere, (3) Layer is thin compared to radius of sphere, (4) Constant properties.

ANALYSIS: The thermal resistance to heat transfer from the sphere is due to the dielectric layer and the convection coefficient. That is,

$$R'' = \frac{\ell}{k} + \frac{1}{h} = \frac{0.002m}{0.04\ W/m\cdot K} + \frac{1}{3300\ W/m^2\cdot K} = (0.050 + 0.0003) = 0.0503\frac{m^2\cdot K}{W},$$

or in terms of an overall coefficient, $U = 1/R'' = 19.88\ W/m^2\cdot K$. The effective Biot number is

$$Bi_e = \frac{UL_c}{k} = \frac{U(r_o/3)}{k} = \frac{19.88\ W/m^2\cdot K \times (0.300/6)m}{48.8\ W/m\cdot K} = 0.0204$$

where the characteristic length is $L_c = r_o/3$ for the sphere. Since $Bi_e < 0.1$, the lumped capacitance approach is applicable. Hence, Eq. 5.5 is appropriate with h replaced by U,

$$t = \frac{r c}{U}\left[\frac{V}{A_s}\right]\ln\frac{q_i}{q_o} = \frac{r c}{U}\left[\frac{V}{A_s}\right]\ln\frac{T(0) - T_\infty}{T(t) - T_\infty}.$$

Substituting numerical values with $(V/A_s) = r_o/3 = D/6$,

$$t = \frac{7832\ kg/m^3 \times 559\ J/kg\cdot K}{19.88\ W/m^2\cdot K}\left[\frac{0.300m}{6}\right]\ln\frac{(500 - 100)^\circ C}{(140 - 100)^\circ C}$$

$$t = 25{,}358s = 7.04h.$$ <

COMMENTS: (1) Note from calculation of R'' that the resistance of the dielectric layer dominates and therefore nearly all the temperature drop occurs across the layer.

PROBLEM 5.8

KNOWN: Thickness, surface area, and properties of iron base plate. Heat flux at inner surface. Temperature of surroundings. Temperature and convection coefficient of air at outer surface.

FIND: Time required for plate to reach a temperature of 135°C. Operating efficiency of iron.

SCHEMATIC:

$T_{sur} = 25^oC$

Iron
$\rho = 2800$ kg/m³   k = 180 W/m-K
c = 900 J/kg-K $\varepsilon = 0.80$

$q''_h = 1.25\times10^4$ W/m²

$A_s = 0.04$ m²

Air

$T_\infty = 25^oC$
h = 10 W/m²-K

L = 0.007 m

ASSUMPTIONS: (1) Radiation exchange is between a small surface and large surroundings, (2) Convection coefficient is independent of time, (3) Constant properties, (4) Iron is initially at room temperature ($T_i = T_\infty$).

ANALYSIS: Biot numbers may be based on convection heat transfer and/or the maximum heat transfer by radiation, which would occur when the plate reaches the desired temperature (T = 135°C). From Eq. (1.9) the corresponding radiation transfer coefficient is $h_r = \varepsilon\sigma(T + T_{sur})\left(T^2 + T_{sur}^2\right) = 0.8 \times 5.67 \times 10^{-8}\ W/m^2{\cdot}K^4\ (408 + 298)\ K\ (408^2 + 298^2)\ K^2 = 8.2\ W/m^2{\cdot}K$. Hence,

$$Bi = \frac{hL}{k} = \frac{10\,W/m^2\cdot K(0.007m)}{180\,W/m\cdot K} = 3.9\times10^{-4}$$

$$Bi_r = \frac{h_r L}{k} = \frac{8.2\,W/m^2\cdot K(0.007m)}{180\,W/m\cdot K} = 3.2\times10^{-4}$$

With convection and radiation considered independently or collectively, Bi, Bi_r, Bi + Bi_r << 1 and the lumped capacitance analysis may be used.

The energy balance, Eq. (5.15), associated with Figure 5.5 may be applied to this problem. With $\dot{E}g = 0$, the integral form of the equation is

$$T - T_i = \frac{A_s}{\rho Vc}\int_0^t \left[q''_h - h\left(T - T_\infty\right) - \varepsilon\sigma\left(T^4 - T_{sur}^4\right)\right]dt$$

Integrating numerically, we obtain, for T = 135°C,

$$t = 168s$$ <

COMMENTS: Note that, if heat transfer is by natural convection, h, like h_r, will vary during the process from a value of 0 at t = 0 to a maximum at t = 168s.

PROBLEM 5.9

KNOWN: Diameter and radial temperature of AISI 1010 carbon steel shaft. Convection coefficient and temperature of furnace gases.

FIND: Time required for shaft centerline to reach a prescribed temperature.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional, radial conduction, (2) Constant properties.

PROPERTIES: AISI 1010 carbon steel, *Table A.1* $\left(\bar{T} = 550\ K\right)$: $r = 7832\ kg/m^3$, k = 51.2 W/m·K, c = 541 J/kg·K, $\alpha = 1.21\times10^{-5}\ m^2/s$.

ANALYSIS: The Biot number is

$$Bi = \frac{hr_o/2}{k} = \frac{100\ W/m^2\cdot K\left(0.05\ m/2\right)}{51.2\ W/m\cdot K} = 0.0488.$$

Hence, the lumped capacitance method can be applied. From Equation 5.6,

$$\frac{T - T_\infty}{T_i - T_\infty} = \exp\left[-\left(\frac{hAs}{r\,Vc}\right)t\right] = \exp\left[-\frac{4h}{r\,cD}t\right]$$

$$\ln\left(\frac{800-1200}{300-1200}\right) = -0.811 = -\frac{4\times100\ W/m^2\cdot K}{7832\ kg/m^3\left(541\ J/kg\cdot K\right)0.1\ m}t$$

$$t = 859\ s.$$

COMMENTS: To check the validity of the foregoing result, use the one-term approximation to the series solution. From Equation 5.49c,

$$\frac{T_o - T_\infty}{T_i - T_\infty} = \frac{-400}{-900} = 0.444 = C_1\ \exp\left(-V_1^2 Fo\right)$$

For Bi = hr_o/k = 0.0976, Table 5.1 yields ς_1 = 0.436 and C_1 = 1.024. Hence

$$\frac{-\left(0.436\right)^2\left(1.2\times10^{-5}\ m^2/s\right)}{\left(0.05\ m\right)^2}t = \ln\left(0.434\right) = -0.835$$

$$t = 915\ s.$$

The results agree to within 6%. The lumped capacitance method underestimates the actual time, since the response at the centerline lags that at any other location in the shaft.

PROBLEM 5.10

KNOWN: Configuration, initial temperature and charging conditions of a thermal energy storage unit.

FIND: Time required to achieve 75% of maximum possible energy storage. Temperature of storage medium at this time.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional conduction, (2) Constant properties, (3) Negligible radiation exchange with surroundings.

PROPERTIES: *Table A-1*, Aluminum, pure $\left(\overline{T} \approx 600\text{K} = 327^\circ\text{C}\right)$: k = 231 W/m·K, c = 1033 J/kg·K, r = 2702 kg/m^3.

ANALYSIS: Recognizing the characteristic length is the half thickness, find

$$\text{Bi} = \frac{\text{hL}}{\text{k}} = \frac{100\ \text{W/m}^2\cdot\text{K}\times 0.025\text{m}}{231\ \text{W/m}\cdot\text{K}} = 0.011.$$

Hence, the lumped capacitance method may be used. From Eq. 5.8,

$$\text{Q} = \left(r\text{Vc}\right)q_{\text{i}}\ \left[1 - \exp\left(-\text{t}/t_{\text{t}}\right)\right] = -\Delta\text{E}_{\text{st}} \qquad (1)$$

$$-\Delta\text{E}_{\text{st,max}} = \left(r\text{Vc}\right)q_{\text{i}}. \qquad (2)$$

Dividing Eq. (1) by (2),

$$\Delta\text{E}_{\text{st}}/\Delta\text{E}_{\text{st,max}} = 1\ - \exp\left(-\text{t}/t_{\text{th}}\right) = 0.75.$$

Solving for $t_{\text{th}} = \dfrac{r\text{Vc}}{\text{hA}_\text{s}} = \dfrac{r\text{Lc}}{\text{h}} = \dfrac{2702\ \text{kg/m}^3\times 0.025\text{m}\times 1033\ \text{J/kg}\cdot\text{K}}{100\ \text{W/m}^2\cdot\text{K}} = 698\text{s}.$

Hence, the required time is

$$-\exp\left(-\text{t}/698\text{s}\right) = -0.25 \qquad \text{or} \qquad \text{t} = 968\text{s}.$$ <

From Eq. 5.6,

$$\frac{\text{T}-\text{T}_\infty}{\text{T}_\text{i}-\text{T}_\infty} = \exp\left(-\text{t}/t_{\text{th}}\right)$$

$$\text{T} = \text{T}_\infty + \left(\text{T}_\text{i} - \text{T}_\infty\right)\exp\left(-\text{t}/t_{\text{th}}\right) = 600^\circ\text{C} - \left(575^\circ\text{C}\right)\exp\left(-968/698\right)$$

$$\text{T} = 456^\circ\text{C}.$$ <

COMMENTS: For the prescribed temperatures, the property temperature dependence is significant and some error is incurred by assuming constant properties. However, selecting properties at 600K was reasonable for this estimate.

PROBLEM 5.11

KNOWN: Diameter, density, specific heat and thermal conductivity of aluminum spheres used in packed bed thermal energy storage system. Convection coefficient and inlet gas temperature.

FIND: Time required for sphere to acquire 90% of maximum possible thermal energy and the corresponding center temperature. Potential advantage of using copper in lieu of aluminum.

SCHEMATIC:

Gas
$T_{g,i}$ = 300°C
h = 75 W/m^2-K
Aluminum sphere
D = 75 mm, T_i = 25°C
ρ = 2700 kg/m^3
c = 950 J/kg-K
k = 240 W/m-K

ASSUMPTIONS: (1) Negligible heat transfer to or from a sphere by radiation or conduction due to contact with other spheres, (2) Constant properties.

ANALYSIS: To determine whether a lumped capacitance analysis can be used, first compute Bi = $h(r_o/3)/k = 75\ W/m^2 \cdot K\ (0.025m)/150\ W/m \cdot K = 0.013 < 0.1$. Hence, the lumped capacitance approximation may be made, and a uniform temperature may be assumed to exist in the sphere at any time. From Eq. 5.8a, achievement of 90% of the maximum possible thermal energy storage corresponds to

$$\frac{Q}{\rho c V \theta_i} = 0.90 = 1 - \exp\left(-t/\tau_t\right)$$

where $\tau_t = \rho V c / hA_s = \rho D c / 6h = 2700\ kg/m^3 \times 0.075m \times 950\ J/kg \cdot K / 6 \times 75\ W/m^2 \cdot K = 427s$. Hence,

$$t = -\tau_t \ln\left(0.1\right) = 427s \times 2.30 = 984s$$ <

From Eq. (5.6), the corresponding temperature at any location in the sphere is

$$T\left(984s\right) = T_{g,i} + \left(T_i - T_{g,i}\right)\exp\left(-6ht/\rho Dc\right)$$

$$T\left(984s\right) = 300°C - 275°C\ \exp\left(-6 \times 75\ W/m^2 \cdot K \times 984s / 2700\ kg/m^3 \times 0.075m \times 950\ J/kg \cdot K\right)$$

$$T\left(984\right)s = 272.5°C$$ <

Obtaining the density and specific heat of copper from Table A-1, we see that $(\rho c)_{Cu} \approx 8900\ kg/m^3 \times 400\ J/kg \cdot K = 3.56 \times 10^6\ J/m^3 \cdot K > (\rho c)_{Al} = 2.57 \times 10^6\ J/m^3 \cdot K$. Hence, for an equivalent sphere diameter, the copper can store approximately 38% more thermal energy than the aluminum.

COMMENTS: Before the packed bed becomes fully charged, the temperature of the gas decreases as it passes through the bed. Hence, the time required for a sphere to reach a prescribed state of thermal energy storage increases with increasing distance from the bed inlet.

PROBLEM 5.12

KNOWN: Wafer, initially at 100°C, is suddenly placed on a chuck with uniform and constant temperature, 23°C. Wafer temperature after 15 seconds is observed as 33°C.

FIND: (a) Contact resistance, R''_{tc}, between interface of wafer and chuck through which helium slowly flows, and (b) Whether R''_{tc} will change if air, rather than helium, is the purge gas.

SCHEMATIC:

Wafer, $T_w(t)$,
$T_w(0) = T_{w,i} = 100\ ^oC$
$w = 0.758$ mm
Helium gas purge
Interface region, greatly exaggerated
Chuck, $T_c = 23\ ^oC$
$T_w(t)$
$\dot{E}_{st}$
q''_{cond}

PROPERTIES: Wafer (silicon, typical values): ρ = 2700 kg/m^3, c = 875 J/kg·K, k = 177 W/m·K.

ASSUMPTIONS: (1) Wafer behaves as a space-wise isothermal object, (2) Negligible heat transfer from wafer top surface, (3) Chuck remains at uniform temperature, (4) Thermal resistance across the interface is due to conduction effects, not convective, (5) Constant properties.

ANALYSIS: (a) Perform an energy balance on the wafer as shown in the Schematic.

$$\dot{E}''_{in} - \dot{E}''_{out} + \dot{E}_g = \dot{E}_{st} \qquad (1)$$

$$-q''_{cond} = \dot{E}''_{st} \qquad (2)$$

$$-\frac{T_w(t) - T_c}{R''_{tc}} = \rho wc \frac{dT_w}{dt} \qquad (3)$$

Separate and integrate Eq. (3)

$$-\int_0^t \frac{dt}{\rho wc R''_{tc}} = \int_{T_{wi}}^{T_w} \frac{dT_w}{T_w - T_c} \qquad (4) \qquad\qquad \frac{T_w(t) - T_c}{T_{wi} - T_c} = \exp\left[-\frac{t}{\rho wc R''_{tc}}\right] \qquad (5)$$

Substituting numerical values for $T_w(15s) = 33°C$,

$$\frac{(33-23)^\circ C}{(100-23)^\circ C} = \exp\left[\frac{15\,s}{2700\,kg/m^3 \times 0.758\times10^{-3} m \times 875\,J/kg\cdot K \times R''_{tc}}\right] \qquad (6)$$

$$R''_{tc} = 0.0041 m^2 \cdot K/W$$ <

(b) R''_{tc} will increase since $k_{air} < k_{helium}$. See Table A.4.

COMMENTS: Note that $Bi = R_{int}/R_{ext} = (w/k)/ R''_{tc} = 0.001$. Hence the spacewise isothermal assumption is reasonable.

PROBLEM 5.13

KNOWN: Inner diameter and wall thickness of a spherical, stainless steel vessel. Initial temperature, density, specific heat and heat generation rate of reactants in vessel. Convection conditions at outer surface of vessel.

FIND: (a) Temperature of reactants after one hour of reaction time, (b) Effect of convection coefficient on thermal response of reactants.

SCHEMATIC:

ASSUMPTIONS: (1) Temperature of well stirred reactants is uniform at any time and is equal to inner surface temperature of vessel ($T = T_{s,i}$), (2) Thermal capacitance of vessel may be neglected, (3) Negligible radiation exchange with surroundings, (4) Constant properties.

ANALYSIS: (a) Transient thermal conditions within the reactor may be determined from Eq. (5.25), which reduces to the following form for $T_i - T_\infty = 0$.

$$T = T_\infty + (b/a)\left[1 - \exp(-at)\right]$$

where $a = UA/\rho Vc$ and $b = \dot{E}_g/\rho Vc = \dot{q}/\rho c$. From Eq. (3.19) the product of the overall heat transfer coefficient and the surface area is $UA = (R_{cond} + R_{conv})^{-1}$, where from Eqs. (3.36) and (3.9),

$$R_{t,cond} = \frac{1}{2\pi k}\left(\frac{1}{D_i} - \frac{1}{D_o}\right) = \frac{1}{2\pi(17\,W/m\cdot K)}\left(\frac{1}{1.0m} - \frac{1}{1.1m}\right) = 8.51\times10^{-4}\,K/W$$

$$R_{t,conv} = \frac{1}{hA_o} = \frac{1}{\left(6\,W/m^2\cdot K\right)\pi(1.1m)^2} = 0.0438\,K/W$$

Hence, UA = 24.4 W/K. It follows that, with $V = \pi D_i^3/6$,

$$a = \frac{UA}{\rho Vc} = \frac{6(22.4\,W/K)}{1100\,kg/m^3 \times \pi(1m)^3\,2400\,J/kg\cdot K} = 1.620\times10^{-5}\,s^{-1}$$

$$b = \frac{\dot{q}}{\rho c} = \frac{10^4\,W/m^3}{1100\,kg/m^3 \times 2400\,J/kg\cdot K} = 3.788\times10^{-3}\,K/s$$

With (b/a) = 233.8°C and t = 18,000s,

$$T = 25°C + 233.8°C\left[1 - \exp\left(-1.62\times10^{-5}s^{-1}\times18{,}000s\right)\right] = 84.1°C$$ <

Neglecting the thermal capacitance of the vessel wall, the heat rate by conduction through the wall is equal to the heat transfer by convection from the outer surface, and from the thermal circuit, we know that

Continued …..

PROBLEM 5.13 (Cont.)

$$\frac{T - T_{s,o}}{T_{s,o} - T_{\infty}} = \frac{R_{t,cond}}{R_{t,conv}} = \frac{8.51 \times 10^{-4}\, K/W}{0.0438\, K/W} = 0.0194$$

$$T_{s,o} = \frac{T + 0.0194\, T_{\infty}}{1.0194} = \frac{84.1°C + 0.0194(25°C)}{1.0194} = 83.0°C$$ <

(b) Representative low and high values of h could correspond to 2 $W/m^2 \cdot K$ and 100 $W/m^2 \cdot K$ for free and forced convection, respectively. Calculations based on Eq. (5.25) yield the following temperature histories.

Forced convection is clearly an effective means of reducing the temperature of the reactants and accelerating the approach to steady-state conditions.

COMMENTS: The validity of neglecting thermal energy storage effects for the vessel may be assessed by contrasting its thermal capacitance with that of the reactants. Selecting values of ρ = 8000 kg/m^3 and c = 475 J/kg·K for stainless steel from Table A-1, the thermal capacitance of the vessel is $C_{t,v} = (\rho V c)_{st} = 6.57 \times 10^5$ J/K, where $V = (\pi/6)\left(D_o^3 - D_i^3\right)$. With $C_{t,r} = (\rho V c)_r = 2.64 \times 10^6$ J/K for the reactants, $C_{t,r}/C_{t,v} \approx 4$. Hence, the capacitance of the vessel is not negligible and should be considered in a more refined analysis of the problem.

PROBLEM 5.14

KNOWN: Volume, density and specific heat of chemical in a stirred reactor. Temperature and convection coefficient associated with saturated steam flowing through submerged coil. Tube diameter and outer convection coefficient of coil. Initial and final temperatures of chemical and time span of heating process.

FIND: Required length of submerged tubing. Minimum allowable steam flowrate.

SCHEMATIC:

$\dot{m}_h$, T_h = 500 K

Tubing
D = 0.02 m
h_o = 2000 W/m^2-K
h_i = 10,000 W/m^2-K

q

Stirred liquid
T_i = 300 K, T(60 min) = 450 K
V = 2.25 m^3
ρ = 1200 kg/m^3
c = 2200 J/kg-K

ASSUMPTIONS: (1) Constant properties, (2) Negligible heat loss from vessel to surroundings, (3) Chemical is isothermal, (4) Negligible work due to stirring, (5) Negligible thermal energy generation (or absorption) due to chemical reactions associated with the batch process, (6) Negligible tube wall conduction resistance, (7) Negligible kinetic energy, potential energy, and flow work changes for steam.

ANALYSIS: Heating of the chemical can be treated as a transient, lumped capacitance problem, wherein heat transfer from the coil is balanced by the increase in thermal energy of the chemical. Hence, conservation of energy yields

$$\frac{dU}{dt} = \rho Vc \frac{dT}{dt} = UA_s\left(T_h - T\right)$$

Integrating, $$\int_{T_i}^{T} \frac{dT}{T_h - T} = \frac{UA_s}{\rho Vc}\int_o^t dt$$

$$-\ln\frac{T_h - T}{T_h - T_i} = \frac{UA_s t}{\rho Vc}$$

$$A_s = -\frac{\rho Vc}{Ut}\ln\frac{T_h - T}{T_h - T_i} \qquad (1)$$

$$U = \left(h_i^{-1} + h_o^{-1}\right)^{-1} = \left[(1/10{,}000) + (1/2000)\right]^{-1} \text{W/m}^2\cdot\text{K}$$

$$U = 1670\,\text{W/m}^2\cdot\text{K}$$

$$A_s = -\frac{\left(1200\,\text{kg/m}^3\right)\left(2.25\text{m}^3\right)\left(2200\,\text{J/kg}\cdot\text{K}\right)}{\left(1670\,\text{W/m}^2\cdot\text{K}\right)\left(3600\text{s}\right)}\ln\frac{500-450}{500-300} = 1.37\text{m}^2$$

$$L = \frac{A_s}{\pi D} = \frac{1.37\text{m}^2}{\pi\left(0.02\text{m}\right)} = 21.8\text{m}$$ <

COMMENTS: Eq. (1) could also have been obtained by adapting Eq. (5.5) to the conditions of this problem, with T_∞ and h replaced by T_h and U, respectively.

PROBLEM 5.15

KNOWN: Thickness and properties of furnace wall. Thermal resistance of film on surface of wall exposed to furnace gases. Initial wall temperature.

FIND: (a) Time required for surface of wall to reach a prescribed temperature, (b) Corresponding value of film surface temperature.

SCHEMATIC:

$T_\infty = 1300K$
$h = 25 W/m^2 \cdot K$
$R''_f = 10^{-2} m^2 \cdot K/W$
$T_{s,o}$
Carbon steel, $T_i = 300K$
$T_{s,i} = 1200K$
$L = 10mm$
T_∞ $T_{s,o}$ $T_{s,i}$
$(1/h)$ R''_f

ASSUMPTIONS: (1) Constant properties, (2) Negligible film thermal capacitance, (3) Negligible radiation.

PROPERTIES: Carbon steel (given): $\rho = 7850\ kg/m^3$, $c = 430\ J/kg{\cdot}K$, $k = 60\ W/m{\cdot}K$.

ANALYSIS: The overall coefficient for heat transfer from the surface of the steel to the gas is

$$U = \left(R''_{tot}\right)^{-1} = \left(\frac{1}{h} + R''_f\right)^{-1} = \left(\frac{1}{25\ W/m^2 \cdot K} + 10^{-2} m^2 \cdot K/W\right)^{-1} = 20\ W/m^2 \cdot K.$$

Hence,

$$Bi = \frac{UL}{k} = \frac{20\ W/m^2 \cdot K \times 0.01\ m}{60\ W/m \cdot K} = 0.0033$$

and the lumped capacitance method can be used.

(a) It follows that

$$\frac{T - T_\infty}{T_i - T_\infty} = \exp(-t/t_t) = \exp(-t/RC) = \exp(-Ut/rLc)$$

$$t = -\frac{rLc}{U} \ln\frac{T - T_\infty}{T_i - T_\infty} = -\frac{7850\ kg/m^3 (0.01\ m)\, 430\ J/kg \cdot K}{20\ W/m^2 \cdot K} \ln\frac{1200 - 1300}{300 - 1300}$$

$$t = 3886s = 1.08h.$$ <

(b) Performing an energy balance at the outer surface (s,o),

$$h\left(T_\infty - T_{s,o}\right) = \left(T_{s,o} - T_{s,i}\right)/R''_f$$

$$T_{s,o} = \frac{hT_\infty + T_{s,i}/R''_f}{h + (1/R''_f)} = \frac{25\ W/m^2 \cdot K \times 1300\ K + 1200\ K/10^{-2} m^2 \cdot K/W}{(25 + 100)\ W/m^2 \cdot K}$$

$$T_{s,o} = 1220\ K.$$ <

COMMENTS: The film increases t_t by increasing R_t but not C_t.

PROBLEM 5.16

KNOWN: Thickness and properties of strip steel heated in an annealing process. Furnace operating conditions.

FIND: (a) Time required to heat the strip from 300 to 600°C. Required furnace length for prescribed strip velocity (V = 0.5 m/s), (b) Effect of wall temperature on strip speed, temperature history, and radiation coefficient.

SCHEMATIC:

$T_{sur} = T_w$

q''_{rad} q''_{conv}

Steel
$T_i = 300\ ^oC$
$T_f = 600\ ^oC$

$\dot{E}st$

$\delta/2 = 0.006$ m

$V = 0.5$ m/s

Combustion gases

$h = 100\ W/m^2 \cdot K$
$T_\infty = T_w$

ASSUMPTIONS: (1) Constant properties, (2) Negligible temperature gradients in transverse direction across strip, (c) Negligible effect of strip conduction in longitudinal direction.

PROPERTIES: Steel: $\rho = 7900\ kg/m^3$, $c_p = 640$ J/kg·K, k = 30 W/m·K, $\varepsilon = 0.7$.

ANALYSIS: (a) Considering a fixed (control) mass of the moving strip, its temperature variation with time may be obtained from an energy balance which equates the change in energy storage to heat transfer by convection and radiation. If the surface area associated with one side of the control mass is designated as A_s, $A_{s,c} = A_{s,r} = 2A_s$ and $V = \delta A_s$ in Equation 5.15, which reduces to

$$\rho c \delta \frac{dT}{dt} = -2\left[h\left(T - T_\infty\right) + \varepsilon\sigma\left(T^4 - T_{sur}^4\right)\right]$$

or, introducing the radiation coefficient from Equations 1.8 and 1.9 and integrating,

$$T_f - T_i = -\frac{1}{\rho c(\delta/2)}\int_o^{t_f}\left[h\left(T - T_\infty\right) + h_r\left(T - T_{sur}\right)\right]dt$$

Using the IHT *Lumped Capacitance Model* to integrate numerically with $T_i = 573$ K, we find that T_f = 873 K corresponds to

$$t_f \approx 209s \qquad <$$

in which case, the required furnace length is

$$L = Vt_f \approx 0.5\,m/s \times 209\,s \approx 105\,m \qquad <$$

(b) For $T_w = 1123$ K and 1273 K, the numerical integration yields $t_f \approx 102s$ and 62s respectively. Hence, for L = 105 m , $V = L/t_f$ yields

$$V\left(T_w = 1123K\right) = 1.03\,m/s$$

$$V\left(T_w = 1273K\right) = 1.69\,m/s \qquad <$$

Continued...

PROBLEM 5.16 (Cont.)

which correspond to increased process rates of 106% and 238%, respectively. Clearly, productivity can be enhanced by increasing the furnace environmental temperature, albeit at the expense of increasing energy utilization and operating costs.

If the annealing process extends from 25°C (298 K) to 600°C (873 K), numerical integration yields the following results for the prescribed furnace temperatures.

As expected, the heating rate and time, respectively, increase and decrease significantly with increasing T_w. Although the radiation heat transfer rate decreases with increasing time, the coefficient h_r increases with t as the strip temperature approaches T_w.

COMMENTS: To check the validity of the lumped capacitance approach, we calculate the Biot number based on a maximum cumulative coefficient of $(h + h_r) \approx 300\ W/m^2 \cdot K$. It follows that $Bi = (h + h_r)(\delta/2)/k = 0.06$ and the assumption is valid.

PROBLEM 5.17

KNOWN: Diameter, resistance and current flow for a wire. Convection coefficient and temperature of surrounding oil.

FIND: Steady-state temperature of the wire. Time for the wire temperature to come within 1°C of its steady-state value.

SCHEMATIC:

Oil
$T_\infty = 25°C$
$h = 500 W/m^2 \cdot K$
$I = 100A$
Wire, D=1mm
x

ASSUMPTIONS: (1) Constant properties, (2) Wire temperature is independent of x.

PROPERTIES: Wire (given): $\rho = 8000\ kg/m^3$, $c_p = 500\ J/kg \cdot K$, $k = 20\ W/m \cdot K$, $R'_e = 0.01\ \Omega/m$.

ANALYSIS: Since

$$Bi = \frac{h(r_o/2)}{k} = \frac{500\ W/m^2 \cdot K \left(2.5 \times 10^{-4} m\right)}{20\ W/m \cdot K} = 0.006 < 0.1$$

the lumped capacitance method can be used. The problem has been analyzed in Example 1.3, and without radiation the steady-state temperature is given by

$$\pi\ Dh(T - T_\infty) = I^2 R'_e .$$

Hence

$$T = T_\infty + \frac{I^2 R'_e}{\pi\ Dh} = 25°C + \frac{(100A)^2\ 0.01\Omega/m}{\pi(0.001\ m)500\ W/m^2 \cdot K} = 88.7°C.$$ <

With no radiation, the transient thermal response of the wire is governed by the expression (Example 1.3)

$$\frac{dT}{dt} = \frac{I^2 R'_e}{\rho c_p \left(\pi D^2/4\right)} - \frac{4h}{\rho\ c_p D}(T - T_\infty).$$

With $T = T_i = 25°C$ at $t = 0$, the solution is

$$\frac{T - T_\infty - \left(I^2 R'_e/\pi\ Dh\right)}{T_i - T_\infty - \left(I^2 R'_e/\pi\ Dh\right)} = \exp\left(-\frac{4h}{\rho\ c_p D} t\right).$$

Substituting numerical values, find

$$\frac{87.7 - 25 - 63.7}{25 - 25 - 63.7} = \exp\left(-\frac{4 \times 500\ W/m^2 \cdot K}{8000\ kg/m^3 \times 500\ J/kg \cdot K \times 0.001\ m} t\right)$$

$$t = 8.31s.$$ <

COMMENTS: The time to reach steady state increases with increasing ρ, c_p and D and with decreasing h.

PROBLEM 5.18

KNOWN: Electrical heater attached to backside of plate while front is exposed to a convection process (T_∞, h); initially plate is at uniform temperature T_∞ before heater power is switched on.

FIND: (a) Expression for temperature of plate as a function of time assuming plate is spacewise isothermal, (b) Approximate time to reach steady-state and $T(\infty)$ for prescribed T_∞, h and q_o'' when wall material is pure copper, (c) Effect of h on thermal response.

SCHEMATIC:

q_o'' = 5000 W/m^2; Heater; T_∞ = 27 °C; h = 50, 100, 200 W/m^2·K; Copper, pure; T_i = 27 °C; x; L = 12 mm; q_o''; $q''_{conv} = h(T-T_\infty)$

ASSUMPTIONS: (1) Plate behaves as lumped capacitance, (2) Negligible loss out backside of heater, (3) Negligible radiation, (4) Constant properties.

PROPERTIES: *Table A-1*, Copper, pure (350 K): k = 397 W/m·K, c_p = 385 J/kg·K, ρ = 8933 kg/m^3.

ANALYSIS: (a) Following the analysis of Section 5.3, the energy conservation requirement for the system is $\dot{E}_{in} - \dot{E}_{out} = \dot{E}_{st}$ or $q_o'' - h(T - T_\infty) = \rho L c_p\, dT/dt$. Rearranging, and with R_t'' = 1/h and C_t'' = $\rho L c_p$,

$$T - T_\infty - q_o''/h = -R_t'' \cdot C_t''\, dT/dt \tag{1}$$

Defining $\theta(t) \equiv T - T_\infty - q_o''/h$ with dθ = dT, the differential equation is

$$\theta = -R_t'' C_t'' \frac{d\theta}{dt}. \tag{2}$$

Separating variables and integrating,

$$\int_{\theta_i}^{\theta} \frac{d\theta}{\theta} = -\int_0^t \frac{dt}{R_t'' C_t''}$$

it follows that

$$\frac{\theta}{\theta_i} = \exp\left(-\frac{t}{R_t'' C_t''}\right) \tag{3}$$ <

where $\theta_i = \theta(0) = T_i - T_\infty - (q_o''/h)$ (4)

(b) For h = 50 W/m^2 ·K, the steady-state temperature can be determined from Eq. (3) with $t \to \infty$; that is,

$$\theta(\infty) = 0 = T(\infty) - T_\infty - q_o''/h \qquad \text{or} \qquad T(\infty) = T_\infty + q_o''/h,$$

giving T(∞) = 27°C + 5000 W/m^2 /50 W/m^2·K = 127°C. To estimate the time to reach steady-state, first determine the thermal time constant of the system,

$$\tau_t = R_t'' C_t'' = \left(\frac{1}{h}\right)(\rho c_p L) = \left(\frac{1}{50\,W/m^2 \cdot K}\right)\left(8933\,kg/m^3 \times 385\,J/kg \cdot K \times 12 \times 10^{-3}\,m\right) = 825\,s$$

Continued...

PROBLEM 5.18 (Cont.)

When t = $3\tau_t$ = 3×825s = 2475s, Eqs. (3) and (4) yield

$$\theta(3\tau_t) = T(3\tau_t) - 27^\circ C - \frac{5000\,W/m^2}{50\,W/m^2 \cdot K} = e^{-3}\left[27^\circ C - 27^\circ C - \frac{5000\,W/m^2}{50\,W/m^2 \cdot K}\right]$$

$T(3\tau_t)$ = 122°C <

(c) As shown by the following graphical results, which were generated using the IHT *Lumped Capacitance Model*, the steady-state temperature and the time to reach steady-state both decrease with increasing h.

COMMENTS: Note that, even for h = 200 W/m^2·K, Bi = hL/k << 0.1 and assumption (1) is reasonable.

PROBLEM 5.19

KNOWN: Electronic device on aluminum, finned heat sink modeled as spatially isothermal object with internal generation and convection from its surface.

FIND: (a) Temperature response after device is energized, (b) Temperature rise for prescribed conditions after 5 min.

SCHEMATIC:

A_s, $\dot{E}_g$, q_{conv}, $T_\infty, \bar{h}$, $T(t), M$

$M = 0.31\,kg$
$T_i = T_\infty = 20°C$
$\dot{E}_g = 60\,W$
$T(\infty) = 100°C$

ASSUMPTIONS: (1) Spatially isothermal object, (2) Object is primarily aluminum, (3) Initially, object is in equilibrium with surroundings at T_∞.

PROPERTIES: *Table A-1*, Aluminum, pure $\left(\bar{T} = (20+100)^\circ C/2 \approx 333K\right)$: c = 918 J/kg·K.

ANALYSIS: (a) Following the general analysis of Section 5.3, apply the conservation of energy requirement to the object,

$$\dot{E}_{in} + \dot{E}_g - \dot{E}_{out} = \dot{E}_{st} \qquad \dot{E}_g - \bar{h}A_s(T - T_\infty) = Mc\frac{dT}{dt} \tag{1}$$

where T = T(t). Consider now steady-state conditions, in which case the storage term of Eq. (1) is zero. The temperature of the object will be T(∞) such that

$$\dot{E}_g = \bar{h}A_s\left(T(\infty) - T_\infty\right). \tag{2}$$

Substituting for $\dot{E}_g$ using Eq. (2) into Eq. (1), the differential equation is

$$\left[T(\infty) - T_\infty\right] - \left[T - T_\infty\right] = \frac{Mc}{\bar{h}A_s}\frac{dT}{dt} \qquad \text{or} \qquad \theta = -\frac{Mc}{\bar{h}A_s}\frac{d\theta}{dt} \tag{3,4}$$

with θ ≡ T - T(∞) and noting that dθ = dT. Identifying $R_t = 1/\bar{h}A_s$ and $C_t = Mc$, the differential equation is integrated with proper limits,

$$\frac{1}{R_tC_t}\int_0^t dt = -\int_{\theta_i}^{\theta}\frac{d\theta}{\theta} \qquad \text{or} \qquad \frac{\theta}{\theta_i} = \exp\left[-\frac{t}{R_tC_t}\right] \tag{5}$$ <

where $\theta_i = \theta(0) = T_i - T(\infty)$ and T_i is the initial temperature of the object.

(b) Using the information about steady-state conditions and Eq. (2), find first the thermal resistance and capacitance of the system,

$$R_t = \frac{1}{\bar{h}A_s} = \frac{T(\infty) - T_\infty}{\dot{E}_g} = \frac{(100-20)^\circ C}{60\ W} = 1.33\ K/W \qquad C_t = Mc = 0.31\ kg \times 918\ J/kg\cdot K = 285\ J/K.$$

Using Eq. (5), the temperature of the system after 5 minutes is

$$\frac{\theta(5min)}{\theta_i} = \frac{T(5min) - T(\infty)}{T_i - T(\infty)} = \frac{T(5min) - 100^\circ C}{(20-100)^\circ C} = \exp\left[-\frac{5\times 60s}{1.33\ K/W \times 285\ J/K}\right] = 0.453$$

$$T(5min) = 100^\circ C + (20-100)^\circ C \times 0.453 = 63.8^\circ C$$ <

COMMENTS: Eq. 5.24 may be used directly for Part (b) with a = hA_s/Mc and $b = \dot{E}_g/Mc$.

PROBLEM 5.20

KNOWN: Spherical coal pellet at 25°C is heated by radiation while flowing through a furnace maintained at 1000°C.

FIND: Length of tube required to heat pellet to 600°C.

SCHEMATIC:

Coal pellet, D=1mm

Air flow

V=3m/s

Tubular furnace at T_{sur}=1000°C

T_o=25°C

x

L

T_L=600°C

ASSUMPTIONS: (1) Pellet is suspended in air flow and subjected to only radiative exchange with furnace, (2) Pellet is small compared to furnace surface area, (3) Coal pellet has emissivity, ε = 1.

PROPERTIES: *Table A-3,* Coal $\left(\overline{T} = (600 + 25)^\circ C/2 = 585K,\ \text{however, only 300K data available}\right)$: ρ = 1350 kg/m^3, c_p = 1260 J/kg·K, k = 0.26 W/m·K.

ANALYSIS: Considering the pellet as spatially isothermal, use the lumped capacitance method of Section 5.3 to find the time required to heat the pellet from T_o = 25°C to T_L = 600°C. From an energy balance on the pellet $\dot{E}_{in} = \dot{E}_{st}$ where

D=1mm

T_{sur}

q_{rad}

T(t)

$$\dot{E}_{in} = q_{rad} = \sigma A_s\left(T_{sur}^4 - T_s^4\right) \qquad \dot{E}_{st} = \rho \forall c_p \frac{dT}{dt}$$

giving
$$A_s\sigma\left(T_{sur}^4 - T_s^4\right) = \rho\forall c_p \frac{dT}{dt}.$$

Separating variables and integrating with limits shown, the temperature-time relation becomes

$$\frac{A_s\sigma}{\rho\forall c_p}\int_0^t dt = \int_{T_o}^{T_L} \frac{dT}{T_{sur}^4 - T^4}.$$

The integrals are evaluated in Eq. 5.18 giving

$$t = \frac{\rho\forall c_p}{4A_s\sigma T_{sur}^3}\left\{\ln\left|\frac{T_{sur}+T}{T_{sur}-T}\right| - \ln\left|\frac{T_{sur}+T_i}{T_{sur}-T_i}\right| + 2\left[\tan^{-1}\left[\frac{T}{T_{sur}}\right] - \tan^{-1}\left[\frac{T_i}{T_{sur}}\right]\right]\right\}.$$

Recognizing that $A_s = \pi D^2$ and $\forall = \pi D^3/6$ or $A_s/\forall = 6/D$ and substituting values,

$$t = \frac{1350\ \text{kg/m}^3\,(0.001\ \text{m})1260\ \text{J/kg}\cdot\text{K}}{24\times 5.67\times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4\,(1273\ \text{K})^3}\left\{\ln\frac{1273+873}{1273-873} - \ln\frac{1273+298}{1273-298}\right.$$

$$\left. +2\left[\tan^{-1}\left(\frac{873}{1273}\right) - \tan^{-1}\left(\frac{298}{1273}\right)\right]\right\} = 1.18\text{s}.$$

Hence, $\qquad L = V\cdot t = 3\text{m/s}\times 1.18\text{s} = 3.54\text{m}.$ <

The validity of the lumped capacitance method requires Bi = h(∀/A_s)k < 0.1. Using Eq. (1.9) for h = h_r and ∀/A_s = D/6, find that when T = 600°C, Bi = 0.19; but when T = 25°C, Bi = 0.10. At early times, when the pellet is cooler, the assumption is reasonable but becomes less appropriate as the pellet heats.

PROBLEM 5.21

KNOWN: Metal sphere, initially at a uniform temperature T_i, is suddenly removed from a furnace and suspended in a large room and subjected to a convection process (T_∞, h) and to radiation exchange with surroundings, T_{sur}.

FIND: (a) Time it takes for sphere to cool to some temperature T, neglecting radiation exchange, (b) Time it takes for sphere to cool to some temperature t, neglecting convection, (c) Procedure to obtain time required if both convection and radiation are considered, (d) Time to cool an anodized aluminum sphere to 400 K using results of Parts (a), (b) and (c).

SCHEMATIC:

Air

$h = 10 \text{W/m}^2\cdot\text{K}$

$T_\infty = 300$ K

$T_{sur} = 300$ K

q''_{conv}

q''_{rad}

Aluminum sphere, $T(t)$

D = 50 mm, $T_i = 800$

ASSUMPTIONS: (1) Sphere is spacewise isothermal, (2) Constant properties, (3) Constant heat transfer convection coefficient, (4) Sphere is small compared to surroundings.

PROPERTIES: *Table A-1*, Aluminum, pure ($\overline{T}$ = [800 + 400] K/2 = 600 K): $\rho = 2702$ kg/m^3, c = 1033 J/kg·K, k = 231 W/m·K, $\alpha = k/\rho c = 8.276 \times 10^{-5}$ m^2/s; Aluminum, anodized finish: $\varepsilon = 0.75$, polished surface: $\varepsilon = 0.1$.

ANALYSIS: (a) Neglecting radiation, the time to cool is predicted by Eq. 5.5,

$$t = \frac{\rho Vc}{hA_s} \ln\frac{\theta_i}{\theta} = \frac{\rho Dc}{6h} \ln\frac{T_i - T_\infty}{T - T_\infty} \qquad (1)$$ <

where $V/A_s = (\pi D^3/6)/(\pi D^2) = D/6$ for the sphere.

(b) Neglecting convection, the time to cool is predicted by Eq. 5.18,

$$t = \frac{\rho Dc}{24\varepsilon\sigma T_{sur}^3}\left\{\ln\left|\frac{T_{sur}+T}{T_{sur}-T}\right| - \ln\left|\frac{T_{sur}+T_i}{T_{sur}-T_i}\right| + 2\left[\tan^{-1}\left(\frac{T}{T_{sur}}\right) - \tan^{-1}\left(\frac{T_i}{T_{sur}}\right)\right]\right\} \qquad (2)$$

where $V/A_{s,r} = D/6$ for the sphere.

(c) If convection and radiation exchange are considered, the energy balance requirement results in Eq. 5.15 (with $q_s'' = \dot{E}_g = 0$). Hence

$$\frac{dT}{dt} = \frac{6}{\rho Dc}\left[h\left(T - T_\infty\right) + \varepsilon\sigma\left(T^4 - T_{sur}^4\right)\right] \qquad (3)$$ <

where $A_{s(c,r)} = A_s = \pi D^2$ and $V/A_{s(c,r)} = D/6$. This relation must be solved numerically in order to evaluate the time-to-cool.

(d) For the aluminum (pure) sphere with an anodized finish and the prescribed conditions, the times to cool from $T_i = 800$ K to T = 400 K are:

Continued...

PROBLEM 5.21 (Cont.)

Convection only, Eq. (1)

$$t = \frac{2702\,\text{kg/m}^3 \times 0.050\,\text{m} \times 1033\,\text{J/kg}\cdot\text{K}}{6 \times 10\,\text{W/m}^2\cdot\text{K}} \ln\frac{800-300}{400-300} = 3743\text{s} = 1.04\text{h} \qquad <$$

Radiation only, Eq. (2)

$$t = \frac{2702\,\text{kg/m}^3 \times 0.050\,\text{m} \times 1033\,\text{J/kg}\cdot\text{K}}{24 \times 0.75 \times 5.67 \times 10^{-8}\,\text{W/m}^2\cdot\text{K}^4 \times (300\,\text{K})^3} \cdot \left\{ \left(\ln\frac{400+300}{400-300} - \ln\frac{800+300}{800-300} \right) + 2\left[\tan^{-1}\frac{400}{300} - \tan^{-1}\frac{800}{300} \right] \right\}$$

$$t = 5.065 \times 10^3 \{1.946 - 0.789 + 2(0.927 - 1.212)\} = 2973\text{s} = 0.826\text{h} \qquad <$$

Radiation and convection, Eq. (3)

Using the IHT *Lumped Capacitance Model*, numerical integration yields

$$t \approx 1600\text{s} = 0.444\text{h}$$

In this case, heat loss by radiation exerts the stronger influence, although the effects of convection are by no means negligible. However, if the surface is polished (ε = 0.1), convection clearly dominates. For each surface finish and the three cases, the temperature histories are as follows.

Temperature, T(K) vs. Time, t(s)

- h = 10 W/m^2.K, eps = 0.75
- h = 0, eps = 0.75
- h = 10 W/m^2.K, eps = 0

Temperature, T(K) vs. Time, t x E-4 (s)

- h = 10 W/m^2.K, eps = 0.1
- h = 10 W/ m^2.K, eps = 0
- h = 0, eps = 0.1

COMMENTS: 1. A summary of the analyses shows the relative importance of the various modes of heat loss:

Active Modes	Time required to cool to 400 K (h)	
	ε = 0.75	ε = 0.1
Convection only	1.040	1.040
Radiation only	0.827	6.194
Both modes	0.444	0.889

2. Note that the spacewise isothermal assumption is justified since Be << 0.1. For the convection-only process,

$$Bi = h(r_o/3)/k = 10\ \text{W/m}^2\cdot\text{K}\ (0.025\ \text{m}/3)/231\ \text{W/m}\cdot\text{K} = 3.6 \times 10^{-4}$$

PROBLEM 5.22

KNOWN: Droplet properties, diameter, velocity and initial and final temperatures.

FIND: Travel distance and rejected thermal energy.

SCHEMATIC:

ASSUMPTIONS: (1) Constant properties, (2) Negligible radiation from space.

PROPERTIES: Droplet (given): ρ = 885 kg/m^3, c = 1900 J/kg·K, k = 0.145 W/m·K, ε = 0.95.

ANALYSIS: To assess the suitability of applying the lumped capacitance method, use Equation 1.9 to obtain the maximum radiation coefficient, which corresponds to $T = T_i$.

$$h_r = \varepsilon\sigma T_i^3 = 0.95\times 5.67\times 10^{-8}\,\text{W/m}^2\cdot\text{K}^4\,(500\ \text{K})^3 = 6.73\ \text{W/m}^2\cdot\text{K}.$$

Hence

$$\text{Bi}_r = \frac{h_r\,(r_o/3)}{k} = \frac{\left(6.73\ \text{W/m}^2\cdot\text{K}\right)\left(0.25\times 10^{-3}\ \text{m}/3\right)}{0.145\ \text{W/m}\cdot\text{K}} = 0.0039$$

and the lumped capacitance method can be used. From Equation 5.19,

$$t = \frac{L}{V} = \frac{\rho c\left(\pi D^3/6\right)}{3\varepsilon\left(\pi D^2\right)\sigma}\left(\frac{1}{T_f^3} - \frac{1}{T_i^3}\right)$$

$$L = \frac{(0.1\ \text{m/s})\,885\ \text{kg/m}^3\,(1900\ \text{J/kg}\cdot\text{K})\,0.5\times 10^{-3}\ \text{m}}{18\times 0.95\times 5.67\times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4}\left(\frac{1}{300^3} - \frac{1}{500^3}\right)\frac{1}{\text{K}^3}$$

$$L = 2.52\ \text{m}.$$ <

The amount of energy rejected by each droplet is equal to the change in its internal energy.

$$E_i - E_f = \rho V c\left(T_i - T_f\right) = 885\ \text{kg/m}^3\,\pi\,\frac{\left(5\times 10^{-4}\,\text{m}\right)^3}{6}\,1900\ \text{J/kg}\cdot\text{K}\,(200\ \text{K})$$

$$E_i - E_f = 0.022\ \text{J}.$$ <

COMMENTS: Because some of the radiation emitted by a droplet will be intercepted by other droplets in the stream, the foregoing analysis overestimates the amount of heat dissipated by radiation to space.

PROBLEM 5.23

KNOWN: Initial and final temperatures of a niobium sphere. Diameter and properties of the sphere. Temperature of surroundings and/or gas flow, and convection coefficient associated with the flow.

FIND: (a) Time required to cool the sphere exclusively by radiation, (b) Time required to cool the sphere exclusively by convection, (c) Combined effects of radiation and convection.

SCHEMATIC:

$T_i = 900^oC$
$T_f = 300^oC$
$T_{sur} = 25^oC$
Niobium
$\rho = 8600$ kg/m^3
c = 290 J/kg-K
k = 63 W/m-K
$\varepsilon = 0.1$ or 0.6
Inert gas
$T_\infty = 25^oC$
h = 200 W/m^2-K
D = 10 mm

ASSUMPTIONS: (1) Uniform temperature at any time, (2) Negligible effect of holding mechanism on heat transfer, (3) Constant properties, (4) Radiation exchange is between a small surface and large surroundings.

ANALYSIS: (a) If cooling is exclusively by radiation, the required time is determined from Eq. (5.18). With $V = \pi D^3/6$, $A_{s,r} = \pi D^2$, and $\varepsilon = 0.1$,

$$t = \frac{8600\ \text{kg/m}^3 (290\,\text{J/kg}\cdot\text{K}) 0.01\text{m}}{24(0.1)5.67\times10^{-8}\text{W/m}^2\cdot\text{K}^4(298\text{K})^3}\left\{\ln\left|\frac{298+573}{298-573}\right| - \ln\left|\frac{298+1173}{298-1173}\right|\right.$$

$$\left. +2\left[\tan^{-1}\left(\frac{573}{298}\right) - \tan^{-1}\left(\frac{1173}{298}\right)\right]\right\}$$

$$t = 6926\text{s}\{1.153 - 0.519 + 2(1.091 - 1.322)\} = 1190\text{s} \qquad (\varepsilon = 0.1)$$ <

If $\varepsilon = 0.6$, cooling is six times faster, in which case,

$$t = 199\text{s} \qquad (\varepsilon = 0.6)$$ <

(b) If cooling is exclusively by convection, Eq. (5.5) yields

$$t = \frac{\rho c D}{6h}\ln\left(\frac{T_i - T_\infty}{T_f - T_\infty}\right) = \frac{8600\ \text{kg/m}^3(290\,\text{J/kg}\cdot\text{K})0.010\text{m}}{1200\,\text{W/m}^2\cdot\text{K}}\ln\left(\frac{875}{275}\right)$$

$$t = 24.1\text{s}$$ <

(c) With both radiation and convection, the temperature history may be obtained from Eq. (5.15).

$$\rho\left(\pi D^3/6\right)c\frac{dT}{dt} = -\pi D^2\left[h\left(T - T_\infty\right) + \varepsilon\sigma\left(T^4 - T_{sur}^4\right)\right]$$

Integrating numerically from T_i = 1173 K at t = 0 to T = 573K, we obtain

$$t = 21.0\text{s}$$ <

Continued …..

PROBLEM 5.23 (Cont.)

Cooling times corresponding to representative changes in ε and h are tabulated as follows

$h(W/m^2 \cdot K)$	200	200	20	500
ε	0.6	1.0	0.6	0.6
t(s)	21.0	19.4	102.8	9.1

For values of h representative of forced convection, the influence of radiation is secondary, even for a maximum possible emissivity of 1.0. Hence, to accelerate cooling, it is necessary to increase h. However, if cooling is by natural convection, radiation is significant. For a representative natural convection coefficient of $h = 20\ W/m^2 \cdot K$, the radiation flux exceeds the convection flux at the surface of the sphere during early to intermediate stages of the transient.

COMMENTS: (1) Even for h as large as $500\ W/m^2 \cdot K$, $Bi = h\,(D/6)/k = 500\ W/m^2 \cdot K\ (0.01m/6)/63\ W/m \cdot K = 0.013 < 0.1$ and the lumped capacitance model is appropriate. (2) The largest value of h_r corresponds to $T_i = 1173\ K$, and for $\varepsilon = 0.6$ Eq. (1.9) yields $h_f = 0.6 \times 5.67 \times 10^{-8}\ W/m^2 \cdot K^4\ (1173 + 298)K\ (1173^2 + 298^2)K^2 = 73.3\ W/m^2 \cdot K$.

PROBLEM 5.24

KNOWN: Diameter and thermophysical properties of alumina particles. Convection conditions associated with a two-step heating process.

FIND: (a) Time-in-flight (t_{i-f}) required for complete melting, (b) Validity of assuming negligible radiation.

SCHEMATIC:

Plasma

Al_2O_3 sphere, $D_p = 50\ \mu m$, $T_i = 300$ K, $k_p = 10.5$ W/m•K, $\rho_p = 3970$ kg/m^3, $c_p = 1560$ J/kg•K, $T_{mp} = 2318$ K, $h_{sf} = 3577$ kJ/kg

$h = 3\times10^4$ W/m^2•K
$T_\infty = 10{,}000$ K

q''_{conv}

ASSUMPTIONS: (1) Particle behaves as a lumped capacitance, (2) Negligible radiation, (3) Constant properties.

ANALYSIS: (a) The two-step process involves (i) the time t_1 to heat the particle to its melting point and (ii) the time t_2 required to achieve complete melting. Hence, $t_{i-f} = t_1 + t_2$, where from Eq. (5.5),

$$t_1 = \frac{\rho_p V c_p}{h A_s} \ln\frac{\theta_i}{\theta} = \frac{\rho_p D_p c_p}{6h} \ln\frac{T_i - T_\infty}{T_{mp} - T_\infty}$$

$$t_1 = \frac{3970\,\text{kg/m}^3\left(50\times10^{-6}\,\text{m}\right)1560\,\text{J/kg}\cdot\text{K}}{6\left(30{,}000\,\text{W/m}^2\cdot\text{K}\right)} \ln\frac{(300-10{,}000)}{(2318-10{,}000)} = 4\times10^{-4}\,\text{s}$$

Performing an energy balance for the second step, we obtain

$$\int_{t_1}^{t_1+t_2} q_{conv}\,dt = \Delta E_{st}$$

where $q_{conv} = hA_s(T_\infty - T_{mp})$ and $\Delta E_{st} = \rho_p V h_{sf}$. Hence,

$$t_2 = \frac{\rho_p D_p}{6h}\frac{h_{sf}}{\left(T_\infty - T_{mp}\right)} = \frac{3970\,\text{kg/m}^3\left(50\times10^{-6}\,\text{m}\right)}{6\left(30{,}000\,\text{W/m}^2\cdot\text{K}\right)} \times \frac{3.577\times10^6\,\text{J/kg}}{(10{,}000-2318)\,\text{K}} = 5\times10^{-4}\,\text{s}$$

Hence $t_{i-f} = 9\times10^{-4}\,\text{s} \approx 1\,\text{ms}$ <

(b) Contrasting the smallest value of the convection heat flux, $q''_{conv,min} = h\left(T_\infty - T_{mp}\right) = 2.3\times10^8\ \text{W/m}^2$ to the largest radiation flux, $q''_{rad,max} = \varepsilon\sigma\left(T_{mp}^4 - T_{sur}^4\right) = 6.5\times10^5$ W/m^2, we conclude that radiation is, in fact, negligible.

COMMENTS: (1) Since Bi = $(hr_p/3)/k \approx 0.05$, the lumped capacitance assumption is good. (2) In an actual application, the droplet should impact the substrate in a superheated condition ($T > T_{mp}$), which would require a slightly larger t_{i-f}.

PROBLEM 5.25

KNOWN: Diameters, initial temperature and thermophysical properties of WC and Co in composite particle. Convection coefficient and freestream temperature of plasma gas. Melting point and latent heat of fusion of Co.

FIND: Times required to reach melting and to achieve complete melting of Co.

SCHEMATIC:

$D_o = 20\ \mu m$
$D_i = 16\ \mu m$
Plasma gas
$T_\infty = 10{,}000$ K
$h = 20{,}000\ W/m^2\text{-}K$
Tungsten carbide
$\rho_c = 16{,}000\ kg/m^3$
$c_c = 300$ J/kg-K
Cobalt
$\rho_s = 8900\ kg/m^3$
$c_s = 750$ J/kg-K
$T_{mp} = 1770$ K
$h_{sf} = 2.59\times10^5$ J/kg

ASSUMPTIONS: (1) Particle is isothermal at any instant, (2) Radiation exchange with surroundings is negligible, (3) Negligible contact resistance at interface between WC and Co, (4) Constant properties.

ANALYSIS: From Eq. (5.5), the time required to reach the melting point is

$$t_1 = \frac{(\rho Vc)_{tot}}{h\pi D_o^2}\ln\frac{T_i - T_\infty}{T_{mp} - T_\infty}$$

where the total heat capacity of the composite particle is

$$(\rho Vc)_{tot} = (\rho Vc)_c + (\rho Vc)_s = 16,000\,kg/m^3\left[\pi\left(1.6\times10^{-5}m\right)^3/6\right]300\,J/kg\cdot K$$

$$+8900\,kg/m^3\left\{\pi/6\left[\left(2.0\times10^{-5}m\right)^3 - \left(1.6\times10^{-5}m\right)^3\right]\right\}750\,J/kg\cdot K$$

$$= \left(1.03\times10^{-8} + 1.36\times10^{-8}\right)J/K = 2.39\times10^{-8}\,J/K$$

$$t_1 = \frac{2.39\times10^{-8}\,J/K}{\left(20,000\,W/m^2\cdot K\right)\pi\left(2.0\times10^{-5}m\right)^2}\ln\frac{(300-10,000)K}{(1770-10,000)K} = 1.56\times10^{-4}s \qquad <$$

The time required to melt the Co may be obtained by applying the first law, Eq. (1.11b) to a control surface about the particle. It follows that

$$E_{in} = h\pi D_o^2\left(T_\infty - T_{mp}\right)t_2 = \Delta E_{st} = \rho_s(\pi/6)\left(D_o^3 - D_i^3\right)h_{sf}$$

$$t_2 = \frac{8900\,kg/m^3(\pi/6)\left[\left(2\times10^{-5}m\right)^3 - \left(1.6\times10^{-5}m\right)^3\right]2.59\times10^5\,J/kg}{\left(20,000\,W/m^2\cdot K\right)\pi\left(2\times10^{-5}m\right)^2(10,000-1770)K} = 2.28\times10^{-5}s \qquad <$$

COMMENTS: (1) The largest value of the radiation coefficient corresponds to $h_r = \varepsilon\sigma\,(T_{mp} + T_{sur})\left(T_{mp}^2 + T_{sur}^2\right)$. For the maximum possible value of $\varepsilon = 1$ and $T_{sur} = 300K$, $h_r = 378\ W/m^2\cdot K << h = 20{,}000\ W/m^2\cdot K$. Hence, the assumption of negligible radiation exchange is excellent. (2) Despite the large value of h, the small values of D_o and D_i and the large thermal conductivities (~ 40 W/m·K and 70 W/m·K for WC and Co, respectively) render the lumped capacitance approximation a good one. (3) A detailed treatment of plasma heating of a composite powder particle is provided by Demetriou, Lavine and Ghoniem (Proc. 5th ASME/JSME Joint Thermal Engineering Conf., March, 1999).

PROBLEM 5.26

KNOWN: Dimensions and operating conditions of an integrated circuit.

FIND: Steady-state temperature and time to come within 1°C of steady-state.

SCHEMATIC:

ASSUMPTIONS: (1) Constant properties, (2) Negligible heat transfer from chip to substrate.

PROPERTIES: Chip material (given): $\rho = 2000 \text{ kg/m}^3$, $c = 700$ J/kg·K.

ANALYSIS: At steady-state, conservation of energy yields

$$-\dot{E}_{out} + \dot{E}_g = 0$$

$$-h\left(L^2\right)\left(T_f - T_\infty\right) + \dot{q}\left(L^2 \cdot t\right) = 0$$

$$T_f = T_\infty + \frac{\dot{q}t}{h}$$

$$T_f = 20°C + \frac{9\times10^6 \text{ W/m}^3 \times 0.001 \text{ m}}{150 \text{ W/m}^2 \cdot \text{K}} = 80°C. \qquad <$$

From the general lumped capacitance analysis, Equation 5.15 reduces to

$$\rho\left(L^2 \cdot t\right)c\frac{dT}{dt} = \dot{q}\left(L^2 \cdot t\right) - h\left(T - T_\infty\right)L^2.$$

With

$$a \equiv \frac{h}{\rho tc} = \frac{150 \text{ W/m}^2 \cdot \text{K}}{\left(2000 \text{ kg/m}^3\right)\left(0.001 \text{ m}\right)\left(700 \text{ J/kg} \cdot \text{K}\right)} = 0.107 \text{ s}^{-1}$$

$$b \equiv \frac{\dot{q}}{\rho c} = \frac{9\times10^6 \text{ W/m}^3}{\left(2000 \text{ kg/m}^3\right)\left(700 \text{ J/kg} \cdot \text{K}\right)} = 6.429 \text{ K/s}.$$

From Equation 5.24,

$$\exp\left(-at\right) = \frac{T - T_\infty - b/a}{T_i - T_\infty - b/a} = \frac{\left(79 - 20 - 60\right)\text{K}}{\left(20 - 20 - 60\right)\text{K}} = 0.01667$$

$$t = -\frac{\ln\left(0.01667\right)}{0.107 \text{ s}^{-1}} = 38.3 \text{ s}. \qquad <$$

COMMENTS: Due to additional heat transfer from the chip to the substrate, the actual values of T_f and t are less than those which have been computed.

PROBLEM 5.27

KNOWN: Dimensions and operating conditions of an integrated circuit.

FIND: Steady-state temperature and time to come within 1°C of steady-state.

SCHEMATIC:

ASSUMPTIONS: (1) Constant properties.

PROPERTIES: Chip material (given): $\rho = 2000$ kg/m^3, $c_p = 700$ J/kg·K.

ANALYSIS: The direct and indirect paths for heat transfer from the chip to the coolant are in parallel, and the equivalent resistance is

$$R_{equiv} = \left[hL^2 + R_t^{-1}\right]^{-1} = \left[\left(3.75\times10^{-3} + 5\times10^{-3}\right)\text{W/K}\right]^{-1} = 114.3\text{ K/W}.$$

The corresponding overall heat transfer coefficient is

$$U = \frac{\left(R_{equiv}\right)^{-1}}{L^2} = \frac{0.00875\text{ W/K}}{(0.005\text{ m})^2} = 350\text{ W/m}^2\cdot\text{K}.$$

To obtain the steady-state temperature, apply conservation of energy to a control surface about the chip.

$$-\dot{E}_{out} + \dot{E}_g = 0 \qquad -UL^2\left(T_f - T_\infty\right) + \dot{q}\left(L^2\cdot t\right) = 0$$

$$T_f = T_\infty + \frac{\dot{q}t}{U} = 20^\circ\text{C} + \frac{9\times10^6\text{ W/m}^3\times0.001\text{ m}}{350\text{ W/m}^2\cdot\text{K}} = 45.7^\circ\text{C}.$$ <

From the general lumped capacitance analysis, Equation 5.15 yields

$$\rho\left(L^2t\right)c\frac{dT}{dt} = \dot{q}\left(L^2t\right) - U\left(T - T_\infty\right)L^2.$$

With

$$a \equiv \frac{U}{\rho\, tc} = \frac{350\text{ W/m}^2\cdot\text{K}}{\left(2000\text{ kg/m}^3\right)(0.001\text{ m})(700\text{ J/kg}\cdot\text{K})} = 0.250\text{ s}^{-1}$$

$$b = \frac{\dot{q}}{\rho\, c} = \frac{9\times10^6\text{ W/m}^3}{\left(2000\text{ kg/m}^3\right)(700\text{ J/kg}\cdot\text{K})} = 6.429\text{ K/s}$$

Equation 5.24 yields

$$\exp(-at) = \frac{T - T_\infty - b/a}{T_i - T_\infty - b/a} = \frac{(44.7 - 20 - 25.7)\text{K}}{(20 - 20 - 25.7)\text{K}} = 0.0389$$

$$t = -\ln(0.0389)/0.250\text{ s}^{-1} = 13.0\text{ s}.$$ <

COMMENTS: Heat transfer through the substrate is comparable to that associated with direct convection to the coolant.

PROBLEM 5.28

KNOWN: Dimensions, initial temperature and thermophysical properties of chip, solder and substrate. Temperature and convection coefficient of heating agent.

FIND: (a) Time constants and temperature histories of chip, solder and substrate when heated by an air stream. Time corresponding to maximum stress on a solder ball. (b) Reduction in time associated with using a dielectric liquid to heat the components.

SCHEMATIC:

Chip
ρ_{ch} = 2300 kg/m^3
c_{ch} = 710 J/kg-K
T_i = 20^{o}C
Solder ball
D = 0.002 mm
ρ_{sd} = 11,000 kg/m^3
c_{sd} = 130 J/kg-K
T_∞ = 80^{o}C
h_1 = 50 W/m^2-K
h_2 = 200 W/m^2-K
L_{ch} = 0.015 m
t_{ch}= 0.002 m
Substrate
ρ_{sb} = 4000 kg/m^3
c_{sb} = 770 J/kg-K
t_{sb} = 0.01 m
L_{sb} = 0.025 m

ASSUMPTIONS: (1) Lumped capacitance analysis is valid for each component, (2) Negligible heat transfer between components, (3) Negligible reduction in surface area due to contact between components, (4) Negligible radiation for heating by air stream, (5) Uniform convection coefficient among components, (6) Constant properties.

ANALYSIS: (a) From Eq. (5.7), $\tau_t = (\rho V c)/hA$

Chip: $V = \left(L_{ch}^2\right)t_{ch} = (0.015\text{m})^2(0.002\text{m}) = 4.50\times10^{-7}\,\text{m}^3,\ A_s = \left(2L_{ch}^2 + 4L_{ch}t_{ch}\right)$

$= 2(0.015\text{m})^2 + 4(0.015\text{m})0.002\text{m} = 5.70\times10^{-4}\,\text{m}^2$

$$\tau_t = \frac{2300\,\text{kg/m}^3\times4.50\times10^{-7}\,\text{m}^3\times710\,\text{J/kg}\cdot\text{K}}{50\,\text{W/m}^2\cdot\text{K}\times5.70\times10^{-4}\,\text{m}^2} = 25.8\text{s} \quad <$$

Solder: $V = \pi D^3/6 = \pi(0.002\text{m})^3/6 = 4.19\times10^{-9}\,\text{m}^3,\ A_s = \pi D^2 = \pi(0.002\text{m})^2 = 1.26\times10^{-5}\,\text{m}^2$

$$\tau_t = \frac{11{,}000\,\text{kg/m}^3\times4.19\times10^{-9}\,\text{m}^3\times130\,\text{J/kg}\cdot\text{K}}{50\,\text{W/m}^2\cdot\text{K}\times1.26\times10^{-5}\,\text{m}^2} = 9.5\text{s} \quad <$$

Substrate: $V = \left(L_{sb}^2\,t_{sb}\right) = (0.025\text{m})^2(0.01\text{m}) = 6.25\times10^{-6}\,\text{m}^3,\ A_s = L_{sb}^2 = (0.025\text{m})^2 = 6.25\times10^{-4}\,\text{m}^2$

$$\tau_t = \frac{4000\,\text{kg/m}^3\times6.25\times10^{-6}\,\text{m}^3\times770\,\text{J/kg}\cdot\text{K}}{50\,\text{W/m}^2\cdot\text{K}\times6.25\times10^{-4}\,\text{m}^2} = 616.0\text{s} \quad <$$

Substituting Eq. (5.7) into (5.5) and recognizing that $(T - T_i)/(T_\infty - T_i) = 1 - (\theta/\theta_i)$, in which case $(T - T_i)/(T_\infty - T_i) = 0.99$ yields $\theta/\theta_i = 0.01$, it follows that the time required for a component to experience 99% of its maximum possible temperature rise is

$$t_{0.99} = \tau\ln(\theta_i/\theta) = \tau\ln(100) = 4.61\tau$$

Hence,

Chip: $t = 118.9\text{s}$, *Solder*: $t = 43.8\text{s}$, *Substrate*: $t = 2840$ <

Continued …..

PROBLEM 5.28 (Cont.)

Histories of the three components and temperature differences between a solder ball and its adjoining components are shown below.

Commensurate with their time constants, the fastest and slowest responses to heating are associated with the solder and substrate, respectively. Accordingly, the largest temperature difference is between these two components, and it achieves a maximum value of 55°C at

$$t(\text{maximum stress}) \approx 40\text{s}$$ <

(b) With the 4-fold increase in h associated with use of a dielectric liquid to heat the components, the time constants are each reduced by a factor of 4, and the times required to achieve 99% of the maximum temperature rise are

Chip: $t = 29.5\text{s}$, *Solder*: $t = 11.0\text{s}$, *Substrate*: $t = 708\text{s}$ <

The time savings is approximately 75%.

COMMENTS: The foregoing analysis provides only a first, albeit useful, approximation to the heating problem. Several of the assumptions are highly approximate, particularly that of a uniform convection coefficient. The coefficient will vary between components, as well as on the surfaces of the components. Also, because the solder balls are flattened, there will be a reduction in surface area exposed to the fluid for each component, as well as heat transfer between components, which reduces differences between time constants for the components.

PROBLEM 5.29

KNOWN: Electrical transformer of approximate cubical shape, 32 mm to a side, dissipates 4.0 W when operating in ambient air at 20°C with a convection coefficient of 10 W/m^2·K.

FIND: (a) Develop a model for estimating the steady-state temperature of the transformer, T(∞), and evaluate T(∞), for the operating conditions, and (b) Develop a model for estimating the temperature-time history of the transformer if initially the temperature is $T_i = T_\infty$ and suddenly power is applied. Determine the time required to reach within 5°C of its steady-state operating temperature.

SCHEMATIC:

ASSUMPTIONS: (1) Transformer is spatially isothermal object, (2) Initially object is in equilibrium with its surroundings, (3) Bottom surface is adiabatic.

ANALYSIS: (a) Under steady-state conditions, for the control volume shown in the schematic above, the energy balance is

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = 0 \qquad 0 - q_{cv} + P_e = -h A_s \left[T(\infty) - T_\infty\right] + P_e = 0 \qquad (1)$$

where $A_s = 5 \times L^2 = 5 \times 0.032\text{m} \times 0.032\text{m} = 5.12 \times 10^{-3}\ \text{m}^2$, find

$$T(\infty) = T_\infty + P_e / h A_s = 20°\text{C} + 4\,\text{W} / \left(10\ \text{W/m}^2 \cdot \text{K} \times 5.12 \times 10^{-3}\text{m}^2\right) = 98.1°\text{C}$$ <

(b) Under transient conditions, for the control volume shown above, the energy balance is

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = \dot{E}_{st} \qquad 0 - q_{cv} + P_e = Mc\frac{dT}{dt} \qquad (2)$$

Substitute from Eq. (1) for P_e, separate variables, and define the limits of integration.

$$-h\left[T(t) - T_\infty\right] + h\left[T(\infty) - T_\infty\right] = Mc\frac{dT}{dt}$$

$$-h\left[T(t) - T(\infty)\right] = Mc\frac{d}{dt}\left(T - T(\infty)\right) \qquad \frac{h}{Mc}\int_0^{t_o} dt = -\int_{\theta_i}^{\theta_o} \frac{d\theta}{\theta}$$

where $\theta = T(t) - T(\infty)$; $\theta_i = T_i - T(\infty) = T_\infty - T(\infty)$; and $\theta_o = T(t_o) - T(\infty)$ with t_o as the time when θ_o = - 5°C. Integrating and rearranging find (see Eq. 5.5),

$$t_o = \frac{Mc}{h A_s}\ell n\frac{\theta_i}{\theta_o}$$

$$t_o = \frac{0.28\ \text{kg} \times 400\ \text{J/kg} \cdot \text{K}}{10\ \text{W/m}^2 \cdot \text{K} \times 5.12 \times 10^{-3}\text{m}^2}\ell n\frac{(20 - 98.1)°\text{C}}{-5°\text{C}} = 1.67\ \text{hour}$$ <

COMMENTS: The spacewise isothermal assumption may not be a gross over simplification since most of the material is copper and iron, and the external resistance by free convection is high. However, by ignoring internal resistance, our estimate for t_o is optimistic.

PROBLEM 5.30

KNOWN: Series solution, Eq. 5.39, for transient conduction in a plane wall with convection.

FIND: Midplane (x*=0) and surface (x*=1) temperatures θ* for Fo=0.1 and 1, using Bi=0.1, 1 and 10 with only the first four eigenvalues. Based upon these results, discuss the validity of the approximate solutions, Eqs. 5.40 and 5.41.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional transient conduction, (2) Constant properties.

ANALYSIS: The series solution, Eq. 5.39a, is of the form,

$$q^* = \sum_{n=1}^{\infty} C_n \exp\left(-z_n^2 Fo\right)\cos\left(z_n x^*\right)$$

where the eigenvalues, z_n, and the constants, C_n, are from Eqs. 5.39b and 5.39c.

$$z_n \tan z_n = Bi \qquad C_n = 4\sin z_n / \left(2z_n + \sin\left(2z_n\right)\right).$$

The eigenvalues are tabulated in Appendix B.3; note, however, that z_1 and C_1 are available from Table 5.1. The values of z_n and C_n used to evaluate θ* are as follows:

Bi	z_1	C_1	z_2	C_2	z_3	C_3	z_4	C_4
0.1	0.3111	1.0160	3.1731	-0.0197	6.2991	0.0050	9.4354	-0.0022
1	0.8603	1.1191	3.4256	-0.1517	6.4373	0.0466	9.5293	-0.0217
10	1.4289	1.2620	4.3058	-0.3934	7.2281	0.2104	10.2003	-0.1309

Using z_n and C_n values, the terms of q^*, designated as q_1^*, q_2^*, q_3^* and q_4^*, are as follows:

Fo=0.1

	Bi=0.1		Bi=1.0		Bi=10	
x*	0	1	0	1	0	1
q_1^*	1.0062	0.9579	1.0393	0.6778	1.0289	0.1455
q_2^*	-0.0072	0.0072	-0.0469	0.0450	-0.0616	0.0244
q_3^*	0.0001	0.0001	0.0007	0.0007	0.0011	0.0006
q_4^*	-2.99×10^{-7}	3.00×10^{-7}	2.47×10^{-6}	2.46×10^{-7}	-3.96×10^{-6}	2.83×10^{-6}
q^*	0.9991	0.9652	0.9931	0.7235	0.9684	0.1705

Continued …..

Fo=1

	Bi=0.1		Bi=1.0		Bi=10	
x*	0	1	0	1	0	1
q_1^*	0.9223	0.8780	0.5339	0.3482	0.1638	0.0232
q_2^*	8.35×10^{-7}	8.35×10^{-7}	-1.22×10^{-5}	1.17×10^{-6}	3.49×10^{-9}	1.38×10^{-9}
q_3^*	7.04×10^{-20}	-	4.70×10^{-20}	-	4.30×10^{-24}	-
q_4^*	4.77×10^{-42}	-	7.93×10^{-42}	-	8.52×10^{-47}	-
q^*	0.9223	0.8780	0.5339	0.3482	0.1638	0.0232

The tabulated results for $q^* = q^*\left(x^*, \text{Bi}, \text{Fo}\right)$ demonstrate that for Fo=1, the first eigenvalue is sufficient to accurately represent the series. However, for Fo=0.1, three eigenvalues are required for accurate representation.

A more detailed analysis would show that a practical criterion for representation of the series solution by one eigenvalue is Fo>0.2. For these situations the approximate solutions, Eqs. 5.40 and 5.41, are appropriate. For the midplane, $x^*=0$, the first two eigenvalues for Fo=0.2 are:

	Fo=0.2	x*=0	
Bi	0.1	1.0	10
q_1^*	0.9965	0.9651	0.8389
q_2^*	-0.00226	-0.0145	-0.0096
q^*	0.9939	0.9506	0.8293
Error,%	+0.26	+1.53	+1.16

The percentage error shown in the last row of the above table is due to the effect of the second term. For Bi=0.1, neglecting the second term provides an error of 0.26%. For Bi=1, the error is 1.53%.

Hence we conclude that the approximate series solutions (with only one eigenvalue) provides systematically high results, but by less than 1.5%, for the Biot number range from 0.1 to 10.

PROBLEM 5.31

KNOWN: One-dimensional wall, initially at a uniform temperature, T_i, is suddenly exposed to a convection process (T_∞, h). For wall #1, the time (t_1 = 100s) required to reach a specified temperature at x = L is prescribed, $T(L_1, t_1)$ = 315°C.

FIND: For wall #2 of different thickness and thermal conditions, the time, t_2, required for $T(L_2, t_2)$ = 28°C.

SCHEMATIC:

$T(x,<0)=T_i$

T_∞, h

$T(L_1, 100s) = 315°C$

$T(L_2, t_2) = 28.5°C$

x L

ASSUMPTIONS: (1) One-dimensional conduction, (2) Constant properties.

ANALYSIS: The properties, thickness and thermal conditions for the two walls are:

Wall	L(m)	$\alpha(m^2/s)$	k(W/m·K)	T_i(°C)	T_∞(°C)	$h(W/m^2\cdot K)$
1	0.10	15×10^{-6}	50	300	400	200
2	0.40	25×10^{-6}	100	30	20	100

The dimensionless functional dependence for the one-dimensional, transient temperature distribution, Eq. 5.38, is

$$q^* = \frac{T(x,t) - T_\infty}{T_i - T_\infty} = f\left(x^*, \text{Bi}, \text{Fo}\right)$$

where

$$x^* = x/L \qquad \text{Bi} = hL/k \qquad \text{Fo} = a\,t/L^2.$$

If the parameters x*, Bi, and Fo are the same for both walls, then $q_1^* = q_2^*$. Evaluate these parameters:

Wall	x*	Bi	Fo	θ*
1	1	0.40	0.150	0.85
2	1	0.40	$1.563\times10^{-4}\ t_2$	0.85

where

$$q_1^* = \frac{315-400}{300-400} = 0.85 \qquad q_2^* = \frac{28.5-20}{30-20} = 0.85.$$

It follows that

$$\text{Fo}_2 = \text{Fo}_1 \qquad 1.563\times10^{-4} t_2 = 0.150$$

$$t_2 = 960\text{s}.$$

<

PROBLEM 5.32

KNOWN: The chuck of a semiconductor processing tool, initially at a uniform temperature of T_i = 100°C, is cooled on its top surface by supply air at 20°C with a convection coefficient of 50 W/m^2·K.

FIND: (a) Time required for the lower surface to reach 25°C, and (b) Compute and plot the time-to-cool as a function of the convection coefficient for the range 10 ≤ h ≤ 2000 W/m^2·K; comment on the effectiveness of the head design as a method for cooling the chuck.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional, transient conduction in the chuck, (2) Lower surface is perfectly insulated, (3) Uniform convection coefficient and air temperature over the upper surface of the chuck, and (4) Constant properties.

PROPERTIES: *Table A.1*, Aluminum alloy 2024 ((25 + 100)°C / 2 = 335 K): ρ = 2770 kg/m^3, c_p = 880 J/kg· K, k = 179 W/m·K.

ANALYSIS: (a) The Biot number for the chuck with h = 50 W/m^2·K is

$$\mathrm{Bi} = \frac{hL}{k} = \frac{50\,\mathrm{W/m^2 \cdot K} \times 0.025\,\mathrm{m}}{179\,\mathrm{W/m \cdot K}} = 0.007 \le 0.1 \qquad (1)$$

so that the lumped capacitance method is appropriate. Using Eq. 5.5, with $V/A_s = L$,

$$t = \frac{\rho V c}{h A_s} \ln \frac{\theta_i}{\theta} \qquad \theta = T - T_\infty \qquad \theta_i = T_i - T_\infty$$

$$t = \left(2770\,\mathrm{kg/m^3} \times 0.025\,\mathrm{m} \times 880\,\mathrm{J/kg \cdot K} / 50\,\mathrm{W/m^2 \cdot K}\right) \ln \frac{(100-20)^\circ \mathrm{C}}{(25-20)^\circ \mathrm{C}}$$

$$t = 3379\,\mathrm{s} = 56.3\,\mathrm{min}$$ <

Continued...

PROBLEM 5.32 (Cont.)

(b) When h = 2000 $W/m^2 \cdot K$, using Eq. (1), find Bi = 0.28 > 0.1 so that the series solution, Section 5.51, for the plane wall with convection must be used. Using the *IHT Transient Conduction, Plane Wall Model*, the time-to-cool was calculated as a function of the convection coefficient. Free convection cooling conduction corresponds to h ≈ 10 $W/m^2 \cdot K$ and the time-to-cool is 282 minutes. With the cooling head design, the time-to-cool can be substantially decreased if the convection coefficient can be increased as shown below.

Time-to-cool, t (min)

Convection coefficient, h (W/m^2.K)

PROBLEM 5.33

KNOWN: Configuration, initial temperature and charging conditions of a thermal energy storage unit.

FIND: Time required to achieve 75% of maximum possible energy storage and corresponding minimum and maximum temperatures.

SCHEMATIC:

$T_\infty = 600°C$
$h = 100 W/m^2 \cdot K$
$L = 0.025 m$
x
Masonry
$\rho = 1900 kg/m^3$
$c = 800 J/kg \cdot K$
$k = 0.7 W/m \cdot K$

ASSUMPTIONS: (1) One-dimensional conduction, (2) Constant properties, (3) Negligible radiation exchange with surroundings.

ANALYSIS: For the system, find first

$$Bi = \frac{hL}{k} = \frac{100 \text{ W/m}^2 \cdot \text{K} \times 0.025\text{m}}{0.7 \text{ W/m} \cdot \text{K}} = 3.57$$

indicating that the lumped capacitance method cannot be used.

Groeber chart, Fig. D.3: $Q/Q_o = 0.75$

$$\alpha = \frac{k}{\rho c} = \frac{0.7 \text{ W/m} \cdot \text{K}}{1900 \text{ kg/m}^3 \times 800 \text{ J/kg} \cdot \text{K}} = 4.605 \times 10^{-7} \text{ m}^2/\text{s}$$

$$Bi^2 Fo = \frac{h^2 \alpha t}{k^2} = \frac{\left(100 \text{ W/m}^2\text{K}\right)^2 \times \left(4.605 \times 10^{-7} \text{m}^2/\text{s}\right) \times t(\text{s})}{\left(0.7 \text{ W/m} \cdot \text{K}\right)^2} = 9.4 \times 10^{-3} t$$

Find $Bi^2 Fo \approx 11$, and substituting numerical values

$$t = 11/9.4 \times 10^{-3} = 1170\text{s}.$$ <

Heisler chart, Fig. D.1: T_{min} is at x = 0 and T_{max} at x = L, with

$$Fo = \frac{\alpha t}{L^2} = \frac{4.605 \times 10^{-7} \text{m}^2/\text{s} \times 1170 \text{ s}}{(0.025\text{m})^2} = 0.86 \qquad Bi^{-1} = 0.28.$$

From Fig. D.1, $q_o^* \approx 0.33$. Hence,

$$T_o \approx T_\infty + 0.33\left(T_i - T_\infty\right) = 600°C + 0.33\left(-575°C\right) = 410°C = T_{min}.$$ <

From Fig. D.2, $\theta/\theta_o \approx 0.33$ at x = L, for which

$$T_{x=L} \approx T_\infty + 0.33\left(T_o - T_\infty\right) = 600°C + 0.33(-190)° C = 537°C = T_{max}.$$ <

COMMENTS: Comparing masonry (m) with aluminum (Al), see Problem 5.10, $(\rho c)_{Al} > (\rho c)_m$ and $k_{Al} > k_m$. Hence, the aluminum can store more energy and can be charged (or discharged) more quickly.

PROBLEM 5.34

KNOWN: Thickness, properties and initial temperature of steel slab. Convection conditions.

FIND: Heating time required to achieve a minimum temperature of 550°C in the slab.

SCHEMATIC:

Combustion gases
$T_\infty = 800°C$
$h = 250\ W/m^2\text{-}K$
$L = 0.05\ m$
Steel, $T_i = 200°C$
$\rho = 7830\ kg/m^3$
$c = 550\ J/kg\text{-}K$
$k = 48\ W/m\text{-}K$
T_∞, h

ASSUMPTIONS: (1) One-dimensional conduction, (2) Negligible radiation effects, (3) Constant properties.

ANALYSIS: With a Biot number of hL/k = (250 W/m^2·K × 0.05m)/48 W/m·K = 0.260, a lumped capacitance analysis should not be performed. At any time during heating, the lowest temperature in the slab is at the midplane, and from the one-term approximation to the transient thermal response of a plane wall, Eq. (5.41), we obtain

$$\theta_o^* = \frac{T_o - T_\infty}{T_i - T_\infty} = \frac{(550-800)°C}{(200-800)°C} = 0.417 = C_1 \exp\left(-\zeta_1^2 Fo\right)$$

With $\zeta_1 \approx 0.488$ rad and $C_1 \approx 1.0396$ from Table 5.1 and $\alpha = k/\rho c = 1.115\times10^{-5} m^2/s$,

$$-\zeta_1^2\left(\alpha t / L^2\right) = \ln(0.401) = -0.914$$

$$t = \frac{0.914 L^2}{\zeta_1^2 \alpha} = \frac{0.841(0.05m)^2}{(0.488)^2\, 1.115\times10^{-5} m^2/s} = 861s$$ <

COMMENTS: The surface temperature at t = 861s may be obtained from Eq. (5.40b), where $\theta^* = \theta_o^* \cos\left(\zeta_1 x^*\right) = 0.417 \cos(0.488\ \text{rad}) = 0.368.$ Hence, $T(L, 792s) \equiv T_s = T_\infty + 0.368(T_i - T_\infty)$ $= 800°C - 221°C = 579°C.$ Assuming a surface emissivity of ε = 1 and surroundings that are at $T_{sur} = T_\infty = 800°C,$ the radiation heat transfer coefficient corresponding to this surface temperature is $h_r = \varepsilon\sigma(T_s + T_{sur})\left(T_s^2 + T_{sur}^2\right) = 205\ W/m^2 \cdot K.$ Since this value is comparable to the convection coefficient, radiation is not negligible and the desired heating will occur well before t = 861s.

PROBLEM 5.35

KNOWN: Pipe wall subjected to sudden change in convective surface condition. See Example 5.4.

FIND: (a) Temperature of the inner and outer surface of the pipe, heat flux at the inner surface, and energy transferred to the wall after 8 min; compare results to the hand calculations performed for the Text Example; (b) Time at which the outer surface temperature of the pipe, T(0,t), will reach 25°C; (c) Calculate and plot on a single graph the temperature distributions, T(x,t) vs. x, for the initial condition, the final condition and the intermediate times of 4 and 8 min; explain key features; (d) Calculate and plot the temperature-time history, T(x,t) vs. t, for the locations at the inner and outer pipe surfaces, x = 0 and L, and for the range $0 \le t \le 16$ min. Use the *IHT | Models | Transient Conduction | Plane Wall* model as the solution tool.

SCHEMATIC:

ASSUMPTIONS: (1) Pipe wall can be approximated as a plane wall, (2) Constant properties, (3) Outer surface of pipe is adiabatic.

ANALYSIS: The IHT model represents the series solution for the plane wall providing temperatures and heat fluxes evaluated at (x,t) and the total energy transferred at the inner wall at (t). Selected portions of the IHT code used to obtain the results tabulated below are shown in the Comments.

(a) The code is used to evaluate the tabulated parameters at t = 8 min for locations x = 0 and L. The agreement is very good between the one-term approximation of the Example and the multiple-term series solution provided by the IHT model.

	Text Ex 5.4	IHT Model
T(L, 8min), °C	45.2	45.4
T(0, 8 min), °C	42.9	43.1
$Q'(8\ \text{min})\times 10^{-7}$, J/m	-2.73	-2.72
$q''_x(L,\ 8\ \text{min})$, W/m^2	-7400	-7305

(b) To determine the time t_o for which T(0,t) = 25°C, the IHT model is solved for t_o after setting x = 0 and T_xt = 25°C. Find, t_o = 4.4 min. <

(c) The temperature distributions, T(x,t) vs x, for the initial condition (t = 0), final condition ($t \to \infty$) and intermediate times of 4 and 8 min. are shown on the graph below.

Continued …..

PROBLEM 5.35 (Cont.)

The final condition corresponds to the steady-state temperature, $T(x,\infty) = T_\infty$. For the intermediate times, the gradient is zero at the insulated boundary (x = 0, the pipe exterior). As expected, the temperature at x = 0 will be less than at the boundary experiencing the convection process with the hot oil, x = L. Note, however, that the difference is not very significant. The gradient at the inner wall, x = L, decreases with increasing time.

(d) The temperature history T(x,t) for the locations at the inner and outer pipe surfaces are shown in the graph below. Note that the temperature difference between the two locations is greatest at the start of the transient process and decreases with increasing time. After a 16 min. duration, the pipe temperature is almost uniform, but yet 3 or 4°C from the steady-state condition.

Temperature-time history, T(x,t) vs. t

Temperature, T(x,t) (C)

60
40
20
0
-20

0 2 4 6 8 10 12 14 16

Time, t (min)

Outer surface, x = 0
Inner surface, x = L

COMMENTS: (1) Selected portions of the IHT code for the plane wall model are shown below. Note the relation for the pipe volume, vol, used in calculating the total heat transferred per unit length over the time interval t.

```
// Models | Transient Conduction | Plane Wall
// The temperature distribution is
T_xt = T_xt_trans("Plane Wall",xstar,Fo,Bi,Ti,Tinf)    // Eq 5.39
//T_xt = 25                          // Part (b) surface temperature, x = 0
// The heat flux in the x direction is
q''_xt = qdprime_xt_trans("Plane Wall",x,L,Fo,Bi,k,Ti,Tinf)   // Eq 2.6

// The total heat transfer from the wall over the time interval t is
QoverQo = Q_over_Qo_trans("Plane Wall",Fo,Bi)     // Eq 5.45
Qo = rho * cp * vol * (Ti - Tinf)     // Eq 5.44
//vol = 2 * As * L                 // Appropriate for wall of 2L thickness
vol = pi * D * L                   // Pipe wall of diameter D, thickness L and unit length
Q = QoverQo * Qo                   // Total energy transfered per unit length
```

(2) Can you give an explanation for why the inner and outer surface temperatures are not very different? What parameter provides a measure of the temperature non-uniformity in a system during a transient conduction process?

PROBLEM 5.36

KNOWN: Thickness, initial temperature and properties of furnace wall. Convection conditions at inner surface.

FIND: Time required for outer surface to reach a prescribed temperature. Corresponding temperature distribution in wall and at intermediate times.

SCHEMATIC:

Fire-clay brick:
$T_i = 20^oC$
$T_f(0,t) = 750^oC$

$\rho = 2600\ kg/m^3$
$c_p = 1000\ J/kg\text{-}K$
$k = 1.5\ W/m\text{-}K$

Combustion gases

$T_\infty = 950^oC$
$h = 100\ W/m^2\text{-}K$

x   L = 0.15 m

ASSUMPTIONS: (1) One-dimensional conduction in a plane wall, (2) Constant properties, (3) Adiabatic outer surface, (4) Fo > 0.2, (5) Negligible radiation from combustion gases.

ANALYSIS: The wall is equivalent to one-half of a wall of thickness 2L with symmetric convection conditions at its two surfaces. With $Bi = hL/k = 100\ W/m^2\cdot K \times 0.15m/1.5\ W/m\cdot K = 10$ and Fo > 0.2, the one-term approximation, Eq. 5.41 may be used to compute the desired time, where $\theta_o^* = (T_o - T_\infty)/(T_i - T_\infty) = 0.215$. From Table 5.1, $C_1 = 1.262$ and $\zeta_1 = 1.4289$. Hence,

$$Fo = -\frac{\ln\left(\theta_o^*/C_1\right)}{\zeta_1^2} = -\frac{\ln(0.215/1.262)}{(1.4289)^2} = 0.867$$

$$t = \frac{Fo\ L^2}{\alpha} = \frac{0.867(0.15m)^2}{\left(1.5\,W/m\cdot K/2600\ kg/m^3 \times 1000\ J/kg\cdot K\right)} = 33{,}800\,s$$ <

The corresponding temperature distribution, as well as distributions at t = 0, 10,000, and 20,000 s are plotted below

Temperature, C
1000
800
600
400
200
0
0 0.2 0.4 0.6 0.8 1
Dimensionless location, x/L

t=0 s
t=10,000 s
t=20,000 s
t=33,800 s

COMMENTS: Because Bi >>1, the temperature at the inner surface of the wall increases much more rapidly than at locations within the wall, where temperature gradients are large. The temperature gradients decrease as the wall approaches a steady-state for which there is a uniform temperature of 950°C.

PROBLEM 5.37

KNOWN: Thickness, initial temperature and properties of steel plate. Convection conditions at both surfaces.

FIND: Time required to achieve a minimum temperature.

SCHEMATIC:

Steel plate:
$T_i = 300^oC$
$T(0,t_f) = 550^oC$
$\rho = 7800$ kg/m^3
$c_p = 500$ J/kg-K
k = 45 W/m-K
Combustion gases
$T_\infty = 700^oC$
h = 100 W/m^2-K
x
L = 50 mm

ASSUMPTIONS: (1) One-dimensional conduction in plate, (2) Symmetric heating on both sides, (3) Constant properties, (4) Negligible radiation from gases, (5) Fo > 0.2.

ANALYSIS: The smallest temperature exists at the midplane and, with Bi = hL/k = 500 W/m^2·K × 0.050m/45 W/m·K = 0.556 and Fo > 0.2, may be determined from the one-term approximation of Eq. 5.41. From Table 5.1, C_1 = 1.076 and ζ_1 = 0.682. Hence, with θ_o^* = $(T_o - T_\infty)/(T_i - T_\infty)$ = 0.375,

$$\text{Fo} = -\frac{\ln\left(\theta_o^* / C_1\right)}{\zeta_1^2} = -\frac{\ln(0.375/1.076)}{(0.682)^2} = 2.266$$

$$t = \frac{\text{Fo } L^2}{\alpha} = \frac{2.266(0.05\text{m})^2}{\left(45\,\text{W/m}\cdot\text{K}/7800\text{ kg/m}^3 \times 500\text{ J/kg}\cdot\text{K}\right)} = 491\text{s}$$ <

COMMENTS: From Eq. 5.40b, the corresponding surface temperature is

$$T_s = T_\infty + (T_i - T_\infty)\theta_o^* \cos(\zeta_1) = 700°\text{C} - 400°\text{C} \times 0.375 \times 0.776 = 584°\text{C}$$

Because Bi is not much larger than 0.1, temperature gradients in the steel are moderate.

PROBLEM 5.38

KNOWN: Plate of thickness 2L = 25 mm at a uniform temperature of 600°C is removed from a hot pressing operation. Case 1, cooled on both sides; case 2, cooled on one side only.

FIND: (a) Calculate and plot on one graph the temperature histories for cases 1 and 2 for a 500-second cooling period; use the *IHT* software; Compare times required for the maximum temperature in the plate to reach 100°C; and (b) For both cases, calculate and plot on one graph, the variation with time of the maximum temperature difference in the plate; Comment on the relative magnitudes of the temperature gradients within the plate as a function of time.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional conduction in the plate, (2) Constant properties, and (3) For case 2, with cooling on one side only, the other side is adiabatic.

PROPERTIES: Plate (*given*): $\rho = 3000$ kg/m^3, c = 750 J/kg·K, k = 15 W/m·K.

ANALYSIS: (a) From *IHT*, call up *Plane Wall, Transient Conduction* from the *Models* menu. For case 1, the plate thickness is 25 mm; for case 2, the plate thickness is 50 mm. The plate center (x = 0) temperature histories are shown in the graph below. The times required for the center temperatures to reach 100°C are

$t_1 = 164$ s $\qquad\qquad t_2 = 367$ s $\qquad\qquad$ <

(b) The plot of T(0, t) – T(1, t), which represents the maximum temperature difference in the plate during the cooling process, is shown below.

COMMENTS: (1) From the plate center-temperature history graph, note that it takes more than twice as long for the maximum temperature to reach 100°C with cooling on only one side.

(2) From the maximum temperature-difference graph, as expected, cooling from one side creates a larger maximum temperature difference during the cooling process. The effect could cause microstructure differences, which could adversely affect the mechanical properties within the plate.

PROBLEM 5.39

KNOWN: Properties and thickness L of ceramic coating on rocket nozzle wall. Convection conditions. Initial temperature and maximum allowable wall temperature.

FIND: (a) Maximum allowable engine operating time, t_{max}, for L = 10 mm, (b) Coating inner and outer surface temperature histories for L = 10 and 40 mm.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional conduction in a plane wall, (2) Constant properties, (3) Negligible thermal capacitance of metal wall and heat loss through back surface, (4) Negligible contact resistance at wall/ceramic interface, (5) Negligible radiation.

ANALYSIS: (a) Subject to assumptions (3) and (4), the maximum wall temperature corresponds to the ceramic temperature at x = 0. Hence, for the ceramic, we wish to determine the time t_{max} at which T(0,t) = $T_o(t)$ = 1500 K. With Bi = hL/k = 5000 W/m^2·K(0.01 m)/10 W/m·K = 5, the lumped capacitance method cannot be used. Assuming Fo > 0.2, obtaining ζ_1 = 1.3138 and C_1 = 1.2402 from Table 5.1, and evaluating $\theta_o^* = (T_o - T_\infty)/(T_i - T_\infty)$ = 0.4, Equation 5.41 yields

$$\text{Fo} = -\frac{\ln\left(\theta_o^*/C_1\right)}{\zeta_1^2} = -\frac{\ln(0.4/1.2402)}{(1.3138)^2} = 0.656$$

confirming the assumption of Fo > 0.2. Hence,

$$t_{max} = \frac{\text{Fo}\left(L^2\right)}{\alpha} = \frac{0.656(0.01\,\text{m})^2}{6\times10^{-6}\,\text{m}^2/\text{s}} = 10.9\,\text{s}$$ <

(b) Using the IHT *Lumped Capacitance Model* for a *Plane Wall*, the inner and outer surface temperature histories were computed and are as follows:

Continued...

PROBLEM 5.39 (Cont.)

The increase in the inner (x = 0) surface temperature lags that of the outer surface, but within t ≈ 45s both temperatures are within a few degrees of the gas temperature for L = 0.01 m. For L = 0.04 m, the increased thermal capacitance of the ceramic slows the approach to steady-state conditions. The thermal response of the inner surface significantly lags that of the outer surface, and it is not until t ≈ 137s that the inner surface reaches 1500 K. At this time there is still a significant temperature difference across the ceramic, with $T(L,t_{max})$ = 2240 K.

COMMENTS: The allowable engine operating time increases with increasing thermal capacitance of the ceramic and hence with increasing L.

PROBLEM 5.40

KNOWN: Initial temperature, thickness and thermal diffusivity of glass plate. Prescribed surface temperature.

FIND: (a) Time to achieve 50% reduction in midplane temperature, (b) Maximum temperature gradient at that time.

SCHEMATIC:

2L=20mm

Glass, $\alpha = 6\times10^{-7}$ m²/s, T_i

$T_i - T_s = 300°C$

T_s T_s

ASSUMPTIONS: (1) One-dimensional conduction, (2) Constant properties.

ANALYSIS: Prescribed surface temperature is analogous to h → ∞ and $T_\infty = T_s$. Hence, Bi = ∞. Assume validity of one-term approximation to series solution for T (x,t).

(a) At the midplane,

$$\theta_o^* = \frac{T_o - T_s}{T_i - T_s} = 0.50 = C_1 \exp\left(-\zeta_1^2 Fo\right)$$

$$\zeta_1 \tan \zeta_1 = Bi = \infty \rightarrow \zeta_1 = \pi/2.$$

Hence

$$C_1 = \frac{4\sin\zeta_1}{2\zeta_1 + \sin(2\zeta_1)} = \frac{4}{\pi} = 1.273$$

$$Fo = -\frac{\ln\left(\theta_o^*/C_1\right)}{\zeta_1^2} = 0.379$$

$$t = \frac{Fo L^2}{\alpha} = \frac{0.379(0.01\text{ m})^2}{6\times10^{-7}\text{ m}^2/\text{s}} = 63\text{ s.}$$ <

(b) With $\theta^* = C_1 \exp\left(-\zeta_1^2 Fo\right) \cos \zeta_1 x^*$

$$\frac{\partial T}{\partial x} = \frac{(T_i - T_s)}{L}\frac{\partial \theta^*}{\partial x^*} = -\frac{(T_i - T_s)}{L}\zeta_1 C_1 \exp\left(-\zeta_1^2 Fo\right)\sin \zeta_1 x^*$$

$$\partial T/\partial x\Big|_{max} = \partial T/\partial x\Big|_{x^*=1} = -\frac{300°C}{0.01\text{ m}}\frac{\pi}{2}0.5 = -2.36\times10^4\ °C/m.$$ <

COMMENTS: Validity of one-term approximation is confirmed by Fo > 0.2.

PROBLEM 5.41

KNOWN: Thickness and properties of rubber tire. Convection heating conditions. Initial and final midplane temperature.

FIND: (a) Time to reach final midplane temperature. (b) Effect of accelerated heating.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional conduction in a plane wall, (2) Constant properties, (3) Negligible radiation.

ANALYSIS: (a) With Bi = hL/k = 200 W/m^2·K(0.01 m)/0.14 W/m·K = 14.3, the lumped capacitance method is clearly inappropriate. Assuming Fo > 0.2, Eq. (5.41) may be used with C_1 = 1.265 and $\zeta_1 \approx$ 1.458 rad from Table 5.1 to obtain

$$\theta_o^* = \frac{T_o - T_\infty}{T_i - T_\infty} = C_1 \exp\left(-\zeta_1^2 Fo\right) = 1.265 \exp\left(-2.126\, Fo\right)$$

With $\theta_o^* = \left(T_o - T_\infty\right)/\left(T_i - T_\infty\right)$ = (-50)/(-175) = 0.286, $Fo = -\ln\left(0.286/1.265\right)/2.126 = 0.70 = \alpha t_f / L^2$

$$t_f = \frac{0.7\left(0.01\text{m}\right)^2}{6.35\times10^{-8}\ \text{m}^2/\text{s}} = 1100\,\text{s}$$ <

(b) The desired temperature histories were generated using the IHT *Transient Conduction Model* for a *Plane Wall*, with h = 5 × 10^4 W/m^2·K used to approximate imposition of a surface temperature of 200°C.

The fact that imposition of a constant surface temperature (h → ∞) does not significantly accelerate the heating process should not be surprising. For h = 200 W/m^2·K, the Biot number is already quite large (Bi = 14.3), and limits to the heating rate are principally due to conduction in the rubber and not to convection at the surface. Any increase in h only serves to reduce what is already a small component of the total thermal resistance.

COMMENTS: The heating rate could be accelerated by increasing the steam temperature, but an upper limit would be associated with avoiding thermal damage to the rubber.

PROBLEM 5.42

KNOWN: Stack or book comprised of 11 metal plates (p) and 10 boards (b) each of 2.36 mm thickness and prescribed thermophysical properties.

FIND: Effective thermal conductivity, k, and effective thermal capacitance, (ρc_p).

SCHEMATIC:

L = 49.56 mm

Book, M = 11 plates, N = 10 boards

Plate (p)

Board (b), 2.36 mm

Plate (p), 2.36 mm

ASSUMPTIONS: (1) One-dimensional conduction, (2) Negligible contact resistance between plates and boards.

PROPERTIES: Metal plate (p, given): $\rho_p = 8000\ kg/m^3$, $c_{p,p} = 480\ J/kg \cdot K$, $k_p = 12\ W/m \cdot K$; Circuit boards (b, given): $\rho_b = 1000\ kg/m^3$, $c_{p,b} = 1500\ J/kg \cdot K$, $k_b = 0.30\ W/m \cdot K$.

ANALYSIS: The thermal resistance of the book is determined as the sum of the resistance of the boards and plates,

$$R''_{tot} = NR''_b + MR''_p$$

where M,N are the number of plates and boards in the book, respectively, and $R''_i = L_i / k_i$ where L_i and k_i are the thickness and thermal conductivities, respectively.

$$R''_{tot} = M\left(L_p / k_p\right) + N\left(L_b / k_b\right)$$
$$R''_{tot} = 11\left(0.00236\ m/12\ W/m \cdot K\right) + 10\left(0.00236\ m/0.30\ W/m \cdot K\right)$$
$$R''_{tot} = 2.163 \times 10^{-3} + 7.867 \times 10^{-2} = 8.083 \times 10^{-2}\ K/W.$$

The effective thermal conductivity of the book of thickness (10 + 11) 2.36 mm is

$$k = L/R''_{tot} = \frac{0.04956\ m}{8.083 \times 10^{-2}\ K/W} = 0.613\ W/m \cdot K. \qquad <$$

The thermal capacitance of the stack is

$$C''_{tot} = M\left(\rho_p L_p c_p\right) + N\left(\rho_b L_b c_b\right)$$
$$C''_{tot} = 11\left(8000\ kg/m^3 \times 0.00236\ m \times 480\ J/kg \cdot K\right) + 10\left(1000\ kg/m^3 \times 0.00236\ m \times 1500\ J/kg \cdot K\right)$$
$$C''_{tot} = 9.969 \times 10^4 + 3.540 \times 10^4 = 1.35 \times 10^5\ J/m^2 \cdot K.$$

The effective thermal capacitance of the book is

$$\left(\rho c_p\right) = C''_{tot} / L = 1.351 \times 10^5\ J/m^2 \cdot K/0.04956\ m = 2.726 \times 10^6\ J/m^3 \cdot K. \qquad <$$

COMMENTS: The results of the analysis allow for representing the stack as a homogeneous medium with *effective* properties: $k = 0.613\ W/m \cdot K$ and $\alpha = (k/\rho c_p) = 2.249 \times 10^{-7}\ m^2/s$. See for example, Problem 5.38.

PROBLEM 5.43

KNOWN: Stack of circuit board-pressing plates, initially at a uniform temperature, is subjected by upper/lower platens to a higher temperature.

FIND: (a) Elapsed time, t_e, required for the mid-plane to reach cure temperature when platens are suddenly changed to T_s = 190°C, (b) Energy removal from the stack needed to return its temperature to T_i.

SCHEMATIC:

Platen, $T_s(t \geq 0) = T(L, t > 0) = 190°C$

Stack, $T(x,0) = T_i$ = 15°C, L = 25 mm

x

2L

PROPERTIES: Stack (given): k = 0.613 W/m·K, $\rho c_p = 2.73\times10^6$ J/m^3·K; $\alpha = k/\rho c_p = 2.245\times10^{-7}$ m^2/s.

ANALYSIS: (a) Recognize that sudden application of surface temperature corresponds to $h \to \infty$, or $Bi^{-1} = 0$ (Heisler chart) or $Bi \to \infty$ (100, Table 5.1). With $T_s = T_\infty$,

$$\theta_o^* = \frac{T(0,t) - T_s}{T_i - T_s} = \frac{(170-190)^\circ C}{(15-190)^\circ C} = 0.114.$$

Using Eq. 5.41 with values of $\zeta_1 = 1.552$ and $C_1 = 1.2731$ at $Bi = 100$ (Table 5.1), find Fo

$$\theta_o^* = C_1 \exp\left(-\zeta_1^2 Fo\right)$$

$$Fo = -\frac{1}{\zeta_1^2}\ln\left(\theta_o^*/C_1\right) = -\frac{1}{(1.552)^2}\ln(0.114/1.2731) = 1.002$$

where Fo = $\alpha t/L^2$,

$$t = \frac{FoL^2}{\alpha} = \frac{1.002\left(25\times10^{-3}\ m\right)^2}{2.245\times10^{-7}\ m^2/s} = 2.789\times10^3 s = 46.5\ min. \qquad <$$

The Heisler chart, Figure D.1, could also be used to find Fo from values of θ_o^* and $Bi^{-1} = 0$.

(b) The energy removal is equivalent to the energy gained by the stack per unit area for the time interval $0 \to t_e$. With Q_o'' corresponding to the maximum amount of energy that could be transferred,

$$Q_o'' = \rho c(2L)(T_i - T_\infty) = 2.73\times10^6\ J/m^3\cdot K\left(2\times25\times10^{-3}\ m\right)(15-190)K = -2.389\times10^7\ J/m^2.$$

Q'' may be determined from Eq. 5.46,

$$\frac{Q''}{Q_o''} = 1 - \frac{\sin\zeta_1}{\zeta_1}\theta_o^* = 1 - \frac{\sin(1.552 rad)}{1.552 rad}\times0.114 = 0.795$$

We conclude that the energy to be removed from the stack per unit area to return it to T_i is

$$Q'' = 0.795 Q_o'' = 0.795\times2.389\times10^7\ J/m^2 = 1.90\times10^7\ J/m^2. \qquad <$$

PROBLEM 5.44

KNOWN: Car windshield, initially at a uniform temperature of -20°C, is suddenly exposed on its interior surface to the defrost system airstream at 30°C. The ice layer on the exterior surface acts as an insulating layer.

FIND: What airstream convection coefficient would allow the exterior surface to reach 0°C in 60 s?

SCHEMATIC:

Insulating ice layer

Windshield, $T(x,0) = T_i = -20\ ^oC$, $T(0, 60s) = 0\ ^oC$

$T_\infty = 30\ ^oC, h$

Airstream

x $L = 5$ mm

ASSUMPTIONS: (1) One-dimensional, transient conduction in the windshield, (2) Constant properties, (3) Exterior surface is perfectly insulated.

PROPERTIES: Windshield (Given): $\rho = 2200\ kg/m^3$, $c_p = 830$ J/kg·K and k = 1.2 W/m·K.

ANALYSIS: For the prescribed conditions, from Equations 5.31 and 5.33,

$$\frac{\theta(0,60s)}{\theta_i} = \frac{\theta_o}{\theta_i} = \frac{T(0,60s)-T_\infty}{T_i - T_\infty} = \frac{(0-30)^\circ C}{(-20-30)^\circ C} = 0.6$$

$$Fo = \frac{kt}{\rho c L^2} = \frac{1.2\,W/m\cdot K\times 60}{2200\,kg/m^3 \times 830\,J/kg\cdot K\times(0.005\,m)^2} = 1.58$$

The single-term series approximation, Eq. 5.41, along with Table 5.1, requires an iterative solution to find an appropriate Biot number. Alternatively, the Heisler charts, Appendix D, Figure D.1, for the midplane temperature could be used to find

$$Bi^{-1} = k/hL = 2.5$$

$$h = 1.2\,W/m\cdot K/2.5\times 0.005\,m = 96\,W/m^2\cdot K$$ <

COMMENTS: Using the *IHT*, *Transient Conduction*, *Plane Wall Model*, the convection coefficient can be determined by solving the model with an assumed h and then sweeping over a range of h until the T(0,60s) condition is satisfied. Since the model is based upon multiple terms of the series, the result of h = 99 $W/m^2\cdot K$ is more precise than that found using the chart.

PROBLEM 5.45

KNOWN: Thickness, initial temperature and properties of plastic coating. Safe-to-touch temperature. Convection coefficient and air temperature.

FIND: Time for surface to reach safe-to-touch temperature. Corresponding temperature at plastic/wood interface.

SCHEMATIC:

Air
$T_\infty = 25^\circ C$
$h = 200\ W/m^2\text{-}K$
Plastic
$T_i = 200^\circ C$
$k = 0.250\ W/m\text{-}K$
$\alpha = 1.20 \times 10^{-7}\ m^2/s$
x
L = 0.002 m
Wood

ASSUMPTIONS: (1) One-dimensional conduction in coating, (2) Negligible radiation, (3) Constant properties, (4) Negligible heat of reaction, (5) Negligible heat transfer across plastic/wood interface.

ANALYSIS: With Bi = hL/k = 200 W/m^2·K × 0.002m/0.25 W/m·K = 1.6 > 0.1, the lumped capacitance method may not be used. Applying the approximate solution of Eq. 5.40a, with C_1 = 1.155 and ζ_1 = 0.990 from Table 5.1,

$$\theta_s^* = \frac{T_s - T_\infty}{T_i - T_\infty} = \frac{(42-25)^\circ C}{(200-25)^\circ C} = 0.0971 = C_1 \exp\left(-\zeta_1^2 Fo\right)\cos\left(\zeta_1 x^*\right) = 1.155 \exp(-0.980\ Fo)\cos(0.99)$$

Hence, for $x^* = 1$,

$$Fo = -\ln\left(\frac{0.0971}{1.155\cos(0.99)}\right) / (0.99)^2 = 1.914$$

$$t = \frac{Fo\, L^2}{\alpha} = \frac{1.914(0.002m)^2}{1.20 \times 10^{-7}\, m^2/s} = 63.8\, s$$ <

From Eq. 5.41, the corresponding interface temperature is

$$T_o = T_\infty + (T_i - T_\infty)\exp\left(-\zeta_1^2 Fo\right) = 25^\circ C + 175^\circ C \exp(-0.98 \times 1.914) = 51.8^\circ C$$ <

COMMENTS: By neglecting conduction into the wood and radiation from the surface, the cooling time is overpredicted and is therefore a conservative estimate. However, if energy generation due to solidification of polymer were significant, the cooling time would be longer.

PROBLEM 5.46

KNOWN: Inlet and outlet temperatures of steel rods heat treated by passage through an oven.

FIND: Rod speed, V.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional radial conduction (axial conduction is negligible), (2) Constant properties, (3) Negligible radiation.

PROPERTIES: *Table A-1,* AISI 1010 Steel $\left(\bar{T} \approx 600\text{K}\right)$: k = 48.8 W/m·K, ρ = 7832 kg/m^3, c_p = 559 J/kg·K, α = (k/ρc_p) = 1.11×10^{-5}m^2/s.

ANALYSIS: The time needed to traverse the rod through the oven may be found from Fig. D.4.

$$\theta_o^* = \frac{T_o - T_\infty}{T_i - T_\infty} = \frac{600-750}{50-750} = 0.214$$

$$Bi^{-1} \equiv \frac{k}{hr_o} = \frac{48.8\ \text{W/m}\cdot\text{K}}{125\ \text{W/m}^2\cdot\text{K}\left(0.025\text{m}\right)} = 15.6.$$

Hence,

$$Fo = \alpha\ t/r_o^2 \approx 12.2$$

$$t = 12.2\left(0.025\text{m}\right)^2 / 1.11\times10^{-5}\text{m}^2/\text{s} = 687\ \text{s}.$$

The rod velocity is

$$V = \frac{L}{t} = \frac{5\text{m}}{687\text{s}} = 0.0073\ \text{m/s}.$$

COMMENTS: (1) Since (h r_o/2)/k = 0.032, the lumped capacitance method could have been used. From Eq. 5.5 it follows that t = 675 s.

(2) Radiation effects decrease t and hence increase V, assuming there is net radiant transfer from the oven walls to the rod.

(3) Since Fo > 0.2, the approximate analytical solution may be used. With Bi = hr_o/k =0.0641, Table 5.1 yields $\zeta_1 = 0.3549$ rad and C_1 = 1.0158. Hence from Eq. 5.49c

$$Fo = -\left(\zeta_1^2\right)^{-1}\ \ln\left[\frac{\theta_o^*}{C_1}\right] = 12.4,$$

which is in good agreement with the graphical result.

PROBLEM 5.47

KNOWN: Hot dog with prescribed thermophysical properties, initially at 6°C, is immersed in boiling water.

FIND: Time required to bring centerline temperature to 80°C.

SCHEMATIC:

ASSUMPTIONS: (1) Hot dog can be treated as infinite cylinder, (2) Constant properties.

ANALYSIS: The Biot number, based upon Eq. 5.10, is

$$Bi \equiv \frac{h\,L_c}{k} = \frac{h\,r_o/2}{k} = \frac{100\ W/m^2\cdot K\left(10\times10^{-3} m/2\right)}{0.52\ W/m\cdot K} = 0.96$$

Since Bi > 0.1, a lumped capacitance analysis is not appropriate. Using the Heisler chart, Figure D.4 with

$$Bi \equiv \frac{hr_o}{k} = \frac{100 W/m^2\cdot K \times 10\times10^{-3} m}{0.52\ W/m\cdot K} = 1.92 \quad \text{or} \quad Bi^{-1} = 0.52$$

and $$\theta_o^* = \frac{\theta_o}{\theta_i} = \frac{T(0,t) - T_\infty}{T_i - T_\infty} = \frac{(80-100)^\circ C}{(6\text{-}100)^\circ C} = 0.21 \qquad (1)$$

find $$Fo = t^* = \frac{\alpha\, t}{r_o^2} = 0.8 \qquad t = \frac{r_o^2}{\alpha}\cdot Fo = \frac{\left(10\times10^{-3} m\right)^2}{1.764\times10^{-7}\,m^2/s}\times 0.8 = 453.5 s = 7.6\ min \qquad <$$

where $$\alpha = k/\rho\, c = 0.52\ W/m\cdot K/880\ kg/m^3 \times 3350\ J/kg\cdot K = 1.764\times10^{-7}\,m^2/s.$$

COMMENTS: (1) Note that $L_c = r_o/2$ when evaluating the Biot number for the lumped capacitance analysis; however, in the Heisler charts, $Bi \equiv hr_o/k$.

(2) The surface temperature of the hot dog follows from use of Figure D.5 with $r/r_o = 1$ and $Bi^{-1} = 0.52$; find $\theta(1,t)/\theta_o \approx 0.45$. From Eq. (1), note that $\theta_o = 0.21\ \theta_i$ giving

$$\theta(1,t) = T(r_o,t) - T_\infty = 0.45\theta_o = 0.45\left(0.21\left[T_i - T_\infty\right]\right) = 0.45\times0.21\left[6-100\right]^\circ C = -8.9^\circ C$$

$$T(r_o,t) = T_\infty - 8.9^\circ C = (100-8.9)^\circ C = 91.1^\circ C$$

(3) Since $Fo \ge 0.2$, the approximate solution for θ^*, Eq. 5.49, is valid. From Table 5.1 with Bi = 1.92, find that $\zeta_1 = 1.3245$ rad and $C_1 = 1.2334$. Rearranging Eq. 5.49 and substituting values,

$$Fo = -\frac{1}{\zeta_1^2}\ln\left(\theta_o^*/C_1\right) = \frac{1}{(1.3245\ rad)^2}\ln\left[\frac{0.213}{1.2334}\right] = 1.00$$

This result leads to a value of t = 9.5 min or 20% higher than that of the graphical method.

PROBLEM 5.48

KNOWN: Long rod with prescribed diameter and properties, initially at a uniform temperature, is heated in a forced convection furnace maintained at 750 K with a convection coefficient of h = 1000 $W/m^2 \cdot K$.

FIND: (a) The corresponding center temperature of the rod, $T(0, t_o)$, when the surface temperature $T(r_o, t_o)$ is measured as 550 K, (b) Effect of h on centerline temperature history.

SCHEMATIC:

D = 60 mm
$T_\infty = 750$ K
$100 \le h \le 1000$ $W/m^2 \cdot K$
$\rho = 8000$ kg/m^3
c = 500 J/kg·K
k = 50 W/m·K
$T(r_o, t_o) = 550$ K (Part a)

ASSUMPTIONS: (1) One-dimensional, radial conduction in rod, (2) Constant properties, (3) Rod, when initially placed in furnace, had a uniform (but unknown) temperature, (4) Fo ≥ 0.2.

ANALYSIS: (a) Since the rod was initially at a uniform temperature and Fo ≥ 0.2, the approximate solution for the infinite cylinder is appropriate. From Eq. 5.49b,

$$\theta^*\left(r^*, Fo\right) = \theta_o^*(Fo) J_0\left(\zeta_1 r^*\right) \tag{1}$$

where, for $r^* = 1$, the dimensionless temperatures are, from Eq. 5.31,

$$\theta^*(1, Fo) = \frac{T(r_o, t_o) - T_\infty}{T_i - T_\infty} \qquad \theta_o^*(Fo) = \frac{T(0, t_o) - T_\infty}{T_i - T_\infty} \tag{2,3}$$

Combining Eqs. (2) and (3) with Eq. (1) and rearranging,

$$\frac{T(r_o, t_o) - T_\infty}{T_i - T_\infty} = \frac{T(0, t_o) - T_\infty}{T_i - T_\infty} J_0(\zeta_1 \cdot 1)$$

$$T(0, t_o) = T_\infty + \frac{1}{J_0(\zeta_1)}\left[T(r_o, t_o) - T_\infty\right] \tag{4}$$

The eigenvalue, $\zeta_1 = 1.0185$ rad, follows from Table 5.1 for the Biot number

$$Bi = \frac{hr_o}{k} = \frac{1000\, W/m^2 \cdot K\,(0.060\, m/2)}{50\, W/m \cdot K} = 0.60\,.$$

From Table B-4, with $\zeta_1 = 1.0185$ rad, $J_0(1.0185) = 0.7568$. Hence, from Eq. (4)

$$T(0, t_o) = 750\, K + \frac{1}{0.7568}[550 - 750]\, K = 486\, K$$ <

(b) Using the IHT *Transient Conduction Model* for a *Cylinder*, the following temperature histories were generated.

Continued...

PROBLEM 5.48 (Cont.)

The times required to reach a centerline temperature of 500 K are 367, 85 and 51s, respectively, for h = 100, 500 and 1000 $W/m^2 \cdot K$. The corresponding values of the Biot number are 0.06, 0.30 and 0.60. Hence, even for h = 1000 $W/m^2 \cdot K$, the convection resistance is not negligible relative to the conduction resistance and significant reductions in the heating time could still be effected by increasing h to values considerably in excess of 1000 $W/m^2 \cdot K$.

COMMENTS: For Part (a), recognize why it is not necessary to know T_i or the time t_o. We require that $Fo \geq 0.2$, which for this sphere corresponds to $t \geq 14s$. For this situation, the time dependence of the surface and center are the same.

PROBLEM 5.49

KNOWN: A long cylinder, initially at a uniform temperature, is suddenly quenched in a large oil bath.

FIND: (a) Time required for the surface to reach 500 K, (b) Effect of convection coefficient on surface temperature history.

SCHEMATIC:

Bath
D = 30 mm
$T(r,0) = T_i = 1000$ K
$T_\infty = 350$ K
$h = 50, 250$ W/m^2•K
$\rho = 400$ kg/m^3
c = 1600 J/kg•K
k = 1.7 W/m•K
$T(r_o,t) = 500$ K

ASSUMPTIONS: (1) One-dimensional radial conduction, (2) Constant properties, (3) Fo > 0.2.

ANALYSIS: (a) Check first whether lumped capacitance method is applicable. For h = 50 W/m^2·K,

$$Bi_c = \frac{hL_c}{k} = \frac{h(r_o/2)}{k} = \frac{50\,W/m^2 \cdot K(0.015\,m/2)}{1.7\,W/m \cdot K} = 0.221.$$

Since $Bi_c > 0.1$, method is not suited. Using the approximate series solution for the infinite cylinder,

$$\theta^*\left(r^*, Fo\right) = C_1 \exp\left(-\zeta_1^2 Fo\right) \times J_0\left(\zeta_1 r^*\right) \tag{1}$$

Solving for Fo and setting $r^* = 1$, find

$$Fo = -\frac{1}{\zeta_1^2} \ln\left[\frac{\theta^*}{C_1 J_0(\zeta_1)}\right]$$

where $\theta^* = (1, Fo) = \dfrac{T(r_o, t_o) - T_\infty}{T_i - T_\infty} = \dfrac{(500-350)K}{(1000-350)K} = 0.231$.

From Table 5.1, with Bi = 0.441, find $\zeta_1 = 0.8882$ rad and $C_1 = 1.1019$. From Table B.4, find $J_0(\zeta_1) = 0.8121$. Substituting numerical values into Eq. (2),

$$Fo = -\frac{1}{(0.8882)^2} \ln[0.231/1.1019 \times 0.8121] = 1.72.$$

From the definition of the Fourier number, Fo = $\alpha t / r_o^2$, and $\alpha = k/\rho c$,

$$t = Fo\frac{r_o^2}{\alpha} = Fo \cdot r_o^2 \frac{\rho c}{k}$$

$$t = 1.72(0.015\,m)^2 \times 400\,kg/m^3 \times 1600\,J/kg \cdot K / 1.7\,W/m \cdot K = 145s.$$ <

(b) Using the IHT *Transient Conduction Model* for a *Cylinder*, the following surface temperature histories were obtained.

Continued...

PROBLEM 5.49 (Cont.)

Increasing the convection coefficient by a factor of 5 has a significant effect on the surface temperature, greatly accelerating its approach to the oil temperature. However, even with h = 250 $W/m^2 \cdot K$, Bi = 1.1 and the convection resistance remains significant. Hence, in the interest of accelerated cooling, additional benefit could be achieved by further increasing the value of h.

COMMENTS: For Part (a), note that, since Fo = 1.72 > 0.2, the approximate series solution is appropriate.

PROBLEM 5.50

KNOWN: Long pyroceram rod, initially at a uniform temperature of 900 K, and clad with a thin metallic tube giving rise to a thermal contact resistance, is suddenly cooled by convection.

FIND: (a) Time required for rod centerline to reach 600 K, (b) Effect of convection coefficient on cooling rate.

SCHEMATIC:

Fluid
$T_\infty = 300$ K
$100 \le h \le 1000$ W/m²·K
D = 20 mm
Thin metal tube
Thermal contact resistance, $R'_{t,c} = 0.12$ m·K/W
Pyroceram rod
$T(x,0) = T_i = 900$
$T(0,t) = 600$ K
Part (a)

ASSUMPTIONS: (1) One-dimensional radial conduction, (2) Thermal resistance and capacitance of metal tube are negligible, (3) Constant properties, (4) Fo ≥ 0.2.

PROPERTIES: *Table A-2,* Pyroceram ($\bar{T}$ = (600 + 900)K/2 = 750 K): $\rho = 2600$ kg/m^3, c = 1100 J/kg·K, k = 3.13 W/m·K.

ANALYSIS: (a) The thermal contact and convection resistances can be combined to give an overall heat transfer coefficient. Note that $R'_{t,c}$ [m·K/W] is expressed per unit length for the outer surface. Hence, for h = 100 W/m^2·K,

$$U = \frac{1}{1/h + R'_{t,c}(\pi D)} = \frac{1}{1/100\,\text{W/m}^2\cdot\text{K} + 0.12\,\text{m}\cdot\text{K/W}\,(\pi \times 0.020\,\text{m})} = 57.0\,\text{W/m}^2\cdot\text{K}.$$

Using the approximate series solution, Eq. 5.50c, the Fourier number can be expressed as

$$\text{Fo} = -\left(1/\zeta_1^2\right)\ln\left(\theta_o^*/C_1\right).$$

From Table 5.1, find $\zeta_1 = 0.5884$ rad and $C_1 = 1.0441$ for

$$\text{Bi} = Ur_o/k = 57.0\,\text{W/m}^2\cdot\text{K}\,(0.020\,\text{m}/2)/3.13\,\text{W/m}\cdot\text{K} = 0.182.$$

The dimensionless temperature is

$$\theta_o^*(0,\text{Fo}) = \frac{T(0,t) - T_\infty}{T_i - T_\infty} = \frac{(600-300)\text{K}}{(900-300)\text{K}} = 0.5.$$

Substituting numerical values to find Fo and then the time t,

$$\text{Fo} = \frac{-1}{(0.5884)^2}\ln\frac{0.5}{1.0441} = 2.127$$

$$t = \text{Fo}\frac{r_o^2}{\alpha} = \text{Fo}\cdot r_o^2\,\frac{\rho c}{k}$$

$$t = 2.127(0.020\,\text{m}/2)^2\,2600\,\text{kg/m}^3 \times 1100\,\text{J/kg}\cdot\text{K}/3.13\,\text{W/m}\cdot\text{K} = 194\text{s}.$$ <

(b) The following temperature histories were generated using the IHT *Transient conduction Model* for a *Cylinder*.

Continued...

PROBLEM 5.50 (Cont.)

While enhanced cooling is achieved by increasing h from 100 to 500 $W/m^2 \cdot K$, there is little benefit associated with increasing h from 500 to 1000 $W/m^2 \cdot K$. The reason is that for h much above 500 $W/m^2 \cdot K$, the contact resistance becomes the dominant contribution to the total resistance between the fluid and the rod, rendering the effect of further reductions in the convection resistance negligible. Note that, for h = 100, 500 and 1000 $W/m^2 \cdot K$, the corresponding values of U are 57.0, 104.8 and 117.1 $W/m^2 \cdot K$, respectively.

COMMENTS: For Part (a), note that, since Fo = 2.127 > 0.2, Assumption (4) is satisfied.

PROBLEM 5.51

KNOWN: Sapphire rod, initially at a uniform temperature of 800K is suddenly cooled by a convection process; after 35s, the rod is wrapped in insulation.

FIND: Temperature rod reaches after a long time following the insulation wrap.

SCHEMATIC:

$T_\infty = 300K$
$h = 1600\ W/m^2 \cdot K$
Rod, $r_o = 20 mm$
$T(x,0) = T_i = 800K$

ASSUMPTIONS: (1) One-dimensional radial conduction, (2) Constant properties, (3) No heat losses from the rod when insulation is applied.

PROPERTIES: *Table A-2*, Aluminum oxide, sapphire (550K): ρ = 3970 kg/m^3, c = 1068 J/kg·K, k = 22.3 W/m·K, $\alpha = 5.259\times10^{-5}$ m^2/s.

ANALYSIS: First calculate the Biot number with $L_c = r_o/2$,

$$\text{Bi} = \frac{h\,L_c}{k} = \frac{h\,(r_o/2)}{k} = \frac{1600\ W/m^2 \cdot K\,(0.020\ m/2)}{22.3\ W/m \cdot K} = 0.72.$$

Since Bi > 0.1, the rod cannot be approximated as a lumped capacitance system. The temperature distribution during the cooling process, $0 \le t \le 35s$, and for the time following the application of insulation, t > 35s, will appear as

T_i, $T(0,t)$, $T(0,35s)$, $T(r,t)$, $T(r_o,t)$, $\bar{T}(\infty)$, T_∞, $T(r_o,35s)$, 0, 35, Time, s

Eventually ($t \to \infty$), the temperature of the rod will be uniform at $\bar{T}(\infty)$. To find $\bar{T}(\infty)$, write the conservation of energy requirement for the rod on a *time interval* basis, $E_{in} - E_{out} = \Delta E \equiv E_{final} - E_{initial}$.

Using the nomenclature of Section 5.5.3 and basing energy relative to T_∞, the energy balance becomes

$$-Q = r\ cV\left(\bar{T}(\infty) - T_\infty\right) - Q_o$$

where $Q_o = \rho cV(T_i - T_\infty)$. Dividing through by Q_o and solving for $\bar{T}(\infty)$, find

$$\bar{T}(\infty) = T_\infty + (T_i - T_\infty)(1 - Q/Q_o).$$

From the Groeber chart, Figure D.6, with

$$\text{Bi} = \frac{hr_o}{k} = \frac{1600\ W/m^2 \cdot K \times 0.020m}{22.3\ W/m \cdot K} = 1.43$$

$$\text{Bi}^2\text{Fo} = \text{Bi}^2\left(a\ t/r_o^2\right) = (1.43)^2\ \left(5.259\times10^{-6}\ m^2/s \times 35s/(0.020m)^2\right) = 0.95.$$

find $Q/Q_o \approx 0.57$. Hence,

$$\bar{T}(\infty) = 300K + (800 - 300)K\ (1\text{-}0.57) = 515\ K.$$ <

COMMENTS: From use of Figures D.4 and D.5, find T(0,35s) = 525K and $T(r_o,35s)$ = 423K.

PROBLEM 5.52

KNOWN: Long bar of 70 mm diameter, initially at 90°C, is suddenly immersed in a water bath (T_∞ = 40°C, h = 20 W/m^2·K).

FIND: (a) Time, t_f, that bar should remain in bath in order that, when removed and allowed to equilibrate while isolated from surroundings, it will have a uniform temperature T(r, ∞) = 55°C.

SCHEMATIC:

Water bath
T_∞=40°C
h=20 W/m²·K
T(r,0)=T_i=90°C
Removed at t_f, insulated and equilibrates at T(r,∞)=55°C
Bar, r_o=35mm

ASSUMPTIONS: (1) One-dimensional radial conduction, (2) Constant properties.

PROPERTIES: Bar (given): ρ = 2600 kg/m^3, c = 1030 J/kg·K, k = 3.50 W/m·K, α = k/ρc = 1.31×10^{-6} m^2/s.

ANALYSIS: Determine first whether conditions are space-wise isothermal

$$Bi = \frac{hL_c}{k} = \frac{h(r_o/2)}{k} = \frac{20\ W/m^2\cdot K(0.035\ m/2)}{3.50\ W/m\cdot K} = 0.10$$

and since Bi ≥ 0.1, a Heisler solution is appropriate.

(a) Consider an overall energy balance on the bar during the time interval Δt = t_f (the time the bar is in the bath).

$$E_{in} - E_{out} = \Delta E$$
$$0 - Q = E_{final} - E_{initial} = Mc(T_f - T_\infty) - Mc(T_i - T_\infty)$$
$$-Q = Mc(T_f - T_\infty) - Q_o$$
$$\frac{Q}{Q_o} = 1 - \frac{T_f - T_\infty}{T_i - T_\infty} = 1 - \frac{(55-40)^\circ C}{(90-40)^\circ C} = 0.70$$

where Q_o is the initial energy in the bar (relative to T_∞; Eq. 5.44). With Bi = hr_o/k = 0.20 and Q/Q_o = 0.70, use Figure D.6 to find Bi^2Fo = 0.15; hence Fo = $0.15/Bi^2$ = 3.75 and

$$t_f = Fo\cdot r_o^2/a = 3.75(0.035\ m)^2/1.31\times10^{-6}\ m^2/s = 3507\ s. \qquad <$$

(b) To determine $T(r_o, t_f)$, use Figures D.4 and D.5 for $\theta(r_o,t)/\theta_i$ (Fo = 3.75, Bi^{-1} = 5.0) and θ_o/θ_i (Bi^{-1} = 5.0, r/r_o = 1, respectively, to find

$$T(r_o, t_f) = T_\infty + \frac{q(r_o,t)}{q_o}\cdot\frac{q_o}{q_i}\cdot q_i = 40^\circ C + 0.25\times0.90(90-50)^\circ C = 49^\circ C. \qquad <$$

PROBLEM 5.53

KNOWN: Long plastic rod of diameter D heated uniformly in an oven to T_i and then allowed to convectively cool in ambient air (T_∞, h) for a 3 minute period. Minimum temperature of rod should not be less than 200°C and the maximum-minimum temperature within the rod should not exceed 10°C.

FIND: Initial uniform temperature T_i to which rod should be heated. Whether the 10°C internal temperature difference is exceeded.

SCHEMATIC:

Rod, r_o=15mm
Ambient air
T_∞=25°C
h=8W/m²·K
T(0,t)
T(r,0)=T_i
T(r_o,3 min)=200°C, worst case condition

ASSUMPTIONS: (1) One-dimensional radial conduction, (2) Constant properties, (3) Uniform and constant convection coefficients.

PROPERTIES: Plastic rod (given): k = 0.3 W/m·K, ρc_p = 1040 kJ/m^3·K.

ANALYSIS: For the worst case condition, the rod cools for 3 minutes and its outer surface is at least 200°C in order that the subsequent pressing operation will be satisfactory. Hence,

$$Bi = \frac{hr_o}{k} = \frac{8\ W/m^2 \cdot K \times 0.015\ m}{0.3\ W/m \cdot K} = 0.40$$

$$Fo = \frac{a\ t}{r_o^2} = \frac{k}{r\ c_p} \cdot \frac{t}{r_o^2} = \frac{0.3\ W/m \cdot K}{1040 \times 10^3\ J/m^3 \cdot K} \times \frac{3 \times 60s}{(0.015\ m)^2} = 0.2308.$$

Using Eq. 5.49a and $z_1 = 0.8516$ rad and C_1 = 1.0932 from Table 5.1,

$$q^* = \frac{T(r_o,t) - T_\infty}{T_i - T_\infty} = C_1 J_0\left(z_1 r_o^*\right) \exp\left(-z_1^2 Fo\right).$$

With $r_o^* = 1$, from Table B.4, $J_0(z_1 \times 1) = J_o(0.8516) = 0.8263$, giving

$$\frac{200 - 25}{T_i - 25} = 1.0932 \times 0.8263 \exp\left(-0.8516^2 \times 0.2308\right) \qquad T_i = 254°C. \qquad <$$

At this time (3 minutes) what is the difference between the center and surface temperatures of the rod? From Eq. 5.49b,

$$\frac{q^*}{q_o} = \frac{T(r_o,t) - T_\infty}{T(0,t) - T_\infty} = \frac{200 - 25}{T(0,t) - 25} = J_0\left(z_1 r_o^*\right) = 0.8263$$

which gives T(0,t) = 237°C. Hence,

$$\Delta T = T(0,180s) - T(r_o,180s) = (237 - 200)^\circ C = 37°C. \qquad <$$

Hence, the desired max-min temperature difference sought (10°C) is not achieved.

COMMENTS: ΔT could be reduced by decreasing the cooling rate; however, h can not be made much smaller. Two solutions are (a) increase ambient air temperature and (b) non-uniformly heat rod in oven by controlling its residence time.

PROBLEM 5.54

KNOWN: Diameter and initial temperature of roller bearings. Temperature of oil bath and convection coefficient. Final centerline temperature. Number of bearings processed per hour.

FIND: Time required to reach centerline temperature. Cooling load.

SCHEMATIC:

Stainless steel
$T_i = 500^oC$
$T(0,t_f) = 50^oC$
N = 10

L = 1 m

D = 0.10 m

Oil bath
$T_\infty = 30^oC$
$h = 500\ W/m^2\text{-}K$

ASSUMPTIONS: (1) One-dimensional, radial conduction in rod, (2) Constant properties.

PROPERTIES: *Table A.1*, St. St. 304 $\left(\overline{T} = 548\,K\right)$: ρ=7900 kg/m^3, k = 19.0 W/m·K, c_p = 546 J/kg·K, α = 4.40 × 10^{-6} m^2/s.

ANALYSIS: With Bi = h (r_o/2)/k = 0.658, the lumped capacitance method can not be used. From the one-term approximation of Eq. 5.49 c for the centerline temperature,

$$\theta_o^* = \frac{T_o - T_\infty}{T_i - T_\infty} = \frac{50-30}{500-30} = 0.0426 = C_1 \exp\left(-\zeta_1^2 Fo\right) = 1.1382 \exp\left[-(0.9287)^2\ Fo\right]$$

where, for Bi = hr_o/k = 1.316, C_1 = 1.1382 and ζ_1 = 0.9287 from Table 5.1.

$$Fo = -\ell n(0.0374)/0.863 = 3.81$$

$$t_f = Fo\, r_o^2/\alpha = 3.81(0.05\,m)^2/4.40\times10^{-6} = 2162\,s = 36\,min$$ <

From Eqs. 5.44 and 5.51, the energy extracted from a single rod is

$$Q = \rho c V\left(T_i - T_\infty\right)\left[1 - \frac{2\theta_o^*}{\zeta_1} J_1(\zeta_1)\right]$$

With J_1 (0.9287) = 0.416 from Table B.4,

$$Q = 7900\,kg/m^3 \times 546\,J/kg\cdot K\left[\pi(0.05\,m)^2\,1m\right]470\,K\left[1 - \frac{0.0852\times0.416}{0.9287}\right] = 1.53\times10^7\ J$$

The nominal cooling load is

$$\overline{q} = \frac{N\,Q}{t_f} = \frac{10\times1.53\times10^7\ J}{2162\,s} = 70{,}800\,W = 7.08\ kW$$ <

COMMENTS: For a centerline temperature of 50°C, Eq. 5.49b yields a surface temperature of

$$T(r_o, t) = T_\infty + (T_i - T_\infty)\theta_o^*\ J_o(\zeta_1) = 30°C + 470°C\times0.0426\times0.795 = 45.9°C$$

PROBLEM 5.55

KNOWN: Long rods of 40 mm- and 80-mm diameter at a uniform temperature of 400°C in a curing oven, are removed and cooled by forced convection with air at 25°C. The 40-mm diameter rod takes 280 s to reach a *safe-to-handle* temperature of 60°C.

FIND: Time it takes for a 80-mm diameter rod to cool to the same safe-to-handle temperature. Comment on the result? Did you anticipate this outcome?

SCHEMATIC:

$T(r_o, t_o) = 60^oC$
$t_o = 280$ s

Air
$T_\infty = 25^oC$

$T(r_o, t_o) = 60^oC$
$t_o = ?$

40-mm diameter rod (ρ, c, k)

80-mm diameter rod (ρ, c, k)

ASSUMPTIONS: (1) One-dimensional radial (cylindrical) conduction in the rods, (2) Constant properties, and (3) Convection coefficient same value for both rods.

PROPERTIES: Rod (*given*): $\rho = 2500$ kg/m^3, c = 900 J/kg·K, k = 15 W/m·K.

ANALYSIS: Not knowing the convection coefficient, the Biot number cannot be calculated to determine whether the rods behave as spacewise isothermal objects. Using the relations from Section 5.6, Radial Systems with Convection, for the infinite cylinder, Eq. 5.50, evaluate $Fo = \alpha t / r_o^2$, and knowing $T(r_o, t_o)$, a trial-and-error solution is required to find $Bi = h r_o/k$ and hence, h. Using the *IHT Transient Conduction* model for the *Cylinder*, the following results are readily calculated for the 40-mm rod. With $t_o = 280$ s,

$$Fo = 4.667 \qquad Bi = 0.264 \qquad h = 197.7 \ W/m^2 \cdot K$$

For the 80-mm rod, with the foregoing value for h, with $T(r_o, t_o) = 60°C$, find

$$Bi = 0.528 \qquad Fo = 2.413 \qquad t_o = 579 \ s$$ <

COMMENTS: (1) The time-to-cool, t_o, for the 80-mm rod is slightly more than twice that for the 40-mm rod. Did you anticipate this result? Did you believe the times would be proportional to the diameter squared?

(2) The simplest approach to explaining the relationship between t_o and the diameter follows from the lumped capacitance analysis, Eq. 5.13, where for the same θ/θ_i, we expect $Bi \cdot Fo_o$ to be a constant. That is,

$$\frac{h \cdot r_o}{k} \times \frac{\alpha t_o}{r_o^2} = C$$

yielding $t_o \sim r_o$ (not r_o^2).

PROBLEM 5.56

KNOWN: Initial temperature, density, specific heat and diameter of cylindrical rod. Convection coefficient and temperature of air flow. Time for centerline to reach a prescribed temperature. Dependence of convection coefficient on flow velocity.

FIND: (a) Thermal conductivity of material, (b) Effect of velocity and centerline temperature and temperature histories for selected velocities.

SCHEMATIC:

D = 0.04 m
Specimen
ρ = 1200 kg/m^3
c = 1250 J/kg-K
T_i = 100°C
T(0,1136 s) = 40°C
Air
T_∞ = 25°C
h = CV$^{0.618}$

ASSUMPTIONS: (1) Lumped capacitance analysis can not be used but one-term approximation for an infinite cylinder is appropriate, (2) One-dimensional conduction in r, (3) Constant properties, (4) Negligible radiation, (5) Negligible effect of thermocouple hole on conduction.

ANALYSIS: (a) With $\theta_o^* = [T_o(0,1136s) - T_\infty]/(T_i - T_\infty) = (40 - 25)/(100 - 25) = 0.20$, Eq. 5.49c yields

$$Fo = \frac{\alpha t}{r_o^2} = \frac{k\ t}{\rho c_p r_o^2} = \frac{k(1136\,s)}{1200\ kg/m^3 \times 1250\,J/kg\cdot K \times (0.02\,m)^2} = -\ln(0.2/C_1)/\zeta_1^2 \qquad (1)$$

Because C_1 and ζ_1 depend on Bi = hr_o/k, a trial-and-error procedure must be used. For example, a value of k may be assumed and used to calculate Bi, which may then be used to obtain C_1 and ζ_1 from Table 5.1. Substituting C_1 and ζ_1 into Eq. (1), k may be computed and compared with the assumed value. Iteration continues until satisfactory convergence is obtained, with

$$k \approx 0.30\ W/m\cdot K$$ <

and, hence, Bi = 3.67, C_1 = 1.45, ζ_1 = 1.87 and Fo = 0.568. For the above value of k,
$-\ln(0.2/C_1)/\zeta_1^2 = 0.567$, which equals the Fourier number, as prescribed by Eq. (1).

(b) With h = 55 W/m^2·K for V = 6.8 m/s, h = CV$^{0.618}$ yields a value of C = 16.8 W·s$^{0.618}$/m$^{2.618}$·K. The desired variations of the centerline temperature with velocity (for t = 1136 s) and time (for V = 3, 10 and 20 m/s) are as follows:

Continued …..

With increasing V from 3 to 20 m/s, h increases from 33 to 107 $W/m^2 \cdot K$, and the enhanced cooling reduces the centerline temperature at the prescribed time. The accelerated cooling associated with increasing V is also revealed by the temperature histories, and the time required to achieve thermal equilibrium between the air and the cylinder decreases with increasing V.

COMMENTS: (1) For the smallest value of $h = 33\ W/m^2 \cdot K$, $Bi \equiv h\,(r_o/2)/k = 1.1 >> 0.1$, and use of the lumped capacitance method is clearly inappropriate.

(2) The *IHT* Transient Conduction Model for a cylinder was used to perform the calculations of Part (b). Because the model is based on the exact solution, Eq. 5.47a, it is accurate for values of $Fo < 0.2$, as well as $Fo > 0.2$. Although in principle, the model may be used to calculate the thermal conductivity for the conditions of Part (a), convergence is elusive and may only be achieved if the initial guesses are close to the correct results.

PROBLEM 5.57

KNOWN: Diameter, initial temperature and properties of stainless steel rod. Temperature and convection coefficient of coolant.

FIND: Temperature distributions for prescribed convection coefficients and times.

SCHEMATIC:

Stainless steel
$T_i = 325^oC$
$\rho = 8000$ kg/m^3
$c_p = 475$ J/kg-K
k = 15 W/m-K
D = 30 mm
Fluid
$T_\infty = 25^oC$
h = 100, 1000, 5000 W/m^2-K

ASSUMPTIONS: (1) One-dimensional radial conduction, (2) Constant properties.

ANALYSIS: The *IHT* model is based on the exact solution to the heat equation, Eq. 5.47. The results are plotted as follows

h=100 W/m^2-K
Temperature, C
325
275
225
175
125
75
25
0
0.2
0.4
0.6
0.8
1
Dimensionless radius, r*
t=0
t=100 s
t=500 s

h=1000 W/m^2-K
Temperature, C
325
275
225
175
125
75
25
0
0.2
0.4
0.6
0.8
1
Dimensionless radius, r*
t=0 s
t=10 s
t=50 s

h=5000 W/m^2-K
Temperature, C
325
275
225
175
125
75
25
0
0.2
0.4
0.6
0.8
1
Dimensionless radius, r*
t=0 s
t=1 s
t=5 s
t=25 s

For h = 100 W/m^2·K, Bi = hr_o/k = 0.1, and as expected, the temperature distribution is nearly uniform throughout the rod. For h = 1000 W/m^2·K (Bi = 1), temperature variations within the rod are not negligible. In this case the centerline-to-surface temperature difference is comparable to the surface-to-fluid temperature difference. For h = 5000 W/m^2·K (Bi = 5), temperature variations within the rod are large and $[T(0,t) - T(r_o,t)]$ is substantially larger than $[T(r_o,t) - T_\infty]$.

COMMENTS: With increasing Bi, conduction within the rod, and not convection from the surface, becomes the limiting process for heat loss.

PROBLEM 5.58

KNOWN: A ball bearing is suddenly immersed in a molten salt bath; heat treatment to harden occurs at locations with T > 1000K.

FIND: Time required to harden outer layer of 1mm.

SCHEMATIC:

ρ = 7800 kg/m³, c = 500 J/kg·K, k = 50 W/m·K; T(r,0) = 300K; T(9mm,t) = 1000K; D = 20mm; Salt bath, T_∞ = 1300K, h = 5000 W/m²·K

ASSUMPTIONS: (1) One-dimensional radial conduction, (2) Constant properties, (3) Fo ≥ 0.2.

ANALYSIS: Since any location within the ball whose temperature exceeds 1000K will be hardened, the problem is to find the time when the location r = 9mm reaches 1000K. Then a 1mm outer layer will be hardened. Begin by finding the Biot number.

$$\mathrm{Bi} = \frac{h\,r_o}{k} = \frac{5000\ \mathrm{W/m^2\cdot K}\,(0.020\mathrm{m}/2)}{50\ \mathrm{W/m\cdot K}} = 1.00.$$

Using the one-term approximate solution for a sphere, find

$$\mathrm{Fo} = -\frac{1}{\zeta_1^2}\ln\left[\theta^*/C_1\frac{1}{\zeta_1 r^*}\sin\left(\zeta_1 r^*\right)\right].$$

From Table 5.1 with Bi = 1.00, for the sphere find $\zeta_1 = 1.5708$ rad and C_1 = 1.2732. With r* = r/r_o = (9mm/10mm) = 0.9, substitute numerical values.

$$\mathrm{Fo} = \frac{-1}{(1.5708)^2}\ln\left[\frac{(1000-1300)\mathrm{K}}{(300-1300)\mathrm{K}}/1.2732\frac{1}{1.5708\times0.9}\sin(1.5708\times0.9\ \mathrm{rad})\right] = 0.441.$$

From the definition of the Fourier number with α = k/ρc,

$$t = \mathrm{Fo}\frac{r_o^2}{\alpha} = \mathrm{Fo}\cdot r_o^2\frac{\rho\,c}{k} = 0.441\times\left[\frac{0.020\mathrm{m}}{2}\right]^2 7800\frac{\mathrm{kg}}{\mathrm{m^3}}\times500\frac{\mathrm{J}}{\mathrm{kg\cdot K}}/50\ \mathrm{W/m\cdot K} = 3.4\mathrm{s}.$$ <

COMMENTS: (1) Note the very short time required to harden the ball. At this time it can be easily shown the center temperature is T(0,3.4s) = 871 K.

(2) The Heisler charts can also be used. From Fig. D.8, with Bi^{-1} = 1.0 and r/r_o = 0.9, read θ/θ_o = 0.69(±0.03). Since

$$\theta = T - T_\infty = 1000 - 1300 = -300\mathrm{K} \qquad \theta_i = T_i - T_\infty = -1000\mathrm{K}$$

it follows that

$$\frac{\theta}{\theta_i} = 0.30. \quad \text{Since} \quad \frac{\theta}{\theta_i} = \frac{\theta}{\theta_o}\cdot\frac{\theta_o}{\theta_i}, \quad \text{then} \quad \frac{\theta}{\theta_i} = 0.69\,\frac{\theta_o}{\theta_i}$$

and $\theta_o/\theta_i = 0.30/0.69 = 0.43\ (\pm0.02)$.

From Fig. D.7 at θ_o/θ_i=0.43, Bi^{-1}=1.0, read Fo = 0.45 (±0.03) and t = 3.5 (±0.2)s. Note the use of tolerances associated with reading the charts to ±5%.

PROBLEM 5.59

KNOWN: An 80mm sphere, initially at a uniform elevated temperature, is quenched in an oil bath with prescribed T_∞, h.

FIND: The center temperature of the sphere, T(0,t) at a certain time when the surface temperature is $T(r_o,t) = 150°C$.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional radial conduction, (2) Initial uniform temperature within sphere, (3) Constant properties, (4) Fo ≥ 0.2.

ANALYSIS: Check first to see if the sphere is spacewise isothermal.

$$\mathrm{Bi}_c = \frac{hL_c}{k} = \frac{h(r_o/3)}{k} = \frac{1000\ \mathrm{W/m^2 \cdot K} \times 0.040\mathrm{m}/3}{50\ \mathrm{W/m \cdot K}} = 0.26.$$

Since $\mathrm{Bi}_c > 0.1$, lumped capacitance method is not appropriate. Recognize that when Fo ≥ 0.2, the time dependence of the temperature at any point within the sphere will be the same as the center. Using the Heisler chart method, Fig. D.8 provides the relation between $T(r_o,t)$ and T(0,t). Find first the Biot number,

$$\mathrm{Bi} = \frac{hr_o}{k} = \frac{1000\ \mathrm{W/m^2 \cdot K} \times 0.040\mathrm{m}}{50\ \mathrm{W/m \cdot K}} = 0.80.$$

With $\mathrm{Bi}^{-1} = 1/0.80 = 1.25$ and $r/r_o = 1$, read from Fig. D.8,

$$\frac{q}{q_o} = \frac{T(r_o,t) - T_\infty}{T(0,t) - T_\infty} = 0.67.$$

It follows that

$$T(0,t) = T_\infty + \frac{1}{0.67}\left[T(r_o,t) - T_\infty\right] = 50^\circ\mathrm{C} + \frac{1}{0.67}[150 - 50]^\circ\mathrm{C} = 199^\circ\mathrm{C}. \qquad <$$

COMMENTS: (1) There is sufficient information to evaluate Fo; hence, we require that the time be sufficiently long after the start of quenching for this solution to be appropriate.

(2)The approximate series solution could also be used to obtain T(0,t). For Bi = 0.80 from Table 5.1, $z_1 = 1.5044$ rad. Substituting numerical values, $r^* = 1$,

$$\frac{q^*}{q_o^*} = \frac{T(r_o,t) - T_\infty}{T(0,t) - T_\infty} = \frac{1}{z_1 r^*}\sin\left(z_1 r^*\right) = \frac{1}{1.5044}\sin(1.5044\ \mathrm{rad}) = 0.663.$$

It follows that T(0,t) = 201°C.

PROBLEM 5.60

KNOWN: Steel ball bearings at an initial, uniform temperature are to be cooled by convection while passing through a refrigerated chamber; bearings are to be cooled to a temperature such that 70% of the thermal energy is removed.

FIND: Residence time of the balls in the 5m-long chamber and recommended drive velocity for the conveyor.

SCHEMATIC:

ASSUMPTIONS: (1) Negligible conduction between ball and conveyor surface, (2) Negligible radiation exchange with surroundings, (3) Constant properties, (4) Uniform convection coefficient over ball's surface.

ANALYSIS: The Biot number for the lumped capacitance analysis is

$$\mathrm{Bi} \equiv \frac{hL_c}{k} = \frac{h(r_o/3)}{k} = \frac{1000\ \mathrm{W/m^2 \cdot K}\ (0.1\mathrm{m}/3)}{50\ \mathrm{W/m \cdot K}} = 0.67.$$

Since Bi > 0.1, lumped capacitance analysis is not appropriate. In Figure D.9, the internal energy change is shown as a function of Bi and Fo. For

$$\frac{Q}{Q_o} = 0.70 \qquad \text{and} \qquad \mathrm{Bi} = \frac{hr_o}{k} = \frac{1000\ \mathrm{W/m^2 \cdot K} \times 0.1\mathrm{m}}{50\ \mathrm{W/m \cdot K}} = 2.0,$$

find $\mathrm{Bi}^2\,\mathrm{Fo} \approx 1.2$. The Fourier number is

$$\mathrm{Fo} = \frac{\alpha\, t}{r_o^2} = \frac{2\times10^{-5}\mathrm{m^2/s} \times t}{(0.1\ \mathrm{m})^2} = 2.0\times10^{-3}t$$

giving

$$t = \frac{\mathrm{Fo}}{2.0\times10^{-3}} = \frac{1.2/\mathrm{Bi}^2}{2.0\times10^{-3}} = \frac{1.2/(2.0)^2}{2.0\times10^{-3}} = 150\mathrm{s}.$$

The velocity of the conveyor is expressed in terms of the length L and residence time t. Hence

$$V = \frac{L}{t} = \frac{5\mathrm{m}}{150\mathrm{s}} = 0.033\mathrm{m/s} = 33\mathrm{mm/s}.$$ <

COMMENTS: Referring to Eq. 5.10, note that for a sphere, the characteristic length is

$$L_c = V/A_s = \frac{4}{3}\pi\, r_o^3 / 4\pi\, r_o^2 = \frac{r_o}{3}.$$

However, when using the Heisler charts, note that $\mathrm{Bi} \equiv h\, r_o/k$.

PROBLEM 5.61

KNOWN: Diameter and initial temperature of ball bearings to be quenched in an oil bath.

FIND: (a) Time required for surface to cool to 100°C and the corresponding center temperature, (b) Oil bath cooling requirements.

SCHEMATIC:

St. St. (AISI 304) ball
D=20mm, T_i=850°C
Oil
T_∞ = 40°C
h=1000 W/m²·K
Q
10^4 Q/h

ASSUMPTIONS: (1) One-dimensional radial conduction in ball bearings, (2) Constant properties.

PROPERTIES: *Table A-1*, St. St., AISI 304, (T ≈ 500°C): k = 22.2 W/m·K, c_p = 579 J/kg·K, ρ = 7900 kg/m^3, α = 4.85×10^{-6} m^2/s.

ANALYSIS: (a) To determine whether use of the lumped capacitance method is suitable, first compute

$$\mathrm{Bi} = \frac{h(r_o/3)}{k} = \frac{1000\ \mathrm{W/m^2\cdot K}(0.010\mathrm{m}/3)}{22.2\ \mathrm{W/m\cdot K}} = 0.15.$$

We conclude that, although the lumped capacitance method could be used as a first approximation, the Heisler charts should be used in the interest of improving accuracy. Hence, with

$$\mathrm{Bi}^{-1} = \frac{k}{hr_o} = \frac{22.2\ \mathrm{W/m\cdot K}}{1000\ \mathrm{W/m^2\cdot K}(0.01\mathrm{m})} = 2.22 \qquad \text{and} \qquad \frac{r}{r_o} = 1,$$

Fig. D.8 gives

$$\frac{q(r_o,t)}{q_o(t)} \approx 0.80.$$

Hence, with

$$\frac{q(r_o,t)}{q_i} = \frac{T(r_o,t)-T_\infty}{T_i-T_\infty} = \frac{100-40}{850-40} = 0.074,$$

Continued …..

it follows that

$$\frac{q_o}{q_i} = \frac{q\left(r_o, t\right)/q_i}{q\left(r_o, t\right)/q_o} = \frac{0.074}{0.80} = 0.093.$$

From Fig. D.7, with $q_o / q_i = 0.093$ and $Bi^{-1} = k/hr_o = 2.22$, find

$$t^* = Fo \approx 2.0$$

$$t = \frac{r_o^2 Fo}{a} = \frac{(0.01m)^2 (2.0)}{4.85\times10^{-6} m^2 / s} = 41s.$$ <

Also,

$$q_o = T_o - T_\infty = 0.093\left(T_i - T_\infty\right) = 0.093\left(850 - 40\right) = 75^\circ C$$

$$T_o = 115^\circ C$$ <

(b) With $Bi^2 Fo = (1/2.2)^2 \times 2.0 = 0.41$, where $Bi \equiv (hr_o/k) = 0.45$, it follows from Fig. D.9 that for a single ball

$$\frac{Q}{Q_o} \approx 0.93.$$

Hence, from Eq. 5.44,

$$Q = 0.93\ r c_p V\left(T_i - T_\infty\right)$$

$$Q = 0.93 \times 7900\ kg/m^3 \times 579\ J/kg \cdot K \times \frac{p}{6}\left(0.02m\right)^3 \times 810^\circ C$$

$$Q = 1.44\times10^4 J$$

is the amount of energy transferred from a single ball during the cooling process. Hence, the oil bath cooling rate must be

$$q = 10^4 Q/3600s$$

$$q = 4\times10^4 W = 40\ kW.$$ <

COMMENTS: If the lumped capacitance method is used, the cooling time, obtained from Eq. 5.5, would be t = 39.7s, where the ball is assumed to be uniformly cooled to 100°C. This result, and the fact that $T_o - T(r_o) = 15^\circ C$ at the conclusion, suggests that use of the lumped capacitance method would have been reasonable. Note that, when using the Heisler charts, accuracy to better than 5% is seldom possible.

PROBLEM 5.62

KNOWN: Diameter and initial temperature of hailstone falling through warm air.

FIND: (a) Time, t_m, required for outer surface to reach melting point, $T(r_o,t_m) = T_m = 0°C$, (b) Centerpoint temperature at that time, (c) Energy transferred to the stone.

SCHEMATIC:

D = 0.005m
Ice, T_i = -30°C, T_m = 0°C
Air
T_∞ = 5°C, h = 250 W/m²·K

ASSUMPTIONS: (1) One-dimensional radial conduction, (2) Constant properties.

PROPERTIES: *Table A-3*, Ice (253K): $\rho = 920\ \text{kg/m}^3$, $k = 2.03$ W/m·K, $c_p = 1945$ J/kg·K; $\alpha = k/\rho c_p = 1.13 \times 10^{-6}\ \text{m}^2/\text{s}$.

ANALYSIS: (a) Calculate the lumped capacitance Biot number,

$$\text{Bi} = \frac{h(r_o/3)}{k} = \frac{250\ \text{W/m}^2\cdot\text{K}(0.0025\text{m}/3)}{2.03\ \text{W/m}\cdot\text{K}} = 0.103.$$

Since Bi > 0.1, use the Heisler charts for which

$$\frac{\theta(r_o,t_m)}{\theta_i} = \frac{T(r_o,t_m) - T_\infty}{T_i - T_\infty} = \frac{0-5}{-30-5} = 0.143$$

$$\text{Bi}^{-1} = \frac{k}{hr_o} = \frac{2.03\ \text{W/m}\cdot\text{K}}{250\ \text{W/m}^2\cdot\text{K} \times 0.0025\text{m}} = 3.25.$$

From Fig. D.8, find $\quad \dfrac{\theta(r_o,t_m)}{\theta_o(t_m)} \approx 0.86.$

It follows that $\quad \dfrac{\theta_o(t_m)}{\theta_i} = \dfrac{\theta(r_o,t_m)/\theta_i}{\theta(r_o,t_m)/\theta_o(t_m)} \approx \dfrac{0.143}{0.86} \approx 0.17.$

From Fig. D.7 find Fo ≈ 2.1. Hence,

$$t_m \approx \frac{\text{Fo}\, r_o^2}{\alpha} = \frac{2.1(0.0025)^2}{1.13\times10^{-6}\,\text{m}^2/\text{s}} = 12\text{s}. \qquad <$$

(b) Since $(\theta_o/\theta_i) \approx 0.17$, find

$$T_o - T_\infty \approx 0.17(T_i - T_\infty) \approx 0.17\ (-30-5) \approx -6.0^\circ\text{C}$$

$$T_o(t_m) \approx -1.0^\circ\text{C}. \qquad <$$

(c) With $\text{Bi}^2\text{Fo} = (1/3.25)^2 \times 2.1 = 0.2$, from Fig. D.9, find $Q/Q_o \approx 0.82$. From Eq. 5.44,

$$Q_o = \rho V c_p \theta_i = \left(920\ \text{kg/m}^3\right)\ (\pi/6)(0.005\text{m})^3 1945\,(\text{J/kg}\cdot\text{K})(-35\text{K}) = -4.10\ \text{J}$$

$$Q = 0.82\ Q_o = 0.82(-4.10\ \text{J}) = -3.4\ \text{J}. \qquad <$$

PROBLEM 5.63

KNOWN: Sphere quenching in a constant temperature bath.

FIND: (a) Plot $T(0,t)$ and $T(r_o,t)$ as function of time, (b) Time required for surface to reach 415 K, t', (c) Heat flux when $T(r_o, t') = 415$ K, (d) Energy lost by sphere in cooling to $T(r_o, t') = 415$ K, (e) Steady-state temperature reached after sphere is insulated at $t = t'$, (f) Effect of h on center and surface temperature histories.

SCHEMATIC:

Sphere
$D = 30$mm
$T_i = 800$ K
$\rho = 400$ kg/m^3
$c_p = 1600$ J/kg·K
$k = 1.7$ W/m·K
$T_\infty = 320$ K
$h = 75, 200$ W/m^2·K
Large bath
$T(r,t)$
T_i
$T(0,t)$
$T(r_o,t)$
T_∞
Time, t

ASSUMPTIONS: (1) One-dimensional radial conduction, (2) Constant properties, (3) Uniform initial temperature.

ANALYSIS: (a) Calculate Biot number to determine if sphere behaves as spatially isothermal object,

$$\mathrm{Bi} = \frac{hL_c}{k} = \frac{h(r_o/3)}{k} = \frac{75\,\mathrm{W/m^2\cdot K}\,(0.015\,\mathrm{m}/3)}{1.7\,\mathrm{W/m\cdot K}} = 0.22\,.$$

Hence, temperature gradients exist in the sphere and $T(r,t)$ vs. t appears as shown above.

(b) The Heisler charts may be used to find t' when $T(r_o, t') = 415$ K. Using Fig. D.8 with $r/r_o = 1$ and $\mathrm{Bi}^{-1} = k/hr_o = 1.7$ W/m·K/(75 W/m^2·K $\times$ 0.015 m) = 1.51, $\theta(1,t')/\theta_o \approx 0.72$. In order to enter Fig. D.7, we need to determine θ_o/θ_i, which is

$$\frac{\theta_o}{\theta_i} = \frac{\theta(1,t')}{\theta_i} \bigg/ \frac{\theta(1,t')}{\theta_o} \approx \frac{(415-320)\mathrm{K}}{(800-320)\mathrm{K}} \bigg/ 0.72 = 0.275$$

Hence, for $\mathrm{Bi}^{-1} = 1.51$, $\mathrm{Fo} \equiv \alpha t'/r_o^2 \approx 0.87$ and

$$t' = \mathrm{Fo}\frac{r_o^2}{\alpha} = \mathrm{Fo}\cdot\frac{\rho c_p}{k}\cdot r_o^2 \approx 0.87\frac{400\,\mathrm{kg/m^3}\times 1600\,\mathrm{J/kg\cdot K}}{1.7\,\mathrm{W/m\cdot K}}\times(0.015\,\mathrm{m})^2 = 74\mathrm{s}$$ <

(c) The heat flux at the outer surface at time t' is given by Newton's law of cooling

$$q'' = h\left[T(r_o,t') - T_\infty\right] = 75\,\mathrm{W/m^2\cdot K}\,[415-320]\,\mathrm{K} = 7125\,\mathrm{W/m^2}..$$ <

The manner in which q'' is calculated indicates that energy is leaving the sphere.

(d) The energy lost by the sphere during the cooling process from t = 0 to t' can be determined from the Groeber chart, Fig. D.9. With Bi = 1/1.51 = 0.67 and $\mathrm{Bi}^2\mathrm{Fo} = (1/1.51)^2 \times 0.87 \approx 0.4$, the chart yields $Q/Q_o \approx 0.75$. The energy loss by the sphere with $V = (\pi D^3)/6$ is therefore

$$Q \approx 0.85 Q_o = 0.85\rho\left(\pi D^3/6\right)c_p\left(T_i - T_\infty\right)$$

$$Q \approx 0.85 \times 400\,\mathrm{kg/m^3}\left(\pi\,[0.030\,\mathrm{m}]^3/6\right)1600\,\mathrm{J/kg\cdot K}\,(800-320)\,\mathrm{K} = 3691\,\mathrm{J}$$ <

Continued...

PROBLEM 5.63 (Cont.)

(e) If at time t' the surface of the sphere is perfectly insulated, eventually the temperature of the sphere will be uniform at $T(\infty)$. Applying conservation of energy to the sphere over a *time interval*, $E_{in} - E_{out} = \Delta E \equiv E_{final} - E_{initial}$. Hence, $-Q = \rho cV[T(\infty) - T_\infty] - Q_o$, where $Q_o \equiv \rho cV[T_i - T_\infty]$. Dividing by Q_o and regrouping, we obtain

$$T(\infty) = T_\infty + (1 - Q/Q_o)(T_i - T_\infty) \approx 320\,K + (1 - 0.75)(800 - 320)\,K = 440\,K \qquad <$$

(f) Using the IHT *Transient Conduction Model* for a *Sphere*, the following graphical results were generated.

Temperature, T(K)
800
700
600
500
400
300
0 50 100 150
Time, t (s)

h = 75 W/m^2.K, r = ro
h = 75 W/m^2.K, r = 0
h = 200 W/m^2.K, r = ro
h = 200 W/m^2.K, r = 0

Heat flux, q''(ro,t) (W/m^2.K)
90000
60000
30000
0
0 50 100 150
Time, t(s)

h = 75 W/m^2.K
h = 200 W/m^2.K

The quenching process is clearly accelerated by increasing h from 75 to 200 $W/m^2 \cdot K$ and is virtually completed by $t \approx 100s$ for the larger value of h. Note that, for both values of h, the temperature difference $[T(0,t) - T(r_o,t)]$ decreases with increasing t. Although the surface heat flux for h = 200 $W/m^2 \cdot K$ is initially larger than that for h = 75 $W/m^2 \cdot K$, the more rapid decline in $T(r_o,t)$ causes it to become smaller at $t \approx 30s$.

COMMENTS: 1. There is considerable uncertainty associated with reading Q/Q_o from the Groeber chart, Fig. D.9, and it would be better to use the one-term approximation solutions of Section 5.6.2. With Bi = 0.662, from Table 5.1, find $\zeta_1 = 1.319$ rad and $C_1 = 1.188$. Using Eq. 5.50, find Fo = 0.852 and $t' = 72.2$ s. Using Eq. 5.52, find $Q/Q_o = 0.775$ and $T(\infty) = 428$ K.

2. Using the *Transient Conduction/Sphere* model in *IHT* based upon multiple-term series solution, the following results were obtained: $t' = 72.1$ s; $Q/Q_o = 0.7745$, and $T(\infty) = 428$ K.

PROBLEM 5.64

KNOWN: Two spheres, A and B, initially at uniform temperatures of 800K and simultaneously quenched in large, constant temperature baths each maintained at 320K; properties of the spheres and convection coefficients.

FIND: (a) Show in a qualitative manner, on T-t coordinates, temperatures at the center and the outer surface for each sphere; explain features of the curves; (b) Time required for the outer surface of each sphere to reach 415K, (c) Energy gained by each bath during process of cooling spheres to a surface temperature of 415K.

SCHEMATIC:

$T(r_o,t) = 415K$, r_o, $T_i = 800K$, Fluid $T_\infty = 320K$

	Sphere A	Sphere B
r_o (mm)	150	15
ρ (kg/m^3)	1600	400
c (J/kg·K)	400	1600
k (W/m·K)	170	1.7
h ($W/m^2 \cdot K$)	5	50

ASSUMPTIONS: (1) One-dimensional radial conduction, (2) Uniform properties, (3) Constant convection coefficient.

ANALYSIS: (a) From knowledge of the Biot number and the thermal time constant, it is possible to qualitatively represent the temperature distributions. From Eq. 5.10, with $L_c = r_o/3$, find

$$Bi_A = \frac{5\ W/m^2 \cdot K\,(0.150m/3)}{170\ W/m \cdot K} = 1.47 \times 10^{-3} \quad (1)$$

$$Bi = \frac{h(r_o/3)}{k}$$

$$Bi_B = \frac{50\ W/m^2 \cdot K\,(0.015m/3)}{1.7\ W/m \cdot K} = 0.147 \quad (2)$$

The thermal time constant for a lumped capacitance system from Eq. 5.7 is

$$\tau = \left[\frac{1}{hA_s}\right](\rho Vc) \qquad \tau_A = \frac{1600\ kg/m^3 \times (0.150m)400\ J/kg \cdot K}{3 \times 5\ W/m^2 \cdot K} = 6400s \quad (3)$$

$$\tau = \frac{\rho\, r_o\, c}{3h} \qquad \tau_B = \frac{400\ kg/m^3 \times (0.015m)1600\ J/kg \cdot K}{3 \times 50\ W/m^2 \cdot K} = 64s \quad (4)$$

When Bi << 0.1, the sphere will cool in a spacewise isothermal manner (Sphere A). For sphere B, Bi > 0.1, hence gradients will be important. Note that the thermal time constant of A is much larger than for B; hence, A will cool much slower. See sketch for these features.

T_i, $T(r,t)$, T_∞, $T(r_o,t)$, $T(0,t)$ } Sphere B, Sphere A - spacewise isothermal, Time

(b) Recognizing that $Bi_A < 0.1$, Sphere A can be treated as spacewise isothermal and analyzed using the lumped capacitance method. From Eq. 5.6 and 5.7, with T = 415 K

$$\frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = \exp(-t/\tau) \quad (5)$$

Continued …..

PROBLEM 5.64 (Cont.)

$$t_A = -\tau_A \left[\ln \frac{T - T_\infty}{T_i - T_\infty} \right] = -6400s \left[\ln \frac{415 - 320}{800 - 320} \right] = 10,367s = 2.88h. \quad <$$

Note that since the sphere is nearly isothermal, the surface and inner temperatures are approximately the same.

Since $Bi_B > 0.1$, *Sphere B* must be treated by the Heisler chart method of solution beginning with Figure D.8. Using

$$Bi_B \equiv \frac{hr_o}{k} = \frac{50\ W/m^2 \cdot K \times (0.015m)}{1.7\ W/m \cdot K} = 0.44 \qquad or \qquad Bi_B^{-1} = 2.27,$$

find that for $r/r_o = 1$,

$$\frac{\theta(1,t)}{\theta_o} = \frac{T(r_o,t) - T_\infty}{\theta_o} = \frac{(415 - 320)}{\theta_o} = 0.8. \qquad (6)$$

Using Eq. (6) and Figure D.7, find the Fourier number,

$$\frac{\theta_o}{\theta_i} = \frac{\left(T(r_o,t) - T_\infty\right)/0.8}{T_i - T_\infty} = \frac{(415 - 320)K/0.8}{(800 - 320)K} = 0.25 \qquad Fo = \frac{\alpha t}{r_o^2} = 1.3.$$

$$t_B = \frac{Fo\ r_o^2}{\alpha} = \frac{1.3\ (0.015m)^2}{2.656 \times 10^{-6} m^2/s} = 110s = 1.8\ min \quad <$$

where $\alpha = k/\rho c = 1.7\ W/m \cdot K / 400\ kg/m^3 \times 1600\ J/kg \cdot K = 2.656 \times 10^{-6} m^2/s$.

(c) To determine the energy change by the spheres during the cooling process, apply the conservation of energy requirement on a time interval basis.

Sphere A:

$$E_{in} - E_{out} = \Delta E \qquad -Q_A = \Delta E = E(t) - E(0).$$

$$Q_A = \rho\ cV\left[T(t) - T_i\right] = 1600 kg/m^3 \times 400 J/kg \cdot K \times (4/3)\pi (0.150m)^3 [415 - 800]K$$

$$Q_A = 3.483 \times 10^6\ J. \quad <$$

Note that this simple expression is a consequence of the spacewise isothermal behavior.

Sphere B: $\quad E_{in} - E_{out} = \Delta E \qquad -Q_B = E(t) - E(0).$

For the nonisothermal sphere, the Groeber chart, Figure D.9, can be used to evaluate Q_B. With Bi = 0.44 and $Bi^2 Fo = (0.44)^2 \times 1.3 = 2.52$, find $Q/Q_o = 0.74$. The energy transfer from the sphere during the cooling process, using Eq. 5.44, is

$$Q_B = 0.74\ Q_o = 0.74\left[\rho\ cV\left(T_i - T_\infty\right)\right]$$

$$Q_B = 0.75 \times 400 kg/m^3 \times 1600 J/kg \cdot K (4/3)\pi (0.015m)^3 (800 - 320)K = 3257\ J. \quad <$$

COMMENTS: (1) In summary:

Sphere	$Bi = hr_o/k$	$\tau(s)$	t(s)	Q(J)
A	4.41×10^{-3}	6400	10,370	3.48×10^6
B	0.44	64	110	3257

PROBLEM 5.65

KNOWN: Spheres of 40-mm diameter heated to a uniform temperature of 400°C are suddenly removed from an oven and placed in a forced-air bath operating at 25°C with a convection coefficient of 300 W/m^2·K.

FIND: (a) Time the spheres must remain in the bath for 80% of the thermal energy to be removed, and (b) Uniform temperature the spheres will reach when removed from the bath at this condition and placed in a carton that prevents further heat loss.

SCHEMATIC:

Air bath
T_∞ = 25°C
h = 300 W/m^2-K
Sphere, D = 40 mm
T(r,0) = T_i = 400°C
(ρ, c, k)
r
Energy out, Q(t)

ASSUMPTIONS: (1) One-dimensional radial conduction in the spheres, (2) Constant properties, and (3) No heat loss from sphere after removed from the bath and placed into the packing carton.

PROPERTIES: Sphere (*given*): ρ = 3000 kg/m^3, c = 850 J/kg·K, k = 15 W/m·K.

ANALYSIS: (a) From Eq. 5.52, the fraction of thermal energy removed during the time interval Δt = t_o is

$$\frac{Q}{Q_o} = 1 - 3\theta_o^* / \zeta_1^3 \left[\sin(\zeta_1) - \zeta_1 \cos(\zeta_1)\right] \qquad (1)$$

where Q/Q_o = 0.8. The Biot number is

$$Bi = hr_o / k = 300\ W/m^2 \cdot K \times 0.020\, m / 15\ W/m \cdot K = 0.40$$

and for the one-term series approximation, from Table 5.1,

$$\zeta_1 = 1.0528 \text{ rad} \qquad C_1 = 1.1164 \qquad (2)$$

The dimensionless temperature θ_o^*, Eq. 5.31, follows from Eq. 5.50.

$$\theta_o^* = C_1 \exp\left(-\zeta_1^2 Fo\right) \qquad (3)$$

where $Fo = \alpha t_o / r_o^2$. Substituting Eq. (3) into Eq. (1), solve for Fo and t_o.

$$\frac{Q}{Q_o} = 1 - 3\, C_1 \exp\left(-\zeta_1^2 Fo\right) / \zeta_1^3 \left[\sin(\zeta_1) - \zeta_1 \cos(\zeta_1)\right] \qquad (4)$$

$$Fo = 1.45 \qquad t_o = 98.6 \text{ s}$$ <

(b) Performing an overall energy balance on the sphere during the interval of time $t_o \le t \le \infty$,

$$E_{in} - E_{out} = \Delta E = E_f - E_i = 0 \qquad (5)$$

where E_i represents the thermal energy in the sphere at t_o,

$$E_i = (1 - 0.8) Q_o = (1 - 0.8) \rho c V (T_i - T_\infty) \qquad (6)$$

and E_f represents the thermal energy in the sphere at t = ∞,

$$E_f = \rho c V \left(T_{avg} - T_\infty\right) \qquad (7)$$

Combining the relations, find the average temperature

$$\rho c V \left[\left(T_{avg} - T_\infty\right) - (1 - 0.8)(T_i - T_\infty)\right] = 0$$

$$T_{avg} = 100°C$$ <

PROBLEM 5.66

KNOWN: Diameter, density, specific heat and thermal conductivity of Pyrex spheres in packed bed thermal energy storage system. Convection coefficient and inlet gas temperature.

FIND: Time required for sphere to acquire 90% of maximum possible thermal energy and the corresponding center temperature.

SCHEMATIC:

Gas
$T_{g,i}$ = 300°C
h = 75 W/m²-K

Pyrex sphere
D = 75 mm, T_i = 25°C
ρ = 2225 kg/m³
c = 835 J/kg-K
k = 1.4 W/m-K

ASSUMPTIONS: (1) One-dimensional radial conduction in sphere, (2) Negligible heat transfer to or from a sphere by radiation or conduction due to contact with adjoining spheres, (3) Constant properties.

ANALYSIS: With Bi ≡ $h(r_o/3)/k$ = 75 W/m²·K (0.0125m)/1.4 W/m·K = 0.67, the approximate solution for one-dimensional transient conduction in a sphere is used to obtain the desired results. We first use Eq. (5.52) to obtain θ_o^*.

$$\theta_o^* = \frac{\zeta_1^3}{3\left[\sin(\zeta_1) - \zeta_1 \cos(\zeta_1)\right]}\left(1 - \frac{Q}{Q_o}\right)$$

With Bi ≡ hr_o/k = 2.01, $\zeta_1 \approx 2.03$ and $C_1 \approx 1.48$ from Table 5.1. Hence,

$$\theta_o^* = \frac{0.1(2.03)^3}{3\left[0.896 - 2.03(-0.443)\right]} = \frac{0.837}{5.386} = 0.155$$

The center temperature is therefore

$$T_o = T_{g,i} + 0.155\left(T_i - T_{g,i}\right) = 300°C - 42.7°C = 257.3°C$$ <

From Eq. (5.50c), the corresponding time is

$$t = -\frac{r_o^2}{\alpha \zeta_1^2} \ln\left(\frac{\theta_o^*}{C_1}\right)$$

where $\alpha = k/\rho c = 1.4\,W/m \cdot K / \left(2225\,kg/m^3 \times 835\,J/kg \cdot K\right) = 7.54 \times 10^{-7}\,m^2/s.$

$$t = -\frac{(0.0375m)^2 \ln(0.155/1.48)}{7.54 \times 10^{-7}\,m^2/s\,(2.03)^2} = 1,020s$$ <

COMMENTS: The surface temperature at the time of interest may be obtained from Eq. (5.50b). With $r^* = 1$,

$$T_s = T_{g,i} + \left(T_i - T_{g,i}\right)\frac{\theta_o^* \sin(\zeta_1)}{\zeta_1} = 300°C - 275°C\left(\frac{0.155 \times 0.896}{2.03}\right) = 280.9°C$$ <

PROBLEM 5.67

KNOWN: Initial temperature and properties of a solid sphere. Surface temperature after immersion in a fluid of prescribed temperature and convection coefficient.

FIND: (a) Time to reach surface temperature, (b) Effect of thermal diffusivity and conductivity on thermal response.

SCHEMATIC:

D = 0.1 m
$T(r_o,t) = 60\ ^oC$
$1.5 \le k \le 150$ W/m•K
$T_\infty = 75\ ^oC$
$10^{-6} \le \alpha \le 10^{-4}\ m^2/s$
$h = 300\ W/m^2$•K
$T_i = 25\ ^oC$

ASSUMPTIONS: (1) One-dimensional, radial conduction, (2) Constant properties.

ANALYSIS: (a) For k = 15 W/m·K, the Biot number is

$$Bi = \frac{h(r_o/3)}{k} = \frac{300\,W/m^2 \cdot K(0.05\,m/3)}{15\,W/m \cdot K} = 0.333.$$

Hence, the lumped capacitance method cannot be used. From Equation 5.50a,

$$\frac{T - T_\infty}{T_i - T_\infty} = C_1 \exp\left(-\zeta_1^2 Fo\right) \frac{\sin\left(\zeta_1 r^*\right)}{\zeta_1 r^*}.$$

At the surface, $r^* = 1$. From Table 5.1, for Bi = 1.0, $\zeta_1 = 1.5708$ rad and $C_1 = 1.2732$. Hence,

$$\frac{60 - 75}{25 - 75} = 0.30 = 1.2732 \exp\left(-1.5708^2 Fo\right) \frac{\sin 90^\circ}{1.5708}$$

exp(-2.467Fo) = 0.370

$$Fo = \frac{\alpha t}{r_o^2} = 0.403$$

$$t = 0.403 \frac{r_o^2}{\alpha} = 0.403 \frac{(0.05\,m)^2}{10^{-5}\,m^2/s} = 100s$$ <

(b) Using the IHT *Transient Conduction Model* for a *Sphere* to perform the parametric calculations, the effect of α is plotted for k = 15 W/m·K.

Continued...

PROBLEM 5.67 (Cont.)

For fixed k and increasing α, there is a reduction in the thermal capacity (ρc_p) of the material, and hence the amount of thermal energy which must be added to increase the temperature. With increasing α, the material therefore responds more quickly to a change in the thermal environment, with the response at the center lagging that of the surface.

The effect of k is plotted for $\alpha = 10^{-5}$ m^2/s.

With increasing k for fixed alpha, there is a corresponding increase in ρc_p, and the material therefore responds more slowly to a thermal change in its surroundings. The thermal response of the center lags that of the surface, with temperature differences, $T(r_o,t) - T(0,t)$, during early stages of solidification increasing with decreasing k.

COMMENTS: Use of this technique to determine h from measurement of $T(r_o)$ at a prescribed t requires an interative solution of the governing equations.

PROBLEM 5.68

KNOWN: Properties, initial temperature, and convection conditions associated with cooling of glass beads.

FIND: (a) Time required to achieve a prescribed center temperature, (b) Effect of convection coefficient on center and surface temperature histories.

SCHEMATIC:

Air

$T_\infty = 15\ ^oC$

$100 \le h \le 1000\ W/m^2 \cdot K$

Glass beads

$T_i = 477\ ^oC,\ T(0,t_f) = 80\ ^oC$

$\rho = 2200\ kg/m^3,\ c_p = 800\ J/kg \cdot K$

$k = 1.4\ W/m \cdot K$

$D = 0.003\ m$

ASSUMPTIONS: (1) One-dimensional conduction in r, (2) Constant properties, (3) Negligible radiation, (4) Fo ≥ 0.2.

ANALYSIS: (a) With h = 400 $W/m^2 \cdot K$, Bi ≡ $h(r_o/3)/k$ = 400 $W/m^2 \cdot K$(0.0005 m)/1.4 W/m·K = 0.143 and the lumped capacitance method should not be used. From the one-term approximation for the center temperature, Eq. 5.50c,

$$\theta_o^* \equiv \frac{T_o - T_\infty}{T_i - T_\infty} = \frac{80 - 15}{477 - 15} = 0.141 = C_1 \exp\left(-\zeta_1^2 Fo\right)$$

For Bi ≡ hr_o/k = 0.429, Table 5.1 yields ζ_1 = 1.101 rad and C_1 = 1.128. Hence,

$$Fo = -\frac{1}{\zeta_1^2} \ln\left(\frac{\theta_o^*}{C_1}\right) = -\frac{1}{(1.101)^2} \ln\left(\frac{0.141}{1.128}\right) = 1.715$$

$$t = 1.715 r_o^2 \frac{\rho c_p}{k} = 1.715(0.0015\,m)^2 \frac{2200\,kg/m^3 \times 800\,J/kg \cdot K}{1.4\,W/m \cdot K} = 4.85s \qquad <$$

From Eq. 5.50b, the corresponding surface (r^* = 1) temperature is

$$T(r_o, t) = T_\infty + (T_i - T_\infty)\theta_o^* \frac{\sin\zeta_1}{\zeta_1} = 15^\circ C + \left(462^\circ C\right)0.141 \frac{0.892}{1.101} = 67.8^\circ C \qquad <$$

(b) The effect of h on the surface and center temperatures was determined using the IHT *Transient Conduction Model* for a *Sphere*.

Center temperature, T(C) vs. Time, t(s)

h = 100 W/m^2.K, r = 0
h = 400 W/m^2.K, r = 0
h = 1000 W/m^2.K, r = 0

Surface temperature, T(C) vs. Time, t(s)

h = 100 W/m^2.K, r = ro
h = 400 W/m^2.K, r = ro
h = 1000 W/m^2.K, r = ro

Continued...

PROBLEM 5.68 (Cont.)

The cooling rate increases with increasing h, particularly from 100 to 400 $W/m^2 \cdot K$. The temperature difference between the center and surface decreases with increasing t and, during the early stages of solidification, with decreasing h.

COMMENTS: Temperature gradients in the glass are largest during the early stages of solidification and increase with increasing h. Since thermal stresses increase with increasing temperature gradients, the propensity to induce defects due to crack formation in the glass increases with increasing h. Hence, there is a value of h above which product quality would suffer and the process should not be operated.

PROBLEM 5.69

KNOWN: Temperature requirements for cooling the spherical material of Ex. 5.4 in air and in a water bath.

FIND: (a) For step 1, the time required for the center temperature to reach T(0,t) = 335°C while cooling in air at 20°C with h = 10 $W/m^2 \cdot K$; find the Biot number; do you expect radial gradients to be appreciable?; compare results with hand calculations in Ex. 5.4; (b) For step 2, time required for the center temperature to reach T(0,t) = 50°C while cooling in water bath at 20°C with h = 6000 $W/m^2 \cdot K$; and (c) For step 2, calculate and plot the temperature history, T(x,t) vs. t, for the center and surface of the sphere; explain features; when do you expect the temperature gradients in the sphere to the largest? Use the IHT *Models* | *Transient Conduction* | *Sphere* model as your solution tool.

SCHEMATIC:

Air
T_∞ = 20°C
h = 10 W/m^2-K
Step 1
T_i = 400°C
$T_a(0, t_a)$ = 335°C

Water
T_∞ = 20°C
h = 6000 W/m^2-K
Step 2
T_i = 335°C
$T_w(0, t_w)$ = 50°C

Spheres, r_o = 5 mm
ρ = 3000 kg/m^3
c = 1 kJ/kg-K
k = 20 W/m-K
$\alpha = 6.6 \times 10^{-6}$ m^2/s

ASSUMPTIONS: (1) One-dimensional conduction in the radial direction, (2) Constant properties.

ANALYSIS: The IHT model represents the series solution for the sphere providing the temperatures evaluated at (r,t). A selected portion of the IHT code used to obtain results is shown in the Comments.

(a) Using the IHT model with step 1 conditions, the time required for $T(0,t_a)$ = T_xt = 335°C with r = 0 and the Biot number are:

$$t_a = 94.2 \text{ s} \qquad\qquad Bi = 0.0025$$ <

Radial temperature gradients will not be appreciable since Bi = 0.0025 << 0.1. The sphere behaves as space-wise isothermal object for the air-cooling process. The result is identical to the lumped-capacitance analysis result of the Text example.

(b) Using the IHT model with step 2 conditions, the time required for $T(0,t_w)$ = T_xt = 50°C with r = 0 and T_i = 335°C is

$$t_w = 3.0 \text{ s}$$ <

Radial temperature gradients will be appreciable, since Bi = 1.5 >> 0.1. The sphere does not behave as a space-wise isothermal object for the water-cooling process.

(c) For the step 2 cooling process, the temperature histories for the center and surface of the sphere are calculated using the IHT model.

Continued …..

PROBLEM 5.69 (Cont.)

At early times, the difference between the center and surface temperature is appreciable. It is in this time region that thermal stresses will be a maximum, and if large enough, can cause fracture. Within 6 seconds, the sphere has a uniform temperature equal to that of the water bath.

COMMENTS: Selected portions of the IHT sphere model codes for steps 1 and 2 are shown below.

```
/* Results, for part (a), step 1,  air cooling; clearly negligible gradient
Bi          Fo       t        T_xt     Ti       r        ro
0.0025      25.13    94.22    335      400      0        0.005  */

// Models | Transient Conduction | Sphere - Step 1, Air cooling
// The temperature distribution T(r,t) is
T_xt = T_xt_trans("Sphere",rstar,Fo,Bi,Ti,Tinf)    // Eq 5.47
T_xt = 335              // Surface temperature



/* Results, for part (b), step 2, water cooling; Ti = 335 C
Bi     Fo       t        T_xt     Ti       r        ro
1.5    0.7936   2.976    50       335      0        0.005  */

// Models | Transient Conduction | Sphere - Step 2, Water cooling
// The temperature distribution T(r,t) is
T_xt = T_xt_trans("Sphere",rstar,Fo,Bi,Ti,Tinf)    // Eq 5.47
//T_xt = 335              // Surface temperature from Step 1; initial temperature for Step 2
T_xt = 50                 // Center temperature, end of Step 2
```

PROBLEM 5.70

KNOWN: Two large blocks of different materials – like copper and concrete – at room temperature, 23°C.

FIND: Which block will feel cooler to the touch?

SCHEMATIC:

Finger, 37°C
Block, 23°C
T_s=37°C
Material, k,ρ,c
T(x,t)
t
T_i=23°C
x

ASSUMPTIONS: (1) Blocks can be treated as semi-infinite solid, (2) Hand or finger temperature is 37°C.

PROPERTIES: *Table A-1*, Copper (300K): ρ = 8933 kg/m^3, c = 385 J/kg·K, k = 401 W/m·K; *Table A-3*, Concrete, stone mix (300K): ρ = 2300 kg/m^3, c = 880 J/kg·K, k = 1.4 W/m·K.

ANALYSIS: Considering the block as a semi-infinite solid, the heat transfer situation corresponds to a sudden change in surface temperature, Case 1, Figure 5.7. The sensation of coolness is related to the heat flow from the hand or finger to the block. From Eq. 5.58, the surface heat flux is

$$q_s''(t) = k(T_s - T_i)/(\pi\alpha\, t)^{1/2} \tag{1}$$

or

$$q_s''(t) \sim (k\rho\, c)^{1/2} \qquad \text{since} \quad \alpha = k/\rho\, c. \tag{2}$$

Hence for the same temperature difference, $T_s - T_i$, and elapsed time, it follows that the heat fluxes for the two materials are related as

$$\frac{q''_{s,copper}}{q''_{s,concrete}} = \frac{(k\rho\, c)^{1/2}_{copper}}{(k\rho\, c)^{1/2}_{concrete}} = \frac{\left[401\dfrac{W}{m\cdot K}\times 8933\dfrac{kg}{m^3}\times 385\dfrac{J}{kg\cdot K}\right]^{1/2}}{\left[1.4\dfrac{W}{m\cdot K}\times 2300\dfrac{kg}{m^3}\times 880\dfrac{J}{kg\cdot K}\right]^{1/2}} = 22.1$$

Hence, the heat flux to the copper block is more than 20 times larger than to the concrete block. The *copper* block will therefore feel noticeably cooler than the concrete one.

PROBLEM 5.71

KNOWN: Asphalt pavement, initially at 50°C, is suddenly exposed to a rainstorm reducing the surface temperature to 20°C.

FIND: Total amount of energy removed (J/m^2) from the pavement for a 30 minute period.

SCHEMATIC:

ASSUMPTIONS: (1) Asphalt pavement can be treated as a semi-infinite solid, (2) Effect of rainstorm is to suddenly reduce the surface temperature to 20°C and is maintained at that level for the period of interest.

PROPERTIES: *Table A-3*, Asphalt (300K): ρ = 2115 kg/m^3, c = 920 J/kg·K, k = 0.062 W/m·K.

ANALYSIS: This solution corresponds to Case 1, Figure 5.7, and the surface heat flux is given by Eq. 5.58 as

$$q_s''(t) = k(T_s - T_i)/(\pi\alpha\ t)^{1/2} \tag{1}$$

The energy into the pavement over a period of time is the integral of the surface heat flux expressed as

$$Q'' = \int_0^t q_s''(t)\,dt. \tag{2}$$

Note that $q_s''(t)$ is into the solid and, hence, Q represents energy into the solid. Substituting Eq. (1) for $q_s''(t)$ into Eq. (2) and integrating find

$$Q'' = k(T_s - T_i)/(\pi\alpha)^{1/2}\int_0^t t^{-1/2}dt = \frac{k(T_s - T_i)}{(\pi\alpha)^{1/2}}\times 2\,t^{1/2}. \tag{3}$$

Substituting numerical values into Eq. (3) with

$$\alpha = \frac{k}{\rho c} = \frac{0.062\ \text{W/m}\cdot\text{K}}{2115\ \text{kg/m}^3 \times 920\ \text{J/kg}\cdot\text{K}} = 3.18\times10^{-8}\text{m}^2/\text{s}$$

find that for the 30 minute period,

$$Q'' = \frac{0.062\ \text{W/m}\cdot\text{K}\ (20-50)\text{K}}{\left(\pi\times3.18\times10^{-8}\text{m}^2/\text{s}\right)^{1/2}}\times 2(30\times60\text{s})^{1/2} = -4.99\times10^5\ \text{J/m}^2. \quad <$$

COMMENTS: Note that the sign for Q'' is negative implying that energy is removed from the solid.

PROBLEM 5.72

KNOWN: Thermophysical properties and initial temperature of thick steel plate. Temperature of water jets used for convection cooling at one surface.

FIND: Time required to cool prescribed interior location to a prescribed temperature.

SCHEMATIC:

$T_s = T_\infty$

Water jets, $T_\infty = 25^oC$

Steel, $T_i = 300^oC$
$\rho = 7800$ kg/m^3
c = 480 J/kg-K
k = 50 W/m-K

x

0.025 m
T(0.025 m, t) = 50^oC

ASSUMPTIONS: (1) One-dimensional conduction in slab, (2) Validity of semi-infinite medium approximation, (3) Negligible thermal resistance between water jets and slab surface ($T_s = T_\infty$), (4) Constant properties.

ANALYSIS: The desired cooling time may be obtained from Eq. (5.57). With T(0.025m, t) = 50°C,

$$\frac{T(x,t)-T_s}{T_i-T_s} = \frac{(50-25)°C}{(300-25)°C} = 0.0909 = \operatorname{erf}\left(\frac{x}{2\sqrt{\alpha t}}\right)$$

$$\frac{x}{2\sqrt{\alpha t}} = 0.0807$$

$$t = \frac{x^2}{(0.0807)^2\, 4\alpha} = \frac{(0.025\text{m})^2}{0.0261\left(1.34\times10^{-5}\text{m}^2/\text{s}\right)} = 1793\text{s}$$ <

where $\alpha = k/\rho c$ = 50 W/m·K/(7800 kg/m^3 × 480 J/kg·K) = 1.34×10^{-5} m^2/s.

COMMENTS: (1) Large values of the convection coefficient ($h \sim 10^4$ W/m^2·K) are associated with water jet impingement, and it is reasonable to assume that the surface is immediately quenched to the temperature of the water. (2) The surface heat flux may be determined from Eq. (5.58). In principle, the flux is infinite at t = 0 and decays as $t^{1/2}$.

PROBLEM 5.73

KNOWN: Temperature imposed at the surface of soil initially at 20°C. See Example 5.5.

FIND: (a) Calculate and plot the temperature history at the burial depth of 0.68 m for selected soil thermal diffusivity values, $\alpha \times 10^7 = 1.0$, 1.38, and 3.0 m^2/s, (b) Plot the temperature distribution over the depth $0 \le x \le 1.0$ m for times of 1, 5, 10, 30, and 60 days with $\alpha = 1.38 \times 10^{-7}\ m^2/s$, (c) Plot the surface heat flux, $q''_x(0,t)$, and the heat flux at the depth of the buried main, $q''_x(0.68\text{m},t)$, as a function of time for a 60 day period with $\alpha = 1.38 \times 10^{-7}\ m^2/s$. Compare your results with those in the Comments section of the example. Use the IHT *Models* | *Transient Conduction* | *Semi-infinite Medium* model as the solution tool.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional conduction in the x-direction, (2) Soil is a semi-infinite medium, and (3) Constant properties.

ANALYSIS: The IHT model corresponds to the case 1, constant surface temperature sudden boundary condition, Eqs. 5.57 and 5.58. Selected portions of the IHT code used to obtain the graphical results below are shown in the Comments.

(a) The temperature history T(x,t) for x = 0.68 m with selected soil thermal diffusivities is shown below. The results are directly comparable to the graph shown in the Ex. 5.5 comments.

Continued …..

(b) The temperature distribution T(x,t) for selected times is shown below. The results are directly comparable to the graph shown in the Ex. 5.5 comments.

alpha = 1.38e-7 m^2/s, T(0,t) = -15C, T(x,0) = 20 C

T(x,t) (C)

Depth, x (m)

1 day
5 days
10 days
30 days
60 days

(c) The heat flux from the soil, $q_x''(0,t)$, and the heat flux at the depth of the buried main, $q_x''(0.68\text{m},t)$, are calculated and plotted for the time period $0 \le t \le 60$ days.

alpha = 1.38e-7 m^2/s, k = 0.52 W/m-K, T(0,t) = -15 C

Heat flux, q''(x,t) (W/m^2)

Time, t (days)

Surface heat flux, x = 0
Buried-main depth, x = 0.68 m

Both the surface and buried-main heat fluxes have a negative sign since heat is flowing in the negative x-direction. The surface heat flux is initially very large and, in the limit, approaches that of the buried-main heat flux. The latter is initially zero, and since the effect of the sudden change in surface temperature is delayed for a time period, the heat flux begins to slowly increase.

Continued …..

COMMENTS: (1) Can you explain why the surface and buried-main heat fluxes are nearly the same at $t = 60$ days? Are these results consistent with the temperature distributions? What happens to the heat flux values for times much greater than 60 days? Use your IHT model to confirm your explanation.

(2) Selected portions of the IHT code for the semi-infinite medium model are shown below.

```
//  Models | Transient Conduction | Semi-infinite Solid | Constant temperature Ts
/* Model: Semi-infinite solid, initially with a uniform temperature T(x,0) = Ti, suddenly subjected to
prescribed surface boundary conditions.  */
// The temperature distribution (Tx,t) is
T_xt = T_xt_semi_CST(x,alpha,t,Ts,Ti)   // Eq 5.55
// The heat flux in the x direction is
q''_xt = qdprime_xt_semi_CST(x,alpha,t,Ts,Ti,k)    //Eq 5.56

// Input parameters
/* The independent variables for this system and their assigned numerical values are */
Ti =  20                    // initial temperature, C
k = 0.52                    // thermal conductivity, W/m.K; base case condition
alpha = 1.38e-7        // thermal diffusivity, m^2/s; base case
//alpha = 1.0e-7
//alpha = 3.0e-7

// Calculating at x-location and time t,
x = 0                                              // m, surface
// x = 0.68                                        // m, burial depth
t = t_day * 24 * 3600                       // seconds to days time covnersion
//t_day = 60
//t_day = 1
//t_day = 5
//t_day = 10
//t_day = 30
t_day = 20

// Surface condition: constant surface temperature
Ts = -15              // surface temperature, K
```

PROBLEM 5.74

KNOWN: Tile-iron, 254 mm to a side, at 150°C is suddenly brought into contact with tile over a subflooring material initially at T_i = 25°C with prescribed thermophysical properties. Tile adhesive softens in 2 minutes at 50°C, but deteriorates above 120°C.

FIND: (a) Time required to lift a tile after being heated by the tile-iron and whether adhesive temperature exceeds 120°C, (2) How much energy has been removed from the tile-iron during the time it has taken to lift the tile.

SCHEMATIC:

ASSUMPTIONS: (1) Tile and subflooring have same thermophysical properties, (2) Thickness of adhesive is negligible compared to that of tile, (3) Tile-subflooring behaves as semi-infinite solid experiencing one-dimensional transient conduction.

PROPERTIES: Tile-subflooring (given): k = 0.15 W/m·K, $\rho c_p = 1.5 \times 10^6$ J/m^3·K, $\alpha = k/\rho c_p = 1.00 \times 10^{-7}$ m^2/s.

ANALYSIS: (a) The tile-subflooring can be approximated as a semi-infinite solid, initially at a uniform temperature T_i = 25°C, experiencing a sudden change in surface temperature $T_s = T(0,t)$ = 150°C. This corresponds to Case 1, Figure 5.7. The time required to heat the adhesive (x_o = 4 mm) to 50°C follows from Eq. 5.57

$$\frac{T(x_o, t_o) - T_s}{T_i - T_s} = \text{erf}\left(\frac{x_o}{2(\alpha t_o)^{1/2}}\right)$$

$$\frac{50-150}{25-150} = \text{erf}\left(\frac{0.004\,\text{m}}{2\left(1.00\times10^{-7}\,\text{m}^2/\text{s}\times t_o\right)^{1/2}}\right)$$

$$0.80 = \text{erf}\left(6.325 t_o^{-1/2}\right)$$

t_o = 48.7s = 0.81 min

using error function values from Table B.2. Since the softening time, Δt_s, for the adhesive is 2 minutes, the time to lift the tile is

$$t_\ell = t_o + \Delta t_s = (0.81 + 2.0)\,\text{min} = 2.81\,\text{min}.$$ <

To determine whether the adhesive temperature has exceeded 120°C, calculate its temperature at t_ℓ = 2.81 min; that is, find $T(x_o, t_\ell)$

$$\frac{T(x_o, t_\ell) - 150}{25-150} = \text{erf}\left(\frac{0.004\,\text{m}}{2\left(1.0\times10^{-7}\,\text{m}^2/\text{s}\times 2.81\times 60\text{s}\right)^{1/2}}\right)$$

Continued...

$$T(x_o, t_\ell) - 150 = -125\,\text{erf}(0.4880) = 125 \times 0.5098$$

$$T(x_o, t_\ell) = 86^\circ\text{C}$$ <

Since $T(x_o,\ t_\ell) < 120°C$, the adhesive will not deteriorate.

(b) The energy required to heat a tile to the lift-off condition is

$$Q = \int_0^{t_\ell} q''_x(0,t) \cdot A_s dt\ .$$

Using Eq. 5.58 for the surface heat flux $q''_s(t) = q''_x(0,t)$, find

$$Q = \int_0^{t_\ell} \frac{k(T_s - T_i)}{(\pi\alpha)^{1/2}} A_s \frac{dt}{t^{1/2}} = \frac{2k(T_s - T_i)}{(\pi\alpha)^{1/2}} A_s t_\ell^{1/2}$$

$$Q = \frac{2 \times 0.15\,\text{W/m} \cdot \text{K}\,(150 - 25)^\circ\text{C}}{\left(\pi \times 1.00 \times 10^{-7}\,\text{m}^2/\text{s}\right)^{1/2}} \times (0.254\,\text{m})^2 \times (2.81 \times 60\text{s})^{1/2} = 56\,\text{kJ}$$ <

COMMENTS: (1) Increasing the tile-iron temperature would decrease the time required to soften the adhesive, but the risk of burning the adhesive increases.

(2) From the energy calculation of part (b) we can estimate the size of an electrical heater, if operating continuously during the 2.81 min period, to maintain the tile-iron at a near constant temperature. The power required is

$$P = Q/t_\ell = 56\,\text{kJ}/2.81 \times 60\text{s} = 330\,\text{W}\ .$$

Of course a much larger electrical heater would be required to initially heat the tile-iron up to the operating temperature in a reasonable period of time.

PROBLEM 5.75

KNOWN: Heat flux gage of prescribed thickness and thermophysical properties (ρ, c_p, k) initially at a uniform temperature, T_i, is exposed to a sudden change in surface temperature $T(0,t) = T_s$.

FIND: Relationships for time constant of gage when (a) backside of gage is insulated and (b) gage is imbedded in semi-infinite solid having the same thermophysical properties. Compare with equation given by manufacturer, $\tau = \left(4d^2 \rho\, c_p\right)/\pi^2 k$.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional conduction, (2) Constant properties.

ANALYSIS: The time constant τ is defined as the time required for the gage to indicate, following a sudden step change, a signal which is 63.2% that of the steady-state value. The manufacturer's relationship for the time constant

$$\tau = \left(4d^2 \rho\, c_p\right)/\pi^2 k$$

can be written in terms of the Fourier number as

$$Fo = \frac{\alpha\tau}{d^2} = \frac{k}{\rho\, c_p} \cdot \frac{\tau}{d^2} = \frac{4}{\pi^2} = 0.4053.$$

The Fourier number can be determined for the two different installations.

(a) For the gage having its backside insulated, the surface and backside temperatures are T_s and $T(0,t)$, respectively. From the sketch it follows that

$$\theta_o^* = \frac{T(0,\tau) - T_s}{T_i - T_s} = 0.368.$$

From Eq. 5.41,

$$\theta_o^* = 0.368 = C_1 \exp\left(-\zeta_1^2 Fo\right)$$

Using Table 5.1 with Bi = 100 (as the best approximation for Bi = hd/k $\rightarrow \infty$, corresponding to sudden surface temperature change with h $\rightarrow \infty$), ζ_1 = 1.5552 rad and C_1 = 1.2731. Hence,

$$0.368 = 1.2731 \exp(-1.5552^2 \times Fo_a)$$

$$Fo_a = 0.513.$$ <

Continued …..

PROBLEM 5.75 (Cont.)

(b) For the gage imbedded in a semi-infinite medium having the same thermophysical properties, Table 5.7 (case 1) and Eq. 5.57 yield

$$\frac{T(x,\tau)-T_s}{T_i-T_s}=0.368=\text{erf}\left[d/2(\alpha\tau)^{1/2}\right]$$

$$d/2(\alpha\tau)^{1/2}=0.3972$$

$$Fo_b=\frac{\alpha\tau}{d^2}=\frac{1}{(2\times 0.3972)^2}=1.585$$ <

COMMENTS: Both models predict higher values of Fo than that suggested by the manufacturer. It is understandable why $Fo_b > Fo_a$ since for (b) the gage is thermally connected to an infinite medium, while for (a) it is isolated. From this analysis we conclude that the gage's transient response will depend upon the manner in which it is applied to the surface or object.

PROBLEM 5.76

KNOWN: Procedure for measuring convection heat transfer coefficient, which involves melting of a surface coating.

FIND: Melting point of coating for prescribed conditions.

SCHEMATIC:

$T_\infty = 300°C$, $h = 200 W/m^2 \cdot K$
Coating, T_m
Insulator, $k \sim 0$
Copper rod, $T_i = 25°C$
Melting @ $t_m = 400s$

ASSUMPTIONS: (1) One-dimensional conduction in solid rod (negligible losses to insulation), (2) Rod approximated as semi-infinite medium, (3) Negligible surface radiation, (4) Constant properties, (5) Negligible thermal resistance of coating.

PROPERTIES: Copper rod (Given): k = 400 W/m·K, $\alpha = 10^{-4}$ m^2/s.

ANALYSIS: Problem corresponds to transient conduction in a semi-infinite solid. Themal response is given by

$$\frac{T(x,t)-T_i}{T_\infty - T_i} = \operatorname{erfc}\left(\frac{x}{2(\alpha t)^{1/2}}\right) - \left[\exp\left(\frac{hx}{k}+\frac{h^2\alpha t}{k^2}\right)\right]\left[\operatorname{erfc}\left(\frac{x}{2(\alpha t)^{1/2}}+\frac{h(\alpha t)^{1/2}}{k}\right)\right].$$

For x = 0, erfc(0) = 1 and T(x,t) = T(0,t) = T_s. Hence

$$\frac{T_s - T_i}{T_\infty - T_i} = 1 - \exp\left(\frac{h^2\alpha t}{k^2}\right)\operatorname{erfc}\left(\frac{h(\alpha t)^{1/2}}{k}\right)$$

with

$$\frac{h(\alpha t_m)^{1/2}}{k} = \frac{200\ W/m^2 \cdot K\left(10^{-4} m^2/s \times 400\ s\right)^{1/2}}{400\ W/m \cdot K} = 0.1$$

$$T_s = T_m = T_i + (T_\infty - T_i)\left[1 - \exp(0.01)\operatorname{erfc}(0.1)\right]$$

$$T_s = 25°C + 275°C[1\text{-}1.01 \times 0.888] = 53.5°C.$$ <

COMMENTS: Use of the procedure to evaluate h from measurement of t_m necessitates iterative calculations.

PROBLEM 5.77

KNOWN: Irreversible thermal injury (cell damage) occurs in living tissue maintained at T ≥ 48°C for a duration Δt ≥ 10s.

FIND: (a) Extent of damage for 10 seconds of contact with machinery in the temperature range 50 to 100°C, (b) Temperature histories at selected locations in tissue (x = 0.5, 1, 5 mm) for a machinery temperature of 100°C.

SCHEMATIC:

Tissue, T_i = 37 °C

T_s, suddenly raised by contact with machinery at T_s

x

T(x,t)

T_s

T_b = 48 °C

T_i = 37 °C

t

x x_b 0

ASSUMPTIONS: (1) Portion of worker's body modeled as semi-infinite medium, initially at a uniform temperature, 37°C, (2) Tissue properties are constant and equivalent to those of water at 37°C, (3) Negligible contact resistance.

PROPERTIES: *Table A-6*, Water, liquid (T = 37°C = 310 K): $\rho = 1/v_f = 993.1\ kg/m^3$, c = 4178 J/kg·K, k = 0.628 W/m·K, $\alpha = k/\rho c = 1.513 \times 10^{-7}\ m^2/s$.

ANALYSIS: (a) For a given surface temperature -- suddenly applied -- the analysis is directed toward finding the skin depth x_b for which the tissue will be at T_b ≥ 48°C for more than 10s? From Eq. 5.57,

$$\frac{T(x_b,t)-T_s}{T_i-T_s} = \text{erf}\left[x_b/2(\alpha t)^{1/2}\right] = \text{erf}[w].$$

For the two values of T_s, the left-hand side of the equation is

$$T_s = 100^\circ C: \quad \frac{(48-100)^\circ C}{(37-100)^\circ C} = 0.825 \qquad\qquad T_s = 50^\circ C: \quad \frac{(48-50)^\circ C}{(37-50)^\circ C} = 0.154$$

The burn depth is

$$x_b = [w]2(\alpha t)^{1/2} = [w]2\left(1.513\times10^{-7}\ m^2/s\times t\right)^{1/2} = 7.779\times10^{-4}[w]t^{1/2}.$$

Continued...

PROBLEM 5.77 (Cont.)

Using Table B.2 to evaluate the error function and letting t = 10s, find x_b as

$T_s = 100°C$: $x_b = 7.779 \times 10^{-4}\,[0.96](10s)^{1/2} = 2.362 \times 10^{3}\text{ m} = 2.36\text{ mm}$ <

$T_s = 50°C$: $x_b = 7.779 \times 10^{-4}\,[0.137](10s)^{1/2} = 3.37 \times 10^{3}\text{ m} = 0.34\text{ mm}$ <

Recognize that tissue at this depth, x_b, has not been damaged, but will become so if T_s is maintained for the next 10s. We conclude that, for $T_s = 50°C$, only superficial damage will occur for a contact period of 20s.

(b) Temperature histories at the prescribed locations are as follows.

The critical temperature of 48°C is reached within approximately 1s at x = 0.5 mm and within 7s at x = 2 mm.

COMMENTS: Note that the burn depth x_b increases as $t^{1/2}$.

PROBLEM 5.78

KNOWN: Thermocouple location in thick slab. Initial temperature. Thermocouple measurement two minutes after one surface is brought to temperature of boiling water.

FIND: Thermal conductivity of slab material.

SCHEMATIC:

$T(0,t) = T_s = 100°C$

$T(x,0) = T_i = 30°C$

$\rho = 2200 kg/m^3$

$c = 700 J/kg \cdot K$

Thermocouple, $T(0.01m, 120s) = 65°C$

$x = 10mm$

ASSUMPTIONS: (1) One-dimensional conduction in x, (2) Slab is semi-infinite medium, (3) Constant properties.

PROPERTIES: Slab material (given): $\rho = 2200$ kg/m^3, c = 700 J/kg·K.

ANALYSIS: For the semi-infinite medium from Eq. 5.57,

$$\frac{T(x,t)-T_s}{T_i-T_s} = \text{erf}\left[\frac{x}{2(\alpha t)^{1/2}}\right]$$

$$\frac{65-100}{30-100} = \text{erf}\left[\frac{0.01m}{2(\alpha \times 120s)^{1/2}}\right]$$

$$\text{erf}\left[\frac{0.01m}{2(\alpha \times 120s)^{1/2}}\right] = 0.5.$$

From Appendix B, find for erf w = 0.5 that w = 0.477; hence,

$$\frac{0.01m}{2(\alpha \times 120s)^{1/2}} = 0.477$$

$$(\alpha \times 120)^{1/2} = 0.0105$$

$$\alpha = 9.156 \times 10^{-7} m^2/s.$$

It follows that since $\alpha = k/\rho c$,

$$k = \alpha \rho c$$

$$k = 9.156 \times 10^{-7} m^2/s \times 2200 kg/m^3 \times 700 J/kg \cdot K$$

$$k = 1.41 W/m \cdot K.$$

<

PROBLEM 5.79

KNOWN: Initial temperature, density and specific heat of a material. Convection coefficient and temperature of air flow. Time for embedded thermocouple to reach a prescribed temperature.

FIND: Thermal conductivity of material.

SCHEMATIC:

Air
$T_\infty = 25^oC$
$h = 200\ W/m^2\text{-}K$
Material of unknown k
$T_i = 100^oC$
$\rho = 950\ kg/m^3$
$c_p = 1100\ J/kg\text{-}K$
$x_m = 0.01\ m$
x
Thermocouple junction
T(0.01 m, 5 min)

ASSUMPTIONS: (1) One-dimensional conduction in x, (2) Sample behaves as a semi-infinite modium, (3) Constant properties.

ANALYSIS: The thermal response of the sample is given by Case 3, Eq. 5.60,

$$\frac{T(x,t)-T_i}{T_\infty - T_i} = \operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right) - \left[\exp\left(\frac{hx}{k} + \frac{h^2 \alpha t}{k^2}\right)\right]\left[\operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha t}} + \frac{h\sqrt{\alpha t}}{k}\right)\right]$$

where, for x = 0.01m at t = 300 s, $[T(x,t) - T_i]/(T_\infty - T_i) = 0.533$. The foregoing equation must be solved iteratively for k, with $\alpha = k/\rho c_p$. The result is

$$k = 0.45\ W/m \cdot K$$ <

with $\alpha = 4.30 \times 10^{-7}\ m^2/s$.

COMMENTS: The solution may be effected by inserting the *Transient Conduction*/*Semi-infinite Solid*/*Surface Conduction Model* of *IHT* into the work space and applying the *IHT Solver*. However, the ability to obtain a converged solution depends strongly on the initial guesses for k and α.

PROBLEM 5.80

KNOWN: Very thick plate, initially at a uniform temperature, T_i, is suddenly exposed to a surface convection cooling process (T_∞,h).

FIND: (a) Temperatures at the surface and 45 mm depth after 3 minutes, (b) Effect of thermal diffusivity and conductivity on temperature histories at x = 0, 0.045 m.

SCHEMATIC:

Coolant; $T_\infty = 15\ ^oC$; $h = 100\ W/m^2\cdot K$; x; Thick plate: $T_i = 325\ ^oC$, $5.6\times10^{-7} \le \alpha \le 5.6\times10^{-5}\ m^2/s$, $2 \le k \le 200\ W/m\cdot K$

ASSUMPTIONS: (1) One-dimensional conduction, (2) Plate approximates semi-infinite medium, (3) Constant properties, (4) Negligible radiation.

ANALYSIS: (a) The temperature distribution for a semi-infinite solid with surface convection is given by Eq. 5.60.

$$\frac{T(x,t)-T_i}{T_\infty - T_i} = \text{erfc}\left(\frac{x}{2(\alpha t)^{1/2}}\right) - \left[\exp\left(\frac{hx}{k}+\frac{h^2\alpha t}{k^2}\right)\right]\left[\text{erfc}\left(\frac{x}{2(\alpha t)^{1/2}}+\frac{h(\alpha t)^{1/2}}{k}\right)\right].$$

At the surface, x = 0, and for t = 3 min = 180s,

$$\frac{T(0,180s)-325^\circ C}{(15-325)^\circ C} = \text{erfc}(0) - \left[\exp\left(0+\frac{100^2\ W^2/m^4 K^2 \times 5.6\times10^{-6}\ m^2/s\times180s}{(20\ W/m\cdot K)^2}\right)\right]$$

$$\times\left[\text{erfc}\left(0+\frac{100\ W/m^2\cdot K\left(5.6\times10^{-6}\ m^2/s\times180s\right)^{1/2}}{20\ W/m\cdot K}\right)\right]$$

$$= 1-\left[\exp(0.02520)\right]\times\left[\text{erfc}(0.159)\right] = 1-1.02552\times(1-0.178)$$

$$T(0,180s) = 325^\circ C - (15-325)^\circ C\cdot(1-1.0255\times0.822)$$

$$T(0,180s) = 325^\circ C - 49.3^\circ C = 276^\circ C .$$ <

At the depth x = 0.045 m, with t = 180s,

$$\frac{T(0.045m,180s)-325^\circ C}{(15-325)^\circ C} = \text{erfc}\left(\frac{0.045\ m}{2\left(5.6\times10^{-6}\ m^2/s\times180s\right)^{1/2}}\right) - \left[\exp\left(\frac{100\ W/m^2\cdot K\times0.045\ m}{20\ W/m\cdot K}+0.02520\right)\right]$$

$$\times\left[\text{erfc}\left(\frac{0.045\ m}{2\left(5.6\times10^{-6}\ m^2/s\times180s\right)^{1/2}}+0.159\right)\right]$$

$$= \text{erfc}(0.7087) + \left[\exp(0.225+0.0252)\right]\times\left[\text{erfc}(0.7087+0.159)\right].$$

$$T(0.045m,180s) = 325^\circ C + (15-325)^\circ C\left[(1-0.684)-1.284(1-0.780)\right] = 315^\circ C$$ <

Continued...

PROBLEM 5.80 (Cont.)

(b) The IHT *Transient Conduction Model* for a *Semi-Infinite Solid* was used to generate temperature histories, and for the two locations the effects of varying α and k are as follows.

k = 20 W/m.K, alpha = 5.6E-5 m^2/s, x = 0
k = 20 W/m.K, alpha = 5.6E-6m^2/s, x = 0
k = 20 W/m.K, alpha = 5.6E-7m^2/s, x = 0

k = 2 W/m.K, alpha = 5.6E-6m^2/s, x = 0
k = 20 W/m.K, alpha = 5.6E-6m^2/s, x = 0
k = 200 W/m.K, alpha = 5.6E-6m^2/s, x = 0

k = 20 W/m.K, alpha = 5.6E-5 m^2.K, x = 45 mm
k = 20 W/m.K, alpha = 5.6E-6m^2.K, x = 45 mm
k = 20 W/m.K, alpha = 5.6E-7m^2.K, x = 45mm

k = 2 W/m.K, alpha = 5.6E-6m^2/s, x = 45 mm
k = 20 W/m.K, alpha = 5.6E-6m^2/s, x = 45 mm
k = 200 W/m.K, alpha = 5.6E-6m^2/s, x = 45 mm

For fixed k, increasing alpha corresponds to a reduction in the thermal capacitance per unit volume (ρc_p) of the material and hence to a more pronounced reduction in temperature at both surface and interior locations. Similarly, for fixed α, decreasing k corresponds to a reduction in ρc_p and hence to a more pronounced decay in temperature.

COMMENTS: In part (a) recognize that Fig. 5.8 could also be used to determine the required temperatures.

PROBLEM 5.81

KNOWN: Thick oak wall, initially at a uniform temperature of 25°C, is suddenly exposed to combustion products at 800°C with a convection coefficient of 20 $W/m^2 \cdot K$.

FIND: (a) Time of exposure required for the surface to reach an ignition temperature of 400°C, (b) Temperature distribution at time t = 325s.

SCHEMATIC:

T_∞ = 800 °C
$T(0,t)$ = 400 °C
Wood, oak
$T(x,t)$
Combustion products
$T(x,t)$ T_i = 25 °C
T_∞ = 800 °C
h = 20 $W/m^2 \cdot K$
x

ASSUMPTIONS: (1) Oak wall can be treated as semi-infinite solid, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible radiation.

PROPERTIES: *Table A-3*, Oak, cross grain (300 K): ρ = 545 kg/m^3, c = 2385 $J/kg \cdot K$, k = 0.17 $W/m \cdot K$, $\alpha = k/\rho c$ = 0.17 $W/m \cdot K$/545 $kg/m^3 \times$ 2385 $J/kg \cdot K$ = 1.31×10^{-7} m^2/s.

ANALYSIS: (a) This situation corresponds to Case 3 of Figure 5.7. The temperature distribution is given by Eq. 5.60 or by Figure 5.8. Using the figure with

$$\frac{T(0,t)-T_i}{T_\infty - T_i} = \frac{400-25}{800-25} = 0.48 \qquad \text{and} \qquad \frac{x}{2(\alpha t)^{1/2}} = 0$$

we obtain $h(\alpha t)^{1/2}/k \approx 0.75$, in which case $t \approx (0.75k/h\alpha^{1/2})^2$. Hence,

$$t \approx \left(0.75 \times 0.17\,W/m \cdot K \Big/ 20\,W/m^2 \cdot K \left(1.31 \times 10^{-7}\,m^2/s\right)^{1/2}\right)^2 = 310s \qquad <$$

(b) Using the IHT *Transient Conduction Model* for a *Semi-infinite Solid*, the following temperature distribution was generated for t = 325s.

Temperature, T(C)
400
325
250
175
100
25
0 0.005 0.01 0.015 0.02 0.025 0.03
Distance from the surface, x(m)

The temperature decay would become more pronounced with decreasing α (decreasing k, increasing ρc_p) and in this case the penetration depth of the heating process corresponds to $x \approx 0.025$ m at 325s.

COMMENTS: The result of part (a) indicates that, after approximately 5 minutes, the surface of the wall will ignite and combustion will ensue. Once combustion has started, the present model is no longer appropriate.

PROBLEM 5.82

KNOWN: Thickness, initial temperature and thermophysical properties of concrete firewall. Incident radiant flux and duration of radiant heating. Maximum allowable surface temperatures at the end of heating.

FIND: If maximum allowable temperatures are exceeded.

SCHEMATIC:

x L = 0.25 m
$q_s'' = 10^4$ W/m^2
Concrete, $T_i = 25°C$
$\rho = 2300$ kg/m^3
c = 880 J/kg-K
k = 1.4 W/m-K
$\alpha_s = 1.0$
$T_{max} = 325°C$
$T_{max} = 25°C$

ASSUMPTIONS: (1) One-dimensional conduction in wall, (2) Validity of semi-infinite medium approximation, (3) Negligible convection and radiative exchange with the surroundings at the irradiated surface, (4) Negligible heat transfer from the back surface, (5) Constant properties.

ANALYSIS: The thermal response of the wall is described by Eq. (5.60)

$$T(x,t) = T_i + \frac{2\, q_o''\,(\alpha t/\pi)^{1/2}}{k}\exp\left(\frac{-x^2}{4\alpha t}\right) - \frac{q_o'' x}{k}\operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right)$$

where, $\alpha = k/\rho c_p = 6.92\times10^{-7}\,m^2/s$ and for $t = 30\ min = 1800s$, $2q_o''(\alpha t/\pi)^{1/2}/k = 284.5\,K$. Hence, at x = 0,

$$T(0, 30\ min) = 25°C + 284.5°C = 309.5°C < 325°C \qquad <$$

At $x = 0.25m$, $\left(-x^2/4\alpha t\right) = -12.54$, $q_o'' x/k = 1{,}786K$, and $x/2(\alpha t)^{1/2} = 3.54$. Hence,

$$T(0.25m,\ 30\,min) = 25°C + 284.5°C\left(3.58\times10^{-6}\right) - 1786°C\times(\sim 0) \approx 25°C \qquad <$$

Both requirements are met.

COMMENTS: The foregoing analysis is conservative since heat transfer at the irradiated surface due to convection and net radiation exchange with the environment have been neglected. If the emissivity of the surface and the temperature of the surroundings are assumed to be ε = 1 and T_{sur} = 298K, radiation exchange at T_s = 309.5°C would be $q''_{rad} = \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right) = 6{,}080\ W/m^2\cdot K$, which is significant (~ 60% of the prescribed radiation).

PROBLEM 5.83

KNOWN: Initial temperature of copper and glass plates. Initial temperature and properties of finger.

FIND: Whether copper or glass feels cooler to touch.

SCHEMATIC:

Finger, $T_i = 310K$

Copper or glass, $T_i = 300K$

ASSUMPTIONS: (1) The finger and the plate behave as semi-infinite solids, (2) Constant properties, (3) Negligible contact resistance.

PROPERTIES: Skin (given): $\rho = 1000\ \text{kg/m}^3$, c = 4180 J/kg·K, k = 0.625 W/m·K; *Table A-1* (T = 300K), Copper: $\rho = 8933\ \text{kg/m}^3$, c = 385 J/kg·K, k = 401 W/m·K; *Table A-3* (T = 300K), Glass: $\rho = 2500\ \text{kg/m}^3$, c = 750 J/kg·K, k = 1.4 W/m·K.

ANALYSIS: Which material feels cooler depends upon the contact temperature T_s given by Equation 5.63. For the three materials of interest,

$$(k\rho c)^{1/2}_{skin} = (0.625 \times 1000 \times 4180)^{1/2} = 1{,}616\ \text{J/m}^2 \cdot \text{K} \cdot \text{s}^{1/2}$$
$$(k\rho c)^{1/2}_{cu} = (401 \times 8933 \times 385)^{1/2} = 37{,}137\ \text{J/m}^2 \cdot \text{K} \cdot \text{s}^{1/2}$$
$$(k\rho c)^{1/2}_{glass} = (1.4 \times 2500 \times 750)^{1/2} = 1{,}620\ \text{J/m}^2 \cdot \text{K} \cdot \text{s}^{1/2}.$$

Since $(k\rho c)^{1/2}_{cu} >> (k\rho c)^{1/2}_{glass}$, the copper will feel much cooler to the touch. From Equation 5.63,

$$T_s = \frac{(k\rho c)^{1/2}_A T_{A,i} + (k\rho c)^{1/2}_B T_{B,i}}{(k\rho c)^{1/2}_A + (k\rho c)^{1/2}_B}$$

$$T_{s(cu)} = \frac{1{,}616(310) + 37{,}137(300)}{1{,}616 + 37{,}137} = 300.4\ \text{K}$$ <

$$T_{s(glass)} = \frac{1{,}616(310) + 1{,}620(300)}{1{,}616 + 1{,}620} = 305.0\ \text{K}.$$ <

COMMENTS: The extent to which a material's temperature is affected by a change in its thermal environment is inversely proportional to $(k\rho c)^{1/2}$. Large k implies an ability to *spread* the effect by conduction; large ρc implies a large capacity for thermal energy *storage*.

PROBLEM 5.84

KNOWN: Initial temperatures, properties, and thickness of two plates, each insulated on one surface.

FIND: Temperature on insulated surface of one plate at a prescribed time after they are pressed together.

SCHEMATIC:

L = 0.02 m
x
T_i = 400K
T_i = 300K
Stainless steel

ASSUMPTIONS: (1) One-dimensional conduction, (2) Constant properties, (3) Negligible contact resistance.

PROPERTIES: Stainless steel (given): ρ = 8000 kg/m^3, c = 500 J/kg·K, k = 15 W/m·K.

ANALYSIS: At the instant that contact is made, the plates behave as semi-infinite slabs and, since the (ρkc) product is the same for the two plates, Equation 5.63 yields a surface temperature of

$$T_s = 350\text{K}.$$

The interface will remain at this temperature, even after thermal effects penetrate to the insulated surfaces. The transient response of the hot wall may therefore be calculated from Equations 5.40 and 5.41. At the insulated surface (x^* = 0), Equation 5.41 yields

$$\frac{T_o - T_s}{T_i - T_s} = C_1 \exp\left(-z_1^2 \text{Fo}\right)$$

where, in principle, $h \to \infty$ and $T_\infty \to T_s$. From Equation 5.39c, $\text{Bi} \longrightarrow \infty$ yields z_1 = 1.5707, and from Equation 5.39b

$$C_1 = \frac{4 \sin z_1}{2z_1 + \sin(2z_1)} = 1.273$$

Also, $$\text{Fo} = \frac{\alpha t}{L^2} = \frac{3.75\times10^{-6}\,\text{m}^2/\text{s}\,(60\text{s})}{(0.02\text{ m})^2} = 0.563.$$

Hence, $$\frac{T_o - 350}{400 - 350} = 1.273 \exp\left(-1.5707^2 \times 0.563\right) = 0.318$$

$$T_o = 365.9\text{ K}.$$ <

COMMENTS: Since Fo > 0.2, the one-term approximation is appropriate.

PROBLEM 5.85

KNOWN: Thickness and properties of liquid coating deposited on a metal substrate. Initial temperature and properties of substrate.

FIND: (a) Expression for time required to completely solidify the liquid, (b) Time required to solidify an alumina coating.

SCHEMATIC:

Alumina
$\rho = 3970 \text{ kg/m}^3$
$h_{sf} = 3.577\times10^6$ J/kg

Liquid, Solid, Substrate, $k_s = 120$ W/m·K, $\alpha_s = 4\times10^{-5}$ m^2/s, $\rho h_{sf}\, dS/dt$, $S(t)$, q''_{cond}, Tungsten, $\delta = 2$ mm, $T_f = 2318$ K, $T_i = 300$ K, 0, T, x

ASSUMPTIONS: (1) Substrate may be approximated as a semi-infinite medium in which there is one-dimensional conduction, (2) Solid and liquid alumina layers remain at fusion temperature throughout solidification (negligible resistance to heat transfer by conduction through solid), (3) Negligible contact resistance at the coating/substrate interface, (4) Negligible solidification contraction, (5) Constant properties.

ANALYSIS: (a) Performing an energy balance on the solid layer, whose thickness S increases with t, the latent heat released at the solid/liquid interface must be balanced by the rate of heat conduction into the solid. Hence, per unit surface area,

$$\rho h_{sf}\frac{dS}{dt} = q''_{cond}$$

where, from Eq. 5.58, $q''_{cond} = k\left(T_f - T_i\right)/\left(\pi\alpha t\right)^{1/2}$. It follows that

$$\rho h_{sf}\frac{dS}{dt} = \frac{k_s\left(T_f - T_i\right)}{\left(\pi\alpha_s t\right)^{1/2}}$$

$$\int_o^\delta dS = \frac{k_s\left(T_f - T_i\right)}{\rho h_{sf}\left(\pi\alpha_s\right)^{1/2}}\int_o^t \frac{dt}{t^{1/2}}$$

$$\delta = \frac{2k_s}{\left(\pi\alpha_s\right)^{1/2}}\left(\frac{T_f - T_i}{\rho h_{sf}}\right)t^{1/2}$$

$$t = \frac{\pi\alpha_s}{4k_s^2}\left(\frac{\delta\rho h_{sf}}{T_f - T_i}\right)^2$$ <

(b) For the prescribed conditions,

$$t = \frac{\pi\left(4\times10^{-5}\,\text{m}^2/\text{s}\right)}{4\left(120\,\text{W/m}\cdot\text{K}\right)^2}\left(\frac{0.002\,\text{m}\times3970\,\text{kg/m}^3\times3.577\times10^6\,\text{J/kg}}{2018\,\text{K}}\right)^2 = 0.43\text{s}$$ <

COMMENTS: Such solidification processes occur over short time spans and are typically termed *rapid solidification*.

PROBLEM 5.86

KNOWN: Properties of mold wall and a solidifying metal.

FIND: (a) Temperature distribution in mold wall at selected times, (b) Expression for variation of solid layer thickness.

SCHEMATIC:

ASSUMPTIONS: (1) Mold wall may be approximated as a semi-infinite medium in which there is one-dimensional conduction, (2) Solid and liquid metal layers remain at fusion temperature throughout solidification (negligible resistance to heat transfer by conduction through solid), (3) Negligible contact resistance at mold/metal interface, (4) Constant properties.

ANALYSIS: (a) As shown in schematic (b), the temperature remains nearly uniform in the metal (at T_f) throughout the process, while both the temperature and temperature penetration increase with time in the mold wall.

(b) Performing an energy balance for a control surface about the solid layer, the latent energy released due to solidification at the solid/liquid interface is balanced by heat conduction into the solid, $q''_{lat} = q''_{cond}$, where $q''_{lat} = \rho h_{sf} \, dS/dt$ and q''_{cond} is given by Eq. 5.58. Hence,

$$\rho h_{sf} \frac{dS}{dt} = \frac{k_w \left(T_f - T_i\right)}{\left(\pi \alpha_w t\right)^{1/2}}$$

$$\int_o^S dS = \frac{k_w \left(T_f - T_i\right)}{\rho h_{sf} \left(\pi \alpha_w\right)^{1/2}} \int_o^t \frac{dt}{t^{1/2}}$$

$$S = \frac{2 k_w \left(T_f - T_i\right)}{\rho h_{sf} \left(\pi \alpha_w\right)^{1/2}} t^{1/2}$$

<

COMMENTS: The analysis of part (b) would only apply until the temperature field penetrates to the exterior surface of the mold wall, at which point, it may no longer be approximated as a semi-infinite medium.

PROBLEM 5.87

KNOWN: Steel (plain carbon) billet of square cross-section initially at a uniform temperature of 30°C is placed in a soaking oven and subjected to a convection heating process with prescribed temperature and convection coefficient.

FIND: Time required for billet center temperature to reach 600°C.

SCHEMATIC:

$2L_2 = 0.3m$, $T(0,0,t) = 600°C$, $T_i = 30°C$, Billet, 0.3m x 0.3m, Oven, x_1, x_2, $T_\infty = 750°C$, $h = 100 W/m^2 \cdot K$, $2L_1 = 0.3m$, T_∞, h

ASSUMPTIONS: (1) Two-dimensional conduction in x_1 and x_2 directions, (2) Constant properties, (3) Heat transfer to billet is by convection only.

PROPERTIES: *Table A-1*, Steel, plain carbon (T = (30+600)°C/2 = 588K = ≈ 600K): ρ = 7854 kg/m^3, c_p = 559 J/kg·K, k = 48.0 W/m·K, $\alpha = k/\rho c_p = 1.093 \times 10^{-5}$ m^2/s.

ANALYSIS: The billet corresponds to Case (e), Figure 5.11 (infinite rectangular bar). Hence, the temperature distribution is of the form

$$\theta^*(x_1, x_2, t) = P(x_1, t) \times P(x_2, t)$$

where P(x,t) denotes the distribution corresponding to the plane wall. Because of symmetry in the x_1 and x_2 directions, the P functions are identical. Hence,

$$\frac{\theta(0,0,t)}{\theta_i} = \left[\frac{\theta_o(0,t)}{\theta_i}\right]^2_{\text{Plane wall}} \qquad \text{where} \begin{cases} \theta = T - T_\infty \\ \theta_i = T_i - T_\infty \\ \theta_o = T(0,t) - T_\infty \end{cases} \qquad \text{and } L = 0.15\text{m}.$$

Substituting numerical values, find

$$\frac{\theta_o(0,t)}{\theta_i} = \left[\frac{T(0,0,t) - T_\infty}{T_i - T_\infty}\right]^{1/2} = \left[\frac{(600-750)°C}{(30-750)°C}\right]^{1/2} = 0.46.$$

Consider now the Heisler chart for the plane wall, Figure D.1. For the values

$$\theta_o^* = \frac{\theta_o}{\theta_i} \approx 0.46 \qquad Bi^{-1} = \frac{k}{hL} = \frac{48.0 \text{ W/m}\cdot\text{K}}{100 \text{ W/m}^2\cdot\text{K} \times 0.15\text{m}} = 3.2$$

find

$$t^* = Fo = \frac{\alpha t}{L^2} \approx 3.2.$$

Hence,

$$t = \frac{3.2 L^2}{\alpha} = \frac{3.2 \ (0.15\text{m})^2}{1.093 \times 10^{-5} \text{m}^2/\text{s}} = 6587\text{s} = 1.83\text{h}.$$

<

PROBLEM 5.88

KNOWN: Initial temperature of fire clay brick which is cooled by convection.

FIND: Center and corner temperatures after 50 minutes of cooling.

SCHEMATIC:

ASSUMPTIONS: (1) Homogeneous medium with constant properties, (2) Negligible radiation effects.

PROPERTIES: *Table A-3*, Fire clay brick (900K): $\rho = 2050\ \text{kg/m}^3$, $k = 1.0\ \text{W/m}\cdot\text{K}$, $c_p = 960\ \text{J/kg}\cdot\text{K}$. $\alpha = 0.51 \times 10^{-6}\ \text{m}^2/\text{s}$.

ANALYSIS: From Fig. 5.11(h), the center temperature is given by

$$\frac{T(0,0,0,t) - T_\infty}{T_i - T_\infty} = P_1(0,t) \times P_2(0,t) \times P_3(0,t)$$

where P_1, P_2 and P_3 must be obtained from Fig. D.1.

$$L_1 = 0.03\text{m}: \qquad Bi_1 = \frac{h\,L_1}{k} = 1.50 \qquad Fo_1 = \frac{\alpha\,t}{L_1^2} = 1.70$$

$$L_2 = 0.045\text{m}: \qquad Bi_2 = \frac{h\,L_2}{k} = 2.25 \qquad Fo_2 = \frac{\alpha\,t}{L_2^2} = 0.756$$

$$L_3 = 0.10\text{m}: \qquad Bi_3 = \frac{h\,L_3}{k} = 5.0 \qquad Fo_3 = \frac{\alpha\,t}{L_3^2} = 0.153$$

Hence from Fig. D.1,

$$P_1(0,t) \approx 0.22 \qquad P_2(0,t) \approx 0.50 \qquad P_3(0,t) \approx 0.85.$$

Hence,
$$\frac{T(0,0,0,t) - T_\infty}{T_i - T_\infty} \approx 0.22 \times 0.50 \times 0.85 = 0.094$$

and the center temperature is

$$T(0,0,0,t) \approx 0.094(1600 - 313)\text{K} + 313\text{K} = 434\text{K}.$$ <

Continued …..

The corner temperature is given by

$$\frac{T(L_1, L_2, L_3, t) - T_\infty}{T_i - T_\infty} = P(L_1, t) \times P(L_2, t) \times P(L_3, t)$$

where

$$P(L_1, t) = \frac{\theta(L_1, t)}{\theta_o} \cdot P_1(0, t), \text{ etc.}$$

and similar forms can be written for L_2 and L_3. From Fig. D.2,

$$\frac{\theta(L_1, t)}{\theta_o} \approx 0.55 \qquad \frac{\theta(L_2, t)}{\theta_o} \approx 0.43 \qquad \frac{\theta(L_3, t)}{\theta_o} \approx 0.25.$$

Hence,

$$P(L_1, t) \approx 0.55 \times 0.22 = 0.12$$
$$P(L_2, t) \approx 0.43 \times 0.50 = 0.22$$
$$P(L_3, t) \approx 0.85 \times 0.25 = 0.21$$

and

$$\frac{T(L_1, L_2, L_3, t) - T_\infty}{T_i - T_\infty} \approx 0.12 \times 0.22 \times 0.21 = 0.0056$$

or

$$T(L_1, L_2, L_3, t) \approx 0.0056(1600 - 313)\text{K} + 313\text{K}.$$

The corner temperature is then

$$T(L_1, L_2, L_3, t) \approx 320\text{K}.$$ <

COMMENTS: (1) The foregoing temperatures are overpredicted by ignoring radiation, which is significant during the early portion of the transient.

(2) Note that, if the time required to reach a certain temperature were to be determined, an iterative approach would have to be used. The foregoing procedure would be used to compute the temperature for an assumed value of the time, and the calculation would be repeated until the specified temperature were obtained.

PROBLEM 5.89

KNOWN: Cylindrical copper pin, 100mm long × 50mm diameter, initially at 20°C; end faces are subjected to intense heating, suddenly raising them to 500°C; at the same time, the cylindrical surface is subjected to a convective heating process (T_∞,h).

FIND: (a) Temperature at center point of cylinder after a time of 8 seconds from sudden application of heat, (b) Consider parameters governing transient diffusion and justify simplifying assumptions that could be applied to this problem.

SCHEMATIC:

ASSUMPTIONS: (1) Two-dimensional conduction, (2) Constant properties and convection heat transfer coefficient.

PROPERTIES: *Table A-1*, Copper, pure $\left(\bar{T} \approx (500+20)^\circ C/2 \approx 500K\right)$: ρ = 8933 kg/m^3, c = 407 J/kg·K, k = 386 W/m·K, α = k/ρc = 386 W/m·K/8933 kg/m^3 × 407 J/kg·K = 1.064 × 10^{-4}m^2/s.

ANALYSIS: (1) The pin can be treated as a two-dimensional system comprised of an infinite cylinder whose surface is exposed to a convection process (T_∞,h) and of a plane wall whose surfaces are maintained at a constant temperature (T_e). This configuration corresponds to the short cylinder, Case (i) of Fig. 5.11,

$$\frac{\theta(r,x,t)}{\theta_i} = C(r,t) \times P(x,t). \quad (1)$$

For the infinite cylinder, using Fig. D.4, with

$$Bi = \frac{hr_o}{k} = \frac{100\ W/m^2 \cdot K\left(25\times10^{-3}m\right)}{385\ W/m\cdot K} = 6.47\times10^{-3} \quad \text{and} \quad Fo = \frac{\alpha\, t}{r_o^2} = \frac{1.064\times10^{-4}\ \frac{m^2}{s}\times 8s}{\left(25\times10^{-3}m\right)^2} = 1.36,$$

find
$$C(0,8s) = \left.\frac{\theta(0,8s)}{\theta_i}\right]_{cyl} \approx 1. \quad (2)$$

For the infinite plane wall, using Fig. D.1, with

$$Bi = \frac{hL}{k} \to \infty \quad \text{or} \quad Bi^{-1} \to 0 \quad \text{and} \quad Fo = \frac{\alpha\, t}{L^2} = \frac{1.064\times10^{-4}\, m^2/s \times 8s}{\left(50\times10^{-3}m\right)^2} = 0.34,$$

find
$$P(0,8s) = \left.\frac{\theta(0,8s)}{\theta_i}\right]_{wall} \approx 0.5. \quad (3)$$

Combining Eqs. (2) and (3) with Eq. (1), find $\frac{\theta(0,0,8s)}{\theta_i} = \frac{T(0,0,8s) - T_\infty}{T_i - T_\infty} \approx 1\times0.5 = 0.5$

$$T(0,0,8s) = T_\infty + 0.5\left(T_i - T_\infty\right) = 500 + 0.5(20-500) = 260^\circ C.$$
<

(b) The parameters controlling transient conduction with convective boundary conditions are the Biot and Fourier numbers. Since Bi << 0.1 for the cylindrical shape, we can assume radial gradients are negligible. That is, we need only consider conduction in the x-direction.

PROBLEM 5.90

KNOWN: Cylindrical-shaped meat roast weighing 2.25 kg, initially at 6°C, is placed in an oven and subjected to convection heating with prescribed (T_∞,h).

FIND: Time required for the center to reach a done temperature of 80°C.

SCHEMATIC:

ASSUMPTIONS: (1) Two-dimensional conduction in x and r directions, (2) Uniform and constant properties, (3) Properties approximated as those of water.

PROPERTIES: *Table A-6*, Water, liquid $\left(\bar{T} = (80+6)^\circ C/2 \approx 315K\right)$:◁ $\rho = 1/v_f = 1/1.009 \times 10^{-3} m^3/kg = 991.1\ kg/m^3$, $c_{p,f} = 4179\ J/kg \cdot K$, $k = 0.634\ W/m \cdot K$, $\alpha = k/\rho c = 1.531 \times 10^{-7} m^2/s$.

ANALYSIS: The dimensions of the roast are determined from the requirement $r_o = L$ and knowledge of its weight and density,

$$M = \rho V = \rho \cdot 2L \cdot \pi r_o^2 \quad \text{or} \quad r_o = L = \left[\frac{M}{2\pi\rho}\right]^{1/3} = \left[\frac{2.25\ kg}{2\pi 991.1\ kg/m^3}\right]^{1/3} = 0.0712 m. \quad (1)$$

The roast corresponds to Case (i), Figure 5.11, and the temperature distribution may be expressed as the product of one-dimensional solutions, $\frac{T(x,r,t) - T_\infty}{T_i - T_\infty} = P(x,t) \times C(r,t)$, where P(x,t) and C(r,t) are defined by Eqs. 5.65 and 5.66, respectively. For the center of the cylinder,

$$\frac{T(0,0,t) - T_\infty}{T_i - T_\infty} = \frac{(80-175)^\circ C}{(6-175)^\circ C} = 0.56. \quad (2)$$

In terms of the product solutions,

$$\frac{T(0,0,t) - T_\infty}{T_i - T_\infty} = 0.56 = \left.\frac{T(0,t) - T_\infty}{T_i - T_\infty}\right]_{wall} \times \left.\frac{T(0,t) - T_\infty}{T_i - T_\infty}\right]_{cylinder} \quad (3)$$

For each of these shapes, we need to find values of θ_o/θ_i such that their product satisfies Eq. (3). For both shapes,

$$Bi = \frac{h\ r_o}{k} = \frac{hL}{k} = \frac{15\ W/m^2 \cdot K \times 0.0712m}{0.634\ W/m \cdot K} = 1.68 \quad \text{or} \quad Bi^{-1} \approx 0.6$$

$$Fo = \alpha\ t/r_o^2 = \alpha\ t/L^2 = 1.53 \times 10^{-7} m^2/s \times t/(0.0712m)^2 = 3.020 \times 10^{-5} t.$$

Continued …..

A trial-and-error solution is necessary. Begin by assuming a value of Fo; obtain the respective θ_o/θ_i values from Figs. D.1 and D.4; test whether their product satisfies Eq. (3). Two trials are shown as follows:

Trial	Fo	t(hrs)	$\theta_o/\theta_i)_{wall}$	$\theta_o/\theta_i)_{cyl}$	$\left.\frac{\theta_o}{\theta_i}\right]_w \times \left.\frac{\theta_o}{\theta_i}\right]_{cyl}$
1	0.4	3.68	0.72	0.50	0.36
2	0.3	2.75	0.78	0.68	0.53

For Trial 2, the product of 0.53 agrees closely with the value of 0.56 from Eq. (2). Hence, it will take approximately 2 ¾ hours to roast the meat.

PROBLEM 5.91

KNOWN: A long alumina rod, initially at a uniform temperature of 850K, is suddenly exposed to a cooler fluid.

FIND: Temperature of the rod after 30s, at an exposed end, T(0,0,t), and at an axial distance 6mm from the end, T(0, 6mm, t).

SCHEMATIC:

ASSUMPTIONS: (1) Two-dimensional conduction in (r,x) directions, (2) Constant properties, (3) Convection coefficient is same on end and cylindrical surfaces.

PROPERTIES: *Table A-2*, Alumina, polycrystalline aluminum oxide (assume $\bar{T} \approx (850 + 600)K/2 = 725K$): ρ = 3970 kg/m^3, c = 1154 J/kg·K, k = 12.4 W/m·K.

ANALYSIS: First, check if system behaves as a lumped capacitance. Find

$$Bi = \frac{hL_c}{k} = \frac{h(r_o/2)}{k} = \frac{500\ W/m \cdot K(0.010m/2)}{12.4\ W/m \cdot K} = 0.202.$$

Since Bi > 0.1, rod does not behave as spacewise isothermal object. Hence, treat rod as a semi-infinite cylinder, the multi-dimensional system Case (f), Fig. 5.11.

The product solution can be written as

$$\theta^*(r,x,t) = \frac{\theta(r,x,t)}{\theta_i} = \frac{\theta(r,t)}{\theta_i} \times \frac{\theta(x,t)}{\theta_i} = C(r^*, t^*) \times S(x^*, t^*)$$

Infinite cylinder, C(r*,t*). Using the Heisler charts with r* = r = 0 and

$$Bi^{-1} = \left[\frac{h\, r_o}{k}\right]^{-1} = \left[\frac{500\ W/m^2 \cdot K \times 0.01m}{12.4\ W/m \cdot K}\right]^{-1} = 2.48.$$

Evaluate α = k/ρc = 2.71 × 10^{-6}m^2/s, find $Fo = \alpha\ t/r_o^2 = 2.71\times10^{-6} m^2/s\ \times 30s/(0.01m)^2$ = 0.812. From the Heisler chart, Fig. D.4, with Bi^{-1} = 2.48 and Fo = 0.812, read C(0,t*) = θ(0,t)/θ$_i$ = 0.61.

Continued …..

PROBLEM 5.91 (Cont.)

Semi-infinite medium, S(x*,t*). Recognize this as Case (3), Fig. 5.7. From Eq. 5.60, note that the LHS needs to be transformed as follows,

$$\frac{T-T_i}{T_\infty - T_i} = 1 - \frac{T-T_\infty}{T_i - T_\infty} \qquad S(x,t) = \frac{T-T_\infty}{T_i - T_\infty}.$$

Thus,

$$S(x,t) = 1 - \left\{ \operatorname{erfc}\left[\frac{x}{2(\alpha t)^{1/2}}\right] - \left[\exp\left[\frac{hx}{k} + \frac{h^2 \alpha t}{k^2}\right]\right]\left[\operatorname{erfc}\left[\frac{x}{2(\alpha t)^{1/2}} + \frac{h(\alpha t)^{1/2}}{k}\right]\right]\right\}.$$

Evaluating this expression at the surface (x = 0) and 6mm from the exposed end, find

$$S(0,30s) = 1 - \left\{ \operatorname{erfc}(0) - \left[\exp\left[0 + \frac{\left(500\ W/m^2 \cdot K\right)^2 2.71\times10^{-6} m^2/s\times30s}{(12.4\ W/m\cdot K)^2}\right]\right]\right.$$

$$\left.\left[\operatorname{erfc}\left[0 + \frac{500\ W/m^2\cdot K\left(2.71\times10^{-6} m^2/s\times30s\right)^{1/2}}{12.4\ W/m\cdot K}\right]\right]\right\}$$

$$S(0,30s) = 1 - \{1 - [\exp(0.1322)][\operatorname{erfc}(0.3636)]\} = 0.693.$$

Note that Table B.2 was used to evaluate the complementary error function, erfc(w).

$$S(6mm,30s) = 1 - \left\{ \operatorname{erfc}\left[\frac{0.006m}{2\left(2.71\times10^{-6} m^2/s\times30s\right)^{1/2}}\right] - \right.$$

$$\left.\left[\exp\left[\frac{500\ W/m^2\cdot K\times0.006m}{12.4\ W/m\cdot K} + 0.1322\right]\right]\left[\operatorname{erfc}(0.3327+0.3636)\right]\right\} = 0.835.$$

The product solution can now be evaluated for each location. At (0,0),

$$\theta^*(0,0,t) = \frac{T(0,0,30s) - T_\infty}{T_i - T_\infty} = C\left(0,t^*\right)\times S\left(0,t^*\right) = 0.61\times0.693 = 0.423.$$

Hence, $T(0,0,30s) = T_\infty + 0.423(T_i - T_\infty) = 350K + 0.423(850-350)K = 561K.$ <

At (0,6mm),

$$\theta^*(0,6mm,t) = C\left(0,t^*\right)\times S\left(6mm,t^*\right) = 0.61\times0.835 = 0.509$$

$$T(0,6mm,30s) = 604K.$$ <

COMMENTS: Note that the temperature at which the properties were evaluated was a good estimate.

PROBLEM 5.92

KNOWN: Stainless steel cylinder of Ex. 5.7, 80-mm diameter by 60-mm length, initially at 600 K, suddenly quenched in an oil bath at 300 K with h = 500 W/m^2·K. Use the *Transient Conduction, Plane Wall* and *Cylinder* models of *IHT* to obtain the following solutions.

FIND: (a) Calculate the temperatures T(r,x,t) after 3 min: at the cylinder center, T(0, 0, 3 min), at the center of a circular face, T(0, L, 3 min), and at the midheight of the side, T(r_o, 0, 3 min); compare your results with those in the example; (b) Calculate and plot temperature histories at the cylinder center, T(0, 0, t), the mid-height of the side, T(r_o, 0, t), for $0 \le t \le 10$ min; comment on the gradients and what effect they might have on phase transformations and thermal stresses; and (c) For $0 \le t \le 10$ min, calculate and plot the temperature histories at the cylinder center, T(0, 0, t), for convection coefficients of 500 and 1000 W/m^2·K.

SCHEMATIC:

ASSUMPTIONS: (1) Two-dimensional conduction in r- and x-coordinates, (2) Constant properties.

PROPERTIES: Stainless steel (*Example 5.7*): ρ = 7900 kg/m^3, c = 526 J/kg·K, k = 17.4 W/m·K.

ANALYSIS: The following results were obtained using the *Transient Conduction* models for the *Plane Wall* and *Cylinder* of *IHT*. Salient portions of the code are provided in the Comments.

(a) Following the methodology for a product solution outlined in Example 5.7, the following results were obtained at t = t_o = 3 min

(r, x, t)	P(x, t)	C(r, t)	T(r, x, t)-IHT (K)	T(r, x, t)-Ex (K)
0, 0, t_o	0.6357	0.5388	402.7	405
0, L, t_o	0.4365	0.5388	370.5	372
r_o, 0, t_o	0.6357	0.3273	362.4	365

Continued …..

PROBLEM 5.92 (Cont.)

The temperatures from the one-term series calculations of the Example 5.7 are systematically higher than those resulting from the *IHT* multiple-term series model, which is the more accurate method.

(b) The temperature histories for the center and mid-height of the side locations are shown in the graph below. Note that at early times, the temperature difference between these locations, and hence the gradient, is large. Large differences could cause variations in microstructure and hence, mechanical properties, as well as induce residual thermal stresses.

(c) Effect of doubling the convection coefficient is to increase the quenching rate, but much less than by a factor of two as can be seen in the graph below.

Quenching with h = 500 W/m^2.K

T(x, r, t) (C)

Time, t (min)

Mid-height of side (0,ro)
Center (0, 0)

Effect of increased conv. coeff. on quenching rate

T(0, 0, t) (C)

Time, t (min)

h = 500 W/m^2.K
h = 1000 W/m^2.K

COMMENTS: From *IHT* menu for *Transient Conduction* | *Plane Wall* and *Cylinder*, the models were combined to solve the product solution. Key portions of the code, less the input variables, are copied below.

```
// Plane wall temperature distribution
// The temperature distribution is
T_xtP = T_xt_trans("Plane Wall",xstar,FoP,BiP,Ti,Tinf)    // Eq 5.39
// The dimensionless parameters are
xstar = x / L
BiP = h * L / k                // Eq 5.9
FoP= alpha * t / L^2       // Eq 5.33
alpha = k/ (rho * cp)
// Dimensionless representation, P(x,t)
P_xt = (T_xtP - Tinf ) / (Ti - Tinf)

// Cylinder temperature distribution
// The temperature distribution T(r,t) is
T_rtC = T_xt_trans("Cylinder",rstar,FoC,BiC,Ti,Tinf)    // Eq 5.47
// The dimensionless parameters are
rstar = r / ro
BiC = h * ro / k
FoC= alpha * t / ro^2
// Dimensionless representation, C(r,t)
C_rt= (T_rtC - Tinf ) / (Ti - Tinf)

// Product solution temperature distribution
(T_xrt - Tinf) / (Ti - Tinf) = P_xt * C_rt
```

PROBLEM 5.93

KNOWN: Stability criterion for the explicit method requires that the coefficient of the T_m^p term of the one-dimensional, finite-difference equation be zero or positive.

FIND: For Fo > 1/2, the finite-difference equation will predict values of T_m^{p+1} which violate the Second law of thermodynamics. Consider the prescribed numerical values.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional conduction in x, (2) Constant properties, (3) No internal heat generation.

ANALYSIS: The explicit form of the finite-difference equation, Eq. 5.73, for an interior node is

$$T_m^{p+1} = Fo\left(T_{m+1}^p + T_{m-1}^p\right) + (1-2\ Fo)T_m^p.$$

The stability criterion requires that the coefficient of T_m^p be zero or greater. That is,

$$(1-2\ Fo) \geq 0 \qquad \text{or} \qquad Fo \leq \frac{1}{2}.$$

For the prescribed temperatures, consider situations for which Fo = 1, ½ and ¼ and calculate T_m^{p+1}.

$$Fo = 1 \qquad T_m^{p+1} = 1(100+100)^\circ C + (1-2\times1)50^\circ C = 250^\circ C$$
$$Fo = 1/2 \qquad T_m^{p+1} = 1/2(100+100)^\circ C + (1-2\times1/2)50^\circ C = 100^\circ C$$
$$Fo = 1/4 \qquad T_m^{p+1} = 1/4(100+100)^\circ C + (1-2\times1/4)50^\circ C = 75^\circ C.$$

Plotting these distributions above, note that when Fo = 1, T_m^{p+1} is greater than 100°C, while for Fo = ½ and ¼ , $T_m^{p+1} \leq 100°C$. The distribution for Fo = 1 is thermodynamically impossible: heat is flowing into the node during the time period Δt, causing its temperature to rise; yet heat is flowing in the direction of increasing temperature. This is a violation of the Second law. When Fo = ½ or ¼, the node temperature increases during Δt, but the temperature gradients for heat flow are proper. This will be the case when Fo ≤ ½, verifying the stability criterion.

PROBLEM 5.94

KNOWN: Thin rod of diameter D, initially in equilibrium with its surroundings, T_{sur}, suddenly passes a current I; rod is in vacuum enclosure and has prescribed electrical resistivity, ρ_e, and other thermophysical properties.

FIND: Transient, finite-difference equation for node m.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional, transient conduction in rod, (2) Surroundings are much larger than rod, (3) Properties are constant and evaluated at an average temperature, (4) No convection within vacuum enclosure.

ANALYSIS: The finite-difference equation is derived from the energy conservation requirement on the control volume, $A_c\Delta x$, where $A_c = \pi D^2/4$ and $P = \pi D$.

The energy balance has the form

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = \dot{E}_{st} \qquad q_a + q_b - q_{rad} + I^2 R_e = \rho\ cV\frac{T_m^{p+1} - T_m^p}{\Delta t}.$$

where $\dot{E}_g = I^2 R_e$ and $R_e = \rho_e \Delta x / A_c$. Using Fourier's law to express the conduction terms, q_a and q_b, and Eq. 1.7 for the radiation exchange term, q_{rad}, find

$$kA_c\frac{T_{m-1}^p - T_m^p}{\Delta x} + kA_c\frac{T_{m+1}^p - T_m^p}{\Delta x} - \varepsilon P\Delta x\sigma\left(T_m^{4,p} - T_{sur}^4\right) + I^2\frac{\rho_e \Delta x}{A_c} = \rho\ cA_c\Delta x\frac{T_m^{p+1} - T_m^p}{\Delta t}.$$

Divide each term by $\rho c A_c\ \Delta x/\Delta t$, solve for T_m^{p+1} and regroup to obtain

$$T_m^{p+1} = \frac{k}{\rho\ c}\cdot\frac{\Delta t}{\Delta x^2}\left(T_{m-1}^p + T_{m+1}^p\right) - \left[2\cdot\frac{k}{\rho\ c}\cdot\frac{\Delta t}{\Delta x^2} - 1\right]T_m^p$$

$$-\frac{\varepsilon P\sigma}{A_c}\cdot\frac{\Delta t}{\rho\ c}\left(T_m^{4,p} - T_{sur}^4\right) + \frac{I^2\rho_e}{A_c^2}\cdot\frac{\Delta t}{\rho\ c}.$$

Recognizing that $Fo = \alpha\ \Delta t/\Delta x^2$, regroup to obtain

$$T_m^{p+1} = Fo\left(T_{m-1}^p + T_{m+1}^p\right) + (1 - 2\ Fo)T_m^p - \frac{\varepsilon P\sigma\Delta x^2}{kA_c}\cdot Fo\left(T_m^{4,p} - T_{sur}^4\right) + \frac{I^2\rho_e\Delta x^2}{kA_c^2}\cdot Fo.$$

The stability criterion is based upon the coefficient of the T_m^p term written as

$$Fo \le ½. \qquad <$$

COMMENTS: Note that we have used the forward-difference representation for the time derivative; see Section 5.9.1. This permits convenient treatment of the non-linear radiation exchange term.

PROBLEM 5.95

KNOWN: One-dimensional wall suddenly subjected to uniform volumetric heating and convective surface conditions.

FIND: Finite-difference equation for node at the surface, x = -L.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional transient conduction, (2) Constant properties, (3) Uniform $\dot{q}$.

ANALYSIS: There are two types of finite-difference equations for the *explicit* and *implicit* methods of solution. Using the energy balance approach, both types will be derived.

Explicit Method. Perform an energy balance on the surface node shown above,

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = \dot{E}_{st} \qquad q_{conv} + q_{cond} + \dot{q}V = \rho\, cV \frac{T_o^{p+1} - T_o^p}{\Delta t} \tag{1}$$

$$h(1\cdot 1)\left(T_\infty - T_o^p\right) + k(1\cdot 1)\frac{T_1^p - T_o^p}{\Delta x} + \dot{q}\left[1\cdot 1\cdot \frac{\Delta x}{2}\right] = \rho\, c\left[1\cdot 1\cdot \frac{\Delta x}{2}\right]\frac{T_o^{p+1} - T_o^p}{\Delta t}. \tag{2}$$

For the explicit method, the temperatures on the LHS are evaluated at the *previous* time (p). The RHS provides a *forward*-difference approximation to the time derivative. Divide Eq. (2) by $\rho c\Delta x/2\Delta t$ and solve for T_o^{p+1}.

$$T_o^{p+1} = 2\frac{h\Delta t}{\rho\, c\Delta x}\left(T_\infty - T_o^p\right) + 2\frac{k\Delta t}{\rho\, c\Delta x^2}\left(T_1^p - T_o^p\right) + \dot{q}\frac{\Delta t}{\rho\, c} + T_o^p.$$

Introducing the Fourier and Biot numbers,

$$Fo \equiv (k/\rho\, c)\Delta t/\Delta x^2 \qquad Bi \equiv h\Delta x/k$$

$$T_o^{p+1} = 2\ Fo\left[T_1^p + Bi\cdot T_\infty + \frac{\dot{q}\Delta x^2}{2k}\right] + \left(1 - 2\ Fo - 2\ Fo\cdot Bi\right)T_o^p. \tag{3}$$

The stability criterion requires that the coefficient of T_o^p be positive. That is,

$$\left(1 - 2\ Fo - 2\ Fo\cdot Bi\right) \ge 0 \qquad \text{or} \qquad Fo \le 1/2(1 + Bi). \tag{4}$$ <

Implicit Method. Begin as above with an energy balance. In Eq. (2), however, the temperatures on the LHS are evaluated at the *new* (p+1) time. The RHS provides a *backward*-difference approximation to the time derivative.

$$h\left(T_\infty - T_o^{p+1}\right) + k\frac{T_1^{p+1} - T_o^{p+1}}{\Delta x} + \dot{q}\left[\frac{\Delta x}{2}\right] = \rho\, c\left[\frac{\Delta x}{2}\right]\frac{T_o^{p+1} - T_o^p}{\Delta t} \tag{5}$$

$$\left(1 + 2\ Fo(Bi + 1)\right)T_o^{p+1} - 2\ Fo\cdot T_1^{p+1} = T_o^p + 2Bi\cdot Fo\cdot T_\infty + Fo\frac{\dot{q}\Delta x^2}{k}. \tag{6}$$ <

COMMENTS: Compare these results (Eqs. 3, 4 and 6) with the appropriate expression in Table 5.2.

PROBLEM 5.96

KNOWN: Plane wall, initially having a linear, steady-state temperature distribution with boundaries maintained at $T(0,t) = T_1$ and $T(L,t) = T_2$, suddenly experiences a uniform volumetric heat generation due to the electrical current. Boundary conditions T_1 and T_2 remain fixed with time.

FIND: (a) On T-x coordinates, sketch the temperature distributions for the following cases: initial conditions ($t \leq 0$), steady-state conditions ($t \rightarrow \infty$) assuming the maximum temperature exceeds T_2, and two intermediate times; label important features; (b) For the three-nodal network shown, derive the finite-difference equation using either the implicit or explicit method; (c) With a time increment of Δt = 5 s, obtain values of T_m for the first 45s of elapsed time; determine the corresponding heat fluxes at the boundaries; and (d) Determine the effect of mesh size by repeating the foregoing analysis using grids of 5 and 11 nodal points.

SCHEMATIC:

ASSUMPTIONS: (1) Two-dimensional, transient conduction, (2) Uniform volumetric heat generation for $t \geq 0$, (3) Constant properties.

PROPERTIES: Wall (Given): $\rho = 4000\ kg/m^3$, c = 500 J/kg·K, k = 10 W/m·K.

ANALYSIS: (a) The temperature distribution on T-x coordinates for the requested cases are shown below. Note the following key features: (1) linear initial temperature distribution, (2) non-symmetrical parabolic steady-state temperature distribution, (3) gradient at x = L is first positive, then zero and becomes negative, and (4) gradient at x = 0 is always positive.

(b) Performing an energy balance on the control volume about node m above, for unit area, find

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = \dot{E}_{st}$$

$$k(1)\frac{T_2 - T_m}{\Delta x} + k(1)\frac{T_1 - T_m}{\Delta x} + \dot{q}(1)\Delta x = \rho(1)c\Delta x\frac{T_m^{p+1} - T_m^p}{\Delta t}$$

$$Fo\left[T_1 + T_2 - 2T_m\right] + \frac{\dot{q}\Delta t}{\rho c_p} = T_m^{p+1} - T_m^p$$

For the T_m term in brackets, use "p" for explicit and "p+1" for implicit form,

Explicit: $$T_m^{p+1} = Fo\left(T_1^p + T_2^p\right) + (1 - 2Fo)T_m^p + \dot{q}\Delta t/\rho c_p \quad (1)$$ <

Implicit: $$T_m^{p+1} = \left[Fo\left(T_1^{p+1} + T_2^{p+1}\right) + \dot{q}\,\Delta t/\rho c_p + T_m^p\right]\Big/(1 + 2Fo) \quad (2)$$ <

Continued...

PROBLEM 5.96 (Cont.)

(c) With a time increment $\Delta t = 5s$, the FDEs, Eqs. (1) and (2) become

Explicit: $$T_m^{p+1} = 0.5T_m^p + 75 \qquad (3)$$

Implicit: $$T_m^{p+1} = \left(T_m^p + 75\right)/1.5 \qquad (4)$$

where

$$Fo = \frac{k\Delta t}{\rho c \Delta x^2} = \frac{10\,W/m\cdot K \times 5\,s}{4000\,kg/m^3 \times 500\,J/kg\cdot K\left(0.010\,m\right)^2} = 0.25$$

$$\frac{\dot{q}\Delta t}{\rho c} = \frac{2\times 10^7\,W/m^3 \times 5\,s}{4000\,kg/m^3 \times 500\,J/kg\cdot K} = 50\,K$$

Performing the calculations, the results are tabulated as a function of time,

p	t (s)	T_1 (°C)	T_m (°C) Explicit	T_m (°C) Implicit	T_2 (°C)
0	0	0	50	50	100
1	5	0	100.00	83.33	100
2	10	0	125.00	105.55	100
3	15	0	137.50	120.37	100
4	20	0	143.75	130.25	100
5	25	0	146.88	136.83	100
6	30	0	148.44	141.22	100
7	35	0	149.22	144.15	100
8	40	0	149.61	146.10	100
9	45	0	149.80	147.40	100

<

The heat flux at the boundaries at t = 45s follows from the energy balances on control volumes about the boundary nodes, using the explicit results for T_m^p,

Node 1: $$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = \dot{E}_{st}$$

$$q''_x(0,t) + k\frac{T_m^p - T_1}{\Delta x} + \dot{q}(\Delta x/2) = 0$$

$$q''_x(0,t) = -k\left(T_m^p - T_1\right)/\Delta x - \dot{q}\Delta x/2 \qquad (5)$$

$$q''_x(0,45s) = -10\,W/m\cdot K\,(149.8 - 0)\,K/0.010\,m - 2\times 10^7\,W/m^3 \times 0.010\,m/2$$

$$q''_x(0,45s) = -149,800\,W/m^2 - 100,000\,W/m^2 = -249,800\,W/m^2$$

<

Node 2: $$k\frac{T_m^p - T_2}{\Delta x} - q''_x(L,t) + \dot{q}(\Delta x/2) = 0$$

$$q''_x(L,t) = k\left(T_m^p - T_2\right)/\Delta x + \dot{q}\Delta x/2 = 0 \qquad (6)$$

Continued...

PROBLEM 5.96 (Cont.)

$$q''_x(L,t) = 10\,W/m\cdot K(149.80 - 100)C/0.010\,m + 2\times10^7\,W/m^3 \times 0.010\,m/2$$

$$q''_x(L,t) = 49{,}800\,W/m^2 + 100{,}000\,W/m^2 = +149{,}800\,W/m^2$$ <

(d) To determine the effect of mesh size, the above analysis was repeated using grids of 5 and 11 nodal points, Δx = 5 and 2 mm, respectively. Using the *IHT Finite-Difference Equation Tool*, the finite-difference equations were obtained and solved for the temperature-time history. Eqs. (5) and (6) were used for the heat flux calculations. The results are tabulated below for t = 45s, where T_m^p (45s) is the center node,

Mesh Size Δx (mm)	T_m^p (45s) (°C)	q''_x (0,45s) kW/m^2	q''_x (L,45s) kW/m^2
10	149.8	-249.8	+149.8
5	149.3	-249.0	+149.0
2	149.4	-249.1	+149.0

COMMENTS: (1) The center temperature and boundary heat fluxes are quite insensitive to mesh size for the condition.

(2) The copy of the IHT workspace for the 5 node grid is shown below.

```
// Mesh size - 5 nodes, deltax = 5 mm
// Nodes a, b(m), and c are interior nodes

// Finite-Difference Equations Tool - nodal
equations
/* Node a: interior node; e and w labeled b and
1. */
rho*cp*der(Ta,t) =
fd_1d_int(Ta,Tb,T1,k,qdot,deltax)
/* Node b: interior node; e and w labeled c and
a. */
rho*cp*der(Tb,t) =
fd_1d_int(Tb,Tc,Ta,k,qdot,deltax)
/* Node c: interior node; e and w labeled 2 and
b. */
rho*cp*der(Tc,t) =
fd_1d_int(Tc,T2,Tb,k,qdot,deltax)

// Assigned Variables:
deltax = 0.005
k = 10
rho = 4000
cp = 500
qdot = 2e7
T1 = 0
T2 = 100

/* Initial Conditions:
Tai = 25
Tbi = 50
Tci = 75   */

/* Data Browser Results - Nodal
temperatures at 45s
Ta          Tb            Tc       t
99.5        149.3         149.5    45  */

// Boundary Heat Fluxes - at t = 45s
q''x0 = - k * (Taa - T1 ) / deltax - qdot
* deltax / 2
q''xL = k * (Tcc - T2 ) / deltax + qdot *
deltax /2
//where Taa = Ta (45s), Tcc =
Tc(45s)
Taa = 99.5
Tcc = 149.5
/* Data Browser results
q''x0              q''xL
-2.49E5   1.49E5     */
```

PROBLEM 5.97

KNOWN: Solid cylinder of plastic material ($\alpha = 6 \times 10^{-7}$ m^2/s), initially at uniform temperature of T_i = 20°C, insulated at one end (T_4), while other end experiences heating causing its temperature T_0 to increase linearly with time at a rate of a = 1°C/s.

FIND: (a) Finite-difference equations for the 4 nodes using the explicit method with Fo = 1/2 and (b) Surface temperature T_0 when T_4 = 35°C.

SCHEMATIC:

$T_0 = T_i + at$, a = 1 °C/s

$T_i(x,<0) = 20$ °C, $\alpha = 6\text{x}10^{-7}$ m^2/s

T_1, T_2, T_3, T_4; Δx = 6 mm; L = 24 mm; x

T_3, T_4, q_b, q_a, $\Delta x/2$

ASSUMPTIONS: (1) One-dimensional, transient conduction in cylinder, (2) Constant properties, and (3) Lateral and end surfaces perfectly insulated.

ANALYSIS: (a) The finite-difference equations using the *explicit* method for the interior nodes (m = 1, 2, 3) follow from Eq. 5.73 with Fo = 1/2,

$$T_m^{p+1} = Fo\left(T_{m+1}^p + T_{m-1}^p\right) + (1-2Fo)T_m^p = 0.5\left(T_{m+1}^p + T_{m-1}^p\right) \tag{1}$$

From an energy balance on the control volume node 4 as shown above yields with Fo = 1/2

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = \dot{E}_{st} \qquad q_a + q_b + 0 = \rho c V\left(T_4^{p+1} - T_4^p\right)\Big/\Delta t$$

$$0 + k\left(T_3^p - T_4^p\right)\Big/\Delta x = \rho c(\Delta x/2)\left(T_4^{p+1} - T_4^p\right)\Big/\Delta t$$

$$T_4^{p+1} = 2Fo T_3^p + (1-2Fo)T_4^p = T_3^p \tag{2}$$

(b) Performing the calculations, the temperature-time history is tabulated below, where $T_0 = T_i + a \cdot t$ where a = 1°C/s and $t = p \cdot \Delta t$ with,

$$Fo = \alpha\Delta t\big/\Delta x^2 = 0.5 \qquad \Delta t = 0.5(0.006\,\text{m})^2\Big/6\times10^{-7}\ \text{m}^2\big/\text{s} = 30\text{s}$$

p	t (s)	T_0 (°C)	T_1 (°C)	T_2 (°C)	T_3 (°C)	T_4 (°C)
0	0	20	20	20	20	20
1	30	50	20	20	20	20
2	60	80	35	20	20	20
3	90	110	50	27.5	20	20
4	120	140	68.75	35	23.75	20
5	150	170	87.5	46.25	27.5	23.75
6	180	200	108.1	57.5	35	27.5
7	210	230	-	-	-	35

When T_4(210s, p = 7) = 35°C, find T_0(210s) = 230°C. <

PROBLEM 5.98

KNOWN: A 0.12 m thick wall, with thermal diffusivity 1.5×10^{-6} m^2/s, initially at a uniform temperature of 85°C, has one face suddenly lowered to 20°C while the other face is perfectly insulated.

FIND: (a) Using the explicit finite-difference method with space and time increments of Δx = 30 mm and Δt = 300s, determine the temperature distribution within the wall 45 min after the change in surface temperature; (b) Effect of Δt on temperature histories of the surfaces and midplane.

SCHEMATIC:

T_i = 85 °C
α = 1.5x10^{-6} m^2/s
$T(L,0)$ = 20 °C
x
L = 0.12 m
T_0 T_1 T_2 T_3 T_L = 20 °C
Δx = 0.03 m
Δt = 300 s, 75 s

ASSUMPTIONS: (1) One-dimensional transient conduction, (2) Constant properties.

ANALYSIS: (a) The finite-difference equations for the interior points, nodes 0, 1, 2, and 3, can be determined from Eq. 5.73,

$$T_m^{p+1} = Fo\left(T_{m-1}^p + T_{m+1}^p\right) + (1-2\,Fo)T_m^p \qquad (1)$$

with

$$Fo = \alpha\Delta t/\Delta x^2 = 1.5\times10^{-6}\,m^2/s\times300s/(0.03\,m)^2 = 1/2. \qquad (2)$$

Note that the stability criterion, Eq. 5.74, Fo ≤ 1/2, is satisfied. Hence, combining Eqs. (1) and (2), $T_m^{p+1} = 1/2\left(T_{m-1}^p + T_{m+1}^p\right)$ for m = 0, 1, 2, 3. Since the adiabatic plane at x = 0 can be treated as a symmetry plane, $T_{m-1} = T_{m+1}$ for node 0 (m = 0). The finite-difference solution is generated in the table below using t = p·Δt = 300 p (s) = 5 p (min).

p	t(min)	T_0	T_1	T_2	T_3	T_L(°C)
0	0	85	85	85	85	20
1		85	85	85	52.5	20
2	10	85	85	68.8	52.5	20
3		85	76.9	68.8	44.4	20
4	20	76.9	76.9	60.7	44.4	20
5		76.9	68.8	60.7	40.4	20
6	30	68.8	68.8	54.6	40.4	20
7		68.8	61.7	54.6	37.3	20
8	40	61.7	61.7	49.5	37.3	20
9	45	61.7	55.6	49.5	34.8	20

<

The temperature distribution can also be determined from the Heisler charts. For the wall,

$$Fo = \frac{\alpha t}{L^2} = \frac{1.5\times10^{-6}\,m^2/s\times(45\times60)s}{(0.12\,m)^2} = 0.28 \qquad \text{and} \qquad Bi^{-1} = \frac{k}{hL} = 0.$$

Continued...

PROBLEM 5.98 (Cont.)

From Figure D.1, for $Bi^{-1} = 0$ and $Fo = 0.28$, find $\theta_o/\theta_i \approx 0.55$. Hence, for $x = 0$

$$\frac{T_o - T_\infty}{T_i - T_\infty} = \frac{\theta_o}{\theta_i} \quad \text{or} \quad T_o = T(0,t) = T_\infty + \frac{\theta_o}{\theta_i}(T_i - T_\infty) = 20^\circ C + 0.55(85-20)^\circ C = 55.8^\circ C.$$

This value is to be compared with 61.7°C for the finite-difference method.

(b) Using the IHT *Finite-Difference Equation* Tool Pad for *One-Dimensional Transient Conduction*, temperature histories were computed and results are shown for the insulated surface (T0) and the midplane, as well as for the chilled surface (TL).

Apart from small differences during early stages of the transient, there is excellent agreement between results obtained for the two time steps. The temperature decay at the insulated surface must, of course, lag that of the midplane.

PROBLEM 5.99

KNOWN: Thickness, initial temperature and thermophysical properties of molded plastic part. Convection conditions at one surface. Other surface insulated.

FIND: Surface temperatures after one hour of cooling.

SCHEMATIC:

L = 60 mm
Δx = 6 mm
x
Air jets
$T_\infty = 20^oC$
$h = 100\ W/m^2\text{-}K$
T_{10} T_9 T_8 T_2 T_1 T_0
Plastic product, $T_i = 80^oC$
$\rho = 1200\ kg/m^3$
$c = 1500\ J/kg\text{-}K$
$k = 0.03\ W/m\text{-}K$

ASSUMPTIONS: (1) One-dimensional conduction in product, (2) Negligible radiation, at cooled surface, (3) Negligible heat transfer at insulated surface, (4) Constant properties.

ANALYSIS: Adopting the implicit scheme, the finite-difference equation for the cooled surface node is given by Eq. (5.88), from which it follows that

$$(1+2\,Fo+2\,FoBi)T_{10}^{p+1}-2Fo\,T_9^{p+1}=2\,FoBi\,T_\infty+T_{10}^p$$

The general form of the finite-difference equation for any interior node (1 to 9) is given by Eq. (5.89),

$$(1+2\,Fo)T_m^{p+1}-Fo\left(T_{m-1}^{p+1}+T_{m+1}^{p+1}\right)=T_m^p$$

The finite-difference equation for the insulated surface node may be obtained by applying the symmetry requirement to Eq. (5.89); that is, $T_{m+1}^p = T_{m-1}^p$. Hence,

$$(1+2\,Fo)T_o^{p+1}-2\,Fo\,T_1^{p+1}=T_o^p$$

For the prescribed conditions, Bi = hΔx/k = 100 W/m^2·K (0.006m)/0.30 W/m·K = 2. If the explicit method were used, the most restrictive stability requirement would be given by Eq. (5.79). Hence, for Fo (1+Bi) ≤ 0.5, Fo ≤ 0.167. With Fo = $\alpha\Delta t/\Delta x^2$ and $\alpha = k/\rho c = 1.67\times10^{-7}\ m^2/s$, the corresponding restriction on the time increment would be $\Delta t \le 36s$. Although no such restriction applies for the implicit method, a value of Δt = 30s is chosen, and the set of 11 finite-difference equations is solved using the *Tools* option designated as *Finite-Difference Equations, One-Dimensional*, and *Transient* from the IHT Toolpad. At t = 3600s, the solution yields:

$$T_{10}(3600s)=24.1°C \qquad T_0(3600s)=71.5°C$$ <

COMMENTS: (1) More accurate results may be obtained from the one-term approximation to the exact solution for one-dimensional, transient conduction in a plane wall. With Bi = hL/k = 20, Table 5.1 yields ζ_1 = 1.496 rad and C_1 = 1.2699. With Fo = $\alpha t/L^2$ = 0.167, Eq. (5.41) then yields $T_o = T_\infty$ + $(T_i - T_\infty)\, C_1 \exp\left(-\zeta_1^2 Fo\right) = 72.4°C$, and from Eq. (5.40b), $T_s = T_\infty + (T_i - T_\infty)\cos\left(\zeta_1\right)$ = 24.5°C.

Since the finite-difference results do not change with a reduction in the time step (Δt < 30s), the difference between the numerical and analytical results is attributed to the use of a coarse grid. To improve the accuracy of the numerical results, a smaller value of Δx should be used.

Continued …..

PROBLEM 5.99 (Cont.)

(2) Temperature histories for the front and back surface nodes are as shown.

Although the surface temperatures rapidly approaches that of the coolant, there is a significant lag in the thermal response of the back surface. The different responses are attributable to the small value of α and the large value of Bi.

PROBLEM 5.100

KNOWN: Plane wall, initially at a uniform temperature T_i = 25°C, is suddenly exposed to convection with a fluid at T_∞ = 50°C with a convection coefficient h = 75 W/m^2·K at one surface, while the other is exposed to a constant heat flux q''_o = 2000 W/m^2. See also Problem 2.43.

FIND: (a) Using spatial and time increments of Δx = 5 mm and Δt = 20s, compute and plot the temperature distributions in the wall for the initial condition, the steady-state condition, and two intermediate times, (b) On q''_x-x coordinates, plot the heat flux distributions corresponding to the four temperature distributions represented in part (a), and (c) On q''_x-t coordinates, plot the heat flux at x = 0 and x = L.

SCHEMATIC:

Heater q''_o = 2000 W/m^2

T(x,<0) = Ti = 25°C

k = 1.5 W/m·K

α = 7.5x10^{-6} m^2/s

T_0 T_1 T_2 T_3 T_9 T_{10}

q''_x(L,t)

T_∞ = 10,000 K

h = 75W/m^2·K

x

Δx = 5mm

L = 50 mm

T_{m-1} T_m T_{m+1}

$q''_{x,a}$ $q''_{x,b}$

ASSUMPTIONS: (1) One-dimensional, transient conduction and (2) Constant properties.

ANALYSIS: (a) Using the *IHT Finite-Difference Equations*, *One-Dimensional*, *Transient Tool*, the equations for determining the temperature distribution were obtained and solved with a spatial increment of Δx = 5 mm. Using the *Lookup Table* functions, the temperature distributions were plotted as shown below.

(b) The heat flux, q''_x(x,t), at each node can be evaluated considering the control volume shown with the schematic above

$$q''_x(m,p) = \left(q''_{x,a} + q''_{x,b}\right)/2 = \left[k(1)\frac{T^p_{m-1} - T^p_m}{\Delta x} + k(1)\frac{T^p_m - T^p_{m+1}}{\Delta x}\right]\Big/2 = k\left(T^p_{m-1} - T^p_{m+1}\right)\Big/2\Delta x$$

From knowledge of the temperature distribution, the heat flux at each node for the selected times is computed and plotted below.

Temperature, T(x,t) (C)

160, 140, 120, 100, 80, 60, 40, 20

0, 10, 20, 30, 40, 50

Wall coordinate, x (mm)

Initial condition, t<=0s

Time = 150s

Time = 300s

Steady-state conditions, t>1200s

Heat flux, q''x(x,t) (W/m^2)

2000, 1500, 1000, 500, 0

0, 10, 20, 30, 40, 50

Wall coordinate, x (mm)

Initial condition, t<=0s

Time = 150s

Time = 300s

Steady-state conditions, t>1200s

(c) The heat fluxes for the locations x = 0 and x = L, are plotted as a function of time. At the x = 0 surface, the heat flux is constant, q''_o = 2000 W/m^2. At the x = L surface, the heat flux is given by Newton's law of cooling, q''_x(L,t) = h[T(L,t) - T_∞]; at t = 0, q''_x(L,0) = -1875 W/m^2. For steady-state conditions, the heat flux $q''_x(x,\infty)$ is everywhere constant at q''_o.

Continued...

PROBLEM 5.100 (Cont.)

2000
1000
0
-1000
-2000
Heat flux (W/m^2)
0 200 400 600 800 1000 1200
Elapsed time, t (s)
q''x(0,t) - Heater flux
q''x(L,t) - Convective flux

Comments: The IHT workspace using the *Finite-Difference Equations Tool* to determine the temperature distributions and heat fluxes is shown below. Some lines of code were omitted to save space on the page.

```
// Finite-Difference Equations, One-Dimensional, Transient Tool:
// Node 0 - Applied heater flux
/* Node 0: surface node (w-orientation); transient conditions; e labeled 1. */
rho * cp * der(T0,t) = fd_1d_sur_w(T0,T1,k,qdot,deltax,Tinf0,h0,q''a0)
q''a0 = 2000        // Applied heat flux, W/m^2;
Tinf0 = 25          // Fluid temperature, C; arbitrary value since h0 is zero; no convection process
h0 = 1e-20          // Convection coefficient, W/m^2.K; made zero since no convection process

// Interior Nodes 1 - 9:
/* Node 1: interior node; e and w labeled 2 and 0. */
rho*cp*der(T1,t) = fd_1d_int(T1,T2,T0,k,qdot,deltax)
/* Node 2: interior node; e and w labeled 3 and 1. */
rho*cp*der(T2,t) = fd_1d_int(T2,T3,T1,k,qdot,deltax)
......
......
/* Node 9: interior node; e and w labeled 10 and 8. */
rho*cp*der(T9,t) = fd_1d_int(T9,T10,T8,k,qdot,deltax)

// Node 10 - Convection process:
/* Node 10: surface node (e-orientation); transient conditions; w labeled 9. */
rho * cp * der(T10,t) = fd_1d_sur_e(T10,T9,k,qdot,deltax,Tinf,h,q''a)
q''a = 0        // Applied heat flux, W/m^2; zero flux shown

// Heat Flux Distribution at Interior Nodes, q''m:
q''1 = k / deltax * (T0 - T2) / 2
q''2 = k / deltax * (T1 - T3) / 2
......
......
q''9 = k / deltax * (T8 - T10) / 2

// Heat flux at boundary x= L, q''10
q''xL = h * (T10 - Tinf)

// Assigned Variables:
deltax = 0.005              // Spatial increment, m
k = 1.5                     // thermal conductivity, W/m.K
alpha = 7.5e-6              // Thermal diffusivity, m^2/s
cp = 1000                   // Specific heat, J/kg.K; arbitrary value
alpha = k / (rho * cp)      // Defintion from which rho is calculated
qdot = 0                    // Volumetric heat generation rate, W/m^3
Ti = 25                     // Initial temperature, C; used also for plotting initial distribution
Tinf = 50                   // Fluid temperature, K
h = 75                      // Convection coefficient, W/m^2.K

// Solver Conditions: integrated t from 0 to 1200 with 1 s step, log every 2nd value
```

PROBLEM 5.101

KNOWN: Plane wall, initially at a uniform temperature $T_o = 25°C$, has one surface (x = L) suddenly exposed to a convection process with $T_\infty = 50°C$ and $h = 1000\ W/m^2 \cdot K$, while the other surface (x = 0) is maintained at T_o. Also, the wall suddenly experiences uniform volumetric heating with $\dot{q} = 1 \times 10^7\ W/m^3$. See also Problem 2.44.

FIND: (a) Using spatial and time increments of $\Delta x = 4$ mm and $\Delta t = 1s$, compute and plot the temperature distributions in the wall for the initial condition, the steady-state condition, and two intermediate times, and (b) On q''_x-t coordinates, plot the heat flux at x = 0 and x = L. At what elapsed time is there zero heat flux at x = L?

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional, transient conduction and (2) Constant properties.

ANALYSIS: (a) Using the *IHT Finite-Difference Equations*, *One-Dimensional*, *Transient Tool*, the temperature distributions were obtained and plotted below.

(b) The heat flux, $q''_x(L,t)$, can be expressed in terms of Newton's law of cooling,

$$q''_x(L,t) = h\left(T_{10}^p - T_\infty\right).$$

From the energy balance on the control volume about node 0 shown above,

$$q''_x(0,t) + \dot{E}_g + q''_a = 0 \qquad\qquad q''_x(0,t) = -\dot{q}(\Delta x/2) - k\left(T_1^p - T_o\right)\Big/\Delta x$$

From knowledge of the temperature distribution, the heat fluxes are computed and plotted.

COMMENTS: The steady-state analytical solution has the form of Eq. 3.40 where $C_1 = 6500$ m-1/°C and $C_2 = 25°C$. Find $q''_x(0,\infty) = -3.25 \times 10^5\ W/m^2$ and $q''_x(L) = +7.5 \times 10^4\ W/m^2$. Comparing with the graphical results above, we conclude that steady-state conditions are not reached in 600 x.

PROBLEM 5.102

KNOWN: Fuel element of Example 5.8 is initially at a uniform temperature of 250°C with no internal generation; suddenly a uniform generation, $\dot{q} = 10^8 \text{W/m}^3$, occurs when the element is inserted into the core while the surfaces experience convection (T_∞,h).

FIND: Temperature distribution 1.5s after element is inserted into the core.

SCHEMATIC:

Fuel element
$\alpha = 5\times10^{-6}$ m²/s
k = 30 W/m·K
Coolant
T_∞ = 250°C
h = 1100 W/m²·K
T(x,0) = 250°C
$\dot{q} = 10^8$ W/m³ at t > 0
Coolant T_∞,h
L = 10mm
x
0 1 2 3 4 5
T_∞,h
Δx = 2mm

ASSUMPTIONS: (1) One-dimensional transient conduction, (2) Constant properties, (3) $\dot{q} = 0$, initially; at t > 0, $\dot{q}$ is uniform.

ANALYSIS: As suggested, the explicit method with a space increment of 2mm will be used. Using the nodal network of Example 5.8, the same finite-difference equations may be used.

Interior nodes, m = 1, 2, 3, 4

$$T_m^{p+1} = \text{Fo}\left[T_{m-1}^p + T_{m+1}^p + \frac{\dot{q}(\Delta x)^2}{2}\right] + (1-2\ \text{Fo})T_m^p. \quad (1)$$

Midplane node, m = 0

Same as Eq. (1), but with $T_{m-1}^p = T_{m+1}^p$.

Surface node, m = 5

$$T_5^{p+1} = 2\ \text{Fo}\left[T_4^p + \text{Bi}\cdot T_\infty + \frac{\dot{q}(\Delta x)^2}{2k}\right] + (1-2\text{Fo}-2\text{Bi}\cdot\text{Fo})T_5^p. \quad (2)$$

The most restrictive stability criterion is associated with Eq. (2), Fo(1+Bi) ≤ 1/2. Consider the following parameters:

$$\text{Bi} = \frac{h\Delta x}{k} = \frac{1100\text{W/m}^2\cdot\text{K}\times(0.002\text{m})}{30\text{W/m}\cdot\text{K}} = 0.0733$$

$$\text{Fo} \le \frac{1/2}{(1+\text{Bi})} = 0.466$$

$$\Delta t \le \frac{\text{Fo}(\Delta x)^2}{\alpha} = 0.466\frac{(0.002\text{m})^2}{5\times10^{-6}\text{m}^2/\text{s}} = 0.373\text{s}.$$

Continued …..

PROBLEM 5.102 (Cont.)

To be well within the stability limit, select $\Delta t = 0.3s$, which corresponds to

$$Fo = \frac{\alpha \Delta t}{\Delta x^2} = \frac{5\times10^{-6} m^2/s \times 0.3s}{(0.002m)^2} = 0.375$$

$$t = p\Delta t = 0.3p(s).$$

Substituting numerical values with $\dot{q} = 10^8 W/m^3$, the nodal equations become

$$T_0^{p+1} = 0.375\left[2T_1^p + 10^8 W/m^3 (0.002m)^2/30W/m\cdot K\right] + (1-2\times0.375)T_0^p$$

$$T_0^{p+1} = 0.375\left[2T_1^p + 13.33\right] + 0.25\ T_0^p \quad (3)$$

$$T_1^{p+1} = 0.375\left[T_0^p + T_2^p + 13.33\right] + 0.25\ T_1^p \quad (4)$$

$$T_2^{p+1} = 0.375\left[T_1^p + T_3^p + 13.33\right] + 0.25\ T_2^p \quad (5)$$

$$T_3^{p+1} = 0.375\left[T_2^p + T_4^p + 13.33\right] + 0.25\ T_3^p \quad (6)$$

$$T_4^{p+1} = 0.375\left[T_3^p + T_5^p + 13.33\right] + 0.25\ T_4^p \quad (7)$$

$$T_5^{p+1} = 2\times0.375\left[T_4^p + 0.0733\times250 + \frac{13.33}{2}\right] + (1-2\times0.375-2\times0.0733\times0.375)T_5^p$$

$$T_5^{p+1} = 0.750\ \left[T_4^p + 24.99\right] + 0.195\ T_5^p. \quad (8)$$

The initial temperature distribution is $T_i = 250°C$ at all nodes. The marching solution, following the procedure of Example 5.8, is represented in the table below.

p	t(s)	T_0	T_1	T_2	T_3	T_4	T_5(°C)
0	0	250	250	250	250	250	250
1	0.3	255.00	255.00	255.00	255.00	255.00	254.99
2	0.6	260.00	260.00	260.00	260.00	260.00	259.72
3	0.9	265.00	265.00	265.00	265.00	264.89	264.39
4	1.2	270.00	270.00	270.00	269.96	269.74	268.97
5	1.5	275.00	275.00	274.98	274.89	274.53	273.50

<

The desired temperature distribution T(x, 1.5s), corresponds to p = 5.

COMMENTS: Note that the nodes near the midplane (0,1) do not feel any effect of the coolant during the first 1.5s time period.

PROBLEM 5.103

KNOWN: Conditions associated with heat generation in a rectangular fuel element with surface cooling. See Example 5.8.

FIND: (a) The temperature distribution 1.5 s after the change in operating power; compare your results with those tabulated in the example, (b) Calculate and plot temperature histories at the mid-plane (00) and surface (05) nodes for $0 \leq t \leq 400$ s; determine the new steady-state temperatures, and approximately how long it will take to reach the new steady-state condition after the step change in operating power. Use the IHT *Tools | Finite-Difference Equations | One-Dimensional | Transient* conduction model builder as your solution tool.

SCHEMATIC:

Symmetry adiabat
00 01 02 03 04 05
x
L = 10 mm
$T_\infty = 250^\circ C$
$h = 1100\ W/m^2\text{-}K$
Coolant
Fuel element
$\dot{q}_1 = 1\times10^7\ W/m^3$
$\dot{q}_2 = 2\times10^7\ W/m^3$
$\alpha = 5\times10^{-6}\ m^2/s$
$k = 30\ W/m\text{-}K$

ASSUMPTIONS: (1) One dimensional conduction in the x-direction, (2) Uniform generation, and (3) Constant properties.

ANALYIS: The IHT model builder provides the transient finite-difference equations for the implicit method of solution. Selected portions of the IHT code used to obtain the results tabulated below are shown in the Comments.

(a) Using the IHT code, the temperature distribution (°C) as a function of time (s) up to 1.5 s after the step power change is obtained from the summarized results copied into the workspace

	t	T00	T01	T02	T03	T04	T05
1	0	357.6	356.9	354.9	351.6	346.9	340.9
2	0.3	358.1	357.4	355.4	352.1	347.4	341.4
3	0.6	358.6	357.9	355.9	352.6	347.9	341.9
4	0.9	359.1	358.4	356.4	353.1	348.4	342.3
5	1.2	359.6	358.9	356.9	353.6	348.9	342.8
6	1.5	360.1	359.4	357.4	354.1	349.3	343.2

(b) Using the code, the mid-plane (00) and surface (05) node temperatures are plotted as a function of time.

Temperature history after step change in power
Temperature, T(x,t) (C)
480
440
400
360
320
0
100
200
300
400
Time, t (s)
T00, Mid-plane, x = 0
T05, Surface, x = L

Continued …..

Note that at t ≈ 240 s, the wall has nearly reached the new steady-state condition for which the nodal temperatures (°C) were found as:

T00	T01	T02	T03	T04	T05
465	463.7	459.7	453	443.7	431.7

COMMENTS: (1) Can you validate the new steady-state nodal temperatures from part (b) by comparison against an analytical solution?

(2) Will using a smaller time increment improve the accuracy of the results? Use your code with Δt = 0.15 s to justify your explanation.

(3) Selected portions of the IHT code to obtain the nodal temperature distribution using spatial and time increments of Δx = 2 mm and Δt = 0.3 s, respectively, are shown below. For the solve-integration step, the initial condition for each of the nodes corresponds to the steady-state temperature distribution with $\dot{q}_1$.

```
// Tools | Finite-Difference Equations | One-Dimensional | Transient
/* Node 00: surface node (w-orientation); transient conditions; e labeled 01. */
rho * cp * der(T00,t) = fd_1d_sur_w(T00,T01,k,qdot,deltax,Tinf01,h01,q''a00)
q''a00 = 0        // Applied heat flux, W/m^2; zero flux shown
Tinf01 = 20       // Arbitrary value
h01 = 1e-8        // Causes boundary to behave as adiabatic
/* Node 01: interior node; e and w labeled 02 and 00. */
rho*cp*der(T01,t) = fd_1d_int(T01,T02,T00,k,qdot,deltax)
/* Node 02: interior node; e and w labeled 03 and 01. */
rho*cp*der(T02,t) = fd_1d_int(T02,T03,T01,k,qdot,deltax)
/* Node 03: interior node; e and w labeled 04 and 02. */
rho*cp*der(T03,t) = fd_1d_int(T03,T04,T02,k,qdot,deltax)
/* Node 04: interior node; e and w labeled 05 and 03. */
rho*cp*der(T04,t) = fd_1d_int(T04,T05,T03,k,qdot,deltax)
/* Node 05: surface node (e-orientation); transient conditions; w labeled 04. */
rho * cp * der(T05,t) = fd_1d_sur_e(T05,T04,k,qdot,deltax,Tinf05,h05,q''a05)
q''a05 = 0          // Applied heat flux, W/m^2; zero flux shown
Tinf05 = 250        // Coolant temperature, C
h05 = 1100          // Convection coefficient, W/m^2.K

// Input parameters
qdot = 2e7          // Volumetric rate, W/m^3, step change
deltax = 0.002      // Space increment
k = 30              // Thermophysical properties
alpha = 5e-6
rho = 1000
alpha = k / (rho * cp)

/* Steady-state conditions,  with qdot1 = 1e7 W/m^3; initial conditions for step change
T_x = 16.67 * (1 - x^2/L^2) + 340.91          // See text
Seek T_x for x = 0, 2, 4, 6, 8, 10 mm; results used for Ti are
Node    T_x
00      357.6
01      356.9
02      354.9
03      351.6
04      346.9
05      340.9      */
```

PROBLEM 5.104

KNOWN: Conditions associated with heat generation in a rectangular fuel element with surface cooling. See Example 5.8.

FIND: (a) The temperature distribution 1.5 s after the change in the operating power; compare results with those tabulated in the Example, and (b) Plot the temperature histories at the midplane, x = 0, and the surface, x = L, for $0 \le t \le 400$ s; determine the new steady-state temperatures, and approximately how long it takes to reach this condition. Use the finite-element software *FEHT* as your solution tool.

SCHEMATIC:

Symmetry adiabat

$T(x, 0) = T_1(x, \dot{q}_1)$

$T_\infty = 250^\circ C$

$h = 1100\ W/m^2\text{-}K$

Coolant

x

L = 10 mm

Fuel element

$\dot{q}_1 = 1\times10^7\ W/m^3$

$\dot{q}_2 = 2\times10^7\ W/m^3$

$\alpha = 5\times10^{-6}\ m^2/s$

k = 30 W/m-K

ASSUMPTIONS: (1) One-dimensional conduction in the x-direction, (2) Uniform generation, (3) Constant properties.

ANALYSIS: Using *FEHT*, an outline of the fuel element is drawn of thickness 10 mm in the x-direction and arbitrary length in the y-direction. The boundary conditions are specified as follows: on the y-planes and the x = 0 plane, treat as adiabatic; on the x = 10 mm plane, specify the convection option. Specify the material properties and the internal generation with $\dot{q}_1$. In the *Setup* menu, click on *Steady-state*, and then *Run* to obtain the temperature distribution corresponding to the initial temperature distribution, $T_i(x,0) = T(x,\dot{q}_1)$, before the change in operating power to $\dot{q}_2$.

Next, in the *Setup* menu, click on *Transient*; in the *Specify | Internal Generation* box, change the value to $\dot{q}_2$; and in the *Run* command, click on *Continue* (not *Calculate*).

(a) The temperature distribution 1.5 s after the change in operating power from the FEHT analysis and from the FDE analysis in the Example are tabulated below.

x/L	0	0.2	0.4	0.6	0.8	1.0
T(x/L, 1.5 s)						
FEHT (°C)	360.1	359.4	357.4	354.1	349.3	343.2
FDE (°C)	360.08	359.41	357.41	354.07	349.37	343.27

The mesh spacing for the FEHT analysis was 0.5 mm and the time increment was 0.005 s. For the FDE analyses, the spatial and time increments were 2 mm and 0.3 s. The agreement between the results from the two numerical methods is within 0.1°C.

(b) Using the FEHT code, the temperature histories at the mid-plane (x = 0) and the surface (x = L) are plotted as a function of time.

Continued …..

PROBLEM 5.104 (Cont.)

From the distribution, the steady-state condition (based upon 98% change) is approached in 215 s. The steady-state temperature distributions after the step change in power from the FEHT and FDE analysis in the Example are tabulated below. The agreement between the results from the two numerical methods is within 0.1°C

x/L	0	0.2	0.4	0.6	0.8	1.0
$T(x/L, \infty)$						
FEHT (°C)	465.0	463.7	459.6	453.0	443.6	431.7
FDE (°C)	465.15	463.82	459.82	453.15	443.82	431.82

COMMENTS: (1) For background information on the *Continue* option, see the *Run* menu in the *FEHT Help* section. Using the *Run/Calculate* command, the steady-state temperature distribution was determined for the $\dot{q}_1$ operating power. Using the *Run*|*Continue* command (after re-setting the generation to $\dot{q}_2$ and clicking on *Setup* | *Transient*), this steady-state distribution automatically becomes the initial temperature distribution for the $\dot{q}_2$ operating power. This feature allows for conveniently prescribing a non-uniform initial temperature distribution for a transient analysis (rather than specifying values on a node-by-node basis).

(2) Use the *View* | *Tabular Output* command to obtain nodal temperatures to the maximum number of significant figures resulting from the analysis.

(3) Can you validate the new steady-state nodal temperatures from part (b) (with $\dot{q}_2$, $t \rightarrow \infty$) by comparison against an analytical solution?

PROBLEM 5.105

KNOWN: Thickness, initial temperature, speed and thermophysical properties of steel in a thin-slab continuous casting process. Surface convection conditions.

FIND: Time required to cool the outer surface to a prescribed temperature. Corresponding value of the midplane temperature and length of cooling section.

SCHEMATIC:

L = 100 mm
Δx = 10 mm
V = 15 mm/s
x
Water jets
T_∞ = 50°C
h = 5000 W/m²-K
T_{10} T_9 T_8 T_2 T_1 T_0
Symmetry plane
Cast steel, T_i = 1400°C
ρ = 7800 kg/m³
c = 700 J/kg-K
k = 30 W/m-K

ASSUMPTIONS: (1) One-dimensional conduction, (2) Negligible radiation at quenched surfaces, (3) Symmetry about the midplane, (4) Constant properties.

ANALYSIS: Adopting the implicit scheme, the finite-difference equaiton for the cooled surface node is given by Eq. (5.88), from which it follows that

$$(1+2\,Fo+2\,FoBi)T_{10}^{p+1}-2\,Fo\,T_9^{p+1}=2\,FoBi\,T_\infty+T_{10}^{p}$$

The general form of the finite-difference equation for any interior node (1 to 9) is given by Eq. (5.89),

$$(1+2\,Fo)T_m^{p+1}-Fo\left(T_{m-1}^{p+1}+T_{m+1}^{p+1}\right)=T_m^{p}$$

The finite-difference equation for the midplane node may be obtained by applying the symmetry requirement to Eq. (5.89); that is, $T_{m+1}^{p}=T_{m-1}^{p}$. Hence,

$$(1+2\,Fo)T_0^{p+1}-2\,Fo\,T_1^{p+1}=T_0^{p}$$

For the prescribed conditions, Bi = hΔx/k = 5000 W/m^2·K (0.010m)/30 W/m·K = 1.67. If the explicit method were used, the stability requirement would be given by Eq. (5.79). Hence, for Fo(1 + Bi) ≤ 0.5, Fo ≤ 0.187. With Fo = $\alpha\Delta t/\Delta x^2$ and $\alpha = k/\rho c = 5.49 \times 10^{-6}$ m^2/s, the corresponding restriction on the time increment would be Δt ≤ 3.40s. Although no such restriction applies for the implicit method, a value of Δt = 1s is chosen, and the set of 11 finite-difference equations is solved using the *Tools* option designated as *Finite-Difference Equations*, *One-Dimensional* and *Transient* from the IHT Toolpad. For T_{10} (t) = 300°C, the solution yields

$$t = 161s$$ <

Continued …..

PROBLEM 5.105 (Cont.)

$$T_0(t) = 1364°C$$ <

With a casting speed of V = 15 mm/s, the length of the cooling section is

$$L_{cs} = Vt = 0.015\,m/s(161s) = 2.42m$$ <

COMMENTS: (1) With Fo = $\alpha t/L^2$ = 0.088 < 0.2, the one-term approximation to the exact solution for one-dimensional conduction in a plane wall cannot be used to confirm the foregoing results. However, using the exact solution from the *Models*, *Transient Conduction*, *Plane Wall* Option of IHT, values of T_0 = 1366°C and T_s = 200.7°C are obtained and are in good agreement with the finite-difference predictions. The accuracy of these predictions could still be improved by reducing the value of Δx.

(2) Temperature histories for the surface and midplane nodes are plotted for 0 < t < 600s.

While T_{10} (600s) = 124°C, T_o (600s) has only dropped to 879°C. The much slower thermal response at the midplane is attributable to the small value of α and the large value of Bi = 16.67.

PROBLEM 5.106

KNOWN: Very thick plate, initially at a uniform temperature, T_i, is suddenly exposed to a convection cooling process (T_∞,h).

FIND: Temperatures at the surface and a 45mm depth after 3 minutes using finite-difference method with space and time increments of 15mm and 18s.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional transient conduction, (2) Plate approximates semi-infinite medium, (3) Constant properties.

ANALYSIS: The grid network representing the plate is shown above. The finite-difference equation for node 0 is given by Eq. 5.82 for one-dimensional conditions or Eq. 5.77,

$$T_0^{p+1} = 2\,Fo\left(T_1^p + Bi \cdot T_\infty\right) + \left(1 - 2\,Fo - 2\,Bi \cdot Fo\right) T_0^p. \qquad (1)$$

The numerical values of Fo and Bi are

$$Fo = \frac{\alpha \Delta t}{\Delta x^2} = \frac{5.6\times10^{-6}\,m^2/s \times 18s}{(0.015m)^2} = 0.448$$

$$Bi = \frac{h\Delta x}{k} = \frac{100\;W/m^2\cdot K \times \left(15\times10^{-3}m\right)}{20\;W/m\cdot K} = 0.075.$$

Recognizing that T_∞ = 15°C, Eq. (1) has the form

$$T_0^{p+1} = 0.0359\,T_0^p + 0.897\,T_1^p + 1.01. \qquad (2)$$

It is important to satisfy the stability criterion, Fo (1+Bi) ≤ 1/2. Substituting values, 0.448 (1+0.075) = 0.482 ≤ 1/2, and the criterion is satisfied.

The finite-difference equation for the interior nodes, m = 1, 2…, follows from Eq. 5.73,

$$T_m^{p+1} = Fo\left(T_{m+1}^p + T_{m-1}^p\right) + (1 - 2Fo)\,T_m^p. \qquad (3)$$

Recognizing that the stability criterion, Fo ≤ 1/2, is satisfied with Fo = 0.448,

$$T_m^{p+1} = 0.448\left(T_{m+1}^p + T_{m-1}^p\right) + 0.104\,T_m^p. \qquad (4)$$

Continued …..

The time scale is related to p, the number of steps in the calculation procedure, and Δt, the time increment,

$$t = p\Delta t. \qquad (5)$$

The finite-difference calculations can now be performed using Eqs. (2) and (4). The results are tabulated below.

p	t(s)	T_0	T_1	T_2	T_3	T_4	T_5	T_6	T_7(K)
0	0	325	325	325	325	325	325	325	325
1	18	304.2	324.7	325	325	325	325	325	325
2	36	303.2	315.3	324.5	325	325	325	325	325
3	54	294.7	313.7	320.3	324.5	325	325	325	325
4	72	293.0	307.8	318.9	322.5	324.5	325	325	325
5	90	287.6	305.8	315.2	321.5	323.5	324.5	325	325
6	108	285.6	301.6	313.5	319.3	322.7	324.0	324.5	325
7	126	281.8	299.5	310.5	317.9	321.4	323.3	324.2	
8	144	279.8	296.2	308.6	315.8	320.4	322.5		
9	162	276.7	294.1	306.0	314.3	319.0			
10	180	274.8	291.3	304.1	312.4				

Hence, find

$$T(0,\ 180s) = T_0^{10} = 275^\circ C \qquad T(45mm,\ 180s) = T_3^{10} = 312^\circ C. \qquad <$$

COMMENTS: (1) The above results can be readily checked against the analytical solution represented in Fig. 5.8 (see also Eq. 5.60). For x = 0 and t = 180s, find

$$\frac{x}{2(\alpha t)^{1/2}} = 0$$

$$\frac{h(\alpha t)^{1/2}}{k} = \frac{100\ W/m^2 \cdot K\left(5.60\times10^{-6} m^2/s \times 180s\right)^{1/2}}{20\ W/m\cdot K} = 0.16$$

for which the figure gives

$$\frac{T - T_i}{T_\infty - T_i} = 0.15$$

so that,

$$T(0,\ 180s) = 0.15(T_\infty - T_i) + T_i = 0.15(15 - 325)^\circ C + 325^\circ C$$

$$T(0,\ 180s) = 278^\circ C.$$

For x = 45mm, the procedure yields T(45mm, 180s) = 316°C. The agreement with the numerical solution is nearly within 1%.

PROBLEM 5.107

KNOWN: Sudden exposure of the surface of a thick slab, initially at a uniform temperature, to convection and to surroundings at a high temperature.

FIND: (a) Explicit, finite-difference equation for the surface node in terms of Fo, Bi, Bi_r, (b) Stability criterion; whether it is more restrictive than that for an interior node and does it change with time, and (c) Temperature at the surface and at 30mm depth for prescribed conditions after 1 minute exposure.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional transient conduction, (2) Thick slab may be approximated as semi-infinite medium, (3) Constant properties, (4) Radiation exchange is between small surface and large surroundings.

ANALYSIS: (a) The explicit form of the FDE for the surface node may be obtained by applying an energy balance to a control volume about the node.

$$\dot{E}''_{in} - \dot{E}''_{out} = q''_{conv} + q''_{rad} + q''_{cond} = \dot{E}''_{st}$$

$$h\left(T_\infty - T_o^p\right) + h_r\left(T_{sur} - T_o^p\right) + k \cdot 1 \cdot \frac{T_1^p - T_o^p}{\Delta x}$$

$$= \rho\, c\left[\frac{\Delta x}{2} \cdot 1\right]\frac{T_o^{p+1} - T_o^p}{\Delta t} \qquad (1)$$

where the radiation process has been linearized, Eq. 1.8. (See also Comment 4, Example 5.9),

$$h_r = h_r^p\left(T_o^p,\ T_{sur}\right) = \varepsilon\sigma\left(T_o^p + T_{sur}\right)\left(\left[T_0^p\right]^2 + T_{sur}^2\right). \qquad (2)$$

Divide Eq. (1) by $\rho c\Delta x/2\Delta t$ and regroup using these definitions to obtain the FDE:

$$Fo \equiv \left(k/\rho\, c\right)\Delta t/\Delta x^2 \qquad Bi \equiv h\Delta x/k \qquad Bi_r \equiv h_r\Delta x/k \qquad (3,4,5)$$

$$T_o^{p+1} = 2Fo\left(Bi \cdot T_\infty + Bi_r \cdot T_{sur} + T_1^p\right) + \left(1 - 2\,Bi \cdot Fo - 2Bi_r \cdot Fo - 2Fo\right)T_o^p. \qquad (6)$$ <

(b) The stability criterion for Eq. (6) requires that the coefficient of T_o^p be positive.

$$1 - 2Fo\left(Bi + Bi_r + 1\right) \geq 0 \qquad \text{or} \qquad Fo \leq 1/2\left(Bi + Bi_r + 1\right). \qquad (7)$$ <

The stability criterion for an interior node, Eq. 5.74, is $Fo \leq 1/2$. Since $Bi + Bi_r > 0$, the stability criterion of the surface node is more restrictive. Note that Bi_r is not constant but depends upon h_r which increases with increasing T_o^p (time). Hence, the restriction on Fo increases with increasing T_o^p (time).

Continued …..

PROBLEM 5.107 (Cont.)

(c) Consider the prescribed conditions with negligible convection (Bi = 0). The FDEs for the thick slab are:

Surface (0) $$T_o^{p+1} = 2Fo\left(Bi \cdot Fo + Bi_r \cdot T_{sur} + T_1^p\right) + \left(1 - 2Bi \cdot Fo - 2Bi_r \cdot Fo - 2Fo\right)T_o^p \qquad (8)$$

Interior (m≥1) $$T_m^{p+1} = Fo\left(T_{m+1}^p + T_{m-1}^p\right) + \left(1 - 2Fo\right)T_m^p \qquad (9,5,7,3)$$

The stability criterion from Eq. (7) with Bi = 0 is,

$$Fo \le 1/2\left(1 + Bi_r\right) \qquad (10)$$

To proceed with the explicit, marching solution, we need to select a value of Δt (Fo) that will satisfy the stability criterion. A few trial calculations are helpful. A value of Δt = 15s provides Fo = 0.105, and using Eqs. (2) and (5), h_r(300K, 1000K) = 72.3 W/m^2·K and Bi_r = 0.482. From the stability criterion, Eq. (10), find Fo ≤ 0.337. With increasing T_o^p, h_r and Bi_r increase: h_r(800K, 1000K) = 150.6 W/m^2·K, Bi_r = 1.004 and Fo ≤ 0.249. Hence, if $T_o^p < 800K$, $\Delta t = 15s$ or $Fo = 0.105$ satisfies the stability criterion.

Using Δt = 15s or Fo = 0.105 with the FDEs, Eqs. (8) and (9), the results of the solution are tabulated below. Note how h_r^p and Bi_r^p are evaluated at each time increment. Note that t = p·Δt, where Δt = 15s.

p	t(s)	T_o / h_r^p / Bi_r	T1(K)	T_2	T_3	T_4	….
0	0	300 72.3 0.482	300	300	300	300	
1	15	370.867 79.577 0.5305	300	300	300	300	
2	30	426.079 85.984 0.5733	307.441	300	300	300	
3	45	470.256 91.619 0.6108	319.117	300.781	300	300	
4	60	502.289	333.061	302.624	300.082	300	

After 60s(p = 4), T_o(0, 1 min) = 502.3K and T_3(30mm, 1 min) = 300.1K. <

COMMENTS: (1) The form of the FDE representing the surface node agrees with Eq. 5.82 if this equation is reduced to one-dimension.

(2) We should recognize that the Δt = 15s time increment represents a coarse step. To improve the accuracy of the solution, a smaller Δt should be chosen.

PROBLEM 5.108

KNOWN: Thick slab of copper, initially at a uniform temperature, is suddenly exposed to a constant net radiant flux at one surface. See Example 5.9.

FIND: (a) The nodal temperatures at nodes 00 and 04 at t = 120 s; that is, T00(0, 120 s) and T04(0.15 m, 120 s); compare results with those given by the exact solution in Comment 1; will a time increment of 0.12 s provide more accurate results?; and, (b) Plot the temperature histories for x = 0, 150 and 600 mm, and explain key features of your results. Use the *IHT Tools | Finite-Difference Equations | One-Dimensional | Transient* conduction model builder to obtain the implicit form of the FDEs for the interior nodes. Use space and time increments of 37.5 mm and 1.2 s, respectively, for a 17-node network. For the surface node 00, use the FDE derived in Section 2 of the Example.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional conduction in the x-direction, (2) Slab of thickness 600 mm approximates a semi-infinite medium, and (3) Constant properties.

ANALYSIS: The IHT model builder provides the implicit-method FDEs for the interior nodes, 01 – 15. The +x boundary condition for the node-16 control volume is assumed adiabatic. The FDE for the surface node 00 exposed to the net radiant flux was derived in the Example analysis. Selected portions of the IHT code used to obtain the following results are shown in the Comments.

(a) The 00 and 04 nodal temperatures for t = 120 s are tabulated below using a time increment of Δt = 1.2 s and 0.12 s, and compared with the results given from the exact analytical solution, Eq. 5.59.

Node	FDE results (°C)		Analytical result (°C)
	Δt = 1.2 s	Δt = 0.12 s	Eq. 5.59
00	119.3	119.4	120.0
04	45.09	45.10	45.4

The numerical FDE-based results with the different time increments agree quite closely with one another. At the surface, the numerical results are nearly 1 °C less than the result from the exact analytical solution. This difference represents an error of -1% (-1 °C / (120 – 20) °C x 100). At the x = 150 mm location, the difference is about -0.4 °C, representing an error of –1.5%. For this situation, the smaller time increment (0.12 s) did not provide improved accuracy. To improve the accuracy of the numerical model, it would be necessary to reduce the space increment, in addition to using the smaller time increment.

(b) The temperature histories for x = 0, 150 and 600 mm (nodes 00, 04, and 16) for the range 0 ≤ t ≤ 150 s are as follows.

Continued …..

PROBLEM 5.108 (Cont.)

Temperature histories for Nodes 00, 04, and 16

Temperature, T(x,t)

120
80
40
0

0 50 100 150

Time, t (s)

T00 = T(0, t)
T04 = T(150 mm, t)
T00 = T(600 mm, t)

As expected, the surface temperature, T00 = T(0,t), increases markedly at early times. As thermal penetration increases with increasing time, the temperature at the location x = 150 mm, T04 = T(150 mm, t), begins to increase after about 20 s. Note, however, the temperature at the location x = 600 mm, T16 = T(600 mm, t), does not change significantly within the 150 s duration of the applied surface heat flux. Our assumption of treating the +x boundary of the node 16 control volume as adiabatic is justified. A copper plate of 600-mm thickness is a good approximation to a semi-infinite medium at times less than 150 s.

COMMENTS: Selected portions of the *IHT* code with the nodal equations to obtain the temperature distribution are shown below. Note how the FDE for node 00 is written in terms of an energy balance using the *der (T,t)* function. The FDE for node 16 assumes that the "east" boundary is adiabatic.

```
// Finite-difference equation, node 00; from Examples solution derivation; implicit method
q''o + k * (T01 - T00) / deltax = rho * (deltax / 2) *cp * der (T00,t)

// Finite-difference equations, interior nodes 01-15; from Tools
/* Node 01: interior node; e and w labeled 02 and 00. */
rho*cp*der(T01,t) = fd_1d_int(T01,T02,T00,k,qdot,deltax)
rho*cp*der(T02,t) = fd_1d_int(T02,T03,T01,k,qdot,deltax)
…..
…..
rho*cp*der(T14,t) = fd_1d_int(T14,T15,T13,k,qdot,deltax)
rho*cp*der(T15,t) = fd_1d_int(T15,T16,T14,k,qdot,deltax)

// Finite-difference equation node 16; from Tools, adiabatic surface
 /* Node 16: surface node (e-orientation); transient conditions; w labeled 15. */
rho * cp * der(T16,t) = fd_1d_sur_e(T16,T15,k,qdot,deltax,Tinf16,h16,q''a16)
q''a16 = 0          // Applied heat flux, W/m^2; zero flux shown
Tinf16 = 20         // Arbitrary value
h16 = 1e-8          // Causes boundary to behave as adiabatic
```

PROBLEM 5.109

KNOWN: Thick slab of copper as treated in Example 5.9, initially at a uniform temperature, is suddenly exposed to large surroundings at 1000°C (instead of a net radiant flux).

FIND: (a) The temperatures T(0, 120 s) and T(0.15 m, 120s) using the finite-element software *FEHT* for a surface emissivity of 0.94 and (b) Plot the temperature histories for x = 0, 150 and 600 mm, and explain key features of your results.

SCHEMATIC:

T_{sur} = 1000°C

ε = 0.94

x

T(x, 0) = T_i = 20°C

ρ = 8933 kg/m^3

c = 385 J/kg-K

k = 401 W/m-K

ASSUMPTIONS: (1) One-dimensional conduction in the x-direction, (2) Slab of thickness 600 mm approximates a semi-infinite medium, (3) Slab is small object in large, isothermal surroundings.

ANALYSIS: (a) Using FEHT, an outline of the slab is drawn of thickness 600 mm in the x-direction and arbitrary length in the y-direction. Click on *Setup* | *Temperatures in K*, to enter all temperatures in kelvins. The boundary conditions are specified as follows: on the y-planes and the x = 600 mm plane, treat as adiabatic; on the surface (0,y), select the convection coefficient option, enter the linearized radiation coefficient after Eq. 1.9 written as

0.94 * 5.67e-8 * (T + 1273) * (T^2 + 1273^2)

and enter the surroundings temperature, 1273 K, in the fluid temperature box. See the Comments for a view of the input screen. From *View*|*Temperatures*, find the results:

T(0, 120 s) = 339 K = 66°C   T(150 mm, 120 s) = 305K = 32°C   <

(b) Using the *View* | *Temperatures* command, the temperature histories for x = 0, 150 and 600 mm (10 mm mesh, Nodes 18, 23 and 15, respectively) are plotted. As expected, the surface temperature increases markedly at early times. As thermal penetration increases with increasing time, the temperature at the location x = 150 mm begins to increase after about 30 s. Note, however, that the temperature at the location x = 600 mm does not change significantly within the 150 s exposure to the hot surroundings. Our assumption of treating the boundary at the x = 600 mm plane as adiabatic is justified. A copper plate of 600 mm is a good approximation to a semi-infinite medium at times less than 150 s.

Continued …..

PROBLEM 5.109 (Cont.)

Temperature history after sudden exposure to surroundings

Temperature [K]

Time [sec]

Surface, x = 0, node 18

x = 150 mm, node 23

x = 600 mm, node 15

COMMENTS: The annotated *Input* screen shows the outline of the slab, the boundary conditions, and the triangular mesh before using the *Reduce-mesh* option.

Slab outline, coarse triangular mesh

Adiabatic BC

Linearized rad coeff
(Tf = 1273 K, eps = 0.94)

o Node 18

o Node 23

Ti = 293 K

Adiabatic BC

Node 15 o

x = 0

x = 150 mm

Adiabatic BC

x = 600 mm

PPROBLEM 5.110

KNOWN: Electric heater sandwiched between two thick plates whose surfaces experience convection. Case 2 corresponds to steady-state operation with a loss of coolant on the x = -L surface. Suddenly, a second loss of coolant condition occurs on the x = +L surface, but the heater remains energized for the next 15 minutes. Case 3 corresponds to the eventual steady-state condition following the second loss of coolant event. See Problem 2.53.

FIND: Calculate and plot the temperature time histories at the plate locations x = 0, ±L during the transient period between steady-state distributions for Case 2 and Case 3 using the finite-element approach with *FEHT* and the finite-difference method of solution with *IHT* (Δx = 5 mm and Δt = 1 s).

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional conduction, (2) Constant properties, (3) Heater has negligible thickness, and (4) Negligible thermal resistance between the heater surfaces and the plates.

PROPERTIES: Plate material (*given*); ρ = 2500 kg/m^3, c = 700 J/kg·K, k = 5 W/m·K.

ANALYSIS: The temperature distribution for Case 2 shown in the above graph represents the initial condition for the period of time following the second loss of coolant event. The boundary conditions at x = ±L are adiabatic, and the heater flux is maintained at q_o'' = 4000 W/m^2 for $0 \le t \le 15$ min.

Using *FEHT*, the heater is represented as a plate of thickness L_h = 0.5 mm with very low thermal capacitance (ρ = 1 kg/m and c = 1 J/kg·K), very high thermal conductivity (k= 10,000 W/m·K), and a uniform volumetric generation rate of $\dot{q} = q_o'' / L_h = 4000\ \text{W/m}^2 / 0.0005\ \text{m} = 8.0 \times 10^6\ \text{W/m}^3$ for $0 \le t \le$ 900 s. In the *Specify* | *Generation* box, the generation was prescribed by the *lookup file* (see *FEHT* Help): 'hfvst',1,2,Time. This *Notepad* file is comprised of four lines, with the values on each line separated by a single tab space:

```
0	8e6
900	8e6
901	0
5000	0
```

The temperature-time histories are shown in the graph below for the surfaces x = - L (lowest curve, 13) and x = +L (19) and the center point x = 0 (highest curve, 14). The center point experiences the maximum temperature of 89°C at the time the heater is deactivated, t = 900 s.

Continued …..

PROBLEM 5.110

For the finite-difference method of solution, the nodal arrangement for the system is shown below. The *IHT* model builder *Tools | Finite-Difference Equations | One Dimensional* can be used to obtain the FDEs for the internal nodes (02-04, 07-10) and the adiabatic boundary nodes (01, 11).

Heater, $q_o'' = 4000$ W/m^2, $0 \le t \le 900$ s

$T(x,t \le 0) = T_2(x)$ Case 2

$\Delta x = 5$ mm

Nodal arrangement, FDE solution

$\dot{E}_{gen}'' = q_o''$ $(0 \le t \le t_{off})$

Control volume, node 06

For the heater-plate interface node 06, the FDE for the implicit method is derived from an energy balance on the control volume shown in the schematic above.

$$\dot{E}_{in}'' - \dot{E}_{out}'' + \dot{E}_{gen}'' = \dot{E}_{st}''$$

$$q_a'' + q_b'' + q_o'' = \dot{E}_{st}''$$

$$k\frac{T_{05}^{p+1} - T_{06}^{p+1}}{\Delta x} + k\frac{T_{07}^{p+1} - T_{06}^{p+1}}{\Delta x} + q_o'' = \rho c \Delta x \frac{T_{06}^{p+1} - T_{06}^{p}}{\Delta t}$$

The *IHT* code representing selected nodes is shown below for the adiabatic boundary node 01, interior node 02, and the heater-plates interface node 06. Note how the foregoing derived finite-difference equation in implicit form is written in the *IHT Workspace*. Note also the use of a *Lookup Table* for representing the heater flux *vs*. time.

Continued …..

PROBLEM 5.110 (Cont.)

```
//  Finite-difference equations from Tools, Nodes 01, 02
/* Node 01: surface node (w-orientation); transient conditions; e labeled 02. */
rho * cp * der(T01,t) = fd_1d_sur_w(T01,T02,k,qdot,deltax,Tinf01,h01,q''a01)
q''a01 = 0            // Applied heat flux, W/m^2; zero flux shown
qdot = 0              // No internal generation
Tinf01 = 20           // Arbitrary value
h01 = 1e-6              // Causes boundary to behave as adiabatic

/* Node 02: interior node; e and w labeled 03 and 01. */
rho*cp*der(T02,t) = fd_1d_int(T02,T03,T01,k,qdot,deltax)

//  Finite-difference equation from energy balance on CV, Node 06
k * (T05 - T06) / deltax + k * (T07 - T06)/ deltax + q''h = rho * cp * deltax * der(T06,t)
q''h = LOOKUPVAL(qhvst,1,t,2)          // Heater flux, W/m^2; specified by Lookup Table

/* See HELP (Solver, Lookup Tables).  The Look-up table file name "qhvst" contains
        0       4000
        900     4000
        900.5   0
        5000    0               */
```

The temperature-time histories using the *IHT* code for the plate locations x = 0, ±L are shown in the graphs below. We chose to show expanded presentations of the histories at early times, just after the second loss of coolant event, t = 0, and around the time the heater is deactivated, t = 900 s.

COMMENTS: (1) The maximum temperature during the transient period is at the center point and occurs at the instant the heater is deactivated, T(0, 900s) = 89°C. After 300 s, note that the two surface temperatures are nearly the same, and never rise above the final steady-state temperature.

(2) Both the FEHT and IHT methods of solution give identical results. Their steady-state solutions agree with the result of an energy balance on a time interval basis yielding T_{ss} = 86.1°C.

PROBLEM 5.111

KNOWN: Plane wall of thickness 2L, initially at a uniform temperature, is suddenly subjected to convection heat transfer.

FIND: The mid-plane, T(0,t), and surface, T(L,t), temperatures at t = 50, 100, 200 and 500 s, using the following methods: (a) the one-term series solution; determine also the Biot number; (b) the lumped capacitance solution; and (c) the two- and 5-node finite-difference numerical solutions. Prepare a table summarizing the results and comment on the relative differences of the predicted temperatures.

SCHEMATIC:

(a) Plane wall, thickness 2L **(b) Nodal networks**

ASSUMPTIONS: (1) One-dimensional conduction in the x-direction, and (2) Constant properties.

ANALYSIS: (a) The results are tabulated below for the mid-plane and surface temperatures using the one-term approximation to the series solution, Eq. 5.40 and 5.41. The Biot number for the heat transfer process is

$$\mathrm{Bi} = h\ L/k = 500\ \mathrm{W/m^2 \cdot K} \times 0.020\ \mathrm{m} / 15\ \mathrm{W/m \cdot K} = 0.67$$

Since Bi >> 0.1, we expect an appreciable temperature difference between the mid-plane and surface as the tabulated results indicate (Eq. 5.10).

(b) The results are tabulated below for the wall temperatures using the lumped capacitance method (LCM) of solution, Eq. 5.6. The LCM neglects the internal conduction resistance and since Bi = 0.67 >> 0.1, we expect this method to predict systematically lower temperatures (faster cooling) at the midplane compared to the one-term approximation.

Solution method/Time(s)	50	100	200	500
Mid-plane, T(0,t) (°C)				
One-term, Eqs. 5.40, 5.41	207.1	160.5	99.97	37.70
Lumped capacitance	181.7	133.9	77.69	30.97
2-node FDE	210.6	163.5	100.5	37.17
5-node FDE	207.5	160.9	100.2	37.77
Surface, T(L,t) (°C)				
One-term, Eqs. 5.40, 5.41	160.1	125.4	80.56	34.41
Lumped capacitance	181.7	133.9	77.69	30.97
2-node FDE	163.7	125.2	79.40	33.77
5-node FDE	160.2	125.6	80.67	34.45

(c) The 2- and 5-node nodal networks representing the wall are shown in the schematic above. The implicit form of the finite-difference equations for the mid-plane, interior (if present) and surface nodes can be derived from energy balances on the nodal control volumes. The time-rate of change of the temperature is expressed in terms of the *IHT* integral intrinsic function, *der*(*T*,*t*).

Continued …..

Mid-plane node

$$k\left(T2-T1\right)/\Delta x=\rho c\left(\Delta x/2\right)\cdot der\left(T1,t\right)$$

Interior node (5-node network)

$$k\left(T1-T2\right)/\Delta x+k\left(T3-T2\right)/\Delta x=\rho c\,\Delta x\cdot der\left(T2,t\right)$$

Surface node (shown for 5-node network)

$$k\left(T4-T5\right)/\Delta x+h\left(T\inf-T5\right)=\rho c\left(\Delta x/2\right)\cdot der\left(T5,t\right)$$

With appropriate values for Δx, the foregoing FDEs were entered into the *IHT* workspace and solved for the temperature distributions as a function of time over the range $0 \le t \le 500$ s using an integration time step of 1 s. Selected portions of the *IHT* codes for each of the models are shown in the Comments. The results of the analysis are summarized in the foregoing table.

COMMENTS: (1) Referring to the table above, we can make the following observations about the relative differences and similarities of the estimated temperatures: (a) The one-term series model estimates are the most reliable, and can serve as the benchmark for the other model results; (b) The LCM model over estimates the rate of cooling, and poorly predicts temperatures since the model neglects the effect of internal resistance and Bi = 0.67 >> 0.1; (c) The 5-node model results are in excellent agreement with those from the one-term series solution; we can infer that the chosen space and time increments are sufficiently small to provide accurate results; and (d) The 2-node model under estimates the rate of cooling for early times when the time-rate of change is high; but for late times, the agreement is improved.

(2) See the *Solver | Intrinsic Functions* section of *IHT|Help* or the *IHT Examples* menu (Example 5.3) for guidance on using the *der(T,t)* function.

(3) Selected portions of the *IHT* code for the 2-node network model are shown below.

```
// Writing the finite-difference equations – 2-node model
//  Node 1
k * (T2 - T1)/ deltax  = rho * cp * (deltax / 2) * der(T1,t)
// Node 2
k * (T1 - T2)/ deltax  + h * (Tinf - T2) = rho * cp * (deltax / 2) * der(T2,t)

// Input parameters
L = 0.020
deltax = L
rho = 7800          // density, kg/m^3
cp =   440          // specific heat, J/kg·K
k = 15              // thermal conductivity, W/m.K
h =   500           // convection coefficient, W/m^2·K
Tinf = 25           // fluid temperature, K
```

(4) Selected portions of the IHT code for the 5-node network model are shown below.

```
// Writing the finite-difference equations – 5-node model
//  Node 1 - midplane
k * (T2 - T1)/ deltax  = rho * cp * (deltax / 2) * der(T1,t)
// Interior nodes
k * (T1 - T2)/ deltax +  k * (T3 - T2 )/ deltax = rho * cp * deltax  * der(T2,t)
k * (T2 - T3)/ deltax +  k * (T4 - T3 )/ deltax = rho * cp * deltax  * der(T3,t)
k * (T3 - T4)/ deltax +  k * (T5 - T4 )/ deltax = rho * cp * deltax  * der(T4,t)
// Node5 - surface
k * (T4 - T5)/ deltax + h * (Tinf - T5) = rho * cp * (deltax / 2) * der(T5,t)

// Input parameters
L = 0.020
deltax = L / 4
……..
……..
```

PROBLEM 5.112

KNOWN: Plastic film on metal strip initially at 25°C is heated by a laser (85,000 W/m^2 for Δt_{on} = 10 s), to cure adhesive; convection conditions for ambient air at 25°C with coefficient of 100 W/m^2·K.

FIND: Temperature histories at center and film edge, T(0,t) and T(x_1,t), for 0 ≤ t ≤ 30 s, using an implicit, finite-difference method with Δx = 4mm and Δt = 1 s; determine whether adhesive is cured (T_c ≥ 90°C for Δt_c = 10s) and whether the degradation temperature of 200°C is exceeded.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional conduction, (2) Constant properties, (3) Uniform convection coefficient on upper and lower surfaces, (4) Thermal resistance and mass of plastic film are negligible, (5) All incident laser flux is absorbed.

PROPERTIES: Metal strip (given): ρ = 7850 kg/m^3, c_p = 435 J/kg·K, k = 60 W/m·K, α = k/ρc_p = 1.757 × 10^{-5} m^2/s.

ANALYSIS: (a) Using a space increment of Δx = 4mm, set up the nodal network shown below. Note that the film half-length is 22mm (rather than 20mm as in Problem 3.97) to simplify the finite-difference equation derivation.

Consider the general control volume and use the conservation of energy requirement to obtain the finite-difference equation.

$$\dot{E}_{in} - \dot{E}_{out} = \dot{E}_{st}$$

$$q_a + q_b + q_{laser} + q_{conv} = Mc_p \frac{T_m^{p+1} - T_m^p}{\Delta t}$$

Continued …..

PROBLEM 5.112 (Cont.)

$$k(d\cdot 1)\frac{T_{m-1}^{p+1}-T_m^{p+1}}{\Delta x}+k(d\cdot 1)\frac{T_{m+1}^{p+1}-T_m^{p+1}}{\Delta x}$$

$$+q_o''(\Delta x\cdot 1)+2h(\Delta x\cdot 1)\left(T_\infty-T_m^{p+1}\right)=\rho(\Delta x\cdot d\cdot 1)c_p\frac{T_m^{p+1}-T_m^p}{\Delta t}$$

$$T_m^p=(1+2Fo+2Fo\cdot Bi)T_m^{p+1} \tag{1}$$

$$-Fo\left(T_{m+1}^{p+1}+T_{m+1}^{p+1}\right)-2Fo\cdot Bi\cdot T_\infty-Fo\cdot Q$$

where

$$Fo=\frac{\alpha\Delta t}{\Delta x^2}=\frac{1.757\times 10^{-5}\,m^2/s\times 1s}{(0.004\ m)^2}=1.098 \tag{2}$$

$$Bi=\frac{h\left(\Delta x^2/d\right)}{k}=\frac{100\ W/m^2\cdot K\left(0.004^2/0.00125\right)m}{60\ W/m\cdot K}=0.0213 \tag{3}$$

$$Q=\frac{q_o''\left(\Delta x^2/d\right)}{k}=\frac{85,000\ W/m^2\left(0.004^2/0.0015\right)m}{60\ W/m\cdot K}=18.133. \tag{4}$$

The results of the matrix inversion numerical method of solution (Δx = 4mm, Δt = 1s) are shown below. The temperature histories for the center (m = 1) and film edge (m = 5) nodes, T(0,t) and T(x_1,t), respectively, permit determining whether the adhesive has cured (T ≥ 90°C for 10 s).

Certainly the center region, T(0,t), is fully cured and furthermore, the degradation temperature (200°C) has not been exceeded. From the T(x_1,t) distribution, note that $\Delta t_c \approx 8$ sec, which is 20% less than the 10 s interval sought. Hence, the laser exposure (now 10 s) should be slightly increased and quite likely, the maximum temperature will not exceed 200°C.

PROBLEM 5.113

KNOWN: Insulated rod of prescribed length and diameter, with one end in a fixture at 200°C, reaches a uniform temperature. Suddenly the insulating sleeve is removed and the rod is subjected to a convection process.

FIND: (a) Time required for the mid-length of the rod to reach 100°C, (b) Temperature history $T(x,t \leq t_1)$, where t_1 is time at which the midlength reaches 50°C. Temperature distribution at 0, 200s, 400s and t_1.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional transient conduction in rod, (2) Uniform h along rod and at end, (3) Negligible radiation exchange between rod and surroundings, (4) Constant properties.

ANALYSIS: (a) Choosing Δx = 0.016 m, the finite-difference equations for the interior and end nodes are obtained.

Interior Point, m:
$$q_a + q_b + q_c = \rho \cdot A_c \Delta x \cdot c_p \cdot \frac{T_m^{p+1} - T_m^p}{\Delta t}$$

$$k \cdot A_c \frac{T_{m-1}^p - T_m^p}{\Delta x} + kA_c \frac{T_{m+1}^p - T_m^p}{\Delta x} + hP\Delta x \left(T_\infty - T_m^p\right) = \rho A_c \Delta x c_p \frac{T_m^{p+1} - T_m^p}{\Delta t}$$

Regrouping,
$$T_m^{p+1} = T_m^p \left(1 - 2Fo - Bi \cdot Fo\right) + Fo\left(T_{m-1}^p + T_{m+1}^p\right) + Bi \cdot Fo T_\infty \qquad (1)$$

where
$$Fo = \frac{\alpha \Delta t}{\Delta x^2} \qquad (2) \qquad\qquad Bi = h\left[\Delta x^2 / \left(A_c/P\right)\right] / k . \qquad (3)$$

From Eq. (1), recognize that the stability of the numerical solution will be assured when the first term on the RHS is positive; that is

Continued...

$$\left(1-2\text{Fo}-\text{Bi}\cdot\text{Fo}\right)\geq 0 \qquad \text{or} \qquad \text{Fo}\leq 1\big/\left(2+\text{Bi}\right). \tag{4}$$

Nodal Point 1: Consider Eq. (1) for the special case that $T_{m-1}^{p} = T_o$, which is independent of time. Hence,

$$T_1^{p+1} = T_1^p\left(1-2\text{Fo}-\text{Bi}\cdot\text{Fo}\right)+\text{Fo}\left(T_o+T_2^p\right)+\text{Bi}\cdot\text{Fo}T_\infty . \tag{5}$$

End Nodal Point 10: $\quad q_a + q_b + q_c = \rho\cdot A_c\dfrac{\Delta x}{2}\cdot c_p\dfrac{T_{10}^{p+1}-T_{10}^{p}}{\Delta t}$

$$k\cdot A_c\frac{T_9^p-T_{10}^p}{\Delta x}+hA_c\left(T_\infty-T_{10}^p\right)+hP\frac{\Delta x}{2}\left(T_\infty-T_{10}^p\right)=\rho A_c\frac{\Delta x}{2}c_p\frac{T_{10}^{p+1}-T_{10}^p}{\Delta t}$$

Regrouping, $T_{10}^{p+1}=T_{10}^p\left(1-2\text{Fo}-2\text{N}\cdot\text{Fo}-\text{Bi}\cdot\text{Fo}\right)+2\text{Fo}T_9^p+T_\infty\left(2\text{N}\cdot\text{Fo}+\text{Bi}\cdot\text{Fo}\right)$ (6)

where $N = h\Delta x/k$. (7)

The stability criterion is $\text{Fo} \leq 1/2(1 + N + \text{Bi}/2)$. (8)

With the finite-difference equations established, we can now proceed with the numerical solution. Having already specified $\Delta x = 0.016$ m, Bi can now be evaluated. Noting that $A_c = \pi D^2/4$ and $P = \pi D$, giving $A_c/P = D/4$, Eq. (3) yields

$$\text{Bi}=30\,\text{W}\big/\text{m}^2\cdot\text{K}\left[\left(0.016\,\text{m}\right)^2\Big/\frac{0.010\,\text{m}}{4}\right]\Big/14.8\,\text{W/m}\cdot\text{K}=0.208 \tag{9}$$

From the stability criteria, Eqs. (4) and (8), for the finite-difference equations, it is recognized that Eq. (8) requires the greater value of Fo. Hence

$$\text{Fo}=\frac{1}{2}\left(1+0.0324+\frac{0.208}{2}\right)=0.440 \tag{10}$$

where from Eq. (7), $N=\dfrac{30\,\text{W}\big/\text{m}^2\cdot\text{K}\times 0.016\,\text{m}}{14.8\,\text{W/m}\cdot\text{K}}=0.0324$. (11)

From the definition of Fo, Eq. (2), we obtain the time increment

$$\Delta t=\frac{\text{Fo}\left(\Delta x\right)^2}{\alpha}=0.440\left(0.016\,\text{m}\right)^2\Big/3.63\times10^{-6}\,\text{m}^2\big/\text{s}=31.1\text{s} \tag{12}$$

and the time relation is $t = p\Delta t = 31.1t$. (13)

Using the numerical values for Fo, Bi and N, the finite-difference equations can now be written (°C).

Nodal Point m ($2 \leq m \leq 9$):

$$T_m^{p+1}=T_m^p\left(1-2\times0.440-0.208\times0.440\right)+0.440\left(T_{m-1}^p+T_{m+1}^p\right)+0.208\times0.440\times25$$

$$T_m^{p+1}=0.029T_m^p+0.440\left(T_{m-1}^p+T_{m+1}^p\right)+2.3 \tag{14}$$

Nodal Point 1:

$$T_1^{p+1}=0.029T_1^p+0.440\left(200+T_2^p\right)+2.3=0.029T_1^p+0.440T_2^p+90.3 \tag{15}$$

Nodal Point 10:

$$T_{10}^{p+1}=0\times T_{10}^p+2\times0.440T_9^p+25\left(2\times0.0324\times0.440+0.208\times0.440\right)=0.880T_9^p+3.0 \tag{16}$$

Continued...

PROBLEM 5.113 (Cont.)

Using finite-difference equations (14-16) with Eq. (13), the calculations may be performed to obtain

p	t(s)	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	T_{10}(°C)
0	0	200	200	200	200	200	200	200	200	200	200
1	31.1	184.1	181.8	181.8	181.8	181.8	181.8	181.8	181.8	181.8	179.0
2	62.2	175.6	166.3	165.3	165.3	165.3	165.3	165.3	165.3	164.0	163.0
3	93.3	168.6	154.8	150.7	150.7	150.7	150.7	150.7	149.7	149.2	147.3
4	124.4	163.3	145.0	138.8	137.0	137.0	137.0	136.5	136.3	135.0	134.3
5	155.5	158.8	137.1	128.1	125.3	124.5	124.3	124.2	123.4	123.0	121.8
6	186.6	155.2	130.2	119.2	114.8	113.4	113.0	112.6	112.3	111.5	111.2
7	217.7	152.1	124.5	111.3	105.7	103.5	102.9	102.4			
8	248.8	145.1	119.5	104.5	97.6	94.8					

Using linear interpolation between rows 7 and 8, we obtain $T(L/2, 230s) = T_5 \approx 100°C$. <

(b) Using the option concerning *Finite-Difference Equations* for *One-Dimensional Transient Conduction* in *Extended Surfaces* from the IHT Toolpad, the desired temperature histories were computed for $0 \le t \le t_1 = 930s$. A *Lookup Table* involving data for T(x) at t = 0, 200, 400 and 930s was created.

t(s)/x(mm)	0	16	32	48	64	80	96	112	128	144	160
0	200	200	200	200	200	200	200	200	200	200	200
200	200	157.8	136.7	127.0	122.7	121.0	120.2	119.6	118.6	117.1	114.7
400	200	146.2	114.9	97.32	87.7	82.57	79.8	78.14	76.87	75.6	74.13
930	200	138.1	99.23	74.98	59.94	50.67	44.99	41.53	39.44	38.2	37.55

and the *LOOKUPVAL2* interpolating function was used with the *Explore* and *Graph* feature of IHT to create the desired plot.

Temperatures decrease with increasing x and t, and except for early times (t < 200s) and locations in proximity to the fin tip, the magnitude of the temperature gradient, |dT/dx|, decreases with increasing x. The slight increase in |dT/dx| observed for t = 200s and $x \rightarrow 160$ mm is attributable to significant heat loss from the fin tip.

COMMENTS: The steady-state condition may be obtained by extending the finite-difference calculations in time to $t \approx 2650s$ or from Eq. 3.70.

PROBLEM 5.114

KNOWN: Tantalum rod initially at a uniform temperature, 300K, is suddenly subjected to a current flow of 80A; surroundings (vacuum enclosure) and electrodes maintained at 300K.

FIND: (a) Estimate time required for mid-length to reach 1000K, (b) Determine the steady-state temperature distribution and estimate how long it will take to reach steady-state. Use a finite-difference method with a space increment of 10mm.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional, transient conduction in rod, (2) Surroundings are much larger than rod, (3) Properties are constant and evaluated at an average temperature.

PROPERTIES: *Table A-1*, Tantalum $\left(\overline{T} = (300+1000)\text{K}/2 = 650\text{K}\right)$: ρ = 16,600 kg/m^3, c = 147 J/kg·K, k = 58.8 W/m·K, and α = k/ρc = 58.8 W/m·K/16,600 kg/m^3 × 147 J/kg·K = 2.410×10^{-5} m^2/s.

ANALYSIS: From the derivation of the previous problem, the finite-difference equation was found to be

$$T_m^{p+1} = Fo\left(T_{m-1}^p + T_{m+1}^p\right) + (1-2Fo)T_m^p - \frac{\varepsilon P \sigma \Delta x^2}{kA_c} Fo\left(T_m^{4,p} - T_{sur}^4\right) + \frac{I^2 \rho_e \Delta x^2}{kA_c^2} \cdot Fo \qquad (1)$$

where $\quad Fo = \alpha\Delta t/\Delta x^2 \qquad A_c = \pi\ D^2/4 \qquad P = \pi\ D. \qquad (2,3,4)$

From the stability criterion, let Fo = 1/2 and numerically evaluate terms of Eq. (1).

$$T_m^{p+1} = \frac{1}{2}\left(T_{m-1}^p + T_{m+1}^p\right) - \frac{0.1 \times 5.67 \times 10^{-8}\,\text{W/m}^2 \cdot \text{K}^4 \times (0.01\text{m})^2\, 4}{58.8\ \text{W/m} \cdot \text{K} \times (0.003\text{m})} \cdot \frac{1}{2}\left(T_m^{4,p} - [300\text{K}]^4\right) +$$
$$+ \frac{(80\text{A})^2 \times 95 \times 10^{-8}\,\Omega \cdot \text{m}\,(0.01\text{m})^2}{58.8\ \text{W/m} \cdot \text{K}\left(\pi\,[0.003\text{m}]^2/4\right)^2} \cdot \frac{1}{2}$$

$$T_m^{p+1} = \frac{1}{2}\left(T_{m-1}^p + T_{m+1}^p\right) - 6.4285 \times 10^{-12}\, T_m^{4,p} + 103.53. \qquad (5)$$

Note that this form applies to nodes 0 through 5. For node 0, $T_{m-1} = T_{m+1} = T_1$. Since Fo = 1/2, using Eq. (2), find that

$$\Delta t = \Delta x^2 Fo/\alpha = (0.01\text{m})^2 \times 1/2/2.410 \times 10^{-5}\,\text{m}^2/\text{s} = 2.07\text{s}. \qquad (6)$$

Hence, t = pΔt = 2.07p. $\qquad$ (7)

Continued …..

PROBLEM 5.114 (Cont.)

(a) To estimate the time required for the mid-length to reach 1000K, that is T_o = 1000K, perform the forward-marching solution beginning with T_i = 300K at p = 0. The solution, as tabulated below, utilizes Eq. (5) for successive values of p. Elapsed time is determined by Eq. (7).

P	t(s)	T_0	T_1	T_2	T_3	T_4	T_5	T_6(°C)
0	0	300	300	300	300	300	300	300
1		403.5	403.5	403.5	403.5	403.5	403.5	300
2		506.9	506.9	506.9	506.9	506.9	455.1	300
3		610.0	610.0	610.0	610.0	584.1	506.7	300
4		712.6	712.6	712.6	699.7	661.1	545.2	300
5	10.4	814.5	814.5	808.0	788.8	724.7	583.5	300
6		915.2	911.9	902.4	867.4	787.9	615.1	300
7		1010.9	1007.9	988.9	945.0	842.3	646.6	300
8		1104.7	1096.8	1073.8	1014.0	896.1	673.6	300
9		1190.9	1183.5	1150.4	1081.7	943.2	700.3	300
10	20.7	1274.1	1261.6	1224.9	1141.5	989.4	723.6	300
11		1348.2	1336.7	1290.6	1199.8	1029.9	746.5	300
12		1419.7	1402.4	1353.9	1250.5	1069.4	766.5	300
13		1479.8	1465.5	1408.4	1299.8	1103.6	786.0	300
14		1542.6	1538.2	1460.9	1341.2	1136.9	802.9	300
15	31.1	1605.3	1569.3	1514.0	1381.6	1164.8	819.3	300

Note that, at p ≈ 6.9 or t = 6.9 × 2.07 = 14.3s, the mid-point temperature is T_o ≈ 1000K. <

(b) The steady-state temperature distribution can be obtained by continuing the marching solution until only small changes in T_m are noted. From the table above, note that at p = 15 or t = 31s, the temperature distribution is still changing with time. It is likely that at least 15 more calculation sets are required to see whether steady-state is being approached.

COMMENTS: (1) This problem should be solved with a computer rather than a hand-calculator. For such a situation, it would be appropriate to decrease the spatial increment in order to obtain better estimates of the temperature distribution.

(2) If the rod were very long, the steady-state temperature distribution would be very flat at the mid-length x = 0. Performing an energy balance on the small control volume shown to the right, find

$$\dot{E}_g - \dot{E}_{out} = 0$$
$$I^2 \frac{\rho_e \Delta x}{A_c} - \varepsilon\sigma P \Delta x \left(T_o^4 - T_{sur}^4\right) = 0.$$

Substituting numerical values, find T_o = 2003K. It is unlikely that the present rod would ever reach this steady-state, maximum temperature. That is, the effect of conduction along the rod will cause the center temperature to be less than this value.

PROBLEM 5.115

KNOWN: Support rod spanning a channel whose walls are maintained at T_b = 300 K. Suddenly the rod is exposed to cross flow of hot gases with T_∞ = 600 K and h = 75 W/m^2·K. After the rod reaches steady-state conditions, the hot gas flow is terminated and the rod cools by free convection and radiation exchange with surroundings.

FIND: (a) Compute and plot the midspan temperature as a function of elapsed heating time; compare the steady-state temperature distribution with results from an analytical model of the rod and (b) Compute the midspan temperature as a function of elapsed cooling time and determine the time required for the rod to reach the safe-to-touch temperature of 315 K.

SCHEMATIC:

Symmetry plane

T_∞ = 600 K, h = 75 W/m^2•K

T_b = 300 K

T_∞, h

1 2 3 9 10

Rod, T_i = 300 K

D = 15 mm, L = 100 mm

k =15 W/m•K, α = 4x10^{-6} m^2/s

Δx = 5 mm

x

50 mm

ASSUMPTIONS: (1) One-dimensional, transient conduction in rod, (2) Constant properties, (3) During heating process, uniform convection coefficient over rod, (4) During cooling process, free convection coefficient is of the form h = CΔT^n where C = 4.4 W/m^2·K$^{1.188}$ and n = 0.188, and (5) During cooling process, surroundings are large with respect to the rod.

ANALYSIS: (a) The finite-difference equations for the 10-node mesh shown above can be obtained using the *IHT Finite-Difference Equation, One-Dimensional, Transient Extended Surfaces Tool*. The temperature-time history for the midspan position T_{10} is shown in the plot below. The steady-state temperature distribution for the rod can be determined from Eq. 3.75, Case B, Table 3.4. This case is treated in the *IHT Extended Surfaces Model, Temperature Distribution and Heat Rate, Rectangular Pin Fin*, for the adiabatic tip condition. The following table compares the steady-state temperature distributions for the numerical and analytical methods.

Method	Temperatures (K) vs. Position x (mm)					
	0	10	20	30	40	50
Analytical	300	386.1	443.4	479.5	499.4	505.8
Numerical	300	386.0	443.2	479.3	499.2	505.6

The comparison is excellent indicating that the nodal mesh is sufficiently fine to obtain precise results.

Midspan temperature, T10 (K)

600 500 400 300

0 100 200 300 400 500 600

Elapsed heating time, t (s)

Continued...

PROBLEM 5.115 (Cont.)

(b) The same finite-difference approach can be used to model the cooling process. In using the IHT tool, the following procedure was used: (1) Set up the FDEs with the convection coefficient expressed as $h_m = h_{fc,m} + h_{r,m}$, the sum of the free convection and linearized radiation coefficients based upon nodal temperature T_m.

$$h_{fc,m} = C\left(T_m^p - T_\infty\right)$$

$$h_{r,m} = \varepsilon\sigma\left(T_m^p + T_{sur}\right)\left(\left(T_m^p\right)^2 + T_{sur}^2\right)$$

(2) For the initial solve, set $h_{fc,m} = h_{r,m} = 5\ W/m^2 \cdot K$ and solve, (3) Using the solved results as the Initial Guesses for the next solve, allow $h_{fc,m}$ and $h_{r,m}$ to be unknowns. The temperature-time history for the midspan during the cooling process is shown in the plot below. The time to reach the safe-to-touch temperature, $T_{10}^p = 315\,K$, is

t = 550 s <

PROBLEM 5.116

KNOWN: Thin metallic foil of thickness, w, whose edges are thermally coupled to a sink at temperature, T_{sink}, initially at a uniform temperature $T_i = T_{sink}$, is suddenly exposed on the top surface to an ion beam heat flux, q''_s, and experiences radiation exchange with the vacuum enclosure walls at T_{sur}. Consider also the situation when the foil is operating under steady-state conditions when suddenly the ion beam is deactivated.

FIND: (a) Compute and plot the midspan temperature-time history during the *heating* process; determine the elapsed time that this point on the foil reaches a temperature within 1 K of the steady-state value, and (b) Compute and plot the midspan temperature-time history during the *cooling* process from steady-state operation; determine the elapsed time that this point on the foil reaches the *safe-to-touch* temperature of 315 K.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional, transient conduction in the foil, (2) Constant properties, (3) Upper and lower surfaces of foil experience radiation exchange with the large surroundings, (4) Ion beam incident on upper surface only, (4) Foil is of unit width normal to the page.

ANALYSIS: (a) The finite-difference equations for the 10-node mesh shown above can be obtained using the *IHT Finite-Difference Equation, One-Dimensional, Transient, Extended Surfaces Tool*. In formulating the energy-balance functions, the following steps were taken: (1) the FDE function coefficient h must be identified for each node, e.g., h_1 and (2) coefficient can be represented by the linearized radiation coefficient, e.g., $h_1 = \varepsilon\sigma\left(T_1 + T_{sur}\right)\left(T_1^2 + T_{sur}^2\right)$, (3) set $q''_a = q''_o/2$ since the ion beam is incident on only the top surface of the foil, and (4) when solving, the initial condition corresponds to T_i = 300 K for each node. The temperature-time history of the midspan position is shown below. The time to reach within 1 K of the steady-state temperature (374.1 K) is

$$T_{10}\left(t_h\right) = 373\,K \qquad t_h = 136\,s$$ <

(b) The same IHT workspace may be used to obtain the temperature-time history for the cooling process by taking these steps: (1) set $q''_s = 0$, (2) specify the initial conditions as the steady-state temperature (K) distribution tabulated below,

T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	T_{10}
374.1	374.0	373.5	372.5	370.9	368.2	363.7	356.6	345.3	327.4

(3) when performing the integration of the independent time variable, set the start value as 200 s and (4) save the results for the heating process in Data Set A. The temperature-time history for the heating and cooling processes can be made using Data Browser results from the Working and A Data Sets. The time required for the midspan to reach the *safe-to-touch* temperature is

$$T_{10}\left(t_c\right) = 315\,K \qquad t_c = 73\,s$$ <

Continued...

Midpsan temperature, T1 (K)
400
380
360
340
320
300
0 100 200 300 400 500
Heating or cooling time, t (s)
Heating process
Cooling process

COMMENTS: The IHT workspace using the Finite-Difference Equations Tool to determine the temperature-time distributions is shown below. Some of the lines of code were omitted to save space on the page.

```
// Finite Difference Equations Tool: One-Dimensional, Transient, Extended Surface
/* Node 1: extended surface interior node;  transient conditions; e and w labeled 2 and 2. */
rho * cp * der(T1,t) = fd_1d_xsur_i(T1,T2,T2,k,qdot,Ac,P,deltax,Tinf, h1,q''a)
q''a1 = q''s / 2                    // Applied heat flux, W/m^2;  on the upper surface only
h1 = eps * sigma * (T1 + Tsur) * (T1^2 + Tsur^2)
sigma = 5.67e-8                     // Boltzmann constant, W/m^2.K^4
/* Node 2: extended surface interior node;  transient conditions; e and w labeled 3 and 1. */
rho * cp * der(T2,t) = fd_1d_xsur_i(T2,T3,T1,k,qdot,Ac,P,deltax,Tinf, h2,q''a2)
q''a2 = 0                           // Applied heat flux, W/m^2; zero flux shown
h2 = eps * sigma * (T2+ Tsur) * (T2^2 + Tsur^2)
.......
.......
/* Node 10: extended surface interior node;  transient conditions; e and w labeled sk and 9. */
rho * cp * der(T10,t) = fd_1d_xsur_i(T10,Tsk,T9,k,qdot,Ac,P,deltax,Tinf, h10,q''a)
q''a10 = 0                          // Applied heat flux, W/m^2; zero flux shown
h10 = eps * sigma * (T10 + Tsur) * (T10^2 + Tsur^2)

// Assigned variables
deltax = L / 10                     // Spatial increment, m
Ac = w * 1                          // Cross-sectional area, m^2
P = 2 * 1                           // Perimeter, m
L = 0.150                           // Overall length, m
w = 0.00025                         // Foil thickness, m
eps = 0.45                          // Foil emissivity
Tinf = Tsur                         // Fluid temperature, K
Tsur = 300                          // Surroundings temperature, K
k = 40                              // Foil thermal conductivity
Tsk = 300                           // Sink temperature, K
q''s = 600                          // Ion beam heat flux, W/m^2; for heating process
q''s = 0                            // Ion beam heat flux, W/m^2; for cooling process
qdot = 0                            // Foil volumetric generation rate, W/m^3
alpha = 3e-5                        // Thermal diffusivity, m^2/s
rho = 1000                          // Density, kg.m^3; arbitrary value
alpha = k / (rho * cp)              // Definition
```

PROBLEM 5.117

KNOWN: Stack or book of steel plates (sp) and circuit boards (b) subjected to a prescribed platen heating schedule $T_p(t)$. See Problem 5.42 for other details of the book.

FIND: (a) Using the implicit numerical method with Δx = 2.36mm and Δt = 60s, find the mid-plane temperature T(0,t) of the book and determine whether curing will occur (> 170°C for 5 minutes), (b) Determine how long it will take T(0,t) to reach 37°C following reduction of the platen temperature to 15°C (at t = 50 minutes), (c) Validate code by using a sudden change of platen temperature from 15 to 190°C and compare with the solution of Problem 5.38.

SCHEMATIC:

Platen (p)
Steel plate (sp)
Circuit board, s
x
T_p (°C)
190
160
15
0
20
40
60
t(min)

ASSUMPTIONS: (1) One-dimensional conduction, (2) Negligible contact resistance between plates, boards and platens.

PROPERTIES: Steel plates (sp, given): ρ_{sp} = 8000 kg/m^3, $c_{p,sp}$ = 480 J/kg·K, k_{sp} = 12 W/m·K; Circuit boards (b, given): ρ_b = 1000 kg/m^3, $c_{p,b}$ = 1500 J/kg·K, k_b = 0.30 W/m·K.

ANALYSIS: (a) Using the suggested space increment Δx = 2.36mm, the model grid spacing treating the steel plates (sp) and circuit boards (b) as discrete elements, we need to derive the nodal equations for the interior nodes (2-11) and the node next to the platen (1). Begin by defining appropriate control volumes and apply the conservation of energy requirement.

Effective thermal conductivity, k_e: Consider an adjacent steel plate-board arrangement. The thermal resistance between the nodes i and j is

$$R''_{ij} = \frac{\Delta x}{k_e} = \frac{\Delta x/2}{k_b} + \frac{\Delta x/2}{k_{sp}}$$

$$k_e = \frac{2}{1/k_{b+} + 1/k_{sp}} = \frac{2}{1/0.3 + 1/12} \text{W/m} \cdot \text{K}$$

$$k_e = 0.585 \text{ W/m} \cdot \text{K}.$$

Δx i k_b
Δx j k_{sp}

Odd-numbered nodes, 3 £ m £ 11 - steel plates (sp): Treat as interior nodes using Eq. 5.89 with

$$a_{sp} \equiv \frac{k_e}{r_{sp} c_{sp}} = \frac{0.585 \text{ W/m} \cdot \text{K}}{8000 \text{ kg/m}^3 \times 480 \text{ J/kg} \cdot \text{K}} = 1.523 \times 10^{-7} \text{ m}^2/\text{s}$$

$$Fo_m = \frac{a_{sp} \Delta t}{\Delta x^2} = \frac{1.523 \times 10^{-7} \text{ m}^2/\text{s} \times 60\text{s}}{(0.00236 \text{ m})^2} = 1.641$$

Continued …..

to obtain, with m as odd-numbered,

$$\left(1+2Fo_m\right)T_m^{p+1} - Fo_m\left(T_{m-1}^{p+1} + T_{m+1}^{p+1}\right) = T_m^p \qquad (1)$$

Even-numbered nodes, 2 £ n £ 10 - circuit boards (b): Using Eq. 5.89 and evaluating α_b and Fo_n

$$a_b = \frac{k_e}{r_b c_b} = 3.900\times10^{-7}\ m^2/s \qquad Fo_n = 4.201$$

$$\left(1+2Fo_n\right)T_n^{p+1} - Fo_n\left(T_{n-1}^{p+1} + T_{n+1}^{p+1}\right) = T_n^p \qquad (2)$$

Plate next to platen, n = 1 - steel plate (sp): The finite-difference equation for the plate node (n = 1) next to the platen follows from a control volume analysis.

$$\dot{E}_{in} - \dot{E}_{out} = \dot{E}_{st}$$

$$q''_a + q''_b = r_{sp}\Delta x c_{sp}\frac{T_1^{p+1} - T_1^p}{\Delta t}$$

where

$$q''_a = k_{sp}\frac{T_p(t) - T_1^{p+1}}{\Delta x/2} \qquad q''_b = k_e\frac{T_2^{p+1} - T_1^{p+1}}{\Delta x}$$

and $T_p(t) = T_p(p)$ is the platen temperature which is changed with time according to the heating schedule. Regrouping find,

$$\left(1+Fo_m\left(1+\frac{2k_{sp}}{k_e}\right)\right)T_1^{p+1} - Fo_m T_2^{p+1} - \frac{2k_{sp}}{k_e}Fo_m T_p(p) = T_1^p \qquad (3)$$

where $2k_{sp}/k_e = 2 \times 12$ W/m·K/0.585 W/m·K = 41.03.

Using the nodal Eqs. (1) -(3), an inversion method of solution was effected and the temperature distributions are shown on the following page.

Temperature distributions - discussion: As expected, the temperatures of the nodes near the center of the book considerably lag those nearer the platen. The criterion for cure is T ≥ 170°C = 443 K for Δt_c = 5 min = 300 sec. From the temperature distributions, note that node 10 just reaches 443 K after 50 minutes and will not be cured. It appears that the region about node 5 will be cured.

(b) The time required for the book to reach 37°C = 310 K can likewise be seen from the temperature distribution results. The plates/boards nearest the platen will cool to the safe handling temperature with 1000 s = 16 min, but those near the center of the stack will require in excess of 2000 s = 32 min.

Continued …..

PROBLEM 5.117 (Cont.)

(c) It is important when validating computer codes to have the program work a "problem" which has an exact analytical solution. You should select the problem such that all features of the code are tested.

PROBLEM 5.118

KNOWN: Reaction and composite clutch plates, initially at a uniform temperature, $T_i = 40°C$, are subjected to the frictional-heat flux shown in the engagement energy curve, q''_f vs. t .

FIND: (a) On T-t coordinates, sketch the temperature histories at the mid-plane of the reaction plate, at the interface between the clutch pair, and at the mid-plane of the composite plate; identify key features; (b) Perform an energy balance on the clutch pair over a time interval basis and calculate the steady-state temperature resulting from a clutch engagement; (c) Obtain the temperature histories using the finite-element approach with *FEHT* and the finite-difference method of solution with *IHT* ($\Delta x = 0.1$ mm and $\Delta t = 1$ ms). Calculate and plot the frictional heat fluxes to the reaction and composite plates, q''_{rp} and q''_{cp}, respectively, as a function of time. Comment on the features of the temperature and frictional-heat flux histories.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional conduction, (2) Negligible heat transfer to the surroundings.

PROPERTIES: Steel, $\rho_s = 7800$ kg/m^3, $c_s = 500$ J/kg·K, $k_s = 40$ W/m·K; Friction material, $\rho_{fm} = 1150$ kg/m^3, $c_{fm} = 1650$ J/kg·K, and $k_{fm} = 4$ W/m·K.

ANALYSIS: (a) The temperature histories for specified locations in the system are sketched on T-t coordinates below.

Initially, the temperature at all locations is uniform at T_i. Since there is negligible heat transfer to the surroundings, eventually the system will reach a uniform, steady-state temperature $T(\infty)$. During the engagement period, the interface temperature increases much more rapidly than at the mid-planes of the reaction (rp) and composite (cp) plates. The interface temperature should be the maximum within the system and could occur before lock-up, $t = t_{lu}$.

Continued …..

PROBLEM 5.118 (Cont.)

(b) To determine the steady-state temperature following the engagement period, apply the conservation of energy requirement on the clutch pair on a time-interval basis, Eq. 1.11b.

$q''_f(t) = 1.6 \times 10^7$ W/m^2 $\quad 0 \le t \le t_{lu}$

rp cp (s) (fm) (s)

$T_i = 40°C$

$T(t \to \infty) = T(\infty)$

Clutch pair - energy balance

The final and initial states correspond to uniform temperatures of $T(\infty)$ and T_i, respectively. The energy input is determined from the engagement energy curve, q''_f vs. t .

$$E''_n - E''_{out} + E''_{gen} = \Delta E''_{st} \qquad\qquad E''_{in} = E''_{out} = 0$$

$$\int_0^{t_{lu}} q''_f(t)dt = E''_f - E''_i = \left[\rho_s c_s \left(L_{rp}/2 + L_{cp}/2\right) + \rho_{fm} c_{fm} L_{fm}\right]\left(T_f - T_i\right)$$

Substituting numerical values, with $T_i = 40°C$ and $T_f = T(\infty)$.

$$0.5\, q''_o\, t_{lu} = \left[\rho_s c_s \left(L_{rp}/2 + L_{cp}/2\right) + \rho_{fm} c_{fm} L_{fm}\right]\left(T(\infty) - T_i\right)$$

$$0.5 \times 1.6 \times 10^7 \text{ W/m}^2 \times 0.100 \text{ s} = \Big[7800 \text{ kg/m}^3 \times 500 \text{ J/kg}\cdot\text{K}\left(0.001 + 0.0005\right)\text{m}$$

$$+1150 \text{ kg/m}^3 \times 1650 \text{ J/kg}\cdot\text{K} \times 0.0005 \text{ m}\Big]\left(T(\infty) - 40\right)°C$$

$$T(\infty) = 158°C$$ <

(c) *Finite-element method of solution, FEHT*. The clutch pair is comprised of the reaction plate (1 mm), an interface region (0.1 mm), and the composite plate (cp) as shown below.

Scale: 1 cm = 0.1 mm | **Interface region, 0.1 mm, qgen(t)**

(steel) (friction material) (steel)

Reaction plate (rp) **Composite plate (cp)**

Continued (2)...

PROBLEM 5.118 (Cont.)

The external boundaries of the system are made adiabatic. The interface region provides the means to represent the frictional heat flux, specified with negligible thermal resistance and capacitance. The generation rate is prescribed as

$$\dot{q} = 1.6 \times 10^{11} (1 - \text{Time}/0.1)\, \text{W/m}^3 \qquad 0 \le \text{Time} \le t_{lu}$$

where the first coefficient is evaluated as $q_o'' / 0.1 \times 10^{-3}\,\text{m}$ and the 0.1 mm parameter is the thickness of the region. Using the *Run* command, the integration is performed from 0 to 0.1 s with a time step of 1×10^{-6} s. Then, using the *Specify|Generation* command, the generation rate is set to zero and the *Run|Continue* command is executed. The temperature history is shown below.

(c) *Finite-difference method of solution*, IHT. The nodal arrangement for the clutch pair is shown below with $\Delta x = 0.1$ mm and $\Delta t = 1$ ms. Nodes 02-10, 13-16 and 18-21 are interior nodes, and their finite-difference equations (FDE) can be called into the *Workspace* using *Tools|Finite Difference Equations|One-Dimenisonal|Transient*. Nodes 01 and 22 represent the mid-planes for the reaction and composite plates, respectively, with adiabatic boundaries. The FDE for node 17 is derived from an energy balance on its control volume (CV) considering different properties in each half of the CV. The FDE for node 11 and 12 are likewise derived using energy balances on their CVs. At the interface, the following conditions must be satisfied

$$T_{11} = T_{12} \qquad q_f'' = q_{rp}'' + q_{cp}''$$

The frictional heat flux is represented by a *Lookup Table*, which along with the FDEs, are shown in the *IHT* code listed in Comment 2.

Nodal arrangement, FDE solution

Continued (3)...

PROBLEM 5.118 (Cont.)

The temperature and heat flux histories are plotted below. The steady-state temperature was found as 156.5° C, which is in reasonable agreement with the energy balance result from part (a).

Temperature history for clutch pair, 100 ms lock-up time

Temperature, T (C)

Engagement time, t (ms)

Midplane, reaction plate, T01
Interface, T11 or T12
Midplane, composite plate, T22

Heat flux histories for clutch pair during engagement

Heat flux, q'' (W/m^2 * 10^7)

Engagement time, t (ms)

Frictional heat flux, q''f
Reaction plate, q''rp
composite plate, q''cp

COMMENTS: (1) The temperature histories resulting from the FEHT and IHT based solutions are in agreement. The interface temperature peaks near 225°C after 75 ms, and begins dropping toward the steady-state condition. The mid-plane of the reaction plate peaks around 100 ms, nearly reaching 200°C. The temperature of the mid-plane of the composite plate increases slowly toward the steady-state condition.

(2) The calculated temperature-time histories for the clutch pair display similar features as expected from our initial sketches on T vs. t coordinates, part a. The maximum temperature for the composite is very high, subjecting the bonded frictional material to high thermal stresses as well as accelerating deterioration. For the reaction steel plate, the temperatures are moderate, but there is a significant gradient that could give rise to thermal stresses and hence, warping. Note that for the composite plate, the steel section is nearly isothermal and is less likely to experience warping.

(2) The *IHT* code representing the FDE for the 22 nodes and the frictional heat flux relation is shown below. Note use of the *Lookup Table* for representing the frictional heat flux *vs*. time boundary condition for nodes 11 and 12.

```
// Nodal equations, reaction plate (steel)
/* Node 01: surface node (w-orientation); transient conditions; e labeled 02. */
rhos * cps * der(T01,t) = fd_1d_sur_w(T01,T02,ks,qdot,deltax,Tinf01,h01,q''a01)
q''a01 = 0            // Applied heat flux, W/m^2; zero flux shown
Tinf01 = 40           // Arbitrary value
h01 = 1e-5            // Causes boundary to behave as adiabatic
qdot = 0
/* Node 02: interior node; e and w labeled 03 and 01. */
rhos*cps*der(T02,t) = fd_1d_int(T02,T03,T01,ks,qdot,deltax)
……………………………
/* Node 10: interior node; e and w labeled 11 and 09. */
rhos*cps*der(T10,t) = fd_1d_int(T10,T11,T09,ks,qdot,deltax)
/* Node 11: From an energy on the CV about node 11 */
ks * (T10 - T11) / deltax + q''rp =  rhos * cps * deltax / 2 * der(T11,t)
```

Continued (4)...

PROBLEM 5.118 (Cont.)

```
//  Friction-surface interface conditions
T11 = T12
q''f = LOOKUPVAL(HFVST16,1,t,2)        // Applied heat flux, W/m^2; specified by Lookup Table
/*  See HELP (Solver, Lookup Tables).  The look-up table, file name "HFVST16'  contains
            0         16e6
            0.1       0
            100       0              */
q''rp + q''cp = q''f                   // Frictional heat flux

// Nodal equations - composite plate
// Frictional material, nodes 12-16
/* Node 12: From an energy on the CV about node 12 */
 kfm * (T13 - T12) / deltax + q''cp =  rhofm * cpfm * deltax / 2 * der(T12,t)
/* Node 13: interior node; e and w labeled 08 and 06. */
rhofm*cpfm*der(T13,t) = fd_1d_int(T13,T14,T12,kfm,qdot,deltax)
……………………………….
/* Node 16: interior node; e and w labeled 11 and 09. */
rhofm*cpfm*der(T16,t) = fd_1d_int(T16,T17,T15,kfm,qdot,deltax)
// Interface between friction material and steel, node 17
/* Node 17: From an energy on the CV about node 17 */
kfm * (T16 - T17) / deltax + ks * (T18 - T17) / deltax = RHS
RHS = ( (rhofm * cpfm * deltax /2) + (rhos * cps * deltax /2) ) * der(T17,t)
// Steel, nodes 18-22
/* Node 18: interior node; e and w labeled 03 and 01. */
rhos*cps*der(T18,t) = fd_1d_int(T18,T19,T17,ks,qdot,deltax)
……………………………….
/* Node 22: interior node; e and w labeled 21 and 21.  Symmetry condition. */
rhos*cps*der(T22,t) = fd_1d_int(T22,T21,T21,ks,qdot,deltax)
// qdot = 0

// Input variables
// Ti = 40                      // Initial temperature; entered during Solve
deltax = 0.0001
rhos = 7800                     // Steel properties
cps = 500
ks = 40
rhofm = 1150                    //Friction material properties
cpfm = 1650
kfm = 4

// Conversions, to facilitate graphing
t_ms = t * 1000
qf_7 = q''f / 1e7
qrp_7 = q''rp / 1e7
qcp_7 = q''cp / 1e7
```

PROBLEM 5.119

KNOWN: Hamburger patties of thickness 2L = 10, 20 and 30 mm, initially at a uniform temperature T_i = 20°C, are grilled on both sides by a convection process characterized by T_∞ = 100°C and h = 5000 W/m^2·K.

FIND: (a) Determine the relationship between *time-to-doneness*, t_d, and patty thickness. Doneness criteria is 60°C at the center. Use *FEHT* and the *IHT Models|Transient Conduction|Plane Wall.* (b) Using the results from part (a), estimate the *time-to-doneness* if the initial temperature is 5 °C rather than 20°C. Calculate values using the *IHT* model, and determine the relationship between time-to-doneness and initial temperature.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional conduction, and (2) Constant properties are approximated as those of water at 300 K.

PROPERTIES: *Table A-6*, Water (300K), ρ = 1000 kg/m^3, c = 4179 J/kg·K, k = 0.613 W/m·K.

ANALYSIS: (a) To determine $T(0, t_d)$, the center point temperature at the *time-to-doneness* time, t_d, a one-dimensional shape as shown in the *FEHT* screen below is drawn, and the material properties, boundary conditions, and initial temperature are specified. With the *Run|Calculate* command, the early integration steps are made very fine to accommodate the large temperature-time changes occurring near x = L. Use the *Run|Continue* command (see *FEHT HELP*) for the second and subsequent steps of the integration. This sequence of *Start-(Step)-Stop* values was used: 0 (0.001) 0.1 (0.01) 1 (0.1) 120 (1.0) 840 s.

Continued …..

PROBLEM 5.119 (Cont.)

Using the *View|Temperature vs. Time* command, the temperature-time histories for the x/L = 0 (center), 0.5, and 1.0 (grill side) are plotted and shown below for the 2L = 10 mm thick patty.

Time-to-doneness (60C) for 10-mm thick hamburger

Grill side, x/L = 1
Mid-point, x/L = 0.5
Center, x/L = 0
Temperature [°C]
Time [sec]

Using the *View|Temperatures* command, the time slider can be adjusted to read t_d, when the center point, x = 0, reaches 60°C. See the summary table below.

The *IHT ready-to-solve* model in *Models|Transient Conduction|Plane Wall* is based upon Eq. 5.40 and permits direct calculation of t_d when $T(0,t_d) = 60°C$ for patty thickness 2L = 10, 20 and 30 mm and initial temperatures of 20 and 5°C. The *IHT* code is provided in Comment 3, and the results are tabulated below.

	Solution method	Time-to-doneness, t (s)			T_i (C)
		Patty thickness, 2L (mm)			
		10	20	30	
	FEHT	66.2	264.5	591	20
	IHT	67.7	264.5	590.4	20
		80.2	312.2	699.1	5
	Eq. 5.40 (see	x	x		5
	Comment 4)		x	x	20

Considering the *IHT* results for T_i = 20°C, note that when the thickness is doubled from 10 to 20 mm, t_d is (264.5/67.7=) 3.9 times larger. When the thickness is trebled, from 10 to 30 mm, t_d is (590.4/67.7=) 8.7 times larger. We conclude that, t_d is nearly proportional to L^2, rather than linearly proportional to thickness.

Continued …..

PROBLEM 5.119 (Cont.)

(b) The temperature span for the cooking process ranges from T_∞ = 100 to T_i = 20 or 5°C. The differences are (100-20 =) 80 or (100-5 =) 95°C. If t_d is proportional to the overall temperature span, then we expect t_d for the cases with T_i = 5°C to be a factor of (95/80 =) 1.19 higher (approximately 20%) than with T_i = 20°C. From the tabulated results above, for the thickness 2L = 10, 20 and 30 mm, the t_d with T_i = 5°C are (80.2/67.7 =) 1.18, (312 / 264.5 =) 1.18, and (699.1/590.4 =) 1.18, respectively, higher than with T_i = 20°C. We conclude that t_d is nearly proportional to the temperature span (T_∞ - T_i).

COMMENTS: (1) The results from the *FEHT* and *IHT* calculations are in excellent agreement. For this analysis, the *FEHT* model is more convenient to use as it provides direct calculations of the time-to-doneness. The *FEHT* tool allows the user to watch the cooking process. Use the *View*|*Temperature Contours* command, click on the *from start-to-stop* button, and observe how color band changes represent the temperature distribution as a function of time.

(2) It is good practice to check software tool analyses against hand calculations. Besides providing experience with the basic equations, you can check whether the tool was used or functioned properly. Using the one-term series solution, Eq. 5.40:

$$\theta_o^* = \frac{T(0, t_d) - T_\infty}{T_i - T_\infty} = C_1 \exp\left(-\zeta^2 Fo\right)$$

$$Fo = \alpha t_d / L^2 \qquad C_1, \zeta = (Bi), \text{ Table 5.1}$$

T_i (°C)	2L (mm)	θ_o^*	Bi	C_1	ζ_1	Fo	t_d (s)
20	10	0.5000	24.47	1.2707	1.5068	0.4108	70.0
5	30	0.4211	73.41	1.2729	1.5471	0.4622	709

The results are slightly higher than those from the *IHT* model, which is based upon a multiple- rather than single-term series solution.

(3) The *IHT* code used to obtain the tabulated results is shown below. Note that T_xt_trans is an intrinsic heat transfer function dropped into the *Workspace* from the *Models* window (see *IHT Help|Solver|Intrinsic Functions|Heat Transfer Functions*).

```
// Models | Transient Conduction | Plane Wall
/* Model: Plane wall of thickness 2L, initially with a uniform temperature T(x,0) = Ti, suddenly subjected
to convection conditions (Tinf,h).  */
// The temperature distribution is
T_xt = T_xt_trans("Plane Wall",xstar,Fo,Bi,Ti,Tinf)          // Eq 5.39
// The dimensionless parameters are
xstar = x / L
Bi = h * L / k                  // Eq 5.9
Fo= alpha * t / L^2             // Eq 5.33
alpha = k/ (rho * cp)

// Input parameters
x = 0               // Center point of meat
L = 0.005           // Meat half-thickness, m
//L = 0.010
//L = 0.015
T_xt = 60            // Doneness temperature requirement at center, x = 0; C
Ti = 20              // Initial uniform temperature
//Ti = 5
rho = 1000          // Water properties at 300 K
cp = 4179
k = 0.613
h = 5000             // Convection boundary conditions
Tinf = 100
```

PROBLEM 5.120

KNOWN: A process mixture at 200°C flows at a rate of 207 kg/min onto a 1-m wide conveyor belt traveling with a velocity of 36 m/min. The underside of the belt is cooled by a water spray.

FIND: The surface temperature of the mixture at the end of the conveyor belt, $T_{e,s,}$ using (a) *IHT* for writing and solving the FDEs, and (b) *FEHT*. Validate your numerical codes against an appropriate analytical method of solution.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional conduction in the x-direction at any z-location, (2) Negligible heat transfer from mixture upper surface to ambient air, and (3) Constant properties.

PROPERTIES: Process mixture (m), $\rho_m = 960$ kg/m^3, $c_m = 1700$ J/kg·K, and $k_m = 1.5$ W/m·K; Conveyor belt (b), $\rho_b = 8000$ kg/ m^3, $c_b = 460$ J/kg·K, and $k_b = 15$ W/m·K.

ANALYSIS: From the conservation of mass requirement, the thickness of the mixture on the conveyor belt can be determined.

$$\dot{m} = \rho_m A_c V \qquad \text{where} \qquad A_c = W\,L_m$$

$$207\ \text{kg/min} \times 1\ \text{min}/60\,\text{s} = 960\ \text{kg/m}^3 \times 1\,\text{m} \times L_m \times 36\,\text{m/min} \times 1\text{min}/60\,\text{s}$$

$$L_m = 0.0060\,\text{m} = 6\ \text{mm}$$

The time that the mixture is in contact with the steel conveyor belt, referred to as the residence time, is

$$t_{res} = L_c / V = 30\ \text{m} / \left(36\ \text{m/min} \times 1\ \text{min}/60\ \text{s}\right) = 50\ \text{s}$$

The composite system comprised of the belt, $L_b = 3$ mm, and mixture, $L_m = 6$ mm, as represented in the schematic above, is initially at a uniform temperature $T(x,0) = T_i = 200°C$ while at location z = 0, and suddenly is exposed to convection cooling (T_∞, h). We will calculate the mixture upper surface temperature after 50 s, $T(0, t_{res}) = T_{e,s}$.

(a) The nodal arrangement for the composite system is shown in the schematic below. The *IHT* model builder *Tools|Finite-Difference Equations|Transient* can be used to obtain the FDEs for nodes 01-12 and 14-19.

Continued …..

PROBLEM 5.120 (Cont.)

For the mixture-belt interface node 13, the FDE for the implicit method is derived from an energy balance on the control volume about the node as shown above.

$$\dot{E}''_{in} - \dot{E}''_{out} = \dot{E}''_{st}$$

$$q''_a + q''_b = \dot{E}''_{st,m} + \dot{E}''_{st,b}$$

$$k_m \frac{T_{12}^{p+1} - T_{13}^{p+1}}{\Delta x} + k_b \frac{T_{14}^{p+1} - T_{13}^{p+1}}{\Delta x} = (\rho_m c_m + \rho_b c_b)(\Delta x / 2) \frac{T_{13}^{p+1} - T_{13}^{p}}{\Delta t}$$

IHT code representing selected FDEs, nodes 01, 02, 13 and 19, is shown in Comment 4 below (Δx = 0.5 mm, Δt = 0.1 s). Note how the FDE for node 13 derived above is written in the *Workspace*. From the analysis, find

$T_{e,s}$ = T(0, 50s) = 54.8°C <

(b) Using *FEHT*, the composite system is drawn and the material properties, boundary conditions, and initial temperature are specified. The screen representing the system is shown below in Comment 5 with annotations on key features. From the analysis, find

$T_{e,s}$ = T(0, 50s) = 54.7°C <

COMMENTS: (1) Both numerical methods, *IHT* and *FEHT*, yielded the same result, 55°C. For the safety of plant personnel working in the area of the conveyor exit, the mixture exit temperature should be lower, like 43°C.

(2) By giving both regions of the composite the same properties, the analytical solution for the plane wall with convection, Section 5.5, Eq. 5.40, can be used to validate the *IHT* and *FEHT* codes. Using the *IHT Models|Transient Conduction|Plane Wall* for a 9-mm thickness wall with mixture thermophysical properties, we calculated the temperatures after 50 s for three locations: T(0, 50s) = 91.4°C; T(6 mm, 50s) = 63.6°C; and T(3 mm, 50s) = 91.4°C. The results from the *IHT* and *FEHT* codes agreed exactly.

(3) In view of the high heat removal rate on the belt lower surface, it is reasonable to assume that negligible heat loss is occurring by convection on the top surface of the mixture.

Continued …..

PROBLEM 5.120 (Cont.)

(4) The *IHT* code representing selected FDEs, nodes 01, 02, 13 and 19, is shown below. The FDE for node 13 was derived from an energy balance, while the others are written from the *Tools* pad.

```
// Finite difference equations from Tools, Nodes 01 -12 (mixture) and 14-19 (belt)
/* Node 01: surface node (w-orientation); transient conditions; e labeled 02. */
rhom * cm * der(T01,t) = fd_1d_sur_w(T01,T02,km,qdot,deltax,Tinf01,h01,q''a01)
q''a01 = 0        // Applied heat flux, W/m^2; zero flux shown
qdot = 0
Tinf01 = 20      // Arbitrary value
h01 = 1e-6       // Causes boundary to behave as adiabatic

/* Node 02: interior node; e and w labeled 03 and 01. */
rhom*cm*der(T02,t) = fd_1d_int(T02,T03,T01,km,qdot,deltax)

/* Node 19: surface node (e-orientation); transient conditions; w labeled 18. */
rhob * cb * der(T19,t) = fd_1d_sur_e(T19,T18,kb,qdot,deltax,Tinf19,h19,q''a19)
q''a19 = 0        // Applied heat flux, W/m^2; zero flux shown
Tinf19 = 30
h19 = 3000

// Finite-difference equation from energy balance on CV, Node 13
km*(T12 - T13)/deltax + kb*(T14 - T13)/deltax = (rhom*cm + rhob*cb) *(deltax/2)*der(T13,t)
```

(5) The screen from the *FEHT* analysis is shown below. It is important to use small time steps in the integration at early times. Use the *View|Temperatures* command to find the temperature of the mixture surface at t_{res} = 50 s.

PROBLEM 5.121

KNOWN: Thin, circular-disc subjected to induction heating causing a uniform heat generation in a prescribed region; upper surface exposed to convection process.

FIND: (a) Transient finite-difference equation for a node in the region subjected to induction heating, (b) Sketch the steady-state temperature distribution on T-r coordinates; identify important features.

SCHEMATIC:

ASSUMPTIONS: (1) Thickness $w << r_o$, such that conduction is one-dimensional in r-direction, (2) In prescribed region, $\dot{q}$ is uniform, (3) Bottom surface of disc is insulated, (4) Constant properties.

ANALYSIS: (a) Consider the nodal point arrangement for the region subjected to induction heating. The size of the control volume is $V = 2\pi\ r_m \cdot \Delta r \cdot w$. The energy conservation requirement for the node m has the form

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = \dot{E}_{st}$$

with $\quad q_a + q_b + q_{conv} + \dot{q}V = \dot{E}_{st}$.

Recognizing that q_a and q_b are conduction terms and q_{conv} is the convection process,

$$k\left[2\pi\left[r_m - \frac{\Delta r}{2}\right]w\right]\frac{T_{m-1}^p - T_m^p}{\Delta r} + k\left[2\pi\left[r_m + \frac{\Delta r}{2}\right]w\right]\frac{T_{m+1}^p - T_m^p}{\Delta r}$$

$$+h\left[2\pi\ r_m \cdot \Delta r\right]\left(T_\infty - T_m^p\right) + \dot{q}\left[2\pi\ r_m \cdot \Delta r \cdot w\right] = \rho\ c_p\left[2\pi\ r_m \cdot \Delta r \cdot w\right]\frac{T_m^{p+1} - T_m^p}{\Delta t}.$$

Upon regrouping, the finite-difference equation has the form,

$$T_m^{p+1} = Fo\left[\left[1 - \frac{\Delta r}{2r_m}\right]T_{m-1}^p + \left[1 + \frac{\Delta r}{2r_m}\right]T_{m+1}^p + Bi\left[\frac{\Delta r}{w}\right]T_\infty + \frac{\dot{q}\Delta r^2}{k}\right] + \left[1 - 2Fo - Bi \cdot Fo\left[\frac{\Delta r}{w}\right]\right]T_m^p \qquad <$$

where $\qquad Fo = \alpha\Delta t/\Delta r^2 \qquad Bi = h\Delta r/k.$

(b) The steady-state temperature distribution has these features:

1. Zero gradient at r = 0, r_0
2. No discontinuity at r_1, r_2
3. T_{max} occurs in region $r_1 < r < r_2$

Note also, distribution will not be linear anywhere; distribution is <u>not</u> parabolic in $r_1 < r < r_2$ region.

PROBLEM 5.122

KNOWN: An electrical cable experiencing uniform volumetric generation; the lower half is well insulated while the upper half experiences convection.

FIND: (a) Explicit, finite-difference equations for an interior node (m,n), the center node (0,0), and an outer surface node (M,n) for the convective and insulated boundaries, and (b) Stability criterion for each FDE; identify the most restrictive criterion.

SCHEMATIC:

ASSUMPTIONS: (1) Two-dimensional (r,ϕ), transient conduction, (2) Constant properties, (3) Uniform $\dot{q}$.

ANALYSIS: The explicit, finite-difference equations may be obtained by applying energy balances to appropriate control volumes about the node of interest. Note the coordinate system defined above where (r,ϕ) $\rightarrow$ (mΔr, n$\Delta\phi$). The stability criterion is determined from the coefficient associated with the node of interest.

Interior Node (m,n). The control volume for an interior node is

$$V = r_m \Delta\phi \cdot \Delta r \cdot \ell$$

(with $r_m = m\Delta r$, $\ell = 1$) where ℓ is the length normal to the page. The conservation of energy requirement is

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = \dot{E}_{st}$$

$$\left(q_1 + q_2\right)_r + \left(q_3 + q_4\right)_\theta + \dot{q}V = \rho\, cV \frac{T_{m,n}^{p+1} - T_{m,n}^p}{\Delta t}$$

$$k \cdot \left[m - \frac{1}{2}\right]\Delta r \cdot \Delta\phi \cdot \frac{T_{m-1,n}^p - T_{m,n}^p}{\Delta r} + k \cdot \left[m + \frac{1}{2}\right]\Delta r \cdot \Delta\phi \cdot \frac{T_{m+1,n}^p - T_{m,n}^p}{\Delta r} + k \cdot \Delta r \cdot \frac{T_{m,n+1}^p - T_{m,n}^p}{(m\Delta r)\Delta\phi}$$

$$+ k \cdot \Delta r \cdot \frac{T_{m,n-1}^p - T_{m,n}^p}{(m\Delta r)\Delta\phi} + \dot{q}\left(m\Delta r \cdot \Delta\phi\right) \cdot \Delta r = \rho\, c\left(m\Delta r \cdot \Delta\phi\right) \cdot \Delta r \cdot \frac{T_{m,n}^{p+1} - T_{m,n}^p}{\Delta t} \qquad (1)$$

Define the Fourier number as

$$Fo = \frac{k}{\rho\, c} \cdot \frac{\Delta t}{\Delta r^2} = \frac{\alpha \Delta t}{\Delta r^2} \qquad (2)$$

and then regroup the terms of Eq. (1) to obtain the FDE,

$$T_{m,n}^{p+1} = Fo\left\{\frac{m - 1/2}{m} T_{m-1,n}^p + \frac{m + 1/2}{m} T_{m+1,n}^p + \frac{1}{(m\Delta\phi)^2}\left(T_{m,n+1}^p + T_{m,n-1}^p\right) + \frac{\dot{q}}{k}\Delta r^2\right\}$$

$$+ \left\{-Fo\left[2 + \frac{2}{(m\Delta\phi)^2}\right] + 1\right\} T_{m,n}^p. \qquad (3) <$$

Continued …..

PROBLEM 5.122 (Cont.)

The stability criterion requires that the last term on the right-hand side in braces be positive. That is, the coefficient of $T^{p}_{m,n}$ must be positive and the stability criterion is

$$\text{Fo} \le 1/2\left[1 + 1/\left(m\Delta\phi\right)^2\right] \qquad (4)$$

Note that, for m >> 1/2 and $(m\Delta\phi)^2$ >>1, the FDE takes the form of a 1-D cartesian system.

Center Node (0,0). For the control volume,

$V = \pi\left(\Delta r/2\right)^2 \cdot 1$. The energy balance is

$\dot{E}'_{in} - \dot{E}'_{out} + \dot{E}'_{g} = \dot{E}'_{st}$ where $\dot{E}'_{in} = \Sigma q'_n$.

$$\sum_{n=0}^{N} k\cdot\left[\frac{\Delta r}{2}\Delta\phi\right]\cdot\frac{T^{p}_{1,n} - T^{p}_{o}}{\Delta r} + \dot{q}\pi\left[\frac{\Delta r}{2}\right]^2$$

$$= \rho\, c\cdot\pi\left[\frac{\Delta r}{2}\right]^2 \frac{T^{p+1}_{o} - T^{p}_{o}}{\Delta t} \qquad (5)$$

where N = $(2\pi/\Delta\phi) - 1$, the total number of q_n. Using the definition of Fo, find

$$T^{p+1}_{o} = 4\text{Fo}\left\{\frac{1}{N+1}\sum_{n=0}^{N} T^{p}_{1,n} + \frac{\dot{q}}{4k}\Delta r^2\right\} + \left(1 - 4\text{Fo}\right)T^{p}_{o}. \quad <$$

By inspection, the stability criterion is Fo ≤ 1/4. (7)

Surface Nodes (M,n). The control volume for the surface node is V = (M - ¼)ΔrΔϕ·Δr/2.1. From the energy balance,

$$\dot{E}'_{in} - \dot{E}'_{out} + \dot{E}'_{g} = \left(q'_1 + q'_2\right)_r + \left(q'_3 + q'_4\right)_\phi + \dot{q}V = \dot{E}'_{st}$$

$$k\cdot\left(M - 1/2\right)\Delta r\cdot\Delta\phi\frac{T^{p}_{M-1,n} - T^{p}_{M,n}}{\Delta r} + h\left(M\Delta r\cdot\Delta\phi\right)\left(T_\infty - T^{p}_{M,n}\right) + k\cdot\frac{\Delta r}{2}\cdot\frac{T^{p}_{M,n+1} - T^{p}_{M,n}}{\left(M\Delta r\right)\Delta\phi}$$

$$+k\cdot\frac{\Delta r}{2}\cdot\frac{T^{p}_{M,n-1} - T^{p}_{M,n}}{\left(M\Delta r\right)\Delta\phi} + \dot{q}\left[\left(M - 1/4\right)\Delta r\cdot\Delta\phi\cdot\frac{\Delta r}{2}\right] = \rho\, c\left[\left(M - 1/4\right)\Delta r\cdot\Delta\phi\cdot\frac{\Delta r}{2}\right]\frac{T^{p+1}_{M,n} - T^{p}_{M,n}}{\Delta t}.$$

Regrouping and using the definitions for Fo = $\alpha\Delta t/\Delta r^2$ and Bi = $h\Delta r/k$,

$$T^{p+1}_{m,n} = \text{Fo}\left\{2\frac{M - 1/2}{M - 1/4}T^{p}_{M-1,n} + \frac{1}{(M\text{-}1/4)M\left(\Delta\phi\right)^2}\left(T^{p}_{M,n+1} - T^{p}_{M,n-1}\right) + 2\text{Bi}\cdot T_\infty + \frac{\dot{q}}{k}\Delta r^2\right\}$$

$$+\left\{1 - 2\text{Fo}\left[\frac{M\text{-}1/2}{M\text{-}1/4} + \text{Bi}\cdot\frac{M}{M - 1/4} + \frac{1}{\left(M - 1/4\right)M\left(\Delta\phi\right)^2}\right]\right\}T^{p}_{M,n}. \qquad (8) \quad <$$

The stability criterion is

$$\text{Fo} \le \frac{1}{2}\left[\frac{M - 1/2}{M - 1/4} + \text{Bi}\frac{M}{M - 1/4} + \frac{1}{\left(M - 1/4\right)M\left(\Delta\phi\right)^2}\right]. \qquad (9)$$

To determine which stability criterion is most restrictive, compare Eqs. (4), (7) and (9). The most restrictive (lowest Fo) has the largest denominator. For small values of m, it is not evident whether Eq. (7) is more restrictive than Eq. (4); Eq. (4) depends upon magnitude of Δϕ. Likewise, it is not clear whether Eq. (9) will be more or less restrictive than Eq. (7). Numerical values must be substituted.

PROBLEM 5.123

KNOWN: Initial temperature distribution in two bars that are to be soldered together; interface contact resistance.

FIND: (a) Explicit FDE for $T_{4,2}$ in terms of Fo and $Bi = \Delta x/k\, R''_{t,c}$; stability criterion, (b) $T_{4,2}$ one time step after contact is made if Fo = 0.01 and value of Δt; whether the stability criterion is satisfied.

SCHEMATIC:

Interface with solder and flux
Copper, pure
Steel AISI 1010
Thermal resistance layer, $R''_{t,c} = 2\times10^{-5}\ m^2\cdot K/W$
1,3 2,3 3,3 4,3 5,3 6,3
1,2 2,2 3,2 4,2 5,2 6,2
1,1 2,1 3,1 4,1 5,1 6,1
y,m x,m Δx Δy
$\Delta x = \Delta y = 20$ mm

PROPERTIES: *Table A-1*, Steel, AISI 1010 (1000K): k = 31.3 W/m·K, c = 1168 J/kg·K, ρ = 7832 kh/m^3.

ASSUMPTIONS: (1) Two-dimensional transient conduction, (2) Constant properties, (3) Interfacial solder layer has negligible thickness.

ANALYSIS: (a) From an energy balance on the control volume V = (Δx/2)·Δy·1.

$T_{4,3}$ q_b $\Delta x/2$ Δy $T_{3,2}$ $T_{4,2}$ $T_{5,2}$ q_a q_c q_d Copper Steel $R''_{t,c}$ $T_{4,1}$

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = \dot{E}_{st}$$

$$q_a + q_b + q_c + q_d = \rho\, cV \frac{T_{4,2}^{p+1} - T_{4,2}^{p}}{\Delta t}.$$

Note that $q_a = \left(\Delta T / R''_{t,c}\right) A_c$ while the remaining q_i are conduction terms,

$$\frac{1}{R''_{t,c}}\left(T_{3,2}^p - T_{4,2}^p\right)\Delta y + k\left(\Delta x/2\right)\frac{\left(T_{4,3}^p - T_{4,2}^p\right)}{\Delta y} + k\left(\Delta y\right)\frac{\left(T_{5,2}^p - T_{4,2}^p\right)}{\Delta x}$$

$$+ k\left(\Delta x/2\right)\frac{\left(T_{4,1}^p - T_{4,2}^p\right)}{\Delta y} = \rho\, c\left[\left(\Delta x/2\right)\cdot\Delta y\right]\frac{T_{4,2}^{p+1} - T_{4,2}^{p}}{\Delta t}.$$

Defining $Fo \equiv \left(k/\rho\, c\right)\Delta t/\Delta x^2$ and $Bi_c \equiv \Delta y / R''_{t,c} k$, regroup to obtain

$$T_{4,2}^{p+1} = Fo\left(T_{4,3}^p + 2T_{5,2}^p + T_{4,1}^p + 2Bi\, T_{3,2}^p\right) + \left(1 - 4Fo - 2FoBi\right)T_{4,2}^p. \qquad <$$

The stability criterion requires the coefficient of the $T_{4,2}^p$ term be zero or positive,

$$\left(1 - 4Fo - 2FoBi\right) \ge 0 \qquad \text{or} \qquad Fo \le 1/\left(4 + 2Bi\right) \qquad <$$

(b) For $Fo = 0.01$ and $Bi = 0.020m/\left(2\times10^{-5} m^2\cdot K/W \times 31.3 W/m\cdot K\right) = 31.95$,

$$T_{4,2}^{p+1} = 0.01\left(1000 + 2\times900 + 1000 + 2\times31.95\times700\right)K + \left(1 - 4\times0.01 - 2\times0.01\times31.95\right)1000K$$

$$T_{4,2}^{p+1} = 485.30K + 321.00K = 806.3K. \qquad <$$

With Fo = 0.01, the time step is

$$\Delta t = Fo\, \Delta x^2\left(\rho\, c/k\right) = 0.01\left(0.020m\right)^2\left(7832 kg/m^3 \times 1168 J/kg\cdot K / 31.3 W/m\cdot K\right) = 1.17s. \qquad <$$

With Bi = 31.95 and Fo = 0.01, the stability criterion, Fo ≤ 0.015, is satisfied. <

PROBLEM 5.124

KNOWN: Stainless steel cylinder of Ex. 5.7, 80-mm diameter by 60-mm length, initially at 600 K, suddenly quenched in an oil bath at 300 K with h = 500 $W/m^2 \cdot K$. Use the ready-to-solve model in the *Examples* menu of *FEHT* to obtain the following solutions.

FIND: (a) Calculate the temperatures T(r, x ,t) after 3 min: at the cylinder center, T(0, 0, 3 mm), at the center of a circular face, T(0, L, 3 min), and at the midheight of the side, $T(r_o, 0, 3$ min); compare your results with those in the example; (b) Calculate and plot temperature histories at the cylinder center, T(0, 0, t), the mid-height of the side, $T(r_o, 0, t)$, for $0 \le t \le 10$ min; use the *View/Temperature vs. Time* command; comment on the gradients and what effect they might have on phase transformations and thermal stresses; (c) Using the results for the total integration time of 10 min, use the *View/Temperature Contours* command; describe the major features of the cooling process shown in this display; create and display a 10-isotherm temperature distribution for t = 3 min; and (d) For the locations of part (a), calculate the temperatures after 3 min if the convection coefficient is doubled (h = 1000 $W/m^2 \cdot K$); for these two conditions, determine how long the cylinder needs to remain in the oil bath to achieve a safe-to touch surface temperature of 316 K. Tabulate and comment on the results of your analysis.

SCHEMATIC:

ASSUMPTIONS: (1) Two-dimensional conduction in r- and x-coordinates, (2) Constant properties.

PROPERTIES: Stainless steel (*Example 5.7*): ρ = 7900 kg/m^3, c = 256 J/kg·K, k = 17.4 W/m·K.

ANALYSIS: (a) The *FEHT ready-to-solve* model for Example 5.7 is accessed through the *Examples* menu and the annotated *Input* page is shown below. The following steps were used to obtain the solution: (1) Use the *Draw* | *Reduce Mesh* command three times to create the 512-element mesh; (2) In *Run*, click on *Check*, (3) In *Run*, press *Calculate* and hit *OK* to initiate the solver; and (4) Go to the *View* menu, select *Tabular Output* and read the nodal temperatures 4, 1, and 3 at t = t_o = 180 s. The tabulated results below include those from the n-term series solution used in the *IHT* software.

Continued …..

PROBLEM 5.124 (Cont.)

(r, x, t_o)	FEHT node	$T(r, x, t_o)$ (K) *FEHT*	$T(r, x, t_o)$ (K) 1-term series	$T(r, x, t_o)$ (K) n-term series
$0, 0, t_o$	4	402.7	405	402.7
$0, L, t_o$	1	368.7	372	370.5
$ro, 0, t_o$	3	362.5	365	362.4

Note that the one-term series solution results of Example 5.7 are systematically lower than those from the 512-element, finite-difference *FEHT* analyses. The *FEHT* results are in excellent agreement with the *IHT* n-term series solutions for the x = 0 plane nodes (4,3), except for the x = L plane node (1).

(b) Using the *View Temperature vs. Time* command, the temperature histories for nodes 4, 1, and 3 are plotted in the graph shown below. There is very small temperature difference between the locations on the surface, (node 1; 0, L) and (node 3; r_o, 0). But, the temperature difference between these surface locations and the cylinder center (node 4; 0, 0) is large at early times. Such differences wherein locations cool at considerably different rates could cause variations in microstructure and hence, mechanical properties, as well as induce thermal stresses.

Continued …..

PROBLEM 5.124 (Cont.)

(c) Use the *View|Temperature Contours* command with the shaded band option for the isotherm contours. Selecting the *From Start to Stop* time option, see the display of the contours as the cylinder cools during the quench process. The "movie" shows that cooling initiates at the corner (r_o,L,t) and the isotherms quickly become circular and travel toward the center (0,0,t). The 10-isotherm distribution for t = 3 min is shown below.

10 isothermal contours: minimum 348.8 K, maximum 402.7 K

(d) Using the *FEHT* model with convection coefficients of 500 and 1000 $W/m^2 \cdot K$, the temperatures at $t = t_o = 180$ s for the three locations of part (a) are tabulated below.

	$h = 500\ W/m^2 \cdot K$	$h = 1000\ W/m^2 \cdot K$
$T(0, 0, t_o)$, K	402.7	352.8
$T(0, L, t_o)$, K	368.7	325.8
$T(r_o, 0, t_o)$, K	362.5	322.1

Note that the effect of doubling the convection coefficient is to reduce the temperature at these locations by about 40°C. The time the cylinder needs to remain in the oil bath to achieve the *safe-to-touch* surface temperature of 316 K can be determined by examining the temperature history of the location (node1; 0, L). For the two convection conditions, the results are tabulated below. Doubling the coefficient reduces the cooling process time by 40 %.

$T(0, L, t_o)$	$h\ (W/m^2 \cdot K)$	t_o (s)
316	500	370
316	1000	219

PROBLEM 5.125

KNOWN: Flue of square cross-section, initially at a uniform temperature is suddenly exposed to hot flue gases. See Problem 4.57.

FIND: Temperature distribution in the wall 5, 10, 50 and 100 hours after introduction of gases using the *implicit* finite-difference method.

SCHEMATIC:

ASSUMPTIONS: (1) Two-dimensional transient conduction, (2) Constant properties.

PROPERTIES: Flue (given): $k = 0.85$ W/m·K, $\alpha = 5.5 \times 10^{-7}$ m^2/s.

ANALYSIS: The network representing the flue cross-sectional area is shown with $\Delta x = \Delta y = 50$mm. Initially all nodes are at $T_i = 25°C$ when suddenly the interior and exterior surfaces are exposed to convection processes, ($T_{\infty,i}$, h_i) and ($T_{\infty,o}$, h_o), respectively Referring to the network above, note that there are four types of nodes: interior (02, 03, 06, 07, 10, 11, 14, 15, 17, 18, 20); plane surfaces with convection (interior – 01, 05, 09); interior corner with convection (13), plane surfaces with convection (exterior – 04, 08, 12, 16, 19, 21); and, exterior corner with convection. The system of finite-difference equations representing the network is obtained using *IHT|Tools|Finite-difference equations|Two-dimensional|Transient*. The *IHT* code is shown in Comment 2 and the results for t = 5, 10, 50 and 100 hour are tabulated below.

Node 17 $$(1+4\text{Fo})T_{17}^{p+1} - \text{Fo}\left(T_{18}^{p+1} + T_{14}^{p+1} + T_{18}^{p+1} + T_{14}^{p+1}\right) = T_{17}^{p}$$

Node 13 $$\left[1+4\text{Fo}\left[1+\frac{1}{3}\text{Bi}_i\right]\right]T_{13}^{p+1} - \frac{2}{3}\text{Fo}\left(2T_{14}^{p+1} + T_9 + 2T_{14}^{p+1} + T_9^{p+1}\right) = T_{13}^{p} + \frac{4}{3}\text{Bi}_i \cdot \text{Fo} \cdot T_{\infty,i}$$

Node 12 $$\left(1+2\text{Fo}\left(2+\text{Bi}_o\right)\right)T_{12}^{p+1} - \text{Fo}\left(2T_{11}^{p+1} + T_{16}^{p+1} + T_{8}^{p+1}\right) = T_{12}^{p} + 2\text{Bi}_o \cdot \text{Fo} \cdot T_{\infty,o}$$

Node 22 $$\left(1+4\text{Fo}\left(1+\text{Bi}_o\right)\right)T_{22}^{p+1} - 2\text{Fo}\left(T_{21}^{p+1} + T_{21}^{p+1}\right) = T_{22}^{p} + 4\text{Bi}_o \cdot \text{Fo} \cdot T_{\infty,o}$$

Numerical values for the relevant parameters are:

$$\text{Fo} = \frac{\alpha \Delta t}{\Delta x^2} = \frac{5.5\times10^{-6}\text{m}^2/\text{s}\times3600\text{s}}{(0.050\text{m})^2} = 7.92000$$

$$\text{Bi}_o = \frac{h_o \Delta x}{k} = \frac{5\ \text{W/m}^2\cdot\text{K}\times0.050\text{m}}{0.85\ \text{W/m}\cdot\text{K}} = 0.29412$$

$$\text{Bi}_i = \frac{h_i \Delta x}{k} = \frac{100\ \text{W/m}^2\cdot\text{K}\times0.050\text{m}}{0.85\ \text{W/m}\cdot\text{K}} = 5.88235$$

The system of FDEs can be represented in matrix notation, [A][T] = [C]. The coefficient matrix [A] and terms for the right-hand side matrix [C] are given on the following page.

Continued …..

	The coefficient matrix [A]																						*RHS matrix [C]*
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	
1	E	2	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	$-0.12626T_1^p - 7331.1765$
2	1	F	1	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	$-0.12626T_2^p$
3	0	1	F	1	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	$-0.12626T_3^p$
4	0	0	2	G	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	$-0.12626T_4^p - 175.38235$
5	1	0	0	0	E	2	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	$-0.12626T_5^p - 7331.1765$
6	0	1	0	0	1	F	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	$-0.12626T_6^p$
7	0	0	1	0	0	1	F	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	$-0.12626T_7^p$
8	0	0	0	1	0	0	2	G	0	0	0	1	0	0	0	0	0	0	0	0	0	0	$-0.12626T_8^p - 175.37235$
9	0	0	0	0	1	0	0	0	E	2	0	0	1	0	0	0	0	0	0	0	0	0	$-0.12626T_9^p - 7331.1765$
10	0	0	0	0	0	1	0	0	1	F	1	0	0	1	0	0	0	0	0	0	0	0	$-0.12626T_{10}^p$
11	0	0	0	0	0	0	1	0	0	1	F	1	0	0	1	0	0	0	0	0	0	0	$-0.12626T_{11}^p$
12	0	0	0	0	0	0	0	1	0	0	2	G	0	0	0	1	0	0	0	0	0	0	$-0.12626T_{12}^p - 175.38235$
13	0	0	0	0	0	0	0	0	4	0	0	0	H	8	0	0	0	0	0	0	0	0	$-0.37879T_{13}^p - 14{,}658.824$
14	0	0	0	0	0	0	0	0	0	1	0	0	1	F	1	0	1	0	0	0	0	0	$-0.12626T_{14}^p$
15	0	0	0	0	0	0	0	0	0	0	1	0	0	1	F	1	0	1	0	0	0	0	$-0.12626T_{15}^p$
16	0	0	0	0	0	0	0	0	0	0	0	1	0	0	2	G	0	0	1	0	0	0	$-0.12626T_{16}^p - 175.38235$
17	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	F	2	0	0	0	0	$-0.12626T_{17}^p$
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	F	1	1	0	0	$-0.12626T_{18}^p$
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	2	G	0	1	0	$-0.12626T_{19}^p - 175.38235$
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	F	2	0	$-0.12626T_{20}^p$
21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	2	G	1	$-0.12626T_{21}^p - 175.38235$
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	K	$-0.12626T_{22}^p - 350.76471$

E = −15.89096   F = −4.12626   G = −4.71450   H = −35.90819   K = −5.30274

For this problem a stock computer program was used to obtain the solution matrix [T]. The initial temperature distribution was $T_m^0 = 298\text{K}$. The results are tabulated below.

Node/time (h)	T(m,n) (C) 0	5	10	50	100
T01	25	335.00	338.90	340.20	340.20
T02	25	248.00	274.30	282.90	282.90
T03	25	179.50	217.40	229.80	229.80
T04	25	135.80	170.30	181.60	181.60
T05	25	334.50	338.50	339.90	339.90
T06	25	245.30	271.90	280.80	280.80
T07	25	176.50	214.60	227.30	227.30
T08	25	133.40	168.00	179.50	179.50
T09	25	332.20	336.60	338.20	338.20
T10	25	235.40	263.40	273.20	273.20
T11	25	166.40	205.40	219.00	219.00
T12	25	125.40	160.40	172.70	172.70
T13	25	316.40	324.30	327.30	327.30
T14	25	211.00	243.00	254.90	254.90
T15	25	146.90	187.60	202.90	202.90
T16	25	110.90	146.70	160.20	160.20
T17	25	159.80	200.50	216.20	216.20
T18	25	117.40	160.50	177.50	177.50
T19	25	90.97	127.40	141.80	141.80
T20	25	90.62	132.20	149.00	149.00
T21	25	72.43	106.70	120.60	120.60
T22	25	59.47	87.37	98.89	98.89

COMMENTS: (1) Note that the steady-state condition is reached by t = 5 hours; this can be seen by comparing the distributions for t = 50 and 100 hours. Within 10 hours, the flue is within a few degrees of the steady-state condition.

Continued …..

PROBLEM 5.125 (Cont.)

(2) The *IHT* code for performing the numerical solution is shown in its entirety below. Use has been made of symmetry in writing the FDEs. The tabulated results above were obtained by copying from the *IHT Browser* and pasting the desired columns into EXCEL.

```
// From Tools|Finite-difference equations|Two-dimensional|Transient
// Interior surface nodes, 01, 05, 09, 13
/* Node 01: plane surface node, s-orientation; e, w, n labeled 05, 05, 02 . */
rho * cp * der(T01,t) = fd_2d_psur_s(T01,T05,T05,T02,k,qdot,deltax,deltay,Tinfi,hi,q''a)
q''a = 0          // Applied heat flux, W/m^2; zero flux shown
qdot = 0
rho * cp * der(T05,t) = fd_2d_psur_s(T05,T09,T01,T06,k,qdot,deltax,deltay,Tinfi,hi,q''a)
rho * cp * der(T09,t) = fd_2d_psur_s(T09,T13,T05,T10,k,qdot,deltax,deltay,Tinfi,hi,q''a)
/* Node 13: internal corner node, w-s orientation; e, w, n, s labeled 14, 09, 14, 09. */
rho * cp * der(T13,t) = fd_2d_ic_ws(T13,T14,T09,T14,T09,k,qdot,deltax,deltay,Tinfi,hi,q''a)

// Interior nodes, 02, 03, 06, 07, 10, 11, 14, 15, 18, 20
/* Node 02: interior node; e, w, n, s labeled 06, 06, 03, 01. */
rho * cp * der(T02,t) = fd_2d_int(T02,T06,T06,T03,T01,k,qdot,deltax,deltay)
rho * cp * der(T03,t) = fd_2d_int(T03,T07,T07,T04,T02,k,qdot,deltax,deltay)
rho * cp * der(T06,t) = fd_2d_int(T06,T10,T02,T07,T05,k,qdot,deltax,deltay)
rho * cp * der(T07,t) = fd_2d_int(T07,T11,T03,T08,T06,k,qdot,deltax,deltay)
rho * cp * der(T10,t) = fd_2d_int(T10,T14,T06,T11,T09,k,qdot,deltax,deltay)
rho * cp * der(T11,t) = fd_2d_int(T11,T15,T07,T12,T10,k,qdot,deltax,deltay)
rho * cp * der(T14,t) = fd_2d_int(T14,T17,T10,T15,T13,k,qdot,deltax,deltay)
rho * cp * der(T15,t) = fd_2d_int(T15,T18,T11,T16,T14,k,qdot,deltax,deltay)
rho * cp * der(T17,t) = fd_2d_int(T17,T18,T14,T18,T14,k,qdot,deltax,deltay)
rho * cp * der(T18,t) = fd_2d_int(T18,T20,T15,T19,T17,k,qdot,deltax,deltay)
rho * cp * der(T20,t) = fd_2d_int(T20,T21,T18,T21,T18,k,qdot,deltax,deltay)

// Exterior surface nodes, 04, 08, 12, 16, 19, 21, 22
/* Node 04: plane surface node, n-orientation; e, w, s labeled 08, 08, 03. */
rho * cp * der(T04,t) = fd_2d_psur_n(T04,T08,T08,T03,k,qdot,deltax,deltay,Tinfo,ho,q''a)
rho * cp * der(T08,t) = fd_2d_psur_n(T08,T12,T04,T07,k,qdot,deltax,deltay,Tinfo,ho,q''a)
rho * cp * der(T12,t) = fd_2d_psur_n(T12,T16,T08,T11,k,qdot,deltax,deltay,Tinfo,ho,q''a)
rho * cp * der(T16,t) = fd_2d_psur_n(T16,T19,T12,T15,k,qdot,deltax,deltay,Tinfo,ho,q''a)
rho * cp * der(T19,t) = fd_2d_psur_n(T19,T21,T16,T18,k,qdot,deltax,deltay,Tinfo,ho,q''a)
rho * cp * der(T21,t) = fd_2d_psur_n(T21,T22,T19,T20,k,qdot,deltax,deltay,Tinfo,ho,q''a)
/* Node 22: external corner node, e-n orientation; w, s labeled 21, 21. */
rho * cp * der(T22,t) = fd_2d_ec_en(T22,T21,T21,k,qdot,deltax,deltay,Tinfo,ho,q''a)

// Input variables
deltax = 0.050
deltay = 0.050
Tinfi = 350
hi = 100
Tinfo = 25
ho = 5
k = 0.85
alpha = 5.55e-7
alpha = k / (rho * cp)
rho = 1000                    // arbitrary value
```

(3) The results for t = 50 hour, representing the steady-state condition, are shown below, arranged according to the coordinate system.

	Tmn (C)						
x/y (mm)	0	50	100	150	200	250	300
0	181.60	179.50	172.70	160.20	141.80	120.60	98.89
50	229.80	227.30	219.00	202.90	177.50	149.00	
100	282.90	280.80	273.20	172.70	216.20		
150	340.20	339.90	338.20	327.30			

In Problem 4.57, the temperature distribution was determined using the FDEs written for steady-state conditions, but with a finer network, $\Delta x = \Delta y = 25$ mm. By comparison, the results for the coarser network are slightly higher, within a fraction of 1°C, along the mid-section of the flue, but notably higher in the vicinity of inner corner. (For example, node 13 is 2.6°C higher with the coarser mesh.)

PROBLEM 5.126

KNOWN: Electrical heating elements embedded in a ceramic plate as described in Problem 4.75; initially plate is at a uniform temperature and suddenly heaters are energized.

FIND: Time required for the difference between the surface and initial temperatures to reach 95% of the difference for steady-state conditions using the implicit, finite-difference method.

SCHEMATIC:

ASSUMPTIONS: (1) Two-dimensional conduction, (2) Constant properties, (3) No internal generation except for Node 7, (4) Heating element approximates a line source; wire diameter is negligible.

ANALYSIS: The grid for the symmetry element above consists of 12 nodes. Nodes 1-3 are points on a surface experiencing convection; nodes 4-12 are interior nodes; node 7 is a special case with internal generation and because of symmetry, $q'_{ht} = 25$ W/m. Their finite-difference equations are derived as follows

Surface Node 2. From an energy balance on the prescribed control volume with $\Delta x/\Delta y = 3$,

$$\dot{E}_{in} = \dot{E}_{st} = q'_a + q'_b + q'_c + q'_d = \rho\, cV \frac{T_2^{p+1} - T_2^p}{\Delta t}$$

$$k\frac{\Delta y}{2}\frac{T_1^{p+1} - T_2^{p+1}}{\Delta x} + h\Delta x\left(T_\infty - T_2^{p+1}\right)$$

$$+k\frac{\Delta y}{2}\frac{T_3^{p+1} - T_2^{p+1}}{\Delta x} + k\Delta x\frac{T_5^{p+1} - T_2^{p+1}}{\Delta y} = \rho\, c\left[\Delta x\frac{\Delta y}{2}\right]\frac{T_2^{p+1} - T_2^p}{\Delta t}.$$

Continued …..

Divide by k, use the following definitions, and regroup to obtain the finite-difference equations.

$$N \equiv h\Delta x/k = 100\ \text{W/m}^2\cdot\text{K}\times 0.006\text{m}/2\ \text{W/m}\cdot\text{K} = 0.3000 \quad (1)$$

$$Fo \equiv (k/\rho\ c)\Delta t/\Delta x\cdot\Delta y = \alpha\Delta t/\Delta x\cdot\Delta y =$$

$$1.5\times 10^{-6}\text{m}^2/\text{s}\times 1\text{s}/(0.006\times 0.002)\text{m}^2 = 0.1250 \quad (2)$$

$$\frac{1}{2}\left[\frac{\Delta y}{\Delta x}\right]\left(T_1^{p+1}-T_2^{p+1}\right)+N\left(T_\infty-T_2^{p+1}\right)+\frac{1}{2}\left[\frac{\Delta y}{\Delta x}\right]\left(T_3^{p+1}-T_2^{p+1}\right)$$

$$+\left[\frac{\Delta x}{\Delta y}\right]\left(T_5^{p+1}-T_2^{p+1}\right)=\frac{1}{2Fo}\left(T_2^{p+1}-T_2^{p}\right)$$

$$\frac{1}{2}\left[\frac{\Delta y}{\Delta x}\right]T_1^{p+1}-\left[\left[\frac{\Delta x}{\Delta y}\right]+N+\left[\frac{\Delta y}{\Delta x}\right]+\frac{1}{2Fo}\right]T_2^{p+1}+\frac{1}{2}\left[\frac{\Delta x}{\Delta y}\right]T_3^{p+1}$$

$$+\left[\frac{\Delta x}{\Delta y}\right]T_5^{p+1} = -NT_\infty-\frac{1}{2Fo}T_2^{p}. \quad (3)$$

Substituting numerical values for Fo and N, and using T_∞ = 30°C and $\Delta x/\Delta y$ = 3, find

$$0.16667T_1^{p+1}-7.63333T_2^{p+1}+0.16667T_3^{p+1}+3.00000T_5^{p+1}=9.0000-4.0000T_2^{p}. \quad (4)$$

By inspection and use of Eq. (3), the FDEs for Nodes 1 and 3 can be inferred.

Interior Node 7. From an energy balance on the prescribed control volume with $\Delta x/\Delta y$ = 3,

$$\dot{E}'_{in}+\dot{E}'_g=\dot{E}'_{st}$$

where $\dot{E}'_g = 2q'_{ht}$ and $\dot{E}'_{in}$ represents the conduction terms $-q'_a+q'_b+q'_c+q'_d$,

$$k\Delta y\frac{T_8^{p+1}-T_7^{p+1}}{\Delta x}+k\Delta x\frac{T_4^{p+1}-T_7^{p+1}}{\Delta y}+k\Delta y\frac{T_8^{p+1}-T_7^{p+1}}{\Delta x}$$

$$+k\Delta x\frac{T_{10}^{p+1}-T_7^{p+1}}{\Delta y}+2q'_{ht}=\rho\ c(\Delta x\cdot\Delta y)\frac{T_7^{p+1}-T_7^{p}}{\Delta t}.$$

Using the definition of Fo, Eq. (2), and regrouping, find

$$\frac{1}{2}\left[\frac{\Delta x}{\Delta y}\right]T_4^{p+1}-\left[\left[\frac{\Delta x}{\Delta y}\right]+\left[\frac{\Delta y}{\Delta x}\right]+\frac{1}{2Fo}\right]T_7^{p+1}$$

$$+\left[\frac{\Delta y}{\Delta x}\right]T_8^{p+1}+\frac{1}{2}\left[\frac{\Delta x}{\Delta y}\right]T_{10}^{p+1}=-\frac{q'_{ht}}{k}-\frac{1}{2Fo}T_7^{p} \quad (5)$$

$$1.50000T_4^{p+1}-7.33333T_7^{p+1}+0.33333T_8^{p+1}+1.50000T_{10}^{p+1}=-12.5000-4.0000T_7^{p}. \quad (6)$$

Continued …..

PROBLEM 5.126 (Cont.)

Recognizing the form of Eq. (5), it is a simple matter to infer the FDE for the remaining interior points for which $\dot{q}_{ht} = 0$. In matrix notation [A][T] = [C], the coefficient matrix [A] and RHS matrix [C] are:

THE COEFFICIENT MATRIX, [A] **[C]**

												[C]
-7.633330	0.333330	0	3.000000	0	0	0	0	0	0	0	0	$-4.0T_1^p - 9.0$
0.166670	-7.633330	0.166670	0	3.000000	0	0	0	0	0	0	0	$-4.0T_2^p - 9.0$
0	0.333330	-7.633330	0	0	3.000000	0	0	0	0	0	0	$-4.0T_3^p - 9.0$
1.500000	0	0	-7.333330	0.333330	0	1.500000	0	0	0	0	0	$-4.0T_4^p$
0	3.000000	0	0.333330	-14.666670	0.333330	0	3.000000	0	0	0	0	$-8.0T_5^p$
0	0	1.500000	0	0.333330	-7.333330	0	0	1.500000	0	0	0	$-4.0T_6^p$
0	0	0	1.500000	0	0	-7.333330	0.333330	0	1.500000	0	0	$-4.0T_7^p - 12.5$
0	0	0	0	3.000000	0	0.333330	-14.666670	0.333330	0	3.000000	0	$-8.0T_8^p$
0	0	0	0	0	1.500000	0	0.333330	-7.333330	0	0	1.500000	$-4.0T_9^p$
0	0	0	0	0	0	3.000000	0	0	-7.333330	0.333330	0	$-4.0T_{10}^p$
0	0	0	0	0	0	0	3.000000	0	0.166670	-7.333330	0.166670	$-4.0T_{11}^p$
0	0	0	0	0	0	0	0	3.000000	0	0.333330	-7.333330	$-4.0T_{12}^p$

Recall that the problem asks for the time required to reach 95% of the difference for steady-state conditions. This provides information on approximately how long it takes for the plate to come to a steady operating condition. If you worked Problem 4.71, you know the steady-state temperature distribution. Then you can proceed to find the T_m^p values with increasing time until the *first* node reaches the required limit. We should not expect the nodes to reach their limit at the same time.

Not knowing the steady-state temperature distribution, use the implicit FDE in matrix form above to step through time $\rightarrow \infty$ to the steady-state solution; that is, proceed to p $\rightarrow$ 10,20…100 until the solution matrix [T] does not change. The results of the analysis are tabulated below. Column 1 labeled $T_m(\infty)$ is the steady-state distribution. Column 2, $T_m(95\%)$, is the 95% limit being sought as per the graph directly above. The third column is the temperature distribution at t = to = 248s, $T_m(248s)$; at this elapsed time, Node 1 has reached its limit. Can you explain why this node was the first to reach this limit? Which nodes will be the last to reach their limits?

$T_m(\infty)$	$T_m(95\%)$	$T_m(248s)$
55.80	54.51	54.51
49.93	48.93	48.64
47.67	46.78	46.38
59.03	57.58	57.64
51.72	50.63	50.32
49.19	48.23	47.79
63.89	62.20	62.42
52.98	51.83	51.52
50.14	49.13	48.68
62.84	61.20	61.35
53.35	52.18	51.86
50.46	49.43	48.98

<

PROBLEM 5.127

KNOWN: Nodal network and operating conditions for a water-cooled plate.

FIND: Transient temperature response.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-sate conditions, (2) Two-dimensional conduction.

ANALYSIS: The energy balance method must be applied to each nodal region. Grouping similar regions, the following results are obtained.

Nodes 1 and 5:

$$\left(1+\frac{2\alpha\Delta t}{\Delta x^2}+\frac{2\alpha\Delta t}{\Delta y^2}\right)T_1^{p+1}-\frac{2\alpha\Delta t}{\Delta x^2}T_2^{p+1}-\frac{2\alpha\Delta t}{\Delta y^2}T_6^{p+1}=T_1^p$$

$$\left(1+\frac{2\alpha\Delta t}{\Delta x^2}+\frac{2\alpha\Delta t}{\Delta y^2}\right)T_5^{p+1}-\frac{2\alpha\Delta t}{\Delta x^2}T_4^{p+1}-\frac{2\alpha\Delta t}{\Delta y^2}T_{10}^{p+1}=T_5^p$$

Nodes 2, 3, 4:

$$\left(1+\frac{2\alpha\Delta t}{\Delta x^2}+\frac{2\alpha\Delta t}{\Delta y^2}\right)T_{m,n}^{p+1}-\frac{\alpha\Delta t}{\Delta x^2}T_{m-1,n}^{p+1}-\frac{\alpha\Delta t}{\Delta x^2}T_{m+1,n}^{p+1}-\frac{2\alpha\Delta t}{\Delta y^2}T_{m,n-1}^{p+1}=T_{m,n}^p$$

Nodes 6 and 14:

$$\left(1+\frac{2\alpha\Delta t}{\Delta x^2}+\frac{2\alpha\Delta t}{\Delta y^2}+\frac{2h\alpha\Delta t}{k\Delta y}\right)T_6^{p+1}-\frac{2\alpha\Delta t}{\Delta y^2}T_1^{p+1}-\frac{2\alpha\Delta t}{\Delta x^2}T_7^{p+1}=\frac{2h\alpha\Delta t}{k\Delta y}T_\infty+T_6^p$$

$$\left(1+\frac{2\alpha\Delta t}{\Delta x^2}+\frac{2\alpha\Delta t}{\Delta y^2}+\frac{2h\alpha\Delta t}{k\Delta y}\right)T_{14}^{p+1}-\frac{2\alpha\Delta t}{\Delta x^2}T_{15}^{p+1}-\frac{2\alpha\Delta t}{\Delta y^2}T_{19}^{p+1}=\frac{2h\alpha\Delta t}{k\Delta y}T_\infty+T_{14}^p$$

Continued …..

Nodes 7 and 15:

$$\left(1+\frac{2\alpha\Delta t}{\Delta x^2}+\frac{2\alpha\Delta t}{\Delta y^2}+\frac{2h\alpha\Delta t}{k\Delta y}\right)T_7^{p+1}-\frac{2\alpha\Delta t}{\Delta y^2}T_2^{p+1}-\frac{\alpha\Delta t}{\Delta x^2}T_6^{p+1}-\frac{\alpha\Delta t}{k\Delta x^2}T_8^{p+1}=\frac{2h\alpha\Delta t}{k\Delta y}T_\infty+T_7^p$$

$$\left(1+\frac{2\alpha\Delta t}{\Delta x^2}+\frac{2\alpha\Delta t}{\Delta y^2}+\frac{2h\alpha\Delta t}{k\Delta y}\right)T_{15}^{p+1}-\frac{\alpha\Delta t}{\Delta x^2}T_{14}^{p+1}-\frac{\alpha\Delta t}{\Delta x^2}T_{16}^{p+1}-\frac{2\alpha\Delta t}{\Delta y^2}T_{20}^{p+1}=\frac{2h\alpha\Delta t}{k\Delta y}T_\infty+T_{15}^p$$

Nodes 8 and 16:

$$\left(1+\frac{2\alpha\Delta t}{\Delta x^2}+\frac{2\alpha\Delta t}{\Delta y^2}+\frac{2}{3}\;\frac{h\alpha\Delta t}{k\Delta x}+\frac{2}{3}\frac{h\alpha\Delta t}{k\Delta y}\right)T_8^{p+1}-\frac{4}{3}\frac{\alpha\Delta t}{\Delta y^2}T_3^{p+1}-\frac{2}{3}\frac{\alpha\Delta t}{\Delta x^2}T_7^{p+1}$$
$$-\frac{4}{3}\frac{\alpha\Delta t}{\Delta x^2}T_9^{p+1}-\frac{2}{3}\frac{\alpha\Delta t}{\Delta y^2}T_{11}^{p+1}=\frac{2}{3}\frac{h\alpha\Delta t}{k}\left(\frac{1}{\Delta x}+\frac{1}{\Delta y}\right)T_\infty+T_8^p$$

$$\left(1+\frac{2\alpha\Delta t}{\Delta x^2}+\frac{2\alpha\Delta t}{\Delta y^2}+\frac{2}{3}+\frac{h\alpha\Delta t}{k\Delta x}+\frac{2}{3}\frac{h\alpha\Delta t}{k\Delta y}\right)T_{16}^{p+1}-\frac{2}{3}\frac{\alpha\Delta t}{\Delta y^2}T_{11}^{p+1}-\frac{2}{3}\frac{\alpha\Delta t}{\Delta x^2}T_{15}^{p+1}$$
$$-\frac{4}{3}\frac{\alpha\Delta t}{\Delta x^2}T_{17}^{p+1}-\frac{4}{3}\frac{\alpha\Delta t}{\Delta y^2}T_{21}^{p+1}=\frac{2}{3}\frac{h\alpha\Delta t}{k}\left(\frac{1}{\Delta x}+\frac{1}{\Delta y}\right)T_\infty+T_{16}^p$$

Node 11:

$$\left(1+\frac{2\alpha\Delta t}{\Delta x^2}+\frac{2\alpha\Delta t}{\Delta y^2}+\frac{2h\alpha\Delta t}{k\Delta x}\right)T_{11}^{p+1}-\frac{\alpha\Delta t}{\Delta y^2}T_8^{p+1}-2\alpha\frac{\Delta t}{\Delta x^2}T_{12}^{p+1}-\frac{\alpha\Delta t}{\Delta y^2}T_{16}^{p+1}=\frac{2h\alpha\Delta t}{k\Delta x}T_\infty+T_{11}^p$$

Nodes 9, 12, 17, 20, 21, 22:

$$\left(1+\frac{2\alpha\Delta t}{\Delta x^2}+\frac{2\alpha\Delta t}{\Delta y^2}\right)T_{m,n}^{p+1}-\frac{\alpha\Delta t}{\Delta y^2}\left(T_{m,n+1}^{p+1}+T_{m,n-1}^{p+1}\right)-\frac{\alpha\Delta t}{\Delta x^2}\left(T_{m-1,n}^{p+1}+T_{m+1,n}^{p+1}\right)=T_{m,n}^p$$

Nodes 10, 13, 18, 23:

$$\left(1+\frac{2\alpha\Delta t}{\Delta x^2}+\frac{2\alpha\Delta t}{\Delta y^2}\right)T_{m,n}^{p+1}-\frac{\alpha\Delta t}{\Delta y^2}\left(T_{m,n+1}^{p+1}+T_{m,n-1}^{p+1}\right)-\frac{2\alpha\Delta t}{\Delta x^2}T_{m-1,n}^{p+1}=T_{m,n}^p$$

Node 19:

$$\left(1+\frac{2\alpha\Delta t}{\Delta x^2}+\frac{2\alpha\Delta t}{\Delta y^2}\right)T_{19}^{p+1}-\frac{\alpha\Delta t}{\Delta y^2}\left(T_{14}^{p+1}+T_{24}^{p+1}\right)-\frac{2\alpha\Delta t}{\Delta x^2}T_{20}^{p+1}=T_{19}^p$$

Nodes 24, 28:

$$\left(1+\frac{2\alpha\Delta t}{\Delta x^2}+\frac{2\alpha\Delta t}{\Delta y^2}\right)T_{24}^{p+1}-\frac{2\alpha\Delta t}{\Delta y^2}T_{19}^{p+1}-\frac{2\alpha\Delta t}{\Delta x^2}T_{25}^{p+1}=\frac{2q_o''\alpha\Delta t}{k\Delta y}+T_{24}^p$$

$$\left(1+\frac{2\alpha\Delta t}{\Delta x^2}+\frac{2\alpha\Delta t}{\Delta y^2}\right)T_{28}^{p+1}-\frac{2\alpha\Delta t}{\Delta y^2}T_{23}^{p+1}-\frac{2\alpha\Delta t}{\Delta x^2}T_{27}^{p+1}=\frac{2q_o''\alpha\Delta t}{k\Delta y}+T_{28}^p$$

Continued …..

Nodes 25, 26, 27:

$$\left(1+\frac{2\alpha\Delta t}{\Delta x^2}+\frac{2\alpha\Delta t}{\Delta y^2}\right)T_{m,n}^{p+1}-\frac{2\alpha\Delta t}{\Delta y^2}T_{m,n+1}^{p+1}-\frac{\alpha\Delta t}{\Delta x^2}\left(T_{m-1,n}^{p+1}+T_{m+1,n}^{p+1}\right)=\frac{2q_o''\alpha\Delta t}{k\Delta y}+T_{m,n}^{p+1}$$

The convection heat rate is

$$q'_{conv}=h\left[(\Delta x/2)(T_6-T_\infty)+\Delta x(T_7-T_\infty)+(\Delta x+\Delta y)(T_8-T_\infty)/2+\Delta y(T_{11}-T_\infty)+(\Delta x+\Delta y)(T_{16}-T_\infty)/2+\Delta x(T_{15}-T_\infty)+(\Delta x/2)(T_{14}-T_\infty)\right]=q_{out}.$$

The heat input is

$$q'_{in}=q_o''(4\Delta x)$$

and, on a percentage basis, the ratio is

$$n\equiv(q'_{conv}/q'_{in})\times 100.$$

Results of the calculations (in °C) are as follows:

Time: 5.00 sec; n = 60.57%

19.612	19.712	19.974	20.206	20.292
19.446	19.597	20.105	20.490	20.609
		21.370	21.647	21.730
24.217	24.074	23.558	23.494	23.483
25.658	25.608	25.485	25.417	25.396
27.581	27.554	27.493	27.446	27.429

Time: 10.00 sec; n = 85.80%

22.269	22.394	22.723	23.025	23.137
21.981	22.167	22.791	23.302	23.461
		24.143	24.548	24.673
27.216	27.075	26.569	26.583	26.598
28.898	28.851	28.738	28.690	28.677
30.901	30.877	30.823	30.786	30.773

Time: 15.0 sec; n = 94.89%

23.228	23.363	23.716	24.042	24.165
22.896	23.096	23.761	24.317	24.491
		25.142	25.594	25.733
28.294	28.155	27.652	27.694	27.719
30.063	30.018	29.908	29.867	29.857
32.095	32.072	32.021	31.987	31.976

Time: 20.00 sec; n = 98.16%

23.574	23.712	24.073	24.409	24.535
23.226	23.430	24.110	24.682	24.861
		25.502	25.970	26.115
28.682	28.543	28.042	28.094	28.122
30.483	30.438	30.330	30.291	30.282
32.525	32.502	32.452	32.419	32.409

Time: 23.00 sec; n = 99.00%

23.663	23.802	24.165	24.503	24.630
23.311	23.516	24.200	24.776	24.957
		25.595	26.067	26.214
28.782	28.644	28.143	28.198	28.226
30.591	30.546	30.438	30.400	30.392
32.636	32.613	32.563	32.531	32.520

COMMENTS: Temperatures at t = 23 s are everywhere within 0.13°C of the final steady-state values.

PROBLEM 5.128

KNOWN: Cubic-shaped furnace, with prescribed operating temperature and convection heat transfer on the exterior surfaces.

FIND: Time required for the furnace to cool to a safe working temperature corresponding to an inner wall temperature of 35°C considering convection cooling on (a) the exterior surfaces and (b) on both the exterior and interior surfaces.

SCHEMATIC:

ASSUMPTIONS: (1) Two-dimensional conduction through the furnace walls and (2) Constant properties.

ANALYSIS: Assuming two-dimensional conduction through the walls and taking advantage of symmetry for the cubical shape, the analysis considers the quarter section shown in the schematic above. For part (a), with no cooling on the interior during the cool-down process, the inner surface boundary condition is adiabatic. For part (b), with cooling on both the exterior and interior, the boundary conditions are prescribed by the convection process. The boundaries through the centerline of the wall and the diagonal through the corner are symmetry planes and considered as adiabatic. We have chosen to use the finite-element software *FEHT* as the solution tool.

Using *FEHT*, an outline of the symmetrical wall section is drawn, and the material properties are specified. To determine the initial conditions for the cool-down process, we will first find the temperature distribution for steady-state operation. As such, specify the boundary condition for the inner surface as a constant temperature of 900°C; the other boundaries are as earlier described. In the *Setup* menu, click *on Steady-State*, and then *Run* to obtain the steady-state temperature distribution. This distribution represents the initial temperature distribution, T_i (x, y, 0), for the wall at the onset of the cool-down process.

Next, in the *Setup* menu, click on *Transient*; for the nodes on the inner surface, in the *Specify | Boundary Conditions* menu, deselect the *Temperature* box (900°C) and set the *Flux* box to zero for the adiabatic condition (part (a)); and, in the *Run* command, click on *Continue* (not *Calculate*). Be sure to change the integration time scale from *seconds* to *hours*.

Because of the high ratio of wall section width (nearly 8.5 m) to the thickness (1 m), the conduction heat transfer through the section is nearly one-dimensional. We chose the x,y-section 1 m to the right of the centerline (1 m, y) as the location for examining the temperature-time history, and determining the cool-down time for the inner surface to reach the safe working temperature of 35°C.

Continued …..

PROBLEM 5.128 (Cont.)

Temperature history during cool down at x,y-section (1 m, y)

Adiabatic inner surface

Time-to-cool, Part (a), Adiabatic inner surface. From the above temperature history, the cool-down time, t_a, corresponds to the condition when T_a (1 m, 0, t_a) = 35°C. As seen from the history, this location is the last to cool. From the *View | Tabular Output*, find that

$$t_a = 1306 \text{ h} = 54 \text{ days} \qquad <$$

Temperature history during cool down at x,y-section (1 m, y)

Cooled inner surface

Continued …..

PROBLEM 5.128 (Cont.)

Time-to-cool, Part (b), Cooled inner surface. From the above temperature history, note that the center portion of the wall, and not the inner surface, is the last to cool. The inner surface cools to 35°C in approximately 175 h or 7 days. However, if the cooling process on the inner surface were discontinued, its temperature would increase and eventually exceed the desired safe working temperature. To assure the safe condition will be met, estimate the cool down time as, t_b, corresponding to the condition when T_b (1 m, 0.75 m, t_b) = 35°C. From the *View* | *Tabular Output*, find that

$$t_b = 311 \text{ h} = 13 \text{ days}$$ <

COMMENTS: (1) Assuming the furnace can be approximated by a two-dimensional symmetrical section greatly simplifies our analysis by not having to deal with three-dimensional corner effects. We justify this assumption on the basis that the corners represent a much shorter heat path than the straight wall section. Considering corner effects would reduce the cool-down time estimates; hence, our analysis provides a conservative estimate.

(2) For background information on the *Continue* option, see the *Run* menu in the *FEHT Help* section. Using the *Run* | *Calculate* command, the steady-state temperature distribution was determined for the normal operating condition of the furnace. Using the *Run* | *Continue* command (after clicking on *Setup* | *Transient*), this steady-state distribution automatically becomes the initial temperature distribution for the cool-down transient process. This feature allows for conveniently prescribing a non-uniform initial temperature distribution for a transient analysis (rather than specifying values on a node-by-node basis.

PROBLEM 5.129

KNOWN: Door panel with ribbed cross-section, initially at a uniform temperature of 275°C, is ejected from the hot extrusion press and experiences convection cooling with ambient air at 25°C and a convection coefficient of 10 $W/m^2 \cdot K$.

FIND: (a) Using the *FEHT View|Temperature vs. Time* command, create a graph with temperature-time histories of selected locations on the panel surface, T(x,0,t). Comment on whether you see noticeable differential cooling in the region above the rib that might explain the appearance defect; and Using the *View|Temperature Contours* command with the shaded-band option for the isotherm contours, select the *From start to stop* time option, and view the temperature contours as the panel cools. Describe the major features of the cooling process you have seen. Use other options of this command to create a 10-isotherm temperature distribution at some time that illustrates important features. How would you re-design the ribbed panel in order to reduce this thermally induced paint defect situation, yet retain the stiffening function required of the ribs?

SCHEMATIC:

ASSUMPTIONS: (1) Two-dimensional conduction in the panel, (2) Uniform convection coefficient over the upper and lower surfaces of the panel, (3) Constant properties.

PROPERTIES: Door panel material (*given*): ρ = 1050 kg/m^3, c = 800 J/kg·K, k = 0.5 W/m·K.

ANALYSIS:

(a) Using the *Draw* command, the shape of the symmetrical element of the panel (darkened region in schematic) was generated and elements formed as shown below. The symmetry lines represent adiabatic surfaces, while the boundary conditions for the exposed web and rib surfaces are characterized by (T_∞, h).

Continued …..

PROBLEM 5.129 (Cont.)

After running the calculation for the time period 0 to 400 s with a 1-second time step, the temperature-time histories for three locations were obtained and the graph is shown below.

As expected, the region directly over the rib (0,0) cooled the slowest, while the extreme portion of the web (0, 13 mm) cooled the fastest. The largest temperature differences between these two locations occur during the time period 50 to 150 s. The maximum difference does not exceed 25°C.

Continued …..

PROBLEM 5.129 (Cont.)

(b) It is possible that the temperature gradients within the web-rib regions – rather than just the upper surface temperature differentials – might be important for understanding the panel's response to cooling. Using the *Temperature Contours* command (with the *From start to stop* option), we saw that the center portion of the web and the end of the rib cooled quickly, but that the region on the rib centerline (0, 3-5 mm), was the hottest region. The isotherms corresponding to t = 100 s are shown below. For this condition, the temperature differential is about 21°C.

Tinf, h
10 isothermal lines
min 147.2 C, max 168.8 C
Tinf, h

From our analyses, we have identified two possibilities to consider. First, there is a significant surface temperature distribution across the panel during the cooling process. Second, the web and the extended portion of the rib cool at about the same rate, and with only a modest normal temperature gradient. The last region to cool is at the location where the rib is thickest (0, 3-5 mm). The large temperature gradient along the centerline toward the surface may be the cause of microstructure variations, which could influence the adherence of paint. An obvious re-design consideration is to reduce the thickness of the rib at the web joint, thereby reducing the temperature gradients in that region. This fix comes at the expense of decreasing the spacing between the ribs.

PROBLEM 6.1

KNOWN: Variation of h_x with x for laminar flow over a flat plate.

FIND: Ratio of average coefficient, $\bar{h}_x$, to local coefficient, h_x, at x.

SCHEMATIC:

Thermal boundary layer, $h_x = Cx^{-1/2}$, where C is a constant

x

ANALYSIS: The average value of h_x between 0 and x is

$$\bar{h}_x = \frac{1}{x}\int_0^x h_x dx = \frac{C}{x}\int_0^x x^{-1/2} dx$$

$$\bar{h}_x = \frac{C}{x} 2x^{1/2} = 2Cx^{-1/2}$$

$$\bar{h}_x = 2h_x.$$

Hence, $$\frac{\bar{h}_x}{h_x} = 2.$$ <

COMMENTS: Both the local and average coefficients decrease with increasing distance x from the leading edge, as shown in the sketch below.

h

$\bar{h}_x(x)$

$h_x(x)$

x

PROBLEM 6.2

KNOWN: Variation of local convection coefficient with x for free convection from a vertical heated plate.

FIND: Ratio of average to local convection coefficient.

SCHEMATIC:

Ts; T∞; Boundary layer, $h_x = Cx^{-1/4}$, where C is a constant; x

ANALYSIS: The average coefficient from 0 to x is

$$\bar{h}_x = \frac{1}{x}\int_0^x h_x dx = \frac{C}{x}\int_0^x x^{-1/4} dx$$

$$\bar{h}_x = \frac{4}{3}\frac{C}{x}x^{3/4} = \frac{4}{3}C\,x^{-1/4} = \frac{4}{3}h_x.$$

Hence,
$$\frac{\bar{h}_x}{h_x} = \frac{4}{3}.$$
<

The variations with distance of the local and average convection coefficients are shown in the sketch.

$h_x, \bar{h}_x$; C; 0.5C; 0; 1; 2; 3; x; $\bar{h}_x = \frac{4}{3}h_x$; $h_x = Cx^{-1/4}$

COMMENTS: Note that $\bar{h}_x / h_x = 4/3$ is independent of x. Hence the average coefficient for an entire plate of length L is $\bar{h}_L = \frac{4}{3}h_L$, where h_L is the local coefficient at x = L. Note also that the average *exceeds* the local. Why?

PROBLEM 6.3

KNOWN: Expression for the local heat transfer coefficient of a circular, hot gas jet at T_∞ directed normal to a circular plate at T_s of radius r_o.

FIND: Heat transfer rate to the plate by convection.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Flow is axisymmetric about the plate, (3) For h(r), a and b are constants and $n \neq -2$.

ANALYSIS: The convective heat transfer rate to the plate follows from Newton's law of cooling

$$q_{conv} = \int_A dq_{conv} = \int_A h(r) \cdot dA \cdot (T_\infty - T_s).$$

The local heat transfer coefficient is known to have the form,

$$h(r) = a + br^n$$

and the differential area on the plate surface is

$$dA = 2\pi r\, dr.$$

Hence, the heat rate is

$$q_{conv} = \int_0^{r_o} \left(a + br^n\right) \cdot 2\pi\ r\, dr \cdot (T_\infty - T_s)$$

$$q_{conv} = 2\pi (T_\infty - T_s) \left[\frac{a}{2} r^2 + \frac{b}{n+2} r^{n+2}\right]_0^{r_o}$$

$$q_{conv} = 2\pi \left[\frac{a}{2} r_o^2 + \frac{b}{n+2} r_o^{n+2}\right] (T_\infty - T_s).$$

<

COMMENTS: Note the importance of the requirement, $n \neq -2$. Typically, the radius of the jet is much smaller than that of the plate.

PROBLEM 6.4

KNOWN: Distribution of local convection coefficient for obstructed parallel flow over a flat plate.

FIND: Average heat transfer coefficient and ratio of average to local at the trailing edge.

SCHEMATIC:

ANALYSIS: The average convection coefficient is

$$\bar{h}_L = \frac{1}{L}\int_0^L h_x dx = \frac{1}{L}\int_0^L \left(0.7 + 13.6x - 3.4x^2\right)dx$$

$$\bar{h}_L = \frac{1}{L}\left(0.7L + 6.8L^2 - 1.13L^3\right) = 0.7 + 6.8L - 1.13L^2$$

$$\bar{h}_L = 0.7 + 6.8(3) - 1.13(9) = 10.9 \text{ W/m}^2 \cdot \text{K.}$$ <

The local coefficient at x = 3m is

$$h_L = 0.7 + 13.6(3) - 3.4(9) = 10.9 \text{ W/m}^2 \cdot \text{K.}$$

Hence,

$$\bar{h}_L / h_L = 1.0.$$ <

COMMENTS: The result $\bar{h}_L / h_L = 1.0$ is unique to x = 3m and is a consequence of the existence of a maximum for $h_x(x)$. The maximum occurs at x = 2m, where $(dh_x / dx) = 0$ and $\left(d^2h_x / dx^2 < 0.\right)$

PROBLEM 6.5

KNOWN: Temperature distribution in boundary layer for air flow over a flat plate.

FIND: Variation of local convection coefficient along the plate and value of average coefficient.

SCHEMATIC:

$T(x,y) = 20 + 70e^{-600xy}$

$T_\infty = 20\ ^oC$

$L = 5$ m

ANALYSIS: From Eq. 6.17,

$$h = -\frac{k\,\partial T/\partial y|_{y=0}}{\left(T_s - T_\infty\right)} = +\frac{k\left(70 \times 600x\right)}{\left(T_s - T_\infty\right)}$$

where $T_s = T(x,0) = 90°C$. Evaluating k at the arithmetic mean of the freestream and surface temperatures, $\overline{T} = (20 + 90)°C/2 = 55°C = 328$ K, Table A.4 yields k = 0.0284 W/m·K. Hence, with $T_s - T_\infty = 70°C = 70$ K,

$$h = \frac{0.0284\,W/m\cdot K\left(42,000x\right)K/m}{70\,K} = 17x\left(W/m^2\cdot K\right)$$ <

and the convection coefficient increases linearly with x.

The average coefficient over the range 0 ≤ x ≤ 5 m is

$$\overline{h} = \frac{1}{L}\int_0^L h\,dx = \frac{17}{5}\int_0^5 x\,dx = \frac{17}{5}\frac{x^2}{2}\Bigg|_0^5 = 42.5\,W/m^2\cdot K$$ <

PROBLEM 6.6

KNOWN: Variation of local convection coefficient with distance x from a heated plate with a uniform temperature T_s.

FIND: (a) An expression for the average coefficient $\bar{h}_{12}$ for the section of length $(x_2 - x_1)$ in terms of C, x_1 and x_2, and (b) An expression for $\bar{h}_{12}$ in terms of x_1 and x_2, and the average coefficients $\bar{h}_1$ and $\bar{h}_2$, corresponding to lengths x_1 and x_2, respectively.

SCHEMATIC:

ASSUMPTIONS: (1) Laminar flow over a plate with uniform surface temperature, T_s, and (2) Spatial variation of local coefficient is of the form $h_x = Cx^{-1/2}$, where C is a constant.

ANALYSIS: (a) The heat transfer rate per unit width from a longitudinal section, $x_2 - x_1$, can be expressed as

$$q'_{12} = \bar{h}_{12}\left(x_2 - x_1\right)\left(T_s - T_\infty\right) \qquad (1)$$

where $\bar{h}_{12}$ is the average coefficient for the section of length $(x_2 - x_1)$. The heat rate can also be written in terms of the local coefficient, Eq. (6.3), as

$$q'_{12} = \int_{x_1}^{x_2} h_x dx\left(T_s - T_\infty\right) = \left(T_s - T_\infty\right)\int_{x_1}^{x_2} h_x dx \qquad (2)$$

Combining Eq. (1) and (2),

$$\bar{h}_{12} = \frac{1}{\left(x_2 - x_1\right)}\int_{x_1}^{x_2} h_x dx \qquad (3)$$

and substituting for the form of the local coefficient, $h_x = Cx^{-1/2}$, find that

$$\bar{h}_{12} = \frac{1}{\left(x_2 - x_1\right)}\int_{x_1}^{x_2} Cx^{-1/2}dx = \frac{C}{x_2 - x_1}\left[\frac{x^{1/2}}{1/2}\right]_{x_1}^{x_2} = 2C\frac{x_2^{1/2} - x_1^{1/2}}{x_2 - x_1} \qquad (4)$$ <

(b) The heat rate, given as Eq. (1), can also be expressed as

$$q'_{12} = \bar{h}_2 x_2\left(T_s - T_\infty\right) - \bar{h}_1 x_1\left(T_s - T_\infty\right) \qquad (5)$$

which is the difference between the heat rate for the plate over the section $(0 - x_2)$ and over the section $(0 - x_1)$. Combining Eqs. (1) and (5), find,

$$\bar{h}_{12} = \frac{\bar{h}_2 x_2 - \bar{h}_1 x_1}{x_2 - x_1} \qquad (6)$$ <

COMMENTS: (1) Note that, from Eq. 6.6,

$$\bar{h}_x = \frac{1}{2}\int_0^x h_x dx = \frac{1}{x}\int_0^x Cx^{-1/2}dx = 2Cx^{-1/2} \qquad (7)$$

or $\bar{h}_x = 2h_x$. Substituting Eq. (7) into Eq. (6), see that the result is the same as Eq. (4).

PROBLEM 6.7

KNOWN: Radial distribution of local convection coefficient for flow normal to a circular disk.

FIND: Expression for average Nusselt number.

SCHEMATIC:

$Nu_D = Nu_o[1 + a(r/r_o)^n]$
$Nu_o = 0.814 Re_D^{1/2} Pr^{0.36}$

V, r_o, D

ASSUMPTIONS: Constant properties

ANALYSIS: The average convection coefficient is

$$\bar{h} = \frac{1}{A_s}\int_{A_s} h dA_s$$

$$\bar{h} = \frac{1}{\pi r_o^2}\int_0^{r_o} \frac{k}{D} Nu_o \left[1 + a(r/r_o)^n\right] 2\pi \, rdr$$

$$\bar{h} = \frac{kNu_o}{r_o^3}\left[\frac{r^2}{2} + \frac{ar^{n+2}}{(n+2)r_o^n}\right]_0^{r_o}$$

where Nu_o is the Nusselt number at the stagnation point (r = 0). Hence,

$$\overline{Nu}_D = \frac{\bar{h}D}{k} = 2Nu_o\left[\frac{(r/r_o)^2}{2} + \frac{a}{(n+2)}\left(\frac{r}{r_o}\right)^{n+2}\right]_0^{r_o}$$

$$\overline{Nu}_D = Nu_o\left[1 + 2a/(n+2)\right]$$

$$\overline{Nu}_D = \left[1 + 2a/(n+2)\right] 0.814 Re_D^{1/2} Pr^{0.36}.$$ <

COMMENTS: The increase in h(r) with r may be explained in terms of the sharp turn which the boundary layer flow must make around the edge of the disk. The boundary layer accelerates and its thickness decreases as it makes the turn, causing the local convection coefficient to increase.

PROBLEM 6.8

KNOWN: Convection correlation and temperature of an impinging air jet. Dimensions and initial temperature of a heated copper disk. Properties of the air and copper.

FIND: Effect of jet velocity on temperature decay of disk following jet impingement.

SCHEMATIC:

ASSUMPTIONS: (1) Validity of lumped capacitance analysis, (2) Negligible heat transfer from sides and bottom of disk, (3) Constant properties.

ANALYSIS: Performing an energy balance on the disk, it follows that
$\dot{E}_{st} = \rho Vc\, dT/dt = -A_s\left(q''_{conv} + q''_{rad}\right)$. Hence, with $V = A_s L$,

$$\frac{dT}{dt} = -\frac{\bar{h}\left(T - T_\infty\right) + h_r\left(T - T_{sur}\right)}{\rho c L}$$

where, $h_r = \varepsilon\sigma\left(T + T_{sur}\right)\left(T^2 + T_{sur}^2\right)$ and, from the solution to Problem 6.7,

$$\bar{h} = \frac{k}{D}\overline{Nu}_D = \frac{k}{D}\left(1 + \frac{2a}{n+2}\right)0.814\, Re_D^{1/2}\, Pr^{0.36}$$

With a = 0.30 and n = 2, it follows that

$$\bar{h} = \left(k/D\right)0.936\, Re_D^{1/2}\, Pr^{0.36}$$

where $Re_D = VD/\nu$. Using the *Lumped Capacitance Model* of IHT, the following temperature histories were determined.

Continued …..

PROBLEM 6.8 (Cont.)

The temperature decay becomes more pronounced with increasing V, and a final temperature of 400 K is reached at t = 2760, 1455 and 976s for V = 4, 20 and 50 m/s, respectively.

COMMENTS: The maximum Biot number, Bi = $\left(\overline{h}+h_r\right)L/k_{Cu}$, is associated with V = 50 m/s (maximum $\overline{h}$ of 169 W/m^2·K) and t = 0 (maximum h_r of 64 W/m^2·K), in which case the maximum Biot number is Bi = (233 W/m^2·K)(0.025 m)/(386 W/m·K) = 0.015 < 0.1. Hence, the lumped capacitance approximation is valid.

PROBLEM 6.9

KNOWN: Local convection coefficient on rotating disk. Radius and surface temperature of disk. Temperature of stagnant air.

FIND: Local heat flux and total heat rate. Nature of boundary layer.

SCHEMATIC:

ASSUMPTIONS: (1) Negligible heat transfer from back surface and edge of disk.

ANALYSIS: If the local convection coefficient is independent of radius, the local heat flux at every point on the disk is

$$q'' = h\left(T_s - T_\infty\right) = 20\,W/m^2 \cdot K\left(50-20\right)°C = 600\,W/m^2 \qquad <$$

Since h is independent of location, $\bar{h} = h = 20\,W/m^2 \cdot K$ and the total power requirement is

$$P_{elec} = q = \bar{h}A_s\left(T_s - T_\infty\right) = \bar{h}\pi r_o^2\left(T_s - T_\infty\right)$$

$$P_{elec} = \left(20\,W/m^2 \cdot K\right)\pi\left(0.1m\right)^2\left(50-20\right)°C = 18.9\ W \qquad <$$

If the convection coefficient is independent of radius, the boundary layer must be of uniform thickness δ. Within the boundary layer, air flow is principally in the circumferential direction. The circumferential velocity component u_θ corresponds to the rotational velocity of the disk at the surface (y = 0) and increases with increasing r ($u_\theta = \Omega r$). The velocity decreases with increasing distance y from the surface, approaching zero at the outer edge of the boundary layer ($y \to \delta$).

PROBLEM 6.10

KNOWN: Form of the velocity and temperature profiles for flow over a surface.

FIND: Expressions for the friction and convection coefficients.

SCHEMATIC:

ANALYSIS: The shear stress at the wall is

$$\tau_s = \mu \left.\frac{\partial u}{\partial y}\right]_{y=0} = \mu \left[A + 2By - 3Cy^2\right]_{y=0} = A\mu.$$

Hence, the friction coefficient has the form,

$$C_f = \frac{\tau_s}{\rho u_\infty^2/2} = \frac{2A\mu}{\rho u_\infty^2}$$

$$C_f = \frac{2A\nu}{u_\infty^2}.$$ <

The convection coefficient is

$$h = \frac{-k_f\left(\partial T/\partial y\right)_{y=0}}{T_s - T_\infty} = \frac{-k_f\left[E + 2Fy - 3Gy^2\right]_{y=0}}{D - T_\infty}$$

$$h = \frac{-k_f E}{D - T_\infty}.$$ <

COMMENTS: It is a simple matter to obtain the important surface parameters from knowledge of the corresponding boundary layer profiles. However, it is rarely a simple matter to determine the form of the profile.

PROBLEM 6.11

KNOWN: Surface temperatures of a steel wall and temperature of water flowing over the wall.

FIND: (a) Convection coefficient, (b) Temperature gradient in wall and in water at wall surface.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat transfer in x, (3) Constant properties.

PROPERTIES: *Table A-1*, Steel Type AISI 1010 (70°C = 343K), k_s = 61.7 W/m·K; *Table A-6*, Water (32.5°C = 305K), k_f = 0.62 W/m·K.

ANALYSIS: (a) Applying an energy balance to the control surface at x = 0, it follows that

$$q''_{x,cond} - q''_{x,conv} = 0$$

and using the appropriate rate equations,

$$k_s \frac{T_{s,2} - T_{s,1}}{L} = h\left(T_{s,1} - T_\infty\right).$$

Hence,

$$h = \frac{k_s}{L}\frac{T_{s,2} - T_{s,1}}{T_{s,1} - T_\infty} = \frac{61.7\ \text{W/m}\cdot\text{K}}{0.35\text{m}}\frac{60^\circ\text{C}}{15^\circ\text{C}} = 705\ \text{W/m}^2\cdot\text{K}.$$ $<$

(b) The gradient in the wall at the surface is

$$\left(dT/dx\right)_s = -\frac{T_{s,2} - T_{s,1}}{L} = -\frac{60^\circ\text{C}}{0.35\text{m}} = -171.4^\circ\text{C/m}.$$

In the water at x = 0, the definition of h gives

$$\left(dT/dx\right)_{f,x=0} = -\frac{h}{k_f}\left(T_{s,1} - T_\infty\right)$$

$$\left(dT/dx\right)_{f,x=0} = -\frac{705\ \text{W/m}^2\cdot\text{K}}{0.62\ \text{W/m}\cdot\text{K}}\left(15^\circ\text{C}\right) = -17{,}056^\circ\text{C/m}.$$ $<$

COMMENTS: Note the relative magnitudes of the gradients. Why is there such a large difference?

PROBLEM 6.12

KNOWN: Boundary layer temperature distribution.

FIND: Surface heat flux.

SCHEMATIC:

Air, $T_\infty = 400K$
$u_\infty/\nu = 5000 m^{-1}$
$Pr = 0.7$

$\frac{T(y) - T_s}{T_\infty - T_s} = 1 - exp(-Pr \frac{u_\infty y}{\nu})$

$T_s = 300K$

PROPERTIES: *Table A-4*, Air (T_s = 300K): k = 0.0263 W/m·K.

ANALYSIS: Applying Fourier's law at y = 0, the heat flux is

$$q_s'' = -k \frac{\partial T}{\partial y}\bigg|_{y=0} = -k(T_\infty - T_s)\left[Pr \frac{u_\infty}{\nu}\right] \exp\left[-Pr \frac{u_\infty y}{\nu}\right]\bigg|_{y=0}$$

$$q_s'' = -k(T_\infty - T_s) Pr \frac{u_\infty}{\nu}$$

$$q_s'' = -0.0263 \text{ W/m} \cdot \text{K}(100\text{K}) 0.7 \times 5000 \text{ 1/m}.$$

$$q_s'' = -9205 \text{ W/m}^2.$$ <

COMMENTS: (1) Negative flux implies convection heat transfer to the surface.

(2) Note use of k at T_s to evaluate q_s'' from Fourier's law.

PROBLEM 6.13

KNOWN: Air flow over a flat plate of length L = 1 m under conditions for which transition from laminar to turbulent flow occurs at $x_c = 0.5$m based upon the critical Reynolds number, $Re_{x,c} = 5\times 10^5$. Forms for the local convection coefficients in the laminar and turbulent regions.

FIND: (a) Velocity of the air flow using thermophysical properties evaluated at 350 K, (b) An expression for the average coefficient $\overline{h}_{lan}(x)$, as a function of distance from the leading edge, x, for the laminar region, $0 \le x \le x_c$, (c) An expression for the average coefficient $\overline{h}_{turb}(x)$, as a function of distance from the leading edge, x, for the turbulent region, $x_c \le x \le L$, and (d) Compute and plot the local and average convection coefficients, h_x and $\overline{h}_x$, respectively, as a function of x for $0 \le x \le L$.

SCHEMATIC:

ASSUMPTIONS: (1) Forms for the local coefficients in the laminar and turbulent regions, $h_{lam} = C_{lam}x^{-0.5}$ and $h_{tirb} = C_{turb}x^{-0.2}$ where $C_{lam} = 8.845\ W/m^{3/2}\cdot K$, $C_{turb} = 49.75\ W/m^2\cdot K^{0.8}$, and x has units (m).

PROPERTIES: *Table A.4*, Air (T = 350 K): k = 0.030 W/m·K, $\nu = 20.92 \times 10^{-6}\ m^2/s$, Pr = 0.700.

ANALYSIS: (a) Using air properties evaluated at 350 K with $x_c = 0.5$ m,

$$Re_{x,c} = \frac{u_\infty x_c}{\nu} = 5\times10^5 \qquad u_\infty = 5\times10^5\,\nu/x_c = 5\times10^5 \times 20.92\times10^{-6}\,m^2/s/0.5\,m = 20.9\,m/s \quad <$$

(b) From Eq. 6.5, the average coefficient in the laminar region, $0 \le x \le x_c$, is

$$\overline{h}_{lam}(x) = \frac{1}{x}\int_0^x h_{lam}(x)dx = \frac{1}{x}C_{lam}\int_o^x x^{-0.5}dx = \frac{1}{x}C_{lam}x^{0.5} = 2C_{lam}x^{-0.5} = 2h_{lam}(x) \qquad (1) \quad <$$

(c) The average coefficient in the turbulent region, $x_c \le x \le L$, is

$$\overline{h}_{turb}(x) = \frac{1}{x}\left[\int_0^{x_c} h_{lam}(x)dx + \int_{x_c}^{x} h_{turb}(x)dx\right] = \left[C_{lam}\left.\frac{x^{0.5}}{0.5}\right|_0^{x_c} + C_{turb}\left.\frac{x^{0.8}}{0.8}\right|_{x_c}^{x}\right]$$

$$\overline{h}_{turb}(x) = \frac{1}{x}\left[2C_{lam}x_c^{0.5} + 1.25C_{turb}\left(x^{0.8} - x_c^{0.8}\right)\right] \qquad (2) \quad <$$

(d) The local and average coefficients, Eqs. (1) and (2) are plotted below as a function of x for the range $0 \le x \le L$.

PROBLEM 6.14

KNOWN: Air speed and temperature in a wind tunnel.

FIND: (a) Minimum plate length to achieve a Reynolds number of 10^8, (b) Distance from leading edge at which transition would occur.

SCHEMATIC:

ASSUMPTIONS: (1) Isothermal conditions, $T_s = T_\infty$.

PROPERTIES: *Table A-4*, Air (25°C = 298K): $\nu = 15.71 \times 10^{-6} m^2/s$.

ANALYSIS: (a) The Reynolds number is

$$Re_x = \frac{\rho\, u_\infty x}{\mu} = \frac{u_\infty x}{\nu}.$$

To achieve a Reynolds number of 1×10^8, the minimum plate length is then

$$L_{min} = \frac{Re_x \nu}{u_\infty} = \frac{1\times10^8 \left(15.71\times10^{-6} m^2/s\right)}{50\ m/s}$$

$$L_{min} = 31.4\ m.$$ <

(b) For a transition Reynolds number of 5×10^5

$$x_c = \frac{Re_{x,c} \nu}{u_\infty} = \frac{5\times10^5 \left(15.71\times10^{-6} m^2/s\right)}{50\ m/s}$$

$$x_c = 0.157\ m.$$ <

COMMENTS: Note that

$$\frac{x_c}{L} = \frac{Re_{x,c}}{Re_L}$$

This expression may be used to quickly establish the location of transition from knowledge of $Re_{x,c}$ and Re_L.

PROBLEM 6.15

KNOWN: Transition Reynolds number. Velocity and temperature of atmospheric air, water, engine oil and mercury flow over a flat plate.

FIND: Distance from leading edge at which transition occurs for each fluid.

SCHEMATIC:

ASSUMPTIONS: Transition Reynolds number is $Re_{x,c} = 5 \times 10^5$.

PROPERTIES: For the fluids at T = 300K;

Fluid	*Table*	*$\nu(m^2/s)$*
Air (1 atm)	A-4	15.89×10^{-6}
Water	A-6	0.858×10^{-6}
Engine Oil	A-5	550×10^{-6}
Mercury	A-5	0.113×10^{-6}

ANALYSIS: The point of transition is

$$x_c = Re_{x,c} \frac{\nu}{u_\infty} = \frac{5 \times 10^5}{1\ m/s} \nu.$$

Substituting appropriate viscosities, find

Fluid	*$x_c(m)$*
Air	7.95
Water	0.43
Oil	275
Mercury	0.06

<

COMMENTS: Due to the effect which viscous forces have on attenuating the instabilities which bring about transition, the distance required to achieve transition increases with increasing ν.

PROBLEM 6.16

KNOWN: Two-dimensional flow conditions for which v = 0 and T = T(y).

FIND: (a) Verify that u = u(y), (b) Derive the x-momentum equation, (c) Derive the energy equation.

SCHEMATIC:

Pressure & shear forces

Energy fluxes

ASSUMPTIONS: (1) Steady-state conditions, (2) Incompressible fluid with constant properties, (3) Negligible body forces, (4) v = 0, (5) T = T(y) or $\partial T/\partial x = 0$, (6) Thermal energy generation occurs only by viscous dissipation.

ANALYSIS: (a) From the mass continuity equation, it follows from the prescribed conditions that $\partial u/\partial x = 0$. Hence u = u(y).

(b) From Newton's second law of motion, $\Sigma F_x =$ (Rate of increase of fluid momentum)$_x$,

$$\left[p - \left[p + \frac{\partial p}{\partial x}dx\right]\right] dy \cdot 1 + \left[-\tau + \left[\tau + \frac{\partial \tau}{\partial y}dy\right]\right] dx \cdot 1 = \left\{(\rho u)u + \frac{\partial}{\partial x}[(\rho u)u]dx\right\} dy \cdot 1 - (\rho u)u\, dy \cdot 1$$

Hence, with $\tau = \mu(\partial u/\partial y)$, it follows that

$$-\frac{\partial p}{\partial x} + \frac{\partial \tau}{\partial y} = \frac{\partial}{\partial x}[(\rho u)u] = 0 \qquad \frac{\partial p}{\partial x} = \mu \frac{\partial^2 u}{\partial y^2}.$$ <

(c) From the conservation of energy requirement and the prescribed conditions, it follows that $\dot{E}_{in} - \dot{E}_{out} = 0$, or

$$\left[pu + \rho u\left(e + u^2/2\right)\right] dy \cdot 1 + \left[-k\frac{\partial T}{\partial y} + \tau u + \frac{\partial (\tau u)}{\partial y}dy\right] dx \cdot 1$$

$$-\left\{pu + \frac{\partial}{\partial x}(pu)dx + \rho u\left(e + u^2/2\right) + \frac{\partial}{\partial x}\left[\rho u\left(e + u^2/2\right)\right] dx\right\} dy \cdot 1 - \left[\tau u - k\frac{\partial T}{\partial y} + \frac{\partial}{\partial y}\left[-k\frac{\partial T}{\partial y}\right] dy\right] dx \cdot 1 = 0$$

or,

$$\frac{\partial(\tau u)}{\partial y} - \frac{\partial}{\partial x}(pu) - \frac{\partial}{\partial x}\left[\rho u\left(e + u^2/2\right)\right] + \frac{\partial}{\partial y}\left[k\frac{\partial T}{\partial y}\right] = 0$$

$$\tau\frac{\partial u}{\partial y} + u\frac{\partial \tau}{\partial y} - u\frac{\partial p}{\partial x} + k\frac{\partial^2 T}{\partial y^2} = 0.$$

Noting that the second and third terms cancel from the momentum equation,

$$\mu\left[\frac{\partial u}{\partial y}\right]^2 + k\left[\frac{\partial^2 T}{\partial y^2}\right] = 0.$$ <

PROBLEM 6.17

KNOWN: Oil properties, journal and bearing temperatures, and journal speed for a lightly loaded journal bearing.

FIND: Maximum oil temperature.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Incompressible fluid with constant properties, (3) Clearance is much less than journal radius and flow is Couette.

ANALYSIS: The temperature distribution corresponds to the result obtained in the text Example on Couette flow,

$$T(y) = T_0 + \frac{\mu}{2k}U^2\left[\frac{y}{L} - \left[\frac{y}{L}\right]^2\right].$$

The position of maximum temperature is obtained from

$$\frac{dT}{dy} = 0 = \frac{\mu}{2k}U^2\left[\frac{1}{L} - \frac{2y}{L^2}\right]$$

or, $\qquad y = L/2.$

The temperature is a maximum at this point since $d^2T/dy^2 < 0$. Hence,

$$T_{max} = T(L/2) = T_0 + \frac{\mu}{2k}U^2\left[\frac{1}{2} - \frac{1}{4}\right] = T_0 + \frac{\mu U^2}{8k}$$

$$T_{max} = 40^\circ C + \frac{10^{-2} kg/s \cdot m (10 m/s)^2}{8 \times 0.15 \text{ W/m} \cdot K}$$

$$T_{max} = 40.83^\circ C.$$ <

COMMENTS: Note that T_{max} *increases* with *increasing* μ and U, *decreases* with *increasing* k, and is *independent* of L.

PROBLEM 6.18

KNOWN: Diameter, clearance, rotational speed and fluid properties of a lightly loaded journal bearing. Temperature of bearing.

FIND: (a) Temperature distribution in the fluid, (b) Rate of heat transfer from bearing and operating power.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Incompressible fluid with constant properties, (3) Couette flow.

PROPERTIES: Oil (Given): $\rho = 800\ kg/m^3$, $\nu = 10^{-5}\ m^2/s$, $k = 0.13\ W/m{\cdot}K$; $\mu = \rho\nu = 8 \times 10^{-3}$ kg/s·m.

ANALYSIS: (a) For Couette flow, the velocity distribution is linear, u(y) = U(y/L), and the energy equation and general form of the temperature distribution are

$$k\frac{d^2T}{dy^2} = -\mu\left[\frac{du}{dy}\right]^2 = -\mu\left[\frac{U}{L}\right]^2 \qquad T = -\frac{\mu}{2k}\left[\frac{U}{L}\right]^2 y^2 + \frac{C_1}{k}y + C_2.$$

Considering the boundary conditions $dT/dy)_{y=L} = 0$ and $T(0) = T_0$, find $C_2 = T_0$ and $C_1 = \mu U^2/L$. Hence,

$$T = T_0 + \left(\mu U^2\right)/k\left[\left(y/L\right) - 1/2\left(y/L\right)^2\right]. \qquad <$$

(b) Applying Fourier's law at y = 0, the rate of heat transfer per unit length to the bearing is

$$q' = -k\left(\pi\ D\right)\frac{dT}{dy}\bigg]_{y=0} = -\left(\pi\ D\right)\frac{\mu U^2}{L} = -\left(\pi \times 75\times10^{-3}m\right)\frac{8\times10^{-3}kg/s\cdot m\ \left(14.14\ m/s\right)^2}{0.25\times10^{-3}m} = -1507.5\ W/m$$

where the velocity is determined as

$$U = \left(D/2\right)\omega = 0.0375m\times3600\ rev/min\ \left(2\pi\ rad/rev\right)/\left(60\ s/min\right) = 14.14\ m/s.$$

The journal power requirement is

$$P' = F'_{(y=L)}U = \tau_{s(y=L)}\cdot\pi\ D\cdot U$$

$$P' = 452.5kg/s^2\cdot m\ \left(\pi\times75\times10^{-3}m\right)14.14m/s = 1507.5kg\cdot m/s^3 = 1507.5W/m \qquad <$$

where the shear stress at y = L is

$$\tau_{s(y=L)} = \mu\left(\partial\ u/\partial\ y\right)_{y=L} = \mu\frac{U}{L} = 8\times10^{-3}kg/s\cdot m\left[\frac{14.14\ m/s}{0.25\times10^{-3}m}\right] = 452.5\ kg/s^2\cdot m.$$

COMMENTS: Note that $q' = P'$, which is consistent with the energy conservation requirement.

PROBLEM 6.19

KNOWN: Conditions associated with the Couette flow of air or water.

FIND: (a) Force and power requirements per unit surface area, (b) Viscous dissipation, (c) Maximum fluid temperature.

SCHEMATIC:

$L = 5\,mm$
$u = U(y/L)$

L, y, 0; T_L; T_0; $\rightarrow u = U = 200\,m/s$; Air or water; $u = 0$; $T_L = T_0 = 27°C$

ASSUMPTIONS: (1) Fully-developed Couette flow, (2) Incompressible fluid with constant properties.

PROPERTIES: *Table A-4*, Air (300K): $\mu = 184.6 \times 10^{-7} N\cdot s/m^2$, $k = 26.3 \times 10^{-3} W/m\cdot K$; *Table A-6*, Water (300K): $\mu = 855 \times 10^{-6} N\cdot s/m^2$, $k = 0.613\ W/m\cdot K$.

ANALYSIS: (a) The force per unit area is associated with the shear stress. Hence, with the linear velocity profile for Couette flow, $\tau = \mu(du/dy) = \mu(U/L)$.

Air: $$\tau_{air} = 184.6 \times 10^{-7}\ N\cdot s/m^2 \times \frac{200\ m/s}{0.005\ m} = 0.738\ N/m^2$$ <

Water: $$\tau_{water} = 855 \times 10^{-6}\ N\cdot s/m^2 \times \frac{200\ m/s}{0.005\ m} = 34.2\ N/m^2.$$

With the required power given by P/A = $\tau \cdot U$,

Air: $$(P/A)_{air} = \left(0.738\ N/m^2\right) 200\ m/s = 147.6\ W/m^2$$ <

Water: $$(P/A)_{water} = \left(34.2\ N/m^2\right) 200\ m/s = 6840\ W/m^2.$$

(b) The viscous dissipation is $\mu\Phi = \mu(du/dy)^2 = \mu(U/L)^2$. Hence,

Air: $$(\mu\Phi)_{air} = 184.6 \times 10^{-7} \frac{N\cdot s}{m^2}\left[\frac{200\ m/s}{0.005\ m}\right]^2 = 2.95 \times 10^4\ W/m^3$$ <

Water: $$(\mu\Phi)_{water} = 855 \times 10^{-6} \frac{N\cdot s}{m^2}\left[\frac{200\ m/s}{0.005\ m}\right]^2 = 1.37 \times 10^6 W/m^3.$$

(c) From the solution to Part 4 of the text Example, the location of the maximum temperature corresponds to $y_{max} = L/2$. Hence, $T_{max} = T_0 + \mu U^2/8k$ and

Air: $$(T_{max})_{air} = 27°C + \frac{184.6 \times 10^{-7}\ N\cdot s/m^2 (200\ m/s)^2}{8 \times 0.0263\ W/m\cdot K} = 30.5°C$$ <

Water: $$(T_{max})_{water} = 27°C + \frac{855 \times 10^{-6}\ N\cdot s/m^2 (200\ m/s)^2}{8 \times 0.613\ W/m\cdot K} = 34.0°C.$$

COMMENTS: (1) The viscous dissipation associated with the entire fluid layer, $\mu\Phi(LA)$, must equal the power, P. (2) Although $(\mu\Phi)_{water} >> (\mu\Phi)_{air}$, $k_{water} >> k_{air}$. Hence, $T_{max,water} \approx T_{max,air}$.

PROBLEM 6.20

KNOWN: Velocity and temperature difference of plates maintaining Couette flow. Mean temperature of air, water or oil between the plates.

FIND: (a) Pr·Ec product for each fluid, (b) Pr·Ec product for air with plate at sonic velocity.

SCHEMATIC:

T_L → $U = 10$ m/s or c

$T_0 - T_L = 25°C$ Air, water, or engine oil, $T = 300K$

T_0

ASSUMPTIONS: (1) Steady-state conditions, (2) Couette flow, (3) Air is at 1 atm.

PROPERTIES: *Table A-4*, Air (300K, 1atm), c_p = 1007 J/kg·K, Pr = 0.707, γ = 1.4, R= 287.02 J/kg·K; *Table A-6*, Water (300K): c_p = 4179 J/kg·K, Pr = 5.83; *Table A-5*, Engine oil (300K), c_p = 1909 J/kg·K, Pr = 6400.

ANALYSIS: The product of the Prandtl and Eckert numbers is dimensionless,

$$\Pr \cdot Ec = \Pr \frac{U^2}{c_p \Delta T} \frown \frac{m^2/s^2}{(J/kg \cdot K)K} \frown \frac{m^2/s^2}{\left(kg \cdot m^2/s^2\right)/kg}.$$

Substituting numerical values, find

	Air	*Water*	*Oil*
Pr·Ec	0.0028	0.0056	13.41

<

(b) For an ideal gas, the speed of sound is

$$c = (\gamma \, R \, T)^{1/2}$$

where R, the gas constant for air, is R_u/M = 8.315 kJ/kmol·K/(28.97 kg/kmol) = 287.02 J/kg·K. Hence, at 300K for air,

$$U = c = (1.4 \times 287.02 \text{ J/kg} \cdot \text{K} \times 300\text{K})^{1/2} = 347.2 \text{ m/s}.$$

For sonic velocities, it follows that

$$\Pr \cdot Ec = 0.707 \frac{(347.2 \text{ m/s})^2}{1007 \text{J/kg} \cdot \text{K} \times 25\text{K}} = 3.38. \quad <$$

COMMENTS: From the above results it follows that viscous dissipation effects must be considered in the high speed flow of gases and in oil flows at moderate speeds. For Pr·Ec to be less than 0.1 in air with ΔT = 25°C, U should be $\lesssim$ 60 m/s.

PROBLEM 6.21

KNOWN: Couette flow with moving plate isothermal and stationary plate insulated.

FIND: Temperature of stationary plate and heat flux at the moving plate.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Incompressible fluid with constant properties, (3) Couette flow.

ANALYSIS: The energy equation is given by

$$0 = k\left[\frac{\partial^2 T}{\partial y^2}\right] + \mu\left[\frac{\partial u}{\partial y}\right]^2$$

Integrating twice find the general form of the temperature distribution,

$$\frac{\partial^2 T}{\partial y^2} = -\frac{\mu}{k}\left[\frac{U}{L}\right]^2 \qquad \frac{\partial T}{\partial y} = -\frac{\mu}{k}\left[\frac{U}{L}\right]^2 y + C_1$$

$$T(y) = -\frac{\mu}{2k}\left[\frac{U}{L}\right]^2 y^2 + C_1 y + C_2.$$

Consider the boundary conditions to evaluate the constants,

$$\partial T/\partial y\Big|_{y=0} = 0 \;\rightarrow\; C_1 = 0 \;\text{ and }\; T(L) = T_L \;\rightarrow\; C_2 = T_L + \frac{\mu}{2k}U^2.$$

Hence, the temperature distribution is

$$T(y) = T_L + \left[\frac{\mu U^2}{2k}\right]\left[1 - \left[\frac{y}{L}\right]^2\right].$$

The temperature of the lower plate (y = 0) is

$$T(0) = T_L + \left[\frac{\mu U^2}{2k}\right]. \qquad <$$

The heat flux to the upper plate (y = L) is

$$q''(L) = -k\frac{\partial T}{\partial y}\Big|_{y=L} = \frac{\mu U^2}{L}. \qquad <$$

COMMENTS: The heat flux at the top surface may also be obtained by integrating the viscous dissipation over the fluid layer height. For a control volume about a unit area of the fluid layer,

$$\dot{E}''_g = \dot{E}''_{out} \qquad \int_0^L \mu\left[\frac{\partial u}{\partial y}\right]^2 dy = q''(L) \qquad q''(L) = \frac{\mu U^2}{L}.$$

PROBLEM 6.22

KNOWN: Couette flow with heat transfer. Lower (insulated) plate moves with speed U and upper plate is stationary with prescribed thermal conductivity and thickness. Outer surface of upper plate maintained at constant temperature, $T_{sp} = 40°C$.

FIND: (a) On T-y coordinates, sketch the temperature distribution in the oil and the stationary plate, and (b) An expression for the temperature at the lower surface of the oil film, $T(0) = T_o$, in terms of the plate speed U, the stationary plate parameters (T_{sp}, k_{sp}, L_{sp}) and the oil parameters (μ, k_o, L_o). Determine this temperature for the prescribed conditions.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Fully developed Couette flow and (3) Incompressible fluid with constant properties.

ANALYSIS: (a) The temperature distribution is shown above with these key features: linear in plate, parabolic in oil film, discontinuity at plate-oil interface, and zero gradient at lower plate surface.

(b) From Example 6.4, the general solution to the conservation equations for the temperature distribution in the oil film is

$$T_o(y) = -Ay^2 + C_3 y + C_4 \qquad \text{where} \qquad A = \frac{\mu}{2k_o}\left(\frac{U}{L_o}\right)^2$$

and the boundary conditions are,

At y = 0, insulated boundary $\qquad \left.\frac{dT_o}{dy}\right)_{y=0} = 0; \qquad C_3 = 0$

At y = L_o, heat fluxes in oil and plate are equal, $\qquad q''_o(L_o) = q''_{sp}(L_o)$

Continued...

$$-k\frac{dT_o}{dy}\bigg)_{y=L_o} = \frac{T_o(L_o)-T_{sp}}{R_{sp}} \qquad \begin{cases} \dfrac{dT_o}{dy}\bigg)_{y=L} = -2AL_o \\ R_{sp} = L_{sp}/k_{sp} \end{cases} \qquad T_o(L) = -AL_o^2 + C_4$$

$$C_4 = T_{sp} + AL_o^2\left[1 + 2\frac{k_o}{L_o}\frac{L_{sp}}{k_{sp}}\right]$$

Hence, the temperature distribution at the lower surface is

$$T_o(0) = -A \cdot 0 + C_4$$

$$T_o(0) = T_{sp} + \frac{\mu}{2k_o}U^2\left[1 + 2\frac{k_o}{L_o}\frac{L_{sp}}{k_{sp}}\right]$$ <

Substituting numerical values, find

$$T_o(0) = 40^\circ C + \frac{0.799\, N\cdot s/m^2}{2\times 0.145\, W/m\cdot K}(5\,m/s)^2\left[1 + 2\frac{0.145}{5}\times\frac{3}{1.5}\right] = 116.9^\circ C$$ <

COMMENTS: (1) Give a physical explanation about why the maximum temperature occurs at the lower surface.

(2) Sketch the temperature distribution if the upper plate moved with a speed U while the lower plate is stationary and all other conditions remain the same.

PROBLEM 6.23

KNOWN: Shaft of diameter 100 mm rotating at 9000 rpm in a journal bearing of 70 mm length. Uniform gap of 1 mm separates the shaft and bearing filled with lubricant. Outer surface of bearing is water-cooled and maintained at T_{wc} = 30°C.

FIND: (a) Viscous dissipation in the lubricant, $\mu\Phi(W/m^3)$, (b) Heat transfer rate from the lubricant, assuming no heat lost through the shaft, and (c) Temperatures of the bearing and shaft, T_b and T_s.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Fully developed Couette flow, (3) Incompressible fluid with constant properties, and (4) Negligible heat lost through the shaft.

ANALYSIS: (a) The viscous dissipation, $\mu\Phi$, Eq. 6.40, for Couette flow from Example 6.4, is

$$\mu\Phi = \mu\left(\frac{du}{dy}\right)^2 = \mu\left(\frac{U}{L}\right)^2 = 0.03\,\mathrm{N\cdot s/m^2}\left(\frac{47.1\,\mathrm{m/s}}{0.001\mathrm{m}}\right)^2 = 6.656\times10^7\ \mathrm{W/m^3}$$ <

where the velocity distribution is linear and the tangential velocity of the shaft is

$$U = \pi DN = \pi(0.100\,\mathrm{m})\times 9000\,\mathrm{rpm}\times(\mathrm{min}/60\mathrm{s}) = 47.1\,\mathrm{m/s}.$$

(b) The heat transfer rate from the lubricant volume $\forall$ through the bearing is

$$q = \mu\Phi\cdot\forall = \mu\Phi(\pi D\cdot L\cdot\ell) = 6.65\times10^7\ \mathrm{W/m^3}(\pi\times0.100\,\mathrm{m}\times0.001\,\mathrm{m}\times0.070\,\mathrm{m}) = 1462\,\mathrm{W}$$ <

where ℓ = 70 mm is the length of the bearing normal to the page.

Continued...

PROBLEM 6.23 (Cont.)

(c) From Fourier's law, the heat rate through the bearing material of inner and outer diameters, D_i and D_o, and thermal conductivity k_b is, from Eq. (3.27),

$$q_r = \frac{2\pi \ell k_b \left(T_b - T_{wc}\right)}{\ln\left(D_o / D_i\right)}$$

$$T_b = T_{wc} + \frac{q_r \ln\left(D_o / D_i\right)}{2\pi \ell k_b}$$

$$T_b = 30^\circ C + \frac{1462\, W \ln\left(200/100\right)}{2\pi \times 0.070\, m \times 45\, W/m \cdot K} = 81.2^\circ C$$ <

To determine the temperature of the shaft, $T(0) = T_s$, first the temperature distribution must be found beginning with the general solution, Example 6.4,

$$T(y) = -\frac{\mu}{2k}\left(\frac{U}{L}\right)^2 y^2 + C_3 y + C_4$$

The boundary conditions are, at y = 0, the surface is adiabatic

$$\left.\frac{dT}{dy}\right)_{y=0} = 0 \qquad C_3 = 0$$

and at y = L, the temperature is that of the bearing, T_b

$$T(L) = T_b = -\frac{\mu}{2k}\left(\frac{U}{L}\right)^2 L^2 + 0 + C_4 \qquad C_4 = T_b + \frac{\mu}{2k} U^2$$

Hence, the temperature distribution is

$$T(y) = T_b + \frac{\mu}{2k} U^2 \left(1 - \frac{y^2}{L^2}\right)$$

and the temperature at the shaft, y = 0, is

$$T_s = T(0) = T_b + \frac{\mu}{2k} U^2 = 81.3^\circ C + \frac{0.03\, N \cdot s/m^2}{2 \times 0.15\, W/m \cdot K}\left(47.1\, m/s\right)^2 = 303^\circ C$$ <

PROBLEM 6.24

KNOWN: Couette flow with heat transfer.

FIND: (a) Dimensionless form of temperature distribution, (b) Conditions for which top plate is adiabatic, (c) Expression for heat transfer to lower plate when top plate is adiabatic.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) incompressible fluid with constant properties, (3) Negligible body forces, (4) Couette flow.

ANALYSIS: (a) From Example 6.4, the temperature distribution is

$$T = T_0 + \frac{\mu}{2k} U^2 \left[\frac{y}{L} - \left(\frac{y}{L} \right)^2 \right] + \left(T_L - T_0 \right) \frac{y}{L}$$

$$\frac{T - T_0}{T_L - T_0} = \frac{\mu U^2}{2k \left(T_L - T_0 \right)} \left[\frac{y}{L} - \left(\frac{y}{L} \right)^2 \right] + \frac{y}{L}$$

or, with

$$\theta \equiv \left(T - T_0 \right) / T_L - T_0 , \qquad \eta \equiv y/L ,$$

$$\Pr \equiv c_p \mu / k , \qquad Ec \equiv U^2 / c_p \left(T_L - T_0 \right)$$

$$\theta = \frac{\Pr \cdot Ec}{2} \left(\eta - \eta^2 \right) + \eta = \eta \left[1 + \frac{1}{2} \Pr \cdot Ec \left(1 - \eta \right) \right] \qquad (1)$$ <

(b) For there to be zero heat transfer at the top plate, $dT/dy)_{y=L} = 0$. Hence,

$$\left. \frac{d\theta}{d\eta} \right)_{\eta=1} \cdot \frac{T_L - T_0}{L} = \frac{\Pr \cdot Ec}{2} \left(1 - 2\eta \right) \Big|_{\eta=1} + 1 = -\frac{\Pr \cdot Ec}{2} + 1 = 0$$

There is no heat transfer at the top plate if,

$$Ec \cdot \Pr = 2. \qquad (2)$$ <

(c) The heat transfer rate to the lower plate (per unit area) is

$$q_0'' = -k \frac{dT}{dy} \Big|_{y=0} = -k \frac{\left(T_L - T_0 \right)}{L} \frac{d\theta}{d\eta} \Big|_{\eta=0}$$

$$q_o'' = -k \frac{T_L - T_0}{L} \left[\frac{\Pr \cdot Ec}{2} \left(1 - 2\eta \right) \Big|_{\eta=0} + 1 \right]$$

$$q_0'' = -k \frac{T_L - T_0}{L} \left(\frac{\Pr \cdot Ec}{2} + 1 \right) = -2k \left(T_L - T_0 \right) / L$$ <

Continued...

PROBLEM 6.24 (Cont.)

(d) Using Eq. (1), the dimensionless temperature distribution is plotted as a function of dimensionless distance, $\eta = y/L$. When Pr·Ec = 0, there is no dissipation and the temperature distribution is linear, so that heat transfer is by conduction only. As Pr ·Ec increases, viscous dissipation becomes more important. When Pr·Ec = 2, heat transfer to the upper plate is zero. When Pr·Ec > 2, the heat rate is out of the oil film at both surfaces.

theta = (T(y)-T0)/(TL-T0)

eta = y/L

Pr*Ec = 0, conduction
Pr*Ec = 1
Pr*Ec = 2, adiabatic at y=L
Pr*Ec = 4
Pr*Ec = 10

PROBLEM 6.25

KNOWN: Steady, incompressible, laminar flow between infinite parallel plates at different temperatures.

FIND: (a) Form of continuity equation, (b) Form of momentum equations and velocity profile. Relationship of pressure gradient to maximum velocity, (c) Form of energy equation and temperature distribution. Heat flux at top surface.

SCHEMATIC:

ASSUMPTIONS: (1) Two-dimensional flow (no variations in z) between infinite, parallel plates, (2) Negligible body forces, (3) No internal energy generation, (4) Incompressible fluid with constant properties.

ANALYSIS: (a) For two-dimensional, steady conditions, the continuity equation is

$$\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}=0.$$

Hence, for an incompressible fluid (constant ρ) in parallel flow (v = 0),

$$\frac{\partial u}{\partial x}=0.$$ <

The flow is fully developed in the sense that, irrespective of y, u is independent of x.

(b) With the above result and the prescribed conditions, the momentum equations reduce to

$$0=-\frac{\partial p}{\partial x}+\mu\frac{\partial^2 u}{\partial y^2} \qquad 0=-\frac{\partial p}{\partial y}$$ <

Since p is independent of y, ∂p/∂x = dp/dx is independent of y and

$$\mu\frac{\partial^2 u}{\partial y^2}=\mu\frac{d^2u}{dy^2}=\frac{dp}{dx}.$$

Since the left-hand side can, at most, depend only on y and the right-hand side is independent of y, both sides must equal the same constant C. That is,

$$\mu\frac{d^2u}{dy^2}=C.$$

Hence, the velocity distribution has the form

$$u(y)=\frac{C}{2\mu}y^2+C_1y+C_2.$$

Using the boundary conditions to evaluate the constants,

$$u(0)=0 \quad\rightarrow\quad C_2=0 \quad \text{and} \quad u(L)=0 \quad\rightarrow\quad C_1=-CL/2\mu.$$

Continued …..

PROBLEM 6.25 (Cont.)

The velocity profile is $u(y) = \frac{C}{2\mu}\left(y^2 - Ly\right).$

The profile is symmetric about the midplane, in which case the maximum velocity exists at y = L/2. Hence,

$$u(L/2) = u_{max} = \frac{C}{2\mu}\left[-\frac{L^2}{4}\right] \qquad \text{or} \qquad u_{max} = -\frac{L^2}{8\mu}\frac{dp}{dx}. \qquad <$$

(c) For fully developed thermal conditions, $(\partial T/\partial x) = 0$ and temperature depends only on y. Hence with v = 0, $\partial u/\partial x = 0$, and the prescribed assumptions, the energy equation becomes

$$\rho u \frac{\partial i}{\partial x} = k\frac{d^2T}{dy^2} + u\frac{dp}{dx} + \mu\left[\frac{du}{dy}\right]^2.$$

With $i = e + p/\rho$, $\quad \frac{\partial i}{\partial x} = \frac{\partial e}{\partial x} + \frac{1}{\rho}\frac{dp}{dx} \quad$ where $\quad \frac{\partial e}{\partial x} = \frac{\partial e}{\partial T}\frac{\partial T}{\partial x} + \frac{\partial e}{\partial \rho}\frac{\partial \rho}{\partial x} = 0.$

Hence, the energy equation becomes $\qquad 0 = k\frac{d^2T}{dy^2} + \mu\left[\frac{du}{dy}\right]^2. \qquad <$

With du/dy = (C/2μ) (2y - L), it follows that

$$\frac{d^2T}{dy^2} = -\frac{C^2}{4k\mu}\left(4y^2 - 4Ly + L^2\right).$$

Integrating twice,

$$T(y) = -\frac{C^2}{4k\mu}\left[\frac{y^4}{3} - \frac{2Ly^3}{3} + \frac{L^2y^2}{2}\right] + C_3y + C_4$$

Using the boundary conditions to evaluate the constants,

$$T(0) = T_2 \quad \rightarrow \quad C_4 = T_2 \quad \text{and} \quad T(L) = T_1 \quad \rightarrow \quad C_3 = \frac{C^2L^3}{24k\mu} + \frac{(T_1 - T_2)}{L}.$$

Hence, $\qquad T(y) = T_2 + \left[\frac{y}{L}\right](T_1 - T_2) - \frac{C^2}{4k\mu}\left[\frac{y^4}{3} - \frac{2Ly^3}{3} + \frac{L^2y^2}{2} - \frac{L^3y}{6}\right]. \qquad <$

From Fourier's law,

$$q''(L) = -k\frac{\partial T}{\partial y}\bigg|_{y=L} = \frac{k}{L}(T_2 - T_1) + \frac{C^2}{4\mu}\left[\frac{4}{3}L^3 - 2L^3 + L^3 - \frac{L^3}{6}\right]$$

$$q''(L) = \frac{k}{L}(T_2 - T_1) + \frac{C^2L^3}{24\mu}. \qquad <$$

COMMENTS: The third and second terms on the right-hand sides of the temperature distribution and heat flux, respectively, represents the effects of viscous dissipation. If C is large (due to large μ or u_{max}), viscous dissipation is significant. If C is small, conduction effects dominate.

PROBLEM 6.26

KNOWN: Pressure independence of μ, k and c_p.

FIND: Pressure dependence of ν and α for air at 350K and p = 1, 10 atm.

ASSUMPTIONS: Perfect gas behavior for air.

PROPERTIES: *Table A-4*, Air (350K, 1 atm): $\nu = 20.92 \times 10^{-6} m^2/s$, $\alpha = 29.9 \times 10^{-6} m^2/s$.

ANALYSIS: The kinematic viscosity and thermal diffusivity are, respectively,

$$n = m / r \qquad a = k/r\, c_p.$$

Hence, ν and α are inversely proportional to ρ.

For an *incompressible liquid,* ρ is constant.

Hence ν and α are independent of pressure. <

For a *perfect gas*, ρ = p/RT.

Hence, ρ is directly proportional to p, in which case ν and α vary inversely with

pressure. It follows that ν and α are inversely proportional to pressure. <

To calculate ν or α for a *perfect* gas at p ≠ 1 atm,

$$n(p) = n(1\text{ atm}) \cdot \frac{1}{p}$$

$$a(p) = a(1\text{ atm}) \cdot \frac{1}{p}$$

Hence, for air at 350K,

p(atm)	$\nu(m^2/s)$	$\alpha(m^2/s)$
1	20.92×10^{-6}	29.9×10^{-6}
10	2.09×10^{-6}	2.99×10^{-6}

COMMENTS: For the incompressible liquid and the perfect gas, Pr = ν/α is independent of pressure.

PROBLEM 6.27

KNOWN: Characteristic length, surface temperature and average heat flux for an object placed in an airstream of prescribed temperature and velocity.

FIND: Average convection coefficient if characteristic length of object is increased by a factor of five and air velocity is decreased by a factor of five.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties.

ANALYSIS: For a particular geometry,

$$\overline{Nu}_L = f\left(Re_L, Pr\right).$$

The Reynolds numbers for each case are

Case 1: $$Re_{L,1} = \frac{V_1 L_1}{\nu_1} = \frac{(100\,m/s)1m}{\nu_1} = \frac{100\ m^2/s}{\nu_1}$$

Case 2: $$Re_{L,2} = \frac{V_2 L_2}{\nu_2} = \frac{(20\,m/s)5m}{\nu_2} = \frac{100\ m^2/s}{\nu_2}.$$

Hence, with $\nu_1 = \nu_2$, $Re_{L,1} = Re_{L,2}$. Since $Pr_1 = Pr_2$, it follows that

$$\overline{Nu}_{L,2} = \overline{Nu}_{L,1}.$$

Hence,

$$\bar{h}_2 L_2 / k_2 = \bar{h}_1 L_1 / k_1$$

$$\bar{h}_2 = \bar{h}_1 \frac{L_1}{L_2} = 0.2\,\bar{h}_1.$$

For *Case 1*, using the rate equation, the convection coefficient is

$$q_1 = \bar{h}_1 A_1 \left(T_s - T_\infty\right)_1$$

$$\bar{h}_1 = \frac{(q_1/A_1)}{(T_s - T_\infty)_1} = \frac{q_1''}{(T_s - T_\infty)_1} = \frac{20,000\ W/m^2}{(400-300)K} = 200\ W/m^2 \cdot K.$$

Hence, it follows that for *Case 2*

$$\bar{h}_2 = 0.2 \times 200\ W/m^2 \cdot K = 40\ W/m^2 \cdot K.$$ <

COMMENTS: If $Re_{L,2}$ were *not* equal to $Re_{L,1}$, it would be necessary to know the specific form of $f(Re_L, Pr)$ before $\bar{h}_2$ could be determined.

PROBLEM 6.28

KNOWN: Heat transfer rate from a turbine blade for prescribed operating conditions.

FIND: Heat transfer rate from a larger blade operating under different conditions.

SCHEMATIC:

Air $T_\infty = 35°C$, $V_1 = 100$ m/s; $q_1 = 1500$ W; A_1; $T_{s,1} = 300°C$; $L_1 = 0.15$ m

Air $T_\infty = 35°C$, $V_2 = 50$ m/s; q_2; A_2; $T_{s,2} = 400°C$; $L_2 = 0.3$ m

ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Surface area A is directly proportional to characteristic length L, (4) Negligible radiation, (5) Blade shapes are geometrically similar.

ANALYSIS: For a prescribed geometry,

$$\overline{Nu} = \frac{\overline{h}L}{k} = f\left(Re_L, Pr\right).$$

The Reynolds numbers for the blades are

$$Re_{L,1} = \left(V_1 L_1 / \nu\right) = 15/\nu \qquad Re_{L,2} = \left(V_2 L_2 / \nu\right) = 15/\nu.$$

Hence, with constant properties, $Re_{L,1} = Re_{L,2}$. Also, $Pr_1 = Pr_2$. Therefore,

$$\overline{Nu}_2 = \overline{Nu}_1$$
$$\left(\overline{h}_2 L_2 / k\right) = \left(\overline{h}_1 L_1 / k\right)$$
$$\overline{h}_2 = \frac{L_1}{L_2}\overline{h}_1 = \frac{L_1}{L_2}\frac{q_1}{A_1\left(T_{s,1} - T_\infty\right)}.$$

Hence, the heat rate for the *second blade* is

$$q_2 = \overline{h}_2 A_2\left(T_{s,2} - T_\infty\right) = \frac{L_1}{L_2}\;\frac{A_2}{A_1}\;\frac{\left(T_{s,2} - T_\infty\right)}{\left(T_{s,1} - T_\infty\right)} q_1$$
$$q_2 = \frac{T_{s,2} - T_\infty}{T_{s,1} - T_\infty} q_1 = \frac{(400 - 35)}{(300 - 35)}(1500 \text{ W})$$

$$q_2 = 2066 \text{ W}.$$ <

COMMENTS: The slight variation of ν from Case 1 to Case 2 would cause $Re_{L,2}$ to differ from $Re_{L,1}$. However, for the prescribed conditions, this non-constant property effect is small.

PROBLEM 6.29

KNOWN: Experimental measurements of the heat transfer coefficient for a square bar in cross flow.

FIND: (a) $\bar{h}$ for the condition when L = 1m and V = 15m/s, (b) $\bar{h}$ for the condition when L = 1m and V = 30m/s, (c) Effect of defining a side as the characteristic length.

SCHEMATIC:

V
Air
L = 0.5m
Results
$V_1 = 20 m/s$ $\bar{h}_1 = 50 W/m^2 \cdot K$
$V_2 = 15 m/s$ $\bar{h}_2 = 40 W/m^2 \cdot K$

ASSUMPTIONS: (1) Functional form $\overline{Nu} = CRe^m Pr^n$ applies with C, m, n being constants, (2) Constant properties.

ANALYSIS: (a) For the experiments and the condition L = 1m and V = 15m/s, it follows that Pr as well as C, m, and n are constants. Hence

$$\bar{h}L \ \alpha \ (VL)^m .$$

Using the experimental results, find m. Substituting values

$$\frac{\bar{h}_1 L_1}{\bar{h}_2 L_2} = \left[\frac{V_1 L_1}{V_2 L_2}\right]^m \qquad \frac{50 \times 0.5}{40 \times 0.5} = \left[\frac{20 \times 0.5}{15 \times 0.5}\right]^m$$

giving m = 0.782. It follows then for L = 1m and V = 15m/s,

$$\bar{h} = \bar{h}_1 \frac{L_1}{L}\left[\frac{V \cdot L}{V_1 \cdot L_1}\right]^m = 50 \frac{W}{m^2 \cdot K} \times \frac{0.5}{1.0}\left[\frac{15 \times 1.0}{20 \times 0.5}\right]^{0.782} = 34.3 W/m^2 \cdot K. \qquad <$$

(b) For the condition L = 1m and V = 30m/s, find

$$\bar{h} = \bar{h}_1 \frac{L_1}{L}\left[\frac{V \cdot L}{V_1 \cdot L_1}\right]^m = 50 \frac{W}{m^2 \cdot K} \times \frac{0.5}{1.0}\left[\frac{30 \times 1.0}{20 \times 0.5}\right]^{0.782} = 59.0 W/m^2 \cdot K. \qquad <$$

(c) If the characteristic length were chosen as a side rather than the diagonal, the value of C would change. However, the coefficients m and n would not change.

COMMENTS: The foregoing Nusselt number relation is used frequently in heat transfer analysis, providing appropriate scaling for the effects of length, velocity, and fluid properties on the heat transfer coefficient.

PROBLEM 6.30

KNOWN: Local Nusselt number correlation for flow over a roughened surface.

FIND: Ratio of average heat transfer coefficient to local coefficient.

SCHEMATIC:

V

$Nu_x = 0.04 Re_x^{0.9} Pr^{1/3}$

x

ANALYSIS: The local convection coefficient is obtained from the prescribed correlation,

$$h_x = Nu_x \frac{k}{x} = 0.04 \frac{k}{x} Re_x^{0.9} Pr^{1/3}$$

$$h_x = 0.04\ k \left[\frac{V}{\nu}\right]^{0.9} Pr^{1/3} \frac{x^{0.9}}{x} \equiv C_1 x^{-0.1}.$$

To determine the average heat transfer coefficient for the length zero to x,

$$\bar{h}_x \equiv \frac{1}{x}\int_0^x h_x\ dx = \frac{1}{x} C_1 \int_0^x x^{-0.1} dx$$

$$\bar{h}_x = \frac{C_1}{x}\frac{x^{0.9}}{0.9} = 1.11\ C_1\ x^{-0.1}.$$

Hence, the ratio of the average to local coefficient is

$$\frac{\bar{h}_x}{h_x} = \frac{1.11\ C_1\ x^{-0.1}}{C_1\ x^{-0.1}} = 1.11.$$ <

COMMENTS: Note that $\overline{Nu}_x / Nu_x$ is also equal to 1.11. Note, however, that

$$\overline{Nu}_x \neq \frac{1}{x}\int_0^x Nu_x\ dx.$$

PROBLEM 6.31

KNOWN: Freestream velocity and average convection heat transfer associated with fluid flow over a surface of prescribed characteristic length.

FIND: Values of $\overline{Nu}_L, Re_L, Pr, \bar{j}_H$ for (a) air, (b) engine oil, (c) mercury, (d) water.

SCHEMATIC:

V = 1 m/s
Air, engine oil, mercury or water
L = 1 m
$\bar{h}$ = 100 W/m²·K
T = 300 K

PROPERTIES: For the fluids at 300K:

Fluid	Table	$\nu(m^2/s)$	k(W/m·K)	$\alpha(m^2/s)$	Pr
Air	A.4	15.89×10^{-6}	0.0263	22.5×10^{-7}	0.71
Engine Oil	A.5	550×10^{-6}	0.145	0.859×10^{-7}	6400
Mercury	A.5	0.113×10^{-6}	8.54	45.30×10^{-7}	0.025
Water	A.6	0.858×10^{-6}	0.613	1.47×10^{-7}	5.83

ANALYSIS: The appropriate relations required are

$$\overline{Nu}_L = \frac{\bar{h}L}{k} \qquad Re_L = \frac{VL}{\nu} \qquad Pr = \frac{\nu}{\alpha} \qquad j_H = \overline{St}Pr^{2/3} \qquad \overline{St} = \frac{\overline{Nu}_L}{Re_L\, Pr}$$

Fluid	$\overline{Nu}_L$	Re_L	Pr	$\bar{j}_H$
Air	3802	6.29×10^{4}	0.71	0.068
Engine Oil	690	1.82×10^{3}	6403	0.0204
Mercury	11.7	8.85×10^{6}	0.025	4.52×10^{-6}
Water	163	1.17×10^{6}	5.84	7.74×10^{-5}

<

COMMENTS: Note the wide range of Pr associated with the fluids.

PROBLEM 6.32

KNOWN: Variation of h_x with x for flow over a flat plate.

FIND: Ratio of average Nusselt number for the entire plate to the local Nusselt number at x = L.

SCHEMATIC:

Thermal boundary layer, $h_x = Cx^{-1/2}$ where C is a constant

ANALYSIS: The expressions for the local and average Nusselt numbers are

$$Nu_L = \frac{h_L L}{k} = \frac{\left(CL^{-1/2}\right)L}{k} = \frac{CL^{1/2}}{k}$$

$$\overline{Nu}_L = \frac{\overline{h}_L L}{k}$$

where

$$\overline{h}_L = \frac{1}{L}\int_0^L h_x \, dx = \frac{C}{L}\int_0^L x^{-1/2} dx = \frac{2C}{L} L^{1/2} = 2\, CL^{-1/2}.$$

Hence,

$$\overline{Nu}_L = \frac{2\, CL^{-1/2}(L)}{k} = \frac{2\, CL^{1/2}}{k}$$

and

$$\frac{\overline{Nu}_L}{Nu_L} = 2.$$ <

COMMENTS: Note the manner in which $\overline{Nu}_L$ is defined in terms of $\overline{h}_L$. Also note that

$$\overline{Nu}_L \neq \frac{1}{L}\int_0^L Nu_x \, dx.$$

PROBLEM 6.33

KNOWN: Laminar boundary layer flow of air at 20°C and 1 atm having $\delta_t = 1.13\ \delta$.

FIND: Ratio δ / δ_t when fluid is ethylene glycol for same conditions.

SCHEMATIC:

ASSUMPTIONS: (1) Laminar flow.

PROPERTIES: *Table A-4,* Air (293K, 1 atm): Pr = 0.709; *Table A-5,* Ethylene glycol (293K): Pr = 211.

ANALYSIS: The Prandtl number strongly influences relative growth of the velocity, δ, and thermal, δ_t, boundary layers. For laminar flow, the approximate relationship is given by

$$\mathrm{Pr}^n \approx \frac{\delta}{\delta_t}$$

where n is a positive coefficient. Substituting the values for air

$$(0.709)^n = \frac{1}{1.13}$$

find that n = 0.355. Hence, for ethylene glycol it follows that

$$\frac{\delta}{\delta_t} = \mathrm{Pr}^{0.355} = 211^{0.355} = 6.69.$$ <

COMMENTS: (1) For laminar flow, generally we find n = 0.33. In which case, $\delta / \delta_t = 5.85$.

(2) Recognize the physical importance of $\nu > \alpha$, which gives large values of the Prandtl number, and causes $\delta > \delta_t$.

PROBLEM 6.34

KNOWN: Air, water, engine oil or mercury at 300K in laminar, parallel flow over a flat plate.

FIND: Sketch of velocity and thermal boundary layer thickness.

ASSUMPTIONS: (1) Laminar flow.

PROPERTIES: For the fluids at 300K:

Fluid	Table	Pr
Air	A.4	0.71
Water	A.6	5.83
Engine Oil	A.5	6400
Mercury	A.5	0.025

ANALYSIS: For laminar, boundary layer flow over a flat plate.

$$\frac{\delta}{\delta_t} \sim \text{Pr}^n$$

where n > 0. Hence, the boundary layers appear as shown below.

Air:

δ_t

δ

Water:

δ

δ_t

Engine Oil:

δ

δ_t

Mercury:

δ_t

δ

COMMENTS: Although Pr strongly influences relative boundary layer development in laminar flow, its influence is weak for turbulent flow.

PROBLEM 6.35

KNOWN: Expression for the local heat transfer coefficient of air at prescribed velocity and temperature flowing over electronic elements on a circuit board and heat dissipation rate for a 4 × 4 mm chip located 120mm from the leading edge.

FIND: Surface temperature of the chip surface, T_s.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Power dissipated within chip is lost by convection across the upper surface only, (3) Chip surface is isothermal, (4) The average heat transfer coefficient for the chip surface is equivalent to the local value at x = L.

PROPERTIES: *Table A-4*, Air (assume $T_s = 45°C$, $T_f = (45 + 25)/2 = 35°C = 308K$, 1atm): $\nu = 16.69 \times 10^{-6} m^2/s$, $k = 26.9 \times 10^{-3}$ W/m·K, Pr = 0.703.

ANALYSIS: From an energy balance on the chip (see above),

$$q_{conv} = \dot{E}_g = 30W. \tag{1}$$

Newton's law of cooling for the upper chip surface can be written as

$$T_s = T_\infty + q_{conv} / \bar{h}\, A_{chip} \tag{2}$$

where $A_{chip} = \ell^2$. Assume that the *average* heat transfer coefficient $(\bar{h})$ over the chip surface is equivalent to the *local* coefficient evaluated at x = L. That is, $\bar{h}_{chip} \approx h_x(L)$ where the local coefficient can be evaluated from the special correlation for this situation,

$$Nu_x = \frac{h_x x}{k} = 0.04\left[\frac{Vx}{\nu}\right]^{0.85} Pr^{1/3}$$

and substituting numerical values with x = L, find

$$h_x = 0.04\frac{k}{L}\left[\frac{VL}{\nu}\right]^{0.85} Pr^{1/3}$$

$$h_x = 0.04\left[\frac{0.0269\ W/m\cdot K}{0.120\ m}\right]\left[\frac{10\ m/s \times 0.120\ m}{16.69\times10^{-6}\ m^2/s}\right]^{0.85}(0.703)^{1/3} = 107\ W/m^2\cdot K.$$

The surface temperature of the chip is from Eq. (2),

$$T_s = 25°C + 30\times10^{-3}\ W/107\ W/m^2\cdot K\times(0.004m)^2 = 42.5°C.$$ <

COMMENTS: (1) Note that the estimated value for T_f used to evaluate the air properties was reasonable. (2) Alternatively, we could have evaluated $\bar{h}_{chip}$ by performing the integration of the local value, h(x).

PROBLEM 6.36

KNOWN: Location and dimensions of computer chip on a circuit board. Form of the convection correlation. Maximum allowable chip temperature and surface emissivity. Temperature of cooling air and surroundings.

FIND: Effect of air velocity on maximum power dissipation, first without and then with consideration of radiation effects.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state, (2) Negligible temperature variations in chip, (3) Heat transfer exclusively from the top surface of the chip, (4) The local heat transfer coefficient at x = L provides a good approximation to the average heat transfer coefficient for the chip surface.

PROPERTIES: *Table A.4*, air ($\overline{T} = (T_\infty + T_c)/2$ = 328 K): ν = 18.71 × 10^{-6} m^2/s, k = 0.0284 W/m·K, Pr = 0.703.

ANALYSIS: Performing an energy balance for a control surface about the chip, we obtain $P_c = q_{conv} + q_{rad}$, where $q_{conv} = \overline{h}A_s(T_c - T_\infty)$, $q_{rad} = h_r A_s(T_c - T_{sur})$, and $h_r = \varepsilon\sigma(T_c + T_{sur})(T_c^2 + T_{sur}^2)$. With $\overline{h} \approx h_L$, the convection coefficient may be determined from the correlation provided in Problem 6.35 ($Nu_L = 0.04\,Re_L^{0.85}\,Pr^{1/3}$). Hence,

$$P_c = \ell^2\left[0.04(k/L)Re_L^{0.85}\,Pr^{1/3}(T_c - T_\infty) + \varepsilon\sigma(T_c + T_{sur})\left(T_c^2 + T_{sur}^2\right)(T_c - T_{sur})\right]$$

where $Re_L = VL/\nu$. Computing the right side of this expression for ε = 0 and ε = 0.85, we obtain the following results.

Since h_L increases as $V^{0.85}$, the chip power must increase with V in the same manner. Radiation exchange increases P_c by a fixed, but small (6 mW) amount. While h_L varies from 14.5 to 223 $W/m^2 \cdot K$ over the prescribed velocity range, h_r = 6.5 $W/m^2 \cdot K$ is a constant, independent of V.

COMMENTS: Alternatively, $\overline{h}$ could have been evaluated by integrating h_x over the range 118 ≤ x ≤ 122 mm to obtain the appropriate average. However, the value would be extremely close to $h_{x=L}$.

PROBLEM 6.37

KNOWN: Form of Nusselt number for flow of air or a dielectric liquid over components of a circuit card.

FIND: Ratios of time constants associated with intermittent heating and cooling. Fluid that provides faster thermal response.

PROPERTIES: Prescribed. Air: k = 0.026 W/m·K, $\nu = 2 \times 10^{-5}$ m^2/s, Pr = 0.71. Dielectric liquid: k = 0.064 W/m·K, $\nu = 10^{-6}$ m^2/s, Pr = 25.

ANALYSIS: From Eq. 5.7, the thermal time constant is

$$\tau_t = \frac{\rho \forall c}{\overline{h} A_s}$$

Since the only variable that changes with the fluid is the convection coefficient, where

$$\overline{h} = \frac{k}{L}\overline{Nu}_L = \frac{k}{L} C \, Re_L^m \, Pr^n = \frac{k}{L} C \left(\frac{VL}{\nu}\right)^m Pr^n$$

the desired ratio reduces to

$$\frac{\tau_{t,air(a)}}{\tau_{t,dielectric(d)}} = \frac{\overline{h}_d}{\overline{h}_a} = \frac{k_d}{k_a}\left(\frac{\nu_a}{\nu_d}\right)^m \left(\frac{Pr_d}{Pr_a}\right)^n$$

$$\frac{\tau_{t,a}}{\tau_{t,d}} = \frac{0.064}{0.026}\left(\frac{2\times 10^{-5}}{10^{-6}}\right)^{0.8}\left(\frac{25}{0.71}\right)^{0.33} = 88.6$$

Since its time constant is nearly two orders of magnitude smaller than that of the air, the dielectric liquid is clearly the fluid of choice.

COMMENTS: The accelerated testing procedure suggested by this problem is commonly used to test the durability of electronic packages.

PROBLEM 6.38

KNOWN: Form of the Nusselt number correlation for forced convection and fluid properties.

FIND: Expression for figure of merit F_F and values for air, water and a dielectric liquid.

PROPERTIES: Prescribed. Air: k = 0.026 W/m·K, $\nu = 1.5 \times 10^{-5}$ m^2/s, Pr = 0.70. Water: k = 0.600 W/m·K, $\nu = 10^{-6}$ m^2/s, Pr = 5.0. Dielectric liquid: k = 0.064 W/m·K, $\nu = 10^{-6}$ m^2/s, Pr = 25

ANALYSIS: With $Nu_L \sim Re_L^m Pr^n$, the convection coefficient may be expressed as

$$h \sim \frac{k}{L}\left(\frac{VL}{\nu}\right)^m Pr^n \sim \frac{V^m}{L^{1-m}}\left(\frac{kPr^n}{\nu^m}\right)$$

The figure of merit is therefore

$$F_F = \frac{kPr^n}{\nu^m} \qquad <$$

and for the three fluids, with m = 0.80 and n = 0.33,

$F_F\left(W \cdot s^{0.8}/m^{2.6} \cdot K\right)$	Air	Water	Dielectric	
	167	64,400	11,700	<

Water is clearly the superior heat transfer fluid, while air is the least effective.

COMMENTS: The figure of merit indicates that heat transfer is enhanced by fluids of large k, large Pr and small ν.

PROBLEM 6.39

KNOWN: Ambient, interior and dewpoint temperatures. Vehicle speed and dimensions of windshield. Heat transfer correlation for external flow.

FIND: Minimum value of convection coefficient needed to prevent condensation on interior surface of windshield.

SCHEMATIC:

$\bar{h}_o$
$T_{\infty,o} = -15^oC$
$T_{s,i} \geq 10^oC$
$\bar{h}_i$
$T_{\infty,i} = 50^oC$
t = 6 mm
L = 800 mm
$T_{\infty,o}$ $T_{s,i}$ $T_{\infty,i}$ q"
$1/\bar{h}_o$ t/k_g $1/\bar{h}_i$

ASSUMPTIONS: (1) Steady-state, (2) One-dimensional heat transfer, (3) Constant properties.

PROPERTIES: *Table A-3*, glass: k_g = 1.4 W/m·K. Prescribed, air: k = 0.023 W/m·K, $\nu = 12.5 \times 10^{-6}$ m^2/s, Pr = 0.70.

ANALYSIS: From the prescribed thermal circuit, conservation of energy yields

$$\frac{T_{\infty,i} - T_{s,i}}{1/\bar{h}_i} = \frac{T_{s,i} - T_{\infty,o}}{t/k_g + 1/\bar{h}_o}$$

where $\bar{h}_o$ may be obtained from the correlation

$$\overline{Nu}_L = \frac{\bar{h}_o L}{k} = 0.030\, Re_L^{0.8}\, Pr^{1/3}$$

With V = (70 mph × 1585 m/mile)/3600 s/h = 30.8 m/s, Re_D = (30.8 m/s × 0.800 m)/12.5 × 10^{-6} m^2/s = 1.97 × 10^6 and

$$\bar{h}_o = \frac{0.023\,W/m\cdot K}{0.800\ m} 0.030\left(1.97\times10^6\right)^{0.8}(0.70)^{1/3} = 83.1\,W/m^2\cdot K$$

From the energy balance, with $T_{s,i} = T_{dp}$ = 10°C

$$\bar{h}_i = \frac{\left(T_{s,i} - T_{\infty,o}\right)}{\left(T_{\infty,i} - T_{s,i}\right)}\left(\frac{t}{k_g} + \frac{1}{\bar{h}_o}\right)^{-1}$$

$$\bar{h}_i = \frac{(10+15)°C}{(50-10)°C}\left(\frac{0.006\ m}{1.4\,W/m\cdot K} + \frac{1}{83.1\,W/m^2\cdot K}\right)^{-1}$$

$$\bar{h}_i = 38.3\,W/m^2\cdot K$$ <

COMMENTS: The output of the fan in the automobile's heater/defroster system must maintain a velocity for flow over the inner surface that is large enough to provide the foregoing value of $\bar{h}_i$. In addition, the output of the heater must be sufficient to maintain the prescribed value of $T_{\infty,i}$ at this velocity.

PROBLEM 6.40

KNOWN: Drag force and air flow conditions associated with a flat plate.

FIND: Rate of heat transfer from the plate.

SCHEMATIC:

$u_\infty = 40$ m/s, $T_\infty = 20°C$, $p = 1$ atm; $T_s = 120°C$; 0.2 m × 0.2 m plate; $F_D = 0.075$ N

ASSUMPTIONS: (1) Chilton-Colburn analogy is applicable.

PROPERTIES: *Table A-4,* Air (70°C,1 atm): $\rho = 1.018$ kg/m^3, $c_p = 1009$ J/kg·K, Pr = 0.70, $\nu = 20.22 \times 10^{-6}$ m^2/s.

ANALYSIS: The rate of heat transfer from the plate is

$$q = 2\bar{h}\left(L^2\right)\left(T_s - T_\infty\right)$$

where $\bar{h}$ may be obtained from the Chilton-Colburn analogy,

$$\bar{j}_H = \frac{\bar{C}_f}{2} = \bar{S}t\,Pr^{2/3} = \frac{\bar{h}}{\rho\, u_\infty\, c_p} Pr^{2/3}$$

$$\frac{\bar{C}_f}{2} = \frac{1}{2}\frac{\bar{\tau}_s}{\rho\, u_\infty^2/2} = \frac{1}{2}\frac{(0.075\text{ N}/2)/(0.2\text{m})^2}{1.018\text{ kg/m}^3(40\text{ m/s})^2/2} = 5.76\times10^{-4}.$$

Hence,

$$\bar{h} = \frac{C_f}{2}\rho\, u_\infty\, c_p\, Pr^{-2/3}$$

$$\bar{h} = 5.76\times10^{-4}\left(1.018\text{kg/m}^3\right)40\text{m/s}\,\left(1009\text{J/kg}\cdot\text{K}\right)\,(0.70)^{-2/3}$$

$$\bar{h} = 30\text{ W/m}^2\cdot\text{K}.$$

The heat rate is

$$q = 2\left(30\text{ W/m}^2\cdot\text{K}\right)(0.2\text{m})^2\,(120-20)^\circ\text{C}$$

$$q = 240\text{ W}.$$ <

COMMENTS: Although the flow is laminar over the entire surface ($Re_L = u_\infty L/\nu = 40$ m/s × 0.2m/20.22 × 10^{-6}m^2/s = 4.0×10^5), the pressure gradient is zero and the Chilton-Colburn analogy is applicable to *average*, as well as *local,* surface conditions. Note that the only contribution to the drag force is made by the surface shear stress.

PROBLEM 6.41

KNOWN: Air flow conditions and drag force associated with a heater of prescribed surface temperature and area.

FIND: Required heater power.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Reynolds analogy is applicable, (3) Bottom surface is adiabatic.

PROPERTIES: *Table A-4*, Air (T_f = 350K, 1atm): ρ = 0.995 kg/m^3, c_p = 1009 J/kg·K, Pr = 0.700.

ANALYSIS: The average shear stress and friction coefficient are

$$\overline{\tau}_s = \frac{F_D}{A} = \frac{0.25\ \text{N}}{0.25\ \text{m}^2} = 1\ \text{N/m}^2$$

$$\overline{C}_f = \frac{\overline{\tau}_s}{\rho\, u_\infty^2/2} = \frac{1\ \text{N/m}^2}{0.995\ \text{kg/m}^3 (15\text{m/s})^2/2} = 8.93\times10^{-3}.$$

From the Reynolds analogy,

$$\overline{St} = \frac{\overline{h}}{\rho\, u_\infty c_p} = \frac{\overline{C}_f}{2} Pr^{-2/3}.$$

Solving for $\overline{h}$ and substituting numerical values, find

$$\overline{h} = 0.995\ \text{kg/m}^3 (15\text{m/s})\ 1009\ \text{J/kg}\cdot\text{K}\ \left(8.93\times10^{-3}/2\right)\ (0.7)^{-2/3}$$

$$\overline{h} = 85\ \text{W/m}^2\cdot\text{K}.$$

Hence, the heat rate is

$$q = \overline{h}\, A\ (T_s - T_\infty) = 85\text{W/m}^2\cdot\text{K}\ \left(0.25\text{m}^2\right)\ (140-15)^\circ\ \text{C}$$

$$q = 2.66\ \text{kW}.$$ <

COMMENTS: Due to bottom heat losses, which have been assumed negligible, the actual power requirement would exceed 2.66 kW.

PROBLEM 6.42

KNOWN: Heat transfer correlation associated with parallel flow over a rough flat plate. Velocity and temperature of air flow over the plate.

FIND: Surface shear stress l m from the leading edge.

SCHEMATIC:

ASSUMPTIONS: (1) Modified Reynolds analogy is applicable, (2) Constant properties.

PROPERTIES: *Table A-4*, Air (300K, 1atm): $\nu = 15.89 \times 10^{-6} m^2/s$, Pr = 0.71, $\rho = 1.16$ kg/m^3.

ANALYSIS: Applying the Chilton-Colburn analogy

$$\frac{C_f}{2} = St_x\, Pr^{2/3} = \frac{Nu_x}{Re_x\, Pr} Pr^{2/3} = \frac{0.04\, Re_x^{0.9}\, Pr^{1/3}}{Re_x\, Pr} Pr^{2/3}$$

$$\frac{C_f}{2} = 0.04\, Re_x^{-0.1}$$

where

$$Re_x = \frac{u_\infty x}{\nu} = \frac{50\ m/s \times 1m}{15.89 \times 10^{-6} m^2/s} = 3.15 \times 10^6.$$

Hence, the friction coefficient is

$$C_f = 0.08\left(3.15 \times 10^6\right)^{-0.1} = 0.0179 = \tau_s / \left(\rho\, u_\infty^2/2\right)$$

and the surface shear stress is

$$\tau_s = C_f\left(\rho\, u_\infty^2/2\right) = 0.0179 \times 1.16 kg/m^3 (50\ m/s)^2/2$$

$$\tau_s = 25.96\ kg/m \cdot s^2 = 25.96\ N/m^2.$$ <

COMMENTS: Note that turbulent flow will exist at the designated location.

PROBLEM 6.43

KNOWN: Nominal operating conditions of aircraft and characteristic length and average friction coefficient of wing.

FIND: Average heat flux needed to maintain prescribed surface temperature of wing.

SCHEMATIC:

Air
$T_\infty = -23^oC$
$V = 100$ m/s
Electrical heater
Airfoil
$T_s = 5^oC$
$\bar{C}_f = 0.0025$
$L = 2$ m

ASSUMPTIONS: (1) Applicability of modified Reynolds analogy, (2) Constant properties.

PROPERTIES: Prescribed, Air: $\nu = 16.3 \times 10^{-6}$ m^2/s, k = 0.022 W/m·K, Pr = 0.72.

ANALYSIS: The average heat flux that must be maintained over the surface of the air foil is $\bar{q}'' = \bar{h}\left(T_s - T_\infty\right)$, where the average convection coefficient may be obtained from the modified Reynolds analogy.

$$\frac{\bar{C}_f}{2} = \text{St}\,\text{Pr}^{2/3} = \frac{\overline{\text{Nu}}_L}{\text{Re}_L\,\text{Pr}}\text{Pr}^{2/3} = \frac{\overline{\text{Nu}}_L}{\text{Re}_L\,\text{Pr}^{1/3}}$$

Hence, with $\text{Re}_L = VL/\nu = 100\,\text{m/s}\left(2\text{m}\right)/16.3\times10^{-6}\,\text{m}^2/\text{s} = 1.23\times10^7$,

$$\overline{\text{Nu}}_L = \frac{0.0025}{2}\left(1.23\times10^7\right)\left(0.72\right)^{1/3} = 13{,}780$$

$$\bar{h} = \frac{k}{L}\overline{\text{Nu}}_L = \frac{0.022\,\text{W/m}\cdot\text{K}}{2\text{m}}\left(13{,}780\right) = 152\,\text{W/m}^2\cdot\text{K}$$

$$\bar{q}'' = 152\,\text{W/m}^2\cdot\text{K}\left[5-\left(-23\right)\right]^\circ\text{C} = 4260\,\text{W/m}^2 \qquad <$$

COMMENTS: If the flow is turbulent over the entire airfoil, the modified Reynolds analogy provides a good measure of the relationship between surface friction and heat transfer. The relation becomes more approximate with increasing laminar boundary layer development on the surface and increasing values of the magnitude of the pressure gradient.

PROBLEM 6.44

KNOWN: Average frictional shear stress of $\overline{\tau}_s = 0.0625$ N/m^2 on upper surface of circuit board with densely packed integrated circuits (ICs)

FIND: Allowable power dissipation from the upper surface of the board if the average surface temperature of the ICs must not exceed a rise of 25°C above ambient air temperature.

SCHEMATIC:

Air
T_∞
$u_\infty = 2$ m/s
$\overline{\tau}_s = 0.0625$ N/m^2
q_{cv}
$T_s - T_\infty \le 25°C$
$A_s = 120 \times 120$ mm
P_e
Circuit board with densely packed ICs

ASSUMPTIONS: (1) Steady-state conditions, (2) The modified Reynolds analogy is applicable, (3) Negligible heat transfer from bottom side of the circuit board, and (4) Thermophysical properties required for the analysis evaluated at 300 K,

PROPERTIES: *Table A-4*, Air (T_f = 300 K, 1 atm): ρ = 1.161 kg/m^3, c_p = 1007 J/kg·K, Pr = 0.707.

ANALYSIS: The power dissipation from the circuit board can be calculated from the convection rate equation assuming an excess temperature ($T_s - T_\infty$) = 25°C.

$$q = \overline{h} A_s \left(T_s - T_\infty\right) \qquad (1)$$

The average convection coefficient can be estimated from the Reynolds analogy and the measured average frictional shear stress $\overline{\tau}_s$.

$$\frac{\overline{C}_f}{2} = \overline{St}\,Pr^{2/3} \qquad \overline{C}_f = \frac{\overline{\tau}_s}{\rho V^2/2} \qquad \overline{St} = \frac{\overline{h}}{\rho V c_p} \qquad (2,3,4)$$

With V = u_∞ and substituting numerical values, find $\overline{h}$.

$$\frac{\tau_s}{\rho V^2} = \frac{\overline{h}}{\rho V c_p} Pr^{2/3}$$

$$\overline{h} = \frac{\overline{\tau}_s c_p}{V} Pr^{-2/3}$$

$$\overline{h} = \frac{0.0625\ N/m^2 \times 1007\ J/kg\cdot K}{2\ m/s}(0.707)^{-2/3} = 39.7\ W/m^2\cdot K$$

Substituting this result into Eq. (1), the allowable power dissipation is

$$q = 39.7\ W/m^2\cdot K \times (0.120 \times 0.120) m^2 \times 25\ K = 14.3\ W$$ <

COMMENTS: For this analyses using the modified or Chilton-Colburn analogy, we found C_f = 0.0269 and St = 0.0170. Using the Reynolds analogy, the results are slightly different with $\overline{h} = 31.5\ W/m^2\cdot K$ and q = 11.3 W.

PROBLEM 6.45

KNOWN: Evaporation rate of water from a lake.

FIND: The convection mass transfer coefficient, $\bar{h}_m$.

SCHEMATIC:

ASSUMPTIONS: (1) Equilibrium at water vapor-liquid surface, (2) Isothermal conditions, (3) Perfect gas behavior of water vapor, (4) Air at standard atmospheric pressure.

PROPERTIES: *Table A-6*, Saturated water vapor (300K): $p_{A,sat} = 0.03531$ bar, $\rho_{A,sat} = 1/v_g = 0.02556$ kg/m^3.

ANALYSIS: The convection mass transfer (evaporation) rate equation can be written in the form

$$\bar{h}_m = \frac{n''_A}{\left(\rho_{A,s} - \rho_{A,\infty}\right)}$$

where

$$\rho_{A,s} = \rho_{A,sat},$$

the saturation density at the temperature of the water and

$$\rho_{A,\infty} = \phi \rho_{A,sat}$$

which follows from the definition of the relative humidity, $\phi = p_A/p_{A,sat}$ and perfect gas behavior. Hence,

$$\bar{h}_m = \frac{n''_A}{\rho_{A,sat}\left(1-\phi\right)}$$

and substituting numerical values, find

$$\bar{h}_m = \frac{0.1 \text{ kg/m}^2 \cdot \text{h} \times 1/3600 \text{ s/h}}{0.02556 \text{ kg/m}^3 \left(1-0.3\right)} = 1.55 \times 10^{-3} \text{ m/s.}$$ <

COMMENTS: (1) From knowledge of $p_{A,sat}$, the perfect gas law could be used to obtain the saturation density.

$$\rho_{A,sat} = \frac{p_{A,sat} M_A}{\Re T} = \frac{0.03531 \text{ bar} \times 18 \text{ kg/kmol}}{8.314 \times 10^{-2} \text{m}^3 \cdot \text{bar/kmol} \cdot \text{K}\left(300\text{K}\right)} = 0.02548 \text{ kg/m}^3.$$

This value is within 0.3% of that obtained from Table A-6.

(2) Note that psychrometric charts could also be used to obtain $\rho_{A,sat}$ and $\rho_{A,\infty}$.

PROBLEM 6.46

KNOWN: Evaporation rate from pan of water of prescribed diameter. Water temperature. Air temperature and relative humidity.

FIND: (a) Convection mass transfer coefficient, (b) Evaporation rate for increased relative humidity, (c) Evaporation rate for increased temperature.

SCHEMATIC:

Air
$T = 23°C$ or $47°C$
$\phi_\infty = 0$ or 0.5
$\dot{m}_{evap} = 1.5\times10^{-5}$ kg/s
$\rho_{A,s}$
Water, $D = 0.23$m
$T_s = 23°C$ or $47°C$

ASSUMPTIONS: (1) Water vapor is saturated at liquid interface and may be approximated as a perfect gas.

PROPERTIES: *Table A-6*, Saturated water vapor (T_s = 296K): $\rho_{A,sat} = v_g^{-1} = (49.4\ m^3/kg)^{-1} =$ $0.0202\ kg/m^3$; (T_s = 320 K): $\rho_{A,sat} = v_g^{-1} = \left(13.98\ m^3/kg\right)^{-1} = 0.0715\ kg/m^3$.

ANALYSIS: (a) Since evaporation is a convection mass transfer process, the rate equation has the form $\dot{m}_{evap} = \overline{h}_m A\left(\rho_{A,s} - \rho_{A,\infty}\right)$ and the mass transfer coefficient is

$$\overline{h}_m = \frac{\dot{m}_{evap}}{\left(\pi\ D^2/4\right)\left(\rho_{A,s} - \rho_{A,\infty}\right)} = \frac{1.5\times10^{-5}\,kg/s}{(\pi/4)(0.23\ m)^2\ 0.0202\ kg/m^3} = 0.0179\ m/s$$ <

with $T_S = T_\infty = 23°C$ and $\phi_\infty = 0$.

(b) If the relative humidity of the ambient air is increased to 50%, the ratio of the evaporation rates is

$$\frac{\dot{m}_{evap}\left(\phi_\infty = 0.5\right)}{\dot{m}_{evap}\left(\phi_\infty = 0\right)} = \frac{\overline{h}_m A\left[\rho_{A,s}\left(T_s\right) - \phi_\infty \rho_{A,s}\left(T_\infty\right)\right]}{\overline{h}_m A\ \rho_{A,s}\left(T_s\right)} = 1 - \phi_\infty \frac{\rho_{A,s}\left(T_\infty\right)}{\rho_{A,s}\left(T_s\right)}.$$

Hence, $\dot{m}_{evap}\left(\phi_\infty = 0.5\right) = 1.5\times10^{-5}\,kg/s\left[1 - 0.5\dfrac{0.0202\ kg/m^3}{0.0202\ kg/m^3}\right] = 0.75\times10^{-5}\,kg/s.$

(c) If the temperature of the ambient air is increased from 23°C to 47°C, with $\phi_\infty = 0$ for both cases, the ratio of the evaporation rates is

$$\frac{\dot{m}_{evap}\left(T_s = T_\infty = 47^\circ C\right)}{\dot{m}_{evap}\left(T_s = T_\infty = 23^\circ C\right)} = \frac{\overline{h}_m A \rho_{A,s}\left(47^\circ C\right)}{\overline{h}_m A \rho_{A,s}\left(23^\circ C\right)} = \frac{\rho_{A,s}\left(47^\circ C\right)}{\rho_{A,s}\left(23^\circ C\right)}.$$

Hence, $\dot{m}_{evap}\left(T_s = T_\infty = 47^\circ C\right) = 1.5\times10^{-5}\,kg/s\dfrac{0.0715\ kg/m^3}{0.0202\ kg/m^3} = 5.31\times10^{-5}\,kg/s.$ <

COMMENTS: Note the highly nonlinear dependence of the evaporation rate on the water temperature. For a 24°C rise in T_s, $\dot{m}_{evap}$ increases by 350%.

PROBLEM 6.47

KNOWN: Water temperature and air temperature and relative humidity. Surface recession rate.

FIND: Mass evaporation rate per unit area. Convection mass transfer coefficient.

SCHEMATIC:

ASSUMPTIONS: (1) Water vapor may be approximated as a perfect gas, (2) No water inflow; outflow is only due to evaporation.

PROPERTIES: *Table A-6*, Saturated water: Vapor (305K), $\rho_g = v_g^{-1} = 0.0336$ kg/m^3; Liquid (305K), $\rho_f = v_f^{-1} = 995$ kg/m^3.

ANALYSIS: Applying conservation of species to a control volume about the water,

$$-\dot{M}_{A,out} = \dot{M}_{A,st}$$
$$-\dot{m}''_{evap} A = \frac{d}{dt}(\rho_f V) = \frac{d}{dt}(\rho_f AH) = \rho_f A \frac{dH}{dt}.$$

Substituting numerical values, find

$$\dot{m}''_{evap} = -\rho_f \frac{dH}{dt} = -995\text{kg/m}^3 \left(-10^{-4}\text{m/h}\right) \left(1/3600 \text{ s/h}\right)$$

$$\dot{m}''_{evap} = 2.76\times10^{-5}\text{kg/s}\cdot\text{m}^2.$$ <

Because evaporation is a convection mass transfer process, it also follows that

$$\dot{m}''_{evap} = n''_A$$

or in terms of the rate equation,

$$\dot{m}''_{evap} = h_m\left(\rho_{A,s} - \rho_{A,\infty}\right) = h_m\left[\rho_{A,sat}(T_s) - \phi_\infty \rho_{A,sat}(T_\infty)\right]$$
$$\dot{m}''_{evap} = h_m \rho_{A,sat}(305\text{K})\ (1-\phi_\infty),$$

and solving for the convection mass transfer coefficient,

$$h_m = \frac{\dot{m}''_{evap}}{\rho_{A,sat}(305\text{K})\ (1-\phi_\infty)} = \frac{2.76\times10^{-5}\text{kg/s}\cdot\text{m}^2}{0.0336 \text{ kg/m}^3(1-0.4)}$$

$$h_m = 1.37\times10^{-3}\text{m/s}.$$ <

COMMENTS: Conservation of species has been applied in exactly the same way as a conservation of energy. Note the sign convention.

PROBLEM 6.48

KNOWN: CO_2 concentration in air and at the surface of a green leaf. Convection mass transfer coefficient.

FIND: Rate of photosynthesis per unit area of leaf.

SCHEMATIC:

ANALYSIS: Assuming that the CO_2 (species A) is consumed as a reactant in photosynthesis at the same rate that it is transferred across the atmospheric boundary layer, the rate of photosynthesis per unit leaf surface area is given by the rate equation,

$$n''_A = h_m\left(\rho_{A,\infty} - \rho_{A,s}\right).$$

Substituting numerical values, find

$$n''_A = 10^{-2}\,\text{m/s}\left(6\times10^{-4} - 5\times10^{-4}\right)\text{kg/m}^3$$

$$n''_A = 10^{-6}\,\text{kg/s}\cdot\text{m}^2.$$ <

COMMENTS: (1) It is recognized that CO_2 transport is from the air to the leaf, and ($\rho_{A,s}$ - $\rho_{A,\infty}$) in the rate equation has been replaced by ($\rho_{A,\infty}$ - $\rho_{A,s}$).

(2) The atmospheric concentration of CO_2 is known to be increasing by approximately 0.3% per year. This increase in $\rho_{A,\infty}$ will have the effect of increasing the photosynthesis rate and hence plant biomass production.

PROBLEM 6.49

KNOWN: Species concentration profile, $C_A(y)$, in a boundary layer at a particular location for flow over a surface.

FIND: Expression for the mass transfer coefficient, h_m, in terms of the profile constants, $C_{A,\infty}$ and D_{AB}. Expression for the molar convection flux, N''_A.

SCHEMATIC:

Profile at given x, $C_A(y) = Dy^2 + Ey + F$

B

$C_{A,\infty}$

y

x

Surface, $C_{A,s} = F$ species A

ASSUMPTIONS: (1) Parameters D, E, and F are constants at any location x, (2) D_{AB}, the mass diffusion coefficient of A through B, is known.

ANALYSIS: The convection mass transfer coefficient is defined in terms of the concentration gradient at the wall,

$$h_m(x) = -D_{AB} \frac{\partial C_A / \partial y)_{y=0}}{\left(C_{A,s} - C_{A,\infty}\right)}.$$

The gradient at the surface follows from the profile, $C_A(y)$,

$$\left.\frac{\partial C_A}{\partial y}\right]_{y=0} = \frac{\partial}{\partial y}\left(Dy^2 + Ey + F\right)_{y=0} = +E.$$

Hence,

$$h_m(x) = -\frac{D_{AB}E}{\left(C_{A,s} - C_{A,\infty}\right)} = \frac{-D_{AB}E}{\left(F - C_{A,\infty}\right)}. \qquad <$$

The molar flux follows from the rate equation,

$$N''_A = h_m\left(C_{A,s} - C_{A,\infty}\right) = \frac{-D_{AB}E}{\left(C_{A,s} - C_{A,\infty}\right)} \cdot \left(C_{A,s} - C_{A,\infty}\right).$$

$$N''_A = -D_{AB}E. \qquad <$$

COMMENTS: It is important to recognize that the influence of species B is present in the property D_{AB}. Otherwise, all the parameters relate to species A.

PROBLEM 6.50

KNOWN: Steady, incompressible flow of binary mixture between infinite parallel plates with different species concentrations.

FIND: Form of species continuity equation and concentration distribution. Species flux at upper surface.

SCHEMATIC:

ASSUMPTIONS: (1) Two-dimensional flow, (2) No chemical reactions, (3) Constant properties.

ANALYSIS: For fully developed conditions, $\partial C_A/\partial x = 0$. Hence with v = 0, the species conservation equation reduces to

$$\frac{d^2 C_A}{dy^2} = 0.$$ <

Integrating twice, the general form of the species concentration distribution is

$$C_A(y) = C_1 y + C_2.$$

Using appropriate boundary conditions and evaluating the constants,

$$C_A(0) = C_{A,2} \quad \rightarrow \quad C_2 = C_{A,2}$$
$$C_A(L) = C_{A,1} \quad \rightarrow \quad C_1 = \left(C_{A,1} - C_{A,2}\right)/L,$$

the concentration distribution is

$$C_A(y) = C_{A,2} + (y/L)\left(C_{A,1} - C_{A,2}\right).$$ <

From Fick's law, the species flux is

$$N''_A(L) = -D_{AB} \left.\frac{dC_A}{dy}\right|_{y=L}$$

$$N''_A(L) = \frac{D_{AB}}{L}\left(C_{A,2} - C_{A,1}\right).$$ <

COMMENTS: An analogy between heat and mass transfer exists if viscous dissipation is negligible. The energy equation is then $d^2T/dy^2 = 0$. Hence, both heat and species transfer are influenced only by diffusion. Expressions for T(y) and $q''(L)$ are analogous to those for $C_A(y)$ and $N''_A(L)$.

PROBLEM 6.51

KNOWN: Flow conditions between two parallel plates, across which vapor transfer occurs.

FIND: (a) Variation of vapor molar concentration between the plates and mass rate of water production per unit area, (b) Heat required to sustain the process.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Fully developed, incompressible flow with constant properties, (3) Negligible body forces, (4) No chemical reactions, (5) All work interactions, including viscous dissipation, are negligible.

ANALYSIS: (a) The flow will be fully developed in terms of the vapor concentration field, as well as the velocity and temperature fields. Hence

$$\frac{\partial C_A}{\partial x} = 0 \qquad \text{or} \qquad C_A(x,y) = C_A(y).$$

Also, with $\partial C_A/\partial t = 0$, $\dot{N}_A = 0$, $v = 0$ and constant D_{AB}, the species conservation equation reduces to

$$\frac{d^2 C_A}{dy^2} = 0.$$

Separating and integrating twice,

$$C_A(y) = C_1(y) + C_2.$$

Applying the boundary conditions,

$$C_A(0) = C_{A,0} \qquad \rightarrow \qquad C_2 = C_{A,0}$$

$$C_A(L) = C_{A,L} \qquad \rightarrow \qquad C_{A,L} = C_1 L + C_2 \qquad C_1 = -\frac{C_{A,0} - C_{A,L}}{L}$$

find the species concentration distribution,

$$C_A(y) = C_{A,0} - \left(C_{A,0} - C_{A,L}\right)(y/L).$$ <

From Fick's law, Eq. 6.19, the species transfer rate is

$$N''_A = N''_{A,s} = -D_{AB} \frac{\partial C_A}{\partial y}\bigg]_{y=0} = D_{AB} \frac{C_{A,0} - C_{A,L}}{L}.$$

Continued …..

Multiplying by the molecular weight of water vapor, M_A, the mass rate of water production per unit area is

$$n''_A = M_A N''_A = M_A D_{AB} \frac{C_{A,0} - C_{A,L}}{L}. \qquad <$$

(b) Heat must be supplied to the bottom surface in an amount equal to the latent and sensible heat transfer from the surface,

$$q'' = q''_{lat} + q''_{sen}$$
$$q'' = n''_{A,s}\ h_{fg} + \left[-k\frac{dT}{dy}\right]_{y=0}.$$

The temperature distribution may be obtained by solving the energy equation, which, for the prescribed conditions, reduces to

$$\frac{d^2T}{dy^2} = 0.$$

Separating and integrating twice,

$$T(y) = C_1 y + C_2.$$

Applying the boundary conditions,

$$T(0) = T_0 \qquad \rightarrow \qquad C_2 = T_0$$
$$T(L) = T_L \qquad \rightarrow \qquad C_1 = (T_1 - T_0)/L$$

find the temperature distribution,

$$T(y) = T_0 - (T_0 - T_L)\, y/L.$$

Hence,

$$-k\frac{dT}{dy}\bigg]_{y=0} = k\frac{(T_0 - T_L)}{L}.$$

Accordingly,

$$q'' = M_A D_{AB} \frac{C_{A,0} - C_{A,L}}{L} h_{fg} + k\frac{(T_0 - T_L)}{L}. \qquad <$$

COMMENTS: Despite the existence of the flow, species and energy transfer across the air are uninfluenced by advection and transfer is only by diffusion. If the flow were not fully developed, advection would have a significant influence on the species concentration and temperature fields and hence on the rate of species and energy transfer. The foregoing results would, of course, apply in the case of no air flow. The physical condition is an example of Poiseuille flow with heat and mass transfer.

PROBLEM 6.52

KNOWN: The conservation equations, Eqs. E.24 and E.31.

FIND: (a) Describe physical significance of terms in these equations, (b) Identify approximations and special conditions used to reduce these equations to the boundary layer equations, Eqs. 6.33 and 6.34, (c) Identify the conditions under which these two boundary layer equations have the same form and, hence, an analogy will exist.

ANALYSIS: (a) The energy conservation equation, Eq. E.24, has the form

$$\underset{1a}{\rho u\frac{\partial i}{\partial x}}+\underset{1b}{\rho v\frac{\partial i}{\partial y}}=\underset{2a}{\frac{\partial}{\partial x}\left[k\frac{\partial T}{\partial x}\right]}+\underset{2b}{\frac{\partial}{\partial y}\left[k\frac{\partial T}{\partial y}\right]}+\underset{3}{\left[u\frac{\partial p}{\partial x}+v\frac{\partial p}{\partial y}\right]}+\underset{4}{\mu\Phi}+\underset{5}{\dot{q}}.$$

The terms, as identified, have the following phnysical significance:

1. Change of enthalpy (thermal + flow work) advected in x and y directions, <
2. Change of conduction flux in x and y directions,
3. Work done by static pressure forces,
4. Word done by viscous stresses,
5. Rate of energy generation.

The species mass conservation equation for a constant total concentration has the form

$$\underset{1a}{u\frac{\partial C_A}{\partial x}}+\underset{1b}{v\frac{\partial C_A}{\partial y}}=\underset{2a}{\frac{\partial}{\partial x}\left[D_{AB}\frac{\partial C_A}{\partial x}\right]}+\underset{2b}{\frac{\partial}{\partial y}\left[D_{AB}\frac{\partial C_A}{\partial y}\right]}+\underset{3}{\dot{N}_A}$$

1. Change in species transport due to advection in x and y directions, <
2. Change in species transport by diffusion in x and y directions, and
3. Rate of species generation.

(b) The special conditions used to reduce the above equations to the boundary layer equations are: *constant properties, incompressible flow, non-reacting species* $\left(\dot{N}_A=0\right)$, *without internal heat generation* $\left(\dot{q}=0\right)$, *species diffusion has negligible effect on the thermal boundary layer,* $u(\partial p/\partial x)$ *is negligible.* The approximations are,

Velocity boundary layer $\quad \left\{u \gg v \qquad \dfrac{\partial u}{\partial y} \gg \dfrac{\partial u}{\partial x},\ \dfrac{\partial v}{\partial y},\ \dfrac{\partial v}{\partial x}\right.$

Thermal b.l.: $\quad \left\{\dfrac{\partial T}{\partial y} \gg \dfrac{\partial T}{\partial x}\right.$ $\qquad$ Concentration b.l.: $\quad \left\{\dfrac{\partial C_A}{\partial y} \gg \dfrac{\partial C_A}{\partial x}\right.$.

The resulting simplified boundary layer equations are

$$\underset{1a}{u\frac{\partial T}{\partial x}}+\underset{1b}{v\frac{\partial T}{\partial y}}=\underset{2a}{\alpha\frac{\partial^2 T}{\partial y^2}}+\underset{3}{\frac{\nu}{c}\left[\frac{\partial u}{\partial y}\right]^2} \qquad \underset{1c}{u\frac{\partial C_A}{\partial x}}+\underset{1d}{v\frac{\partial C_A}{\partial y}}=\underset{2b}{D_{AB}\frac{\partial^2 C_A}{\partial y^2}} \qquad <$$

where the terms are: 1. Advective transport, 2. Diffusion, and 3. Viscous dissipation.

(c) When viscous dissipation effects are negligible, the two boundary layer equations have identical form. If the boundary conditions for each equation are of the same form, an analogy between heat and mass (species) transfer exists.

PROBLEM 6.53

KNOWN: Thickness and inclination of a liquid film. Mass density of gas in solution at free surface of liquid.

FIND: (a) Liquid momentum equation and velocity distribution for the x-direction. Maximum velocity, (b) Continuity equation and density distribution of the gas in the liquid, (c) Expression for the local Sherwood number, (d) Total gas absorption rate for the film, (e) Mass rate of NH_3 removal by a water film for prescribed conditions.

SCHEMATIC:

NH_3 (A) – Water (B)
$L = 2m$
$\delta = 1$ mm
$D = 0.05m$
$W = \pi D = 0.157m$
$\rho_{A,o} = 25$ kg/m^3
$D_{AB} = 2 \times 10^{-9}$ m^2/s
$\phi = 0°$

ASSUMPTIONS: (1) Steady-state conditions, (2) The film is in fully developed, laminar flow, (3) Negligible shear stress at the liquid-gas interface, (4) Constant properties, (5) Negligible gas concentration at x = 0 and y = δ, (6) No chemical reactions in the liquid, (7) Total mass density is constant, (8) Liquid may be approximated as semi-infinite to gas transport.

PROPERTIES: *Table A-6*, Water, liquid (300K): $\rho_f = 1/v_f = 997$ kg/m^3, $\mu = 855 \times 10^{-6}$ N·s/m^2, $\nu = \mu/\rho_f = 0.855 \times 10^{-6}$ m^2/s.

ANALYSIS: (a) For fully developed flow ($v = w = 0$, $\partial u/\partial x = 0$), the x-momentum equation is

$$0 = \partial\tau_{yx}/\partial y + X \quad \text{where} \quad \tau_{yx} = \mu(\partial u/\partial y) \quad \text{and} \quad X = (\rho g)\cos\phi.$$

That is, the momentum equation reduces to a balance between gravitational and shear forces. Hence,

$$\mu\left(\partial^2 u/\partial y^2\right) = -(\rho g)\cos\phi.$$

Integrating,
$$\partial u/\partial y = -(g\cos\phi/\nu)y + C_1 \qquad u = -(g\cos\phi/2\nu)y^2 + C_1 y + C_2.$$

Applying the boundary conditions,

$$\partial u/\partial y)_{y=0} = 0 \quad \rightarrow \quad C_1 = 0$$

$$u(\delta) = 0 \quad \rightarrow \quad C_2 = g\cos\phi\,\frac{\delta^2}{2\nu}.$$

Hence,
$$u = \frac{g\cos\phi}{2\nu}\left(\delta^2 - y^2\right) = \frac{g\cos\phi\,\delta^2}{2\nu}\left[1-(y/\delta)^2\right] \qquad <$$

and the maximum velocity exists at y = 0,

$$u_{max} = u(0) = \left(g\cos\phi\,\delta^2\right)/2\nu. \qquad <$$

(b) Species transport within the liquid is influenced by diffusion in the y-direction and convection in the x-direction. Hence, the species continuity equation with u assumed equal to u_{max} throughout the region of gas penetration is

Continued …..

$$u\frac{\partial \rho_A}{\partial x} = D_{AB}\frac{\partial^2 \rho_A}{\partial y^2} \qquad \frac{\partial^2 \rho_A}{\partial y^2} = \frac{u_{max}}{D_{AB}}\frac{\partial \rho_A}{\partial x}.$$

Appropriate boundary conditions are: $\rho_A(x,0) = \rho_{A,o}$ and $\rho_A(x,\infty) = 0$ and the entrance condition is: $\rho_A(0,y) = 0$. The problem is therefore analogous to transient conduction in a semi-infinite medium due to a sudden change in surface temperature. From Section 5.7, the solution is then

$$\frac{\rho_A - \rho_{A,o}}{0-\rho_{A,o}} = \text{erf}\frac{y}{2\left(D_{AB}x/u_{max}\right)^{1/2}} \qquad \rho_A = \rho_{A,o}\text{erfc}\frac{y}{2\left(D_{AB}x/u_{max}\right)^{1/2}} \quad <$$

(c) The Sherwood number is defined as

$$Sh_x = \frac{h_{m,x}x}{D_{AB}} \quad \text{where} \quad h_{m,x} \equiv \frac{n''_{A,x}}{\rho_{A,o}} = \frac{-D_{AB}\partial\rho_A/\partial y)_{y=0}}{\rho_{A,o}}$$

$$\left.\frac{\partial \rho_A}{\partial y}\right|_{y=0} = -\rho_{A,o}\frac{2}{(\pi)^{1/2}}\exp\left[-\frac{y^2 u_{max}}{4D_{AB}x}\right]\left.\frac{1}{2\left(D_{AB}x/u_{max}\right)^{1/2}}\right|_{y=0} = -\rho_{A,o}\left[\frac{u_{max}}{\pi D_{AB}x}\right]^{1/2}.$$

Hence,

$$h_{m,x} = \left[\frac{u_{max}D_{AB}}{\pi x}\right]^{1/2} \qquad Sh_x = \frac{1}{(\pi)^{1/2}}\left[\frac{u_{max}x}{D_{AB}}\right]^{1/2} = \frac{1}{(\pi)^{1/2}}\left[\frac{u_{max}x}{\nu}\right]^{1/2}\left[\frac{\nu}{D_{AB}}\right]^{1/2}$$

and with $Re_x \equiv u_{max}x/\nu$,

$$Sh_x = \left[1/(\pi)^{1/2}\right]Re_x^{1/2}\,Sc^{1/2} = 0.564\,Re_x^{1/2}\,Sc^{1/2}. \quad <$$

(d) The total gas absorption rate may be expressed as

$$n_A = \overline{h}_{m,x}(W\cdot L)\rho_{A,o}$$

where the average mass transfer convection coefficient is

$$\overline{h}_{m,x} = \frac{1}{L}\int_0^L h_{m,x}dx = \frac{1}{L}\left[\frac{u_{max}D_{AB}}{\pi}\right]^{1/2}\int_0^L\frac{dx}{x^{1/2}} = \left[\frac{4u_{max}D_{AB}}{\pi L}\right]^{1/2}.$$

Hence, the absorption rate per unit width is

$$n_A/W = \left(4u_{max}D_{AB}L/\pi\right)^{1/2}\rho_{A,o}. \quad <$$

(e) From the foregoing results, it follows that the ammonia absorption rate is

$$n_A = \left[\frac{4u_{max}D_{AB}L}{\pi}\right]^{1/2}W\rho_{A,o} = \left[\frac{4g\cos\phi\delta^2 D_{AB}L}{2\pi\nu}\right]^{1/2}W\rho_{A,o}.$$

Substituting numerical values,

$$n_A = \left[\frac{4\times 9.8\text{ m/s}^2\times 1\times\left(10^{-3}\text{m}\right)^2\left(2\times 10^{-9}\text{m}^2/\text{s}\right)2\text{m}}{2\pi\times 0.855\times 10^{-6}\text{m}^2/\text{s}}\right]^{1/2}(0.157\text{m})\,25\text{ kg/m}^3 = 6.71\times 10^{-4}\text{kg/s}. \quad <$$

COMMENTS: Note that $\rho_{A,o} \neq \rho_{A,\infty}$, where $\rho_{A,\infty}$ is the mass density of the gas phase. The value of $\rho_{A,o}$ depends upon the pressure of the gas and the solubility of the gas in the liquid.

PROBLEM 6.54

KNOWN: Cross flow of gas X over object with prescribed characteristic length L, Reynolds number, and average heat transfer coefficient. Thermophysical properties of gas X, liquid Y, and vapor Y.

FIND: Average mass transfer coefficient for same object when impregnated with liquid Y and subjected to same flow conditions.

SCHEMATIC:

ASSUMPTIONS: (1) Heat and mass transfer analogy is applicable, (2) Vapor Y behaves as perfect gas

PROPERTIES:

(Given)	$\nu(m^2/s)$	k(W/m·K)	$\alpha(m^2/s)$
Gas X	21×10^{-6}	0.030	29×10^{-6}
Liquid Y	3.75×10^{-7}	0.665	1.65×10^{-7}
Vapor Y	4.25×10^{-5}	0.023	4.55×10^{-5}

Mixture of gas X - vapor Y: Sc = 0.72

ANALYSIS: The heat-mass transfer analogy may be written as

$$\overline{Nu}_L = \frac{\bar{h}_L L}{k} = f\left(Re_L,\ Pr\right) \qquad \overline{Sh}_L = \frac{\bar{h}_{m,L} L}{D_{AB}} = f\left(Re_L, Sc\right)$$

The flow conditions are the same for both situations. Check values of Pr and Sc. For Pr, the properties are those for gas X (B).

$$Pr = \frac{\nu_B}{\alpha_B} = \frac{21\times10^{-6} m^2/s}{29\times10^{-6} m^2/s} = 0.72$$

while Sc = 0.72 for the gas X (B) - vapor Y (A) mixture. It follows for this situation

$$\overline{Nu}_L = \frac{\bar{h}_L L}{k} = \overline{Sh}_L = \frac{\bar{h}_{m,L} L}{D_{AB}} \qquad \text{or} \qquad \bar{h}_{m,L} = \bar{h}_L \frac{D_{AB}}{k}.$$

Recognizing that

$$D_{AB} = \nu_B / Sc = 21.6\times10^{-6} m^2/s \big/ (0.72) = 30.0\times10^{-6} m^2/s$$

and substituting numerical values, find

$$\bar{h}_{m,L} = 25\ W/m^2 \cdot K \times \frac{30.0\times10^{-6} m^2/s}{0.030\ W/m\cdot K} = 0.0250\ m/s.$$ <

COMMENTS: Note that none of the thermophysical properties of liquid or vapor Y are required for the solution. Only the gas X properties and the Schmidt number (gas X - vapor Y) are required.

PROBLEM 6.55

KNOWN: Free stream velocity and average convection mass transfer coefficient for fluid flow over a surface of prescribed characteristic length.

FIND: Values of $\overline{Sh}_L$, Re_L, Sc and $\bar{j}_m$ for (a) air flow over water, (b) air flow over naphthalene, and (c) warm glycerol over ice.

SCHEMATIC:

(B)
Air or glycerol
T= 300K
V= 1 m/s

L = 1m

$\bar{h}_m = 10^{-2}$ m/s

(A)
Water (v),
naphthalene (v),
or water (ℓ)

PROPERTIES: For the fluids at 300K:

Table	*Fluid(s)*	$\nu(m^2/s)\times 10^{-6}$	$D_{AB}(m^2/s)$
A-4	Air	15.89	-
A-5	Glycerin	634	-
A-8	Water vapor - Air	-	0.26×10^{-4}
A-8	Naphthalene - Air	-	0.62×10^{-5}
A-8	Water - Glycerol	-	0.94×10^{-9}

ANALYSIS: (a) *Water (vapor) - Air:*

$$\overline{Sh}_L = \frac{\bar{h}_m L}{D_{AB}} = \frac{(0.01\text{m/s})1\text{m}}{0.26\times 10^{-4}\text{m}^2/\text{s}} = 385$$

$$Re_L = \frac{VL}{\nu} = \frac{(1\text{ m/s})1\text{m}}{15.89\times 10^{-6}\text{m}^2/\text{s}} = 6.29\times 10^4$$

$$Sc = \frac{\nu}{D_{AB}} = \frac{0.16\times 10^{-6}\text{m}^2/\text{s}}{0.26\times 10^{-6}\text{m}^2/\text{s}} = 0.62$$

$$\bar{j}_m = St_m Sc^{2/3} = \frac{h_m}{V} Sc^{2/3} = \frac{0.01\text{ m/s}}{1\text{ m/s}}(0.62)^{2/3} = 0.0073. \quad <$$

(b) *Naphthalene (vapor) - Air:*

$$\overline{Sh}_L = 1613 \qquad Re_L = 6.29\times 10^4 \qquad Sc = 2.56 \qquad \bar{j}_m = 0.0187. \quad <$$

(c) *Water (1iquid) - Glycerol:*

$$\overline{Sh}_L = 1.06\times 10^7 \qquad Re_L = 1577 \qquad Sc = 6.74\times 10^5 \qquad \bar{j}_m = 76.9. \quad <$$

COMMENTS: Note the association of ν with the freestream fluid B.

PROBLEM 6.56

KNOWN: Characteristic length, surface temperature, average heat flux and airstream conditions associated with an object of irregular shape.

FIND: Whether similar behavior exists for alternative conditions, and average convection coefficient for similar cases.

SCHEMATIC:

Case:	1	2	3	4	5
L,m	1	2	2	2	2
V, m/s	100	50	50	50	250
p, atm	1	1	0.2	1	0.2
T∞, K	275	275	275	300	300
Ts, K	325	325	325	300	300
q'', W/m^2	12,000	-	-	-	-
$\bar{h}$, W/m$^2\cdot$K	240	-	-	-	-
$D_{AB}\times10^{+4}$, m^2/s	-	-	-	1.12	1.12

ASSUMPTIONS: (1) Heat and mass transfer analogy is applicable; that is, $f(Re_L,Pr) = f(Re_L,Sc)$, see Eqs. 6.57 and 6.61.

PROPERTIES: *Table A-4*, Air (300K, 1 atm): $\nu_1 = 15.89\times10^{-6}\,m^2/s$, $Pr_1 = 0.71$, $k_1 = 0.0263\,W/m\cdot K$.

ANALYSIS: $Re_{L,1} = V_1L_1/\nu_1 = (100\,m/s\times1m)/15.89\times10^{-6}\,m^2/s = 6.29\times10^6$ and $Pr_1 = 0.71$.

Case 2: $$Re_{L,2} = \frac{V_2L_2}{\nu_2} = \frac{50\,m/s\times2m}{15.89\times10^{-6}m^2/s} = 6.29\times10^6, \qquad Pr_2 = 0.71.$$

From Eq. 6.57 it follows that Case 2 is analogous to Case 1. Hence $\overline{Nu}_2 = \overline{Nu}_1$ and

$$\bar{h}_2 = \frac{\bar{h}_1L_1}{k_1}\frac{k_2}{L_2} = \bar{h}_1\frac{L_1}{L_2} = 240\frac{W}{m^2\cdot K}\frac{1m}{2m} = 120\,W/m^2\cdot K.$$ <

Case 3: With p = 0.2 atm, $\nu_3 = 79.45\times10^{-6}\,m^2/s$ and $Re_{L,3} = \frac{V_3L_3}{\nu_3} = \frac{50\,m/s\times2m}{79.45\times10^{-6}m^2/s} = 1.26\times10^6$, $Pr_3 = 0.71$.

Since $Re_{L,3} \neq Re_{L,1}$, Case 3 is not analogous to Case 1. <

Case 4: $$Re_{L,4} = Re_{L,1},\; Sc_4 = \frac{\nu_4}{D_{AB,4}} = \frac{15.89\times10^{-6}m^2/s}{1.12\times10^{-4}m^2/s} = 0.142 \neq Pr_1.$$

Hence, Case 4 is not analogous to Case 1. <

Case 5: $$Re_{L,5} = \frac{V_5L_5}{\nu_5} = \frac{250\,m/s\times2m}{79.45\times10^{-6}m^2/s} = 6.29\times10^6 = Re_{L,1}$$

$$Sc_5 = \frac{\nu_5}{D_{AB,5}} = \frac{79.45\times10^{-6}m^2/s}{1.12\times10^{-4}m^2/s} = 0.71 = Pr_1.$$

Hence, conditions are analogous to Case 1, and with $\overline{Sh}_5 = \overline{Nu}_1$,

$$h_{m,5} = h_1\frac{L_1}{L_5}\frac{D_{AB,5}}{k_1} = 240\frac{W}{m^2\cdot K}\times\frac{1m}{2m}\times\frac{1.12\times10^{-4}m^2/s}{0.0263\,W/m\cdot K} = 0.51\,m/s.$$ <

COMMENTS: Note that Pr, k and Sc are independent of pressure, while ν and D_{AB} vary inversely with pressure.

PROBLEM 6.57

KNOWN: Surface temperature and heat loss from a runner's body on a cool, spring day. Surface temperature and ambient air-conditions for a warm summer day.

FIND: (a) Water loss on summer day, (b) Total heat loss on summer day.

SCHEMATIC:

V, $T_\infty = 10°C$ → Case 1: $T_s = 30°C, A_s$; $q_1 = 500 W$

V, $T_\infty = 30°C$, $\phi_\infty = 0.6$ → Case 2: $T_s = 35°C, A_s$; q, n_A

ASSUMPTIONS: (1) Heat and mass transfer analogy is applicable. Hence, from Eqs. 6.57 and 6.61, $f(Re_L,Pr)$ is of same form as $f(Re_L,Sc)$, (2) Negligible surface evaporation for Case 1, (3) Constant properties, (4) Water vapor is saturated for Case 2 surface and may be approximated as a perfect gas.

PROPERTIES: Air (given): $\nu = 1.6\times10^{-5}$ m^2/s, k = 0.026 W/m·K, Pr = 0.70; Water vapor - air (given): $D_{AB} = 2.3\times10^{-5} m^2/s$; *Table A-6*, Saturated water vapor (T_∞ = 303K): $\rho_{A,sat} = v_g^{-1} = 0.030\ kg/m^3$; $(T_s = 308K)$: $\rho_{A,sat} = v_g^{-1} = 0.039\ kg/m^3$, $h_{fg} = 2419$ kJ/kg.

ANALYSIS: (a) With $Re_{L,2} = Re_{L,1}$ and $Sc = \nu/D_{AB} = 1.6\times10^{-5} m^2/s / 2.3\times10^{-5} m^2/s = 0.70 = Pr$, it follows that $\overline{Sh}_L = \overline{Nu}_L$. Hence

$$\overline{h}_m L/D_{AB} = \overline{h}L/k$$

$$\overline{h}_m = \overline{h}\frac{D_{AB}}{k} = \frac{q_1}{A_s\left(T_s - T_\infty\right)_1}\ \frac{D_{AB}}{k} = \frac{500\ W}{A_s\left(20K\right)}\ \frac{2.3\times10^{-5} m^2/s}{0.026\ W/m\cdot K} = \left[\frac{0.0221}{A_s}\right] m/s.$$

Hence, from the rate equation, with A_s as the wetted surface

$$n_A = \overline{h}_m A_s\left(\rho_{A,s} - \rho_{A,\infty}\right) = \left[\frac{0.0221}{A_s}\right]\ \frac{m}{s} A_s\left[\rho_{A,sat}\left(T_{s,2}\right) - \phi_\infty \rho_{A,sat}\left(T_{\infty,2}\right)\right]$$

$$n_A = 0.0221\ m^3/s\left(0.039 - 0.6\times0.030\right) kg/m^3 = 4.64\times10^{-4} kg/s.$$ <

(b) The total heat loss for Case 2 is comprised of sensible and latent contributions, where

$$q_2 = q_{sen} + q_{lat} = \overline{h}A_s\left(T_{s,2} - T_{\infty,2}\right) + n_A\ h_{fg}.$$

Hence, with $\overline{h}A_s = q_1/\left(T_{s,1} - T_{\infty,1}\right) = 25$ W/K,

$$q_2 = 25\ W/K\ \left(35 - 30\right)^\circ C + 4.64\times10^{-4} kg/s \times 2.419\times10^6\ J/kg$$

$$q_2 = 125\ W\ +\ 1122\ W\ =\ 1247\ W.$$ <

COMMENTS: Note the significance of the evaporative cooling effect.

PROBLEM 6.58

KNOWN: Heat transfer results for an irregularly shaped object.

FIND: (a) The concentration, C_A, and partial pressure, p_A, of vapor in an airstream for a drying process of an object of similar shape, (b) Average mass transfer flux, $n''_A \left(\text{kg/s}\cdot\text{m}^2\right)$.

SCHEMATIC:

Air
V=120m/s
P_∞=1atm
T_∞=0°C
T(x*,y*)=80°C
T_s=100°C
q''=2000W/m²
L_1=1m

C_A(x*,y*)
Air
V=60m/s
P_∞=1atm
T_∞=50°C
$C_{A,\infty}$=0
$P_{A,s}$=0.0323atm
T_s=50°C
n''_A
L_2=2m

Case 1: Heat Transfer Case 2: Mass Transfer

ASSUMPTIONS: (1) Heat-mass transfer analogy applies, (b) Perfect gas behavior.

PROPERTIES: *Table A-4*, Air (323K, 1 atm): $\nu = 18.20\times10^{-6}\,\text{m}^2/\text{s}$, Pr = 0.703, $k = 28.0\times10^{-3}\,\text{W/m}\cdot\text{K}$; Plastic vapor (given): $\mathcal{M}_A$ = 82 kg/kmol, $p_{sat}(50°\text{C})$ = 0.0323 atm, $D_{AB} = 2.6\times^{-5}\,\text{m}^2/\text{s}$.

ANALYSIS: (a) Calculate Reynolds numbers

$$Re_1 = \frac{V_1 L_1}{n} = \frac{120\text{ m/s}\times 1\text{m}}{18.2\times 10^{-6}\text{m}^2/\text{s}} = 6.59\times10^6$$

$$Re_2 = \frac{60\text{ m/s}\times 2\text{m}}{18.2\times10^{-6}\text{m}^2/\text{s}} = 6.59\times10^6.$$

Note that

$$Pr_1 = 0.703 \qquad Sc_2 = \frac{n}{D_{AB}} = \frac{18.2\times10^{-6}\text{m}^2/\text{s}}{2.6\times10^{-5}\text{m}^2/\text{s}} = 0.700.$$

Since $Re_1 = Re_2$ and $Pr_1 = Sc_2$, the dimensionless solutions to the energy and species equations are identical. That is, from Eqs. 6.54 and 6.58,

$$T^*\left(x^*, y^*\right) = C_A^*\left(x^*, y^*\right)$$

$$\frac{T - T_s}{T_\infty - T_s} = \frac{C_A - C_{A,s}}{C_{A,\infty} - C_{A,s}} \tag{1}$$

where T^* and C_A^* are defined by Eqs. 6.37 and 6.38, respectively. Now, determine

$$C_{A,s} = \frac{p_{A,sat}}{\Re T} = \left(0.0323\text{ atm}/8.205\times10^{-2}\text{m}^3\cdot\text{atm/kmol}\cdot\text{K}\times(273+50)\text{K}\right)$$

$$C_{A,s} = 1.219\times10^{-3}\text{ kmol/kg.}$$

Continued …..

Substituting numerical values in Eq. (1),

$$C_A = C_{A,s} + \left(C_{A,\infty} - C_{A,s}\right)\frac{T - T_s}{T_\infty - T_s}$$

$$C_A = 1.219\times10^{-3}\ \text{kmol/m}^3 + \left(0 - 1.219\times10^{-3}\right)\ \text{kmol/m}^3\,\frac{(80-100)^\circ\text{C}}{(0-100)^\circ\text{C}}$$

$$C_A = 0.975\times10^{-3}\ \text{kmol/m}^3. \qquad <$$

The vapor pressure is then

$$p_A = C_A\,\Re T = 0.0258\ \text{atm}. \qquad <$$

(b) For case 1, $q'' = 2000\ \text{W/m}^2$. The rate equations are

$$q'' = \bar{h}\left(T_s - T_\infty\right) \tag{2}$$

$$n''_A = \bar{h}_m\left(C_{A,s} - C_{A,\infty}\right)\mathcal{M}_A. \tag{3}$$

From the analogy

$$\overline{Nu}_L = \overline{Sh}_L \qquad \rightarrow \qquad \frac{\bar{h}\,L_1}{k} = \frac{\bar{h}_m\,L_2}{D_{AB}} \quad \text{or} \quad \frac{\bar{h}}{\bar{h}_m} = \frac{L_2}{L_1}\,\frac{k}{D_{AB}}. \tag{4}$$

Combining Eqs. (2) - (4),

$$n''_A = q''\frac{\bar{h}_m}{\bar{h}}\,\frac{\left(C_{A,s} - C_{A,\infty}\right)\mathcal{M}_A}{\left(T_s - T_\infty\right)} = q''\frac{L_1 D_{AB}}{L_2 k}\,\frac{\left(C_{A,s} - C_{A,\infty}\right)\mathcal{M}_A}{\left(T_s - T_\infty\right)}$$

which numerically gives

$$n''_A = 2000\ \text{W/m}^2\ \frac{1\text{m}\left(2.6\times10^{-5}\text{m}^2/\text{s}\right)}{2\text{m}\left(28\times10^{-3}\text{W/m}\cdot\text{K}\right)}\,\frac{\left(1.219\times10^{-3} - 0\right)\text{kmol/m}^3\ \left(82\ \text{kg/kmol}\right)}{(100\text{-}0)\ \text{K}}$$

$$n''_A = 9.28\times10^{-4}\text{kg/s}\cdot\text{m}^2. \qquad <$$

COMMENTS: Recognize that the analogy between heat and mass transfer applies when the conservation equations and boundary conditions are of the same form.

PROBLEM 6.59

KNOWN: Convection heat transfer correlation for flow over a contoured surface.

FIND: (a) Evaporation rate from a water film on the surface, (b) Steady-state film temperature.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (b) Constant properties, (c) Negligible radiation, (d) Heat and mass transfer analogy is applicable.

PROPERTIES: *Table A-4*, Air (300K, 1 atm): k = 0.0263 W/m·K, $\nu = 15.89\times10^{-6} m^2/s$, Pr = 0.707; *Table A-6*, Water ($T_s \approx 280K$): $v_g = 130.4\ m^3/kg$, $h_{fg} = 2485$ kJ/kg; *Table A-8*, Water-air (T ≈ 298K): $D_{AB} = 0.26\times10^{-4} m^2/s$.

ANALYSIS: (a) The mass evaporation rate is

$$\dot{m}_{evap} = n_A = \bar{h}_m\ A\left[\rho_{A,sat}(T_s) - \phi_\infty\ \rho_{A,sat}(T_\infty)\right] = \bar{h}_m\ A\ \rho_{A,sat}(T_s).$$

From the heat and mass transfer analogy: $\overline{Sh}_L = 0.43\ Re_L^{0.58}\ Sc^{0.4}$

$$Re_L = \frac{VL}{\nu} = \frac{(10\ m/s)\ 1m}{15.89\times10^{-6} m^2/s} = 6.29\times10^5 \qquad Sc = \frac{\nu}{D_{AB}} = \frac{15.89\times10^{-6} m^2/s}{26\times10^{-6} m^2/s} = 0.61$$

$$\overline{Sh}_L = 0.43\left(6.29\times10^5\right)^{0.58}(0.61)^{0.4} = 814$$

$$\bar{h}_m = \frac{D_{AB}}{L}\overline{Sh}_L = \frac{0.26\times10^{-4} m^2/s}{1m}(814) = 0.0212\ m/s$$

$$\rho_{A,sat}(T_s) = v_g(T_s)^{-1} = 0.0077\ kg/m^3.$$

Hence, $$\dot{m}_{evap} = 0.0212 m/s \times 1m^2 \times 0.0077 kg/m^3 = 1.63\times10^{-4} kg/s.$$ <

(b) From a surface energy balance, $q''_{conv} = q''_{evap}$, or

$$\bar{h}_L(T_\infty - T_s) = \dot{m}''_{evap} h_{fg} \qquad T_s = T_\infty - \frac{\left(\dot{m}''_{evap} h_{fg}\right)}{\bar{h}_L}.$$

With $$\overline{Nu}_L = 0.43\left(6.29\times10^5\right)^{0.58}(0.707)^{0.4} = 864$$

$$\bar{h}_L = \frac{k}{L}\overline{Nu}_L = \frac{0.0263\ W/m\cdot K}{1m}864 = 22.7\ W/m^2\cdot K.$$

Hence, $$T_s = 300K - \frac{1.63\times10^{-4} kg/s\cdot m^2\left(2.485\times10^6\ J/kg\right)}{22.7\ W/m^2\cdot K} = 282.2K.$$ <

COMMENTS: The saturated vapor density, $\rho_{A,sat}$, is strongly temperature dependent, and if the initial guess of T_s needed for its evaluation differed from the above result by more than a few degrees, the density would have to be evaluated at the new temperature and the calculations repeated.

PROBLEM 6.60

KNOWN: Surface area and temperature of a coated turbine blade. Temperature and pressure of air flow over the blade. Molecular weight and saturation vapor pressure of the naphthalene coating. Duration of air flow and corresponding mass loss of naphthalene due to sublimation.

FIND: Average convection heat transfer coefficient.

SCHEMATIC:

Air flow
$p = 1$ atm
$T_\infty = 27^oC$

Naphthalene coating
$T_s = 27^oC$, $A_s = 0.05$ m^2
$p_{A,sat} = 1.33 \times 10^{-4}$ bar
$\Delta m = 8$ g for $\Delta t = 30$ min

ASSUMPTIONS: (1) Applicability of heat and mass transfer analogy, (2) Negligible change in A_s due to mass loss, (3) Naphthalene vapor behaves as an ideal gas, (4) Solid/vapor equilibrium at surface of coating, (5) Negligible vapor density in freestream of air flow.

PROPERTIES: *Table A-4*, Air (T = 300K): $\rho = 1.161$ kg/m^3, $c_p = 1007$ J/kg·K, $\alpha = 22.5 \times 10^{-6}$ m^2/s. *Table A-8*, Naphthalene vapor/air (T = 300K): $D_{AB} = 0.62 \times 10^{-5}$ m^2/s.

ANALYSIS: From the rate equation for convection mass transfer, the average convection mass transfer coefficient may be expressed as

$$\overline{h}_m = \frac{n_a}{A_s\left(\rho_{A,s} - \rho_{A,\infty}\right)} = \frac{\Delta m / \Delta t}{A_s \ \rho_{A,s}}$$

where

$$\rho_{A,s} = \rho_{A,sat}\left(T_s\right) = \frac{\mathcal{M}_a \, p_{A,sat}}{\Re T_s} = \frac{(128.16\,\text{kg/kmol})1.33\times10^{-4}\,\text{bar}}{0.08314\,\text{m}^3\cdot\text{bar/kmol}\cdot\text{K}\ (300\text{K})} = 6.83\times10^{-4}\,\text{kg/m}^3$$

Hence,

$$\overline{h}_m = \frac{0.008\,\text{kg}/\left(30\,\text{min}\times 60\text{s/min}\right)}{0.05\text{m}^2\left(6.83\times10^{-4}\,\text{kg/m}^3\right)} = 0.13\,\text{m/s}$$

Using the heat and mass transfer analogy with n = 1/3, we then obtain

$$\overline{h} = \overline{h}_m \rho c_p \text{Le}^{2/3} = \overline{h}_m \rho c_p \left(\frac{\alpha}{D_{AB}}\right)^{2/3} = 0.130\,\text{m/s}\left(1.161\,\text{kg/m}^3\right)\times$$

$$1007\,\text{J/kg}\cdot\text{K}\left(22.5\times10^{-6}/0.62\times10^{-5}\right)^{2/3} = 359\,\text{W/m}^2\cdot\text{K}$$

<

COMMENTS: The naphthalene sublimation technique has been used extensively to determine convection coefficients associated with complex flows and geometries.

PROBLEM 6.61

KNOWN: Mass transfer experimental results on a half-sized model representing an engine strut.

FIND: (a) The coefficients C and m of the correlation $\overline{Sh}_L = CRe_L^m Sc^{1/3} \dfrac{n!}{r!(n-r)!}$ for the mass transfer results, (b) Average heat transfer coefficient, $\overline{h}$, for the full-sized strut with prescribed operating conditions, (c) Change in total heat rate if characteristic length L_H is doubled.

SCHEMATIC:

Air, $T_\infty = 27°C$, $p_\infty = 1atm$; Naphthalene; $L_M = L_H/2$

$\overline{Sh}_L$	Re_L
282	60,000
491	120,000
568	144,000
989	288,000

Air, $T_\infty = 184°C$, $V = 60 m/s$, $p_\infty = 1atm$; Strut; $A_s = 2.2 L_H \cdot \ell$; $T_s = 70°C$; $L_H = 60mm$

Mass transfer　　　　Heat transfer

ASSUMPTIONS: Analogy exists between heat and mass transfer.

PROPERTIES: *Table A-4*, Air $\left(\overline{T} = (T_\infty + T_s)/2 = 400K, 1\ atm\right)$: $\nu = 26.41\times10^{-6} m^2/s$, $k = 0.0338$ W/m·K, Pr = 0.690; $\left(\overline{T} = 300K\right)$: $\nu_B = 15.89\times10^{-6}\ m^2/s$; *Table A-8*, Naphthalene-air (300K, 1 atm): $D_{AB} = 0.62\times10^{-5} m^2/s$, $Sc = \nu_B/D_{AB} = 15.89\times10^{-6} m^2/s / 0.62\times10^{-5} m^2/s = 2.56$.

ANALYSIS: (a) The correlation for the mass transfer experimental results is of the form $\overline{Sh}_L = CRe_L^m\ Sc^{1/3}$. The constants C,m may be evaluated from two data sets of $\overline{Sh}_L$ and Re_L; choosing the middle sets (2,3):

$$\frac{\left(\overline{Sh}_L\right)_2}{\left(\overline{Sh}_L\right)_3} = \frac{(Re_L)_2^m}{(Re_L)_3^m} \quad \text{or } m = \frac{\log\left[\overline{Sh}_L)_2 / \overline{Sh}_L)_3\right]}{\log\left[Re_L)_2 / Re_L)_3\right]} = \frac{\log\ [491/568]}{\log\ [120{,}000/144{,}000]} = 0.80. \qquad <$$

Then, using set 2, find

$$C = \frac{\overline{Sh}_L)_2}{Re_L^m)_2\ Sc^{1/3}} = \frac{491}{(120{,}000)^{0.8}\ 2.56^{1/3}} = 0.031. \qquad <$$

(b) For the heat transfer analysis of the strut, the correlation will be of the form $\overline{Nu}_L = \overline{h}_L \cdot L_H/k = 0.031\ Re_L^{0.8} Pr^{1/3}$ where $Re_L = V L_H/\nu$ and the constants C,m were determined in Part (a). Substituting numerical values,

$$\overline{h}_L = \overline{Nu}_L \cdot \frac{k}{L_H} = 0.031 \left[\frac{60\ m/s \times 0.06\ m}{26.41\times10^{-6} m^2/s}\right]^{0.8} 0.690^{1/3}\ \frac{0.0338\ W/m\cdot K}{0.06\ m} = 198\ W/m^2\cdot K. \qquad <$$

(c) The total heat rate for the strut of characteristic length L_H is $q = \overline{h}\ A_s\left(T_s - T_\infty\right)$, where A_s = 2.2 L_H·l and

$$\overline{h} \sim \overline{Nu}_L \cdot L_H^{-1} \sim RE_L^{0.8} \cdot L_H^{-1} \sim L_H^{0.8} \cdot L_H^{-1} \sim L_H^{-0.2} \quad A_s \sim L_H$$

Hence, $q \sim \overline{h} \cdot A_s \sim \left(L_H^{-0.2}\right)\ \left(L_H\right) \sim L_H^{0.8}$. If the characteristic length were doubled, the heat rate would be increased by a factor of $(2)^{0.8} = 1.74$. $<$

PROBLEM 6.62

KNOWN: Boundary layer temperature distribution for flow of dry air over water film.

FIND: Evaporative mass flux and whether net energy transfer is to or from the water.

SCHEMATIC:

ASSUMPTIONS: (1) Heat and mass transfer analogy is applicable, (2) Water is well insulated from below.

PROPERTIES: *Table A-4*, Air (T_s = 300K, 1 atm): k = 0.0263 W/m·K; *Table A-6*, Water vapor (T_s = 300K): $\rho_{A,s} = v_g^{-1} = 0.0256\ \text{kg/m}^3,\ h_{fg} = 2.438\times10^6\ \text{J/kg}$; *Table A-8*, Air-water vapor $(T_s = 300\text{K})$: $D_{AB} = 0.26\times10^{-4}\text{m}^2/\text{s}$.

ANALYSIS: From the heat and mass transfer analogy,

$$\frac{\rho_A - \rho_{A,s}}{\rho_{A,\infty} - \rho_{A,s}} = 1 - \exp\left[-Sc\frac{u_\infty y}{\nu}\right].$$

Using Fick's law at the surface (y = 0), the species flux is

$$n''_A = -D_{AB}\frac{\partial \rho_A}{\partial y}\bigg|_{y=0} = +\rho_{A,s}\ D_{AB}\ Sc\ \frac{u_\infty}{\nu}$$

$$n''_A = 0.0256\ \text{kg/m}^3 \times 0.26\times10^{-4}\text{m}^2/\text{s} \times (0.6)5000\ \text{m}^{-1} = 2.00\times10^{-3}\ \text{kg/s}\cdot\text{m}^2.$$

The net heat flux to the water has the form

$$q''_{net} = q''_{conv} - q''_{evap} = +k\ \frac{\partial T}{\partial y}\bigg|_{y=0} - n''_A\ h_{fg} = k(T_\infty - T_s)\ Pr\ \frac{u_\infty}{\nu} - n''_A\ h_{fg}$$

and substituting numerical values, find

$$q''_{net} = 0.0263\frac{\text{W}}{\text{m}\cdot\text{K}}(100\text{K})\ 0.7 \times 5000\ \text{m}^{-1} - 2\times10^{-3}\frac{\text{kg}}{\text{s}\cdot\text{m}^2}\times2.438\times10^6\ \text{J/kg}$$

$$q''_{net} = 9205\ \text{W/m}^2 - 4876\ \text{W/m}^2 = 4329\ \text{W/m}^2.$$

Since $q''_{net} > 0$, the net heat transfer is to the water. <

COMMENTS: Note use of properties (D_{AB} and k) evaluated at T_s to determine surface fluxes.

PROBLEM 6.63

KNOWN: Distribution of local convection heat transfer coefficient for obstructed flow over a flat plate with surface and air temperatures of 310K and 290K, respectively.

FIND: Average convection mass transfer coefficient.

SCHEMATIC:

Air
T_∞, V
Naphthalene
$h_x = 0.7 + 13.6x - 3.4x^2$
$L = 3m$

ASSUMPTIONS: Heat and mass transfer analogy is applicable.

PROPERTIES: *Table A-4*, Air $\left(T_f = (T_s + T_\infty)/2 = (310 + 290)K/2 = 300 \text{ K, 1 atm}\right)$: $k = 0.0263 \text{ W/m}\cdot\text{K}$, $\nu = 15.89\times10^{-6}\text{m}^2/\text{s}$, $Pr = 0.707$. *Table A-8*, Air-napthalene (300K, 1 atm): $D_{AB} = 0.62\times10^{-5}\text{m}^2/\text{s}$, $Sc = \nu/D_{AB} = 2.56$.

ANALYSIS: The average heat transfer coefficient is

$$\bar{h}_L = \frac{1}{L}\int_0^L h_x dx = \frac{1}{L}\int_0^L \left(0.7 + 13.6x - 3.4x^2\right)dx$$

$$\bar{h}_L = 0.7 + 6.8L - 1.13L^2 = 10.9 \text{ W/m}^2\cdot\text{K}.$$

Applying the heat and mass transfer analogy with n = 1/3, Equation 6.66 yields

$$\frac{\overline{Nu}_L}{Pr^{1/3}} = \frac{\overline{Sh}_L}{Sc^{1/3}}$$

Hence,

$$\frac{\bar{h}_{m,L}L}{D_{AB}} = \frac{\bar{h}_L L}{k}\frac{Sc^{1/3}}{Pr^{1/3}}$$

$$\bar{h}_{m,L} = \bar{h}_L \frac{D_{AB}}{k}\frac{Sc^{1/3}}{Pr^{1/3}} = 10.9 \text{ W/m}^2\cdot\text{K}\,\frac{0.62\times10^{-5}\text{m}^2/\text{s}}{0.0263 \text{ W/m}\cdot\text{K}}\left(\frac{2.56}{0.707}\right)^{1/3}$$

$$\bar{h}_{m,L} = 0.00395 \text{ m/s.}$$ <

COMMENTS: The napthalene sublimation method provides a useful tool for determining local convection coefficients.

PROBLEM 6.64

KNOWN: Radial distribution of local Sherwood number for uniform flow normal to a circular disk.

FIND: (a) Expression for average Nusselt number. (b) Heat rate for prescribed conditions.

SCHEMATIC:

Air
$T_\infty = 25°C$, V
$Re_D = 5\times10^4$
r_o
D = 20mm
$Sh_D = Sh_o[1 + a(r/r_o)^n]$
$Sh_o = 0.814 Re_D^{1/2} Sc^{0.36}$
n = 5.5, a = 1.2
$T_s = 125°C$

ASSUMPTIONS: (1) Constant properties, (2) Applicability of heat and mass transfer analogy.

PROPERTIES: *Table A-4*, Air $\left(\bar{T} = 75°C = 348\ K\right)$: k = 0.0299 W/m·K, Pr = 0.70.

ANALYSIS: (a) From the heat and mass transfer analogy, Equation 6.66,

$$\frac{\overline{Nu}_D}{Pr^{0.36}} = \frac{\overline{Sh}_D}{Sc^{0.36}}$$

where

$$\overline{Sh}_D = \frac{1}{A_s}\int_{A_s} Sh_D(r)dA_s = \frac{Sh_o}{\pi\ r_o^2}2\pi\int_0^{r_o}\left[1 + a\left(r/r_o\right)^n\right]rdr$$

$$\overline{Sh}_D = \frac{2Sh_o}{r_o^2}\left[\frac{r^2}{2} + \frac{ar^{n+2}}{(n+2)r_o^n}\right]_0^{r_o} = Sh_o\left[1 + 2a/(n+2)\right]$$

Hence,

$$\overline{Nu}_D = 0.814\left[1 + 2a/(n+2)\right]Re_D^{1/2}Pr^{0.36}.$$ <

(b) The heat rate for these conditions is

$$q = \bar{h}A\left(T_s - T_\infty\right) = 0.814\left[1 + 2a/(n+2)\right]\frac{k}{D}Re_D^{1/2}Pr^{0.36}\frac{\left(\pi\ D^2\right)}{4}\left(T_s - T_\infty\right)$$

$$q = 0.814(1 + 2.4/7.5)0.0299\ W/m\cdot K\left(\pi 0.02\ m/4\right)\left(5\times10^4\right)^{1/2}(0.07)^{0.36}\left(100°C\right)$$

$$q = 9.92\ W.$$ <

COMMENTS: The increase in h(r) with r may be explained in terms of the sharp turn which the boundary layer flow must make around the edge of the disk. The boundary layer accelerates and its thickness decreases as it makes the turn, causing the local convection coefficient to increase.

PROBLEM 6.65

KNOWN: Convection heat transfer correlation for wetted surface of a sand grouse. Initial water content of surface. Velocity of bird and ambient air conditions.

FIND: Flight distance for depletion of 50% of initial water content.

SCHEMATIC:

Air
$T_\infty = 310K$
$V = 30 m/s$
$\phi_\infty = 0.25$
$A_s = 0.04 m^2$
$T_s = 305K$
$M_i = 0.05 kg$

ASSUMPTIONS: (1) Steady-state conditions, (2) Vapor behaves as a perfect gas, (3) Constant properties, (4) Applicability of heat and mass transfer analogy.

PROPERTIES: Air (given): $\nu = 16.7\times10^{-6} m^2/s$; Air-water vapor (given): $D_{AB} = 0.26\times10^{-4} m^2/s$; *Table A-6*, Water vapor (T_s = 305 K): v_g = 29.74 m^3/kg; (T_s = 310 K), v_g = 22.93 m^3/kg.

ANALYSIS: The maximum flight distance is

$$X_{max} = V t_{max}$$

where the time to deplete 50% of the initial water content ΔM is

$$t_{max} = \frac{\Delta M}{\dot{m}_{evap}} = \frac{\Delta M}{\overline{h}_m A_s \left(\rho_{A,s} - \rho_{A,\infty}\right)}.$$

The mass transfer coefficient is

$$\overline{h}_m = \overline{Sh}_L \frac{D_{AB}}{L} = 0.034 Re_L^{4/5} Sc^{1/3} \frac{D_{AB}}{L}$$

$$Sc = \nu / D_{AB} = 0.642, \qquad L = (A_s)^{1/2} = 0.2 \text{ m}$$

$$Re_L = \frac{VL}{\nu} = \frac{30 \text{ m/s} \times 0.2 \text{ m}}{16.7\times10^{-6} m^2/s} = 3.59\times10^5$$

$$\overline{h}_m = 0.034\left(3.59\times10^5\right)^{4/5} (0.642)^{1/3} \left(0.26\times10^{-4} m^2/s / 0.2 \text{ m}\right) = 0.106 \text{ m/s}.$$

Hence,

$$t_{max} = \frac{0.025 \text{ kg}}{0.106 \text{ m/s}\left(0.04 \text{ m}^2\right)\left[(29.74)^{-1} - 0.25(22.93)^{-1}\right] \text{kg/m}^3} = 259 \text{ s}$$

$$X_{max} = 30 \text{ m/s}\left(259 \text{ s}\right) = 7785 \text{ m} = 7.78 \text{ km}.$$ <

COMMENTS: Evaporative heat loss is balanced by convection heat transfer from air.

Hence, $T_s < T_\infty$.

PROBLEM 6.66

KNOWN: Water-soaked paper towel experiences simultaneous heat and mass transfer while subjected to parallel flow of air, irradiation from a radiant lamp bank, and radiation exchange with surroundings. Average convection coefficient estimated as $\overline{h}$ = 28.7 W/m^2·K.

FIND: (a) Rate at which water evaporates from the towel, n_A (kg/s), and (b) The net rate of radiation transfer, q_{rad} (W), to the towel. Determine the irradiation G (W/m^2).

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Vapor behaves as an ideal gas, (3) Constant properties, (4) Towel experiences radiation exchange with the large surroundings as well as irradiation from the lamps, (5) Negligible heat transfer from the bottom side of the towel, and (6) Applicability of the heat-mass transfer analogy.

PROPERTIES: *Table A.4*, Air (T_f = 300 K): ρ = 1.1614 kg/m^3, c_p = 1007 J/kg·K, $\alpha = 22.5 \times 10^{-6}$ m^2/s; *Table A.6*, Water (310 K): $\rho_{A,s} = \rho_g = 1/v_g = 1/22.93 = 0.0436$ kg/m^3, h_{fg} = 2414 kJ/kg.

ANALYSIS: (a) The evaporation rate from the towel is

$$n_A = \overline{h}_m A_s \left(\rho_{A,s} - \rho_{A,\infty}\right)$$

where $\overline{h}_m$ can be determined from the heat-mass transfer analogy, Eq. 6.92, with n = 1/3,

$$\frac{h}{h_m} = \rho c_p Le^{2/3} = \rho c_p \left(\frac{\alpha}{D_{AB}}\right)^{2/3} = 1.614\,kg/m^3 \times 1007\,J/kg\cdot K\left(\frac{22.5\times10^{-6}}{0.26\times10^{-4}}\right)^{2/3} = 1062\,J/m^3\cdot K$$

$$h_m = 28.7\,W/m^2\cdot K/1062\,J/m^3\cdot K = 0.0270\,m/s$$

The evaporation rate is

$$n_A = 0.0270\,m/s \times (0.0925\times0.0925)m^2\,(0.0436-0)kg/m^3 = 1.00\times10^{-5}\,kg/s \qquad <$$

(b) Performing an energy balance on the towel considering processes of evaporation, convection and radiation, find

$$\dot{E}_{in} - \dot{E}_{out} = q_{conv} - q_{evap} + q_{rad} = 0$$

$$\overline{h}A_s\left(T_\infty - T_s\right) - n_A h_{fg} + q_{rad} = 0$$

$$q_{rad} = 1.00\times10^{-5}\,kg/s \times 2414\times10^3\,J/kg - 27.8\,W/m^2\,(0.0925\,m)^2\,(290-310)K$$

$$q_{rad} = 2414\,W + 4.76\,W = 28.9\,W \qquad <$$

Continued...

The net radiation heat transfer to the towel is comprised of the absorbed irradiation and the net exchange between the surroundings and the towel,

$$q_{rad} = \alpha G A_s + \varepsilon A_s \sigma \left(T_{sur}^4 - T_s^4\right)$$

$$28.9\,\text{W} = 0.96G\,(0.0925\,\text{m})^2 + 0.96 \times (0.0925\,\text{m})^2\, 5.67 \times 10^{-8}\,\text{W/m}^2 \cdot \text{K}^4 \left(300^4 - 310^4\right)\text{K}^4$$

Solving, find the irradiation from the lamps,

$G = 3583\ \text{W/m}^2$. <

COMMENTS: (1) From the energy balance in Part (b), note that the heat rate by convection is considerably smaller than that by evaporation.

(2) As we'll learn in Chapter 12, the lamp irradiation found in Part (c) is nearly 3 times that of solar irradiation to the earth's surface.

PROBLEM 6.67

KNOWN: Thin layer of water on concrete surface experiences evaporation, convection with ambient air, and radiation exchange with the sky. Average convection coefficient estimated as $\overline{h} = 53\ W/m^2 \cdot K$.

FIND: (a) Heat fluxes associated with convection, q''_{conv}, evaporation, q''_{evap}, and radiation exchange with the sky, q''_{rad}, (b) Use results to explain why the concrete is wet instead of dry, and (c) Direction of heat flow and the heat flux by conduction into or out of the concrete.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Vapor behaves as an ideal gas, (3) Constant properties, (4) Water surface is small compare to large, isothermal surroundings (sky), and (4) Applicability of the heat-mass transfer analogy.

PROPERTIES: *Table A.4*, Air ($T_f = (T_\infty + T_s)/2 = 282.5$ K): $\rho = 1.243\ kg/m^3$, $c_p = 1007\ J/kg \cdot K$, $\alpha = 2.019 \times 10^5\ m^2/s$; *Table A.8*, Water-air ($T_f = 282.5$ K): $D_{AB} = 0.26 \times 10^{-4}\ m^2/s\ (282.5/298)^{3/2} = 0.24 \times 10^{-4}\ m^2/s$; *Table A.6*, Water ($T_s = 275$ K): $\rho_{A,s} = \rho_g = 1/v_g = 1/181.7 = 0.0055\ kg/m^3$, $h_{fg} = 2497$ kJ/kg; *Table A.6*, Water ($T_\infty = 290$ K): $\rho_{A,s} = 1/69.7 = 0.0143\ kg/m^3$.

ANALYSIS: (a) The heat fluxes associated with the processes shown on the schematic are

Convection:

$$q''_{conv} = \overline{h}\left(T_\infty - T_s\right) = 53\, W/m^2 \cdot K\left(290 - 275\right)K = +795\, W/m^2$$ <

Radiation Exchange:

$$q''_{rad} = \varepsilon\sigma\left(T_s^4 - T_{sky}\right) = 0.96 \times 5.76 \times 10^{-8}\, W/m^2 \cdot K^4\left(275^4 - 240^4\right)K^4 = +131\, W/m^2$$ <

Evaporation:

$$q''_{evap} = n''_A h_{fg} = -2.255 \times 10^{-4}\ kg/s \cdot m^2 \times 2497 \times 10^3\ J/kg = -563.1\, W/m^2$$ <

where the evaporation rate *from* the surface is

$$n''_A = \overline{h}_m\left(\rho_{A,s} - \rho_{A,\infty}\right) = 0.050\, m/s\left(0.0055 - 0.7 \times 0.0143\right)kg/m^3 = -2.255 \times 10^{-4}\, kg/s \cdot m^2$$

Continued...

and where the mass transfer coefficient is evaluated from the heat-mass transfer analogy, Eq. 6.92, with n = 1/3,

$$\frac{\overline{h}}{\overline{h}_m} = \rho c_p Le^{2/3} = \rho c_p \left(\frac{\alpha}{D_{AB}}\right)^{2/3} = 1.243\,kg/m^3 \times 1007\,J/kg\cdot K \left(\frac{2.019\times10^{-5}}{0.26\times10^{-4}}\right)^{2/3}$$

$$\frac{\overline{h}}{\overline{h}_m} = 1058\,J/m^3\cdot K$$

$$\overline{h}_m = 53\,W/m^2\cdot K / 1058\,J/m^3\cdot K = 0.050\,m/s$$

(b) From the foregoing evaporation calculations, note that water vapor from the air is condensing on the liquid water layer. That is, vapor is being transported to the surface, explaining why the concrete surface is wet, even without rain.

(c) From an overall energy balance on the water film considering conduction in the concrete as shown in the schematic,

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$q''_{conv} - q''_{evap} - q''_{rad} - q''_{cond} = 0$$

$$q''_{cond} = q''_{conv} - q''_{evap} - q''_{rad}$$

$$q''_{cond} = 1795\,W/m^3 - \left(-563.1\,W/m^2\right) - \left(+131\,W/m^2\right) = 1227\,W/m^2 \qquad <$$

The heat flux by conduction is *into* the concrete.

PROBLEM 6.68

KNOWN: Heater power required to maintain wetted (water) plate at 27°C, and average convection coefficient for specified dry air temperature, case (a).

FIND: Heater power required to maintain the plate at 37°C for the same dry air temperature if the convection coefficients remain unchanged, case (b).

SCHEMATIC:

Air
$T_\infty = 32^oC$
$\bar{h} = 20\ W/m^2\text{-}K$
$\phi_\infty = 0$
Water film
$A_s = 0.2\ m^2$
q''_{cv} q''_{evap}
$T_{s,a} = 27^oC$
Case (a)
$P_{e,a} = 432\ W$
q''_{cv} q''_{evap}
$T_{s,b} = 37^oC$
Case (b)
$P_{e,b} = ?$

ASSUMPTIONS: (1) Steady-state conditions, (2) Convection coefficients unchanged for different plate temperatures, (3) Air stream is dry at atmospheric pressure, and (4) Negligible heat transfer from the bottom side of the plate.

PROPERTIES: *Table A-6*, Water ($T_{s,a}$ = 27°C = 300 K): $\rho_{A,s} = 1/v_g = 0.02556\ kg/m^3$, $h_{fg} = 2.438 \times 10^6$ J/kg; Water ($T_{s,b}$ = 37°C = 310 K): $\rho_{A,s} = 1/v_g = 0.04361\ kg/m^3$, $h_{fg} = 2.414 \times 10^6$ J/kg.

ANALYSIS: For *case (a)* with T_s = 27°C and P_e = 432 W, perform an energy balance on the plate to determine the mass transfer coefficient h_m.

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$P_{e,a} - \left(q''_{evap} + q''_{cv}\right) A_s = 0$$

Substituting the rate equations and appropriate properties,

$$P_{e,a} - \left[\bar{h}_m\left(\rho_{A,s} - \rho_{A,\infty}\right)h_{fg} + \bar{h}\left(T_{s,a} - T_\infty\right)\right]A_s = 0$$

$$432\ W + \Big[\bar{h}_m\left(0.0256\ kg/m^3 - 0\right)\times 2.438\times 10^6\ J/kg +$$
$$20\ W/m^2\cdot K\left(27-32\right)K\Big]\times 0.2\ m^2 = 0$$

where $\rho_{A,s}$ and h_{fg} are evaluated at T_s = 27°C = 300 K. Find,

$$\bar{h}_m = 0.0363\ m/s$$

For case (b), with T_s = 37°C and the same values for $\bar{h}$ and $\bar{h}_m$, perform an energy balance to determine the heater power required to maintain this condition.

$$P_{e,b} - \left[\bar{h}_m\left(\rho_{A,s} - 0\right)h_{fg} + \bar{h}\left(T_{s,b} - T_\infty\right)\right]A_s = 0$$

$$P_{e,b} - \Big[0.0363\ m/s\left(0.04361 - 0\right)kg/m^3 \times 2.414\times 10^6\ J/kg +$$
$$20\ W/m^2\cdot K\left(37-32\right)\Big]\times 0.2\ m^2 = 0$$

$$P_{e,b} = 784\ W$$ <

where $\rho_{A,a}$ and h_{fg} are evaluated at T_s = 37°C = 310 K.

PROBLEM 6.69

KNOWN: Dry air at 32°C flows over a wetted plate of width 1 m maintained at a surface temperature of 27°C by an embedded heater supplying 432 W.

FIND: (a) The evaporation rate of water from the plate, n_A (kg/h) and (b) The plate temperature T_s when all the water is evaporated, but the heater power remains the same.

SCHEMATIC:

$T_\infty = 32\ ^oC$ Dry air q_{conv} q_{evap} $T_s = 27\ ^oC$ x $P_e = 432$ W L

(a) Wetted plate

q_{conv} T_s x $P_e = 432$ W $L = 200$ mm

(a) Dry plate

ASSUMPTIONS: (1) Steady-state conditions, (2) Vapor behaves as an ideal gas, (3) Constant properties, and (4) Applicability of the heat-mass transfer analogy.

PROPERTIES: *Table A.4*, Air (T_f = (32 + 27)°C/2 = 302.5 K): $\rho = 1.153$ kg/m^3, $c_p = 1007$ J/kg·K, α = 2.287 $\times 10^5$ m^2/s; *Table A.8*, Water-air ($T_f \approx 300$ K): $D_{AB} = 0.26 \times 10^{-4}$ m^2/s; *Table A.6*, Water (T_s = 27°C = 300 K): $\rho_{A,s} = 1/v_g = 1/39.13 = 0.0256$ kg/m^3, $h_{fg} = 2438$ kJ/kg.

ANALYSIS: (a) Perform an energy balance on the wetted plate to obtain the evaporation rate, n_A.

$$\dot{E}_{in} - \dot{E}_{out} = 0 \qquad\qquad P_e + q_{conv} - q_{evap} = 0$$

$$P_e + \overline{h}A_s\left(T_\infty - T_s\right) - n_A h_{fg} = 0 \tag{1}$$

In order to find $\overline{h}$, invoke the heat-mass transfer analogy, Eq. (6.92) with n = 1/3,

$$\frac{\overline{h}}{\overline{h}_m} = \rho c_p Le^{2/3} = \rho c_p\left(\frac{\alpha}{D_{AB}}\right)^{2/3} = 1.153\,kg/m^3 \times 1007\,J/kg\cdot K\left(\frac{2.287\times10^{-5}}{0.26\times10^{-4}}\right)^{2/3} = 1066\,J/m^3\cdot K \tag{2}$$

The evaporation rate equation

$$n_A = \overline{h}_m A_s\left(\rho_{A,s} - \rho_{A,\infty}\right)$$

Substituting Eqs. (2) and (3) into Eq. (1), find $\overline{h}_m$

$$P_e + \left(1066\,J/m^3\cdot K\,h_m\right)A_s\left(T_\infty - T_s\right) - h_m A_s\left(\rho_{A,s} - \rho_{A,\infty}\right)h_{fg} = 0 \tag{4}$$

$$432\,W + \left[1066\,J/m^3\cdot K\left(32-27\right)K - \left(0.0256-0\right)kg/m^3\times2438\times10^3\,J/kg\right]\left(0.200\times1\right)m^2\cdot h_m = 0$$

$432 + [5330 - 62{,}413] \times 0.20\, h_m = 0$

$h_m = 0.0378$ m/s

Using Eq. (3), find

$$n_A = 0.0378\,m/s\left(0.200\times1\right)m^2\left(0.0256-0\right)kg/m^3 = 1.94\times10^{-4}\,kg/s = 0.70\,kg/h \qquad <$$

(b) When the plate is dry, all the power must be removed by convection,

$P_e = q_{conv} = \overline{h}\, A_s(T_s - T_\infty)$

Assuming $\overline{h}$ is the same as for conditions with the wetted plate,

$$T_s = T_\infty + P_e/\overline{h}\,A_s = T_\infty + P_e/\left(1066h_m\right)A_s$$

$$T_s = 32^\circ C + 432\,W\Big/\left(1066\times0.0378\,W/m^2\cdot K\times0.200\,m^2\right) = 85.6^\circ C \qquad <$$

PROBLEM 6.70

KNOWN: Surface temperature of a 20-mm diameter sphere is 32°C when dissipating 2.51 W in a dry air stream at 22°C.

FIND: Power required by the imbedded heater to maintain the sphere at 32°C if its outer surface has a thin porous covering saturated with water for the same dry air temperature.

SCHEMATIC:

Air
q_{cv}
$T_s = 32°C$
$P_{e,d} = 2.51$ W
Sphere, D = 20 mm
$T_\infty = 22°C$
$\phi_\infty = 0$
Dry condition (d)
q_{cv}
$T_s = 32°C$
q_{evap}
$P_{e,w} = ?$
Water film
Wet condition (w)

ASSUMPTIONS: (1) Steady-state conditions, (2) Heat and mass transfer analogy is applicable, (3) Heat transfer convection coefficient is the same for the dry and wet condition, and (3) Properties of air and the diffusion coefficient of the air-water vapor mixture evaluated at 300 K.

PROPERTIES: *Table A-4*, Air (300 K, 1 atm): $\rho = 1.1614$ kg/m^3, $c_p = 1007$ J/kg·K, $\alpha = 22.5 \times 10^{-6}$ m^2/s; *Table A-8*, Water-air mixture (300 K, 1 atm): $D_{A-B} = 0.26 \times 10^{-4}$ m^2/s; *Table A-4*, Water (305 K, 1 atm): $\rho_{A,s} = 1/v_g = 0.03362$ kg/m^3, $h_{fg} = 2.426 \times 10^6$ J/kg.

ANALYSIS: For the *dry case (d)*, perform an energy balance on the sphere and calculate the heat transfer convection coefficient.

$$\dot{E}_{in} - \dot{E}_{out} = P_{e,d} - q_{cv} = 0 \qquad P_{e,d} - \overline{h}\,A_s\left(T_s - T_\infty\right) = 0$$

$$2.51\ \text{W} - \overline{h}\pi\left(0.020\ \text{m}\right)^2 \times \left(32-22\right)\text{K} = 0 \qquad \overline{h} = 200\ \text{W/m}^2\cdot\text{K}$$

Use the heat-mass analogy, Eq. (6.67) with n = 1/3, to determine $\overline{h}_m$.

$$\frac{\overline{h}}{\overline{h}_m} = \rho\, c_p \left(\frac{\alpha}{D_{AB}}\right)^{2/3}$$

$$\frac{200\ \text{W/m}^2\cdot\text{K}}{\overline{h}_m} = 1.1614\ \text{kg/m}^3 \times 1007\ \text{J/kg}\cdot\text{K}\left(\frac{22.5\times10^{6}\,\text{m}^2/\text{s}}{0.26\times10^{6}\,\text{m}^2/\text{s}}\right)^{2/3}$$

$$\overline{h}_m = 0.188\ \text{m/s}$$

For the *wet case* (*w*), perform an energy balance on the wetted sphere using values for $\overline{h}$ and $\overline{h}_m$ to determine the power required to maintain the same surface temperature.

$$\dot{E}_{in} - \dot{E}_{out} = P_{e,w} - q_{cv} - q_{evap} = 0$$

$$P_{e,w} - \left[\overline{h}\left(T_s - T_\infty\right) + \overline{h}_m\left(\rho_{A,s} - \rho_{A,\infty}\right)h_{fg}\right]A_s = 0$$

$$P_{e,w} - \Big[200\ \text{W/m}^2\cdot\text{K}\left(32-22\right)\text{K} +$$

$$0.188\ \text{m/s}\left(0.03362-0\right)\text{kg/m}^3 \times 2.426\times10^6\,\text{J/kg}\Big]\pi\left(0.020\,\text{m}\right)^2 = 0$$

$$P_{e,w} = 21.8\ \text{W}$$ <

COMMENTS: Note that $\rho_{A,s}$ and h_{fg} for the mass transfer rate equation are evaluated at T_s = 32°C = 305 K, not 300 K. The effect of evaporation is to require nearly 8.5 times more power to maintain the same surface temperature.

PROBLEM 6.71

KNOWN: Operating temperature, ambient air conditions and make-up water requirements for a hot tub.

FIND: Heater power required to maintain prescribed conditions.

SCHEMATIC:

ASSUMPTIONS: (1) Side wall and bottom are adiabatic, (2) Heat and mass transfer analogy is applicable.

PROPERTIES: *Table A-4*, Air $\left(\bar{T} = 300K, 1 \text{ atm}\right)$: r = 1.161 kg/m^3, c_p = 1007 J/kg·K, α = 22.5× 10^{-6}m^2/s; *Table A-6*, Sat. water vapor (T = 310K): h_{fg} = 2414 kJ/kg, $\rho_{A,sat}(T) = 1/v_g$ = (22.93m^3/kg)$^{-1}$ = 0.0436 kg/m^3; (T_∞ = 290K): $\rho_{A,sat}(T_\infty) = 1/v_g$ = (69.7 m^3/kg)$^{-1}$ = 0.0143 kg/m^3; *Table A-8*, Air-water vapor (298K): D_{AB} = 26 × 10^{-6}m^2/s.

ANALYSIS: Applying an energy balance to the control volume,

$$q_{elec} = q_{conv} + q_{evap} = \bar{h}\,A\left(T - T_\infty\right) + \dot{m}_{evap}\,h_{fg}\left(T\right).$$

Obtain $\bar{h}\,A$ from Eq. 6.67 with n = 1/3,

$$\frac{\bar{h}}{\bar{h}_m} = \frac{\bar{h}\,A}{\bar{h}_m A} = r\ c_p Le^{2/3}$$

$$\bar{h}\,A = \bar{h}_m A\ r\ c_p Le^{2/3} = \frac{\dot{m}_{evap}}{r_{A,sat}\left(T\right) - f_\infty\ r_{A,sat}\left(T_\infty\right)}\ r\ c_p Le^{2/3}.$$

Substituting numerical values,

$$Le = a/D_{AB} = \left(22.5\times10^{-6}m^2/s\right)/26\times10^{-6}m^2/s = 0.865$$

$$\bar{h}A = \frac{10^{-3}\text{ kg/s}}{\left[0.0436 - 0.3\times0.0143\right]\text{ kg/m}^3}1.161\frac{kg}{m^3}\times1007\frac{J}{kg\cdot K}\left(0.865\right)^{2/3}$$

$$\bar{h}A = 27.0\text{ W/K}.$$

Hence, the required heater power is

$$q_{elec} = 27.0\text{W/K}\left(310 - 290\right)\text{K} + 10^{-3}\text{kg/s}\times2414\text{kJ/kg}\times1000\text{J/kJ}$$

$$q_{elec} = \left(540 + 2414\right)\ \text{W} = 2954\text{ W}.$$ <

COMMENTS: The evaporative heat loss is dominant.

PROBLEM 6.72

KNOWN: Water freezing under conditions for which the air temperature exceeds 0°C.

FIND: (a) Lowest air temperature, T_∞, before freezing occurs, neglecting evaporation, (b) The mass transfer coefficient, h_m, for the evaporation process, (c) Lowest air temperature, T_∞, before freezing occurs, including evaporation.

SCHEMATIC:

No evaporation   With evaporation

ASSUMPTIONS: (1) Steady-state conditions, (2) Water insulated from ground, (3) Water surface has ε = 1, (4) Heat-mass transfer analogy applies, (5) Ambient air is dry.

PROPERTIES: *Table A-4*, Air (T_f ≈ 2.5°C ≈ 276K, 1 atm): $\rho = 1.2734\ kg/m^3$, $c_p = 1006\ J/kg{\cdot}K$, $\alpha = 19.3 \times 10^{-6} m^2/s$; *Table A-6*, Water vapor (273.15K): $h_{fg} = 2502\ kJ/kg$, $\rho_g = 1/v_g = 4.847 \times 10^{-3} kg/m^3$; *Table A-8*, Water vapor - air (298K): $D_{AB} = 0.26 \times 10^{-4} m^2/s$.

ANALYSIS: (a) Neglecting evaporation and performing an energy balance,

$$q''_{conv} - q''_{rad} = 0$$

$$h\left(T_\infty - T_s\right) - es\left(T_s^4 - T_{sky}^4\right) = 0 \quad \text{or} \quad T_\infty = T_s + \left(es/h\right)\left(T_s^4 - T_{sky}^4\right)$$

$$T_\infty = 0^\circ C + \frac{1 \times 5.667 \times 10^{-8}\ W/m^2 \cdot K^4}{25\ W/m^2 \cdot K}\left[\left(0 + 273\right)^4 - \left(-30 + 273\right)^4\right] = 4.69^\circ C. \qquad <$$

(b) Invoking the heat-mass transfer analogy in the form of Eq. 6.67 with n = 1/3,

$$\frac{h}{h_m} = r\ c_p Le^{2/3} \quad \text{or} \quad h_m = h/r\ c_p Le^{2/3} \quad \text{where} \quad Le = a/D_{AB}$$

$$h_m = \left(25\ W/m^2 \cdot K\right)/1.273\ kg/m^3\left(1006\ J/kg \cdot K\right)\left[\frac{19.3 \times 10^{-6} m^2/s}{0.26 \times 10^{-4} m^2/s}\right]^{2/3} = 0.0238\ m/s. \qquad <$$

(c) Including evaporation effects and performing an energy balance gives $q''_{conv} - q''_{rad} - q''_{evap} = 0$ where $q''_{evap} = \dot{m}'' h_{fg} = h_m\left(r_{A,s} - r_{A,\infty}\right)h_{fg}$, $r_{A,s} = r_g$ and $r_{A,\infty} = 0$. Hence,

$$T_\infty = T_s + \left(es/h\right)\left(T_s^4 - T_{sky}^4\right) + \left(h_m/h\right)\left(r_g - 0\right)h_{fg}$$

$$T_\infty = 4.69^\circ C + \frac{0.0238\ m/s}{25\ W/m^2 \cdot K} \times 4.847 \times 10^{-3} kg/m^3 \times 2.502 \times 10^6 J/kg$$

$$T_\infty = 4.69^\circ C + 11.5^\circ C = 16.2^\circ C. \qquad <$$

PROBLEM 6.73

KNOWN: Wet-bulb and dry-bulb temperature for water vapor-air mixture.

FIND: (a) Partial pressure, p_A, and relative humidity, ϕ, using Carrier's equation, (b) p_A and ϕ using psychrometric chart, (c) Difference between air stream, T_∞, and wet bulb temperatures based upon evaporative cooling considerations.

SCHEMATIC:

T_∞ → Air (B) + water vapor (A),
→ $T_{DB} = 37.8°C$
q''_{conv} q''_{evap} $T_{WB} = 21.1°C, T_s$
Water

ASSUMPTIONS: (1) Evaporative cooling occurs at interface, (2) Heat-mass transfer analogy applies, (3) Species A and B are perfect gases.

PROPERTIES: *Table A-6*, Water vapor: $p_{A,sat}$ (21.1°C) = 0.02512 bar, $p_{A,sat}$ (37.8°C) = 0.06603 bar, h_{fg} (21.1°C) = 2451 kJ/kg; *Table A-4*, Air ($T_{am} = [T_{WB} + T_{DB}]/2 \cong 300K$, 1 atm): $\alpha = 22.5 \times 10^{-6}\ m^2/s$, c_p = 1007 J/kg·K; *Table A-8*, Air-water vapor (298K): $D_{AB} = 0.26 \times 10^{-4}\ m^2/s$.

ANALYSIS: (a) Carrier's equation has the form

$$p_v = p_{gw} - \frac{\left(p - p_{gw}\right)\ \left(T_{DB} - T_{WB}\right)}{1810 - T_{WB}}$$

where p_v = partial pressure of vapor in air stream, bar
p_{gw} = sat. pressure at TWB = 21.1°C, 0.02512 bar
p = total pressure of mixture, 1.033 bar
T_{DB} = dry bulb temperature, 37.8°C
T_{WB} = wet bulb temperature, 21.1°C.

Hence,

$$p_v = 0.02512\ \text{bar} - \frac{\left(1.013 - 0.02512\right)\text{bar} \times \left(37.8 - 21.1\right)^\circ \text{C}}{1810 - \left(21.1 + 273.1\right)\text{K}} = 0.0142\ \text{bar}.$$

The relative humidity, ϕ, is then

$$\phi \equiv \frac{p_A}{p_{A,sat}} = \frac{p_v}{p_A\left(37.8^\circ \text{C}\right)} = \frac{0.0142\ \text{bar}}{0.06603\ \text{bar}} = 0.214.$$ <

(b) Using a psychrometric chart

$$\left.\begin{array}{l} T_{WB} = 21.1^\circ\text{C} = 70^\circ\text{F} \\ T_{DB} = 37.8^\circ\text{C} = 100^\circ\text{F} \end{array}\right\} \qquad \phi \approx 0.225$$ <

$$p_v = \phi\, p_{sat} = 0.225 \times 0.06603\ \text{bar} = 0.0149\ \text{bar}.$$ <

Continued …..

PROBLEM 6.73 (Cont.)

(c) An application of the heat-mass transfer analogy is the process of evaporative cooling which occurs when air flows over water. The change in temperature is estimated by Eq. 6.73.

$$(T_\infty - T_s) = \frac{(M_A / M_B) h_{fg}}{c_p Le^{2/3}} \left[\frac{p_{A,sat}(T_s)}{p} - \frac{p_{A,\infty}}{p} \right]$$

where c_p and Le are evaluated at $T_{am} = (T_\infty + T_s)/2$ and $p_{A,\infty} = p_v$, as determined in Part (a). Substituting numerical values, using $Le = \alpha/D_{AB}$,

$$(T_\infty - T_s) = \frac{(18 \text{ kg/kmol}/29 \text{ kg/kmol}) \times 2451 \times 10^3 \frac{J}{kg}}{1007 \text{ J/kg} \cdot K \left[\frac{22.5 \times 10^{-6} m^2/s}{0.26 \times 10^{-4} m^2/s} \right]^{2/3}} \left[\frac{0.02491 \text{ bar}}{1.013 \text{ bar}} - \frac{0.0149 \text{ bar}}{1.013 \text{ bar}} \right]$$

$$(T_\infty - T_s) = 17.6^\circ C.$$ <

Note that c_p and α are associated with the air.

COMMENTS: The following table compares results from the two calculation methods.

	Carrier's Eq.	*Psychrometric Chart*
p_v (bar)	0.0142	0.0149
ϕ	0.214	0.225

Evaporative Cooling $T_\infty - T_s = 17.6°C$

Observed Difference $T_{DB} - T_{WB} = 16.7°C$

% Difference: $\frac{17.6 - 16.7}{16.7} \times 100 = 5.4\%$.

PROBLEM 6.74

KNOWN: Wet and dry bulb temperatures.

FIND: Relative humidity of air.

SCHEMATIC:

ASSUMPTIONS: (1) Perfect gas behavior for vapor, (2) Steady-state conditions, (3) Negligible radiation, (4) Negligible conduction along thermometer.

PROPERTIES: *Table A-4*, Air (308K, 1 atm): $\rho = 1.135$ kg/m^3, $c_p = 1007$ J/kg·K, $\alpha = 23.7 \times 10^{-6}$ m^2/s; *Table A-6*, Saturated water vapor (298K): $v_g = 44.25$ m^3/kg, $h_{fg} = 2443$ kJ/kg; (318K): $v_g = 15.52$ m^3/kg; *Table A-8*, Air-vapor (1 atm, 298K): $D_{AB} = 0.26 \times 10^{-4}$ m^2/s, D_{AB} (308K) = 0.26×10^{-4} m^2/s $\times (308/298)^{3/2} = 0.27 \times 10^{-4}$ m^2/s, $Le = \alpha/D_{AB} = 0.88$.

ANALYSIS: From an energy balance on the wick, Eq. 6.71 follows from Eq. 6.68. Dividing Eq. 6.71 by $\rho_{A,sat}(T_\infty)$,

$$\frac{T_\infty - T_s}{r_{A,sat}(T_\infty)} = h_{fg}\left[\frac{h_m}{h}\right]\left[\frac{r_{A,sat}(T_s)}{r_{A,sat}(T_\infty)} - \frac{r_{A,\infty}}{r_{A,sat}(T_\infty)}\right].$$

With $\left[r_{A,\infty}/r_{A,sat}(T_\infty)\right] \approx f_\infty$ for a perfect gas and h/h_m given by Eq. 6.67,

$$f_\infty = \frac{r_{A,sat}(T_s)}{r_{A,sat}(T_\infty)} - \frac{r\ c_p}{Le^{2/3}\, r_{A,sat}(T_\infty)\, h_{fg}}(T_\infty - T_s).$$

Using the property values, evaluate

$$\frac{r_{A,sat}(T_s)}{r_{A,sat}(T_\infty)} = \frac{v_g T_\infty}{v_g(T_s)} = \frac{15.52}{44.25} = 0.351$$

$$r_{A,sat}(T_\infty) = \left(15.52\ \text{m}^3/\text{kg}\right)^{-1} = 0.064\ \text{kg/m}^3.$$

Hence,

$$f_\infty = 0.351\ -\ \frac{1.135\ \text{kg/m}^3\ (1007\ \text{J/kg}\cdot\text{K})}{(0.88)^{2/3}\ 0.064\ \text{kg/m}^3\ \left(2.443\times10^6\ \text{J/kg}\right)}(45-25)\text{K}$$

$$f_\infty = 0.351 - 0.159 = 0.192.$$

<

COMMENTS: Note that latent heat must be evaluated at the surface temperature (evaporation occurs at the surface).

PROBLEM 6.75

KNOWN: Heat transfer correlation for a contoured surface heated from below while experiencing air flow across it. Flow conditions and steady-state temperature when surface experiences evaporation from a thin water film.

FIND: (a) Heat transfer coefficient and convection heat rate, (b) Mass transfer coefficient and evaporation rate (kg/h) of the water, (c) Rate at which heat must be supplied to surface for these conditions.

SCHEMATIC:

Heat transfer correlation: $\overline{Nu}_L = 0.43 Re_L^{0.58} Pr^{0.4}$

Air

$T_\infty = 290K$

$V = 10 m/s$

$\phi_\infty = 0$

Water film

Surface, $T_s = 310K$, $L = 1m$, $A_s = 1m^2$

ASSUMPTIONS: (1) Steady-state conditions, (2) Heat-mass transfer analogy applies, (3) Correlation requires properties evaluated at $T_f = (T_s + T_\infty)/2$.

PROPERTIES: *Table A-4*, Air ($T_f = (T_s + T_\infty)/2 = (290 + 310)K/2 = 300$ K, 1 atm): $\nu = 15.89 \times 10^{-6} m^2/s$, $k = 0.0263$ W/m·K, Pr = 0.707; *Table A-8*, Air-water mixture (300 K, 1 atm): $D_{AB} = 0.26 \times 10^{-4} m^2/s$; *Table A-6*, Sat. water (T_s = 310 K): $\rho_{A,sat} = 1/v_g = 1/22.93\ m^3/kg = 0.04361\ kg/m^3$, $h_{fg} = 2414$ kJ/kg.

ANALYSIS: (a) To characterize the flow, evaluate Re_L at T_f

$$Re_L = \frac{VL}{n} = \frac{10\ m/s \times 1\ m}{15.89 \times 10^{-6} m^2/s} = 6.293 \times 10^5$$

and substituting into the prescribed correlation for this surface, find

$$\overline{Nu}_L = 0.43\left(6.293 \times 10^5\right)^{0.58} (0.707)^{0.4} = 864.1$$

$$\overline{h}_L = \frac{\overline{Nu}_L \cdot k}{L} = \frac{864.1 \times 0.0263\ W/m \cdot K}{1\ m} = 22.7\ W/m^2 \cdot K.$$ <

Hence, the convection heat rate is

$$q_{conv} = \overline{h}_L A_s (T_s - T_\infty)$$

$$q_{conv} = 22.7\ W/m^2 \cdot K \times 1\ m^2 (310 - 290) K = 454\ W$$ <

(b) Invoking the heat-mass transfer analogy

$$\overline{Sh}_L = \frac{\overline{h}_m L}{D_{AB}} = 0.43 Re_L^{0.58} Sc^{0.4}$$

where

$$Sc = \frac{n}{D_{AB}} = \frac{15.89 \times 10^{-6} m^2/s}{0.26 \times 10^{-4} m^2/s} = 0.611$$

and ν is evaluated at T_f. Substituting numerical values, find

Continued …..

$$\overline{Sh}_L = 0.43\left(6.293\times10^5\right)^{0.58}(0.611)^{0.4} = 815.2$$

$$\bar{h}_m = \frac{\overline{Sh}_L \cdot D_{AB}}{L} = \frac{815.2\times0.26\times10^{-4}\,m^2/s}{1\ m} = 2.12\times10^{-2}\ m/s. \qquad <$$

The evaporation rate, with $\rho_{A,s} = \rho_{A,sat}(T_s)$, is

$$\dot{m} = \bar{h}_m A_s\left(\rho_{A,s} - \rho_{A,\infty}\right)$$
$$\dot{m} = 2.12\times10^{-2}\ m/s\times1\ m^2(0.04361\ -\ 0)kg/m^3$$

$$\dot{m} = 9.243\times10^{-4} kg/s = 3.32\ kg/h. \qquad <$$

(c) The rate at which heat must be supplied to the plate to maintain these conditions follows from an energy balance.

$$\dot{E}_{in} - \dot{E}_{out} = 0$$
$$q_{in} - q_{conv} - q_{evap} = 0$$

where q_{in} is the heat supplied to sustain the losses by convection and evaporation.

$$q_{in} = q_{conv} + q_{evap}$$
$$q_{in} = \bar{h}_L A_s\left(T_s - T_\infty\right) + \dot{m}h_{fg}$$
$$q_{in} = 454\ W\ +\ 9.243\times10^{-4}\ kg/s\times2414\times10^3\ J/kg$$

$$q_{in} = (254 + 2231)W = 2685\ W. \qquad <$$

COMMENTS: Note that the loss from the surface by evaporation is nearly 5 times that due to convection.

PROBLEM 6.76

KNOWN: Thickness, temperature and evaporative flux of a water layer. Temperature of air flow and surroundings.

FIND: (a) Convection mass transfer coefficient and time to completely evaporate the water, (b) Convection heat transfer coefficient, (c) Heater power requirement per surface area, (d) Temperature of dry surface if heater power is maintained.

SCHEMATIC:

T_{sur} = 300 K

q"

n''_A = 0.03 kg/s-m²

Air

Water, T_w = 340 K

ε_w = 0.95

T_∞ = 300 K

Plate, ε_p = 0.60

δ_i = 2 mm

P_{elec}

ASSUMPTIONS: (1) Steady-state, (2) Applicability of heat and mass transfer analogy with n = 1/3, (3) Radiation exchange at surface of water may be approximated as exchange between a small surface and large surroundings, (4) Air is dry ($\rho_{A,\infty}$ = 0), (5) Negligible heat transfer from unwetted surface of the plate.

PROPERTIES: *Table A-6*, Water (T_w = 340K): ρ_f = 979 kg/m³, $\rho_{A,sat} = v_g^{-1} = 0.174\,\text{kg/m}^3$, $h_{fg} = 2342\,\text{kJ/kg}$. Prescribed, Air: ρ = 1.08 kg/m³, c_p = 1008 J/kg·K, k = 0.028 W/m·K. Vapor/Air: $D_{AB} = 0.29 \times 10^{-4}$ m²/s. Water: ε_w = 0.95. Plate: ε_p = 0.60.

ANALYSIS: (a) The convection mass transfer coefficient may be determined from the rate equation $n''_A = h_m\left(\rho_{A,s} - \rho_{A,\infty}\right)$, where $\rho_{A,s} = \rho_{A,sat}\left(T_w\right)$ and $\rho_{A,\infty} = 0$. Hence,

$$h_m = \frac{n''_A}{\rho_{A,sat}} = \frac{0.03\,\text{kg/s}\cdot\text{m}^2}{0.174\,\text{kg/m}^3} = 0.172\,\text{m/s} \qquad <$$

The time required to completely evaporate the water is obtained from a mass balance of the form $-n''_A = \rho_f\, d\delta/dt$, in which case

$$\rho_f \int_{\delta_i}^{0} d\delta = -n''_A \int_0^t dt$$

$$t = \frac{\rho_f \delta_i}{n''_A} = \frac{979\,\text{kg/m}^3\left(0.002\text{m}\right)}{0.03\,\text{kg/s}\cdot\text{m}^2} = 65.3\text{s} \qquad <$$

(b) With n = 1/3 and Le = $\alpha/D_{AB} = k/\rho c_p D_{AB}$ = 0.028 W/m·K/(1.08 kg/m³ × 1008 J/kg·K × 0.29 × 10^{-4} m²/s) = 0.887, the heat and mass transfer analogy yields

$$h = \frac{k\,h_m}{D_{AB}Le^{1/3}} = \frac{0.028\,\text{W/m}\cdot\text{K}\left(0.172\,\text{m/s}\right)}{0.29\times10^{-4}\text{m}^2/\text{s}\left(0.887\right)^{1/3}} = 173\,\text{W/m}^2\cdot\text{K} \qquad <$$

The electrical power requirement per unit area corresponds to the rate of heat loss from the water. Hence,

Continued …..

$$P''_{elec} = q''_{evap} + q''_{conv} + q''_{rad} = n''_A h_{fg} + h\left(T_w - T_\infty\right) + \varepsilon_w \sigma \left(T_w^4 - T_{sur}^4\right)$$

$$P''_{elec} = 0.03\,kg/s\cdot m^2\left(2.342\times10^6\ J/kg\right) + 173\,W/m^2\cdot K\left(40K\right) + 0.95\times5.67\times10^{-8}\,W/m^2\cdot K^4\left(340^4 - 300^4\right)$$

$$P''_{elec} = 70,260\,W/m^2 + 6920\,W/m^2 + 284\,W/m^2 = 77,464\,W/m^2$$ <

(c) After complete evaporation, the steady-state temperature of the plate is determined from the requirement that

$$P''_{elec} = h\left(T_p - T_\infty\right) + \varepsilon_p \sigma \left(T_p^4 - T_{sur}^4\right)$$

$$77,464\,W/m^2 = 173\,W/m^2\cdot K\left(T_p - 300\right) + 0.60\times5.67\times10^{-8}\,W/m^2\cdot K^4\left(T_p^4 - 300^4\right)$$

$$T_p = 702K = 429°C$$ <

COMMENTS: The evaporative heat flux is the dominant contributor to heat transfer from the water layer, with convection of sensible energy being an order of magnitude smaller and radiation exchange being negligible. Without evaporation (a dry surface), convection dominates and is approximately an order of magnitude larger than radiation.

PROBLEM 6.77

KNOWN: Heater power required to maintain water film at prescribed temperature in dry ambient air and evaporation rate.

FIND: (a) Average mass transfer convection coefficient $\overline{h}_m$, (b) Average heat transfer convection coefficient $\overline{h}$, (c) Whether values of $\overline{h}_m$ and $\overline{h}$ satisfy the heat-mass analogy, and (d) Effect on evaporation rate and disc temperature if relative humidity of the ambient air were increased from 0 to 0.5 but with heater power maintained at the same value.

SCHEMATIC:

ASSUMPTIONS: (1) Water film and disc are at same temperature; (2) Mass and heat transfer coefficient are independent of ambient air relative humidity, (3) Constant properties.

PROPERTIES: *Table A-6*, Saturated water (305 K): $v_g = 29.74\ m^3/kg$, $h_{fg} = 2426 \times 10^3\ J/kg$; *Table A-4*, Air $(\overline{T} = 300\ K,\ 1\ atm)$: $k = 0.0263\ W/m{\cdot}K$, $\alpha = 22.5 \times 10^{-6}\ m^2/s$, *Table A-8*, Air-water vapor (300 K, 1 atm): $D_{AB} = 0.26 \times 10^{-4}\ m^2/s$.

ANALYSIS: (a) Using the mass transfer convection rate equation,

$$n_A = \overline{h}_m A_s \left(\rho_{A,s} - \rho_{A,\infty}\right) = \overline{h}_m A_s\, \rho_{A,sat}\left(1 - \phi_\infty\right)$$

and evaluating $\rho_{A,s} = \rho_{A,sat}$ (305 K) $= 1/v_g$ (305 K) with $\phi_\infty \sim \rho_{A,\infty} = 0$, find

$$\overline{h}_m = \frac{n_A}{A_s\left(\rho_{A,s} - \rho_{A,\infty}\right)}$$

$$\overline{h}_m = \frac{2.55\times10^{-4}\,\text{kg/hr}/\left(3600\text{s/hr}\right)}{\left(\pi\left(0.020\ \text{m}\right)^2/4\right)\left(1/29.74 - 0\right)\text{kg/m}^3} = 6.71\times10^{-3}\ \text{m/s}.$$ <

(b) Perform an overall energy balance on the disc,

$$q = q_{conv} + q_{evap} = \overline{h}A_s\left(T_s - T_\infty\right) + n_A h_{fg}$$

and substituting numerical values with h_{fg} evaluated at T_s, find $\overline{h}$:

$$200\times10^{-3}\ \text{W} = \overline{h}\pi\left(0.020\ \text{m}\right)^2/4\left(305 - 295\right)\text{K} + 7.083\times10^{-8}\ \text{kg/s}\times2426\times10^3\ \text{J/kg}$$

$$\overline{h} = 8.97\ \text{W/m}^2\cdot\text{K}.$$ <

Continued …..

(c) The heat-mass transfer analogy, Eq. 6.67, requires that

$$\frac{\bar{h}}{h_m} \stackrel{?}{=} \frac{k}{D_{AB}}\left(\frac{D_{AB}}{\alpha}\right)^{1/3}.$$

Evaluating k and D_{AB} at $\bar{T} = (T_s + T_\infty)/2 = 300\text{ K}$ and substituting numerical values,

$$\frac{8.97\text{ W/m}^2\cdot\text{K}}{6.71\times10^{-3}\text{ m/s}} = 1337 \neq \frac{0.0263\text{ W/m}\cdot\text{K}}{0.26\times10^{-4}\text{ m}^2/\text{s}}\left(\frac{0.26\times10^{-4}\text{ m}^2/\text{s}}{22.5\times10^{-6}\text{m}^2/\text{s}}\right)^{1/3} = 1061$$

Since the equality is not satisfied, we conclude that, for this situation, the analogy is only approximately met (≈ 30%).

(d) If $\phi_\infty = 0.5$ instead of 0.0 and q is unchanged, n_A will decrease by nearly a factor of two, as will $n_A h_{fg} = q_{evap}$. Hence, since q_{conv} must increase and $\bar{h}$ remains nearly constant, $T_s - T_\infty$ must increase. Hence, T_s will increase.

COMMENTS: Note that in part (d), with an increase in T_s, h_{fg} decreases, but only slightly, and $\rho_{A,sat}$ increases. From a trial-and-error solution assuming constant values for $\bar{h}_m$ and h, the disc temperature is 315 K for $\phi_\infty = 0.5$.

PROBLEM 6.78

KNOWN: Power-time history required to completely evaporate a droplet of fixed diameter maintained at 37°C.

FIND: (a) Average mass transfer convection coefficient when droplet, heater and dry ambient air are at 37°C and (b) Energy required to evaporate droplet if the dry ambient air temperature is 27°C.

SCHEMATIC:

Air
T_∞
$\phi_\infty = 0$
$T_s = 37°C$
Droplet
D=4mm

20
P(mW)
0 50 100 t(min)
t_e

ASSUMPTIONS: (1) Wetted surface area of droplet is of fixed diameter D, (2) Heat-mass transfer analogy is applicable, (3) Heater controlled to operate at constant temperature, $T_s = 37°C$, (4) Mass of droplet same for part (a) and (b), (5) Mass transfer coefficients for parts (a) and (b) are the same.

PROPERTIES: *Table A-6*, Saturated water (37°C = 310 K): $h_{fg} = 2414$ kJ/kg, $\rho_{A,sat} = 1/v_g = 1/22.93 = 0.04361\ kg/m^3$; *Table A-8*, Air-water vapor (T_s = 37°C = 310 K, 1 atm): $D_{AB} = 0.26 \times 10^{-6}\ m^2/s(310/289)^{3/2} = 0.276 \times 10^{-6}\ m^2/s$; *Table A-4*, Air ($\bar{T}$ = (27 + 37)°C/2 = 305 K, 1 atm): $\rho = 1.1448\ kg/m^3$, $c_p = 1008$ J/kg·K, $\nu = 16.39 \times 10^{-6}\ m^2/s$, Pr = 0.706.

ANALYSIS: (a) For the isothermal conditions (37°C), the electrical energy Q required to evaporate the droplet during the interval of time $\Delta t = t_e$ follows from the area under the P-t curve above,

$$Q = \int_0^{t_e} P dt = \left[20\times10^{-3}\ W\times(50\times60)s + 0.5\times20\times10^{-3}\ W(100-50)\times60s\right]$$
$$Q = 90\ J.$$

From an overall energy balance during the interval of time $\Delta t = t_e$, the mass loss due to evaporation is

$$Q = Mh_{fg} \quad \text{or} \quad M = Q/h_{fg}$$
$$M = 90\ J/2414\times10^3\ J/kg = 3.728\times10^{-5}\ kg.$$

To obtain the average mass transfer coefficient, write the rate equation for an interval of time $\Delta t = t_e$,

$$M = \dot{m}\cdot t_e = \bar{h}_m A_s\left(\rho_{A,s} - \rho_{A,\infty}\right)\cdot t_e = \bar{h}_m A_s \rho_{A,s}\left(1-\phi_\infty\right)\cdot t_e$$

Substituting numerical values with $\phi_\infty = 0$, find

$$3.278\times10^{-5}\ kg = \bar{h}_m\left(\pi(0.004\ m)^2/4\right)0.04361\ kg/m^3\times(100\times60)s$$

Continued …..

PROBLEM 6.78 (Cont.)

$\overline{h}_m = 0.0113$ m/s. <

(b) The energy required to evaporate the droplet of mass M = 3.728×10^{-5} kg follows from an overall energy balance,

$$Q = Mh_{fg} + \overline{h}A_s\left(T_s - T_\infty\right)$$

where $\overline{h}$ is obtained from the heat-mass transfer analogy, Eq. 6.67, using n = 1/3,

$$\frac{\overline{h}}{h_m} = \frac{k}{D_{AB}Le^n} = r\ c_p Le^{2/3}$$

where

$$Sc = \frac{n}{D_{AB}} = \frac{16.39\times10^{-6}\ m^2/s}{0.276\times10^{-4}\ m^2/s} = 0.594$$

$$Le = \frac{Sc}{Pr} = \frac{0.594}{0.706} = 0.841.$$

Hence,

$$\overline{h} = 0.0113\ m/s \times 1.1448\ kg/m^3 \times 1008\ J/kg\cdot K\left(0.841\right)^{2/3} = 11.62\ W/m^2\cdot K.$$

and the energy requirement is

$$Q = 3.728\times10^{-5}\ kg \times 2414\ kJ/kg\ +\ 11.62\ W/m^2\cdot K\left(p\left(0.004\ m\right)^2/4\right)\left(37-27\right)^\circ C$$

$$Q\ =\ \left(90.00 + 0.00145\right)J\ =\ 90\ J.$$ <

The energy required to meet the convection heat loss is very small compared to that required to sustain the evaporative loss.

PROBLEM 6.79

KNOWN: Initial plate temperature T_p (0) and saturated air temperature (T_∞) in a dishwasher at the start of the dry cycle. Thermal mass per unit area of the plate Mc/A_s = 1600 J/m^2·K.

FIND: (a) Differential equation to predict plate temperature as a function of time during the dry cycle and (b) Rate of change in plate temperature at the start of the dry cycle assuming the average convection heat transfer coefficient is 3.5 W/m^2·K.

SCHEMATIC:

Plate, M, c, A_s
$Mc/A_s = 1600$ J/m^2·K
$T_p(t)$, $T_p(0) = 65°C$
q''_{conv}
q''_{evap}
Ambient air
$T_\infty = 55°C$
$\phi_\infty = 1$

ASSUMPTIONS: (1) Plate is spacewise isothermal, (2) Negligible thermal resistance of water film on plate, (3) Heat-mass transfer analogy applies.

PROPERTIES: *Table A-4*, Air ($\overline{T}$ =(55 + 65)°C/2 = 333 K, 1 atm): ρ = 1.0516 kg/m^3, c_p = 1008 J/kg·K, Pr = 0.703, $\nu = 19.24\times 10^{-6}$ m^2/s; *Table A-6*, Saturated water vapor, (T_s = 65°C = 338 K): ρ_A = $1/v_g$ = 0.1592 kg/m^3, h_{fg} = 2347 kJ/kg; (T_s = 55°C = 328 K): ρ_A = $1/v_g$ = 0.1029 kg/m^3; *Table A-8*, Air-water vapor (T_s = 65°C = 338 K, 1 atm): $D_{AB} = 0.26 \times 10^{-4}$ m^2/s $(338/298)^{3/2} = 0.314 \times 10^{-4}$ m^2/s.

ANALYSIS: (a) Perform an energy balance on a rate basis on the plate,

$$\dot{E}_{in} - \dot{E}_{out} = \dot{E}_{st} \qquad q''_{conv} - q''_{evap} = \left(Mc/A_s\right)\left(dT_p/dt\right).$$

Using the rate equations for the heat and mass transfer fluxes, find

$$\overline{h}\left[T_\infty - T_p(t)\right] - \overline{h}_m\left[r_{A,s}(T_s) - r_{A,\infty}(T_\infty)\right]h_{fg} = \left(Mc/A_s\right)\left(dT/dt\right). \qquad <$$

(b) To evaluate the change in plate temperature at t = 0, the start of the drying process when T_p (0) = 65°C and T_∞ = 55°C, evaluate $\overline{h}_m$ from knowledge of $\overline{h} = 3.5\ \text{W/m}^2\cdot\text{K}$ using the heat-mass transfer analogy, Eq. 6.67, with n = 1/3,

$$\frac{\overline{h}}{\overline{h}_m} = r\ c_p Le^{2/3} = r\ c_p\left(\frac{Sc}{Pr}\right)^{2/3} = r\ c_p\left(\frac{n/D_{AB}}{Pr}\right)^{2/3}$$

and evaluating thermophysical properties at their appropriate temperatures, find

$$\frac{3.5\ \text{W/m}^2\cdot\text{K}}{\overline{h}_m} = 1.0516\ \text{kg/m}^3 \times 1008\ \text{J/kg}\cdot\text{K}\left(\frac{19.24\times10^{-6}\text{m}^2/\text{s}/0.314\times10^{-4}\text{m}^2/\text{s}}{0.703}\right)^{2/3} \qquad \overline{h}_m = 3.619\times10^{-3}\text{m/s}.$$

Substituting numerical values into the conservation expression of part (a), find

$$3.5\ \text{W/m}^2\cdot\text{K}(55-65)^\circ\text{C} - 3.619\times10^{-3}\text{m/s}(0.1592-0.1029)\text{kg/m}^3\times2347\times10^3\ \text{J/kg} = 1600\ \text{J/m}^2\cdot\text{K}\left(dT_p/dt\right)$$

$$dT_p/dt = -\left[35.0 + 478.2\right]\text{W/m}^2\cdot\text{K}/1600\ \text{J/m}^2\cdot\text{K} = -0.32\ \text{K/s}. \qquad <$$

COMMENTS: This rate of temperature change will not be sustained for long, since, as the plate cools, the rate of evaporation (which dominates the cooling process) will diminish.

PROBLEM 7.1

KNOWN: Temperature and velocity of fluids in parallel flow over a flat plate.

FIND: (a) Velocity and thermal boundary layer thicknesses at a prescribed distance from the leading edge, and (b) For each fluid plot the boundary layer thicknesses as a function of distance.

SCHEMATIC:

ASSUMPTIONS: (1) Transition Reynolds number is 5×10^5.

PROPERTIES: *Table A.4*, Air (300 K, 1 atm): $\nu = 15.89 \times 10^{-6}$ m²/s, Pr = 0.707; *Table A.6*, Water (300 K): $\nu = \mu/\rho = 855 \times 10^{-6}$ N·s/m²/997 kg/m³ $= 0.858 \times 10^{-6}$ m²/s, Pr = 5.83; *Table A.5*, Engine Oil (300 K): $\nu = 550 \times 10^{-6}$ m²/s, Pr = 6400; *Table A.5*, Mercury (300 K): $\nu = 0.113 \times 10^{-6}$ m²/s, Pr = 0.0248.

ANALYSIS: (a) If the flow is laminar, the following expressions may be used to compute δ and δ_t, respectively,

$$\delta = \frac{5x}{Re_x^{1/2}} \qquad \delta_t = \frac{\delta}{Pr^{1/3}}$$

where

$$Re_x = \frac{u_\infty x}{\nu} = \frac{1\,m/s\,(0.04\,m)}{\nu} = \frac{0.04\,m^2/s}{\nu}$$

Fluid	Re_x	δ (mm)	δ_t (mm)
Air	2517	3.99	4.48
Water	4.66×10^4	0.93	0.52
Oil	72.7	23.5	1.27
Mercury	3.54×10^5	0.34	1.17

<

(b) Using IHT with the foregoing equations, the boundary layer thicknesses are plotted as a function of distance from the leading edge, x.

COMMENTS: (1) Note that $\delta \approx \delta_t$ for air, $\delta > \delta_t$ for water, $\delta >> \delta_t$ for oil, and $\delta < \delta_t$ for mercury. As expected, the boundary layer thicknesses increase with increasing distance from the leading edge.

(2) The value of δ_t for mercury should be viewed as a rough approximation since the expression for δ/δ_t was derived subject to the approximation that Pr > 0.6.

PROBLEM 7.2

KNOWN: Temperature and velocity of engine oil. Temperature and length of flat plate.

FIND: (a) Velocity and thermal boundary layer thickness at trailing edge, (b) Heat flux and surface shear stress at trailing edge, (c) Total drag force and heat transfer per unit plate width, and (d) Plot the boundary layer thickness and local values of the shear stress, convection coefficient, and heat flux as a function of x for $0 \le x \le 1$ m.

SCHEMATIC:

ASSUMPTIONS: (1) Critical Reynolds number is 5×10^5, (2) Flow over top and bottom surfaces.

PROPERTIES: *Table A.5*, Engine Oil (T_f = 333 K): $\rho = 864\ kg/m^3$, $\nu = 86.1 \times 10^{-6}\ m^2/s$, k = 0.140 W/m·K, Pr = 1081.

ANALYSIS: (a) Calculate the Reynolds number to determine nature of the flow,

$$Re_L = \frac{u_\infty L}{\nu} = \frac{0.1\,m/s \times 1\,m}{86.1 \times 10^{-6}\,m^2/s} = 1161$$

Hence the flow is laminar at x = L, from Eqs. 7.19 and 7.24, and

$$\delta = 5L\,Re_L^{-1/2} = 5(1\,m)(1161)^{-1/2} = 0.147\,m \qquad <$$

$$\delta_t = \delta\,Pr^{-1/3} = 0.147\,m(1081)^{-1/3} = 0.0143\,m \qquad <$$

(b) The local convection coefficient, Eq. 7.23, and heat flux at x = L are

$$h_L = \frac{k}{L} 0.332\,Re_L^{1/2}\,Pr^{1/3} = \frac{0.140\,W/m\cdot K}{1m} 0.332(1161)^{1/2}(1081)^{1/3} = 16.25\,W/m^2\cdot K$$

$$q_x'' = h_L(T_s - T_\infty) = 16.25\,W/m^2\cdot K(20-100)^\circ C = -1300\,W/m^2 \qquad <$$

Also, the local shear stress is, from Eq. 7.20,

$$\tau_{s,L} = \frac{\rho u_\infty^2}{2} 0.664\,Re_L^{-1/2} = \frac{864\,kg/m^3}{2}(0.1\,m/s)^2\,0.664(1161)^{-1/2}$$

$$\tau_{s,L} = 0.0842\,kg/m\cdot s^2 = 0.0842\,N/m^2 \qquad <$$

(c) With the drag force per unit width given by $D' = 2L\overline{\tau}_{s,L}$ where the factor of 2 is included to account for both sides of the plate, it follows that

$$D' = 2L\left(\rho u_\infty^2/2\right)1.328\,Re_L^{-1/2} = (1m)864\,kg/m^3\,(0.1\,m/s)^2 / 2\ 1.328(1161)^{-1/2} = 0.337\,N/m \qquad <$$

For laminar flow, the average value $\overline{h}_L$ over the distance 0 to L is twice the local value, h_L,

$$\overline{h}_L = 2h_L = 32.5\,W/m^2\cdot K$$

The total heat transfer rate per unit width of the plate is

$$q' = 2L\overline{h}_L(T_s - T_\infty) = 2(1m)32.5\,W/m^2\cdot K(20-100)^\circ C = -5200\,W/m \qquad <$$

Continued...

PROBLEM 7.2 (Cont.)

(c) Using IHT with the foregoing equations, the boundary layer thickness, and local values of the convection coefficient and heat flux were calculated and plotted as a function of x.

deltax*10, hx*100, -q''x

Distance from leading edge, x (m)

BL thickness, deltax * 10 (mm)
Convection coefficient, hx * 100 (N/m^2)
Heat flux, - q''x (W/m^2)

COMMENTS: (1) Note that since Pr >> 1, $\delta >> \delta_t$. That is, for the high Prandtl liquids, the velocity boundary layer will be much thicker than the thermal boundary layer.

(2) A copy of the *IHT Workspace* used to generate the above plot is shown below.

```
// Boundary layer thickness, delta
delta = 5 * x * Rex ^-0.5
delta_mm = delta * 1000
delta_plot = delta_mm * 10          // Scaling parameter for convenience in  plotting

// Convection coefficient and heat flux, q''x
q''x = hx * (Ts - Tinf)
Nux = 0.332 * Rex^0.5 * Pr^(1/3)
Nux = hx * x / k
hx_plot = 100 * hx                  // Scaling parameter for convenience in plotting
q''x_plot = ( -1 ) * q''x           // Scaling parameter for convenience in plotting

// Reynolds number
Rex = uinf * x / nu

// Properties Tool: Engine oil
// Engine Oil property functions : From Table A.5
// Units: T(K)
rho = rho_T("Engine Oil",Tf)            // Density, kg/m^3
cp = cp_T("Engine Oil",Tf)              // Specific heat, J/kg·K
nu = nu_T("Engine Oil",Tf)              // Kinematic viscosity, m^2/s
k = k_T("Engine Oil",Tf)                // Thermal conductivity, W/m·K
Pr = Pr_T("Engine Oil",Tf)              // Prandtl number


// Assigned variables
Tf = (Ts + Tinf) / 2                    // Film temperature, K
Tinf = 100 + 273                        // Freestream temperature, K
Ts = 20 + 273                           // Surface temperature, K
uinf = 0.1                              // Freestream velocity, m/s
x = 1                                   // Plate length, m
```

PROBLEM 7.3

KNOWN: Velocity and temperature of air in parallel flow over a flat plate.

FIND: (a) Velocity boundary layer thickness at selected stations. Distance at which boundary layers merge for plates separated by H = 3 mm. (b) Surface shear stress and v(δ) at selected stations.

SCHEMATIC:

ASSUMPTIONS: (1) Steady flow, (2) Boundary layer approximations are valid, (3) Flow is laminar.

PROPERTIES: *Table A-4*, Air (300 K, 1 atm): $\rho = 1.161\ \text{kg/m}^3$, $\nu = 15.89 \times 10^{-6}\ \text{m}^2\text{/s}$.

ANALYSIS: (a) For laminar flow,

$$\delta = \frac{5x}{Re_x^{1/2}} = \frac{5}{\left(u_\infty/\nu\right)^{1/2}} x^{1/2} = \frac{5x^{1/2}}{\left(25\ \text{m/s}/15{,}89\times10^{-6}\ \text{m}^2/\text{s}\right)^{1/2}} = 3.99\times10^{-3}\ x^{1/2}.$$

x (m)	0.001	0.01	0.1
δ (mm)	0.126	0.399	1.262

Boundary layer merger occurs at $x = x_m$ when δ = 1.5 mm. Hence

$$x_m^{1/2} = \frac{0.0015\ \text{m}}{3.99\times10^{-3}\ \text{m}^{1/2}} = 0.376\ \text{m}^{1/2} \qquad x_m = 141\ \text{mm}.$$ <

(b) The shear stress is

$$\tau_{s,x} = 0.664\frac{\rho u_\infty^2/2}{Re_x^{1/2}} = \frac{\rho u_\infty^2/2}{\left(u_\infty/\nu\right)^{1/2} x^{1/2}} = \frac{0.664\times1.161\ \text{kg/m}^3\left(25\ \text{m/s}\right)^2/2}{\left(25\ \text{m/s}/15.89\times10^{-6}\ \text{m}^2/\text{s}\right)^{1/2} x^{1/2}} = \frac{0.192}{x^{1/2}}\left(\text{N/m}^2\right).$$

x(m)	0.001	0.01	0.1
$\tau_{s,x}$ (N/m²)	6.07	1.92	0.61

The velocity distribution in the boundary layer is $v = (1/2)\left(\nu u_\infty/x\right)^{1/2}\left(\eta\, df/d\eta - f\right)$. At y = δ, η ≈ 5.0, f ≈ 3.24, df/dη ≈ 0.991.

$$v = \frac{0.5}{x^{1/2}}\left(15.89\times10^{-6}\ \text{m}^2/\text{s}\times 25\ \text{m/s}\right)^{1/2}\left(5.0\times0.991-3.28\right) = \left(0.0167/x^{1/2}\right)\text{m/s}.$$

x(m)	0.001	0.01	0.1
v(m/s)	0.528	0.167	0.053

COMMENTS: (1) $v \ll u_\infty$ and $\delta \ll x$ are consistent with BL approximations. Note, $v \to \infty$ as $x \to 0$ and approximations breakdown very close to the leading edge. (2) Since $Re_{x_m} = 2.22\times10^5$, laminar BL model is valid. (3) Above expressions are approximations for flow between parallel plates, since $du_\infty/dx > 0$ and $dp/dx < 0$.

PROBLEM 7.4

KNOWN: Liquid metal in parallel flow over a flat plate.

FIND: An expression for the local Nusselt number.

SCHEMATIC:

ASSUMPTIONS: (1) Steady, incompressible flow, (2) $\delta << \delta_t$, hence $u(y) \approx u_\infty$, (3) Boundary layer approximations are valid, (4) Constant properties.

ANALYSIS: The boundary layer energy equation is

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2}.$$

Assuming $u(y) = u_\infty$, it follows that $v = 0$ and the energy equation becomes

$$u_\infty \frac{\partial T}{\partial x} = \alpha \frac{\partial^2 T}{\partial y^2} \qquad \text{or} \qquad \frac{\partial T}{\partial x} = \frac{\alpha}{u_\infty} \frac{\partial^2 T}{\partial y^2}.$$

Boundary Conditions: $T(x,0) = T_s$, $T(x,\infty) = T_\infty$.

Initial Condition: $T(0,y) = T_\infty$.

The differential equation is analogous to that for transient one-dimensional conduction in a plane wall, and the conditions are analogous to those of Fig. 5.7, Case (1). Hence the solution is given by Eqs. 5.57 and 5.58. Substituting y for x, x for t, T_∞ for T_i, and α/u_∞ for α, the boundary layer temperature and the surface heat flux become

$$\frac{T(x,y) - T_s}{T_\infty - T_s} = \text{erf}\left[\frac{y}{2\left(\alpha x/u_\infty\right)^{1/2}}\right]$$

$$q_s'' = \frac{k\left(T_s - T_\infty\right)}{\left(\pi \alpha x/u_\infty\right)^{1/2}}.$$

Hence, with

$$Nu_x \equiv \frac{h x}{k} = \frac{q_s'' x}{\left(T_s - T_\infty\right)k}$$

find

$$Nu_x = \frac{x}{\left(\pi \alpha x/u_\infty\right)^{1/2}} = \frac{\left(x u_\infty\right)^{1/2}}{\pi^{1/2}\left(k/\rho c_p\right)^{1/2}} = \frac{1}{\pi^{1/2}}\left[\frac{\rho u_\infty x}{\mu} \cdot \frac{c_p \mu}{k}\right]^{1/2}$$

$$Nu_x = 0.564 \left(Re_x Pr\right)^{1/2} = 0.564\, Pe^{1/2}$$

where $Pe = Re \cdot Pr$ is the Peclet number.

COMMENTS: Because k is very large, axial conduction effects may not be negligible. That is, the $\alpha\, \partial^2 T/\partial x^2$ term of the energy equation may be important.

PROBLEM 7.5

KNOWN: Form of velocity profile for flow over a flat plate.

FIND: (a) Expression for profile in terms of u_∞ and δ, (b) Expression for δ(x), (c) Expression for $C_{f,x}$.

SCHEMATIC:

ASSUMPTIONS: (1) Steady state conditions, (2) Constant properties, (3) Incompressible flow, (4) Boundary layer approximations are valid.

ANALYSIS: (a) From the boundary conditions

$$u(x,0) = 0 \rightarrow C_1 = 0 \qquad \text{and} \qquad u(x,d) = u_\infty \rightarrow C_2 = u_\infty / d.$$

Hence, $\qquad u = u_\infty (y/d).$ <

(b)From the momentum integral equation for a flat plate

$$\frac{d}{dx}\int_0^d (u_\infty - u)\,u\,dy = t_s / r$$

$$u_\infty^2 \frac{d}{dx}\int_0^d \left(1-\frac{u}{u_\infty}\right)\frac{u}{u_\infty}\,dy = \frac{m}{r}\left.\frac{¶\,u}{¶\,y}\right)_{y=0} = \frac{n\,u_\infty}{d}$$

$$u_\infty^2 \frac{d}{dx}\int_0^d \left(1-\frac{y}{d}\right)\frac{y}{d}dy = \frac{n\,u_\infty}{d}$$

$$u_\infty^2 \frac{d}{dx}\left[\left(\frac{y^2}{2d}-\frac{y^3}{3d^2}\right)\Bigg|_0^d\right] = \frac{m\,u_\infty}{d} \qquad \text{or} \qquad \frac{u_\infty}{6}\frac{dd}{dx} = \frac{n}{d}.$$

Separating and integrating, find

$$\int_0^d d\,d\,d = \frac{6n}{u_\infty}\int_0^x dx \qquad d = \left(\frac{12\,n\,x}{u_\infty}\right)^{1/2} = 3.46\,x\left(\frac{n}{u_\infty x}\right)^{1/2} = 3.46\,x\,Re_x^{-1/2}.$$ <

(c) The shear stress at the wall is

$$t_s = m\left.\frac{¶\,u}{¶\,y}\right)_{y=0} = m\frac{u_\infty}{d} = \frac{m\,u_\infty}{3.46\,x}Re_x^{1/2}$$

and the friction coefficient is

$$C_{f,x} = \frac{t_s}{r\,u_\infty^2/2} = \frac{m}{r\,u_\infty x}\,\frac{2}{3.46}Re_x^{1/2} = 0.578\,Re_x^{-1/2}.$$ <

COMMENTS: The foregoing results underpredict those associated with the exact solution $\left(d = 4.96\,x\,Re_x^{-1/2},\ C_{f,x} = 0.664\,Re_x^{-1/2}\right)$ and the cubic profile $\left(d = 4.64\,x\,Re_x^{-1/2},\right.$ $\left.C_{f,x} = 0.646\,Re_x^{-1/2}\right)$.

PROBLEM 7.6

KNOWN: Velocity and temperature profiles and shear stress-boundary layer thickness relation for turbulent flow over a flat plate.

FIND: (a) Expressions for hydrodynamic boundary layer thickness and average friction coefficient, (b) Expressions for local and average Nusselt numbers.

SCHEMATIC:

ASSUMPTIONS: (1) Steady flow, (2) Constant properties, (3) Fully turbulent boundary layer, (4) Incompressible flow, (5) Isothermal plate, (6) Negligible viscous dissipation, (7) $\delta \approx \delta_t$.

ANALYSIS: (a) The momentum integral equation is

$$r\ u_\infty^2\ \frac{d}{dx}\int_0^d \left(1-\frac{u}{u_\infty}\right)\frac{u}{u_\infty}\,dy\ =\ t_s.$$

Substituting the expression for the wall shear stress

$$r\ u_\infty^2 \frac{d}{dx}\int_0^d \left[1-\left(\frac{y}{d}\right)^{1/7}\right]\left(\frac{y}{d}\right)^{1/7} dy\ =\ 0.0228\ r\ u_\infty^2\left(\frac{u_\infty d}{n}\right)^{-1/4}$$

$$\frac{d}{dx}\int_0^d \left[\left(\frac{y}{d}\right)^{1/7}-\left(\frac{y}{d}\right)^{2/7}\right] dy\ =\ \frac{d}{dx}\left(\frac{7}{8}\frac{y^{8/7}}{d^{1/7}}-\frac{7}{9}\frac{y^{9/7}}{d^{2/7}}\right)\Bigg|_0^d$$

$$\frac{d}{dx}\left(\frac{7}{8}d-\frac{7}{9}d\right)\ =\ 0.0228\left(\frac{u_\infty d}{n}\right)^{-1/4}$$

$$\frac{7}{72}\frac{d\,d}{dx} = 0.0228\left(\frac{n}{u_\infty}\right)^{1/4} d^{-1/4} \qquad \frac{7}{72}\int_0^d d^{1/4}\,dd = 0.0228\left(\frac{n}{u_\infty}\right)^{1/4}\int_0^x dx$$

$$\frac{7}{72}\times\frac{4}{5}d^{5/4} = 0.0228\left(\frac{n}{u_\infty}\right)^{1/4} x, \qquad d = 0.376\left(\frac{n}{u_\infty}\right)^{1/5} x^{4/5},\ \ \frac{d}{x} = 0.376 Re_x^{-1/5}. <$$

Knowing δ, it follows

$$t_s = 0.0228\ r u_\infty^2\left(\frac{u_\infty}{n}\right)^{-1/4}\left[0.376\ x\ Re_x^{-1/5}\right]^{-1/4}$$

$$C_{f,x} = \frac{t_s}{r\ u_\infty^2/2} = 0.0456\left[0.376\ \frac{u_\infty}{n}\left(\frac{u_\infty}{n}\right)^{-1/5} x\ x^{-1/5}\right]^{-1/4} = 0.0592\ Re_x^{-1/5}.$$

Continued …..

PROBLEM 7.6 (Cont.)

The average friction coefficient is then

$$\bar{C}_{f,x} = \frac{1}{x}\int_0^x C_{f,x}\,dx = \frac{1}{x}0.0592\left(\frac{u_\infty}{\nu}\right)^{-1/5}\int_0^x x^{-1/5}\,dx$$

$$\bar{C}_{f,x} = \frac{1}{x}0.0592\left(\frac{u_\infty}{\nu}\right)^{-1/5} x^{4/5}\left(\frac{5}{4}\right) = 0.074\,Re_x^{-1/5}.$$ <

(b) The energy integral equation for turbulent flow is

$$\frac{d}{dx}\int_0^{\delta_t} u\left(T_\infty - T\right)dy = \frac{q_s''}{\rho\,c_p} = -\frac{h}{\rho\,c_p}\left(T_s - T_\infty\right).$$

Hence,

$$u_\infty\frac{d}{dx}\int_0^{\delta_t}\frac{u}{u_\infty}\,\frac{T - T_\infty}{T_s - T_\infty}\,dy = u_\infty\frac{d}{dx}\int_0^{\delta_t}\left(y/\delta\right)^{1/7}\left[1 - \left(y/\delta_t\right)^{1/7}\right]dy = \frac{h}{\rho\,c_p}$$

$$u_\infty\frac{d}{dx}\left[\frac{7}{8}\frac{\delta_t^{8/7}}{\delta^{1/7}} - \frac{7}{9}\frac{\delta_t^{8/7}}{\delta^{1/7}}\right] = \frac{h}{\rho\,c_p}$$

or, with $\xi \equiv \delta_t/\delta$,

$$u_\infty\frac{d}{dx}\left[\frac{7}{8}\delta\xi^{8/7} - \frac{7}{9}\delta\xi^{8/7}\right) = \frac{h}{\rho\,c_p} \qquad u_\infty\frac{d}{dx}\left\{\frac{7}{72}\delta\xi^{8/7}\right\} = \frac{h}{\rho\,c_p}.$$

Hence, with $\xi \approx 1$ and $\delta/x = 0.376\,Re_x^{-1/5}$,

$$\frac{7}{72}u_\infty(0.376)\left(\frac{u_\infty}{\nu}\right)^{-1/5}\frac{d\left(x^{4/5}\right)}{dx} = \frac{h}{\rho\,c_p}$$

$$h = 0.0292\,\rho\,c_p u_\infty\,Re_x^{-1/5} = 0.0292\,\frac{k}{x}\,\frac{\nu}{\alpha}\,\frac{u_\infty x}{\nu}\,Re_x^{-1/5}$$

$$Nu_x = \frac{hx}{k} = 0.0292\,Re_x^{4/5}\,Pr.$$ <

Hence,

$$\bar{h}_x = \frac{1}{x}\int_0^x h\,dx = \frac{0.0292\,Pr}{x}k\left(\frac{u_\infty}{\nu}\right)^{4/5}\int_0^x x^{-1/5}\,dx = 0.0292\,\frac{k}{x}\,Pr\left(\frac{u_\infty x}{\nu}\right)^{4/5}\frac{5}{4}$$

$$\overline{Nu}_x = \frac{\bar{h}_x x}{k} = 0.037\,Re_x^{4/5}\,Pr.$$ <

COMMENTS: (1) The foregoing results are in excellent agreement with empirical correlations, except that use of $Pr^{1/3}$ instead of Pr, would be more appropriate.

(2) Note that the 1/7 profile breaks down at the surface. For example,

$$\left.\frac{\partial\left(u/u_\infty\right)}{\partial\,y}\right)_{y=0} = \frac{1}{7}\delta^{-1/7}\,y^{-6/7} = \infty$$

or $\tau_s = \infty$. Despite this unrealistic characteristic of the profile, its use with integral methods provides excellent results.

PROBLEM 7.7

KNOWN: Parallel flow over a flat plate and two locations representing a short span x_1 to x_2 where $(x_2 - x_1) << L$.

FIND: Three different expressions for the average heat transfer coefficient over the short span x_1 to x_2, $\bar{h}_{1-2}$.

SCHEMATIC:

ASSUMPTIONS: (1) Parallel flow over a flat plate.

ANALYSIS: The heat rate per unit width for the span can be written as

$$q'_{1-2} = \bar{h}_{1-2}\left(x_2 - x_1\right)\left(T_s - T_\infty\right) \quad (1)$$

where $\bar{h}_{1-2}$ is the average heat transfer coefficient over the span and can be evaluated in either of the following three ways:

(a) *Local coefficient at* $\bar{x} = \left(x_1 + x_2\right)/2$. If the span is very short, it is reasonable to assume that

$$\bar{h}_{1-2} \approx h_{\bar{x}} \quad (2)$$

where $h_{\bar{x}}$ is the local convection coefficient at the mid-point of the span.

(b) *Local coefficients at x_1 and x_2.* If the span is very short it is reasonable to assume that $\bar{h}_{1-2}$ is the average of the local values at the ends of the span,

$$\bar{h}_{1-2} \approx \left[h_{x1} + h_{x2}\right]/2. \quad (3)$$

(c) *Average coefficients for x_1 and x_2.* The heat rate for the span can also be written as

$$q'_{1-2} = q'_{0-2} - q'_{0-1} \quad (4)$$

where the rate q_{0-x} denotes the heat rate for the plate over the distance from 0 to x. In terms of heat transfer coefficients, find

$$\bar{h}_{1-2} \cdot \left(x_2 - x_1\right) = \bar{h}_2 \cdot x_2 - \bar{h}_1 \cdot x_1$$

$$\bar{h}_{1-2} = \bar{h}_2 \frac{x_2}{x_2 - x_1} - \bar{h}_1 \frac{x_1}{x_2 - x_1} \quad (5)$$

where $\bar{h}_1$ and $\bar{h}_2$ are the average coefficients from 0 to x_1 and x_2, respectively.

COMMENTS: Eqs. (2) and (3) are approximate and work better when the span is small and the flow is turbulent rather than laminar ($h_x \sim x^{-0.2}$ vs $h_x \sim x^{-0.5}$). Of course, we require that $x_c < x_1$, x_2 or $x_c > x_1$, x_2; that is, the approximations are inappropriate around the transition region. Eq. (5) is an exact relationship, which applies under any conditions.

PROBLEM 7.8

KNOWN: Flat plate comprised of rectangular modules of surface temperature T_s, thickness a and length b cooled by air at 25°C and a velocity of 30 m/s. Prescribed thermophysical properties of the module material.

FIND: (a) Required power generation for the module positioned 700 mm from the leading edge of the plate and (b) Maximum temperature in this module.

SCHEMATIC:

ASSUMPTIONS: (1) Laminar flow at leading edge of plate, (2) Transition Reynolds number of 5×10^5, (3) Heat transfer is one-dimensional in y-direction within each module, (4) $\dot{q}$ is uniform within module, (5) Negligible radiation heat transfer.

PROPERTIES: Module material (given): k = 5.2 W/m·K, c_p = 320 J/kg·K, ρ = 2300 kg/m^3; *Table A-4,* Air $\left(\overline{T}_f = (T_s + T_\infty)/2 = 360\text{ K, 1 atm}\right)$: k = 0.0308 W/m·K, ν = 22.02 × 10^{-6} m^2/s, Pr = 0.698.

ANALYSIS: (a) The module power generation follows from an energy balance on the module surface,

$$q''_{conv} = q''_{gen}$$

$$\overline{h}(T_s - T_\infty) = \dot{q} \cdot a \qquad \text{or} \qquad \dot{q} = \frac{\overline{h}(T_s - T_\infty)}{a}.$$

To select a convection correlation for estimating $\overline{h}$, first find the Reynolds numbers at x = L.

$$Re_L = \frac{u_\infty L}{n} = \frac{30\text{ m/s} \times 0.70\text{ m}}{22.02 \times 10^{-6}\text{m}^2/\text{s}} = 9.537 \times 10^5.$$

Since the flow is turbulent over the module, the approximation $\overline{h} \approx h_x\ (L + b/2)$ is appropriate, with

$$Re_{L+b/2} = \frac{30\text{ m/s} \times (0.700 + 0.050/2)\text{ m}}{22.02 \times 10^{-6}\text{m}^2/\text{s}} = 9.877 \times 10^5.$$

Using the turbulent flow correlation with x = L + b/2 = 0.725 m,

$$Nu_x = \frac{h_x x}{k} = 0.0296 Re_x^{4/5} Pr^{1/3}$$

$$Nu_x = 0.0296\left(9.877 \times 10^5\right)^{4/5} (0.698)^{1/3} = 1640$$

$$\overline{h} \approx h_x = \frac{Nu_x k}{x} = \frac{1640 \times 0.0308\text{ W/m} \cdot \text{K}}{0.725} = 69.7\text{ W/m}^2 \cdot \text{K}.$$

Continued …..

Hence,

$$\dot{q} = \frac{69.7\ \text{W/m}^2 \cdot \text{K}\left(150 - 25\right)\text{K}}{0.010\ \text{m}} = 8.713 \times 10^5\ \text{W/m}^3. \qquad <$$

(b) The maximum temperature within the module occurs at the surface next to the insulation (y = 0). For one-dimensional conduction with thermal energy generation, use Eq. 3.42 to obtain

$$T\left(0\right) = \frac{\dot{q}a^2}{2k} + T_s = \frac{8.713 \times 10^5\ \text{W/m}^3 \times \left(0.010\ \text{m}\right)^2}{2 \times 5.2\ \text{W/m} \cdot \text{K}} + 150^\circ\text{C} = 158.4^\circ\text{C}. \qquad <$$

COMMENTS: An alternative approach for estimating the average heat transfer coefficient for the module follows from the relation

$$q_{module} = q_{0 \to L+b} - q_{0 \to L}$$

$$\bar{h} \cdot b = \bar{h}_{L+b} \cdot \left(L + b\right) - \bar{h}_L \cdot L \qquad \text{or} \qquad \bar{h} = \bar{h}_{L+b}\frac{L+b}{b} - \bar{h}_L\frac{L}{b}.$$

Recognizing that laminar and turbulent flow conditions exist, the appropriate correlation is

$$\overline{Nu}_x = \left(0.037 Re_x^{4/5} - 871\right) Pr^{1/3}$$

With x = L + b and x = L, find

$$\bar{h}_{L+b} = 54.81\ \text{W/m}^2 \cdot \text{K} \qquad \text{and} \qquad \bar{h}_L = 53.73\ \text{W/m}^2 \cdot \text{K}.$$

Hence,

$$\bar{h} = \left[54.81\frac{0.750}{0.050} - 53.73\frac{0.700}{0.05}\right]\text{W/m}^2 \cdot \text{K} = 69.9\ \text{W/m}^2 \cdot \text{K}.$$

which is in excellent agreement with the approximate result employed in part (a).

PROBLEM 7.9

KNOWN: Dimensions and surface temperature of electrically heated strips. Temperature and velocity of air in parallel flow.

FIND: (a) Rate of convection heat transfer from first, fifth and tenth strips as well as from all the strips, (b) For air velocities of 2, 5 and 10 m/s, determine the convection heat rates for all the locations of part (a), and (c) Repeat the calculations of part (b), but under conditions for which the flow is fully turbulent over the entire array of strips.

SCHEMATIC:

w = 0.2 m
L = 25 Δ L = 0.25 m
Air
u_∞ = 2 m/s
T_∞ = 25 °C
p = 1 atm
q″
T_s = 500 °C
Δ L = 0.01 m

ASSUMPTIONS: (1) Top surface is smooth, (2) Bottom surface is adiabatic, (3) Critical Reynolds number is 5×10^5, (4) Negligible radiation.

PROPERTIES: *Table A.4*, Air (T_f = 535 K, 1 atm): $\nu = 43.54 \times 10^{-6}$ m²/s, k = 0.0429 W/m·K, Pr = 0.683.

ANALYSIS: (a) The location of transition is determined from

$$x_c = 5\times10^5 \frac{\nu}{u_\infty} = 5\times10^5 \frac{43.54\times10^{-6}\,m^2/s}{2\,m/s} = 10.9\,m$$

Since $x_c >> L$ = 0.25 m, the air flow is laminar over the entire heater. For the *first* strip, $q_1 = \bar{h}_1(\Delta L \times w)(T_s - T_\infty)$ where $\bar{h}_1$ is obtained from

$$\bar{h}_1 = \frac{k}{\Delta L} 0.664\, Re_x^{1/2}\, Pr^{1/3}$$

$$\bar{h}_1 = \frac{0.0429\,W/m\cdot K}{0.01m} \times 0.664 \left(\frac{2\,m/s \times 0.01\,m}{43.54\times10^{-6}\,m^2/s}\right)^{1/2} (0.683)^{1/3} = 53.8\,W/m^2\cdot K$$

$$q_1 = 53.8\,W/m^2\cdot K\,(0.01m \times 0.2\,m)(500-25)^\circ C = 51.1\,W \qquad <$$

For the *fifth* strip, $q_5 = q_{0-5} - q_{0-4}$,

$$q_5 = h_{0-5}(5\Delta L \times w)(T_s - T_\infty) - \bar{h}_{0-4}(4\Delta L \times w)(T_s - T_\infty)$$

$$q_5 = (5\bar{h}_{0-5} - 4\bar{h}_{0-4})(\Delta L \times w)(T_s - T_\infty)$$

Hence, with $x_5 = 5\Delta L$ = 0.05 m and $x_4 = 4\Delta L$ = 0.04 m, it follows that $\bar{h}_{0-5}$ = 24.1 W/m²·K and $\bar{h}_{0-4}$ = 26.9 W/m²·K and

$$q_5 = (5\times24.1 - 4\times26.9)\,W/m^2\cdot K\,(0.01\times0.2)m^2\,(500-25)K = 12.2\,W. \qquad <$$

Similarly, where $\bar{h}_{0-10}$ = 17.00 W/m²·K and $\bar{h}_{0-9}$ = 17.92 W/m²·K.

$$q_{10} = (10\bar{h}_{0-10} - 9\bar{h}_{0-9})(\Delta L \times w)(T_s - T_\infty)$$

$$q_{10} = (10\times17.00 - 9\times17.92)\,W/m^2\cdot K\,(0.01\times0.2)m^2\,(500-25)K = 8.3\,W \qquad <$$

Continued...

PROBLEM 7.9 (Cont.)

For the entire heater,

$$\overline{h}_{0-25} = \frac{k}{L} 0.664\, Re_L^{1/2}\, Pr^{1/3} = \frac{0.0429}{0.25} \times 0.664 \left(\frac{2 \times 0.25}{43.54 \times 10^{-6}} \right)^{1/2} (0.683)^{1/3} = 10.75\, W/m^2 \cdot K$$

and the heat rate over all 25 strips is

$$q_{0-25} = \overline{h}_{0-25}\,(L \times w)(T_s - T_\infty) = 10.75\, W/m^2 \cdot K\,(0.25 \times 0.2)\,m^2\,(500 - 25)^\circ C = 255.3\, W \quad <$$

(b,c) Using the *IHT Correlations Tool*, *External Flow*, for *Laminar or Mixed Flow Conditions*, and following the same method of solution as above, the heat rates for the first, fifth, tenth and all the strips were calculated for air velocities of 2, 5 and 10 m/s. To evaluate the heat rates for fully turbulent conditions, the analysis was performed setting $Re_{s,c} = 1 \times 10^{-6}$. The results are tabulated below.

Flow conditions	u_∞ (m/s)	q_1 (W)	q_5 (W)	q_{10} (W)	q_{0-25} (W)
Laminar	2	51.1	12.1	8.3	256
	5	80.9	19.1	13.1	404
	10	114	27.0	18.6	572
Fully turbulent	2	17.9	10.6	9.1	235
	5	37.3	22.1	19.0	490
	10	64.9	38.5	33.1	853

COMMENTS: (1) An alternative approach to evaluating the heat loss from a single strip, for example, strip 5, would take the form $q_5 = \overline{h}_5\,(\Delta L \times w)(T_s - T_\infty)$, where $h_5 \approx h_{x=4.5\Delta L}$ or $\overline{h}_5 \approx \left(h_{x=5\Delta L} + h_{x=4\Delta L}\right)/2$.

(2) From the tabulated results, note that for both flow conditions, the heat rate for each strip and the entire heater, increases with increasing air velocity. For both flow conditions and for any specified velocity, the strip heat rates decrease with increasing distance from the leading edge.

(3) The effect of flow conditions, laminar vs. fully turbulent flow, on strip heat rates shows some unexpected behavior. For the u_∞ = 5 m/s condition, the effect of turbulent flow is to increase the heat rates for the entire heater and the tenth and fifth strips. For the u_∞ = 10 m/s, the effect of turbulent flow is to increase the heat rates at all locations. This behavior is a consequence of low Reynolds number ($Re_x = 2.3 \times 10^4$) at x = 0.25 m with u_∞ = 10 m/s.

(4) To more fully appreciate the effects due to laminar vs. turbulent flow conditions and air velocity, it is useful to examine the local coefficient as a function of distance from the leading edge. How would you use the results plotted below to explain heat rate behavior evident in the summary table above?

PROBLEM 7.10

KNOWN: Speed and temperature of atmospheric air flowing over a flat plate of prescribed length and temperature.

FIND: Rate of heat transfer corresponding to $Re_{x,c} = 10^5$, 5×10^5 and 10^6.

SCHEMATIC:

Air
$u_\infty = 25$ m/s
$T_\infty = 25°C$
$p = 1$ atm
q
$T_s = 125°C$
x
L = 1m

ASSUMPTIONS: (1) Flow over top and bottom surfaces.

PROPERTIES: *Table A-4*, Air (T_f = 348K, 1 atm): ρ = 1.00 kg/m^3, ν = 20.72 × 10^{-6} m^2/s, k = 0.0299 W/m·K, Pr = 0.700.

ANALYSIS: With

$$Re_L = \frac{u_\infty L}{n} = \frac{25 \text{ m/s} \times 1\text{m}}{20.72 \times 10^{-6} \text{m}^2/\text{s}} = 1.21 \times 10^6$$

the flow becomes turbulent for each of the three values of $Re_{x,c}$. Hence,

$$\overline{Nu}_L = \left(0.037\, Re_L^{4/5} - A\right) Pr^{1/3}$$
$$A = 0.037\, Re_{x,c}^{4/5} - 0.664\, Re_{x,c}^{1/2}$$

$Re_{x,c}$	10^5	5×10^5	10^6
A	160	871	1671
$\overline{Nu}_L$	2272	1641	931
$\bar{h}_L \left(W/m^2 \cdot K\right)$	67.9	49.1	27.8
$q'(W/m)$	13,580	9820	5560

where $q' = 2\,\bar{h}_L L\,(T_s - T_\infty)$ is the total heat loss per unit width of plate.

COMMENTS: Note that $\bar{h}_L$ decreases with increasing $Re_{x,c}$, as more of the surface becomes covered with a laminar boundary layer.

PROBLEM 7.11

KNOWN: Velocity and temperature of air in parallel flow over a flat plate of 1-m length.

FIND: (a) Calculate and plot the variation of the local convection coefficient, $h_x(x)$, with distance for flow conditions corresponding to transition Reynolds numbers of 5×10^5, 2.5×10^5 and 0 (fully turbulent), (b) Plot the variation of the average convection coefficient, $\overline{h}_x(x)$, for the three flow conditions of part (a), and (c) Determine the average convection coefficients for the entire plate, $\overline{h}_L$, for the three flow conditions of part (a).

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Constant surface temperature, and (3) Critical Reynolds depends upon prescribed flow conditions.

PROPERTIES: *Table A.4*, Air (T_f = 300 K, 1 atm): $\nu = 15.89 \times 10^{-6}$ m^2/s, k = 0.0263 W/m·K, Pr = 0.707.

ANALYSIS: (a) The Reynolds number for the plate (L = 1 m) is

$$Re_L = \frac{u_\infty L}{\nu} = \frac{10\,m/s \times 1\,m}{15.89 \times 10^{-6}\,m^2/s} = 6.29 \times 10^5 .$$

Hence, the boundary layer conditions are mixed with $Re_{x,c} = 5 \times 10^5$,

$$x_c = L\left(Re_{x,c}/Re_L\right) = 1\,m\,\frac{5 \times 10^5}{6.29 \times 10^5} = 0.795\,m$$

Using the *IHT Correlation Tool*, *External Flow*, *Local* coefficients for *Laminar* or *Turbulent Flow*, $h_x(x)$ was evaluated and plotted with critical Reynolds numbers of 5×10^5, 2.5×10^5 and 0 (fully turbulent). Note the location of the laminar-turbulent transition for the first two flow conditions.

Continued...

PROBLEM 7.11 (Cont.)

(b) Using the *IHT Correlation Tool*, *External Flow*, *Average* coefficient for *Laminar* or *Mixed Flow*, $\bar{h}_x(x)$ was evaluated and plotted for the three flow conditions. Note that the change in $\bar{h}_x(x)$ at the critical length, x_c, is rather gradual, compared to the abrupt change for the local coefficient, $h_x(x)$.

(c) The average convection coefficients for the plate can be determined from the above plot since $\bar{h}_L = \bar{h}_x(L)$. The values for the three flow conditions are, respectively,

$$\bar{h}_L = 17.4,\ 27.5 \text{ and } 37.8\,\mathrm{W/m^2 \cdot K}$$ <

COMMENTS: A copy of the *IHT Workspace* used to generate the above plots is shown below.

```
// Method of Solution:  Use the Correlation Tools, External Flow, Flat Plate, for (i) Local, laminar or turbulent
flow and (ii) Average, laminar or mixed flow, to evaluate the local and average convection coefficients as a
function of position on the plate.  In each of these tools, the value of the critical Reynolds number, Rexc, can
be set corresponding to the special flow conditions.

// Correlation Tool: External Flow, Plate Plate, Local, laminar or turbulent flow.
Nux = Nux_EF_FP_LT(Rex,Rexc,Pr)      // Eq 7.23,37
Nux = hx * x / k
Rex = uinf * x / nu
Rexc =1e-10
// Evaluate properties at the film temperature, Tf.
//Tf = (Tinf + Ts) / 2

/*  Correlation description: Parallel external flow (EF) over a flat plate (FP), local coefficient; laminar flow (L) for
Rex<Rexc, Eq 7.23; turbulent flow (T) for Rex>Rexc, Eq 7.37; 0.6<=Pr<=60. See Table 7.9.  */

// Correlation Tool: External Flow, Plate Plate, Average, laminar or mixed flow.
NuLbar = NuL_bar_EF_FP_LM(Rex,Rexc,Pr)      // Eq 7.31, 7.39, 7.40
NuLbar = hLbar * x / k                      // Changed variable from L to x
//ReL = uinf * x / nu
//Rexc = 5.0E5

/*  Correlation description: Parallel external flow (EF) over a flat plate (FP), average coefficient; laminar (L) if
ReL<Rexc, Eq 7.31; mixed (M) if ReL>Rexc, Eq 7.39 and 7.40; 0.6<=Pr<=60. See Table 7.9.  */

// Properties Tool - Air:
// Air property functions : From Table A.4
// Units: T(K); 1 atm pressure
nu = nu_T("Air",Tf)              // Kinematic viscosity, m^2/s
k = k_T("Air",Tf)                // Thermal conductivity, W/m·K
Pr = Pr_T("Air",Tf)              // Prandtl number

// Assigned Variables:
x = 1                            // Distance from leading edge; 0 <= x <= 1 m
uinf = 10                        // Freestream velocity, m/s
Tf = 300                         // Film temperature, K
```

PROBLEM 7.12

KNOWN: Velocity and temperature of water in parallel flow over a flat plate of 1-m length.

FIND: (a) Calculate and plot the variation of the local convection coefficient, h_x (x), with distance for flow conditions corresponding to transition Reynolds numbers of 5×10^5, 3×10^5 and 0 (fully turbulent), (b) Plot the variation of the average convection coefficient, $\overline{h}_x(x)$, for the three flow conditions of part (a), and (c) Determine the average convection coefficients for the entire plate, $\overline{h}_L$, for the three flow conditions of part (a).

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Constant surface temperature, and (3) Critical Reynolds depends upon prescribed flow conditions.

PROPERTIES: *Table A.6*, Water (300 K): $\rho = 997$ kg/m^3, $\mu = 855 \times 10^{-6}$ N·s/m^2, $\nu = \mu/\rho = 0.858 \times 10^{-6}$ m^2/s, k = 0.613 W/m·K, Pr = 583.

ANALYSIS: (a) The Reynolds number for the plate (L = 1 m) is

$$Re_L = \frac{u_\infty L}{\nu} = \frac{2\,\text{m/s} \times 1\,\text{m}}{0.858 \times 10^{-6}\,\text{m}^2/\text{s}} = 2.33 \times 10^6 .$$

and the boundary layer is mixed with $Re_{x,c} = 5 \times 10^5$,

$$x_c = L\left(Re_{x,c}/Re_L\right) = 1\,\text{m}\,\frac{5 \times 10^5}{2.33 \times 10^6} = 0.215\,\text{m}$$

Using the *IHT Correlation Tool*, *External Flow*, *Local* coefficients for *Laminar* or *Turbulent Flow*, $h_x(x)$ was evaluated and plotted with critical Reynolds numbers of 5×10^5, 3.0×10^5 and 0 (fully turbulent). Note the location of the laminar-turbulent transition for the first two flow conditions.

Continued...

PROBLEM 7.12 (Cont.)

(b) Using the *IHT Correlation Tool*, *External Flow*, *Average* coefficient for *Laminar* or *Mixed Flow*, $\overline{h}_x(x)$ was evaluated and plotted for the three flow conditions. Note that the change in $\overline{h}_x(x)$ at the critical length, x_c, is rather gradual, compared to the abrupt change for the local coefficient, $h_x(x)$.

Average cofficient, hxbar (W/m^2.K)

Distance from the leading edge, x (m)

Rexc = 5e5, Mixed flow
Rexc = 3e5, Mixed flow
Rexc = 0, Fully turbulent flow

(c) The average convection coefficients for the plate can be determined from the above plot since $\overline{h}_L = \overline{h}_x(L)$. The values for the three flow conditions are

$$\overline{h}_L = 4110,\ 4490 \text{ and } 5072\ \text{W/m}^2 \cdot \text{K}$$ <

COMMENTS: A copy of the *IHT Workspace* used to generate the above plot is shown below.

```
/* Method of Solution:  Use the Correlation Tools, External Flow, Flat Plate, for (i) Local, laminar or turbulent
flow and (ii) Average, laminar or mixed flow, to evaluate the local and average convection coefficients as a
function of position on the plate.  In each of these tools, the value of the critical Reynolds number, Rexc, can
be set corresponding to the special flow conditions.   */

// Correlation Tool: External Flow, Plate Plate, Local, laminar or turbulent flow.
Nux = Nux_EF_FP_LT(Rex,Rexc,Pr)     // Eq 7.23,37
Nux = hx * x / k
Rex = uinf * x / nu
Rexc = 1e-10
// Evaluate properties at the film temperature, Tf.
//Tf = (Tinf + Ts) / 2

/*  Correlation description: Parallel external flow (EF) over a flat plate (FP), local coefficient; laminar flow (L) for
Rex<Rexc, Eq 7.23; turbulent flow (T) for Rex>Rexc, Eq 7.37; 0.6<=Pr<=60. See Table 7.9.  */

// Correlation Tool: External Flow, Plate Plate, Average, laminar or mixed flow.
NuLbar = NuL_bar_EF_FP_LM(Rex,Rexc,Pr)     // Eq 7.31, 7.39, 7.40
NuLbar = hLbar * x / k                                  // Changed variable from L to x
//ReL = uinf * x / nu
//Rexc = 5.0E5

/*  Correlation description: Parallel external flow (EF) over a flat plate (FP), average coefficient; laminar (L) if
ReL<Rexc, Eq 7.31; mixed (M) if ReL>Rexc, Eq 7.39 and 7.40; 0.6<=Pr<=60. See Table 7.9.  */

// Properties Tool - Water:
// Water property functions :T dependence, From Table A.6
// Units: T(K), p(bars);
xf =  0                          // Quality (0=sat liquid or 1=sat vapor);  "x" is used as spatial coordinate
p = psat_T("Water", Tf)          // Saturation pressure, bar
nu = nu_Tx("Water",Tf,x)         // Kinematic viscosity, m^2/s
k = k_Tx("Water",Tf,x)           // Thermal conductivity, W/m·K
Pr = Pr_Tx("Water",Tf,x)         // Prandtl number

// Assigned Variables:
x = 1                            // Distance from leading edge; 0 <= x <= 1 m
uinf = 2                         // Freestream velocity, m/s
Tf = 300                         // Film temperature, K
```

PROBLEM 7.13

KNOWN: Two plates of length L and 2L experience parallel flow with a critical Reynolds number of 5×10^5.

FIND: Reynolds numbers for which the total heat transfer rate is independent of orientation.

SCHEMATIC:

$Re_{x,c} = 5 \times 10^5$

ASSUMPTIONS: (1) Plate temperatures and flow conditions are equivalent.

ANALYSIS: The total heat transfer rate would be the same ($q_L = q_{2L}$), if the convection coefficients were equal, $\bar{h}_L = \bar{h}_{2L}$. Conditions for which such an equality is possible may be inferred from a sketch of $\bar{h}_L$ versus Re_L.

For laminar flow $\left(Re_L < Re_{x,c}\right)$, $\bar{h}_L \; a \; L^{-1/2}$, and for mixed laminar and turbulent flow $\left(Re_L > Re_{x,c}\right)$, $\bar{h}_L = C_1 L^{-1/5} - C_2 L^{-1}$. Hence $\bar{h}_L$ varies with Re_L as shown, and two possibilities are suggested.

Case (a): Laminar flow exists on the shorter plate, while mixed flow conditions exist on the longer plate.

Case (b): Mixed boundary layer conditions exist on both plates.

In both cases, it is required that

$$\bar{h}_L = \bar{h}_{2L} \qquad \text{and} \qquad Re_{2L} = 2\, Re_L .$$

Continued …..

Case (a): From expressions for $\overline{h}_L$ in laminar and mixed flow

$$0.664\ \frac{k}{L}\mathrm{Re}_L^{1/2}\ \mathrm{Pr}^{1/3} = \frac{k}{2L}\left(0.037\ \mathrm{Re}_{2L}^{4/5} - 871\right)\mathrm{Pr}^{1/3}$$
$$0.664\ \mathrm{Re}_L^{1/2} = 0.032\ \mathrm{Re}_L^{4/5} - 435.$$

Since $\mathrm{Re}_L < 5 \times 10^5$ and $\mathrm{Re}_{2L} = 2\ \mathrm{Re}_L > 5 \times 10^5$, the required value of Re_L may be narrowed to the range

$$2.5 \times 10^5 \quad < \quad \mathrm{Re}_L \quad < \quad 5 \times 10^5.$$

From a trial-and-error solution, it follows that

$$\mathrm{Re}_L \approx 3.2 \times 10^5.$$ <

Case (b): For mixed flow on both plates

$$\frac{k}{L}\left(0.037\ \mathrm{Re}_L^{4/5} - 871\right)\ \mathrm{Pr}^{1/3} = \frac{k}{2L}\left(0.037\ \mathrm{Re}_{2L}^{4/5} - 871\right)\ \mathrm{Pr}^{1/3}$$

or

$$0.037\ \mathrm{Re}_L^{4/5} - 871 = 0.032\ \mathrm{Re}_L^{4/5} - 435$$
$$0.005\ \mathrm{Re}_L^{4/5} = 436$$

$$\mathrm{Re}_L \approx 1.50 \times 10^6.$$ <

COMMENTS: (1) Note that it is impossible to satisfy the requirement that $\overline{h}_L = \overline{h}_{2L}$ if $\mathrm{Re}_L < 0.25 \times 10^5$ (laminar flow for both plates).

(2) The results are independent of the nature of the fluid.

PROBLEM 7.14

KNOWN: Water flowing over a flat plate under specified conditions.

FIND: (a) Heat transfer rate per unit width, $q'(\mathrm{W/m})$, evaluating properties at $T_f = (T_s + T_\infty)/2$, (b) Error in q' resulting from evaluating properties at T_∞, (c) Heat transfer rate, q', if flow is assumed turbulent at leading edge, x = 0.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions.

PROPERTIES: *Table A-6*, Water (T_∞ = 4°C = 277K): $\rho_f = 1000\ \mathrm{kg/m^3}$, $\mu_f = 1560 \times 10^{-6}\ \mathrm{N{\cdot}s/m^2}$, $\nu_f = \mu_f/\rho_f = 1.560 \times 10^{-6}\ \mathrm{m^2/s}$, $k_f = 0.577$ W/m·K, Pr = 11.44; Water (T_f = 295K): $\nu = 0.961 \times 10^{-6}\ \mathrm{m^2/s}$, k = 0.606 W/m·K, Pr = 6.62; Water (T_s = 40°C = 313K): $\mu = 657 \times 10^{-6}\ \mathrm{N{\cdot}s/m^2}$.

ANALYSIS: (a) The heat rate is given as $q' = \overline{h}L\left(T_s - T_\infty\right)$, and $\overline{h}$ must be estimated by the proper correlation. Using properties evaluated at T_f, the Reynolds number is

$$Re_L = \frac{u_\infty L}{n} = \frac{0.6\ \mathrm{m/s} \times 1.5\mathrm{m}}{0.961 \times 10^{-6}\mathrm{m^2/s}} = 9.365 \times 10^5.$$

Hence flow is mixed and the appropriate correlation and convection coefficient are

$$\overline{Nu}_L = \left[0.037\ Re_L^{4/5} - 871\right] Pr^{1/3} = \left[0.037\left(9.365 \times 10^5\right)^{4/5} - 871\right] 6.62^{1/3} = 2522$$

$$\overline{h}_L \equiv \frac{\overline{Nu}_L k}{L} = \frac{2522 \times 0.606\ \mathrm{W/m \cdot K}}{1.5\mathrm{m}} = 1019\ \mathrm{W/m^2 \cdot K}.$$

The heat rate is then

$$q' = 1019\ \mathrm{W/m^2 \cdot K} \times 1.5\mathrm{m}\left(40 - 4\right)^\circ \mathrm{C} = 55.0\ \mathrm{kW/m}. \qquad <$$

(b) Evaluating properties at the free stream temperature, T_∞,

$$Re_L = \frac{0.6\mathrm{m/s} \times 1.5\mathrm{m}}{1.560 \times 10^{-6}\mathrm{m^2/s}} = 5.769 \times 10^5$$

The flow is still mixed, giving

$$\overline{Nu}_L = \left[0.037\left(5.769 \times 10^5\right)^{4/5} - 871\right] 11.44^{1/3} = 1424$$

$$\overline{h}_L = 1424 \times 0.577\ \mathrm{W/m \cdot K/1.5m} = 575\ \mathrm{W/m \cdot K}$$

$$q' = 575\ \mathrm{W/m \cdot K} \times 1.5\mathrm{m}\left(40 - 4\right)^\circ \mathrm{C} = 31.1\ \mathrm{kW/m}. \qquad <$$

Continued …..

(c) If flow were tripped at the leading edge, the flow would be turbulent over the full length of the plate, in which case,

$$\overline{Nu}_L = 0.037\, Re_L^{4/5}\, Pr^{1/3} = 0.037\left(9.365\times10^5\right)^{4/5} 6.62^{1/3} = 4157$$

$$\bar{h}_L = \overline{Nu}_L\, k/L = 4157\times0.606\ W/m\cdot K/1.5m = 1679\ W/m^2\cdot K$$

$$q' = \bar{h}_L L\left(T_s - T_\infty\right) = 1679\ W/m^2\cdot K\times1.5m\ \left(40-4\right)^\circ C = 90.7\ kW/m.$$ <

COMMENTS: Comparing results:

Flow	Part	Property Evaluation	q' (kW/m)	Difference (%)
mixed	(a)	T_f	55.0	--
mixed	(b)	T_∞	31.1	-43
turbulent	(c)	T_f	90.7	--

The heat rate is significantly underpredicted if the properties are incorrectly evaluated at T_∞ instead of T_f.

PROBLEM 7.15

KNOWN: Temperature, pressure and Reynolds number for air flow over a flat plate of uniform surface temperature.

FIND: (a) Rate of heat transfer from the plate, (b) Rate of heat transfer if air velocity is doubled and pressure is increased to 10 atm.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Uniform surface temperature, (3) Negligible radiation, (4) $Re_{x_c} = 5\times10^5$.

PROPERTIES: *Table A-4*, Air (T_f = 348K, 1 atm): k = 0.0299 W/m·K, Pr = 0.70.

ANALYSIS: (a) The heat rate is

$$q = \bar{h}_L (w\times L)\ (T_s - T_\infty).$$

Since the flow is laminar over the entire plate for $Re_L = 4\times10^4$, it follows that

$$\overline{Nu}_L = \frac{\bar{h}_L L}{k} = 0.664\ Re_L^{1/2}\ Pr^{1/3} = 0.664(40{,}000)^{1/2}(0.70)^{1/3} = 117.9.$$

Hence $$\bar{h}_L = 117.9\frac{k}{L} = 117.9\frac{0.0299\ W/m\cdot K}{0.2m} = 17.6\ W/m^2\cdot K$$

and $$q = 17.6\frac{W}{m^2\cdot K}(0.1m\times0.2m)\ (100-50)^\circ C = 17.6\ W.$$ <

(b) With $p_2 = 10\ p_1$, it follows that $\rho_2 = 10\ \rho_1$ and $\nu_2 = \nu_1/10$. Hence

$$Re_{L,2} = \left(\frac{u_\infty L}{n}\right)_2 = 2\times10\left(\frac{u_\infty L}{n}\right)_1 = 20\ Re_{L,1} = 8\times10^5$$

and mixed boundary layer conditions exist on the plate. Hence

$$\overline{Nu}_L = \frac{\bar{h}_L L}{k} = \left(0.037\ Re_L^{4/5} - 871\right)\ Pr^{1/3} = \left[0.037\times\left(8\times10^5\right)^{4/5} - 871\right]\ (0.70)^{1/3}$$

$$\overline{Nu}_L = 961.$$

Hence, $$\bar{h}_L = 961\frac{0.0299\ W/m\cdot K}{0.2m} = 143.6\ W/m^2\cdot K$$

$$q = 143.6\frac{W}{m^2\cdot K}(0.1m\times0.2m)\ (100-50)^\circ C = 143.6\ W.$$ <

COMMENTS: Note that, in calculating $Re_{L,2}$, ideal gas behavior has been assumed. It has also been assumed that k, μ and Pr are independent of pressure over the range considered.

PROBLEM 7.16

KNOWN: Length and surface temperature of a rectangular fin.

FIND: (a) Heat removal per unit width, q', when air at a prescribed temperature and velocity is in parallel, turbulent flow over the fin, and (b) Calculate and plot q' for motorcycle speeds ranging from 10 to 100 km/h.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible radiation, (3) Turbulent flow over entire surface.

PROPERTIES: *Table A.4*, Air (412 K, 1 atm): $\nu = 27.85 \times 10^{-6}$ m²/s, k = 0.0346 W/m·K, Pr = 0.69.

ANALYSIS: (a) The heat loss per unit width is

$$q' = 2 \times \left[\bar{h}_L L \left(T_s - T_\infty\right)\right]$$

where $\bar{h}$ is obtained from the correlation, Eq. 7.41 but with turbulent flow over the entire surface,

$$\overline{Nu}_L = 0.037\,Re_L^{4/5}\,Pr^{1/3} = 0.037\left[\frac{80\,km/h \times 1000\,m/km \times 1/3600\,h/s \times 0.15\,m}{27.85 \times 10^{-6}\,m^2/s}\right]^{4/5} (0.69)^{1/3} = 378$$

Hence,

$$\bar{h}_L = \frac{k}{L}\overline{Nu}_L = \frac{0.0346\,W/m\cdot K}{0.15\,m}378 = 87\,W/m^2\cdot K$$

$$q' = 2 \times \left[87\,W/m^2\cdot K \times 0.15\,m\,(523 - 300)\,K\right] = 5826\,W/m\,.$$ <

(b) Using the foregoing equations in the IHT Workspace, q' as a function of speed was calculated and is plotted as shown.

COMMENTS: (1) Radiation emission from the fin is not negligible. With an assumed emissivity of ε = 1, the rate of emission per unit width at 80 km/h would be $q' = \left(\sigma T_s^4\right)2L$ = 1273 W/m. If the fin received negligible radiation from its surroundings, its loss by radiation would then be approximately 20% of that by convection.

(2) From the correlation and heat rate expression, it follows that $q' \sim u_\infty^{4/5}$. That is, q' vs. u_∞ is nearly linear as evident from the above plot.

PROBLEM 7.17

KNOWN: Wall of a metal building experiences a 10 mph (4.47 m/s) breeze with air temperature of 90°F (32.2°C) and solar insolation of 400 W/m^2. The length of the wall in the wind direction is 10 m and the emissivity is 0.93.

FIND: Estimate the average wall temperature.

SCHEMATIC:

T_∞ = 90°F (32.2°C)
u_∞ = 5 mph (4.47 m/s)
Warm breeze
G_S = 400 W/m^2
Building wall
$\overline{T}_s$, ε = 0.93, α_S = 1.0
L = 10 m
x

ASSUMPTIONS: (1) Steady-state conditions, (2) The solar absorptivity of the wall is unity, (3) Sky irradiation is negligible, (4) Wall is isothermal at the average temperature T_s, (5) Flow is fully turbulent over the wall, and (6) Negligible heat transfer into the building.

PROPERTIES: *Table A-4*, Air (assume T_f = 305 K, 1 atm): $\nu = 16.27 \times 10^{6}$ m^2/s, k = 0.02658 W/m·K, Pr = 0.707.

ANALYSIS: Perform an energy balance on the wall surface considering convection, absorbed irradiation and emission. On a per unit width,

$$\dot{E}'_{in} - \dot{E}'_{out} = 0$$

$$-q'_{cv} + \left(\alpha_S G_S - E_s\right) L = 0$$

$$-\overline{h}_L\, L\left(T_s - T_\infty\right) + \left(\alpha_S G_S - \varepsilon\sigma T_s^4\right) L = 0 \qquad (1)$$

The average convection coefficient is estimated using Eq. 7.41 assuming fully turbulent flow over the length of the wall in the direction of the breeze.

$$\overline{Nu}_L = \frac{\overline{h}_L\, L}{k} = 0.037\ Re_L^{4/5}\, Pr^{1/3} \qquad (2)$$

$$Re_L = u_\infty L/\nu = 4.47\,\text{m/s} \times 10\,\text{m}/16.27 \times 10^{6}\,\text{m}^2/\text{s} = 2.748 \times 10^{6}$$

$$\overline{h}_L = \left(0.02658\ \text{W/m} \cdot \text{K}/10\,\text{m}\right) \times 0.037\left(2.748 \times 10^{6}\right)^{4/5}\left(0.707\right)^{1/3} = 12.4\ \text{W/m}^2 \cdot \text{K}$$

Substituting numerical values into Eq. (1), find T_s.

$$-12.4\ \text{W/m}^2 \times 10\,\text{m}\left[T_s - \left(32.2 + 273\right)\right]\text{K}$$

$$+\left[1.0 \times 400\ \text{W/m}^2 - 0.93 \times 5.67 \times 10^{-8}\,\text{W/m}^2 \cdot \text{K}^4 T_s^4\right] \times 10\,\text{m} = 0$$

$$T_s = 302.2\ \text{K} = 29°\text{C}$$ <

COMMENTS: (1) The properties for the correlation should be evaluated at $T_f = (T_s + T_\infty)/2$ = 304 K. The assumption of 305 K was reasonable.

(2) Is the heat transfer by the emission process significant? Would application of a low emissive coating be effective in reducing the wall temperature, assuming α_S remained unchanged? Or, should a low solar absorbing coating be considered?

PROBLEM 7.18

KNOWN: Square solar panel with an area of 0.09 m^2 has solar-to-electrical power conversion efficiency of 12%, solar absorptivity of 0.85, and emissivity of 0.90. Panel experiences a 4 m/s breeze with an air temperature of 25°C and solar insolation of 700 W/m^2.

FIND: Estimate the temperature of the solar panel for: (a) The operating condition *(on)* described above when the panel is producing power, and (b) The *off* condition when the solar array is inoperative. Will the panel temperature increase, remain the same or decrease, all other conditions remaining the same?

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) The backside of the panel experiences no heat transfer, (3) Sky irradiation is negligible, and (4) Wind is in parallel, fully turbulent flow over the panel.

PROPERTIES: *Table A-4*, Air (Assume T_f = 300 K, 1 atm): $\nu = 15.89 \times 10^{-6}$ m^2/s, k = 0.0263 W/m·K, Pr = 0.707.

ANALYSIS: (a) Perform an energy balance on the panel as represented in the schematic above considering convection, absorbed insolation, emission and generated electrical power.

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = 0$$

$$-q_{cv} + \left[\alpha_S G_S - \varepsilon\sigma T_s^4\right] A_s - P_{elec} = 0 \qquad (1)$$

Using the convection rate equation and power conversion efficiency,

$$q_{cv} = \overline{h}_L A_s \left(T_s - T_\infty\right) \qquad P_{elec} = \eta_e \alpha_S G_S A_s \qquad (2,3)$$

The average convection coefficient for fully turbulent conditions is

$$\overline{Nu}_L = \overline{h}L/k = 0.037\ Re_L^{4/5}\ Pr^{1/3}$$

$$Re_L = u_\infty L/\nu = 4\,m/s \times 0.3\,m/15.89\times10^{-6} m^2/s = 7.49\times10^4$$

$$\overline{h}_L = \left(0.0263\ W/m\cdot K/0.3m\right)\times 0.037 \times \left(7.49\times10^4\right)^{4/5}\left(0.707\right)^{1/3}$$

$$\overline{h}_L = 23.0\ W/m^2\cdot K$$

Substituting numerical values in Eq. (1) using Eqs. (2 and 3) and dividing through by A_s, find T_s.

Continued …..

PROBLEM 7.18 (Cont.)

$$23\ \mathrm{W/m^2\cdot K}\left(T_s - 298\right)\mathrm{K} + 0.85\times 700\ \mathrm{W/m^2} - 0.90\times 5.67\times 10^{-8}\,\mathrm{W/m^2\cdot K^4}\,T_s^4$$

$$-0.12\left[0.85\times 700\ \mathrm{W/m^2}\right] = 0 \qquad (4)$$

$$T_s = 302.2\ \mathrm{K} = 29.2°\mathrm{C}$$ <

(b) If the solar array becomes inoperable *(off)* for reason of wire bond failures or the electrical circuit to the battery is opened, the P_{elec} term in the energy balance of Eq. (1) is zero. Using Eq. (4) with $\eta_e = 0$, find

$$T_s = 31.7°\mathrm{C}$$ <

COMMENTS: (1) Note how the electrical power P_{elec} is represented by the $\dot{E}_{gen}$ term in the energy balance. Recall from Section 1.2 that $\dot{E}_{gen}$ is associated with conversion *from* some form of energy *to* thermal energy. Hence, the solar-to-electrical power conversion (P_{elec}) will have a negative sign in Eq. (1).

(2) It follows that when the solar array is *on*, a fraction (η_e) of the absorbed solar power (thermal energy) is converted to electrical energy. As such, the array surface temperature will be higher in the *off* condition than in the *on* condition.

(3) Note that the assumed value for T_f at which to evaluate the properties was reasonable.

PROBLEM 7.19

KNOWN: Ambient air conditions and absorbed solar flux for an aircraft wing of prescribed length and speed.

FIND: (a) Steady-state temperature of wing and (b) Calculate and plot the steady-state temperature for plane speeds 100 to 250 m/s.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Uniform wing temperature, (3) Negligible radiation emission from surface.

PROPERTIES: *Table A.4*, Air ($T_f \approx 270$ K, p = 0.7 bar): k = 0.0239 W/m·K, Pr = 0.715, $\nu = 13.22 \times 10^{-6}\ m^2/s$ (1.0133 bar/0.7 bar) = $19.14 \times 10^{-6}\ m^2/s$.

ANALYSIS: From an energy balance on the airfoil

$$q''_{S,abs} A_S = 2q_{conv} = 2\bar{h}_L A_S \left(T_S - T_\infty\right) \qquad T_S = T_\infty + q''_{S,abs} \big/ 2\bar{h}_L \qquad (1,2)$$

Since

$$Re_L = u_\infty L/\nu = (100\,m/s)\,2.5\,m \big/ 19.14 \times 10^{-6}\ m^2/s = 1.31 \times 10^7 \qquad (3)$$

and $Re_{s,c} = 5 \times 10^5$, the flow may be approximated as turbulent over the entire plate. Hence, from Eq. 7.41,

$$\overline{Nu}_L = 0.037\,Re_L^{4/5}\,Pr^{1/3} = 0.037\left(1.31 \times 10^7\right)^{4/5}(0.715)^{1/3} = 1.63 \times 10^4 \qquad (4)$$

$$\bar{h}_L = \frac{\overline{Nu}_L k}{L} = \frac{1.63 \times 10^4\,(0.0239\,W/m \cdot K)}{2.5\,m} = 156\,W/m^2 \cdot K \qquad (5)$$

Hence, from the energy balance

$$T_S = 263\,K + 800\,W/m^2 \big/ 2 \times 156\,W/m^2 \cdot K = 266\,K \qquad <$$

(b) Using the energy balance relation for T_s, Eq. (1), and the *IHT Correlations Tool, External Flow, Average* coefficient for *Laminar* or *Turbulent Flow*, T_s as a function of u_∞ was evaluated.

COMMENTS: (1) Radiation emission from the wing surface would *decrease* T_s, while radiation incident from the earth's surface and the sky would act to *increase* T_s. The net effect on T_s is likely to be small.

(2) How do you explain that the effect of aircraft speed on T_s appears to be only slight? How does $\bar{h}_L$ dependent upon u_∞? What is the limit of T_s with increasing speed?

PROBLEM 7.20

KNOWN: Material properties, inner surface temperature and dimensions of roof of refrigerated truck compartment. Truck speed and ambient temperature. Solar irradiation.

FIND: (a) Outer surface temperature of roof and rate of heat transfer to compartment, (b) Effect of changing radiative properties of outer surface, (c) Effect of eliminating insulation.

SCHEMATIC:

$q''_S = 750\ W/m^2$ $\quad q''_{conv}$ $\quad E$ $\quad T_{s,o},\ \varepsilon = \alpha_S = 0.5$, $W = 3.5m$

$u_\infty = 105\ km/h$, $T_\infty = 32^oC$

Urethane foam $k_i = 0.026\ W/m\text{-}K$ $\quad q''_{cond}$

Aluminum alloy $k_p = 180\ W/m\text{-}K$ $\quad T_{s,i} = -10^oC$

$t_1 = 5\ mm$, $t_2 = 50\ mm$, $t_3 = 5\ mm$

ASSUMPTIONS: (1) Negligible irradiation from the sky, (2) Turbulent flow over entire outer surface, (3) Average convection coefficient may be used to estimate average surface temperature, (4) Constant properties.

PROPERTIES: *Table A-4*, air (p = 1 atm, $T_f \approx 300K$): $\nu = 15.89 \times 10^{-6}\ m^2/s$, k = 0.0263 W/m·K, Pr = 0.707.

ANALYSIS: (a) From an energy balance for the outer surface,

$$\alpha_S G_S + q''_{conv} - E = q''_{cond} = \frac{T_{s,o} - T_{s,i}}{R''_{tot}}$$

$$\alpha_S G_S + \bar{h}\left(T_\infty - T_{s,o}\right) - \varepsilon\sigma T_{s,o}^4 = \frac{T_{s,o} - T_{s,i}}{2R''_p + R''_i}$$

where $R''_p = \left(t_1 / k_p\right) = 2.78 \times 10^{-5} m^2 \cdot K/W$, $R''_i = \left(t_2 / k_i\right) = 1.923\, m^2 \cdot K/W$, and with $Re_L = u_\infty L/\nu$ $= 29.2\, m/s \times 10m / 15.89 \times 10^{-6} m^2/s = 1.84 \times 10^7$,

$$\bar{h} = \frac{k}{L} 0.037\, Re_L^{4/5}\, Pr^{1/3} = \frac{0.0263\, W/m \cdot K}{10m} 0.037\left(1.84 \times 10^7\right)^{4/5}\left(0.707\right)^{1/3} = 56.2\, W/m^2 \cdot K$$

Hence,

$$0.5\left(750\, W/m^2 \cdot K\right) + 56.2\, W/m^2 \cdot K\left(305 - T_{s,o}\right) - 0.5 \times 5.67 \times 10^{-8}\, W/m^2 \cdot K^4\, T_{s,o}^4 = \frac{T_{s,o} - 263K}{\left(5.56 \times 10^{-5} + 1.923\right) m^2 \cdot K/W}$$

Solving, we obtain

$$T_{s,o} = 306.8K = 33.8°C \qquad <$$

Hence, the heat load is

$$q = \left(W \cdot L\right) q''_{cnd} = \left(3.5m \times 10m\right) \frac{\left(33.8 + 10\right)°C}{1.923 m^2 \cdot K/W} = 797\, W \qquad <$$

(b) With the special surface finish $\left(\alpha_S = 0.15,\ \varepsilon = 0.8\right)$,

Continued …..

PROBLEM 7.20 (Cont.)

$$T_{s,o} = 301.1\text{K} = 27.1°\text{C}$$ <

$$q = 675.3\text{W}$$ <

(c) Without the insulation (t_2 = 0) and with $\alpha_S = \varepsilon = 0.5$,

$$T_{s,o} = 263.1\text{K} = -9.9°\text{C}$$ <

$$q = 90,630\text{W}$$ <

COMMENTS: (1) Use of the special surface finish reduces the solar input, while increasing radiation emission from the surface. The cumulative effect is to reduce the heat load by 15%. (2) The thermal resistance of the aluminum panels is negligible, and without the insulation, the heat load is *enormous*.

$\alpha_S q''_S$ $T_{s,o}$ $T_{s,i}$
E q''_{cond}
q''_{conv} t_1/k_p t_2/k_i t_1/k_p

PROBLEM 7.21

KNOWN: Surface characteristics of a flat plate in an air stream.

FIND: Orientation which minimizes convection heat transfer.

SCHEMATIC:

ASSUMPTIONS: (1) Surface B is sufficiently rough to trip the boundary layer when in the upstream position (Configuration 2).

PROPERTIES: *Table A-4*, Air (T_f = 333K, 1 atm): $\nu = 19.2 \times 10^{-6}\ m^2/s$, $k = 28.7 \times 10^{-3}$ W/m·K, Pr = 0.7.

ANALYSIS: Since Configuration (2) results in a turbulent boundary layer over the entire surface, the lowest heat transfer is associated with Configuration (1). Find

$$Re_L = \frac{u_\infty L}{\nu} = \frac{20\ m/s \times 1m}{19.2 \times 10^{-6} m^2/s} = 1.04 \times 10^6.$$

Hence in Configuration (1), transition will occur just before the rough surface (x_c = 0.48m). Note that

$$\overline{Nu}_{L,1} = \left[0.037\left(1.04 \times 10^6\right)^{4/5} - 871\right] 0.7^{1/3} = 1366$$

$$\overline{Nu}_{L,2} = 0.037\left(1.04 \times 10^6\right)^{4/5}(0.7)^{1/3} = 2139 > \overline{Nu}_{L,1}.$$

For Configuration (1): $$\frac{\overline{h}_{L,1} L}{k} = \overline{Nu}_{L,1} = 1366.$$

Hence

$$\overline{h}_{L,1} = 1366\left(28.7 \times 10^{-3}\ W/m \cdot K\right)/1m = 39.2\ W/m^2 \cdot K$$

and

$$q_1 = \overline{h}_{L,1} A\left(T_s - T_\infty\right) = 39.2\ W/m^2 \cdot K\left(0.5m \times 1m\right)\left(100 - 20\right)K$$

$$q_1 = 1568\ W.$$ <

PROBLEM 7.22

KNOWN: Heat rate from and surface temperature of top surface of an oven under quiescent room air conditions (Case A).

FIND: (a) Heat rate when air at 15 m/s is blown across surface, (b) Surface temperature, T_s, achieved with the forced convection condition, and (c) Calculate and plot T_s as a function of room air velocity for $5 \le u_\infty \le 30$ m/s.

SCHEMATIC:

ASSUMPTIONS: (1) Surface has uniform temperature under both conditions, (2) Negligible radiation effects, (3) Air is blown parallel to edge, and (4) Thermal resistance due to oven wall and internal convection are the same for both conditions.

PROPERTIES: *Table A.4*, Air ($\overline{T}_f = (T_s + T_\infty)/2 \approx (37 + 17)/2 = 27$ °C = 300 K): k = 0.0263 W/m·K, $\nu = 15.89 \times 10^{-6}$ m²/s, Pr = 0.707.

ANALYSIS: (a) For Case A, we can determine the thermal resistance due to the wall and internal convection as,

$$R_{t,i} = \frac{T_i - T_s}{q_A} = \frac{(150-47)^\circ C}{40\,W} = 2.575\,K/W \quad (1)$$

which remains constant for case B. Hence, for Case B with forced convection, the heat rate is

$$q_B = UA(T_i - T_\infty) \quad (2)$$

where

$$(UA)^{-1} = R_{t,i} + \left(1/\overline{h}_o A_s\right) \quad (3)$$

To estimate $\overline{h}_o$, find

$$Re_L = \frac{u_\infty L}{\nu} = \frac{20\,m/s \times 0.5\,m}{15.89\times10^{-6}\,m^2/s} = 6.293\times10^5.$$

Assuming $Re_{s,c} = 5 \times 10^5$, flow conditions are mixed; hence

$$\overline{Nu}_L = \frac{\overline{h}_o L}{k} = \left(0.037\,Re_L^{4/5} - 871\right)Pr^{1/3} = \left(0.037\left(6.293\times10^5\right)^{0.8} - 871\right)(0.707)^{1/3} = 660.0$$

$$\overline{h}_o = 660.0 \times 0.0263\,W/m\cdot K/0.5\,m = 34.7\,W/m^2\cdot K.$$

Using Eq. (3) for $(UA)^{-1}$ and Eq. (2) for q_B, find

$$(UA)^{-1} = 2.575\,K/W + \left(1/34.7\,W/m^2\cdot K\,(0.5\,m)^2\right) = (2.575 + 0.115) = 2.690\,K/W$$

Continued...

PROBLEM 7.22 (Cont.)

$$q_B = (1/2.690\,K/W)(150-17)K = 49.4\,W\,.$$ <

(b) From the rate equation at the surface,

$$T_s = T_\infty + q/\bar{h}_o A_s \qquad (4)$$

$$T_s = 17^\circ C + 49.4\,W \Big/ \left(34.7\,W/m^2 \cdot K \times (0.5\,m)^2\right)$$

$$T_s = (17+5.7)^\circ C = 22.7^\circ C$$ <

(c) Using Eqs. (2), (3) and (4), and evaluating $\bar{h}_o$ using *IHT Correlations Tool*, *External Flow*, *Average* coefficient for *Laminar* or *Mixed Flow*, the surface temperature was evaluated as a function of room air velocity and is plotted below.

COMMENTS: (1) Note that in part (a), $T_f = (T_s + T_\infty)/2 = (22.7 + 17)/2 = 19.8°C = 293$ K compared to the assumed value of 300 K. Performing an iterative solution with IHT, find $T_f = 293$ with $T_s = 22.4°C$ suggesting the approximate value for T_f was satisfactory.

(2) From the plot, as expected, T_s decreases with increasing air velocity. What is the cause of the inflection in the curve at $u_\infty = 15$ m/s? As u_∞ increases, what is the limit for T_s?

PROBLEM 7.23

KNOWN: Prevailing wind with prescribed speed blows past ten window panels, each of 1-m length, on a penthouse tower.

FIND: (a) Average convection coefficient for the first, third and tenth window panels when the wind speed is 5 m/s; evaluate thermophysical properties at 300 K, but determine suitability when ambient air temperature is in the range $-15 \le T_\infty \le 38°C$; (b) Compute and plot the average coefficients for the same panels with wind speeds for the range $5 \le u_\infty \le 100$ km/h; explain features and relative magnitudes.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Wind over panels approximates parallel flow over a smooth flat plate, and (4) Transition Reynolds number is $Re_{s,c} = 5 \times 10^5$.

PROPERTIES: *Table A.4*, Air (T_f = 300 K, 1 atm): $\nu = 15.89 \times 10^{-5}$ m^2/s, k = 26.3×10^{-3} W/m·K, Pr = 0.707.

ANALYSIS: (a) The average convection coefficients for the first, third and tenth panels are

$$\bar{h}_1 \qquad \bar{h}_{2-3} = \frac{\bar{h}_3 x_3 - \bar{h}_2 x_2}{x_3 - x_2} \qquad \bar{h}_{9-10} = \frac{\bar{h}_{10} x_{10} - \bar{h}_9 x_9}{x_{10} - x_9} \qquad (1,2,3)$$

where $\bar{h}_2 = \bar{h}_2(x_2)$, etc. If $Re_{x,c} = 5 \times 10^5$, with properties evaluated at T_f = 300 K, transition occurs at

$$x_c = \frac{\nu}{u_\infty} Re_{x,c} = \frac{15.89\times10^{-6}\,\text{m}^2/\text{s}}{5\,\text{m/s}} \times 5\times10^5 = 1.59\,\text{m}$$

The flow over the first panel is laminar, and $\bar{h}_1$ can be estimated using Eq. (7.31).

$$\overline{Nu}_{x1} = \frac{\bar{h}_1 x_1}{k} = 0.664\, Re_x^{1/2}\, Pr^{1/3}$$

$$\bar{h}_1 = (0.0263\,\text{W/m}\cdot\text{K}\times0.664/1\text{m})\left(5\,\text{m/s}\times1\text{m}/15.89\times10^{-6}\,\text{m}^2/\text{s}\right)^{1/2}(0.707)^{1/3} = 8.73\,\text{W/m}^2\cdot\text{K}$$ <

The flow over the third and tenth panels is mixed, and $\bar{h}_2$, $\bar{h}_3$, $\bar{h}_9$ and $\bar{h}_{10}$ can be estimated using Eq. (7.41). For the third panel with x_3 = 3 m and x_2 = 2 m,

$$\overline{Nu}_{x3} = \frac{\bar{h}_3 x_3}{k} = \left(0.037\, Re_x^{4/5} - 871\right) Pr^{1/3}$$

$$\bar{h}_3 = (0.0263\,\text{W/m}\cdot\text{K}/3\text{m}) \times\left[0.037\left(5\,\text{m/s}\times3\text{m}/15.89\times10^{-6}\,\text{m}^2/\text{s}\right)^{4/5} - 871\right](0.707)^{1/3} = 10.6\,\text{W/m}^2\cdot\text{K}$$

Continued...

PROBLEM 7.23 (Cont.)

$$\bar{h}_2 = (0.0263\,\mathrm{W/m\cdot K}/2\mathrm{m})$$
$$\times\left[0.037\left(5\,\mathrm{m/s}\times 2\mathrm{m}\big/15.89\times10^{-6}\,\mathrm{m^2/s}\right)^{4/5} - 871\right](0.707)^{1/3} = 8.68\,\mathrm{W/m^2\cdot K}$$

From Eq. (2),

$$\bar{h}_{2-3} = \frac{10.61\,\mathrm{W/m^2\cdot K}\times 3\mathrm{m} - 8.68\,\mathrm{W/m^2\cdot K}\times 2\mathrm{m}}{(3-2)\mathrm{m}} = 14.5\,\mathrm{W/m^2\cdot K}$$ <

Following the same procedure for the tenth panel, find $\bar{h}_{10}$ = 11.64 W/m^2·K and $\bar{h}_9$ = 11.71 W/m^2·K, and

$$\bar{h}_{9-10} = 11.1\,\mathrm{W/m^2\cdot K}$$ <

Assuming that the window panel temperature will always be close to room temperature, T_s = 23°C = 296 K. If T_∞ ranges from -15 to 38°C, the film temperature, $T_f = (T_s + T_\infty)/2$, will vary from 275 to 310 K. We'll explore the effect of T_f subsequently.

(b) Using the *IHT Tool*, *Correlations*, *External Flow*, *Flat Plate*, results were obtained for the average coefficients $\bar{h}$. Using Eqs. (2) and (3), average coefficients for the panels as a function of wind speed were computed and plotted.

Average coefficient, hbar (W/m^2.K)
Wind speed, uinf (km/h)
First panel
Third panel
Tenth panel

COMMENTS: (1) The behavior of the panel average coefficients as a function of wind speed can be explained from the behavior of the local coefficient as a function of distance for difference velocities as plotted below.

Local coefficient, hx (W/m^2.K)
Distance from leading edge, x (m)
uinf = 5 km/h
uinf = 15 km/h
uinf = 25 km/h
uinf = 50 km/h

Continued...

PROBLEM 7.23 (Cont.)

For low wind speeds, transition occurs near the mid-panel, making $\overline{h}_1$ and $\overline{h}_{9-10}$ nearly equal and very high because of leading-edge and turbulence effects, respectively. As the wind speed increases, transition occurs closer to the leading edge. Notice how $\overline{h}_{2-3}$ increases rather abruptly, subsequently becoming greater than $\overline{h}_{9-10}$. The abrupt increase in $\overline{h}_1$ around 30 km/h is a consequence of transition occurring with x < 1m.

(2) Using the IHT code developed for the foregoing analysis with u_∞ = 5 m/s, the effect of T_f is tabulated below

T_f (K)	275	300	310
$\overline{h}_1$ (W/m²·K)	8.72	8.73	8.70
$\overline{h}_{2-3}$ (W/m²·K)	15.1	14.5	14.2
$\overline{h}_{9-10}$ (W/m²·K)	11.6	11.1	10.8

The overall effect of T_f on estimates for the average panel coefficient is slight, less than 5%.

PROBLEM 7.24

KNOWN: Design of an anemometer comprised of a thin metallic strip supported by stiff rods serving as electrodes for passage of heating current. Fine-wire thermocouple on trailing edge of strip.

FIND: (a) Relationship between electrical power dissipation per unit width of the strip in the transverse direction, P' (mW/mm), and airstream velocity u_∞ when maintained at constant strip temperature, T_s; show the relationship graphically; (b) The uncertainty in the airstream velocity if the accuracy with which the strip temperature can be measured and maintained constant is ±0.2°C; (c) Relationship between strip temperature and airstream velocity u_∞ when the strip is provided with a constant power, P' = 30 mW/mm; show the relationship graphically. Also, find the uncertainty in the airstream velocity if the accuracy with which the strip temperature can be measured is ±0.2°C; (d) Compare features associated with each of the operating nodes.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Strip has uniform temperature in the midspan region of the strip, (4) Negligible conduction in the transverse direction in the midspan region, and (5) Airstream over strip approximates parallel flow over two sides of a smooth flat plate.

ANALYSIS: (a) In the midspan region of uniform temperature T_s with no conduction in the transverse direction, all the dissipated electrical power is transferred by convection to the airstream,

$$P' = 2\bar{h}_L L\left(T_s - T_\infty\right) \qquad (1)$$

where P' is the power per unit width (transverse direction). Using the *IHT Correlation Tool* for *External Flow-Flat Plate* the power as a function of airstream velocity was determined and is plotted below. The IHT tool uses the flat plate correlation, Eq. 7.31 since the flow is laminar over this velocity range.

Continued...

PROBLEM 7.24 (Cont.)

(b) By differentiation of Eq. (1), the relative uncertainties of the convection coefficient and strip temperature are, assuming the power remains constant,

$$\frac{\Delta \bar{h}_L}{\bar{h}} = -\frac{\Delta T_s}{T_s - T_\infty} \qquad (2)$$

Since the flow was laminar for the range of airstream velocities, Eq. 7.31,

$$\bar{h}_L \sim u_\infty^{1/2} \qquad \text{or} \qquad \frac{\Delta \bar{h}_L}{\bar{h}_L} = 0.5\frac{\Delta u_\infty}{u_\infty} \qquad (3)$$

Hence, the relative uncertainty in the air velocity due to uncertainty in T_s, $\Delta T_s = \pm 0.2^\circ C$

$$\frac{\Delta u_\infty}{u_\infty} = 2\frac{\Delta T_s}{T_s - T_\infty} = 2\frac{\pm 0.2^\circ C}{(35-25)^\circ C} = \pm 4\% \qquad (4)$$ <

(c) Using the IHT workspace setting $P' = 30$ mW/mm, the strip temperature T_s as a function of the airstream velocity was determined and plotted. Note that the slope of the T_s vs. u_∞ curve is steep for low velocities and relatively flat for high velocities. That is, the technique is more sensitive at lower velocities. Using Eq. (4), but with T_s dependent upon u_∞, the relative uncertainty in u_∞ can be determined.

(d) For the constant power mode of operation, part (a), the uncertainty in u_∞ due to uncertainty in temperature measurement was found as 4%, independent of the magnitude u_∞. For the constant-temperature mode of operation, the uncertainty in u_∞ is less than 4% for velocities less than 30 m/s, with a value of 1% around 2 m/s. However, in the upper velocity range, the error increases to 5%.

PROBLEM 7.25

KNOWN: Plate dimensions and initial temperature. Velocity and temperature of air in parallel flow over plates.

FIND: Initial rate of heat transfer from plate. Rate of change of plate temperature.

SCHEMATIC:

ASSUMPTIONS: (1) Negligible radiation, (2) Negligible effect of conveyor velocity on boundary layer development, (3) Plates are isothermal, (4) Negligible heat transfer from sides of plate, (5) $Re_{x,c} = 5\times10^5$, (6) Constant properties.

PROPERTIES: *Table A-1*, AISI 1010 steel (573K): k_p = 49.2 W/m·K, c = 549 J/kg·K, ρ = 7832 kg/m^3. *Table A-4*, Air (p = 1 atm, T_f = 433K): ν = 30.4 × 10^{-6} m^2/s, k = 0.0361 W/m·K, Pr = 0.688.

ANALYSIS: The initial rate of heat transfer from a plate is

$$q = 2\bar{h}\,A_s\left(T_i - T_\infty\right) = 2\bar{h}\,L^2\left(T_i - T_\infty\right)$$

With $Re_L = u_\infty L/\nu = 10\,m/s \times 1m/30.4\times10^{-6}m^2/s = 3.29\times10^5$, flow is laminar over the entire surface and

$$\overline{Nu}_L = 0.664\,Re_L^{1/2}\,Pr^{1/3} = 0.664\left(3.29\times10^5\right)^{1/2}\left(0.688\right)^{1/3} = 336$$

$$\bar{h} = \left(k/L\right)\overline{Nu}_L = \left(0.0361\,W/m\cdot K/1m\right)336 = 12.1\,W/m^2\cdot K$$

Hence,

$$q = 2\times12.1\,W/m^2\cdot K\left(1m\right)^2\left(300-20\right)°C = 6780\,W$$ <

Performing an energy balance at an instant of time for a control surface about the plate, $-\dot{E}_{out} = \dot{E}_{st}$, we obtain (Eq. 5.2),

$$\rho\,\delta L^2 c\left.\frac{dT}{dt}\right|_i = -\bar{h}\,2L^2\left(T_i - T_\infty\right)$$

$$\left.\frac{dT}{dt}\right|_i = -\frac{2\left(12.1\,W/m^2\cdot K\right)\left(300-20\right)°C}{7832\,kg/m^3\times0.006m\times549\,J/kg\cdot K} = -0.26°C/s$$ <

COMMENTS: (1) With $Bi = \bar{h}\left(\delta/2\right)/k_p = 7.4\times10^{-4}$, use of the lumped capacitance method is appropriate. (2) Despite the large plate temperature and the small convection coefficient, if adjoining plates are in close proximity, radiation exchange with the surroundings will be small and the assumption of negligible radiation is justifiable.

PROBLEM 7.26

KNOWN: Velocity, initial temperature, and dimensions of aluminum strip on a production line. Velocity and temperature of air in counter flow over top surface of strip.

FIND: (a) Differential equation governing temperature distribution along the strip and expression for outlet temperature, (b) Value of outlet temperature for prescribed conditions.

SCHEMATIC:

ASSUMPTIONS: (1) Negligible variation of sheet temperature across its thickness, (2) Negligible effect of conduction along length (x) of sheet, (3) Negligible radiation, (4) Turbulent flow over entire top surface, (5) Negligible effect of sheet velocity on boundary layer development, (6) Negligible heat transfer from bottom surface and sides, (7) Constant properties.

PROPERTIES: *Table A-1*, Aluminum, 2024-T6 $(\overline{T}_{AL} \approx 500\text{K})$: $\rho = 2770\,\text{kg/m}^3$, $c_p = 983\,\text{J/kg}\cdot\text{K}$, k=186 W/m·K. *Table A-4*, Air $(p = 1\,\text{atm},\ T_f \approx 400\text{K})$: $\nu = 26.4\times10^{-6}\,\text{m}^2/\text{s}$, $k = 0.0338\,\text{W/m}\cdot\text{K}$, $\text{Pr} = 0.69$

ANALYSIS: (a) Applying conservation of energy to a stationary control surface, through which the sheet moves, steady-state conditions exist and $\dot{E}_{in} - \dot{E}_{out} = 0$. Hence, with *inflow* due to *advection* and outflow due to *advection* and *convection*,

$$\rho V A_c c_p (T + dT) - \rho V A_c c_p T - dq = 0$$

$$+\rho V \delta W c_p dT - h_x (dx \cdot W)(T - T_\infty) = 0$$

$$\frac{dT}{dx} = +\frac{h_x}{\rho V \delta c_p}(T - T_\infty) \qquad (1) \quad <$$

Alternatively, if the control surface is fixed to the sheet, conditions are transient and the energy balance is of the form, $-\dot{E}_{out} = \dot{E}_{st}$, or

$$-h_x (dx \cdot W)(T - T_\infty) = \rho (dx \cdot W \cdot \delta) c_p \frac{dT}{dt}$$

$$\frac{dT}{dt} = -\frac{h_x}{\rho \delta c_p}(T - T_\infty)$$

Dividing the left- and right-hand sides of the equation by dx/dt and dx/dt = - V, respectively, equation (1) is obtained. The equation may be integrated from x = 0 to x = L to obtain

$$\int_{T_o}^{T_i} \frac{dT}{T - T_\infty} = \frac{L}{\rho V \delta c_p}\left[\frac{1}{L}\int_0^L h_x\, dx\right]$$

Continued …..

PROBLEM 7.26 (Cont.)

where $h_x = (k/x)0.0296\,Re_x^{4/5}\,Pr^{1/3}$ and the bracketed term on the right-hand side of the equation reduces to $\overline{h}_L = (k/L)0.037\,Re_L^{4/5}\,Pr^{1/3}$.

Hence,

$$\ln\left(\frac{T_i - T_\infty}{T_o - T_\infty}\right) = \frac{L\,\overline{h}_L}{\rho V \delta c_p}$$

$$\frac{T_o - T_\infty}{T_i - T_\infty} = \exp\left(-\frac{L\,\overline{h}_L}{\rho V \delta c_p}\right)$$ <

(b) For the prescribed conditions, $Re_L \approx u_\infty L/\nu = 20\,m/s \times 5m / 26.4 \times 10^{-6} m^2/s = 3.79 \times 10^6$ and

$$\overline{h}_L = \left(\frac{0.0338\,W/m\cdot K}{5m}\right)0.037\left(3.79\times10^6\right)^{4/5}(0.69)^{1/3} = 40.5\,W/m^2\cdot K$$

$$T_o = 20°C + (280°C)\exp\left(-\frac{5m \times 40.5\,W/m^2\cdot K}{2770\,kg/m^3 \times 0.1\,m/s \times 0.002m \times 983\,J/kg\cdot K}\right) = 213°C$$ <

COMMENTS: (1) With $T_o = 213°C$, $\overline{T}_{Al} = 530K$ and $T_f = 411K$ are close to values used to determine the material properties, and iteration is not needed. (2) For a representative emissivity of $\varepsilon = 0.2$ and $T_{sur} = T_\infty$, the maximum value of the radiation coefficient is

$h_r = \varepsilon\sigma\,(T_i + T_{sur})\left(T_i^2 + T_{sur}^2\right) = 4.1\,W/m^2\cdot K << \overline{h}_L$. Hence, the assumption of negligible radiation is appropriate.

PROBLEM 7.27

KNOWN: Velocity, initial temperature, properties and dimensions of steel strip on a production line. Velocity and temperature of air in cross flow over top and bottom surfaces of strip. Temperature of surroundings.

FIND: (a) Differential equation governing temperature distribution along the strip, (b) Exact solution for negligible radiation and corresponding value of outlet temperature for prescribed conditions, (c) Effect of radiation on outlet temperature, and parametric effect of sheet velocity on temperature distribution.

SCHEMATIC:

ASSUMPTIONS: (1) Negligible variation of sheet temperature across its width and thickness, (2) Negligible effect of conduction along length (x) of sheet, (3) Constant properties, (4) Radiation exchange between small surface (both sides of sheet) and large surroundings, (5) Turbulent flow over top and bottom surfaces of sheet, (6) Motion of sheet has a negligible effect on the convection coefficient, ($V << u_\infty$), (7) Negligible heat transfer from sides of sheet.

PROPERTIES: Prescribed. Steel: $\rho = 7850\,\text{kg/m}^3, c_p = 620\,\text{J/kg}\cdot\text{K}, \varepsilon = 0.70.$ Air: k = 0.044 W/m·K, $\nu = 4.5\times10^{-5}\,\text{m}^2/\text{s}$, Pr = 0.68.

ANALYSIS: (a) Applying conservation of energy to a stationary differential control surface, through which the sheet passes, conditions are steady and $\dot{E}_{in} - \dot{E}_{out} = 0$. Hence, with *inflow* due to *advection* and *outflow* due to *advection*, *convection* and *radiation*

$$\rho V A_c c_p T - \rho V A_c c_p (T + dT) - 2\,dq = 0$$

$$-\rho V \delta W c_p dT - 2(W dx)\left[\bar{h}_W (T - T_\infty) + \varepsilon\sigma\left(T^4 - T_{sur}^4\right)\right] = 0$$

$$\frac{dT}{dx} = -\frac{2}{\rho V \delta c_p}\left[\bar{h}_W (T - T_\infty) + \varepsilon\sigma\left(T^4 - T_{sur}^4\right)\right] \qquad (1) \quad <$$

Alternatively, if the control surface is fixed to the sheet, conditions are transient and the energy balance is of the form, $-\dot{E}_{out} = \dot{E}_{st}$, or

$$-2(W dx)\left[\bar{h}_W (T - T_\infty) + \varepsilon\sigma\left(T^4 - T_{sur}^4\right)\right] = \rho (W \delta\, dx) c_p \frac{dT}{dt}$$

$$\frac{dT}{dt} = -\frac{2}{\rho \delta c_p}\left[\bar{h}_W (T - T_\infty) + \varepsilon\sigma\left(T^4 - T_{sur}^4\right)\right]$$

Dividing the left- and right-hand sides of the equation by dx/dt and V = dx/dt, respectively, Eq. (1) is obtained.

(b) Neglecting radiation, separating variables and integrating, Eq. (1) becomes

$$\int_{T_i}^{T} \frac{dT}{T - T_\infty} = -\frac{2\bar{h}_W}{\rho V \delta c_p}\int_0^x dx$$

Continued …..

PROBLEM 7.27 (Cont.)

$$\ln\left(\frac{T-T_\infty}{T_i-T_\infty}\right)=-\frac{2\bar{h}_W x}{\rho V \delta c_p}$$

$$T=T_\infty+\left(T_i-T_\infty\right)\exp\left(-\frac{2\bar{h}_W x}{\rho V \delta c_p}\right) \qquad (2) \quad <$$

With $Re_W = u_\infty W/\nu = 20\,m/s \times 1m / 4\times10^{-5} m^2/s = 5\times10^5$, the correlation for turbulent flow over a flat plate yields

$$\overline{Nu}_W = 0.037\,Re_W^{4/5}\,Pr^{1/3} = 0.037\left(5\times10^5\right)^{4/5}(0.68)^{1/3} = 1179$$

$$\bar{h}_W = \frac{k}{W}\overline{Nu}_W = \frac{0.044\,W/m\cdot K}{1m}1179 = 51.9\,W/m^2\cdot K$$

Hence, applying Eq. (2) at x = L = 10m,

$$T_o = 20°C + (480°C)\exp\left(-\frac{2\times51.9\,W/m^2\cdot K\times10m}{7850\,kg/m^3\times0.1m/s\times0.003m\times620\,J/kg\cdot K}\right) = 256°C \quad <$$

(c) Using the DER function of IHT, Eq. (1) may be numerically integrated from x = 0 to x = L = 10m to obtain

$$T_o = 210°C \quad <$$

Contrasting this result with that of Part (b), it's clear that radiation makes a discernable contribution to cooling of the sheet. IHT was also used to determine the effect of the sheet velocity on the temperature distribution.

The sheet velocity has a significant influence on the temperature distribution. The temperature decay decreases with increasing V due to the increasing effect of advection on energy transfer in the x direction.

COMMENTS: (1) A critical parameter in the production process is the *coiling temperature*, that is, the temperature at which the wire may be safely coiled for subsequent storage or shipment. The larger the production rate (V), the longer the cooling distance needed to achieve a desired coiling temperature. (2) Cooling may be enhanced by increasing the cross stream velocity u_∞.

PROBLEM 7.28

KNOWN: Length, thickness, speed and temperature of steel strip.

FIND: Rate of change of strip temperature 1 m from leading edge and at trailing edge. Location of minimum cooling rate.

SCHEMATIC:

ASSUMPTIONS: (1) Constant properties, (2) Negligible radiation, (3) Negligible longitudinal conduction in strip, (4) Critical Reynolds number is 5×10^5.

PROPERTIES: Steel (given): ρ = 7900 kg/m^3, c_p = 640 J/kg·K. *Table A-4*, Air $\left(\overline{T} = 750\text{K, 1 atm}\right)$: $\nu = 76.4 \times 10^{-6}$ m^2/s, k = 0.0549 W/m·K, Pr = 0.702.

ANALYSIS: Performing an energy balance for a control mass of unit surface area A_s riding with the strip,

$$-\dot{E}_{out} = dE_{st}/dt$$

$$-2h_x A_s\left(T - T_\infty\right) = rd A_s c_p\left(dT/dt\right)$$

$$dT/dt) = \frac{-2h_x\left(T - T_\infty\right)}{rd c_p} = -\frac{2(900\text{K})h_x}{7900\ \text{kg/m}^3(0.003\ \text{m})640\ \text{J/kg}\cdot\text{K}} = -0.119 h_x\ (\text{K/s}).$$

At x = 1 m, $\quad Re_x = \frac{Vx}{n} = \frac{20\ \text{m/s}\,(1\text{m})}{76.4\times10^{-6}\ \text{m}^2/\text{s}} = 2.62\times10^5 < Re_{x,c}$. Hence,

$$h_x = (k/x)0.332 Re_x^{1/2} Pr^{1/3} = \frac{0.0549\ \text{W/m}\cdot\text{K}}{1\ \text{m}}(0.332)\left(2.62\times10^5\right)^{1/2}(0.702)^{1/3} = 8.29\text{W/m}^2\cdot\text{K}$$

and at x = 1m, $\quad$ dT/dt) = -0.987 K/s. <

At the trailing edge, $Re_x = 2.62\times10^7 > Re_{x,c}$. Hence

$$h_x = (k/x)0.0296 Re_x^{4/5} Pr^{1/3} = \frac{0.0549\ \text{W/m}\cdot\text{K}}{100\ \text{m}}(0.0296)\left(2.62\times10^7\right)^{4/5}(0.702)^{1/3} = 12.4\text{W/m}^2\cdot\text{K}$$

and at x = 100 m, $\quad$ dT/dt) = -1.47 K/s. <

The minimum cooling rate occurs just before transition; hence, for $Re_{x,c} = 5\times10^5$

$$x_c = 5\times10^5\left(n/V\right) = \frac{5\times10^5\times76.4\times10^{-6}\text{m}^2/\text{s}}{20\ \text{m/s}} = 1.91\ \text{m}$$ <

COMMENTS: The cooling rates are very low and would remain low even if radiation were considered. For this reason, hot strip metals are quenched by water and not by air.

PROBLEM 7.29

KNOWN: Finned heat sink used to cool a power diode.

FIND: (a) Operating temperature T_d of the diode for prescribed conditions, (b) Options for reducing T_d.

SCHEMATIC:

Contact resistance, $R''_{t,c} = 10^{-5}$ m^2 °C/W, $D = 5$ mm

5 mm

$L_f(fin) = 10$ mm

1.5 mm t

10.5 mm

Diode, $R_{t,i} = 0.8$ °C/W, 5W

Air, $T_\infty = 17$ °C, $u_\infty = 10$ m/s

L_p(plate) = 20 mm

$q = 5$ W, T_d, $R_{t,i} = 0.8$ °C/W, $R_{t,c}$, $R_{t,s}$, T_b, $R_{t,o}$, $T_\infty = 17$ °C

ASSUMPTIONS: (1) All diode power is rejected from the four fins, (2) Diode behaves as an isothermal disk on a semi-infinite medium, (3) Fin tips are adiabatic, (4) Fins behave as flat plates with regard to forced convection (boundary layer thickness between fins is less than 1.5 mm/2), (5) Negligible heat loss from fin edges and prime (exposed base) surfaces.

PROPERTIES: *Table A-1*, Aluminum alloy 2024 ($\bar{T} \approx 300$ K): k = 177 W/m·K; *Table A-4*, Air (T_f = $(T_s + T_\infty)/2$ ≈ 300 K, 1 atm): $\nu = 15.89 \times 10^{-6}$ m^2/s, k = 0.0263 W/m·K.

ANALYSIS: (a) From the thermal circuit for the system, $T_d = T_\infty + qR_{tot}$ where $R_{tot} = R_{t,i} + R_{t,c} + R_{t,s} + R_{t,o}$.

Thermal contact resistance, $R_{t,c}$:

$$R_{t,c} = R''_{t,c}/A_d = 10^{-5}\text{m}^2\cdot{}^\circ\text{C/W}\Big/(\pi/4)(0.005\,\text{m})^2 = 0.509^\circ\,\text{C/W}.$$

Spreading thermal resistance, $R_{t,s}$: This resistance is due to conduction between the diode (an isothermal disk) and the heat sink (semi-infinite medium). From Table 4.1, the conduction shape factor is S = 2D. Hence,

$$R_{t,s} = 1/k(2D) = 1/177\,\text{W/m}\cdot\text{K}(2\times 0.005\,\text{m}) = 0.565^\circ\,\text{C/W}.$$

Thermal resistance of the fin array, $R_{t,o}$: From Table 3.4 for the fin with insulated tip,

$$R_{t,f} = \frac{\theta_b}{q_f} = \frac{1}{M^*\cdot\tanh(mL_f)}$$

where

$$m^2 = \left(\bar{h}P/kA_c\right) \qquad M^* = \left(\bar{h}PkA_c\right)^{1/2}.$$

To estimate the average heat transfer coefficient, consider the fin as a flat plate in parallel flow along the length, L_p = 20 mm. The Reynolds number is

$$Re_L = \frac{u_\infty L_p}{\nu} = \frac{10\,\text{m/s}\times 0.020\,\text{m}}{15.89\times 10^{-6}\,\text{m}^2/\text{s}} = 1.259\times 10^4.$$

Continued...

PROBLEM 7.29 (Cont.)

The flow is laminar, in which case

$$\overline{Nu}_L = \frac{\bar{h}L}{k} = 0.664\, Re_L^{1/2}\, Pr^{1/3}$$

$$\bar{h} = \frac{0.0263\, W/m\cdot K}{0.020\, m} \times 0.664 \left(1.259\times 10^4\right)^{1/2} (0.707)^{1/3} = 87.3\, W/m^2\cdot K\, .$$

With $P = (2t + 2L_p) = 0.046$ and $A_c = tL_p = 3 \times 10^{-5}\ m^2$,

$$m = \left[87.3\, W/m^2\cdot K \times 0.046\, m / 177\, W/m\cdot K \times 3\times 10^{-5}\, m^2\right]^{1/2} = 27.52\, m^{-1}$$

$$M^* = \left[87.3\, W/m^2\cdot K \times 0.046\, m \times 177\, W/m\cdot K \times 3\times 10^{-5}\right]^{1/2} = 0.146\, W/K\, .$$

Hence, with $L_f = 10$ mm,

$$R_{t,f} = 1/0.146\, W/K \times \tanh\left(27.52\, m^{-1}\times 0.010\, m\right) = 25.51^\circ\, C/W\, .$$

With $R_{t,o} \approx R_{t,f}/4$, the diode temperature is

$$T_d = 17^\circ C + 5\, W\left[0.80 + 0.509 + 0.565 + 0.25(27.3)\right]^\circ C/W \approx 58^\circ C$$ <

(b) The IHT *Extended Surfaces* Model for an *Array of Straight, Rectangular Fins* was used with the *External Flow, Flat Plate* option from the *Correlations* Tool Pad to assess the effects of varying u_∞ and L_f.

Clearly, there are benefits to increasing both quantities, with T_d reduced from approximately 58°C (u_∞ = 10 m/s, L_f = 10 mm) to 47.2°C (u_∞ = 25 m/s, L_f = 10 mm) to 38.5°C (u_∞ = 25 m/s, L_f = 20 mm). For u_∞ = 25 m/s and L_f = 20 mm, the fin efficiency remains large (η_f = 0.87), suggesting that, air flow and space limitations permitting, significant reduction, in T_d could still be gained by going to even larger values of L_f.

Subject to the constraint that the spacing between fins between remain at or larger than 1.5 mm, there is no advantage to reducing the fin thickness. For a thickness of 0.5 mm, it would be possible to add only one more fin (N = 5), yielding T_d = 44.4°C for u_∞ = 25 m/s and L_f = 20 mm.

COMMENTS: Note that the fin resistance makes the dominant contribution to the total resistance. Hence, efforts to reduce the total resistance should focus on reducing the fin resistance.

PROBLEM 7.30

KNOWN: Dimensions of aluminum heat sink. Temperature and velocity of coolant (water) flow through the heat sink. Power dissipation of electronic package attached to the heat sink.

FIND: Base temperature of heat sink.

SCHEMATIC:

ASSUMPTIONS: (1) Average convection coefficient association with flow over fin surfaces may be approximated as that for a flat plate in parallel flow, (2) All of the electric power is dissipated by the heat sink, (3) Transition Reynolds number of $Re_{x,c} = 5 \times 10^5$, (4) Constant properties.

PROPERTIES: Given. Aluminum: $k_{hs} = 180$ W/m·K. Water: $k_w = 0.62$ W/m·K, $\nu = 7.73 \times 10^{-7}$ m^2/s, Pr = 5.2.

ANALYSIS: From the thermal circuit,

$$q = P_{elec} = \frac{T_b - T_\infty}{R_b + R_{t,o}}$$

where $R_b = L_b / k_{hs}(w_1 \times w_2) = 0.02\text{m}/180\,\text{W/m}\cdot\text{K}(0.11\text{m})^2 = 9.18\times10^{-3}\,\text{K/W}$ and, from Eqs. 3.102 and 3.103,

$$R_{t,o} = \left\{\bar{h} A_t \left[1 - \frac{NA_f}{A_t}(1-\eta_f)\right]\right\}^{-1}$$

The fin and total surface area of the array are $A_f = 2w_2(L_f + t/2) = 0.22\text{m}(0.065\text{m}) = 0.0143\text{m}^2$ and
$A_t = NA_f + A_b = NA_f + (N-1)(S-t)w_2 = 6(0.0143\text{m}^2) + 5(0.01\text{m})0.11\text{m} = (0.0858 + 0.0055) = 0.0913\text{m}^2$.

With $Re_{w_2} = u_\infty w_2/\nu = 3\,\text{m/s}\times0.11\text{m}/7.73\times10^{-7}\,\text{m}^2/\text{s} = 4.27\times10^5$, laminar flow may be assumed over the entire surface. Hence

$$\bar{h} = \left(\frac{k_w}{w_2}\right)0.664\,Re_{w_2}^{1/2}\,Pr^{1/3} = \left(\frac{0.62\,\text{W/m}\cdot\text{K}}{0.11\text{m}}\right)0.664\left(4.27\times10^5\right)^{1/2}(5.2)^{1/3} = 4236\,\text{W/m}^2\cdot\text{K}$$

With $m = (2\bar{h}/k_{hs}t)^{1/2} = \left(8472\,\text{W/m}^2\cdot\text{K}/180\,\text{W/m}\cdot\text{K}\times0.01\text{m}\right)^{1/2} = 68.6\,\text{m}^{-1}$, $mL_c = 68.6\,\text{m}^{-1}$ $(0.065\text{m}) = 4.46$ and $\tanh mL_c = 0.9997$, Eq. 3.89 yields

$$\eta_f = \frac{\tanh mL_c}{mL_c} = \frac{0.9997}{4.46} = 0.224$$

Continued …..

Hence,

$$R_{t,o} = \left\{4236\,W/m^2 \cdot K \times 0.0913\,m^2\left[1-\frac{0.0858\,m^2}{0.0913\,m^2}(0.776)\right]\right\}^{-1} = 9.55\times10^{-3}\,K/W \qquad <$$

and

$$T_b = T_\infty + P_{elec}\left(R_b + R_{t,o}\right) = 17°C + 1200\,W\left(0.0187\,K/W\right) = 39.5°C \qquad <$$

COMMENTS: (1) The boundary layer thickness at the trailing edge of the fin is $\delta = 5w_2/\left(Re_{w_2}\right)^{1/2} = 0.84\text{ mm} << (S-t)$. Hence, the assumption of parallel flow over a flat plate is reasonable. (2) If a finned heat sink is not employed and heat transfer is simply by convection from the $w_2 \times w_2$ base surface, the corresponding convection resistance would be 0.0195 K/W, which is only twice the resistance associated with the fin array. The small enhancement by the array is attributable to the large value of $\bar{h}$ and the correspondingly small value of η_f. Were a fluid such as air or a dielectric liquid used as the coolant, the much smaller thermal conductivity would yield a smaller $\bar{h}$, a larger η_f and hence a larger effectiveness for the array.

PROBLEM 7.31

KNOWN: Plate dimensions and freestream conditions. Maximum allowable plate temperature.

FIND: (a) Maximum allowable power dissipation for electrical components attached to bottom of plate, (b) Effect of air velocity and fins on maximum allowable power dissipation.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Negligible heat loss form sides and bottom, (4) Transition Reynolds number is 5×10^5, (5) Isothermal plate.

PROPERTIES: *Table A.1*, Aluminum (T ≈ 350 K): k ≈ 240 W/m·K; *Table A.4*, Air (T_f = 325 K, 1 atm): $\nu = 18.4 \times 10^{-6}$ m^2/s, k = 0.028 W/m·K, Pr = 0.70.

ANALYSIS: (a) The heat transfer from the plate by convection is

$$P_{elec} = q = \bar{h} A_s \left(T_s - T_\infty\right).$$

For u_∞ = 15 m/s,

$$Re_L = \frac{u_\infty L}{\nu} = \frac{15\,\text{m/s} \times 1.2\,\text{m}}{18.41 \times 10^{-6}\,\text{m}^2/\text{s}} = 9.78 \times 10^5 > Re_{x,c}.$$

Hence, transition occurs on the plate and

$$\overline{Nu}_L = \left(0.037\,Re_L^{4/5} - 871\right)Pr^{1/3} = \left[0.037\left(9.78 \times 10^5\right)^{4/5} - 871\right]\left(0.70\right)^{1/3} = 1263$$

$$\bar{h} = Nu_L \frac{k}{L} = 1263 \frac{0.028\,\text{W/m} \cdot \text{K}}{1.2\,\text{m}} = 29.7\,\text{W/m}^2 \cdot \text{K}$$

The heat rate is

$$q = 29.7\,\text{W/m}^2 \cdot \text{K}\left(1.2\,\text{m}\right)^2\left(350 - 300\right)\text{K} = 2137\,\text{W}.$$ <

(b) The effect of the freestream velocity was considered by combining the *Correlations* Toolpad for the average coefficient associated with flow over a flat plate with the *Explore* and *Graph* options of IHT.

Continued...

PROBLEM 7.31 (Cont.)

The effect of increasing u_∞ is significant, particularly following transition at $u_\infty \approx 7.7$ m/s. A maximum heat rate of q = 3876 W is obtained for u_∞ = 25 m/s, which corresponds to $\bar{h} \approx 54$ W/m^2·K and $Re_L = 1.63 \times 10^6$.

The *Extended Surfaces* Model for an *Array* of *Straight Rectangular Fins* was used with the *Correlations* Toolpad to determine the effect of adding fins, and a copy of the program is appended. With L_f = 25 mm, w = 1.2 m, t = 0.005 m, S = 0.015 m, N = 80 and u_∞ = 25 m/s, the solution yields

$$q = 16{,}480 \text{ W}$$ <

which is more than a four-fold increase relative to the unfinned case.

COMMENTS: (1) With a fin efficiency of $\eta_f = 0.978$, there is significant latitude for yet further enhancement in heat transfer, as, for example, by increasing the fin length, L_f.

(2) The *IHT* code below includes the model for the *Extended Surface, Array of Straight Fins* and the *Correlation* for the convection coefficient of a flat plate with mixed flow conditions.

```
/* Fin analysis results, uinf = 25 m/s
Ab      Acb      Af      Ap          At      Aw      etaf    etaoc   m       qt      R"tc
0.96    0.006    0.066   0.0001375           6.24    1.44    0.978   0.9814  9.471
        1.648E4  0       */

/* Correlation results and air thermophysical properties at Tf
NuLbar  Pr       ReL     Tf      hLbar   k        nu        uinf
2294    0.7035   1.63E6  325     53.82   0.02815  1.841E-5  25      */

// IHT Model, Extended Surfaces, Array of Straight Rectangular Fins
/* Model: Fin array with straight fins of rectangular profile, thickness t, width w and length L. Array has N
fins with spacing S. */

/* Find: Array heat rate and performance parameters */

/* Assumptions:(1) Steady-state conditions, (2) One-dimensional conduction along the fin, (3) Constant
properties, (4) Negligible radiation exchange with surroundings, (5) Uniform convection coefficient over
fins and base, (6) Insulated tip, Lc = L + t / 2 */

// The total heat rate for the array
qt = (Tb - Tinf) / (Rtoc)  // Eq 3.104
/* where the fin array thermal resistance, including thermal contact resistance, R"tc,  at the fin base is */
Rtoc = 1 / (etaoc * h * At)

// The overall surface efficiency is
etaoc = 1 - (N * Af / At) * (1 - etaf / C1)          // Eq 3.105
C1 = 1 + etaf * h * Af * (R"tc / Acb)
// where N is the total number of fins, and the surface area of a single fin is
Af = 2 * w * Lc
// where the equivalent length, accounting for the adiabatic tip, is
Lc = Lf + (t / 2)

/* The surface area associated with the fins and the exposed portion of the base (referred to also as the
prime surface, Ab) is */
At = N * Af + Ab
Ab = Aw - N * Acb

// The total area of the base surface follows from the schematic
Aw = w * N * S
// where S is the fin spacing. The base area for a single fin is
Acb = t * w

// The fin efficiency for a single fin is:
etaf = (tanh(m * Lc)) / (m * Lc)                      // Eq 3.89
// where
m = sqrt(2 * h / (kf * t))
```

PROBLEM 7.31 (Cont.)

```
/* The input (independent) values for this system are:
Fin characteristics */
Tb =   350                      // base temperature, K
t =    0.005                    // thickness, m
w =  1.2                        // spacing width, m
Lf =  0.025                     // length, m
S =   0.015                     // fin spacing, m
N =   80                        // number of fins
kf =   240                      // thermal conductivity, W/m·K

// Convection conditions
Tinf =  300                     // fluid temperature, K
h =   hLbar                     // convection coefficient,W/m^2·K

/* Thermal contact resistance per unit area at fin base. Set equal to zero if not present. */
R''tc =   0               // thermal resistance per unit area, K·m^2/W
```

// Correlation, External flow, Flate Plate, Laminar or Mixed Flow

```
NuLbar = NuL_bar_EF_FP_LM(ReL,Rexc,Pr)     // Eq 7.31, 7.39, 7.40
NuLbar = hLbar * L / k
ReL = uinf * L / nu
Rexc = 5.0E5
// Evaluate properties at the film temperature, Tf.
Tf = (Tinf + Tb) / 2
/*  Correlation description: Parallel external flow (EF) over a flat plate (FP), average coefficient; laminar (L)
if ReL<Rexc, Eq 7.31; mixed (M) if ReL>Rexc, Eq 7.39 and 7.40; 0.6<=Pr<=60. See Table 7.9.  */
```

// Air property functions : From Table A.4

```
// Units: T(K); 1 atm pressure
nu = nu_T("Air",Tf)             // Kinematic viscosity, m^2/s
k = k_T("Air",Tf)               // Thermal conductivity, W/m·K
Pr = Pr_T("Air",Tf)             // Prandtl number
```

// Input variables, correlation

```
uinf = 25                       // freestream velocity, m/s
L = 1.2                         // plate width, m
```

PROBLEM 7.32

KNOWN: Operating power of electrical components attached to one side of copper plate. Contact resistance. Velocity and temperature of water flow on opposite side.

FIND: (a) Plate temperature, (b) Component temperature.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Negligible heat loss from sides and bottom, (4) Turbulent flow throughout.

PROPERTIES: Water (given): $\nu = 0.96 \times 10^{-6}$ m^2/s, k = 0.620 W/m·K, Pr = 5.2.

ANALYSIS: (a) From the convection rate equation,

$$T_s = T_\infty + q/\bar{h}A$$

where $q = Nq_c$ = 2500 W and $A = L^2 = 0.04$ m^2. The convection coefficient is given by the turbulent flow correlation

$$\bar{h} = \overline{Nu}_L\,(k/L) = 0.037 Re_L^{4/5} Pr^{1/3} (k/L)$$

where

$$Re_L = (u_\infty L/n) = (2\text{ m/s} \times 0.2\text{m})/0.96 \times 10^{-6}\text{m}^2/\text{s} = 4.17 \times 10^5$$

and hence

$$\bar{h} = 0.037\left(4.17 \times 10^5\right)^{4/5}(5.2)^{1/3}(0.62\text{ W/m}\cdot\text{K}/0.2\text{ m}) = 6228\text{ W/m}^2\cdot\text{K}.$$

The plate temperature is then

$$T_s = 17^\circ\text{C} + 2500\text{ W}/\left(6228\text{ W/m}^2\cdot\text{K}\right)(0.20\text{ m})^2 = 27^\circ\text{C}.$$ <

(b) For an individual component, a rate equation involving the component's contact resistance can be used to find its temperature,

$$q_c = (T_c - T_s)/R_{t,c} = (T_c - T_s)/\left(R''_{t,c}/A_c\right)$$

$$T_c = T_s + q_c R''_{t,c}/A_c = 27^\circ\text{C} + 25\text{ W}\left(2 \times 10^{-4}\text{ m}^2\cdot\text{K/W}\right)/10^{-4}\text{ m}^2$$

$$T_c = 77^\circ\text{C}.$$ <

COMMENTS: With $Re_L = 4.17 \times 10^5$, the boundary layer would be laminar over the entire plate without the boundary layer trip, causing T_s and T_c to be appreciably larger.

PROBLEM 7.33

KNOWN: Air at 27°C with velocity of 10 m/s flows turbulently over a series of electronic devices, each having dimensions of 4 mm × 4 mm and dissipating 40 mW.

FIND: (a) Surface temperature T_s of the fourth device located 15 mm from the leading edge, (b) Compute and plot the surface temperatures of the first four devices for the range $5 \le u_\infty \le 15$ m/s, and (c) Minimum free stream velocity u_∞ if the surface temperature of the hottest device is not to exceed 80°C.

SCHEMATIC:

ASSUMPTIONS: (1) Turbulent flow, (2) Heat from devices leaving through top surface by convection only, (3) Device surface is isothermal, and (4) The average coefficient for the devices is equal to the local value at the mid position, i.e. $\bar{h}_4 = h_x(L)$.

PROPERTIES: *Table A.4*, Air (assume T_s = 330 K, $\bar{T} = (T_s + T_\infty)/2$ = 315 K, 1 atm): k = 0.0274 W/m·K, $\nu = 17.40 \times 10^{-6}$ m^2/s, $\alpha = 24.7 \times 10^{-6}$ m^2/s, Pr = 0.705.

ANALYSIS: (a) From Newton's law of cooling,

$$T_s = T_\infty + q_{conv}/\bar{h}_4 A_s \qquad (1)$$

where $\bar{h}_4$ is the average heat transfer coefficient over the 4th device. Since flow is turbulent, it is reasonable and convenient to assume that

$$\bar{h}_4 = h_x(L = 15\,\text{mm}). \qquad (2)$$

To estimate h_x, use the turbulent correlation evaluating thermophysical properties at $\bar{T}_f$ = 315 K (assume T_s = 330 K),

$$Nu_x = 0.0296\,Re_x^{4/5}\,Pr^{1/3}$$

where

$$Re_x = \frac{u_\infty L}{\nu} = \frac{10\,\text{m/s} \times 0.015\,\text{m}}{17.4 \times 10^{-6}\,\text{m}^2/\text{s}} = 8621$$

giving

$$Nu_x = \frac{h_x L}{k} = 0.0296(8621)^{4/5}(0.705)^{1/3} = 37.1$$

$$\bar{h}_4 = h_x = \frac{Nu_x k}{L} = \frac{37.1 \times 0.0274\,\text{W/m}\cdot\text{K}}{0.015\,\text{m}} = 67.8\,\text{W/m}^2\cdot\text{K}$$

Hence, with A_s = 4 mm × 4 mm, the surface temperature is

$$T_s = 300\,\text{K} + \frac{40 \times 10^{-3}\,\text{W}}{67.8\,\text{W/m}^2\cdot\text{K} \times (4 \times 10^{-3}\,\text{m})^2} = 337\,\text{K} = 64^\circ\text{C}.$$ <

Continued...

PROBLEM 7.33 (Cont.)

(b) The surface temperature for each of the four devices (i = 1, 2, 3 4) follows from Eq. (1),

$$T_{s,i} = T_\infty + q_{conv} / \bar{h}_i A_s \qquad (3)$$

For devices 2, 3 and 4, $\bar{h}_i$ is evaluated as the local coefficient at the mid-positions, Eq. (2), x_2 = 6.5 mm, x_3 = 10.75 mm and x_4 = 15 mm. For device 1, $\bar{h}_1$ is the average value 0 to x_1, where evaluated $x_1 = L_1$ = 4.25 mm. Using Eq. (3) in the *IHT Workspace* along with the *Correlations Tool*, *External Flow*, *Local Coefficient* for *Laminar* or *Turbulent Flow*, the surface temperatures $T_{s,i}$ are determined as a function of the free stream velocity.

(c) Using the *Explore* option on the *Plot Window* associated with the IHT code of part (b), the minimum free stream velocity of

$$u_\infty = 6.6 \text{ m/s}$$ <

will maintain device 4, the hottest of the devices, at a temperature $T_{s,4}$ = 80°C.

COMMENTS: (1) Note that the thermophysical properties were evaluated at a reasonable assumed film temperature in part (a).

(2) From the $T_{s,i}$ vs. u_∞ plots, note that, as expected, the surface temperatures of the devices increase with distance from the leading edge.

PROBLEM 7.34

KNOWN: Convection correlation for irregular surface due to electronic elements mounted on a circuit board experiencing forced air cooling with prescribed temperature and velocity

FIND: Surface temperature when heat dissipation rate is 30 mW for chip of prescribed area located a specific distance from the leading edge.

SCHEMATIC:

ASSUMPTIONS: (1) Situation approximates parallel flow over a flat plate with prescribed correlation, (2) Heat rate is from top surface of chip.

PROPERTIES: *Table A-4*, Air (assume $T_s \approx 45°C$, then $\overline{T} = (45 + 25)°C/2 \approx 310$ K, 1 atm): k = 0.027 W/m·K, $\nu = 16.90 \times 10^{-6}$ m^2/s, Pr = 0.706.

ANALYSIS: For the chip upper surface, the heat rate is

$$q_{chip} = \overline{h}_{chip} A_s \left(T_s - T_\infty\right) \qquad \text{or} \qquad T_s = T_\infty + q_{chip} / \overline{h}_{chip} A_s$$

Assuming the average convection coefficient over the chip length to be equal to the local value at the center of the chip ($x = x_o$), $\overline{h}_{chip} \approx h_x\left(x_o\right)$, where

$$Nu_x = 0.04 Re_x^{0.85} Pr^{0.33}$$

$$Nu_x = 0.04\left(10 \text{ m/s} \times 0.120 \text{ m}/16.90 \times 10^{-6} \text{ m}^2/\text{s}\right)^{0.85} \left(0.706\right)^{0.33} = 473.4$$

$$h_x = \frac{Nu_x k}{x_o} = \frac{473.4 \times 0.027 \text{ W/m}\cdot\text{K}}{0.120 \text{ m}} = 107 \text{ W/m}^2\cdot\text{K}$$

Hence,

$$T_s = 25°C + 30 \times 10^{-3} \text{ W}/107 \text{ W/m}^2\cdot\text{K} \times \left(4 \times 10^{-3} \text{m}\right)^2 = \left(25 + 17.5\right)° C = 42.5°C. \quad <$$

COMMENTS: (1) Note that the assumed value of $\overline{T}$ used to evaluate the thermophysical properties was reasonable. (2) We could have evaluated $\overline{h}_{chip}$ by two other approaches. In one case the average coefficient is approximated as the arithmetic mean of local values at the leading and trailing edges of the chip.

$$\overline{h}_{chip} \approx \left[h_{x2}\left(x_2\right) + h_{x1}\left(x_1\right)\right]/2 = 107 \text{ W/m}^2\cdot\text{K}.$$

The exact approach is of the form

$$\overline{h}_{chip} \cdot \ell = \overline{h}_{x2} \cdot x_2 - \overline{h}_{x1} \cdot x_1$$

Recognizing that $h_x \sim x^{-0.15}$, it follows that

$$\overline{h}_x = \frac{1}{x}\int_0^x h_x \cdot dx = 1.176 h_x$$

and $\overline{h}_{chip} = 108 \text{ W/m}^2\cdot\text{K}$. Why do results for the two approximate methods and the exact method compare so favorably?

PROBLEM 7.35

KNOWN: Air at atmospheric pressure and a temperature of 25°C in parallel flow at a velocity of 5 m/s over a 1-m long flat plate with a uniform heat flux of 1250 W/m^2.

FIND: (a) Plate surface temperature, $T_s(L)$, and local convection coefficient, $h_x(L)$, at the trailing edge, x = L, (b) Average temperature of the plate surface, $\overline{T}_s$, (c) Plot the variation of the plate surface temperature, $T_s(x)$, and the convection coefficient, $h_x(x)$, with distance on the same graph; explain key features of these distributions.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Flow is fully turbulent, and (3) Constant properties.

PROPERTIES: *Table A-4*, Air (assume T_f = 325 K, 1 atm): $\nu = 18.76 \times 10^{-6}$ m^2/s; k = 0.0284 W/m·K; Pr = 0.703

ANALYSIS: (a) At the trailing edge, x = L, the convection rate equation is

$$q_s'' = q_{cv}'' = h_x(L)\left[T_s(L) - T_\infty\right] \quad (1)$$

where the local convection coefficient, assuming turbulent flow, follows from Eq. 7.51.

$$Nu_x = \frac{h_x x}{k} = 0.0308\ Re_x^{4/5}\ Pr^{1/3} \quad (2)$$

With x = L = 1m, find

$$Re_x = u_\infty L/\nu = 5\ m/s \times 1\ m/18.76\times10^{-6}\ m^2/s = 2.67\times10^5$$

$$h_x(L) = (0.0284\ W/m\cdot K/1m)\times 0.0308\left(2.67\times10^5\right)^{4/5}(0.703)^{1/3} = 17.1\ W/m^2\cdot K$$

Substituting numerical values into Eq. (1),

$$T_s(L) = 25°C + 1250\ W/m^2/17.1\ W/m^2\cdot K = 98.3°C$$ <

(b) The average surface temperature $\overline{T}_s$ follows from the expression

$$\overline{T}_s - T_\infty = \frac{1}{L}\int_0^L (T_s - T_\infty)dx = \frac{q_s''}{L}\int_0^L \frac{x}{k\,Nu_x}dx \quad (3)$$

where Nu_x is given by Eq. (2). Using the *Integral* function in *IHT* as described in Comment (3) find

$$\overline{T}_s = 86.1°C.$$ <

(c) The variation of the plate surface temperature $T_s(x)$ and convection coefficient, $h_x(x)$, shown in the graph are calculated using Eqs. (1) and (2).

Continued …..

COMMENTS: (1) To avoid performing the integration of part (b), it is reasonable to use the approximate, simpler Eqs. 7.53a and integrating Eq. 7.51,

$$\overline{Nu}_L = 0.0385\ Re_L^{4/5}\, Pr^{1/3} = 0.0385\left(2.67\times10^5\right)^{4/5}(0.703)^{1/3} = 751$$

$$\bar{h}_L = \overline{Nu}_L\, k/L = 751\times0.0284\ W/m\cdot K/1\ m = 213\ W/m^2\cdot K$$

$$\bar{T}_s = T_\infty + \frac{q_s'' L}{k\,\overline{Nu}_L} = 83.6°C.$$

(2) The properties for the correlation should be evaluated at $T_f = \left(\bar{T}_s + T_\infty\right)/2$. From the foregoing analyses, T_f = (86.1 + 25)°/2 = 55.5°C = 329 K. Hence, the assumed value of 325 K was reasonable.

(3) The IHT code, excluding the input variables and air property functions, used to evaluate the integral of Eq. (3) and generate the graphs in part (c) is shown below.

```
/* Programming note:  when using the INTEGRAL function, the value of the independent variable
must not be specified as an input variable.  If done so, this error message will appear:
"Redefinition of a constant variable." */

// Turbulent flow correlation, Eq. 7.50, local values
Nu_x = 0.0308 * Re_x^0.8 * Pr^0.333
Nu_x = h_x * x / k
Re_x = uinf * x / nu

// Plate temperatures
// Local
Ts_x = Tinf + q''s / h_x
// Average
Ts_avg - Tinf = q''s / L * INTEGRAL (y,x)
delT_avg = Ts_avg - Tinf
y = x / (k * Nu_x)
```

PROBLEM 7.36

KNOWN: Conditions for airflow over isothermal plate with optional unheated starting length.

FIND: (a) local coefficient, h_x, at leading and trailing edges with and without an unheated starting length, $\xi = 1$ m.

SCHEMATIC:

$u_\infty = 2$ m/s
$T_\infty = 300$ K
Air
T_∞
$T_s = 350$ K
x
$\xi = 1$ m
2 m
x
$\xi = 1$ m
Unheated starting length
Heated length, $\xi = 0$

PROPERTIES: *Table A.4*, Air ($T_f = 325$ K, 1 atm): $\nu = 18.4 \times 10^{-6}$ m^2/s, Pr = 0.703, k = 0.0282 W/m·K.

ANALYSIS: (a) The Reynolds number at $\xi = 1$ m is

$$Re_\xi = \frac{u_\infty \xi}{\nu} = \frac{2\,\text{m/s} \times 1\,\text{m}}{18.4 \times 10^{-6}\,\text{m}^2/\text{s}} = 1.087 \times 10^5$$

If $Re_{x,c} = 5 \times 10^5$, flow is laminar over the entire plate (with or without the starting length). In general,

$$Nu_x = \frac{0.332\, Re_x^{1/2}\, Pr^{1/3}}{\left[1 - (\xi/x)^{3/4}\right]^{1/3}} \tag{1}$$

$$h_x = \frac{\left(0.332 k\, Pr^{1/3}\right) Re_x^{1/2}}{x\left[1 - (\xi/x)^{3/4}\right]^{1/3}} = 0.00832\,\text{W/m}\cdot\text{K} \frac{Re_x^{1/2}}{x\left[1 - (\xi/x)^{3/4}\right]^{1/3}}.$$

With Unheated Starting Length: Leading edge (x = 1 m): $Re_x = Re_\xi$, $\xi/x = 1$, $h_x = \infty$ <

Trailing Edge (x = 2 m): $Re_x = 2\, Re_\xi = 2.17 \times 10^5$, $\xi/x = 0.5$

$$h_x = 0.00832\,\text{W/m}\cdot\text{K} \frac{\left(2.17 \times 10^5\right)^{1/2}}{2\,\text{m}\left[1 - (0.5)^{3/4}\right]^{1/3}} = 2.61\,\text{W/m}^2\cdot\text{K}$$ <

Without Unheated Starting Length: Leading edge (x = 0): $h_x = \infty$ <

Trailing edge (x = 1 m): $Re_x = 1.087 \times 10^5$

$$h_x = 0.00832\,\text{W/m}\cdot\text{K} \frac{\left(1.087 \times 10^5\right)^{1/2}}{1\,\text{m}} = 2.74\,\text{W/m}^2\cdot\text{K}$$ <

(b) The average convection coefficient $\bar{h}_L$ for the two cases in the schematic are, from Eq. 6.6,

Continued...

PROBLEM 7.36 (Cont.)

$$\bar{h}_L = \frac{1}{L}\int_{\xi=0}^{L} h_x(x)\,dx \qquad (2)$$

where L is the x location at the end of the heated section. Substituting Eq. (1) into Eq. (2) and numerically integrate, the results are tabulated below:

ξ (m)	h_x (L)(W/m^2·K)	$\bar{h}_L$ (W/m^2·K)
0	2.74	5.41
1	2.61	4.22

<

(c) The variation of the local convection coefficient over the plate, with and without the unheated starting length, using Eq. (1) is shown below. The abscissa is x - ξ.

COMMENTS: (1) When the velocity and thermal boundary layers grow simultaneously (*without starting length*), we expect the local and average coefficients to be larger than when the velocity boundary layer is thicker (*with starting length*).

(2) When ξ = 0, $\bar{h}_L = 2h_L$, when ξ = 1, $\bar{h}_L < 2h_L$. From Eq. (7.49), $\bar{h}_L = 4.25\ \text{W/m}^2\cdot\text{K}$.

(3) The numerical integration of Eq. (2) was performed using the INTEGRAL (f,x) operation in IHT as shown in the Workspace below.

```
// Average Coefficient:
hbarL =  1 / (L - zeta ) * INTEGRAL (hx,x)

// Local Coefficient With Unheated Starting Length:
hx  = ( k / x) *  0.332 *  Rex^0.5 * Pr^0.3333 /  ( 1 - (zeta / x)^(3/4) )^(1/3)
Rex = uinf * x / nu

// Properties Tool - Air:
// Air property functions : From Table A.4
// Units: T(K); 1 atm pressure
nu = nu_T("Air",Tf)                    // Kinematic viscosity, m^2/s
k = k_T("Air",Tf)                      // Thermal conductivity, W/m·K
Pr = Pr_T("Air",Tf)                    // Prandtl number
Tf = 325                               // Film temperature, K

// Assigned Variables:
uinf =  2                              // Airstream velocity, m/s
x = 1                                  // Distance from leading edge, m
L = 2                                  // Full length of plate, m
zeta = 1                               // Starting length, m
xzeta = x - zeta                       // Difference
```

PROBLEM 7.37

KNOWN: Cover plate dimensions and temperature for flat plate solar collector. Air flow conditions.

FIND: (a) Heat loss with simultaneous velocity and thermal boundary layer development, (b) Heat loss with unheated starting length.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible radiation, (3) Boundary layer is not disturbed by roof-plate interface, (4) $Re_{x,c} = 5\times10^5$.

PROPERTIES: *Table A-4*, Air (T_f = 285.5K, 1 atm): $\nu = 14.6 \times 10^{-6}\ m^2/s$, k = 0.0251 W/m·K, Pr = 0.71.

ANALYSIS: (a) The Reynolds number for the plate of L = 1m is

$$Re_L = \frac{u_\infty L}{n} = \frac{2\ m/s \times 1m}{14.6\times10^{-6}\ m^2/s} = 1.37\times10^5 < Re_{x,c}.$$

For laminar flow

$$\overline{Nu}_L = 0.664\ Re_L^{1/2}\ Pr^{1/3} = 0.664\left(1.37\times10^5\right)^{1/2}(0.71)^{1/3} = 219.2$$

$$q = \frac{k}{L}\overline{Nu}_L\ A_s\left(T_s - T_\infty\right) = \frac{0.0251\ W/m\cdot K}{1m}219.2\left(2m^2\right)5^\circ C = 55\ W. \qquad <$$

(b) The Reynolds number for the roof and collector of length L = 3m is

$$Re_L = \frac{2\ m/s\times3m}{14.6\times10^{-6}\ m^2/s} = 4.11\times10^5 < Re_{x,c}.$$

Hence, laminar boundary layer conditions exist throughout and the heat rate is

$$q = \int_x^L q''\,dA = \left(T_s - T_\infty\right)0.332\left(\frac{u_\infty}{n}\right)^{1/2} Pr^{1/3} kW \int_x^L \frac{x^{-1/2}dx}{\left[1-(x/x)^{3/4}\right]^{1/3}}$$

$$q = \left(5^\circ C\right)0.332\left(\frac{2\ m/s}{14.6\times10^{-6}\ m^2/s}\right)^{1/2}(0.71)^{1/3}\ 0.0251\frac{W}{m\cdot K}2m\int_x^L \frac{x^{-1/2}dx}{\left[1-(x/x)^{3/4}\right]^{1/3}}$$

Using a numerical technique to evaluate the integral,

$$q = 27.50\int_2^3 \frac{x^{-1/2}dx}{\left[1-(2.0/x)^{3/4}\right]^{1/3}} = 27.50\times1.417 = 39\ W \qquad <$$

COMMENTS: Values of $\bar{h}$ with and without the unheated starting length are 3.9 and 5.5 $W/m^2\cdot K$. Prior development of the velocity boundary layer decreases $\bar{h}$.

PROBLEM 7.38

KNOWN: Surface dimensions for an array of 10 silicon chips. Maximum allowable chip temperature. Air flow conditions.

FIND: Maximum allowable chip electrical power (a) without and (b) with a turbulence promoter at the leading edge.

SCHEMATIC:

Air
$u_\infty = 40$ m/s
$T_\infty = 24°C$
p = 1 atm
10mm
$T_s \le 80°C$
10th chip
x
L=0.1m

ASSUMPTIONS: (1) Steady-state conditions, (2) Film temperature of 52°C, (3) Negligible radiation, (4) Negligible heat loss through insulation, (5) Uniform heat flux at chip interface with air, (6) $Re_{x,c} = 5\times10^5$.

PROPERTIES: *Table A-4*, Air (T_f = 325K, 1 atm): $\nu = 18.4 \times 10^{-6}$ m^2/s, k = 0.0282 W/m·K, Pr = 0.703.

ANALYSIS: $Re_L = u_\infty L/n = 40\text{ m/s}\times0.1\text{ m}/18.4\times10^{-6}\text{ m}^2/\text{s} = 2.174\times10^5$. Hence, flow is laminar over all chips without the promoter.

(a) For *laminar flow*, the minimum h_x exists on the last chip. Approximating the average coefficient for Chip 10 as the local coefficient at x = 95 mm, $\bar{h}_{10} = h_{x=0.095m}$.

$$\bar{h}_{10} = 0.453\frac{k}{x}Re_x^{1/2}Pr^{1/3}$$

$$Re_x = \frac{u_\infty x}{n} = \frac{40\text{ m/s}\times0.095\text{ m}}{18.4\times10^{-6}\text{ m}^2/\text{s}} = 2.065\times10^5$$

$$\bar{h}_{10} = 0.453\frac{0.0282\text{ W/m}\cdot\text{K}}{0.095}\left(2.065\times10^5\right)^{1/2}(0.703)^{1/3} = 54.3\text{W/m}^2\cdot\text{K}$$

$$q_{10} = \bar{h}_{10}A\left(T_s - T_\infty\right) = 54.3\frac{\text{W}}{\text{m}^2\cdot\text{K}}(0.01\text{ m})^2(80-24)^\circ\text{C} = 0.30\text{ W}.$$

Hence, if all chips are to dissipate the same power and T_s is not to exceed 80°C.

$$q_{max} = 0.30\text{ W}. \qquad <$$

(b) For *turbulent flow*,

$$\bar{h}_{10} = 0.0308\frac{k}{x}Re_x^{4/5}Pr^{1/3} = 0.308\frac{0.0282\text{W/m}\cdot\text{K}}{0.095\text{ m}}\left(2.065\times10^5\right)^{4/5}(0.703)^{1/3} = 145\text{W/m}^2\cdot\text{K}$$

$$q_{10} = \bar{h}_{10}A\left(T_s - T_\infty\right) = 1452\frac{\text{W}}{\text{m}^2\cdot\text{K}}(0.01\text{ m})^2(80-24)^\circ\text{C} = 0.81\text{ W}.$$

Hence, $q_{max} = 0.81$ W. $<$

COMMENTS: It is far better to orient array normal to the air flow. Since $\bar{h}_1 > \bar{h}_{10}$, more heat could be dissipated per chip, and the same heat could be dissipated from each chip.

PROBLEM 7.39

KNOWN: Dimensions and maximum allowable temperature of a silicon chip. Air flow conditions.

FIND: Maximum allowable power with or without unheated starting length.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) T_f = 52°C, (3) Negligible radiation, (4) Negligible heat loss through insulation, (5) Uniform heat flux at chip-air interface, (6) $Re_{x,c} = 5 \times 10^5$.

PROPERTIES: *Table A-4*, Air (T_f = 325K, 1 atm): $\nu = 18.41 \times 10^{-6}$ m^2/s, k = 0.0282 W/m·K, Pr = 0.703.

ANALYSIS: For uniform heat flux, maximum T_s corresponds to minimum h_x. Without unheated starting length,

$$Re_L = \frac{u_\infty L}{n} = \frac{20\text{ m/s} \times 0.01\text{ m}}{18.41 \times 10^{-6}\text{ m}^2/\text{s}} = 10{,}864.$$

With the unheated starting length, L = 0.03 m, Re_L = 32,591. Hence, the flow is laminar in both cases and the minimum h_x occurs at the trailing edge (x = L).

Without unheated starting length,

$$h_L = \frac{k}{L} 0.453 Re_L^{1/2} Pr^{1/3} = \frac{0.0282\text{ W/m}\cdot\text{K}}{0.01\text{ m}} 0.453 (10{,}864)^{1/2} (0.703)^{1/3} \leq$$

$$h_L = 118\text{ W/m}^2\cdot\text{K}$$

$$q''(L) = h_L (T_s - T_\infty) = 118\text{ W/m}^2\cdot\text{K} (80-24)^\circ\text{C} = 6630\text{ W/m}^2$$

$$q_{max} = A_s q'' = \left(10^{-2}\text{m}\right)^2 6630\text{ W/m}^2 = 0.66\text{ W}.$$ <

With the unheated starting length,

$$h_L = \frac{k}{L} 0.453 \frac{Re_L^{1/2} Pr^{1/3}}{\left[1-(x/L)^{3/4}\right]^{1/3}} = \frac{0.0282\text{ W/m}\cdot\text{K}}{0.03\text{ m}} 0.453 \frac{(32{,}951)^{1/2} (0.703)^{1/3}}{\left[1-(0.02/0.03)^{3/4}\right]^{1/3}}$$

$$h_L = 107\text{ W/m}^2\cdot\text{K}$$

$$q''(L) = h_L (T_s - T_\infty) = 107\text{ W/m}^2\cdot\text{K} (80-24)^\circ\text{C} = 6013\text{ W/m}^2$$

$$q_{max} = A_s q'' = 10^{-4}\text{m}^2 \times 6013\text{ W/m}^2 = 0.60\text{ W}.$$ <

COMMENTS: Prior velocity boundary layer development on the unheated starting section decreases h_x, although the effect diminishes with increasing x.

PROBLEM 7.40

KNOWN: Experimental apparatus providing nearly uniform airstream over a flat *test plate*. Temperature history of the pre-heated plate for airstream velocities of 3 and 9 m/s were fitted to a fourth-order polynomial.

FIND: (a) Convection coefficient for the two cases assuming the plate behaves as a spacewise isothermal object and (b) Coefficients C and m for a correlation of the form $\overline{Nu}_L = C\,Re^m\,Pr^{1/3}$; compare result with a standard-plate correlation and comment on the goodness of the comparison; explain any differences.

SCHEMATIC:

u_∞ (m/s)	3	9
Δt (s)	300	160
a (°C)	56.87	57.00
b (°C/s)	-0.1472	-0.2641
c (°C/s^2)	3×10^{-4}	9×10^{-4}
d (°C/s^3)	-4×10^{-7}	-2×10^{-6}
e (°C/s^4)	2×10^{-10}	1×10^{-9}

ASSUMPTIONS: (1) Airstream over the *test plate* approximates parallel flow over a flat plate, (2) Plate is spacewise isothermal, (3) Negligible radiation exchange between plate and surroundings, (4) Constant properties, and (5) Negligible heat loss from the bottom surface or edges of the test plate.

PROPERTIES: *Table A.4*, Air (T_f = (T_s - T_∞)/2 ≈ 310 K, 1atm): k_a = 0.0269 W/m·K, $\nu = 1.669 \times 10^{-5}$ m^2/s, Pr = 0.706. Test plate (Given): ρ = 2770 kg/m^3, c_p = 875 J/kg·K, k = 177 W/m·K.

ANALYSIS: (a) Using the lumped-capacitance method, the energy balance on the plate is

$$-\overline{h}_L A_s\left[T_s(t) - T_\infty\right] = \rho V c_p \frac{dT}{dt} \qquad (1)$$

and the average convection coefficient can be determined from the temperature history, $T_s(t)$,

$$\overline{h}_L = \frac{\rho V c_p}{A_s}\frac{(dT/dt)}{T_s(t) - T_\infty} \qquad (2)$$

where the temperature-time derivative is

$$\frac{dT_s}{dt} = b + 2ct + 3dt^2 + 4et^3 \qquad (3)$$

The temperature time history plotted below shows the experimental behavior of the observed data.

Continued...

PROBLEM 7.40 (Cont.)

Consider now the integrated form of the energy balance, Eq. (5.6), expressed as

$$\ln \frac{T_s(t) - T_\infty}{T_i - T_\infty} = -\left(\frac{\bar{h}_L A_s}{\rho V c} \right) t \qquad (4)$$

If we were to plot the LHS vs t, the slope of the curve would be proportional to $\bar{h}_L$. Using IHT, plots were generated of $\bar{h}_L$ vs. T_s, Eq. (1), and $\ln\left[\left(T_s(t) - T_\infty\right)/\left(T_i - T_\infty\right)\right]$ vs. t, Eq. (4). From the latter plot, recognize that the regions where the slope is constant corresponds to early times (≤ 100s when u_∞ = 3 m/s and ≤ 50s when u_∞ = 5 m/s).

Selecting two elapsed times at which to evaluate $\bar{h}_L$, the following results were obtained

u_∞ (m/s)	t (s)	T_s (t), (°C)	$\bar{h}_L$ (W/m^2·K)	$\overline{Nu}_L$	Re_L
3	100	44.77	30.81	152.4	2.39×10^4
9	50	45.80	56.7	280.4	7.17×10^4

where the dimensionless parameters are evaluated as

$$\overline{Nu}_L = \frac{\bar{h}_L L}{k_a} \qquad Re_L = \frac{u_\infty L}{\nu} \qquad (5,6)$$

where k_a, ν are thermophysical properties of the airstream.

(b) Using the above pairs of $\overline{Nu}_L$ and Re_L, C and m in the correlation can be evaluated,

$$\overline{Nu}_L = C\, Re_L^m\, Pr^{1/3} \qquad (7)$$

$$152.4 = C(2.39 \times 10^4)^m (0.706)^{1/3}$$

$$280.4 = C(7.17 \times 10^4)^m (0.706)^{1/3}$$

Solving, find

$$C = 0.633 \qquad m = 0.555 \qquad (8,9)$$ <

Continued...

PROBLEM 7.40 (Cont.)

The plot below compares the experimental correlation (C = 0.633, m = 0.555) with those for laminar flow (C = 0.664, m = 0.5) and fully turbulent flow (C = 0.037, m = 0.8). The experimental correlation yields $\overline{Nu}_L$ values which are 25% higher than for the correlation. The most likely explanation for this unexpected trend is that the airstream reaching the plate is not parallel, but with a slight impingement effect and/or the flow is very highly turbulent at the leading edge.

350
250
150
50
Nusselt number, NuLbar
20000 40000 60000 80000
Reynold's number, ReLbar
exp
lam
turb

COMMENTS: (1) A more extensive analysis of the experimental observations would involve determining $\overline{Nu}_L$ for the full range of elapsed time (rather than at two selected times) and using a fitting routine to determine values for C and m.

PROBLEM 7.41

KNOWN: Cylinder diameter and surface temperature. Temperature and velocity of fluids in cross flow.

FIND: (a) Rate of heat transfer per unit length for the fluids: atmospheric air and saturated water, and engine oil, for velocity V = 5 m/s, using the Churchill-Bernstein correlation, and (b) Compute and plot q' as a function of the fluid velocity $0.5 \le V \le 10$ m/s.

SCHEMATIC:

Air, water, or oil
$T_\infty = 20\ ^oC$
$V = 5$ m/s
$T_s = 50\ ^oC$
$D = 0.01$ m

ASSUMPTIONS: (1) Steady-state conditions, (2) Uniform cylinder surface temperature.

PROPERTIES: *Table A.4*, Air (T_f = 308 K, 1 atm): $\nu = 16.69 \times 10^{-6}\ m^2/s$, k = 0.0269 W/m·K, Pr = 0.706; *Table A.6*, Saturated Water (T_f = 308 K): $\rho = 994\ kg/m^3$, $\mu = 725 \times 10^{-6}\ N \cdot s/m^2$, k = 0.625 W/m·K, Pr = 4.85; *Table A.5*, Engine Oil (T_f = 308 K): $\nu = 340 \times 10^{-6}\ m^2/s$, k = 0.145 W/m·K, Pr = 4000.

ANALYSIS: (a) For each fluid, calculate the Reynolds number and use the Churchill-Bernstein correlation, Eq. 7.57,

$$\overline{Nu}_D = \frac{\bar{h}D}{k} = 0.3 + \frac{0.62\,Re_D^{1/2}\,Pr^{1/3}}{\left[1+(0.4/Pr)^{2/3}\right]^{1/4}}\left[1+\left(\frac{Re_D}{282,000}\right)^{5/8}\right]^{4/5}$$

Fluid: Atmospheric Air

$$Re_D = \frac{VD}{\nu} = \frac{(5\,m/s)\,0.01\,m}{16.69\times10^{-6}\,m^2/s} = 2996$$

$$\overline{Nu}_D = 0.3 + \frac{0.62(2996)^{1/2}(0.706)^{1/3}}{\left[1+(0.4/0.706)^{2/3}\right]^{1/4}}\left[1+\left(\frac{2996}{282,000}\right)^{5/8}\right]^{4/5} = 28.1$$

$$\bar{h} = \frac{k}{D}\overline{Nu}_D = \frac{0.0269\,W/m\cdot K}{0.01\,m}28.1 = 75.5\,W/m^2\cdot K$$

$$q' = \bar{h}\pi D(T_s - T_\infty) = 75.5\,W/m^2\cdot K\,\pi\,(0.01\,m)(50-20)^\circ C = 71.1\,W/m$$ <

Fluid: Saturated Water

$$Re_D = \frac{VD}{\nu} = \frac{(5\,m/s)\,0.01\,m}{725\times10^{-6}\,N\cdot s/m^2/994\,kg/m^3} = 68,552$$

$$\overline{Nu}_D = 0.3 + \frac{0.62(68,552)^{1/2}(4.85)^{1/3}}{\left[1+(0.4/4.85)^{2/3}\right]^{1/4}}\left[1+\left(\frac{68,552}{282,000}\right)^{5/8}\right]^{4/5} = 347$$

Continued...

PROBLEM 7.41 (Cont.)

$$\bar{h} = \frac{k}{D}\overline{Nu}_D = \frac{0.625\,W/m\cdot K}{0.01\,m}347 = 21{,}690\,W/m^2\cdot K \qquad q' = 20{,}438\,W/m$$ <

Fluid: Engine Oil

$$Re_D = \frac{VD}{\nu} = \frac{(5\,m/s)\,0.01\,m}{340\times10^{-6}\,m^2/s} = 147$$

$$\overline{Nu}_D = 0.3 + \frac{0.62(147)^{1/2}(4000)^{1/3}}{\left[1+(0.4/4000)^{2/3}\right]^{1/4}}\left[1+\left(\frac{147}{282,000}\right)^{5/8}\right]^{4/5} = 120$$

$$\bar{h} = \frac{k}{D}\overline{Nu}_D = \frac{0.145\,W/m\cdot K}{0.01\,m}120 = 1740\,W/m^2\cdot K \qquad q' = 1639\,W/m$$ <

(b) Using the *IHT Correlations Tool*, *External Flow*, *Cylinder*, along with the *Properties Tool* for each of the fluids, the heat rates, q', were calculated for the range $0.5 \le V \le 10$ m/s. Note the q' scale multipliers for the air and oil fluids which permit easy comparison of the three curves.

COMMENTS: (1) Note the inapplicability of the Zhukauskas relation, Eq. 7.56, since $Pr_{oil} > 500$.

(2) In the plot above, recognize that the heat rate for the water is more than 10 times that with oil and 300 times that with air. How do changes in the velocity affect the heat rates for each of the fluids?

PROBLEM 7.42

KNOWN: Conditions associated with air in cross flow over a pipe.

FIND: (a) Drag force per unit length of pipe, (b) Heat transfer per unit length of pipe.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Uniform cylinder surface temperature, (3) Negligible radiation effects.

PROPERTIES: *Table A-4*, Air (T_f = 335 K, 1 atm): $\nu = 19.31 \times 10^{-6}\ m^2/s$, $\rho = 1.048\ kg/m^3$, k = 0.0288 W/m·K, Pr = 0.702.

ANALYSIS: (a) From the definition of the drag coefficient with A_f = DL, find

$$F_D = C_D A_f \frac{r V^2}{2}$$

$$F_D' = C_D D \frac{r V^2}{2}.$$

With

$$Re_D = \frac{VD}{n} = \frac{15\ m/s \times (0.025\ m)}{19.31 \times 10^{-6}\ m^2/s} = 1.942 \times 10^4$$

from Fig. 7.8, $C_D \approx 1.1$. Hence

$$F_D = 1.1(0.025\ m)\ 1.048\ kg/m^3\ (15\ m/s)^2/2 = 3.24\ N/m.$$ <

(b) Using Hilpert's relation, with C = 0.193 and m = 0.618 from Table 7.2,

$$\bar{h} = \frac{k}{D} C\, Re_D^m\, Pr^{1/3} = \frac{0.0288\ W/m \cdot K}{0.025\ m} \times 0.193\left(1.942 \times 10^4\right)^{0.618} (0.702)^{1/3}$$

$$\bar{h} = 88\ W/m^2 \cdot K.$$

Hence, the heat rate per unit length is

$$q' = \bar{h}\,(p D)\ (T_s - T_\infty) = 88\ W/m^2 \cdot K\,(p \times 0.025\ m)\ (100 - 25)^\circ C = 520\ W/m.$$ <

COMMENTS: Using the Zhukauskas correlation and evaluating properties at T_∞ ($\nu = 15.71 \times 10^{-6}\ m^2/s$, k = 0.0261 W/m·K, Pr = 0.707), but with Pr_s = 0.695 at T_s,

$$\bar{h} = \frac{0.0261}{0.025} 0.26 \left(\frac{15 \times 0.025}{15.71 \times 10^{-6}}\right)^{0.6} (0.707)^{0.37} (0.707/0.695)^{1/4} = 102\ W/m^2 \cdot K.$$

This result agrees with that obtained from Hilpert's relation to within the uncertainty normally associated with convection correlations.

PROBLEM 7.43

KNOWN: Initial temperature, power dissipation, diameter, and properties of heating element. Velocity and temperature of air in cross flow.

FIND: (a) Steady-state temperature, (b) Time to come within 10°C of steady-state temperature.

SCHEMATIC:

q'_{conv} T
$P'_{elec} = 1000$ W/m
Heating element
$T_i = 300$ K
$k = 240$ W/m•K
$\rho = 2700$ kg/m^3
$c_p = 900$ J/kg•K
Air
$D = 0.01$ m
$V = 10$ m/s
$T_\infty = 300$ K

ASSUMPTIONS: (1) Uniform heater temperature, (2) Negligible radiation.

PROPERTIES: *Table A.4*, air (assume $T_f \approx 450$ K): $\nu = 32.39 \times 10^{-6}$ m^2/s, k = 0.0373 W/m·K, Pr = 0.686.

ANALYSIS: (a) Performing an energy balance for steady-state conditions, we obtain

$$q'_{conv} = \bar{h}(\pi D)(T - T_\infty) = P'_{elec} = 1000\,\text{W/m}$$

With

$$Re_D = \frac{VD}{\nu} = \frac{(10\,\text{m/s})0.01\,\text{m}}{32.39 \times 10^{-6}\,\text{m}^2/\text{s}} = 3,087$$

the Churchill and Bernstein correlation, Eq. 7.57, yields

$$\overline{Nu}_D = 0.3 + \frac{0.62\,Re_D^{1/2}\,Pr^{1/3}}{\left[1 + (0.4/Pr)^{2/3}\right]^{1/4}}\left[1 + \left(\frac{Re_D}{282,000}\right)^{5/8}\right]^{4/5}$$

$$\overline{Nu}_D = 0.3 + \frac{0.62(3087)^{1/2}(0.686)^{1/3}}{\left[1 + (0.4/0.686)^{2/3}\right]^{1/4}}\left[1 + \left(\frac{3087}{282,000}\right)^{5/8}\right]^{4/5} = 28.2$$

$$\bar{h} = \frac{k}{D}\overline{Nu}_D = \frac{0.0373\,\text{W/m}\cdot\text{K}}{0.010\,\text{m}}28.2 = 105.2\,\text{W/m}^2\cdot\text{K}$$

Hence, the steady-state temperature is

$$T = T_\infty + \frac{P'_{elec}}{\pi D\bar{h}} = 300\,\text{K} + \frac{1000\,\text{W/m}}{\pi(0.01\,\text{m})105.2\,\text{W/m}^2\cdot\text{K}} = 603\,\text{K}$$ <

(b) With Bi = $\bar{h}r_o/k$ = 105.2 W/m^2·K(0.005 m)/240 W/m·K = 0.0022, a lumped capacitance analysis may be performed. The time response of the heater is given by Eq. 5.25, which, for $T_i = T_\infty$, reduces to

$$T = T_\infty + (b/a)\left[1 - \exp(-at)\right]$$

Continued...

where a = $4\bar{h}/D\rho c_p = (4\times105.2\,W/m^2\cdot K)/(0.01\,m\times2700\,kg/m^3\times900\,J/kg\cdot K)$ = 0.0173 s^{-1} and b/a = $P'_{elec}/\pi D\bar{h}$ = $1000\,W/m/\pi(0.01\,m\times105.2\,W/m^2\cdot K)$ = 302.6 K. Hence,

$$\left[1-\exp(-0.0173t)\right]=\frac{(593-300)K}{302.6\,K}=0.968$$

$$t \approx 200s$$ <

COMMENTS: (1) For T = 603 K and a representative emissivity of ε = 0.8, net radiation exchange between the heater and surroundings at $T_{sur} = T_\infty$ = 300 K would be $q'_{rad} = \varepsilon\sigma(\pi D)\left(T^4 - T_{sur}^4\right)$ = 0.8 $\times$ 5.67 $\times$ 10^{-8} $W/m^2\cdot K^4$ ($\pi \times 0.01$ m)(603^4 - 300^4)K^4 = 177 W/m. Hence, although small, radiation exchange is not negligible. The effects of radiation are considered in Problem 7.46.

(2) The assumed value of T_f is very close to the actual value, rendering the selected air properties accurate.

PROBLEM 7.44

KNOWN: Initial temperature, power dissipation, diameter, and properties of a heating element. Velocity and temperature of air in cross flow. Temperature of surroundings.

FIND: (a) Steady-state temperature, (b) Time to come within 10°C of steady-state temperature, (c) Variation of power dissipation required to maintain a fixed heater temperature of 275°C over a range of velocities.

SCHEMATIC:

Heating element
$T_i = 300$ K
$k = 240$ W/m·K
$\rho = 2700$ kg/m^3
$c_p = 900$ J/kg·K
q'_{rad}
q'_{conv}
$T, \varepsilon = 0.8$
$P'_{elec} = 1000$ W/m
$T_{sur} = 300$ K
Air
$D = 0.01$ m
$5 \le V \le 10$ m/s
$T_\infty = 300$ K

ASSUMPTIONS: Uniform heater surface temperature.

ANALYSIS: (a) Performing an energy balance for steady-state conditions, we obtain

$$q'_{conv} + q'_{rad} = P'_{elec}$$

$$\bar{h}(\pi D)(T - T_\infty) + \varepsilon\sigma(\pi D)\left(T^4 - T_{sur}^4\right) = P'_{elec}$$

$$\bar{h}(\pi \times 0.01\,m)(T - 300)K + 0.8\left(5.67\times10^{-8}\,W/m^2\cdot K\right)\left(T^4 - 300^4\right)K^4 = 1000\,W/m$$

Using the *IHT Energy Balance Model* for an *Isothermal Solid Cylinder* with the *Correlations* Tool Pad for a *Cylinder* in *Crossflow* and the *Properties* Tool Pad for Air, we obtain

$$T = 562.4\ K$$ <

where $\bar{h} = 105.4\,W/m^2\cdot K$, $h_r = 15.9\,W/m^2\cdot K$, $q'_{conv} = 868.8\,W/m$, and $q'_{rad} = 131.2\,W/m$.

(b) With Bi = $\left(\bar{h} + h_r\right)r_o/k$ = (121.3 W/m^2·K)0.005 m/240 W/m·K = 0.0025, the transient behavior may be analyzed using the lumped capacitance method. Using the *IHT Lumped Capacitance Model* to perform the numerical integration, the following temperature histories were obtained.

Temperature, T(K)
600
550
500
450
400
350
300
0
50
100
150
200
250
300
Time, t(s)
Without radiation
With radiation, eps = 0.8

Continued...

PROBLEM 7.44 (Cont.)

The agreement between predictions with and without radiation for $t < 50$s implies negligible radiation. However, as the heater temperature increases with time, radiation becomes significant, yielding a reduced heater temperature. Steady-state temperatures correspond to 562.4 K and 602.8 K, with and without radiation, respectively. The time required for the heater to reach 552.4 K (with radiation) is $t \approx 155$s.

(c) If the heater temperature is to be maintained at a fixed value in the face of velocity excursions, provision must be made for adjusting the heater power. Using the *Explore* and *Graph* options of IHT with the model of part (a), the following results were obtained.

Power dissipation, P'elec
950
900
850
800
750
700
650
5 6 7 8 9 10
Freestream velocity, V(m/s)

For T = 275°C = 548 K, the controller would compensate for velocity reductions from 10 to 5 m/s by reducing the power from approximately 935 to 690 W/m.

COMMENTS: Although convection heat transfer substantially exceeds radiation heat transfer, radiation is not negligible and should be included in the analysis. If it is neglected, T = 603 K would be predicted for P'_{elec} = 1000 W/m, in contrast 562 K from the results of part (a).

PROBLEM 7.45

KNOWN: Pin fin of 10 mm diameter dissipates 30 W by forced convection in cross-flow of air with $Re_D = 4000$.

FIND: Fin heat rate if diameter is doubled while all conditions remain the same.

SCHEMATIC:

$q_{conv} = 30W$

T_b

Infinitely long pin fin, $D = 10 mm$

Air $Re_D = 4000$

ASSUMPTIONS: (1) Pin behaves as infinitely long fin, (2) Conditions of flow, as well as base and air temperatures, remain the same for both situations, (3) Negligible radiation heat transfer.

ANALYSIS: For an infinitely long pin fin, the fin heat rate is

$$q_f = q_{conv} = \left(\overline{h} P k A_c\right)^{1/2} \theta_b$$

where $P = \pi D$ and $A_c = \pi D^2/4$. Hence,

$$q_{conv} \sim \left(\overline{h} \cdot D \cdot D^2\right)^{1/2}.$$

For forced convection cross-flow over a cylinder, an appropriate correlation for estimating the dependence of $\overline{h}$ on the diameter is

$$\overline{Nu}_D = \frac{\overline{h}D}{k} = CRe_D^m\, Pr^{1/3} = C\left(\frac{VD}{n}\right)^m Pr^{1/3}.$$

From Table 7.2 for $Re_D = 4000$, find $m = 0.466$ and

$$\overline{h} \sim D^{-1}(D)^{0.466} = D^{-0.534}.$$

It follows that

$$q_{conv} \sim \left(D^{-0.534} \cdot D \cdot D^2\right)^{1/2} = D^{1.23}.$$

Hence, with $q_1 \rightarrow D_1$ (10 mm) and $q_2 \rightarrow D_2$ (20 mm), find

$$q_2 = q_1\left(\frac{D_2}{D_1}\right)^{1.23} = 30\ W\left(\frac{20}{10}\right)^{1.23} = 70.4\ W.$$ <

COMMENTS: The effect of doubling the diameter, with all other conditions remaining the same, is to increase the fin heat rate by a factor of 2.35. The effect is nearly linear, with enhancements due to the increase in surface and cross-sectional areas ($D^{1.5}$) exceeding the attenuation due to a decrease in the heat transfer coefficient ($D^{-0.267}$). Note that, with increasing Reynolds number, the exponent m increases and there is greater heat transfer enhancement due to increasing the diameter.

PROBLEM 7.46

KNOWN: Pin fin installed on a surface with prescribed heat rate and temperature.

FIND: (a) Maximum heat removal rate possible, (b) Length of the fin, (c) Effectiveness, ε_f, (d) Percentage increase in heat rate from surface due to fin.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Conditions over A_s are uniform for both situations, (3) Conditions over fin length are uniform, (4) Flow over pin fin approximates cross-flow.

PROPERTIES: *Table A-4*, Air ($T_f = (T_\infty + T_s)/2 = (27 + 127)°C/2 = 350$ K): $\nu = 20.92 \times 10^{-6}$ m^2/s, k = 30.0×10^{-3} W/m·K, Pr = 0.700. *Table A-1*, SS AISI304 ($\overline{T} = T_f = 350$ K): k = 15.8 W/m·K.

ANALYSIS: (a) Maximum heat rate from fin occurs when fin is infinitely long,

$$q_f = M = \left(\overline{h}PkA_c\right)^{1/2}\theta_b \qquad (1)$$

from Eq. 3.80. Estimate convection heat transfer coefficient for cross-flow over cylinder,

$$Re_D = \frac{VD}{\nu} = 5\text{ m/s} \times 0.005\text{ m}/20.92\times10^{-6}\text{m}^2/\text{s} = 1195.$$

Using the Hilpert correlation, Eq. 7.55, with Table 7.2,find

$$\overline{h} = \frac{k}{D}CRe_D^m Pr = (0.030\text{W/m}\cdot\text{K}/0.005\text{m})\,0.683(1195)^{0.466}(0.700)^{1/3} = 98.9\text{W/m}^2\cdot\text{K}$$

From Eq. (1), with $P = \pi D$, $A_c = \pi D^2/4$, and $\theta_b = T_s - T_\infty$, find

$$q_f = \left(98.9\text{ W/m}^2\cdot\text{K}\times\pi\,(0.005\text{ m})\times15.8\text{ W/m}\cdot\text{K}\times\pi\,(0.005\text{ m})^2/4\right)^{1/2}(127-27)\text{K} = 2.20\text{ W.} \quad <$$

(b) From Example 3.8, $L \approx L_\infty = 2.65(kA_c/hP)^{1/2}$. Hence,

$$L \approx L_\infty = 2.65\left[15.8\text{ W/m}\cdot\text{K}\times\pi\,(0.005\text{ m})^2/4/98.9\text{ W/m}^2\cdot\text{K}\times\pi\,(0.005\text{ m})\right]^{1/2} = 37.4\text{ mm.} \quad <$$

(c) From Eq. 3.81, with h_s used for the base area A_s, the effectiveness is

$$\varepsilon_f = \frac{q_f}{h_s A_{c,b}\theta_b} = \frac{q_f}{q_{wo}}\frac{A_s}{A_{c,b}} = \frac{2.2\text{ W}}{0.5\text{ W}}\cdot\frac{(0.020\times0.020)\text{m}^2}{\pi\,(0.005\text{ m})^2/4} = 89.6 \quad <$$

where $\quad h_s = q_{wo}/A_s\theta_b$.

(d) The percentage increase in heat rate with the installed fin (w) is

$$\frac{q_w - q_{wo}}{q_{wo}}\times100 = \left(\left[q_f + h_s\left(A_s - \pi D^2/4\right)(T_s - T_\infty)\right] - q_{wo}\right)\times100/q_{wo}$$

$$\Delta q/q = \left\{\left[2.2\text{ W} + 12.5\text{ W/m}^2\cdot\text{K}\left([0.02\text{ m}]^2 - (\pi/4)(0.005\text{ m})^2\right)100\text{ K} - 0.5\text{ W}\right\}\times100/0.5\text{ W}$$

$$\Delta q/q = 435\%. \quad <$$

PROBLEM 7.47

KNOWN: Dimensions of chip and pin fin. Chip temperature. Free stream velocity and temperature of air coolant.

FIND: (a) Average pin convection coefficient, (b) Pin heat transfer rate, (c) Total heat rate, (d) Effect of velocity and pin diameter on total heat rate.

SCHEMATIC:

Air
$10 \le V \le 40$ m/s
$T_\infty = 300$ K
$L = 12$ mm
$2 \le D \le 4$ mm
W = 4 mm
$T_b = 350$ K

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in pin, (3) Constant properties, (4) Convection coefficients on pin surface (tip and side) and chip surface correspond to single cylinder in cross flow, (5) Negligible radiation.

PROPERTIES: *Table A.1*, Copper (350 K): k = 399 W/m·K; *Table A.4*, Air ($T_f \approx$325 K, 1 atm): ν = 18.41×10^{-6} m^2/s, k = 0.0282 W/m·K, Pr = 0.704.

ANALYSIS: (a) With V = 10 m/s and D = 0.002 m,

$$Re_D = \frac{VD}{\nu} = \frac{10\,\text{m/s} \times 0.002\,\text{m}}{18.41 \times 10^{-6}\,\text{m}^2/\text{s}} = 1087$$

Using the Churchill and Bernstein correlations, Eq. (7.57),

$$\overline{Nu}_D = 0.3 + \frac{0.62\,Re_D^{1/2}\,Pr^{1/3}}{\left[1 + (0.4/Pr)^{2/3}\right]^{1/4}}\left[1 + \left(\frac{Re_D}{282,000}\right)^{5/8}\right]^{4/5} = 16.7$$

$$\bar{h} = \left(\overline{Nu}_D k/D\right) = \left(16.7 \times 0.0282\,\text{W/m}\cdot\text{K}/0.002\,\text{m}\right) = 235\,\text{W/m}^2\cdot\text{K}$$ <

(b) For the fin with tip convection and

$$M = \left(\bar{h}\pi D k \pi D^2/4\right)^{1/2}\theta_b = (\pi/2)\left[235\,\text{W/m}^2\cdot\text{K}\,(0.002\,\text{m})^3\,399\,\text{W/m}\cdot\text{K}\right]^{1/2} 50\,\text{K} = 2.15\,\text{W}$$

$$m = \left(\bar{h}P/kA_c\right)^{1/2} = \left(4 \times 235\,\text{W/m}^2\cdot\text{K}/399\,\text{W/m}\cdot\text{K} \times 0.002\,\text{m}\right)^{1/2} = 34.3\,\text{m}^{-1}$$

$$mL = 34.3\,\text{m}^{-1}(0.012\,\text{m}) = 0.412$$

$$\left(\bar{h}/mk\right) = \left(235\,\text{W/m}^2\cdot\text{K}/34.3\,\text{m}^{-1} \times 399\,\text{W/m}\cdot\text{K}\right) = 0.0172.$$

The fin heat rate is

$$q_f = M\frac{\sinh mL + \left(\bar{h}/mk\right)\cosh mL}{\cosh mL + \left(\bar{h}/mk\right)\sinh mL} = 0.868\,\text{W}.$$ <

Continued...

PROBLEM 7.47 (Cont.)

(c) The total heat rate is that from the base and through the fin,

$$q = q_b + q_f = \bar{h}\left(W^2 - \pi D^2/4\right)\theta_b + q_f = (0.151 + 0.868)\,W = 1.019\,W\,.$$ <

(d) Using the IHT Extended Surface Model for a Pin Fin with the Correlations Tool Pad for a Cylinder in crossflow and Properties Tool Pad for Air, the following results were generated.

Clearly, there is significant benefit associated with increasing V which increases the convection coefficient and the total heat rate. Although the convection coefficient decreases with increasing D, the increase in the total heat transfer surface area is sufficient to yield an increase in q with increasing D. The maximum heat rate is q = 2.77 W for V = 40 m/s and D = 4 mm.

COMMENTS: Radiation effects should be negligible, although tip and base convection coefficients will differ from those calculated in parts (a) and (d).

PROBLEM 7.48

KNOWN: Diameter, resistivity, thermal conductivity and emissivity of Nichrome wire. Electrical current. Temperature of air flow and surroundings. Velocity of air flow.

FIND: (a) Surface and centerline temperatures of the wire, (b) Effect of flow velocity and electric current on temperatures.

SCHEMATIC:

T_{sur} = 50°C
T_s
T_o
Nichrome
k = 25 W/m-K
$\rho_e = 10^{-6}$ Ω-m
ε = 0.20
D = 1 mm
Air
T_∞ = 50°C
V = 5 m/s

ASSUMPTIONS: (1) Steady-state, (2) Radiation exchange with large surroundings, (3) Constant Nichrome properties, (4) Uniform surface temperature.

PROPERTIES: Prescribed, Nichrome: k = 25 W/m·K, $\rho_e = 10^{-6}\,\Omega\cdot\text{m}$, $\varepsilon = 0.2$. *Table A-4*, air $\left(T_f \approx 800\text{K}: k_a = 0.057\text{ W/m}\cdot\text{K},\ \nu = 8.5\times10^{-5}\text{m}^2/\text{s},\ \text{Pr} = 0.71\right)$.

ANALYSIS: (a) The surface temperature may be obtained from Eq. 3.55, with $\overline{h} = \overline{h}_c + h_r$ and

$\dot{q} = I^2 R_e / \forall = I^2 \rho_e / A_c^2 = I^2 \rho_e / \left(\pi D^2/4\right)^2 = 1.013\times10^9\text{ W/m}^3$.

$$T_s = T_\infty + \frac{\dot{q}(D/2)}{2\left(\overline{h}_c + h_r\right)} \tag{1}$$

The convection coefficient is obtained from the Churchill and Bernstein correlation

$$\overline{h}_c = \frac{k_a}{D}\left\{0.3 + \frac{0.62\,Re_D^{1/2}\,Pr^{1/3}}{\left[1+(0.4/Pr)^{2/3}\right]^{1/4}}\left[1+\left(\frac{Re_D}{282,000}\right)^{5/8}\right]^{4/5}\right\} = 230\text{ W/m}^2\cdot\text{K}$$

where $Re_D = VD/\nu = 58.8$, and the radiation coefficient is obtained from Eq. 1.9

$$h_r = \varepsilon\sigma\left(T_s + T_{sur}\right)\left(T_s^2 + T_{sur}^2\right) \tag{2}$$

From an iterative solution of Eqs. (1) and (2), we obtain

$$T_s \approx 1285\text{K} = 1012°\text{C}$$ <

From Eq. 3.53, the centerline temperature is

$$T_o = \frac{\dot{q}(D/2)^2}{4k} + T_s = \frac{1.013\times10^9\text{ W/m}^3(0.0005\text{m})^2}{100\text{ W/m}\cdot\text{K}} + 1012°\text{C} \approx 1014°\text{C}$$ <

The centerline temperature is only approximately 2°C larger than the surface temperature, and the wire may be assumed to be isothermal.

Continued …..

PROBLEM 7.48 (Cont.)

(b) Over the range $1 \le V < 100$ m/s for I = 25A, $\overline{h}_c$ varies from approximately $114\,W/m^2 \cdot K$ to $1050\,W/m^2 \cdot K$, while h_r varies from approximately $69\,W/m^2 \cdot K$ to $4\,W/m^2 \cdot K$. The effect on the surface temperature is shown below.

Maximum and minimum values of $T_s = 1433°C$ and $T_s = 290°C$ are associated with the smallest and largest velocities respectively, while the difference between the centerline and surface temperatures remains at $(T_o - T_s) \approx 2°C$.

For V = 5 m/s, the effect on T_s of varying the current over the range from 1 to 30A is shown below.

From a value of $T_s \approx 52°C$ at 1A, T_s increases to 1320°C at 30A. Over this range the temperature difference $(T_o - T_s)$ increases from approximately 0.01°C to 3°C.

COMMENTS: (1) The radiation coefficient for the conditions of Part (a) is $h_r = 32\,W/m^2 \cdot K$, which is approximately 1/8 of the total coefficient $\overline{h}$. Hence, except for small values of V less than approximately 5 m/s, radiation is negligible compared with convection. (2) The small wire diameter and large thermal conductivity are responsible for maintaining nearly isothermal conditions within the wire. (3) The calculations of Part (b) were performed using the IHT solver with the function $T_f = T_{fluid_avg}(T_s, T_{inf})$ used to account for the effect of temperature on the air properties.

PROBLEM 7.49

KNOWN: Temperature and heat dissipation in a wire of diameter D.

FIND: (a) Expression for flow velocity over wire, (b) Velocity of airstream for prescribed conditions.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Uniform wire temperature, (3) Negligible radiation.

PROPERTIES: *Table A-4*, Air (T_∞ = 298 K, 1 atm): $\nu = 15.8 \times 10^{-6}$ m^2/s, k = 0.0262 W/m·K, Pr = 0.71; (T_s = 313 K, 1 atm): Pr = 0.705.

ANALYSIS: (a) The rate of heat transfer per unit cylinder length is

$$q' = (q/L) = \bar{h}(\pi D)\ (T_s - T_\infty)$$

where, from the Zhukauskas relation, with $Pr \approx Pr_s$,

$$\bar{h} = \frac{k}{D} C\, Re_D^m\, Pr^n = \frac{k}{D}\, C \left(\frac{VD}{\nu}\right)^m Pr^n$$

Hence,

$$V = \left[\frac{q'}{(k/D) C\, Pr^n (\pi D)\ (T_s - T_\infty)}\right]^{1/m} \left(\frac{\nu}{D}\right).$$ <

(b) Assuming ($10^3 < Re_D < 2 \times 10^5$), C = 0.26, m = 0.6 from Table 7.3. Hence,

$$V = \left[\frac{35 \text{ W/m}}{0.0262 \text{ W/m} \cdot \text{K} \times 0.26 (0.71)^{0.37}\, \pi\, (40-25)^\circ \text{C}}\right]^{1/0.6} \left(\frac{15.8 \times 10^{-6} \text{m}^2/\text{s}}{5 \times 10^{-4} \text{m}}\right)$$

$$V = 97 \text{ m/s}.$$ <

To verify the assumption of the Reynolds number range, calculate

$$Re_D = \frac{VD}{\nu} = \frac{97 \text{ m/s} \left(5 \times 10^{-4} \text{m}\right)}{15.8 \times 10^{-6} \text{m}^2/\text{s}} = 3074.$$

Hence the assumption was correct.

COMMENTS: The major uncertainty associated with using this method to determine V is that associated with use of the correlation for $\overline{Nu}_D$.

PROBLEM 7.50

KNOWN: Platinum wire maintained at a constant temperature in an airstream to be used for determining air velocity changes.

FIND: (a) Relationship between fractional changes in current to maintain constant wire temperature and fractional changes in air velocity and (b) Current required when air velocity is 10 m/s.

SCHEMATIC:

Air
T_∞
V
T_s
Platinum wire, D = 0.5mm

ASSUMPTIONS: (1) Steady-state conditions, (2) Cross-flow of air on wire with $40 < Re_D < 1000$, (3) Radiation effects negligible, (4) Wire is isothermal.

PROPERTIES: Platinum wire (given): Electrical resistivity, $\rho_e = 17.1 \times 10^{-5}$ Ohm·m; *Table A-4*, Air (T_∞ = 27°C = 300 K, 1 atm): $\nu = 15.89 \times 10^{-6}$ m²/s, k = 0.0263 W/m·K, Pr = 0.707; (T_s = 77°C = 350 K, 1 atm): $Pr_s = 0.700$.

ANALYSIS: (a) From an energy balance on a unit length of the platinum wire,

$$q'_{elec} - q'_{conv} = I^2 R'_e - \bar{h}P\left(T_s - T_\infty\right) = 0 \qquad (1)$$

where the electrical resistance per unit length is $R'_e = \rho_e / A_c$, $P = \pi D$, and $A_c = \pi D^2/4$. Hence,

$$I = \left[\frac{\bar{h}PA_c}{\rho_e}\left(T_s - T_\infty\right)\right]^{1/2} = \left[\frac{\pi^2 \bar{h} D^3}{4\rho_e}\left(T_s - T_\infty\right)\right]^{1/2} \qquad (2)$$

For the range $40 < Re_D < 1000$, using the Zhukauskas correlation for cross-flow over a cylinder with C = 0.51 and m = 0.5,

$$\overline{Nu}_D = \frac{\bar{h}D}{k} = 0.51\, Re_D^{0.5}\, Pr^{0.37}\left(\frac{Pr}{Pr_s}\right)^{1/4} = 0.51\left(\frac{VD}{\nu}\right)^{0.5} Pr^{0.37}\left(\frac{Pr}{Pr_s}\right)^{1/4} \qquad (3)$$

note that $\bar{h} \sim V^{0.5}$, which, when substituted into Eq. (2) yields

$$I \sim \bar{h}^{1/2} = \left(V^{0.5}\right)^{1/2} = V^{1/4}.$$

Differentiating the proportionality and dividing the result by the proportionality, it follows that

$$\frac{\Delta I}{I} \approx \frac{1}{4}\frac{\Delta V}{V}. \qquad (4)$$ <

(b) For air at T_∞ = 27°C and V = 10 m/s, the current required to maintain the wire of D = 0.5 mm at T_s = 77°C follows from Eq. (2) with $\bar{h}$ evaluated by Eq. (3)

Continued …..

PROBLEM 7.50 (Cont.)

$$\overline{h} = \frac{0.0263 \text{ W/m}\cdot\text{K}}{0.0005 \text{ m}} \times 0.51 \left(\frac{10 \text{ m/s} \times 0.0005 \text{ m}}{15.89 \times 10^{-6} \text{ m}^2/\text{s}} \right)^{0.5} (0.707)^{0.37} \left(\frac{0.707}{0.700} \right)^{1/4}$$

$$\overline{h} = 420 \text{ W/m}^2 \cdot \text{K}$$

where $Re_D = 315$. Hence the required current is

$$I = \left[\frac{p^2 \times 420 \text{ W/m}^2 \cdot \text{K} (0.0005 \text{ m})^3}{4 \times 17.1 \times 10^{-5} \Omega \cdot \text{m}} (77 - 27) \text{K} \right]^{1/2} = 195 \text{ mA}. \qquad (5)$$

COMMENTS: (1) To measure 1% fractional velocity change, a 0.25% fractional change in current must be measured according to Eq. (4). From Eq. (5), this implies that $\Delta I = 0.0025I = 0.0025 \times 195$ mA = 488 μA. An electronic circuit with such measurement sensitivity requires care in its design.

(2) Instruments built on this principle to measure air velocities are called *hot-wire anemometers*. Generally, the wire diameters are much smaller (3 to 30 μm vs 500 μm of this problem) in order to have faster response times.

(3) What effect would the presence of radiation exchange between the wire and its surroundings have?

PROBLEM 7.51

KNOWN: Temperature sensor of 12.5 mm diameter experiences cross-flow of water at 80°C and velocity, $0.005 < V < 0.20$ m/s. Sensor temperature may vary over the range $20 < T_s < 80°C$.

FIND: Expression for convection heat transfer coefficient as a function of T_s and V.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Sensor-water flow approximates a cylinder in cross-flow, (3) Prandtl number varies linearly with temperature over the range of interest.

PROPERTIES: *Table A-6*, Sat. water (T_∞ = 80°C = 353 K): k = 0.670 W/m·K, $\nu = \mu v_f = 352 \times 10^{-6}\ N\cdot s/m^2 \times 1.029 \times 10^{-3}\ m^3/kg = 3.621 \times 10^{-7}\ m^2/s$; Pr_s values for $20 \le T_s \le 80°C$:

T (K)	293	300	325	350	353
Pr	7.00	5.83	3.42	2.29	2.20

ANALYSIS: Using the Zhukauskus correlation for the range $40 < Re_D < 4000$ with C = 0.51 and m = 0.5,

$$\overline{Nu}_D = \frac{\bar{h}D}{k} = 0.51 Re_D^{0.5}\, Pr^{0.37}\left(\frac{Pr}{Pr_s}\right)^{1/4}.$$

with $Re_D = VD/\nu$, the thermophysical properties of interest are k, ν and Pr, which are evaluated at T_∞ = 80°C, and Pr_s which varies markedly with T_s for the range $20 < T_s < 80°C$. Assuming Pr_s to vary linearly with T_s and using the extreme values to find the relation,

$$Pr_s = 7.00 + \frac{(2.20 - 7.00)}{(353 - 293)\,K}(T_s - 293)\,K = 7.00 - 0.0800(T_s - 293)$$

where the units of T_s are [K]. Substituting numerical values, find

$$\bar{h}(T_s) = \frac{0.670\ W/m\cdot K}{0.0125\ m}0.51\left(\frac{V \times 0.0125\ m}{3.621\times10^{-7}\ m^2/s}\right)^{0.5}(2.20)^{0.37}\left(\frac{2.20}{7.00 - 0.080(T_s - 293)}\right)^{1/4}$$

$$\bar{h}(T_s) = 6810 V^{0.5}\left[3.182 - 0.0364(T_s - 293)\right]^{-1/4}.$$ <

COMMENTS: (1) From the Pr_s vs T_s graph above, a linear fit is seen to be poor for this temperature range. However, because the Pr_s dependence is to the ¼ power, the discrepancy may be acceptable.

PROBLEM 7.52

KNOWN: Diameter, electrical resistance and current for a high tension line. Velocity and temperature of ambient air.

FIND: (a) Surface and (b) Centerline temperatures of the wire, (c) Effect of air velocity on surface temperature.

SCHEMATIC:

D = 0.025 m
q'
$\dot{q} = q'/(\pi D^2)/4$
$R'_e = 10^{-4}$ Ω/m
$I = 1000$ A
Air $T_\infty = 10\ ^oC$, $1 \le V \le 10$ m/s

ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) One-dimensional radial conduction.

PROPERTIES: *Table A.4*, Air ($T_f \approx 300$ K, 1 atm): $\nu = 15.89 \times 10^{-6}$ m²/s, k = 0.0263 W/m·K, Pr = 0.707; *Table A.1*, Copper (T ≈ 300 K): k = 400 W/m·K.

ANALYSIS: (a) Applying conservation of energy to a control volume of unit length,

$$\dot{E}'_g = I^2 R'_e = q' = \bar{h}\pi D\left(T_s - T_\infty\right)$$

With

$$Re_D = \frac{VD}{\nu} = \frac{10\,m/s\left(0.025\,m\right)}{15.89\times10^{-6}\,m^2/s} = 15,733$$

the Churchill and Bernstein correlation, yields

$$\overline{Nu}_D = 0.3 + \frac{0.62\,Re_D^{1/2}\,Pr^{1/3}}{\left[1+\left(0.4/Pr\right)^{2/3}\right]^{1/4}}\left[1+\left(\frac{Re_D}{282,000}\right)^{5/8}\right]^{4/5} = 69.0$$

Hence,

$$\bar{h} = \overline{Nu}_D\,\frac{k}{D} = 69.0\,\frac{0.0263\,W/m\cdot K}{0.025\,m} = 72.6\,W/m^2\cdot K$$

and

$$T_s = T_\infty + \frac{I^2 R'_e}{\bar{h}\pi D} = 10^\circ C + \frac{\left(1000\,A\right)^2 10^{-4}\,\Omega/m}{\left(72.6\,W/m^2\cdot K\right)\pi\left(0.025\,m\right)} = 10^\circ C + 17.6^\circ C = 27.6^\circ C$$ <

(b) With $\dot{q} = \dot{E}'_g\Big/\left(\pi D^2/4\right) = 4\left(1000\,A\right)^2\left(10^{-4}\,\Omega/m\right)\Big/\pi\left(0.025\,m\right)^2 = 2.04\times10^5\,W/m^3$, Equation 3.53 yields

$$T(0) = \frac{\dot{q}r_o^2}{4k} + T_s = \frac{2.041\times10^5\,W/m^3\left(0.0125\,m\right)^2}{1600\,W/m\cdot K} + 27.6^\circ C = 0.02^\circ C + 27.6^\circ C \approx 27.6^\circ C$$ <

Continued...

PROBLEM 7.52 (Cont.)

(c) The effect of V on the surface temperature was determined using the *Correlations* and *Properties* Tool Pads of IHT.

Surface temperature, Ts(C)
75
65
55
45
35
25
1 2 3 4 5 6 7 8 9 10
Freestream velocity, V (m/s)

The effect is significant, with a surface temperature of $T_s \approx 70°C$ corresponding to V = 1 m/s. For velocities of 1 and 10 m/s, respectively, convection coefficients are 21.1 and 72.8 $W/m^2 \cdot K$ and film temperatures are 313.2 and 291.7 K.

COMMENTS: The small values of $\dot{q}$ and r_o and the large value of k render the wire approximately isothermal.

PROBLEM 7.53

KNOWN: Aluminum transmission line with a diameter of 20 mm having an electrical resistance of $R' = 2.636\times10^{-4}$ ohm/m carrying a current of 700 A subjected to severe cross winds. To reduce potential fire hazard when adjacent lines make contact and spark, insulation is to be applied.

FIND: (a) The bare conductor temperature when the air temperature is 20°C and the line is subjected to cross flow with a velocity of 10 m/s; (b) The conductor temperature for the same conditions, but with an insulation covering of 2 mm thickness and thermal conductivity of 0.15 W/m·K; and (c) Plot the conductor temperatures of the bare and insulated conductors for wind velocities in the range of 2 to 20 m/s. Comment on the features of the curves and the effect that wind velocity has on the conductor operating temperatures.

SCHEMATIC:

$T_{c,bare}$
Conductor (bare)
D = 20 mm
R'_e = 2.636x10^{-4} Ω/m
I = 700 A
Air
T_∞ = 20°C
V = 10 m/s
$T_{c,ins}$
T_s
Insulation
t = 2 mm thick
k = 0.15 W/m-K
Cross-sectional view with insulation

ASSUMPTIONS: (1) Steady-state conditions, (2) Uniform surface temperatures, (3) Negligible solar irradiation and radiation exchange, and (4) Constant properties.

PROPERTIES: *Table A-4*, Air ($T_f = (T_s + T_\infty)/2$, 1 atm): evaluated using the *IHT Properties* library with a *Correlation* function; see Comment 2.

ANALYSIS: (a) For the *bare* conductor the energy balance per unit length is

$$\dot{E}'_{in} - \dot{E}'_{out} + \dot{E}'_{gen} = 0$$

$$0 - q'_{cv} + \dot{q}\,A_c = 0 \tag{1}$$

where the crossectional area of the conductor is $A_c = \pi D^2/4$ and the generation rate is

$$\dot{q} = I^2 R'_e / A_c = (700\,\text{A})^2 \times 2.636\times10^{-4}\,\Omega/\text{m} \Big/ \left(\pi\,(0.020\,\text{m})^2/4\right) \tag{2}$$

$$\dot{q} = 4.111\times10^5\ \text{W/m}^3$$

The convection rate equation can be expressed as

$$q'_{cv} = \left(T_{c,bare} - T_\infty\right)/R'_t \qquad\qquad R'_t = 1/\left(\bar{h}_D \times \pi D\right) \tag{3,4}$$

and the convection coefficient is estimated using the Churchill-Bernstein correlation, Eq. 7.57, with $Re_D = VD/\nu$,

$$\overline{Nu}_L = \frac{\bar{h}_D D}{k} = 0.3 + \frac{0.62\,Re_D^{1/2}\,Pr^{1/3}}{\left[1+(0.4/Pr)^{2/3}\right]^{1/4}}\left[1+\left(\frac{Re_D}{282,000}\right)^{5/8}\right]^{4/5} \tag{4}$$

(b) For the conductor *with insulation* thickness t = 2 mm, the energy balance per unit length is

$$\dot{E}'_{in} - \dot{E}'_{out} + \dot{E}'_{gen} = 0$$

$$0 - \left(T_{c,ins} - T_\infty\right)/R'_t + I^2 R'_e / A_c = 0 \tag{5}$$

Continued …..

where R'_t is the sum of the insulation conduction and convection process thermal resistances,

$$R'_t = \ell n\left[(D+2t)/D\right]/(2\pi k)+1/\left[\bar{h}_{D+2t}\pi(0+2t)\right]$$
(6)

The results of the analysis using *IHT* are tabulated below.

Condition	V (m/s)	d (mm)	Re_d	$\overline{Nu}_d$	$\bar{h}_d$ ($W/m^2 \cdot K$)	R'_t ($m \cdot K/W$)	T_c (°C)
bare	10	20	1.214×10^4	59.6	79.6	0.1998	45.8
insulated	10	24	1.468×10^4	66.3	73.6	0.3736	68.3

(c) Using the *IHT* code with the foregoing relations, the conductor temperatures $T_{c,base}$ and $T_{c,ins}$ for the bare and insulated conditions are calculated and plotted for the wind velocity range of 2 to 20 m/s.

COMMENTS: (1) The effect of the 2-mm thickness insulation is to increase the conductor operating temperature by (68.3 – 46.1)°C = 22°C. While we didn't account for an increase in the electrical resistivity with increasing temperature, the adverse effect is to increase the I^2R loss, which represents a loss of revenue to the power provider. From the graph, note that the conductor temperature increases markedly with decreasing wind velocity, and the effect of insulation is still around +20°C.

(2) Because of the tediousness of hand calculations required in using the convection correlation without fore-knowledge of T_f at which to evaluate properties, we used the *IHT Correlation* function treating T_f as one of the unknowns in the system of equations. Salient portions of the *IHT* code and property values are provided below.

Continued …..

```
// Forced convection, cross flow, cylinder
NuDbar = NuD_bar_EF_CY(ReD,Pr)              // Eq 7.57
NuDbar = hDbar * Do / k
ReD = V * Do / nu                           // Outer diameter; bare or with insulation

// Evaluate properties at the film temperature, Tf.
Tf =  Tfluid_avg (Tinf,Ts)              // Ts is the outer surface temperature
/*  Correlation description: External cross flow (EF) over cylinder (CY), average coefficient,
ReD*Pr>0.2, Churchill-Bernstein correlation, Eq 7.57. See Table 7.9.  */

// Air property functions : From Table A.4
// Units: T(K); 1 atm pressure
nu = nu_T("Air",Tf)                     // Kinematic viscosity, m^2/s
k = k_T("Air",Tf)                       // Thermal conductivity, W/m·K
Pr = Pr_T("Air",Tf)                     // Prandtl number
```

(3) Is the temperature gradient within the conductor significant?

PROBLEM 7.54

KNOWN: Diameter and length of a copper rod, with fixed end temperatures, inserted in an airstream of prescribed velocity and temperature.

FIND: (a) Midplane temperature of rod, (b) Rate of heat transfer from the rod.

SCHEMATIC:

$T_b = 90°C$ — L=50mm — Copper — D=10mm — $T_b = 90°C$

Air, $V=25\,m/s$, $T_\infty = 25°C$

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in rod, (3) Negligible contact resistance, (4) Negligible radiation, (5) Constant properties.

PROPERTIES: *Table A-1*, Copper (T ≈ 80°C = 353 K): k = 398 W/m·K; *Table A-4*, Air (T_∞ = 25°C ≈ 300K, 1 atm): $\nu = 15.8 \times 10^{-6}\ m^2/s$, k = 0.0263 W/m·K, Pr = 0.707; *Table A-4*, Air (T_s ≈ 80°C ≈ 350K, 1 atm): $Pr_s = 0.700$.

ANALYSIS: (a) For case B of Table 3.4, $\dfrac{q}{q_b} = \dfrac{\cosh m(L-x)}{\cosh(mL)} = \dfrac{T - T_\infty}{T_b - T_\infty}$ where

$m = \left(\bar{h}P/kA_c\right)^{1/2} = \left(4\bar{h}/kD\right)^{1/2}$. Using the Zhukauskas correlation with n = 0.37,

$$\overline{Nu}_D = C\, Re_D^m\, Pr^n \left(Pr/Pr_s\right)^{1/4}$$

$$Re_D = \frac{VD}{\nu} = \frac{25\ m/s\,(0.01\ m)}{15.8\times10^{-6}\ m^2/s} = 15{,}823$$

and C = 0.26, m = 0.6 from Table 7-4. Hence

$$\overline{Nu}_D = 0.26(15{,}823)^{0.6}(0.707)^{0.37}(0.707/0.700)^{1/4} = 75.8$$

$$\bar{h} = \frac{k}{D}\overline{Nu}_D = \frac{0.0263\ W/m\cdot K}{0.01\ m}(75.8) = 199\ W/m^2\cdot K$$

$$m = \left(\frac{4\times199\ W/m^2\cdot K}{398\ W/m\cdot K\times0.01\ m}\right)^{1/2} = 14.2\ m^{-1}.$$

Hence,
$$\frac{T(L)-T_\infty}{T_b-T_\infty} = \frac{\cosh(0)}{\cosh\left(14.2\ m^{-1}\times0.05\ m\right)} = \frac{1}{1.26} = 0.79$$

$$T(L) = 25°C + 0.79(90-25) = 76.6°C. \qquad <$$

(b) From Eq. 3.76, $q = 2q_f = 2M \tanh mL$,

$$M = \left(\bar{h}PkA_c\right)^{1/2}q_b = \left[199\frac{W}{m^2\cdot K}(\pi\times0.01\ m)\left(398\frac{W}{m\cdot K}\right)\frac{\pi}{4}(0.01\ m)^2\right]^{1/2} 65°C$$

$$M = 28.7\ W \qquad q = 2(28.7\ W)\tanh\left(14.2\ m^{-1}\times0.05\ m\right) = 35\ W. \qquad <$$

COMMENTS: Note adiabatic condition associated with symmetry about midplane.

PROBLEM 7.55

KNOWN: Diameter, thickness and thermal conductivity of steel pipe. Temperature of water flow in pipe. Temperature and velocity of air in cross flow over pipe. Cost of producing hot water.

FIND: (a) Cost of daily heat loss from an uninsulated pipe, (b) Savings associated with insulating the pipe.

SCHEMATIC:

Air
T_∞ = -5°C
V = 3 m/s
Water
T = 50°C
Insulation
D_o = 120 mm
k_i = 0.026 W/m-K
Steel pipe
D_i = 84 mm
D_o = 100 mm
k_p = 60 W/m-K
T
T_∞
$R'_{cnd,p}$
$R'_{cnd,i}$
R'_{cnv}
q'

ASSUMPTIONS: (1) Steady-state, (2) Negligible convection resistance for water flow, (3) Negligible contact resistance between insulation and pipe, (4) Negligible radiation.

PROPERTIES: *Table A-4*, air $\left(p = 1\,\text{atm},\ T_f \approx 300\text{K}\right)$: $k_a = 0.0263\,\text{W/m}\cdot\text{K}$, $\nu = 15.89\times10^{-6}\,\text{m}^2/\text{s}$, $\text{Pr} = 0.707$.

ANALYSIS: (a) With $\text{Re}_D = VD_o/\nu = 3\,\text{m/s}\times0.1\text{m}/15.89\times10^{-6}\,\text{m}^2/\text{s} = 18{,}880$, application of the Churchill-Bernstein correlation yields

$$\overline{\text{Nu}}_D = 0.3 + \frac{0.62(18{,}800)^{1/2}(0.707)^{1/3}}{\left[1+(0.4/0.707)^{2/3}\right]^{1/4}}\left[1+\left(\frac{18{,}880}{282{,}000}\right)^{5/8}\right]^{4/5} = 76.6$$

$$\bar{h} = \frac{k_a}{D_o}\overline{\text{NU}}_D = \frac{0.0263\,\text{W/m}\cdot\text{K}}{0.1\text{m}}76.6 = 20.1\,\text{W/m}^2\cdot\text{K}$$

Without the insulation, the total thermal resistance and heat loss per length of pipe are then

$$R'_{tot(wo)} = \frac{\ln(D_o/D_i)}{2\pi k_p} + \frac{1}{\pi D_o \bar{h}} = \frac{\ln(100/84)}{2\pi\times60\,\text{W/m}\cdot\text{K}} + \frac{1}{\pi(0.1\text{m})20.1\,\text{W/m}^2\cdot\text{K}}$$

$$= \left(4.63\times10^{-4} + 0.158\right)\text{m}\cdot\text{K/W} = 0.159\,\text{m}\cdot\text{K/W}$$

$$q'_{wo} = \frac{T - T_\infty}{R'_{tot(wo)}} = \frac{55°\text{C}}{0.159\,\text{m}\cdot\text{K/W}} = 346\,\text{W/m} = 0.346\,\text{kW/m}$$

The corresponding daily energy loss is

$$Q'_{wo} = 0.346\,\text{kW/m}\times24\,\text{h/d} = 8.3\,\text{kW}\cdot\text{h/m}\cdot\text{d}$$

and the associated cost is

$$C'_{wo} = (8.3\,\text{kW}\cdot\text{h/m}\cdot\text{d})(\$0.05/\text{kW}\cdot\text{h}) = \$0.415/\text{m}\cdot\text{d} \quad <$$

(b) The conduction resistance of the insulation is

Continued …..

$$R'_{cnd} = \frac{\ln(D_o / D_i)}{2\pi k_i} = \frac{\ln(120/100)}{2\pi(0.026\,W/m\cdot K)} = 1.116\,m\cdot K/W$$

Using the Churchill-Bernstein correlation with an outside diameter of $D_o = 0.12m$, $Re_D = 22,660$, $\overline{Nu}_D = 83.9$ and $\bar{h} = 18.4\,W/m^2\cdot K$. The convection resistance is then

$$R'_{cnv} = \frac{1}{\pi D_o \bar{h}} = \frac{1}{\pi(0.12m)18.4\,W/m^2\cdot K} = 0.144\,m\cdot K/W$$

and the total resistance is

$$R'_{tot(w)} = \left(4.63\times10^{-4} + 1.116 + 0.144\right) m\cdot K/W = 1.261\,m\cdot K/W$$

The heat loss and cost are then

$$q'_w = \frac{T - T_\infty}{R'_{tot(w)}} = \frac{55°C}{1.261\,m\cdot K/W} = 43.6\,W/m = 0.0436\,kW/m$$

$$C'_w = 0.0436\,kW/m \times 24h/d \times \$0.05/kW\cdot h = \$0.052/m\cdot d$$

The daily savings is then

$$S' = C'_{wo} - C'_w = \$0.363/m\cdot d$$ <

COMMENTS: (1) The savings are significant, and the pipe should be insulated. (2) Assuming a negligible temperature drop across the pipe wall, a pipe emissivity of $\varepsilon_p = 0.6$ and surroundings at $T_{sur} = 268K$, the radiation coefficient associated with the uninsulated pipe is $h_r = \varepsilon\sigma(T + T_{sur})(T^2 + T_{sur}^2) = 0.6\times5.67\times10^{-8}\,W/m^2\cdot K^4\,(591K)\,(323^2 + 268^2)K^2 = 3.5\,W/m^2\cdot K$. Accordingly, radiation increases the heat loss estimate of Part (a) by approximately 17%.

PROBLEM 7.56

KNOWN: Diameter and surface temperature of an uninsulated steam pipe. Velocity and temperature of air in cross flow.

FIND: (a) Heat loss per unit length, (b) Effect of insulation thickness on heat loss.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Uniform surface temperature, (3) Negligible radiation.

PROPERTIES: *Table A.4*, Air ($T_f \approx 350$ K, 1 atm): $\nu = 20.9 \times 10^{-6}$ m^2/s, k = 0.030 W/m·K, Pr = 0.70.

ANALYSIS: (a) Without the insulation, the heat loss per unit length is

$$q' = \bar{h}\pi D\left(T_{s,i} - T_\infty\right)$$

where $\bar{h}$ may be obtained from the Churchill-Bernstein relation. With

$$Re_D = \frac{VD}{\nu} = \frac{5\,m/s \times 0.5\,m}{20.9 \times 10^{-6}\,m^2/s} = 1.196 \times 10^5$$

$$\overline{Nu}_D = 0.3 + \frac{0.62\,Re_D^{1/2}\,Pr^{1/3}}{\left[1 + (0.4/Pr)^{2/3}\right]^{1/4}}\left[1 + \left(\frac{Re_D}{282,000}\right)^{5/8}\right]^{4/5} = 242$$

$$\bar{h} = \overline{Nu}_D \frac{k}{D} = 242 \frac{0.030\,W/m\cdot K}{0.5\,m} = 14.5\,W/m^2\cdot K$$

The heat rate is then

$$q' = 14.5\,W/m^2\cdot K\,\pi\,(0.5\,m)\left(150 - (-10)\right)^\circ C = 3644\,W/m\,.$$ <

(b) With the insulation, the heat loss may be expressed as

$$q' = U_i \pi D\left(T_{s,i} - T_\infty\right)$$

where, from Eq. 3.31,

$$U_i = \left[\frac{(D/2)}{k_i}\ln\bar{r} + \frac{1}{\bar{r}\bar{h}}\right]^{-1}$$

and $\bar{r} \equiv (D/2 + \delta)/(D/2)$. The outer diameter, $D_o = D + 2\delta$, as well as the film temperature, $T_f = (T_{s,o} + T_\infty)/2$, must now be used to evaluate the convection coefficient, where

Continued...

$$\frac{T_{s,i}-T_{s,o}}{T_{s,i}-T_{\infty}}=\frac{R'_{cond}}{R'_{tot}}=\frac{\left(\ln \bar{r}\right)/k_i}{\left(\ln \bar{r}\right)/k_i+1/\left(D/2\right)\bar{r}\bar{h}}$$

Using the IHT *Correlations* and *Properties* Tool Pads to evaluate $\bar{h}$, the following results were obtained.

The insulation is extremely effective, with a thickness of only 10 mm yielding a 7-fold reduction in heat loss and decreasing the outer surface temperature from 150 to 10°C. For δ = 50 mm, U_i = 0.56 W/m^2·K, q' = 140 W/m and $T_{s,o}$ = -5.2°C.

COMMENTS: The dominant contribution to the total thermal resistance is made by the insulation.

PROBLEM 7.57

KNOWN: Person, approximated as a cylinder, is subjected to prescribed convection conditions.

FIND: Heat rate from body for prescribed temperatures.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Person can be approximated by cylindrical form having uniform surface temperature, (3) Negligible heat loss from cylinder top and bottom surfaces, (4) Negligible radiation effects.

PROPERTIES: *Table A-4*, Air (T_∞ = 268 K, 1 atm): $\nu = 13.04 \times 10^{-6}$ m^2/s, $k = 23.74 \times 10^{-3}$ W/m·K, Pr = 0.725; (T_s = 297 K, 1 atm): Pr = 0.707.

ANALYSIS: The heat transfer rate from the cylinder, approximating the person, is given as

$$q = \overline{h} A_s \left(T_s - T_\infty \right)$$

where $A_s = \pi D \ell$ and $\overline{h}$ must be estimated from a correlation appropriate to cross-flow over a cylinder. Use the Zhukauskas relation,

$$\overline{Nu}_D = \frac{\overline{h} D}{k} = C\, Re_D^m\, Pr^n \left(Pr/Pr_s \right)^{1/4}$$

and calculate the Reynold's number,

$$Re_D = \frac{VD}{\nu} = \frac{15 \text{m/s} \times 0.3 \text{ m}}{13.04 \times 10^{-6} \text{ m}^2/\text{s}} = 345{,}092.$$

From Table 7-4, find C = 0.076 and m = 0.7. Since Pr < 10, n = 0.37, giving

$$\overline{Nu}_D = 0.076 \left(345{,}092\right)^{0.7} 0.725^{0.37} \left(\frac{0.725}{0.707} \right)^{1/4} = 511$$

$$\overline{h} = \overline{Nu}_D \frac{k}{D} = \frac{511 \times 23.74 \times 10^{-3} \text{W/m} \cdot \text{K}}{0.3 \text{ m}} = 40.4 \text{ W/m}^2 \cdot \text{K}.$$

The heat transfer rate is

$$q = 40.4 \text{W/m}^2 \cdot \text{K} \left(\pi \times 0.3 \text{ m} \times 1.8 \text{ m}\right) \left(24 - (-5)\right)^\circ \text{C} = 1988 \text{ W}.$$ <

COMMENTS: Note the temperatures at which properties are evaluated for the Zhukauskas correlation.

PROBLEM 7.58

KNOWN: Dimensions and thermal conductivity of a thermocouple well. Temperatures at well tip and base. Air velocity.

FIND: Air temperature.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) One-dimensional conduction along well, (4) Uniform convection coefficient, (5) Negligible radiation.

PROPERTIES: Steel (given): k = 35 W/m·K; Air (given): ρ = 0.774 kg/m^3, $\mu = 251 \times 10^{-7}$ N·s/m^2, k = 0.0373 W/m·K, Pr = 0.686.

ANALYSIS: Applying Equation 3.70 at the well tip (x = L), where T = T_1,

$$\frac{T_1 - T_\infty}{T_2 - T_\infty} = \left[\cosh mL + \left(\bar{h}/mk\right)\sinh mL\right]^{-1}$$

$$m = \left(\bar{h}P/kA_c\right)^{1/2} \qquad P = \pi D_o = \pi\,(0.010\text{ m}) = 0.0314\text{ m}$$

$$A_c = (\pi/4)\left(D_o^2 - D_i^2\right) = (\pi/4)\left(0.010^2 - 0.005^2\right)\text{m}^2 = 5.89\times10^{-5}\text{ m}^2.$$

With $$Re_D = \frac{\rho VD}{\mu} = \frac{0.744\text{ kg/m}^3(3\text{m/s})\,0.01\text{ m}}{251\times10^{-7}\text{ N}\cdot\text{s/m}^2} = 925$$

C = 0.51, m = 0.5, n = 0.37 and the Zhukauskas correlation yields

$$\overline{Nu}_D = 0.51 Re_D^{0.5} Pr^{0.37}\left(Pr/Pr_s\right)^{1/4} \approx 0.51(925)^{0.5}(0.686)^{0.37}\times1 = 13.5$$

$$\bar{h} = \overline{Nu}_D\,\frac{k}{D_o} = 13.5\,\frac{0.0373\text{ W/m}\cdot\text{K}}{0.01\text{ m}} = 50.4\text{ W/m}^2\cdot\text{K}.$$

Hence

$$m = \left[\frac{\left(50.4\text{ W/m}^2\cdot\text{K}\right)0.0314\text{ m}}{(35\text{ W/m}\cdot\text{K})\,5.89\times10^{-5}\text{ m}^2}\right]^{1/2} = 27.7\text{ m}^{-1} \qquad mL = \left(27.7\text{ m}^{-1}\right)0.15\text{ m} = 4.15.$$

With

$$\left(\bar{h}/mk\right) = \left(50.4\text{ W/m}^2\cdot\text{K}\right)/\left(27.7\text{ m}^{-1}\right)(35\text{ W/m}\cdot\text{K}) = 0.0519$$

find $$\frac{T_1 - T_\infty}{T_2 - T_\infty} = \left[32.62 + (0.0519)\,32.61\right]^{-1} = 0.0291 \qquad T_\infty = 452.2\text{ K}.$$ <

COMMENTS: Heat conduction along the wall to the base at 375 K is balanced by convection from the air.

PROBLEM 7.59

KNOWN: Mercury-in-glass thermometer mounted on duct wall used to measure air temperature.

FIND: (a) Relationship for the immersion error, $\Delta T_i = T(L) - T_\infty$ as a function of air velocity, thermometer diameter and length, (b) Length of insertion if ΔT_i is not to exceed 0.25°C when the air velocity is 10 m/s, (c) For the length of part (b), calculate and plot ΔT_i as a function of air velocity for 2 to 20 m/s, and (d) For a given insertion length, will ΔT_i increase or decrease with thermometer diameter increase; is ΔT_i more sensitive to diameter or velocity changes?

SCHEMATIC:

Wall
$T_w = T_b = 15\,^oC$
Glass thermometer, D = 4 mm
T_L
L
Air
$T_\infty = 77\,^oC$
$2 \le V \le 20$ m/s
x

ASSUMPTIONS: (1) Steady-state conditions, (2) Thermometer approximates a one-dimensional (glass) fin with an *adiabatic* tip, (3) Convection coefficient is uniform over length of thermometer.

PROPERTIES: *Table A.3*, Glass (300 K): k_g = 1.4 W/m·K; *Table A.4*, Air (T_f = (15 + 77)°C/2 ≈ 320 K, 1 atm): k = 0.0278 W/m·K, $\nu = 17.90 \times 10^{-6}$ m/s^2, Pr = 0.704.

ANALYSIS: (a) From the analysis of a one-dimensional fin, see Table 3.4,

$$\frac{T_L - T_\infty}{T_b - T_\infty} = \frac{1}{\cosh(mL)} \qquad m^2 = \frac{\bar{h}P}{k_g A_c} = \frac{4\bar{h}}{k_g D} \qquad (1)$$

where $P = \pi D$ and $A_c = \pi D^2/4$. Hence, the immersion error is

$$\Delta T_i = T(L) - T_\infty = (T_b - T_\infty)/\cosh(mL). \qquad (2)$$

Using the Hilpert correlation for the circular cylinder in cross flow,

$$\bar{h} = \frac{k}{D} C\, Re_D^m\, Pr^{1/3} = \frac{k}{D} C \left(\frac{VD}{\nu}\right)^m Pr^{1/3} = \frac{k\, Pr^{1/3}}{\nu^m} \cdot C \cdot V^m \cdot D^{m-1} \qquad (3)$$

$$\bar{h} = N \cdot V^m \cdot D^{m-1} \qquad \text{where} \qquad N = \frac{k\, Pr^{1/3}}{\nu^m} C \qquad (4,5)$$

Substituting into Eq. (2), the immersion error is

$$\Delta T_i(V, D, L) = (T_b - T_\infty)/\cosh\left\{\left[(4/k_g) N \cdot V^m \cdot D^{m-2}\right]^{1/2} L\right\} \qquad (6)$$ <

where k_g is the thermal conductivity of the glass thermometer.

(b) When the air velocity is 10 m/s, find

$$Re_D = \frac{VD}{\nu} = \frac{10\,\text{m/s} \times 0.004\,\text{m}}{17.9 \times 10^{-6}\,\text{m/s}^2} = 2235$$

Continued...

PROBLEM 7.59 (Cont.)

with C = 0.683 and m = 0.466 from Table 7.2 for the range $40 < Re_D < 4000$. From Eqs. (5) and (6),

$$N = \frac{0.0278\,W/m\cdot K\,(0.704)^{1/3}}{\left(17.9\times10^{-6}\,m/s^2\right)^{0.466}} \times 0.683 = 2.753$$

$$\Delta T_i = (15-77)K/\cosh\left\{\left[\frac{4}{1.4\,W/m\cdot K}\times 2.753(10\,m/s)^{0.466}(0.004\,m)^{0.466-2}\right]^{1/2} L\right\}$$

and when ΔT_i = -0.25°C, find

$$L = 18.7\text{ mm}$$ <

(c) For the air velocity range 2 to 20 m/s, find $447 \le Re_D \le 4470$ for which the previous values of C and m of the Hilpert correlation are appropriate. Hence, the immersion error for an insertion length of L = 18.7 mm, part (b), find

$$\Delta T_i = (15-77)K/\cosh\left\{\left[\frac{4}{1.4\,W/m\cdot K}\times 2.753\times V^{0.466}(0.004\,m)-1.534\right]^{1/2} 0.0187\right\}$$

$$\Delta T_i = -62^\circ C/\cosh\left(3.629V^{0.233}\right)$$ <

where the units of V are [m/s]. Entering the above equation into the IHT Workspace the plot shown below was generated.

V(m/s)	ΔT_i (°C)
2	-1.74
5	-0.63
10	-0.25
15	-0.14
20	-0.08

(d) For a given insertion length, the immersion error will *increase* if the diameter of the thermometer were *increased*. This follows from Eq. (6) written as

$$\Delta T_i \sim 1/\cosh\left(A\cdot D^{(m-2)/2}\right) \quad (7)$$

where A is a constant depending on variables other than D. For a given insertion length and air velocity, from Eq. (6)

$$\Delta T_i \sim 1/\cosh\left(B\cdot V^{m/2}\right) \quad (8)$$

where B is a constant. From Eq. (7) we see ΔT_i relates to change in *diameter* as $D^{-0.767}$ and to change in *velocity* as $V^{0.233}$. That is, to reduce the immersion error decrease D and increase V (both cause $\overline{h}$ to increase!). Based upon the exponents of each parameter, however, diameter change is the more influential.

PROBLEM 7.60

KNOWN: Hot film sensor on a quartz rod maintained at $T_s = 50°C$.

FIND: (a) Compute and plot the convection coefficient as a function of velocity for water, $0.5 \le V_w \le 5$ m/s, and air, $1 \le V_a \le 20$ m/s with $T_\infty = 20°C$ and (b) Suitability of using the hot film sensor for the two fluids based upon Biot number considerations.

SCHEMATIC:

ASSUMPTIONS: (1) Cross-flow over a smooth cylinder, (2) Steady-state conditions, (3) Uniform surface temperature.

PROPERTIES: *Table A.6*, Water (T_f = 308 K, sat liquid); *Table A.4*, Air (T_f = 308 K, 1 atm).

ANALYSIS: (a) Using the *IHT Tool*, *Correlations*, *Cylinder*, along with the *Properties Tool* for *Air* and *Water*, results were obtained for the convection coefficients as a function of velocity.

(b) The Biot number, hD/2k, is the ratio of the internal to external thermal resistances. When Bi >> 1, the thin film is thermally coupled well to the fluid. When $Bi \le 1$, significant power from the heater is dissipated axially by conduction in the rod. The Biot numbers for the fluids as a function of velocity are shown below.

We conclude that the sensor is well suited for use with water, but not so for use with air.

Continued...

COMMENTS: A copy of the IHT workspace developed to generate the above plots is shown below.

```
// Problem 7.61

// Correlation Tool:  External Flow, Cylinder
/*  Correlation description: External cross flow (EF) over cylinder (CY), average coefficient, ReD*Pr>0.2,
Churchill-Bernstein correlation, Eq 7.57. See Table 7.9.  */
// Air flow  (a)
NuDbara = NuD_bar_EF_CY(ReDa,Pra)              // Eq 7.57
NuDbara = hDbara * D / ka
ReDa = Va * D / nua
// Evaluate properties at the film temperature, Tfa.
Tf = (Tinf + Ts) / 2
Bia = hDbara * D / (2 * k)                    // Biot number
// Properties Tool: Air
// Air property functions : From Table A.4
// Units: T(K); 1 atm pressure
nua = nu_T("Air",Tf)                          // Kinematic viscoty, m^2/s
ka = k_T("Air",Tf)                            // Thermal conductivity, W/m·K
Pra = Pr_T("Air",Tf)                          // Prandtl number

// Water flow (w)
NuDbarw = NuD_bar_EF_CY(ReDw,Prw)             // Eq 7.57
NuDbarw = hDbarw * D / kw
ReDw = Vw * D / nuw
// Evaluate properties at the film temperature, Tfw.
//Tfw = (Tinfw + Tsw) / 2
Biw = hDbarw * D / (2 * k )                   // Biot number
// Properties Tool: Water
// Water property functions :T dependence, From Table A.6
// Units: T(K), p(bars); x = quality (0=sat liquid or 1=sat vapor)
xf = 0
nuw = nu_Tx("Water",Tf,xf)                    // Kinematic viscosity, m^2/s
kw = k_Tx("Water",Tf,xf)                      // Thermal conductivity, W/m·K
Prw = Pr_Tx("Water",Tf,xf)                    // Prandtl number

// Assigned Variables:
Va = 1                                        // Air velocity, m/s; range 1 to 20 m/s
Vw = 0.5                                      // Water velocity, m/s; range 0.5 to 5 m/s
k = 1.4                                       // Thermal conductivity, W/m.K; quartz rod
D = 0.0015                                    // Diameter, m
Ts = 30 + 273                                 // Surface temperature, K
Tinf = 20 + 273                               // Fluid temperature, K

/* Solve, Explore and Graph:  After solving, separate Explore sweeps for 1 <= Va <= 20 and
0.5 <= Vw <= 5 m/s were performed saving results in different Data Sets.  Four separate
plot windows were generated.  */
```

PROBLEM 7.61

KNOWN: Diameter, temperature and heat flux of a hot-film sensor. Fluid temperature. Thickness and thermal conductivity of deposit.

FIND: (a) Fluid velocity, (b) Heat flux if sensor is coated by a deposit.

SCHEMATIC:

Water
V, $T_\infty = 17^oC$
$T_{s,hf} = 22^oC$
Deposit
$\delta = 0.1$ mm
$k_d = 2$ W/m-K
D = 1.5 mm
T_∞ — $1/\bar{h}$ — δ/k_d — $T_{s,hf}$ ← q''_{hf}

ASSUMPTIONS: (1) Steady-state, (2) Constant properties, (3) Thickness of hot film sensor is negligible, (4) Applicability of Churchill-Bernstein correlation for uniform surface heat flux, (5) $Re_D \ll 282,000$, (6) Deposit may be approximated as a plane layer.

PROPERTIES: *Table A-6*, water $(T_f = 292.5K): k = 0.602\ W/m \cdot K,\ \nu = 1.02 \times 10^{-6} m^2/s,\ Pr = 7.09.$

ANALYSIS: (a) With $Re_D << 282,000$ and $\bar{h} = q''_{hf}/(T_{s,hf} - T_\infty)$, Eq. (7.57) reduces to

$$\overline{Nu}_D = \frac{q''_{hf} D}{k(T_{s,hf} - T_\infty)} \approx 0.3 + \frac{0.62 Re_D^{1/2} Pr^{1/3}}{\left[1+(0.4/Pr)^{2/3}\right]^{1/4}} \qquad (1)$$

Substituting for D, $(T_{s,hf} - T_\infty)$, k and Pr,

$$4.98 \times 10^{-4} q''_{hf} \approx 0.3 + 1.15 Re_D^{1/2}$$

or, with $Re_D^{1/2} = (D/\nu)^{1/2} V^{1/2} = 38.3 V^{1/2}$,

$$4.98 \times 10^{-4} q''_{hf} \approx 0.3 + 44.1 V^{1/2} \qquad (2)$$

Substituting for q''_{hf},

$$V = 0.20\,m/s$$ <

(b) For a fixed value of $T_{s,hf}$, the thermal resistance of the deposit reduces q''_{hf}. From the thermal circuit.

$$q''_{hf} = \frac{T_{s,hf} - T_\infty}{(1/\bar{h}) + (\delta/k_d)}$$

Using Eq. (1) to evaluate $\bar{h}$,

Continued …..

$$\overline{h} \approx \frac{k}{(D+\delta)} \left\{ 0.3 + \frac{0.62\,Re_D^{1/2}\,Pr^{1/3}}{\left[1+(0.4/Pr)^{2/3}\right]^{1/4}} \right\}$$

where, with V = 0.20 m/s, $Re_D = V(D+\delta)/\nu = 314$, we obtain

$$\overline{h} \approx \frac{0.602\,W/m\cdot K}{0.0016m}\{20.7\} = 7,780\,W/m^2\cdot K$$

Hence, $$q''_{hf} = \frac{5°C}{\left(1.285\times10^{-4} + 0.5\times10^{-4}\right)m^2\cdot K/W} = 2.80\times10^4\,W/m^2 \quad <$$

With the foregoing heat flux applied to the sensor and use of the model for Part (a), the sensor would indicate a velocity predicted from Eq. (2), or

$$V = \left[\left(4.98\times10^{-4}\times2.80\times10^4 - 0.3\right)/44.1\right]^2 = 0.096\,m/s$$

The error in the velocity measurement is therefore

$$\%\,Error \equiv \frac{V_{(a)} - V_{(b)}}{V_{(a)}}(100\%) = \frac{0.20-0.096}{0.20}\times100 = 52\% \quad <$$

COMMENTS: (1) The accuracy of the hot-film sensor is strongly influenced by the deposit, and in any such application it is important to maintain a clean surface. (2) The Reynolds numbers are much less than 282,000 and assumption 5 is valid.

PROBLEM 7.62

KNOWN: Long coated plastic, 20-mm diameter rod, initially at a uniform temperature of T_i = 25°C, is suddenly exposed to the cross-flow of air at T_∞ = 350°C and V = 50 m/s.

FIND: (a) Time for the surface of the rod to reach 175°C, the temperature above which the special coating cures, and (b) Compute and plot the time-to-reach 175°C as a function of air velocity for 5 ≤ V ≤ 50 m/s.

SCHEMATIC:

$T(o) = T_i = 25\,^{o}C$
$T_\infty = 350\,^{o}C$
V = 50 m/s
Air
Plastic rod, D = 20 mm

ASSUMPTIONS: (a) One-dimensional, transient conduction in the rod, (2) Constant properties, and (3) Evaluate thermophysical properties at $T_f = [(T_s + T_i)/2 + T_\infty] = [(175 + 25)/2 + 350]$°C = 225°C = 500 K.

PROPERTIES: Rod (Given): ρ = 2200 kg/m^3, c = 800 J/kg·K, k = 1 W/m·K, α = k/ρc = 5.68 × 10^{-7} m^2/s; *Table A.4*, Air (T_f ≈ 500 K, 1 atm): ν = 38.79 × 10^{-6} m^2/s, k = 0.0407 W/m·K, Pr = 0.684.

ANALYSIS: (a) To determine whether the lumped capacitance method is valid, determine the Biot number

$$Bi_{lc} = \frac{\bar{h}(r_o/2)}{k} \qquad (1)$$

The convection coefficient can be estimated using the Churchill-Bernstein correlation, Eq. 7.57,

$$\overline{Nu}_D = \frac{\bar{h}D}{k} = 0.3 + \frac{0.63 Re_D^{1/2}\, Pr^{1/3}}{\left[1+(0.4/Pr)^{2/3}\right]^{1/4}}\left[1+\left(\frac{Re_D}{282,000}\right)^{5/8}\right]^{4/5}$$

$$Re_D = \frac{VD}{\nu} = 50\,m/s \times 0.020\,m \Big/ 38.79\times10^{-6}\,m^2\Big/s = 25,780$$

$$\bar{h} = \frac{0.0407\,W/m\cdot K}{0.020\,m}\left\{0.3 + \frac{0.63(25,780)^{1/2}(0.684)^{1/3}}{\left[1+(0.4/0.684)^{2/3}\right]^{1/4}}\left[1+\left(\frac{25,780}{282,000}\right)^{5/8}\right]^{4/5}\right\} = 184\ W/m^2\cdot K \quad (2)$$

Substituting for $\bar{h}$ from Eq. (2) into Eq. (1), find

$$Bi_{lc} = 184\,W\Big/m^2\cdot K\,(0.010\,m/2)\Big/1\,W/m\cdot K = 0.92 \qquad >> 0.1$$

Hence, the lumped capacitance method is inappropriate. Using the one-term series approximation, Section 5.6.2, Eqs. 5.49 with Table 5.1,

$$\theta^* = C_1 \exp\left(-\zeta_1^2 Fo\right) J_o\left(\zeta_1 r^*\right) \qquad r^* = r/r_o = 1$$

$$\theta^* = \frac{T(r_o,t) - T_\infty}{T_i - T_\infty} = \frac{(175-350)^\circ C}{(25-350)^\circ C} = 0.54$$

$$Bi = \bar{h}r_o/k = 1.84 \qquad \zeta_1 = 1.5308\ rad \qquad C_1 = 1.3384$$

Continued...

PROBLEM 7.62 (Cont.)

$0.54 = 1.3384\exp[-(1.5308\text{rad})^2 Fo] J_o(1.5308 \times 1)$

Using Table B.4 to evaluate $J_o(1.5308) = 0.4944$, find Fo = 0.0863 where

$$Fo = \frac{\alpha t_o}{r_o^2} = \frac{5.68\times10^{-7}\ \text{m}^2/\text{s}\times t_o}{(0.010\,\text{m})^2} = 5.68\times10^{-3} t_o \qquad (6)$$

$t_o = 15.2\text{s}$ <

(b) Using the *IHT Model*, *Transient Conduction*, *Cylinder*, and the *Tool*, *Correlations*, *External Flow*, *Cylinder*, results for the time-to-reach a surface temperature of 175°C as a function of air velocity V are plotted below.

COMMENTS: (1) Using the *IHT Tool*, *Correlations*, *External Flow*, *Cylinder*, the effect of the film temperature T_f on the estimated convection coefficient with V = 50 m/s can be readily evaluated.

T_f (K)	460	500	623
$\bar{h}$ (W/m²·K)	187	184	176

At early times, $\bar{h}$ = 184 W/m²·K is a good estimate, while as the cylinder temperature approaches the airsteam temperature, the effect starts to be noticeable (10% decrease).

(2) The IHT analysis performed for part (b) was developed in two parts. Using a known value for $\bar{h}$, the *Transient Conduction*, *Cylinder Model* was tested. Separately, the *Correlation Tools* was assembled and tested. Then, the two files were merged to give the workspace for determining the time-to-reach 175°C as a function of velocity V.

PROBLEM 7.63

KNOWN: Velocity, diameter, initial temperature and properties of extruded wire. Temperature and velocity of air. Temperature of surroundings.

FIND: (a) Differential equation for temperature distribution T(x), (b) Exact solution for negligible radiation and corresponding value of temperature at prescribed length of wire, (c) Effect of radiation on temperature of wire at prescribed length. Effect of wire velocity and emissivity on temperature distribution.

SCHEMATIC:

D = 5 mm
dq_{conv}
dq_{rad}
$T_{sur} = 25^oC$
$V_e = 0.2$ m/s
dx
Air
$T_\infty = 25^oC$
V = 5 m/s

ASSUMPTIONS: (1) Negligible variation of wire temperature in radial direction, (2) Negligible effect of axial conduction along the wire, (3) Constant properties, (4) Radiation exchange between small surface and large enclosure, (5) Motion of wire has a negligible effect on the convection coefficient ($V_e << V$).

PROPERTIES: Prescribed. Copper: $\rho = 8900\,\text{kg/m}^3$, $c_p = 400\,\text{J/kg}\cdot\text{K}$, $\varepsilon = 0.55$. Air: $k = 0.037\,\text{W/m}\cdot\text{K}$, $\nu = 3\times10^{-5}\,\text{m}^2/\text{s}$, $\text{Pr} = 0.69$.

ANALYSIS: (a) Applying conservation of energy to a stationary control surface, through which the wire moves, steady-state conditions exist and $\dot{E}_{in} - \dot{E}_{out} = 0$. Hence, with *inflow* due to *advection* and *outflow* due to *advection*, *convection* and *radiation*,

$$\rho V_e A_c c_p T - \rho V_e A_c c_p (T + dT) - dq_{conv} - dq_{rad} = 0$$

$$-\rho V_e \left(\pi D^2/4\right) c_p dT - \pi D dx\left[\bar{h}(T - T_\infty) + \varepsilon\sigma\left(T^4 - T_{sur}^4\right)\right] = 0$$

$$\frac{dT}{dx} = -\frac{4}{\rho V_e D c_p}\left[\bar{h}(T - T_\infty) + \varepsilon\sigma\left(T^4 - T_{sur}^4\right)\right] \qquad (1)$$

<

Alternatively, if the control surface is fixed to the wire, conditions are transient and the energy balance is of the form, $-\dot{E}_{out} = \dot{E}_{st}$, or

$$-\pi D\,dx\left[\bar{h}(T - T_\infty) + \varepsilon\sigma\left(T^4 - T_{sur}^4\right)\right] = \rho\left(\frac{\pi D^2}{4}dx\right)c_p\frac{dT}{dt}$$

$$\frac{dT}{dt} = -\frac{4}{\rho D c_p}\left[\bar{h}(T - T_\infty) + \varepsilon\sigma\left(T^4 - T_{sur}^4\right)\right]$$

Dividing the left- and right-hand sides of the equation by dx/dt and $V_e = dx/dt$, respectively, Eq. (1) is obtained.

(b) Neglecting radiation, separating variables and integrating, Eq. (1) becomes

$$\int_{T_i}^{T}\frac{dT}{T - T_\infty} = -\frac{4\bar{h}}{\rho V_e D c_p}\int_0^x dx$$

Continued …..

PROBLEM 7.63 (Cont.)

$$\ln\left(\frac{T-T_\infty}{T_i-T_\infty}\right)=-\frac{4\bar{h}\,x}{\rho V_e\, D\, c_p}$$

$$T=T_\infty+\left(T_i-T_\infty\right)\exp\left(-\frac{4\bar{h}x}{\rho V_e\, D\, c_p}\right) \qquad (2) \qquad <$$

With $Re_D = VD/\nu = 5\,m/s \times 0.005\,m/3\times10^{-5}\,m^2/s = 833$, the Churchill-Bernstein correlation yields

$$\overline{Nu}_D = 0.3+\frac{0.62(833)^{1/2}(0.69)^{1/3}}{\left[1+(0.4/0.69)^{2/3}\right]^{1/4}}\left[1+\left(\frac{833}{282{,}000}\right)^{5/8}\right]^{4/5} = 14.4$$

$$\bar{h}=\frac{k}{D}\overline{Nu}_D=\frac{0.037\,W/m\cdot K}{0.005m}14.4=107\,W/m^2\cdot K$$

Hence, applying Eq. (2) at x = L,

$$T_o = 25°C+\left(575°C\right)\exp\left(-\frac{4\times107\,W/m^2\cdot K\times5m}{8900\,kg/m^3\times0.2\,m/s\times0.005m\times400\,J/kg\cdot K}\right)$$

$$T_o = 340°C \qquad <$$

(c) Using the DER function of IHT, Eq. (1) may be numerically integrated from x = 0 to x = L = 5.0m to obtain

$$T_o = 309°C \qquad <$$

Hence, radiation makes a discernable contribution to cooling of the wire. IHT was also used to obtain the following distributions.

Wire temperature, T(C)

Distance from extruder exit, x(m)

Ve=0.5 m/s
Ve=0.2 m/s
Ve=0.1 m/s

Wire temperature, T(C)

Distance from extruder exit, x(m)

eps=0.8
eps=0.55
eps=0

The speed with which the wire is drawn from the extruder has a significant influence on the temperature distribution. The temperature decay decreases with increasing V_e due to the increasing effect of advection on energy transfer in the x direction. The effect of the surface emissivity is less pronounced, although, as expected, the temperature decay becomes more pronounced with increasing ε.

COMMENTS: (1) A critical parameter in wire extrusion processes is the *coiling temperature*, that is, the temperature at which the wire may be safely coiled for subsequent storage or shipment. The larger the production rate (V_e), the longer the cooling distance needed to achieve a desired coiling temperature. (2) Cooling may be enhanced by increasing the cross-flow velocity, and the specific effect of V may also be explored.

PROBLEM 7.64

KNOWN: Experimental apparatus comprised of a flat plate subjected to an airstream in parallel flow. Electrical patch heater on backside dissipates 15.5 W for all conditions. Pin fins fabricated from brass with prescribed diameter and length can be firmly attached to the plate. Fin tip and base temperatures observed for five different configurations (N, number of fins).

FIND: (a) The thermal resistance between the plate and airstream for the five configurations, (b) Model of the plate-fin system using appropriate convection correlations to predict the thermal resistances for the five configurations; compare predictions and observations; explain differences, and (b) Predict thermal resistances when the airstream velocity is doubled.

SCHEMATIC:

Experimental observations:

N	T_{tip} (°C)	T_s (°C)
0	--	70.2
1	40.6	67.4
2	39.5	64.7
5	36.4	57.4
8	34.2	52.1

ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible effect of flow interactions between pins, (3) Negligible radiation exchange with surroundings, (4) All heater power is transferred to airstream, and (5) Constant properties.

PROPERTIES: *Table A.4*, Air (T_f = 310 K, 1 atm): k = 0.0270 W/m·K, $\nu = 1.69 \times 10^{-5}$ m²/s, Pr = 0.706; *Table A.1*, Brass (T = 300 K): k = 110 W/m·K.

ANALYSIS: (a) The thermal resistance between the plate and the airstream is defined as

$$R_{tot} = \frac{T_s - T_\infty}{q} \qquad (1)$$

The heat rate is 15.6 W for all configurations and using T_s values from the above table with T_∞ = 20°C, find

Continued...

N	0	1	2	5	8
R_{tot} (K/W)	3.24	3.06	2.88	2.41	2.07

<

(b) The thermal resistance of the plate-fin system can be expressed as

$$R_{tot} = [1/R_{base} + N/R_{fin}]^{-1} \tag{2}$$

where the thermal resistance of the exposed portion of the base, A_b, is

$$R_{base} = \frac{1}{\bar{h}_b A_b} \tag{3}$$

$$A_b = A_s - NA_c \tag{4}$$

where the A_c is the cross-sectional area of a fin and A_s is the plate surface area. Approximating the airstream over the plate as parallel flow over a plate, use the *IHT Correlation Tool*, *External Flow*, *Flat Plate* assuming the flow is turbulated by the leading edge, to find

$$\bar{h}_b = 51\,\mathrm{W/m^2 \cdot K}\,.$$

From the experimental observation with no fins (N = 0), the convection coefficient was measured as

$$\bar{h}_{b,cxp} = \frac{q}{A_s\left(T_s - T_\infty\right)} = \frac{15.5\,\mathrm{W}}{(0.0259\,\mathrm{m})^2\,(70.2-20)^\circ\,\mathrm{C}} = 460\,\mathrm{W/m^2 \cdot K}$$

Since the predicted coefficient is nearly an order of magnitude lower, we chose to use the experimental value in our subsequent analyses to predict overall system thermal resistance.

Approximating the airstream over a pin fin as cross-flow over a cylinder, use the *IHT Correlation Tool*, *External Flow*, *Cylinder* to find

$$\bar{h}_{fin} = 118\,\mathrm{W/m^2 \cdot K}\,.$$

Using the *IHT Extended Surface Model* for the *Rectangular Pin Fin* (*Temperature Distribution and Heat Rate*) with a convection tip condition, the following fin thermal resistance was found as

$$R_{fin} = 25.4\,\mathrm{K/W}$$

Using the foregoing values for R_{fin} and $\bar{h}_b$, the thermal resistances of the plate-fin system are tabulated below.

N	0	1	2	4	8
R_{base} (K/W)	3.241	3.331	3.426	3.746	4.133
R_{fin} (K/W)	--	25.4	12.7	5.08	3.18
R_{tot} (K/W)	3.24	2.95	2.70	2.16	1.80

<

By comparison with the experimental results of part (a), note that we assured agreement for the N = 0 condition by using the measured rather than estimated (correlation) convection coefficient. The predicted thermal resistances are systematically lower than the experimental values, with the worst case (N = 8) being 13% lower.

Continued...

(c) The effect of doubling the velocity, from u_∞ = 6 to 12 m/s, will cause the fin convection coefficient to increase from $\overline{h}_{fin}$ = 118 to 169 W/m^2·K. For the base convection coefficient, we'll assume the flow is fully turbulent so that $\overline{h} \sim (u_\infty)^{0.8}$ according to Eq. 7.41, hence

$$\overline{h}_b\left(12\,\text{m/s}\right) = \overline{h}_b\left(6\,\text{m/s}\right)\left(\frac{12}{6}\right)^{0.8} = 460\,\text{W/m}^2\cdot\text{K}\,(2)^{0.8} = 800\,\text{W/m}^2\cdot\text{K}$$

Using the same procedure as above, find

N	0	1	2	4	8
R_{base} (K/W)	1.863	1.915	1.970	2.154	2.376
R_{fin} (K/W)	--	18.96	9.480	4.740	2.370
R_{tot} (K/W)	1.86	1.74	1.63	1.48	1.19

The effect of doubling the airstream velocity is to reduce the thermal resistance by approximately 35%.

PROBLEM 7.65

KNOWN: Temperature and velocity of water flowing over a sphere of prescribed temperature and diameter.

FIND: (a) Drag force, (b) Rate of heat transfer.

SCHEMATIC:

Water
$T_\infty = 20°C$
$V = 5\,m/s$
$D = 0.02\,m$
$T_s = 60°C$
F_D
q

ASSUMPTIONS: (1) Steady-state conditions, (2) Uniform surface temperature.

PROPERTIES: *Table A-6*, Saturated Water (T_∞ = 293K): ρ = 998 kg/m^3, $\mu = 1007 \times 10^{-6}$ N·s/m^2, k = 0.603 W/m·K, Pr = 7.00; (T_s = 333 K): $\mu = 467 \times 10^{-6}$ N·s/m^2; (T_f = 313 K): ρ = 992 kg/m^3, $\mu = 657 \times 10^{-6}$ N·s/m^2.

ANALYSIS: (a) Evaluating μ and ρ at the film temperature,

$$Re_D = \frac{\rho VD}{\mu} = \frac{\left(992\text{ kg/m}^3\right)5\text{ m/s}\ (0.02\text{ m})}{657\times10^{-6}\text{ N}\cdot\text{s/m}^2} = 1.51\times10^5$$

and from Fig. 7.8, C_D = 0.42. Hence

$$F_D = C_D\frac{\pi D^2}{4}\rho\frac{V^2}{2} = 0.42\frac{\pi\,(0.02\text{ m})^2}{4}992\frac{\text{kg}}{\text{m}^3}\frac{(5\text{ m/s})^2}{2} = 1.64\text{ N.} \qquad <$$

(b) With the Reynolds number evaluated at the free stream temperature,

$$Re_D = \frac{\rho VD}{\mu} = \frac{998\text{ kg/m}^3(5\text{ m/s})\ (0.02\text{ m})}{1007\times10^{-6}\text{ N}\cdot\text{s/m}^2} = 9.91\times10^4$$

it follows from the Whitaker relation that

$$\overline{Nu}_D = 2 + \left[0.4Re_D^{1/2} + 0.06Re_D^{2/3}\right]Pr^{0.4}\left(\frac{\mu}{\mu_s}\right)^{1/4}$$

$$\overline{Nu}_D = 2 + \left[0.4\left(9.91\times10^4\right)^{1/2} + 0.06\left(9.91\times10^4\right)^{2/3}\right](7.0)^{0.4}\left(\frac{1007}{467}\right)^{1/4} = 673.$$

Hence, the convection coefficient and heat rate are

$$\overline{h} = \frac{k}{D}\overline{Nu}_D = \frac{0.603\text{ W/m}\cdot\text{K}}{0.02\text{ m}}673 = 20{,}300\text{ W/m}^2\cdot\text{K}$$

$$q = \overline{h}\left(\pi D^2\right)\ \left(T_s - T_\infty\right) = 20{,}300\frac{\text{W}}{\text{m}^2\cdot\text{K}}\pi\ (0.02\text{ m})^2\ \ (60-20)^\circ\text{C} = 1020\text{ W.} \qquad <$$

COMMENTS: Compare the foregoing value of $\overline{h}$ with that obtained in the text example under similar conditions. The significant increase in $\overline{h}$ is due to the much larger value of k and smaller value of ν for the water. Note that Re_D is slightly beyond the range of the correlation.

PROBLEM 7.66

KNOWN: Temperature and velocity of air flow over a sphere of prescribed surface temperature and diameter.

FIND: (a) Drag force, (b) Heat transfer rate with air velocity of 25 m/s; and (c) Compute and plot the heat rate as a function of air velocity for the range $1 \le V \le 25$ m/s.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Uniform surface temperature, (3) Negligible radiation exchange with surroundings.

PROPERTIES: *Table A.4*, Air (T_∞ = 298 K, 1 atm): $\mu = 184 \times 10^{-7}$ N·s/m^2; $\nu = 15.71 \times 10^{-6}$ m^2/s, k = 0.0261 W/m·K, Pr = 0.71; (T_s = 348 K): $\mu = 208 \times 10^{-7}$ N·s/m^2; (T_f = 323 K): $\nu = 18.2 \times 10^{-6}$ m^2/s, ρ = 1.085 kg/m^3.

ANALYSIS: (a) Working with properties evaluated at T_f

$$Re_D = \frac{VD}{\nu} = \frac{25\,m/s\,(0.01\,m)}{18.2\times10^{-6}\,m^2/s} = 1.37\times10^4$$

and from Fig. 7.8, find $C_D \approx 0.4$. Hence

$$F_D = C_D\left(\pi D^2/4\right)\left(\rho V^2/2\right) = 0.4(\pi/4)(0.01\,m)^2\,1.085\,kg/m^3\,(25\,m/s)^2/2 = 0.011\,N \quad <$$

(b) With

$$Re_D = \frac{VD}{\nu} = \frac{25\,m/s\,(0.01\,m)}{15.71\times10^{-6}\,m^2/s} = 1.59\times10^4$$

it follows from the Whitaker relation that

$$\overline{Nu}_D = 2 + \left[0.4\,Re_D^{1/2} + 0.06\,Re_D^{2/3}\right]Pr^{0.4}\left(\frac{\mu}{\mu_s}\right)^{1/4}$$

$$\overline{Nu}_D = 2 + \left[0.4\left(1.59\times10^4\right)^{1/2} + 0.06\left(1.59\times10^4\right)^{2/3}\right](0.71)^{0.4}\left(\frac{184}{208}\right)^{1/4} = 76.7$$

Hence, the convection coefficient and convection heat rate are

$$\bar{h} = \overline{Nu}_D\,\frac{k}{D} = 76.7\,\frac{0.0261\,W/m\cdot K}{0.01\,m} = 200\,W/m^2\cdot K$$

$$q = \bar{h}\pi D^2\left(T_s - T_\infty\right) = 200\,W/m^2\cdot K\times\pi\,(0.01\,m)^2\,(75-25)^\circ C = 3.14\,W \quad <$$

Continued...

PROBLEM 7.66 (Cont.)

(c) Using the *IHT Correlation Tool*, *External Flow*, *Sphere*, the average coefficient and heat rate were calculated and are plotted below.

Convection coefficient, hbar (W/m^2.K)

Air velocity, V (m/s)

COMMENTS: (1) A copy of the IHT Workspace used to generate the above plot is shown below.

```
// Correlation Tool - External Flow, Sphere:
NuDbar = NuL_bar_EF_SP(ReD,Pr,mu,mus)     // Eq 7.58
NuDbar = hbar * D / k
ReD = V * D / nu
/* Evaluate properties at Tinf and the surface temperature, Ts. */

/*  Correlation description: External flow (EF) over a sphere (SP), average coefficient, 3.5<ReD<7.6x10^4,
0.71<Pr<380, 1.0<(mu/mus)<3.2, Whitaker correlation, Eq 7.59. See Table 7.9.  */

// Properties Tool - Air:
// Air property functions : From Table A.4
// Units: T(K); 1 atm pressure
mu = mu_T("Air",Tinf)              // Viscosity,  N·s/m^2
mus = mu_T("Air",Ts)               // Viscosity,  N·s/m^2
nu = nu_T("Air",Tinf)              // Kinematic viscosity, m^2/s
k = k_T("Air",Tinf)                          // Thermal conductivity, W/m·K
Pr = Pr_T("Air",Tinf)              // Prandtl number

// Heat Rate Equation:
q = hbar * pi * D^2 * (Ts - Tinf)

// Assigned Variables:
D = 0.01                           // Sphere diameter, m
Ts = 75 + 273                      // Surface temperature, K
V = 25                             // Airstream velocity, m/s
Tinf = 25 + 273                    // Airstream temperature, K
```

PROBLEM 7.67

KNOWN: Sphere with a diameter of 20 mm and a surface temperature of 60°C that is immersed in a fluid at a temperature of 30°C with a velocity of 2.5 m/s.

FIND: The drag force and the heat rate when the fluid is (a) water and (b) air at atmospheric pressure. Explain why the results for the two fluids are so different.

SCHEMATIC:

Water
$T_\infty = 30^oC$
V = 2.5 m/s
Air
$q_{cv,w}$
$q_{cv,a}$
Sphere
$T_s = 60^oC$
D = 20 mm
$F_{D,w}$ or $F_{D,a}$

ASSUMPTIONS: (1) Flow over a smooth sphere, (2) Constant properties.

PROPERTIES: *Table A-6*, Water (T_∞ = 30°C = 303 K): $\mu = 8.034 \times 10^{-4}$ N·s/m^2, $\nu = 8.068 \times 10^{-7}$ m^2/s, k = 0.6172 W/m·K, Pr = 5.45; Water (T_s = 333 K): $\mu_s = 4.674 \times 10^{-4}$ N·s/m^2; *Table A-4*, Air (T_∞ = 30°C = 303 K, 1 atm): $\mu = 1.86 \times 10^{-5}$ N·s/m^2, $\nu = 1.619 \times 10^{-5}$ m^2/s, k = 0.0265 W/m·K, Pr = 0.707; Air (T_∞ = 333 K): $\mu_s = 2.002 \times 10^{-5}$ N·s/m^2.

ANALYSIS: The drag force, F_o, for the sphere is determined from the drag coefficient, Eq. 7.54,

$$C_D = \frac{F_D}{A_f\left(\rho V^2/2\right)}$$

where $A_f = \pi D^2/4$ is the frontal area. C_D is a function of the Reynolds number $Re_D = VD/\nu$ as represented in Figure 7.8. For the convection rate equation**,**

$$q = \overline{h}_D A_s \left(T_s - T_\infty\right)$$

where $A_s = \pi D^2$ is the surface area and the convection coefficient is estimated using the Whitaker correlation, Eq. 7.59,

$$\overline{Nu}_D = 2 + \left[0.4 Re_D^{1/2} + 0.06 Re_D^{2/3}\right] Pr^{0.4} \left(\mu/\mu_s\right)^{1/4}$$

where all properties except μ_s are evaluated at T_∞. For convenience we will evaluate properties required for the drag force at T_∞. The results of the analyses for the two fluids are tabulated below.

Fluid	Re_D	C_D	F_D (N)	$\overline{Nu}_D$	$\overline{h}_D\left(W/m^2\cdot K\right)$	q(W)
water	6.198×10^4	0.5	0.489	439	13,540	510
air	3.088×10^3	0.4	0.452×10^{-3}	31.9	42.3	1.59

The frontal and surface areas, respectively, are $A_f = 3.142 \times 10^{-4}$ m^2 and $A_s = 1.257 \times 10^{-3}$ m^2.

COMMENTS: The Reynolds number is the ratio of inertia to viscous forces. We associate higher viscous shear and heat transfer with larger Reynolds numbers. The drag force also depends upon the fluid density, which further explains why F_D for water is much larger, by a factor of 1000, than for air. Nu_D is dependent upon Re_D^n where n is 1/2 to 2/3, and represents the dimensionless temperature gradient at the surface. Since the thermal conductivity of water is nearly 20 times that of air, we expect a significant difference between $\overline{h}_D$ and q for the two fluids.

PROBLEM 7.68

KNOWN: Conditions associated with airflow over a spherical light bulb of prescribed diameter and surface temperature.

FIND: Heat loss by convection.

SCHEMATIC:

Air
V = 0.5 m/s
T_∞ = 25°C
p = 1 atm
D = 0.05 m
T_s = 140°C
50-W bulb,
$A = 4\pi r^2 = \pi D^2$
q

ASSUMPTIONS: (1) Steady-state conditions, (2) Uniform surface temperature.

PROPERTIES: *Table A-4*, Air (T_f = 25°C, 1 atm): $\nu = 15.71 \times 10^{-6}$ m^2/s, k = 0.0261 W/m·K, Pr = 0.71, $\mu = 183.6 \times 10^{-7}$ N·s/m^2; *Table A-4*, Air (T_s = 140°C, 1 atm): $\mu = 235.5 \times 10^{-7}$ N·s/m^2.

ANALYSIS: The heat rate by convection is

$$q = \overline{h}\left(\pi D^2\right)\ \left(T_s - T_\infty\right)$$

where $\overline{h}$ may be estimated from the Whitaker relation

$$\overline{h} = \frac{k}{D}\left[2 + \left(0.4\ Re_D^{1/2} + 0.06\ Re_D^{2/3}\right)\ Pr^{0.4}\left(\mu/\mu_s\right)^{1/4}\right]$$

where

$$Re_D = \frac{VD}{\nu} = \frac{0.5\text{ m/s} \times 0.05\text{ m}}{15.71 \times 10^{-6}\text{ m}^2/\text{s}} = 1591.$$

Hence,

$$\overline{h} = \frac{0.0261\text{ W/m}\cdot\text{K}}{0.05\text{ m}}\ \left\{2 + \left[0.4(1591)^{1/2} + 0.06(1591)^{2/3}\right]\ (0.71)^{0.4}\left(\frac{183.6}{235.5}\right)^{1/4}\right\}$$

$$\overline{h} = 11.4\text{ W/m}^2\cdot\text{K}$$

and the heat rate is

$$q = 11.4\frac{\text{W}}{\text{m}^2\cdot\text{K}}\pi(0.05\text{ m})^2\ \ (140-25)^\circ\text{C} = 10.3\text{ W}.$$ <

COMMENTS: (1) The low value of $\overline{h}$ suggests that heat transfer by free convection may be significant and hence that the total loss by convection exceeds 10.3 W.

(2) The surface of the bulb also dissipates heat to the surrounding by radiation. Further, in an actual light bulb, there is also heat loss by conduction through the socket.

PROBLEM 7.69

KNOWN: Diameter, properties and initial temperature of niobium sphere. Velocity and temperature of nitrogen. Temperature of surroundings.

FIND: (a) Time for sphere to cool to prescribed temperature if radiation is neglected, (b) Cooling time if radiation is considered. Effect of flow velocity.

SCHEMATIC:

T_{sur} = 25°C

Nitrogen

T_∞ = 25°C

V = 5 m/s

Niobium sphere, D = 10 mm

T_i = 900°C, T_f = 300°C

ρ = 8600 kg/m³ c = 290 J/kg-K

k = 63 W/m-K ε = 0.6

ASSUMPTIONS: (1) Lumped capacitance method is valid, (2) Constant properties, (3) Radiation exchange with large surroundings.

PROPERTIES: *Table A-4*, nitrogen $(T_\infty = 298\text{K})$: $\mu = 177\times10^{-7}\ \text{N}\cdot\text{s/m}^2$, $\nu = 15.7\times10^{-6}\text{m}^2/\text{s}$, $k = 0.0257\ \text{W/m}\cdot\text{K}$, $\Pr = 0.716$. *Table A-4*, nitrogen $(\bar{T}_s = 873\text{K})$: $\mu_s = 368\times10^{-7}\ \text{N}\cdot\text{s/m}^2$.

ANALYSIS: (a) Neglecting radiation, the cooling time may be determined from Eq. (5.5),

$$t = \frac{\rho\left(\pi D^3/6\right)c}{\bar{h}\pi D^2}\ln\frac{\theta_i}{\theta} = \frac{\rho c D}{6\bar{h}}\ln\frac{T_i - T_\infty}{T_f - T_\infty}$$

The convection coefficient is obtained from the Whitaker correlation with $Re_D = VD/\nu$ $= 5\,\text{m/s}\times0.01\text{m}/15.7\times10^{-6}\text{m}^2/\text{s} = 3185$. Hence,

$$\overline{Nu}_D = \left(\bar{h}D/k\right) = 2 + \left(0.4\,Re_D^{1/2} + 0.06\,Re_D^{2/3}\right)Pr^{0.4}\left(\mu/\mu_s\right)^{1/4}$$

$$\bar{h} = \frac{0.0257\ \text{W/m}\cdot\text{K}}{0.01\text{m}}\left\{2 + \left[0.4\left(3185\right)^{1/2} + 0.06\left(3185\right)^{2/3}\right]\left(0.716\right)^{0.4}\left(\frac{177}{368}\right)^{0.25}\right\} = 71.8\ \text{W/m}^2\cdot\text{K}$$

$$t = \frac{8600\,\text{kg/m}^3\times290\,\text{J/kg}\cdot\text{K}\times0.01\text{m}}{6\times71.8\,\text{W/m}^2\cdot\text{K}}\ln\frac{(900-25)}{(300-25)} = 67\,\text{s}$$ <

(b) If the effect of radiation is considered, the cooling time can be obtained by integrating Eq. (5.15). With $A_s/V = \pi D^2/\left(\pi D^3/6\right) = 6/D$, the appropriate form of the equation is

$$\frac{dT}{dt} = -\frac{6}{\rho c D}\left[\bar{h}\left(T - T_\infty\right) + \varepsilon\sigma\left(T^4 - T_{sur}^4\right)\right]$$

Using the DER function of IHT to integrate this equation over the limits from $T_i = 1173\text{K}$ to $T_f = 573\text{K}$, we obtain

$$t = 48\,\text{s}$$ <

Continued …..

PROBLEM 7.69 (Cont.)

For V = 1.0 and 25.0 m/s, the cooling times are t ≈ 80 and 24 s, respectively. Temperature histories for the three velocities are shown below.

COMMENTS: The cooling time is significantly affected by the flow velocity.

PROBLEM 7.70

KNOWN: An underwater instrument pod having a spherical shape with a diameter of 85 mm dissipating 300 W.

FIND: Estimate the surface temperature of the pod for these conditions: (a) when submersed in a bay where the water temperature is 15°C and the current is 1 m/s, and (b) after being hauled out of the water *without deactivating the power* and suspended in the ambient where the air temperature is 15°C and the wind speed is 3 m/s.

SCHEMATIC:

Water
T_∞ = 15°C
V = 1 m/s
q_{cv}
$T_{s,w}$
Instrument pod
D = 85 mm
P_e = 300 W

Ambient air
T_∞ = 15°C
V = 3 m/s
q_{cv}
$T_{s,a}$
P_e

ASSUMPTIONS: (1) Steady-state conditions, (2) Flow over a smooth sphere, (3) Uniform surface temperatures, (4) Negligible radiation heat transfer for air (a) condition, and (5) Constant properties.

PROPERTIES: *Table A-6*, Water (T_∞ = 15°C = 288 K): μ = 0.001053 $N\cdot s/m^2$, $\nu = 1.139 \times 10^{-6}$ m^2/s, k = 0.5948 W/m·K, Pr = 8.06; *Table A-4*, Air (T_∞ = 288 K, 1 atm): $\mu = 1.788 \times 10^{-5}$ $N\cdot s/m^2$, $\nu = 1.482 \times 10^{-5}$ m^2/s, k = 0.02534 W/m·K; Air (T_s = 945 K): $\mu_s = 4.099 \times 10^{-5}$ $N\cdot s/m^2$, Pr = 0.710.

ANALYSIS: The energy balance for the submersed-in-water (w) and suspended-in-air (a) conditions are represented in the schematics above and have the form

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = -q_{cv} + P_e = 0 \tag{1}$$

$$-\overline{h}_D A_s (T_s - T_\infty) + P_e = 0$$

where $A_s = \pi D^2$ and $\overline{h}_D$ is estimated using the Whitaker correlation, Eq. 7.59,

$$\overline{Nu}_D = 2 + \left[0.4 Re_D^{1/2} + 0.06 Re_D^{2/3}\right] Pr^{0.4} (\mu / \mu_s)^{1/4} \tag{2}$$

where all properties except μ_s are evaluated at T_∞. The results are tabulated below.

Condition	Re_D	$\overline{Nu}_D$	$\overline{h}_D$ ($W/m^2\cdot K$)	T_s (°C)
(w) water	7.465×10^4	509	3559	18.7
(a) air	1.72×10^4	67.5	20.1	672

COMMENTS: (1) While submerged and dissipating 300 W, the pod is safely operating at a temperature slightly above that of the water. When hauled from the water and suspended in air, the pod temperature increases to a destruction temperature (672°C). The pod gets smoked!

(2) The assumption that $\mu/\mu_s \approx 1$ is appropriate for the water (w) condition. For the air (a) condition, $\mu/\mu_s = 0.436$ and the final term of the correlation is significant. Recognize that radiation exchange with the surroundings for the air condition should be considered for an improved estimate.

Continued …..

PROBLEM 7.70 (Cont.)

(3) Why such a difference in T_s for the water (w) and air (a) conditions? From the results table note that the Re_D, Nu_D, and $\bar{h}_D$ are, respectively, 4x, 7x and 170x times larger for water compared to air. Water, because of its thermophysical properties which drive the magnitude of $\bar{h}_D$, is a much better coolant than air for similar flow conditions.

```
/* Comment:  Because Ts is much larger than Tinf for the in-air operation, the ratio of mu / mus
exceeds the limits for the correlation.  Hence, a warning message comes with the IHT solution.  */

/* Results  - operation in air
As      NuDbar   Pr       ReD      Tinf     Ts       Ts_C     hbar     k        mu
        mus      nu       D        Pelec    Tinf_C   V
0.0227  67.5     0.7101   1.72E4   288      944.8    671.8    20.12    0.02534  1.786E-5
        4.099E-5 1.482E-5 0.085    300      15       3  */

// Correlation, sphere
NuDbar = NuL_bar_EF_SP(ReD,Pr,mu,mus)     // Eq 7.59
NuDbar = hbar * D / k
ReD = V * D / nu
/* All properties except mus are evaluated at Tinf. */
/*  Correlation description: External flow (EF) over a sphere (SP), average coefficient,
3.5<ReD<7.6x10^4, 0.71<Pr<380, 1.0<(mu/mus)<3.2, Whitaker correlation, Eq 7.59. See Table 7.9.  */

// Energy balance
Pelec - hbar * As * (Ts - Tinf) = 0
As = pi * D^2

// Input variables
D = 0.085
//V = 1.0                // Water current
V = 3                    // Wind speed
Tinf_C = 15
Pelec = 300

// Conversions
Tinf = Tinf_C + 273
Ts = Ts_C + 273

// Air property functions : From Table A.4
// Units: T(K); 1 atm pressure
mu = mu_T("Air",Tinf)         // Viscosity,  N·s/m^2
mus = mu_T("Air",Ts)          // Viscosity,  N·s/m^2
// mus = mu
nu = nu_T("Air",Tinf)         // Kinematic viscosity, m^2/s
k = k_T("Air",Tinf)           // Thermal conductivity, W/m·K
Pr = Pr_T("Air",Tinf)         // Prandtl number
```

PROBLEM 7.71

KNOWN: Air cooling requirements for lead pellets in the molten slate.

FIND: Height of tower from which pellets must be dropped to convert from liquid to solid state.

SCHEMATIC:

Lead pellet, ρ_ℓ, h_{sf}; F_g; F_D; $D = 3$ mm; T_s; q_{conv}; E_s; V; Air ρ_a, ν, k, $T_\infty = 15°C$

ASSUMPTIONS: (1) Pellet remains at melting point temperature throughout process, (2) Density of lead, ρ_ℓ, remains constant (at density of molten lead) throughout process, (3) Radiation effects are negligible.

PROPERTIES: *Table A-7*, Lead (M.P. = T_s = 327.2°C): $\rho_\ell \approx 10{,}600\ \text{kg/m}^3$; *Handbook Chemistry and Physics*: Latent heat of fusion, h_{sf} = 24.5 kJ/kg; *Table A-4*, Air (T_∞ = 15°C): ρ_a = 1.22 kg/m^3, $\nu = 14.8 \times 10^{-6}$ m^2/s, $k = 25.3 \times 10^{-3}$ W/m·K, Pr = 0.71, $\mu = 178.6 \times 10^{-7}$ N·s/m^2; (T_s = 327°C): $\mu_s = 306 \times 10^{-7}$ N·s/m^2.

ANALYSIS: Conservation of energy dictates that the energy released to solidification must be given off to the air by convection. Applying the conservation of energy requirement on a time interval basis,

$$-E_{out} = \Delta E_{st} \qquad \text{where} \qquad E_{out} = q_{conv} \cdot t_s$$

and t_s is the time required to completely solidify a pellet. Hence,

$$-\bar{h}\left(\pi D^2\right)\left(T_s - T_\infty\right) \cdot t_s = -h_{sf}\ \rho_\ell\left(\pi D^3/6\right).$$

With the pellet moving at the terminal velocity, V, the height of the tower must be

$$H = V \cdot t_s = \frac{V h_{sf}\ \rho_\ell D}{6\bar{h}\left(T_s - T_\infty\right)}$$

The terminal velocity may be obtained from a force balance on the pellet. Equating the drag and gravity forces,

$$F_g = F_D$$

where $F_g = \rho_\ell\left(\pi D^3/6\right)g$ and F_D is obtained from the drag coefficient

$$\rho_\ell\left(\pi D^3/6\right)g = C_D\left(\pi D^2/4\right)\left(\rho_a V^2/2\right)$$

$$V = \left(\frac{4}{3}\ \frac{\rho_\ell}{\rho_a}\ \frac{gD}{C_D}\right)^{1/2} - \left[\frac{4}{3}\frac{10{,}600\ \text{kg/m}^3}{1.22\ \text{kg/m}^3}\frac{\left(9.8\ \text{m/s}^2\right)\left(0.003\ \text{m}\right)}{C_D}\right]^{1/2}$$

$$V\left(\text{m/s}\right) = 18.5/C_D^{1/2}. \quad <$$

Continued …..

The drag coefficient may be obtained from Fig. 7.8 and knowledge of the Reynolds number, where

$$Re_D = \frac{VD}{n} = \frac{V(0.003\ m)}{14.8\times10^{-6}\ m^2/s} = 202.7\ V(m/s).$$

In a trial-and-error procedure which involves guessing a value of V, calculating Re_D, obtaining C_D from Fig. 7.8, and computing V from Eq. (1), it was found that

$$V \approx 29\ m/s \qquad Re_D \approx 5900.$$

From the Whitaker correlation, it follows that

$$\overline{Nu}_D = 2 + \left[0.4 Re_D^{1/2} + 0.06 Re_D^{2/3}\right] Pr^{0.4}\left(\frac{m}{m_s}\right)^{1/4}$$

$$\overline{Nu}_D = 2 + \left[0.4(5900)^{1/2} + 0.06(5900)^{2/3}\right]\ (0.71)^{0.4}\left(\frac{178.6\times10^{-7}}{306\times10^{-7}}\right)^{1/4} = 40.4$$

$$\overline{h} = \overline{Nu}_D\left(\frac{k}{D}\right) = 40.4\left(\frac{25.3\times10^{-3}\ W/m\cdot K}{0.003\ m}\right) = 341\ W/m^2\cdot K.$$

Accordingly,

$$H = \frac{29\ m/s\times24{,}500\ J/kg\times10{,}600\ kg/m^3\times0.003\ m}{6\times341\ W/m^2\cdot K\times(327.2-15)^\circ C} = 35\ m.$$ <

COMMENTS: (1) In a free fall from such a height (H = 35 m), the pellet will not have sufficient time to reach the terminal velocity (its maximum velocity on impacting the water would be 28.7 m/s). Accordingly, V has been overestimated and the required value of H has been overpredicted. A more accurate treatment would involve applying the energy balance at successive times from the initiation of the fall, using the pellet velocity appropriate to each time.

(2) Accounting for radiation effects would further diminish the required value of H.

(3) The correlation has been used outside its range of applicability, since $\mu/\mu_s < 1$.

PROBLEM 7.72

KNOWN: A spherical workpiece of pure copper with a diameter of 15 mm and emissivity of 0.5 is suspended in a large furnace with walls at a uniform temperature of 600°C. The air flow over the workpiece has a temperature of 900°C with a velocity of 7.5 m/s.

FIND: (a) The steady-state temperature of the workpiece; (b) Estimate the time required for the workpiece to reach within 5°C of the steady-state temperature if its initial, uniform temperature is 25°C; (c) Estimate the steady-state temperature of the workpiece if the air velocity is doubled with all other conditions remaining the same; also, determine the time required for the workpiece to reach within 5°C of this value. Plot on the same graph the workpiece temperature histories for the two air velocity conditions.

SCHEMATIC:

$T_{sur} = 600°C$

Air

$T_\infty = 900°C$

$V = 7.5$ m/s

Sphere, T_s

D = 15 mm

$\varepsilon = 0.5$

pure copper

q_{rad}

q_{cv}

T_s, ε

$\dot{E}_{st} = 0$

Energy balance, steady-state

ASSUMPTIONS: (1) Flow over a smooth sphere, (2) Sphere behaves as spacewise isothermal object; lumped capacitance method is valid, (3) Sphere is small object in large, isothermal surroundings, and (4) Constant properties.

PROPERTIES: *Table A-4*, Air (T_∞ = 1173 K, 1 atm): $\mu = 4.665 \times 10^{-5}$ $N\cdot s/m^2$, $\nu = 0.0001572$ m^2/s, k = 0.075 W/m·K, Pr = 0.728; Air (T_s = 1010 K, 1 atm): $\mu_s = 4.268 \times 10^{-5}$ $N\cdot s/m^2$.

ANALYSIS: (a) The steady-state temperature is determined from the energy balance on the sphere as represented in the schematic above.

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = 0 \qquad -q_{cv} - q_{rad} + 0 = 0$$

$$-\overline{h}_D A_s \left(T_s - T_\infty\right) - \varepsilon A_s \sigma \left(T_s^4 - T_{sur}^4\right) = 0 \tag{1}$$

where $A_s = \pi D^2/4$. The convection coefficient can be estimated using the Whitaker correlation, Eq. 7.59, where all properties except μ_s are evaluated at T_∞. Assume T_s = 737°C = 1010 K to evaluate μ_s.

$$\overline{Nu}_D = 2 + \left[0.4 Re_D^{1/2} + 0.06 Re_D^{2/3}\right] Pr^{0.4} \left(\mu / \mu_s\right)^{1/4} \tag{2}$$

See the table below for results of the correlation calculations. From the energy balance, canceling out A_s, with numerical values, find T_s.

$$-79.8\ W/m^2\cdot K\left(T_s - 1173\right)K - 0.5 \times 5.67 \times 10^{-8} W/m^2\cdot K^4\left(T_s^4 - 873^4\right)K^4$$

$$T_s = 1010\ K = 737°C.$$ <

(b) The time required for the sphere initially at T_i = 25°C to reach within 5°C of the steady-state temperature can be determined from the energy balance for the transient condition.

Continued …..

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = \dot{E}_{st}$$

$$-\overline{h}_D A_s \left(T_s - T_\infty\right) - \varepsilon A_s \sigma \left(T_s^4 - T_{sur}^4\right) = \rho c \left(\pi D^3 / 6\right) \frac{dT}{dt} \qquad (3)$$

Recognize that $\overline{h}_D$ is not constant, but depends upon $T_s(t)$. Using *IHT* to perform the integration, evaluate $\overline{h}_D$, and provide pure copper properties ρ and c as a function of T_s, the time t_o for $T(t_o)$ = (737 – 5)°C = 732°C is

$$t_o = 274 \text{ s}$$ <

See Comments 1 and 2 for details on the *IHT* calculation method.

(c) Use Eq. (1) and (2) to find the steady-state temperature when the air velocity is doubled, V = 2 × 7.5 ms = 15 m/s. The results are tabulated below along with those from part (a).

Part	V (m/s)	Re_D	$\overline{Nu}_D$	$\overline{h}_D$ (W/m^2·K)	T_s (°C)
a	7.5	715.6	15.96	79.8	737
b	15	1431	22.42	112.1	760

As expected, increasing the air velocity will cause the sphere temperature to increase toward T_∞. Note that $\overline{h}_D$ increases by a factor of 1.4 as the air velocity is doubled. From correlation Eq. (2) note that $\overline{h}_D$ is approximately proportional to V^n where n is in the range 1/2 to 2/3. Using the *IHT* code for the lumped capacitance analysis, the time for $T(t_o)$ = (760 – 5)°C = 755°C is

$$t_o = 230 \text{ s}$$ <

The temperature histories for the two air velocity conditions are calculated using the foregoing transient analyses in the *IHT* workspace.

Workpiece temperature history

Sphere temperature, Ts (C)

Elapsed time, t (s)

V = 7.5 m/s, air velocity
V = 15 m/s

Continued …..

PROBLEM 7.72 (Cont.)

COMMENTS: (1) The portion of the *IHT* code for performing the energy balance and evaluating the convection correlation function using the properties function follows.

```
// Convection correlation, sphere
NuDbar = NuL_bar_EF_SP(ReD,Pr,mu,mus)     // Eq 7.59
NuDbar = hbar * D / k
ReD = V * D / nu
/* All properties except mus are evaluated at Tinf. */
/*  Correlation description: External flow (EF) over a sphere (SP), average coefficient,
3.5<ReD<7.6x10^4, 0.71<Pr<380, 1.0<(mu/mus)<3.2, Whitaker correlation, Eq 7.59. See Table 7.9.  */

// Energy balance, steady-state temperature
-hbar * As * (Ts - Tinf) - eps * sigma * (Ts^4 - Tsur^4) * As = 0
As = pi * D^2
sigma = 5.67e-8

// Air property functions : From Table A.4
// Units: T(K); 1 atm pressure
mu = mu_T("Air",Tinf)          // Viscosity,  N·s/m^2
mus = mu_T("Air",Ts)           // Viscosity,  N·s/m^2
nu = nu_T("Air",Tinf)          // Kinematic viscosity, m^2/s
k = k_T("Air",Tinf)            // Thermal conductivity, W/m·K
Pr = Pr_T("Air",Tinf)          // Prandtl number

// Input variables
D = 0.015
eps = 0.5
V = 7.5
Tinf = 900 + 273
Tsur = 600 + 273
```

(2) Two modifications can be made to the code above to perform the lumped capacitance method for the transient analysis: (a) include the storage term in the energy balance and (b) provide the properties function for copper. The initial condition, T_i = 288 K, is entered as the initial condition when the solver performs the integration.

```
// Energy balance, steady-state; equilibrium temperature
-hbar * As * (Ts - Tinf) - eps * sigma * (Ts^4 - Tfur^4) * As = M * ccu * der(Ts,t)
As = pi * D^2
sigma = 5.67e-8
M = rhocu * pi * D^3 / 6

// Copper (pure) property functions : From Table A.1
// Units: T(K)
rhocu = rho_300K("Copper")           // Density, kg/m^3
kcu = k_T("Copper",Ts)       // Thermal conductivity,W/m·K
ccu = cp_T("Copper",Ts)      // Specific heat, J/kg·K
```

(3) Show that the lumped capacitance method is valid for this application.

PROBLEM 7.73

KNOWN: Diameter and initial and final temperatures of copper spheres quenched in a water bath.

FIND: (a) Terminal velocity in the bath, (b) Tank height.

SCHEMATIC:

Water
$T_\infty = 280K$

Copper sphere, D=20mm,
$T_i(r,0) = 360K$, $T_f(0,t_f) = 320K$

$F_D = C_D A_f \frac{\rho V^2}{2}$

q_{conv}

V

$F_g = (\rho_{cu} - \rho)\, g\, \frac{\pi D^3}{6}$

ASSUMPTIONS: (1) Sphere descends at terminal velocity, (2) Uniform, but time varying surface, temperature.

PROPERTIES: *Table A-1*, Copper (350K): ρ = 8933 kg/m^3, k = 398 W/m·K, c_p = 387 J/kg·K; *Table A-6*, Water (T_∞ = 280 K): ρ = 1000 kg/m^3, $\mu = 1422 \times 10^{-6}$ N·s/m^2, k = 0.582 W/m·K, Pr = 10.26; ($T_s \approx$ 340 K): $\mu_s = 420 \times 10^{-5}$ N·s/m^2.

ANALYSIS: A force balance gives $C_D\left(p\,D^2/4\right)\, r\,V^2/2 = \left(r_{cu} - r\right)\, g\, p\,D^3/6$,

$$C_D\ V^2 = \frac{4D}{3}\frac{r_{cu} - r}{r}g = \frac{4 \times 0.02\ \text{m}}{3} \cdot \frac{8933 - 1000}{1000} 9.8\ \text{m/s}^2 = 2.07\ \text{m}^2/\text{s}^2.$$

An iterative solution is needed, where C_D is obtained from Figure 7.8 with $Re_D = VD/\nu$ = 0.02 m V/1.42 × 10^{-6} m^2/s = 14,085 V (m/s). Convergence is achieved with

$$V \approx 2.1\ \text{m/s}$$ <

for which Re_D = 29,580 and $C_D \approx 0.46$. Using the Whitaker expression

$$\overline{Nu}_D = 2 + \left(0.4 \times 29{,}850^{1/2} + 0.06 \times 29{,}850^{2/3}\right)\ (10.26)^{0.4}(1422/420)^{1/4} = 439$$

$$\bar{h} = \overline{Nu}_D\ k/D = 439 \times 0.582\ \text{W/m}\cdot\text{K}/0.02\ \text{m} = 12{,}775\ \text{W/m}^2\cdot\text{K}.$$

To determine applicability of lumped capacitance method, find $Bi = \bar{h}\left(r_o/3\right)/\ k_{cu} = 12{,}775$ $\text{W/m}^2\cdot\text{K}\ \left(0.01\ \text{m}/3\right)/398\text{W/m}\cdot\text{K} = 0.11.$ Applicability is marginal. Use Heisler charts,

$$q_o^* = \frac{T_o - T_\infty}{T_i - T_\infty} = \frac{320 - 280}{360 - 280} = 0.5, \quad Bi^{-1} = \frac{k}{\bar{h} r_o} = 3.12, \quad Fo \approx 0.88 = \frac{a\,t_f}{r_o^2}.$$

With $\alpha_{cu} = k/\rho c_p$ = 398 W/m·K/(8933 kg/m^3) (387 J/kg·K) = 1.15 × 10^{-4} m^2/s, find

$$t_f = 0.88\ \left(0.01\ \text{m}\right)^2/1.15 \times 10^{-4}\text{m}^2/\text{s} = 0.77\ \text{s}.$$

Required tank height is

$$H = t_f \cdot V = 0.77\ \text{s} \times 2.1\ \text{m/s} = 1.6\ \text{m}.$$ <

COMMENTS: If t_f is evaluated from the approximate series solution, $q_o^* = C_1 \exp\left(-z_1^2\ Fo\right)$, we obtain t_f = 0.76 s. Note that the terminal velocity is not reached immediately. Reduced V implies reduced $\bar{h}$ and increased t_f.

PROBLEM 7.74

KNOWN: Diameter and initial and final temperatures of copper spheres quenched in an oil bath.

FIND: (a) Terminal velocity in bath, (b) Bath height.

SCHEMATIC:

Oil $T_\infty = 300K$

Cu, D=20mm, $T_i = 360K$, $T_f(0,t_f) = 320K$

$F_D = C_D A_f \frac{\rho V^2}{2}$

q_{conv}

V

$F_g = (\rho_{cu} - \rho) g \frac{\pi D^3}{6}$

ASSUMPTIONS: (1) Sphere descends at terminal velocity, (2) Uniform, but time varying, surface temperature.

PROPERTIES: *Table A-1*, Copper (350K): ρ_{cu} =8933 kg/m^3, k = 398 W/m·K, c_p = 387 J/kg·K; *Table A-5*, Oil (T_∞ = 300K): ρ = 884 kg/m^3, μ = 0.486 N·s/m^2, k = 0.145 W/m·K, Pr = 6400; ($T_s \approx$ 340K): μ = 0.0531 N·s/m^2.

ANALYSIS: (a) Force balance gives $C_D\left(\pi D^2/4\right)\rho V^2/2 = (\rho_{cu} - \rho)\ g\ \pi D^3/6$,

$$C_D\ V^2 = \frac{4D}{3}\frac{\rho_{cu}-\rho}{\rho}\ g = \frac{4\times 0.02\text{ m}}{3}\frac{8933-884}{884}9.8\frac{\text{m}}{\text{s}^2} = 2.38\text{m}^2/\text{s}^2.$$

An iterative solution is needed, where C_D is obtained from Fig. 7.8 with

$$Re_D = \frac{VD}{\nu} = \frac{0.02\text{ m }(V)}{(0.486/884)\text{ m}^2/\text{s}} = 36.4\text{ V}(\text{m/s}).$$

Convergence is achieved for $V \approx 1.1$ m/s <

for which Re_D = 40 and $C_D \approx 1.97$. Using the Whitaker expression

$$\overline{Nu}_D = 2 + \left(0.4\,Re_D^{1/2} + 0.06\,Re_D^{2/3}\right)Pr^{0.4}\left(\mu/\mu_s\right)^{1/4}$$

$$\overline{Nu}_D = 2 + \left(0.4\times 40^{1/2} + 0.06\times 40^{2/3}\right)\ (6400)^{0.4}\left(0.486/0.0531\right)^{1/4} = 189.2$$

$$\overline{h} = \overline{Nu}_D\ \ k/D = 189.2\times 0.145/0.02 = 1357\text{ W/m}^2\cdot\text{K}.$$

To determine applicability of the lumped capacitance method, find $Bi = \overline{h}\left(r_o/3\right)/k_{cu}$ =

$1357\text{ W/m}^2\cdot\text{K}\ (0.01\text{ m/3})/398\text{ W/m}\cdot\text{K} = 0.011$. Hence lumped capacitance method can be used; from Eq. 5.5,

$$t_f = \frac{(\rho\ c)_{cu}\pi D^3/6}{\overline{h}\,\pi D^2}\ell n\frac{T_i - T_\infty}{T_f - T_\infty}$$

$$t_f = \frac{8933\text{ kg/m}^3\times 387\text{ J/kg}\cdot\text{K}}{1357\text{ W/m}^2\cdot\text{K}}\frac{0.02\text{ m}}{6}\ell n\frac{60}{20} = 9.33\text{ s}.$$

Required tank height is $H = t_f \cdot V$ = 9.33 s × 1.1 m/s = 10.3 m. <

COMMENTS: (1) Whitaker correlation has been used well beyond its limits (Pr >> 380). Hence estimate of $\overline{h}$ is uncertain. (2) Since terminal velocity is not reached immediately,

$\overline{h} < 1357\text{ W/m}^2\cdot\text{K}$ and $t_f > 9.33$ s.

PROBLEM 7.75

KNOWN: Velocity of plasma jet and initial particle velocity in a plasma spray coating process. Distance from particle injection to impact.

FIND: (a) Particle velocity and distance of travel as a function of time. Time-in-flight and particle impact velocity, (b) Convection heat transfer coefficient and time required to heat particle to melting point and to subsequently melt it.

SCHEMATIC:

L = 100 mm

Alumina particle (m_p, V_p, D_p = 50 μm)
T_{mp} = 2318 K, h_{sf} = 3577 kJ/kg
ρ_p = 3970 kg/m^3, k_p = 10.5 W/m·K,
c_p = 1560 J/kg K

Plasma jet
V = 400 m/s
T_∞= 10,000 K
k = 0.671 W/m·K, c_p = 1480 J/kg·K
μ = 2.70x10^{-4} kg/s·m
ν = 5.6x10^{-3} m^2/s

ASSUMPTIONS: (1) Applicability of Stokes' law, (2) Constant particle and plasma properties, (3) Negligible influence of viscosity ratio in Whitaker correlation, (4) Negligible radiation effects, (5) Validity of lumped capacitance approximation.

ANALYSIS: (a) From Eqs. 7.54 and 7.58,

$$C_D \equiv \frac{F_D}{A_f\left(\rho \bar{V}^2/2\right)} = \frac{24}{Re_D} = \frac{24}{\rho \bar{V} D_p/\mu}$$

where $\bar{V} \equiv V - V_p$ is the relative velocity and $A_f = \pi D_p^2/4$. Hence, the drag force on the particle is

$$F_D = 3\pi\mu D_p \bar{V} = m_p\left(dV_p/dt\right) = -m_p\left(d\bar{V}/dt\right)$$

Separating variables and integrating from the nozzle exit, where $V_p = 0$, $\bar{V} = V$ and $t = 0$,

$$\int_V^{\bar{V}} \frac{d\bar{V}}{V} = -\frac{3\pi\mu D_p}{m_p}\int_0^t dt$$

$$\ln\frac{\bar{V}}{V} = -\frac{3\pi\mu D_p t}{m_p}$$

$$\bar{V} = V\exp\left(-3\pi\mu D_p t/m_p\right) = V - V_p$$

Hence,

$$V_p(t) = V\left[1 - \exp\left(-3\pi\mu D_p t/m_p\right)\right]$$ <

With $V_p = dx_p/dt$, it follows that

$$\int_o^L dx_p = \int_o^{t_f} V\left[1 - \exp\left(-3\pi\mu D_p t/m_p\right)\right]dt$$

Continued...

PROBLEM 7.75 (Cont.)

$$L = Vt_f - \frac{Vm_p}{3\pi\mu D_p}\left[1-\exp\left(-3\pi\mu D_p t_f / m_p\right)\right]$$ <

Substituting the prescribed values of D_p, L, V and the material properties, the foregoing equations yield

$$V_p = 166.7\,\text{m/s} \qquad t_f = 0.0011\,\text{s}$$ <

(b) Assuming an average value of $\bar{V}$ = 315 m/s, the Reynolds number is

$$Re_D = \frac{315\,\text{m/s} \times 50\times10^{-6}\,\text{m}}{5.6\times10^{-3}\,\text{m}^2/\text{s}} = 2.81$$

From the Whitaker correlation,

$$\overline{Nu}_D = 2 + \left(0.4\,Re_D^{1/2} + 0.06\,Re_D^{2/3}\right)Pr^{0.4}$$

$$\overline{Nu}_D = 2 + \left(0.4\times2.81^{1/2} + 0.06\times2.81^{2/3}\right)(0.60)^{0.4} = 2.64$$

$$\bar{h} = 2.64k/D_p = 2.64(0.671\,\text{W/m}\cdot\text{K})/50\times10^{-6}\,\text{m} = 35,400\,\text{W/m}^2\cdot\text{K}$$ <

The two-step melting process involves (i) the time t_1 to heat the particle to its melting point and (ii) the time t_2 required to achieve complete melting. Hence, $t_m = t_1 + t_2$, where from Eq. 5.5,

$$t_1 = \frac{\rho_p D_p c_p}{6\bar{h}} \ln\frac{T_i - T_\infty}{T_{mp} - T_\infty}$$

$$t_1 = \frac{3970\,\text{kg/m}^3\left(50\times10^{-6}\,\text{m}\right)1560\,\text{J/kg}\cdot\text{K}}{6\left(35,400\,\text{W/m}^2\cdot\text{K}\right)} \ln\frac{(300-10,000)}{(2318-10,000)} = 3.4\times10^{-4}\,\text{s}$$

Performing an energy balance for the second step, we obtain

$$\int_{t_1}^{t_m} q_{conv}dt = \Delta E_{st} = \rho_p \forall h_{sf}$$

Hence,

$$t_2 = \frac{\rho_p D_p}{6\bar{h}} \frac{h_{sf}}{\left(T_\infty - T_{mp}\right)} = \frac{3970\,\text{kg/m}^3\left(50\times10^{-6}\,\text{m}\right)}{6\left(35,400\,\text{W/m}^2\cdot\text{K}\right)} \times \frac{3.577\times10^6\,\text{J/kg}}{(10,000-2318)\text{K}} = 4.4\times10^{-4}\,\text{s}$$

Hence,

$$t_m = \left(3.4\times10^{-4} + 4.4\times10^{-4}\right)\text{s} = 7.8\times10^{-4}\,\text{s}$$ <

and the prescribed value of L is sufficient to insure complete melting before impact.

COMMENTS: (1) Since Bi = $\left(\bar{h}\,r_p/3\right)/k_p \approx 0.03$, use of the lumped capacitance approach is appropriate.

(2) With Re_D = 2.81, conditions are slightly outside the ranges associated with Stokes' law and the Whitaker correlation.

PROBLEM 7.76

KNOWN: Diameter, velocity, initial temperature and melting point of molten aluminum droplets. Temperature of helium atmosphere.

FIND: Maximum allowable separation between droplet injector and substrate.

SCHEMATIC:

Helium

$T_\infty = 300$ K

Molten aluminum sphere

$D = 5\times10^{-4}$ m

$T_i = 1100$ K, $T_f \geq 933$ K

$V = 3$ m/s

ASSUMPTIONS: (1) Lumped capacitance approximation is valid, (2) Constant properties, (3) Negligible radiation.

PROPERTIES: *Table A-4*, Helium $(T_\infty = 300\text{K})$: $\nu = 122\times10^{-6}\,\text{m}^2/\text{s}$, $\mu = 199\times10^{-7}\,\text{N}\cdot\text{s/m}^2$, $k = 0.152\,\text{W/m}\cdot\text{K}$, $\text{Pr} = 0.68$. Helium $(T_s \approx 1000\text{K})$: $\mu_s = 446\times10^{-7}\,\text{N}\cdot\text{s/m}^2$. Given, Aluminum: $\rho = 2500\,\text{kg/m}^3$, $c = 1200\,\text{J/kg}\cdot\text{K}$, $k = 200\,\text{W/m}\cdot\text{K}$.

ANALYSIS: With $\text{Re}_D = VD/\nu = 3\,\text{m/s}\left(5\times10^{-4}\,\text{m}\right)/122\times10^{-6}\,\text{m}^2/\text{s} = 12.3$, the Whitaker correlation yields

$$\bar{h} = \frac{k}{D}\left[2 + \left(0.4\,\text{Re}_D^{1/2} + 0.06\,\text{Re}_D^{2/3}\right)\right]\text{Pr}^{0.4}\left(\mu/\mu_s\right)^{1/4}$$

$$\bar{h} = \frac{0.152\,\text{W/m}\cdot\text{K}}{0.0005\text{m}}\left\{2 + \left[0.4(12.3)^{1/2} + 0.06(12.3)^{2/3}\right](0.68)^{0.4}\left(\frac{199}{446}\right)^{1/4}\right\} = 975\,\text{W/m}^2\cdot\text{K}$$

The *time-of-flight* for the droplet to cool from 1100K to 933K may be obtained from Eq. 5.5.

$$t = \frac{\rho\forall c}{\bar{h}A_s}\ln\frac{\theta_i}{\theta} = \frac{\rho c D}{6\bar{h}}\ln\frac{T_i - T_\infty}{T_f - T_\infty}$$

$$t = \frac{\left(2500\,\text{kg/m}^3\right)1200\,\text{J/kg}\cdot\text{K}\left(0.0005\text{m}\right)}{6\times975\,\text{W/m}^2\cdot\text{K}}\ln\left(\frac{800}{633}\right) = 0.06\,\text{s}$$

The maximum separation is therefore

$$L = V\times t = 3\,\text{m/s}\times0.06\,\text{s} = 0.18\text{m} = 180\ \text{mm}$$ <

COMMENTS: (1) With $\text{Bi} = \bar{h}(D/6)/k = 4\times10^{-4}$, the lumped capacitance approximation is excellent. (2) With the surroundings assumed to be at $T_{sur} = T_\infty$ and a representative emissivity of ε = 0.1 for molten aluminum, $h_r \leq \varepsilon\sigma\left(T_i + T_\infty\right)\left(T_i^2 + T_\infty^2\right) \approx 10\,\text{W/m}^2\cdot\text{K} << \bar{h} = 975\,\text{W/m}^2\cdot\text{K}$. Hence, radiation is, in fact, negligible.

PROBLEM 7.77

KNOWN: Diameter, initial temperature and properties of glass beads suspended in an airstream of prescribed temperature.

FIND: (a) Velocity of airstream, (b) Time required to cool the beads from 477 to 80°C.

SCHEMATIC:

Glass beads
$T_i = 750$ K, $T_f = 353$ K
$\rho_g = 2200$ kg/m³, $c_g = 800$ J/kg·K
$k_g = 1.4$ W/m·K, $\varepsilon_g = 0.8$
$T_{sur} = 288$ K
q_{rad}
F_d
F_g
q_{conv}
D = 0.003 m
Air
V, $T_\infty = 288$ K

ASSUMPTIONS: (1) Lumped capacitance approximation may be used, (2) Constant properties, (3) Radiation exchange is with large surroundings at $T_{sur} = T_\infty$.

PROPERTIES: *Table A.4*, Air (T_∞ = 288 K): $\rho = 1.21$ kg/m³, $\nu = 14.8 \times 10^{-6}$ m²/s, $\mu = 179 \times 10^{-7}$ N·s/m², k = 0.0253 W/m·K, Pr = 0.71.

ANALYSIS: (a) Using Eq. 7.44 with the force balance, $F_g = F_d$,

$$\rho_g \left(\pi D^3/6\right) g = C_D \left(\pi D^2/4\right)\left(\rho V^2/2\right)$$

$$V = \left(\frac{4}{3} \times \frac{\rho_g}{\rho} \times \frac{gD}{C_D}\right)^{1/2} = \left(\frac{4}{3} \times \frac{2200}{1.21} \times \frac{9.8\,\text{m/s}^2 \times 0.003\,\text{m}}{C_D}\right)^{1/2} = \frac{8.44}{C_D^{1/2}}$$

Also,

$$Re_D = \frac{VD}{\nu} = \frac{V(0.003\,\text{m})}{14.8 \times 10^{-6}\,\text{m}^2/\text{s}} = 202.7\,V$$

From Fig. 7.8, the foregoing results yield $C_D \approx 0.4$, for which

$$V \approx 13.3 \text{ m/s}$$ <

and $Re_D \approx 2700$.

(b) Applying an energy balance to a control surface about the bead, Eq. 5.15 may be obtained, with $\dot{E}_g$ = 0, $q''_s = 0$, $A_{s(c,r)} = \pi D^2$, and $\forall = \pi D^3/6$. Hence,

$$\rho_g c_g \frac{dT}{dt} = -(6/D)\left[\bar{h}(T - T_\infty) + \varepsilon_g \sigma \left(T^4 - T_{sur}^4\right)\right]$$

where $\bar{h}$ is given by the Whitaker correlation,

$$\overline{Nu}_D = 2 + \left(0.4\,Re_D^{1/2} + 0.06\,Re_D^{2/3}\right) Pr^{0.4} \left(\mu/\mu_s\right)^{1/4}$$

Using the *IHT Lumped Capacitance Model* with the appropriate *Correlations* and *Properties* Tool Pads, the foregoing integration was evaluated numerically, and the following temperature history was obtained.

Continued...

The desired temperature of T = 80°C = 353 K is obtained at t = 7s, and at t = 20s the temperature is within 1.5°C of ambient conditions.

COMMENTS: (1) With $Bi = \left(\bar{h} + h_{rad}\right) r_o / k$ = (218 + 30) $W/m^2 \cdot K$(0.0005 m)/1.4 W/m·K = 0.089 at T = 750 K, the lumped capacitance assumption is satisfactory and becomes increasingly better as h_{rad} decreases with decreasing T.

(2) The small bead diameter and large velocity provide a large convection coefficient, which insures rapid cooling to the desired temperature. Even at the maximum temperature (T = 750 K), h_{rad} = 30 $W/m^2 \cdot K$ makes a small contribution to the cooling process.

PROBLEM 7.78

KNOWN: Velocity and temperature of combustion gases. Diameter and emissivity of thermocouple junction. Combustor temperature.

FIND: (a) Time to achieve 98% of maximum thermocouple temperature rise, (b) Steady-state thermocouple temperature, (c) Effect of gas velocity and thermocouple emissivity on measurement error.

SCHEMATIC:

q_{conv} q_{rad}
$1 \le V \le 25$ m/s
$T_\infty = 1000$ K
$D = 0.001$ m
$T_c = 400$ K

ASSUMPTIONS: (1) Validity of lumped capacitance analysis, (2) Constant properties, (3) Negligible conduction through lead wires, (4) Radiation exchange between small surface and a large enclosure (parts b and c).

PROPERTIES: Thermocouple (given): $0.1 \le \varepsilon \le 1.0$, k = 100 W/m·K, c = 385 J/kg·K, $\rho = 8920$ kg/m^3; Gases (given): k = 0.05 W/m·K, $\nu = 50 \times 10^{-6}$ m^2/s, Pr = 0.69.

ANALYSIS: (a) If the lumped capacitance analysis may be used, it follows from Equation 5.5 that

$$t = \frac{\rho V c}{\bar{h} A_s} \ln \frac{T_i - T_\infty}{T - T_\infty} = \frac{D\rho c}{6\bar{h}} \ln(50).$$

Neglecting the viscosity ratio correlation for variable property effects, use of V = 5 m/s with the Whitaker correlation yields

$$\overline{Nu}_D = \left(\bar{h}D/k\right) = 2 + \left(0.4\,Re_D^{1/2} + 0.06\,Re_D^{2/3}\right)Pr^{0.4} \qquad Re_D = \frac{VD}{\nu} = \frac{5\,m/s\,(0.001\,m)}{50\times10^{-6}\,m^2/s} = 100$$

$$\bar{h} = \frac{0.05\,W/m\cdot K}{0.001\,m}\left[2 + \left(0.4(100)^{1/2} + 0.06(100)^{2/3}\right)(0.69)^{0.4}\right] = 328\,W/m^2\cdot K$$

Since Bi = $\bar{h}(r_o/3)/k$ = 5.5×10^{-4}, the lumped capacitance method may be used. Hence,

$$t = \frac{0.001\,m\left(8920\,kg/m^3\right)385\,J/kg\cdot K}{6\times 328\,W/m^2\cdot K}\ln(50) = 6.83\,s$$ <

(b) Performing an energy balance on the junction and evaluating radiation exchange from Equation 1.7, $q_{conv} = q_{rad}$. Hence, with $\varepsilon = 0.5$,

$$\bar{h}A_s\left(T_\infty - T\right) = \varepsilon A_s \sigma\left(T^4 - T_c^4\right)$$

$$(1000 - T)K = \frac{0.5\times5.67\times10^{-8}\,W/m^2\cdot K^4}{328\,W/m^2\cdot K}\left[T^4 - (400)^4\right]K^4.$$

T = 936 K <

(c) Using the *IHT First Law Model* for a *Solid Sphere* with the appropriate *Correlation* for external flow from the Tool Pad, parametric calculations were performed to determine the effects of V and ε_g, and the following results were obtained.

Continued...

PROBLEM 7.78 (Cont.)

Since the temperature recorded by the thermocouple junction increases with increasing V and decreasing ε, the measurement error, T_∞ - T, decreases with increasing V and decreasing ε. The error is due to net radiative transfer from the junction (which depresses T) and hence should decrease with decreasing ε. For a prescribed heat loss, the temperature difference (T_∞ - T) decreases with decreasing convection resistance, and hence with increasing h(V).

COMMENTS: To infer the actual gas temperature (1000 K) from the measured result (936 K), correction would have to be made for radiation exchange with the cold surroundings.

PROBLEM 7.79

KNOWN: Diameter, emissivity and temperature of a thermocouple junction exposed to hot gases flowing through a duct of prescribed surface temperature.

FIND: (a) Relative magnitudes of gas and thermocouple temperatures if the duct surface temperature is less than the gas temperature, (b) Gas temperature for prescribed conditions, (c) Effect of Velocity and emissivity on measurement error.

SCHEMATIC:

Duct

$T_s = 175\ ^oC$

Hot gases

T_g

$V = 3$ m/s

q_{conv}

q_{rad}

Thermocouple junction,

$T_j = 320\ ^oC$, $D = 0.002$ m, $\varepsilon = 0.6$

ASSUMPTIONS: (1) Steady-state conditions, (2) Junction is diffuse-gray, (3) Duct forms a large enclosure about the junction, (4) Negligible heat transfer by conduction through the thermocouple leads, (5) Gas properties are those of atmospheric air.

PROPERTIES: *Table A-4*, Air ($T_g \approx 650$ K, 1 atm): $\nu = 60.21 \times 10^{-6}$ m^2/s, k = 0.0497 W/m·K, Pr = 0.690, $\mu = 322.5 \times 10^{-7}$ N·s/m^2; Air ($T_j = 593$ K, 1 atm): $\mu = 304 \times 10^{-7}$ N·s/m^2.

ANALYSIS: (a) From an energy balance on the thermocouple junction, $\underset{(g\to j)}{q_{conv}} = \underset{(j\to s)}{q_{rad}}$. Hence,

$$\bar{h}A\left(T_g - T_j\right) = \varepsilon\sigma A\left(T_j^4 - T_s^4\right) \qquad \text{or} \qquad T_g - T_j = \frac{\varepsilon}{\bar{h}}\sigma\left(T_j^4 - T_s^4\right).$$

If $T_s < T_j$, it follows that $T_j < T_g$. <

(b) Neglecting the variable property correction, $\left(\mu/\mu_s\right)^{1/4} = \left(322.5/304\right)^{1/4}$ = 1.01 ≈ 1.00, and using

$$Re_D = \frac{VD}{\nu} = \frac{3\,\text{m/s}\left(0.002\,\text{m}\right)}{60.21\times10^{-6}\,\text{m}^2/\text{s}} = 100$$

the Whitaker correlation for a sphere gives

$$\bar{h} = \frac{0.0497\,\text{W/m}\cdot\text{K}}{0.002\,\text{m}}\left\{2 + \left[0.4\left(100\right)^{1/2} + 0.06\left(100\right)^{2/3}\right]\left(0.69\right)^{0.4}\right\} = 163\,\text{W/m}^2\cdot\text{K}.$$

Hence

$$\left(T_g - 593\,\text{K}\right) = \frac{0.6}{163\,\text{W/m}^2\cdot\text{K}}5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left[\left(593\,\text{K}\right)^4 - \left(448\,\text{K}\right)^4\right] = 17\,\text{K}$$

$$T_g = 610\,\text{K} = 337^\circ\text{C}.$$ <

(c) With T_g fixed at 610 K, the IHT *First Law* Model was used with the *Correlations* and *Properties* Tool Pads to compute the measurement error as a function of V and ε.

Continued...

PROBLEM 7.79 (Cont.)

Since the convection resistance decreases with increasing V, the junction temperature will approach the gas temperature and the measurement error will decrease. Since the depression in the junction temperature is due to radiation losses from the junction to the duct wall, a reduction in ε will reduce the measurement error.

COMMENTS: In part (b), calculations could be improved by evaluating properties at 610 K (instead of 650 K).

PROBLEM 7.80

KNOWN: Diameter and emissivity of a thermocouple junction exposed to hot gases of prescribed velocity and temperature flowing through a duct of prescribed surface temperature.

FIND: (a) Thermocouple reading for gas at atmospheric pressure, (b) Thermocouple reading when gas pressure is doubled.

SCHEMATIC:

Hot gases
$T_g = 500°C, V = 3 m/s$
$p = 1 atm$ or $2 atm$
q_{conv}
q_{rad}
$T_s = 200°C$
Thermocouple junction,
$T_j, D = 0.002 m, \varepsilon = 0.6$

ASSUMPTIONS: (1) Steady-state conditions, (2) Junction is diffuse-gray, (3) Duct forms a large enclosure about junction, (4) Negligible heat loss by conduction through thermocouple leads, (5) Gas properties are those of air, (6) Perfect gas behavior.

PROPERTIES: *Table A-4*, Air (T_g = 773 K, 1 atm): $\nu = 80.5 \times 10^{-6}$ m^2/s, k = 0.0561 W/m·K, Pr = 0.705.

ANALYSIS: (a) Performing an energy balance on the junction

$$\underset{(g \to j)}{q_{conv}} = \underset{(j \to s)}{q_{rad}}$$

$$\bar{h}A\left(T_g - T_j\right) = es\ A\left(T_j^4 - T_s^4\right).$$

Neglecting the variable property correction, $(\mu/\mu_s)^{1/4}$, and using

$$Re_D = \frac{VD}{n} = \frac{3\ m/s \times 0.002\ m}{80.5 \times 10^{-6}\ m^2/s} = 74.5$$

the Whitaker correlation for a sphere gives,

$$\bar{h} = \frac{0.0561\ W/m \cdot K}{0.002\ m}\left\{2 + \left[0.4(74.5)^{1/2} + 0.06(74.5)^{2/3}\right](0.705)^{0.4}\right\} = 166\ W/m^2 \cdot K.$$

$$166\left(773 - T_j\right) = 0.6 \times 5.67 \times 10^{-8}\left[T_j^4 - (473)^4\right]$$

and from a trial-and-error solution,

$$T_j \approx 726\ K.$$ <

(b) Assuming all properties other than ν to remain constant with a change in pressure, ↑ p by 2 will ↓ ν by 2 and hence ↑ Re_D by 2, giving Re_D = 149. Hence

$$\bar{h} = \frac{0.0561}{0.002}\left\{2 + \left[0.4(149)^{1/2} + 0.06(149)^{2/3}\right](0.705)^{0.4}\right\} = 216\ W/m^2 \cdot K.$$

$$216\left(773 - T_j\right) = 0.6 \times 5.67 \times 10^{-8}\left[T_j^4 - (473)^4\right]$$

and from a trial-and-error solution

$$T_j \approx 735\ K.$$ <

COMMENTS: The thermocouple error will ↓ with ↑ h, which ↑ with ↑p.

PROBLEM 7.81

KNOWN: Velocity and temperature of helium flow over graphite coated uranium oxide pellets. Pellet and coating diameters and thermal conductivity. Surface temperature of coating.

FIND: (a) Rate of heat transfer, (b) Volumetric generation rate in pellet and pellet surface temperature, (c) Radial temperature distribution in pellet, (d) Effect of gas velocity on center and surface temperatures.

SCHEMATIC:

$D_o = 0.012$ m
$D_i = 0.01$ m
Helium
$T_{s,o} = 1300$ K
$V = 20$ m/s
$T_\infty = 500$ K
$k_g = 2$ W/m·K
$T_{s,i}$
$\dot{q}$, $k_p = 2$ W/m·K

ASSUMPTIONS: (1) One-dimensional, steady conduction in the radial direction, (2) Uniform generation, (3) Constant properties, (4) Negligible radiation, (5) Negligible contact resistance.

PROPERTIES: *Table A.4*, Helium (T_∞ = 500 K, 1 atm): $\nu = 290 \times 10^{-6}$ m^2/s, k = 0.22 W/m·K, Pr = 0.67, $\mu = 283 \times 10^{-7}$ N·s/m^2; ($T_{s,o}$ = 1300 K, with extrapolation): $\mu = 592 \times 10^{-7}$ N·s/m^2.

ANALYSIS: (a) The heat transfer rate is $q = \bar{h} A_s \left(T_{s,o} - T_\infty\right)$, where the convection coefficient can be estimated from $\overline{Nu}_D = 2 + \left(0.4\,Re_D^{1/2} + 0.06\,Re_D^{2/3}\right) Pr^{0.4} \left(\mu_\infty / \mu_s\right)^{1/4}$, where

$$Re_D = \frac{VD_o}{\nu} = \frac{20\,m/s \times 0.012\,m}{290 \times 10^{-6}\,m^2/s} = 828$$

$$\overline{Nu}_D = 2 + \left[0.4(828)^{1/2} + 0.06(828)^{2/3}\right](0.67)^{0.4}(283/592)^{1/4} = 13.9$$

$$\bar{h} = \frac{k}{D_o}\overline{Nu}_D = \frac{0.22\,W/m \cdot K}{0.012\,m} \times 13.9 = 255\,W/m^2 \cdot K.$$

Hence, $q = 255\,W/m^2 \cdot K \times \pi (0.012\,m)^2 (1300 - 500)K = 92.2\,W.$ <

(b) The volumetric heat rate in the pellet is

$$\dot{q} = \frac{q}{\pi D_i^3/6} = \frac{6 \times 92.2\,W}{\pi (0.01\,m)^3} = 1.76 \times 10^8\,W/m^3$$ <

The inner surface temperature of the coating is equal to the pellet surface temperature,

$$T_{s,i} - T_{s,o} = q \frac{1}{4\pi k_g}\left(\frac{1}{r_i} - \frac{1}{r_o}\right) = \frac{92.2\,W}{4\pi (2\,W/m \cdot K)}\left(\frac{1}{0.005\,m} - \frac{1}{0.006\,m}\right) = 122.3\,K$$

$$T_{s,i} = 1300\,K + 122.3\,K = 1422\,K.$$ <

(c) The heat equation for the spherical pellet reduces to

$$\frac{k_p}{r^2}\frac{d}{dr}\left(r^2 \frac{dT}{dr}\right) = -\dot{q}$$

Integrating twice,

$$r^2 \frac{dT}{dr} = -\frac{\dot{q}}{3k_p} r^3 + C_1 \qquad \frac{dT}{dr} = -\frac{\dot{q}}{3k_p} r + \frac{C_1}{r^2}$$

Continued...

$$T = -\frac{\dot{q}}{6k_p} r^2 - \frac{C_1}{r} + C_2 .$$

Applying boundary conditions,

$r = 0$: $\quad dT/dr)_{r=0} = 0 \quad \rightarrow \quad C_1 = 0$

$r = r_i$: $\quad T(r_i) = T_{s,i} \quad \rightarrow \quad C_2 = T_{s,i} + \left(\dot{q}/6k_p\right) r_i^2 .$

Hence the temperature distribution is

$$T(r) = T_{s,i} + \left(\dot{q}/6k_p\right)\left(r_i^2 - r^2\right) = T(0) - \left(\dot{q}/6k_p\right) r^2$$ <

where the temperature at the pellet center is $T(0) = T_{s,i} + \left(\dot{q}/6k_p\right) r_i^2$.

For the prescribed conditions,

$$T(0) = 1422\,\text{K} + \left(1.76 \times 10^8\ \text{W/m}^3 / 6 \times 2\ \text{W/m} \cdot \text{K}\right)(0.005\,\text{m})^2 = 1789\,\text{K} .$$

(d) With $\dot{q} = 1.5 \times 10^8\ \text{W/m}^3$, parametric calculations were performed using the IHT Model for *One-Dimensional*, *Steady-State Conduction* in a sphere, with the surface condition, $q''(r_i) = \left(T_{s,i} - T_\infty\right)/R''_{t,i}$, where the total thermal resistance, $R_{t,i} = R''_{t,i}/4\pi r_i^2$, is

$$R_{t,i} = R_{tcnd} + R_{tcnv} = \frac{(1/r_i) - (1/r_o)}{4\pi k_p} + \frac{1}{4\pi r_o^2 \overline{h}}$$

The *Correlations* and *Properties* Tool Pads were used to evaluate the convection coefficient, and the following results were obtained.

Temperature, T(K)
2200
1800
1400
1000
5
10
15
20
Gas velocity, V(m/s)
Center temperature, T(0)
Inner surface temperature, Tsi
Outer surface temperature, Tso

Thermal resistances, (K/W)
16
12
8
4
0
5
10
15
20
Velocity, V(m/s)
Conduction, Rtcnd
Convection, Rtcnv

As expected, all temperatures increase with decreasing V, while fixed values of $\dot{q}$, and hence $q(r_i)$, and R_{tcnd} provide fixed values of $(T(0) - T_{s,i})$ and $(T_{s,i} - T_{s,o})$, respectively.

COMMENTS: In a more detailed analysis, radiation heat transfer, which would decrease the temperatures, should be considered.

PROBLEM 7.82

KNOWN: Initial temperature, dimensions and properties of chip and solder connectors. Velocity, temperature and properties of liquid.

FIND: (a) Ratio of time constants (chip-to-solder), (b) Chip-to-solder temperature difference after 0.25s of heating.

SCHEMATIC:

L = 10 mm
t = 1 mm
Silicon chip
Dielectric liquid
$T_i = 20^oC$
$T_\infty = 80^oC$
V = 0.2 m/s
Solder ball, D = 1 mm

ASSUMPTIONS: (1) Solder balls and chips are spatially isothermal, (2) Negligible heat transfer from sides of chip, (3) Top and bottom surfaces of chip act as flat plates in turbulent parallel flow, (4) Heat transfer from solder balls may be approximated as that from an isolated sphere, (5) Constant properties.

PROPERTIES: Given. Dielectric liquid: $k = 0.064\,W/m\cdot K$, $\nu = 10^{-6}\,m^2/s$, $Pr = 25$; Silicon chip: $k = 150\,W/m\cdot K$, $\rho = 2300\,kg/m^3$, $c_p = 700\,J/kg\cdot K$; Solder ball: $k = 40\,W/m\cdot K$, $\rho = 10{,}000\,kg/m^3$, $c_p = 150\,J/kg\cdot K$.

ANALYSIS: (a) From Eq. 5.7, the thermal time constant is $\tau_t = (\rho \forall c/\bar{h} A_s)$. Hence,

$$\frac{\tau_{t,ch}}{\tau_{t,sld}} = \frac{(\rho c)_{ch}(L^2 t)}{2\bar{h}_{ch}L^2}\,\frac{\bar{h}_{sld}(\pi D^2)}{(\rho c)_{sld}(\pi D^3/6)} = 3\frac{t}{D}\,\frac{(\rho c)_{ch}}{(\rho c)_{sld}}\,\frac{\bar{h}_{sld}}{\bar{h}_{ch}}$$

The convection coefficient for the chip may be obtained from Eq. 7.44, with $Re_L = VL/\nu = 0.2\,m/s \times 0.01m/10^{-6}\,m^2/s = 2000$.

$$\bar{h}_{ch} = \frac{0.064\,W/m\cdot K}{0.01m}(0.037)(2000)^{4/5}(25)^{1/3} = 302\,W/m^2\cdot K$$

The convection coefficient for the solder may be obtained from Eq. 7.59, with $Re_D = VD/\nu = 0.2\,m/s \times 0.001m/10^{-6}\,m^2/s = 200$. Neglecting the effect of the viscosity ratio,

$$\bar{h}_{sld} = \frac{0.064\,W/m\cdot K}{0.001m}\left\{2 + \left[0.4(200)^{1/2} + 0.06(200)^{2/3}\right](25)^{0.4}\right\} = 1916\,W/m^2\cdot K$$

Hence,
$$\frac{\tau_{t,ch}}{\tau_{t,sld}} = 3\left(\frac{2300\,kg/m^3 \times 700\,J/kg\cdot K}{10{,}000\,kg/m^3 \times 150\,J/kg\cdot K}\right)\frac{1916\,W/m^2\cdot K}{302\,W/m^2\cdot K} = 20.4 \quad <$$

Hence, the solder responds much more quickly to the convective heating.

(b) From Eq. 5.6, the chip-to-solder temperature difference may be expressed as

Continued …..

PROBLEM 7.82 (Cont.)

$$T_{ch} - T_{sld} = (T_i - T_\infty)\left\{\exp\left[-\left(\frac{2\bar{h}}{\rho c t}\right)_{ch} t\right] - \exp\left[-\left(\frac{6\bar{h}}{\rho c D}\right)_{sld} t\right]\right\}$$

$$T_{ch} - T_{sld} = 60°C\left\{\exp\left[-\frac{604\,W/m^2\cdot K}{1610\,J/m^2\cdot K}0.25\,s\right] - \exp\left[-\frac{11,496\,W/m^2\cdot K}{1500\,J/m^2\cdot K}0.25\,s\right]\right\}$$

$$T_{ch} - T_{sld} = 60°C\{0.910 - 0.147\} = 45.8°C$$ <

COMMENTS: (1) The foregoing process is used to subject soldered chip connections (a major reliability issue) to rapid and intense thermal stresses. (2) Some heat transfer by conduction will occur between the chip and solder balls, thereby reducing the temperature difference and thermal stress. (3) Constriction of flow between the chip and substrate will reduce $\bar{h}_{sld}$, as well as $\bar{h}_{ch}$ at the lower surface of the chip, relative to values predicted by the correlations. The corresponding time constants would be increased accordingly. (4) With $Bi_{ch} = \bar{h}_{ch}(t/2)/k_{chip} = 0.001 << 1$ and $Bi_{sld} = \bar{h}_{sld}(D/6)/k_{sld} = 0.008 << 1$, the lumped capacitance analysis is appropriate for both components.

PROBLEM 7.83

KNOWN: Conditions associated with Example 7.6, but with reduced longitudinal and transverse pitches.

FIND: (a) Air side convection coefficient, (b) Tube bundle pressure drop, (c) Heat rate.

SCHEMATIC:

S_T=20.5mm
Air
V=6 m/s
T_∞=15°C
S_L = 20.5mm
Tube, D=16.4 mm
T_s = 70°C
N_L= 7, N_T =8

ASSUMPTIONS: (1) Steady-state conditions, (2) Uniform tube surface temperature.

PROPERTIES: *Table A-4*, Atmospheric air (T_∞ = 288 K): ρ = 1.217 kg/m^3, $\nu = 14.82 \times 10^{-6}$ m^2/s, k = 0.0253 W/m·K, Pr = 0.71, c_p = 100.7 J/kg·K; (T_s = 343 K): Pr = 0.701.

ANALYSIS: (a) From the tube pitches, find

$$S_D = \left[S_L^2 + \left(S_T/2\right)^2\right]^{1/2} = \left[(20.5)^2 + (10.25)^2\right]^{1/2} = 22.91 \text{ mm}$$

$$\left(S_T + D\right)/2 = \left(20.5+16.4\right)/2 = 18.45 \text{ mm.}$$

Hence, the maximum velocity occurs on the transverse plane, and

$$V_{max} = \frac{S_T}{S_T - D} V = \frac{20.5 \text{ mm}}{(20.5-16.4) \text{ mm}} 6 \text{ m/s} = 30 \text{ m/s.}$$

With $$Re_{D,max} = \frac{V_{max} D}{n} = \frac{30 \text{ m/s}\left(0.0164 \text{ m}\right)}{14.82\times10^{-6} \text{ m}^2/\text{s}} = 3.32\times10^4$$

and $(S_T/S_L) = 1 < 2$, it follows from Table 7.7 that

$$C = 0.35 \qquad m = 0.60.$$

Hence, from the Zhukauskas correlation and Table 7.8 (C_2 = 0.95),

$$\overline{Nu}_D = (0.95)\, 0.35\, Re_{D,max}^{0.6}\, Pr^{0.36} \left(Pr/Pr_s\right)^{1/4}$$

$$\overline{Nu}_D = (0.95)\, 0.35 \left(3.32\times10^4\right)^{0.6} (0.71)^{0.36} (0.71/0.701)^{1/4} = 152$$

$$\overline{h} = \overline{Nu}_D \frac{k}{D} = 152 \times \frac{0.0253 \text{ W/m}\cdot\text{K}}{0.0164 \text{ m}} = 234 \text{ W/m}^2\cdot\text{K.}$$ <

(b) From the Zhukauskas relation

$$\Delta p = N_L c \left(\frac{r V_{max}^2}{2}\right) f.$$

With $Re_{D,max} = 3.32 \times 10^4$, $P_T = (S_T/D) = 1.25$ and $(P_T/P_L) = 1$, it follows from Fig. 7.14 that

$$\chi \approx 1.02 \qquad f \approx 0.38.$$

Continued …..

PROBLEM 7.83 (Cont.)

Hence

$$\Delta p = 7 \times 1.02 \frac{1.217\ \text{kg/m}^3 (30\ \text{m/s})^2}{2} 0.38 = 1490\ \text{N/m}^2$$

$$\Delta p = 0.0149\ \text{bar.}$$ <

(c) The air outlet temperature is obtained from

$$T_s - T_o = (T_s - T_i)\exp\left(-\frac{p\, D N \bar{h}}{r\, V N_t S_t c_p}\right)$$

$$T_s - T_o = 55^\circ\text{C}\ \exp\left(\frac{-p\ (0.0164\ \text{m})\, 56 \left(234\ \text{W/m}^2 \cdot \text{K}\right)}{1.217\ \text{kg/m}^3 \times 6\ \text{m/s} \times 8 \times 0.0205\ \text{m} \times 1007\ \text{J/kg} \cdot \text{K}}\right)$$

$$T_s - T_o = 31.4^\circ\text{C}$$

$$T_o = 38.5^\circ\text{C.}$$ <

The log mean temperature difference is

$$\Delta T_{\ell m} = \frac{\Delta T_i - \Delta T_o}{\ell n\left(\Delta T_i / \Delta T_o\right)} = \frac{(55 - 31.4)^\circ\ \text{C}}{\ell n\,(55/31.4)} = 42.1^\circ\text{C}$$

$$q' = N \bar{h} p\, D \Delta T_{\ell m} = 56 \left(234\ \text{W/m}^2 \cdot \text{K}\right) p\ (0.0164\ \text{m})\ 42.1^\circ\text{C}$$

$$q' = 28.4\ \text{kW/m.}$$ <

COMMENTS: Making the tube bank more compact has the desired effect of increasing the convection coefficient and therefore the heat transfer rate. However, it has the adverse effect of increasing the pressure drop and hence the fan power requirement. Note that the convection coefficient increases by a factor of (234/135.6) = 1.73, while the pressure drop increases by a factor of (1490/246) = 6.1. This disparity is a consequence of the fact that $\bar{h} \sim V_{max}^{0.6}$, while $\Delta p \sim V_{max}^2$. Hence any increase in V_{max}, which would result from a more closely spaced arrangement, would more adversely affect Δp than favorably affect $\bar{h}$.

PROBLEM 7.84

KNOWN: Surface temperature and geometry of a tube bank. Velocity and temperature of air in cross flow.

FIND: (a) Total heat transfer, (b) Air flow pressure drop.

SCHEMATIC:

S_T=15mm
Air
V=5 m/s
T_∞=25°C=T_i
p=1atm
S_L=15mm
T_o
Tube, D=10mm
T_s=100°C
N_L=14 rows, N_T=14 tubes/row
L=1m, P_T=P_L=1.5

ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible radiation, (3) Uniform surface temperature.

PROPERTIES: *Table A-4*, Atmospheric air (T_∞ = 298 K): $\nu = 15.8 \times 10^{-6}$ m^2/s, k = 0.0263 W/m·K, Pr = 0.707, c_p = 1007 J/kg·K, ρ = 1.17 kg/m^3; (T_s = 373 K): Pr = 0.695.

ANALYSIS: (a) The total heat transfer rate is

$$q = \bar{h} N p DL \frac{(T_s - T_i) - (T_s - T_o)}{\ell n\left[(T_s - T_i)/(T_s - T_o)\right]} = \bar{h} N p DL\, \Delta T_{\ell m}.$$

With $V_{max} = \frac{S_T}{S_T - D} V = \frac{15 \text{ mm}}{5 \text{ mm}} 5 \text{ m/s} = 15 \text{ m/s}$, $Re_{D,max} = \frac{15 \text{ m/s}(0.01 \text{ m})}{15.8 \times 10^{-6} \text{ m}^2/\text{s}} = 9494$. Tables 7.7 and 7.8 give C = 0.27, m = 0.63 and $C_2 \approx 0.99$. Hence, from the Zhukauskas correlation

$$\overline{Nu}_D = 0.99 \times 0.27 (9494)^{0.63} (0.707)^{0.36} (0.707/0.695)^{1/4} = 75.9$$

$$\bar{h} = \overline{Nu}_D \; k/D = 75.9 \times 0.0263 \text{ W/m} \cdot \text{K}/0.01 \text{ m} = 200 \text{ W/m}^2 \cdot \text{K}$$

$$T_s - T_o = (T_s - T_i)\exp\left(-\frac{p D N \bar{h}}{r V N_T S_T c_p}\right) = 75^\circ\text{C} \exp\left(-\frac{p \times 0.01 \text{ m} \times 196 \times 200 \text{ W/m}^2 \cdot \text{K}}{1.17 \text{ kg/m}^3 \times 5 \text{ m/s} \times 14 \times 0.015 \text{ m} \times 1007 \text{ J/kg} \cdot \text{K}}\right)$$

$$T_s - T_o = 27.7^\circ\text{C}.$$

Hence

$$q = 200 \text{ W/m}^2 \cdot \text{K} \times 196 p\,(0.01 \text{ m})\; 1 \text{ m} \frac{75^\circ\text{C} - 27.7^\circ\text{C}}{\ell n(75/27.7)} = 58.5 \text{ kW}. \qquad <$$

(b) With $Re_{D,max}$ = 9494, $(P_T - 1)/(P_L - 1) = 1$, Fig. 7.13 yields f ≈ 0.32 and $\chi = 1$. Hence,

$$\Delta p = N c \left(r V_{max}^2/2\right) f = 14 \times 1 \left(\frac{1.17 \text{ kg/m}^3 (15 \text{ m/s})^2}{2}\right) 0.32$$

$$\Delta p = 590 \text{ N/m}^2 = 5.9 \times 10^{-3} \text{ bar}.$$

COMMENTS: The heat transfer rate would have been substantially overestimated (93.3 kW) if the inlet temperature difference (T_s - T_i) had been used in lieu of the log-mean temperature difference.

PROBLEM 7.85

KNOWN: Surface temperature and geometry of a tube bank. Inlet velocity and inlet and outlet temperatures of air in cross flow over the tubes.

FIND: Number of tube rows needed to achieve the prescribed outlet temperature and corresponding pressure of drop of air.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state, (2) Negligible temperature drop across tube wall and uniform outer surface temperature, (3) Constant properties, (4) $C_2 \approx 1$.

PROPERTIES: *Table A-4,* Atmospheric air. $\left(\overline{T} = (T_i + T_o)/2 = 323K\right)$: $\rho = 1.085\,kg/m^3$, $c_p = 1007\,J/kg\cdot K$, $\nu = 18.2\times10^{-6}\,m^2/s$, $k = 0.028\,W/m\cdot K$, $Pr = 0.707$; $\left(T_i = 298K\right)$: $\rho = 1.17\,kg/m^3$; $\left(T_s = 373K\right)$: $Pr_s = 0.695$.

ANALYSIS: The temperature difference $\left(T_s - T\right)$ decreases exponentially in the flow direction, and at the outlet

$$\frac{T_s - T_o}{T_s - T_i} = \exp\left(-\frac{\pi D N_L \overline{h}}{\rho V S_T c_p}\right)$$

where $N_L = N/N_T$. Hence,

$$N_L = -\frac{\rho V S_T c_p}{\pi D \overline{h}} \ell n\left(\frac{T_s - T_o}{T_s - T_i}\right) \qquad (1)$$

With $V_{max} = \left[S_T/\left(S_T - D\right)\right]V = 15\,m/s, Re_{D,max} = V_{max}D/\nu = 8240$. Hence, with $S_T/S_L = 1 > 0.7$, C = 0.27 and m = 0.63 from Table 7.7, and the Zhukauskas correlation yields

$$\overline{Nu}_D = C\,C_2\,Re_{D,max}^m\,Pr^{0.36}\left(\frac{Pr}{Pr_s}\right)^{1/4} = 0.27\times1\left(8240\right)^{0.63}\left(0.707\right)^{0.36}\left(0.707/0.695\right)^{1/4} = 70.1$$

$$\overline{h} = \frac{k}{D}\overline{Nu}_D = \frac{0.028\,W/m\cdot K}{0.01m}70.1 = 196.3\,W/m^2\cdot K$$

Hence,

$$N_L = -\frac{1.17\,kg/m^3\left(5\,m/s\right)0.015m\left(1007\,J/kg\cdot K\right)}{\pi\left(0.01m\right)196.3\,W/m^2\cdot K}\ell n\left(\frac{25}{75}\right) = 15.7$$

and 16 tube rows should be used $\qquad N_L = 16 \qquad$ <

With $Re_{D,max} = 8240$, $P_L = 1.5$ and $\left(P_T - 1\right)/\left(P_L - 1\right) = 1$, $f \approx 0.35$ and $\chi = 1$ from Fig. 7.13. Hence,

$$\Delta p \approx N_L\chi\left(\frac{\rho V_{max}^2}{2}\right)f = 16\left[\frac{1.085\,kg/m^3\times\left(15\,m/s\right)^2}{2}\right]0.35 = 684\,N/m^2 \qquad <$$

COMMENTS: (1) With $C_2 = 0.99$ for $N_L = 16$ from Table 7.8, assumption 4 is appropriate. (2) Note use of the density evaluated at $T_i = 298K$ in Eq. (1).

PROBLEM 7.86

KNOWN: Geometry, surface temperature, and air flow conditions associated with a tube bank.

FIND: Rate of heat transfer per unit length.

SCHEMATIC:

S_T=20mm
Flue gas
V=5m/s
T_∞=700K=T_i
p=1atm
S_L=20mm
Tube, D=10mm
T_s=300K
N_L=10rows, N_T=50 tubes/row
N=500, $S_L/D=S_T/D=2$
T_o

ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible radiation effects, (3) Gas properties are approximately those of air.

PROPERTIES: *Table A-4*, Air (300K, 1 atm): Pr = 0.707; *Table A-4*, Air (700K, 1 atm): $\nu = 68.1 \times 10^{-6}$ m^2/s, k = 0.0524 W/m·K, Pr = 0.695, ρ = 0.498 kg/m^3, c_p = 1075 J/kg·K.

ANALYSIS: The rate of heat transfer per unit length of tubes is

$$q' = \bar{h} N \pi D\, \Delta T_{\ell m} = \bar{h} N \pi D \frac{(T_s - T_i) - (T_s - T_o)}{\ell n\left[(T_s - T_i)/(T_s - T_o)\right]}.$$

With $V_{max} = \frac{S_T}{S_T - D} V = \frac{20}{10}\, 5 \text{ m/s} = 10 \text{ m/s}$, $Re_{D,max} = \frac{V_{max} D}{\nu} = \frac{10 \text{ m/s} \times 0.01 \text{ m}}{68.1 \times 10^{-6} \text{ m}^2/\text{s}} = 1468.$

Tables 7.7 and 7.8 give C = 0.27, m = 0.63 and C_2 = 0.97. Hence from the Zhukauskas correlation,

$$\overline{Nu}_D = C C_2 Re_{D,max}^m Pr^{0.36} (Pr/Pr_s)^{1/4} = 0.26(1468)^{0.63}(0.695)^{0.36}(0.695/0.707)^{1/4}$$

$$\overline{Nu}_D = 22.4 \quad \bar{h} = \frac{k}{D}\overline{Nu}_D = 0.0524 \text{ W/m} \cdot \text{K} \times 22.4/0.01 \text{ m} = 117 \text{ W/m}^2 \cdot \text{K}.$$

Hence,

$$(T_s - T_o) = (T_s - T_i)\exp\left(-\frac{\pi D N \bar{h}}{\rho V N_T S_T c_p}\right) = -400\text{K} \exp\left(-\frac{\pi \times 0.01 \text{ m} \times 500 \times 117 \text{ W/m}^2 \cdot \text{K}}{0.498 \text{ kg/m}^3 (5 \text{ m/s}) 50 (0.02 \text{ m}) 1075 \text{J/kg} \cdot \text{K}}\right)$$

$$T_s - T_o = -201.3\text{K}$$

and the heat rate is

$$q' = \left(117 \text{ W/m}^2 \cdot \text{K}\right) 500\pi \,(0.01 \text{ m}) \frac{(-400 + 201.3)\text{K}}{\ell n\left[(-400)/(-201.3)\right]} = -532 \text{ kW/m} \qquad <$$

COMMENTS: (1) There is a significant decrease in the gas temperature as it passes through the tube bank. Hence, the heat rate would have been substantially overestimated (- 768 kW) if the inlet temperature difference had been used in lieu of the log-mean temperature difference. (2) The negative sign implies heat transfer to the water. (3) If the temperature of the water increases substantially, the assumption of uniform T_s becomes poor. The extent to which the water temperature increases depends on the water flow rate.

PROBLEM 7.87

KNOWN: An air duct heater consists of an aligned arrangement of electrical heating elements with $S_L = S_T = 24$ mm, $N_L = 3$ and $N_T = 4$. Atmospheric air with an upstream velocity of 12 m/s and temperature of 25°C moves in cross flow over the elements with a diameter of 12 mm and length of 250 mm maintained at a surface temperature of 350°C.

FIND: (a) The total heat transfer to the air and the temperature of the air leaving the duct heater, (b) The pressure drop across the element bank and the fan power requirement, (c) Compare the average convection coefficient obtained in part (a) with the value for an isolated (single) element; explain the relative difference between the results; (d) What effect would increasing the longitudinal and transverse pitches to 30 mm have on the exit temperature of the air, the total heat rate, and the pressure drop?

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible radiation effects, (3) Negligible effect of change in air temperature across tube bank on air properties.

PROPERTIES: *Table A-4*, Air (T_i = 298, 1 atm): $\rho = 1.171$ kg/m^3, $c_p = 1007$ J/kg·K; Air ($T_m = (T_i + T_o)/2 = 309$ K, 1 atm): $\rho = 1.130$ kg/m^3, $c_p = 1007$ J/kg·K, $\mu = 1.89 \times 10^{-5}$ N·s/m^2, k = 0.02699 W/m·K, Pr = 0.7057; Air (T_s = 623 K, 1 atm): $Pr_s = 0.687$; Air ($T_f = (T_i + T_o)/2 = 461$ K, 1 atm): $\nu = 3.373 \times 10^{-5}$ m^2/s, k = 0.03801 W/m·K, Pr = 0.686.

ANALYSIS: (a) The total heat transfer to the air is determined from the rate equation, Eq. 7.71,

$$q = N\left(\bar{h}_D \pi D \Delta T_{\ell m}\right) \qquad (1)$$

where the log mean temperature difference, Eq. 7.69, is

$$\Delta T_{\ell m} = \frac{T_s - T_i}{T_s - T_o} \Big/ \ell m \frac{(T_s - T_i)}{(T_s - T_o)} \qquad (2)$$

and from the overall energy balance, Eq. 7.70,

$$\frac{T_s - T_o}{T_s - T_i} = \exp\left(\frac{\pi D N \bar{h}_D}{\rho V N_T S_T c_p}\right) \qquad (3)$$

The properties ρ and c_p in Eq. (3) are evaluated at the inlet temperature T_i. The average convection coefficient using the Zhukaukus correlation, Eq. 7.67 and 7.68,

$$\overline{Nu}_D = \frac{\bar{h}_D}{k} = C\ Re_{D,max}^m\ Pr^{0.36}\left(Pr/Pr_s\right)^{1/4} \qquad (4)$$

where C = 0.27, m = 0.63 are determined from Table 7.7 for the *aligned* configuration with $S_T/S_L = 1 > 0.7$ and $10^3 < Re_{D,max} \le 10^5$. All properties except Pr_s are evaluated at the arithmetic mean temperature $T_m = (T_i + T_o)/2$. The maximum Reynolds number, Eq. 7.62, is

Continued …..

PROBLEM 7.87 (Cont.)

$$Re_{D,max} = \rho V_{max} D / \mu \qquad (5)$$

where for the *aligned* arrangement, the maximum velocity occurs at the transverse plane, Eq. 7.65,

$$V_{max} = \frac{S_T}{S_T - D} V \qquad (6)$$

The results of the analyses for $S_T = S_L = 24$ mm are tabulated below.

V_{max} (m/s)	$Re_{D,max}$	$\overline{Nu}_D$	$\overline{h}_D$ (W/m^2·K)	$\Delta T_{\ell m}$ (°C)	q (W)	T_o (°C)	
24	1.723×104	96.2	216	314	7671	47.6	<

(b) The pressure drop across the tube bundle follows from Eq. 7.72,

$$\Delta p = N_L \chi \left(\rho V_{max}^2 / 2 \right) f \qquad (7)$$

where the friction factor, f, and correction factor, χ, are determined from Fig. 7.13 using $Re_{D,max} = 1.723 \times 10^4$,

$f = 0.2 \qquad \chi = 1$

Substituting numerical values,

$$\Delta p = 3 \times 1 \left[1.171 \text{ kg/m}^3 \times (24 \text{ m/s})^2 / 2 \right] \times 0.2$$

$$\Delta p = 195 \text{ N/m}^2$$ <

The fan power requirement is

$$P = \forall \Delta p = V\, N_T S_T L \Delta p \qquad (8)$$

$$P = 12 \text{ m/s} \times 4 \times 0.024 \text{ m} \times 0.250 \text{ m} \times 195 \text{ N/m}^2$$

$$P = 56 \text{ W}$$ <

where ∀ is the volumetric flow rate. For this calculation, ρ in Eq. (7) was evaluated at T_m.

(c) For a single element in cross flow, the average convection coefficient can be estimated using the Churchill-Bernstein correlation, Eq. 7.57,

$$\overline{Nu}_D = \frac{\overline{h}_D D}{k} = 0.3 + \frac{0.62\, Re_D^{1/2}\, Pr^{1/3}}{\left[1 + (0.4/Pr)^{2/3}\right]^{1/4}} \left[1 + \left(\frac{Re_D}{282{,}000} \right)^{5/8} \right]^{4/5} \qquad (9)$$

where all properties are evaluated at the film temperature, $T_f = (T_i + T_o)/2$. The results of the calculations are

$$Re_D = 4269 \qquad \overline{Nu}_{D,1} = 33.4 \qquad \overline{h}_{D,1} = 106 \text{ W/m}^2 \cdot \text{K}$$ <

Continued …..

PROBLEM 7.87 (Cont.)

For the isolated element, $\bar{h}_{D,1} = 106\ W/m^2 \cdot K$, compared to the average value for the array, $\bar{h}_D = 216\ W/m^2 \cdot K$. Because the first row of the array acts as a turbulence grid, the heat transfer coefficient for the second and third rows will be larger than for the first row. Here, the array value is twice that for the isolated element.

(d) The effect of increasing the longitudinal and transverse pitches to 30 mm, should be to reduce the outlet temperature, heat rate, and pressure drop. The effect can be explained by recognizing that the maximum Reynolds number will be decreased, which in turn will result in lower values for the convection coefficient and pressure drop. Repeating the calculations of part (a) for $S_L = S_T = 30$ mm, find

V_{max} (m/s)	$Re_{D,max}$	$\overline{Nu}_D$	$\bar{h}_D$ ($W/m^2 \cdot K$)	$\Delta T_{\ell m}$ (°C)	q (W)	T_o (°C)
12	1.46×10^4	86.7	193	317	6925	41.3

and part (b) for the pressure drop and fan power, find

$$f = 0.18 \qquad \chi = 1 \qquad \Delta p = 122\ N/m^2 \qquad P = 44\ W$$

PROBLEM 7.88

KNOWN: Surface temperature and geometry of a tube bank. Velocity and temperature of air in cross-flow.

FIND: (a) Air outlet temperature, (b) Pressure drop and fan power requirements.

SCHEMATIC:

S_T=60mm
Air
V=15 m/s
T_∞=300K=T_i
S_L=60mm
Tube, D=30mm
T_s=373K
N_L=10, N_T=7, N=70
T_o

ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible radiation, (3) Air pressure is approximately one atmosphere, (4) Uniform surface temperature.

PROPERTIES: *Table A-4*, Air (300 K, 1 atm): ρ = 1.1614 kg/m^3, c_p = 1007 J/kg·K, ν = 15.89 × 10^{-6} m^2/s, k = 0.0263 W/m·K, Pr = 0.707; (373K): Pr = 0.695.

ANALYSIS: (a) The air temperature increases exponentially, with

$$T_o = T_s - (T_s - T_i)\exp\left(-\frac{p D N \bar{h}}{r V N_T S_T c_p}\right).$$

With $V_{max} = \frac{S_T}{S_T - D} V = \frac{60}{30} 15\frac{m}{s} = 30\frac{m}{s}$; $Re_{D,max} = \frac{30 \text{ m/s} \times 0.03 \text{ m}}{15.89 \times 10^{-6} \text{ m}^2/\text{s}} = 56{,}639.$

Tables 7.7 and 7.8 give C = 0.27, m = 0.63 and C_2 = 0.97. Hence from the Zhukauskas correlation,

$$\overline{Nu}_D = 0.27(0.97)(56{,}639)^{0.63}(0.707)^{0.36}(0.707/0.695)^{1/4} = 229$$

$$\bar{h} = \overline{Nu}_D k/D = 229 \times 0.0263 \text{ W/m}\cdot\text{K}/0.03 \text{ m} = 201 \text{ W/m}^2\cdot\text{K}.$$

Hence,

$$T_o = 373\text{K} - (373 - 300)\text{K} \exp\left(-\frac{p \times 0.03 \text{ m} \times 70 \times 201 \text{ W/m}^2\cdot\text{K}}{1.1614 \text{ kg/m}^3 \times 15 \text{ m/s} \times 7 \times 0.06 \text{ m} \times 1007 \text{ J/kg}\cdot\text{K}}\right)$$

$$T_o = 373\text{K} - 73\text{K} \times 0.835 = 312\text{K} = 39^\circ\text{C}.$$ <

(b) With $Re_{D,max}$ = 5.66 × 10^4, P_L = 2, (P_T - 1)/(P_L - 1) = 1, Fig. 7.13 yields f ≈ 0.19 and χ = 1. Hence,

$$\Delta p = N_L c\left(\frac{r V_{max}^2}{2}\right) f = 10\left(\frac{1.1614 \text{ kg/m}^3 \times (30 \text{ m/s})^2}{2}\right) 0.19 = 993 \text{ N/m}^2 = 0.00993 \text{ bar}.$$ <

The fan power requirement is

$$P = \dot{m}_a \Delta p / r = r V N_T S_T L \, \Delta p / r = 15 \text{ m/s} \times 7 \times 0.06 \text{ m} \times 1\text{m} \times 993 \text{ N/m}^2 = 6.26 \text{ kW}.$$ <

COMMENTS: The heat rate is

$$q = \dot{m}_a c_p (T_o - T_i) = r V N_T S_T L \, c_p (T_o - T_i)$$

$$q = 1.1614 \text{ kg/m}^3 \times 15 \text{ m/s} \times 7 \times 0.06\text{m} \times 1\text{m} \times 1007 \text{ J/kg}\cdot\text{K} \,(312 - 300)\text{K} = 88.4 \text{ kW}.$$

PROBLEM 7.89

KNOWN: Characteristics of pin fin array used to enhance cooling of electronic components. Velocity and temperature of coolant air.

FIND: (a) Average convection coefficient for array, (b) Total heat rate and air outlet temperature.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible radiation, (3) One-dimensional conduction in pins, (4) Uniform plate temperature, (5) Plates have a negligible effect on flow over pins, (6) Uniform convection coefficient over all surfaces, corresponding to average coefficient for flow over a tube bank.

PROPERTIES: Air (300 K, 1 atm): ρ = 1.1614 kg/m^3, Pr = 0.707, c_p = 1007 J/kg·K, μ = 184.6 × 10^{-7} kg/s·m, k = 0.0263 W/m·K. Aluminum (given): k = 240 W/m·K.

ANALYSIS: (a) From the Zhukauskas relation

$$\overline{Nu}_D = C Re_{D,max}^m Pr^{0.36}\left(Pr_\infty / Pr_s\right)^{1/4}$$

$$\left(Pr_\infty / Pr_s\right)^{1/4} \approx 1 \qquad V_{max} = \frac{S_T}{S_T - D} V = \frac{4}{4-2} 10\ \text{m/s} = 20\ \text{m/s}$$

$$Re_{D,max} = \frac{1.164\ \text{kg/m}^3 \times 20\ \text{m/s} \times 0.002\ \text{m}}{184.6\times10^{-7}\ \text{kg/s}\cdot\text{m}} = 2517$$

From Table 7.7 find C = 0.27 and m = 0.63, hence

$$\overline{Nu}_D = 0.27(2517)^{0.63}(0.707)^{0.36} = 33.1$$

$$\bar{h} = \overline{Nu}_D \frac{k}{D} = 33.1\times\frac{0.0263\ \text{W/m}\cdot\text{K}}{0.002\ \text{m}} = 435\ \text{W/m}^2\cdot\text{K}.$$ <

(b) If T_s = 350 K is taken to be the temperature of all of the heat transfer surfaces, correction must be made for the actual temperature drop along the pins. This is done by introducing the overall surface efficiency η_o and replacing $\bar{h}A$ by $\bar{h}A_t h_o$. Hence, to obtain the air outlet temperature, we use

$$\frac{T_s - T_o}{T_s - T_i} = \exp\left(-\frac{\bar{h}A_t h_o}{\dot{m}c_p}\right)$$

where

Continued …..

PROBLEM 7.89 (Cont.)

$$A_t = N\,(p DL) + 2W^2 - 2N\left(p D^2/4\right)$$

$$A_t = 625\left(p \times 0.002\text{ m} \times 0.1\text{ m}\right) + 2\left(0.1\text{ m}\right)^2 - 2 \times 625 p\left(0.002\text{ m}\right)^2/4 = 0.409\ \text{m}^2$$

Also $\quad h_o = 1 - \dfrac{A_f}{A_t}\left(1 - h_f\right)$ where h_f is given by Eq. (3.86). With symmetry about the midplane of the pin, q_f = M tanh (mL/2). Hence

$$h_f = \frac{q}{q_{max}} = \frac{\left(\bar{h} p\, Dk p\, D^2/4\right)^{1/2} q_b \tanh\left(mL/2\right)}{\bar{h} p\, D\left(L/2\right) q_b} = \frac{\tanh\left(mL/2\right)}{\left(\bar{h}/kD\right)^{1/2} L}$$

or, with $m = \left[\bar{h} p\, D/\left(k p\, D^2/4\right)\right]^{1/2} = 2\left(\bar{h}/kD\right)^{1/2}$,

$$h_f = \frac{\tanh\left(mL/2\right)}{mL/2}$$

$$m = 2\left(\frac{435\ \text{W/m}^2\cdot\text{K}}{240\ \text{W/m}\cdot\text{K} \times 0.002\ \text{m}}\right)^{1/2} = 60.2\ \text{m}^{-1}$$

$$mL/2 = 60.2\ \text{m}^{-1} \times 0.05\ \text{m} = 3.01 \qquad \text{and} \qquad \tanh\left(mL/2\right) = 0.995$$

$$h_f = \frac{0.995}{3.01} = 0.331.$$

Hence, $\quad h_o = 1 - \dfrac{625 \times p\left(0.002\text{ m}\right)\left(0.1\text{ m}\right)}{0.409\ \text{m}^2}\left(1 - 0.331\right) = 0.357$

$$\dot{m} = r V L N_T S_T = 1.1614\ \text{kg/m}^3\left(10\ \text{m/s}\right)0.1\ \text{m}\left(25\right)\left(0.004\ \text{m}\right) = 0.116\ \text{kg/s}.$$

Now evaluating the air outlet temperature,

$$\frac{T_s - T_o}{T_s - T_i} = \exp\left(-\frac{435\ \text{W/m}^2\cdot\text{K} \times 0.409\ \text{m}^2 \times 0.357}{0.116\ \text{kg/s} \times 1007\ \text{J/kg}\cdot\text{K}}\right) = 0.581$$

$$T_o = T_s - 0.581\left(T_s - T_i\right) = 350\ \text{K} - 0.581\left(50\ \text{K}\right)$$

$$T_o = 321\ \text{K}.$$ <

The total heat rate is

$$q = \dot{m} c_p\left(T_o - T_i\right) = 0.116\ \text{kg/s}\left(1007\ \text{J/kg}\cdot\text{K}\right)21\ \text{K} = 2453\ \text{W}.$$ <

COMMENTS: (1) The average surface heat flux which can be dissipated by the electronic components is $q/2W^2$ = 122,650 W/m^2, or 12.3 W/cm^2. (2) To check the numerical results, compute

$$\Delta T_{\ell m} = \frac{\Delta T_o - \Delta T_i}{\ln\left(\Delta T_o/\Delta T_i\right)} = \frac{29\ \text{K} - 50\ \text{K}}{\ln\left(29/50\right)} = 38.6\ \text{K}$$

Hence $\quad q = \bar{h} A_t h_o \Delta T_{\ell m} = 435\ \text{W/m}^2\cdot\text{K} \times 0.409\ \text{m}^2 \times 0.357 \times 38.6\ \text{K} = 2449\ \text{W}.$

PROBLEM 7.90

KNOWN: Dimensions and properties of chip, board and pin fin assembly. Convection conditions for chip and board surface. Maximum allowable chip temperature.

FIND: Effect of design and operating conditions on maximum chip power dissipation.

SCHEMATIC:

ASSUMPTIONS: (1) Uniform chip temperature, (2) One-dimensional conduction in pins, (3) Insulated pin tips, (4) Negligible radiation, (5) Uniform convection coefficient over pin and base surfaces.

PROPERTIES: *Table A.1*, copper (T ≈ 340 K): k_p = 397 W/m·K. *Table A.4*, air: properties evaluated using IHT *Properties* Tool Pad.

ANALYSIS: The chip heat rate may be expressed as

$$q_c = \frac{A_c\left(T_c - T_\infty\right)}{\left[R''_{t,c} + \left(L_b/k_b\right) + \left(1/h_b\right)\right]} + q_t$$

where $A_c = W^2$ and q_t is the total heat rate for the fin array. This heat rate must account for the variation of the air temperature across the array. Hence, the appropriate driving potential is $\Delta T_{lm} = \left[\left(T_c - T_i\right) - \left(T_c - T_o\right)\right]/\ln\left[\left(T_c - T_i\right)/\left(T_c - T_o\right)\right]$. However, the total surface area must account for the finite pin length and the exposed base (prime) surface. Hence, from Eqs. 3.101 and 3.102, with ΔT_{lm} replacing θ_b,

$$q_t = \bar{h} A_t \eta_o \Delta T_{lm}$$

where $A_t = N^2 A_f + A_b$, $A_b = A_c - N^2 A_{p,c}$, $A_{p,c} = \pi D_p^2/4$ and

$$\eta_o = 1 - \frac{N^2 A_f}{A_t}\left(1 - \eta_f\right)$$

For an adiabatic tip, Eq. 3.95 yields

$$\eta_f = \frac{\tanh mL_p}{mL_p}$$

where $m = \left(4\bar{h}/k_p D_p\right)^{1/2}$. The air outlet temperature is given by the expression

$$\frac{T_c - T_o}{T_c - T_i} = \exp\left(-\frac{\bar{h} A_t \eta_o}{\dot{m} c_p}\right)$$

Continued...

where $\dot{m} = \rho VWL_p$ and $\overline{h}$ is obtained from the Zhukauskas correlation,

$$\overline{Nu}_D = C_2 C\, Re_{D,max}^m\, Pr^{0.36} \left(Pr/Pr_s\right)^{1/4}$$

The foregoing model, including the convection correlation, was entered from the keyboard into the workspace of IHT and used with the *Properties* Tool Pad to perform the following parametric calculations.

Remaining within the limit $ND_p \leq 9$ mm, there is clearly considerable benefit associated with increasing N from 4 to 6 for $D_p = 1.5$ mm or with increasing D_p from 1.5 to 2.25 mm for N = 4. However, the best configuration corresponds to N = 6 and $D_p = 1.5$ mm (a larger number of smaller diameter pins), for which both A_t and $\overline{h}$ are approximately 50% and 20% larger than values associated with N = 4 and D_p = 2.25 mm. The peak heat rate is $q_c = 64.5$ W for V = 10 m/s, N = 6, and $D_p = 1.5$ mm.

COMMENTS: (1) The heat rate through the board is only $q_b = 0.295$ W and hence a negligible portion of the total heat rate. (2) Values of C = 0.27 and m = 0.63 were used for the entire range of conditions. However, $Re_{D,max}$ was less than 1000 in the mid to low range of V, for which the correlation was therefore used outside its prescribed limits and the results are somewhat approximate. (3) Using the IHT solver, the model was implemented in three stages, beginning with (i) the correlation and the Properties Tool Pad and sequentially adding (ii) expressions for q_t and $(T_c - T_o)/(T_c - T_i)$ without η_o, and (iii) inclusion of η_o in the model. Results computed from one calculation were loaded as initial guesses for the next calculation.

PROBLEM 7.91

KNOWN: Tube geometry and flow conditions for steam condenser. Surface temperature and pressure of saturated steam.

FIND: (a) Coolant outlet temperature, (b) Heat and condensation rates, (c) Effects of reducing longitudinal pitch and change in velocity.

SCHEMATIC:

S_L = 60, 30 mm (N_L = 20, 40)
S_T = 30 mm (N_T = 20)
Air
V = 4 m/s
T_i = 300 K
T_o
Steam
$p = p_{sat}$ = 2.455 bars
D = 20 mm, L =2 m
T_s = 390 K

ASSUMPTIONS: (1) Steady-state, (2) Negligible radiation, (3) Negligible effect of temperature change on air properties, (parts a and b), (4) Applicability of convection correlation outside designated range.

PROPERTIES: *Table A.4*, air (T_i = 300 K): ρ = 1.16 kg/m^3, c_p = 1007 J/kg·K, $\nu = 15.89 \times 10^{-6}$ m^2/s, k = 0.0263 W/m·K, Pr = 0.707. (T_s = 390 K): Pr = 0.692. *Table A.6*, saturated water at 2.455 bars: $h_{fg} = 2.183 \times 10^6$ J/kg.

ANALYSIS: (a) From Section 7.6 of the textbook,

$$T_o = T_s - (T_s - T_i)\exp\left(-\frac{\pi D N \bar{h}}{\rho V N_T S_T c_p}\right)$$

With

$$V_{max} = \frac{S_T}{S_T - D} V = \frac{30}{10} 4\,\text{m/s} = 12\,\text{m/s}$$

$$Re_{D,max} = \frac{V_{max} D}{\nu} = \frac{12\,\text{m/s}\,(0.02\,\text{m})}{15.89\times10^{-6}\,\text{m}^2/\text{s}} = 15{,}104$$

Using the Zhukauskas correlation outside its designated range $(S_T/S_L = 0.5)$, Table 7.7 yields C = 0.27 and m = 0.63. Hence, with C_2 = 1,

$$\overline{Nu}_D = C\,Re_{D,max}^m\,Pr^{0.36}\left(Pr/Pr_s\right)^{1/4} = 0.27(15{,}104)^{0.63}(0.707)^{0.36}\left(\frac{0.707}{0.692}\right)^{1/4} = 103$$

$$\bar{h} = \overline{Nu}_D\,(k/D) = 103(0.0263\,\text{W/m}\cdot\text{K}/0.02\,\text{m}) = 135\,\text{W/m}^2\cdot\text{K}$$

$$T_o = 390\,\text{K} - (90\,\text{K})\exp\left[-\frac{\pi(0.02\,\text{m})400(135\,\text{W/m}^2\cdot\text{K})}{1.16\,\text{kg/m}^3(4\,\text{m/s})20(0.03\,\text{m})1007\,\text{J/kg}\cdot\text{K}}\right] = 363\,\text{K}$$ <

(b) With $q = q' L$,

Continued...

$$q = N\left(\bar{h}\pi DL\Delta T_{lm}\right)$$

where

$$\Delta T_{lm} = \frac{\left(T_s - T_i\right) - \left(T_s - T_o\right)}{\ln\left(\dfrac{T_s - T_i}{T_s - T_o}\right)} = \frac{\left(90 - 27\right)K}{\ln\left(\dfrac{90}{27}\right)} = 52.3\,K$$

Hence $q = 400\left(135\,W/m^2 \cdot K\right)\pi\left(0.02\,m\right)2\,m\left(52.3\,K\right) = 355\,kW$ <

The condensation rate is

$$\dot{m}_{cond} = \frac{q}{h_{fg}} = \frac{3.55\times10^5\,W}{2.183\times10^6\,J/kg} = 0.163\,kg/s$$ <

(c) For S_L = 0.03 m, N_L = 40 and N = 800, using IHT with the foregoing model and the Properties Tool Pad to evaluate air properties at $(T_i + T_o)/2$, we obtain

$$T_o = 383.6\,K, \quad \Delta T_{lm} = 31.6\,C, \quad q = 414\,kW, \quad \dot{m}_{cond} = 0.190\,kg/s$$ <

As expected, q and $\dot{m}_{cond}$ increase with increasing N_L. However, due to a corresponding increase in T_o, and hence a reduction in ΔT_{lm}, the increase is not commensurate with the two-fold increase in surface area for the tube bank.

The effect of velocity is shown below.

The heat rate, and hence condensation rate, is strongly affected by velocity, because in addition to increasing $\bar{h}$, an increase in V decreases T_o, and hence increases ΔT_{lm}.

COMMENTS: (1) The calculations of part (a) should be repeated with air properties evaluated at $(T_i + T_o)/2$. (2) the condensation rate could be increased significantly by using a water-cooled (larger $\bar{h}$), rather than an air-cooled, condenser.

PROBLEM 7.92

KNOWN: Geometry of air jet impingement on a transistor. Jet temperature and velocity. Maximum allowable transistor temperature.

FIND: Maximum allowable operating power.

SCHEMATIC:

$T_e = 15°C$
$V_e = 20 m/s$
$D = 2 mm$
$H = 10 mm$
$T_s = 85°C$
$D_t = 10 mm$ $(r = 5 mm)$

ASSUMPTIONS: (1) Steady-state conditions, (2) Isothermal surface, (3) Bell-shaped nozzle, (4) All of the transistor power is dissipated to the jet.

PROPERTIES: *Table A-4*, Air (T_f = 323 K, 1 atm): $\nu = 18.2 \times 10^{-6}\ m^2/s$, k = 0.028 W/m·K, Pr = 0.704.

ANALYSIS: The maximum power or heat transfer rate by convection is

$$P_{max} = q_{max} = \overline{h}\left(\pi D_t^2/4\right)\left(T_s - T_e\right)_{max}.$$

For a single round nozzle,

$$\frac{\overline{Nu}}{Pr^{0.42}} = G\left(r/D, H/D\right) F_1\left(Re\right)$$

where D/r = 0.4 and

$$G = \frac{D}{r}\ \frac{1-1.1\left(D/r\right)}{1+0.1\left(H/D-6\right)\left(D/r\right)} = 0.4\frac{1-0.44}{1+0.1\left(-1\right)0.4} = 0.233.$$

With

$$Re = \frac{V_e D}{\nu} = \frac{\left(20\ m/s\right)0.002\ m}{18.2\times10^{-6}\ m^2/s} = 2198$$

$$F_1 = 2Re^{1/2}\left(1+0.005Re^{0.55}\right)^{1/2} = 2\left(2198\right)^{1/2}\left[1+0.005\left(2198\right)^{0.55}\right]^{1/2} = 108.7$$

Hence $\overline{h} = \frac{k}{D}GF_1 Pr^{0.42} = \frac{0.028\ W/m\cdot K}{0.002\ m}\left(0.233\right)\left(108.7\right)\left(0.704\right)^{0.42} = 306\ W/m^2\cdot K$

Hence $P_{max} = \left(306\ W/m^2\cdot K\right)\left(\pi/4\right)\left(0.01\ m\right)^2\left(70°C\right) = 1.68\ W.$ <

COMMENTS: (1) All conditions required for use of the correlation are satisfied.

(2) Power dissipation may be enhanced by allowing for heat loss through the side and base of the transistor.

PROBLEM 7.93

KNOWN: Dimensions of heated plate and slot jet array. Jet exit temperature and velocity. Initial plate temperature.

FIND: Initial plate cooling rate.

SCHEMATIC:

ASSUMPTIONS: (a) Negligible variation in h along plate, (b) Negligible heat loss from back surface of plate, (c) Negligible radiation from front surface of plate.

PROPERTIES: *Table A-1*, AISI 304 Stainless steel (1200 K): k = 28.0 W/m·K, c_p = 640 J/kg·K, ρ = 7900 kg/m^3; *Table A-4*, Air ($\overline{T}_f$ = 800 K): ν = 84.9 × 10^{-6} m^2/s, k = 0.0573 W/m·K, Pr = 0.709.

ANALYSIS: Performing an energy balance on a control surface about the plate,

$$-q_{conv} = -\overline{h}A_s\left(T_i - T_e\right) = \dot{E}_{st} = \rho\left(A_s t\right)c_p\left(dT/dt\right)_i \qquad \left.\frac{dT}{dt}\right)_i = -\frac{\overline{h}\left(T_i - T_e\right)}{\rho\, c_p t}.$$

For an array of slot nozzles,

$$\frac{\overline{Nu}}{Pr^{0.42}} = \frac{2}{3}A_{r,o}^{3/4}\left[\frac{2Re}{A_r / A_{r,o} + A_{r,o} / A_r}\right]^{2/3}$$

where A_r = W/S = 0.1

$$A_{r,o} = \left\{60 + 4\left[\left(H/2W\right) - 2\right]^2\right\}^{-1/2} = \left\{60 + 4\left(64\right)\right\}^{-1/2} = 0.0563$$

$$Re = \frac{V_e\left(2W\right)}{\nu} = \frac{30\text{ m/s}\left(0.02\text{ m}\right)}{84.9\times10^{-6}\text{ m}^2/\text{s}} = 7067$$

$$\overline{h} = \frac{0.0573\text{ W/m}\cdot\text{K}}{0.02\text{ m}}\frac{2}{3}\left(0.0563\right)^{3/4}\left[\frac{2\times7067}{1.776+0.563}\right]^{2/3} = 73.2\text{ W/m}^2\cdot\text{K}.$$

Hence,

$$\left.\frac{dT}{dt}\right)_i = -\frac{73.2\text{ W/m}^2\cdot\text{K}\left(800\text{ K}\right)}{\left(7900\text{ kg/m}^3\right)\left(640\text{ J/kg}\cdot\text{K}\right)\left(0.008\text{ m}\right)} = 1.45\text{ K/s}.$$ <

COMMENTS: (1) Bi = $\overline{h}t/k$ = (73.2 W/m^2·K) (0.008 m)/28 W/m·K = 0.02 and use of the lumped capacitance method is justified.

(2) Radiation may be significant.

(3) Conditions required for use of the correlation are satisfied.

PROBLEM 7.94

KNOWN: Air at 10 m/s and 15°C is available for cooling hot plastic plate. An array of slotted nozzles with prescribed width, pitch and nozzle-to-plate separation.

FIND: (a) Improvement in cooling rate achieved using the slotted nozzle arrangement in place of turbulent air in parallel flow over the plate, (b) Change in heat rates if air velocities were doubled, (c) Air mass rate requirement for the slotted nozzle arrangement.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) For parallel flow over plate, flow is turbulent, (3) Negligible radiation effects.

PROPERTIES: *Table A-4*, Air (T_f = (140 + 15)°C/2 = 350 K, 1 atm): ρ = 0.995 kg/m^3, $\nu = 20.92 \times 10^{-6}$ m^2/s, k = 30.3×10^{-3} W/m·K, Pr = 0.700.

ANALYSIS: (a) For turbulent flow over the *plate* of length L with

$$\mathrm{Re_L} = \frac{u_\infty L}{n} = \frac{10\ \mathrm{m/s} \times 0.5\ \mathrm{m}}{20.92 \times 10^{-6}\ \mathrm{m^2/s}} = 2.390 \times 10^5$$

using the turbulent flow correlation, find

$$\overline{\mathrm{Nu}}_\mathrm{L} = \frac{\bar{h}L}{k} = 0.037\mathrm{Re}_\mathrm{L}^{4/5}\,\mathrm{Pr}^{1/3} = 0.037\left(2.390 \times 10^5\right)^{4/5}(0.700)^{1/3} = 659.6$$

$$\bar{h} = \overline{\mathrm{Nu}}_\mathrm{L}\, k/L = 659.6 \times 0.030\ \mathrm{W/m \cdot K}/0.5\ \mathrm{m} = 39.6\ \mathrm{W/m^2 \cdot K}.$$

For an array of *slot nozzles*,

$$\overline{\mathrm{Nu}} = \frac{\bar{h}D}{k} = \frac{2}{3}A_{r,o}^{3/4}\left[\frac{2\mathrm{Re}}{A_r/A_{r,o} + A_{r,o}/A_r}\right]^{2/3}\mathrm{Pr}^{0.42}$$

where
$$\mathrm{Re} = \frac{V_e D_h}{n} = \frac{10\ \mathrm{m/s}\,(2 \times 0.004\ \mathrm{m})}{20.92 \times 10^{-6}\ \mathrm{m^2/s}} = 3824$$

$$A_{r,o} = \left\{60 + 4\left[(H/2W) - 2\right]^2\right\}^{-1/2} = \left\{60 + 4\left[40/2 \times 4 - 2\right]^2\right\}^{-1/2} = 0.1021$$

$$A_r = W/S = 4\ \mathrm{mm}/56\ \mathrm{mm} = 0.0714$$

$$\overline{\mathrm{Nu}} = \frac{2}{3}(0.1021)^{3/4}\left[\frac{2 \times 3824}{0.0714/0.1021 + 0.1021/0.0714}\right]^{2/3}(0.700)^{0.42} = 24.3$$

$$\bar{h} = \overline{\mathrm{Nu}}k/D_h = 24.3 \times 0.030\ \mathrm{W/m \cdot K}/2 \times 0.004\ \mathrm{m} = 91.1\ \mathrm{W/m^2 \cdot K}.$$

Continued …..

The improvement in heat rate with the slot nozzles (sn) over the flat plate (fp) is

$$\frac{q''_{sn}}{q''_{fp}} = \frac{\overline{h}_{sn}}{\overline{h}_{fp}} = \frac{91.1\ \text{W/m}^2 \cdot \text{K}}{39.6\ \text{W/m}^2 \cdot \text{K}} = 2.3. \qquad <$$

(b) If the air velocities were doubled for each arrangement in part (a), the heat transfer coefficients are affected as

$$\overline{h}_{sn} \sim Re^{2/3} \qquad \overline{h}_{fp} \sim Re^{4/5}.$$

Hence

$$\frac{\overline{h}_{sn}}{\overline{h}_{fp}} = 2.3\left(\frac{2^{2/3}}{2^{4/5}}\right) = 2.1. \qquad <$$

That is, comparative advantage of the slot nozzle over the flat plate decreases with increasing velocity.

(c) The mass rate of air flow through the array of slot nozzles is

$$\dot{m} = \rho N A_{c,e} = 0.995\text{kg/m}^3 \times 9\ (0.5\ \text{m} \times 0.004\ \text{m})10\text{m/s} = 0.179\text{kg/s}$$

where the number of slots is determined as

$$N \approx L/S = 0.5\text{m}/0.056\ \text{m} = 8.9 \approx 9. \qquad <$$

COMMENTS: Note, for the slot nozzle, the hydraulic diameter is $D_h = 2W$ and the relative nozzle area ($A_{c,e}/A_{cell}$) is $A_r = W/S$.

PROBLEM 7.95

KNOWN: Air jet velocity and temperature of 10 m/s and 15°C, respectively, for cooling hot plastic plate..

FIND: Design of optimal round nozzle array. Compare cooling rate with results for a slot nozzle array and flow over a flat plate. Discuss features associated with these three methods relevant to selecting one for this application.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible radiation effects.

PROPERTIES: *Table A-4*, Air (T_f = (140 + 15)°C/2 = 350 K, 1 atm): ρ = 0.995 kg/m^3, ν = 20.92 × 10^{-6} m^2/s, k = 30.0 × 10^{-3} W/m·K, Pr = 0.700.

ANALYSIS: To design an *optimal array* of round nozzles, we require that $D_{h,op}$ ≈ 0.2H and S_{op} ≈ 1.4H. Choose H = 40 mm, the nozzle-to-plate separation, hence

$$D_{h,op} = D = 0.2\times 40\text{ mm} = 8\text{ mm} \qquad S_{op} = 1.4\times 40\text{ mm} = 56\text{ mm}.$$

For an array of round nozzles,

$$\overline{Nu} = K\left(A_r, H/D\right)\cdot G\left(A_r, H/D\right)\cdot F_2\left(Re\right)\cdot Pr^{0.42}$$

where for an *in-line* array, see Fig. 7.17,

$$A_r = \frac{p D^2}{4S^2} = \frac{p\left(8\text{ mm}\right)^2}{4\left(56\text{ mm}\right)^2} = 0.0160$$

$$K = \left[1+\left(\frac{H/D}{0.6/A_r^{1/2}}\right)^6\right]^{-0.05} = \left[1+\left(\frac{40/8}{0.6/0.0160^{1/2}}\right)^6\right]^{-0.05} = 0.9577$$

$$G = 2A_r^{1/2}\frac{1-2.2A_r}{1+0.2\left(H/D-6\right)A_r^{1/2}} = 2\times 0.0160^{1/2}\frac{1-2.2\times 0.0160}{1+0.2\left(40/8-6\right)0.0160^{1/2}}$$

$$G = 0.2504$$

$$F_2 = 0.5Re^{2/3} = 0.5\left(\frac{10\text{ m/s}\times 0.008\text{ m}}{20.92\times 10^{-6}\text{ m}^2/\text{s}}\right)^{2/3} = 122.2.$$

The average heat transfer coefficient for the *optimal in-line* (op, il) array of round nozzles is,

$$\overline{h}_{op,il} = \overline{Nu}\,k/D_{h,op} = \frac{0.030\text{ W/m}\cdot\text{K}}{0.008\text{ m}}\times 0.9577\times 0.2504\times 122.2\left(0.700\right)^{0.42}$$

$$\overline{h}_{op,il} = 94.6\text{ W/m}^2\cdot\text{K}.$$

Continued …..

PROBLEM 7.95 (Cont.)

If an *optimal staggered* (op,s) array were used, see Fig. 7.17, with

$$A_r = \frac{\pi D^2}{2(3)^{1/2} S^2} = \frac{\pi \times (8\text{ mm})^2}{2(3)^{1/2}(56\text{ mm})^2} = 0.0185$$

find K = 0.9447, G = 0.2632, F_2 = 122.2 and $\overline{h}_{op,s} = 100.0\text{ W/m}^2 \cdot \text{K}$.

Using the previous results for *parallel flow* (pf) and the *slot nozzle* (sn) array, the heat rates, which are proportional to the average convection coefficients, can be compared.

Arrangement	Flat plate (fp)	Slot nozzle (sn)	Optimal round nozzle (op) In-line (il)	Optimal round nozzle (op) Staggered (s)
$\overline{h}$, $\text{W/m}^2 \cdot \text{K}$	39.6	91.1	94.6	100.0
$\overline{h}/h_{fp}$	1.0	2.30	2.39	2.53
$\dot{m}$, kg/s	---	0.199	0.040	0.046

For these flow conditions, we conclude that there is only slightly improved performance associated with using the round nozzles. As expected, the *staggered* array is better than the *in-line* arrangement, since the former has a higher area ratio (A_r). The air flow requirements for the round nozzle arrays are

$$\dot{m} = \rho N A_{c,e} V_e = \rho \left(A_s / A_{cell}\right) A_{c,e} V_e = \rho A_r A_s V_e$$

where $N = A_s/A_{cell}$ is the number of nozzles and A_s is the area of the plate to be cooled. Substituting numerical values, find

$$\dot{m}_{op,il} = 0.995\text{ kg/m}^3 \times 0.0160\left(0.5 \times 0.5\text{ m}^2\right) \times 10\text{ m/s} = 0.040\text{ kg/s}$$

$$\dot{m}_{op,s} = 0.995\text{ kg/m}^3 \times 0.0185\left(0.5 \times 0.5\text{ m}^2\right) \times 10\text{ m/s} = 0.046\text{ kg/s}.$$

For this application, selection of a nozzle arrangement should be based upon air flow requirements (round nozzles have considerable advantage) and costs associated with fabrication of the arrays (slot nozzle may be easier to form from sheet metal).

PROBLEM 7.96

KNOWN: Exit diameter of plasma generator and radius of jet impingement surface. Temperature and velocity of plasma jet. Temperature of impingement surface. Droplet deposition rate.

FIND: Rate of heat transfer to substrate due to convection and release of latent heat.

SCHEMATIC:

D = 10 mm
Plasma jet
V_e = 400 m/s, T_e = 10,000 K
k = 0.671 W/m·K, ν =5.6 x 10^{-3} m^2/s
Pr = 0.60
Droplets
$\dot{m}''_p$ = 0.02 kg/s·m^2
h_{sf} = 3.577 x 10^6 J/kg
H = 100 mm
r = 25 mm
q
Substrate
T_s = 300 K

ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible radiation, (3) Negligible sensible energy change due to cooling of droplets to T_s.

ANALYSIS: The total heat rate to the substrate is due to convection from the jet and release of the latent heat of fusion due to solidification, $q = q_{conv} + q_{lat}$. With Re = V_eD/ν = (400 m/s)0.01 m/5.6 × 10^{-3} m^2/s = 714, D/r = 0.4, and H/D = 10, $F_1 = 2Re^{1/2}(1 + 0.005\ Re^{0.55})^{1/2}$ = 58.2 and G = (D/r)(1 - 1.1D/r)/[1 + 0.1(H/D - 6)D/r] = 0.193, the correlation for a single round nozzle (Chapter 7.7) yields

$$\overline{Nu} = GF_1\,Pr^{0.42} = 0.193(58.2)\left(0.60^{0.42}\right) = 9.07$$

$$\bar{h} = \overline{Nu}\,(k/D) = 9.07\,(0.671\,W/m\cdot K/0.01\,m) = 6.09\,W/m^2\cdot K$$

Hence,

$$q = \bar{h}A_s\,(T_e - T_s) = 609\,W/m^2\cdot K \times \pi\,(0.025\,m)^2\,(10{,}000 - 300)\,K = 11{,}600\,W$$ <

The release of latent heat is

$$q_{lat} = A_s\dot{m}''_p h_{sf} = \pi\,(0.025\,m)^2\left(0.02\,kg/s\cdot m^2\right)3.577\times10^6\,J/kg = 140\,W$$ <

COMMENTS: (1) The large plasma temperature renders heat transfer due to droplet deposition negligible compared to convection from the plasma. (2) Note that Re = 714 is outside the range of applicability of the correlation, which has therefore been used as an approximation to actual conditions.

PROBLEM 7.97

KNOWN: A round nozzle with a diameter of 1 mm located a distance of 2 mm from the surface mount area with a diameter of 2.5 mm; air jet has a velocity of 70 m/s and a temperature of 400°C.

FIND: (a) Estimate the average convection coefficient over the area of the surface mount, (b) Estimate the time required for the surface mount region on the PCB, modeled as a semi-infinite medium initially at 25°C, to reach 183°C; (c) Calculate and plot the surface temperature of the surface mount region for air jet temperatures of 400, 500 and 600°C as a function time for $0 \le t \le 40$ s. Comment on the outcome of your study, the appropriateness of the assumptions, and the feasibility of using the jet for a soldering application.

SCHEMATIC:

ASSUMPTIONS: (1) Air jet is a single round nozzle, (2) Uniform temperature over the PCB surface, and (3) Surface mount region can be modeled as a one-dimensional semiinfinite medium.

PROPERTIES: *Table A-4*, Air (T_f = 486 K, 1 atm): $\nu = 3.693 \times 10^{-6}$ m^2/s, k = 0.03971 W/m·K, Pr = 0.685; Solder (given): ρ = 8333 kg/m^3, c_p = 188 J/kg·K, and k = 51 W/m·K; eutectic temperature, T_{sol} = 183°C; PCB (given): glass transition temperature, T_{gl} = 250°C.

ANALYSIS: For a single round nozzle, from the correlation of Eqs. 7.79, 7.80 and 7.81b, estimate the convection coefficient,

$$\frac{\overline{Nu}}{Pr^{0.42}} = G\left(\frac{r}{D}, \frac{H}{D}\right) F_1(Re) \qquad \overline{Nu} = \frac{\bar{h}D}{k} \tag{1,2}$$

where

$$F_1 = 2\,Re^{1/2}\left(1 + 0.005\ Re^{0.55}\right)^{1/2} \tag{3}$$

$$G = 2\ A_r^{1/2} \frac{1 - 2.2\ A_r^{1/2}}{1 + 0.2(H/D - 6)A_r^{1/2}} \tag{4}$$

$$A_r = D^2/4r_o^2 \tag{5}$$

The Reynolds number is based on the jet diameter and velocity at the nozzle,

$$Re_D = V_e\,D/\nu \tag{6}$$

and r_o is the radius of the region over which the average coefficient is being evaluated. The thermophysical properties are evaluated at the film temperature, $T_f = (T_e + T_s)/2$. The results of the calculation are tabulated below.

Continued …..

PROBLEM 7.97 (Cont.)

Re	F_1	G	A_r	$\overline{Nu}$	$\bar{h}$ (W/m^2·K)
1895	99.94	0.2667	0.04	22.73	903

<

Consider the surface mount region as a semi-infinite medium, with solder properties, initially at a uniform temperature of 25°C, that experiences sudden exposure to the convection process with the air jet at a temperature T_∞ = 400°C and the convection coefficient as found in part (a). The surface temperature, T(0,t), is determined from Case 3, Fig. 5.7 and Eq. 5.60,

$$\frac{T(0,t)-T_i}{T_\infty - T_i} = -\exp\left(\frac{h^2 \alpha t}{k^2}\right) \times \operatorname{erfc}\left(\frac{h(\alpha t)^{1/2}}{k}\right) \qquad (7)$$

where $\alpha = k/\rho c_p$. With T_i = 25°C and $T_\infty = T_e$, by trial-and-error, or by using the appropriate *IHT* model, find

$$T(0,t_o) = 183°C \qquad t_o = 31.9\ s$$

<

(c) Using the foregoing relations in *IHT*, the surface temperature T(0,t) is calculated and plotted for jet air temperatures of 400, 500 and 600°C for $0 \le t \le 40$ s.

Surface temperature, T(0,t) (C)
Elapsed time, t (s)
Te = 400 C
Te = 500 C
Te = 600 C
Tsol = 183 C, solder temperature
Tgl = 250 C, glass transition

The effect of increasing the jet air temperature is to reduce the time for the surface temperature to reach the solder temperature of 183°C. With the 600°C air jet, it takes about 10 s to reach the solder temperature, and the glass transition temperature is achieved in 27 s. The analysis represents a first-order model giving approximate results only. While the estimates for the average convection coefficients are reasonable, modeling the surface mount region as a semi-infinite medium is an over simplification. The region is of limited extent on the PCB, which is thin and also a poor approximation to an infinite medium. However, the model has provided insight into the conditions under which an air jet could be used for a soldering operation.

COMMENTS: (1) Note that for our application, the round nozzle correlation of part (a) meets the ranges of validity.

(2) The jet convection coefficient is not strongly dependent upon the air temperature. Values for 400, 500, and 600°C, respectively, are 903, 889, and 876 W/m^2·K.

PROBLEM 7.98

KNOWN: Diameter and properties of aluminum spheres used in packed bed. Porosity of bed and velocity and temperature of inlet air.

FIND: Time for sphere to acquire 90% of maximum possible thermal energy.

SCHEMATIC:

Packed bed, $\varepsilon = 0.40$

Aluminum sphere
$D = 75$ mm
$T_i = 25^oC$

PROBLEM 7.99

KNOWN: Overall dimensions of a packed bed of rocks. Rock diameter and thermophysical properties. Initial temperature of rock and bed porosity. Flow rate and upstream temperature of atmospheric air passing through the pile.

FIND: Rate of heat transfer to pile.

SCHEMATIC:

Air, $\dot{m}_a = 1\,kg/s$, $T_{a,i} = 90°C$, V; $D_b = 1\,m$; $L_b = 2\,m$; dx; T_a; $T_a + dT_a$; $T_{a,o}$

Rocks, $\varepsilon = 0.42$, $D_r = 0.03\,m$, $T_s = 25°C$, $\rho = 2300\,kg/m^3$, $c_p = 879\,J/kg \cdot K$

ASSUMPTIONS: (1) Rocks are spherical and at a uniform temperature, (2) Steady-state conditions.

PROPERTIES: *Table A-4*, Atmospheric air (T_∞ = 363K): $\nu = 22.35 \times 10^{-6}\ m^2/s$, $k = 0.031$ W/m·K, $Pr = 0.70$, $\rho = 0.963\ kg/m^3$, $c_p = 1010$ J/kg·K.

ANALYSIS: The heat transfer rate may be expressed as $q = \overline{h} A_{p,t} \Delta T_{\ell m}$ where the total surface area of the rocks is

$$A_{p,t} = V_r \frac{\pi D_r^2}{\pi D_r^3/6} = (1-\varepsilon)\left(\frac{\pi D_b^2}{4} L_b\right)\frac{6}{D_r} = (1-0.42)\ \left(\pi 1\ m^2/4 \times 2m\right) 6/0.03\ m = 182.2\ m^2.$$

The upstream velocity and Reynolds number are

$$V = \frac{\dot{m}_a}{\rho \pi D_b^2/4} = \frac{4 \times 1\ kg/s}{\left(0.963\ kg/m^3\right)\pi\ \ 1m^2} = 1.32\ m/s \quad Re_D = \frac{VD_r}{\nu} = \frac{1.32\ m/s \times 0.03\ m}{22.35 \times 10^{-6}\ m^2/s} = 1772.$$

From Section 7.8, it follows that

$$\varepsilon \overline{j}_H = \varepsilon \overline{St} Pr^{2/3} = \varepsilon \frac{\overline{h}}{\rho V c_p} Pr^{2/3} = 2.06\ Re_D^{-0.575}$$

$$\overline{h} = \frac{2.06}{\varepsilon} \rho V c_p Re_D^{-0.575}\ Pr^{-2/3}$$

$$\overline{h} = \frac{2.06}{0.42} 0.963\ kg/m^3 \times 1.32\ m/s \times 1010 J/kg \cdot K\,(1772)^{-0.575}(0.70)^{-2/3} = 108\ W/m^2 \cdot K.$$

The appropriate form of the mean temperature difference, $\Delta T_{\ell m}$, may be obtained by performing an energy balance on a differential control volume about the rock. That is,

$$\dot{m}_a\ c_p\ T_a - \dot{m}_a\ c_p\left(T_a + dT_a\right) - dq_r = 0$$

where $dq_r = \overline{h} A'_{p,t} dx\left(T_a - T_s\right)$ and $A'_{p,t}$ is the rock surface area per unit length of bed. Hence

$$\dot{m}_a\ c_p\ dT_a = -\overline{h} A'_{p,t} dx\left(T_a - T_s\right) \qquad \frac{d\,T_a}{dx} = -\frac{\overline{h} A'_{p,t}}{\dot{m}_a\ c_p}\left(T_a - T_s\right).$$

Continued …..

PROBLEM 7.99 (cont.)

Integrating between inlet and outlet, it follows that

$$\ell n\left(T_a - T_s\right)\Big|_i^o = -\frac{\bar{h}A'_{p,t}}{\dot{m}_a\ c_p} L_b = -\frac{\bar{h}A_{p,t}}{\dot{m}_a\ cp} \qquad \ell n\frac{T_{a,o} - T_s}{T_{a,i} - T_s} = -\frac{\bar{h}A_{p,t}}{\dot{m}_a\ c_p}.$$

With $\quad q = \dot{m}_a\ c_p\left(T_{a,i} - T_{a,o}\right) = \dot{m}_a\ c_p\left[\left(T_{a,i} - T_s\right) - \left(T_{a,o} - T_s\right)\right]$

it follows that

$$q = \bar{h}A_{p,t}\frac{\left(T_{a,i} - T_s\right) - \left(T_{a,o} - T_s\right)}{\ell n\left[\left(T_{a,i} - T_s\right)/\left(T_{a,o} - T_s\right)\right]} = \bar{h}A_{p,t}\Delta T_{\ell m}$$

where $\quad \Delta T_{lm} = \dfrac{\left(T_{a,i} - T_s\right) - \left(T_{a,o} - T_s\right)}{\ln\left[\left(T_{a,i} - T_s\right)/\left(T_{a,o} - T_s\right)\right]}.$

The air outlet temperature may be obtained from the requirement

$$\frac{T_{a,o} - T_s}{T_{a,i} - T_s} = \exp\left(-\frac{\bar{h}A_{p,t}}{\dot{m}_a\ c_p}\right) = \exp\left(-\frac{108\text{W/m}^2 \cdot \text{K} \times 182.2\ \text{m}^2}{1\ \text{kg/s} \times 1010\ \text{J/kg} \cdot \text{K}}\right) = 3.46 \times 10^{-9}$$

$$T_{a,o} = 25^\circ\text{C} + 65^\circ\text{C}\left(3.46 \times 10^{-9}\right) = 25^\circ\text{C} + 2.25 \times 10^{-7}\ {}^\circ\text{C}$$

$$T_{a,o} \approx T_s = 25^\circ\text{C}.$$

Hence $\quad \Delta T_{\ell m} = \dfrac{65^\circ\text{C} - 2.25 \times 10^{-7}\ {}^\circ\text{C}}{\ell n\left(65^\circ\text{C}/2.25 \times 10^{-7}\ {}^\circ\text{C}\right)} = 3.34^\circ\text{C}$

and $\quad q = 108\text{W/m}^2 \cdot \text{K}\left(182.2\ \text{m}^2\right)3.34^\circ\text{C} = 65.7\ \text{kW}.$ <

COMMENTS: (1) The above result may be checked from the requirement that $q = \dot{m}_a\ c_p\left(T_{a,i} - T_{a,o}\right) = 1\ \text{kg/s} \times 1010\ \text{J/kg} \cdot \text{K} \times 65^\circ\text{C} = 65.7\ \text{kW}$.

(2) The heat rate would be *grossly* overpredicted by using a rate equation of the form $q = \bar{h}A_{p,t}\left(T_{a,i} - T_s\right)$.

(3) The foregoing results are reasonable during the early stages of the heating process; however q would decrease with increasing time as the temperature of the rock increases. The axial temperature distribution of the rock in the pile would be as shown for different times.

PROBLEM 7.100

KNOWN: Dimensions of packed bed of graphite-coated uranium oxide fuel elements. Volumetric generation rate in uranium oxide and upstream velocity and temperature of helium passing through the bed.

FIND: (a) Mean temperature of helium leaving bed, (b) Maximum temperature of uranium oxide.

SCHEMATIC:

Helium, $V = 30\,m/s$, $T_{m,i} = 400K$; $L = 0.3m$; $\varepsilon = 0.4$; $D = 0.1m$; $T_{m,o}$; Graphite; Uranium oxide, $\dot{q} = 4 \times 10^7\,W/m^3$; $T_{s,o}$; $T_{s,i}$; $\delta = 0.001m$; $D_p = 0.01m$

ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Negligible kinetic and potential energy changes, (4) Negligible longitudinal conduction in bed, (5) Bed is insulated from surroundings, (6) One-dimensional conduction in pellets.

PROPERTIES: Helium (given): $\rho = 0.089$ kg/m^3, $c_p = 5193$ J/kg·K, k = 0.236 W/m·K, $\mu = 3 \times 10^{-5}$ kg/s·m, Pr = 0.66; Graphite (given): k = 2 W/m·K; Uranium oxide (given): k = 2 W/m·K.

ANALYSIS: (a) From an energy balance for the entire packed bed

$$q = \dot{m}c_p\left(T_{m,o} - T_{m,i}\right)$$

where the heat rate is due to volumetric generation in the pellets.

$$q = \dot{q} \cdot V_p = \dot{q}\left(p D^2/4\right)L\left(1 - e\right)\left[D_p/\left(D_p + 2d\right)\right]^3$$

$$q = 4 \times 10^7 \text{ W/m}^3 \left(p/4\right)\left(0.1 \text{ m}\right)^2\left(0.3 \text{ m}\right)\left(0.6\right)\left(0.010/0.012\right)^3$$

$$q = 4 \times 10^7 \text{ W/m}^3\left(8.181 \times 10^{-4} \text{ m}^3\right) = 3.272 \times 10^4 \text{ W.}$$

With $\quad \dot{m} = r VA = 0.089 \text{ kg/m}^3 \times 30\text{m/s}\left(p/4\right)\left(0.1 \text{ m}\right)^2 = 0.021 \text{ kg/s}$

the outlet temperature is

$$T_{m,o} = T_{m,i} + \frac{q}{\dot{m}c_p} = 400 \text{ K} + \frac{3.272 \times 10^4 \text{W}}{0.021 \text{ kg/s} \times 5193 \text{ J/kg} \cdot \text{K}} = 700 \text{ K.}$$ <

(b) The maximum temperature occurs at the center of the pellet in the exit plane. Beginning with the heat equation for pellet, find

$$\frac{d}{dr}\left(r^2 \frac{dT}{dr}\right) = -\frac{\dot{q}}{k} r^2$$

$$r^2 \frac{dT}{dr} = -\frac{\dot{q}}{3k} r^3 + C_1$$

$$T\left(r\right) = -\frac{\dot{q}}{6k} r^2 - \frac{C_1}{r} + C_2.$$

Continued …..

PROBLEM 7.100 (Cont.)

Applying boundary conditions:

$$\text{at } r = 0 \qquad dT/dr\big|_{r=0} = 0 \qquad \rightarrow \qquad C_1 = 0$$

$$\text{at } r = r_i \qquad T(r_i) = T_{s,i} \qquad \rightarrow \qquad C_2 = T_{s,i} + \frac{\dot{q}}{6k} r_i^2$$

$$T(r) = T_{s,i} + \frac{\dot{q}}{6k}\left(r_i^2 - r^2\right)$$

$$T(0) = T_{s,i} + \frac{\dot{q} D_p^2}{24k}.$$

For one-dimensional conduction in a spherical shell,

$$T_{s,i} = T_{s,o} + \frac{q_p}{4p\,k}\left[\frac{1}{r_i} - \frac{1}{r_o}\right]$$

where $$T_{s,o} = T_o + \frac{q_p}{\overline{h}p\left(D_p + 2d\right)^2}$$

$$q_p = \dot{q}\left(p D_p^3/6\right) = 4\times10^7 \text{ W/m}^3 (p/6)(0.010 \text{ m})^3 = 20.9 \text{ W}.$$

The convection coefficient may be obtained from

$$e\,\overline{j}_H = 2.06 Re_D^{-0.575}$$

with $$Re_D = VD/n = 30 \text{ m/s}(0.012 \text{ m})\times 0.089 \text{ kg/m}^3/3\times10^{-5} \text{ kg/s}\cdot\text{m} = 1068.$$

Hence $$\overline{h} = \frac{rVc_p}{e}\;\frac{Re_D^{-0.575}}{Pr^{2/3}}\times 2.06$$

$$\overline{h} = \frac{0.089 \text{ kg/m}^3 \times 30 \text{ m/s} \times 5193 \text{ J/kg}\cdot\text{K}}{0.4}\times 2.06(1.068)^{-0.575}/(0.66)^{2/3} = 1709 \text{ W/m}^2\cdot\text{K}.$$

Evaluating the temperatures,

$$T_{s,o} = 700 \text{ K} + \frac{20.9 \text{ W}}{\left(1709 \text{ W/m}^2\cdot\text{K}\right)p\,(0.012 \text{ m})^2} = 727 \text{ K}$$

$$T_{s,i} = 727 \text{ K} + \frac{20.9 \text{ W}}{4p\times 2 \text{ W/m}\cdot\text{K}}\left(\frac{1}{0.005 \text{ m}} - \frac{1}{0.006 \text{ m}}\right) = 755 \text{ K}$$

$$T(0) = 755 \text{ K} + \frac{4\times10^7 \text{ W/m}^3 (0.01)^2}{24\times 2 \text{ W/m}\cdot\text{K}} = 838 \text{ K}.$$ <

COMMENTS: The prescribed conditions provide for operation well below the melting point of uranium oxide. Hence $\dot{q}$ could be substantially increased to achieve a higher helium outlet temperature.

PROBLEM 7.101

KNOWN: Diameter and properties of phase-change material. Dimensions of cylindrical vessel and porosity of packed bed. Inlet temperature and velocity of air.

FIND: (a) Outlet temperature of air and rate of melting, (b) Effect of inlet velocity and capsule diameter on outlet temperature, (c) Location at which complete melting of PCM is first to occur and subsequent variation of outlet temperature.

SCHEMATIC:

ASSUMPTIONS: (1) Negligible thickness (and thermal resistance) of capsule shell, (2) All capsules are at T_{mp}, (3) Constant properties, (4) Negligible heat transfer from surroundings to vessel.

PROPERTIES: Prescribed, PCM: $T_{mp} = 4°C$, $\rho = 1200\,kg/m^3$, $h_{sf} = 165\,kJ/kg$. *Table A-4*, Air (Assume $(T_i + T_o)/2 = 17°C = 290K$): $\rho_a = 1.208\,kg/m^3$, $c_p = 1007\,J/kg\cdot K$, $\nu = 15.00\times10^{-6}\,m^2/s$, Pr = 0.71.

ANALYSIS: (a) For a packed bed (Section 7.8), the outlet temperature is given by

$$T_o = T_{mp} - (T_{mp} - T_i)\exp\left(-\frac{\bar{h} A_{p,t}}{\rho_a V A_{c,b} c_p}\right)$$

where $A_{c,b} = \pi D_v^2/4 = \pi\left(0.40m^2\right)/4 = 0.126m^2$ and $A_{p,t} = (1-\varepsilon)(\forall_v/\forall_c)\left(\pi D_c^2\right) = (1-\varepsilon)$ $\left(1.5\pi L_v D_v^2/D_c\right) = 0.5\left(1.5\pi\times0.4m^3/0.05m\right) = 3.02m^2$. With $Re_D = VD_c/\nu = 1\,m/s\times0.05m/15.00$ $\times10^{-6}m^2/s = 3333$, the convection correlation for a packed bed yields

$$\varepsilon \bar{j}_H = \varepsilon\overline{St}\,Pr^{2/3} = \varepsilon\frac{\bar{h}}{\rho_a V\,c_p}Pr^{2/3} = 2.06\,Re_D^{-0.575}$$

$$\bar{h} = \frac{2.06\,\rho_a V\,c_p}{\varepsilon\,Pr^{2/3}\,Re_D^{0.575}} = \frac{2.06\times1.208\,kg/m^3\times1\,m/s\times1007\,J/kg\cdot K}{0.5(0.71)^{2/3}(3333)^{0.575}} = 59.4\,W/m^2\cdot K$$

Hence,

$$T_o = 4°C + (21°C)\exp\left(-\frac{59.4\,W/m^2\cdot K\times3.02\,m^2}{1.208\,kg/m^3\times1\,m/s\times0.126\,m^2\times1007\,J/kg\cdot K}\right) = 10.5°C \quad <$$

The rate at which PCM in the vessel changes from the solid to liquid state, $\dot{M}(kg/s)$, may be obtained from an energy balance that equates the total rate of heat transfer to the capsules to the rate of increase in latent energy of the PCM. That is

$$q = \frac{d}{dt}(M\,h_{sf}) = h_{sf}\,\dot{M}$$

Continued …..

PROBLEM 7.101 (Cont.)

where M is the total mass of PCM and

$$q = -\overline{h}\,A_{p,t}\,\frac{\left(T_{mp}-T_i\right)-\left(T_{mp}-T_o\right)}{\ell n\left(\dfrac{T_{mp}-T_i}{T_{mp}-T_o}\right)} = -59.4\,W/m^2\cdot K\times 3.02\,m^2\,\frac{-14.5°C}{\ell n\left(\dfrac{-21}{-6.5}\right)} = 2220\,W$$

Hence, $\dot{M} = q/h_{sf} = 2220\,W/165,000\,J/kg = 0.0134\,kg/s$ <

(b) The effect of the inlet velocity and capsule diameter are shown below.

Outlet temperature, To(C) vs. Inlet velocity, V(m/s) — y-axis 4 to 13, x-axis 0.1 to 1.5

Outlet temperature, To(C) vs. Capsule diameter, Dc(m) — y-axis 4 to 16, x-axis 0.01 to 0.08

Despite the reduction in $\overline{h}$ with decreasing V, the reduction in the mass flow rate of air through the vessel and the corresponding increase in the residence time of air in the vessel allow it to more closely achieve thermal equilibrium with the capsules before it leaves the vessel. Hence, T_o decreases with decreasing V, approaching T_{mp} in the limit $V \to 0$. Of course, the production of chilled air in kg/s decreases accordingly. With decreasing capsule diameter, there is an increase in the number of capsules in the vessel and in the total surface area $A_{p,t}$ for heat transfer from the air. Hence, the heat rate increases with decreasing D_c and the outlet temperature of the air decreases.

(c) Because the temperature of the air decreases as it moves through the vessel, heat rates to the capsules are largest and smallest at the entrance and exit, respectively, of the vessel. Hence, complete melting will first occur in capsules at the entrance. After complete melting begins to occur in the capsules, progressing downstream with increasing time, heat transfer from the air will increase the temperatures of the capsules, thereby decreasing the heat rate. With decreasing heat rate, the outlet temperature will increase, approaching the inlet temperature after melting has occurred in all capsules and they achieve thermal equilibrium with the inlet air.

COMMENTS: (1) The estimate of To used to evaluate the properties of air was good, and iteration of the solution is not necessary. (2) The total mass of phase change material in the vessel is $M = N_c\,\rho\,\forall_c = \left[(1-\varepsilon)\forall_v/\forall_c\right]\rho\,\forall_c = (1-\varepsilon)\,\rho\,L_v\left(\pi D_v^2/4\right) = (\pi/4)0.5\times 1200\,kg/m^3\,(0.4m)^3 = 30.2\,kg$. At the maximum possible melting rate of $\dot{M} = 0.0134\,kg/s$, it would therefore take 2250s = 37.5 min to melt all of the PCM in the vessel. Why would it, in fact, take longer to melt all of the PCM?

PROBLEM 7.102

KNOWN: Diameter and properties of phase-change material. Dimensions of cylindrical vessel and porosity of packed bed. Inlet temperature and velocity of air.

FIND: (a) Outlet temperature of air and rate of freezing, (b) Effect of inlet velocity and capsule diameter on outlet temperature, (c) Location at which complete melting of PCM is first to occur and subsequent variation of outlet temperature.

SCHEMATIC:

ASSUMPTIONS: (1) Negligible thickness (and thermal resistance) of capsule shell, (2) All capsules are at T_{mp}, (3) Constant properties, (4) Negligible heat transfer from vessel to surroundings.

PROPERTIES: Prescribed, PCM: T_{mp} = 50°C, $\rho = 900\,\text{kg/m}^3$, $h_{sf} = 200\,\text{kJ/kg}$. *Table A-4*, Air (Assume $(T_i + T_o)/2$ = 30°C = 303K): $\rho_a = 1.151\,\text{kg/m}^3$, $c_p = 1007\,\text{J/kg}\cdot\text{K}$, $\nu = 16.2\times10^{-6}\,\text{m}^2/\text{s}$, $Pr = 0.707$.

ANALYSIS: (a) For a packed bed (Section 7.8), the outlet temperature is given by

$$T_o = T_{mp} - \left(T_{mp} - T_i\right)\exp\left(-\frac{\bar{h}\,A_{p,t}}{\rho_a V\,A_{c,b}\,c_p}\right)$$

where $A_{c,b} = \pi D_v^2/4 = 0.126\,\text{m}^2$ and $A_{p,t} = (1-\varepsilon)(\forall_v/\forall_c)\pi D_c^2 = 3.02\,\text{m}^2$. With $Re_D = VD_c/\nu = 3086$, the convection correlation for a packed bed yields

$$\varepsilon\,\bar{j}_H = \varepsilon\overline{St}\,Pr^{2/3} = \varepsilon\frac{\bar{h}}{\rho_a V c_p}Pr^{2/3} = 2.06\,Re_D^{-0.575}$$

$$\bar{h} = \frac{2.06\,\rho_a V c_p}{\varepsilon\,Pr^{2/3}\,Re_D^{0.575}} = \frac{2.06\times1.151\,\text{kg/m}^3\times1\,\text{m/s}\times1007\,\text{J/kg}\cdot\text{K}}{0.5(0.7)^{2/3}(3086)^{0.575}} = 59.1\,\text{W/m}^2\cdot\text{K}$$

Hence,

$$T_o = 50°\text{C} - (30°\text{C})\exp\left(-\frac{59.1\,\text{W/m}^2\cdot\text{K}\times3.02\,\text{m}^2}{1.151\,\text{kg/m}^3\times1\,\text{m/s}\times0.126\,\text{m}^2\times1007\,\text{J/kg}\cdot\text{K}}\right) = 41.2°\text{C} \quad <$$

The rate at which PCM in the vessel solidifies, $\dot{M}$ (kg/s), may be obtained from an energy balance that equates the total rate of heat transfer from the capsules to the rate at which the latent energy of the PCM decreases. That is,

$$q = \frac{d}{dt}\left(M\,h_{s,f}\right) = h_{sf}\;\dot{M}$$

where M is the total mass of PCM and

Continued …..

PROBLEM 7.102 (Cont.)

$$q = \bar{h} A_{p,t} \frac{(T_{mp} - T_i) - (T_{mp} - T_o)}{\ell n \left(\frac{T_{mp} - T_i}{T_{mp} - T_o} \right)} = 59.1\, W/m^2 \cdot K \times 3.02\, m^2 \frac{21.2°C}{\ell n \left(\frac{30}{8.8} \right)} = 3085\, W$$

Hence, $\dot{M} = q / h_{sf} = 3085\, W / 200{,}000\, J/kg = 0.0154\, kg/s$ <

(b) The effect of V and D_c are shown below

Despite the reduction in $\bar{h}$ with decreasing V, the reduction in the mass flow rate of air in the vessel and the corresponding increase in the residence time of air in the vessel allow it to more closely reach thermal equilibrium with the capsules before it leaves the vessel. Hence, T_o increases with decreasing V, approaching T_{mp} in the limit $V \rightarrow 0$. Of course, the production of warm air in kg/s decreases accordingly. With decreasing capsule diameter, there is an increase in the number of capsules in the vessel and in the total surface area $A_{p,t}$ for heat transfer to the air. Hence, the heat rate and the air outlet temperature increase with decreasing D_c.

(c) Because the air temperature increases as it moves through the vessel, heat rates from the capsules are largest and smallest at the entrance and exit, respectively, of the vessel. Hence, complete freezing will first occur in capsules at the entrance. After complete freezing begins to occur in the capsules, progressing downstream with increasing time, heat transfer to the air will decrease the temperatures of the capsules, thereby decreasing the heat rate. With decreasing heat rate, the outlet temperature will decrease, approaching the inlet temperature after freezing has occurred in all capsules and they achieve thermal equilibrium with the inlet air.

COMMENTS: (1) The estimate of T_o used to evaluate the properties of air was good, and iteration of the solution is not necessary. (2) The total mass of phase change material in the vessel is $M = N_c \rho \forall_c = [(1-\varepsilon)\forall_v / \forall_c] \rho \forall_c = (1-\varepsilon)\rho L_v (\pi D_v^2 / 4) = 22.6\, kg$. At the maximum possible melting rate of $\dot{M} = 0.0154\, kg/s$, it would therefore take 1470s = 24.5 min to freeze all of the PCM in the vessel. Why would it, in fact, take longer to freeze all of the PCM?

PROBLEM 7.103

KNOWN: Flow of air over a flat, smooth wet plate.

FIND: (a) Average mass transfer coefficient, $\overline{h}_m$, (b) Water vapor mass loss rate, n_A (kg/s).

SCHEMATIC:

Dry air
$u_\infty = 35\,m/s$
$T_\infty = 300K$
$p_\infty = 1\,atm$
$\phi_\infty = 0$
Water
$\rho_{A,s} = \rho_{A,sat}(T_s), T_s = 300K$
Width, $w = 3m$
$L = 0.5m$
x

ASSUMPTIONS: (1) Steady-state conditions, (2) Heat-mass transfer analogy applies, (3) $Re_{x,c} = 5 \times 10^5$.

PROPERTIES: *Table A-4*, Air (300K): $\nu = 15.89 \times 10^{-6}\ m^2/s$, Pr = 0.707; *Table A-8*, Water vapor-air (300K, 1 atm): $D_{AB} = 0.26 \times 10^{-4}\ m^2/s$, $Sc = \nu/D_{AB} = 0.611$; *Table A-6*, Water vapor (300K): $\rho_{A,sat} = 1/v_g = 0.0256\ kg/m^3$.

ANALYSIS: (a) The Reynolds number for the plate, x = L, is

$$Re_L = \frac{u_\infty L}{n} = \frac{35\ m/s \times 0.5\ m}{15.89 \times 10^{-6}\ m^2/s} = 1.10 \times 10^6.$$

Hence flow is mixed and the appropriate flat plate convection correlation is given by Eq. 7.42,

$$\overline{Sh}_L = \frac{\overline{h}_m L}{D_{AB}} = \left(0.037\ Re_L^{4/5} - 871\right) Sc^{1/3} = \left(0.037\left[1.10 \times 10^6\right]^{0.8} - 871\right) 0.611^{0.33}$$

giving

$$\overline{Sh}_L = 1399 \qquad \overline{h}_m = \frac{1399 \times 0.26 \times 10^{-4}\ m^2/s}{0.5\ m} = 0.0728\ m/s. \qquad <$$

(b) The evaporative mass loss rate is

$$n_A = \overline{h}_m A_s \left(r_{A,s} - r_{A,\infty}\right)$$

where $A_s = L \cdot w$, $r_{A,\infty} = 0$ (dry air) and $r_{A,s} = r_{A,sat}$. Hence,

$$n_A = 0.0728\ m/s \times (0.5 \times 3)\ m^2\ (0.0256 - 0)\ kg/m^3 = 0.0028\ kg/s. \qquad <$$

PROBLEM 7.104

KNOWN: Air flow conditions over a wetted flat plate of known length and temperature.

FIND: (a) Heat loss and evaporation rate, per unit plate width, q' and n'_A, respectively, (b) Compute and plot q' and n'_A for a range of water temperatures $300 \le T_s \le 350$ K with air velocities of 10, 20 and 35 m/s, and (c) Water temperature T_s at which the heat loss will be zero for the air velocities and temperatures of part (b).

SCHEMATIC:

Vapor, A
n'_A
q'
Wetted surface, T_s = 300 K
Air, B
u_∞ = 35 m/s
p_∞ = 1 atm
T_∞ = 300 K
ϕ_∞ = 0
x
L = 0.5 m

ASSUMPTIONS: (1) Heat and mass transfer analogy is applicable, (2) Constant properties, (3) $Re_{x,c} = 5 \times 10^5$.

PROPERTIES: *Table A.4*, Air (T = 300 K, 1 atm): $\nu = 15.89 \times 10^{-6}\ m^2/s$; *Table A.6*, Water (300 K): $v_g = 39.13\ m^3/kg$, $h_{fg} = 2438$ kJ/kg; *Table A.8*, Water-air (298 K, 1 atm): $D_{AB} = 0.26 \times 10^{-4}\ m^2/s$, Sc = 0.61.

ANALYSIS: (a) The heat loss from the plate is due only to the transfer of latent heat. Per unit width of the plate,

$$q' = n'_A h_{fg} \quad (1)$$

$$n'_A = \bar{h}_m L\left[\rho_{A,sat}(T_s) - \rho_{A,\infty}\right] = \bar{h}_m L \rho_{A,sat}(T_s) \quad (2)$$

With

$$Re_L = \frac{u_\infty L}{\nu} = \frac{35\ m/s \times 0.5\ m}{15.89 \times 10^{-6}\ m^2/s} = 1.10 \times 10^6$$

mixed boundary layer condition exists and the appropriate correlation is Eq. 7.42,

$$\overline{Sh}_L = \left(0.037\, Re_L^{4/5} - 871\right) Sc^{1/3} = \left[0.037\left(1.10 \times 10^6\right)^{4/5} - 871\right](0.61)^{1/3} \quad (3)$$

giving $\overline{Sh}_L$ = 1398 and

$$\bar{h}_m = \overline{Sh}_L \frac{D_{AB}}{L} = 1398 \frac{0.26 \times 10^{-4}\ m^2/s}{0.5\ m} = 0.0727\ m/s\,.$$

with $\rho_{A,sat}(T_s) = v_g^{-1} = 0.0256\ kg/m^3$,

$$n'_A = 0.0727\ m/s\,(0.5\ m)\left(0.0256\ kg/m^3\right) = 9.29 \times 10^{-4}\ kg/s \cdot m$$ <

Hence, the evaporative heat loss per unit plate width is

$$q' = n'_A h_{fg} = 9.29 \times 10^{-4}\ kg/s \cdot m\left(2.438 \times 10^6\ J/kg\right) = 2265\ W/m$$ <

Continued...

PROBLEM 7.104 (Cont.)

Heat would have to be applied to the plate in the amount of 2265 W/m to maintain its temperature at 300 K with the evaporative heat loss.

(b) When T_s and T_∞ are different, convection heat transfer will also occur, and the heat loss from the water surface is

$$q'_{loss} = q'_{conv} + q'_{evap} = \overline{h}L\left(T_s - T_\infty\right) + n'_A h_{fg} \tag{4}$$

Invoking the heat-mass analogy, Eq. 6.92 with n = 1/3,

$$\overline{h}/\overline{h}_m = \rho c\left(\alpha/D_{AB}\right)^{2/3} \tag{5}$$

where $\overline{h}_m$ and n'_A are evaluated using Eqs. (3) and (2), respectively. Using the foregoing relations in the *IHT Workspace*, but evaluating $\overline{h}$ (rather than $\overline{h}_m$) with the *Correlations Tool, External Flow*, for the *Average* coefficient for *Laminar* or *Mixed Flow*, q'_{loss} was evaluated as a function of u_∞ with T_∞ = 300 K.

(c) To determine the water temperature T_s at which the heat loss is zero, the foregoing IHT model was run with q'_{loss} = 0 with the result that, for all velocities,

T_s = 281 K $\quad <$

COMMENTS: Why is the result for part (c) independent of the air velocity?

PROBLEM 7.105

KNOWN: Flow over a heated flat plate coated with a volatile substance.

FIND: Electric power required to maintain surface at $T_s = 134°C$.

SCHEMATIC:

Air, B
$u_\infty = 8\ m/s$
$T_\infty = 20°C$
$\rho_{A,\infty} = 0,\ p = 1\ atm$
q_{conv}
q_{evap}
$\rho_{A,s}$, $T_s = 134°C$
Volatile coating (A)
Plate with electrical heater, $W = 0.25m$
q_{elec}
$L = 4\ m$
x

ASSUMPTIONS: (1) Steady-state conditions, (2) Heat-mass transfer analogy is applicable, (3) Transition occurs at $Re_{xc} = 5 \times 10^5$, (4) Perfect gas behavior of vapor A, (5) Upstream air is dry, $\rho_{A,\infty} = 0$.

PROPERTIES: *Table A-4*, Air (T_f = (134 + 20)°C/2 = 350 K, 1 atm): $\nu = 20.92 \times 10^{-6}\ m^2/s$, k = 0.030 W/m·K, Pr = 0.700; Substance A (given): $\mathcal{M}_A$ = 150 kg/kmol, $p_{A,sat}$ (134°C) = 0.12 atm, $D_{AB} = 7.75 \times 10^{-7}\ m^2/s$, $h_{fg} = 5.44 \times 10^6$ J/kg.

ANALYSIS: From an overall energy balance on the plate, the power required to maintain T_s is

$$q_{elec} = q_{conv} + q_{evap} = \overline{h}_L A_s \left(T_s - T_\infty\right) + \overline{h}_{m,L} A_s \left(\rho_{A,s} - \rho_{A,\infty}\right) h_{fg}. \qquad (1)$$

To estimate $\overline{h}_L$, first determine Re_L,

$$Re_L = u_\infty L/\nu = 8\ m/s \times 4\ m/20.92 \times 10^{-6}\ m^2/s = 1.530 \times 10^6.$$

Hence the flow is mixed and the appropriate correlation:

$$\overline{Nu}_L = \overline{h}_L L/k = \left(0.037\ Re_L^{4/5} - 871\right) Pr^{1/3}$$

$$\overline{h}_L = \left(0.030\ W/m \cdot K/4m\right) \left(0.037\left(1.530 \times 10^6\right)^{4/5} - 871\right) \left(0.700\right)^{1/3} = 16.0\ W/m^2 \cdot K.$$

To estimate $\overline{h}_{m,L}$, invoke the heat-mass analogy, with $Sc = \nu_B/D_{AB}$,

$$\overline{h}_{m,L} = \overline{h}_L \frac{D_{AB}}{k}\left(\frac{Sc}{Pr}\right)^{1/3} = 16.0 \frac{W}{m^2 \cdot K} \left(\frac{7.75 \times 10^{-7} m^2/s}{0.030\ W/m \cdot K}\right) \left(\frac{20.92 \times 10^{-6} m^2/s}{7.75 \times 10^{-7} m^2/s} /0.700\right)^{1/3} = 0.00140 \frac{m}{s}.$$

The density of species A at the surface, $\rho_{A,s}(T_s)$, follows from the perfect gas law,

$$\rho_{A,s} = p_{A,s} / \frac{\Re}{\mathcal{M}_A} T_s = 0.12\ atm / \frac{8.205 \times 10^{-2}\ m^3 \cdot atm/kmol \cdot K}{150\ kg/kmol} \cdot \left(134 + 273\right) K = 0.539 \frac{kg}{m^3}.$$

Using values calculated for $\overline{h}_L$, $\overline{h}_{m,L}$ and $\rho_{A,s}$ in Eq. (1), find

$$q_{elec} = \left(4m \times 0.25m\right) \left[16.0 \frac{W}{m^2 \cdot K}\left(134 - 20\right)^\circ C + 0.00140 \frac{m}{s}\left(0.539 - 0\right) \frac{kg}{m^3} \times 5.44 \times 10^6 \frac{J}{kg}\right]$$

$$q_{elec} = 1.0\ m^2\ [1{,}824 + 4{,}105]\ W/m^2 = 5.93\ kW.$$

<

COMMENTS: For these conditions, nearly 70% of the heat loss is by evaporation.

PROBLEM 7.106

KNOWN: Flow of dry air over a water-saturated plate for prescribed flow conditions and mixed temperature.

FIND: (a) Mass rate of evaporation per unit plate width, $n''_A \left(kg/s\cdot m\right)$, and (b) Calculate and plot n'_A as a function of velocity for the range $1 \le u_\infty \le 25$ m/s for air and water temperatures of $T_s = T_\infty = 300$, 325, and 350 K.

SCHEMATIC:

Dry Air, B

$u_\infty = 25$ m/s, $T_\infty = 350$ K

$\rho_{A,\infty} = 0$, $p = 1$ atm

Water (A)

$\rho_{A,s}$, $T = 350$ K

x

$L = 1$ m

ASSUMPTIONS: (1) Water surface is smooth, (2) Heat and mass transfer analogy is applicable, (3) $Re_{x,c} = 5 \times 10^5$.

PROPERTIES: *Table A.6*, Water vapor ($T_s = 350$ K, 1 atm): $\rho_{A,s} = 1/v_g = 1/3.846\ m^3/kg = 0.2600\ kg/m^3$; *Table A.4*, Air ($T_f = T_\infty = 350$ K, 1 atm): $\nu = 20.92 \times 10^{-6}\ m^2/s$, $\alpha = 29.9 \times 10^{-6}\ m^2/s$; *Table A.8*, Air-water ($T_f = T_\infty = 350$ K, 1 atm): $D_{AB} = 0.26 \times 10^{-4}\ m^2/s\ (350\ K/293\ K)^{3/2} = 0.339 \times 10^{-4}\ m^2/s$.

ANALYSIS: (a) Determine the nature of the air flow by calculating Re_L. With L = 1 m,

$$Re_L = \frac{u_\infty L}{\nu} = \frac{25\,m/s \times 1\,m}{20.92\times10^{-6}\,m^2/s} = 1.195\times10^6. \qquad (1)$$

Since $Re_L > 5 \times 10^5$, it follows that the flow is mixed, and with Eq. 7.42 using $Sc = \nu/D_{AB}$,

$$\overline{Sh}_L = \frac{\overline{h}_m L}{D_{AB}} = \left(0.037\,Re_L^{4/5} - 871\right)Sc^{1/3}. \qquad (2)$$

$$\overline{Sh}_L = \left(0.037\left[1.195\times10^6\right]^{4/5} - 871\right)\left(\frac{20.92\times10^{-6}\,m^2/s}{0.339\times10^{-4}\,m^2/s}\right)^{1/3} = 1550$$

The average mass transfer coefficient for the entire plate is

$$\overline{h}_m = \overline{Sh}_L \frac{D_{AB}}{L} = 1550\,\frac{0.339\times10^{-4}\,m^2/s}{1\,m} = 0.0526\,m/s.$$

The mass rate of water evaporation per unit plate width is

$$n'_A = \overline{h}_m L\left(\rho_{A,s} - \rho_{A,\infty}\right) = 0.0526\,m/s\times1\,m\left(0.260-0\right)kg/m^3 = 0.0137\,kg/s\cdot m \qquad <$$

(b) Using Eq. (1) and (3) in the IHT Workspace with the *Correlations Tool*, *External Flow*, *Flat Plate*, *Average* coefficient for *Laminar* or *Mixed Flow*, replacing heat transfer with mass transfer parameters, the evaporation rate as a function of a velocity for selected air-water velocities was calculated and is plotted below.

Continued...

PROBLEM 7.106 (Cont.)

Evaporation rate, n'A (kg/s.m)

0.014
0.012
0.01
0.008
0.006
0.004
0.002
0

0 5 10 15 20 25

Air velocity, uinf (m/s)

Air-water temperature, Ts = Tinf = 300 K
Ts = Tinf = 325 K
Ts = Tinf = 350 K

COMMENTS: (1) Note carefully the use of the heat-mass transfer analogy, recognizing that air is species B.

(2) How do you explain the abrupt slope changes in the evaporation rate as a function of velocity in the above plot?

PROBLEM 7.107

KNOWN: Temperature of water bath used to dissipate heat from 100 integrated circuits. Air flow conditions.

FIND: Heat dissipation per circuit.

SCHEMATIC:

ASSUMPTIONS: (1) Heat and mass transfer analogy is applicable, (2) Vapor may be approximated as a perfect gas, (3) Turbulent boundary layer over entire surface, (4) All heat loss is across air-water interface.

PROPERTIES: *Table A-4*, Air (325 K, 1 atm): $\nu = 18.4 \times 10^{-6}$ m^2/s, k = 0.0282 W/m·K, Pr = 0.704; *Table A-8*, Air-vapor (325 K, 1 atm): $D_{AB} = 0.26 \times 10^{-4}$ m^2/s$(325/298)^{3/2} = 0.296 \times 10^{-4}$ m^2/s, Sc = ν/D_{AB} = 0.622; *Table A-6*, Saturated water vapor (T_b = 350 K): ρ_g = 0.260 kg/m^3, $h_{fg} = 2.32 \times 10^6$ J/kg; (T_∞ = 300 K): ρ_g = 0.026 kg/m^3.

ANALYSIS: The heat rate is

$$q_1 = \frac{q}{N} = \frac{L^2}{N}\left[q'' + n''_A h_{fg}(T_b)\right].$$

Evaluate the heat and mass transfer convection coefficients with

$$Re_L = \frac{u_\infty L}{n} = \frac{10\text{ m/s} \times 0.1\text{ m}}{18.4 \times 10^{-6}\text{ m}^2/\text{s}} = 54{,}348$$

$$\bar{h} = (k/L)\,0.037 Re_L^{4/5} Pr^{1/3} = (0.0282\text{ W/m}\cdot\text{K}/0.1\text{ m})\,0.037(54{,}348)^{4/5}(0.704)^{1/3} = 57\text{ W/m}^2\cdot\text{K}$$

$$\bar{h}_m = (D_{AB}/L)\,0.037 Re_L^{4/5} Sc^{1/3} = \left(0.296 \times 10^{-4}\text{ m}^2/\text{s}/0.1\text{ m}\right)0.037(54{,}348)^{4/5}(0.622)^{1/3} = 0.0574\text{ m/s.}$$

The convection heat transfer rate is

$$q'' = \bar{h}(T_b - T_\infty) = 57\text{ W/m}^2\cdot\text{K}(350 - 300)\text{K} = 2850\text{ W/m}^2$$

and the evaporative cooling rate is

$$n''_A h_{fg} = \bar{h}_m\left[r_{A,sat}(T_b) - f_\infty r_{A,sat}(T_\infty)\right]h_{fg}(T_b)$$

$$n''_A h_{fg} = 0.0574\text{ m/s}[0.260 - 0.5 \times 0.026]\text{kg/m}^3 \times 2.32 \times 10^6\text{ J/kg} = 32{,}890\text{ W/m}^2$$

Hence

$$q_1 = \frac{(0.1\text{ m})^2}{100}(2850 + 32{,}890)\text{ W/m}^2 = 3.57\text{ W.}$$

<

COMMENTS: Heat loss due to evaporative cooling is approximately an order of magnitude larger than that due to the convection of sensible energy.

PROBLEM 7.108

KNOWN: Dry air flows at 300 K over water-filled trays, each 222 mm long, with velocity of 15 m/s while radiant heaters maintain the surface temperature at 330 K.

FIND: (a) Evaporative flux (kg/s·m^2) at a distance 1 m from leading edge, (b) Radiant flux at this distance required to maintain water temperature at 330 K, (c) Evaporation rate from the tray at location L = 1 m, n'_A (kg/s·m) and (d) Irradiation which should be applied to each of the first four trays such that their rates are identical to that found in part (c).

SCHEMATIC:

Radiant heaters
Dry air
T_∞ = 300 K
Water-filled tray
T_s = 330 K
x
L = 1 m
q''_{rad}
q''_{evap}
q''_{conv}

ASSUMPTIONS: (1) Steady-state conditions, (2) Heat-mass transfer analogy applicable, (3) Water vapor behaves as perfect gas, (4) All incident radiant power absorbed by water, (5) Critical Reynolds number is 5×10^5.

PROPERTIES: *Table A.4*, Air (T_f = 315 K, 1 atm): $\nu = 17.40 \times 10^{-6}$ m^2/s, k = 0.0274 W/m·K, Pr = 0.705; *Table A.8*, Water vapor-air (T_f = 315 K): $D_{AB} = 0.26 \times 10^{-4}$ m^2/s $(315/298)^{3/2} = 0.28 \times 10^{-4}$ m^2/s, Sc = ν/D_{AB} = 0.616; *Table A.6*, Saturated water vapor (T_s = 330 K): $\rho_{A,sat} = 1/v_g = 0.1134$ kg/m^3, h_{fg} = 2366 kJ/kg.

ANALYSIS: (a) The evaporative flux of water vapor (A) at location x is

$$\dot{n}''_{A,x} = h_{m,x}\left(\rho_{A,s} - \rho_{A,\infty}\right) = h_{m,x}\left[\rho_{A,sat}\left(T_s\right) - \phi_\infty \rho_{A,sat}\left(T_\infty\right)\right] \quad (1)$$

Evaluate Re_x to determine the nature of the flow and then select the proper correlation.

$$Re_x = \frac{u_\infty x}{\nu} = 15\,\text{m/s} \times 1\,\text{m} \Big/ 17.40 \times 10^{-6}\,\text{m}^2/\text{s} = 8.621 \times 10^5.$$

Hence, the flow is turbulent, and invoking the heat-mass analogy with Eq. 7.45,

$$Sh_x = \frac{h_m x}{D_{AB}} = 0.0296\,Re_x^{4/5}\,Sc^{1/3}$$

$$h_m = \frac{0.28 \times 10^{-4}\,\text{m}^2/\text{s}}{1\text{m}} \times 0.0296\left(8.621 \times 10^5\right)^{4/5}(0.616)^{1/3} = 3.952 \times 10^{-2}\,\text{m/s}.$$

Hence, the evaporative flux at x = 1 m is

$$\dot{n}''_{A,x} = 3.952 \times 10^{-2}\,\text{m/s}\left(0.1134\,\text{kg/m}^3 - 0\right) = 4.48 \times 10^{-3}\,\text{kg/s}\cdot\text{m}^2 \quad (2) \;<$$

(b) From an energy balance on the differential element at x = 1 m,

$$q''_{rad} = q''_{conv} + q''_{evap} = h_x\left(T_s - T_\infty\right) + \dot{n}''_{A,x} h_{fg} \quad (3)$$

Continued...

To estimate h_x, invoke the heat-mass analogy using the correlation, Eq. 7.45,

$$Nu_x / Sh_x = (Pr/Sc)^{1/3} \qquad \text{or} \qquad h_x = h_{m,x}\, k/D_{AB}\,(Pr/Sc)^{1/3} \tag{4}$$

$$h_x = 3.95\times10^{-2}\ \text{kg/s}\cdot\text{m}^2\left(0.0274\ \text{W/m}\cdot\text{K}\big/0.28\times10^{-4}\ \text{m}^2/\text{s}\right)(0.705/0.616)^{1/3} = 40.45\ \text{W/m}^2\cdot\text{K}$$

Hence, the required radiant flux is

$$q''_{rad} = 40.45\ \text{W/m}^2\cdot\text{K}\,(330-300)\,\text{K} + 4.48\times10^{-3}\ \text{kg/s}\cdot\text{m}^2 \times 2366\times10^3\ \text{J/kg}$$

$$q''_{rad} = 1,214\ \text{W/m}^2 + 10,600\ \text{W/m}^2 = 11,813\ \text{W/m}^2$$ <

(c) The flow is turbulent over tray 5 having its mid-length at x = 1 m, so that it is reasonable to assume,

$$\bar{h}_5 \approx h_x(1\,\text{m}) \tag{5}$$

so that the evaporation rate can be determined from the evaporative flux as,

$$n'_A = n''_{A,x}\Delta L = 4.48\times10^{-3}\ \text{kg/s}\cdot\text{m}^2 \times 0.222\ \text{m} = 9.95\times10^{-4}\ \text{kg/s}\cdot\text{m}$$ <

(d) For tray 5, following the form of Eq. (3), the energy balance is

$$q''_{rad,5}\Delta L = \bar{h}_5\Delta L\left(T_{s,5} - T_\infty\right) + n'_{A,5}h_{fg} \tag{6}$$

and the evaporation rate for the tray is

$$n'_{A,5} = \bar{h}_{m,5}\Delta L\left(\rho_{A,s} - 0\right) \tag{7}$$

While $\bar{h}_5$ and $\bar{h}_{m,5}$ represent tray averages, Eq. (4) is still applicable. Using the *IHT Correlation Tool*, *External Flow*, *Average* coefficient for *Laminar*, or *Mixed Flow*, $\bar{h}_5$ is evaluated as

$$\bar{h}_5 = \left[\bar{h}_x(1.10\,\text{m})L_5 - \bar{h}_x(0.880\,\text{m})L_4\right]/\Delta L \tag{8}$$

where $\Delta L = L_5 - L_4 = 0.22$ m. The same relations can be applied to trays 2, 3 and 4. For tray 1, $\bar{h}_1$ = $\bar{h}$ (0.22 m)·L_1, where $L_1 = \Delta L$. With Eqs. (3, 6, 7 and 8) in the IHT Workspace, along with the *Correlations* and *Properties Tools*, the following results were obtained with the requirement that the evaporation rate for each tray is equal at $n'_{A,5}$ = 10.01 × 10^{-4} kg/s·m.

Tray	1	2	3	4	5
T_s	342.7	357	348.1	329	330
q''_{rad}	11,920	11,150	11,400	11,950	11,920

COMMENTS: (1) Note carefully at which temperatures the thermophysical properties are evaluated.

(2) Recognize that in part (d), if we require equal evaporation rates for each tray, $n'_{A,5}$, the water temperature, T_s, and radiant flux, q''_{rad}, for each tray must be different since the convection coefficients $\bar{h}_x$ and $\bar{h}_{m,x}$ are different for each of the trays. How do you explain the changes in T_s? Which tray has the highest $\bar{h}$? The lowest $\bar{h}$?

(3) For tray 5, using Eq. (5) we found $\bar{h}_5$ = 40.45 W/m^2·K; using the more accurate formulation, Eq. (8), the result is 40.49 W/m^2·K. If the flow were laminar or mixed over the tray, Eq. (5) would be inappropriate.

PROBLEM 7.109

KNOWN: Irradiation on sequential water-filled trays of prescribed length and width. Temperature and velocity of airflow over the trays.

FIND: Rate of water loss from first, third and fourth trays and temperature of water in each tray.

SCHEMATIC:

Dry air
$T_\infty = 300$ K
$u_\infty = 15$ m/s
Radiant heaters
$G = 10^4$ W/m^2
G, q''_{evap}, q''_{conv}
x
$\Delta x = 0.25$ m ($W = 1$ m)

ASSUMPTIONS: (1) Steady-state conditions, (2) Uniform irradiation of each container, (3) Complete absorption of irradiation by water, (4) Negligible heat transfer between containers and from bottom of containers, (5) Validity of heat-mass transfer analogy, (6) Applicability of convection correlations for an isothermal surface, (7) $Re_{x,c} = 5 \times 10^5$.

PROPERTIES: *Table A.4*, air (1 atm, *assume* $T_f = 315$ K): $\nu = 17.4 \times 10^{-6}$ m^2/s, k = 0.0274 W/m·K, Pr = 0.705. *Table A.8*, vapor/air (1 atm, 315 K): $D_{AB} = 0.26 \times 10^{-4}$ m^2/s $(315/298)^{3/2} = 0.28 \times 10^{-4}$ m^2/s, Sc $= \nu/D_{AB} = 0.616$.

ANALYSIS: The temperature of each tray is determined by a balance between the absorbed radiation and the convection and evaporative losses. Hence,

$$G = q''_{conv} + q''_{evap} = \bar{h}\left(T_s - T_\infty\right) + \bar{h}_m \rho_{A,sat} h_{fg}$$

where, assuming an exponent of n = 1/3, the heat-mass transfer analogy yields

$$\bar{h}_m = \left(D_{AB}/k\right)\left(Sc/Pr\right)^{1/3}\bar{h} = \left(0.26\times10^{-4}\ m^2/s/0.0274\ W/m\cdot K\right)\left(0.616/0.705\right)^{1/3}\bar{h} = \left(9.07\times10^{-4}\ m^3\cdot K/W\cdot s\right)\bar{h}$$

Hence,

$$G = \bar{h}\left[\left(T_s - T_\infty\right) + 9.07\times10^{-4}\rho_{A,sat}h_{fg}\right]$$

With $Re_N = u_\infty N\Delta x/\nu = 15$ m/s$(N \times 0.25$ m$)/17.4 \times 10^{-6}$ m^2/s $= (2.155 \times 10^5)N$, the flow is laminar for N = 1,2 with transition to turbulence occurring for N = 3.

For tray 1,

$$\bar{h} = \left(k/\Delta x\right)0.664\,Re_1^{1/2}\,Pr^{1/3}$$
$$= \left(0.0274\ W/m\cdot K/0.25\ m\right)0.664\left(2.155\times10^5\right)^{1/2}\left(0.705\right)^{1/3} = 30.1\ W/m^2\cdot K$$

For tray 4, with x = 0.875 m (N = 7/2),

$$\bar{h}_4 \approx \left(k/x\right)0.0296\,Re_{7/2}^{4/5}\,Pr^{1/3}$$
$$= \left(0.0274\ W/m\cdot K/0.875\ m\right)0.0296\left(7.543\times10^5\right)^{4/5}\left(0.705\right)^{1/3} = 41.5\ W/m^2\cdot K$$

Continued...

PROBLEM 7.109 (Cont.)

For tray 3, $\bar{h}_3 = \left(\bar{h}_{1-3}L_3 - \bar{h}_{1-2}L_2\right)/\Delta x$, where

$$\bar{h}_{1-3}L_3 = k\left(0.037\,Re_3^{4/5} - 871\right)Pr^{1/3}$$
$$= 0.0274\,W/m\cdot K\,(0.037\times 44{,}510 - 871)(0.705)^{1/3} = 18.9\,W/m\cdot K$$

$$\bar{h}_{1-2}L_2 = k\left(0.664\,Re_2^{1/2}\,Pr^{1/3}\right)$$
$$= 0.0274\,W/m\cdot K\,(0.664\times 656.5)(0.705)^{1/3} = 10.6\,W/m\cdot K$$

$$\bar{h}_3 = (18.9 - 10.6)\,W/m\cdot K/0.25\,m = 33.1\,W/m^2\cdot K$$

For tray 1, the energy balance yields

$$10^4\,W/m^2 = 30.1\,W/m^2\cdot K\left[(T_s - T_\infty) + 9.07\times 10^{-4}\rho_{A,sat}h_{fg}\right]$$

Since $\rho_{A,sat}$ depends strongly on T_s, the solution to this equation must be obtained by trial-and-error, with $\rho_{A,sat}$ (and h_{fg}) determined from Table A.6. The solution yields

$$T_{s,1} \approx 334.7\,K$$ <

Similarly, for trays 3 and 4

$$T_{s,3} \approx 332.8\,K \qquad T_{s,4} \approx 327.1\,K$$ <

The evaporation rate for tray N is

$$\dot{m}_{evap} = \bar{h}_m\rho_{A,sat}(W\Delta x) = 2.27\times 10^{-4}\bar{h}\rho_{A,sat}$$

from which it follows that

$$\dot{m}_{evap,1} \approx 9.5\times 10^{-4}\,kg/s, \qquad \dot{m}_{evap,3} \approx 9.5\times 10^{-4}\,kg/s, \qquad \dot{m}_{evap,4} \approx 9.3\times 10^{-4}\,kg/s$$ <

COMMENTS: (1) The largest convection coefficient is associated with the tray for which the entire flow is turbulent. (2) The temperature of the water varies inversely with the average convection coefficient for its tray.

PROBLEM 7.110

KNOWN: Apparatus as described in Problem 7.40 providing a nearly uniform airstream over a flat *test plate* to experimentally determine the heat and mass transfer coefficients. Temperature history of the pre-heated plate for airstream velocities of 3 and 9 m/s were fitted to a fourth-order polynomial for determining the heat transfer coefficient. Water mass loss observations from a water-saturated paper over the plate and its surface temperature for determining the heat transfer coefficient.

FIND: (a) From the temperature-time history, determine the heat transfer coefficients and evaluate the constants C and m for a correlation of the form $\overline{Nu}_L = C\,Re^m\,Pr^{1/3}$; compare results with a standard-plate correlation and comment on the goodness of the comparison; explain any differences; (b) From the water mass loss observations, determine the mass transfer coefficients for the two flow conditions; evaluate the constants C and m for a correlation of the form $\overline{Sh}_L = C\,Re^m\,Sc^{1/3}$; and (c) Using the heat-mass analogy, compare the experimental results with each other and against standard correlations; comment on the goodness of the comparison; explain any differences.

SCHEMATIC:

Air
T_∞= 20 °C
u_∞= 3 or 5 m/s
$T_s(t) = a + bt + ct^2 + dt^3 + et^4$
Test plate
$A_s = L \times L$
x
L = 133 mm

(a) Heat transfer experiment

Temperature Observations

u_∞ (m/s)	3	9
Δt (s)	300	160
a (°C)	56.87	57.00
b (°C/s)	-0.1472	-0.2641
c (°C/s²)	3×10^{-4}	9×10^{-4}
d (°C/s³)	-4×10^{-7}	-2×10^{-6}
e (°C/s⁴)	2×10^{-10}	1×10^{-9}

Air
T_{db} = 27 °C
T_{wb} = 13 °C
V = 3 or 9 m/s
Water-saturated paper, 133 X 133 mm
$\overline{T}_s$
x
L = 133 mm

(b) Mass transfer experiment

Mass Loss Observations

V (m/s)	$\overline{T}_s$ (°C)	m (t) (g)	m (t + Δt) (g)	Δt (s)
3	15.3	55.62	54.45	475
9	16.0	55.60	54.50	240

ASSUMPTIONS: (1) Airstream over the test plate approximates parallel flow over a flat plate, (2) Plate is spacewise isothermal, (3) Negligible radiation exchange between plate and surroundings, (4) Constant properties, and (5) Negligible heat loss from the bottom surface or edges of the test plate.

PROPERTIES: *Heat transfer coefficient*, *Table A.4*, Air ($T_f = \left(\overline{T}_s - T_\infty\right)/2$ = 310 K, 1 atm): k_a = 0.0269 W/m·K, ν = 1.669 × 10⁻⁵ m²/s, Pr = 0.706. *Test plate* (*Given*): ρ = 2770 kg/m³, c_p = 875 J/kg·K, k = 177 W/m·K. *Mass transfer coefficient*: *Table A.6*, Water vapor ($\overline{T}_s$ = 15.3°C = 288.3 K): $\rho_{A,sat} = 1/v_g$ = 79.81 m³/kg = 0.01253 kg/m³; *Table A.6*, Water vapor ($\overline{T}_s$ = 16.0°C = 289 K): $\rho_{A,sat}$ = 0.01322 kg/m³; *Table A.6*, Water vapor (T_{inf} = 27°C = 300 K): $\rho_{A,sat}$ = 0.02556 kg/m³; *Table A.8*, Water vapor-air [$T_f = \left(\overline{T}_s + T_\infty\right)/2 \approx 295$ K]: $D_{AB} = 0.26 \times 10^{-4}$ m²/s $(295/298)^{1.5} = 0.256 \times 10^{-4}$ m²/s.

ANALYSIS: (a) Using the lumped-capacitance method, the energy balance on the plate is

$$-\overline{h}_L A_s \left[T_s(t) - T_\infty\right] = \rho V c_p \frac{dT}{dt} \quad (1)$$

Continued...

PROBLEM 7.110 (Cont.)

and the average convection coefficient can be determined from the temperature history, $T_s(t)$,

$$\bar{h}_L = \frac{\rho V c_p}{A_s} \frac{(dT/dt)}{T_s(t) - T_\infty} \tag{2}$$

where the temperature-time derivative is

$$\frac{dT_s}{dt} = b + 2ct + 3dt^2 + 4et^3 \tag{3}$$

The temperature time history plotted below shows the experimental behavior of the observed data.

Coefficient, hLbar (W/m^2.K)

80
60
40
20

30 40 50 60

Plate temperature, Ts (C)

unif = 3 m/s
unif = 5 m/s

Consider now the integrated form of the energy balance, Eq. (5.6), expressed as

$$\ln \frac{T_s(t) - T_\infty}{T_i - T_\infty} = -\left(\frac{\bar{h}_L A_s}{\rho V c}\right) t \tag{4}$$

If we were to plot the LHS vs t, the slope of the curve would be proportional to $\bar{h}_L$. Using IHT, plots were generated of $\bar{h}_L$ vs. T_s, Eq. (1), and $\ln\left[\left(T_s(t) - T_\infty\right)/\left(T_i - T_\infty\right)\right]$ vs. t, Eq. (4). From the latter plot, recognize that the regions where the slope is constant corresponds to early times (≤ 100s when u_∞ = 3 m/s and ≤ 50s when u_∞ = 5 m/s).

ln(theta/thetai)

0
-1
-2

0 50 100 150 200 250 300

Elapsed time, t (s)

uinf = 3 m/s
uinf = 5 m/s

Temperatures, Ts (C)

60
50
40
30

0 50 100 150 200 250 300

Elapsed time, t (s)

uinf = 3 m/s
uinf = 5 m/s

Continued...

Selecting two elapsed times at which to evaluate $\bar{h}_L$, the following results were obtained

u_∞ (m/s)	t (s)	T_s (t), (°C)	$\bar{h}_L$ (W/m^2·K)	$\overline{Nu}_L$	Re_L
3	100	44.77	30.81	152.4	2.39×10^4
9	50	45.80	56.7	280.4	7.17×10^4

where the dimensionless parameters are evaluated as

$$\overline{Nu}_L = \frac{\bar{h}_L L}{k_a} \qquad Re_L = \frac{u_\infty L}{\nu} \qquad (5,6)$$

where k_a, ν are thermophysical properties of the airstream.

(b) Using the above pairs of $\overline{Nu}_L$ and Re_L, C and m in the correlation can be evaluated,

$$\overline{Nu}_L = C\, Re_L^m\, Pr^{1/3} \qquad (7)$$

$$152.4 = C(2.39 \times 10^4)^m (0.706)^{1/3} \qquad 280.4 = C(7.17 \times 10^4)^m (0.706)^{1/3}$$

Solving, find C = 0.633 m = 0.555 (8,9) <

The plot below compares the experimental correlation (C = 0.633, m = 0.555) with those for laminar flow (C = 0.664, m = 0.5) and fully turbulent flow (C = 0.037, m = 0.8). The experimental correlation yields $\overline{Nu}_L$ values which are 25% higher than for the correlation. The most likely explanation for this unexpected trend is that the airstream reaching the plate is not parallel, but with a slight impingement effect and/or the flow is very highly turbulent at the leading edge.

(b) From the convection mass transfer rate equation,

$$n_A = \bar{h}_{m,L} A_s \left(\rho_{A,s} - \rho_{A,\infty}\right) \qquad (10)$$

where the evaporation rate can be determined from the paper mass and time interval observations,

$$n_A = \left[m(t + \Delta t) - m(t)\right] / \Delta t \qquad (11)$$

and the species densities, $\rho_{A,s}$ and $\rho_{A,\infty}$, correspond to $\rho_{A,sat}(\bar{T}_s)$ and $\phi_\infty \rho_{A,sat}(T_\infty)$, respectively.

Using the ASHRAE psychrometric chart (1 atm) with T_{wb} = 13°C and T_{db} = 27°C, find the relative humidity as ϕ_∞ = 0.17. The correlation dimensionless parameters are evaluated as

$$\overline{Sh}_L = \frac{\bar{h}_{m,L} L}{D_{AB}} \qquad Re_L = \frac{u_\infty L}{\nu} \qquad Sc = \frac{\nu}{D_{AB}} \qquad (12,13,14)$$

Continued...

PROBLEM 7.110 (Cont.)

where all the properties are evaluated at $T_f = \left(\overline{T}_s + T_\infty\right)/2$. The results of the analyses are summarized in the following table.

u_∞ (m/s)	n_A kg/s	$\overline{h}_{m,L}$ (m/s)	$\overline{Sh}_L$	Re_L	Sc
3	2.463×10^{-6}	0.0168	87.58	2.594×10^4	0.603
9	4.583×10^{-6}	0.0288	150	7.767×10^4	0.603

Using the two sets of tabulated values for $\overline{Sh}_L$, Re_L and Sc and the standard correlation of the form,

$$\overline{Sh}_L = C\, Re_L^m\, Sc^{1/3} \tag{15}$$

$$87.58 = C\left(2.594 \times 10^4\right)^m (0.603)^{1/3} \qquad 150 = C\left(7.767 \times 10^4\right)^m (0.603)^{1/3}$$

solve simultaneously to find C = 0.711 m = 0.490 (16,17)

From the heat-mass analogy, we expect the constants C and m in Eq. (7) for heat transfer and in Eq. (13) for mass transfer to be the same. From the two experiments, we found

	C	m
Heat transfer	0.633	0.555
Mass transfer	0.711	0.490

In the plot below, the parameters $\overline{Sh}_L / Sc^{1/3}$ or $\overline{Nu}_L / Pr^{1/3}$ are plotted against Re_L using Eq. (15) or (7). Note that the curves are nearly parallel on the log-log axes since their "m" constants are of similar value. The mass transfer results are, however, nearly 50% higher than those for heat transfer. We have no way to explain this systematic difference without more information on the apparatus, observation procedures and repeated observations. However, overall the results support the general form of the heat-mass analogy.

PROBLEM 7.111

KNOWN: Dry air at prescribed temperature and velocity flowing over a wetted plate of length 500 mm and width 150 mm. Imbedded electrical heater maintains the surface at T_s = 20°C.

FIND: (a) Water evaporation rate (kg/h) and electrical power P_e (W) required to maintain steady-state conditions, and (b) The temperature of the plate after all the water has evaporated, for the same airstream conditions and heater power of part (a).

SCHEMATIC:

Dry air
T_∞ = 35 °C
u_∞ = 20 m/s
q_{conv} q_{evap} T_s = 20 °C
Water film, plate 150 mm width
P_e x L = 500 m
(a) Water film
q_{conv} T_s P_e
(a) Dry surface

ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties and (3) Heat-mass transfer analogy is applicable.

PROPERTIES: *Table A.4*, Air ($T_f = (T_s + T_\infty)/2$ = 300 K, 1 atm): ρ = 1.16 kg/m³, c_p = 1007 J/kg·K, k = 0.0263 W/m·K, ν = 15.94 × 10^{-6} m²/s, α = 2.257 × 10^{-5} m²/s, *Table A.6*, Water (T_s = 20°C = 293 K): $\rho_{A,s} = 1/v_g$ = 1/59.04 = 0.0169 kg/m³, h_{fg} = 2454 kJ/K; *Table A.8*, Water-air (T_f = 300 K): D_{AB} = 0.26 × 10^{-4} m²/s.

ANALYSIS: (a) Perform an energy balance on the plate,

$$\dot{E}_{in} - \dot{E}_{out} = 0 \qquad P_e - q_{conv} - q_{evap} = 0 \tag{1}$$

where the convection and evaporation rate equations are,

$$q_{conv} = \bar{h}_L A_s (T_s - T_\infty) \tag{2}$$

$$q_{evap} = n_A h_{fg} = \bar{h}_m A_s (\rho_{A,s} - \rho_{A,\infty}) - h_{fg} \tag{3}$$

The Reynolds number for the plate length L is

$$Re_L = \frac{u_\infty L}{\nu} = \frac{20\,\text{m/s} \times 0.50\,\text{m}}{15.94 \times 10^{-6}\,\text{m}^2/\text{s}} = 6.274 \times 10^5$$

so that the flow is mixed and Eq. 7.41 is appropriate to estimate $\bar{h}_L$,

$$\overline{Nu}_L = \frac{\bar{h}_L L}{k} = \left(0.037\, Re_D^{4/5} - 871\right) Pr^{1/3}$$

$$\bar{h}_L = \frac{0.0263\,\text{W/m}\cdot\text{K}}{0.5\,\text{m}} \left(0.037\left[6.274 \times 10^5\right]^{4/5} - 871\right)(0.707)^{1/3} = 34.5\,\text{W/m}^2\cdot\text{K}$$

Evoking the heat-mass analogy, Eq. 6.92, with n = 1/3

$$\frac{\bar{h}_L}{\bar{h}_m} = \rho c_p \left(\frac{\alpha}{D_{AB}}\right)^{-2/3} = 1.16\,\text{kg/m}^3 \times 1007\,\text{J/kg}\cdot\text{K}\left(\frac{2.257 \times 10^{-5}\,\text{m}^2/\text{s}}{0.26 \times 10^{-4}\,\text{m}^2/\text{s}}\right)^{-2/3} = 1284\,\text{J/m}^3\cdot\text{K}$$

Continued...

$$\overline{h}_m = 34.5\,W/m^2\cdot K / 1284\,J/m^3\cdot K = 0.0269\,m/s$$

Substituting numerical values, the energy balance, Eq. (1), with $A_s = 0.5\ m \times 0.15\ m = 0.075\ m^2$,

$$P_e - 34.5\,W/m^2\cdot K \times 0.075\,m^2\,(20-35)K$$
$$-0.0269\,m/s \times 0.075\,m^2\,(0.0169-0)\,kg/m^3 \times 2454\times10^3\ J/kg\cdot K = 0$$

$$P_e = -38.8\,W + 83.7 = 44.9\,W$$ <

The evaporation rate is

$$n_A = \overline{h}_m A_s\left(\rho_{A,s} - \rho_{A,\infty}\right) = 0.0269\,m/s \times 0.075\,m^2 \times 0.0169\,kg/m^3 = 0.123\,kg/h$$ <

(b) When the plate is dry, the energy balance is

$$P_e = \overline{h}_L A_s\left(T_s - T_\infty\right)$$

and with P_e and $\overline{h}_L$ as determined in part (a),

$$T_s = T_\infty + P_e/\overline{h}_L A_s = 35^\circ C + 44.9\,W/34.5\,W/m^2\cdot K \times 0.075\,m^2 = 52.3^\circ C$$ <

COMMENTS: Using *IHT Correlations Tool*, *External Flow*, *Flat Plate*, the calculation of part (b) was performed using the proper film temperature, $T_f = 318$ K, to find $\overline{h}_L = 32.7\ W/m^2\cdot K$ and $T_s = 53.3°C$.

PROBLEM 7.112

KNOWN: Convection mass transfer with turbulent flow over a flat plate (van roof).

FIND: (a) Location on van that will dry last, (b) Evaporation rate at trailing edge, $kg/s{\cdot}m^2$.

SCHEMATIC:

Air, B
$u_\infty = 90 km/h$
$T_\infty = 27°C$
$\phi_\infty = 0.8$
$p = 1 atm$
$T_s = 27°C$, Water (A)
x
L = 6m

ASSUMPTIONS: (1) Turbulent flow over entire plate (van top), (2) Heat-mass transfer analogy is applicable, (3) Perfect gas behavior for water vapor (A).

PROPERTIES: *Table A-4*, Air (300 K, 1 atm): $\nu = 15.89 \times 10^{-6}\ m^2/s$, k = 0.0263 W/m·K, Pr = 0.707; *Table A-8*, Air-water vapor (25°C): $D_{AB} = 0.26 \times 10^{-4}\ m^2/s$; *Table A-6*, Saturated water vapor (300K): $r_{A,sat} = v_g^{-1} = 0.0256\ kg/m^3$.

ANALYSIS: (a) The mass transfer coefficient, $h_m(x)$, will be largest at x = 0 and smallest at x = L for turbulent flow conditions. Hence, the trailing edge will dry last.

(b) The evaporation rate on a per unit area basis, at the trailing edge where x = L, is given by the rate equation,

$$n''_A = h_{m,L}\left(r_{A,s} - r_{A,\infty}\right) = h_{m,L}\ r_{A,sat}\left(1 - f_\infty\right)$$

For turbulent flow the appropriate correlation for estimating $h_{m,L}$ is of the form

$$Sh_x = h_{m,x} x/D_{AB} = 0.0296\, Re_x^{4/5}\ Sc^{1/3}.$$

Substituting numerical values,

$$Re_L = \frac{u_\infty L}{n_B} = \frac{90 \times 10^3\ m/h}{3600\ s/h} \times 6m/15.89 \times 10^{-6}\ m^2/s = 9.44 \times 10^6$$

$$Sc = \frac{n_B}{D_{AB}} = 15.89 \times 10^{-6}\ m^2/s/0.26 \times 10^{-4}\ m^2/s = 0.611$$

$$h_{m,L} = \left(0.26 \times 10^{-4}\ m^2/s/6m\right) \times 0.0296\ \left(9.44 \times 10^6\right)^{4/5} (0.611)^{1/3} = 0.0414\ m/s.$$

Hence, the evaporation flux (rate per unit area) is

$$n''_A = 0.0414\ m/s \times 0.0256\ kg/m^3\ (1 - 0.8) = 2.12 \times 10^{-4}\ kg/s \cdot m^2.$$ <

COMMENTS: Recognize how the heat-mass analogy is utilized and the appropriate correlation selected from Table 7.9.

PROBLEM 7.113

KNOWN: Length and thickness of a layer of benzene. Velocity and temperature of air in parallel flow over the layer.

FIND: Time required for complete evaporation.

SCHEMATIC:

Air, B
$u_\infty = 1$ m/s
$T_\infty = 25°C$
$p = 1$ atm
n_A
Benzene, $\rho_\ell = 900$ kg/m³
$\rho_{A,sat} = 0.417$ kg/m³
$\delta_i = 1$ mm
x
L = 2 m

ASSUMPTIONS: (1) Steady-state conditions, (2) Smooth liquid surface and negligible free-stream turbulence, (3) Heat and mass transfer analogy is applicable, (4) Negligible benzene vapor concentration in free-stream air, (5) Isothermal conditions at 25°C.

PROPERTIES: *Table A-4*, Air (25°C, 1 atm): $\nu = 15.7 \times 10^{-6}$ m²/s; *Table A-8*, Benzene-air, (25°C, 1 atm): $D_{AB} = 0.88 \times 10^{-5}$ m²/s, Sc = 1.78.

ANALYSIS: Applying conservation of mass to a control volume about the liquid,

$$\frac{dM}{dt} = \frac{d(\rho_\ell V)}{dt} = -n_A.$$

For a unit width, $V = L \cdot \delta$. Hence

$$\rho_\ell L \frac{d\delta}{dt} = -n_A' = -\overline{h}_m L\left(\rho_{A,sat} - \rho_{A,\infty}\right)$$

and integrating

$$\int_{\delta_i}^{0} d\delta = -\frac{\overline{h}_m}{\rho_\ell}\rho_{A,sat}\int_0^t dt$$

$$t = \frac{\delta_i \rho_\ell}{\overline{h}_m \rho_{A,sat}}.$$

With $\quad Re_L = \dfrac{u_\infty L}{\nu} = \dfrac{1 \text{ m/s} \times 2 \text{ m}}{15.7 \times 10^{-6} \text{ m}^2/\text{s}} = 1.27 \times 10^5,$

the flow is laminar throughout and from Eq. 7.32,

$$\overline{h}_m = \frac{D_{AB}}{L} 0.664\, Re_L^{1/2} Sc^{1/3} = \frac{0.88 \times 10^{-5} \text{ m}^2/\text{s}}{2 \text{ m}} \times 0.664\left(1.27 \times 10^5\right)^{1/2} (1.78)^{1/3}$$

$$\overline{h}_m = 1.26 \times 10^{-3} \text{ m/s}$$

and

$$t = \frac{0.001 \text{ m}\left(900 \text{ kg/m}^3\right)}{\left(1.26 \times 10^{-3} \text{ m/s}\right)\left(0.417 \text{ kg/m}^3\right)} = 1713 \text{ s} = 28.6 \text{ min.}$$

<

PROBLEM 7.114

KNOWN: Parallel air flow over a series of water-filled trays.

FIND: Power required to maintain each of the first three trays at 300K.

SCHEMATIC:

ASSUMPTIONS: (a) Steady-state conditions, (2) Heat-mass transfer analogy applicable, (3) Perfect gas behavior for water vapor, (4) $Re_{x,c} = 5 \times 10^5$.

PROPERTIES: *Table A-4*, Air (300 K, 1 atm): $\nu = \nu_B = 15.89 \times 10^{-6}\ m^2/s$; *Table A-8*, Water vapor-air (300K): $D_{AB} = 0.26 \times 10^{-4}\ m^2/s$, $Sc = \nu_B/D_{AB} = 0.611$; *Table A-6*, Saturated water vapor (300K): $\rho_{A,sat} = v_g^{-1} = 0.02556\ kg/m^3$, $h_{fg} = 2438$ kJ/kg.

ANALYSIS: Since $T_s = T_\infty$, there is no convective heat transfer, hence,

$$q_{tray} = \dot{m}_{tray} h_{fg} = \overline{h}_m \cdot A_s \cdot \rho_{A,sat}(1 - \phi_\infty) h_{fg} \qquad (1)$$

where

$$\phi_\infty \equiv \rho_{A,\infty} / \rho_{A,sat} \text{ and } \rho_{A,s} = \rho_{A,sat}(T_s).$$ Calculate the Reynolds number at x_3,

$$Re_{x3} = u_\infty x_3 / \nu_B = 12\ m/s \times 1.5 m / 15.89 \times 10^{-6}\ m^2/s = 1.133 \times 10^6$$

finding that transition occurs at x = 0.662 m, a location on tray 2. The average mass transfer coefficients $\overline{h}_m$ and heat rates for each tray are as follows:

Tray 1: The flow is laminar and the appropriate correlation for $\overline{h}_{m,1}$ and heat rate are

$$\overline{Sh}_{x1} = \overline{h}_{m,1} x_1 / D_{AB} = 0.664\ Re_{x1}^{1/2}\ Sc^{1/3}$$

$$\overline{h}_{m,1} = \left(0.26 \times 10^{-4}\ m^2/s / 0.5\ m\right) \times 0.664 \left(\frac{12\ m/s \times 0.5\ m}{15.89 \times 10^{-6}\ m^2/s}\right)^{1/2} (0.611)^{1/3} = 1.800 \times 10^{-2}\ m/s$$

$$q_1' = 1.800 \times 10^{-2}\ m/s \times 0.5\ m \times 0.02556\ kg/m^3\ (1 - 0.40) \times 2438 \times 10^3\ J/kg = 337\ W/m.$$ <

Tray 2: Since transition occurs over the span of tray 2, the rate equation has the form

$$q_2' = \left[x_2 \overline{h}_{m,0-2} - x_1 \overline{h}_{m,0-1}\right]\ \rho_{A,sat}(1 - \phi_\infty) h_{fg}. \qquad (2)$$

Continued ….

Note that $\overline{h}_{m,0-1} = \overline{h}_{m,1}$ from above and that $\overline{h}_{m,0-2}$ is evaluated using the correlation

$$\overline{Sh}_x = \left(0.037\ Re_x^{4/5} - 871\right) Sc^{1/3}$$

$$\overline{h}_{m,0-2} = 2.193 \times 10^{-2}\ m/s \qquad q_2' = 483\ W/m. \qquad <$$

Tray 3: The rate equation is of the same form as Eq. (2). Alternatively, an approximation can be used,

$$q_3' = h_m(\overline{x})\ (x_3 - x_2)\ r_{A,sat}\ (1 - f_\infty)\ h_{fg}$$

where $h_m(\overline{x})$ is the *local* value at the midspan, $\overline{x} = (x_2 + x_3)/2$. Using

$$Sh_x = 0.0296\ Re_x^{4/5}\ Sc^{1/3}$$

and substituting numerical values, find

$$h_{m(\overline{x})} = 3.148 \times 10^{-2}\ m/s \qquad q_3' = 589\ W/m. \qquad <$$

PROBLEM 7.115

KNOWN: Air and surface conditions for a drying process in which photographic plates are aligned in the direction of the air flow.

FIND: (a) Variation of local mass transfer convection coefficient, (b) Drying rate for fastest drying plate, (c) Heat addition needed to maintain the plate temperature.

SCHEMATIC:

ASSUMPTIONS: (1) Heat and mass transfer analogy is applicable, (2) Critical Reynolds number is $Re_{x,c} = 5 \times 10^5$, (3) Radiation effects are negligible.

PROPERTIES: *Table A-4*, Air (50°C = 323K): $\nu = 18.2 \times 10^{-6}\ m^2/s$; *Table A-6*, Water vapor (50°C = 323K): $\rho_{A,sat} = 0.082\ kg/m^3$, $h_{fg} = 2383\ kJ/kg$; *Table A-8*, Water vapor-air (25°C = 298K) $D_{AB} = 0.26 \times 10^{-4}\ m^2/s$; since $D_{AB} \propto T^{3/2}$, $D_{AB}(50°C = 323K) = 0.26 \times 10^{-4} (323/298)^{3/2} = 0.29 \times 10^{-4}\ m^2/s$, $Sc = \nu/D_{AB} = 0.62$.

ANALYSIS: (a) With $Re_{x,c} = u_\infty x_c/\nu = 5 \times 10^5$, the point of transition is

$$x_c = \frac{5\times10^5 \left(18.2\times10^{-6}\ m^2/s\right)}{9.1\ m/s} = 1\ m$$

and the variation of the local mass transfer coefficient is as shown below

<

(b) The largest evaporation will be associated with either the first plate or the fifth plate. For the *first* plate,

$$n_{A,1} = \overline{h}_{m,1} A_{s,1} \left(\rho_{A,s} - \rho_{A,\infty}\right)$$

where $\rho_{A,\infty} = 0$ since the upstream air is dry. Since the boundary layer is laminar over the entire plate, with

$$Re_{x,1} = (9.1\ m/s)\ (0.25\ m)/\left(18.2\times10^{-6}\ m^2/s\right) = 1.25\times10^5$$

Continued …..

Eq. 7.32 may be used to obtain

$$\bar{h}_{m,1} = \left(\frac{D_{AB}}{x_1}\right) 0.664\, Re_{x,1}^{1/2}\, Sc^{1/3} = \left(\frac{0.29\times10^{-4}\ m^2/s}{0.25\ m}\right) 0.664\, \left(1.25\times10^5\right)^{1/2} (0.62)^{1/3}$$

$\bar{h}_{m,1} = 0.0232$ m/s.

Hence $\qquad n_{A,1} = 0.0232 m/s (0.25\ m \times 1\ m)\ \left(0.082 kg/m^3\right) = 4.72\times10^{-4}\ kg/s\cdot m.$

For the *fifth* plate,

$$n_{A,5} = n_{A,0-5} - n_{A,0-4} = \left[\left(\bar{h}_m A_s\right)_{0-5} - \left(\bar{h}_m A_s\right)_{0-4}\right]\ \left(\rho_{A,s} - \rho_{A,\infty}\right).$$

With $Re_{x,5} = 6.25 \times 10^5$, Eq. 7.42 gives

$$\bar{h}_{m,0-5} = \left(\frac{D_{AB}}{x_5}\right) \left[0.037\, Re_{x,5}^{4/5} - 871\right]\, Sc^{1/3}$$

$$\bar{h}_{m,0-5} = \left(\frac{0.29\times10^{-4}\ m^2/s}{1.25\ m}\right) \left[0.037\left(6.25\times10^5\right)^{4/5} - 871\right]\, (0.62)^{1/3}$$

$$\bar{h}_{m,0-5} = 0.0145\ m/s.$$

With $Re_{x,4} = 5 \times 10^5$, Eq. 7.32 gives

$$\bar{h}_{m,0-4} = \left(\frac{D_{AB}}{x_4}\right) 0.664\, Re_{x,4}^{1/4}\, Sc^{1/3}$$

$$\bar{h}_{m,0-4} = \left(\frac{0.29\times10^{-4}\ m^2/s}{1\ m}\right) \left[0.664\, \left(5\times10^5\right)^{1/2}\ (0.62)^{1/3}\right]$$

$$\bar{h}_{m,0-4} = 0.0116\ m/s.$$

Hence,

$$n_{A,5} = [0.0145 m/s \times 1.25\ m \times 1\ m - 0.0116 m/s \times 1\ m \times 1\ m]\ \left(0.082 kg/m^3\right)$$

$$n_{A,5} = 5.35\times10^{-4}\ kg/s\cdot m.$$ <

Hence the evaporation rate is largest for Plate 5.

(c) Heat would have to be supplied to each plate at a rate which is equal to the evaporative cooling rate in order to maintain the prescribed temperature. Hence

$$q_5 = n_{A,5} h_{fg} = 5.35\times10^{-4} kg/s\cdot m \times 2.383\times10^6\ J/kg = 1.275\ kW/m.$$ <

COMMENTS: The large value of q_5 is a consequence of the significant evaporative cooling effect.

PROBLEM 7.116

KNOWN: Dimensions and temperature of a cooling pond. Conditions of air flow.

FIND: Daily make-up water requirement.

SCHEMATIC:

Cooling pond, water (A), T=27°C
Air, B
u_∞ = 2 m/s
T_∞ = 27°C
ϕ_∞ = 0
p = 1 atm
W = 500m
L = 1000 m

ASSUMPTIONS: (1) Steady-state conditions, (2) Turbulent boundary layer over the entire surface, (3) Heat and mass transfer analogy is applicable.

PROPERTIES: *Table A-4*, Air (T = 300K, 1 atm): $\nu = 15.89 \times 10^{-6}$ m^2/s, k = 0.0263 W/m·K, Pr = 0.707; *Table A-6*, Water vapor (300K): $r_{A,sat} = v_g^{-1} = 0.0256$ kg/m^3; *Table A-8*, Water vapor-air (300K): $D_{AB} = 0.26 \times 10^{-4}$ m^2/s, Sc = ν/D_{AB} = 0.61.

ANALYSIS: The make-up water requirement must equal the daily water loss due to evaporation,

$$\Delta M = \dot{m}_{evap}\Delta t = \overline{h}_m (W \cdot L) \left[r_{A,sat}(T_s) - f_\infty r_{A,sat}(T_\infty) \right] \cdot \Delta t.$$

From Eq. 7.45, $\overline{Sh}_L = 0.037\, Re_L^{4/5}\, Sc^{1/3}$, with

$$Re_L = \frac{u_\infty L}{n} = \frac{2\text{ m/s} \times 1000\text{ m}}{15.89 \times 10^{-6}\text{ m}^2/\text{s}} = 1.26 \times 10^8$$

$$\overline{Sh}_L = 0.037\left(1.26 \times 10^8\right)^{4/5} (0.61)^{1/3} = 9.48 \times 10^4$$

$$\overline{h}_{m,L} = \frac{D_{AB}\overline{Sh}_L}{L} = \frac{0.26 \times 10^{-4}\text{ m}^2/\text{s} \times 9.48 \times 10^4}{1000\text{ m}}$$

$$\overline{h}_{m,L} = 2.47 \times 10^{-3}\text{ m/s}.$$

Hence, the make-up water requirement is

$$\Delta M = 2.47 \times 10^{-3}\text{ m/s}\ (500\text{ m} \times 1000\text{ m})\ 0.0256\text{ kg/m}^3 (24\text{h} \times 3600\text{ s/h})$$

$$\Delta M = 2.73 \times 10^6\text{ kg/day.}$$ <

PROBLEM 7.117

KNOWN: Dimensions and initial temperature of plate covered by liquid film. Properties of liquid. Velocity and temperature of air flow over the plates.

FIND: Initial rate of heat transfer from plate and rate of change of plate temperature.

SCHEMATIC:

ASSUMPTIONS: (1) Negligible effect of conveyor velocity on boundary layer development, (2) Plates are isothermal and at same temperature as liquid film, (3) Negligible heat transfer from sides of plate, (4) Smooth air-liquid interface, (5) Applicability of heat/mass transfer analogy, (6) Negligible solvent vapor in free stream, (7) $Re_{x,c} = 5 \times 10^5$, (8) Constant properties.

PROPERTIES: *Table A-1*, AISI 1010 steel (313K): c = 441 J/kg·K, $\rho = 7832\ \text{kg/m}^3$. *Table A-4*, Air (p = 1 atm, T_f = 303K): $\nu = 16.2 \times 10^{-6}\ \text{m}^2/\text{s}$, $k = 0.0265\ \text{W/m}\cdot\text{K}$, $Pr = 0.707$. Prescribed: Solvent: $\rho_{A,sat} = 0.75\ \text{kg/m}^3$, $D_{AB} = 10^{-5}\ \text{m}^2/\text{s}$, $h_{fg} = 9 \times 10^5\ \text{J/kg}$.

SOLUTION: The initial rate of heat transfer from the plate is due to both convection and evaporation.

$$q = q_{conv} + q_{evap} = \bar{h} A_s (T_i - T_\infty) + n_A h_{fg} = \bar{h} A_s (T_i - T_\infty) + \bar{h}_m A_s \rho_{A,sat} h_{fg}$$

With $Re_L = u_\infty L/\nu = 1\ \text{m/s} \times 1\text{m}/16.2 \times 10^{-6}\ \text{m}^2/\text{s} = 6.17 \times 10^4$, flow is laminar over the entire surface. Hence,

$$\overline{Nu}_L = 0.664\, Re_L^{1/2} Pr^{1/3} = 0.664 \left(6.17 \times 10^4\right)^{1/2} (0.707)^{1/3} = 147$$

$$\bar{h} = (k/L)\overline{Nu}_L = (0.0265\ \text{W/m}\cdot\text{K}/1\text{m})147 = 3.9\ \text{W/m}^2\cdot\text{K}$$

Also, with $Sc = \nu/D_{AB} = 16.2 \times 10^{-6}\ \text{m}^2/\text{s}/10^{-5}\ \text{m}^2/\text{s} = 1.62$,

$$\overline{Sh}_L = 0.664\, Re_L^{1/2} Sc^{1/3} = 0.664 \left(6.17 \times 10^4\right)^{1/2} (1.62)^{1/3} = 194$$

$$\bar{h}_m = (D_{AB}/L)\overline{Sh}_L = \left(10^{-5}\ \text{m}^2/\text{s}/1\text{m}\right)194 = 0.00194\ \text{m/s}$$

Hence, with $A_s = 2\,L^2 = 2\ \text{m}^2$,

$$q = 2\,\text{m}^2 \left[3.9\ \text{W/m}^2\cdot\text{K}\,(20°\text{C}) + 0.00194\ \text{m/s} \times 0.75\ \text{kg/m}^3 \times 9 \times 10^5\ \text{J/kg}\right] = 156\ \text{W} + 2619\ \text{W} = 2775\ \text{W} \quad <$$

Performing an energy balance at an instant of time for a control surface about the plate, $-\dot{E}_{out} = \dot{E}_{st}$, we obtain (Eq. 5.2),

$$\left.\frac{dT}{dt}\right|_i = -\frac{q}{\rho\,\delta\,L^2 c} = -\frac{2775\ \text{W}}{7832\ \text{kg/m}^3 \times 0.006\text{m}\,(1\text{m})^2\ 441\ \text{J/kg}\cdot\text{K}} = -0.13°\text{C/s} \quad <$$

COMMENTS: (1) Heat transfer by evaporation exceeds that due to convection by more than an order of magnitude, (2) The total heat rate is small enough to render the lumped capacitance approximation excellent.

PROBLEM 7.118

KNOWN: Dimensions of round jet array. Jet exit velocity and temperature. Temperature of paper.

FIND: Drying rate per unit surface area.

SCHEMATIC:

D = 20 mm; S = 100 mm; H = 200 mm; T_e = 300 K, V_e = 20 m/s; T_s = 300 K; u = 0.2 m/s

ASSUMPTIONS: (1) Applicability of heat and mass transfer analogy. (2) Paper motion has a negligible effect on convection (u << V_e), (3) Air is dry.

PROPERTIES: *Table A-4*, Air (300K, 1 atm): $\nu = 15.89 \times 10^{-6}\ \mathrm{m^2/s}$; *Table A-6*, Saturated water (300K): $\rho_{A,sat} = v_g^{-1} = 0.0256\ \mathrm{kg/m^3}$; *Table A-8*, water vapor-air (300K): $D_{AB} = 0.26 \times 10^{-4}\ \mathrm{m^2/s}$, Sc = 0.61.

ANALYSIS: The average mass evaporation flux is

$$n''_A = \overline{h}_m \left(\rho_{A,s} - \rho_{A,e}\right) = \overline{h}_m\, \rho_{A,s}$$

For an array of round nozzles,

$$\overline{Sh} = 0.5\, K\, G\, Re^{2/3}\, Sc^{0.42}$$

where $Re = V_e D/\nu = 20\ \mathrm{m/s} \times 0.02\mathrm{m}/15.89 \times 10^{-6}\ \mathrm{m^2/s} = 25{,}170$ and, with H/D = 10 and $A_r = \pi D^2/4S^2 = 0.0314$,

$$K = \left[1 + \left(\frac{H/D}{0.6/A_r^{1/2}}\right)^6\right]^{-0.05} = \left[1 + \left(\frac{10}{3.39}\right)^6\right]^{-0.05} = 0.723$$

$$G = 2A_r^{1/2}\,\frac{1 - 2.2A_r^{1/2}}{1 + 0.2(H/D - 6)A_r^{1/2}} = 0.354\,\frac{1 - 0.390}{1 + 0.2(4)0.177} = 0.189$$

Hence,

$$\overline{h}_m = \frac{D_{AB}}{D}\overline{Sh} = \frac{0.26 \times 10^{-4}\ \mathrm{m^2/s}}{0.02\mathrm{m}}\left[0.5 \times 0.723 \times 0.189\,(25{,}170)^{2/3}(0.61)^{0.42}\right] = 0.062\ \mathrm{m/s}$$

The average evaporative flux is then

$$n''_A = 0.062\ \mathrm{m/s}\left(0.0256\ \mathrm{kg/m^3}\right) = 0.0016\ \mathrm{kg/s \cdot m^2}$$ <

COMMENTS: Note that, for maximum evaporation, the ratio D/H = 0.1 is less than the optimum of $D/H)_{op} \approx 0.2$, as is S/H = 0.5 less than $S/H)_{op} \approx 1.4$. If H is reduced by a factor of 2 and S is increased by 40%, a near optimal condition could be achieved.

PROBLEM 7.119

KNOWN: Paper mill process using radiant heat for drying.

FIND: (a) Evaporative flux at a distance 1 m from roll edge; corresponding irradiation, G (W/m^2), required to maintain surface at T_s = 300 K, and (b) Compute and plot variations of $h_{m,x}(x)$, N''_A (x), and G(x) for the range $0 \le x \le 1$ m when the velocity and temperature are increased to 10 m/s and 340 K, respectively.

SCHEMATIC:

x = 1 m
Slurry, T_s = 330 K
u_∞
Air, B
u_∞ = 5 m/s
T_∞ = 300 K
ϕ_∞ = 0
p = 1 atm
T_s = 330 K, $\rho_{A,s}$
q''_{conv} q''_{rad} m''_A, q''_{evap}
Water (A)
x
1 m

ASSUMPTIONS: (1) Steady-state conditions, (2) Heat-mass transfer analogy, (3) Paper slurry (water-fiber mixture) has water properties, (4) Water vapor behaves as perfect gas, (5) All irradiation absorbed by slurry, (6) Negligible emission from the slurry, (7) $Re_{x,c} = 5 \times 10^5$.

PROPERTIES: *Table A.4,* Air (T_f = 315 K, 1 atm): $\nu = 17.40 \times 10^{-6}$ m^2/s, k = 0.0274 W/m·K, Pr = 0.705; *Table A.8,* Water vapor-air (T_f = 315 K): $D_{AB} = 0.26 \times 10^{-4}$ m^2/s $(315/298)^{3/2} = 0.28 \times 10^{-4}$ m^2/s, $Sc = \nu_B/D_{AB} = 0.616$; *Table A.6,* Saturated water vapor (T_s = 330 K): $\rho_{A,sat} = 1/v_g = 0.1134$ kg/m^3, h_{fg} = 2366 kJ/kg.

ANALYSIS: (a) Recognize that the drying process can be modeled as flow over a flat plate with heat and mass transfer. For a unit area at x = 1 m,

$$n''_{A,x} = h_{m,x}\left(\rho_{A,s} - \rho_{A,\infty}\right) = h_{m,x}\left[\rho_{A,sat}(T_s) - \phi_\infty \rho_{A,sat}(T_\infty)\right] \quad (1)$$

Evaluate Re_x to determine the nature of flow, select a correlation to estimate $h_{m,x}$,

$$Re_x = u_\infty x/\nu_B = (5\,m/s \times 1\,m)\Big/17.40\times10^{-6}\,m^2\big/s = 2.874\times10^5.$$

Since $Re_x < 5 \times 10^5$, the flow is laminar. Invoking the heat-mass analogy,

$$Sh_x = \frac{h_{m,x}x}{D_{AB}} = 0.332\,Re_x^{1/2}\,Sc^{1/3} \quad (2)$$

$$h_{m,x} = \left(0.28\times10^{-4}\,m^2/s/1\,m\right)\times0.332\left(2.874\times10^5\right)^{1/2}(0.616)^{1/3} = 4.24\times10^{-3}\,m/s.$$

Hence, the evaporative flux at x = 1 m is

$$n''_{A,x} = 4.24\times10^{-3}\,m/s\left(0.1134\,kg/m^3 - 0\right) = 4.81\times10^{-4}\,kg/s\cdot m^2 \quad <$$

From an energy balance on the differential element at x = 1 m (see above),

$$G = q''_{conv} + q''_{evap} = h_x\left(T_s - T_\infty\right) + n''_{A,x}h_{fg}. \quad (3)$$

Continued...

PROBLEM 7.119 (Cont.)

To estimate h_x, invoke the heat-mass transfer analogy using the correlation of Eq. (2),

$$h_x = h_{m,x} \frac{k}{D_{AB}} \left(\frac{Pr}{Sc}\right)^{1/3} = 4.24\times10^{-3}\ \text{m/s} \left(\frac{0.0274\ \text{W/m}\cdot\text{K}}{0.28\times10^{-4}\ \text{m}^2/\text{s}}\right)\left(\frac{0.705}{0.616}\right)^{1/3} = 4.34\ \text{W/m}^2\cdot\text{K} \qquad (4)$$

Hence, from Eq. (3), the radiant power required to maintain the slurry at T_s = 330 K is

$$G = 4.34\ \text{W/m}^2\cdot\text{K}\,(330-300)\,\text{K} + 4.81\times10^{-4}\ \text{kg/s}\cdot\text{m}^2 \times 2366\times10^3\ \text{J/kg}$$

$$G = (130+1138)\ \text{W/m}^2 = 1268\ \text{W/m}^2 .$$

<

(b) Equations (1), (3) and (4) were entered into the *IHT Workspace*. The *Correlations Tool*, *External Flow*, *Local* coefficients for *Laminar* or *Turbulent Flow* was used to estimate the heat transfer convection coefficient. The results for $h_{m,x}(x)$, $n''_{A,x}$ (x) and G(x) were evaluated, and are plotted below for T_s = 340 K and u_∞ = 10 m/s.

Local coeff, hmx (m/s)
Distance from the leading edge, x (m)

Evaporative flux, n''Ax (kg/s.m^2)
Distance from the leading edge, x (m)

Irradiation, G (W/m^2)
Distance from the leading edge, x(m)

COMMENTS: (1) The abrupt change in the parameter plots occurs at the transition, x_c = 0.9 m.

PROBLEM 7.120

KNOWN: Geometry and air flow conditions for a water storage channel.

FIND: (a) Evaporation rate, (b) Expression for rate of change of water layer depth and time required for complete evaporation.

SCHEMATIC:

$L = 25m$
$W = 2z$
$A_s = wL = 2zL$

Air flow, $u_\infty = 5$ m/s, $\phi_\infty = 0.50$

Water, $T_s = T_\infty = 25°C$

$\pi/2$, $z = 1m$, w

n_A, $\dot{M}_{A,st}$, $V = z^2 L$

ASSUMPTIONS: (1) Steady-stae conditions, (2) Smooth water surface and negligible free stream turbulence, (3) Heat and mass transfer analogy is applicable, (4) $Re_{x,c} = 5 \times 10^5$, (5) Perfect gas behavior for water vapor.

PROPERTIES: *Table A-4* Air (25°C = 298K): $\nu = 15.71 \times 10^{-6}$ m^2/s; *Table A-6*, Water (25°C = 298K): $\rho_{A,sat} = v_g^{-1} = 0.0226$ kg/m^3, $\rho_f = v_f^{-1} = 997$ kg/m^3; *Table A-8*, Water vapor-air (25°C = 298K): $D_{AB} = 0.26 \times 10^{-4}$ m^2/s, $Sc = \nu/D_{AB} = 0.60$.

ANALYSIS: (a) The evaporation rate is $n_A = \overline{h}_m A_s \left(r_{A,sat} - r_{A,\infty}\right) = \overline{h}_m (w \times L)\, r_{A,sat}$ $\left(1 - f_\infty\right)$. With

$$Re_L = u_\infty L/n = 5 \text{ m/s} \times 25 \text{ m}/15.71 \times 10^{-6} \text{ m}^2/\text{s} = 7.96 \times 10^6$$

Eq. 7.42 yields $\overline{Sh}_L = \left[0.037\left(7.96 \times 10^6\right)^{4/5} - 871\right] (0.6)^{1/3} = 9616$

$$\overline{h}_m = 9616\ D_{AB}/L = 9616 \times 0.26 \times 10^{-4} \text{ m}^2/\text{s}/(25 \text{ m}) = 0.010 \text{ m/s}.$$

With w = 2z = 2m,

$$n_A = 0.01 \text{ m/s } (2\text{m} \times 25\text{m})\, 0.0226 \text{ kg/m}^3\, (0.5) = 0.00565 \text{ kg/s} = 20.3 \text{ kg/h}. \qquad <$$

(b) Performing a mass balance on a control volume about the water,

$$-n_A = \dot{m}_{A,st} = \frac{d}{dt}\left(r_f V\right) \qquad -\overline{h}_m (2\, z\, L)\ r_{A,sat} \left(1 - f_\infty\right) = \frac{d}{dt}\left(r_f\ z^2 L\right)$$

$$\frac{dz}{dt} = -\overline{h}_m \frac{r_{A.sat}}{r_f}\left(1 - f_\infty\right).$$

Integrating, $\displaystyle\int_z^0 dz = -\overline{h}_m \frac{r_{A,sat}}{r_f}\left(1 - f_\infty\right) \int_0^t dt$

$$t = \frac{z\, r_f}{\overline{h}_m\, r_{A,sat}} \frac{1}{1 - f_\infty} = \frac{1 \text{ m} \times 997 \text{ kg/m}^3}{0.01 \text{ m/s} \times 0.0226 \text{ kg/m}^3\ (1 - 0.5)} = 8.82 \times 10^6 \text{ s} = 2451 \text{ h} = 102 \text{ d}. \qquad <$$

COMMENTS: Although the evaporation rate decreases with increasing time due to decreasing A_s, dz/dt remains constant and the water depth decreases linearly.

PROBLEM 7.121

KNOWN: Mass change for a given time period of a solid naphthalene cylinder subjected to cross flow of air for prescribed conditions.

FIND: (a) Mass transfer coefficient, $\overline{h}_m$, based upon experimental observations and (b) $\overline{h}_m$ based upon appropriate correlation.

SCHEMATIC:

Air
$V = 12$ m/s
$T_\infty = 26°C$
$\rho_{A,\infty} = 0$
$p = 750.6$ mm Hg

$\rho_{A,s}$, $T = 26°C$
Naphthalene (A)
n_A
Mass loss $= 0.35 \times 10^{-3}$ kg
Time period $= 39$ min
Cylinder, $D = 18.4$ mm, $\ell = 88.9$ mm

ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible naphthalene vapor in free stream, (3) Heat-mass transfer analogy applies.

PROPERTIES: *Table A-4*, Air (299K, 1 atm): $\nu = 15.80 \times 10^{-6}$ m^2/s; *Table A-6*, Naphthalene vapor-air (298K, 1 atm): $D_{AB} = 0.62 \times 10^{-5}$ m^2/s; Naphthalene (given): $\mathcal{M} = 128.16$ kg/kmol, $p_{sat} = p \times 10^E$ where E = 8.67 - (3766/T) with p[bar] and T[K].

ANALYSIS: (a) The rate equation for the sublimation of naphthalene vapor from the solid naphthalene can be written in terms of the mass transfer coefficient.

$$\overline{h}_m = \frac{n_A}{A_s\left(\rho_{A,s} - \rho_{A,\infty}\right)} \tag{1}$$

where $A_s = \pi D \ell$. From the mass loss and time observations

$$n_A = \frac{\Delta m}{\Delta t} = \frac{0.35 \times 10^{-3}\,\text{kg}}{39 \times 60\text{ s}} = 1.50 \times 10^{-7}\text{ kg/s}.$$

The saturation density of the vapor at the solid surface, $\rho_{A,s}$, can be determined from the perfect gas relation,

$$\rho_{A,s} = C_{A,s}\mathcal{M}_A = \frac{p_{sat}\left(T_s\right)}{\left(\Re/\mathcal{M}_A\right)T_s}. \tag{2}$$

The saturation pressure, p_{sat}, is given by

$$p_{sat} = p \times 10^E \tag{3}$$

where $E = 8.67 - (3766/T) = 8.67 - (3766/299\text{K}) = -3.925$

$$p = 750.6\text{ mm Hg} \times \frac{1\text{ N/m}^2}{2.953 \times 10^{-4}\text{in Hg}} \times \frac{1\text{ in}}{25.4\text{ mm}} \times \frac{1\text{ bar}}{1 \times 10^5\text{ N/m}^2} = 1.001\text{ bar}$$

or $p_{sat} = 1.001\text{ bar} \times 10^{-3.925} = 1.190 \times 10^{-4}\,\text{bar}.$

Continued …..

Substituting into Eq. (2),

$$r_{A,s} = 1.190\times10^{-4}\text{bar}/\frac{8.314\times10^{-2}\ \text{m}^3\cdot\text{bar/kmol}\cdot\text{K}}{128.16\ \text{kg/kmol}}\times299\text{K} = 6.135\times10^{-4}\text{kg/m}^3.$$

Using the parameters required for Eq. (1), the mass transfer coefficient is

$$\overline{h}_m = \frac{1.50\times10^{-7}\text{kg/s}}{p\left(18.4\times10^{-3}\text{m}\right)\ \left(88.9\times10^{-3}\text{m}\right)\left[6.135\times10^{-4}-0\right]\ \text{kg/m}^3}$$

$$\overline{h}_m = 4.76\times10^{-2}\text{m/s}.$$ <

(b) Invoking the heat-mass transfer analogy and assuming a Prandtl number ratio of unity, Eq. 7.56 can be used to estimate $\overline{h}_m$,

$$\overline{Sh}_D = \frac{\overline{h}_m D}{D_{AB}} = C\ Re_D^m\ Sc^n.$$

With

$$Re_D = \frac{VD}{n} = 12\ \text{m/s}\left(18.4\times10^{-3}\right)\text{m}/15.80\times10^{-6}\text{m}^2/\text{s} = 13{,}975$$

it follows from Table 7.4 that C = 0.26 and m = 0.6. With

$$Sc = n/D_{AB} = 15.80\times10^{-6}\text{m}^2\ \text{/s}/0.62\times10^{-5}\text{m}^2/\text{s} = 2.55$$

n = 0.37 and

$$\overline{Sh}_D = 0.26(13{,}975)^{0.6}(2.55)^{0.37} = 112.9$$

and

$$\overline{h}_m = \overline{Sh}_D\ \frac{D_{AB}}{D} = 112.9\times\frac{0.62\times10^{-5}\ \text{m}^2/\text{s}}{18.4\times10^{-3}\ \text{m}} = 3.80\times10^{-2}\ \text{m/s}.$$ <

COMMENTS: The result from the correlation is 20% less than the experimental result. This may be considered reasonable in view of the uncertainties associated with the observations and the approximate nature of the correlation.

PROBLEM 7.122

KNOWN: Flow of dry air over a cylindrical medium saturated with water.

FIND: (a) Mass rate of water vapor evaporated per unit length n'_A, when water-air is at 300 K, (b) Briefly explain change in mass rate if temperatures are at 325 K.

SCHEMATIC:

Air
$V_\infty = 15$ m/s
$T_\infty = 300$ K
$\rho_{A,\infty} = 0$
$p_\infty = 1$ atm
$\rho_{A,s}$, $T_s = 300$ K
Water, A
$D = 40$ mm

ASSUMPTIONS: (1) Steady-state conditions, (2) Heat-mass transfer analogy.

PROPERTIES: *Table A.4*, Air (300 K, 1 atm): $\nu = 15.89 \times 10^{-6}$ m^2/s, Pr = 0.707; Air (325 K, 1 atm): $\nu = 18.41 \times 10^{-6}$ m^2/s, Pr = 0.703; *Table A.8,* Water vapor-air (300 K): $D_{AB} = 0.26 \times 10^{-4}$ m^2/s; *Table A.6,* Water vapor (300 K, 1 atm): $\rho_{A,sat} = (v_g)^{-1} = (39.13\ m^3/kg)^{-1} = 0.0256$ kg/m^3; Water vapor (325 K, 1 atm): $\rho_{A,sat} = (v_g)^{-1} = (11.06\ m^3/kg)^{-1} = 0.0904$ kg/m^3.

ANALYSIS: (a) For cross-flow over a cylinder, Eq. 7.55,

$$\overline{Sh}_D = C\,Re^m\,Sc^{1/3} \qquad (1)$$

where m,n are taken from Table 7.2. Calculate the Reynolds number, $Re_D = VD/\nu = 15\ m/s \times 0.04\ m/15.89 \times 10^{-6}\ m^2/s = 37{,}760$. With C = 0.193, m = 0.618, and $Sc \equiv \nu/D_{AB}$,

$$\overline{Sh}_D = \frac{\overline{h}_m D}{D_{AB}} = 0.193(37,760)^{0.618}\left[15.89\times10^{-6}\ m^2/s\big/0.26\times10^{-4}\ m^2/s\right]^{1/3} = 110.4 \qquad (2)$$

$$\overline{h}_m = \overline{Sh}_D\, D_B/D = 110.4\times0.26\times10^{-4}\ m^2/s\big/0.04\,m = 0.0717\ m/s$$

The evaporation rate, with $A_s = \pi D\cdot\ell$, is

$$n_A = \overline{h}_m A_s\left(\rho_{A,s}-\rho_{A,\infty}\right) \qquad n'_A = n_A/\ell = \overline{h}_m \pi D\left(\rho_{A,s}-\rho_{A,\infty}\right) \qquad (3)$$

$$n'_A = 0.0717\ m/s(\pi\times0.04\,m)(0.0256-0)\,kg/m^3 = 2.31\times10^{-4}\ kg/s\cdot m \qquad <$$

(b) The foregoing equations were entered into the *IHT Workspace*, and using the *Properties Tools* for air and water vapor thermophysical properties, the evaporation rate n'_A was calculated as a function of air-water temperatures ($T_s = T_{inf}$).

Continued...

PROBLEM 7.122 (Cont.)

Evap rate, n'Ax10^4 (kg/s.m)

0 10 20 30

300 310 320 330 340 350

Air-water temperature, Ts, Tinf (K)

As expected, the evaporation rate increased with increasing temperature markedly. For a 50 K increase, the evaporation rate increased by a factor of approximately 12.

COMMENTS: (1) What parameters cause this high sensitivity of n'_A to T_s? From the IHT analysis, we observed only modest changes in D_{AB} (0.26 to 0.33×10^{-4} m^2/s) and $\overline{h}_m$ (0.07273 to 0.0779 m/s) over the range 300 to 350 K. The density of water vapor, $\rho_{A,s}$, however, is highly temperature dependent as can be seen by examining the steam tables, Table A.6. Find $\rho_{A,s}$ (300 K) = 0.02556 kg/m^3 while $\rho_{A,s}$ (350 K) = 0.260 kg/m^3, which accounts for more than a factor of 10 change.

(2) A copy of the IHT Workspace used to perform the analysis is shown below.

```
// The Mass Transfer Rate Equation:
n'A = hmbar * pi * D * (rhoAs - 0 )       // Eq (3)
n'A_plot = 1e4 * n'A                       // Scale change for plotting

// Mass Transfer Coefficient Correlation:
ShDbar = C * ReD^m * Sc^(1/3)             // Eq (1,2)
ShDbar = hmbar * D / DAB
C = 0.193                      // Table 7.2,  4000 <= ReD <= 40000
m = 0.618
ReD = uinf * D / nu
Sc = nu / DAB

// Properties Tool - Water Vapor:
// Water property functions :T dependence, From Table A.6
// Units: T(K), p(bars);
xs = 1                         // Quality (0=sat liquid or 1=sat vapor)
rhoAs = rho_Tx("Water",Ts,xs)             // Density, kg/m^3

// Properties Tool - Air:
// Air property functions : From Table A.4
// Units: T(K); 1 atm pressure
nu = nu_T("Air",Tf)            // Kinematic viscosity, m^2/s

// Properties, Table A.8, Water Vapor - Air:
DAB = 0.26e-4 * ( Tf / 298 )^1.5          // Table A.8
Tf = (Ts + Tinf ) / 2

// Assigned Variables:
Ts = 300                       // Surface temperature, K
D = 0.040                      // Diameter, m
uinf = 15                      // Airstream velocity, m/s
Tinf = Ts                      // Airstream temperature, K
```

PROBLEM 7.123

KNOWN: Dry air at prescribed temperature and velocity flowing over a long, wetted cylinder of diameter 20 mm. Imbedded electrical heater maintains the surface at $T_s = 20°C$.

FIND: (a) Water evaporation rate per unit length (kg/h·m) and electrical power per unit length P'_e (W/m) required to maintain steady-state conditions, and (b) The temperature of the cylinder after all the water has evaporated for the same airstream conditions and heater power of part (a).

SCHEMATIC:

(a) Water film (b) Dry surface

ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties and (3) Heat-mass transfer analogy is applicable.

PROPERTIES: *Table A.4*, Air ($T_f = (T_s + T_\infty)/2$ = 300 K, 1 atm): $\rho = 1.16$ kg/m^3, $c_p = 1007$ J/kg·K, k = 0.0263 W/m·K, $\nu = 15.94 \times 10^{-6}$ m^2/s, $\alpha = 2.257 \times 10^{-5}$ m^2/s, *Table A.6*, Water (T_s = 20°C = 293 K): $\rho_{A,s} = 1/v_g = 1/59.04 = 0.0169$ kg/m^3, h_{fg} = 2454 kJ/K; *Table A.8*, Water-air (T_f = 300 K): $D_{AB} = 0.26 \times 10^{-4}$ m^2/s.

ANALYSIS: (a) Perform an energy balance on the cylinder,

$$\dot{E}_{in} - \dot{E}_{out} = 0 \qquad P'_e - q'_{conv} - q'_{evap} = 0 \tag{1}$$

where the convection and evaporation rate equations are,

$$q'_{conv} = \overline{h}_D \pi D (T_s - T_\infty) \tag{2}$$

$$q_{evap} = n_A h_{fg} = \overline{h}_m \pi D (\rho_{A,s} - \rho_{A,\infty}) h_{fg} \tag{3}$$

The convection coefficient can be estimated from the Churchill-Bernstein correlation, Eq. 7.57,

$$\overline{Nu}_D = 0.3 + \frac{0.62\, Re_D^{1/2}\, Pr^{1/3}}{\left[1 + (0.4/Pr)^{2/3}\right]^{1/4}} \left[1 + \left(\frac{Re_D}{282,000}\right)^{3/8}\right]^{4/5}$$

$$Re_D = \frac{VD}{\nu} = \frac{15\,m/s \times 0.020\,m}{15.94 \times 10^{-6}\,m^2/s} = 18,821$$

$$\overline{Nu}_D = 0.3 + \frac{0.62(18,821)^{1/2}(0.707)^{1/3}}{\left[1 + (0.4/0.707)^{2/3}\right]^{1/4}} \left[1 + \left(\frac{18,821}{282,000}\right)^{3/8}\right]^{4/5} = 76.5$$

$$\overline{h}_D = \frac{k}{D}\overline{Nu}_D = \frac{0.0263\,W/m\cdot K}{0.020\,m} \times 76.5 = 101\,W/m^2\cdot K$$

Continued...

Evoking the heat-mass analogy, Eq. 6.92, with n = 1/3

$$\frac{\overline{h}_D}{\overline{h}_m} = \rho c_p \left(\frac{\alpha}{D_{AB}} \right)^{-2/3} = 1.16\,kg/m^3 \times 1007\,J/kg\cdot K \left(\frac{2.257\times10^{-5}\,m^2/s}{0.26\times10^{-4}\,m^2/s} \right)^{-2/3} = 1284\,J/m^3\cdot K$$

$$\overline{h}_m = 101\,W/m^2\cdot K / 1284\,J/m^3\cdot K = 0.0787\,m/s$$

Substituting numerical values, the energy balance, Eq. (1),

$$P_e - 101\,W/m^2\cdot K \times \pi \times 0.020\,m\,(20-35)\,K$$
$$- 0.0787\,m/s \times \pi \times 0.020\,m\,(0.0169-0)\,kg/m^3 \times 2454\times10^3\,J/kg\cdot K = 0$$

$$P_e = -95.1\,W/m + 205.1\,W/m = 110\,W/m \quad <$$

The evaporation rate is

$$n_A = \overline{h}_m \pi D \left(\rho_{A,s} - \rho_{A,\infty} \right) = 0.0787\,m/s\,\pi \times 0.0020\,m\,(0.0169-0)\,kg/m^3 = 0.301\,kg/h\cdot m \quad <$$

(b) When the cylinder is dry, the energy balance is

$$P_e' = \overline{h}_D \pi D \left(T_s - T_\infty \right)$$

$$T_s = T_\infty + P_e'/\overline{h}_D \pi D = 35^\circ C + 110\,W/m \Big/ \left(101\,W/m^2\cdot K \pi \times 0.020\,m \right) = 52.3^\circ C \quad <$$

COMMENTS: Using *IHT Correlations Tool, External Flow, Cylinder*, the calculation of part (b) was performed using the proper film temperature, T_f = 316.8 K, to find $\overline{h}_D$ = 99.4 W/m^2·K and T_s = 52.6°C.

PROBLEM 7.124

KNOWN: Dry air at prescribed temperature and velocity flows over a rod covered with a thin porous coating saturated with water. The ends of the rod are attached to heat sinks maintained at a constant temperature.

FIND: Temperature at the midspan of the rod and evaporation rate from the surface using a steady-state, finite-difference analysis. Validate your code, without the evaporation process, by comparing the temperature distribution with the analytical solution of a fin.

SCHEMATIC:

Rod with porous coating, water saturated, $D = 20$ mm, $k = 175$ W/m·K

$T_b = 35\ ^oC$

T_1 T_2 T_3 T_9 T_{10}

x

$L_o = 125$ mm

Dry air

$T_\infty = 20\ ^oC$
$V = 15$ m/s
$\phi_\infty = 0$

T_{m-1} T_m T_{m+1}

q_c q_a q_b q_{evap}

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in rod, (3) Constant properties, and (4) Heat-mass transfer analogy is applicable.

PROPERTIES: *Table A.4*, Air ($\overline{T}_f$, see Eq. (2); 1 atm): ρ, c_p, k, α, Pr; *Table A.6*, Water ($T_m = T_{sat,m}$, 1 atm): $\rho_{A,sat} = 1/v_g$, h_{fg}; *Table A.8*, Water Vapor-Air ($\overline{T}_f$, 1 atm): $D_{AB} = D_{AB}(298\text{ K}) \times (\overline{T}_f/298)^{1.5}$.

ANALYSIS: As suggested, the 10-node network shown above represents the half-length of the system. Performing an energy balance on the control volume about the m-th node, the finite-difference equation for the system is derived.

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$q_a - q_{evap} + q_b + q_c = 0$$

$$kA_c \frac{T_{m+1} - T_m}{\Delta x} = n_{A,m} h_{fg,m} + \overline{h} P \Delta x \left(T_\infty - T_m\right) + kA_c \frac{T_{m-1} - T_m}{\Delta x} = 0 \qquad (1)$$

where the cross-sectional area and perimeter are $A_c = \pi D^2/4$ and $P = \pi D$, respectively. The average heat transfer coefficient $\overline{h}$ can be evaluated using the Churchill-Bernstein correlation, Eq. 7.57, evaluating thermophysical properties at an average film temperature for the system,

$$\overline{T}_f = \left[\left(T_1 + T_b\right)/2 + T_\infty\right]/2 \qquad (2)$$

The evaporation rate from Eq. (1) can be expressed as

$$n_{A,m} = \overline{h}_{D,m} P \Delta x \left(\rho_{A,s,m} - 0\right) \qquad (3)$$

where $\overline{h}_{D,m}$ can be determined from the heat-mass analogy, Eq. 6.92, with n = 1/3,

$$\frac{\overline{h}}{\overline{h}_m} = \rho c_p \left(\frac{\alpha}{D_{AB}}\right)^{-2/3} \qquad (4)$$

Continued...

PROBLEM 7.124 (Cont.)

where all properties are evaluated at $\overline{T}_f$. The density of water vapor, $\rho_{A,s,m}$, as well as the heat of vaporization, $h_{fg,m}$, must be evaluated at the nodal temperature T_m.

Using the *IHT Correlation Tool*, *External Flow*, *Cylinder*, an estimate of $\overline{h}_D = 101\ W/m^2{\cdot}K$ was obtained with $\overline{T}_f = 298.5$ K (based upon assumed value of $T_1 = 27°C$). From the analogy, Eq. (4), find that $\overline{h}_{D,m} = 0.0772$ m/s. Using the *IHT Workspace*, the finite-difference equations, Eq. (1), were entered and the temperature distribution (K, Case 1) determined as tabulated below. Using this same code with $\overline{h}_{D,m} = 1.0 \times 10^{-10}$ m/s, the temperature distribution (K, Case 2) was obtained. The results compared identically with the analytical solution for a fin with an adiabatic tip using the *IHT Model*, *Extended Surface*, *Rectangular Pin Fin*.

Case	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	T_{10}	T_b
1	287	287.2	287.6	288.3	289.4	290.9	292.9	295.4	298.6	302.7	308
2	300.3	300.4	300.6	300.9	301.4	302.1	302.8	303.8	305	306.4	308

<

The evaporation rate obtained by summing rates from each nodal element including node b is

$$n_{A,tot} = 1.08 \times 10^{-5}\ kg/s$$ <

COMMENTS: A copy of the *IHT Workspace* used to perform the above analysis is shown below.

```
// Nodal finite-difference equations (Only Nodes 1, 2 and 10 shown):
k * Ac * (T2 - T1) / delx - mdot1 * hfg1 + hbar * P * delx * (Tinf - T1) + k * Ac * (T2 - T1) / delx = 0
mdot1 = hmbar * P * delx * rhoA1
k * Ac * (T3 - T2) / delx - mdot2 * hfg2 + hbar * P * delx * (Tinf - T2) + k * Ac * (T1 - T2) / delx = 0
mdot2 = hmbar * P * delx * rhoA2
........
........
k * Ac * (Tb - T10) / delx - mdot10 * hfg10 + hbar * P * delx * (Tinf - T10) + k * Ac * (T9 - T10) / delx = 0
mdot10 = hmbar * P * delx * rhoA10

// Evaporation Rate:
mtot = mdot1/2 + mdot2 + mdot3 + mdot4 + mdot5 + mdot6 + mdot7 + mdot8 + mdot9 + mdot10 + mdotb
mdotb = hmbar * P * delx/2 * rhoAb

// Properties Tool - Water Vapor, rhoAm and hfgm
// Water property functions :T dependence, From Table A.6
// Units: T(K), p(bars);
x = 1                               // Quality (0=sat liquid or 1=sat vapor)
rhoA1 = rho_Tx("Water",T1,x)        // Density, kg/m^3
hfg1 = hfg_T("Water",T1)            // Heat of vaporization, J/kg
rhoA2 = rho_Tx("Water",T2,x)        // Density, kg/m^3
hfg2 = hfg_T("Water",T2)            // Heat of vaporization, J/kg
.....
.....
rhoA10 = rho_Tx("Water",T10,x)      // Density, kg/m^3
hfg10 = hfg_T("Water",T10)          // Heat of vaporization, J/kg
rhoAb = rho_Tx("Water",Tb,x)        // Density, kg/m^3
hfgb = hfg_T("Water",Tb)            // Heat of vaporization, J/kg

// Assigned Variables
Ac = pi * D^2 /4                    // Cross-sectional area, m^2
P = pi * D                          // Perimeter, m
D = 0.020                           // Diameter, m
delx = 0.125 /10                    // Spatial increment, m
k = 175                             // Thermal conductivity, W/m.K
Tb = 35 + 273                       // Base temperature, K
Tinf = 20 + 273                     // Fluid temperature, K
hmbar = 0.07719                     // Average mass transfer coefficient, m/s
hbar = 101                          // Average heat transfer coefficient, W/m^2.K
```

PROBLEM 7.125

KNOWN: The dimensions of a cylinder which approximates the human body.

FIND: (a) Heat loss by forced convection to ambient air, (b) Total heat loss when a water film covers the surface.

SCHEMATIC:

Air, B
V = 10 m/s
T_∞ = 20°C
ϕ = 0.6

D = 0.3 m
L = 1.75 m
Approximation to human body with and without water film (A), T_s = 30°C

ASSUMPTIONS: (1) Direct contact between skin and air (no clothing), (2) Negligible radiation effects, (3) Heat and mass transfer analogy is applicable, (4) Water vapor is an ideal gas.

PROPERTIES: *Table A-6*, Water (30°C = 303 K): $\rho_{A,sat} = v_g^{-1} = 0.0336$ kg/m^3, h_{fg} = 2431 kJ/kg; Water (20°C = 293K): $\rho_{A,sat} = 0.017$ kg/m^3; *Table A-4*, Air: (T_∞ = 20°C = 293K): ν = 15.27 $\times 10^{-6}$ m^2/s, k = 25.7 $\times 10^{-3}$ W/m·K, Pr = 0.71; *Table A-8*, Water vapor-air (300K): $D_{AB} = 26 \times 10^{-6}$ m^2/s, Sc = ν/D_{AB} = 0.59.

ANALYSIS: (a) The heat rate is

$$q = \overline{h}(p DL)\ (T_s - T_\infty).$$

With

$$Re_D = \frac{VD}{n} = \frac{(10\text{ m/s})\ (0.3\text{ m})}{15.27\times10^{-6}\text{ m}^2/\text{s}} = 1.96\times10^5$$

obtain $\overline{h}$ from Eq. 7.56, where n = 0.37 and, from Table 7.4, C = 0.26 and m = 0.6,

$$\overline{Nu}_D = 0.6\left(1.96\times10^5\right)^{0.6}\ (0.71)^{0.37}\ (0.71/0.71)^{0.25} = 343.$$

Hence $\overline{h} = \overline{Nu}_D\ \dfrac{k}{D} = 343\times\dfrac{25.7\times10^{-3}\text{ W/m}\cdot\text{K}}{0.3\text{ m}} = 29.4\text{ W/m}^2\cdot\text{K}$

and $q = 29.4\text{ W/m}^2\cdot\text{K}\ (p\times0.3\text{ m}\times1.75\text{ m})\ (30-20)^\circ\text{C} = 485\text{ W}.$ <

(b) The total heat loss with the water film includes latent, as well as sensible, contributions and may be expressed as

$$q = \overline{h}(p DL)\ (T_s - T_\infty) + \dot{n}_A\ h_{fg}$$

where $\dot{n}_A = \overline{h}_m\ (p DL)\left[r_{A,sat}(T_s) - r_{A,\infty}\right]$

$$r_{A,sat}(T_s) = 0.0336\text{ kg/m}^3 \qquad r_{A,\infty} \approx f r_{A,sat}(T_\infty) = 0.6(0.017) = 0.010\text{ kg/m}^3.$$

Continued …..

PROBLEM 7.125 (Cont.)

The convection mass transfer coefficient may be obtained from Eq. 6.92 or by expressing the mass transfer analog of Eq. 7.56. Neglecting the Pr ratio, the analogous form is

$$\overline{Sh}_D = 0.26\ Re_D^{0.6}\ Sc^{0.37}$$
$$\overline{Sh}_D = 0.26\ \left(1.96\times10^5\right)^{0.6}\left(0.59\right)^{0.37} = 320.$$

Hence

$$\overline{h}_m = 320\frac{D_{AB}}{D} = \frac{320\times0.26\times10^{-4}\ m^2/s}{0.3\ m} = 0.028\ m/s.$$

The evaporation rate is then

$$\dot{n}_A = 0.028 m/s\ \left(p\times0.3\ m\times1.75\ m\right)\ \left[0.0336-0.010\right]\ kg/m^3$$
$$\dot{n}_A = 1.09\times10^{-3}\ kg/s.$$

Hence,

$$q = 485\ W + 1.09\times10^{-3} kg/s\times2.431\times10^6\ J/kg$$

$$q = 485\ W + 2650\ W = 3135\ W. \qquad <$$

COMMENTS: The evaporative (latent) heat loss dominates over the sensible heat loss. Its effect is often felt when stepping out of a swimming pool or other body of water.

PROBLEM 7.126

KNOWN: Horizontal tube exposed to transverse stream of dry air.

FIND: Equation to determine heat transfer enhancement due to wetting. Evaluate enhancement for prescribed conditions.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Heat-mass transfer analogy applicable, (3) Water vapor behaves as perfect gas.

PROPERTIES: *Table A-4*, Air (310K, 1 atm): $\rho = 1.1281\ \text{kg/m}^3$, $c_p = 1007.4$ J/kg, $\nu = 16.90 \times 10^{-6}\ \text{m}^2/\text{s}$, Pr = 0.706; *Table A-8*, Air-water vapor mixture (310K): $D_{AB} \approx 0.26 \times 10^{-4}\ \text{m}^2/\text{s}$, Sc = $\nu_B/D_{AB} = 0.650$; *Table A-6*, Saturated water vapor (320K): $\rho_{A,sat} = 1/v_g = 0.07153\ \text{kg/m}^3$, h_{fg} = 2390 kJ/kg.

ANALYSIS: The enhancement due to wetting can be expressed as the ratio of the wet-to-dry cylinder heat fluxes.

$$\frac{q''_w}{q''_d} = \frac{q''_{conv} + q''_{evap}}{q''_{conv}} = 1 + \frac{q''_{evap}}{q''_{conv}}$$

where

$$q''_{conv} = \bar{h}\left(T_s - T_\infty\right) \qquad q''_{evap} = \dot{m}''_A h_{fg} = \bar{h}_m \left(\rho_{A,s} - \rho_{A,\infty}\right) h_{fg} = \bar{h}_m\ \rho_{A,sat}\ h_{fg}.$$

Invoking the heat-mass transfer analogy, using Eq. 6.92, find

$$\frac{\bar{h}}{\bar{h}_m} = \left(\rho\ c_p\right)_B\ Le^{1-n} = \left(\rho\ c_p\right)_B\ (Sc/Pr)^{2/3}$$

assuming n = 1/3 with $\rho_{A,\infty} = 0$, find

$$\frac{q''_w}{q''_d} = 1 + \left[\left(\rho\ c_p\right)_B (Sc/Pr)^{2/3}\right]^{-1} \frac{\rho_{A,sat}\ h_{fg}}{\left(T_s - T_\infty\right)}. \qquad <$$

Substituting numerical values, the enhancement is

$$\frac{q''_w}{q''_d} = 1 + \left[\left(1.1281\frac{\text{kg}}{\text{m}^3} \times 1007.4\frac{\text{J}}{\text{kg}}\right)\left(\frac{0.650}{0.706}\right)^{2/3}\right]^{-1} \frac{0.07153\ \text{kg/m}^3 \times 2390 \times 10^3\ \text{J/kg}}{\left(320 - 300\right)\text{K}} = 9.0. \qquad <$$

COMMENTS: For the prescribed conditions, the effect of wetting is to enhance the heat transfer by nearly an order of magnitude. Will the enhancement increase or decrease with increasing T_s?

PROBLEM 7.127

KNOWN: Moisture-soaked paper is cylindrical form maintained at given temperature by imbedded heaters. Dry air at prescribed velocity and temperature in cross flow over cylinder.

FIND: (a) Required electrical power and the evaporation rate per unit length, q'_{evap} and n'_A, respectively, and (b) Calculate and plot q' and n'_A as a function of dry air velocity $5 \le V \le 20$ m/s and paper temperatures of 65, 70 and 75°C.

SCHEMATIC:

Dry air

$T_\infty = 20\,^oC$

V = 10 m/s

$\phi_\infty = 0$

q'_{evap}, q'_{conv}, q'_{elec}

Moisture-soaked paper cylinder, $D = 0.15$ m, $T_s = 70\,^oC$

ASSUMPTIONS: (1) Steady-state conditions, (2) Heat-mass transfer analogy applicable, (3) Negligible radiation effects.

PROPERTIES: *Table A.4*, Air (T_∞ = 20°C = 293 K, 1 atm): $\rho = 1.1941$ kg/m^3, $c_p = 1007$ J/kg·K, k = 25.7×10^{-3} W/m·K, $\nu = 15.26 \times 10^{-6}$ m^2/s, Pr = 0.709; (T_s = 70°C = 343 K): $Pr_s = 0.701$; *Table A.6*, Sat. water vapor (T_s = 70°C = 343 K): $\rho_{A,s} = 1/v_g = 0.196$ kg/m^3, $h_{fg} = 2334 \times 10^3$ J/kg; *Table A.8*, Air-water vapor mixture ($\overline{T}_f = (T_\infty + T_s)/2 = 318$ K, 1 atm): $D_{AB} = 0.26 \times 10^{-4}$ m^2/s$(318/298)^{3/2} = 0.29 \times 10^{-4}$ m^2/s.

ANALYSIS: (a) From an energy balance on the cylinder on a per unit length basis,

$$q'_{elec} = q'_{conv} + q'_{evap} \qquad q'_{elec} = \pi D\left[\overline{h}\left(T_s - T_\infty\right) + \overline{h}_m\left(\rho_{A,s} - \rho_{A,\infty}\right)h_{fg}\right] \quad (1)$$

where $\rho_{A,\infty} = 0$, the freestream air is dry, and $\rho_{A,s} = \rho_{A,sat}(T_s)$. To estimate $\overline{h}$, find

$$Re_D = \frac{VD}{\nu} = \frac{10\,m/s \times 0.15\,m}{15.26 \times 10^{-6}\,m^2/s} = 98,296 \quad (2)$$

and using the Zhukauskus correlation, from Table 7.4: C = 0.26, m = 0.6, and n = 0.37,

$$\overline{Nu}_D = \frac{\overline{h}D}{k} = 0.26\,Re^{0.6}\,Pr^{0.37}\left(Pr/Pr_s\right)^{0.25} \quad (3)$$

$$\overline{h} = \frac{0.0257\,W/m \cdot K}{0.15\,m} \times 0.26\left(98,296\right)^{0.6}\left(0.709\right)^{0.37}\left(0.709/0.701\right)^{0.25} = 38.9\,W/m^2 \cdot K.$$

Using the heat-mass analogy with n = 1/3, find

$$\overline{h}/\overline{h}_m = \left(\rho c_p\right)_B \left(Sc/Pr\right)^{2/3} = \left(\rho c_p\right)_B \left(\nu/D_{AB}/Pr\right)^{2/3} \quad (4)$$

$$\overline{h}_m = 38.9\,W/m^2 \cdot K \Big/ \left(1.1941\,kg/m^3 \times 1007\,J/kg \cdot K\right)\left[\frac{15.26 \times 10^{-6}\,m^2/s / 0.29 \times 10^{-4}\,m^2/s}{0.709}\right]^{2/3}$$

$$\overline{h}_m = 0.03946\,m/s.$$

Hence, the electric power requirement is

$$q'_{elec} = \pi \times 0.15\,m\left[38.9\,W/m^2 \cdot K\left(70 - 20\right)K + 0.03946\,m/s\left(0.196 - 0\right)kg/m^3 \times 2334 \times 10^3\,J/kg\right]$$

Continued...

PROBLEM 7.127 (Cont.)

$$q'_{elec} = (917 + 8507)\,\text{W/m} = 9424\,\text{W/m} \quad (5)$$ <

(b) The foregoing equations were entered into the IHT Workspace, and using the *Properties Tools*, for air and water vapor required thermophysical properties, the required electrical power, q', and evaporation rate, n'_A, were calculated as a function of dry air velocity for selected water temperatures.

Evap rate, n'A*10^3 (kg/s.m)
Dry air velocity, V (m/s)
Ts = 65 C
Ts = 70 C
Ts = 75 C

Elec power, q'elec (W/m)
Dry air velocity, V (m/s)
Ts = 65 C
Ts = 70 C
Ts = 75 C

COMMENTS: (1) Note at which temperatures the thermophysical properties are evaluated.

(2) From Equation (5), note that the evaporation heat rate far exceeds that due to convection.

(3) From the plots, note that both q'_{elec} and n'_A are nearly proportional to air velocity, and increase with increasing water temperature.

PROBLEM 7.128

KNOWN: Dry-and wet-bulb temperatures associated with a moist airflow through a large diameter duct of prescribed surface temperature.

FIND: Temperature and relative humidity of airflow.

SCHEMATIC:

Dry-bulb thermometer, $T_{db} = 45\ °C$, $D_{db} = 3$ mm, $\varepsilon_g = 0.95$

Moist air T_∞, ϕ_∞, $p = 1$ atm, $V = 5$ m/s

q_{conv}, q_{rad}, $T_s = 35\ °C$, q_{evap}, Wick

Wet-bulb thermometer, $T_{wb} = 25\ °C$, $D_{wb} = 4$ mm, $\varepsilon_w = 0.95$

ASSUMPTIONS: (1) Steady-state conditions, (2) Conduction along the thermometers is negligible, (3) Duct wall forms a large enclosure about the thermometers.

PROPERTIES: *Table A-4*, Air (318K, 1 atm): $\nu = 17.7 \times 10^{-6}\ m^2/s$, k = 0.0276 W/m·K, Pr = 0.70; *Table A-4*, Air (298K, 1 atm): $\nu = 15.7 \times 10^{-6}\ m^2/s$, k = 0.0261 W/m·K, Pr = 0.71; *Table A-6*, Saturated water vapor (298K): $v_g = 44.3\ m^3/kg$, $h_{fg} = 2442$ kJ/kg; Saturated water vapor (318.5K): $v_g = 15.5\ m^3/kg$; *Table A-8*, Water vapor-air (298K): $D_{AB} = 0.26 \times 10^{-4}\ m^2/s$, Sc = 0.60.

ANALYSIS: *Dry-bulb Thermometer*: Since $T_{db} > T_s$, there is net radiation transfer from the surface of the dry-bulb thermometer to the duct wall. Hence to maintain steady-state conditions, the thermometer temperature must be less than that of the air ($T_{db} < T_\infty$) to allow for convection heat transfer from the air. Hence, from application of a surface energy balance to the thermometer, $q_{conv} = q_{rad}$, or, from Eqs. 6.4 and 1.7,

$$\bar{h}A_{db}\left(T_\infty - T_{db}\right) = \varepsilon_g A_{db}\sigma\left(T_{db}^4 - T_s^4\right).$$

The air temperature is then

$$T_\infty = T_{db} + \left(\varepsilon_g\sigma / \bar{h}\right)\left(T_{db}^4 - T_s^4\right) \quad (1)$$

where $\bar{h}$ may be obtained from Eq. 7.56.

Wet-bulb Temperature: The relative humidity may be obtained by performing an energy balance on the wet-bulb thermometer. In this case convection heat transfer to the wick is balanced by evaporative and radiative heat losses from the wick,

$$q_{conv} = q_{evap} + q_{rad} \qquad q_{evap} = n''_A A_{wb} h_{fg} = \bar{h}_m\left[\rho_{A,sat}\left(T_{wb}\right) - \phi_\infty\rho_{A,sat}\left(T_\infty\right)\right]A_{wb}h_{fg}.$$

$$\bar{h}A_{wb}\left(T_\infty - T_{wb}\right) = \bar{h}_m\left[\rho_{A,sat}\left(T_{wb}\right) - \phi_\infty\rho_{A,sat}\left(T_\infty\right)\right]A_{wb}h_{fg} + \varepsilon_w A_{wb}\sigma\left(T_{wb}^4 - T_s^4\right)$$

$$\phi_\infty = \left\{\rho_{A,sat}\left(T_{wb}\right) + \left[\varepsilon_w\sigma\left(T_{wb}^4 - T_s^4\right) - \bar{h}\left(T_\infty - T_{wb}\right)\right] / h_{fg}\bar{h}_m\right\} / \rho_{A,sat}\left(T_\infty\right) \quad (2)$$

where $\bar{h}_m$ may be determined from the mass transfer analog of Eq. 7.56.

Continued …..

Convection Calculations: For the prescribed conditions, the Reynolds number associated with the dry-bulb thermometer is

$$Re_{D(db)} = VD_{bd}/\nu = 5\text{ m/s}\times 0.003\text{ m}/17.7\times 10^{-6}\text{ m}^2/\text{s} = 847.$$

Approximating the Prandtl number ratio as unity, from Eq. 7.56 and Table 7.4,

$$\overline{Nu}_{D(db)} = CRe_{D(db)}^{m}Pr^{n} = 0.51(847)^{0.5}(0.70)^{0.37} = 13.01$$

$$\bar{h} = 13.01\frac{k}{D_{db}} = 13.01\frac{0.0276\text{ W/m}\cdot\text{K}}{0.003\text{ m}} = 120\text{ W/m}^2\cdot\text{K}.$$

From Eq. (1) the air temperature is

$$T_\infty = 45^\circ\text{C} + \frac{0.95\times 5.67\times 10^{-8}\text{W/m}^2\cdot\text{K}^4}{120\text{ W/m}^2\cdot\text{K}}\left(318^4 - 308^4\right)\text{K}^4 = 45^\circ\text{C} + 0.55^\circ\text{C} = 45.6^\circ\text{C}. \quad <$$

The relative humidity may now be obtained from Eq. (2). The Reynolds number associated with the wet-bulb thermometer is

$$Re_{D(wb)} = VD_{wb}/\nu = 5\text{ m/s}\times 0.004\text{ m}/15.7\times 10^{-6}\text{ m}^2/\text{s} = 1274.$$

From Eq. 7.56 and Table 7.4, it follows that

$$\overline{Nu}_{D(wb)} = 0.26(1274)^{0.6}(0.71)^{0.37} = 16.71$$

$$\bar{h} = 16.71\frac{k}{D_{wb}} = 16.71\frac{0.0261\text{ W/m}\cdot\text{K}}{0.004\text{ m}} = 109\text{ W/m}^2\cdot\text{K}.$$

Using the mass transfer analog of Eq. 7.56, it also follows that

$$\overline{Sh}_{D(wb)} = 0.26Re_{D(wb)}^{0.6}Sc^{0.37} = 0.26(1274)^{0.6}(0.6)^{0.37} = 15.7$$

$$\bar{h}_m = 15.7\frac{D_{AB}}{D_{wb}} = \frac{15.7\times 0.26\times 10^{-4}\text{ m}^2/\text{s}}{0.004\text{ m}} = 0.102\text{ m/s}.$$

Also, $\rho_{A,sat}(T_{wb}) = v_g(298\text{ K})^{-1} = \left(44.3\text{ m}^3/\text{kg}\right)^{-1} = 0.0226\text{ kg/m}^3$

$$\rho_{A,sat}(T_\infty) = v_g(318.5\text{ K})^{-1} = \left(15.5\text{ m}^3/\text{kg}\right)^{-1} = 0.0645\text{ kg/m}^3.$$

Hence the relative humidity is, from Eq. (2)

$$\phi_\infty = \left(0.0226\text{ kg/m}^3 + \frac{\left[0.95\times 5.67\times 10^{-8}\text{W/m}^2\cdot\text{K}^4\left(298^4 - 308^4\right)\text{K}^4 - 109\text{W/m}^2\cdot\text{K}\left(45.55 - 25\right)\text{K}\right]}{\left(2.442\times 10^6\text{ J/kg}\right)\left(0.102\text{ m/s}\right)}\right)/0.0645\text{ kg/m}^3$$

$$\phi_\infty = 0.21 \quad <$$

COMMENTS: (1) The effect of radiation exchange between the duct wall and the thermometers is small. For this reason $T_\infty = T_{db}$. (2) The evaporative heat loss is significant due to the small value of ϕ_∞, causing T_{wb} to be significantly less than T_∞.

PROBLEM 7.129

KNOWN: Velocity, diameter and temperature of a spherical droplet. Conditions of surroundings.

FIND: (a) Expressions for droplet evaporation and cooling rates, (b) Evaporation and cooling rates for prescribed conditions.

SCHEMATIC:

D=3mm; Air, $T_\infty = 25°C$, $\phi_\infty = 0.6$; $\dot{m}_{evap}$; q_{conv}; q_{evap}; q_{rad}; $\dot{E}_{st}$; Water, T=40°C; V=7m/s; T_{sur}=15°C

ASSUMPTIONS: (1) Negligible temperature gradients in the drop, (2) Heat and mass transfer analogy is applicable, (3) Perfect gas behavior for vapor.

PROPERTIES: *Table A-4*, Air (T_∞ = 298K, 1 atm): $\nu = 15.71 \times 10^{-6}$ m^2/s, k = 0.0261 W/m·K, Pr = 0.71; *Table A-6*, Water (T = 40°C): $\rho_{A,sat}$ = 0.050 kg/m^3, h_{fg} = 2407 kJ/kg, ρ_ℓ = 992 kg/m^3, $c_{p,\ell}$ = 4179 J/kg·K; (T_∞ = 25°C): $\rho_{A,sat}$ = 0.023 kg/m^3; *Table A-8*, Water vapor-air (298K): D_{AB} = 0.26 $\times 10^{-4}$ m^2/s.

ANALYSIS: (a) The evaporation rate is given by

$$\dot{m}_{evap} = \overline{h}_m A_s \left(\rho_{A,s} - \rho_{A,\infty}\right) = \overline{h}_m \pi D^2 \left[\rho_{A,sat}(T) - \phi_\infty \rho_{A,sat}(T_\infty)\right]. \quad <$$

The cooling rate is obtained from an energy balance performed for a control surface about the droplet,

$$\dot{E}_{st} = -q_{out} = -\left(q_{conv} + q_{rad} + q_{evap}\right)$$

or

$$\frac{d}{dt}\left(\rho_\ell \frac{\pi D^3}{6} c_{p,\ell} T\right) = -A_s\left[\overline{h}\left(T_s - T_\infty\right) + \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right) + \dot{m}''_{evap} h_{fg}\right].$$

With $A_s = \pi D^2$, it follows that

$$\frac{dT}{dt} = -\frac{6}{\rho_\ell c_{p,\ell} D}\left[\overline{h}\left(T_s - T_\infty\right) + \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right) + \dot{m}''_{evap} h_{fg}\right]. \quad <$$

(b) To obtain $\overline{h}_m$, the mass transfer analog of the Ranz-Marshall correlation gives

$$\overline{Sh}_D = 2 + 0.6 Re_D^{1/2} Sc^{1/3}$$

where

$$Re_D = \frac{VD}{\nu} = \frac{7 \text{ m/s} \times 0.003 \text{ m}}{15.71 \times 10^{-6} \text{ m}^2/\text{s}} = 1337, \qquad Sc = \frac{\nu}{D_{AB}} = \frac{15.71 \times 10^{-6}}{26 \times 10^{-6}} = 0.60.$$

Continued …..

Hence

$$\overline{Sh}_D = 2 + 0.6(1337)^{1/2}(0.6)^{1/3} = 20.5$$

$$\bar{h}_m = \overline{Sh}_D \frac{D_{AB}}{D} = 20.5 \frac{0.26\times10^{-4}\ m^2/s}{0.003\ m} = 0.18\ m/s$$

$$\dot{m}_{evap} = 0.18\ m/s\ p\ (0.003\ m)^2 [0.05 - 0.6\times.023] kg/m^3 = 1.82\times10^{-7} kg/s. \qquad <$$

The evaporative heat flux is then

$$q''_{evap} = \frac{q_{evap}}{A_s} = \frac{\dot{m}_{evap} h_{fg}}{p D^2} = \frac{1.82\times10^{-7}\ kg/s\left(2.407\times10^6\ J/kg\right)}{p\ (0.003\ m)^2}$$

$$q''_{evap} = 15{,}494\ W/m^2.$$

Using the heat transfer correlation, the Nusselt number is

$$\overline{Nu}_D = 2 + 0.6 Re_D^{1/2} Pr^{1/3} = 2 + 0.6(1337)^{1/2}(0.71)^{1/3} = 21.58.$$

Hence $$\bar{h} = \overline{Nu}_D \frac{k}{D} = 21.58 \frac{0.0261\ W/m\cdot K}{0.003\ m} = 188\ W/m^2\cdot K$$

and the sensible heat flux is

$$q''_{conv} = \bar{h}(T - T_\infty) = 188\ W/m^2\cdot K\,(40-25)^\circ C$$

$$q''_{conv} = 2815\ W/m^2.$$

The net radiative flux is

$$q''_{rad} = e s \left(T^4 - T_{sur}^4\right) - 0.95\times5.67\times10^{-8}\ W/m^2\cdot K^4 \left[313^4 - 288^4\right] K^4$$

$$q''_{rad} = 146\ W/m^2\cdot K.$$

Hence $$\frac{dT}{dt} = -\frac{6}{992\ kg/m^3\times4179\ J/kg\cdot K(0.003\ m)}(2815 + 146 + 15{,}494)\ W/m^2$$

$$\frac{dT}{dt} = -8.9\ K/s. \qquad <$$

COMMENTS: (1) Evaporative cooling provides the dominant heat loss from the drop. (2) To test the validity of assuming negligible temperature gradients in the drop, calculate

$$Bi \approx \frac{h_{eff}(r_o/3)}{k_\ell}, \quad \text{where} \quad h_{eff} \equiv \frac{q''_{tot}}{T - T_\infty} = \frac{18{,}455}{25} = 738\ W/m^2\cdot K.$$

From Table A-6, $k_\ell = 0.631\ W/m\cdot K$, hence

$$Bi \approx \frac{738\ W/m^2\cdot K(0.0005\ m)}{0.631\ W/m\cdot K} = 0.58.$$

Hence, although suspect, the assumption is not totally unreasonable.

PROBLEM 7.130

KNOWN: Cranberries with an average diameter of 15 mm rolling over a fine screen. Thickness of the water film is 0.2 mm.

FIND: Time required to dry the berries exposed to heated air with a velocity of 2 m/s and temperature of 30°C.

SCHEMATIC:

Berry, D = 15 mm, water film, 0.2 mm thick

Fine screen

Air

$T_\infty = 30^oC$
V = 2 m/s

ASSUMPTIONS: (1) Steady-state conditions, (2) Air stream is dry, (3) Water film on the berries is also at 30°C, (4) Convection process is uniform over the exposed surface, and (5) Heat-mass analogy is applicable.

PROPERTIES: *Table A-6*, Water (T_f = 30°C = 303 K): $\rho_{A,f} = 995.8\ \text{kg/m}^3$, $\rho_{A,g}$ = 0.02985 kg/m^3; *Table A-8*, Water-air (T_f = 303 K, 1 atm): $D_{AB} = 0.26 \times 10^{-4}\ \text{m}^2/\text{s}\ (303/298)^{1.5}$ = $2.67 \times 10^{-5}\ \text{m}^2/\text{s}$; *Table A-4*, Air ($T_f$ = 303 K, 1 atm): $\mu = \mu_s = 1.86 \times 10^{-5}\ \text{N·s/m}^2$, $\nu = 1.619 \times 10^{-5}\ \text{m}^2/\text{s}$, $\alpha = 2.294 \times 10^{-5}\ \text{m}^2/\text{s}$, k = 0.02652 W/m·K, Pr = 0.861.

ANALYSIS: The evaporation rate of water from the berry surface is given by the rate equation,

$$n = \bar{h}_m A_s \left(\rho_{A,s} - \rho_{A,\infty}\right) \tag{1}$$

where $A_s = \pi D^2$ and $\bar{h}_m$ is determined using the heat-mass analogy, Eq. 6.67,

$$\frac{\bar{h}}{\bar{h}_m} = \frac{k}{D_{AB}} Le^{-n} \tag{2}$$

where $Le = \alpha/D_{AB}$ and typically n = 1/3. The heat transfer coefficient $\bar{h}$ is estimated with the Whitaker correlation, Eq. 7.59,

$$\overline{Nu}_D = \frac{\bar{h}D}{k} = 2 + \left[0.4\ Re_D^{1/2} + 0.06\ Re_D^{2/3}\right] Pr^{0.4} \left(\mu/\mu_s\right)^{1/4} \tag{3}$$

Substituting numerical values, find

$$Re_D = \frac{VD}{\nu} = \frac{2\ \text{m/s} \times 0.015\ \text{m}}{1.86 \times 10^{-5} \text{m}^2/\text{s}} = 1853$$

$$Nu_D = 2 + \left[0.4(1853)^{1/2} + 0.06(1853)^{2/3}\right] \times (0.707)^{0.4} \times 1 = 24.9$$

$$\bar{h} = 24.9 \times 0.02652\ \text{W/m} \cdot \text{K} / 0.015\ \text{m} = 43.4\ \text{W/m}^2 \cdot \text{K}$$

and using the heat-mass analogy,

$$\bar{h}_m = 43.4\ \text{W/m}^2 \cdot \text{K} \times \left(2.67 \times 10^{-5} \text{m}^2/\text{s} \big/ 0.02652\ \text{W/m} \cdot \text{K}\right) \times (0.861)^{1/3}$$

$$\bar{h}_m = 0.0420\ \text{m/s}$$

Continued …..

PROBLEM 7.130 (Cont.)

where

$$Le = \alpha / D_{AB} = 2.294 \times 10^{-5} m^2/s \big/ 2.667 \times 10^{-5} m^2/s = 0.861$$

Using Eq. (1), the evaporation rate is

$$n = 0.0420 \text{ m/s} \times \left(\pi (0.015 \text{ m})^2 / 4\right)(0.02985 - 0) \text{kg/m}^3 = 8.87 \times 10^{-7} \text{ kg/s}$$

The time, t_o, required to evaporate the water film of thickness $\delta = 0.2$ mm is

$$nt_o = M_{film} = \rho_{A,\ell} (\pi D) \delta$$

$$t_o = 995.8 \text{ kg/m}^3 (\pi \times 0.015 \text{ m}) \times 0.0002 \text{ m} / 8.87 \times 10^{-7} \text{ kg/s}$$

$$t_o = 159 \text{ s}$$ $<$

PROBLEM 7.131

KNOWN: Spherical droplet at prescribed temperature and velocity falling in still, hotter dry air.

FIND: Instantaneous rate of evaporation.

SCHEMATIC:

Spherical water droplet, $T_s = 60°C$, $D = 0.5mm$

Dry air

$T_\infty = 100°C$
$V = 2.15m/s$
$\phi_\infty = 0$

ASSUMPTIONS: (1) Steady-state conditions, (2) Heat-mass transfer analogy applicable.

PROPERTIES: *Table A-4*, Air (T_∞ = 100°C = 373K, 1atm): $\rho = 0.9380\ kg/m^3$, $c_p = 1011\ J/kg\cdot K$, $k = 0.0317\ W/m\cdot K$, $\nu = 23.45 \times 10^{-6}\ m^2/s$, Pr = 0.695; *Table A-6*, Sat. water (T_s = 60°C = 333 K): $\rho_\ell = 1/v_\ell = 983\ kg/m^3$, $\rho_{A,s} = 1/v_f = 0.129\ kg/m^3$; *Table A-8*, Air-water vapor mixture (T_∞ = 373K, 1 atm): $D_{AB} = 0.267 \times 10^{-4}\ m^2/s\ (373/298)^{3/2} = 0.36 \times 10^{-4}\ m^2/s$.

ANALYSIS: The instantaneous evaporation rate is

$$\dot{n}_A = \bar{h}_m A_s \left(\rho_{A,s} - \rho_{A,\infty}\right)$$

where $A_s = \pi D^2$, $\rho_{A,\infty} = 0$ and $\rho_{A,s} = \rho_{A,sat}(T_s)$. To estimate $\bar{h}_m$ use the Whitaker correlation, written in terms of mass transfer parameters and with $\mu/\mu_s \approx 1$,

$$\overline{Sh}_D = \frac{\bar{h}_m D}{D_{AB}} = 2 + \left(0.4 Re_D^{1/2} + 0.06 Re_D^{2/3}\right) Sc^{0.4}$$

$$\bar{h}_m = \frac{0.36\times10^{-4}\ m^2/s}{0.0005\ m}\left[2 + \left(0.4(45.8)^{1/2} + 0.06(45.8)^{2/3}\right)\times 0.651^{0.4}\right] = 0.355\ m/s$$

where
$$Re_D = \frac{VD}{\nu} = \frac{2.15\ m/s\times0.0005\ m}{23.45\times10^{-6}\ m^2/s} = 45.8$$

$$Sc = \nu/D_{AB} = 23.45\times10^{-6}\ m^2/s / 0.36\times10^{-4}\ m^2/s = 0.651.$$

Hence, the evaporation rate is

$$\dot{n}_A = 0.355\ m/s\times\pi\,(0.0005\ m)^2(0.129-0)\,kg/m^3 = 3.60\times10^{-8}\ kg/s. \quad <$$

COMMENTS: If this evaporation rate were to remain constant with time, the droplet of mass M would be completely evaporated in

$$\Delta t = M/\dot{n}_A = \frac{\rho_\ell\left(4\pi D^3/3\right)}{\dot{n}_A} = \frac{983\ kg/m^3\left(4\pi\,(0.0005\ m)^3/3\right)}{3.60\times10^{-8}\ kg/s} = 14.3\ s.$$

To determine whether the droplet temperature will increase or decrease with time, it is necessary to compare convective heat and evaporation rates. Hence it is not clear whether the time to completely evaporate will be less or greater than 14.3 s.

PROBLEM 7.132

KNOWN: Diameter, velocity and surface vapor concentration of alcohol droplet falling in quiescent air. Latent heat of vaporization and diffusion coefficient. Air temperature.

FIND: Droplet surface temperature

SCHEMATIC:

q''_{conv}, q''_{evap}, T_s

Alchohol (A) $\left\{ \begin{array}{l} D_{AB} = 10^{-5}\ m^2/s \\ h_{fg} = 8.42 \times 10^5\ J/kg \end{array} \right.$

$\rho_{a,s} = 0.0573\ kg/m^3$

Air (B) $T_\infty = 300$ K

$D = 0.5$ mm

$V = 1.8$ m/s

ASSUMPTIONS: (1) Steady-state conditions, (2) Applicability of heat and mass transfer analogy, (3) Negligible radiation, (4) Negligible vapor concentration in air ($\rho_{A,\infty}$ = 0).

PROPERTIES: *Table A.4*, air (T_∞ = 300 K): $\nu = 15.89 \times 10^{-6}\ m^2/s$, k = 0.0263 W/m·K, Pr = 0.707.

ANALYSIS: Application of a surface energy balance yields

$$q''_{evap} = q''_{conv}$$

$$\bar{h}_m \left(\rho_{A,s} - \rho_{A,\infty}\right) h_{fg} = \bar{h}\left(T_\infty - T_s\right)$$

$$T_s = T_\infty - \frac{\bar{h}_m}{\bar{h}} \rho_{A,s} h_{fg}$$

With $Re_D = VD/\nu = 1.8\,m/s \times 5 \times 10^{-4}\,m \big/ 15.89 \times 10^{-6}\,m^2/s$ = 56.6 and Sc = ν/D_{AB} = 1.59, the Ranz-Marshall correlation yields

$$\overline{Nu}_D = 2 + 0.6 Re_D^{1/2} Pr^{1/3} = 2 + 0.6(56.6)^{1/2}(0.707)^{1/3} = 6.02$$

$$\overline{Sh}_D = 2 + 0.6 Re_D^{1/2} Sc^{1/3} = 2 + 0.6(56.6)^{1/2}\left(1.59^{1/3}\right) = 7.27$$

With $\bar{h}_m/\bar{h}$ = $\overline{Sh}_D\left(D_{AB}/D\right)\big/\overline{Nu}_D\left(k/D\right)$,

$$\frac{\bar{h}_m}{\bar{h}} = \frac{\overline{Sh}_D\left(D_{AB}\right)}{\overline{Nu}_D\left(k\right)} = \frac{7.27 \times 10^{-5}\,m^2/s}{6.02 \times 0.0263\,W/m \cdot K} = 4.59 \times 10^{-4}\,m^3 \cdot K/J$$

Hence,

$$T_s = 300\,K - 4.59 \times 10^{-4}\,m^3 \cdot K/J\left(0.0573\,kg/m^3\right)\left(8.42 \times 10^5\,J/kg\right) = 277.9\,K$$ <

COMMENTS: The large vapor density, $\rho_{A,s}$, renders the *evaporative cooling* effect significant.

PROBLEM 7.133

KNOWN: Diameter, velocity and temperature of water droplets in air of known temperature.

FIND: Evaporation rate for a single drop.

SCHEMATIC:

Air, B
$V = 5$ m/s
$T_\infty = 35°C$
$\phi = 0$
$p = 1$ atm
$T_s = 25°C$
$D = 3$ mm

ASSUMPTIONS: (a) Steady-state conditions, (b) Dry air, (c) Drop oscillations and distortions are negligible.

PROPERTIES: *Table A-4*, Air (35°C = 308K): $\nu = 16.7 \times 10^{-6}$ m^2/s; *Table A-8*, Water vapor-air (35°C = 308K): $D_{AB} = 0.26 \times 10^{-4}$ m^2/s; *Table A-6*, Saturated water vapor (25°C = 298K): v_g = 44.3 m^3/kg.

ANALYSIS: The mass evaporation rate is

$$n_A = \overline{h}_m \left(\pi D^2\right)\left(\rho_{A,s} - \rho_{A,\infty}\right)$$

where $\rho_{A,s} = v_g^{-1} = 0.023$ kg/m^3 and $\rho_{A,\infty} = 0$. From Eq. 7.58,

$$\overline{Sh}_D = 2 + \left(0.4\,Re_D^{1/2} + 0.06\,Re_D^{2/3}\right) Sc^{0.4}$$

where

$$Re_D = \frac{VD}{\nu} = \frac{(5\text{ m/s})\left(3\times10^{-3}\text{ m}\right)}{16.7\times10^{-6}\text{ m}^2/\text{s}} = 898 \qquad Sc = \frac{\nu}{D_{AB}} = 0.64$$

$$\overline{Sh}_D = 2 + \left[0.4(898)^{1/2} + 0.06(898)^{2/3}\right](0.64)^{0.4} = 16.7$$

$$\overline{h}_m = \overline{Sh}_D\,\frac{D_{AB}}{D} = 16.7\frac{0.26\times10^{-4}\text{ m}^2/\text{s}}{3\times10^{-3}\text{ m}} = 0.145\text{ m/s.}$$

Hence,

$$n_A = 0.145\text{ m/s}\;\pi\left(3\times10^{-3}\text{ m}\right)^2 \times 0.023\text{ kg/m}^3 = 9.43\times10^{-8}\text{ kg/s.} \qquad <$$

COMMENTS: For the small difference between T_s and T_∞, it is reasonable to neglect the viscosity ratio in Eq. 7.58. Use of Eq. 7.59 gives $\overline{h}_m$ = 0.152 m/s, which is in good agreement with the result from Eq. 7.58.

PROBLEM 7.134

KNOWN: Humidity and temperature of air entering heater; temperature of air leaving heater. Diameter, temperature and relative velocity of injected droplets.

FIND: Droplet evaporation rate.

SCHEMATIC:

Water droplet
D = 1 mm
V = 15 m/s
T_s = 320 K

Air
T_i = 290 K
ϕ_i = 0.7
Heater
$\rho_{A,\infty}$, T_∞ = 320 K
T_o = 320 K

ASSUMPTIONS: (1) Negligible change in droplet diameter due to evaporation, (2) Negligible cooling of droplet due to evaporation, (3) Applicability of heat/mass transfer analogy, (4) Ideal gas behavior for vapor.

PROPERTIES: *Table A.4*, air ($T_\infty = T_o$ = 320 K): $\nu = 17.90 \times 10^{-6}$ m^2/s, k = 0.0278 W/m·K, Pr = 0.705. *Table A.6*, saturated water (T_i = 290 K): p_{sat} = 0.01917 bars; (T_o = 320 K): p_{sat} = 0.1053 bars, v_g = 13.98 m^3/kg. *Table A.8*, H_2O/air (T = 320 K): $D_{AB} = 0.26 \times 10^{-4}$ m^2/s $(320/298)^{3/2} = 0.289 \times 10^{-4}$ m^2/s.

ANALYSIS: Due to an increase in temperature, the air leaves the heater with a smaller relative humidity. With ϕ_i = 0.7 and $p_{sat,i}$ = 0.01917 bars, the vapor pressure at the heater inlet is $p_i = \phi_i\, p_{sat,i}$ = 0.7(0.01917 bars) = 0.0134 bars. Since the vapor pressure doesn't change with passage through the heater,

$$\phi_o = \frac{p_i}{p_{sat,o}} = \frac{0.0134\,\text{bars}}{0.1053\,\text{bars}} = 0.127$$

The vapor density associated with air flow around the droplets is therefore

$$\rho_{A,\infty} = \phi_o \rho_{A,sat}(T_o) = \phi_o v_g(T_o)^{-1} = 0.127 \times 0.0715\,\text{kg/m}^3 = 0.0091\,\text{kg/m}^3$$

The droplet evaporation rate is

$$\dot{m}_{evap} = \overline{h}_m A_s \left[\rho_{A,sat}(T_s) - \rho_{A,\infty}\right]$$

where $\overline{h}_m$ may be obtained from the mass transfer analog to the Whitaker correlation. With Re_D = VD/ν = 15 m/s × 0.001 m/17.9 × 10^{-6} m^2/s = 838, Sc = ν/D_{AB} = 17.9 × 10^{-6} m^2/s/28.9 × 10^{-6} m^2/s = 0.62, and μ/μ_s = 1,

$$\overline{Sh}_D = 2 + \left(0.4\,Re_D^{1/2} + 0.06\,Re_D^{2/3}\right)Sc^{0.4} = 2 + \left[0.4(838)^{1/2} + 0.06(838)^{2/3}\right](0.62)^{0.4} = 16.0$$

$$\overline{h}_m = \overline{Sh}_D\,(D_{AB}/D) = 16\left(0.289 \times 10^{-4}\,\text{m}^2/\text{s}/0.001\,\text{m}\right) = 0.462\,\text{m/s}$$

$$\dot{m}_{evap} = (0.462\,\text{m/s})\pi(0.001\,\text{m})^2(0.0715 - 0.0091)\,\text{kg/m}^3 = 9.06 \times 10^{-8}\,\text{kg/s} \qquad <$$

COMMENTS: The energy required for evaporation must be supplied by convection heat transfer from the heated air to the droplet. Hence, in actuality, the droplet temperature T_s must be less than that of the freestream air, T_∞, which in turn will decrease from the value T_o at the heater outlet.

PROBLEM 7.135

KNOWN: Diameter and temperature of sphere wetted with kerosene. Air flow conditions.

FIND: (a) Minimum kerosene flow rate, (b) Air temperature required to maintain wetted surface at 300K.

SCHEMATIC:

Air, B
$V = 15\ m/s$
$T_\infty = 300K$
$D = 1\ mm$
Kerosene (A), $T_s = 300K$
q_{evap}
q_{conv}

ASSUMPTIONS: (1) Steady-state conditions, (2) Sphere mount has a negligible influence on the flow field and hence on $\bar{h}$, (3) Negligible kerosene vapor concentration in free stream.

PROPERTIES: *Table A-4*, Air (300K): $\nu = 15.89 \times 10^{-6}\ m^2/s$, k = 0.0263 W/m·K, ρ = 1.161 kg/m^3, Pr = 0.707; Kerosene (given): $\rho_{A,sat} = 0.015\ kg/m^3$, h_{fg} = 300 kJ/kg; Kerosene vapor-air (given): $D_{AB} = 10^{-5}\ m^2/s$.

ANALYSIS: (a) The kerosene flowrate is $n_A = \bar{h}_m A\left(\rho_{A,sat} - \rho_{A,\infty}\right)$. Using the mass transfer analog of Eq. 7.58 and neglecting the viscosity ratio,

$$\overline{Sh}_D = 2 + \left(0.4\ Re_D^{1/2} + 0.06\ Re_D^{2/3}\right)\ Sc^{0.4}$$

with $$Re_D = \frac{VD}{\nu} = \frac{15\ m/s \times 0.001\ m}{15.89 \times 10^{-6}\ m^2/s} = 944 \qquad Sc = \frac{\nu}{D_{AB}} = \frac{15.89 \times 10^{-6}}{10 \times 10^{-6}} = 1.59$$

$$\overline{Sh}_D = 2 + \left(0.4 \times 944^{1/2} + 0.06944^{2/3}\right)\ (1.59)^{0.4} = 23.7$$

$$\bar{h}_m = \overline{Sh}_D\ D_{AB}/D = 23.7 \times 10^{-5}\ m^2/s/0.001\ m = 0.237\ m/s$$

$$n_A = 0.237\ m/s\ \pi\left(10^{-3}\ m\right)^2\ 0.015\ kg/m^3 = 1.12 \times 10^{-8}\ kg/s. \qquad <$$

(b) An energy balance on the sphere yields $n_A\ h_{fg} = \bar{h}A\ \left(T_\infty - T_s\right)$. Using the Whitaker correlation and neglecting the viscosity ratio,

$$\overline{Nu}_D = 2 + \left(0.4 \times 944^{1/2} + 0.06 \times 944^{2/3}\right)\ (0.707)^{0.4} = 17.72$$

$$\bar{h} = \overline{Nu}_D k/D = 17.72 \times 0.0263\ W/m \cdot K/0.001\ m = 466\ W/m^2 \cdot K$$

$$T_\infty = T_s + \frac{n_A\ h_{fg}}{\bar{h}\pi D^2} = 300K + \frac{1.12 \times 10^{-8}\ kg/s \times 3 \times 10^5\ J/kg}{466\ W/m^2 \cdot K \times \pi\ (0.001\ m)^2}$$

$$T_\infty = 300K + 2.3K = 302.3K \qquad <$$

or $$T_\infty - T_s = 2.3K.$$

COMMENTS: The small temperature excess (2.3K) is due to comparatively small values of $\rho_{A,sat}$ and h_{fg} for kerosene.

PROBLEM 7.136

KNOWN: Geometry and surface temperature of a tube bank with or without wetted surfaces. Temperature, velocity and flowrate associated with air in cross flow.

FIND: (a) Ratio of air cooling with water film to that without film, (b) Air outlet temperature and specific humidity for prescribed conditions.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Heat and mass transfer analogy is applicable, (3) Air is dry, (4) Heat and mass transfer driving potentials are $T_{a,i} - T_s$ and $\rho_{A,sat}(T_s)$, (5) Vapor has negligible effect on flowrate.

PROPERTIES: *Table A-4*, Air (assume $\overline{T}_a \approx 305K$): $\rho = 1.1448\ kg/m^3$, $c_p = 1007\ J/kg \cdot K$, $\nu = 16.39 \times 10^{-6}\ m^2/s$, $k = 0.0267\ W/m \cdot K$, $Pr = 0.706$, $\alpha = 23.2 \times 10^{-6}\ m^2/s$; *Table A-6*, Water vapor ($T_s = 10°C$): $v_g = 111.8\ m^3/kg$, $\rho_{A,sat} = 8.94 \times 10^{-3}\ kg/m^3$, $h_{fg} = 2.478 \times 10^6\ J/kg$; *Table A-8*, Water vapor-air ($T_f \approx 298K$): $D_{AB} = 0.26 \times 10^{-4}\ m^2/s$, $Sc = (\nu/D_{AB}) = 0.642$.

ANALYSIS: (a) The rate of heat loss from the air may be expressed as

$$q = \dot{m}_a c_{p,a} \left(T_{a,i} - T_{a,o}\right)$$

in which case, the amount of air cooling is

$$\left(T_{a,i} - T_{a,o}\right) = \frac{q}{\dot{m}_a c_{p,a}}. \qquad (1)$$

Without the water film, $q_{wo} \approx \overline{h}A\left(T_{a,i} - T_s\right)$ (2)

With the film,

$$q_w \approx \overline{h}A\left(T_{a,i} - T_s\right) + \dot{m}_{evap}\, h_{fg}$$

$$q_w \approx \overline{h}A\left(T_{a,i} - T_s\right) + \overline{h}_m A\left(\rho_{A,sat} - \rho_{A,\infty}\right) h_{fg} \qquad (3)$$

where $\rho_{A,\infty} = 0$. Hence

$$\frac{\left(T_{a,i} - T_{a,o}\right)_w}{\left(T_{a,i} - T_{a,o}\right)_{wo}} \approx 1 + \frac{\overline{h}_m \rho_{A,sat} h_{fg}}{\overline{h}\left(T_{a,i} - T_s\right)}$$

or substituting from Eq. 6.92, with $Le = \alpha/D_{AB}$ and a value of $n = 0.33$,

$$\frac{\left(T_{a,i} - T_{a,o}\right)_w}{\left(T_{a,i} - T_{a,o}\right)_{wo}} \approx 1 + \frac{\left(D_{AB}/\alpha\right)^{0.67}}{\rho\ c_p} \frac{\rho_{A,sat}\ h_{fg}}{\left(T_{a,i} - T_s\right)}$$ <

Continued …..

PROBLEM 7.136 (Cont.)

For the prescribed conditions,

$$\frac{\left(T_{a,i}-T_{a,o}\right)_{w}}{\left(T_{a,i}-T_{a,o}\right)_{wo}} \approx 1+\frac{\left(\frac{0.26\times10^{-4}m^{2}/s}{0.232\times10^{-4}m^{2}/s}\right)^{0.67}}{1.145\text{ kg/m}^{3}\times1007\text{ J/kg}\cdot K}\times\frac{8.94\times10^{-3}\text{kg/m}^{3}\times2.478\times10^{6}\text{J/kg}}{(35-10)^{\circ}C}\approx 1.83.$$ <

(b) $T_{a,o}$ may be obtained from Eq. (1), where q is approximated by Eq. (2) or Eq. (3). With S_D = 26.83 mm > (S_T + D)/2 = 16, V_{max} is at the transverse plane. Hence

$$V_{max}=\frac{S_T}{S_T-D}V=\frac{24}{16}\times3\text{m/s}=4.5\text{m/s} \qquad Re_{D,max}=\frac{4.5\text{ m/s}\times0.008\text{ m}}{16.39\times10^{-6}\text{ m}^{2}/s}=2196.$$

From Tables 7.7 and 7.8, C = 0.35, m = 0.60, C_2 = 0.98 and the Zhukauskas relation gives

$$\overline{Nu}_D=0.35(0.98)(2196)^{0.6}(0.706)^{0.36}=30.6$$

where$(Pr/Pr_s)^{1/4}$ is 1.00. Hence

$$\overline{h}=\overline{Nu}_D\ k/D=30.6(0.0267\text{ W/m}\cdot K)/0.008\text{ m}=102\text{ W/m}^{2}\cdot K.$$

Also $$\overline{h}_m=\overline{h}\frac{(D_{AB}/a)^{0.67}}{r\ c_p}=102\frac{W}{m^{2}\cdot K}\frac{(0.26/0.232)^{0.67}}{1.145\text{ kg/m}^{3}\times1007\text{ J/kg}\cdot K}=0.0956\text{ m/s}.$$

Hence

$$q_{conv}\approx\overline{h}A\left(T_{a,i}-T_s\right)=102\text{W/m}^{2}\cdot K\times p\ (0.008\text{ m})0.5\text{ m}\times60(35-10)^{\circ}C=1923\text{ W}$$

$$q_{evap}=n_A h_{fg}=\overline{h}_m A\ r_{A,sat}\ h_{fg}$$

$$q_{evap}=0.0956\text{ m/s}\times p\ (0.008\text{ m})0.5\text{ m}\times60\left(8.94\times10^{-3}\text{kg/m}^{3}\right)2.478\times10^{6}\text{ J/kg}$$

$$q_{evap}\approx1597W.$$

With water film,

$$T_{a,o}=T_{a,i}-\frac{q_{conv}+q_{evap}}{\dot{m}_a\ c_{p,a}}\approx35^{\circ}C-\frac{(1923+1597)W}{0.5\text{ kg/s}\times1007\text{ J/kg}\cdot K}=28.0^{\circ}C.$$ <

The specific humidity of the outlet air is

$$w_o=\frac{n_A}{\dot{m}_a}=\frac{\overline{h}_m 60p\,DL\ r_{A,sat}}{\dot{m}_a}$$

$$w_o=\frac{0.0956\text{ m/s}(60p)\ (0.008\text{ m})0.5\text{ m}\left(8.94\times10^{-3}\text{kg/m}^{3}\right)}{0.5\text{ kg/s}}=0.00129.$$ <

COMMENTS: (1) Enhancement of air cooling by evaporation is significant ($T_{a,o}$ = Ta,i $-q_{conv}/\dot{m}_a\ c_{p,a}\approx31.1^{\circ}C$ without the film). (2) Small value of ω_o justifies neglecting effect of evaporation on $\dot{m}_a$. (3) q_{conv} has been overestimated by using ($T_{a,i}$ - T_s) as the driving potential for convection heat transfer. A more accurate determination involves $\Delta T_{\ell m}$ rather than ($T_{a,i}$ - T_s). (4) Apparently the air properties were evaluated at an appropriate $\overline{T}_a$.

PROBLEM 7.137

KNOWN: Dimensions of slot jet array. Jet exit velocity and temperature. Temperature of paper.

FIND: Drying rate per unit surface area.

SCHEMATIC:

ASSUMPTIONS: (1) Applicability of heat and mass transfer analogy, (2) Paper motion has negligible effect on convection ($U << V_e$).

PROPERTIES: *Table A-4*, Air (300 K, 1 atm): $\nu = 15.89 \times 10^{-6}$ m^2/s; *Table A-6*, Saturated water (300 K): $\rho_{A,sat} = v_g^{-1} = 0.0256$ kg/m^3; *Table A-8*, Water vapor-air (300 K): $D_{AB} = 0.26 \times 10^{-4}$ m^2/s, Sc = 0.61.

ANALYSIS: The mass evaporation flux is

$$n''_A = \overline{h}_m \left(r_{A,s} - r_{A,e} \right) = \overline{h}_m r_{A,sat}$$

For an array of slot nozzles,

$$\frac{\overline{Sh}}{Sc^{0.42}} = \frac{2}{3} A_{r,o}^{3/4} \left(\frac{2Re}{A_r / A_{r,o} + A_{r,o} / A_r} \right)^{2/3}$$

where

$$A_r = W/S = 0.1$$

$$A_{r,o} = \left\{ 60 + 4\left[\left(H/2W \right) - 2 \right]^2 \right\}^{1/2} = \left\{ 60 + 4(64) \right\}^{-1/2} = 0.0563$$

$$Re = \frac{V_e (2W)}{n} = \frac{20 \text{ m/s} (0.02 \text{ m})}{15.89 \times 10^{-6} \text{m}^2/\text{s}} = 25{,}173.$$

Hence

$$\frac{\overline{Sh}}{Sc^{0.42}} = 0.667 (0.0563)^{3/4} \left(\frac{50{,}346}{1.776 + 0.563} \right)^{2/3} = 59.6$$

$$\overline{h}_m = \frac{D_{AB}}{2W} 59.6 Sc^{0.42} = \frac{0.26 \times 10^{-4} \text{ m}^2/\text{s}}{0.02 \text{ m}} 59.6 (0.61)^{0.42} = 0.063 \text{m/s}.$$

The evaporative flux is then

$$n''_A = 0.063 \text{ m/s} \left(0.0256 \text{ kg/m}^3 \right) = 0.0016 \text{ kg/s} \cdot \text{m}^2.$$ <

COMMENTS: The mass fraction of water vapor to air leaving the sides of the dryer is $n''_A \left(S \times L \right) / r_{air} V_e \left(W \times L \right) = 7 \times 10^{-4}$. Hence, the assumption of dry air throughout the dryer is reasonable.

PROBLEM 8.1

KNOWN: Flowrate and temperature of water in fully developed flow through a tube of prescribed diameter.

FIND: Maximum velocity and pressure gradient.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Isothermal flow.

PROPERTIES: *Table A-6*, Water (300K): $\rho = 998\ \text{kg/m}^3$, $\mu = 855 \times 10^{-6}\ \text{N·s/m}^2$.

ANALYSIS: From Eq. 8.6,

$$\text{Re}_D = \frac{4\dot{m}}{\pi D \mu} = \frac{4 \times 0.01\ \text{kg/s}}{\pi\,(0.025\text{m})\ 855 \times 10^{-6}\ \text{kg}\cdot\text{m/s}} = 596.$$

Hence the flow is laminar and the velocity profile is given by Eq. 8.15,

$$\frac{u(r)}{u_m} = 2\left[1 - (r/r_o)^2\right].$$

The maximum velocity is therefore at r = 0, the centerline, where

$$u(0) = 2\ u_m.$$

From Eq. 8.5

$$u_m = \frac{\dot{m}}{\rho\pi\ D^2/4} = \frac{4 \times 0.01\ \text{kg/s}}{998\ \text{kg/m}^3 \times \pi\ (0.025\text{m})^2} = 0.020\ \text{m/s},$$

hence

$$u(0) = 0.041\ \text{m/s}.$$

Combining Eqs. 8.16 and 8.19, the pressure gradient is

$$\frac{dp}{dx} = -\frac{64}{\text{Re}_D}\ \frac{\rho u_m^2}{2D}$$

$$\frac{dp}{dx} = -\frac{64}{596} \times \frac{998\ \text{kg/m}^3\ (0.020\ \text{m/s})^2}{2 \times 0.025\ \text{m}} = -0.86\ \text{kg/m}^2 \cdot \text{s}^2$$

$$\frac{dp}{dx} = -0.86 \text{N/m}^2 \cdot \text{m} = -0.86 \times 10^{-5}\ \text{bar/m}.$$ <

PROBLEM 8.2

KNOWN: Temperature and mean velocity of water flow through a cast iron pipe of prescribed length and diameter.

FIND: Pressure drop.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Fully developed flow, (3) Constant properties.

PROPERTIES: *Table A-6*, Water (300K): $\rho = 997$ kg/m^3, $\mu = 855 \times 10^{-6}$ N·s/m^2.

ANALYSIS: From Eq. 8.22, the pressure drop is

$$\Delta p = f \frac{\rho u_m^2}{2D} L.$$

With

$$Re_D = \frac{\rho u_m D}{\mu} = \frac{997 \text{ kg/m}^3 \times 0.2 \text{ m/s} \times 0.15 \text{ m}}{855 \times 10^{-6} \text{ N} \cdot \text{s/m}^2} = 3.50 \times 10^4$$

the flow is turbulent and with $e = 2.6 \times 10^{-4}$ m for cast iron (see Fig. 8.3), it follows that $e/D = 1.73 \times 10^{-3}$ and

$$f \approx 0.027.$$

Hence,

$$\Delta p = 0.027 \frac{997 \text{ kg/m}^3 (0.2 \text{ m/s})^2}{2 \times 0.15 \text{ m}} (600\text{m})$$

$$\Delta p = 2154 \text{ kg/s}^2 \cdot \text{m} = 2154 \text{ N/m}^2$$

$$\Delta p = 0.0215 \text{ bar}.$$ <

COMMENTS: For the prescribed geometry, $L/D = (600/0.15) = 4000 >> (x_{fd,h}/D)_{turb} \approx 10$, and the assumption of fully developed flow throughout the pipe is justified.

PROBLEM 8.3

KNOWN: Temperature and velocity of water flow in a pipe of prescribed dimensions.

FIND: Pressure drop and pump power requirement for (a) a smooth pipe, (b) a cast iron pipe with a clean surface, and (c) smooth pipe for a range of mean velocities 0.05 to 1.5 m/s.

SCHEMATIC:

Water
$u_m = 1$ m/s
$T_m = 300$ K
$L = 1000$ m
Cast iron pipe, $D = 0.25$ m

ASSUMPTIONS: (1) Steady, fully developed flow.

PROPERTIES: *Table A.6*, Water (300 K): $\rho = 997$ kg/m^3, $\mu = 855 \times 10^{-6}$ N·s/m^2, $\nu = \mu/\rho = 8.576 \times 10^{-7}$ m^2/s.

ANALYSIS: From Eq. 8.22a and 8.22b, the pressure drop and pump power requirement are

$$\Delta p = f\,\frac{\rho u_m^2}{2D}L \qquad\qquad P = \Delta p\dot{V} = \Delta p\left(\pi D^2/4\right)u_m \qquad (1,2)$$

The friction factor, f, may be determined from Figure 8.3 for different relative roughness, e/D, surfaces or from Eq. 8.21 for the smooth condition, $3000 \le Re_D \le 5 \times 10^6$,

$$f = \left(0.790\ln\left(Re_D\right) - 1.64\right)^{-2} \qquad (3)$$

where the Reynolds number is

$$Re_D = \frac{u_m D}{\nu} = \frac{1\,\text{m/s}\times 0.25\,\text{m}}{8.576\times 10^{-7}\,\text{m}^2/\text{s}} = 2.915\times 10^5 \qquad (4)$$

(a) *Smooth surface:* from Eqs. (3), (1) and (2),

$$f = \left(0.790\ln\left(2.915\times 10^5\right) - 1.64\right)^{-2} = 0.01451$$

$$\Delta p = 0.01451\left(997\,\text{kg/m}^3 \times 1\,\text{m}^2/\text{s}^2 / 2\times 0.25\,\text{m}\right)1000\,\text{m} = 2.89\times 10^4\,\text{kg/s}^2\cdot\text{m} = 0.289\,\text{bar} \quad <$$

$$P = 2.89\times 10^4\,\text{N/m}^2\left(\pi\times 0.25^2\,\text{m}^2/4\right)1\,\text{m/s} = 1418\,\text{N}\cdot\text{m/s} = 1.42\,\text{kW} \quad <$$

(b) *Cast iron clean surface:* with e = 260 μm, the relative roughness is $e/D = 260 \times 10^{-6}$ m/0.25 m = 1.04×10^{-3}. From Figure 8.3 with $Re_D = 2.92 \times 10^5$, find f = 0.021. Hence,

$\Delta p = 0.419$ bar $\qquad\qquad P = 2.06$ kW $\quad <$

(c) *Smooth surface:* Using IHT with the expressions of part (a), the pressure drop and pump power requirement as a function of mean velocity, u_m, for the range $0.05 \le u_m \le 1.5$ m/s are computed and plotted below.

Continued...

PROBLEM 8.3 (Cont.)

deltap (bar) or P (kW)

Mean velocity, um (m/s)

Pressure drop, deltap (bar)
Pump power, P (kW)

The pressure drop is a strong function of the mean velocity. So is the pump power since it is proportional to both Δp and the mean velocity.

COMMENTS: (1) Note that $L/D = 4000 >> (x_{fg,h}/D) \approx 10$ for turbulent flow and the assumption of fully developed conditions is justified.

(2) Surface fouling results in increased surface roughness and increases operating costs through increasing pump power requirements.

(3) The *IHT Workspace* used to generate the graphical results follows.

```
// Pressure drop:
deltap = f * rho * um^2 * L / ( 2 * D )          // Eq (1); Eq 8.22a
deltap_bar = deltap / 1.00e5                     // Conversion, Pa to bar units
Power = deltap * ( pi * D^2 / 4 ) * um           // Eq (2); Eq 8.22b
Power_kW = Power / 1000                          // Useful for scaling graphical result

// Reynolds number and friction factor:
ReD = um * D / nu                                // Eq (3)
f =  (0.790 * ln (ReD) - 1.64 ) ^ (-2)           // Eq (4); Eq 8.21, smooth surface condition

// Properties Tool - Water:
// Water property functions :T dependence, From Table A.6
// Units: T(K), p(bars);
x = 0                          // Quality (0=sat liquid or 1=sat vapor)
rho = rho_Tx("Water",Tm,x)     // Density, kg/m^3
nu = nu_Tx("Water",Tm,x)       // Kinematic viscosity, m^2/s

// Assigned variables:
um = 1                         // Mean velocity, m/s
Tm = 300                       // Mean temperature, K
D = 0.25                       // Tube diameter, m
L = 1000                       // Tube length, m
```

PROBLEM 8.4

KNOWN: Temperature and mass flow rate of various liquids moving through a tube of prescribed diameter.

FIND: Mean velocity and hydrodynamic and thermal entry lengths.

SCHEMATIC:

ASSUMPTIONS: Constant properties.

PROPERTIES: (T = 300K)

Liquid	*Table*	*r(kg/m3)*	*m(N·s/m2)*	*n(m2/s)*	*Pr*
Engine oil	A-5	884	0.486	550×10^{-6}	6400
Mercury	A-5	13,529	0.152×10^{-2}	0.113×10^{-6}	0.0248
Water	A-6	1000	0.855×10^{-3}	0.855×10^{-6}	5.83

ANALYSIS: The mean velocity is given by

$$u_m = \frac{\dot{m}}{r A_c} = \frac{0.03\ \text{kg/s}}{r p\,(0.025\text{m})^2 / 4} = \frac{61.1\ \text{kg/s}\cdot\text{m}^2}{r}.$$

The hydrodynamic and thermal entry lengths depend on Re_D,

$$Re_D = \frac{4\dot{m}}{p\ D m} = \frac{4\times 0.03\ \text{kg/s}}{p\,(0.025\text{m})\, m} = \frac{1.53\ \text{kg/s}\cdot\text{m}}{m}.$$

Hence, even for water ($\mu = 0.855 \times 10^{-3}$ N·s/m^2), $Re_D < 2300$ and the flow is laminar. From Eqs. 8.3 and 8.23 it follows that

$$x_{fd,h} = 0.05\ D\ Re_D = \frac{1.91\times 10^{-3}\ \text{kg/s}}{m}$$

$$x_{fd,t} = 0.05\ D\ Re_D\ Pr = \frac{\left(1.91\times 10^{-3}\ \text{kg/s}\right) Pr}{m}.$$

Hence:

Liquid	u_m*(m/s)*	$x_{fd,h}$*(m)*	$x_{fd,t}$*(m)*
Oil	0.069	0.0039	25.2
Mercury	0.0045	1.257	0.031
Water	0.061	2.234	13.02

COMMENTS: Note the effect of viscosity on the hydrodynamic entry length and the effect of Pr on the thermal entry length.

PROBLEM 8.5

KNOWN: Number, diameter and length of tubes and flow rate for an engine oil cooler.

FIND: Pressure drop and pump power (a) for flow rate of 24 kg/s and (b) as a function of flow rate for the range $10 \le \dot{m} \le 30$ kg/s.

SCHEMATIC:

ASSUMPTIONS: (1) Fully developed flow throughout the tubes.

PROPERTIES: *Table A.5*, Engine oil (300 K): $\rho = 884$ kg/m^3, $\mu = 0.486$ kg/s·m.

ANALYSIS: (a) Considering flow through a single tube, find

$$\mathrm{Re}_D = \frac{4\dot{m}}{\pi D \mu} = \frac{4(24\,\mathrm{kg/s})}{25\pi(0.010\,\mathrm{m})0.486\,\mathrm{kg/s\cdot m}} = 251.5 \qquad (1)$$

Hence, the flow is laminar and from Equation 8.19,

$$f = \frac{64}{\mathrm{Re}_D} = \frac{64}{251.5} = 0.2545. \qquad (2)$$

With

$$u_m = \frac{\dot{m}_1}{\rho\left(\pi D^2/4\right)} = \frac{(25/25)\,\mathrm{kg/s}\,(4)}{\left(884\,\mathrm{kg/m^3}\right)\pi(0.010\,\mathrm{m})^2} = 13.8\ \mathrm{m/s} \qquad (3)$$

Equation 8.22a yields

$$\Delta p = f\,\frac{\rho u_m^2}{2D}L = 0.2545\,\frac{\left(884\,\mathrm{kg/m^3}\right)(13.8\,\mathrm{m/s})^2}{2(0.010\,\mathrm{m})}2.5\,\mathrm{m} = 5.38\times10^6\ \mathrm{N/m^2} = 53.8\ \mathrm{bar} \qquad (4)$$ <

The pump power requirement from Equation 8.23b,

$$P = \Delta p\cdot\dot{V} = \Delta p\cdot\frac{\dot{m}}{\rho} = 5.38\times10^6\ \mathrm{N/m^2}\ \frac{24\,\mathrm{kg/s}}{884\,\mathrm{kg/m^3}} = 1.459\times10^5\ \mathrm{N\cdot m/s} = 146\ \mathrm{W}. \qquad (5)$$ <

(b) Using IHT with the expressions of part (a), the pressure drop and pump power requirement as a function of flow rate, $\dot{m}$, for the range $10 \le \dot{m} \le 30$ kg/s are computed and plotted below.

Continued...

PROBLEM 8.5 (Cont.)

In the plot above, note that the pressure drop is linear with the flow rate since, from Eqs. (2), the friction factor is inversely dependent upon mean velocity. The pump power, however, is quadratic with the flow rate.

COMMENTS: (1) If there is a hydrodynamic entry region, the average friction factor for the entire tube length would exceed the fully developed value, thereby increasing Δp and P.

(2) The *IHT Workspace* used to generate the graphical results follows.

```
/* Results: base case, part (a)
P_kW          ReD       deltap_bar          f         mu        rho       um        D         N
        mdot
145.9         251.5     53.75               0.2545    0.486     884.1     13.83     0.01      25
        24               */

// Reynolds number and friction factor
ReD = 4 * mdot1 / (pi * D * mu)       // Reynolds number, Eq (1)
f = 64 / ReD                                  // Friction factor, laminar flow, Eq. 8.19, Eq. (2)

// Average velocity and flow rate
mdot1 = rho * Ac * um          // Flow rate, kg/s; single tube
mdot = mdot1 * N               // Total flow rate, kg/s; N tubes
Ac = pi * D^2 / 4              // Tube cross-sectional area, m^2

// Pressure drop and power
deltap = f * rho * um^2 * L / (2 * D)          // Pressure drop, N/m^2
deltap_bar = deltap * 1e-5           // Pressure drop, bar
P = deltap * mdot / rho                        // Power, W
P_kW = P / 1000                      // Power, kW

// Input variables
D = 0.01              // Diameter, m
mdot = 24             // Total flow rate, kg/s
L = 2.5               // Tube length, m
N = 25                // Number of tubes
Tm = 300              // Mean temperature of oil, K

// Engine Oil property functions : From Table A.5
rho = rho_T("Engine Oil",Tm)         // Density, kg/m^3
mu = mu_T("Engine Oil",Tm)           // Viscosity,  N·s/m^2
```

PROBLEM 8.6

KNOWN: The x-momentum equation for fully developed laminar flow in a parallel-plate channel

$$\frac{dP}{dx} = constant = \mu \frac{d^2 u}{dy^2}$$

FIND: Following the same approach as for the circular tube in Section 8.1: (a) Show that the velocity profile, *u(y)*, is parabolic of the form

$$u(y) = \frac{3}{2} u_m \left[1 - \frac{y^2}{(a/2)^2} \right]$$

where u_m is the mean velocity expressed as

$$u_m = \frac{a^2}{12\mu} \left(-\frac{dP}{dx} \right)$$

and $-dp/dx = \Delta p/L$ where Δp is the pressure drop across the channel of length L; (b) Write the expression defining the friction factor, f, using the hydraulic diameter as the characteristic length, D_h; What is the hydraulic diameter for the parallel-plate channel? (c) The friction factor is estimated from the expression $f = C / Re_{D_h}$ where C depends upon the flow cross-section as shown in Table 8.1; What is the coefficient C for the parallel-plate channel ($b/a \to \infty$) ? (d) Calculate the mean air velocity and the Reynolds number for air at atmospheric pressure and 300 K in a parallel-plate channel with separation of 5 mm and length of 100 mm subjected to a pressure drop of $\Delta P = 3.75\ N/m^2$; Is the assumption of fully developed flow reasonable for this application? If not, what effect does this have on the estimate for u_m?

SCHEMATIC:

Fluid
u_m
Δp
y
+a/2
-a/2
x
L

Parallel plate channel
a = 5 mm
$\Delta p = 3.75\ N/m^2$
L = 100 mm
T = 300 K

ASSUMPTIONS: (1) Fully developed laminar flow, (2) Parallel-plate channel, a << b.

PROPERTIES: *Table A-4*, Air (300 K, 1 atm): $\mu = 184.6 \times 10^{-7}\ N \cdot s/m^2$, $\nu = 15.89 \times 10^{-6}\ m^2/s$.

ANALYSIS: (a) The x-momentum equation for fully developed laminar flow is

$$\mu \left(\frac{d^2 u}{dy^2} \right) = \frac{dp}{dx} = \text{constant} \qquad (1)$$

Since the longitudinal pressure gradient is constant, separate variables and integrate twice,

$$\frac{d}{dy}\left(\frac{du}{dy}\right) = \frac{1}{\mu}\left(\frac{dp}{dx}\right) \qquad \frac{du}{dy} = \frac{1}{\mu}\left(\frac{dp}{dx}\right) y + C_1$$

$$u = \frac{1}{2\mu}\left(\frac{dp}{dx}\right) y^2 + C_1 y + C_2$$

Continued …..

PROBLEM 8.6 (Cont.)

The integration constants are determined from the boundary conditions,

$$\left.\frac{du}{dy}\right|_{y=0} = 0 \qquad u(a/2) = 0$$

to find

$$C_1 = 0 \qquad C_2 = -\frac{1}{2\mu}\left(\frac{dp}{dx}\right)(a/2)^2$$

giving

$$u(y) = -\frac{(a/2)^2}{2\mu}\left(\frac{dp}{dx}\right)\left[1 - \frac{y^2}{(a/2)^2}\right] \qquad (2)$$

The mean velocity is

$$u_m = \frac{2}{a}\int_0^{a/2} u(y)\,dy = -\frac{2}{a}\frac{(a/2)^2}{2\mu}\left(\frac{dp}{dx}\right)\left[y - \frac{y^3/3}{(a/2)^2}\right]_0^{a/2}$$

$$u_m = \frac{a^2}{12\mu}\left(-\frac{dp}{dx}\right) \qquad (3)$$

Substituting Eq. (3) for dp/dx into Eq. (2) find the velocity distribution in terms of the mean velocity

$$u(y) = \frac{3}{2}u_m\left[1 - \frac{y^2}{(a/2)^2}\right] \qquad < \quad (4)$$

(b) The friction factor follows from its definition, Eq. 8.16,

$$f = \frac{-(dp/dx)D_h}{\rho \cdot u_m^2/2} \qquad (5)$$

where the hydraulic diameter for the channel using Eq. 8.67 is

$$D_h = \frac{4 \cdot A_c}{P} = \frac{4(a \times b)}{2(a+b)} = 2a \qquad < \quad (6)$$

since a << b.

(c) Substituting for the pressure gradient, Eq. (3), and rearranging, find using Eq. (6),

$$f = \frac{u_m}{a^2/12\mu}\frac{D_h}{\rho u_m^2/2} = \frac{96}{u_m D_h/\nu} = \frac{96}{Re_{D_h}} \qquad < \quad (7)$$

where the Reynolds number is

$$Re_{D_h} = u_m D_h/\nu \qquad (8)$$

Continued …..

PROBLEM 8.6 (Cont.)

This result is in agreement with Table 8.1 for the cross-section with b/a $\rightarrow \infty$ where

C = 96. <

(d) For the conditions shown in the schematic, with air properties evaluated at 300 K, using Eqs. (3) and (8), find

$$u_m = \frac{(0.005\ \text{m})^2}{12 \times 184.6 \times 10^{-7}\ \text{N} \cdot \text{s/m}^2}\left(\frac{3.75\ \text{N/m}^2}{0.100\ \text{m}}\right) = 1.06\ \text{m/s}$$

$$Re_D = \frac{1.06\ \text{m/s} \times 2 \times 0.005\ \text{m}}{15.89 \times 10^{-6}\ \text{m}^2/\text{s}} = 667$$

The flow is laminar as $Re_{D_h} < 2300,$ and from Eq. 8.3, the entry length is

$$\left(\frac{x_{fd,h}}{D_h}\right)_{\ell am} = 0.05\ Re_{D_h}$$

$$x_{fd,h} = 2 \times 0.005\,\text{m} \times 0.05 \times 667 = 0.334\,\text{m} = 334\ \text{mm}$$

We conclude that the flow is not fully developed, and the friction factor in the entry region will be higher than for fully developed conditions. Hence, for the same pressure drop, the mean velocity will be less than our estimate.

PROBLEM 8.7

KNOWN: Mean velocity and temperature of oil, water and mercury flowing through a tube of prescribed diameter.

FIND: Corresponding hydrodynamic and thermal entry lengths.

SCHEMATIC:

Oil, water, or mercury
$u_m = 5 \times 10^{-3}$ m/s
$T_m = 27°C$
D=0.025m

ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties.

PROPERTIES: (T_m = 300K)

Liquid	*Table*	$\rho (kg/m^3)$	$\mu (N \cdot s/m^2)$	*Pr*
Engine Oil	A-5	884	0.486	6400
Mercury	A-5	13,529	0.152×10^{-2}	0.0248
Water	A-6	997	0.855×10^{-3}	5.83

ANALYSIS: With

$$Re_D = \frac{\rho\, u_m D}{\mu} = \frac{\rho}{\mu} \times 5 \times 10^{-3} m/s \times 0.025 m = 1.25 \times 10^{-4}\, m^2/s \frac{\rho}{\mu}$$

It follows that

	Oil	*Mercury*	*Water*
Re_D	0.227	1113	146

Hence for each fluid, the flow is laminar and from Eqs. 8.3 and 8.23,

$$x_{fd,h} = 0.05\, Re_d \qquad x_{fd,t} = 0.5\, D\, Re_D\, Pr.$$

Hence:

Liquid	$x_{fd,h} (m)$	$x_{fd,t} (m)$
Oil	2.84×10^{-4}	1.82
Mercury	1.39	0.0345
Water	0.183	1.06

<

COMMENTS: Note the effect of viscosity on the hydrodynamic entry length and the effect of Prandtl number on the thermal entry length.

PROBLEM 8.8

KNOWN: Velocity and temperature profiles for laminar flow in a tube of radius r_o = 10 mm.

FIND: Mean (or bulk) temperature, T_m, at this axial position.

SCHEMATIC:

u(r), T(r)

Fluid

r_o

r

ASSUMPTIONS: (1) Laminar incompressible flow, (2) Constant properties.

ANALYSIS: The prescribed velocity and temperature profiles, (m/s and K, respectively) are

$$u(r) = 0.1\,[1-(r/r_o)^2] \qquad T(r) = 344.8 + 75.0\,(r/r_o)^2 - 18.8\,(r/r_o)^4 \qquad (1,2)$$

For incompressible flow with constant c_v in a circular tube, from Eq. 8.27, the mean temperature and u_m, the mean velocity, from Eq. 8.8 are, respectively,

$$T_m = \frac{2}{u_m r_o^2}\int_0^{r_o} u(r)\cdot T(r)\cdot r\cdot dr \qquad u_m = \frac{2}{r_o^2}\int_0^{r_o} u(r)\cdot r\cdot dr \qquad (3,4)$$

Substituting the velocity profile, Eq. (1), into Eq. (4) and integrating, find

$$u_m = \frac{2}{r_o^2}r_o^2\int_0^1 0.1\left[1-(r/r_o)^2\right](r/r_o)\,d(r/r_o) = 2\left\{0.1\left[\frac{1}{2}(r/r_o)^2 - \frac{1}{4}(r/r_o)^4\right]\right\}_0^1 = 0.05\,m/s$$

Substituting the profiles and u_m into Eq. (3), find

$$T_m = \frac{2}{(0.05\,m/s)r_o^2}r_o^2\int_0^1\left\{0.1\left[1-(r/r_o)^2\right]\right\}\left\{344.8+75.0(r/r_o)^2-18.8(r/r_o)^4\right\}\cdot(r/r_o)\cdot d(r/r_o)$$

$$T_m = 4\int_0^1\left\{\left[344.8(r/r_o)+75.0(r/r_o)^3-18.8(r/r_o)^5\right]-\left[344.8(r/r_o)^3+75.0(r/r_o)^5-18.8(r/r_o)^7\right]\right\}d(r/r_o)$$

$$T_m = 4\{[172.40+18.75-3.13]-[86.20+12.50-2.35]\} = 367\,K$$

<

The velocity and temperature profiles appear as shown below. Do the values of u_m and T_m found above compare with their respective profiles as you thought? Is the fluid being heated or cooled?

Velocity, u(r) (m/s): 0, 0.02, 0.04, 0.06, 0.08, 0.1

Radial coordinate, r/ro: 0, 0.2, 0.4, 0.6, 0.8, 1

Temperature, T(r) (K): 340, 360, 380, 400, 420, 440

Radial coordinate, r/ro: 0, 0.2, 0.4, 0.6, 0.8, 1

PROBLEM 8.9

KNOWN: Velocity and temperature profiles for laminar flow in a parallel plate channel.

FIND: Mean velocity, u_m, and mean (or bulk) temperature, T_m, at this axial position. Plot the velocity and temperature distributions. Comment on whether values of u_m and T_m appear reasonable.

SCHEMATIC:

y, y_o, 0, $u(y/y_o)$, $T(y/y_o)$, Fluid

ASSUMPTIONS: (1) Laminar incompressible flow, (2) Constant properties.

ANALYSIS: The prescribed velocity and temperature profiles (m/s and °C, respectively) are

$$u(y) = 0.75\left[1-(y/y_o)^2\right] \qquad T(y) = 5.0 + 95.66(y/y_o)^2 - 47.83(y/y_o)^4 \quad (1,2)$$

The mean velocity, u_m, follows from its definition, Eq. 8.7,

$$\dot{m} = \rho A_c u_m = \rho \int_{A_c} u(y)\cdot dA_c$$

where the flow cross-sectional area is $dA_c = 1\cdot dy$, and $A_c = 2y_o$,

$$u_m = \frac{1}{A_c}\int_{A_c} u(y)\cdot dy = \frac{1}{2y_o}\int_{-y_o}^{+y} u(y)dy \quad (3)$$

$$u_m = \frac{1}{2y_o}\cdot y_o \int_{-1}^{+1} 0.75\left[1-(y/y_o)^2\right] d(y/y_o)$$

$$u_m = 1/2\left\{0.75\left[(y/y_o) - 1/3(y/y_o)^3\right]\right\}_{-1}^{+1}$$

$$u_m = 1/2\times 0.75\{[1-1/3]-[-1+1/3]\} = 1/2\times 0.75\times 4/3 = 2/3\times 0.75 = 0.50\,\text{m/s}$$ <

The mean temperature, T_m, follows from its definition, Eq. 8.25,

$$\dot{E}_t = \dot{m} c_v T_m \qquad \text{where} \qquad \dot{m} = \rho A_c u_m$$

$$\rho A_c u_m c_v T_m = \rho c_v \int_{A_c} u(y)\cdot T(y) dA_c$$

Hence, substituting velocity and temperature profiles,

$$T_m = \frac{1}{u_m A_c}\int_{-y_o}^{+y_o} u(y)\cdot T(y) dy \quad (4)$$

$$T_m = \frac{1}{(0.5\,\text{m/s})2y_o} y_o \int_{-1}^{+1}\left\{0.75\left[1-(y/y_o)^2\right]\right\}\left\{5.0+95.66(y/y_o)^2 - 47.83(y/y_o)^4\right\} d(y/y_o)$$

$$T_m = \frac{0.75}{0.5\times 2}\left\{\left[5(y/y_o) + 31.89(y/y_o)^3 - 9.57(y/y_o)^5\right] - \left[1.67(y/y_o)^3 + 19.13(y/y_o)^5 - 6.83(y/y_o)^7\right]\right\}_{-1}^{+1}$$

$$T_m = \frac{0.75}{0.5\times 2}\{[27.32-13.97]-[-27.32-(-13.97)]\} = 20.0^\circ\text{C}$$ <

Continued...

PROBLEM 8.9 (Cont.)

The velocity and temperature profiles along with the u_m and T_m values are plotted below.

Velocity, u(y) (m/s)

Dimensionless coordinate, x/xo

Velocity profile, u(y)
Mean velocity, um = 0.5 m/s

Temperature, T(y) (C)

Dimensionless coordinate, y/yo

Temperature profile, T(y)
Mean temperature, Tm = 20 C

For the velocity profile, the mean velocity is 2/3 that of the centerline velocity, $u_m = 2u(0)/3$. Note that the areas above and below the u_m line appear to be equal. Considering the temperature profile, we'd expect the mean temperature to be closer to the centerline temperature since the velocity profile weights the integral toward the centerline.

COMMENTS: The integrations required to obtain u_m and T_m, Eqs. (3) and (4), could also be performed using the intrinsic function *INTEGRAL (y,x)* in the *IHT Workspace*.

PROBLEM 8.10

KNOWN: Flow rate, inlet temperature and pressure, and outlet pressure of water flowing through a pipe with a prescribed surface heat rate.

FIND: (a) Outlet temperature, (b) Outlet temperature assuming negligible flow work changes.

SCHEMATIC:

ASSUMPTIONS: (1) Negligible kinetic and potential energy changes, (2) Constant properties, (3) Incompressible liquid.

PROPERTIES: *Table A-6*, Water (T = 300K): $\rho = 997\ \text{kg/m}^3$, $c_p = c_v = 4179$ J/kg·K.

ANALYSIS: (a) Accounting for the flow work effect, Eq. 8.35 may be integrated from inlet to outlet to obtain

$$q_{conv} = \dot{m}\left[c_v\left(T_{m,o} - T_{m,i}\right) + (pv)_o - (pv)_i\right]$$

Hence,

$$T_{m,o} = T_{m,i} + \frac{q_{conv}}{\dot{m}c_v} + \frac{1}{r\ c_v}\left(p_i - p_o\right)$$

$$T_{m,o} = 25^\circ\text{C} + \frac{10^5\text{W}}{2\ \text{kg/s} \times 4179\ \ \text{J/kg} \cdot \text{K}} + \frac{(100-2)\ \text{bar}\ \left(10^5\ \ \text{N/m}^2\right)/\text{bar}}{997\ \text{kg/m}^3 \times 4179\ \text{J/kg} \cdot \text{K}}$$

$$T_{m,o} = 25^\circ\text{C} + 12^\circ\text{C} + 2.4^\circ\text{C}$$

$$T_{m,o} = 39.4^\circ\text{C}.$$ <

(b) Neglecting the flow work effect, it follows from Eq. 8.37 that,

$$T_{m,o} = T_{m,i} + \frac{q_{conv}}{\dot{m}c_p} = 25^\circ\text{C} + 12^\circ\text{C}$$

$$T_{m,o} = 37^\circ\text{C}.$$ <

COMMENTS: Even for the large pressure drop of this problem, flow work effects make a small contribution to heating the water. The effects may justifiably be neglected in most practical problems.

PROBLEM 8.11

KNOWN: Internal flow with prescribed wall heat flux as a function of distance.

FIND: (a) Beginning with a properly defined differential control volume, the temperature distribution, $T_m(x)$, (b) Outlet temperature, $T_{m,o}$, (c) Sketch $T_m(x)$, and $T_s(x)$ for fully developed *and* developing flow conditions, and (d) Value of uniform wall flux q_s'' (instead of q_s' = ax) providing same outlet temperature as found in part (a); sketch $T_m(x)$ and $T_s(x)$ for this heating condition.

SCHEMATIC:

Water
$\dot{m}$ = 450 kg/h
$T_{m,i}$ = 27 °C
$q' = ax$ { q' W/m; a 20 W/m^2; x m }
$T_{m,o}$
x
Tube, 0
L = 30 m
dq_{conv}
$T_s(x)$
$T_m(x)$
$\dot{m}\, c_p\, T_m$
$\dot{m}\, c_p(T_m + d\, T_m)$

ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Incompressible flow.

PROPERTIES: *Table A.6*, Water (300 K): c_p = 4.179 kJ/kg·K.

ANALYSIS: (a) Applying energy conservation to the control volume above,

$$dq_{conv} = \dot{m}c_p dT_m \tag{1}$$

where $T_m(x)$ is the mean temperature at any cross-section and $dq_{conv} = q' \cdot dx$. Hence,

$$ax = \dot{m}c_p \frac{dT_m}{dx}. \tag{2}$$

Separating and integrating with proper limits gives

$$a\int_{x=0}^{x} x\,dx = \dot{m}c_p \int_{T_{m,i}}^{T_m(x)} dT_m \qquad T_m(x) = T_{m,i} + \frac{ax^2}{2\dot{m}c_p} \tag{3,4}$$ <

(b) To find the outlet temperature, let x = L, then

$$T_m(L) = T_{m,o} = T_{m,i} + aL^2 / 2\dot{m}c_p. \tag{5}$$

Solving for $T_{m,o}$ and substituting numerical values, find

$$T_{m,o} = 27^\circ C + \frac{20\,W/m^2 \left(30\,m^2\right)}{2\left(450\,kg/h/\left(3600\,s/h\right)\right)\times 4179\,J/kg\cdot K} = 27^\circ C + 17.2^\circ C = 44.2^\circ C.$$ <

(c) For *linear wall heating*, $q_s' = ax$, the fluid temperature distribution along the length of the tube is quadratic as prescribed by Eq. (4). From the convection rate equation,

$$q_s' = h(x)\cdot \pi D\left(T_s(x) - T_m(x)\right) \tag{6}$$

For fully developed flow conditions, h(x) = h is a constant; hence, $T_s(x) - T_m(x)$ increases linearly with x. For developing conditions, h(x) will decrease with increasing distance along the tube eventually achieving the fully developed value.

Continued...

(d) For *uniform wall heat flux heating*, the overall energy balance on the tube yields

$$q = q_s''\pi DL = \dot{m}c_p\left(T_{m,o} - T_{m,i}\right)$$

Requiring that $T_{m,o}$ = 44.2°C from part (a), find

$$q_s'' = \frac{(450/3600)\,\text{kg/s} \times 4179\,\text{J/kg}\cdot\text{K}\,(44.2 - 27)\,\text{K}}{\pi D \times 30\,\text{m}} = 95.3/D\ \text{W/m}^2$$ <

where D is the diameter (m) of the tube which, when specified, would permit determining the required heat flux, q_s''. For uniform heating, Section 3.3.2, we know that $T_m(x)$ will be linear with distance. $T_s(x)$ will also be linear for fully developed conditions and appear as shown below when the flow is developing.

COMMENTS: (1) Note that c_p should be evaluated at T_m = (27 + 44)°C/2 = 309 K.

(2) Why did we show $T_s(0) = T_m(0)$ for both types of history when the flow was developing?

(3) Why must $T_m(x)$ be linear with distance in the case of uniform wall flux heating?

PROBLEM 8.12

KNOWN: Internal flow with constant surface heat flux, q''_s.

FIND: (a) Qualitative temperature distributions, T(x), under developing and fully-developed flow, (b) Exit mean temperature for both situations.

SCHEMATIC:

ASSUMPTIONS: (a) Steady-state conditions, (b) Constant properties, (c) Incompressible flow.

ANALYSIS: Based upon the analysis leading to Eq. 8.40, note for the case of constant surface heat flux conditions,

$$\frac{dT_m}{dx} = \text{constant.}$$

Hence, regardless of whether the hydrodynamic or thermal boundary layer is fully developed, it follows that

$T_m(x)$ is linear and

$T_{m,2}$ will be the same for all flow conditions. <

The surface heat flux can also be written, using Eq. 8.28, as

$$q''_s = h\left[T_s(x) - T_m(x)\right].$$

Under fully-developed flow and thermal conditions, $h = h_{fd}$ is a constant. When flow is developing $h > h_{fd}$. Hence, the temperature distributions appear as below.

<

PROBLEM 8.13

KNOWN: Geometry and coolant flow conditions associated with a nuclear fuel rod. Axial variation of heat generation within the rod.

FIND: (a) Axial variation of local heat flux and total heat transfer rate, (b) Axial variation of mean coolant temperature, (c) Axial variation of rod surface temperature and location of maximum temperature.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Constant fluid properties, (3) Uniform surface convection coefficient, (4) Negligible axial conduction in rod and fluid, (5) Negligible kinetic energy, potential energy and flow work changes, (6) Outer surface is adiabatic.

ANALYSIS: (a) Performing an energy balance for a control volume about the rod,

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = 0 \qquad -dq + \dot{E}_g = 0$$

or

$$-q''(\pi D\,dx) + \dot{q}_o \sin(\pi x/L)\left(\pi D^2/4\right) dx = 0 \qquad q'' = \dot{q}_o (D/4)\sin(\pi x/L). \qquad <$$

The total heat transfer rate is then

$$q = \int_0^L q'' \pi D\,dx = \left(\pi D^2/4\right)\dot{q}_o \int_0^L \sin(\pi x/L)\,dx$$

$$q = \frac{\pi D^2}{4}\dot{q}_o\left(-\frac{L}{\pi}\cos\frac{\pi x}{L}\right)\Bigg|_0^L = \frac{D^2 \dot{q}_o L}{4}(1+1)$$

$$q = \frac{D^2 L}{2}\dot{q}_o. \qquad (1) \quad <$$

(b) Performing an energy balance for a control volume about the coolant,

$$\dot{m}\,c_p\,T_m + dq = \dot{m}\,c_p\,(T_m + dT_m) = 0.$$

Hence

$$\dot{m}\,c_p\,d\,T_m = dq = (\pi D\,dx)\,q''$$

$$\frac{d\,T_m}{dx} = \frac{\pi D}{\dot{m}\,c_p}\,\frac{\dot{q}_o D}{4}\sin\left(\frac{\pi x}{L}\right).$$

Continued …..

PROBLEM 8.13 (Cont.)

Integrating,

$$T_m(x) - T_{m,i} = \frac{\pi D^2}{4} \frac{\dot{q}_o}{\dot{m} c_p} \int_0^x \sin \frac{\pi x}{L} dx$$

$$T_m(x) = T_{m,i} + \frac{L D^2}{4} \frac{\dot{q}_o}{\dot{m} c_p} \left[1 - \cos \frac{\pi x}{L}\right]$$

(2) <

(c) From Newton's law of cooling,

$$q'' = h(T_s - T_m).$$

Hence

$$T_s = \frac{q''}{h} + T_m$$

$$T_s = \frac{\dot{q}_o D}{4h} \sin \frac{\pi x}{L} + T_{m,i} + \frac{LD^2}{4} \frac{\dot{q}_o}{\dot{m} c_p} \left[1 - \cos \frac{\pi x}{L}\right].$$ <

To determine the location of the maximum surface temperature, evaluate

$$\frac{d T_s}{dx} = 0 = \frac{\dot{q}_o D\pi}{4hL} \cos \frac{\pi x}{L} + \frac{LD^2}{4} \frac{\dot{q}_o}{\dot{m} c_p} \frac{\pi}{L} \sin \frac{\pi x}{L}$$

or

$$\frac{1}{hL} \cos \frac{\pi x}{L} + \frac{D}{\dot{m} c_p} \sin \frac{\pi x}{L} = 0.$$

Hence

$$\tan \frac{\pi x}{L} = -\frac{\dot{m} c_p}{D h L}$$

$$x = \frac{L}{\pi} \tan^{-1}\left(-\frac{\dot{m} c_p}{D h L}\right) = x_{max}.$$ <

COMMENTS: Note from Eq. (2) that

$$T_{m,o} = T_m(L) = T_{m,i} + \frac{L D^2 \dot{q}_o}{2 \dot{m} c_p}$$

which is equivalent to the result obtained by combining Eq. (1) and Eq. 8.37.

PROBLEM 8.14

KNOWN: Axial variation of surface heat flux for flow through a tube.

FIND: Axial variation of fluid and surface temperatures.

SCHEMATIC:

ASSUMPTIONS: (1) Convection coefficient is independent of x, (2) Negligible axial conduction and kinetic and potential energy changes, (3) Fluid is an ideal gas or a liquid for which $d(pv) << d(c_v T_m)$.

ANALYSIS: Since Equation 8.38 is applicable,

$$\frac{dT_m}{dx} = \frac{q_s'' P}{\dot{m} c_p} = \frac{(\pi D) q_{s,m}'' \sin(\pi x/L)}{\dot{m} c_p}$$

Separating variables and integrating from x = 0

$$\int_{T_{m,i}}^{T_{m,o}} dT_m = \frac{\pi D q_{s,m}''}{\dot{m} c_p} \int_0^x \sin\frac{\pi x}{L} dx$$

$$T_m(x) - T_{m,i} = -\frac{LDq_{s,m}''}{\dot{m} c_p} \cos\frac{\pi x}{L} \Big|_0^x$$

$$T_m(x) = T_{m,i} + \frac{LDq_{s,m}''}{\dot{m} c_p}(1 - \cos \pi x/L).$$ <

From Newton's law of cooling, Eq. 8.28,

$$T_s(x) = (q_s''/h) + T_m(x)$$

$$T_s(x) = \frac{q_{s,m}''}{h} \sin\frac{\pi x}{L} + T_{m,i} + \frac{LDq_{s,m}''}{\dot{m} c_p}(1 - \cos \pi x/L).$$ <

COMMENTS: For the prescribed surface condition, the flow is not fully developed. Hence, the assumption of constant h should be viewed as a first approximation.

PROBLEM 8.15

KNOWN: Surface heat flux for air flow through a rectangular channel.

FIND: (a) Differential equation describing variation in air mean temperature, (b) Air outlet temperature for prescribed conditions.

SCHEMATIC:

$q''_o = 700 W/m^2$, Cover plate, w = 1m, Absorber plate, Air, $\dot{m} = 0.1 kg/s$, $T_{m,i} = 40°C$, $T_{m,o}$, L = 3m, x, q''_o, $\dot{m} c_p (T_m + dT_m)$, $\dot{m} c_p T_m$, dx

ASSUMPTIONS: (1) Negligible change in kinetic and potential energy of air, (2) No heat loss through bottom of channel, (3) Uniform heat flux at top of channel.

PROPERTIES: *Table A-4*, Air (T ≈ 50°C, 1 atm): c_p = 1008 J/kg·K.

ANALYSIS: (a) For the differential control volume about the air,

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}\, c_p\, T_m + q''_o\, (w \cdot dx) = \dot{m}\, c_p\, (T_m + d\,T_m)$$

$$\frac{d\,T_m}{dx} = \frac{q''_o \cdot w}{\dot{m}\, c_p}$$

Separating and integrating between the limits of x = 0 and x, find

$$T_m(x) = T_{m,i} + \frac{q''_o (w \cdot x)}{\dot{m}\, c_p}$$

$$T_{m,o} = T_{m,i} + \frac{q''_o (w \cdot L)}{\dot{m}\, c_p}. \qquad <$$

(b) Substituting numerical values, the air outlet temperature is

$$T_{m,o} = 40^\circ C + \frac{\left(700\ W/m^2\right)\ (1 \times 3)\, m^2}{0.1\ kg/s\ (1008\ J/kg \cdot K)}$$

$$T_{m,o} = 60.8^\circ C. \qquad <$$

COMMENTS: Due to increasing heat loss with increasing T_m, the net flux q''_o will actually decrease slightly with increasing x.

PROBLEM 8.16

KNOWN: Air inlet conditions and heat transfer coefficient for a circular tube of prescribed geometry. Surface heat flux.

FIND: (a) Tube heat transfer rate, q, air outlet temperature, $T_{m,o}$, and surface inlet and outlet temperatures, $T_{s,i}$ and $T_{s,o}$, for a uniform surface heat flux, q_s''. Air mean and surface temperature distributions. (b) Values of q, $T_{m,o}$, $T_{s,i}$ and $T_{s,o}$ for a linearly varying surface heat flux q_s'' = 500x (m). Air mean and surface temperature distributions, (c) For each type of heating process (a & b), compute and plot the mean fluid and surface temperatures, $T_m(x)$ and $T_s(x)$, respectively, as a function of distance; What is effect of four-fold increase in convection coefficient, and (d) For each type of heating process, heat fluxes required to achieve an outlet temperature of $T_{m,o}$ = 125°C; Plot temperatures.

SCHEMATIC:

$T_{s,o}$ $T_{m,o}$

$T_{s,i}$

L = 3m

Air

$\dot{m}$ = 0.005 kg/s

$T_{m,i}$ = 20 °C

p = 1 atm

x

h = 25 W/m²·K, D = 0.05m

q''_s = 1000 W/m², or q'' = 500 x (W/m²)

ASSUMPTIONS: (1) Fully developed conditions in the tube, (2) Applicability of Eq. 8.36, (3) Heat transfer coefficient is the same for both heating conditions.

PROPERTIES: *Table A.4*, Air (for an *assumed* value of $T_{m,o}$ = 100°C, $\bar{T}_m$ = $(T_{m,i} + T_{m,o})/2$ = 60°C = 333 K): c_p = 1.008 kJ/kg·K.

ANALYSIS: (a) With constant heat flux, from Eq. 8.39,

$$q = q_s''(\pi DL) = 1000\,W/m^2\,(\pi \times 0.05\,m \times 3\,m) = 471\,W\,. \tag{1}$$

From the overall energy balance, Eq. 8.37,

$$T_{m,o} = T_{m,i} + \frac{q}{\dot{m}c_p} = 20^\circ C + \frac{471\,W}{0.005\,kg/s \times 1008\,J/kg\cdot K} = 113.5^\circ C \tag{2}$$ <

From the convection rate equation, it follows that

$$T_{s,i} = T_{m,i} + \frac{q_s''}{h} = 20^\circ C + \frac{1000\,W/m^2}{25\,W/m^2\cdot K} = 60^\circ C \tag{3}$$ <

$$T_{s,o} = T_{m,o} + q_s''/h = 113.5^\circ C + 40^\circ C = 153.5^\circ C$$ <

From Eq. 8.40, (dT_m/dx) is a constant, as is (dT_s/dx) for constant h from Eq. 8.31. In the more realistic case for which h decreases with x in the entry region, (dT_m/dx) is still constant but (dT_s/dx) decreases with increasing x. See the plot below.

(b) From Eq. 8.38,

$$\frac{dT_m}{dx} = \frac{500x(\pi D)}{\dot{m}c_p} = \frac{500x\,W/m^2\,(\pi \times 0.05\,m)}{0.005\,kg/s \times 1008\,J/kg\cdot K} = 15.6x\,K/m\,. \tag{4}$$

Continued...

PROBLEM 8.16 (Cont.)

Integrating from x = 0 to L it follows that

$$T_{m,o} = T_{m,i} + 15.6\int_0^3 x\,dx = 20^\circ C + 15.6\left.\frac{x^2}{2}\right|_0^3 = 20^\circ C + 70.2^\circ C = 90.2^\circ C. \qquad (5)$$ <

The heat rate is

$$q = \int q_s'' dA_s = 500(\pi \times 0.05\,m)\int_0^3 x\,dx = 78.5\left.\frac{x^2}{2}\right|_0^3 = 353\,W$$ <

From Eq. 8.28 it then follows that

$$T_s = T_m + q_s''/h = T_{m,i} + 15.6\frac{x^2}{2} + \frac{500}{25}x = 20^\circ C + 7.8x^2 + 20x \qquad (6)$$

Hence, at the inlet (x = 0) and outlet (x = L),

$$T_{s,i} = T_{m,i} = 20^\circ C \qquad \text{and} \qquad T_{s,o} = 150.2^\circ C$$ <

Note that (dT_s/dx) and (dT_m/dx) both increase linearly with x, but $(dT_s/dx) > (dT_m/dx)$.

(c) The foregoing relations can be used to determine $T_m(x)$ and $T_s(x)$ for the two heating conditions:

Uniform surface flux, q_s''; Eqs. (1-3),

$$T_m(x) = T_{m,i} + q_s''\pi Dx/\dot{m}c_p \qquad T_s(x) = T_m(x) + q_s''/h \qquad (7,8)$$

Linear surface heat flux, $q_s'' = a_o x$, $a_o = 500\ W/m^3$; Eqs. (4-6),

$$T_m(x) = T_{m,i} + \left(a_o\pi D/2\dot{m}c_p\right)x^2 \qquad T_s(x) = T_m(x) + a_o x/h \qquad (9, 10)$$

Using Eqs. (7-10) in IHT, the mean fluid and surface temperatures as a function of distance are evaluated and plotted below. The calculations were repeated with the coefficient increased four-fold, $h = 4 \times 25 = 100\ W/m^2\cdot K$. As expected, the fluid temperature remained unchanged, but the surface temperatures decreased since the thermal resistance between the surface and fluid decreased.

Tm, Ts (C)
Distance from inlet, x (m)
Tm(x); q''s = 1000 W/m^2
Ts(x); h = 25 W/m^2.K
Ts(x); h = 100 W/m^2.K

Tm, Ts (C)
Distance from inlet, x (m)
Tm(x); q''s = 500x W/m^2
Ts(x); h = 25 W/m^2.K
Ts(x); h = 100 W/m^2.K

(d) The foregoing set of equations, Eqs. (7-10), in the IHT model can be used to determine the required heat fluxes for the two heating conditions to achieve $T_{m,o} = 125^\circ C$. The results with $h = 25\ W/m^2\cdot K$ are:

Uniform flux: $q_s'' = 1123\ W/m^2$ *Linear flux:* $q_s'' = 748.7x\ W/m^2$ <

Continued...

PROBLEM 8.16 (Cont.)

The temperature distributions resulting from these heat fluxes are plotted below. The heat rate for both heating processes is 529 W.

Tm, Ts (C)
240
200
160
120
80
40
0
0 1 2 3
Distance from inlet, x (m)
Tm(x); q''s = 1123 W/m^2
Ts(x); h = 25 W/m^2.K

Tm, Ts (C)
240
200
160
120
80
40
0
0 1 2 3
Distance from inlet, x (m)
Tm(x); q''s = 749x (W/m^2)
Ts(x); h = 25 W/m^2.K

T (°C)
150
100
50
0 1 2 3 $x(m)$
$T_s(x)$
Constant h
$T_m(x)$
Entry region effect, $h = f(x)$

T (°C)
150
100
50
0 1 2 3 $x(m)$
$T_s(x)$
$T_m(x)$
Entry region effect, $h = f(x)$

COMMENTS: Note that the assumed value for $T_{m,o}$ (100°C) in determining the specific heat of the air was reasonable.

PROBLEM 8.17

KNOWN: Water at prescribed temperature and flow rate enters a 0.25 m diameter, black thin-walled tube of 8-m length, which passes through a large furnace whose walls and air are at a temperature of $T_{fur} = T_\infty = 700$ K. The convection coefficients for the internal water flow and external furnace air are 300 $W/m^2 \cdot K$ and 50 $W/m^2 \cdot K$, respectively.

FIND: (a) An expression for the linearized radiation coefficient for the radiation exchange process between the outer surface of the pipe and the furnace walls; represent the tube by an average temperature and explain how to calculate this value, and (b) determine the outlet temperature of the water, T_o.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions; (2) Tube is small object with large, isothermal surroundings; (3) Furnace air and walls are at the same temperature; and (3) Tube is thin-walled with black surface.

PROPERTIES: *Table A-6*, Water ($T_m = (T_{m,i} + T_{m,o})/2 = 331$ K): $c_p = 4192$ J/kg·K.

ANALYSIS: (a) The linearized radiation coefficient follows from Eq. 1.9 with ε = 1,

$$\bar{h}_{rad} = \sigma\left(\bar{T}_t + \bar{T}_{fur}\right)\left(\bar{T}_t^2 + T_{fur}^2\right)$$

where $\bar{T}_t$ represents the average tube wall surface temperature, which can be evaluated from an energy balance on the tube as represented by the thermal circuit above.

$$T_m = \left(T_{m,i} + T_{m,o}\right)/2$$

$$R_{tot} = R_{cv,i} + \frac{1}{1/R_{cv,o} + 1/R_{rad}}$$

$$\frac{T_m - \bar{T}_t}{R_{cv,i}} = \frac{\bar{T}_t - T_{fur}}{1/R_{cv,o} + 1/R_{rad}}$$

The thermal resistances, with $A_s = PL = \pi DL$, are

$$R_{cv,i} = 1/h_i A_s \qquad R_{cv,o} = 1/h_o A_s \qquad R_{rad} = 1/\bar{h}_{rad}$$

(b) The outlet temperature can be calculated using the energy balance relation, Eq. 8.46b, with $T_{fur} = T_\infty$,

$$\frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = \exp\left(-\frac{1}{\dot{m}\, c_p\, R_{tot}}\right)$$

where c_p is evaluated at T_m. Using *IHT*, the following results were obtained.

$$R_{cv,i} = 6.631\times10^{-5}\ \text{K/W} \qquad R_{cv,o} = 3.978\times10^{-4}\ \text{K/W} \qquad R_{rad} = 4.724\times10^{-4}\ \text{K/W}$$

$$T_m = 331\ \text{K} \qquad \bar{T}_t = 418\ \text{K}$$

$$T_{m,o} = 362\ \text{K}$$ <

COMMENTS: Since $T_\infty = T_{fur}$, it was possible to use Eq. 8.46b with R_{tot}. How would you write the energy balance relation if $T_\infty \neq T_{fur}$?

PROBLEM 8.18

KNOWN: Laminar, slug flow in a circular tube with uniform surface heat flux.

FIND: Temperature distribution and Nusselt number.

SCHEMATIC:

ASSUMPTIONS: (1) Steady, incompressible flow, (2) Constant properties, (3) Fully developed, laminar flow, (4) Uniform surface heat flux.

ANALYSIS: With v = 0 for fully developed flow and $\partial T/\partial x = dT_m/dx$ = const, from Eqs. 8.33 and 8.40, the energy equation, Eq. 8.48, reduces to

$$u_o \frac{dT_m}{dx} = \frac{\alpha}{r} \frac{\partial}{\partial r}\left(r \frac{\partial T}{\partial r}\right).$$

Integrating twice, it follows that

$$T(r) = \frac{u_o}{\alpha} \frac{dT_m}{dx} \frac{r^2}{4} + C_1 \ell n(r) + C_2.$$

Since T(0) must remain finite, $C_1 = 0$. Hence, with $T(r_o) = T_s$

$$C_2 = T_s - \frac{u_o}{\alpha} \frac{dT_m}{dx} \frac{r_o^2}{4} \qquad T(r) = T_s - \frac{u_o}{4\alpha} \frac{dT_m}{dx}\left(r_o^2 - r^2\right).$$ <

From Eq. 8.27, with $u_m = u_o$,

$$T_m = \frac{2}{r_o^2}\int_0^{r_o} Tr\,dr = \frac{2}{r_o^2}\int_0^{r_o}\left[T_s r - \frac{u_o}{4\alpha}\frac{dT_m}{dx}\left(r r_o^2 - r^3\right)\right]dr$$

$$T_m = \frac{2}{r_o^2}\left[T_s \frac{r_o^2}{2} - \frac{u_o}{4\alpha}\frac{dT_m}{dx}\left(\frac{r_o^4}{2} - \frac{r_o^4}{4}\right)\right] = T_s - \frac{u_o r_o^2}{8\alpha}\frac{dT_m}{dx}.$$

From Eq. 8.28 and Fourier's law,

$$h = \frac{q_s''}{T_s - T_m} = \frac{k \left.\frac{\partial T}{\partial r}\right|_{r_o}}{T_s - T_m}$$

hence,

$$h = \frac{k\left(\frac{u_o r_o}{2\alpha}\right)\frac{dT_m}{dx}}{\frac{u_o r_o^2}{8\alpha}\frac{dT_m}{dx}} = \frac{4k}{r_o} = \frac{8k}{D} \qquad \overline{Nu}_D = \frac{hD}{k} = 8.$$ <

PROBLEM 8.19

KNOWN: Heat transfer between fluid flow over a tube and flow through the tube.

FIND: Axial variation of mean temperature for inner flow.

SCHEMATIC:

ASSUMPTIONS: (1) Negligible change in kinetic and potential energy, (2) Negligible axial conduction, (3) Constant c_p, (4) Uniform T_∞.

ANALYSIS: From Eq. 8.36,

$$dq = \dot{m}\, c_p d\, T_m$$

with

$$dq = UdA\left(T_\infty - T_m\right) = UP\left(T_\infty - T_m\right)dx.$$

The overall heat transfer coefficient may be defined in terms of the inner or outer surface area, with

$$U_i P_i = U_o P_o.$$

For the inner surface, from Eq. 3.31,

$$U_i = \left[\frac{1}{h_i} + \frac{r_i}{k}\ln\frac{r_o}{r_i} + \frac{r_i}{r_o}\frac{1}{h_o}\right]^{-1}.$$

Hence,

$$\frac{d\, T_m}{T_\infty - T_m} = +\frac{UP}{\dot{m}\, c_p}dx$$

or, with $\Delta T \equiv T_\infty - T_m$,

$$\int_{\Delta T_i}^{\Delta T_o} \frac{d\left(\Delta T\right)}{\Delta T} = -\frac{P}{\dot{m}\, c_p}\int_0^L Udx.$$

Hence,

$$\ln\frac{\Delta T_o}{\Delta T_i} = -\frac{PL}{\dot{m}\, c_p}\left(\frac{1}{L}\int_0^L Udx\right)$$

$$\frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = \exp\left(-\frac{PL}{\dot{m}\, c_p}\overline{U}\right).$$ <

COMMENTS: The development and results parallel those for a constant surface temperature, with $\overline{U}$ and T_∞ replacing $\overline{h}$ and T_s.

PROBLEM 8.20

KNOWN: Thin-walled tube experiences sinusoidal heat flux distribution on the wall.

FIND: (a) Total rate of heat transfer from the tube to the fluid, q, (b) Fluid outlet temperature, $T_{m,o}$, (c) Axial distribution of the wall temperature $T_s(x)$ and (d) Magnitude and position of the highest wall temperature, and (e) For prescribed conditions, calculate and plot the mean fluid and surface temperatures, $T_m(x)$ and $T_s(x)$, respectively, as a function of distance along the tube; identify features of the distributions; explore the effect of ±25% changes in the convection coefficient on the distributions.

SCHEMATIC:

$q''_s(x) = q''_o \sin(\pi x/L)$

ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible kinetic and potential energy changes, (3) Turbulent, fully developed flow.

ANALYSIS: (a) The total rate heat transfer from the tube to the fluid is

$$q = \int_0^L q''_s P dx = q''_o \pi D \int_0^D \sin(\pi x/L) dx = q''_o \pi D (L/\pi)\left[-\cos(\pi x/L)\right]_0^L = 2DLq''_o \quad (1) <$$

(b) The fluid outlet temperature follows from the overall energy balance with knowledge of the total heat rate,

$$q = \dot{m}c_p\left(T_{m,o} - T_{m,i}\right) = 2DLq''_o \qquad T_{m,o} = T_{m,i} + \left(2DLq''_o/\dot{m}c_p\right) \quad (2) <$$

(c) The axial distribution of the wall temperature can be determined from the rate equation

$$q''_s = h\left[T_s(x) - T_m(x)\right] \qquad T_{s,x} = T_{m,x}(x) + q''_s/h \quad (3)$$

where, by combining expressions of parts (a) and (b), $T_{m,x}(x)$ is

$$\int_o^x q''_s P dx = \dot{m}c_p\left(T_{m,x} - T_{m,i}\right)$$

$$T_{m,x} = T_{m,i} + \frac{q''_o \pi D}{\dot{m}c_p}\int_0^x \sin(\pi x/L) dx = T_{m,i} + \frac{DLq''_o}{\dot{m}c_p}\left[1 - \cos(\pi x/L)\right] \quad (4)$$

Hence, substituting Eq. (4) into (3), find

$$T_s(x) = T_{m,i} + \frac{DLq''_o}{\dot{m}c_p}\left[1 - \cos(\pi x/L)\right] + \frac{q''_o}{h}\sin(\pi x/L) \quad (5) <$$

(d) To determine the location of the maximum wall temperature x' where $T_x(x') = T_{s,max}$, set

$$\frac{dT_s(x)}{dx} = 0 = \frac{d}{dx}\left\{\frac{DLq''_o}{\dot{m}c_p}\left[1 - \cos(\pi x/L)\right] + \frac{q''_o}{h}\sin(\pi x/L)\right\}$$

$$\frac{DLq''_o}{\dot{m}c_p}\cdot\frac{\pi}{L}\cdot\sin(\pi x'/L) + \frac{q''_o}{h}\cdot\frac{\pi}{L}\cdot\cos(\pi x'/L) = 0 \qquad \tan(\pi x'/L) = -\frac{q''_o/h}{DLq''_o/\dot{m}c_p} = -\frac{\dot{m}c_p}{DLh}$$

Continued...

PROBLEM 8.20 (Cont.)

$$x' = \frac{L}{\pi}\tan^{-1}\left(-\dot{m}c_p / DLh\right) \qquad (6) \quad <$$

At this location, the wall temperature is

$$T_{s,max} = T_s\left(x'\right) = T_{m,i} + \frac{DLq_o''}{\dot{m}c_p}\left[1-\cos\left(\pi x'/L\right)\right] + \frac{q_o''}{h}\sin\left(\pi x'/L\right) \qquad (7) \quad <$$

(e) Consider the prescribed conditions for which to compute and plot $T_m(x)$ and $T_s(x)$,

D = 40 mm	$\dot{m}$ = 0.025 kg/s	h = 1000 W/m^2	q_o'' = 10,000 W/m^2
L = 4 m	c_p = 4180 J/kg·K	$T_{m,i}$ = 25°C	

Using Eqs. (4) and (5) in IHT, the results are plotted below.

Ts, Tm (C)
Distance, x (m)
Tm(x)
Ts(x); h = 1250 W/m^2.K
Ts(x); h = 1000 W/m^2.K
Ts(x); h = 750 W/m^2.K

The effect of a lower convection coefficient is to increase the wall temperature. The position of the maximum temperature, $T_{s,max}$, moves away from the tube exit with decreasing convection coefficient.

COMMENTS: (1) Because the flow is fully developed and turbulent, assuming h is constant along the entire length of the tube is reasonable.

(2) To determine whether the $T_x(x)$ distribution has a maximum (rather than a minimum), you should evaluate $d^2T_s(x)/dx^2$ to show the value is indeed negative.

PROBLEM 8.21

KNOWN: Water is heated in a tube having a wall flux that is dependent upon the wall temperature.

FIND: (a) Beginning with a properly defined differential control volume in the tube, derive expressions that can be used to obtain the temperatures for the water and the wall surface as a function of distance from the inlet, $T_m(x)$ and $T_s(x)$, respectively; (b) Using a numerical integration scheme, calculate and plot the temperature distributions, $T_m(x)$ and $T_s(x)$, on the same graph. Identify and comment on the main features of the distributions; and (c) Calculate the total heat transfer rate to the water.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Fully developed flow and thermal conditions, (3) No losses to the outer surface of the tube, and (3) Constant properties.

PROPERTIES: *Table A-6*, Water $\left(\overline{T}_m = \left(T_{m,i} + T_{m,o}\right)/2 = 300\ K\right)$: $c_p = 4179$ J/kg·K

ANALYSIS: (a) The properly defined control volume of perimeter $P = \pi D$ shown in the above schematic follows from Fig. 8.6. The energy balance on the CV includes advection, convection at the inner tube surface, and the heat flux dissipated in the tube wall. (See Eq. 8.38).

$$\dot{m} c_p \frac{dT_m}{dx} = q_s''(x) P = h P \left[T_s(x) - T_m(x)\right] \tag{1,2}$$

where $q_s''(x)$ is dependent upon $T_s(x)$ according to the relation

$$q_s''(x) = q_{s,o}''\left[1 + \alpha\left(T_s(x) - T_{ref}\right)\right] \tag{3}$$

(b) Eqs. (1 and 2) with Eq. (3) can be solved by numerical integration using the Der function in *IHT* as shown in Comment 1. The temperature distributions for the water and wall surface are plotted below.

Continued …..

(c) The total heat transfer to the water can be evaluated from an overall energy balance on the water,

$$q = \dot{m} c_p \left(T_{m,o} - T_{m,i}\right) \qquad (4)$$

$$q = 0.1 \text{ kg/s} \times 4179 \text{ J/kg} \cdot \text{K} \left(34.4 - 20\right) \text{K} = 6018 \text{ W} \qquad <$$

Alternatively, the heat rate can be evaluated by integration of the heat flux from the tube surface over the length of the tube,

$$q = \int_0^L q_s''(x) P dx \qquad (5)$$

where $q_s''(x)$ is given by Eq. (3), and $T_s(x)$ and $T_m(x)$ are determined from the differential form of the energy equation, Eqs. (1) and (2). The result as shown in the *IHT* code below is 6005 W.

COMMENTS: (1) Note that $T_m(x)$ increases with distance greater than linearly, as expected since $q_s''(x)$ does. Also as expected, the difference, $T_s(x) - T_m(x)$, likewise increases with distance greater than linearly.

(2) In the foregoing analysis, c_p is evaluated at the mean fluid temperature $T_m = (T_{m,i} + T_{m,o})/2$.

(3) The *IHT* code representing the foregoing equations to calculate and plot the temperature distribution and to calculate the total heat rate to the water is shown below.

```
/* Results: integration for distributions; conditions at x = 2 m
F_xTs  Ts      q'     q''s_x    x     Tm
11.64  73.18   5483   1.164E5   2     34.39
3      30      1414   3E4       0     20   */

/* Results: heat rate by energy balances on fluid and tube surface
q_eb   q_hf
6018   6005    */

/* Results: for evaluating cp at Tm
Ts     cp      q''s_x    x     Tm
73.31  4179    1.166E5   2     34.44
30     4179    3E4       0     20      */

// Energy balances
mdot * cp * der(Tm,x) = q'                // Energy balance, Eq. 8.38
q' = q''s_x * P
q''s_x = q''o * F_xTs
q' = h * P * (Ts - Tm)                    // Convection rate equation
P = pi * D

// Surface heat flux specification
F_xTs = (1 + alpha * (Ts -Tref))
alpha = 0.2
Tref = 20

// Overall heat rate
// Energy balance on the fluid
q_eb = mdot * cp * (Tmo - Tmi)
Tmi = 20
Tmo = 34.4                  // From initial solve
// Integration of the surface heat flux
q_hf =  q''o * P * INTEGRAL(F_xTs, x)

// Input variables
mdot = 0.1
D = 0.015
h = 3000
q''o = 1.0e4
// L = 2                          // Limit of integration over x
// Tmi = 20                 // Initial condition for integration

// Water property functions :T dependence, From Table A.6
// Units: T(K), p(bars);
xx = 0               // Quality (0=sat liquid or 1=sat vapor)
cp = cp_Tx("Water",Tmm,xx)        // Specific heat, J/kg·K
Tmm = (20 + 34.4) / 2 + 273
```

PROBLEM 8.22

KNOWN: Flow rate of engine oil through a long tube.

FIND: (a) Heat transfer coefficient, $\bar{h}$, (b) Outlet temperature of oil, $T_{m,o}$.

SCHEMATIC:

Engine oil
$\dot{m} = 0.02 kg/s$
$T_s = 100°C$
$T_{m,i} = 60°C$
Tube, $D = 3 mm$
$L = 30 m$
$A_s = \pi DL$
$T_{m,o}$

ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Combined entry conditions exist.

PROPERTIES: *Table A-5*, Engine Oil (T_s = 100°C = 373K): $\mu_s = 1.73 \times 10^{-2}$ N·s/m^2; *Table A-5*, Engine Oil $\left(\bar{T}_m = 77^\circ C = 350K\right)$: c_p = 2118 J/kg·K, $\mu = 3.56 \times 10^{-2}$ N·s/m^2, k = 0.138 W/m·K, Pr = 546.

ANALYSIS: (a) The overall energy balance and rate equations have the form

$$q = \dot{m}\, c_p\, \left(T_{m,o} - T_{m,i}\right) \qquad q = \bar{h} A_s \Delta T_{lm} \tag{1,2}$$

Using Eq. 8.42b, with P = πD, and Eq. 8.6

$$\frac{\Delta T_o}{\Delta T_i} = \frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp\left(-\frac{PL}{\dot{m}\, c_p} \cdot \bar{h}\right). \tag{3}$$

$$Re_D = \frac{4\dot{m}}{\pi\, D \mu} = \frac{4 \times 0.02 \text{ kg/s}}{\pi \times 3 \times 10^{-3} \text{m} \times 3.56 \times 10^{-2} \text{N} \cdot \text{s/m}^2} = 238.$$

For laminar and combined entry conditions, use Eq. 8.57

$$\overline{Nu}_D = 1.86 \left(\frac{Re_D\, Pr}{L/D}\right)^{1/3} \left(\frac{\mu}{\mu_s}\right)^{0.14} = \left(\frac{238 \times 546}{30\text{m}/3 \times 10^{-3}\text{m}}\right)^{1/3} \left(\frac{3.56}{1.73}\right)^{0.14} = 4.83$$

$$\bar{h} = \overline{Nu}_D\, k/D = 4.83 \times 0.138 \text{ W/m} \cdot \text{K}/3 \times 10^{-3} \text{m} = 222 \text{ W/m}^2 \cdot \text{K}.$$ <

(b) Using Eq. (3) with the foregoing value of $\bar{h}$,

$$\frac{\left(100 - T_{m,o}\right)^\circ C}{(100 - 60)^\circ C} = \exp\left(-\frac{\pi \times 3 \times 10^{-3} \text{m} \times 30 \text{m}}{0.02 \text{ kg/s} \times 2118 \text{ J/kg} \cdot \text{K}} \times 222 \text{W/m}^2 \cdot \text{K}\right) \qquad T_{m,o} = 90.9^\circ C.$$ <

COMMENTS: (1) Note that requirements for the correlation, Eq. 8.57, are satisfied.

(2) The assumption of $\bar{T}_m$ = 77°C for selecting property values was satisfactory.

(3) For thermal entry effect only, Eq. 8.56, $\bar{h} = 201 \text{ W/m}^2 \cdot \text{K}$ and $T_{m,o}$ = 89.5°C.

PROBLEM 8.23

KNOWN: Inlet temperature and flowrate of oil flowing through a tube of prescribed surface temperature and geometry.

FIND: (a) Oil outlet temperature and total heat transfer rate, and (b) Effect of flowrate.

SCHEMATIC:

ASSUMPTIONS: (1) Negligible temperature drop across tube wall, (2) Negligible kinetic energy, potential energy and flow work effects.

PROPERTIES: *Table A.5*, Engine oil (assume $T_{m,o}$ = 140°C, hence $\overline{T}_m$ = 80°C = 353 K): ρ = 852 kg/m^3, ν = 37.5 × 10^{-6} m^2/s, k = 138 × 10^{-3} W/m·K, Pr = 490, μ = ρ·ν = 0.032 kg/m·s, c_p = 2131 J/kg·K.

ANALYSIS: (a) For constant surface temperature the oil outlet temperature may be obtained from Eq. 8.42b. Hence

$$T_{m,o} = T_s - \left(T_s - T_{m,i}\right)\exp\left(-\frac{\pi DL}{\dot{m}c_p}\overline{h}\right)$$

To determine $\overline{h}$, first calculate Re_D from Eq. 8.6,

$$Re_D = \frac{4\dot{m}}{\pi D\mu} = \frac{4\left(0.5\,kg/s\right)}{\pi\left(0.05m\right)\left(0.032\,kg/m\cdot s\right)} = 398$$

Hence the flow is laminar. Moreover, from Eq. 8.23 the thermal entry length is

$$x_{fd,t} \approx 0.05D\,Re_D\,Pr = 0.05\left(0.05\,m\right)\left(398\right)\left(490\right) = 486\,m\,.$$

Since L = 25 m the flow is far from being thermally fully developed. However, from Eq. 8.3, $x_{fd,h} \approx 0.05DRe_D$ = 0.05(0.05 m)(398) = 1 m and it is reasonable to assume fully developed hydrodynamic conditions throughout the tube. Hence $\overline{h}$ may be determined from Eq. 8.56

$$\overline{Nu}_D = 3.66 + \frac{0.0668\left(D/L\right)Re_D\,Pr}{1+0.04\left[\left(D/L\right)Re_D\,Pr\right]^{2/3}}\,.$$

With $(D/L)Re_DPr$ = (0.05/25)398 × 490 = 390, it follows that

$$\overline{Nu}_D = 3.66 + \frac{26}{1+2.14} = 11.95.$$

Hence, $\overline{h} = \overline{Nu}_D\frac{k}{D} = 11.95\frac{0.138\,W/m\cdot K}{0.05\,m} = 33\,W/m^2\cdot K$ and it follows that

Continued...

PROBLEM 8.23 (Cont.)

$$T_{m,o} = 150^\circ C - \left(150^\circ C - 20^\circ C\right)\exp\left[-\frac{\pi(0.05\,m)(25\,m)}{0.5\,kg/s \times 2131\,J/kg\cdot K} \times 33\,W/m^2\cdot K\right]$$

$T_{m,o} = 35°C.$ <

From the overall energy balance, Eq. 8.37, it follows that

$$q = \dot{m}c_p\left(T_{m,o} - T_{m,i}\right) = 0.5\,kg/s \times 2131\,J/kg\cdot K \times (35 - 20)^\circ\,C$$

$q = 15{,}980$ W. <

The value of $T_{m,o}$ has been grossly overestimated in evaluating the properties. The properties should be re-evaluated at $\overline{T}$ = (20 + 35)/2 = 27°C and the calculations repeated. Iteration should continue until satisfactory convergence is achieved between the calculated and assumed values of $T_{m,o}$. Following such a procedure, one would obtain $T_{m,o}$ = 36.4°C, Re_D = 27.8, $\overline{h}$ = 32.8 $W/m^2\cdot K$, and q = 15,660 W. The small effect of reevaluating the properties is attributed to the compensating effects on Re_D (a large decrease) and Pr (a large increase).

(b) The effect of flowrate on $T_{m,o}$ and q was determined by using the appropriate IHT *Correlations* and *Properties* Toolpads.

The heat rate increases with increasing $\dot{m}$ due to the corresponding increase in Re_D and hence $\overline{h}$. However, the increase is not proportional to $\dot{m}$, causing $\left(T_{m,o} - T_{m,i}\right) = q/\dot{m}c_p$, and hence $T_{m,o}$, to decrease with increasing $\dot{m}$. The maximum heat rate corresponds to the maximum flowrate ($\dot{m}$ = 0.20 kg/s).

COMMENTS: Note that significant error would be introduced by assuming fully developed thermal conditions and $\overline{Nu}_D$ = 3.66. The flow remains well within the laminar region over the entire range of $\dot{m}$.

PROBLEM 8.24

KNOWN: Inlet temperature and flowrate of oil moving through a tube of prescribed diameter and surface temperature.

FIND: (a) Oil outlet temperature $T_{m,o}$ for two tube lengths, 5 m and 100 m, and log mean and arithmetic mean temperature differences, (b) Effect of L on $T_{m,o}$ and $\overline{Nu}_D$.

SCHEMATIC:

T_s = 100 °C
$T_{m,o}$
D = 25 mm
$\dot{m}$ = 0.5 kg/s
$T_{m,i}$ = 25 °C
L
5 ≤ L ≤ 100 m

ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible kinetic energy, potential energy and flow work changes, (3) Constant properties.

PROPERTIES: *Table A.4*, Oil (330 K): c_p = 2035 J/kg·K, μ = 0.0836 N·s/m^2, k = 0.141 W/m·K, Pr = 1205.

ANALYSIS: (a) Using Eqs. 8.42b and 8.6

$$T_{m,o} = T_s - \left(T_s - T_{m,i}\right)\exp\left(-\frac{\pi DL}{\dot{m}c_p}\bar{h}\right)$$

$$Re_D = \frac{4\dot{m}}{\pi D\mu} = \frac{4\times 0.5\,kg/s}{\pi\times 0.025\,m\times 0.0836\,N\cdot s/m^2} = 304.6$$

Since entry length effects will be significant, use Eq. 8.56

$$\bar{h} = \frac{k}{D}\left[3.66 + \frac{0.0688(D/L)Re_D\,Pr}{1+0.04\left[(D/L)Re_D\,Pr\right]^{2/3}}\right] = \frac{0.141\,W/m\cdot K}{0.025\,m}\left[3.66 + \frac{2.45\times 10^4\,D/L}{1+205(D/L)^{2/3}}\right]$$

For L = 5 m, $\bar{h} = 5.64(3.66+17.51) = 119\,W/m^2\cdot K$, hence

$$T_{m,o} = 100^\circ C - \left(75^\circ C\right)\exp\left(-\frac{\pi\times 0.025\,m\times 5\,m\times 119\,W/m^2\cdot K}{0.5\,kg/s\times 2035\,J/kg\cdot K}\right) = 28.4^\circ C$$ <

For L = 100 m, $\bar{h} = 5.64(3.66+3.38) = 40\,W/m^2\cdot K$, $T_{m,o}$ = 44.9°C. <

Also, *for* L = 5 m,

$$\Delta T_{\ell m} = \frac{\Delta T_o - \Delta T_i}{\ell n\left(\Delta T_o/\Delta T_i\right)} = \frac{71.6-75}{\ell n\left(71.6/75\right)} = 73.3^\circ C \qquad \Delta T_{am} = \left(\Delta T_o + \Delta T_i\right)/2 = 73.3^\circ C$$ <

For L = 100 m, $\Delta T_{\ell m} = 64.5^\circ C$, $\Delta T_{am} = 65.1^\circ C$ <

(b) The effect of tube length on the outlet temperature and Nusselt number was determined by using the *Correlations* and *Properties* Toolpads of IHT.

Continued...

PROBLEM 8.24 (Cont.)

The outlet temperature approaches the surface temperature with increasing L, but even for L = 100 m, $T_{m,o}$ is well below T_s. Although $\overline{Nu}_D$ decays with increasing L, it is still well above the fully developed value of $Nu_{D,fd} = 3.66$.

COMMENTS: (1) The average, mean temperature, $\overline{T}_m$ = 330 K, was significantly overestimated in part (a). The accuracy may be improved by evaluating the properties at a lower temperature. (2) Use of ΔT_{am} instead of $\Delta T_{\ell m}$ is reasonable for small to moderate values of $(T_{m,i} - T_{m,o})$. For large values of $(T_{m,i} - T_{m,o})$, $\Delta T_{\ell m}$ should be used.

PROBLEM 8.25

KNOWN: Oil at 80°C enters a single-tube preheater of 9-mm diameter and 5-m length; tube surface maintained at 165°C by swirling combustion gases.

FIND: Determine the flow rate and heat transfer rate when the outlet temperature is 95°C.

SCHEMATIC:

$T_s = 165^oC$

Oil

$\dot{m} = ?$

$T_{m,i} = 80^oC$

$T_{m,o} = 95^oC$

x

Tube, D = 9 mm

L = 5 m

ASSUMPTIONS: (1) Combined entry length, laminar flow, (2) Tube wall is isothermal, (3) Negligible kinetic and potential energy, and flow work, (4) Constant properties.

PROPERTIES: *Table A-5*, Engine oil, new ($T_m = (T_{m,i} + T_{m,o})/2 = 361$ K): $\rho = 847.5$ kg/m^3, $c_p = 2163$ J/kg·K, $\nu = 2.931 \times 10^{-5}$ m^2/s, k = 0.1879 W/m·K, Pr = 3902; ($T_s = 430$ K): $\mu_s = 0.047$ N·s/m^2.

ANALYSIS: The overall energy balance, Eq. 8.37, and rate equation, Eq. 8.42b, are

$$q = \dot{m} c_p \left(T_{m,o} - T_{m,i}\right) \tag{1}$$

$$\frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp\left(-\frac{PL\bar{h}}{\dot{m} c_p}\right) \tag{2}$$

Not knowing the flow rate $\dot{m}$, the Reynolds number cannot be calculated. Assume that the flow is laminar, and the combined entry length condition occurs. The average convection coefficient can be estimated using the Sieder-Tate correlation, Eq. 8.57,

$$\overline{Nu}_D = \frac{\bar{h} D}{k} = 1.86\left(\frac{Re_D \, Pr}{L/D}\right)^{1/3}\left(\frac{\mu}{\mu_s}\right)^{0.14} \tag{3}$$

where all properties are evaluated at $T_m = (T_{m,i} + T_{m,o})/2$, except for μ_s at the wall temperature T_s. The Reynolds number follows from Eq. 8.6,

$$Re_D = 4\dot{m}/\pi D \mu \tag{4}$$

A tedious trial-and-error solution is avoided by using *IHT* to solve the system of equations with the following result:

Re_D	$\overline{Nu}_D$	$\bar{h}_D$ (W/m^2·K)	q(W)	$\dot{m}$ (kg/h)	
251	9.54	146	1432	159	<

Note that the flow is laminar, and evaluating x_{fd} using Eq. 8.3, find $x_{fd,h} = 44$ m so the combined entry length condition is appropriate.

PROBLEM 8.26

KNOWN: Ethylene glycol flowing through a coiled, thin walled tube submerged in a well-stirred water bath maintained at a constant temperature.

FIND: Heat rate and required tube length for prescribed conditions.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Tube wall thermal resistance negligible, (3) Convection coefficient on water side infinite; cooling process approximates constant wall surface temperature distribution, (4) KE, PE and flow work changes negligible, (5) Constant properties, (6) Negligible heat transfer enhancement associated with the coiling.

PROPERTIES: *Table A-5*, Ethylene glycol (T_m = (85 + 35)°C/2 = 60°C = 333 K): c_p = 2562 J/kg·K, $\mu = 0.522 \times 10^{-2}$ N·s/m^2, k = 0.260 W/m·K, Pr = 51.3.

ANALYSIS: From an overall energy balance on the tube,

$$q_{conv} = \dot{m}\, c_p \left(T_{m,o} - T_{m,i}\right) = 0.01 \text{ kg/s} \times 2562 \text{ J/kg} \left(35-85\right)^\circ \text{C} = -1281 \text{ W}. \quad (1) <$$

For the constant surface temperature condition, from the rate equation,

$$A_s = q_{conv} / \bar{h} \Delta T_{\ell m} \quad (2)$$

$$\Delta T_{\ell m} = \left(\Delta T_o - \Delta T_i\right) / \ell n \frac{\Delta T_o}{\Delta T_i} = \left[\left(35-25\right)^\circ \text{C} - \left(85-25\right)^\circ \text{C}\right] / \ell n \frac{35-25}{85-25} = 27.9^\circ \text{C}. \quad (3)$$

Find the Reynolds number to determine flow conditions,

$$Re_D = \frac{4\dot{m}}{\pi D \mu} = \frac{4 \times 0.01 \text{ kg/s}}{\pi \times 0.003 \text{ m} \times 0.522 \times 10^{-2} \text{N} \cdot \text{s/m}^2} = 813. \quad (4)$$

Hence, the flow is laminar and, assuming the flow is fully developed, the appropriate correlation is

$$\overline{Nu}_D = \frac{\bar{h} D}{k} = 3.66, \qquad \bar{h} = Nu \frac{k}{D} = 3.66 \times 0.260 \frac{\text{W}}{\text{m} \cdot \text{K}} / 0.003\text{m} = 317 \text{ W/m}^2 \cdot \text{K}. \quad (5)$$

From Eq. (2), the required area, A_s, and tube length, L, are

$$A_s = 1281 \text{ W} / 317 \text{ W/m}^2 \cdot \text{K} \times 27.9^\circ\text{C} = 0.1448 \text{ m}^2$$

$$L = A_s / \pi D = 0.1448\text{m}^2 / \pi \left(0.003\text{m}\right) = 15.4\text{m}. \quad <$$

COMMENTS: Note that for fully developed laminar flow conditions, the requirement is satisfied: Gz^{-1} = (L/D) / Re_D Pr = (15.3/0.003) / (813 × 51.3) = 0.122 > 0.05. Note also the sign of the heat rate q_{conv} when using Eqs. (1) and (2).

PROBLEM 8.27

KNOWN: Inlet and outlet temperatures and velocity of fluid flow in tube. Tube diameter and length.

FIND: Surface heat flux and temperatures at x = 0.5 and 10 m.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Negligible heat loss to surroundings, (4) Negligible potential and kinetic energy changes and axial conduction.

PROPERTIES: Pharmaceutical (given): $\rho = 1000\ \text{kg/m}^3$, $c_p = 4000$ J/kg·K, $\mu = 2 \times 10^{-3}$ kg/s·m, k = 0.48 W/m·K, Pr = 10.

ANALYSIS: With

$$\dot{m} = r\text{VA} = 1000\ \text{kg/m}^3 (0.2\ \text{m/s}) p\ (0.0127\ \text{m})^2 / 4 = 0.0253\ \text{kg/s}$$

Eq. 8.37 yields

$$q = \dot{m}\, c_p \left(T_{m,o} - T_{m,i}\right) = 0.0253\ \text{kg/s}\left(4000\ \text{J/kg}\cdot\text{K}\right) 50\ \text{K} = 5060\ \text{W}.$$

The required heat flux is then

$$q_s'' = q/A_s = 5060\ \text{W}/p\ (0.0127\ \text{m}) 10\ \text{m} = 12{,}682\ \text{W/m}^2. \qquad <$$

With

$$\text{Re}_D = r\text{VD}/m = 1000\ \text{kg/m}^3 (0.2\ \text{m/s}) 0.0127\ \text{m}/2 \times 10^{-3}\ \text{kg/s}\cdot\text{m} = 1270$$

the flow is laminar and Eq. 8.23 yields

$$x_{fd,t} = 0.05 \text{Re}_D\, \text{Pr}\, \text{D} = 0.05(1270)10(0.0127\ \text{m}) = 8.06\ \text{m}.$$

Hence, with fully developed hydrodynamic and thermal conditions at x = 10 m, Eq. 8.53 yields

$$h\,(10\ \text{m}) = \text{Nu}_{D,fd}\,(k/D) = 4.36\,(0.48\ \text{W/m}\cdot\text{K}/0.0127\ \text{m}) = 164.8\ \text{W/m}^2\cdot\text{K}.$$

Hence, from Newton's law of cooling,

$$T_{s,o} = T_{m,o} + \left(q_s''/h\right) = 75^\circ\text{C} + \left(12{,}682\ \text{W/m}^2/164.8\ \text{W/m}^2\cdot\text{K}\right) = 152^\circ\text{C}. \qquad <$$

At x = 0.5 m, $(x/D)/(\text{Re}_D\text{Pr}) = 0.0031$ and Figure 8.9 yields $\text{Nu}_D \approx 8$ for a thermal entry region with uniform surface heat flux. Hence, h(0.5 m) = 302.4 W/m^2·K and, since T_m increases linearly with x, $T_m(x = 0.5\ \text{m}) = T_{m,i} + (T_{m,o} - T_{m,i})\,(x/L) = 27.5^\circ\text{C}$. It follows that

$$T_s\,(x = 0.5\ \text{m}) \approx 27.5^\circ\text{C} + \left(12{,}682\ \text{W/m}^2/302.4\ \text{W/m}^2\cdot\text{K}\right) = 69.4^\circ\text{C}. \qquad <$$

PROBLEM 8.28

KNOWN: Inlet temperature, flow rate and properties of hot fluid. Initial temperature, volume and properties of pharmaceutical. Heat transfer coefficient at outer surface and dimensions of coil.

FIND: (a) Expressions for $T_c(t)$ and $T_{h,o}(t)$, (b) Plots of $T_c(t)$ and $T_{h,o}(t)$ for prescribed conditions. Effect of flow rate on time for pharmaceutical to reach a prescribed temperature.

SCHEMATIC:

$\dot{m}_h = 2.4$ kg/s, $T_{h,i} = 200^oC$

Hot fluid

$c_{p,h} = 2500$ J/kg-K
$\mu_h = 0.002$ N/s-m^2
$k_h = 0.260$ W/m-K
$Pr_h = 20$

$T_{h,o}(t)$

Tubing

D = 0.05 m, L = 40 m
$h_o = 1000$ W/m^2-K

q(t)

$T_c(t)$

Pharmaceutical

$T_{c,i} = 25^oC$, $\forall_c = 1$ m^3
$\rho_c = 1100$ kg/m^3
$c_{v,c} = 2000$ J/kg-K

ASSUMPTIONS: (1) Constant properties, (2) Negligible heat loss from vessel to surroundings, (3) Pharmaceutical is isothermal, (4) Negligible work due to stirring, (5) Negligible thermal energy generation (or absorption) due to chemical reactions associated with the batch process, (6) Negligible kinetic energy, potential energy and flow work changes for the hot fluid, (7) Negligible tube wall conduction resistance.

ANALYSIS: (a) Performing an energy balance for a control surface about the stirred liquid, it follows that

$$\frac{dU_c}{dt} = \frac{d}{dt}\left(\rho_c \forall_c c_{v,c} T_c\right) = \rho_c V_c c_{v,c} \frac{dT_c}{dt} = q(t) \quad (1)$$

where,
$$q(t) = \dot{m}_h c_{p,h}\left(T_{h,i} - T_{h,o}\right) \quad (2)$$

or,
$$q(t) = UA_s \Delta T_{\ell m} \quad (3a)$$

where

$$\Delta T_{\ell m} = \frac{\left(T_{h,i} - T_c\right) - \left(T_{h,o} - T_c\right)}{\ell n\left(\dfrac{T_{h,i} - T_c}{T_{h,o} - T_c}\right)} = \frac{\left(T_{h,i} - T_{h,o}\right)}{\ell n\left(\dfrac{T_{h,i} - T_c}{T_{h,o} - T_c}\right)} \quad (3b)$$

Substituting (3b) into (3a) and equating to (2),

$$\dot{m}_h c_{p,h}\left(T_{h,i} - T_{h,o}\right) = UA_s \frac{\left(T_{h,i} - T_{h,o}\right)}{\ell n\left(\dfrac{T_{h,i} - T_c}{T_{h,o} - T_c}\right)}$$

Hence,
$$\ell n\left(\frac{T_{h,i} - T_c}{T_{h,o} - T_c}\right) = \frac{UA_s}{\dot{m}_h c_{p,h}}$$

or,
$$T_{h,o}(t) = T_c + \left(T_{h,i} - T_c\right)\exp\left(-UA_s / \dot{m}_h c_{p,h}\right) \quad (4)$$ <

Substituting Eqs. (2) and (4) into Eq. (1),

Continued …..

PROBLEM 8.28 (Cont.)

$$\rho_c \forall_c c_{v,c} \frac{dT_c}{dt} = \dot{m}_h c_{p,h} \left[T_{h,i} - T_c - \left(T_{h,i} - T_c \right) \exp\left(-UA_s / \dot{m}_h c_{p,h} \right) \right]$$

$$\frac{dT_c}{dt} = \frac{\dot{m}_h c_{p,h} \left(T_{h,i} - T_c \right)}{\rho_c \forall_c c_{v,c}} \left[1 - \exp\left(-UA_s / \dot{m}_h c_{p,h} \right) \right]$$

$$-\int_{T_{c,i}}^{T_c(t)} \frac{dT_c}{\left(T_c - T_{h,i} \right)} = \frac{\dot{m}_h c_{p,h}}{\rho_c \forall_c c_{v,c}} \left[1 - \exp\left(-UA_s / \dot{m}_h c_{p,h} \right) \right] \int_o^t dt$$

$$-\ell n \left(\frac{T_c - T_{h,i}}{T_{c,i} - T_{h,i}} \right) = \frac{\dot{m}_h c_{p,h}}{\rho_c V_c c_{v,c}} \left[1 - \exp\left(-UA_s / \dot{m}_h c_{p,h} \right) \right] t$$

$$T_c(t) = T_{h,i} - \left(T_{h,i} - T_{c,i} \right) \exp\left\{ -\frac{\dot{m}_h c_{p,h} \left[1 - \exp\left(-UA / \dot{m}_h c_{p,h} \right) \right] t}{\rho_c \forall_c c_{v,c}} \right\} \qquad (5) \quad <$$

Eq. (5) may be used to determine $T_c(t)$ and the result used with (4) to determine $T_{h,o}(t)$.

(b) To evaluate the temperature histories, the overall heat transfer coefficient, $U = \left(h_o^{-1} + h_i^{-1} \right)^{-1}$, must first be determined. With $Re_D = 4\dot{m} / \pi D \mu = 4 \times 2.4\,kg/s / \pi \left(0.05m \right) 0.002\,N \cdot s/m^2 = 30,600$, the flow is turbulent and

$$h_i = \frac{k}{D} Nu_D = \frac{0.260\,W/m \cdot K}{0.05m} \left[0.023 \left(30,600 \right)^{4/5} \left(20 \right)^{0.3} \right] = 1140\,W/m^2 \cdot K$$

Hence, $U = \left[\left(1000 \right)^{-1} + \left(1140 \right)^{-1} \right]^{-1} W/m^2 \cdot K = 532\,W/m^2 \cdot K$. As shown below, the temperature of the pharmaceuticals increases with time due to heat transfer from the hot fluid, approaching the inlet temperature of the hot fluid (and its maximum possible temperature of 200°C) at t = 3600s.

Temperature(C)

Time(s)

Pharmaceutical, Tc
Hot fluid, Th

Continued …..

PROBLEM 8.28 (Cont.)

With increasing T_c, the rate of heat transfer from the hot fluid decreases (from 4.49×10^5 W at t = 0 to 6760 W at 3600s), in which case $T_{h,o}$ increases (from 125.2°C at t = 0 to 198.9°C at 3600s). The time required for the pharmaceuticals to reach a temperature of T_c = 160°C is

$$t_c = 1266\text{s}$$ <

With increasing $\dot{m}_h$, the overall heat transfer coefficient increases due to increasing h_i and the hot fluid maintains a higher temperature as it flows through the tube. Both effects enhance heat transfer to the pharmaceutical, thereby reducing the time to reach 160°C from 2178s for $\dot{m}_h = 1\,\text{kg/s}$ to 906s at 5 kg/s.

For $1 \le \dot{m}_h \le 5\,\text{kg/s},\ 12,700 \le Re_D \le 63,700$ and $565 \le h_i \le 2050\,\text{W/m}^2 \cdot \text{K}$.

COMMENTS: Although design changes involving the length and diameter of the coil can be used to alter the heating rate, process control parameters are limited to $T_{h,i}$ and $\dot{m}_h$.

PROBLEM 8.29

KNOWN: Tubing with glycerin welded to transformer lateral surface to remove dissipated power. Maximum allowable temperature rise of coolant is 6°C.

FIND: (a) Required coolant rate $\dot{m}$, tube length L and lateral spacing S between turns, and (b) Effect of flowrate on outlet temperature and maximum power.

SCHEMATIC:

Transformer, 1000 W
Glycerin, $T_{m,i}$ = 24 °C
T_s = 47 °C
S
H = 500 mm
D = 20 mm
D = 300 mm

ASSUMPTIONS: (1) Steady-state conditions, (2) All heat dissipated by transformer transferred to glycerin, (3) Fully developed flow (part a), (4) Negligible kinetic and potential energy changes, (5) Negligible tube wall thermal resistance.

PROPERTIES: *Table A.5*, Glycerin ($\overline{T}_m \approx 300$ K): ρ = 1259.9 kg/m^3, c_p = 2427 J/kg·K, μ = 79.9 × 10^{-2} N·s/m^2, k = 286 × 10^{-3} W/m·K, Pr = 6780.

ANALYSIS: (a) From an overall energy balance assuming the maximum temperature rise of the glycerin coolant is 6°C, find the flow rate as

$$q = \dot{m}c_p\left(T_{m,o} - T_{m,i}\right) \qquad \dot{m} = q/c_p\left(T_{m,o} - T_{m,i}\right) = 1000\,\text{W}/2427\,\text{J/kg}\cdot\text{K}\,(6\,\text{K}) = 6.87\times10^{-2}\ \text{kg/s} \quad <$$

From Eq. 8.43, the length of tubing can be determined,

$$\frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp\left(-PL\overline{h}/\dot{m}c_p\right)$$

where P = πD. For the tube flow, find

$$Re_D = \frac{4\dot{m}}{\pi D\mu} = \frac{4\times6.87\times10^{-2}\ \text{kg/s}}{\pi\times0.020\,\text{m}\times79.9\times10^{-2}\ \text{N}\cdot\text{s/m}^2} = 5.47$$

which implies laminar flow, and if fully developed,

$$\overline{Nu}_D = \frac{\overline{h}D}{k} = 3.66 \qquad \overline{h} = \frac{3.66\times286\times10^{-3}\ \text{W/m}\cdot\text{K}}{0.020\,\text{m}} = 52.3\,\text{W/m}^2\cdot\text{K}$$

$$\frac{(47-30)^\circ\text{C}}{(47-24)^\circ\text{C}} = \exp\left[-\left(\pi\,(0.020\,\text{m})\times52.3\,\text{W/m}^2\cdot\text{K}\times L\right)\Big/\left(6.87\times10^{-2}\ \text{kg/s}\times2427\,\text{J/kg}\cdot\text{K}\right)\right]$$

L = 15.3 m. <

The number of turns of the tubing, N, is N = L/(πD) = (15.3 m)/π(0.3 m) = 16.2 and hence the spacing S will be

S = H/N = 500 mm/16.2 = 30.8 mm. <

Continued...

PROBLEM 8.29 (Cont.)

(b) Parametric calculations were performed using the IHT *Correlations* Toolpad based on Eq. 8.56 (a thermal entry length condition), and the following results were obtained.

Heat rate, q(W)
3000
2600
2200
1800
1400
1000
0.05 0.09 0.13 0.17 0.21 0.25
Mass flowrate, mdot(kg/s)

Outlet temperature, Tmo(C)
35
33
31
29
27
25
0.05 0.09 0.13 0.17 0.21 0.25
Mass flowrate, mdot(kg/s)

With T_s maintained at 47°C, the maximum allowable transformer power (heat rate) and glycerin outlet temperature increase and decrease, respectively, with increasing $\dot{m}$. The increase in q is due to an increase in $\overline{Nu}_D$ (and hence $\overline{h}$) with increasing Re_D. The value of $\overline{Nu}_D$ increased from 5.3 to 9.4 with increasing $\dot{m}$ from 0.05 to 0.25 kg/s.

COMMENTS: Since $Gz_D^{-1} = (L/D)/Re_D\,Pr$ = (15.3 m/0.02 m)/(5.47 × 6780) = 0.0206 < 0.05, entrance length effects are significant, and Eq. 8.56 should be used to determine $\overline{Nu}_D$.

PROBLEM 8.30

KNOWN: Diameter and length of copper tubing. Temperature of collector plate to which tubing is soldered. Water inlet temperature and flow rate.

FIND: (a) Water outlet temperature and heat rate, (b) Variation of outlet temperature and heat rate with flow rate. Variation of water temperature along tube for the smallest and largest flowrates.

SCHEMATIC:

Water; $D = 0.01$ m; $T_s = 70\ ^oC$; $T_{m,i} = 25\ ^oC$; $0.005 \le \dot{m} \le 0.05$ kg/s; $T_{m,o}$; x; $L = 8$ m

ASSUMPTIONS: (1) Straight tube with smooth surface, (2) Negligible kinetic/potential energy and flow work changes, (3) Negligible thermal resistance between plate and tube inner surface, (4) $Re_{D,c}$ = 2300.

PROPERTIES: *Table A.6*, water (assume $\bar{T}_m = (T_{m,i} + T_s)/2 = 47.5°C = 320.5$ K): $\rho = 986$ kg/m^3, c_p = 4180 J/kg·K, $\mu = 577 \times 10^{-6}$ N·s/m^2, k = 0.640 W/m·K, Pr = 3.77. *Table A.6*, water (T_s = 343 K): μ_s = 400×10^{-6} N·s/m^2.

ANALYSIS: (a) For $\dot{m}$ = 0.01 kg/s, $Re_D = 4\ \dot{m}/\pi D\mu$ = 4(0.01 kg/s)/π(0.01 m)577 × 10^{-6} N·s/m^2 = 2200, in which case the flow may be assumed to be laminar. With $x_{fd,t}/D \approx 0.05 Re_D Pr$ = 0.05(2200)(3.77) = 415 and L/D = 800, the flow is fully developed over approximately 50% of the tube length. With $\left[Re_D\, Pr/(L/D)\right]^{1/3} (\mu/\mu_s)^{0.14}$ = 2.30, Eq. 8.57 may therefore be used to compute the average convection coefficient

$$\overline{Nu}_D = 1.86\left(\frac{Re_D\, Pr}{L/D}\right)^{1/3}\left(\frac{\mu}{\mu_s}\right)^{0.14} = 4.27$$

$$\bar{h} = (k/D)\overline{Nu}_D = 4.27(0.640\,W/m\cdot K)/0.01m = 273\,W/m^2\cdot K$$

From Eq. 8.42b,

$$\frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp\left(-\frac{\pi DL}{\dot{m}c_p}\bar{h}\right) = \exp\left(-\frac{\pi \times 0.01m \times 8m \times 273\,W/m^2\cdot K}{0.01kg/s \times 4180\,J/kg\cdot K}\right)$$

$$T_{m,o} = T_s - 0.194(T_s - T_{m,i}) = 70^\circ C - 8.7^\circ C = 61.3^\circ C \qquad <$$

Hence, $q = \dot{m}c_p\left(T_{m,o} - T_{m,i}\right) = 0.01kg/s\left(4186\,J/kg\cdot K\right)\left(36.3K\right) = 1519\,W \qquad <$

(b) The IHT *Correlations*, *Rate Equations* and *Properties* Tool Pads were used to determine the parametric variations. The effect of $\dot{m}$ was considered in two steps, the first corresponding to $\dot{m}$ < 0.011 kg/s (Re_D < 2300) and the second for $\dot{m}$ > 0.011 kg/s (Re_D > 2300). In the first case, Eq. 8.57 was used to determine $\bar{h}$, while in the second Eq. 8.60 was used. The effects of $\dot{m}$ are as follows.

Continued...

PROBLEM 8.30 (Cont.)

Outlet temperature, Tmo(C)
Mass flowrate, mdot(kg/s)
Laminar flow (ReD < 2300)

Outlet temperature, Tmo(C)
Mass flowrate, mdot(kg/s)
Turbulent flow (ReD>2300)

Heat rate, q(W)
Mass flowrate, mdot(kg/s)
Laminar flow (ReD < 2300)

Heat rate, q(W)
Mass flowrate, mdot(kg/s)
Turbulent flow (ReD>2300)

The outlet temperature decreases with increasing $\dot{m}$, although the effect is more pronounced for laminar flow. If q were independent of $\dot{m}$, ($T_{m,o}$ - $T_{m,i}$) would decrease inversely with increasing $\dot{m}$. In turbulent flow, however, the convection coefficient, and hence the heat rate, increases approximately as $\dot{m}^{0.8}$, thereby attenuating the foregoing effect. In laminar flow, q ~ $\dot{m}^{0.5}$ and this attenuation is not as pronounced.

The temperature distributions were computed from Eq. 8.43, with $\overline{h}$ assumed to be independent of x. For laminar flow ($\dot{m}$ = 0.005 kg/s), $\overline{h}$ was based on the entire tube length (L = 8 m) and computed from Eq. 8.57, while for turbulent flow ($\dot{m}$ = 0.05 kg/s) it was assumed to correspond to the value for fully developed flow and computed from Eq. 8.60. The corresponding temperature distributions are as follows.

Continued...

PROBLEM 8.30 (Cont.)

Mean temperature, Tm(C)

Axial location, x(m)

mdot = 0.005 kg/s

Mean temperature, Tm(C)

Axial location, x(m)

mdot = 0.05 kg/s

The more pronounced increase for turbulent flow is due to the much larger value of $\overline{h}$ (4300 $W/m^2 \cdot K$ for $\dot{m}$ = 0.05 kg/s relative to 217 $W/m^2 \cdot K$ for $\dot{m}$ = 0.05 kg/s).

PROBLEM 8.31

KNOWN: Diameter and surface temperature of ten tubes in an ice bath. Inlet temperature and flowrate per tube. Volume (∀) of container and initial volume fraction, $f_{v,i}$, of ice.

FIND: (a) Tube length required to achieve a prescribed air outlet temperature $T_{m,o}$ and time to completely melt the ice, (b) Effect of mass flowrate on $T_{m,o}$ and suitable design and operating conditions.

SCHEMATIC:

Ice bath
$\forall = 10\ m^3$, $f_{v,i} = 0.8$
Air
$D = 0.05$ m
$T_s = 0\ ^oC$
$N = 10$ tubes
$T_{m,i} = 24\ ^oC$
$0.005 \le \dot{m} \le 0.05$ kg/s
$T_{m,o}$
L

ASSUMPTIONS: (1) Steady-state, (2) Negligible kinetic/potential energy and flow work changes, (3) Constant properties, (4) Fully developed flow throughout each tube, (5) Negligible tube wall thermal resistance.

PROPERTIES: *Table A.4*, air (assume $\overline{T}_m$ = 292 K): c_p = 1007 J/kg·K, $\mu = 180.6 \times 10^{-7}$ N·s/m^2, k = 0.0257 W/m·K, Pr = 0.709; Ice: ρ = 920 kg/m^3, $h_{sf} = 3.34 \times 10^5$ J/kg.

ANALYSIS: (a) With $Re_D = 4\dot{m}/\pi D\mu = 4(0.01\ kg/s)/\pi(0.05\ m)180.6 \times 10^{-7}\ N\cdot s/m^2 = 14,100$ for $\dot{m}$ = 0.01 kg/s, the flow is turbulent, and from Eq. 8.60,

$$\overline{Nu}_D = Nu_D = 0.023 Re_D^{0.8} Pr^{0.3} = 0.023(14,100)^{0.8}(0.709)^{0.3} = 43.3$$

$$\overline{h} = \overline{Nu}_D (k/D) = 43.3(0.0257\,W/m\cdot K/0.05\,m) = 22.2\,W/m^2\cdot K$$

With $T_{m,o}$ = 14°C, the tube length may be obtained from Eq. 8.42b,

$$\frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \frac{-14}{-24} = \exp\left(-\frac{\pi D L \overline{h}}{\dot{m} c_p}\right) = \exp\left[-\frac{\pi(0.05\,m)(22.2\,W/m^2\cdot K)L}{0.01\,kg/s(1007\,J/kg\cdot K)}\right]$$

$$L = 1.56\ m$$ <

The time required to completely melt the ice may be obtained from an energy balance of the form,

$$(-q)t = f_{v,i}\forall(\rho h_{sf})$$

where $q = N\dot{m}c_p(T_{m,i} - T_{m,o}) = 10(0.01\,kg/s)1007\,J/kg\cdot K(10\,K) = 1007\,W$. Hence,

$$t = \frac{0.8(10\,m^3)(920\,kg/m^3)3.34\times10^5\ J/kg}{1007\,W} = 2.44\times10^6 s = 28.3\,days$$ <

(b) Using the appropriate IHT Correlations and Properties Tool Pads, the following results were obtained.

Continued...

Outlet temperature, Tmo(C)
17
16
15
14
13
12
0 0.01 0.02 0.03 0.04 0.05
Mass flowrate per tube, mdot(kg/s)

Although heat extraction from the air passing through each tube increases with increasing flowrate, the increase is not in proportion to the change in $\dot{m}$ and the temperature difference ($T_{m,i}$ - $T_{m,o}$) decreases. If 0.05 kg/s of air is routed through a single tube, the outlet temperature of $T_{m,o}$ = 16.2°C slightly exceeds the desired value of 16°C. The prescribed value could be achieved by slightly increasing the tube length. However, in the interest of reducing pressure drop requirements, it would be better to operate at a lower flowrate per tube. If, for example, air is routed through four of the tubes at 0.01 kg/s per tube and the discharge is mixed with 0.01 kg/s of the available air at 24°C, the desired result would be achieved.

COMMENTS: Since the flow is turbulent and L/D = 31, the assumption of fully developed flow throughout a tube is marginal and the foregoing analysis overestimates the discharge temperature.

PROBLEM 8.32

KNOWN: Thermal conductivity and inner and outer diameters of plastic pipe. Volumetric flow rate and inlet and outlet temperatures of air flow through pipe. Convection coefficient and temperature of water.

FIND: Pipe length and fan power requirement.

SCHEMATIC:

Water $T_\infty = 17^oC$; $h_o = 1500$ W/m²-K; $D_o = 0.17$ m; $D_i = 0.15$ m; $T_{m,o} = 21^oC$; Air; $\dot{\forall}_i = 0.025$ m³/s; $T_{m,i} = 29^oC$; L; Plastic pipe, k = 0.15 W/m-K

ASSUMPTIONS: (1) Steady-state, (2) Negligible heat transfer from air in vertical legs of pipe, (3) Negligible flow work and potential and kinetic energy changes for air flow through pipe, (4) Smooth interior surface, (5) Constant properties.

PROPERTIES: *Table A-4*, Air ($T_{m,i}$ = 29°C): $\rho_i = 1.155\,kg/m^3$. Air $\left(\overline{T}_m = 25°C\right)$: c_p = 1007 J/kg·K, $\mu = 183.6 \times 10^{-7}$ N·s/m², k_a = 0.0261 W/m·K, Pr = 0.707.

ANALYSIS: From Eq. (8.46a)

$$\frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = \exp\left(-\frac{\overline{U}A_s}{\dot{m}\,c_p}\right)$$

where, from Eq. (3.32), $\left(\overline{U}A_s\right)^{-1} = R_{tot} = \dfrac{1}{\overline{h}_i \pi D_i L} + \dfrac{\ln\left(D_o/D_i\right)}{2\pi Lk} + \dfrac{1}{h_o \pi D_o L}$

With $\dot{m} = \rho_i \dot{\forall}_i = 0.0289\,kg/s$ and $Re_D = 4\dot{m}/\pi D_i \mu = 13,350$, flow in the pipe is turbulent. Assuming fully developed flow throughout the pipe, and from Eq. (8.60),

$$\overline{h}_i = \frac{k_a}{D_i} 0.023\, Re_D^{4/5}\, Pr^{0.3} = \frac{0.0261\,W/m\cdot K \times 0.023}{0.15m}(13,350)^{4/5}(0.707)^{0.3} = 7.20\,W/m^2\cdot K$$

$$\left(\overline{U}A_s\right)^{-1} = \frac{1}{L}\left(\frac{1}{7.21\,W/m^2\cdot K \times \pi \times 0.15m} + \frac{\ln(0.17/0.15)}{2\pi \times 0.15\,W/m\cdot K} + \frac{1}{1500\,W/m^2\cdot K \times \pi \times 0.17m}\right)$$

$$\overline{U}A_s = \frac{L}{(0.294 + 0.133 + 0.001)} = 2.335\,L\;\;W/K$$

$$\frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = \frac{17-21}{17-29} = 0.333 = \exp\left(-\frac{2.335\,L}{0.0289\,kg/s \times 1007\,J/kg\cdot K}\right) = \exp(-0.0802)$$

$$L = -\frac{\ln(0.333)}{0.0802} = 13.7m$$ <

From Eqs. (8.22a) and (8.22b) and with $u_{m,i} = \dot{\forall}_i / \left(\pi D_i^2/4\right) = 1.415\,m/s$, the fan power is

$$P = (\Delta p)\dot{\forall} \approx f\,\frac{\rho_i u_{m,i}^2}{2D_i} L\,\dot{\forall}_i = 0.0294\,\frac{1.155\,kg/m^3\,(1.415\,m/s)^2}{2(0.15m)} 13.7m \times 0.025\,m^3/s = 0.078\,W$$ <

where $f = 0.316\,Re_D^{-1/4} = 0.0294$ from Eq. (8.20a).

COMMENTS: (1) With L/Di = 91, the assumption of fully developed flow throughout the pipe is justified. (2) The fan power requirement is small, and the process is economical. (3) The resistance to heat transfer associated with convection at the outer surface is negligible.

PROBLEM 8.33

KNOWN: Flow rate, inlet temperature and desired outlet temperature of water passing through a tube of prescribed diameter and surface temperature.

FIND: (a) Required tube length, L, for prescribed conditions, (b) Required length using tube diameters over the range $30 \le D \le 50$ mm with flow rates $\dot{m}$ = 1, 2 and 3 kg/s; represent this design information graphically, and (c) Pressure gradient as a function of tube diameter for the three flow rates assuming the tube wall is smooth.

SCHEMATIC:

T_s = 100 °C
$T_{m,o}$ = 75 °C
D = 0.04 m
L
Water
$\dot{m}$ = 2 kg/s
$T_{m,i}$ = 25 °C

ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible potential energy, kinetic energy and flow work changes, (3) Constant properties.

PROPERTIES: *Table A.6*, Water ($\overline{T}_m$ = 323 K): c_p = 4181 J/kg·K, $\mu = 547 \times 10^{-6}$ N·s/m^2, k = 0.643 W/m·K, Pr = 3.56.

ANALYSIS: (a) From Eq. 8.6, the Reynolds number is

$$Re_D = \frac{4\dot{m}}{\pi D \mu} = \frac{4 \times 2\,kg/s}{\pi (0.04\,m) 547 \times 10^{-6}\,N \cdot s/m^2} = 1.16 \times 10^5 . \qquad (1)$$

Hence the flow is turbulent, and assuming fully developed conditions throughout the tube, it follows from the Dittus-Boelter correlation, Eq. 8.60,

$$\overline{h} = \frac{k}{D} 0.023\, Re_D^{4/5}\, Pr^{0.4} = \frac{0.643\,W/m \cdot K}{0.04\,m} 0.023 \left(1.16 \times 10^5\right)^{4/5} (3.56)^{0.4} = 6919\,W/m^2 \cdot K \qquad (2)$$

From Eq. 8.42a, we then obtain

$$L = \frac{-\dot{m} c_p \ell n \left(\Delta T_o / \Delta T_i\right)}{\pi D \overline{h}} = -\frac{2\,kg/s\left(4181\,J/kg \cdot K\right) \ell n \left(25^\circ C / 75^\circ C\right)}{\pi (0.04\,m) 6919\,W/m^2 \cdot K} = 10.6\,m .$$

<

(b) Using the *IHT Correlations Tool*, *Internal Flow*, for fully developed *Turbulent Flow*, along with appropriate energy balance and rate equations, the required length L as a function of flow rate is computed and plotted on the right.

Tube length, L (m)
15
10
5
30 35 40 45 50
Tube diameter, D (mm)
Flow rate, mdot = 1 kg/s
mdot = 2 kg/s
mdot = 3 kg/s

Continued...

(c) From Eq. 8.22a the pressure drop is

$$\frac{\Delta p}{\Delta x} = f\,\frac{\rho u_m^2}{2D} \qquad (4)$$

The friction factor, f, for the smooth surface condition, Eq. 8.21 with $3000 \le Re_D \le 5 \times 10^6$, is

$$f = \left(0.790\,\ell n\left(Re_D\right) - 1.64\right)^{-2} \qquad (5)$$

Using IHT with these equations and Eq. (1), the pressure gradient as a function of diameter for the selected flow rates is computed and plotted on the right.

COMMENTS: (1) Since L/D = (10.6/0.040) = 265, the assumption of fully developed conditions throughout is justified.

(2) The IHT Workspace used to generate the graphical results are shown below.

```
// Rate Equation Tool - Tube Flow with Constant Surface Temperature:
/* For flow through a tube with a uniform wall temperature, Fig 8.7b, the
overall energy balance and heat rate equations  are */
q = mdot*cp*(Tmo - Tmi)    // Heat rate, W; Eq 8.37
(Ts - Tmo) / (Ts - Tmi) = exp ( - P * L * hDbar / (mdot * cp))          // Eq 8.42b
// where the fluid and constant tube wall temperatures are
Ts =  100 + 273             // Tube wall temperature, K
Tmi = 25 + 273                      // Inlet mean fluid temperature, K
Tmo = 75 + 273              // Outlet mean fluid temperature, K
// The tube parameters are
P = pi * D                  // Perimeter, m
Ac = pi * (D^2) / 4         // Cross sectional area, m^2
D = 0.040                   // Tube diameter, m
D_mm = D * 1000
// The tube mass flow rate and fluid thermophysical properties are
mdot = rho * um * Ac
mdot = 1                    // Mass flow rate, kg/s

// Correlation Tool - Internal Flow, Fully Developed Turbulent Flow (Assumed):
NuDbar = NuD_bar_IF_T_FD(ReD,Pr,n)     // Eq 8.60
n = 0.4   // n = 0.4 or 0.3 for Ts>Tm or Ts<Tm
NuDbar = hDbar * D / k
ReD = um * D / nu
/* Evaluate properties at the fluid average mean temperature, Tmbar. */
Tmbar = Tfluid_avg (Tmi,Tmo)

// Properties Tool - Water:
// Water property functions :T dependence, From Table A.6
// Units: T(K), p(bars);
x = 0                  // Quality (0=sat liquid or 1=sat vapor)
rho = rho_Tx("Water",Tmbar,x)        // Density, kg/m^3
cp = cp_Tx("Water",Tmbar,x)          // Specific heat, J/kg·K
nu = nu_Tx("Water",Tmbar,x)          // Kinematic viscosity, m^2/s
k = k_Tx("Water",Tmbar,x)  // Thermal conductivity, W/m·K
Pr = Pr_Tx("Water",Tmbar,x)          // Prandtl number

// Pressure Gradient, Equations 8.21, 8.22a:
dPdx = f * rho * um^2 / ( 2 * D )
f = ( 0.790 * ln (ReD) - 1.64 ) ^ -2
```

PROBLEM 8.34

KNOWN: Flow rate and inlet temperature of water passing through a tube of prescribed length, diameter and surface temperature.

FIND: (a) Outlet water temperature and rate of heat transfer to water for prescribed conditions, and (b) Compute and plot the required tube length L to achieve $T_{m,o}$ found in part (a) as a function of the surface temperature for the range $85 \le T_s \le 95°C$.

SCHEMATIC:

q
T_s = 90 °C
$T_{m,o}$
D = 0.04 m
Water
$\dot{m}$ = 2 kg/s
$T_{m,i}$ = 25 °C
L = 4m

ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Negligible kinetic energy, potential energy and flow work effects, (4) Fully developed flow conditions.

PROPERTIES: *Table A.6*, Water ($\overline{T}_m \approx 325$ K): $c_p = 4182$ J/kg·K, $\mu = 528 \times 10^{-6}$ N·s/m^2, k = 0.645 W/m·K, Pr = 3.42.

ANALYSIS: (a) From Eq. 8.6, the Reynolds number is

$$Re_D = \frac{4\dot{m}}{\pi D \mu} = \frac{4 \times 2\,kg/s}{\pi\left(0.04\,m\right)528 \times 10^{-6}\,N \cdot s/m^2} = 1.21 \times 10^5 .$$

Hence the flow is turbulent, and assuming fully developed conditions throughout the tube, it follows from the Dittus-Boelter correlation, Eq. 8.60,

$$\overline{h} = \frac{k}{D} 0.023\,Re_D^{4/5}\,Pr^{0.4} = \frac{0.645\,W/m \cdot K}{0.04\,m} 0.023\left(1.21 \times 10^5\right)^{4/5}\left(3.42\right)^{0.4} = 7064\,W/m^2 \cdot K .$$

From the energy balance relation, Eq. 8.42b,

$$T_{m,o} = T_s - \left(T_s - T_{m,i}\right)\exp\left(-\frac{\pi D L}{\dot{m} c_p}\overline{h}\right)$$

$$T_{m,o} = 90°C - \left(90°C - 25°C\right)\exp\left(-\frac{\pi \times 0.04\,m \times 4\,m}{2\,kg/s \times 4182\,J/kg \cdot K} 7064\,W/m^2 \cdot K\right) = 47.5°C$$ <

From the overall energy balance, Eq. 8.37,

$$q = \dot{m} c_p\left(T_{m,o} - T_{m,i}\right) = 2\,kg/s \times 4182\,J/kg \cdot K\left(47.5 - 25\right)°C = 188\,kW .$$ <

(b) Using the *IHT Correlations Tool*, *Internal Flow*, for fully developed *Turbulent Flow*, along with the energy balance and rate equations used above, the required length, L, to achieve $T_{m,o} = 44.9°C$ (see comment 1 below) as a function of tube surface temperature is computed and plotted below.

Continued...

From the plot, the outlet temperature increases nearly linearly with the surface temperature. The convection coefficient and heat rate show similar behavior for this range of conditions.

COMMENTS: (1) The mean temperature T_m = 325 K was overestimated in part (a). Another iteration is recommended and the results with $\overline{T}_m$ = 309 K are: $\overline{h}$ = 6091 W/m^2·K, $T_{m,o}$ = 44.9°C and q = 167 kW.

(2) The *IHT Workspace* used to generate the graphical results are shown below.

```
// Rate Equation Tool - Tube Flow with Constant Surface Temperature:
/* For flow through a tube with a uniform wall temperature, Fig 8.7b, the overall energy balance and heat
rate equations  are */
q = mdot*cp*(Tmo - Tmi)                          // Heat rate, W; Eq 8.37
(Ts - Tmo) / (Ts - Tmi) = exp ( - P * L * hDbar / (mdot * cp))          // Eq 8.42b
// where the fluid and constant tube wall temperatures are
Ts =  90 + 273              // Tube wall temperature, K
Ts_C = Ts - 273
Tmi = 25 + 273              // Inlet mean fluid temperature, K
//Tmo =                     // Outlet mean fluid temperature, K
Tmo_C = Tmo - 273
// The tube parameters are
P = pi * D                  // Perimeter, m
Ac = pi * (D^2) / 4         // Cross sectional area, m^2
D = 0.040                   // Tube diameter, m
D_mm = D * 1000
L =  4                      // Tube length, m; unknown
// The tube mass flow rate and fluid thermophysical properties are
mdot = rho * um * Ac
mdot = 2                    // Mass flow rate, kg/s

 // Correlation Tool - Internal Flow, Fully Developed Turbulent Flow (Assumed):
NuDbar = NuD_bar_IF_T_FD(ReD,Pr,n)     // Eq 8.60
n = 0.4   // n = 0.4 or 0.3 for Ts>Tm or Ts<Tm
NuDbar = hDbar * D / k
ReD = um * D / nu
/* Evaluate properties at the fluid average mean temperature, Tmbar. */
Tmbar = Tfluid_avg (Tmi,Tmo)

// Properties Tool - Water:
// Water property functions :T dependence, From Table A.6
// Units: T(K), p(bars);
x = 0                               // Quality (0=sat liquid or 1=sat vapor)
rho = rho_Tx("Water",Tmbar,x)       // Density, kg/m^3
cp = cp_Tx("Water",Tmbar,x)         // Specific heat, J/kg·K
mu = mu_Tx("Water",Tmbar,x)         // Viscosity, N·s/m^2
nu = nu_Tx("Water",Tmbar,x)         // Kinematic viscosity, m^2/s
k = k_Tx("Water",Tmbar,x)           // Thermal conductivity, W/m·K
Pr = Pr_Tx("Water",Tmbar,x)         // Prandtl number
```

PROBLEM 8.35

KNOWN: Diameters and thermal conductivity of steel pipe. Temperature and velocity of water flow in pipe. Temperature and velocity of air in cross flow over pipe. Cost of producing hot water.

FIND: Daily cost of heat loss per unit length of pipe.

SCHEMATIC:

Air: $T_\infty = -5^oC$, $V = 3$ m/s
Water: $T_m = 50^oC$, $u_m = 0.5$ m/s
Steel pipe: $D_o = 100$ mm, $D_i = 84$ mm, $k_p = 60$ W/m-K
Thermal circuit: T_m — $R'_{cnv,w}$ — R'_{cnd} — $R'_{cnv,a}$ — T_∞ → q'

ASSUMPTIONS: (1) Steady state, (2) Constant properties, (3) Negligible radiation from outer surface, (4) Fully-developed flow in pipe.

PROPERTIES: *Table A-4*, air (p = 1 atm, $T_f \approx 300K$): $k_a = 0.0263$ W/m·K, $\nu_a = 15.89 \times 10^{-6}$ m^2/s, $Pr_a = 0.707$. *Table A-6*, water ($T_m = 323$ K): $\rho_w = 988$ kg/m^3, $\mu_w = 548 \times 10^{-6}$ N·s/m^2, $k_w = 0.643$ W/m·K, $Pr_w = 3.56$.

ANALYSIS: The heat loss per unit length of pipe is

$$q' = \frac{T_m - T_\infty}{R'_{cnv,w} + R'_{cnd} + R'_{cnv,a}} = \frac{T_m - T_\infty}{(h_w \pi D_i)^{-1} + \frac{\ln(D_o/D_i)}{2\pi k_p} + (h_a \pi D_o)^{-1}}$$

With $Re_{D,w} = \rho_w u_m D_i / \mu_w = 988\,kg/m^3 \times 0.5\,m/s \times 0.084\,m / 548 \times 10^{-6}\,N\cdot s/m^2 = 75,700$, flow is turbulent, and for fully developed conditions, the Dittus-Boelter correlation yields

$$h_w = \frac{k_w}{D_i} 0.023 Re_{D_w}^{0.8} Pr_w^{0.3} = 0.023 \frac{0.643\,W/m\cdot K}{0.084\,m} (75,700)^{0.8} (3.56)^{0.3} = 2060\,W/m^2\cdot K$$

With $Re_{D,a} = VD_o/\nu_a = 3\,m/s \times (0.1m)/15.89 \times 10^{-6} m^2/s = 18,880$, the Churchill-Bernstein correlation yields

$$h_a = \bar{h} = \frac{k_a}{D_o}\left\{0.3 + \frac{0.62 Re_{D,a}^{1/2} Pr_a^{1/3}}{\left[1 + (0.4/Pr_a)^{2/3}\right]^{1/4}}\left[1 + \left(\frac{Re_{D,w}}{282,000}\right)^{5/8}\right]^{4/5}\right\} = 20.1\,W/m^2\cdot K$$

Hence, $$q' = \frac{50°C - (-5°C)}{\left(1.84\times10^{-3} + 0.46\times10^{-3} + 158.3610^{-3}\right)K/W} = 342\,W/m = 0.342\,kW/m$$

The daily energy loss is then $$Q' = 0.346\,kW/m \times 24\,h/d = 8.22\,kW\cdot h/d\cdot m$$

and the associated cost is $$C' = (8.22\,kW\cdot h/d\cdot m)(\$0.05/kW\cdot h) = \$0.411/m\cdot d$$ <

COMMENTS: Because $R'_{cnv,a} >> R'_{cnv,w}$, the convection resistance for the water side of the pipe could have been neglected, with negligible error. The implication is that the temperature of the pipe's inner surface closely approximates that of the water. If $R'_{cnv,w}$ is neglected, the heat loss is $q' = 346\,W/m$.

PROBLEM 8.36

KNOWN: Inner and outer diameter of a steel pipe insulated on the outside and experiencing uniform heat generation. Flow rate and inlet temperature of water flowing through the pipe.

FIND: (a) Pipe length required to achieve desired outlet temperature, (b) Location and value of maximum pipe temperature.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Negligible kinetic energy, potential energy and flow work changes, (4) One-dimensional radial conduction in pipe wall, (5) Outer surface in adiabatic.

PROPERTIES: *Table A-1*, Stainless steel 316 ($T \approx 400K$): $k = 15$ W/m·K; *Table A-6*, Water $(\bar{T}_m = 303K)$: $c_p = 4178$ J/kg·K, $k = 0.617$ W/m·K, $\mu = 803 \times 10^{-6}$ N·s/m^2, Pr = 5.45.

ANALYSIS: (a) Performing an energy balance for a control volume about the inner tube, it follows that

$$\dot{m}\, c_p \left(T_{m,o} - T_{m,i}\right) = q = \dot{q}(p/4)\left(D_o^2 - D_i^2\right)L$$

$$L = \frac{\dot{m}\, c_p \left(T_{m,o} - T_{m,i}\right)}{\dot{q}(p/4)\left(D_o^2 - D_i^2\right)} = \frac{(0.1\text{ kg/s})4178(\text{J/kg}\cdot\text{K})20^\circ\text{C}}{10^6\text{ W/m}^3(p/4)\left[(0.04\text{m})^2 - (0.02\text{m})^2\right]}$$

$$L = 8.87\text{m}.$$ <

(b) The maximum wall temperature exists at the pipe exit ($x = L$) and the insulated surface ($r = r_o$). From Eq. 3.50, the radial temperature distribution in the wall is of the form

$$T(r) = -\frac{\dot{q}}{4k}r^2 + C_1 \ell n\, r + C_2.$$

Considering the boundary conditions;

$$r = r_o: \quad \left.\frac{dT}{dr}\right)_{r=r_o} = 0 = -\frac{\dot{q}}{2k}r_o + \frac{C_1}{r_o} \qquad C_1 = \frac{\dot{q} r_o^2}{2k}$$

Continued …..

PROBLEM 8.36 (Cont.)

$$r = r_i: \quad T(r_i) = T_s = -\frac{\dot{q}}{4k} r_i^2 + \frac{\dot{q}\, r_o^2}{2k} \ell n\, r_i + C_2 \qquad C_2 = \frac{\dot{q}}{4k} r_i^2 - \frac{\dot{q}\, r_o^2}{2k} \ell n\, r_i + T_s.$$

The temperature distribution and the maximum wall temperature ($r = r_o$) are

$$T(r) = -\frac{\dot{q}}{4k}\left(r^2 - r_i^2\right) + \frac{\dot{q}\, r_o^2}{2k} \ell n \frac{r}{r_i} + T_s$$

$$T_{w,max} = T(r_o) = -\frac{\dot{q}}{4k}\left(r_o^2 - r_i^2\right) + \frac{\dot{q}\, r_o^2}{2k} \ell n \frac{r_o}{r_i} + T_s$$

where T_s, the inner surface temperature of the wall at the exit, follows from

$$q_s'' = \frac{\dot{q}(p/4)\left(D_o^2 - D_i^2\right)L}{p\, D_i L} = \frac{\dot{q}\left(D_o^2 - D_i^2\right)}{4\, D_i} = h\left(T_s - T_{m,o}\right)$$

where h is the local convection coefficient at the exit. With

$$Re_D = \frac{4\, \dot{m}}{p\, D_i m} = \frac{4 \times 0.1\ \text{kg/s}}{p\,(0.02\text{m})\, 803 \times 10^{-6}\ \text{N} \cdot \text{s/m}^2} = 7928$$

the flow is turbulent and, with (L/D_i) = (8.87 m/0.02m) = 444 >> (x_{fd}/D) ≈ 10, it is also fully developed. Hence, from the Dittus-Boelter correlation, Eq. 8.60,

$$h = \frac{k}{D_i}\left(0.023\, Re_D^{4/5}\, Pr^{0.4}\right) = \frac{0.617\ \text{W/m} \cdot \text{K}}{0.02\ \text{m}} 0.023(7928)^{4/5} 5.45^{0.4} = 1840\ \text{W/m}^2 \cdot \text{K}.$$

Hence, the inner surface temperature of the wall at the exit is

$$T_s = \frac{\dot{q}\left(D_o^2 - D_i^2\right)}{4\, h\, D_i} + T_{m,o} = \frac{10^6\ \text{W/m}^3\left[(0.04\text{m})^2 - (0.02\text{m})^2\right]}{4 \times 1840\ \text{W/m}^2 \cdot \text{K}\,(0.02\text{m})} + 40^\circ\text{C} = 48.2^\circ\text{C}$$

and

$$T_{w,max} = -\frac{10^6\ \text{W/m}^3}{4 \times 15\ \text{W/m} \cdot \text{K}}\left[(0.02\text{m})^2 - (0.01\text{m})^2\right]$$

$$+ \frac{10^6\ \text{W/m}^3\,(0.02\text{m})^2}{2 \times 15\ \text{W/m} \cdot \text{K}} \ell n \frac{0.02}{0.01} + 48.2^\circ\text{C} = 52.4^\circ\text{C}. \qquad <$$

COMMENTS: The physical situation corresponds to a uniform surface heat flux, and T_m increases linearly with x. In the fully developed region, T_s also increases linearly with x.

T

$T_s\,(x)$

$T_m(x)$

x_{fd}

x

PROBLEM 8.37

KNOWN: Dimensions and thermal conductivity of concrete duct. Convection conditions of ambient air. Flow rate and inlet temperature of water flow through duct.

FIND: (a) Outlet temperature, (b) Pressure drop and pump power requirement, (c) Effect of flow rate and pipe diameter on outlet temperature.

SCHEMATIC:

Air
$T_\infty = 0^oC$
$h = 25\ W/m^2\text{-}K$
D = 0.15 m
w = 0.30 m
$\dot{m} = 2\ kg/s$
$T_{m,i} = 90^oC$
Water
$T_{m,o}$
Concrete
k = 1.4 W/m-K
L = 100 m
T_m $R_{cnv,w}$ R_{cnd} $R_{cnv,a}$ T_∞ q

ASSUMPTIONS: (1) Steady-state, (2) Fully developed flow throughout duct, (3) Negligible pipe wall conduction resistance, (4) Negligible potential energy, kinetic energy and flow work changes for water, (5) Constant properties.

PROPERTIES: *Table A-6*, water $(\overline{T}_m \approx 360\,K)$: $\rho = 967\,kg/m^3$, $c_p = 4203\,J/kg\cdot K$, $\mu = 324\times10^{-6}$ $N\cdot s/m^2$, $k_w = 0.674\,W/m\cdot K$, $Pr = 2.02$.

ANALYSIS: (a) The outlet temperature is given by

$$T_{m,o} = T_\infty + \left(T_{m,i} - T_\infty\right)\exp\left(-UA/\dot{m}\,c_p\right)$$

where

$$UA = \left(R_{tot}\right)^{-1} = \left(R_{cnv,w} + R_{cnd} + R_{cnv,a}\right)^{-1}$$

$$R_{cnd} = \frac{\ln\left(1.08\,w/D\right)}{2\pi kL} = \frac{\ln\left(1.08\times0.30m/0.15m\right)}{2\pi\left(1.4\,W/m\cdot K\right)100m} = 8.75\times10^{-4}\ K/W$$

$$R_{cnv,a} = \left(4\,w\,L\,h\right)^{-1} = \left(4\times0.3m\times100m\times25\,W/m^2\cdot K\right)^{-1} = 3.33\times10^{-4}\,K/W$$

With $Re_D = 4\,\dot{m}/\pi\,D\,\mu = \left(4\times2\,kg/s\right)/\left(\pi\times0.15m\times324\times10^{-6}\,N\cdot s/m^2\right) = 52{,}400,$

$$\overline{h}_w \approx h_{fd} = \frac{k_w}{D}\,0.023\,Re_D^{4/5}\,Pr^{0.3} = \frac{0.674\,W/m\cdot K\times0.023}{0.15m}\left(52{,}400\right)^{4/5}\left(2.02\right)^{0.3} = 761\,W/m^2\cdot K$$

$$R_{cnv,w} = \left(\pi\,D\,L\,\overline{h}_w\right)^{-1} = \left(\pi\times0.15m\times100m\times761\,W/m^2\cdot K\right)^{-1} = 2.79\times10^{-5}\,K/W$$

$$UA = \left[\left(2.79\times10^{-5} + 8.75\times10^{-4} + 3.33\times10^{-4}\right)K/W\right]^{-1} = 809\,W/K$$

$$T_{m,o} = 0°C + 90°C\ \exp\left(-\frac{809\,W/K}{2\,kg/s\times4203\,J/kg\cdot K}\right) = 81.7°C$$ <

Continued …..

(b) With f = 0.0206 from Fig. 8.3 and $u_m = \dot{m}/\rho\pi D^2/4 = 0.117\,m/s$,

$$\Delta p = f\,\frac{\rho u_m^2}{2D}L = 0.0206\,\frac{967\,kg/m^3\,(0.117\,m/s)^2}{2\times 0.15m}100m = 91\,N/m^2 = 8.98\times 10^{-4}\text{ bars} \quad <$$

With $\dot{\forall} = \dot{m}/\rho = 2.07\times 10^{-3}\,m^3/s$, the pump power requirement is

$$P = \Delta p\dot{\forall} = \left(91\,N/m^2\right)2.07\times 10^{-3}\,m^3/s = 0.19\,W \quad <$$

(c) The effects of varying the flowrate and duct diameter were assessed using the IHT software, and results are shown below.

Outlet temperature, Tmo(C)
Mass flow rate, mdot(kg/s)

Outlet temperature, Tmo(C)
Duct diameter, D(m)

Although $R_{cnv,w}$, and hence R_{tot}, decreases with increasing $\dot{m}$, thereby increasing UA, the effect is significantly less than that of $\dot{m}$ to the first power, causing the exponential term, $\exp\left(-UA/\dot{m}c_p\right)$, to approach unity and $T_{m,o}$ to approach $T_{m,i}$. The effect can alternatively be attributed to a reduction in the residence time of the water in the pipe (u_m increases with increasing $\dot{m}$ for fixed D). With increasing D for fixed $\dot{m}$ and w, $T_{m,o}$ decreases due to an increase in the residence time, as well as a reduction in the conduction resistance, R_{cnd}.

COMMENTS: (1) Use of $\overline{T}_m = 360\,K$ to evaluate properties of the water for Parts (a) and (b) is reasonable, and iteration is not necessary. (2) The pressure drop and pump power requirement are small.

PROBLEM 8.38

KNOWN: Water flow through a thick-walled tube immersed in a well stirred, hot reaction tank maintained at 85°C; conduction thermal resistance of the tube wall based upon the inner surface area is $R''_{cd} = 0.002\ m^2 \cdot K/W$.

FIND: (a) The outlet temperature of the process fluid, $T_{m,o}$; assume, and then justify, fully developed flow and thermal conditions within the tube; and (b) Do you expect $T_{m,o}$ to increase or decrease if the combined thermal entry condition exists within the tube? Estimate the outlet temperature of the process fluid for this condition.

SCHEMATIC:

Well-stirred reaction fluid $T_\infty = 85^oC$ $R''_{cd,i} = 0.002\ m^2\text{-}K/W$

Process fluid $\dot{m} = 33$ kg/h $T_{m,i} = 20^oC$ $T_{m,o}$

x Thick-walled tube, D = 12 mm ID L = 8 m

ASSUMPTIONS: (1) Flow is fully developed, part (a), (2) Constant properties, (3) Negligible kinetic and potential energy changes and flow work, and (4) Constant wall temperature heating.

PROPERTIES: *Table A-6, Water* ($T_m = (T_{m,o} + T_{m,i})/2 = 337$ K): $c_p = 4187$ J/kg·K, $\mu = 4.415 \times 10^{-4}$ N·s/m^2, k = 0.6574 W/m·K, Pr = 2.80; ($T_s = 358$ K): $\mu_s = 3.316 \times 10^{-4}$ N·s/m^2.

ANALYSIS: (a) The outlet temperature is determined from the rate equation, Eq. 8.46a, written as

$$\frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp\left(-\frac{\bar{U}A_s}{\dot{m}\,c_p}\right) \tag{1}$$

where the overall coefficient, based upon the inner surface area of the tube is expressed in terms of the convection and conduction thermal resistances,

$$\frac{1}{\bar{U}} = \frac{1}{\bar{h}} + R''_{cd,i} \tag{2}$$

To estimate $\bar{h}$, begin by characterizing the flow

$$Re_D = 4\dot{m}/\pi D\mu \tag{3}$$

$$Re_D = 4(33/3600\ kg/s)/\pi \times 0.012\ m \times 4.415 \times 10^{-4} N \cdot s/m^2 = 2210$$

Consider the flow as laminar, and assuming fully developed conditions, estimate $\bar{h}$ with the correlation of Eq. 8.55,

$$\overline{Nu}_D = \bar{h}D/k = 3.66 \tag{4}$$

$$\bar{h} = 3.66 \times 0.6574\,W/m \cdot K/0.012\ m = 201\ W/m^2 \cdot K$$

From Eq. (2),

$$\bar{U} = \left[1/201\ W/m^2 \cdot K + 0.002\ m^2 \cdot K/W\right]^{-1} = 143.1\ W/m^2 \cdot K$$

and from Eq. (1), with $A_s = \pi DL$, calculate $T_{m,o}$.

Continued …..

$$\frac{85 - T_{m,o}}{85 - 20} = \exp\left(- \frac{143.1\ W/m^2 \cdot K \times \pi \times 0.012\ m \times 8\ m}{33/3600\ kg/s \times 4187\ J/kg \cdot K} \right)$$

$$T_{m,o} = 64°C$$ <

Fully developed flow and thermal conditions are justified if the tube length is much greater than the fully developed length $x_{fd,t}$. From Eq. 8.23,

$$\frac{x_{fd,t}}{D} = 0.05\ Re_D\ Pr$$

$$x_{fd,t} = 0.012\ m \times 0.05 \times 221.0 \times 2.41 = 3.20\ m$$

That is, the length is only twice that required to reach fully developed conditions.

(b) Considering combined entry length conditions, estimate the convection coefficient using the Sieder-Tate correlation, Eq. 8.56,

$$\overline{Nu}_D = 3.66 + \frac{0.0668 (D/L) Re_D\ Pr}{1 + 0.04 \left[(D/L) Re_D\ Pr \right]^{2/3}} \qquad (5)$$

substituting numerical values, find

$$\overline{Nu}_D = 4.05 \qquad \bar{h} = 222\ W/m^2 \cdot K$$

which is a 10% increase over the fully developed analysis result. Using the foregoing relations, find

$$U = 154\ W/m^2 \cdot K \qquad T_{m,o} = 65.5°C$$ <

COMMENTS: (1) The thermophysical properties for the fully developed correlation are evaluated at the mean fluid temperature $T_m = (T_{m,o} + T_{m,i})/2$. The values are shown above in the properties section.

(2) For the Sieder-Tate correlation, the properties are also evaluated at T_m, except for μ_s, which is evaluated at T_s.

(3) For this case where the tube length is about twice $x_{fd,t}$, the average heat transfer coefficient is larger as we would expect, but amounts to only a 10% increase.

PROBLEM 8.39

KNOWN: Flow rate and temperature of atmospheric air entering a duct of prescribed diameter, length and surface temperature.

FIND: (a) Air outlet temperature and duct heat loss for the prescribed conditions and (b) Calculate and plot q and Δp for the range of diameters, $0.1 \le D \le 0.2$ m, maintaining the total surface area, $A_s = \pi DL$, at the same value as part (a). Explain the trade off between the heat transfer rate and pressure drop.

SCHEMATIC:

q
T_s = 15 °C
$T_{m,o}$
D = 0.15 m
L = 10 m
Air
$\dot{m}$ = 0.04 kg/s
$T_{m,i}$ = 60 °C
p = 1 atm

ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Negligible kinetic energy and potential energy changes, (4) Uniform surface temperature, (5) Fully developed flow conditions.

PROPERTIES: *Table A.4*, Air ($\overline{T}_m \approx 310$ K, 1 atm): $\rho = 1.128$ kg/m^3, $c_p = 1007$ J/kg·K, $\mu = 189 \times 10^{-7}$ N·s/m^2, k = 0.027 W/m·K, Pr = 0.706.

ANALYSIS: (a) With

$$Re_D = \frac{4\dot{m}}{\pi D \mu} = \frac{4 \times 0.04\,\text{kg/s}}{\pi (0.15\,\text{m}) 189 \times 10^{-7}\,\text{N}\cdot\text{s/m}^2} = 17{,}965$$

the flow is turbulent. Assuming fully developed conditions throughout the tube, it follows from the Dittus-Boelter correlation, Eq. 8.60, that

$$\overline{h} = \frac{k}{D} 0.023 Re_D^{4/5} Pr^{0.4} = \frac{0.027\,\text{W/m}\cdot\text{K}}{0.15\,\text{m}} 0.023 (17{,}965)^{4/5} (0.706)^{0.4} = 9.44\,\text{W/m}^2\cdot\text{K}\,.$$

Hence, from the energy balance relation, Eq. 8.42b,

$$T_{m,o} = T_s - (T_s - T_{m,i}) \exp\left(-\frac{\pi DL}{\dot{m} c_p} \overline{h} \right)$$

$$T_{m,o} = 15^\circ\text{C} + 45^\circ\text{C} \exp\left(-\frac{\pi (0.15\,\text{m}) 10\,\text{m} \left(9.44\,\text{W/m}^2\cdot\text{K}\right)}{0.04\,\text{kg/s} (1007\,\text{J/kg}\cdot\text{K})} \right) = 29.9^\circ\text{C}$$ <

From the overall energy balance, Eq. 8.37, it follows that

$$q = \dot{m} c_p (T_{m,o} - T_{m,i}) = 0.04\,\text{kg/s} \times 1007\,\text{J/kg}\cdot\text{K} (29.9 - 60)^\circ\text{C} = -1212\,\text{W}\,.$$ <

From Eq. 8.22a, the pressure drop is

$$\Delta p = f \frac{\rho u_m^2}{2D} L$$

Continued...

PROBLEM 8.39 (Cont.)

and for the smooth surface conditions, Eq. 8.21 can be used to evaluate the friction factor,

$$f = \left(0.790 \ln\left(Re_D\right) - 1.64\right)^{-2} = \left(0.790 \ln\left(17,965\right) - 1.64\right)^{-2} = 0.0269$$

Hence, the pressure drop is

$$\Delta p = 0.0269 \frac{1.128\,\text{kg}/\text{m}^3 \left(2.0\,\text{m/s}\right)^2}{2 \times 0.15\,\text{m}} \times 10\,\text{m} = 4.03\,\text{N}/\text{m}^2$$ <

where $u_m = \dot{m}/\rho A_c = 0.04\,\text{kg/s} \Big/ 1.128\,\text{kg}/\text{m}^3 \times \left(\pi 0.15^2\,\text{m}^2/4\right) = 2.0\,\text{m/s}$.

(b) For the prescribed conditions of part (a), $A_s = \pi DL = \pi(0.15\text{ m}) \times 10\text{ m} = 4.712\text{ m}^2$, using the *IHT Correlations Tool*, *Internal Flow* for fully developed *Turbulent Flow* along with the energy balance equation, rate equation and pressure drop equations used above, the heat rate q and Δp are calculated and plotted below.

From above, as D increases, L decreases so that A_s remains unchanged. The decrease in heat rate with increasing diameter is nearly linear, while the pressure drop decreases markedly. This is the trade off: increased heat rate requires a more significant increase in pressure drop, and hence fan blower power requirements.

COMMENTS: (1) To check the calculations, compute q from Eq. 8.44, where $\Delta T_{\ell m}$ is given by Eq. 8.45. It follows that $\Delta T_{\ell m}$ = -27.1°C and q = -1206 W. The small difference in results may be attributed to round-off error.

(2) For part (a), a slight improvement in accuracy may be obtained by evaluating the properties at $\bar{T}_m$ = 318 K: $\bar{h}$ = 9.42 W/m^2·K, $T_{m,o}$ = 303 K = 30°C, q = -1211 W, f = 0.0271 and Δp = 4.20 N/m^2.

PROBLEM 8.40

KNOWN: Inlet temperature, pressure and flow rate of air. Tube diameter and length. Pressure of saturated steam.

FIND: Outlet temperature and pressure of air. Mass rate of steam condensation.

SCHEMATIC:

p = 2.455 bar — Sat. steam

$\dot{m}_c$ = 0.03 kg/s — Air
$T_{m,i}$ = 17°C
p_i = 5 atm

D_i = 0.05 m
L = 5 m
$T_{m,o}$, p_o, $\dot{m}_c$

ASSUMPTIONS: (1) Steady-state, (2) Outer surface of annulus is adiabatic, (3) Negligible potential energy, kinetic energy and flow work changes for air, (4) Fully-developed flow throughout the tube, (5) Smooth tube surface, (6) Constant properties.

PROPERTIES: *Table A-4*, air $(\overline{T}_m \approx 325\,\text{K},\ p = 5\,\text{atm})$: $\rho = 5 \times \rho(1\,\text{atm}) = 5.391\,\text{kg/m}^3$, $c_p = 1008\,\text{J/kg}\cdot\text{K}$, $\mu = 196.4\times10^{-7}\,\text{N}\cdot\text{s/m}^2$, $k = 0.0281\,\text{W/m}\cdot\text{K}$, $Pr = 0.703$. *Table A-6*, sat. steam (p = 2.455 bars): T_s = 400 K, h_{fg} = 2183 kJ/kg.

ANALYSIS: With a uniform surface temperature, the air outlet temperature is

$$T_{m,o} = T_s - (T_s - T_{m,i})\exp\left(-\frac{\pi D_i L}{\dot{m}\, c_p}\overline{h}\right)$$

With $Re_D = 4\dot{m}/\pi D_i\,\mu = 0.12\,\text{kg/s}/\pi(0.05\text{m})196.4\times10^{-7}\,\text{kg/s}\cdot\text{m} = 38{,}980$, the flow is turbulent, and the Dittus-Boelter correlation yields

$$\overline{h} \approx h_{fd} = \left(\frac{k}{D_i}\right)0.023\,Re_D^{4/5}\,Pr^{0.4} = \left(\frac{0.0281\,\text{W/m}\cdot\text{K}}{0.05\text{m}}\right)0.023(38{,}980)^{4/5}(0.703)^{0.4} = 52.8\,\text{W/m}^2\cdot\text{K}$$

$$T_{m,o} = 127°\text{C} - (110°\text{C})\exp\left(-\frac{\pi\times0.05\text{m}\times5\text{m}\times52.8\,\text{W/m}^2\cdot\text{K}}{0.03\,\text{kg/s}\times1008\,\text{J/kg}\cdot\text{K}}\right) = 99°\text{C}$$ <

The pressure drop is $\Delta p = f\left(\rho u_m^2/2D_i\right)L$, where, with $A_c = \pi D_i^2/4 = 1.963\times10^{-3}\text{m}^2$, $u_m = \dot{m}/\rho A_c = 2.83\,\text{m/s}$, and with $Re_D = 38{,}980$, Fig. 8.3 yields f ≈ 0.022. Hence,

$$\Delta p \approx 0.022\times5.391\,\text{kg/m}^3\,\frac{(2.83\,\text{m/s})^2\,5\text{m}}{2\times0.05\text{m}} = 95\,\text{N/m}^2 = 9.4\times10^{-4}\text{atm}$$ <

The rate of heat transfer to the air is

$$q = \dot{m}\,c_p\left(T_{m,o} - T_{m,i}\right) = 0.03\,\text{kg/s}\times1008\,\text{J/kg}\cdot\text{K}(82°\text{C}) = 2480\,\text{W}$$

and the rate of condensation is then

$$\dot{m}_c = \frac{q}{h_{fg}} = \frac{2480\,\text{W}}{2.183\times10^6\,\text{J/kg}} = 1.14\times10^{-3}\text{kg/s}$$ <

COMMENTS: (1) With $\overline{T}_m = \left(T_{m,i} + T_{m,o}\right)/2 = 331\,\text{K}$, the initial estimate of 325 K is reasonable and iteration is not necessary. (2) For a steam flow rate of 0.01 kg/s, approximately 10% of the outflow would be in the form of saturated liquid, (3) With L/D_i = 100, it is reasonable to assume fully developed flow throughout the tube.

PROBLEM 8.41

KNOWN: Duct diameter and length. Thermal conductivity of insulation. Gas inlet temperature and velocity and minimum allowable outlet temperature. Temperature and velocity of air in cross flow.

FIND: Minimum allowable insulation thickness.

SCHEMATIC:

Ambient air
$V = 15$ m/s
$T_\infty = 250$ K
Insulation
D_o, $k_{ins} = 0.125$ W/m-K
$T_{m,o} \geq 1400$ K
$D_i = 1$ m
Combustion gases
$u_{m,i} = 10$ m/s
$T_{m,i} = 1600$ K
$L = 100$ m

ASSUMPTIONS: (1) Negligible potential and kinetic energy and flow work changes for gas flow through duct, (2) Fully developed flow throughout duct, (3) Negligible duct wall conduction resistance, (4) Negligible effect of insulation thickness on outer convection coefficient and thermal resistance, (5) Properties of gas may be approximated as those of air.

PROPERTIES: *Table A-4*, air (p = 1 atm). $T_{m,i} = 1600$K: ($\rho_i = 0.218$ kg/m^3). $\bar{T}_m = (T_{m,i} + T_{m,o})/2 = 1500$K: ($\rho = 0.232$ kg/m^3, $c_p = 1230$ J/kg·K, $\mu = 557 \times 10^{-7}$ N·s/m^2, k = 0.100 W/m·K, Pr = 0.685). $T_f \approx 300$K (assumed): $\nu = 15.89 \times 10^{-6}$ m^2/s, k = 0.0263 W/m·K, Pr = 0.707.

ANALYSIS: From Eqs. (8.46a) and (3.19),

$$\frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = \frac{-1150\,\text{K}}{-1350\,\text{K}} = 0.852 = \exp\left(-\frac{\bar{U}A_s}{\dot{m}\,c_p}\right) = \exp\left(-\frac{1}{R_{tot}\dot{m}\,c_p}\right)$$

Hence, with $\dot{m} = \left(\rho\, u_m A_c\right)_i = 0.218\,\text{kg/m}^3 \times 10\,\text{m/s} \times \pi\,(1\text{m})^2/4 = 1.712\,\text{kg/s}$,

$$R_{tot} = -\left[\dot{m}\,c_p \ln(0.852)\right]^{-1} = -\left[1.712\,\text{kg/s} \times 1230\,\text{J/kg}\cdot\text{K} \times (-0.160)\right]^{-1} = 2.96 \times 10^{-3}\,\text{K/W}$$

The total thermal resistance is

$$R_{tot} = R_{conv,i} + R_{cond,ins} + R_{conv,o} = \left(h_i \pi D_i L\right)^{-1} + \frac{\ln\left(D_o/D_i\right)}{2\pi k_{ins} L} + \left(h_o \pi D_o L\right)^{-1} \qquad (1)$$

With $Re_{D,i} = 4\dot{m}/\pi D_i \mu = \left(4 \times 1.712\,\text{kg/s}\right)/\left(\pi \times 1\text{m} \times 557 \times 10^{-7}\,\text{N}\cdot\text{s/m}^2\right) = 39{,}130$, the Dittus-Boelter correlation yields

$$h_i = \left(\frac{k}{D}\right) 0.023\, Re_D^{4/5}\, Pr^{1/3} = \left(\frac{0.100\,\text{W/m}\cdot\text{K}}{1\text{m}}\right) 0.023\,(39{,}130)^{4/5}\,(0.685)^{1/3} = 9.57\,\text{W/m}^2\cdot\text{K}$$

The internal resistance is then

$$R_{conv,i} = \left(h_i \pi D_i L\right)^{-1} = \left(9.57\,\text{W/m}^2\cdot\text{K} \times \pi \times 1\text{m} \times 100\text{m}\right)^{-1} = 3.33 \times 10^{-4}\,\text{K/W}$$

With $Re_D \approx VD_i/\nu = 15$ m/s $\times$ 1m/15.89 $\times 10^{-6}$ m^2/s $= 9.44 \times 10^5$, the Churchill-Bernstein correlation yields

Continued …..

$$h_o \approx \left(\frac{k}{D}\right)\left\{0.3+\frac{0.62\,Re_D^{1/2}\,Pr^{1/3}}{\left[1+(0.4/Pr)^{2/3}\right]^{1/4}}\left[1+\left(\frac{Re_D}{282,000}\right)^{5/8}\right]^{4/5}\right\}=30.9\,W/m^2\cdot K$$

$$R_{conv,o} \approx (h_o \pi D_i L)^{-1} = \left(30.9\,W/m^2\cdot K\times\pi\times 1m\times 100m\right)^{-1} = 1.03\times10^{-4}\,K/W$$

Hence, from Eq. (1)

$$\frac{\ln(D_o/D_i)}{2\pi\,k_{ins}\,L} = \left(2.96\times10^{-3} - 3.33\times10^{-4} - 1.03\times10^{-4}\right)K/W = 2.52\times10^{-3}\,K/W$$

$$D_o = D_i\exp\left(2\pi k_{ins} L\times 2.52\times10^{-3}\,K/W\right) = 1m\times\exp\left(1.58\times10^{-2}\,K/W\times 0.125\,W/m\cdot K\times 100m\right) = 1.22m$$

Hence, the minimum insulation thickness is

$$t_{min} = (D_o - D_i)/2 = 0.11m$$ <

COMMENTS: With D_o = 1.22m, use of D_i = 1m to evaluate the outer convection coefficient and thermal resistance is a reasonable approximation. However, improved accuracy may be obtained by using the calculated value of D_o to determine conditions at the outer surface and iterating on the solution.

PROBLEM 8.42

KNOWN: Flow rate, inlet temperature and desired outlet temperature of liquid mercury flowing through a tube of prescribed diameter and surface temperature.

FIND: Required tube length and error associated with use of a correlation for moderate to large Pr fluids.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Negligible kinetic energy, potential energy and flow work effects, (4) Fully developed flow.

PROPERTIES: *Table A-5*, Mercury $(\bar{T}_m = 350\text{K})$: $c_p = 137.7$ J/kg·K, $\mu = 0.1309 \times 10^{-2}$ N·s/m^2, k = 9.18 W/m·K, Pr = 0.0196.

ANALYSIS: The Reynolds and Peclet numbers are

$$Re_D = \frac{4\dot{m}}{\pi D \mu} = \frac{4 \times 0.5 \text{ kg/s}}{\pi (0.05\text{m}) 0.1309 \times 10^{-2} \text{N} \cdot \text{s/m}^2} = 9727$$

$$Pe_D = Re_D \, Pr = 9727(0.0196) = 191.$$

Hence, assuming fully developed turbulent flow throughout the tube, it follows from Eq. 8.66 that

$$\bar{h} = \frac{k}{D}\left(5.0 + 0.025\, Pe_D^{0.8}\right) = \frac{9.18 \text{ W/m} \cdot \text{K}}{0.05 \text{ m}}\left(5.0 + 0.025 \times 191^{0.8}\right) = 1224 \text{ W/m}^2 \cdot \text{K}.$$

From Eq. 8.42a, it follows that

$$L = -\frac{\dot{m}\, c_p}{\pi D \bar{h}} \ln \frac{\Delta T_o}{\Delta T_i} = -\frac{(0.5 \text{ kg/s}) 137.7 \text{ J/kg} \cdot \text{K}}{\pi (0.05 \text{ m}) 1224 \text{ W/m}^2 \cdot \text{K}} \ln \frac{450 - 400}{450 - 300} = 0.39 \text{ m.} \qquad <$$

If the Dittus-Boelter correlation, Eq. 8.60, is used in place of Eq. 8.66,

$$\bar{h} = \frac{k}{D} 0.023\, Re_D^{4/5}\, Pr^{0.4} = \frac{9.18 \text{ W/m}^2 \cdot \text{K}}{0.05 \text{ m}} 0.023 (9727)^{4/5} (0.0196)^{0.4} = 1358 \text{ W/m}^2 \cdot \text{K}$$

and the required tube length is

$$L = -\frac{\dot{m}\, c_p}{\pi D \bar{h}} \ln \frac{\Delta T_o}{\Delta T_i} = -\frac{(0.5 \text{ kg/s}) 137.7 \text{ J/kg} \cdot \text{K}}{\pi (0.05 \text{ m}) 1358 \text{ W/m}^2 \cdot \text{K}} \ln \frac{450 - 400}{450 - 300} = 0.35 \text{ m.} \qquad <$$

COMMENTS: Such good agreement between results does not occur in general. For example, if $Re_D = 2 \times 10^4$, $\bar{h}$ = 1463 from Eq. 8.66 and 2417 from Eq. 8.60. Large errors are usually associated with using conventional (moderate to large Pr) correlations with liquid metals.

PROBLEM 8.43

KNOWN: Surface temperature and diameter of a tube. Velocity and temperature of air in cross flow. Velocity and temperature of air in fully developed internal flow.

FIND: Convection heat flux associated with the external and internal flows.

SCHEMATIC:

Air
V=30 m/s
T_∞=25°C
T_s=100°C
Air
u_m=30 m/s
T_m=25°C
D=0.05m

ASSUMPTIONS: (1) Steady-state conditions, (2) Uniform cylinder surface temperature, (3) Fully developed internal flow.

PROPERTIES: *Table A-4*, Air (298K): $\nu = 15.71 \times 10^{-6}$ m^2/s, k = 0.0261 W/m·K, Pr = 0.71.

ANALYSIS: For the *external* and *internal* flows,

$$Re_D = \frac{VD}{n} = \frac{u_m D}{n} = \frac{30 \text{ m/s} \times 0.05 \text{ m}}{15.71 \times 10^{-6} \text{m}^2/\text{s}} = 9.55 \times 10^4.$$

From the Zhukauskas relation for the *external* flow, with C = 0.26 and m = 0.6,

$$\overline{Nu}_D = C\, Re_D^m\, Pr^n\, (Pr/Pr_s)^{1/4} = 0.26\left(9.55 \times 10^4\right)^{0.6} (0.71)^{0.37} (1)^{1/4} = 223.$$

Hence, the convection coefficient and heat flux are

$$\overline{h} = \frac{k}{D}\overline{Nu}_D = \frac{0.0261 \text{ W/m} \cdot \text{K}}{0.05 \text{ m}} \times 223 = 116.4 \text{ W/m}^2 \cdot \text{K}$$

$$q'' = h\left(T_s - T_\infty\right) = 116.4 \text{W/m}^2 \cdot \text{K}\left(100 - 25\right)^\circ \text{C} = 8.73 \times 10^3 \text{ W/m}^2. \qquad <$$

Using the Dittus-Boelter correlation, Eq. 8.60, for the *internal* flow, which is turbulent,

$$\overline{Nu}_D = 0.023\, Re_D^{4/5}\, Pr^{0.4} = 0.023\left(9.55 \times 10^4\right)^{4/5} (0.71)^{0.4} = 193$$

$$\overline{h} = \frac{k}{D}\overline{Nu}_D = \frac{0.0261 \text{ W/m} \cdot \text{K}}{0.05 \text{ m}} \times 193 = 101 \text{ W/m}^2 \cdot \text{K}$$

and the heat flux is

$$q'' = h\left(T_s - T_m\right) = 101 \text{ W/m}^2 \cdot \text{K}\left(100 - 25\right)^\circ \text{C} = 7.58 \times 10^3 \text{ W/m}^2. \qquad <$$

COMMENTS: Convection effects associated with the two flow conditions are comparable.

PROBLEM 8.44

KNOWN: Diameter, length and surface temperature of condenser tubes. Water velocity and inlet temperature.

FIND: (a) Water outlet temperature evaluating properties at T_m = 300 K, (b) Repeat calculations using properties evaluated at the appropriate temperature, $\overline{T}_m = (T_{m,i} + T_{m,o})/2$, and (c) Coolant mean velocities for the range $4 \le L \le 7$ m which provide the same $T_{m,o}$ as found in part (b).

SCHEMATIC:

$L = 5$ m

$T_{m,i} = 290$ K

$u_m = 1$ m/s

$D = 0.0254$ m

$T_s = 350$ K

ASSUMPTIONS: (1) Negligible tube wall conduction resistance, (2) Negligible kinetic energy, potential energy and flow work changes.

PROPERTIES: *Table A.6*, Water ($\overline{T}_m$ = 300 K): $\rho = 997\ kg/m^3$, $c_p = 4179$ J/kg·K, $\mu = 855 \times 10^{-6}$ kg/s·m, k = 0.613 W/m·K, Pr = 5.83.

ANALYSIS: (a) From Equation 8.42b

$$T_{m,o} = T_s - \left(T_s - T_{m,i}\right)\exp\left[-\left(\pi DL/\dot{m}c_p\right)\overline{h}\right].$$

and evaluating properties at $\overline{T}_m$ = 300 K, find

$$Re_D = \frac{\rho u_m D}{\mu} = \frac{997\,kg/m^3\,(1\,m/s)\,0.0254\,m}{855\times10^{-6}\,kg/s\cdot m} = 29,618$$

The flow is turbulent, and since L/D = 197, it is reasonable to assume fully developed flow throughout the tube. Hence, $\overline{h} \approx h_{fd}$. From the Dittus-Boelter equation,

$$Nu_D = 0.023 Re_D^{4/5} Pr^{0.4} = 0.023(29,618)^{4/5}(5.83)^{0.4} = 176$$

$$\overline{h} = Nu_D\,(k/D) = 176(0.613\,W/m\cdot K/0.0254\,m) = 4248\,W/m^2\cdot K.$$

With

$$\dot{m} = \rho u_m\left(\pi D^2/4\right) = (\pi/4)997\,kg/m^3\,(1\,m/s)(0.0254\,m)^2 = 0.505\,kg/s.$$

Equation 8.42b yields

$$T_{m,o} = 350\,K - (60\,K)\exp\left[-\frac{\pi(0.0254\,m)5\,m\left(4248\,W/m^2\cdot K\right)}{0.505\,kg/s\left(4179\,J/kg\cdot K\right)}\right] = 323\,K = 50^\circ C$$ <

(b) Using the *IHT Correlations Tool*, *Internal Flow*, for fully developed *Turbulent Flow*, along with the energy balance and rate equations above, the calculation of part (a) is repeated with $\overline{T}_m = (T_{m,i} + T_{m,o})/2$ giving these results:

$\overline{T}_m = 307.3\,K$ $\qquad$ $T_{m,o}$ = 51.7°C = 324.7 K <

(c) Using the IHT model developed for the part (b) analysis, the coolant mean velocity, u_m, as a function of tube length L with $T_{m,o}$ = 51.7°C is calculated and the results plotted below.

Continued...

PROBLEM 8.44 (Cont.)

Velocity, um (m/s)

6
5
4
3
2
1
0

4 5 6 7

Tube length, L (m)

COMMENTS: (1) Using $\overline{T}_m$ = 300 K vs. $\overline{T}_m$ = $(T_{m,i} + T_{m,o})/2$ = 307 K for this application resulted in a difference of $T_{m,o}$ = 50°C vs. $T_{m,o}$ = 51.7°C. While the difference is only 1.7°C, it is good practice to use the proper value for $\overline{T}_m$.

(2) Note that u_m must be increased markedly with increasing length in order that $T_{m,o}$ remain fixed.

PROBLEM 8.45

KNOWN: Gas turbine vane approximated as a tube of prescribed diameter and length maintained at a known surface temperature. Air inlet temperature and flowrate.

FIND: (a) Outlet temperature of the air coolant for the prescribed conditions and (b) Compute and plot the air outlet temperature $T_{m,o}$ as a function of flow rate, $0.1 \le \dot{m} \le 0.6$ kg/h. Compare this result with those for vanes having passage diameters of 2 and 4 mm.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible kinetic and potential energy changes.

PROPERTIES: *Table A.4*, Air (assume $\overline{T}_m$ = 780 K, 1 atm): c_p = 1094 J/kg·K, k = 0.0563 W/m·K, μ = 363.7×10^{-7} N·s/m², Pr = 0.706; (T_s = 650°C = 923 K, 1 atm): μ = 404.2×10^{-7} N·s/m².

ANALYSIS: (a) For constant wall temperature heating, from Eq. 8.43,

$$\frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp\left(-\frac{PL\bar{h}}{\dot{m}c_p}\right) \qquad (1)$$

where P = πD. For flow in circular passage,

$$Re_D = \frac{4\dot{m}}{\pi D \mu} = \frac{4 \times 0.18\,\text{kg/h}\,(1/3600\,\text{s/h})}{\pi\,(0.003\,\text{m})\,363.7 \times 10^{-7}\,\text{N}\cdot\text{s/m}^2} = 584\,. \qquad (2)$$

The flow is laminar, and since L/D = 75 mm/3 mm = 25, the Sieder-Tate correlation including combined entry length yields

$$\overline{Nu}_D = \frac{\bar{h}D}{k} = 1.86\left(\frac{Re_D\,Pr}{L/D}\right)^{1/3}\left(\frac{\mu}{\mu_s}\right)^{0.14} \qquad (3)$$

$$\bar{h} = \frac{0.0563\,\text{W/m}\cdot\text{K}}{0.003\,\text{m}}\,1.86\left(\frac{584 \times 0.706}{25}\right)^{1/3}\left(\frac{363.7 \times 10^{-7}}{404.2 \times 10^{-7}}\right)^{0.14} = 87.5\,\text{W/m}^2\cdot\text{K}\,.$$

Hence, the air outlet temperature is

$$\frac{650 - T_{m,o}}{(650 - 427)^\circ\text{C}} = \exp\left(-\frac{\pi\,(0.003\,\text{m}) \times 0.075\,\text{m} \times 87.5\,\text{W/m}^2\cdot\text{K}}{(0.18/3600)\,\text{kg/s} \times 1094\,\text{J/kg}\cdot\text{K}}\right)$$

$$T_{m,o} = 578^\circ\text{C}$$

<

(b) Using the *IHT Correlations Tool*, *Internal Flow*, for *Laminar Flow* with *combined entry length*, along with the energy balance and rate equations above, the outlet temperature $T_{m,o}$ was calculated as a function of flow rate for diameters of D = 2, 3 and 4 mm. The plot below shows that $T_{m,o}$ decreases nearly linearly with increasing flow rate, but is independent of passage diameter.

Continued...

PROBLEM 8.45 (Cont.)

COMMENTS: (1) Based upon the calculation for $T_{m,o}$ = 578°C, $\overline{T}_m$ = 775 K which is in good agreement with our assumption to evaluate the thermophysical properties.

(2) Why is $T_{m,o}$ independent of D? From Eq. (3), note that $\overline{h}$ is inversely proportional to D, $\overline{h} \sim D^{-1}$. From Eq. (1), note that on the right-hand side the product $P \cdot \overline{h}$ will be independent of D. Hence, $T_{m,o}$ will depend only on $\dot{m}$. This is, of course, a consequence of the laminar flow condition and will not be the same for turbulent flow.

PROBLEM 8.46

KNOWN: Gas-cooled nuclear reactor tube of 20 mm diameter and 780 mm length with helium heated from 600 K to 1000 K at 8×10^{-3} kg/s.

FIND: (a) Uniform tube wall temperature required to heat the helium, (b) Outlet temperature and required flow rate to achieve same removal rate and wall temperature if the coolant gas is air.

SCHEMATIC:

Tube, D=20mm, L=780mm
T_s
Helium or air
$\dot m = 8\times10^{-3}$ kg/s
$T_{m,i}$ = 600K
$T_{m,o}$ = 1000K

ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible kinetic energy and potential energy changes, (3) Fully developed conditions.

PROPERTIES: *Table A-4,* Helium $\left(\overline{T}_m = 800\text{K, 1 atm}\right)$: ρ = 0.06272 kg/m^3, c_p = 5193 J/kg·K, k = 0.304 W/m·K, μ = 382 × 10^{-7} N·s/m^2, ν = 6.09 × 10^{-4} m^2/s, Pr = 0.654; Air $\left(\overline{T}_m = 800\text{K, 1 atm}\right)$: ρ = 0.4354 kg/m^3, c_p = 1099 J/kg·K, k = 57.3 × 10^{-3} W/m·K, ν = 84.93 × 10^{-6} m^2/s, Pr = 0.709.

ANALYSIS: (a) For helium and a constant wall temperature, from Eq. 8.46,

$$\frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp\left(-\frac{PL\overline{h}}{\dot m\, c_p}\right)$$

where P = πD. For the circular tube,

$$Re_D = \frac{4\dot m}{p\ Dm} = \frac{4\times8\times10^{-3}\ \text{kg/s}}{p\times0.020\ \text{m}\times382\times10^{-7}\,\text{N}\cdot\text{s/m}^2} = 1.333\times10^4$$

and using the Colburn correlation for turbulent, fully developed flow,

$$Nu = 0.023\, Re_D^{4/5}\, Pr^{1/3} = 0.023\left(1.333\times10^4\right)^{4/5}(0.654)^{1/3} = 39.83$$

$$h = Nu\cdot k/D = 39.83\times0.304\ \text{W/m}\cdot\text{K}/0.02\ \text{m} = 605\ \text{W/m}^2\cdot\text{K}.$$

Hence, the surface temperature is

$$\frac{T_s - 1000\ \text{K}}{T_s - 600\ \text{K}} = \exp\left[-\frac{p\,(0.020\ \text{m})\times0.780\ \text{m}\times605\ \text{W/m}^2\cdot\text{K}}{8\times10^{-3}\ \text{kg/s}\times5193\ \text{J/kg}\cdot\text{K}}\right] = 0.4898$$

$$T_s = 1384\ \text{K}.$$ <

The heat rate with helium coolant is

$$q = \dot m\, c_p\left(T_{m,o} - T_{m,i}\right) = 8\times10^{-3}\ \text{kg/s}\times5193\ \text{J/kg}\cdot\text{K}\,(1000-600)\,\text{K} = 16.62\ \text{kW}.$$

Continued …..

PROBLEM 8.46 (Cont.)

(b) For the same heat removal rate (q) and wall temperature (T_s) with air supplied at $T_{m,i}$, the relevant relations are

$$q = 16{,}620\ \text{W} = \dot{m}_a\ c_p\left(T_{m,o} - T_{m,i}\right) \tag{1}$$

$$\frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp\left[-\frac{PL\bar{h}_a}{\dot{m}_a\ c_p}\right] \tag{2}$$

$$Re = \frac{4\dot{m}_a}{\pi\ D\mu} \qquad \frac{\bar{h}D}{k} = 0.023\ Re_D^{4/5}\ Pr^{1/3} \tag{3,4}$$

where $T_{m,o}$ and $\dot{m}$ are unknown. An iterative solution is required: assume a value of $T_{m,o}$ and find $\dot{m}$ from Eq. (1); use $\dot{m}$ in Eqs. (3) and (4) to find $\bar{h}$ and then Eq. (2) to evaluate $T_{m,o}$; compare results and iterate. Using thermophysical properties of air evaluated at $\bar{T}_m$ = 800K, the above relations, written in the order they would be used in the iteration, become

$$\dot{m}_a = \frac{15.123}{T_{m,o} - 600} \tag{5}$$

$$\bar{h}_a = 5.725 \times 10^3 \dot{m}_a^{4/5} \tag{6}$$

$$T_{m,o} = 1384 - 784\ \exp\left[-4.459 \times 10^{-5}\left(\bar{h}_a / \dot{m}_a\right)\right] \tag{7}$$

Results of the iterative solution are

Trial	$T_{m,o}$ (K) (Assumed)	$\dot{m}$ (kg/s) Eq. (5)	$\bar{h}_a$ $\left(W/m^2 \cdot K\right)$ Eq. (6)	$T_{m,o}$ (K) Eq. (7)
1	1000	3.025×10^{-2}	348.6	915.0
2	950	4.321×10^{-2}	463.7	898.
3	900	5.041×10^{-2}	524.6	891.0
4	890	5.215×10^{-2}	539.0	889.5

Hence, we find

$$\dot{m}_a = 5.22 \times 10^{-2}\ \text{kg/s} \qquad T_{m,o} = 890\ \text{K}.$$

<

COMMENTS: To achieve the same cooling rate with air, the required mass rate is 6.5 times that obtained with helium.

PROBLEM 8.47

KNOWN: Air at prescribed inlet temperature and mean velocity heated by condensing steam on its outer surface.

FIND: (a) Air outlet temperature, pressure drop and heat transfer rate and (b) Effect on parameters of part (a) if pressure were doubled.

SCHEMATIC:

Air
$p = 200 kPa$
$u_m = 6 m/s$
$T_{m,i} = 150°C$
$D = 25 mm$
Condensing steam, 20bar, $T_s = T_{sat} = 485 K$
$T_{m,o}$
x
$L = 2 m$

ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible kinetic and potential energy changes, (3) Thermal resistance of tube wall and condensate film are negligible.

PROPERTIES: *Table A-4*, Air (assume $\overline{T}_m$ = 450K, 1 atm = 101.3 kPa): $\rho = 0.7740\ kg/m^3$, $c_p = 1021\ J/kg\cdot K$, $\mu = 250.7 \times 10^{-7}\ N\cdot s/m^2$, k = 0.0373 W/m·K, $Pr = \mu c_p/k = 0.686$. Note that only ρ is pressure dependent; i.e., ρ α P; *Table A-6,* Saturated water (20 bar): $T_{sat} = T_s = 485K$.

ANALYSIS: (a) For constant wall temperature heating, from Eq. 8.46 but with $U \approx \overline{h}_i$ since $\overline{h}_o >> \overline{h}_i$, where $\overline{h}_o$ is the convection coefficient for the condensing steam,

$$\frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp\left(-\frac{PL}{\dot{m}\, c_p}\overline{h}_i\right)$$

where P = πD. For the air flow, find the mass rate and Reynolds number,

$$\dot{m} = r A_c u_m = 0.7740\ kg/m^3 (200\ kPa/101.3\ kPa)\left(p\,(0.025\ m)^2/4\right) \times 6\ m/s$$

$$\dot{m} = 4.501 \times 10^{-3} kg/s.$$

$$Re_D = \frac{4\dot{m}}{mp\ D} = \frac{4 \times 4.501 \times 10^{-3}\ kg/s}{250.7 \times 10^{-7}\ N\cdot s/m^2 \times p\,(0.025\ m)} = 9.143 \times 10^3.$$

Using the Dittus-Boelter correlation for fully-developed turbulent flow,

$$Nu_D = 0.023 Re^{4/5} Pr^{0.4} = 0.023\left(9.143 \times 10^3\right)^{4/5}(0.682)^{0.4} = 29.12$$

$$h_i = Nu \cdot k/D = 29.12 \times 0.0373\ W/m\cdot K/0.025\ m = 43.4\ W/m^2\cdot K.$$

Hence, the outlet temperature is

$$\frac{212 - T_{m,o}}{(212-150)^\circ C} = \exp\left[-\frac{p\,(0.025\ m) \times 2\ m \times 43.4\ W/m^2\cdot K}{4.501 \times 10^{-3}\ kg/s \times 1021\ J/kg\cdot K}\right]$$

$$T_{m,o} = 198^\circ C.$$ <

Continued …..

The pressure drop follows from Eqs. 8.20 and 8.22,

$$f = 0.316 Re_D^{-1/4} = 0.316\left(9.143\times10^{3}\right)^{-1/4} = 0.0323$$

$$\Delta p = f\,\frac{\rho u_m^2}{2D} L$$

$$\Delta p = 0.0323 \frac{0.7740\ \text{kg/m}^3 (200/101.3)(6\ \text{m/s})^2 \times 2\ \text{m}}{2\times0.025\ \text{m}} = 71.1\ \text{N/m}^2. \qquad <$$

The heat transfer rate is

$$q = \dot{m}\, c_p \left(T_{m,o} - T_{m,i}\right) = 4.501\times10^{-3}\ \text{kg/s}\times1021\ \text{J/kg}\cdot\text{K}\left(198-150\right)\text{K} = 221\ \text{W}. \qquad <$$

(b) If the pressure were doubled, we can see from the above relations, that $\dot{m} \propto \rho$, hence

$$\dot{m} = 2\dot{m}_o$$

$$Re_D = 2Re_{D,o},$$

since

$$h_i \propto (Re)^{4/5} \rightarrow \left(h_i / h_{i,o}\right) = 2^{4/5},$$

$$h_i = 1.74 h_{i,o}.$$

It follows that $T_{m,o}$ = 195°C, so that the effect on temperature is slight. However, the pressure drop increases by the factor $2(2)^{-1/4}$ = 1.68 and the heat rate by 2(195 - 150)/(198 - 150) = 1.88. In summary:

Parameter	p = 200 kPa Part (a)	p = 400 kPa Part (b)	Increase, %
$\dot{m}$, kg/s $\times 10^3$	4.501	9.002	100
h_i, W/m$^2\cdot$K	43.4	86.8	100
T_{mo} - $T_{m,i}$°C	48	45	-6
Δp, N/m^2	71.1	119	68
q, W	221	415	88

COMMENTS: (1) Note that $\overline{T}_m$ = (198 + 150)°C/2 = 447 K agrees well with the assumed value (450 K) used to evaluate the thermophysical properties.

PROBLEM 8.48

KNOWN: Diameter, length and surface temperature of tubes used to heat ambient air. Flowrate and inlet temperature of air.

FIND: (a) Air outlet temperature and heat rate per tube, (b) Effect of flowrate on outlet temperature. Design and operating conditions suitable for providing 1 kg/s of air at 75°C.

SCHEMATIC:

Saturated Steam

Air

$D = 0.05$ m

$T_s = 100\ ^oC$

$T_{m,i} = 20\ ^oC$

$0.005 \le \dot{m} \le 0.05$ kg/s

$T_{m,o}$

$L = 5$ m

ASSUMPTIONS: (1) Steady-state, (2) Negligible kinetic/potential energy and flow work changes, (3) Negligible tube wall thermal resistance.

PROPERTIES: *Table A.4,* air (assume $\overline{T}_m$ = 330 K): c_p = 1008 J/kg·K, $\mu = 198.8 \times 10^{-7}$ N·s/m^2, k = 0.0285 W/m·K, Pr = 0.703.

ANALYSIS: (a) For $\dot{m}$ = 0.01 kg/s, $Re_D = 4\dot{m}/\pi D\mu$ = 0.04 kg/s/π(0.05 m)198.8×10^{-7} N·s/m^2 = 12,810. Hence, the flow is turbulent. If fully developed flow is assumed throughout the tube, the Dittus-Boelter correlation may be used to obtain the average Nusselt number.

$$\overline{Nu}_D \approx Nu_D = 0.023 Re_D^{4/5} Pr^{0.4} = 0.023(12,810)^{0.8}(0.703)^{0.4} = 38.6$$

Hence, $\overline{h} = \overline{Nu}_D(k/D) = 38.6(0.0285\,W/m\cdot K/0.05\,m) = 22.0\,W/m^2\cdot K$

From Eq. 8.42b,

$$\frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp\left(-\frac{\pi D L \overline{h}}{\dot{m} c_p}\right) = \exp\left(-\frac{\pi \times 0.05\,m \times 5\,m \times 22\,W/m^2\cdot K}{0.01\,kg/s \times 1008\,J/kg\cdot K}\right) = 0.180$$

$$T_{m,o} = T_s - 0.180(T_s - T_{m,i}) = 100^\circ C - 0.180(80^\circ C) = 85.6^\circ C$$ <

Hence, $q = \dot{m} c_p (T_{m,o} - T_{m,i}) = 0.01\,kg/s(1008\,J/kg\cdot K)65.6\,K = 661\,W$ <

(b) The effect of flowrate on the outlet temperature was determined by using the IHT *Correlations* and *Properties* Toolpads.

Outlet temperature, Tmo(C) (75–90) vs. Mass flowrate, mdot(kg/s) (0–0.05)

Continued...

PROBLEM 8.48 (Cont.)

Although $\bar{h}$ and hence the heat rate increase with increasing $\dot{m}$, the increase in q is not linearly proportional to the increase in $\dot{m}$ and $T_{m,o}$ decreases with increasing $\dot{m}$.

A flowrate of $\dot{m}$ = 0.05 kg/s is not large enough to provide the desired outlet temperature of 75°C, and to achieve this value, a flowrate of 0.0678 kg/s would be needed. At such a flowrate, N = 1 kg/s/0.0678 kg/s = 14.75 ≈ 15 tubes would be needed to satisfy the process air requirement. Alternatively, a lower flowrate could be supplied to a larger number of tubes and the discharge mixed with ambient air to satisfy the desired conditions. Requirements of this option are that

$$N\dot{m} + \dot{m}_{amb} = 1\,kg/s$$

$$\left(N\dot{m} + \dot{m}_{amb}\right)c_p\left(T_{m,o} - T_{m,i}\right) = 1\,kg/s \times 1008\,J/kg \cdot K\left(75 - 20\right)K = 55,400\,W$$

where $\dot{m}$ is the flowrate per tube. Using a larger number of tubes with a smaller flowrate per tube would reduce flow pressure losses and hence provide for reduced operating costs.

COMMENTS: With L/D = 5 m/0.05 m = 100, the assumption of fully developed conditions throughout the tube is reasonable.

PROBLEM 8.49

KNOWN: Length and diameter of tube submerged in paraffin of prescribed dimensions. Inlet temperature and flow rate of water flowing through tube.

FIND: (a) Outlet temperature, heat rate, and time required for complete melting, and (b) Effect of flowrate on operating conditions.

SCHEMATIC:

D = 0.025 m
$T_{m,o}$
H = 0.25 m
Paraffin
Water
q
$0.1 \leq \dot{m} \leq 0.5$ kg/s
$T_{m,i}$ = 60 °C
L = 3 m
W = 0.25 m

ASSUMPTIONS: (1) Negligible KE/PE and flowwork changes for water, (2) Constant water properties, (3) Negligible tube wall conduction resistance, (4) Negligible convection resistance in melt ($T_s = T_\infty = T_{mp}$), (5) Fully developed flow, (6) No heat loss to the surroundings.

PROPERTIES: Water (given): c_p = 4.185 kJ/kg·K, k = 0.653 W/m·K, $\mu = 467 \times 10^{-6}$ kg/s·m, Pr = 2.99; Paraffin (given): T_{mp} = 27.4°C, h_{sf} = 244 kJ/kg, ρ = 770 kg/m^3.

ANALYSIS: (a) From Eq. 8.42b, $\dfrac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = \exp\left(-\dfrac{\pi D L \bar{h}}{\dot{m} c_p}\right)$. With $Re_D = \dfrac{4\dot{m}}{\pi D \mu} = \dfrac{4 \times 0.1\,\text{kg/s}}{\pi \times 0.025\,\text{m} \times 467 \times 10^{-6}\,\text{kg/s}\cdot\text{m}}$ = 10,906, the flow is turbulent. Assuming fully developed conditions,

$$h = \frac{Nu_D k}{D} = \frac{k}{D} 0.023\, Re_D^{4/5}\, Pr^{0.3} = \frac{0.653\,\text{W/m}\cdot\text{K}}{0.025\,\text{m}} 0.023 (10,906)^{4/5} (2.99)^{0.3} = 1418\,\text{W/m}^2\cdot\text{K}$$

$$T_{m,o} = 27.4^\circ\text{C} - (27.4 - 60)^\circ\text{C} \exp\left(-\frac{\pi \times 0.025\,\text{m} \times 3\,\text{m}}{0.1\,\text{kg/s} \times 4185\,\text{J/kg}\cdot\text{K}} 1418\,\text{W/m}^2\cdot\text{K}\right) = 42.17^\circ\text{C}$$ <

From the overall energy balance,

$$q = \dot{m} c_p \left(T_{m,i} - T_{m,o}\right) = 0.1\,\text{kg/s} \times 4185\,\text{J/kg}\cdot\text{K}\,(60 - 42.17)^\circ\text{C} = 7500\,\text{W}$$ <

Applying an energy balance to a control volume about the paraffin, $E_{in} = \Delta E_{st}$, the time t_m required to melt the paraffin is

$$q t_m = \rho V h_{sf} = \rho L \left(WH - \rho D^2/4\right) h_{sf}$$

$$t_m = \frac{770\,\text{kg/m}^3 \times 3\,\text{m}\left(0.25 \times 0.25\,\text{m}^2 - \pi (0.025\,\text{m})^2/4\right)}{7500\,\text{W}} 2.44 \times 10^5\,\text{J/kg} = 4660\,\text{s} = 1.29\,\text{h}$$ <

Continued...

PROBLEM 8.49 (Cont.)

(b) The effect of $\dot{m}$ on q and $T_{m,o}$ was determined by accessing the *Correlations* Toolpad of IHT, and the results are plotted as follows.

Although q increases with increasing $\dot{m}$ due to the attendant increase in Re_D, and therefore $\bar{h}$, the increase is not linearly proportional to the change in $\dot{m}$. Hence, from the overall energy balance, $q = \dot{m}\, c_p(T_{m,i} - T_{m,o})$, there is a reduction in $(T_{m,i} - T_{m,o})$, which corresponds to an increase in $T_{m,o}$. With the increase in q, there is a reduction in t_m, and for $\dot{m}$ = 0.5 kg/s,

$$t_m = 1167\,\text{s} = 0.324\,\text{h}$$ <

COMMENTS: Heat transfer from the water to the paraffin is also affected by free convection in the melt region around the tube. The effect is to decrease U, increase T_s, and decrease q with increasing time. The actual time to achieve complete melting would exceed values computed in the foregoing analysis.

PROBLEM 8.50

KNOWN: Configuration of microchannel heat sink.

FIND: (a) Expressions for longitudinal distributions of fluid mean and surface temperatures, (b) Coolant and channel surface temperature distributions for prescribed conditions, (c) Effect of heat sink design and operating conditions on the chip heat flux for a prescribed maximum allowable surface temperature.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state, (2) Negligible PE, KE and flow work changes, (3) All of the chip power dissipation is transferred to the coolant, with a uniform surface heat flux, q''_s, (4) Laminar, fully developed flow, (5) Constant properties.

PROPERTIES: *Table A.6*, Water (assume $\overline{T}_m = T_{m,i}$ = 290 K): c_p = 4184 J/kg·K, $\mu = 1080 \times 10^{-6}$ N·s/m^2, k = 0.598 W/m·K, Pr = 7.56.

ANALYSIS: (a) The number of channels passing through the heat sink is N = L/S = L/C_1D, and conservation of energy dictates that

$$q''_c L^2 = N(\pi D L) q''_s = \pi L^2 q''_s / C_1$$

which yields

$$q''_s = \frac{C_1 q''_c}{\pi} \quad (1)$$

With the mass flowrate per channel designated as $\dot{m}_1 = \dot{m}/N$, Eqs. 8.41 and 8.28 yield

$$T_m(x) = T_{m,i} + \frac{q''_s \pi D}{\dot{m}_1 c_p} x = T_{m,i} + \frac{L q''_c}{\dot{m} c_p} x \quad (2) <$$

$$T_s(x) = T_m(x) + \frac{q''_s}{h} = T_m(x) + \frac{C_1 q''_c}{\pi h} \quad (3) <$$

where, for laminar, fully developed flow with uniform q''_s, Eq. 8.53 yields h = 4.36 k/D.

(b) With L = 12 mm, D = 1 mm, C_1 = 2 and $\dot{m}$ = 0.01 kg/s, it follows that S = 2 mm, N = 6 and Re_D = $4\dot{m}_1/\pi D \mu$ = $4(0.01\,\text{kg/s})/6\pi(0.001\,\text{m})1.08\times10^{-3}\,\text{N}\cdot\text{s/m}^2$ = 1965. Hence, the flow is laminar, as assumed, and h = 4.36(0.598 W/m·K/0.001 m) = 2607 W/m^2·K. From Eqs. (2) and (3) the outlet mean and surface temperatures are

$$T_{m,o} = 290\,\text{K} + \frac{(0.012\,\text{m})^2\, 20\times10^4\,\text{W/m}^2}{0.01\,\text{kg/s}\,(4184\,\text{J/kg}\cdot\text{K})} = 290.7\,\text{K} = 17.7^\circ\text{C}$$

$$T_{s,o} = T_{m,o} + \frac{2}{\pi} \times \frac{20\times10^4\,\text{W/m}^2}{2607\,\text{W/m}^2\cdot\text{K}} = 339.5\,\text{K} = 66.5^\circ\text{C}$$

Continued...

PROBLEM 8.50 (Cont.)

The axial temperature distributions are as follows

The flowrate is sufficiently large (and the convection coefficient sufficiently low) to render the increase in T_m and T_s with increasing x extremely small.

(c) The desired constraint of $T_{s,max} \le 50°C$ is not met by the foregoing conditions. An obvious and logical approach to achieving improved performance would involve increasing $\dot{m}_1$ such that turbulent flow is maintained in each channel. A value of $\dot{m}_1 > 0.002$ kg/s would provide $Re_D > 2300$ for D = 0.001. Using Eq. 8.60 with n = 0.4 to evaluate Nu_D and accessing the Correlations Toolpad of IHT to explore the effect of variations in $\dot{m}_1$ for different combinations of D and C_1, the following results were obtained.

We first note that a significant increase in q''_c may be obtained by operating the channels in turbulent flow. In addition, there is an obvious advantage to reducing C_1, thereby increasing the number of channels for a fixed channel diameter. The biggest enhancement is associated with reducing the channel diameter, which significantly increases the convection coefficient, as well as the number of channels for fixed C_1. For $\dot{m}_1 = 0.005$ kg/s, h increases from 32,400 to 81,600 $W/m^2 \cdot K$ with decreasing D from 1.0 to 0.6 mm. However, for fixed $\dot{m}_1$, the mean velocity in a channel increases with decreasing D and care must be taken to maintain the flow pressure drop within acceptable limits.

COMMENTS: Although the distribution computed for $T_m(x)$ in part (b) is correct, the distribution for $T_s(x)$ represents an upper limit to actual conditions due to the assumption of fully developed flow throughout the channel.

PROBLEM 8.51

KNOWN: Cold plate geometry and temperature. Inlet temperature and flow rate of water. Number of circuit boards and temperature and velocity of air in parallel flow over boards.

FIND: (a) Heat dissipation by cold plates, (b) Heat dissipation by air flow.

SCHEMATIC:

Water
$\dot{m}_1$ = 0.2 kg/s, $T_{m,i}$ = 7°C
$T_{s,cp}$ = 32°C
H = 0.75 m
Coolant passage, N = 10
D = 0.01 m
Part (a)

W = 0.35 m
$T_{s,cb}$ = 47°C
L = 0.60 m
Air
p = 1 atm
u_∞ = 10 m/s
T_∞ = 7°C
Part (b)

ASSUMPTIONS: (1) Isothermal cold plate, (2) All heated generated by circuit boards is dissipated by cold plates (Part (a)), (3) Circuit boards may be represented as isothermal at an average surface temperature, (4) Air flow over circuit boards approximates that over a flat plate in parallel flow, (5) Steady operation, (6) Constant properties.

PROPERTIES: *Table A-6*, Water $\left(\overline{T}_m \approx 290\text{K}\right)$: c_p = 4184 J/kg·K, $\mu = 1080\times10^{-6}\ \text{N}\cdot\text{s/m}^2$, k=0.598 W/m·K, Pr = 7.56. *Table A-4*, Air (p = 1 atm, T_f = 300K): $\nu = 15.89\times10^{-6}\ \text{m}^2/\text{s}$, k = 0.0263 W/m·K, Pr = 0.707.

ANALYSIS: (a) With $Re_D = 4\dot{m}_1/\pi D\mu = 4\times0.2\,\text{kg/s}/\pi\times0.01\text{m}\times1080\times10^{-6}\ \text{N}\cdot\text{s/m}^2 = 23,600$, the flow is turbulent, and from Eq. (8.60),

$$h = \frac{k}{D}Nu_D = 0.023\frac{k}{D}Re_D^{4/5}Pr^{0.4} = \frac{0.023\times0.598\,\text{W/m}\cdot\text{K}}{0.01\text{m}}(23,600)^{4/5}(7.56)^{0.4} = 9,730\,\text{W/m}^2\cdot\text{K}$$

With H/D = 0.75/0.01 = 75, it is reasonable to assume fully developed flow throughout the tube. Hence, from Eqs. (8.42b) and (8.37)

$$\frac{T_{s,cp}-T_{m,o}}{T_{s,cp}-T_{m,i}} = \exp\left(-\frac{\pi DH}{\dot{m}_1 c_p}h\right) = \exp\left(-\frac{\pi\times0.01\text{m}\times0.75\text{m}\times9730\,\text{W/m}^2\cdot\text{K}}{0.2\,\text{kg/s}\times4184\,\text{J/kg}\cdot\text{K}}\right) = 0.760$$

$$T_{m,o} = T_{s,cp} - 0.76\left(T_{s,cp}-T_{m,i}\right) = 13°\text{C}$$

$$q_1 = \dot{m}_1 c_p\left(T_{m,o}-T_{m,i}\right) = 0.2\,\text{kg/s}\times4184\,\text{J/kg}\cdot\text{K}\times6°\text{C} = 5021\,\text{W}$$

With a total of 2N = 20 passages, the total heat dissipation is

$$q = 2Nq_1 = 20\times5021\,\text{W} = 100\,\text{kW}$$ <

(b) For the air flow, $Re_D = u_\infty L/\nu = 10\,\text{m/s}\times0.60\text{m}/15.89\times10^{-6}\text{m}^2\text{s} = 378,000$, and the flow is laminar. From Eq. (7.31),

$$\overline{h} = \frac{k}{L}\overline{Nu}_L = 0.664\frac{k}{L}Re_L^{1/2}Pr^{1/3} = \frac{0.664\times0.0263\,\text{W/m}\cdot\text{K}}{0.60\text{m}}(378,000)^{1/2}(0.707)^{1/3} = 15.9\,\text{W/m}^2\cdot\text{K}$$

Heat dissipation to the air from both sides of 10 circuit boards is then

$$q = 2N_{cb}\overline{h}(WL)\left(T_{s,cb}-T_\infty\right) = 20\times15.9\,\text{W/m}^2\cdot\text{K}\times0.21\text{m}^2\times40°\text{C} = 2,670\,\text{W}$$ <

COMMENTS: The cooling capacity of the cold plates far exceeds that of the air flow. However, the challenge would be one of efficiently transferring such a large amount of energy to the cold plates without incurring excessive temperatures on the circuit boards.

PROBLEM 8.52

KNOWN: Flow rate and temperature of Freon passing through a Teflon tube of prescribed inner and outer diameter. Velocity and temperature of air in cross flow over tube.

FIND: Heat transfer per unit tube length.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional radial conduction, (3) Constant properties, (4) Fully developed flow.

PROPERTIES: *Table A-4*, Air (T = 300K, 1 atm): $\nu = 15.89 \times 10^{-6}\ m^2/s$, k = 0.0263 W/m·K, Pr = 0.707; *Table A-5*, Freon (T = 240K): $\mu = 3.85 \times 10^{-4}\ N \cdot s/m^2$, k = 0.069 W/m·K, Pr = 5.0; *Table A-3*, Teflon (T ≈ 300K): k = 0.35 W/m·K.

ANALYSIS: Considering the thermal circuit shown above, the heat rate is

$$q' = \frac{T_\infty - T_m}{\left(1/\bar{h}_o \pi D_o\right) + \left[\ell n\left(D_o / D_i\right)/2\pi k\right] + \left(1/h_i \pi D_i\right)}.$$

$$Re_{D,i} = \frac{4\dot{m}}{\pi D_i \mu} = \frac{0.4\ kg/s}{\pi(0.025 m)\ 3.85\times10^{-4}\ N \cdot s/m^2} = 13{,}228$$

and the flow is turbulent. Hence, from the Dittus-Boelter correlation

$$h_i = \frac{k}{D_i} 0.023\ Re_{D,i}^{4/5}\ Pr^{0.4} = \frac{0.069\ W/m \cdot K}{0.025\ m} 0.023(13{,}228)^{4/5}(5)^{0.4} = 240\ W/m^2 \cdot K.$$

With $Re_{D,o} = \frac{VD_o}{\nu} = \frac{(25\ m/s)0.028\ m}{15.89\times10^{-6}\ m^2/s} = 4.405\times10^4$

it follows from Eq. 7.56 and Table 7.4 that

$$\bar{h}_o = \frac{k}{D} 0.26\ Re_{D,o}^{0.6}\ Pr^{0.37} = \frac{0.0263\ W/m \cdot K}{0.028\ m} 0.26\left(4.405\times10^4\right)^{0.6}(0.707)^{0.37} = 131\ W/m^2 \cdot K.$$

Hence

$$q' = \frac{T_\infty - T_m}{\left(131\ W/m^2 \cdot K \pi\, 0.028\ m\right)^{-1} + \ell n\left(28/25\right)/2\pi\left(0.350\ W/m \cdot K\right) + \left(240\ W/m^2 \cdot K \pi\, 0.025\ m\right)^{-1}}$$

$$q' = \frac{(300 - 240)K}{(0.087 + 0.052 + 0.053)\ K \cdot m/W} = 312\ W/m. \qquad <$$

COMMENTS: The three thermal resistances are comparable. Note that $T_{s,o} = T_\infty - q'/h_o \pi D_o$ = 300K - 312 W/m/131 $W/m^2 \cdot K$ π 0.028 m = 273 K.

PROBLEM 8.53

KNOWN: Oil flowing slowly through a long, thin-walled pipe suspended in a room.

FIND: Heat loss per unit length of the pipe, q'_{conv}.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Tube wall thermal resistance negligible, (3) Fully developed flow, (4) Radiation exchange between pipe and room negligible.

PROPERTIES: *Table A-5*, Unused engine oil (T_m = 150°C = 423K): k = 0.133 W/m·K.

ANALYSIS: The rate equation, for a unit length of the pipe, can be written as

$$q'_{conv} = \frac{(T_m - T_\infty)}{R'_t}$$

where the thermal resistance is comprised of two elements,

$$R'_t = \frac{1}{h_i \pi D} + \frac{1}{h_o \pi D} = \frac{1}{\pi D}\left(\frac{1}{h_i} + \frac{1}{h_o}\right).$$

The convection coefficient for internal flow, h_i, must be estimated from an appropriate correlation. From practical considerations, we recognize that the oil flow rate cannot be large enough to achieve turbulent flow conditions. Hence, the flow is *laminar*, and if the pipe is very long, the flow will be *fully developed*. The appropriate correlation is

$$Nu_D = \frac{h_i D}{k} = 3.66$$

$$h_i = Nu_D\ k/D = 3.66 \times 0.133 \frac{W}{m \cdot K} / 0.030\ m = 16.2\ W/m^2 \cdot K.$$

The heat rate per unit length of the pipe is

$$q'_{conv} = \frac{(150-20)^\circ C}{\dfrac{1}{\pi(0.030m)}\left(\dfrac{1}{16.2} + \dfrac{1}{11}\right)\dfrac{m^2 \cdot K}{W}} = 80.3\ W/m.$$ <

COMMENTS: This problem requires making a judgment that the oil flow will be laminar rather than turbulent. Why is this a reasonable assumption? Recognize that the correlation applies to a constant surface temperature condition.

PROBLEM 8.54

KNOWN: Thin-walled, tall stack discharging exhaust gases from an oven into the environment.

FIND: (a) Outlet gas and stack surface temperatures, $T_{m,o}$ and $T_{s,o}$, and (b) Effect of wind temperature and velocity on $T_{m,o}$.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Wall thermal resistance negligible, (3) Exhaust gas properties approximated as those of atmospheric air, (4) Radiative exchange with surroundings negligible, (5) PE, KE, and flow work changes negligible, (6) Fully developed flow, (7) Constant properties.

PROPERTIES: *Table A.4*, air (assume $T_{m,o}$ = 773 K, $\bar{T}_m$ = 823 K, 1 atm): c_p = 1104 J/kg·K, μ = 376.4 $\times 10^{-7}$ N·s/m^2, k = 0.0584 W/m·K, Pr = 0.712; *Table A.4*, air (assume T_s = 523 K, T_∞ = 4°C = 277 K, T_f = 400 K, 1 atm): $\nu = 26.41 \times 10^{-6}$ m^2/s, k = 0.0338 W/m·K, Pr = 0.690.

ANALYSIS: (a) From Eq. 8.46a,

$$T_{m,o} = T_\infty - \left(T_\infty - T_{m,i}\right)\exp\left[-\frac{PL}{\dot{m}c_p}\bar{U}\right] \qquad U = 1\bigg/\left(\frac{1}{h_i} + \frac{1}{h_o}\right) \tag{1,2}$$

where h_i and h_o are average coefficients for internal and external flow, respectively.

Internal flow: With a Reynolds number of

$$Re_{D_i} = \frac{4\dot{m}}{\pi D \mu} = \frac{4 \times 0.5\,\text{kg/s}}{\pi \times 0.5\,\text{m} \times 376.4 \times 10^{-7}\,\text{N}\cdot\text{s/m}^2} = 33{,}827 \tag{3}$$

and the flow is turbulent. Considering the flow to be fully developed throughout the stack (L/D = 12) and with $T_s < T_m$, the Dittus-Boelter correlation has the form

$$Nu_D = \frac{h_i D}{k} = 0.023\, Re_{D_i}^{4/5}\, Pr^{0.3} \tag{4}$$

$$h_i = \frac{58.4 \times 10^{-3}\,\text{W/m}\cdot\text{K}}{0.5\,\text{m}} \times 0.023(33{,}827)^{4/5}(0.712)^{0.3} = 10.2\,\text{W/m}^2\cdot\text{K}\,.$$

External flow: Working with the Churchill/Bernstein correlation, the Reynolds and Nusselt numbers are

$$Re_{D_o} = \frac{VD}{\nu} = \frac{5\,\text{m/s} \times 0.5\,\text{m}}{26.41 \times 10^{-6}\,\text{m}^2/\text{s}} = 94{,}660 \tag{5}$$

Continued...

PROBLEM 8.54 (Cont.)

$$\overline{Nu}_D = 0.3 + \frac{0.62\,Re_D^{1/2}\,Pr^{1/3}}{\left[1+(0.4/Pr)^{2/3}\right]^{1/4}}\left[1+\left(\frac{Re_D}{282,000}\right)^{5/8}\right]^{4/5} = 205$$

Hence,

$$h_o = (0.0338\,W/m\cdot K/0.5\,m)\times 205 = 13.9\,W/m^2\cdot K \qquad (6)$$

The outlet gas temperature is then

$$T_{m,o} = 4^\circ C - (4-600)^\circ C \exp\left[-\frac{\pi\times 0.5\,m\times 6\,m}{0.5\,kg/s\times 1104\,J/kg\cdot K}\left(\frac{1}{1/10.2+1/13.9}W/m^2\cdot K\right)\right] = 543^\circ C$$ <

The outlet stack surface temperature can be determined from a local surface energy balance of the form, $h_i(T_{m,o} - T_{s,o}) = h_o(T_{s,o} - T_\infty)$, which yields

$$T_{s,o} = \frac{h_i T_{m,o} + h_o T_\infty}{h_i + h_o} = \frac{(10.2\times 543 + 13.9\times 4)\,W/m^2}{(10.2+13.9)\,W/m^2\cdot K} = 232^\circ C$$ <

(b) Using the Correlations and Properties Toolpads of IHT, with a surface temperature of T_s = 523 K assumed solely for the purpose of evaluating properties associated with airflow over the cylinder, the following results were generated.

Due to the elevated temperatures of the gas, the variation in ambient temperature has only a small effect on the gas exit temperature. However, the effect of the freestream velocity is more pronounced. Discharge temperatures of approximately 530 and 560°C would be representative of cold/windy and warm/still atmospheric conditions, respectively.

COMMENTS: If there are constituents in the discharge gas flow that condense or precipitate out at temperatures below $T_{s,o}$, this operating condition should be avoided.

PROBLEM 8.55

KNOWN: Hot fluid passing through a thin-walled tube with coolant in cross flow over the tube. Fluid flow rate and inlet and outlet temperatures.

FIND: Outlet temperature, $T_{m,o}$, if the flow rate is increased by a factor of 2 with all other conditions the same.

SCHEMATIC:

Tube, D = 10 mm
L = 1 m
Coolant
T_∞ = 25 °C
$T_{m,o}$ = 78 °C
Hot fluid
$T_{m,i}$ = 85 °C
$\dot{m}$ = 18 kg/h

ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible kinetic and potential energy changes and axial conduction, (3) Constant properties, (4) Fully developed flow and thermal conditions, (5) Convection coefficients, $\bar{h}_o$ and $\bar{h}_i$, independent of temperature, and (6) Negligible wall thermal resistance.

PROPERTIES: Hot fluid (Given): ρ = 1079 kg/m³, c_p = 2637 J/kg·K, μ = 0.0034 N·s/m², k = 0.261 W/m·K.

ANALYSIS: For conditions prescribed in the Schematic, Eq 8.46a can be used to evaluate the overall convection coefficient with P = πD,

$$\frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = \exp\left(-\frac{PL}{\dot{m}_o cp}\bar{U}\right) \tag{1}$$

$$\frac{(25-78)^\circ C}{(25-85)^\circ C} = \exp\left(-\frac{\pi \times 0.010\,m \times 1m}{(18/3600)kg/s \times 2637\,J/kg\cdot K}\bar{U}\right)$$

$$U = 52.1\,W/m^2\cdot K$$

The overall coefficient can be expressed in terms of the inside and outside coefficients,

$$U = \left(1/\bar{h}_i + 1/\bar{h}_o\right)^{-1} \tag{2}$$

Characterize the internal flow with the Reynolds number, Eq. 8.6,

$$Re_D = \frac{4\dot{m}_o}{\pi D\mu} = \frac{4\times(18/3600)kg/s}{\pi\times 0.010m\times 0.0034\,N\cdot s/m^2} = 187$$

and since the flow is laminar, and assumed to be fully developed, $\bar{h}_i$ will not change when the flow rate is doubled. That is, U = 52.1 W/m²·K when $\dot{m}$ = $2m_o$. Using Eq. (1) again, but with $T_{m,o}$ unknown,

$$\frac{(25-T_{m,o})^\circ C}{(25-85)^\circ C} = \exp\left(-\frac{\pi\times 0.010\,m\times 1m}{2(18/3600)kg/s\times 2637\,J/kg\cdot K}\times 52.1 W/m^2\cdot K\right)$$

$$T_{m,o} = 81.4^\circ C$$ <

COMMENTS: Examine the assumptions and explain why they were necessary in order to affect the solution.

PROBLEM 8.56

KNOWN: Thin walled tube of prescribed diameter and length. Water inlet temperature and flow rate.

FIND: (a) Outlet temperature of the water when the tube surface is maintained at a uniform temperature T_s = 27°C assuming $\overline{T}_m$ = 300 K for evaluating water properties, (b) Outlet temperature of the water when the tube is heated by cross flow of air with V = 10 m/s and T_∞ = 100°C assuming $\overline{T}_f$ = 350 K for evaluating air properties, and (c) Outlet temperature of the water for the conditions of part (b) using properly evaluated properties.

SCHEMATIC:

Water; $\dot{m}$ = 0.2 kg/s; T_s = 27 °C; $T_{m,i}$ = 47 °C; $T_{m,o}$; D = 10 mm, L = 2 m; (a)

Air; V = 10 m/s; $T_{m,o}$; (b)

ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible kinetic and potential energy changes and axial conduction, (3) Fully developed flow and thermal conditions for internal flow, and (4) Negligible tube wall thermal resistance.

PROPERTIES: *Table A.6*, Water ($\overline{T}_m$ = 300 K): ρ = 997 kg/m^3, c_p = 4179 J/kg·K, μ = 855 × 10^{-6} N·s/m^2, k = 0.613 W/m·K, Pr = 5.83; *Table A.4*, Air ($\overline{T}_f$ = 350 K, 1 atm): ν = 20.92 × 10^{-6} m^2/s, k = 0.030 W/m·K, Pr = 0.700.

ANALYSIS: (a) For the constant wall temperature cooling process, T_s = 27°C, the water outlet temperature can be determined from Eq. 8.42b, with P = πD,

$$\frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp\left(-\frac{PL}{\dot{m}c_p}\overline{h}_i\right) \tag{1}$$

To estimate the convection coefficient, characterize the flow evaluating properties at $\overline{T}_m$ = 300 K

$$Re_D = \frac{4\dot{m}}{\pi D\mu} = \frac{4\times 0.2\,\text{kg/s}}{\pi\times 0.010\,\text{m}\times 855\times 10^{-6}\,\text{N}\cdot\text{s/m}^2} = 29,783$$

Hence, the flow is turbulent and assuming fully developed (L/D = 200), and using the Dittus-Boelter correlation, Eq. 8.60, find $\overline{h}_i$,

$$Nu_D = \frac{\overline{h}_i D}{k} = 0.023\,Re_D^{0.8}\,Pr^{0.3} \qquad \overline{h}_i = \frac{0.613\,\text{W/m}\cdot\text{K}}{0.010\,\text{m}}0.023(29,783)^{0.8}(5.83)^{0.3} = 9080\,\text{W/m}^2\cdot\text{K} \tag{2}$$

Substituting this value for $\overline{h}_i$ into Eq. (1), find

$$\frac{(27 - T_{m,o})}{(27-47)^\circ\text{C}} = \exp\left(-\frac{\pi\times 0.010\,\text{m}\times 2\,\text{m}}{0.2\,\text{kg/s}\times 4179\,\text{J/kg}\cdot\text{K}}\times 9080\,\text{W/m}^2\cdot\text{K}\right) \qquad T_{m,o} = 37.1^\circ\text{C} \quad <$$

(b) For the air heating process, T_∞ = 100°C, the water outlet temperature follows from Eq. 8.46a,

$$\frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = \exp\left(-\frac{\pi DL}{\dot{m}c_p}\overline{U}\right) \tag{3}$$

Continued...

PROBLEM 8.56 (Cont.)

where the overall coefficient is $\overline{U} = \left(1/\overline{h}_i + 1/\overline{h}_o\right)$ (4)

To estimate $\overline{h}_o$, use the Churchill-Bernstein correlation, Eq. 7.57, for cross flow over a cylinder using properties evaluated at $\overline{T}_f$ = 350 K.

$$Re_D = \frac{VD}{\nu} = \frac{10\,m/s \times 0.010\,m}{20.92 \times 10^{-6}\,m^2/s} = 4780 \quad (5)$$

$$\overline{Nu}_D = 0.3 + \frac{0.62\,Re_D^{1/2}\,Pr^{1/3}}{\left[1+(0.4/Pr)^{2/3}\right]^{1/4}}\left[1+\left(\frac{Re_D}{282,000}\right)^{5/8}\right]^{4/5} \quad (6)$$

$$\overline{Nu}_D = 0.3 + \frac{0.62(4780)^{1/2}(0.700)^{1/3}}{\left[1+(0.4/0.700)^{2/3}\right]^{1/4}}\left[1+\left(\frac{4780}{282,000}\right)^{5/8}\right]^{4/5} = 35.76$$

$$\overline{h}_o = \frac{\overline{Nu}_D k}{D} = \frac{0.030\,W/m\cdot K}{0.010\,m} \times 35.76 = 107\,W/m^2\cdot K$$

Assuming $\overline{h}_i$ = 9080 W/m^2·K as calculated from part (a), find $\overline{U}$ then $T_{m,o}$,

$$\overline{U} = (1/9080 + 1/107)^{-1}\,W/m^2\cdot K = 106\,W/m^2\cdot K$$

$$\frac{100 - T_{m,o}}{(100-47)^\circ C} = \exp\left(-\frac{\pi \times 0.010\,m \times 2\,m}{0.2\,kg/s \times 4179\,J/kg\cdot K} \times 106\,W/m^2\cdot K\right) \qquad T_{m,o} = 47.4^\circ C \quad <$$

(c) Using the *IHT Correlation Tools* for *Internal Flow* (*Turbulent Flow*) and *External Flow* (over a *Cylinder*) the analyses of part (b) were performed considering the appropriate temperatures to evaluate the thermophysical properties. For internal and external flow, respectively,

$$\overline{T}_m = \left(T_{m,i} + T_{m,o}\right)/2 \qquad \overline{T}_f = \left(\overline{T}_s + T_\infty\right)/2 \quad (7,8)$$

where the average tube wall temperature is evaluated from the thermal circuit,

$$\frac{\overline{T}_m - \overline{T}_s}{1/\overline{h}_i} = \frac{\overline{T}_s - T_\infty}{1/\overline{h}_o} \quad (9)$$

T_m — $1/h_i$ — T_s — $1/h_o$ — T_∞

The results of the analyses are summarized in the table along with the results from parts (a) and (b),

Condition	$\overline{T}_m$ (K)	$\overline{h}_i$ (W/m^2·K)	$\overline{T}_f$ (K)	$\overline{h}_o$ (W/m^2·K)	$\overline{U}$ (W/m^2·K)	$T_{m,o}$ (°C)
T_s = 27°C	300	9080	---	---	---	37.1°C
T_∞ = 100 °C, T_f = 350°C	300	9080	350	107	106	47.4°C
Exact solution	320	11,420	347	107.3	106.3	47.4°C

Note that since $\overline{h}_o << \overline{h}_i$, $\overline{U}$ is controlled by the value of $\overline{h}_o$ which was evaluated near 350 K for both parts (b) and (c). Hence, it follows that $T_{m,o}$ is not very sensitive to $\overline{h}_i$ which, as seen above, is sensitive to the value of $\overline{T}_m$.

PROBLEM 8.57

KNOWN: Diameter of tube through which water of prescribed flow rate and inlet and outlet temperatures flows. Temperature of fluid in cross flow over the tube.

FIND: (a) Required tube length for air in cross flow at prescribed velocity, (b) Required tube length for water in cross flow at a prescribed velocity.

SCHEMATIC:

Water
V = 2 m/s
Air
V = 20 m/s
T_∞ = 15 °C
$T_{m,o}$ = 30 °C
D = 0.05 m
L
Water
$T_{m,i}$ = 70 °C
$\dot{m}$ = 0.215 kg/s

ASSUMPTIONS: (1) Steady-state, (2) Constant properties, (3) Negligible tube wall conduction resistance, (4) Negligible KE, PE and flow work changes.

PROPERTIES: *Table A.6*, water ($\overline{T}_m$ = 50°C = 323 K): c_p = 4181 J/kg·K, $\mu = 548 \times 10^{-6}$ N·s/m^2, k = 0.643 W/m·K, Pr = 3.46. *Table A.4,* air (assume T_f = 300 K): $\nu = 15.89 \times 10^{-6}$ m^2/s, k = 0.0263 W/m·K, Pr = 0.707. *Table A.6*, water (assume T_f = 300 K): $\nu = 0.858 \times 10^{-6}$ m^2/s, k = 0.613 W/m·K, Pr = 5.83.

ANALYSIS: The required heat rate may be determined from the overall energy balance,

$$q = \dot{m}c_p\left(T_{m,i} - T_{m,o}\right) = 0.215\,\text{kg/s}\left(4181\,\text{J/kg}\cdot\text{K}\right)40^\circ\text{C} = 35{,}960\,\text{W}$$

and the required tube length may be determined from the rate equation, Eq. 8.47a,

$$L = \frac{q}{U\pi D\Delta T_{\ell m}}$$

where

$$\Delta T_{\ell m} = \frac{\left(T_{m,i} - T_\infty\right) - \left(T_{m,o} - T_\infty\right)}{\ell n\left(\dfrac{T_{m,i} - T_\infty}{T_{m,o} - T_\infty}\right)} = 30.8^\circ\text{C} \qquad \text{and} \qquad 1/U = 1/h_i + 1/h_o.$$

With

$$Re_{D_i} = 4\dot{m}/\pi D\mu = 0.860\,\text{kg/s}\Big/\pi\left(0.05\,\text{m}\right)548\times10^{-6}\,\text{N}\cdot\text{s}\big/\text{m}^2 = 9991$$

the flow is turbulent and, assuming fully developed flow throughout the tube, the inside convection coefficient is determined from Eq. 8.60

$$Nu_{D_i} = 0.023\,Re_{D_i}^{4/5}\,Pr^{0.3} = 0.023\left(9991\right)^{0.8}\left(3.46\right)^{0.3} = 52.9$$

$$h_i = Nu_{D_i}\,k/D = 52.9\left(0.643\,\text{W/m}\cdot\text{K}\right)/0.05\,\text{m} = 680\,\text{W}\big/\text{m}^2\cdot\text{K}$$

Continued...

(a) For water in cross flow at 20 m/s, Re_{D_o} = VD/ν = 20 m/s(0.05 m)/15.89 × 10^{-6} m^2/s = 62,933. From the Churchill/Bernstein correlation, it follows that

$$Nu_{D_o} = 0.3 + \frac{0.62\, Re_{D_o}^{1/2}\, Pr^{1/3}}{\left[1+(0.4/Pr)^{2/3}\right]^{1/4}} \left[1+\left(\frac{Re_{D_o}}{282,000}\right)^{5/8}\right]^{4/5} = 158.7$$

$$h_o = Nu_{D_o}\, k/D = 158.7\left(0.0263\, W/m\cdot K\right)/0.05\, m = 83.5\, W/m^2\cdot K$$

Hence, U = $\left(1/h_i + 1/h_o\right)^{-1}$ = 74.4 W/m^2·K and

$$L = \frac{35,960\, W}{\left(74.4\, W/m^2\cdot K\right)\pi\,(0.05\, m)\, 30.8^\circ C} = 100\, m$$ <

(b) For water in cross flow at 2 m/s, Re_{D_o} = 2 m/s(0.05 m)/0.858 × 10^{-6} m^2/s = 116,550, and the correlation yields $Nu_{D_o} = 527.3$. Hence,

$$h_o = Nu_{D_o}\, k/D = 527.3\left(0.613\, W/m\cdot K\right)/0.05\, m = 6,465\, W/m^2\cdot K$$

$$U = \left(1/h_i + 1/h_o\right)^{-1} = 615.3\ W/m^2\cdot K$$

Hence,

$$L = \frac{35,960\, W}{\left(615.3\, W/m^2\cdot K\right)\pi\,(0.05\, m)\, 30.8^\circ C} = 12\, m$$ <

COMMENTS: The foregoing results clearly indicate the superiority of water (relative to air) as a heat transfer fluid. Note the dominant contribution made by the smaller convection coefficient to the value of U in each of the two cases.

PROBLEM 8.58

KNOWN: Water flow rate and inlet temperature for a thin-walled tube of prescribed length and diameter.

FIND: Water outlet temperature for each of the following conditions: (a) Tube surface maintained at 27°C, (b) Insulation applied and outer surface maintained at 27°C, (c) Insulation applied and outer surface exposed to ambient air at 27°C.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Fully developed flow throughout the tube, (3) Negligible tube wall conduction resistance, (4) Negligible contact resistance between tube wall and insulation, (5) Uniform outside convection coefficient.

PROPERTIES: Assume water cools to $T_{m,o}$ = 27°C with no insulation but that cooling is negligible ($T_{m,o}$ = 97°C) with insulation. *Table A-4*, Water $\left(\bar{T}_m = 335\text{K}\right)$: c_p = 4186 J/kg·K, μ = 453×10^{-6} N·s/m^2, k =0.656 W/m·K, Pr = 2.88; *Table A-4,* Water ($T_{m,i}$ = 370K): c_p = 4214 J/kg·K, μ = 289×10^{-6} N·s/m^2, k = 0.679 W/m·K, Pr = 1.80.

ANALYSIS: For each of the three cases, heat is transferred from the warm water to a surface (or the air) which is at a fixed temperature (27°C). Accordingly, an expression of the form given by Eq. 8.42b may be used to determine the outlet temperature of the water, so long as the appropriate heat transfer coefficient is used. In particular, each of the cases can be described by Eq. 8.46.

$$\frac{\Delta T_o}{\Delta T_i} = \exp\left(-\frac{\bar{U}A_s}{\dot{m}\,c_p}\right)$$

Referring to the thermal circuit associated with heat transfer from the water,

$$T_m \;—\; \frac{1}{h_i \pi D_i L} \;—\; T_{s,i} \;—\; \frac{\ln(D_o/D_i)}{2\pi k L} \;—\; T_{s,o} \;—\; \frac{1}{h_o \pi D_o L} \;—\; T_\infty \quad q \rightarrow$$

and using Eq. 3.32, the UA product may be evaluated as

$$UA = \left(\Sigma R_t\right)^{-1}.$$

(a) For the first case: $\quad T_{s,i} = 27^\circ\text{C} \quad \Delta T_i = T_{m,i} - T_{s,i} = 70^\circ\text{C} \quad UA = h_i \pi D_i L.$

$$Re_D = \frac{4\dot{m}}{\pi D_i \mu} = \frac{4\times 0.015\text{ kg/s}}{\pi\left(0.003\text{m}\right)453\times 10^{-6}\text{N}\cdot\text{s/m}^2} = 14{,}053.$$

Continued …..

From Eq. 8.60,

$$h_i = \frac{k}{D_i} 0.023\, Re_D^{4/5}\, Pr^{0.30} = \frac{0.656\ W/m\cdot K}{0.003m}(0.023)\ (14{,}053)^{4/5}(2.88)^{0.3} = 14{,}373\ W/m^2\cdot K.$$

$$\Delta T_o = \Delta T_i \exp\left(-\frac{h_i p\ D_i L}{\dot{m}\, c_p}\right) = 70^\circ C \exp\left(-\frac{14{,}373\dfrac{W}{m^2\cdot K} p \times 0.003m \times 1m}{0.015\ kg/s \times 4186\ J/kg\cdot K}\right) = 8.1^\circ C$$

$$T_{m,o} = \Delta T_o + T_{s,i} = 8.1^\circ C + 27^\circ C = 35.1^\circ C.$$ $<$

(b) For the second case: $T_{s,o} = 27°C$ with

$$\Delta T_i = T_{m,i} - T_{s,o} = 70^\circ C \qquad UA = \left[(1/h_i p\ D_i L) + \ell n(D_o/D_i)/2p\ kL\right]^{-1}.$$

With $$Re_D = \frac{4\ \dot{m}}{p\ D_i m} = \frac{4 \times 0.015\ kg/s}{p\,(0.003m)\, 289 \times 10^{-6}\, N\cdot s/m^2} = 22{,}028$$

$$h_i = \frac{k}{D_i} 0.023\, Re_D^{4/5}\, Pr^{0.3} = \frac{0.679\ W/m\cdot K}{0.003m}(0.023)\ (22{,}028)^{4/5}(1.80)^{0.3} = 18{,}511\ W/m^2\cdot K.$$

It follows that

$$UA = \left[\frac{1}{18{,}511\ p \times 0.003} + \frac{\ell n(0.004/0.003)}{2p\,(0.05)}\right]^{-1} = \left[5.73 \times 10^{-3} + 0.916\right]^{-1} = 1.085\ W/K$$

and the outlet temperature is

$$\Delta T_o = 70^\circ C \exp\left(-\frac{1.085\ W/K}{0.015\ kg/s \times 4214\ J/kg\cdot K}\right) = 68.8^\circ C$$

$$T_{m,o} = \Delta T_o + T_{s,o} = 68.8^\circ C + 27^\circ C = 95.8^\circ C.$$ $<$

(c) For the third case: $T_\infty = 27°C$, $\Delta T_i = T_{m,i} - T_\infty = 70°C$ and

$$UA = \left[(1/h_i p\ D_i L) + \ell n(D_o/D_i)/2p\ kL + (1/h_o p\ D_o L)\right]^{-1}$$

$$UA = \left[5.73 \times 10^{-3} + 0.916 + \frac{1}{5p\,(0.004)}\right]^{-1} = \left[5.73 \times 10^{-3} + 0.916 + 15.92\right]^{-1} = 0.0594\ W/K$$

$$\Delta T_o = 70^\circ C \exp\left(-\frac{0.0594\ W/K}{0.015\ kg/s \times 4214\ J/kg\cdot K}\right) = 69.9^\circ C$$

$$T_{m,o} = \Delta T_o + T_\infty = 69.9^\circ C + 27^\circ C = 96.9^\circ C.$$ $<$

COMMENTS: Note that $R_{conv,o} >> R_{cond,insul} >> R_{conv,i}$,

PROBLEM 8.59

KNOWN: Thick-walled pipe of thermal conductivity 60 W/m·K passing hot water with Re_D = 20,000, a mean temperature of 80°C, and cooled externally by air in cross-flow at 20 m/s and 25°C.

FIND: Heat transfer rate per unit pipe length, q'.

SCHEMATIC:

Air
$T_\infty = 25°C$
$V = 20 m/s$
Water
$Re_D = 20,000$
$D_o = 25 mm$
$D_i = 20 mm$
$T_m = 80°C$

ASSUMPTIONS: (1) Steady-state conditions, (2) Internal flow is turbulent and fully developed.

PROPERTIES: *Table A-6*, Water (T_m = 80°C = 353K): k = 0.670 W/m·K, Pr = 2.20; *Table A-4*, Air (T_∞ = 25°C ≈ 300K, 1 atm): $\nu = 15.89 \times 10^{-6}$ m^2/s, k = 0.0263 W/m·K, Pr = 0.707.

ANALYSIS: The heat rate per unit length, considering thermal resistances to internal flow, wall conduction (Eq. 3.28) and external flow, with A = πDL, is

$$q' = \left[1/h_i \pi D_i + (1/2\pi k)\ln(D_o/D_i) + 1/h_o \pi D_o\right]^{-1}(T_m - T_\infty).$$

Internal Flow: Using the Dittus-Boelter correlation with n = 1/3 for turbulent, fully developed flow, where $Re_{D_i} = 20{,}000$

$$h_i = (k/D_i)Nu_D = (k/D_i)0.023Re^{4/5}Pr^{1/3}$$

$$h_i = (0.670 \text{ W/m}\cdot\text{K}/0.020 \text{ m})0.023(20{,}000)^{4/5}2.20^{1/3} = 2765 \text{ W/m}^2\cdot\text{K}.$$

External Flow: Using the Zhukauskas correlation for cross-flow over a circular cylinder with $Pr/Pr_s \approx 1$, find first

$$Re_D = \frac{VD_o}{\nu} = \frac{20 \text{ m/s} \times 0.025 \text{ m}}{15.89 \times 10^{-6} \text{m}^2/\text{s}} = 31{,}466$$

and from Table 7.4, C = 0.26 and m = 0.6, where n = 0.37,

$$Nu_D = \frac{h_o D}{k} = CRe_D^m Pr^n (Pr/Pr_s)^{1/4}$$

$$h_o = (0.0263 \text{ W/m}\cdot\text{K}/0.025 \text{ m})0.26(31{,}466)^{0.6}(0.707)^{0.37} = 120 \text{ W/m}^2\cdot\text{K}.$$

Hence, the heat rate is

$$q' = \left[\left(1/2765 \text{ W/m}^2\cdot\text{K} \times \pi\, 0.020 \text{ m}\right) + (1/2\pi\, 60 \text{ W/m}\cdot\text{K})\ln(25/20) + \left(1/120 \text{ W/m}^2\cdot\text{K} \times \pi\, 0.025 \text{ m}\right)\right]^{-1}(80-25)^\circ\text{C}$$

$$q' = \left[5.756 \times 10^{-3} + 5.919 \times 10^{-4} + 1.061 \times 10^{-1}\right]^{-1} \text{W/m}\cdot\text{K}\,(80-25)^\circ\text{C}$$

$$q' = 489 \text{ W/m.}$$

<

COMMENTS: Note that the external flow represents the major thermal resistance to heat transfer.

PROBLEM 8.60

KNOWN: Reaction vessel with process fluid at 75°C cooled by water at 27°C and 0.12 kg/s through 15 mm tube. High convection coefficient on outside of tube (3000 $W/m^2 \cdot K$) created by vigorous stirring.

FIND: (a) Maximum heat transfer rate if outlet temperature of water cannot exceed $T_{m,o}$ = 47°C, and (b) Required tube length.

SCHEMATIC:

Stirrer
Reaction vessel
Process fluid, $T_\infty = 75°C$, $h_o = 3000 W/m^2 \cdot K$
$T_{m,o}$
$T_{m,i} = 27°C$
Water $\dot{m} = 0.12 kg/s$
$D = 15 mm, L$
$T_\infty = 75°C$
T
$T_{m,o} \le 47°C$
$T_{m,i} = 27°C$

ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible kinetic and potential energy changes, (3) Negligible thermal resistance of tube wall.

PROPERTIES: *Table A-6*, Water $\left(\bar{T}_m = (47+27)^\circ C/2 = 310K\right)$: ρ = $1/v_f$ = 993.1 kg/m^3, c_p = 4178 J/kg·K, μ = 695 × 10^{-6} $N \cdot s/m^2$, k = 0.628 W/m·K, Pr = 4.62.

ANALYSIS: (a) From an overall energy balance on the tube with $T_{m,o}$ = 47°C,

$$q_{max} = \dot{m}\, c_p \left(T_{m,o} - T_{m,i}\right) = 0.12 \text{ kg/s} \times 4178 \text{ J/kg} \cdot \text{K} \left(47-27\right)^\circ \text{C} = 10{,}027 \text{ W}. \quad <$$

(b) For the constant surface temperature heating condition, from Eq. 8.46,

$$\frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = \exp\left(-\frac{PL}{\dot{m}\, c_p}\bar{U}\right) \qquad \text{where} \qquad 1/U = 1/\bar{h}_o + 1/\bar{h}_i .$$

For internal flow in the tube, find

$$Re_D = \frac{4\dot{m}}{\pi D \mu} = \frac{4 \times 0.12 \text{ kg/s}}{\pi \times 0.015 \text{ m} \times 695 \times 10^{-6} \text{N} \cdot \text{s/m}^2} = 14{,}656$$

and the flow is turbulent. Assuming fully developed flow, use the Dittus-Boelter correlation with n = 0.4 (heating),

$$Nu_D = h_i D/k = 0.023 Re_D^{4/5} Pr^{0.4}$$

$$h_i = [0.628 \text{ W/m} \cdot \text{K}/0.015 \text{ m}] \times 0.023(14{,}656)^{4/5}(4.62)^{0.4} = 3822 \text{ W/m}^2 \cdot \text{K}.$$

Hence, 1/U = [1/3000 + 1/3822] $m^2 \cdot K/W$ or U = 1680 $W/m^2 \cdot K$. From the energy balance relation with P = πD, find

$$\frac{(75-47)^\circ C}{(75-27)^\circ C} = \exp\left(-\frac{\pi\,(0.015 \text{ m})\, L \times 1680 \text{ W/m}^2 \cdot \text{K}}{0.12 \text{ kg/s} \times 4178 \text{ J/kg} \cdot \text{K}}\right) \qquad L = 3.4 \text{ m}. \quad <$$

COMMENTS: Note that L/D = 227 and the fully developed flow assumption is appropriate.

PROBLEM 8.61

KNOWN: Water flowing through a tube heated by cross flow of a hot gas. Required to heat water from 15 to 35°C with a flow rate of 0.2 kg/s.

FIND: Design graphs to demonstrate acceptable combinations of tube diameter (D = 20, 30 or 40 mm), tube length (L = 3, 4 or 6 m) and hot gas velocity ($20 \le V \le 40$ m/s) and temperature (T_∞ = 250, 375 or 500°C).

SCHEMATIC:

Hot gases

T_∞ = 250 to 500 °C

D = 20, 40 or 60 mm

$T_{m,o}$ = 35 °C

L = 3, 4 or 6 m

Water

$\dot{m}$ = 0.2 kg/s

$T_{m,i}$ = 15 °C

ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible kinetic and potential energy changes and axial conduction, (3) Fully developed flow and thermal conditions for internal flow, (4) Properties of the hot gas are those of atmospheric air, and (5) Negligible tube wall thermal resistance.

PROPERTIES: *Table A.6*, Water ($\bar{T}_m = (15+35)^\circ C/2 = 298K$); *Table A.4*, Air ($\bar{T}_f = (\bar{T}_s + T_\infty)/2$, 1 atm).

ANALYSIS: *Method of Analysis*: The tube having internal flow of water with cross flow of hot gas can be analyzed by the energy balance relation, Eq. 4.86a

$$\frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = \exp\left(\frac{(\pi DL)}{\dot{m}c_p}\bar{U}\right) \tag{1}$$

where the overall coefficient $\bar{U}$ is

$$\bar{U} = \left(1/\bar{h}_i + 1/\bar{h}_o\right)^{-1} \tag{2}$$

Estimation of the internal flow coefficient, $\bar{h}_i$: Evaluating water properties at the average mean fluid

$$\bar{T}_m = \left(T_{m,i} + T_{m,o}\right)/2, \tag{3}$$

characterize the flow with the Reynolds number,

$$Re_{D,i} = \frac{4\dot{m}}{(\pi D\mu)} \tag{4}$$

and assuming the flow to be both turbulent and fully developed (L/D > 3m/0.07m = 42), use the Dittus-Boelter correlation, Eq. 8.60, to evaluate $\bar{h}_i$,

Continued...

PROBLEM 8.61 (Cont.)

$$\overline{Nu}_{D,i} = \frac{\overline{h}_i D}{k_i} = 0.023\,Re_{D,i}^{0.8}\,Pr^{0.4} \tag{5}$$

Estimation of the external flow coefficient, $\overline{h}_o$: Evaluating gas (air) properties at the average film temperature

$$\overline{T}_f = \left(\overline{T}_s + T_\infty\right)/2 \tag{6}$$

where $\overline{T}_s$ is the average tube wall temperature (see Eq. (9)), characterize the flow

$$Re_{D,o} = \frac{VD}{\nu} \tag{7}$$

and use the Churchill-Bernstein correlation, Eq. 7.57, for cross-flow over a cylinder,

$$Nu_{D,o} = \frac{\overline{h}_o D}{k_o} = 0.3 + \frac{0.62\,Re_{D,o}^{1/2}\,Pr_o^{1/3}}{\left[1+\left(0.4/Pr_o\right)^{2/3}\right]^{1/4}}\left[1+\left(\frac{Re_{D,o}}{282,000}\right)^{5/8}\right]^{4/5} \tag{8}$$

$\overline{T}_m$ $\overline{T}_s$ T_∞

$1/\overline{h}_i$ $1/\overline{h}_o$

The average tube wall temperature, $\overline{T}_s$, follows from the thermal circuit

$$\frac{\overline{T}_m - \overline{T}_s}{1/\overline{h}_i} = \frac{\overline{T}_s - \overline{T}_\infty}{1/\overline{h}_o} \tag{9}$$

The IHT Workspace: Using the *Correlation Tools* for *Internal Flow* (*Turbulen*t flow), and *External Flow* (*Flow over a Cylinder*) and *Properties* for *Air* and *Water*, along with the appropriate energy balances and rate equations, the heater-tube system can be analyzed.

The Design Strategy: We have chosen to generate the design information in the following manner: for a specified gas temperature, T_∞, plot the required length L (limiting the scale to $3 \le L \le 6$m) as a function of gas velocity V ($20 \le V \le 40$ m/s) for tube diameters of D = 20, 30 and 40 mm. Three design graphs corresponding to T_∞ = 250, 375 and 500°C were generated and are shown on the next page.

COMMENTS: (1) The collection of design graphs will allow the contractor to select appropriate combinations of tube D and L and gas stream parameters (T_∞ and V) to achieve the required water heating.

(2) Note from the design graphs that with T_∞ = 250°C, the required heating of the water can be achieved only with a 40-mm diameter by 6 m length tube with gas velocities greater than 32 m/s. This configuration represents a worst case condition of largest tube parameters and highest gas velocity.

(3) Which operating conditions, T_∞ = 375 or 500°C, provides the contractor with more options in selecting combinations of tube parameters and gas velocities? What are the trade-offs in operating at 375 or 500°C? Consider such features as tube life, tubing costs and fan requirements.

(4) The Reynolds numbers for the internal flow are approximately 7,100, 9,460 and 14,200 for the tube diameters of 20, 30 and 40 mm. For the larger tube sizes, the Reynolds numbers are below 10,000, the usual lower limit for turbulent flow.

Continued...

PROBLEM 8.62

KNOWN: Exhaust gasses at 200°C and mass rate 0.03 kg/s enter tube of diameter 6 mm and length 20 m. Tube experiences cross-flow of autumn winds at 15°C and 5 m/s.

FIND: Average heat transfer coefficients for (a) exhaust gas inside tube and (b) air flowing across outside of tube, (c) Estimate overall coefficient and exhaust gas temperature at outlet of tube.

SCHEMATIC:

Air
T = 15°C
V = 5 m/s
Tube
D = 10 mm
Gas
$\dot{m}_1$ = 0.003 kg/s
$T_{m,i}$ = 200°C
L = 20 m
$T_{m,o}$

ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible kinetic and potential energy changes, (3) Negligible tube wall resistance, (4) Exhaust gas properties are those of air, (5) Negligible radiation effects.

PROPERTIES: *Table A-4*, Air (assume $T_{m,o} \approx 15°C$, hence $\overline{T}_m$ = 380K, 1 atm):c_p = 1012 J/kg·K, k = 0.0323 W/m·K, $\mu = 221.6 \times 10^{-7}$ N·s/m^2, Pr = 0.694; Air (T_∞ = 15°C = 288 K, 1 atm): k = 0.0253 W/m·K, $\nu = 14.82 \times 10^{-6}$ m^2/s, Pr = 0.710; Air ($\overline{T}_s \approx 90°C$ = 363 K, 1 atm): Pr = 0.698.

ANALYSIS: (a) For the *internal flow* through the tube assuming a value for $T_{m,o}$ = 15°C, find

$$Re_D = \frac{4\dot{m}}{\pi D\mu} = \frac{4\times 0.003 \text{ kg/s}}{\pi \times 0.006 \text{ m} \times 221.6\times 10^{-7} \text{ N}\cdot\text{s/m}^2} = 2.873\times 10^4.$$

Hence the flow is turbulent and, since L/D >> 10, fully developed. Using the Dittus-Doelter correlation with n = 0.3,

$$Nu_D = 0.023 Re_D^{0.8} Pr^{0.3} = 0.023\left(2.873\times 10^4\right)^{0.8}(0.694)^{0.3} = 76.0$$

$$h_i = Nu\cdot k/D = 76.0\times 0.0323 \text{ W/m}\cdot\text{K}/0.006 \text{ m} = 409 \text{ W/m}^2\cdot\text{K}.$$ <

(b) For *cross-flow* over the circular tube, find using thermophysical properties at T_∞,

$$Re_D = \frac{VD}{\nu} = \frac{5 \text{ m/s}\times 0.006 \text{ m}}{14.82\times 10^{-6} \text{ m}^2/\text{s}} = 2024$$

and using the Zhukauskus correlation with C = 0.26, m = 0.6, and n = 0.37,

$$Nu_D = C Re_D^m Pr^n \left(Pr/Pr_s\right)^{1/4} = 0.26(2024)^{0.6}\, 0.710^{0.37}\,(0.710/0.698)^{0.25} = 23.1$$

where Pr_s is evaluated at $\overline{T}_s$. Hence,

$$h_o = Nu_D\cdot k/D = 23.1\times 0.0253 \text{ W/m}\cdot\text{K}/0.006 \text{ m} = 97.5 \text{ W/m}^2\cdot\text{K}.$$ <

Continued …..

(c) Assuming the thermal resistance of the tube wall is negligible,

$$\frac{1}{U} = \frac{1}{h_o} + \frac{1}{h_i} = \left(\frac{1}{97.5} + \frac{1}{409}\right) m^2 \cdot K/W \qquad U = 78.8 \text{ W/m}^2 \cdot K.$$ <

The gas outlet temperature can be determined from the expression where $P = \pi D$.

$$\frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = \exp\left(-\frac{PUL}{\dot{m}\, c_p}\right) = \exp\left(-\frac{\pi \times 0.006 \text{ m} \times 78.8 \text{ W/m}^2 \cdot K \times 20 \text{ m}}{0.003 \text{ kg/s} \times 1012 \text{ J/kg} \cdot K}\right)$$

$$\frac{15 - T_{m,o}}{(15 - 200)^\circ C} = 0.999$$

$$T_{m,o} = 15^\circ C.$$ <

COMMENTS: (1) With $T_{m,o}$ = 15°C, find $\overline{T}_m$ = 380 K; hence thermophysical properties for the internal flow correlation were evaluated at a reasonable temperature. Note that the gas is cooled from 200°C to the ambient air temperature, $T_{m,o} = T_\infty$, over the 20-m length!

(2) The average wall surface temperature, $\overline{T}_s$, follows from an energy balance on the wall surface,

$$\frac{\overline{T}_m - \overline{T}_s}{\overline{T}_s - T_{inf}} = \frac{h_i}{h_o}$$

and substituting numerical values, find $\overline{T}_s = 90°C = 363$ K, the value we assumed for evaluating Pr_s. Can you draw a thermal circuit to represent this energy balance relation?

(3) When using the Zhukauskus correlation, it is reasonable to evaluate Prs at the $\overline{T}_m$ for the first trial. For gases the assumption is a safe one, but for liquids, especially oils, additional trials will be required since the Prandtl number may be strongly dependent upon temperature.

PROBLEM 8.63

KNOWN: Superheated steam passing through thin-walled pipe covered with insulation and suspended in a quiescent air.

FIND: Point along pipe surface where steam will begin condensing (x_1).

SCHEMATIC:

Insulation, $k = 0.085$ W/m·K
$t = 25$ mm
Quiescent air, $T_\infty = 20\ ^oC$
$h_o = 10\ W/m^2 \cdot K$
Superheated steam
$p = 1$ atm
$u_m = 10$ m/s
$D_i = 0.05$ m
$T_{m,i} = 120\ ^oC$
$T_s(x_1) = 100\ ^oC$
x
x_1

ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible KE, PE and flow work changes, (3) Steam properties may be approximated as those corresponding to saturated conditions.

PROPERTIES: *Table A.6*, Saturated steam ($\overline{T}_m$ = (100 + 120)°C/2 = 110°C ≈ 385 K): $\rho_g = 0.876$ kg/m^3, $c_{p,g} = 2080$ J/kg·K, $\mu_g = 12.49 \times 10^{-6}$ N·S/m^2, $k_g = 0.0258$ W/m·K, $Pr_g = 1.004$.

ANALYSIS: From Eq. 8.46a, where $T_{m,x}$ is the mean temperature at any distance x,

$$\frac{T_\infty - T_{m,x}}{T_\infty - T_{m,i}} = \exp\left(-\frac{Px}{\dot{m}c_p}U\right) \tag{1}$$

The mass flow rate, with $A_c = \pi D^2/4$, is

$$\dot{m} = \rho_g A_c u_m = 0.876\,\text{kg/m}^3\left(\pi\,(0.050\,\text{m})^2/4\right)\times 10\,\text{m/s} = 0.0172\,\text{kg/s}$$

and for the internal flow,

$$Re_D = \frac{4\dot{m}}{\pi D\mu} = \frac{4\times 0.0172\,\text{kg/s}}{\pi\,(0.050\,\text{m})\times 12.49\times 10^{-6}\,\text{N}\cdot\text{s/m}^2} = 35{,}068.$$

Assuming the flow is fully developed, the Dittus-Boelter correlation yields

$$Nu_D = \frac{h_i D}{k} = 0.023(35{,}068)^{4/5}(1.004)^{0.3} = 99.58$$

$$h_i = \frac{0.0258\,\text{W/m}\cdot\text{K}}{0.050\,\text{m}}\times 99.58 = 51.4\,\text{W/m}^2\cdot\text{K}$$

Hence, from Eq. 3.31, the overall coefficient for the inner surface is

$$U_i = \left[\frac{1}{h_i} + \frac{D_i \ln(D_o/D_i)}{2k} + \frac{D_i}{D_o}\frac{1}{h_o}\right]^{-1} = \left[\frac{1}{51.4} + \frac{(0.050)\ln(0.100/0.050)}{2\times 0.085} + \frac{0.050}{0.100}\frac{1}{10}\right]^{-1}\text{W/m}^2\cdot\text{K}$$

$$U_i = \left[1.946\times 10^{-2} + 2.039\times 10^{-1} + 5.000\times 10^{-2}\right]^{-1} = 3.66\,\text{W/m}^2\cdot\text{K}.$$

Continued...

With condensation occurring when the surface temperature reaches 100°C, the corresponding value of T_m may be determined from the local ($x = x_1$) requirement that $U_i(\pi D_i)\left[T_m(x_1) - T_\infty\right]$ $= h_i(\pi D_i)\left[T_m(x_1) - T_s\right]$. Hence,

$$T_m(x_1) = \frac{T_\infty - (h_i/U_i)T_s}{1-(h_i/U_i)} = \frac{20-(51.4/3.66)100^\circ\text{C}}{1-(51.4/3.66)} = 107.7^\circ\text{C}$$

The distance at which the mean steam temperature is 107.7°C can then be estimated from Eq. (1), where $P = \pi D_i$ and $U = U_i$,

$$\frac{(20-107.7)^\circ\text{C}}{(20-120)^\circ\text{C}} = \exp\left(-\frac{\pi(0.050\,\text{m})3.66\,\text{W/m}^2\cdot\text{K}\,(x_1)}{0.0172\,\text{kg/s}\times 2080\,\text{J/kg}\cdot\text{K}}\right)$$

$x_1 = 8.15$ m <

COMMENTS: Note that condensation first occurs at the location for which the surface, and not the mean, temperature reaches 100°C.

PROBLEM 8.64

KNOWN: Length and diameter of air conditioning duct. Inlet temperature of chilled air. Temperature and convection coefficient associated with outer air. Chilled air flowrate.

FIND: Chilled air exit temperature and heat flow rate.

SCHEMATIC:

$L = 15\,m$
$T_{m,i} = 7°C$
$\dot{m} = 0.05\,kg/s$
$D = 0.3\,m$
q
$T_{\infty} = 37°C$
$h_o = 2\,W/m^2\cdot K$
$T_{m,o}$

ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible tube wall conduction resistance, (3) Negligible kinetic and potential energy changes and axial conduction.

PROPERTIES: *Table A-4*, Air (300K, 1 atm): c_p = 1007 J/kg·K, $\mu = 184.6 \times 10^{-7}$ kg/s·m, k = 0.0263 W/m·K, Pr = 0.707.

ANALYSIS: The exit temperature may be obtained from Eq. 8.46, where

$$\bar{U} = \left(h_i^{-1} + h_o^{-1}\right)^{-1}$$

With $$Re_D = (4\dot{m}/\pi D\mu) = \frac{4(0.05\text{ kg/s})}{\pi(0.3\text{ m})184.6\times10^{-7}\text{ kg/s}\cdot\text{m}} = 11{,}495$$

the flow is turbulent and, assuming fully developed conditions over the entire length, the Dittus-Boelter correlation yields

$$Nu_D = 0.023 Re_D^{4/5} Pr^{0.4} = 0.023(11{,}495)^{4/5}(0.707)^{0.4} = 35.5$$

$$h_i = Nu_D(k/D) = 35.5(0.0263\text{ W/m}\cdot\text{K}/0.3\text{ m}) = 31.1\text{ W/m}^2\cdot\text{K}$$

and $$\bar{U} = \left(3.11^{-1} + 2.0^{-1}\right)^{-1}\left(\text{W/m}^2\cdot\text{K}\right) = 1.22\text{ W/m}^2\cdot\text{K}.$$

Eq. 8.46 yields $$T_{m,o} = T_\infty - \left(T_\infty - T_{m,i}\right)\exp\left[-\left(\pi DL/\dot{m}c_p\right)\bar{U}\right]$$

$$T_{m,o} = 37°\text{C} - 30°\text{C}\exp\left[-\frac{\pi(0.3\text{ m})15\text{ m}\left(1.22\text{ W/m}^2\cdot\text{K}\right)}{0.05\text{ kg/s}\left(1007\text{ J/kg}\cdot\text{K}\right)}\right] = 15.7°\text{C}$$ <

and the heat rate is

$$q = \dot{m}c_p\left(T_{m,o} - T_{m,i}\right) = 0.05\text{ kg/s}\left(1007\text{ J/kg}\cdot\text{K}\right)\left(8.7°\text{C}\right) = 438\text{ W}.$$ <

COMMENTS: The temperature rise of the chilled air is excessive, and the outer surface of the duct should be insulated to reduce $\bar{U}$ and thereby $T_{m,o}$ and q.

PROBLEM 8.65

KNOWN: Length, diameter, insulation characteristics and burial depth of a pipe. Ground surface temperature. Inlet temperature, flow rate and properties of oil flowing through pipe.

FIND: (a) An expression for the oil outlet temperature, (b) Oil outlet temperature and pipe heat transfer rate for prescribed conditions, and (c) Design information for trade off between burial depth of pipe (z) and pipe insulation thickness (t) on the heat loss.

SCHEMATIC:

T_s = -40 °C Soil, k_s = 0.50 W/m•K

z = 3 m

Insulation, D_i = 1.2 m
D_o = 1.5 m, k_i = 0.05 W/m•K

Oil
$\dot{m}_o$ = 500 kg/s
$T_{m,i}$ = 120 °C

dq_{conv} $T_{m,o}$ d_x $L = 10^5$ m

ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Two-dimensional conduction in soil, (4) Negligible pipe wall thermal resistance, (5) Total resistance to heat loss is independent of x.

PROPERTIES: Oil (given): $\rho_o = 900\ kg/m^3$, $c_{p,o} = 2000\ J/kg\cdot K$, $\nu_o = 8.5 \times 10^{-4}\ m^2/s$, $k_o = 0.140\ W/m\cdot K$, $Pr_o = 10^4$; Soil (given): $k_s = 0.50\ W/m\cdot K$; Insulation (given): $k_i = 0.05\ W/m\cdot K$.

ANALYSIS: (a) From Eq. 8.36 for a differential control volume in the oil and the rate equation

$$dq_{conv} = \dot{m}_o c_{p,o} dT_m = dq = (T_s - T_m)/R_{tot} \quad (1)$$

where the total resistance is expressed as

$$R_{tot} = R_{conv} + R_{cond,i} + R_{cond,s} = \left(\bar{h}\pi D dx\right)^{-1} + \frac{\ln(D_o/D_i)}{2\pi k_i dx} + \frac{1}{k_s S}$$

$$R_{tot} = \left(\frac{1}{\bar{h}\pi D_i} + \frac{\ln(D_o/D_i)}{2\pi k_i} + \frac{\cosh^{-1}(2z/D_o)}{2\pi k_s}\right)/dx = R'_{tot}/dx \quad (2)$$

where, from Table 4.1,

$$S = 2\pi dx \Big/ \cosh^{-1}(2z/D_o) \quad (3)$$

It follows that

$$\frac{(T_s - T_m)dx}{R'_{tot}} = \dot{m}_o c_{p,o} dT_m \qquad \frac{dT_m}{T_s - T_m} = \frac{dx}{\dot{m}_o c_{p,o} R'_{tot}}$$

Integrating between inlet and outlet conditions

$$\int_{T_{m,i}}^{T_{m,o}} \frac{dT_m}{T_m - T_s} = -\int_0^L \frac{dx}{\dot{m}_o c_{p,o} R'_{tot}}.$$

Assuming R'_{tot} to be independent of x and integrating,

$$\frac{T_{m,o} - T_s}{T_{m,i} - T_s} = \exp\left(-\frac{L}{\dot{m}_o c_{p,o} R'_{tot}}\right). \quad (3) <$$

Continued...

PROBLEM 8.65 (Cont.)

(b) To calculate $T_{m,o}$ for the prescribed conditions, begin by evaluating $\bar{h}$, where

$$Re_D = \frac{4\dot{m}_o}{\pi D_i \rho_o \nu_o} = \frac{4 \times 500\,\text{kg/s}}{\pi(1.2\,\text{m})900\,\text{kg/m}^3 \times 8.5 \times 10^{-4}\,\text{m}^2\text{/s}} = 694 \tag{4}$$

Hence, the flow is laminar, and with a thermal entry length,

$$\overline{Nu}_D = 3.66 + \frac{0.0668(D_i/L)Re_D\,Pr}{1 + 0.04\left[(D_i/L)Re_D\,Pr\right]^{2/3}} \tag{5}$$

$$(D_i/L)Re_D\,Pr = \left(\frac{1.2}{10^5}\right)(694)10^4 = 83.3 \qquad \overline{Nu}_D = 6.82$$

$$\bar{h} = \frac{k}{D_i}6.82 = \frac{0.14\,\text{W/m}\cdot\text{K}}{1.2\,\text{m}}6.82 = 0.80\,\text{W/m}^2\cdot\text{K}$$

From Eq. (2), the overall thermal resistance is

$$R'_{tot} = \frac{1}{0.8\,\text{W/m}^2\cdot\text{K}\pi(1.2\,\text{m})} + \frac{\ln(1.5/1.2)}{2\pi(0.05\,\text{W/m}\cdot\text{K})} + \frac{\cosh^{-1}(4)}{2\pi(0.5\,\text{W/m}\cdot\text{K})}$$

$$R'_{tot} = (0.33 + 0.71 + 0.66)\text{K}\cdot\text{m/W} = 1.70\,\text{K}\cdot\text{m/W}$$

and the oil outlet temperature can be calculated as

$$\frac{T_{m,o} - T_s}{T_{m,i} - T_s} = \exp\left(-\frac{10^5\text{m}}{500\,\text{kg/s} \times 2000\,\text{J/kg}\cdot\text{K} \times 1.7\,\text{K}\cdot\text{m/W}}\right) = 0.943$$

$T_{m,o} = 110.9°C$ <

The total rate of heat transfer *from* the pipeline is then

$$q = \dot{m}_o c_{p,o}\left(T_{m,i} - T_{m,o}\right) \tag{6}$$

$$q = 500\,\text{kg/s} \times 2000\,\text{J/kg}\cdot\text{K}(120 - 110.9)^\circ\text{C} = 9.1 \times 10^6\,\text{W}.$$ <

(c) Using the *IHT Workspace* with the foregoing equations, an analysis was performed to determine the heat loss, q, as a function of burial depth for the range, $1 \le z \le 6$ m, for thicknesses of insulation which are -25%, +25%, +50% and 100% that of the base case, $t = r_o - r_1 = 150$ mm.

Heat loss, q (MW)

Burial depth, z (m)

Insulation thickness, t = 113 mm (-25%)
t = 150 mm (base case)
t = 188 mm (+25%)
t = 225 mm (+50%)
t = 300 mm (+100%)

Continued...

From this information, the operations manager can compare the costs associated with burial depth and insulation thickness with respect to acceptable heat loss.

COMMENTS: (1) Since the thermal entry region is very long, $x_{fd,t} \approx 0.05DRe_DPr = 4.16 \times 10^5$ m, h_x will be changing with x throughout the pipe. A more accurate solution would therefore be one in which Eq. (1) is integrated numerically, in a step-by-step fashion. For example, the integration could involve a step width of $\Delta x = 10^3$ m, with h and R'_t evaluated at each step.

(2) The three contributions to the total thermal resistance are comparable.

(3) IHT version 1.0 doesn't support inverse hyperbolic functions. To determine the shape factor from Eq. (3), use this approach:

```
// Shape factor:
S = 2 * pi / yy                               // Eq (2); Table 4.1
cosh (yy) = 2 * z / Do                        // Inverse hyperbolic function representation
cosh (yy) = 0.5 * ( exp (yy) +  exp (-yy) )   // Definition of the the function
R'conds = 1 / ( ks * S )                      // Thermal resistance
```

PROBLEM 8.66

KNOWN: Length, diameter, insulation characteristics and burial depth of pipe. Ground surface temperature. Inlet temperature, minimum allowable exit temperature, flow rate and properties of oil flow through pipe.

FIND: Effect of soil thermal conductivity and flowrate on heat rate and outlet temperature.

SCHEMATIC:

T_s = -40 °C
Soil, 0.25 ≤ k_s ≤ 1.0 W/m K
z = 3 m
Insulation, D_i = 1.2 m
D_o = 1.5 m, ki = 0.05 W/m·K
Oil
250 ≤ $\dot{m}_o$ ≤ 500 kg/s
$T_{m,i}$ = 120 °C
$T_{m,o}$ ≥ 110 °C
L = 10^5 m

ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Two-dimensional conduction in soil, (4) Negligible pipe wall thermal resistance, (5) Total resistance to heat loss is independent of x.

PROPERTIES: Oil (given): $\rho_o = 900\ kg/m^3$, $c_{p,o} = 2000$ J/kg·K, $\nu_o = 8.5 \times 10^{-4}\ m^2/s$, $k_o = 0.140$ W/m·K, $Pr_o = 10^4$.

ANALYSIS: From the analysis of Problem 8.60, the outlet temperature may be computed from the expression

$$\frac{T_{m,o} - T_s}{T_{m,i} - T_s} = \exp\left(-\frac{L}{\dot{m} c_{p,o} R'_{tot}}\right)$$

where

$$R'_{tot} = \frac{1}{\bar{h}\pi D_i} + \frac{\ln(D_o/D_i)}{2\pi k_i} + \frac{\cosh^{-1}(2z/D_o)}{2\pi k_s}$$

and $\bar{h}$ is determined from Eq. 8.56. The heat rate may then be obtained from the overall energy balance

$$q = \dot{m} c_p \left(T_{m,i} - T_{m,o}\right)$$

Using the *Correlations* Toolpad of IHT to perform the parametric calculations, the following results were obtained.

Outlet temperature, Tmo(C)
120
110
100
90
0.25 0.4 0.55 0.7 0.85 1
Soil thermal conductivity, ks(W/m.K)
mdot = 250 kg/s
mdot = 375 kg/s
mdot = 500 kg/s

Heat rate, q(W)
1.2E7
1E7
8E6
6E6
0.25 0.4 0.55 0.7 0.85 1
Soil thermal conductivity, ks(W/m.K)
mdot = 250 kg/s
mdot = 375 kg/s
mdot = 500 kg/s

Continued...

PROBLEM 8.66 (Cont.)

Due to a reduction in the thermal conduction resistance of the soil with increasing k_s, there is a corresponding increase in the heat rate q from the pipe and a reduction in the oil outlet temperature. The heat rate also increases with increasing $\dot{m}$ (due to an increase in $\bar{h}$ and hence a decrease in the convection resistance), but the increase lags that of the flow rate, causing the outlet temperature to increase with increasing $\dot{m}$. Conditions for which $T_{m,o} \geq 110°C$ cannot be achieved for $\dot{m}$ = 250 kg/s, but can be achieved for $k_s \leq 0.33$ W/m·K and $k_s \leq 0.65$ W/m·K for $\dot{m}$ = 375 kg/s and 500 kg/s, respectively. The worst case condition corresponds to the smallest value of $\dot{m}$ and the largest value of k_s.

Measures to maintain $T_{m,o} \geq 110°C$ could include increasing the burial depth, increasing the insulation thickness, and/or using an insulation of lower thermal conductivity.

COMMENTS: The thermophysical properties of oil depend strongly on temperature, and a more accurate solution would account for the effect of variations in $\bar{T}_m$ on the properties.

PROBLEM 8.67

KNOWN: Water flowing through a thin-walled tube is heated by hot gases moving in cross flow over the tube.

FIND: Outlet temperature of the water, $T_{m,o}$.

SCHEMATIC:

Exhaust gases
V=100 m/s
T_∞=225°C
Water
$\dot{m}$ = 0.25 kg/s
$T_{m,i}$ = 30°C
L = 4m
$T_{m,o}$
Tube, D = 40mm

ASSUMPTIONS: (1) Steady-state conditions, (2) Tube wall thermal resistance negligible, (3) Exhaust gas properties those of atmospheric air, (4) KE, PE, flow work changes negligible, (5) Fully developed internal flow, (6) Constant properties.

PROPERTIES: *Table A-6*, Water $\left(\text{assume } T_{m,o} \approx 50^\circ\text{C}, \ \overline{T}_m = (50+30)^\circ\text{C}/2 \approx 315\text{ K}\right)$: $\rho = 1/v_f = 991.1\text{ kg/m}^3$, $c_p = 4179$ J/kg·K, $\mu = 631 \times 10^{-6}$ N·s/m^2, k = 0.634 W/m·K, Pr = 4.16; *Table A-4*, Air ($T_\infty = 225°C \approx 500$ K, 1 atm): $\nu = 38.79 \times 10^{-6}$ m^2/s, k = 40.7×10^{-3} W/m·K, Pr = 0.684.

ANALYSIS: This situation may be analyzed using Eq. 8.46 with $\overline{U} = 1/\left(1/\overline{h}_i + 1/\overline{h}_o\right)$, where $\overline{h}_i$ and $\overline{h}_o$ correspond to coefficients for internal and external flows, respectively. *Internal flow*: With properties evaluated at $\overline{T}_m$, assuming $T_{m,o}$ = 50°C, $Re_D = 4\dot{m}/\pi D\mu = 4 \times 0.25\text{ kg/s} / \pi \times 0.040\text{ m} \times 631 \times 10^{-6}\text{ N·s/m}^2 = 12,611$. The internal flow is turbulent and fully developed (L/D = 100), and the Dittus-Boelter correlation for heating conditions ($T_s > T_m$) is appropriate,

$$h_i = \frac{k}{D} 0.023 Re_D^{4/5} Pr^{0.4} = \frac{0.634\text{ W/m}\cdot\text{K}}{0.040\text{ m}} 0.023(12,611)^{4/5}(4.16)^{0.4} = 1230\text{ W/m}^2\cdot\text{K}.$$

For the external flow,

$$Re_D = VD/\nu = 100\text{ m/s} \times 0.040\text{ m}/38.79 \times 10^{-6}\text{ m}^2/\text{s} = 1.031 \times 10^5$$

and from Table 7.4, C = 0.26 and m = 0.6; Pr ≤ 10, n = 0.37, and Pr ≈ Pr_s,

$$\overline{h}_o = (k/D) C Re_D^m Pr^n \left(Pr/Pr_s\right)^{1/4}$$

$$\overline{h}_o = \frac{40.7 \times 10^{-3}\text{ W/m}\cdot\text{K}}{0.040\text{ m}} \times 0.26\left(1.031 \times 10^5\right)^{0.6} (0.684)^{0.37} (1)^{1/4} = 234\text{ W/m}^2\cdot\text{K}.$$

Substituting numerical values into Eq. 8.46 with P = πD and U = 197 W/m^2·K,

$$\frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = \exp\left(-PLU/\dot{m}c_p\right) \quad (1)$$

$$T_{m,o} = 225^\circ\text{C} - (225-30)^\circ\text{C} \exp\left[-\frac{\pi \times 0.040\text{ m} \times 4\text{ m}}{0.25\text{ kg/s} \times 4179\text{ J/kg}\cdot\text{K}} \times 197\text{ W/m}^2\cdot\text{K}\right] = 47.6^\circ\text{C}$$ <

COMMENTS: Note the assumed value of $T_{m,o}$ to evaluate water properties was reasonable. Using Eq. (1), replacing T_∞ and U with T_s and h_i, respectively, find T_s = 63.2°C; hence, $Pr_s(T_s) \approx 0.687$. The assumption that Pr ≈ Pr_s in the Zhukauskas relation is reasonable.

PROBLEM 8.68

KNOWN: Single tube heat exchanger for cooling blood.

FIND: (a) Temperature at which properties are evaluated in estimating $\bar{h}$, (b) Prandtl number for the blood, Pr, (c) Flow condition: laminar or turbulent, (d) Average heat transfer coefficient, $\bar{h}$, for blood flow, (e) Total heat rate, q, (f) Required length of tube, L, when U is known.

SCHEMATIC:

$T_i = 40°C$
$D = 2.5\,mm$
Blood
$\dot{V} = 10^{-4}\,m^3/min$
$\bar{U} = 300\,W/m^2\cdot K$
$T_o = 30°C$
i
Water - ice, T_∞
o

PROPERTIES: Blood $\left(\text{Given}, \bar{T}_m\right)$: $\rho = 1000\ kg/m^3$, $\nu = 7 \times 10^{-7}\ m^2/s$, $k = 0.5\ W/m\cdot K$, $c_p = 4000\ J/kg\cdot K$.

ASSUMPTIONS: (1) Flow and thermal conditions fully developed, (2) Thermal resistance of tube material is negligible, (3) Overall heat transfer coefficient between blood and water-ice mixture is U = 300 W/m^2·K, (4) Constant properties, (5) Negligible heat transfer enhancement associated with coiling.

ANALYSIS: (a) Evaluate properties at $\bar{T}_m = \left(T_o + T_i\right)/2 = \left(40 + 30\right)^\circ C/2 = 35^\circ C$. <

(b) The Prandtl number is

$$Pr = \frac{c_p m}{k} = \frac{c_p n r}{k} = \frac{\left(4000\ J/kg\cdot K \times 7 \times 10^{-7} m^2/s \times 1000\ kg/m^3\right)}{0.5\ W/m\cdot K} = 5.60.$$ <

(c) Calculate Reynolds number as

$$Re_D = \frac{4\ \dot{m}}{p D m} = \frac{4\ \dot{V} r}{p\ D n r} = \frac{4\ \dot{V}}{p\ D n} = \frac{4 \times 10^{-4} m^3/min\ \left(1\ min/60s\right)}{p \times 2.5 \times 10^{-3}\ m \times 7 \times 10^{-7}\ m^2/s} = 1213$$

Hence, the flow is laminar, <

(d) For laminar and fully developed conditions, Eq. 8.55 is the proper correlation,

$$Nu_D = \bar{h}D/k = 3.66 \qquad \bar{h} = 3.66 \times 0.5\ W/m\cdot K/2.5 \times 10^{-3}\ m = 732\ W/m^2\cdot K,$$ <

(e) The total heat rate follows from an overall energy balance, Eq. 8.37,

$$q = \dot{m}\, c_p\left(T_o - T_i\right) = r\dot{V}c_p\left(T_o - T_i\right)$$

$$q = 1000\ kg/m^3\left(10^{-4}\ m^3/min/60\ s/min\right)4000\ J/kg\cdot K \times \left(30 - 40\right)^\circ C = -66.7\ W.$$ <

(f) Using the rate equation, Eq. 8.47, solve for L,

$$L = \frac{q}{\bar{U}p\ D\Delta T_{\ell m}} = \frac{66.7\ W}{300\ W/m^2\cdot K\left(p \times 2.5 \times 10^{-3} m\right) \times 34.8^\circ C} = 0.81\ m$$ <

where A = πDL and $\Delta T_{\ell m}$ = [(40 - 0)°C - (30 - 0)°C]/ ℓ n(40/30) = 34.8°C.

PROBLEM 8.69

KNOWN: Flow conditions associated with water passing through a pipe and air flowing over the pipe.

FIND: (a) Differential equation which determines the variation of the mixed-mean temperature of the water, (b) Heat transfer per unit length of pipe at the inlet and outlet temperature of the water.

SCHEMATIC:

ASSUMPTIONS: (1) Negligible temperature drop across the pipe wall, (2) Negligible radiation exchange between outer surface of insulation and surroundings, (3) Fully developed flow throughout pipe, (4) Negligible potential and kinetic energy and flow work effects.

PROPERTIES: *Table A-6*, Water ($T_{m,i}$ = 200°C): $c_{p,w}$ = 4500 J/kg·K, $\mu_w = 134 \times 10^{-6}$ N·s/m^2, k_w = 0.665 W/m·K, Pr_w = 0.91; *Table A-4*, Air (T_∞ = - 10°C): $\nu_a = 12.6 \times 10^{-6}$ m^2/s, k_a = 0.023 W/m·K, Pr_a = 0.71, $Pr_s \approx 0.7$.

ANALYSIS: (a) Following the development of Section 8.3.1 and applying an energy balance to a differential element in the water, we obtain

$$\dot{m}\, c_{p,w}\, T_m - dq - \dot{m}\, c_{p,w}\left(T_m + dT_m\right) = 0.$$

Hence $\qquad dq = -\dot{m}\, c_{p,w}\, dT_m$

where $\qquad dq = U_i dA_i \left(T_m - T_\infty\right) = U_i \pi\, D\, dx \left(T_m - T_\infty\right).$

Substituting into the energy balance, it follows that

$$\frac{d\,T_m}{dx} = -\frac{U_i \pi\, D}{\dot{m}\, c_p}\left(T_m - T_\infty\right). \qquad (1)$$ <

The overall heat transfer coefficient based on the inside surface area may be evaluated from Eq. 3.30 which, for the present conditions, reduces to

$$U_i = \frac{1}{\dfrac{1}{h_i} + \dfrac{D}{2k}\ell n\left(\dfrac{D+2t}{D}\right) + \dfrac{D}{D+2t}\,\dfrac{1}{h_o}}. \qquad (2)$$

For the *inner water flow*, Eq. 8.6 gives

$$Re_D = \frac{4\,\dot{m}}{\pi\, D \mu_w} = \frac{4 \times 2\text{ kg/s}}{\pi\left(1\text{ m}\right) \times 134 \times 10^{-6}\text{ kg/s}\cdot\text{m}} = 19{,}004.$$

Continued …..

PROBLEM 8.69 (Cont.)

Hence, the flow is turbulent. With the assumption of fully developed conditions, it follows from Eq. 8.60 that

$$h_i = \frac{k_w}{D} \times 0.023\ Re_D^{4/5}\ Pr_w^{0.3}. \qquad (3)$$

For the *external air flow*

$$Re_D = \frac{V(D+2t)}{\nu} = \frac{4\ m/s\,(1.3m)}{12.6\times10^{-6}\ m^2/s} = 4.13\times10^5.$$

Using Eq. 7.31 to obtain the outside convection coefficient,

$$h_o = \frac{k_a}{(D+2t)} \times 0.076\ Re_D^{0.7}\ Pr_a^{0.37}\ (Pr_a/Pr_s)^{1/4}. \qquad (4)$$

(b) The heat transfer per unit length of pipe at the inlet is

$$q' = \pi\ D\ U_i\left(T_{m,i} - T_\infty\right). \qquad (5)$$

From Eqs. (3 and 4),

$$h_i = \frac{0.665\ W/m\cdot K}{1\ m} \times 0.023(19{,}004)^{4/5}\ (0.91)^{0.3} = 39.4\ W/m^2\cdot K$$

$$h_o = \frac{0.023\ W/m\cdot K}{(1.3\ m)} \times 0.076\left(4.13\times10^5\right)^{0.7}(0.71)^{0.37}(1)^{1/4} = 10.1\ W/m^2\cdot K.$$

Hence, from Eq. (2)

$$U_i = \left[\frac{1}{39.4\ W/m^2\cdot K} + \frac{1\ m}{0.1\ W/m\cdot K}\ln\left(\frac{1.3}{1}\right) + \frac{1}{1.3}\times\frac{1}{10.1\ W/m^2\cdot K}\right]^{-1} = 0.37\ W/m^2\cdot K$$

and from Eq. (5)

$$q' = \pi\,(1\ m)\ \left(0.37\ W/m^2\cdot K\right)\ (200+10)^\circ C = 244\ W/m. \qquad <$$

Since U_i is a constant, independent of x, Eq. (1) may be integrated from x = 0 to x = L. The result is analogous to Eq. 8.42b and may be expressed as

$$\frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = \exp\left(-\frac{\pi\ DL}{\dot{m}\ c_{p,w}}U_i\right) = \exp\left(-\frac{\pi\times1m\times500m}{2\ kg/s\times4500\ J/kg\cdot K}\times0.37\ W/m^2\cdot K\right)$$

Hence $$\frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = 0.937.$$

$$T_{m,o} = T_\infty + 0.937\left(T_{m,i} - T_\infty\right) = 187^\circ C. \qquad <$$

COMMENTS: The largest contribution to the denominator on the right-hand side of Eq. (2) is made by the conduction term (the insulation provides 96% of the total resistance to heat transfer). For this reason the assumption of fully developed conditions throughout the pipe has a negligible effect on the calculations. Since the reduction in T_m is small (13°C), little error is incurred by evaluating all properties of water at $T_{m,i}$.

PROBLEM 8.70

KNOWN: Inner and outer radii and thermal conductivity of a teflon tube. Flowrate and temperature of confined water. Heat flux at outer surface and temperature and convection coefficient of ambient air.

FIND: Fraction of heat transfer to water and temperature of tube outer surface.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Fully-developed flow, (3) One-dimensional conduction, (4) Negligible tape contact and conduction resistances.

PROPERTIES: *Table A-6*, Water (T_m = 290K): $\mu = 1080 \times 10^{-6}$ kg/s·m, k = 0.598 W/m·K, Pr = 7.56.

ANALYSIS: The outer surface temperature follows from a surface energy balance

$$(2\pi r_o L) q'' = \frac{T_{s,o} - T_\infty}{(h_o 2\pi r_o L)^{-1}} + \frac{T_{s,o} - T_m}{(\ln(r_o / r_i)/2\pi Lk) + (1/2\pi r_i L h_i)}$$

$$q'' = h_o (T_{s,o} - T_\infty) + \frac{T_{s,o} - T_m}{(r_o / k)\ln(r_o / r_i) + (r_o / r_i)/h_i}.$$

With $\quad Re_D = 4\dot{m}/(\pi D\mu) = 4(0.2 kg/s)/\left[\pi (0.02 m) 1080 \times 10^{-6} kg/s \cdot m\right] = 11,789$

the flow is turbulent and Eq. 8.60 yields

$$h_i = (k/D_i) 0.023 Re_D^{4/5} Pr^{0.4} = (0.598 W/m \cdot K/0.02 m)(0.023)(11,789)^{4/5}(7.56)^{0.4} = 2792 W/m^2 \cdot K.$$

Hence

$$2000 W/m^2 = 25 W/m^2 \cdot K (T_{s,o} - 300K) + \frac{T_{s,o} - 290 K}{(0.013 m/0.35 W/m \cdot K)\ln(1.3) + (1.3)/\left(2792 W/m^2 \cdot K\right)}$$

and solving for $T_{s,o}$, $\quad T_{s,o} = 308.3$ K. <

The heat flux to the air is

$$q''_o = h_o (T_{s,o} - T_\infty) = 25 W/m^2 \cdot K (308.3 - 300) K = 207.5 W/m^2.$$

Hence, $\quad q''_i / q'' = (2000 - 207.5) W/m^2 / 2000 W/m^2 = 0.90.$ <

COMMENTS: The resistance to heat transfer by convection to the air substantially exceeds that due to conduction in the teflon and convection in the water. Hence, most of the heat is transferred to the water.

PROBLEM 8.71

KNOWN: Temperature recorded by a thermocouple inserted in a stack containing flue gases with a prescribed flow rate. Diameters and emissivities of thermocouple tube and gas stack. Conditions associated with stack surroundings.

FIND: Equations for predicting thermocouple error and error associated with prescribed conditions.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Flue gas has properties of air at $T_g \approx 327°C$, (3) Stack forms a large enclosure about the thermocouple tube and surroundings form a large enclosure around the stack, (4) Stack surface energy balance is unaffected by heat loss to tube, (5) Gas flow is fully developed, (6) Negligible conduction along thermocouple tube, (7) Stack wall is thin.

PROPERTIES: *Table A-4*, Air ($T_g \approx 600K$, $p_g = 1$ atm): $\rho = 0.58\ kg/m^3$, $\mu = 305.8 \times 10^{-7}\ N{\cdot}s/m^2$, $\nu = 52.7 \times 10^{-6}\ m^2/s$, $k = 0.0469\ W/m{\cdot}K$, $Pr = 0.685$.

ANALYSIS: Determination of the thermocouple error necessitates determining the gas temperature T_g and relating it to the thermocouple temperature T_t. From an energy balance applied to a control surface about the thermocouple,

$$q_{conv} = q_{rad} \qquad \text{or} \qquad h_t A_t \left(T_g - T_t\right) = \varepsilon_t \sigma A_t \left(T_t^4 - T_s^4\right).$$

Hence

$$T_g = T_t + \frac{\varepsilon_t \sigma}{h_t}\left(T_t^4 - T_s^4\right). \qquad (1) \quad <$$

However, T_s is unknown and must be determined from an energy balance on the stack wall.

$$q_{conv,i} = q_{conv,o} + q_{rad}$$

$$h_i A_s \left(T_g - T_s\right) = h_o A_s \left(T_s - T_\infty\right) + \varepsilon_s \sigma A_s \left(T_s^4 - T_{sur}^4\right)$$

or

$$T_g = T_s + \frac{h_o}{h_i}\left(T_s - T_\infty\right) + \frac{\varepsilon_s \sigma}{h_i}\left(T_s^4 - T_{sur}^4\right). \qquad (2) \quad <$$

T_g and T_s may be determined by simultaneously solving Eqs. (1) and (2). For the prescribed conditions

$$Re_{Dt} = \frac{\rho V D_t}{\mu} = \frac{\rho\left(\dot{m}_g / \rho \pi D_s^2 / 4\right) D_t}{\mu} = \frac{4\,\dot{m}_g D_t}{\pi \mu D_s^2} = \frac{4 \times 1\ kg/s \times 0.01\ m}{\pi \times 305.8 \times 10^{-7}\ N\cdot s/m^2\ \left(0.6\ m\right)^2} = 1157.$$

Continued …..

PROBLEM 8.71 (Cont.)

Assuming $(Pr/Pr_s) = 1$, it follows from the Zhukauskus correlation

$$\overline{Nu}_D = 0.26Re_{Dt}^{0.6}\,Pr^{0.37}$$

where C = 0.26 and m = 0.6 from Table 7.4. Hence

$$h_t = \frac{0.0469\ W/m\cdot K}{0.01\ m}(1157)^{0.6}(0.685)^{0.87}\times 0.26 = 73\ W/m^2\cdot K.$$

Hence, from Eq. (1) $$T_g = 573\ K + \frac{0.8\times 5.67\times 10^{-8}\ W/m^2\cdot K^4}{73\ W/m^2\cdot K}\left(573^4 - T_s^4\right)K^4$$

$$T_g = 573\ K + 67\ K - 6.214\times 10^{-10}T_s^4 = 640 - 6.214\times 10^{-10}T_s^4. \qquad (1a)$$

Also, $$Re_{Ds} = \frac{4\,\dot{m}_g}{\pi\ D_s\mu} = \frac{4\times 1\ kg/s}{\pi\,(0.6\ m)305.8\times 10^{-7}\ N\cdot s/m^2} = 6.94\times 10^4$$

and the gas flow is turbulent. Hence from the Dittus-Boelter correlation,

$$h_i = \frac{k}{D_s}0.023Re_{Ds}^{4/5}\,Pr^{0.3} = \frac{0.0469\ W/m\cdot K}{0.6\ m}\times 0.023\left(6.94\times 10^4\right)^{4/5}\times(0.685)^{0.3} = 12\ W/m^2\cdot K.$$

Hence from Eq. (2)

$$T_g = T_s + \frac{25}{12}\left(T_s - 300\ K\right) + \frac{0.8\times 5.67\times 10^{-8}\ W/m^2\cdot K^4}{12\ W/m^2\cdot K}\left[T_s^4 - 300^4\right]K^4$$

$$T_g = T_s + 2.083T_s - 625\ K + 3.78\times 10^{-9}T_s^4 - 30.6\ K = -655.6\ K + 3.083T_s + 3.78\times 10^{-9}\,T_s^4. \qquad (2a)$$

Solve Eqs. (1a) and (2a) by trial-and-error. Assume values for T_s and determine T_g from (1a) and (2a). Continue until values of T_g agree.

T_s (K)	T_g (K) $\rightarrow$ (1a)	T_g (K) $\rightarrow$ (2a)
400	624	674
375	628	575
387	626	622
388	626	626

Hence $T_s = 388$ k, $T_g = 626$ K

and the thermocouple error is $$T_g - T_t = 626\ K - 573\ K = 53^\circ C.$$ <

COMMENTS: The thermocouple error results from radiation exchange between the thermocouple tube and the cooler stack wall. Anything done to ↑ T_s would ↓ this error (e.g., ↓ h_o or ↑ T_∞ and T_{sur}). The error also ↓ with ↑ h_t. The error could be reduced by installing a radiation shield around the tube.

PROBLEM 8.72

KNOWN: Platen heated by hot ethylene glycol flowing through tubing arrangement with spacing S soldered to lower surface. Top surface exposed to convection process.

FIND: Tube spacing S and heating fluid temperature T_m which will maintain the top surface at 45 ± 0.25°C.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions; (2) Lower surface is insulated, all heat transfer from hot fluid is into platen; (3) Copper tube is thick-walled such that interface between solder and platen is isothermal; (4) Fully developed flow conditions in tube.

PROPERTIES: *Table A.4*, Ethylene glycol (T_m = 60°C): $\mu = 0.00522\ N\cdot s/m^2$, k = 0.2603 W/m·K.

ANALYSIS: Begin the analysis by setting up a nodal mesh (9 ×6) to represent the platen experiencing convection on the top surface (T_∞, h) while the two side boundaries are symmetry adiabats. On the lower surface, nodes 46 and 47 represent the isothermal platen-solder interface maintained at T_o by the hot fluid. The remaining nodes (49-54) are insulated on their lower boundary.

The heat rate supplied by the tube to the platen can be expressed as

$$q'_{cv} = 0.5h_o(\pi D_i)(T_m - T_o) \tag{1}$$

From energy balances about nodes 46 and 47, the heat rate into the platen by conduction can be expressed as

$$q'_{cd} = q'_a + q'_b + q'_c \tag{2}$$

$$q'_a = k(\Delta x/2)(T_{46} - T_{37})/\Delta y \tag{3}$$

Continued...

PROBLEM 8.72 (Cont.)

$$q'_b = k(\Delta x)(T_{47} - T_{38})/\Delta y \quad (4)$$

$$q'_c = k(\Delta y/2)(T_{47} - T_{48})/\Delta x \quad (5)$$

and we require that

$$q'_{cd} = q'_{cv} \quad (6)$$

The convection coefficient for internal flow can be estimated from a correlation assuming fully developed flow. First, characterize the flow with

$$Re_D = \frac{4\dot{m}}{\pi D_i \mu} = \frac{4 \times 0.06\,\text{kg/s}}{\pi(0.008\,\text{m})0.00522\,\text{N}\cdot\text{s/m}^2} = 1829$$

and since it is laminar,

$$Nu_D = \frac{h_o D_i}{k} = 3.66$$

$$h_o = 3.66 \times 0.2603\,\text{W/m}\cdot\text{K}/0.008\,\text{m} = 119.1\,\text{W/m}\cdot\text{K}$$

where properties are evaluated at T_m. Using the *IHT Finite-Difference Tool* for *Two-Dimensional Steady-State Conditions* and the *Properties Tool* for *Ethylene Glycol*, along with the foregoing rate equations and energy balances, Eqs. (1-6), a model was developed to solve for the temperature distribution in the platen. In the solution, we determined what hot fluid temperature was required to maintain T_1 = 45°C. Two trials were run. In the first, the nodal arrangement was as shown above (9 × 6) for which S/2 = (9 - 1)Δx = 42.67 mm with Δx = $2D_i/3$ = 5.33 mm and Δy = w/5 = 5 mm. In the second trial, we repositioned the right-hand symmetry adiabat to pass vertically through the nodes 6-51 so that now the nodal mesh is (6 × 6) and S/2 = (6 - 1)Δx = 26.65 mm with Δx and Δy remaining the same. The results of the trials are tabulated below.

Trial	Mesh	T_1 (°C)	T_6 (°C)	T_9 (°C)	T_m (°C)	q'_{cv} (W/m)
1	9 × 6	45.0	43.5	43.0	105	80.5
2	6 × 6	45.0	44.5	---	85	52.6

From the trial 2 results, the surface temperature uniformity is (T_1 - T_6) = 0.5°C which satisfies the ±0.25°C requirement. So that suitable tube spacing and fluid temperature are

S = 53 mm $\qquad$ T_m = 85°C $\qquad$ <

COMMENTS: (1) Recognize that the grid spacing is quite coarse and good practice demands that we repeat the analysis decreasing the nodal spacing until no further changes are seen in T_m.

(2) In the first trial, note that T_m = 105°C which of course, is not possible.

PROBLEM 8.73

KNOWN: Features of tubing used in a ground source heat pump. Temperature of surrounding soil. Fluid inlet temperature and flowrate.

FIND: (a) Effect of tube length on outlet temperature, (b) Recommended tube length and the effect of variations in the flowrate.

SCHEMATIC:

Polyethylene tubing
(t = 8 mm, k_t = 0.47 W/m·K)
T_s = 10 °C
$T_{m,o}$
D_i = 25 mm
L
$T_{m,i}$ = 0 °C
$\dot{m}$ = 0.03 kg/s

ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Negligible conduction resistance in soil, (4) Negligible KE, PE and flow work changes, (5) Fluid properties correspond to those of water.

PROPERTIES: *Table A.6* (assume $\overline{T}_m$ = 277 K): c_p = 4206 J/kg·K, $\mu = 1560 \times 10^{-6}$ N·s/m^2, k = 0.577 W/m·K, Pr = 11.44.

ANALYSIS: (a) For the prescribed conditions, $Re_D = 4\dot{m}/\pi D_i \mu = 4(0.03\,kg/s)/\pi(0.025\,m)1560 \times 10^{-6}\,N \cdot s/m^2$ = 980 and the flow is laminar. Assuming thermal entry length conditions, which is reasonable for Pr = 11.44, Eq. 8.56 may be used to determine the average convection coefficient

$$\overline{Nu}_D = 3.66 + \frac{0.0668(D/L)Re_D\,Pr}{1 + 0.04\left[(D/L)Re_D\,Pr\right]^{2/3}}$$

With T_s used in lieu of T_∞, Eq. 8.46b may be used to determine $T_{m,o}$,

$$\frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp\left(-\frac{L}{\dot{m} c_p R'_{tot}}\right)$$

where R'_{tot} accounts for the convection and tube wall conduction resistances,

$$R'_{tot} = R'_{cnv} + R'_{cnd} = \left(1/\pi D_i \overline{h}\right) + \ln\left(D_o/D_i\right)/2\pi k_t$$

and

$$D_o = D_i + 2t = 41\,mm\,.$$

Using the *Correlations* and *Properties* Toolpads of IHT, the following results were obtained for the effect of the tube length L on $T_{m,o}$.

Continued...

The longer the tube the larger the rate of heat extraction from the soil, and for $\dot{m}$ = 0.030 kg/s, the temperature rise of $\Delta T = (T_{m,o} - T_{m,i}) \approx 6°C$ is well below the maximum possible value of $\Delta T_{max} = 10°C$.

(b) The length should be *at least* 50 m long. If the flowrate were reduced by 50% ($\dot{m}$ = 0.015 kg/s), the corresponding temperature rise would be close to ΔT_{max} and L = 50 m would be close to optimal. However, for the nominal flowrate and a 50% increase from the nominal, the length should exceed 50 m to recover more heat and provide a heat pump inlet temperature which is closer to the maximum possible value.

COMMENTS: In practice, the tube surface temperature would be less than 10°C (if the temperature of the soil well removed from the tube were at 10°C), thereby reducing the heat extraction rate and $T_{m,o}$.

PROBLEM 8.74

KNOWN: Reynolds numbers for fully developed turbulent flow of water in a smooth circular tube.

FIND: Nusselt numbers based on the Colburn, Petukhov and Gnielinski correlations.

SCHEMATIC:

Water

$Re_D = 4000, 10^4, 10^5; Pr = 6$

ANALYSIS: The correlations are

Colburn, Equation 8.59,

$$Nu_D = 0.023 Re_D^{4/5} Pr^{1/3}$$

Petukhov, Equation 8.62,

$$Nu_D = \frac{(f/8) Re_D Pr}{1.07 + 12.7 (f/8)^{1/2} \left(Pr^{2/3} - 1\right)}$$

$$f = \left(1.82 \log_{10} Re_D - 1.64\right)^{-2}$$

Gnielinski, Equation 8.63,

$$Nu_D = \frac{(f/8)(Re_D - 1000) Pr}{1 + 12.7 (f/8)^{1/2} \left(Pr^{2/3} - 1\right)}$$

$$f = \left(0.79 \ln Re_D - 1.64\right)^{-2}$$

It follows that:

	Re_D		
Correlation	4000	10^4	10^5
Colburn	31.8	66.2	417.9
Petukhov	40.0	81.2	549.5
Gnielinski	30.0	75.0	560.0

COMMENTS: The Colburn equation does well in the transitional region ($Re_D \leq 10^4$), where the Gnielinski equation provides the best predictions, but can significantly underpredict the Nusselt number under fully turbulent conditions.

PROBLEM 8.75

KNOWN: Effect of entry length on average Nusselt number for turbulent flow in a tube.

FIND: Ratio of average to fully developed Nusselt numbers for prescribed conditions.

SCHEMATIC:

x $Nu_D(x)$ $Nu_{D,fd}$

ASSUMPTIONS: (1) Sharp edged inlet, (2) Combined entry region.

ANALYSIS: From Eq. 8.64,

$$\frac{\overline{Nu}_D}{Nu_{D,fd}} = 1 + \frac{C}{(x/D)^m}$$

and with $C = 24 Re_D^{-0.23}$ and $m = 0.815 - 2.08 \times 10^{-6} Re_D$,

$$\frac{\overline{Nu}_D}{Nu_{D,fd}} = 1 + \frac{24 Re_D^{-0.23}}{(x/D)^{\left(0.815 - 2.08 \times 10^{-6} Re_D\right)}}.$$

It follows that

$\left(\overline{Nu}_D / Nu_{D,fd}\right)$	Re_D	(x/D)
1.464	10^4	10
1.112	10^4	60
1.420	10^5	10
1.142	10^5	60

COMMENTS: The assumption $\overline{Nu}_D \approx Nu_{fd}$ for $x/D = 10$ would result in underprediction of $\overline{Nu}_D$ by approximately 45%. The underprediction is only approximately 10% for x/D = 60.

PROBLEM 8.76

KNOWN: Fluid enters a thin-walled tube of 5-mm diameter and 2-m length with a flow rate of 0.04 kg/s and temperature of $T_{m,i}$ = 85°C; tube surface temperature is maintained at T_s = 25°C; and, for this *base* operating condition, the outlet temperature is $T_{m,o}$ = 31.1°C.

FIND: The outlet temperature if the flow rate is doubled?

SCHEMATIC:

$T_s = 25^oC$

Fluid

$T_{m,i} = 85^oC$

$\dot{m}_b = 0.04$ kg/s

or

$\dot{m}_n = 2\dot{m}_b$

$T_{m,o})_b = 31.3^oC$

$T_{m,o})_n = ?$

Thin-walled tube

D = 5 mm

L = 2 m

ASSUMPTIONS: (1) Flow is fully developed and turbulent, (2) Fluid properties are independent of temperature, and (3) Constant surface temperature cooling conditions.

ANALYSIS: For the *base* operating condition (b), the rate equation, Eq. 8.42b, with $C = \dot{m}\,c_p$, the capacity rate, is

$$\frac{T_s - T_{m,o})_b}{T_s - T_{m,i}} = \exp\left(-\frac{PL\,\bar{h}_b}{C_b}\right) \tag{1}$$

Substituting numerical values, with P = πD, find the ratio, $\bar{h}_b / C_b$,

$$\frac{25-31.1}{25-85} = \exp\left[\pi \times 0.005 \text{ m} \times 2 \text{ m}\left(\bar{h}_b / C_b\right)\right]$$

$$\bar{h}_b / C_b = 72.77 \text{ m}^{-2}$$

For the *new* operating condition (n), the flow rate is doubled, $C_n = 2C_b$, and the convection coefficient scales according to the Dittus-Boelter relation, Eq. 8.60,

$$\bar{h} \propto Re_D^{0.8} \propto \dot{m}^{0.8}$$

$$\bar{h}_n = 2^{0.8}\bar{h}_b \text{ and } \left(\bar{h}_n / C_n\right) = 2^{0.8}/2\left(\bar{h}_b / C_b\right) \tag{2}$$

Using the rate equation for the new operating condition, find

$$\frac{T_s - T_{m,o})_n}{T_s - T_{m,i}} = \exp\left(-\frac{PL\,\bar{h}_n}{C_n}\right) = \exp\left[-PL \times 0.871\left(\bar{h}_b / C_b\right)\right] \tag{3}$$

$$\frac{25 - T_{m,o})_n}{25-85} = \exp\left[-\pi \times 0.005 \text{ m} \times 2 \text{ m} \times 0.871 \times 72.77 \text{ m}^{-2}\right]$$

$$T_{m,o})_n = 33.2°C$$ <

PROBLEM 8.77

KNOWN: Flow rate and inlet temperature of air passing through a rectangular duct of prescribed dimensions and surface heat flux.

FIND: Air and duct surface temperatures at outlet.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Uniform surface heat flux, (3) Constant properties, (4) Atmospheric pressure, (3) Fully developed conditions at duct exit, (6) Negligible KE, PE and flow work effects.

PROPERTIES: *Table A-4*, Air $\left(\overline{T}_m \approx 300\text{K, 1 atm}\right)$: $c_p = 1007$ J/kg·K, $\mu = 184.6 \times 10^{-7}$ N·s/m^2, k = 0.0263 W/m·K, Pr = 0.707.

ANALYSIS: For this uniform heat flux condition, the heat rate is

$$q = q_s'' \, A_s = q_s''\left[2\left(L \times W\right) + 2\left(L \times H\right)\right]$$

$$q = 600 \text{ W/m}^2\left[2\left(1\text{m} \times 0.016\text{m}\right) + 2\left(1\text{m} \times 0.004\text{m}\right)\right] = 24 \text{ W.}$$

From an overall energy balance

$$T_{m,o} = T_{m,i} + \frac{q}{\dot{m}\, c_p} = 300\text{K} + \frac{24 \text{ W}}{3 \times 10^{-4} \text{ kg/s} \times 1007 \text{ J/kg} \cdot \text{K}} = 379 \text{ K.} \qquad <$$

The surface temperature at the outlet may be determined from Newton's law of cooling, where

$$T_{s,o} = T_{m,o} + q''/h.$$

From Eqs. 8.67 and 8.1

$$D_h = \frac{4\, A_c}{P} = \frac{4\left(0.016\text{m} \times 0.004\text{m}\right)}{2\left(0.016\text{m} + 0.004\text{m}\right)} = 0.0064 \text{ m}$$

$$Re_D = \frac{r\, u_m\, D_h}{m} = \frac{\dot{m}\, D_h}{A_c m} = \frac{3 \times 10^{-4} \text{kg/s}\left(0.0064\text{m}\right)}{64 \times 10^{-6} \text{m}^2\left(184.6 \times 10^{-7} \text{ N} \cdot \text{s/m}^2\right)} = 1625.$$

Hence the flow is laminar, and from Table 8.1

$$h = \frac{k}{D_h} 5.33 = \frac{0.0263 \text{ W/m} \cdot \text{K}}{0.0064 \text{ m}} 5.33 = 22 \text{ W/m}^2 \cdot \text{K}$$

$$T_{s,o} = 379 \text{ K} + \frac{600 \text{ W/m}^2}{22 \text{ W/m}^2 \cdot \text{K}} = 406 \text{ K.} \qquad <$$

COMMENTS: The calculations should be reperformed with properties evaluated at $\overline{T}_m = 340$ K. The change in $T_{m,o}$ would be negligible, and $T_{s,o}$ would decrease slightly.

PROBLEM 8.78

KNOWN: Flow rate and temperature of air entering a triangular duct of prescribed dimensions and surface temperature.

FIND: Air outlet temperature.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Uniform surface temperature, (4) Fully developed conditions throughout, (5) Air is at atmospheric pressure, (6) Negligible potential and kinetic energy and flow work effects.

PROPERTIES: *Table A-4*, Air (assume $\overline{T}_m \approx 325K$, 1 atm): $c_p = 1008$ J/kg·K, $\mu = 196.4 \times 10^{-7}$ N·s/m^2, k = 0.0282 W/m·K, Pr = 0.707.

ANALYSIS: From Eqs. 8.67 and 8.1

$$D_h = \frac{4\ A_c}{P} = \frac{4\left(1.73\times10^{-4}\text{m}^2\right)}{3(0.02\text{m})} = 0.0115\ \text{m}$$

$$Re_D = \frac{r\ u_m D_h}{m} = \frac{\dot{m}\ D_h}{A_c\ m} = \frac{4\times10^{-4}\text{kg/s}(0.0115\ \text{m})}{1.73\times10^{-4}\text{m}^2\left(196.4\times10^{-7}\ \text{N}\cdot\text{s/m}^2\right)} = 1354.$$

Hence the flow is laminar and from Table 8.1,

$$h = \frac{k}{D_h}2.47 = \frac{0.0282\ \text{W/m}\cdot\text{K}}{0.0115\ \text{m}}2.47 = 6.1\ \text{W/m}^2\cdot\text{K}.$$

From Eq. 8.42b it follows that, with P = 3 W,

$$T_{m,o} = T_s - \left(T_s - T_{m,i}\right)\exp\left(-\frac{PL}{\dot{m}\ c_p}\overline{h}\right)$$

$$T_{m,o} = 100^\circ\text{C} - \left(100^\circ\text{C} - 27^\circ\text{C}\right)\exp\left(-\frac{3\times0.02\text{m}\times2\text{m}\times6.1\ \text{W/m}^2\cdot\text{K}}{4\times10^{-4}\text{kg/s}\times1008\ \text{J/kg}\cdot\text{K}}\right)$$

$$T_{m,o} = 88^\circ\text{C}.$$

<

COMMENTS: With $T_{m,o}$ = 88°C, $\overline{T}_m$ = 330K and there is no need to re-evaluate the properties.

PROBLEM 8.79

KNOWN: Temperature and velocity of gas flow between parallel plates of prescribed surface temperature and separation. Thickness and location of plate insert.

FIND: Heat flux to the plates (a) without and (b) with the insert.

SCHEMATIC:

$T_s = 350K$; $u_m = 60 m/s$, $T_m = 1000K$; H; (a); 40mm; 20mm; (b)

ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible radiation, (3) Gas has properties of atmospheric air, (4) Plates are of infinite width W, (5) Fully developed flow.

PROPERTIES: *Table A-4*, Air (1 atm, T_m = 1000K): $\rho = 0.348\ kg/m^3$, $\mu = 424.4 \times 10^{-7}$ kg/s·m, k = 0.0667 W/m·K, Pr = 0.726.

ANALYSIS: (a) Based upon the hydraulic diameter D_h, the Reynolds number is

$$D_h = 4\, A_c / P = 4(H \cdot W)/2(H+W) = 2H = 80\ mm$$

$$Re_{D_h} = \frac{r\ u_m\ D_h}{m} = \frac{0.348\ kg/m^3 (60\ m/s) 0.08\ m}{424.4 \times 10^{-7}\ kg/s \cdot m} = 39{,}360.$$

Since the flow is fully developed and turbulent, use the Dittus-Boelter correlation,

$$Nu_D = 0.023\ Re_D^{4/5}\ Pr^{0.3} = 0.023(39{,}360)^{4/5}(0.726)^{0.3} = 99.1$$

$$h = \frac{k}{D_h} Nu_D = \frac{0.0667\ W/m \cdot K}{0.08\ m} 99.1 = 82.6\ W/m^2 \cdot K$$

$$q'' = h(T_m - T_s) = 82.6\ W/m^2 \cdot K (1000 - 350) K = 53{,}700\ W/m^2.$$ <

(b) From continuity,

$$\dot{m} = (r\ u_m A)_a = (r\ u_m A)_b \qquad u_m)_b = u_m)_a\, (rA)_a / (rA)_b = 60\ m/s\,(40/20) = 120\ m/s.$$

For each of the resulting channels, D_h = 0.02 m and

$$Re_{D_h} = \frac{r\ u_m\ D_h}{m} = \frac{0.348\ kg/m^3 (120\ m/s)\, 0.02\ m}{424.4 \times 10^{-7} kg/s \cdot m} = 19{,}680.$$

Since the flow is still turbulent,

$$Nu_D = 0.023(19{,}680)^{4/5}(0.726)^{0.3} = 56.9 \qquad h = \frac{56.9(0.0667\ W/m \cdot K)}{0.02\ m} = 189.8\ W/m^2 \cdot K$$

$$q'' = 189.8\ W/m^2 \cdot K\ (1000 - 350) K = 123{,}400\ W/m^2.$$ <

COMMENTS: From the Dittus-Boelter equation,

$$h_b / h_a = (u_{m,b} / u_{m,a})^{0.8} (D_{h,a} / D_{h,b})^{0.2} = (2)^{0.8} (4)^{0.2} = 1.74 \times 1.32 = 2.30.$$

Hence, heat transfer enhancement due to the insert is primarily a result of the increase in u_m and secondarily a result of the decrease in D_h.

PROBLEM 8.80

KNOWN: Temperature, pressure and flow rate of air entering a rectangular duct of prescribed dimensions and surface temperature.

FIND: Air outlet temperature and duct heat transfer rate.

SCHEMATIC:

Air
$\dot{m} = 0.1 kg/s$
$T_{m,i} = 285K$
$p = 1atm$
$T_s = 400K$
q
$W = 0.15m$
$T_{m,o}$
$H = 0.075m$
$L = 2m$

ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Uniform surface temperature, (4) Fully developed flow throughout, (5) Negligible potential and kinetic energy and flow work effects.

PROPERTIES: *Table A-4*, Air (assume $T_m \approx 325K$, 1 atm): $c_p = 1008$ J/kg·K, $\mu = 196.4 \times 10^{-7}$ N·s/m^2, k = 0.0282 W/m·K, Pr = 0.707.

ANALYSIS: From Eqs. 8.67 and 8.1,

$$D_h = \frac{4\,A_c}{P} = \frac{4\times(0.15\times0.075)\,m^2}{2(0.15+0.075)\,m} = 0.10\ m$$

$$Re_D = \frac{r\ u_m D_h}{m} = \frac{\dot{m}\ D_h}{A_c m} = \frac{0.1\ kg/s(0.1m)}{(0.15m\times0.075m)196.4\times10^{-7}\ N\cdot s/m^2} = 45{,}260.$$

Hence the flow is turbulent, and from Eq. 8.60

$$h = \frac{k}{D_h} 0.023\ Re_D^{4/5}\ Pr^{0.4} = \frac{0.0282\ W/m\cdot K}{0.10\ m} 0.023(45{,}260)^{4/5}(0.707)^{0.4} = 30\ W/m^2\cdot K.$$

From Eq. 8.42b, with P = 2(W + H),

$$T_{m,o} = T_s - (T_s - T_{m,i})\exp\left(-\frac{PL}{\dot{m}\ c_p}\overline{h}\right)$$

$$T_{m,o} = 400\ K - (400-285)K\ \exp\left[-\frac{2(0.15m+0.075m)2m\left(30W/m^2\cdot K\right)}{0.1\ kg/s\times1008\ J/kg\cdot K}\right]$$

$$T_{m,o} = 312\ K$$ <

and from Eq. 8.37

$$q = \dot{m}\ c_p\left(T_{m,o} - T_{m,i}\right) = 0.1\ kg/s\times1008\ J/kg\cdot K(312-285)K = 2724\ W.$$ <

COMMENTS: (1) The calculations may be checked by determining q from Eqs. 8.44 and 8.45. We obtain $\Delta T_{\ell m} = 101^\circ C$ and q = 2724 W.

(2) $\overline{T}_m$ has been over-estimated. The calculations should be repeated with properties evaluated at $\overline{T}_m$ = 299 K.

PROBLEM 8.81

KNOWN: Dimensions of semi-circular copper tubes in contact at plane surfaces. Thermal contact resistance. Tube flow conditions.

FIND: (a) Heat rate per unit tube length, and (b) The effect on the heat rate when the fluids are ethylene glycol, the exchanger tube is fabricated from an aluminum alloy, or the exchanger tube thickness is increased.

SCHEMATIC:

$\dot{m}_h = 0.2$ kg/s $= \dot{m}_c$
$T_{h,m} = 330$ K
$T_{c,m} = 290$ K

$r_i = 20$ mm; $R''_{t,c} = 10^{-5}$ m²·K/W; $t = 3$ mm; Insulation; Section AA

ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Adiabatic outer surface, (4) Fully developed flow, (5) Negligible heat loss to surroundings.

PROPERTIES: *Table A.1*, Copper (T ≈ 300 K): k = 400 W/m·K; *Table A.6*, Water (290 K): μ = 1080 × 10^{-6} N·s/m², k = 0.598 W/m·K, Pr = 7.56; (330 K): μ = 489 × 10^{-6} N·s/m², k = 0.65 W/m·K, Pr = 3.15; (given): μ = 800 × 10^{-6} kg/s·m, k = 0.625 W/m·K, Pr = 5.35.

ANALYSIS: (a,b) Heat transfer from the hot to cold fluids is *enhanced* by conduction through the semi-circular portions of the tube walls. The walls may be approximated as straight fins with an insulated tip, and the thermal circuit is shown below.

$T_{c,m}$, $T_{b,c}$, $T_{b,h}$, $T_{h,m}$, L

q'; $T_{c,m}$ — $R'_{f,c}$ ∥ $R'_{conv} = 1/h_c 2r_i$ — $T_{b,c}$ — $R'_{cond} = t/k2r_i$ — T_c — $R''_{t,c}/2r_i$ — T_h — $t/k2r_i$ — $T_{b,h}$ — $R'_{f,h}$ ∥ $1/h_h 2r_i$ — $T_{h,m}$

Note that, since each semi-circular surface is insulated on one side, surfaces may be combined to yield a *single* fin of thickness 2t with convection on both sides. Also, due to the equivalent geometry and the assumption of constant properties, there is symmetry on opposite sides of the contact resistance. From the thermal circuit, the heat rate is

$$q' = \frac{T_{h,m} - T_{c,m}}{R'_{tot}} \tag{1}$$

For flow through the semi-circular tube,

$$Re_D = \frac{\rho u_m D_h}{\mu} = \frac{\dot{m} D_h}{A_c \mu} = \frac{4\dot{m} A_c}{A_c P \mu} = \frac{4\dot{m}}{P\mu} = \frac{4\dot{m}}{(2r_i + \pi r_i)\mu} \tag{2}$$

$$Re_D = \frac{4 \times 0.2\,\text{kg/s}}{(2+\pi)0.02\,\text{m} \times 800 \times 10^{-6}\,\text{kg/s}\cdot\text{m}} = 9725$$

the flow is turbulent. Using the Colburn correlation,

$$Nu_D = 0.023 Re_D^{4/5} Pr^{1/3} = 0.023(9725)^{4/5}(5.35)^{1/3} = 62.4 \tag{3}$$

Continued...

$$D_h = \frac{4A_c}{P} = \frac{4\left(\pi r_i^2/2\right)}{(\pi+2)r_i} = \frac{2\pi}{\pi+2}0.02\,\text{m} = 0.0244\,\text{m} \tag{4}$$

$$h = Nu_D \frac{k}{D_h} = 62.4\frac{0.625}{0.0244} = 1600\,\text{W/m}^2\cdot\text{K}. \tag{5}$$

Find now values for the thermal resistance of the circuit.

$$R'_{conv} = \frac{1}{2r_i h} = \frac{1}{(0.04\,\text{m})1600\,\text{W/m}^2\cdot\text{K}} = 0.0156\,\text{m}\cdot\text{K/W} \tag{6}$$

$$R'_{fin} = \frac{\theta_b}{q'_f} = \frac{1}{\left(hP'kA'_c\right)^{1/2}\tanh\left(hP/kA_c\right)L} \tag{7}$$

$$L = \pi r_i/2 = \pi(0.01\,\text{m}) = 0.0314\,\text{m} \qquad A_c = 2t\cdot 1\,\text{m} = 0.006\,\text{m}^2 \qquad P \approx 2.1\text{ m} \quad (8,9,10)$$

$$\left(hP'kA'_c\right)^{1/2} = \left(1600\,\text{W/m}^2\cdot\text{K}\times 2\,\text{m/m}\times 400\,\text{W/m}\cdot\text{K}\times 0.006\,\text{m}^2/\text{s}\right)^{1/2} = 87.6\,\text{W/K}\cdot\text{m}$$

$$\left(hP/kA_c\right)^{1/2}L = \left(1600\,\text{W/m}^2\cdot\text{K}\times 2\,\text{m}/400\,\text{W/m}\cdot\text{K}\times 0.006\,\text{m}^2\right)^{1/2}0.0314\,\text{m} = 1.15$$

$$R'_{fin} = \frac{1}{87.6\,\text{W/m}\cdot\text{K}\,(0.817)} = 0.0140\,\text{m}\cdot\text{K/W} \tag{11}$$

$$R'_{cond} = \frac{t}{2kr_i} = \frac{0.003\,\text{m}}{2(400\,\text{W/m}\cdot\text{K})(0.02\,\text{m})} = 1.875\times10^{-4}\,\text{m}\cdot\text{K/W} \tag{12}$$

$$R'_{t,c} = \frac{R''_{t,c}}{2r_i} = \frac{10^{-5}\,\text{m}^2\cdot\text{K/W}}{2(0.02\,\text{m})} = 2.5\times10^{-4}\,\text{m}\cdot\text{K/W} \tag{13}$$

The equivalent resistance of the parallel circuit is

$$R'_{eq} = \left(R'^{-1}_{fin} + R'^{-1}_{conv}\right)^{-1} = \left(71.4\,\text{W/m}\cdot\text{K} + 64.1\,\text{W/m}\cdot\text{K}\right)^{-1} = 7.38\times10^{-3}\,\text{m}\cdot\text{K/W} \tag{14}$$

Hence

$$R'_{tot} = 2\left(R'_{eq} + R'_{cond}\right) + R'_{t,c} \tag{15}$$

$$R'_{tot} = \left[2\left(7.38\times10^{-3} + 1.875\times10^{-4}\right) + 2.50\times10^{-4}\right]\text{m}\cdot\text{K/W} = 0.0154\,\text{m}\cdot\text{K/W}$$

$$q' = \frac{(330-290)\text{K}}{0.0154\,\text{m}\cdot\text{K/W}} = 2600\,\text{W/m}. \qquad <$$

(c) Using the *IHT Workspace* with the foregoing equations, analyses were performed and the results summarized in the table below. The “Conditions” are described below; the “Change” is relative to the base case condition.

Continued …..

PROBLEM 8.81 (Cont.)

Condition*	$R'_{conv} \times 10^4$ (m·K/W)	$R'_{fin} \times 10^4$ (m·K/W)	$R'_{cond} \times 10^4$ (m·K/W)	$R'_{tot} \times 10^4$ (m·K/W)	$R'_{eq} \times 10^4$ (m·K/W)	q' (W/m)	Change (%)
Base case	156.8	140.1	1.88	154.2	73.96	2594	--
Ethylene glycol	1382	923.1	1.88	1113.0	553.5	359	-86.2
Aluminum alloy	156.8	183.2	4.24	180.0	84.49	2223	-14.3
Thicker tube	156.8	130.6	2.50	150.0	71.25	2667	+2.8

*Conditions: change from base case

Base case - water, copper (k = 400 W/m·K), t = 3 mm
Ethylene glycol - ethylene glycol instead of water
Aluminum alloy - alloy (k = 177 W/m·K) instead of copper
Thicker tube - t = 4 mm instead of 3 mm

As expected, using ethylene glycol as the working fluid would decrease the heat rate. Note that R'_{conv} is the dominate resistance since the convection coefficient is considerably reduced compared to that with water. Using aluminum alloy, rather than copper, as the tube material reduces the heat rate by 14%. Conduction-convection (fin) in the tube wall is important as can be seen by examining the change in R'_{fin} relative to the base condition. Increasing the tube wall thickness for the copper tube exchanger from 3 to 4 mm had only a marginal positive effect on the heat rate.

COMMENTS: A more accurate calculation would account for the absence of symmetry about the contact plane. Evaluation of water properties at $T_{h,m}$ = 330 K and $T_{c,m}$ = 290 K yields h_h = 2060 W/m^2·K and h_c = W/m^2·K.

PROBLEM 8.82

KNOWN: Heat exchanger to warm blood from a storage temperature 10°C to 37° at 200 ml/min. Tubing has rectangular cross-section 6.4 mm × 1.6 mm sandwiched between plates maintained at 40°C.

FIND: (a) Length of tubing and (b) Assessment of assumptions to indicate whether analysis under- or over-estimates length.

SCHEMATIC:

Blood $\dot{m}$=200mℓ/min; $T_{m,i}$; Plate, T_s=40°C; Tube, cross-section 6.4x1.6mm, length L; $T_{m,o}$; T(°C): 40, 37, 15; ΔT_1, ΔT_2.

ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible kinetic and potential energy changes, (3) Blood flow is fully developed, (4) Blood has properties of water, and (5) Negligible tube wall and contact resistance.

PROPERTIES: *Table A-6*, Water ($\bar{T}_m \approx 300$ K): $c_{p,f} = 4179$ J/kg·K, $\rho_f = 1/v_f = 997$ kg/m^3, $\nu_f = \mu_f v_f = 8.58 \times 10^{-7}$ m^2/s, k = 0.613 W/m·K, Pr = 5.83.

ANALYSIS: (a) From an overall energy balance and the rate equation,

$$q = \dot{m}\, c_p \left(T_{m,o} - T_{m,i}\right) = \bar{h} A_s \Delta T_{LMTD} \qquad (1)$$

where

$$\Delta T_{LMTD} = \frac{\Delta T_1 - \Delta T_2}{\ln\left(\Delta T_1 / \Delta T_2\right)} = \frac{(40-15)-(40-37)}{\ln(25/3)} = 10^\circ\text{C}.$$

To estimate $\bar{h}$, find the Reynolds number for the rectangular tube,

$$Re_D = \frac{u_m D_h}{n} = \frac{0.326 \text{ m/s} \times 0.00256 \text{ m}}{8.58 \times 10^{-7} \text{m}^2/\text{s}} = 973$$

where

$$D_h = 4\, A_c / P = 4(6.4 \text{ mm} \times 1.6 \text{ mm}) / 2(6.4 + 1.6)\text{mm} = 2.56 \text{ mm}$$

$$A_c = (6.4 \text{ mm} \times 1.6 \text{ mm}) = 1.024 \times 10^{-5} \text{m}^2$$

$$u_m = \dot{m}/r A_c = \dot{\forall}/A_c = 200 \text{ m}\ell/60 \text{ s}\left(10^{-6}\text{m}^3/\text{m}\ell\right)/1.024 \times 10^{-5}\text{m}^2 = 0.326 \text{ m/s}.$$

Hence the flow is laminar, but assuming fully developed flow with an isothermal surface from Table 8.1 with b/a = 6.4/1.6 = 4,

$$Nu_D = \frac{hD_h}{k} = 4.4 \qquad h = \frac{4.4 \times 0.613 \text{ W/m} \cdot \text{K}}{0.00256 \text{ m}} = 1054 \text{ W/m}^2 \cdot \text{K}.$$

Continued …..

From Eq. (1) with

$$A_s = PL = 2(6.4 + 1.6)\times10^{-3} m \times L = 1.6\times10^{-2} Lz$$

$$\dot{m} = r A_c u_m = 997 \text{ kg/m}^3 \times 1.024\times10^{-5} \text{ m}^2 \times 0.326 \text{ m/s} = 3.328\times10^{-3} \text{ kg/s}$$

the length of the rectangular tubing can be found as

$$3.328\times10^{-3} \text{ kg/s} \times 4179 \text{ J/kg}\cdot\text{K}(37-15)\text{K} = 1054 \text{ W/m}^2\cdot\text{K}\times1.6\times10^{-2} \text{Lm}^2 \times 10 \text{ K}$$

$$L = 1.8 \text{ m.}$$ <

(b) Consider these comments with regard to whether the analysis under- or over-estimates the length,

- ⇒ fully-developed flow - L/D_h = 1.8 m/0.00256 = 700; not likely to have any effect,
- ⇒ negligible tube wall resistance - depends upon materials of construction; if plastic, analysis under predicts length,
- ⇒ negligible thermal contact resistance between tube and heating plate - if present, analysis under predicts length.

PROBLEM 8.83

KNOWN: Coolant flowing through a rectangular channel (gallery) within the body of a mold.

FIND: Convection coefficient when the coolant is process water or ethylene glycol.

SCHEMATIC:

T_s = 140 °C

a = 90 mm

Coolant

$\dot{V} = 1.3 \times 10^{-3}$ m³/s

$T_{m,i}$ = 15 °C

b = 9.5 mm

ASSUMPTIONS: (1) Gallery can be approximated as a rectangular channel with a uniform surface temperature, (2) Fully developed flow conditions.

PROPERTIES: *Table A.6*, Water ($\overline{T}_m$ = (140 + 15)°C/2 = 350 K): ρ = 974 kg/m³, μ = 365 × 10^{-6} n·s/m², ν = μ/ρ = 3.749 × 10^{-7} m²/s, k = 0.668 W/m·K, Pr = 2.29; *Table A.5*, Ethylene glycol ($\overline{T}_m$ = 350 K): ρ = 1079 kg/m³, ν = 3.17 × 10^{-6} m²/s, k = 0.261 W/m·K, Pr = 34.6.

ANALYSIS: The characteristic length of the channel, the hydraulic diameter, Eq. 8.67, is $D_h = 4A_c/P$ where A_c is the cross-sectional flow area and P is the wetted perimeter. For our channel,

$$D_h = \frac{4(a \times b)}{2(a+b)} = \frac{4 \times 0.090\,\text{m} \times 0.0095\,\text{m}}{2(0.090+0.0095)\,\text{m}} = 0.0172\,\text{m}$$

For the *water* coolant, from the continuity equation, find the Reynolds number to characterize the flow

$$u_m = \frac{\dot{V}}{A_c} = \frac{1.3 \times 10^{-3}\,\text{m}^3/\text{s}}{0.090\,\text{m} \times 0.0095\,\text{m}} = 1.52\,\text{m/s}$$

$$Re_{Dh} = \frac{u_m D_h}{\nu} = \frac{1.52\,\text{m/s} \times 0.0172\,\text{m}}{3.749 \times 10^{-7}\,\text{m}^2/\text{s}} = 69,736$$

Since the flow is turbulent, and assuming fully developed conditions, use the Dittus-Boelter correlation, Eq. 8.60, to estimate the convection coefficient,

$$Nu_{Dh} = \frac{hD_h}{k} = 0.023\,Re_{Dh}^{0.8}\,Pr^{0.4} = 0.023(69,736)^{0.8}(2.29)^{0.4} = 240$$

$$h_w = \frac{0.668\,\text{W/m}\cdot\text{K}}{0.0172\,\text{m}} \times 240 = 9326\,\text{W/m}^2\cdot\text{K} \qquad <$$

Repeating the calculations using properties for the *ethylene glycol* coolant, find

$$Re_{Dh} = 8,247 \qquad Nu_{Dh} = 128 \qquad h_{eg} = 1957\,\text{W/m}^2\cdot\text{K} \qquad <$$

Continued...

COMMENTS: (1) The convection coefficient for the *water* coolant is more than 4 times greater than that with the *ethylene glycol* coolant. The corrosion protection afforded by the latter coolant greatly compromises the thermal performance of the gallery. In such situations, it is useful to explore a compromise between corrosion protection and thermal performance by using an aqueous solution of ethylene glycol (50%-50%, for example).

(2) Recognize that for the ethylene glycol coolant calculation the Reynolds number is slightly below the lower limit of applicability of the Dittus-Boelter correlation.

PROBLEM 8.84

KNOWN: Dimensions, surface temperature and thermal conductivity of a *cold* plate. Velocity, inlet temperature, and properties of coolant.

FIND: (a) Model for determining the heat rate q and outlet temperature, $T_{m,o}$, (b) Values of q and $T_{m,o}$ for prescribed conditions.

SCHEMATIC:

δ = 4mm, $h = w$ = 6 mm
H = 10 mm, W = 100 mm
N = 10, $S = W/N$ = 10 mm
u_m = 2 m/s, T_{mi} = 300 K
T_s = 360 K
k_{cp} = 400 W/m·K

ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible KE, PE and flow work changes, (3) Constant properties, (4) Symmetry about the midplane (horizontal) of the cold plate and the midplane (vertical) of each cooling channel, (5) Negligible heat transfer at sidewalls of cold plate, (6) One-dimensional conduction from outer surface of cold plate to base surface of channel and within the channel side walls, which act as extended surfaces.

PROPERTIES: Water (prescribed): ρ = 984 kg/m^3, c_p = 4184 J/kg·K, $\mu = 489 \times 10^{-6}$ N·s/m^2, k = 0.65 W/m·K, Pr = 3.15.

ANALYSIS: (a) The outlet temperature, $T_{m,o}$, may be determined from the energy balance prescribed by Eq. 8.46b,

$$\frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp\left(-\frac{1}{\dot{m}_1 c_p R_{tot}}\right)$$

where $\dot{m}_1 = \rho u_m A_c$ is the flowrate for a single channel and R_{tot} is the total resistance to heat transfer between the cold plate surface and the coolant for a particular channel. This resistance may be determined from the symmetrical section shown schematically, which represents one-half of the cell associated with a full channel. With the number of channels (and cells) corresponding to N = W/S, there are 2N = 2(W/S) symmetrical sections, and the total resistance R_{tot} of a cell is one-half that of a symmetrical section. Hence, $R_{tot} = R_{ss}/2$, where the resistance of the symmetrical section includes the effect of conduction through the outer wall of the cold plate and convection from the inner surfaces. Hence,

$$R_{ss} = \frac{(H-h)/2}{k_{cp}(SW)} + \frac{1}{\eta_o \bar{h} A_t}$$

where $A_t = A_f + A_b = 2(h/2 \times W) + (w \times W)$, $\bar{h}$ is the average convection coefficient for the channel flow, and η_o is the overall surface efficiency.

$$\eta_o = 1 - \frac{A_f}{A_t}(1 - \eta_f)$$

Continued...

The efficiency η_f corresponds to that of a straight, rectangular fin with an adiabatic tip, Eq. 3.89, and L_c = w/2. With $D_h = 4A_c/P = 4w^2/4w = w = 0.006\,m$, $Re_{D_h} = \rho u_m D_h/\mu$ = 984 kg/m^3 × 2 m/s × 0.006 m/489 × 10^{-6} N·s/m^2 = 24,150 and the channel flow is turbulent. Assuming fully-developed flow throughout the channel, the Dittus-Boelter correlation, Eq. 8.60, may therefore be used to evaluate $\bar{h}$, where

$$\overline{Nu}_D \approx Nu_{D,fd} = 0.023 Re_D^{4/5} Pr^{0.4}$$

The total heat rate for the cold plate may be expressed as

$$q = Nq_1 = N\dot{m}_1 c_p \left(T_{m,o} - T_{m,i}\right)$$

(b) For the prescribed conditions,

$$\dot{m}_1 = \rho u_m A_c = 984\,kg/m^3\,(2\,m/s)(0.006\,m)^2 = 0.0708\,kg/s$$

$$\overline{Nu}_D = 0.023(24{,}150)^{4/5}(3.15)^{0.4} = 116.8$$

$$\bar{h} = 116.8\,k/D_h = 116.8(0.65\,W/m\cdot K)/(0.006\,m) = 12{,}650\,W/m^2\cdot K$$

$$A_f = 2(h/2 \times W) = 2(0.003\,m \times 0.1\,m) = 6\times10^{-4}\,m^2$$

$$A_t = A_f + A_b = 6\times10^{-4}\,m^2 + (0.006\,m \times 0.1\,m) = 1.2\times10^{-3}\,m^2$$

With $m = \left(\bar{h}P_f/k_{cp}A_{cf}\right)^{1/2} = \left[\bar{h}(2\delta + 2W)/k_{cp}(\delta W)\right]^{1/2}$ = [12,650 W/m^2·K(0.008 + 0.200)m/400 W/m·K(0.004 × 0.100)m^2]$^{1/2}$ = 128.2 m^{-1}.

$$\eta_f = \frac{\tanh m(h/2)}{m(h/2)} = \frac{\tanh(128.2\times0.003)}{128.2\times0.003} = \frac{0.366}{0.385} = 0.952$$

$$\eta_o = 1 - 0.5(1 - 0.952) = 0.976$$

$$R_{ss} = \frac{(0.010 - 0.006)\,m/2}{400\,W/m\cdot K\,(0.01\,m\times0.1\,m)} + \frac{1}{0.976\left(12650\,W/m^2\cdot K\right)1.2\times10^{-3}\,m^2}$$

$$R_{ss} = (0.005 + 0.0675)\,K/W = 0.0725\,K/W$$

With $R_{tot} = R_{ss}/2$ = 0.0362 K/W,

$$\frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp\left(-\frac{1}{0.0708\,kg/s \times 4184\,J/kg\cdot K \times 0.0362\,K/W}\right) = 0.911$$

$$T_{m,o} = T_s - 0.911\left(T_s - T_{m,i}\right) = 360\,K - 0.911(360 - 300)\,K = 305.3\,K \qquad <$$

The total heat rate is

$$q = N\dot{m}_1 c_p\left(T_{m,o} - T_{m,i}\right) = 10\times0.0708\,kg/s \times 4184\,J/kg\cdot K\,(305.3 - 300)\,K = 15{,}700\,W \qquad <$$

COMMENTS: The prescribed properties correspond to a value of $\bar{T}_m$ which significantly exceeds that obtained from the foregoing solution ($\bar{T}_m$ = 302.6 K). Hence, the calculations should be repeated using more appropriate thermophysical properties (see next problem). From Eq. 3.85, the effectiveness of the extended surface is

$$\varepsilon = R_{t,b}/R_{t,f} = (\bar{h}\delta W)^{-1}/(\bar{h}A_f\eta_f)^{-1} = (A_f\eta_f/\delta W) = \left(6\times10^{-4}\,m^2 \times 0.954\right)/(0.004\,m \times 0.10\,m) = 1.43.$$

Hence, the ribs are only marginally effective in enhancing heat transfer to the coolant.

PROBLEM 8.85

KNOWN: Geometry, surface temperature and thermal conductivity of a *cold plate*. Velocity and inlet temperature of coolant.

FIND: Effect of channel width on total heat rate.

SCHEMATIC:

δ = 4mm, h = 6 mm
H = 10 mm, W = 100 mm
u_m = 2 m/s, T_{mi} = 300 K
T_s = 360 K
k_{cp} = 400 W/m·K

ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible KE, PE and flow work changes, (3) Constant properties, (4) Symmetry about midplane (horizontal) of the cold plate and the midplane (vertical) of each channel, (5) Negligible heat transfer at sidewalls of cold plate, (6) One-dimensional conduction from outer surface of cold plate to base surface of channel and within the channel side walls, which act as extended surfaces.

PROPERTIES: Water: Evaluated at $\overline{T}_m$ using the *Properties* Toolpad of IHT.

ANALYSIS: The model developed for the preceding problem was entered into the workspace of IHT, with the Dittus-Boelter equation and exponential relation accessed from the *Correlations* Toolpad and modified to account for the hydraulic diameter and the total resistance to heat transfer. Calculations were performed for

Case 1: w = 96 mm, N = 1, S = W = 100 mm
Case 2: w = 46 mm, N = 2, S = 50 mm
Case 3: w = 21 mm, N = 4, S = 25 mm
Case 4: w = 6 mm, N = 10, S = 10 mm
Case 5: w = 1 mm, N = 20, S = 5 mm

and the results are tabulated as follows.

Case	N	D_h (m)	Re_D	$\overline{h}\left(W/m^2\cdot K\right)$	$T_{m,o}$ (K)	q (W)
1	1	0.01129	24,820	8269	300.7	3164
2	2	0.01062	25,320	8895	302.3	10,370
3	4	0.00933	22,340	9142	302.6	10,960
4	10	0.00600	14,620	10,070	304.3	12,940
5	20	0.00171	4761	13,740	317.2	17,160

It is clearly beneficial to increase the number of channels, with the total heat rate increasing by approximately a factor of 5 as N increases from 1 to 20. The heat rate may be increased further by increasing u_m, and hence the flowrate per channel, although an upper limit would be associated with the pressure drop, which would increase with decreasing D_h. Could additional heat transfer enhancement be achieved by altering the thickness δ of the channel walls?

COMMENTS: Note that results obtained for Case 4 differ from those of the preceding problem due to different fluid properties. In this case the properties were evaluated at the actual value of $\overline{T}_m$ = 302.6 K, rather than at an assumed (significantly larger) value.

PROBLEM 8.86

KNOWN: Heat sink with 24 passages for air flow removes power dissipation from circuit board.

FIND: Operating temperature of the board and pressure drop across the sink.

SCHEMATIC:

Insulation

Channel, (24 total), 6mm x 25mm, L = 150 mm

Circuit board, 50W dissipation, T_s

Air flow: $T_{m,i} = 27°C$, $Q = 0.060 m^3/s$

ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible kinetic and potential energy changes, (3) Negligible thermal resistance between the circuit boards and air passages, (4) Sink surface and board are isothermal at T_s.

PROPERTIES: *Table A-4*, Air ($\overline{T} \approx 310$ K,1 atm): $\rho = 1.1281$ kg/m^3, $c_p = 1008$ J/kg·K, $\nu = 16.89 \times 10^{-6}$ m^2/s, k = 0.0270 W/m·K, Pr = 0.706.

ANALYSIS: The air outlet temperature follows from Eq. 8.43,

$$\frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp\left(-\frac{PL\overline{h}}{\dot{m}\, c_p}\right).$$

The mass flow rate for the entire sink is

$$\dot{m} = r\dot{\forall} = 1.1281 \text{ kg/m}^3 \times 0.060 \text{ m}^3/\text{s} = 6.77 \times 10^{-2} \text{ kg/s}$$

and the Reynolds number for a rectangular passage is

$$Re_D = \frac{u_m D_h}{n}$$

where $D_h = 4A_c / P = 4(6 \text{ mm} \times 25 \text{ mm})/2(6+25)\text{mm} = 9.68 \text{ mm}$

$$u_m = \frac{\dot{m}/24}{r A_c} = \frac{6.77 \times 10^{-2} \text{ kg/s}/24}{1.1281 \text{ kg/m}^3 (6 \times 25) \times 10^{-6} \text{m}^2} = 16.7 \text{ m/s}$$

giving $Re_D = \dfrac{16.7 \text{ m/s} \times 9.68 \times 10^{-3} \text{ m}}{16.89 \times 10^{-6} \text{ m}^2/\text{s}} = 9571.$

Assume the flow is turbulent and fully developed and using the Dittus-Boelter correlation find

$$Nu_D = 0.023 Re^{4/5} Pr^{0.4} = 0.023(9571)^{4/5}(0.706)^{0.4} = 30.6$$

$$h = \frac{Nu \cdot k}{D_h} = \frac{30.6 \times 0.027 \text{ W/m} \cdot \text{K}}{0.00968 \text{ m}} = 85.4 \text{ W/m}^2 \cdot \text{K}.$$

Continued …..

From an overall energy balance on the sink,

$$q = \dot{m}\, c_p \left(T_{m,o} - T_{m,i}\right) \qquad T_{m,o} = T_{m,i} + q/\dot{m}\, c_p$$

$$T_{m,o} = 27^\circ C + 50\ W/6.77 \times 10^{-2}\ kg/s \times 1008\ J/kg \cdot K = 27.73^\circ C$$

Hence, the operating temperature of the circuit board for these conditions is

$$\frac{T_s - 27.73}{T_s - 27} = \exp\left[-\frac{2\left(6+25\right)\times 10^{-3}\ m \times 0.150 m \times 85.4\ W/m^2 \cdot K}{\left(6.77\times 10^{-2}\ kg/s/24\right)\times 1008\ J/kg \cdot K}\right]$$

$$T_s = 30^\circ C. \qquad <$$

The pressure drop in the rectangular passage for the smooth surface condition follows from Eqs. 8.22 and 8.20

$$\Delta p = f\, \frac{r\ u_m^2}{D_h} L$$

where

$$f = 0.31 Re_D^{1/4} = 0.316\left(9554\right)^{-1/4} = 0.0320.$$

$$\Delta p = 0.0320 \frac{1.1281\ kg/m^3 \left(16.7\ m/s\right)^2}{0.00968\ m} \times 0.150\ m = 156\ N/m^2. \qquad <$$

COMMENTS: (1) The analysis has been simplified by assuming the channel is rectangular and all four sides experience heat transfer. Since the insulated surface is a small portion of the total passage surface area, the effect can't be very large.

(2) The power required to move the air through the heat sink is $P_{elec} = \dot{\forall}\, \Delta p = 0.060\ m^3/s \times 156\ N/m^2 = 9.4\ W$.

PROBLEM 8.87

KNOWN: Channel formed between metallic blades dissipating heat by internal volumetric generation.

FIND: (a) The heat removal rate per blade for the prescribed thermal conditions and (b) Time required to slow a train of mass 10^6 kg from 120 km/h to 50 km/h.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions for channel blades and air flow, (2) The blades form a channel of rectangular (a × b) cross-section and length L, (3) Negligible kinetic energy changes and axial conduction inside the channel, and (4) Fully developed flow conditions in the channel.

PROPERTIES: *Table A.4*, Air ($\overline{T}_m \approx 350$ K, 1 atm): ρ = 0.995 kg/m^3, c_p = 1009 J/kg ·K, ν = 20.92 × 10^{-6} m^2/s, k = 0.030 W/m·K, Pr = 0.700.

ANALYSIS: (a) The heat removal rate by the air from a single channel (one blade) follows from an overall energy balance,

$$q = \dot{m}c_p\left(T_{m,o} - T_{m,i}\right) \tag{1}$$

where the outlet temperature can be determined from Eq. 8.42b,

$$\frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp\left(-\frac{PL}{\dot{m}c_p}\overline{h}\right) \tag{2}$$

The hydraulic diameter, D_h, flows from Eq. 8.67,

$$D_h = \frac{4A_c}{P} = \frac{4(a \times b)}{2(a+b)} = \frac{4(0.220 \times 0.004)\,m^2}{2(0.220 + 0.004)\,m} = 0.0079\,m \tag{3}$$

Assuming $\overline{T}_m = 350K$, the Reynolds number is

$$Re_{Dh} = \frac{u_m D_h}{\nu} = \frac{50\,m/s \times 0.0079m}{20.92 \times 10^{-6}\,m^2/s} = 18,779 \tag{4}$$

Using the Dittus-Boelter correlation, Eq. 8.60,

$$Nu_{Dh} = \frac{\overline{h}D_h}{k} = 0.023\,Re_{Dh}^{0.8}\,P_r^{0.4} = 0.023(18,779)^{0.8}(0.700)^{0.4} = 52.37 \tag{5}$$

Continued...

$$\bar{h} = \frac{0.030\,\mathrm{W/m \cdot K}}{0.0079\mathrm{m}} \times 52.37 = 199\,\mathrm{W/m^2 \cdot K}$$

Hence, the outlet temperature is

$$\frac{600 - T_{m,o}}{(600-25)^\circ\mathrm{C}} = \exp\left(\frac{2(0.220+0.004)\mathrm{m} \times 0.070\mathrm{m}}{0.0438\,\mathrm{kg/s} \times 1009\,\mathrm{J/kg \cdot K}} 199\,\mathrm{W/m^2 \cdot K}\right)$$

$$T_{m,o} = 100.7^\circ\mathrm{C}$$

where

$$\dot{m} = \rho A_c u_m = 0.995\ \mathrm{kg/m^3} \times (0.220 \times 0.004)\ \mathrm{m^2} \times 50\ \mathrm{m/s} = 0.0438\ \mathrm{kg/s}$$

and the rate of heat removal per blade, from Eq. (1), is

$$q = 0.0438\,\mathrm{kg/s} \times 1009\,\mathrm{J/kg \cdot K}\,(100.7 - 25)^\circ\mathrm{C} = 3.346\,\mathrm{kW}$$ <

(b) From an energy balance on the locomotive over an interval of time, Δt, the heat energy transferred to the air stream results in a change in kinetic energy of the train,

$$-E_{out} = \Delta E = KE_f - KE_i \quad (6)$$

$$-(q \times N) \times \Delta t = \frac{1}{2} M \left(V_f^2 - V_i^2\right)$$

$$-3346\,\mathrm{W/blade} \times 2000\mathrm{blades} \times \Delta t\,(\mathrm{s}) = \frac{1}{2} \times 10^{-6}\mathrm{kg}\left[\left(\frac{50,000}{3600}\right)^2 - \left(\frac{120,000}{3600}\right)^2\right]\mathrm{m^2/s^2}$$

$$\Delta t = 69\mathrm{s}$$ <

COMMENTS: (1) For the channel, L/D_h = 0.070 m/0.0079 m = 8.9 < 10 so that the assumption of fully developed conditions may not be satisfied. Recognize that the flow at the channel entrance may be highly turbulent because of the upstream fan swirl and ducting.

(2) What benefits could be realized by increasing the heat transfer coefficient? Aside from increasing velocity, what design changes would you make to increase h?

(3) Our assumption for $\bar{T}_m$ = 350 K at which to evaluate properties is reasonable considering T_m = (100.7 + 25)°C/2 = 335 K.

PROBLEM 8.88

KNOWN: Chip and cooling channel dimensions. Channel flowrate and inlet temperature. Chip temperature.

FIND: Water outlet temperature and chip power.

SCHEMATIC:

$L = 10 mm$

$W = 50 \mu m$

$T_s = 350K$

$H = 200 \mu m$

$L = 10 mm$

$T_{m,i} = 290K, \dot{m}_1 = 10^{-4} kg/s$

$N = 50$ channels

ASSUMPTIONS: (1) Negligible kinetic and potential energy changes for water, (2) Uniform channel surface temperature, (3) $\overline{T}_m$ = 300 K, (4) Fully developed flow.

PROPERTIES: *Table A-6*, Water ($\overline{T}_m$ = 300 K): c_p = 4179 J/kg·K, μ = 855 × 10^{-6} kg/s·m, k = 0.613 W/m·K, Pr = 583.

ANALYSIS: Using the hydraulic diameter, find the Reynolds number,

$$D_h = \frac{4(H \times W)}{2(H+W)} = \frac{2(50 \times 200)mm^2}{250\ mm} 10^{-6}\ m/mm = 8 \times 10^{-5}\ m$$

$$Re_D = \frac{r\ u_m D_h}{m} = \frac{\dot{m}_1 D_h}{A_c m} = \frac{10^{-4}\ kg/s\left(8 \times 10^{-5}\ m\right)}{(50 \times 200)10^{-12}\ m^2\left(855 \times 10^{-6}\ kg/s \cdot m\right)} = 936.$$

Hence, the flow is laminar and, from Table 8.1, Nu_D = 4.44, so that

$$h = Nu_D \frac{k}{D_h} = \frac{4.44(0.613\ W/m \cdot K)}{8 \times 10^{-5}\ m} = 34{,}022\ W/m^2 \cdot K.$$

With P = 2(H + W) = 2(250 μm) 10^{-6} m/μm = 5 ×10^{-4} m, Eq. 8.42b yields

$$\frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \frac{350K - T_{m,o}}{60\ K} = \exp\left(-\frac{PL}{\dot{m}_1\ c_p} h\right) = \exp\left(-\frac{5 \times 10^{-6}\ m^2 \times 34{,}022\ W/m^2 \cdot K}{10^{-4}\ kg/s \times 4179\ J/kg \cdot K}\right)$$

$$T_{m,o} = 350K - 60\ K \exp(-0.407) = 310\ K. \quad <$$

Hence, from Eq. 8.37,

$$q = \dot{m}\ c_p\left(T_{m,o} - T_{m,i}\right) = N\dot{m}_1 c_p\left(T_{m,o} - T_{m,i}\right) = 50 \times 10^{-4}\ kg/s(4179\ J/kg \cdot K)(20\ K) = 418\ W. \quad <$$

COMMENTS: (1) The chip heat flux of 418 W/cm^2 is extremely large and the method provides a very efficient means of heat removal from high power chips. However, clogging of the microchannels is a potential problem which could seriously compromise reliability. (2) L/D_h = 125 and 0.05 $Re_D Pr$ = 272. Hence, fully developed conditions are not realized and $\overline{h}$ > 34,022. The actual power dissipation is therefore greater than 418 W.

PROBLEM 8.89

KNOWN: Chip and cooling channel dimensions. Channel flow rate and inlet temperature. Temperature of chip at base of channel.

FIND: (a) Water outlet temperature and chip power, (b) Effect of channel width and pitch on power dissipation.

SCHEMATIC:

ASSUMPTIONS: (1) Negligible flow work and kinetic and potential energy changes for water, (2) Flow may be approximated as fully developed and channel walls as isothermal for purposes of estimating the convection coefficient, (3) One-dimensional conduction along channel side walls, (4) Adiabatic condition at end of side walls, (5) Heat dissipation is exclusively through fluid flow in channels, (6) Constant properties.

PROPERTIES: *Table A-6*, Water ($\overline{T}_m$ = 300K): c_p = 4179 J/kg·K, $\mu = 855 \times 10^{-6}$ kg/s·m, k = 0.613 W/m·K, Pr = 5.83.

ANALYSIS: (a) The channel sidewalls act as fins, and a *unit* channel/sidewall combination is shown in schematic (a), where the total number of unit cells corresponds to N = L/S. With N = 50 and L = 10 mm, S = 200 μm and δ = S – W = 150 μm. Alternatively, the unit cell may be represented in terms of a single fin of thickness δ, as shown in schematic (b). The thermal resistance of the unit cell may be obtained from the expression for a fin array, Eq. (3.103), $R_{t,o} = (\eta_o h A_t)^{-1}$, where $A_t = A_f + A_b$ = L (2 H + W) = 0.01m $(4 \times 10^{-4} + 0.5 \times 10^{-4})$ m = 4.5×10^{-6} m^2. With D_h = 4 (H × W)/2 (H + W) = 4 (2 × 10^{-4}m × 0.5 × 10^{-4}m)/2 (2.5 × 10^{-4}m) = 8 × 10^{-5}m, the Reynolds number is $Re_D = \rho u_m D_h/\mu = \dot{m}_1 D_h/A_c\mu$ = 10^{-4} kg/s × 8 × 10^{-5}m/(2 × 10^{-4}m × 0.5 × 10^{-4}m) 855 × 10^{-6} kg/s·m = 936. Hence, the flow is laminar, and assuming fully developed conditions throughout a channel with uniform surface temperature, Table 8.1 yields Nu_D = 4.44. Hence,

$$h = \frac{k}{D_h} Nu_D = \frac{0.613\,W/m \cdot K \times 4.44}{8 \times 10^{-5} m} = 34,022\,W/m^2 \cdot K$$

With $m = (2h/k_{ch}\delta)^{1/2} = (68,044\ W/m^2\cdot K/140\ W/m\cdot K \times 1.5 \times 10^{-4} m)^{1/2} = 1800\ m^{-1}$ and mH = 0.36, the fin efficiency is

$$\eta_f = \frac{\tanh\ mH}{mH} = \frac{0.345}{0.36} = 0.958$$

and the overall surface efficiency is

$$\eta_o = 1 - \frac{A_f}{A_t}(1 - \eta_f) = 1 - \frac{4.0 \times 10^{-6}}{4.5 \times 10^{-6}}(1 - 0.958) = 0.963$$

The thermal resistance of the unit cell is then

Continued …..

$$R_{t,o} = (\eta_o h A_t)^{-1} = \left(0.963 \times 34{,}022\,W/m^2 \cdot K \times 4.5 \times 10^{-6} m^2\right)^{-1} = 6.78\,K/W$$

The outlet temperature follows from Eq. (8.46b),

$$T_{m,o} = T_s - \left(T_s - T_{m,i}\right)\exp\left(-\frac{1}{\dot{m}\,c_p\ R_{t,o}}\right) = 350K - (60K) \times$$

$$\exp\left(-\frac{1}{10^{-4} kg/s \times 4179\,J/kg \cdot K \times 6.78\,K/W}\right) = 307.8K$$ <

The heat rate per channel is then

$$q_1 = \dot{m}_1\, c_p \left(T_{m,o} - T_{m,i}\right) = 10^{-4} kg/s \times 4179\,J/kg \cdot K\,(17.8K) = 7.46\,W$$

and the chip power dissipation is

$$q = Nq_1 = 50 \times 7.46\,W = 373\,W$$ <

(b) The foregoing result indicates significant heat transfer from the channel side walls due to the large value of η_f. If the pitch is reduced by a factor of 2 (S = 100 μm), we obtain

$$S = 100\mu m,\ W = 50\mu m,\ \delta = 50\mu m,\ N = 100 : q_1 = 7.04\,W,\ q = 704\,W$$ <

Hence, although there is a reduction in η_f due to the reduction in δ (η_f = 0.89) and therefore a slight reduction in the value of q_l, the effect is more than compensated by the increase in the number of channels. Additional benefit may be derived by further reducing the pitch to whatever minimum value of δ is imposed by manufacturing or structural limitations. There would also be an advantage to increasing the channel hydraulic diameter and or flowrate, such that turbulent flow is achieved with a correspondingly larger value of h.

COMMENTS: (1) Because electronic devices fail by contact with a polar fluid such as water, great care would have to be taken to hermetically seal the devices from the coolant channels. In lieu of water, a dielectric fluid could be used, thereby permitting contact between the fluid and the electronics. However, all such fluids, such as air, are less effective as coolants. (2) With L/D_h = 125 and $L/D_h)_{fd} \approx 0.05\, Re_D\, Pr$ = 273, fully developed flow is not achieved and the value of $h = h_{fd}$ underestimates the actual value of $\bar{h}$ in the channel. The coefficient is also underestimated by using a Nusselt number that presumes heat transfer from all four (rather than three) surfaces of a channel.

PROBLEM 8.90

KNOWN: Chip and cooling channel dimensions. Channel flow rate and inlet temperature. Temperature of chip at base of channel.

FIND: (a) Outlet temperature and chip power dissipation for dielectric liquid, (b) Outlet temperature and chip power dissipation for air.

SCHEMATIC:

ASSUMPTIONS: (1) Negligible flow work and kinetic and potential energy changes, (2) Flow may be approximated as fully developed and channel walls as isothermal for purposes of estimating the convection coefficient, (3) One-dimensional conduction along the channel side walls, (4) Adiabatic condition at end of side walls, (5) Heat dissipation is exclusively through fluid flow in channels, (6) Constant properties.

PROPERTIES: Prescribed. Dielectric liquid: c_p = 1050 J/kg·K, k = 0.065 W/m·K, μ = 0.0012 N·s/m^2, Pr = 15. Air: c_p = 1007 J/kg·K, k = 0.0263 W/m·K, $\mu = 185 \times 10^{-7}$ N·s/m^2, Pr = 0.707.

ANALYSIS: (a) The channel side walls act as fins, and a *unit* channel/sidewall combination is shown in schematic (a), where δ = S – W = 150 μm. Alternatively, the unit cell may be represented in terms of a single fin of thickness δ, as shown in schematic (b). The thermal resistance of the unit cell may be obtained from the expression for a fin array, Eq. (3.103), $R_{t,o} = (\eta_o\, h\, A_t)^{-1}$, where $A_t = A_f + A_b = L\,(2\,H + W) = 4.5 \times 10^{-6}$ m^2. With $A_c = H \times W = 10^{-8}$ m^2 and $D_h = 4\,A_c/2(H + W) = 8 \times 10^{-5}$m, the Reynolds number is $Re_D = \rho u_m D_h/\mu = \dot{m}_1\, D_h/A_c\mu = 667$. Hence, the flow is laminar, and assuming fully developed conditions throughout a channel with uniform surface temperature, Table 8.1 yields $Nu_D = 4.44$. Hence, $h = \frac{k}{D_h} Nu_D = \frac{0.065\,W/m\cdot K \times 4.44}{8 \times 10^{-5} m} = 3608\,W/m^2\cdot K$

With $m = (2\,h/k_{ch}\,\delta)^{1/2} = 586$ m^{-1} and mH = 0.117, the fin efficiency is

$$\eta_f = \frac{\tanh mH}{mH} = \frac{0.1167}{0.117} = 0.995$$

and the overall surface efficiency is

$$\eta_o = 1 - \frac{A_f}{A_t}(1 - \eta_f) = 1 - \frac{4.0 \times 10^{-6}}{4.5 \times 10^{-6}}(1 - 0.995) = 0.996.$$

The thermal resistance of the unit cell is then

$$R_{t,o} = (\eta_o h A_t)^{-1} = \left(0.996 \times 3608\,W/m^2\cdot K \times 4.5 \times 10^{-6} m^2\right)^{-1} = 61.9\,K/W$$

The outlet temperature follows from Eq. (8.46b),

$$T_{m,o} = T_s - (T_s - T_{m,i})\exp\left(-\frac{1}{\dot{m}_1 c_p R_{t,o}}\right) = 350K$$

Continued …..

$$-(60\text{K})\exp\left(-\frac{1}{10^{-4}\,\text{kg/s}\times 1050\,\text{J/kg}\cdot\text{K}\times 61.9\,\text{K/W}}\right)=298.6\text{K} \qquad <$$

The heat rate per channel is then

$$q_1 = \dot{m}_1 c_p \left(T_{m,o} - T_{m,i}\right) = 10^{-4}\,\text{kg/s}\times 1050\,\text{J/kg}\cdot\text{K}\times 8.6\,\text{K} = 0.899\,\text{W}$$

and the chip power dissipation is

$$q = Nq_1 = 50\times 0.899\,\text{W} = 45.0\,\text{W} \qquad <$$

(b) With $\dot{m}_1 = 10^{-6}$ kg/s, $Re_D = \dot{m}_1 D_h / A_c\mu = 432$ and the flow is laminar. Hence, with $Nu_D = 4.44$,

$$h = \frac{k}{D_h} Nu_D = \frac{0.0263\,\text{W/m}\cdot\text{K}\times 4.44}{8\times 10^{-5}\text{m}} = 1460\,\text{W/m}^2\cdot\text{K}$$

With $m = (2\,h/k_{ch}\delta)^{1/2} = 373\ \text{m}^{-1}$ and $mH = 0.0746$, the fin efficiency is

$$\eta_f = \frac{\tanh mH}{mH} = \frac{0.0744}{0.0746} = 0.998$$

and the overall surface efficiency is

$$\eta_o = 1 - \frac{A_f}{A_t}(1-\eta_f) = 1 - \frac{4.0\times 10^{-6}}{4.5\times 10^{-6}}(1-0.998) = 0.998$$

Hence, $$R_{t,o} = (\eta_o h A_t)^{-1} = \left(0.998\times 1460\,\text{W/m}^2\cdot\text{K}\times 4.5\times 10^{-6}\text{m}^2\right)^{-1} = 153\,\text{K/W}$$

The outlet temperature is then

$$T_{m,o} = T_s - \left(T_s - T_{m,i}\right)\exp\left(-\frac{1}{\dot{m}_1 c_p R_{t,o}}\right) = 350\text{K}$$

$$-(60\text{K})\exp\left(-\frac{1}{10^{-6}\,\text{kg/s}\times 1007\,\text{J/kg}\cdot\text{K}\times 153\,\text{K/W}}\right) = 349.9\,\text{K} \qquad <$$

$$q_1 = \dot{m}_1 c_p \left(T_{m,o} - T_{m,i}\right) = 10^{-6}\,\text{kg/s}\times 1007\,\text{J/kg}\cdot\text{K}\times 59.9\,\text{K} = 0.060\,\text{W}$$

$$q = Nq_1 = 3.02\,\text{W} \qquad <$$

COMMENTS: (1) For laminar flow in the channels, there is a clear advantage to using the dielectric liquid instead of air. (2) The prescribed channel geometry is by no means optimized, and the number of fins should be increased by reducing S. Also, channel dimensions and/or flow rates could be increased to achieve turbulent flow and hence much larger values of h. (3) With $L/D_h = 125$ and $L/Dh)_{fd} \approx 0.05\, Re_D\, Pr = 500$ for the dielectric liquid, fully developed flow is not achieved and its assumption yields a conservative (under) estimate of the convection coefficient. The coefficient is also underestimated by using a Nusselt number that presumes heat transfer from all four (rather than three) surfaces of a channel.

PROBLEM 8.91

KNOWN: Arrangement of chips and cooling channels for a substrate. Contact and conduction resistances. Coolant velocity and inlet temperature.

FIND: (a) Coolant temperature rise, (b) Chip and substrate temperatures.

SCHEMATIC:

ASSUMPTIONS: (1) Constant properties, (2) Fully-developed flow, (3) Negligible kinetic and potential energy changes, (4) Heat transfer exclusively to water, (5) Steady-state conditions.

PROPERTIES: Water (given): ρ = 1000 kg/m^3, c_p = 4180 J/kg·K, k = 0.610 W/m·K, Pr = 5.8, μ = 855×10^{-6} kg/s·m.

ANALYSIS: (a) For a single flow channel, the overall energy balance yields

$$T_{m,o} - T_{m,i} = \frac{q}{\dot{m}\, c_p} = \frac{N_L P}{\rho\, u_m A_c c_p} = \frac{10 \times 5\text{ W}}{1000\text{ kg/m}^3 (1\text{ m/s})(0.005\text{ m})^2\ 4180\text{ J/kg}\cdot\text{K}} = 0.48^\circ\text{C}.$$ <

From the thermal circuit,

$$q = \frac{T_o - \overline{T}_m}{R_{t,c} + R_{cond} + R_{conv}} \qquad R_{t,c} = R''_{t,c}/A_s = \left(0.5\times10^{-4}\text{ m}^2\cdot\text{K/W}\right)/10(0.005\text{ m})^2 = 0.2\text{ K/W}.$$

With $D_h = 4A_c/P = 4(0.005\text{ m})^2/4(0.005\text{ m}) = 0.005$ m,

$$Re_D = \frac{\rho\, u_m D_h}{\mu} = \frac{1000\text{ kg/m}^3(1\text{ m/s})\,0.005\text{ m}}{855\times10^{-6}\text{ kg/s}\cdot\text{m}} = 5848.$$

With turbulent flow, the Dittus-Boelter correlation yields

$$h = \frac{k}{D} 0.023 Re_D^{4/5} Pr^{0.4} = \left(\frac{0.61\text{ W/m}\cdot\text{K}}{0.005\text{ m}}\right) 0.023(5848)^{4/5}(5.8)^{0.4} = 5849\text{ W/m}^2\cdot\text{K}$$

$$R_{conv} = (hA_s)^{-1} = \left(5849\text{ W/m}^2\cdot\text{K}\times 4\times0.005\text{ m}\times0.2\text{ m}\right)^{-1} = 0.043\text{ K/W}.$$

Approximating T_m as $(T_{m,i} + T_{m,o})/2$ = 25.24°C,

$$\overline{T}_c = \overline{T}_m + q\left(R_{t,c} + R_{cond} + R_{conv}\right) = 25.24^\circ\text{C} + 50\text{ W}(0.2 + 0.12 + 0.043)\text{K/W} = 43.3^\circ\text{C}.$$ <

Similarly, from the thermal circuits,

$$\overline{T}_s = \overline{T}_m + q\times R_{conv} = 25.24^\circ\text{C} + 50\text{W}\times0.043\text{K/W} = 27.4^\circ\text{C}$$ <

COMMENTS: (1) Since the coolant temperature rise is less than 0.5°C, all chip temperatures will be within 0.5°C of each other. (2) The channel surface temperature may also be obtained from Eq. 8.42b, yielding the same result.

PROBLEM 8.92

KNOWN: Power dissipation of components on each side of a hollow core PCB. Dimensions of PCB. Inlet temperature and flow rate of air.

FIND: Outlet air temperature and inlet and outlet surface temperatures for prescribed flow rates.

SCHEMATIC:

ASSUMPTIONS: (1) Steady flow, (2) Negligible flow work and potential and kinetic energy changes, (3) Channel may be approximated as infinite parallel plates, (4) Uniform surface heat flux, (5) Fully developed flow at exit, (6) Constant properties.

PROPERTIES: *Table A-4*, Air ($\overline{T}_m \approx 310K$): $\rho = 1.128$ kg/m^3, $c_p = 1007$ J/kg·K, $\mu = 189.3 \times 10^{-7}$ N·s/m^2, k = 0.0270 W/m·K, Pr = 0.706.

ANALYSIS: Performing an energy balance for a control surface about the hollow core, $2q = \dot{m} c_p \left(T_{m,o} - T_{m,i}\right)$, in which case

$$T_{m,o} = \frac{2q}{\dot{m} c_p} + T_{m,i} = \frac{80\,W}{0.002\,kg/s \times 1007\,J/kg \cdot K} + 20°C = 59.7°C \qquad <$$

The surface temperatures may be obtained from Newton's law of cooling, $q_s'' = h\left(T_s - T_m\right)$. Hence, with $h \to \infty$ at the entrance, where the thermal boundary layer thickness is zero,

$$T_{s,i} = T_{m,i} = 20°C \qquad <$$

With $Re_D = \rho u_m D_h/\mu = \dot{m} D_h/A_c \mu$, where $D_h = 2H = 0.008$m and $A_c = H \times W = 0.004m \times 0.3m = 0.0012m^2$, $Re_D = (0.002\ kg/s \times 0.008m)/(0.0012m^2 \times 189.3 \times 10^{-7}\ N \cdot s/m^2) = 704$ and the flow is laminar. With a uniform surface heat flux, $q_s'' = q/(W \times L) = 40\ W/(0.3m)^2 = 444\ W/m^2$, Table 8.3 yields $Nu_D = 8.23$. Hence,

$$h = \frac{Nu_D\,k}{D_h} = \frac{8.23 \times 0.027\,W/m \cdot K}{0.008m} = 27.8\,W/m^2 \cdot K$$

$$T_{s,o} = T_{m,o} + \frac{q_s''}{h} = 59.7°C + \frac{444\,W/m^2}{27.8\,W/m^2 \cdot K} = 75.7°C \qquad <$$

If the flowrate is increased by a factor of 5,

$$T_{m,o} = \frac{2q}{\dot{m} c_p} + T_{m,i} = \frac{80\,W}{0.01 kg/s \times 1007\,J/kg \cdot K} + 20°C = 27.9°C \qquad <$$

The surface temperature at the inlet is unchanged,

Continued …..

$$T_{s,i} = 20°C \qquad <$$

but with Re_D = 3520, flow in the channel is now turbulent. Using Eq. (8.60) as a first approximation,

$$h = \left(\frac{k}{D_h}\right)0.023\,Re_D^{4/5}\,Pr^{0.4} = \left(\frac{0.027\,W/m\cdot K}{0.008m}\right)0.023(3520)^{4/5}(0.706)^{0.4} = 46.4\,W/m^2\cdot K$$

$$T_{s,o} = T_{m,o} + \frac{q_s''}{h} = 27.9°C + \frac{444\,W/m^2}{46.4\,W/m^2\cdot K} = 37.5°C \qquad <$$

COMMENTS: (1) With L/D_h = 37.5 and $L/D_h)_{fd} \approx 0.05\,Re_D\,Pr = 25$ for the laminar flow, it is reasonable to assume fully developed conditions at the exit. The same may be said for the turbulent flow condition. (2) The temperature difference, $T_s - T_m$, increases from approximately 0 at the entrance to a maximum value associated with fully developed conditions.

PROBLEM 8.93

KNOWN: Printed-circuit board (PCB) with uniform temperature T_s cooled by laminar, fully developed flow in a parallel-plate channel. The air flow with an inlet temperature of $T_{m,i}$ is driven by a pressure difference, Δp.

FIND: The average heat removal rate per unit area, $\bar{q}_s''\left(W/m^2\right)$, from the PCB.

SCHEMATIC:

Air

PCB, $T_s = 65^oC$, $\bar{q}_s''$

Insulation

a = 10 mm

$T_{m,i} = 20^oC$
$\Delta p = 2\ N/m^2$

$T_{m,o}$

x

L = 150 mm

ASSUMPTIONS: (1) Laminar, fully developed flow, (2) Upper and lower walls of the channel are insulated and of infinite extent in the transverse direction, (3) PCB has uniform surface temperature, (4) Constant properties, (5) Negligible kinetic and potential energy changes and flow work.

PROPERTIES: *Table A-4*, Air (T_m = 293 K, 1 atm): $\rho = 1.192\ kg/m^3$, c_p = 1007 J/kg·K, $\nu = 1.531 \times 10^{-5}\ m^2/s$, k = 0.0258 W/m·K, Pr = 0.709.

ANALYSIS: The energy equations for determining the heat rate from one surface of the board are Eqs. 8.37 and 8.42b

$$q = \dot{m}\, c_p \left(T_{m,o} - T_{m,i}\right) = \bar{q}_s''\, A_s \tag{1}$$

$$\frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp\left(-\frac{PL\,\bar{h}}{\dot{m}\, c_p}\right) \tag{2}$$

where A_s = Lw and P = 2(w + a) where w is the width in the transverse direction. For the fully developed flow condition, the velocity is estimated from the friction pressure drop relation, Eq. 8.22a,

$$\Delta p = f\left(\rho u_m^2/2\right)\left(L/D_h\right) \tag{3}$$

where the hydraulic diameter for the channel cross section is

$$D_h = \frac{4A_c}{P} = \frac{4(w\,a)}{2(w+a)} = 2a \qquad\qquad a \ll w$$

The friction factor f from Table 8.1 for the cross section b/a = ∞ is

$$f \cdot Re_{Dh} = 96 \tag{4}$$

where the Reynolds number is

$$Re_{Dh} = u_m D_h/\nu \tag{5}$$

Continued …..

and the flow rate through one channel is

$$\dot{m} = \rho A_c u_m = \rho (wa) u_m \quad (6)$$

For fully developed laminar flow from Table 8.1.

$$\overline{Nu}_D = \bar{h} D_h / k = 7.54 \quad (7)$$

The above system of equations needs to be solved simultaneously for the unknowns: q, $\dot{m}$, $T_{m,o}$, Re_{Dh}, f, u_m, $\bar{h}$. Using *IHT* with w = 1 m, find these results:

u_m (m/s)	Re_{Dh}	f	$\bar{h}$ (W/m^2·K)	$T_{m,o}$ (°C)	q (W)	$\bar{q}_s''$ (W/m^2)
6.0	7578	0.0127	9.9	20.9	66	442

From Eq. (1), the average heat removal per unit area of the board is

$$\bar{q}_s'' = 442 \text{ W/m}^2$$ <

COMMENTS: (1) The thermophysical properties of the air are evaluated at the average mean temperature, $T_m = (T_{m,i} + T_{m,o})/2$.

(2) The fully developed flow length, $x_{fd,t}$, for the channel follows from Eq. 8.23,

$$x_{fd,t} = D_h \times 0.05 Re_{Dh} Pr$$

$$x_{fd,t} = 2 \times 0.010 \text{ m} \times 0.05 \times 7578 \times 0.707 = 5.4 \text{ m}$$

Since $L << x_{fd,t}$, we conclude that the flow is not likely to be fully developed.

(3) Recognize also that the Reynolds number is larger than the critical value indicating that appreciable turbulence could be present. Considering that $L << x_{fd,t}$ and $Re_{Dh} > 2300$, do you conclude that our estimate for the average heat flux is a conservative or an optimistic one?

PROBLEM 8.94

KNOWN: Inner and outer tube surface conditions for an annulus.

FIND: (a) Velocity profile, (b) Temperature profile and expression for inner surface Nusselt number.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Laminar, fully developed flow, (3) Uniform heat flux at inner surface, (4) Adiabatic outer surface, (5) Constant properties, (6) Negligible potential and kinetic energy changes, flow work, and viscous dissipation.

ANALYSIS: (a) From Section 8.1.3, the general solution to Eq. 8.12, which also applies to annular flow as represented in Figure 8.10, is

$$u(r) = \frac{1}{\mu}\left(\frac{dp}{dx}\right)\frac{r^2}{4} + C_1 \ell n\, r + C_2.$$

Applying the boundary conditions,

$$u(r_i) = 0 \qquad 0 = \frac{1}{\mu}\left(\frac{dp}{dx}\right)\frac{r_i^2}{4} + C_1 \ell n r_i + C_2$$

$$u(r_o) = 0 \qquad 0 = \frac{1}{\mu}\left(\frac{dp}{dx}\right)\frac{r_o^2}{4} + C_1 \ell n r_o + C_2.$$

Hence,

$$C_1 = \frac{\frac{1}{\mu}\left(\frac{dp}{dx}\right)\left(\frac{r_o^2}{4} - \frac{r_i^2}{4}\right)}{\ell n\, r_i / r_o} \qquad C_2 = -\frac{1}{\mu}\left(\frac{dp}{dx}\right)\frac{r_o^2}{4} - \frac{1}{\mu}\left(\frac{dp}{dx}\right)\left(\frac{r_o^2}{4} - \frac{r_i^2}{4}\right)\frac{\ell n\, r_o}{\ell n\,(r_i / r_o)}$$

and the velocity distribution is

$$u(r) = \frac{1}{\mu}\left(\frac{dp}{dx}\right)\left(\frac{r^2}{4} - \frac{r_o^2}{4}\right) + \frac{1}{\mu}\left(\frac{dp}{dx}\right)\left(\frac{r_o^2}{4} - \frac{r_i^2}{4}\right)\frac{\ell n\, r}{\ell n\,(r_i / r_o)}$$

$$- \frac{1}{\mu}\left(\frac{dp}{dx}\right)\left(\frac{r_o^2}{4} - \frac{r_i^2}{4}\right)\frac{\ell n\, r_o}{\ell n\,(r_i / r_o)}$$

$$u(r) = -\frac{r_o^2}{4\mu}\left(\frac{dp}{dx}\right)\left[1 - (r/r_o)^2 + \frac{(r_i / r_o)^2 - 1}{\ell n\,(r_i / r_o)}\ell n\,(r/r_o)\right]. \qquad (1)$$ <

(b) For fully developed conditions with uniform surface heat flux,

$$v = 0 \qquad \partial T / \partial x = dT_m / dx = \text{const.}$$

Continued …..

Hence, from Eq. 8.48, which also applies for laminar flow,

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)=\frac{u}{\alpha}\frac{dT_m}{dx}.k$$

Substituting the velocity distribution, with

$$C_1=-\frac{r_o^2}{4\mu}\left(\frac{dp}{dx}\right) \qquad C_2=\frac{\left(r_i/r_o\right)^2-1}{\ell n\left(r_i/r_o\right)} \tag{2}$$

it follows that $\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)=\frac{C_1}{\alpha}\frac{dT_m}{dx}\left[1-\left(r/r_o\right)^2+C_2\,\ell n\left(r/r_o\right)\right].$

$$r\frac{\partial T}{\partial r}=\frac{C_1}{\alpha}\frac{dT_m}{dx}\int\left[r-\frac{r^3}{r_o^2}+C_2 r\ell n\frac{r}{r_o}\right]dr+C_3$$

$$\frac{\partial T}{\partial r}=\frac{C_1}{\alpha}\frac{dT_m}{dx}\left[\frac{r}{2}-\frac{r^3}{4r_o^2}+C_2\left(\frac{r}{2}\ell n\frac{r}{r_o}-\frac{r}{4}\right)\right]+\frac{C_3}{r}$$

and the temperature distribution is

$$T(r)=\frac{C_1}{\alpha}\frac{dT_m}{dx}\left[\frac{r^2}{4}-\frac{r^4}{16\,r_o^2}+C_2\left(\frac{r^2}{4}\ell n\frac{r}{r_o}-\frac{r^2}{4}\right)\right]+C_3\,\ell n r+C_4. \tag{3}$$ <

From the requirement that $q_o''=0$, it follows that $\partial T/\partial r)_{r_o}=0$. Hence,

$$\frac{C_1}{\alpha}\frac{dT_m}{dx}\left[\frac{r_o}{2}-\frac{r_o}{4}+C_2\left(-\frac{r_o}{4}\right)\right]+\frac{C_3}{r_o}=0$$

$$C_3=\frac{C_1}{\alpha}\frac{dT_m}{dx}\frac{r_o^2}{4}\left(C_2-1\right). \tag{4}$$ <

From the condition that $T(r_i) = T_{s,i}$, it follows that

$$C_4=T_{s,i}-\frac{C_1}{\alpha}\frac{dT_m}{dx}\left[\frac{r_i^2}{4}-\frac{r_i^4}{16\,r_o^2}+C_2\left(\frac{r_i^2}{4}\ell n\frac{r_i}{r_o}-\frac{r_i^2}{4}\right)\right]+C_3\,\ell n\,r_i. \tag{5}$$ <

From Eqs. 8.68 and 8.70, the inner surface Nusselt number is

$$Nu_i=\frac{h_i D_h}{k}=\frac{q_i'' D_h}{k\left(T_{s,i}-T_m\right)}$$

where $D_h = 2(r_o - r_i)$. To obtain a workable form of Nu_i, the mean temperature T_m must be evaluated. This may be done by substituting Eqs. (1) and (3) into Eq. 8.27 and evaluating u_m by substituting Eq. (1) into Eq. 8.8. Since the integrations are long and tedious, they are not provided.

COMMENTS: From an energy balance performed for a differential control volume in the annular region, $dT_m/dx=2r_i q_i''/\rho\, c_p\, u_m\left(r_o^2-r_i^2\right)$.

PROBLEM 8.95

KNOWN: Inlet temperature, pressure and flow rate of air. Annulus length and tube diameters. Pressure of saturated steam.

FIND: Outlet temperature and pressure drop of air. Mass rate of steam condensation.

SCHEMATIC:

$D_o = 0.065$ m
$D_i = 0.050$ m
p = 2.455 bar, Sat. steam
$\dot{m} = 0.03$ kg/s
$T_{m,i} = 17^oC$
$p_i = 5$ atm
Air
L = 5 m
$\dot{m}_c$
$T_{m,o}$
p_o

ASSUMPTIONS: (1) Steady-state, (2) Outer surface of annulus is adiabatic, (3) Negligible potential energy, kinetic energy and flow work changes for air, (4) Fully developed flow throughout annulus, (5) Smooth annulus surfaces, (6) Constant properties.

PROPERTIES: *Table A-4*, air ($\overline{T}_m \approx 325$K, p = 5 atm): $\rho = 5 \times \rho$ (1 atm) = 5.391 kg/m^3, c_p = 1008 J/kg·K, $\mu = 196.4 \times 10^{-7}$ N·s/m^2, k = 0.0281 W/m·K, Pr = 0.703. *Table A-6*, sat. steam (p = 2.455 bars): T_s = 400K, h_{fg} = 2183 kJ/kg.

ANALYSIS: With a uniform surface temperature, the air outlet temperature is

$$T_{m,o} = T_s - \left(T_s - T_{m,i}\right)\exp\left(-\frac{\pi D_i L}{\dot{m}\, c_p}\overline{h}\right)$$

With $A_c = \pi\left(D_o^2 - D_i^2\right)/4 = 1.355\times10^{-3}\text{m}^2$, $D_h = D_o - D_i = 0.015\text{m}$ and $Re_D = \rho u_m D_h/\mu$ $= \dot{m}D_h/A_c\mu = 16,900$, the flow is turbulent and the Dittus-Boelter correlation yields

$$\overline{h} \approx h_{fd} = \left(\frac{k}{D_h}\right)0.023\,Re_D^{4/5}\,Pr^{0.4} = \left(\frac{0.0281\,\text{W/m}\cdot\text{K}}{0.015\text{m}}\right)0.023\left(16,900\right)^{4/5}\left(0.703\right)^{0.4} = 90.3\,\text{W/m}^2\cdot\text{K}$$

$$T_{m,o} = 127°\text{C} - \left(110°\text{C}\right)\exp\left(-\frac{\pi\times0.05\text{m}\times5\text{m}\times90.3\,\text{W/m}^2\cdot\text{K}}{0.03\,\text{kg/s}\times1008\,\text{J/kg}\cdot\text{K}}\right) = 116.5°\text{C}$$ <

The pressure drop is $\Delta p = f\left(\rho u_m^2/2D_h\right)L$, where, with $u_m = \dot{m}/\rho A_c = 0.03\,\text{kg/s}/\left(5.391\,\text{kg/m}^3\times1.355\times10^{-3}\text{m}^2\right) = 4.11\,\text{m/s}$, and with $Re_D = 16,900$, Fig. 8.3 yields f ≈ 0.026. Hence,

$$\Delta p \approx 0.026\times5.391\,\text{kg/m}^3\,\frac{\left(4.11\,\text{m/s}\right)^2 5\text{m}}{2\times0.015\text{m}} = 395\,\text{N/m}^2 = 3.9\times10^{-3}\ \text{atm}$$ <

The rate of heat transfer to the air is

$$q = \dot{m}\,c_p\left(T_{m,o} - T_{m,i}\right) = 0.03\,\text{kg/s}\times1008\,\text{J/kg}\cdot\text{K}\left(99.5°\text{C}\right) = 3009\,\text{W}$$

and the rate of condensation is then

$$\dot{m}_c = \frac{q}{h_{fg}} = \frac{3009\,\text{W}}{2.183\times10^6\ \text{J/kg}} = 1.38\times10^{-3}\ \text{kg/s}$$ <

COMMENTS: (1) With $\overline{T}_m = \left(T_{m,i} + T_{m,o}\right)/2 = 340\text{K}$, the initial estimate of 325K is too low and an iterative solution should be obtained, (2) For a steam flow rate of 0.01 kg/s, approximately 14% of the outflow would be in the form of saturated liquid, (3) With L/D_h = 333, the assumption of fully developed flow throughout the tube is excellent.

PROBLEM 8.96

KNOWN: Dimensions and surface thermal conditions for a concentric tube annulus. Water flow rate and inlet temperature.

FIND: (a) Tube length required to achieve desired outlet temperature, (b) Inner tube surface temperature at outlet.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Uniform heat flux at inner surface, (3) Adiabatic outer surface, (4) Fully developed flow at exit, (5) Constant properties.

PROPERTIES: *Table A-6*, Water $\left(\overline{T}_m = 328\text{K}\right)$: c_p = 4183 J/kg·K; ($T_{m,o}$ = 358K): $\mu = 332 \times 10^{-6}$ N·s/m^2, k = 0.673 W/m·K, Pr = 2.07.

ANALYSIS: (a) From the overall energy balance, Eq. 8.37,

$$q = q_i' L = \dot{m}\, c_p \left(T_{m,o} - T_{m,i}\right)$$

$$L = \frac{\dot{m}\, c_p \left(T_{m,o} - T_{m,i}\right)}{q_i'} = \frac{(0.04\text{ kg/s})\, 4183\text{ J/kg}\cdot\text{K}\,(85-25)^\circ\text{C}}{4000\text{ W/m}} = 2.51\text{ m.}$$ <

(b) From Eqs. 8.1 and 8.5,

$$Re_D = \frac{\rho\, u_m D_h}{\mu} = \frac{\dot{m}\, D_h}{A_c \mu} = \frac{\dot{m}\,(D_o - D_i)}{(\pi/4)\left(D_o^2 - D_i^2\right)\mu} = \frac{4\,\dot{m}}{\pi\,(D_o + D_i)\,\mu}$$

$$Re_D = \frac{4 \times 0.04\text{ kg/s}}{\pi\,(0.075\text{ m})\,332 \times 10^{-6}\text{kg/s}\cdot\text{m}} = 2045.$$

Hence the flow is laminar, and with $D_i/D_o = 0.5$, it follows from Eq. 8.73 and Table 8.3

$$Nu_i = Nu_{ii} = 6.24$$

$$h_i = 6.24\frac{k}{D_h} = 6.24\frac{0.673\text{ W/m}\cdot\text{K}}{0.025\text{ m}} = 168\text{ W/m}^2\cdot\text{K.}$$

From Eq. 8.68,

$$T_{s,i}(L) = T_{m,o} + \frac{q_i''}{h_i} = T_{m,o} + \frac{q_i'/\pi D_i}{h_i}$$

$$T_{s,i}(L) = 85^\circ\text{C} + \frac{4000\text{ W/m}}{\pi\,(0.025\text{m})\;168\text{ W/m}^2\cdot\text{K}} = 388^\circ\text{C.}$$ <

COMMENTS: Unless the water is pressurized, local boiling would occur at the tube surface, causing h_i to be larger.

PROBLEM 8.97

KNOWN: Heat rate per unit length at the inner surface of an annular recuperator of prescribed dimensions. Flow rate and inlet temperature of air passing through annular region.

FIND: (a) Temperature of air leaving the recuperator, (b) Inner pipe temperature at inlet and outlet and outer pipe temperature at inlet.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Uniform heating of recuperator inner surface, (4) Adiabatic outer surface, (5) Negligible kinetic and potential energy changes for air, (6) Fully developed air flow throughout.

PROPERTIES: *Table A-4*, Air $\left(\overline{T}_m = 500\text{K}\right)$: c_p = 1030 J/kg·K, $\mu = 270 \times 10^{-7}$ N·s/m^2, k = 0.041 W/m·K, Pr = 0.68.

ANALYSIS: (a) From an energy balance on the air

$$q_i' \, L = \dot{m}_a \, c_{p,a}\left(T_{a,2} - T_{a,1}\right)$$

$$T_{a,2} = T_{a,1} + \frac{q_i' \, L}{\dot{m}_a \, c_{p,a}} = 300\text{K} + \frac{1.25\times10^5 \text{W/m}\times 7\text{m}}{2.1 \text{ kg/s}\times 1030 \text{ J/kg}\cdot\text{K}} = 704.5\text{K}.$$ <

(b) The surface temperatures may be evaluated from Eqs. 8.68 and 8.69 with

$$Re_D = \frac{\rho \, u_m D_h}{\mu} = \frac{\dot{m}_a \, \left(D_o - D_i\right)}{\left(\pi/4\right)\left(D_o^2 - D_i^2\right)\mu} = \frac{4\,\dot{m}_a}{\pi\left(D_o + D_i\right)\mu} = \frac{4\left(2.1 \text{ kg/s}\right)}{\pi\left(4.05\text{m}\right) \, 270\times10^{-7} \text{N}\cdot\text{s/m}^2}$$

$$Re_D = 24{,}452$$

the flow is turbulent and from Eq. 8.60

$$h_i \approx h_o \approx \frac{k}{D_h} 0.023 \, Re_D^{4/5} \, Pr^{0.4} = \frac{0.041 \text{ W/m}\cdot\text{K}}{0.05 \text{ m}} 0.023\left(24{,}452\right)^{4/5}\left(0.68\right)^{0.4} = 52 \text{ W/m}^2\cdot\text{K}.$$

With $\quad q_i'' = q_i' / \pi \, D_i = 1.25\times10^5 \text{W/m}/\pi\times 2\text{m} = 19{,}900 \text{ W/m}^2$

Eq. 8.68 gives

$$\left(T_{s,i} - T_m\right) = q_i''/h_i = 19{,}900 \text{ W/m}^2/52 \text{ W/m}^2\cdot\text{K} = 383\text{K}$$

$$T_{s,i,1} = 683\text{K} \qquad T_{s,i,2} = 1087\text{K}.$$ <

From Eq. 8.69, with $q_o'' = 0$, $\left(T_{s,o} - T_m\right) = 0$. Hence

$$T_{s,o,1} = T_{a,1} = 300\text{K}.$$ <

PROBLEM 8.98

KNOWN: A concentric tube arrangement for removing heat generated from a biochemical reaction in a settling tank. Water is supplied to the annular region at rate of 0.2 kg/s.

FIND: (a) The inlet temperature of the supply water that will provide for an average tank surface temperature of 37°C; assume and then justify fully developed flow and thermal conditions; and (b) Sketch the water and surface temperatures along the flow direction for two cases: the fully developed conditions of part (a), and when entrance effects are important. Comment on the features of the temperature distributions, with particular attention to the longitudinal gradient on the tank surface. What change to the system or operating conditions would you make to reduce the gradient?

SCHEMATIC:

ASSUMPTIONS: (1) Fully developed flow and thermal conditions, (2) Inner annulus surface has uniform heat flux, while outer surface is insulated, (3) Constant properties, (4) Negligible kinetic and potential energy changes and flow work.

PROPERTIES: *Table A-6*, Water (T_m = 304 K): ρ = 995.6 kg/m^3, c_p = 4178 J/kg·K, ν = 7.987 × 10^{-7} m^2/s, k = 0.618 W/m·K, Pr = 5.39.

ANALYSIS: (a) The overall energy balance on the fluid passing through the concentric tube is

$$q = \dot{m}\, c_p \left(T_{m,i} - T_{m,o}\right) \tag{1}$$

and from an energy balance on the reaction tank,

$$q = q''_{s,i} \cdot A_{s,i} = \dot{q}\left(\pi D_i\right)L. \tag{2}$$

The convection rate equation applied to the inner surface $A_{s,i}$ is

$$q''_{cv} = q''_{s,i} = \bar{h}_i\left(\bar{T}_s - T_m\right) \tag{3}$$

where $\bar{T}_s$ is the average inner surface temperature and

$$T_m = \left(T_{m,i} + T_{m,o}\right)/2. \tag{4}$$

To estimate $\bar{h}$, begin by characterizing the flow with

$$Re_{Dh} = u_m D_h/\nu \qquad D_h = D_o - D_i \qquad \dot{m} = \rho A_c u_m$$

where $A_c = \pi\left(D_o^2 - D_i^4\right)/4$. Substituting numerical values find

$$Re_{Dh} = 1779$$

Assuming fully developed conditions for laminar flow through an annulus, it follows from Table 8.3 and Eq. 8.73 with D_i/D_o = 0.8,

$$\overline{Nu}_i = \bar{h}_i D_h/k = 5.58 \qquad \bar{h}_i = 172\ \mathrm{W/m^2 \cdot K}$$

Continued …..

Using Eq. (3) with $\overline{h}_i$, and $\overline{T}_s = 37°C$, and $q''_{s,i}$ from Eq. (2), find

$$T_m = 30.5°C$$

From Eqs. (1) and (4), calculate

$$T_{m,i} = 30.3°C \qquad T_{m,o} = 30.6°C$$ <

For this annulus, the thermal entry length from Eq. 8.23 is

$$x_{fd,t} = D_h \times 0.05\ Re_{Dh}\ Pr$$

$$x_{fd,t} = (0.100 - 0.080)m \times 0.05 \times 1779 \times 5.39 = 9.59\ m$$

Since L = 1 m, we conclude that entry length effects are significant, and the fully developed flow assumption is approximate.

(b) Since the fluid is being heated by flow over a surface with uniform heat flux, the mean fluid temperature, $T_m(x)$, will increase linearly with longitudinal distance x. Assuming fully developed conditions, the surface temperature $T_s(x)$ will likewise increase linearly with distance as shown in the schematic below. Note that the longitudinal temperature difference is about 0.3°C, and that the inlet mean temperature is 30.3°C.

Considering now entrance length effects, the convection coefficient is no longer uniform, and will be largest near the entrance, and larger than for the fully developed flow everywhere. Hence, we expect the surface temperature near the entrance to be closer to the mean fluid temperature than elsewhere. We also expect the average mean temperature of the fluid will be higher so that the average surface temperature, $\overline{T}_s$, remains at 37°C. However, the rise in temperature of the fluid ($T_{m,o} - T_{m,i}$) will remain the same, about 0.3°C, since the heat removal rate is the same. Increasing the flow rate will tend to minimize the longitudinal gradient by reducing ($T_{m,o} - T_{m,i}$) and increasing h(x). The graph below illustrates the distinctive features of the fully developed flow and entrance length effects.

COMMENTS: The thermophysical properties required in the convection correlation and the energy equations were evaluated at Tm = ($T_{m,i} + T_{m,o}$)/2.

PROBLEM 8.99

KNOWN: Surface thermal conditions and diameters associated with a concentric tube annulus. Water flow rate and inlet temperature.

FIND: (a) Length required to achieve desired outlet temperature, (b) Heat flux from inner tube at outlet.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Fully developed conditions throughout, (3) Adiabatic outer surface, (4) Uniform temperature at inner surface, (5) Constant properties.

PROPERTIES: *Table A-6*, Water $\left(\overline{T}_m = 320\text{K}\right)$: c_p = 4180 J/kg·K, $\mu = 577 \times 10^{-6}$ N·s/m^2, k = 0.640 W/m·K, Pr = 3.77.

ANALYSIS: (a) From Eq. 8.42a,

$$L = -\frac{\dot{m}\, c_p}{P\overline{h}} \ell n \frac{\Delta T_o}{\Delta T_i} = -\frac{\dot{m}\, c_p}{p\ D_i \overline{h}} \ell n \frac{T_s - T_{m,o}}{T_s - T_{m,i}}.$$

With
$$Re_D = \frac{r\ u_m D_h}{m} = \frac{\dot{m}\left(D_o - D_i\right)}{(p/4)\left(D_o^2 - D_i^2\right)m} = \frac{4\,\dot{m}}{p\left(D_o + D_i\right)m}$$

$$Re_D = \frac{4 \times 0.02\ \text{kg/s}}{p\left(0.125\text{m}\right)577 \times 10^{-6}\text{N}\cdot\text{s/m}^2} = 353$$

the flow is laminar. Hence, from Eq. 8.70 and Table 8.2,

$$\overline{h} = h_i = \frac{k}{D_h} Nu_i = \frac{0.64\ \text{W/m}\cdot\text{K}}{\left(0.100 - 0.025\right)\ \text{m}} 7.37 = 63\ \text{W/m}^2\cdot\text{K}$$

and
$$L = -\frac{0.02\ \text{kg/s}\ \left(4180\ \text{J/kg}\cdot\text{K}\right)}{p\left(0.025\text{m}\right)63\ \text{W/m}^2\cdot\text{K}} \ell n \frac{\left(100 - 75\right)^\circ \text{C}}{\left(100 - 20\right)^\circ \text{C}} = 19.7\ \text{m}.$$ <

(b) From Eq. 8.69

$$q_i''(L) = h_i\left(T_{s,i} - T_{m,o}\right) = 63 \frac{\text{W}}{\text{m}^2.\text{K}}\left(100 - 75\right)^\circ \text{C} = 1575\ \text{W/m}^2.$$ <

COMMENTS: The total heat rate to the water is

$$q = \dot{m}\, c_p\ \left(T_{m,o} - T_{m,i}\right) = 0.02\ \text{kg/s} \times 4180\ \text{J/kg}\cdot\text{K}\left(55^\circ\text{C}\right) = 4598\ \text{W}.$$

PROBLEM 8.100

KNOWN: Surface thermal conditions and diameters associated with a concentric tube annulus. Water flow rate and inlet temperature.

FIND: Length required to achieve desired outlet temperature.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Fully developed conditions throughout, (3) Adiabatic outer surface, (4) Uniform temperature at inner surface, (5) Constant properties.

PROPERTIES: *Table A-6*, Water $\left(\overline{T}_m = 320\text{K}\right)$: c_p = 4180 J/kg·K, $\mu = 577 \times 10^{-6}$ N·s/m^2, k = 0.640 W/m·K, Pr = 3.77.

ANALYSIS: From Eq. 8.42a,

$$L = -\frac{\dot{m}\, c_p}{P\,\overline{h}}\,\ell n\frac{\Delta T_o}{\Delta T_i} = -\frac{\dot{m}\, c_p}{\pi\, D_i \overline{h}}\,\ell n\frac{T_s - T_{m,o}}{T_s - T_{m,i}}.$$

With

$$Re_D = \frac{\rho\, u_m D_h}{\mu} = \frac{\dot{m}\left(D_o - D_i\right)}{\left(\pi/4\right)\left(D_o^2 - D_i^2\right)\mu} = \frac{4\,\dot{m}}{\pi\left(D_o + D_i\right)\mu}$$

$$Re_D = \frac{4 \times 0.30\text{ kg/s}}{\pi\left(0.125\text{m}\right)577 \times 10^{-6}\,\text{N}\cdot\text{s/m}^2} = 5296$$

and the flow is turbulent. Hence, from Eq. 8.60,

$$\overline{h} = \frac{k}{D_h}\,Nu_D = 0.023\frac{k}{D_h}\,Re_D^{4/5}\,Pr^{0.4}$$

$$\overline{h} = 0.023\frac{0.640\text{ W/m}\cdot\text{K}}{0.075\text{ m}}\left(5296\right)^{4/5}\left(3.77\right)^{0.4} = 318\text{ W/m}^2\cdot\text{K}$$

and hence the required length is

$$L = -\frac{0.30\text{ kg/s}\left(4180\text{ J/kg}\cdot\text{K}\right)}{\pi\left(0.025\text{m}\right)\,318\text{ W/m}^2\cdot\text{K}}\,\ell n\frac{\left(100-75\right)^\circ\text{C}}{\left(100-20\right)^\circ\text{C}} = 58.4\text{ m.}$$

<

COMMENTS: Increasing $\dot{m}$ by a factor of 15 increases Re_D accordingly, and the flow is turbulent. However, $\overline{h}$ increases by a factor of only 5, from the result of Problem 8.99, in which case the tube length must be a factor of 3 larger than that of Problem 8.99.

PROBLEM 8.101

KNOWN: Dimensions and thermal conductivity of plastic pipe. Volumetric flow rate and temperature of inlet air. Enhancement of inner convection coefficient and friction factor associated with coiled spring. Thermal resistance of coating on outer surface.

FIND: (a) Air outlet temperature and fan power requirement without coating and coiled spring, (b) Effect of coiled spring on air outlet temperature and fan power, (c) Effect of coating on outlet temperature.

SCHEMATIC:

Water $T_\infty = 17^oC$
Thin film $R''_{t,c} = 0.050\ m^2\text{-}K/W$
$D_o = 0.17$ m
$D_i = 0.15$ m
$T_{m,o}$
$\dot{\forall}_i = 0.025\ m^3/s$
$T_{m,i} = 29^oC$
Air
Plastic pipe k = 0.15 W/m-K
Coiled spring $h_{cp} = 2h_i$, $f_{cp} = 1.5f$
L = 15 m
$h_o = 1500\ W/m^2\text{-}K$

ASSUMPTIONS: (1) Steady-state, (2) Negligible heat transfer from air in vertical pipe sections, (3) Negligible flow work and potential and kinetic energy changes for air flow through pipe, (4) Smooth interior surface without spring, (5) Negligible coating thickness, (6) Constant properties.

PROPERTIES: *Table A-4*, Air ($T_{m,i}$ = 29°C): $\rho_i = 1.155\ kg/m^3$. Air ($\overline{T}_m \approx 25°C$): $c_p = 1007$ J/kg·K, $\mu = 183.6 \times 10^{-7}\ N\cdot s/m^2$, $k_a = 0.0261$ W/m·K, Pr = 0.707.

ANALYSIS: (a) From Eq. (8.46a),

$$\frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = \exp\left(-\frac{\overline{U}A_s}{\dot{m}\,c_p}\right)$$

where, from Eq. (3.32),

$$\left(\overline{U}A_s\right)^{-1} = R_{tot} = \frac{1}{\overline{h}_i \pi D_i L} + \frac{\ln(D_o/D_i)}{2\pi L k} + \frac{1}{h_o \pi D_o L}$$

With $\dot{m} = \rho_i \dot{\forall}_i = 0.0289\ kg/s$ and $Re_D = 4\dot{m}/\pi D_i \mu = 13,350$, the pipe flow is turbulent. With L/D_i = 100, we may assume fully developed flow throughout the pipe, and from Eq. (8.60),

$$\overline{h}_i = \frac{ka}{D_i} 0.023 Re_D^{4/5} Pr^{0.3} = \frac{0.0261\ W/m\cdot K}{0.15m} 0.023(13,350)^{4/5}(0.707)^{0.3} = 7.20\ W/m^2\cdot K$$

Hence,
$$R_{tot} = \left(\frac{1}{7.20\times\pi\times0.15\times15} + \frac{\ln(0.17/0.15)}{2\pi\times15\times0.15} + \frac{1}{1500\times\pi\times0.17\times15}\right)\frac{K}{W}$$

$$R_{tot} = (0.0196 + 0.0089 + 0.0001)\ K/W = 0.0286\ K/W$$

Hence, $\overline{U}A_s = R_{tot}^{-1} = 35.0\ W/K$ and

$$T_{m,o} = T_\infty + (T_{m,i} - T_\infty)\exp\left(-\frac{\overline{U}A_s}{\dot{m}\,c_p}\right) = 17°C + (12°C)\exp\left(-\frac{35.0\ W/K}{0.0289\ kg/s \times 1007\ J/kg\cdot K}\right) = 20.6°C$$ <

Continued …..

From Eq. (8.20a), f = 0.316 $Re_D^{-1/4}$ = 0.0294. Hence, from Eqs. (8.22a) and (8.22b), with $u_{m,i} = \dot{\forall}_i / A_c = 1.415\,m/s$,

$$P \approx f\,\frac{\rho_i u_{m,i}^2}{2D_i} L\,\dot{\forall}_i = 0.0294\frac{1.155\,kg/m^3\left(1.415\,m/s\right)^2}{2\left(0.15m\right)}15m \times 0.025\,m^3/s = 0.085\,W \qquad <$$

(b) With $h_{cp} = 2h_i$ = 14.4 W/m^2·K, the inner convection resistance is reduced from 0.0196 K/W to 0.0098 K/W and hence the total resistance from 0.0286 K/W to 0.0188 K/W. It follows that $\overline{U}A_s = 53.2\,W/K$ and

$$T_{m,o} = 18.9°C \qquad <$$

With f_{cp} = 1.5f,

$$P = 0.128\,W \qquad <$$

(c) With the coating of organic matter, there is an additional thermal resistance of the form $R_{t,c}$ = $R''_{t,c}/\left(\pi D_o L\right) = \left(0.05\,m^2 \cdot K/W\right)/\left(\pi \times 0.17m \times 15m\right) = 0.0062\,K/W$. The total resistance is then R_{tot} = 0.0348 K/W and $\overline{U}A_s = 28.7\,W/K$. Hence,

$$T_{m,o} = 21.5°C \qquad <$$

COMMENTS: (1) The fan power requirement is small, and the process is economical, with or without the coiled spring. (2) Heat transfer enhancement associated with the coiled spring is manifested by a 34% reduction in the total thermal resistance and a 1.7°C reduction in the outlet temperature. (3) *Fouling* of the outer surface increases the total resistance by 22% and the outlet temperature by 0.9°C. The penalty is not severe but could be ameliorated by periodic cleaning of the surface.

PROBLEM 8.102

KNOWN: Air flow through a plastic tube in which evaporation occurs.

FIND: Convection mass transfer coefficient, h_m.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Heat-mass transfer analogy applicable, (4) Fully-developed flow and mass transfer conditions.

PROPERTIES: Plastic-air (given, 400K): $Sc = \nu/D_{AB} = 2.0$; *Table A-4*, Air (400K, 1 atm): $\nu = 26.41 \times 10^{-6}\ m^2/s$.

ANALYSIS: For fully-developed flow and thermal conditions with laminar flow and a uniform surface temperature,

$$Nu_D = \frac{h\,D}{k} = 3.66$$

This situation is analogous to the evaporation of plastic vapor into the air stream with the inner surface remaining at a constant concentration of plastic vapor, $C_{A,s}$, along the length of the tube. Invoking the heat-mass transfer analogy,

$$Sh_D = \frac{h_m D}{D_{AB}} = 3.66.$$

Recognizing that $Sc = \nu/D_{AB}$,

$$h_m = 3.66\left(\frac{n}{Sc}\right)\frac{1}{D} = 3.66 \times \frac{26.4\times10^{-6}\,m^2/s}{2.0} \times \frac{1}{2\times10^{-3}\,m} = 2.42\times10^{-2}\,m/s.$$ <

COMMENTS: (1) The heat-mass transfer analogy requires that the vapor (A) have a negligible effect on the flow. Hence, the flow is that of air (B) and $\nu = \nu_B$.

(2) Only the mixture property D_{AB} is required to characterize the plastic vapor for this evaporation process.

PROBLEM 8.103

KNOWN: Air passing upward through a tube having a thin water film on its inside surface.

FIND: Convection mass transfer coefficient.

SCHEMATIC:

D = 30 mm
Thin water film
T_w = 300 K
T_a = 300K $\dot{m}$ = 3 kg/h
Air

ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Heat-mass analogy applicable, and (4) Fully developed flow and thermal conditions.

PROPERTIES: *Table A.4*, Air (300 K, 1 atm): $\mu = 184.6 \times 10^{-7}$ N·s/m², k = 0.0263 W/m·K; *Table A.8*, Water vapor-air (300 K, 1atm): $D_{AB} = 0.26 \times 10^{-4}$ m²/s.

ANALYSIS: Begin by characterizing the air flow with the Reynolds number,

$$Re_D = \frac{4\dot{m}}{\pi D \mu} = \frac{4 \times (3/3600)\,kg/s}{\pi \times 0.030m \times 184.6 \times 10^{-7}\,N/s \cdot m^2} = 1916$$

Since the flow is laminar, and assuming fully developed flow and thermal conditions, Eq. 8.55 is appropriate for the uniform T_s wall condition,

$$Nu_D = \frac{hD}{k} = 3.66 \qquad h = \frac{0.0263\,W/m \cdot K}{0.030m} \times 3.66 = 3.21\,W/m^2 \cdot K$$

Invoking the heat-mass analogy, for laminar flow conditions,

$$Sh_D = \frac{h_m D}{D_{AB}} = Nu_D$$

$$h_m = \frac{D_{AB}}{D} Nu_D = \frac{0.26 \times 10^{-4}\,m^2/s}{0.030m} \times 3.66 = 0.0032\,m/s$$ <

COMMENTS: (1) The heat-mass analogy requires that the water vapor (A) have negligible effect on the velocity boundary layer. It is important to recognize that the vapor is species (A) and the air species (B). Hence the flow is that of air (B) and hence $\mu = \mu_B$.

(2) Note only the mixture property D_{AB} is required to characterize the water vapor for this evaporation process.

PROBLEM 8.104

KNOWN: Temperature and flow rate of air in a tube with a naphthalene coated inner surface.

FIND: Convection mass transfer coefficient under fully developed conditions and velocity and concentration entry lengths.

SCHEMATIC:

Air, B
$\dot{m} = 0.04$ kg/s
$T = 300$K
Tube, $D = 0.05$m
Napthalene coating (A)

ASSUMPTIONS: (1) Heat and mass transfer analogy is applicable, (2) Uniform vapor concentration along inner surface.

PROPERTIES: *Table A-4*, Air (300K, 1 atm): $\mu = 184.6 \times 10^{-7}$ N·s/m^2, $\nu = 15.89 \times 10^{-6}$ m^2/s; *Table A-8*, Naphthalene-air (300K, 1 atm): $D_{AB} = 6.2 \times 10^{-6}$ m^2/s, $Sc = \mu/D_{AB} = 2.56$.

ANALYSIS: For air flow through the tube,

$$Re_D = \frac{4\,\dot{m}}{\pi\, D\mu} = \frac{4 \times 0.04\ \text{kg/s}}{\pi\,(0.05\text{m})\ 184.6 \times 10^{-7}\text{N}\cdot\text{s/m}^2} = 55{,}178.$$

Hence the flow is turbulent and from the Colburn equation, Eq. 8.59,

$$Sh_D = 0.023\ Re_D^{4/5}\ Sc^{1/3} = 0.023(55{,}178)^{4/5}(2.56)^{1/3} = 196$$

$$h_m = \frac{D_{AB}}{D} Sh_D = \frac{6.2 \times 10^{-6}\text{m}^2/\text{s}}{0.05\ \text{m}} 196 = 0.024\ \text{m/s}.$$ <

From Eq. 8.4, it follows that

$$10D \le x_{fd,h} \approx x_{fd,c} \le 60D$$

or

$$0.5\ \text{m} \le x_{fd,h} \approx x_{fd,c} \le 3\ \text{m}.$$ <

An entry length of 0.5m is assumed.

COMMENTS: Note that the flow properties are taken to be those of the air, with the contribution of the naphthalene vapor assumed to be negligible.

PROBLEM 8.105

KNOWN: Air flow over roughened section of tube constructed from naphthalene.

FIND: Mass and heat transfer convection coefficients associated with the roughened section; contrast these results with those for a smooth section.

SCHEMATIC:

L=150mm
D=75mm
Air, B
$\rho_{A,m}=0$
$Re_D=35,000$
300K, 1atm
$\dot{m}_A=0.01\text{kg}/3\text{h}$
Roughened naphthalene section (A)

ASSUMPTIONS: (1) Steady-state conditions, (2) Heat-mass transfer analogy applicable, (3) Negligible naphthalene vapor in airstream, $\rho_{A,m} = 0$, (4) Constant properties, (5) Naphthalene vapor behaves as perfect gas.

PROPERTIES: *Table A-4*, Air (300K, 1 atm): $\nu = 15.89 \times 10^{-6}\ \text{m}^2/\text{s}$, k = 0.0263 W/m·K, Pr = 0.707; *Table A-8*, Naphthalene-air mixture (300K, 1 atm): $D_{AB} = 0.62 \times 10^{-5}\ \text{m}^2/\text{s}$, $Sc = \nu_B/D_{AB} = 2.563$; Naphthalene (given, 300K): $p_{sat,A} = 1.31 \times 10^{-4}$ bar, $M_A = 128.16$ kg/kmol.

ANALYSIS: Using the rate equation with the experimentally observed sublimination rate of naphthalene vapor, the average mass transfer coefficient for the section is

$$h_m = \dot{m}_A / (p\ DL)\ \left(r_{A,s} - r_{A,m}\right)$$

$$r_{A,m} = 0 \qquad r_{A,s} = r_{A,sat}(300\text{K}) = M_A\ p_{sat,A} / \Re T$$

$$r_{A,s} = 128.16\ \text{kg/kmol} \times \frac{1.31\times10^{-4}\text{bar}}{8.314\times10^{-2}\text{m}^3\cdot\text{bar/kmol}\cdot\text{K}\times300\text{K}} = 6.731\times10^{-4}\ \text{kg/m}^3$$

$$h_m = \frac{0.010\ \text{kg}}{3\times3600\ \text{s}} / (p\times0.075\text{m}\times0.150\text{m})\ \left(6.731\times10^{-4} - 0\right)\ \text{kg/m}^3 = 3.89\times10^{-2}\text{m/s}. \qquad <$$

Invoking the heat-mass transfer analogy, the associated heat transfer coefficient is

$$h = h_m \frac{k}{D_{AB}}\left(\frac{Pr}{Sc}\right)^{1/3} = 3.89\times10^{-2}\text{m/s}\frac{0.0263\ \text{W/m}\cdot\text{K}}{0.62\times10^{-5}\text{m}^2/\text{s}}\left(\frac{0.707}{2.563}\right)^{1/3} = 107\ \text{W/m}^2\cdot\text{K}. \qquad <$$

The corresponding convection coefficients for a *smooth* section can be estimated using the Colburn relation,

$$h = \frac{k}{D}0.023\ Re_D^{4/5}\ Pr^{1/3} = (0.0263\ \text{W/m}\cdot\text{K}/0.075\ \text{m})\times0.023\,(35{,}000)^{4/5}\,(0.707)^{1/3} = 31\ \text{W/m}^2\cdot\text{K}. \qquad <$$

Invoking the heat-mass transfer analogy,

$$h_m = (D_{AB}/D)0.023\ Re_D^{4/5}Sc^{1/3} = \left(0.62\times10^{-5}\text{m}^2/\text{s}/0.075\ \text{m}\right)\times0.023(35{,}000)^{4/5}\,(2.563)^{1/3}$$

$$h_m = 1.12 \times 10^{-2}\ \text{m/s} \qquad <$$

COMMENTS: The effect of roughening is to increase the convection coefficients over the corresponding value for the smooth condition; in this case, by a factor of approximately 3.5.

PROBLEM 8.106

KNOWN: Dry air with prescribed velocity and temperature flowing over a thin-walled tube with a water-saturated fibrous coating. Water passes at a prescribed rate through the tube to maintain an approximately uniform surface temperature $T_s = 27°C$.

FIND: (a) Heat rate from the external surface of the tube considering heat and mass transfer processes and (b) For a flow rate of $\dot{m} = 0.025$ kg/s, the inlet temperature, $T_{m,i}$, of the water that must be supplied to the tube.

SCHEMATIC:

Air
$T_\infty = 35\ ^oC$
$V = 10$ m/s
Water
$\dot{m}, T_m, i$
Saturated surface condition
$T_s = 27\ ^oC$

ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Heat-mass analogy applicable, and (4) Negligible kinetic energy and axial conduction in the tube flow.

PROPERTIES: *Table A.4*, Air ($\overline{T}_f = (T_s + T_\infty)/2 = 304$ K): $\rho = 1.148$ kg/m^3, $c_p = 1007$ J/kg·K, $\nu = 16.29 \times 10^{-6}$ m^2/s, k = 0.0266 W/m·K, $\alpha = 23.09 \times 10^{-6}$ m^2/s, Pr = 0.706; *Table A.6*, Water (T_s = 300 K): $\rho_{A,s} = 1/v_g = 0.02556$ kg/m^3, $h_{fg} = 2438$ kJ/kg, $\mu = 855 \times 10^{-6}$ N·s/m^2; *Table A.6*, Water ($\overline{T}_m = 305$ K): $\rho = 995$ kg/m^3, $c_p = 4178$ J/kg·K, $\mu = 769 \times 10^{-6}$ N·s/m^2, k = 0.620 W/m·K, Pr = 5.20; *Table A.8*, Water vapor-air (T_s = 300 K): $D_{AB} = 0.26 \times 10^{-4}$ m^2/s.

ANALYSIS: (a) On the Schematic above, the surface energy balance yields

$$q_{out} = q_{conv} + q_{evap} \tag{1}$$

and substituting the rate equations,

$$q_{conv} = \overline{h}_o A_s \left(T_s - T_\infty\right) \qquad q_{evap} = n_A h_{fg} = \overline{h}_m A_s \left(\rho_{A,s} - \rho_{A,\infty}\right) h_{fg} \tag{2,3}$$

where $\overline{h}_o$ can be estimated from an appropriate correlation and $\overline{h}_m$ from the heat-mass analogy using $\overline{h}_o$.

Estimation of the heat transfer coefficient, $\overline{h}_o$: The Reynolds number, evaluated with properties at $\overline{T}_f = (T_s + T_\infty)/2 = 304$ K, is

$$Re_{D_o} = \frac{VD}{\nu} = \frac{10\,m/s \times 0.020\,m}{1.629 \times 10^{-5}\,m^2/s} = 12,277 \tag{4}$$

Using the Churchill-Bernstein correlation, Eq. 7.57, for cross flow over a cylinder, find $\overline{h}_o$

$$Nu_{D,o} = 0.3 + \frac{0.62\,Re_{D,o}^{1/2}\,Pr_o^{1/3}}{\left[1 + \left(0.4/Pr_o\right)^{2/3}\right]^{1/4}} \left[1 + \left(\frac{Re_{D,o}}{282,000}\right)^{5/8}\right]^{4/5} \tag{5}$$

Continued...

$$\mathrm{Nu}_{D,o} = 0.3 + \frac{0.62(12,277)^{1/2}(0.706)^{1/3}}{\left[1+(0.4/0.706)^{2/3}\right]^{1/4}}\left[1+\left(\frac{12,277}{282,000}\right)^{5/8}\right]^{4/5}$$

$$\bar{h}_o = \frac{k}{D}\mathrm{Nu}_{D,o} = \frac{0.0266\,\mathrm{W/m\cdot K}}{0.020\,\mathrm{m}} \times 60.1 = 80.0\,\mathrm{W/m^2\cdot K}$$

The Heat-Mass Analogy: From Eq. 6.92, with n = 1/3,

$$\frac{\bar{h}_o}{\bar{h}_m} = \rho c_p \mathrm{Le}^{2/3} = \rho c_p \left(\frac{\alpha}{D_{AB}}\right)^{2/3} \tag{6}$$

$$\bar{h}_m = 80.0\,\mathrm{W/m^2\cdot K} \Big/ \left[1.148\,\mathrm{kg/m^3} \times 1007\,\mathrm{J/kg\cdot K}\left(23.09\times10^{-6}\,\mathrm{m^2/s} / 0.26\times10^{-4}\,\mathrm{m^2/s}\right)^{2/3}\right] = 0.0749\,\mathrm{m/s}$$

Hence, the heat rate leaving the tube surface from Eq. (1) is,

$$q_{out} = \left[80\,\mathrm{W/m^2\cdot K}\,(27-35)^\circ\mathrm{C} + 0.0749\,\mathrm{m/s}\,(0.02556-0)\,\mathrm{kg/m^3} \times 2438\times10^3\,\mathrm{J/kg}\right](\pi \times 0.020\,\mathrm{m} \times 0.200\,\mathrm{m})$$

$$q_{out} = -8.04\,\mathrm{W} + 58.65 = 50.6\,\mathrm{W}$$ <

(b) For tube flow analysis, the heat rate and rate equations are

$$q = \dot{m}c_p\left(T_{m,o} - T_{m,i}\right) \qquad \frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp\left(-\frac{\pi DL}{\dot{m}c_p}\right)\bar{h}_i \tag{7,8}$$

where T_s = 27°C, the uniform temperature of the tube surface, and q = -50.6 W according to the analysis of part (a). To estimate $\bar{h}_i$, first characterize the flow,

$$\mathrm{Re}_{D,i} = \frac{4\dot{m}}{\pi D\mu_i} = \frac{4\times0.025\,\mathrm{kg/s}}{\pi\times0.020\,\mathrm{m}\times769\times10^{-6}\,\mathrm{N\cdot s/m^2}} = 2070 \tag{9}$$

using properties evaluated at an assumed mean temperature, $\bar{T}_m$ = 305 K (slightly above T_s). The flow is laminar, and assuming a combined entry region, use the Sieder-Tate correlation, Eq. 8.57,

$$\overline{\mathrm{Nu}}_{D,i} = 1.86\left(\frac{\mathrm{Re}_{D,i}\,\mathrm{Pr}_i}{L/D}\right)^{1/3}\left(\frac{\mu}{\mu_s}\right)^{0.14} \tag{10}$$

$$\overline{\mathrm{Nu}}_{D,i} = 1.86\left(\frac{2070\times5.20}{0.200/0.020}\right)^{1/3}\left(\frac{769\times10^{-6}}{855\times10^{-6}}\right)^{0.14}$$

$$\bar{h}_i = \frac{k_i}{D}\overline{\mathrm{Nu}}_{D,i} = \frac{0.620\,\mathrm{W/m\cdot K}}{0.020\,\mathrm{m}} \times 18.78 = 582\,\mathrm{W/m^2\cdot K}$$

Referring to Eqs. (7) and (8), recognize that there are two unknowns, $T_{m,i}$ and $T_{m,o}$, as we have evaluated both q and $\bar{h}_i$. Using the IHT solver, we found

$$T_{m,i} = 34.2^\circ\mathrm{C} \qquad T_{m,o} = 33.7^\circ\mathrm{C}$$ <

Continued...

COMMENTS: Using the *IHT Rate Equation Tool*, *Rate Equation* for a *Tube*, *Constant Surface Temperature*, and the *Correlation*, *Internal Flow*, *Laminar*, *Combined Entry Length*, a model to perform the analysis for part (b) was developed and is copied below.

```
// Rate Equation Tool - Tube, Constant Surface Temperature:
/* For flow through a tube with a uniform wall temperature, Fig 8.7b, the
overall energy balance and heat rate equations  are */
q = mdot*cp*(Tmo - Tmi)              // Heat rate, W; Eq 8.37
q =  - 50.64                         // Heat rate, W; required to sustain heat loss on outer surface
(Ts - Tmo) / (Ts - Tmi) = exp ( - P * L * h / (mdot * cp))        // Eq 8.42b
// where the fluid and constant tube wall temperatures are
Ts = 27 + 273                        // Tube wall temperature, K
Tmi_C = Tmi -273                     // Inlet mean fluid temperature, K
Tmo_C = Tmo - 273                    // Outlet mean fluid temperature, K
// The tube parameters are
P = pi * D                           // Perimeter, m
Ac = pi * (D^2) / 4                  // Cross sectional area, m^2
D =  0.020                           // Tube diameter, m
L =   0.20                           // Tube length, m
// The tube mass flow rate and fluid thermophysical properties are
mdot = rho * um * Ac
mdot = 0.025

// Properties Tool - Water
// Water property functions :T dependence, From Table A.6
// Units: T(K), p(bars);
x = 0                                // Quality (0=sat liquid or 1=sat vapor)
rho = rho_Tx("Water",Tm,x)           // Density, kg/m^3
cp = cp_Tx("Water",Tm,x)             // Specific heat, J/kg·K
mu = mu_Tx("Water",Tm,x)             // Viscosity, N·s/m^2
mus = mu_Tx("Water",Ts,x)            // Viscosity, N·s/m^2
nu = nu_Tx("Water",Tm,x)             // Kinematic viscosity, m^2/s
k = k_Tx("Water",Tm,x)               // Thermal conductivity, W/m·K
Pr = Pr_Tx("Water",Tm,x)             // Prandtl number
Tm = Tfluid_avg(Tmo, Tmi)            // Average mean temperature, K
//Tm = 300                           // Assigned value, initial solve

// Correlations Tool - Internal Flow, Laminar, combined entry length
NuDbar = NuD_bar_IF_L_CEL_CWT(ReD,Pr,D,L,mu,mus)     // Eq 8.57
NuDbar = h * D / k
ReD = um * D / nu

// Data Browser results:
         Ac                  NuDbar    P          Pr          ReD        Tmi        Tmi_C      Tmo
         Tmo_C     cp        h         k          mu                     mus        nu         rho
         um        D         L         Ts         mdot        q          x          Tm
         0.0003142           18.64     0.06283    4.975       2150       307.2      34.18      306.7
         33.7      4178      580.8     0.6231     0.0007403              0.000855   7.445E-7   994.3
         0.08004   0.02      0.2       300        0.025       -50.64     0          306.9
```

PROBLEM 8.107

KNOWN: Density and flow rate of gas through a tube with evaporation or sublimination at the tube surface.

FIND: (a) Longitudinal distribution of mean vapor density, (b) Total rate of vapor transfer.

SCHEMATIC:

ASSUMPTIONS: (1) Steady, incompressible flow, (2) Flow rate is independent of x, (3) Negligible chemical reactions, (4) Uniform perimeter P.

ANALYSIS: (a) Applying conservation of species to a differential control volume

$$\rho_{A,m}\, u_m\, A_c + dn_A = \left(\rho_{A,m} + \frac{d\rho_{A,m}}{dx}dx\right) u_m\, A_c$$

or, with $\quad u_m A_c = \dot{m}/\rho \quad$ and $\quad dn_A = h_m P\, dx\left(\rho_{A,s} - \rho_{A,m}\right)$,

$$\frac{\dot{m}}{\rho}\,\frac{d\rho_{A,m}}{dx}dx = h_m P\, dx\left(\rho_{A,s} - \rho_{A,m}\right).$$

Separating variables and integrating,

$$\int_{\rho_{A,mi}}^{\rho_{A,m}} \frac{d\rho_{A,m}}{\rho_{A,s} - \rho_{A,m}} = \int_0^x \frac{\rho\, h_m\, P}{\dot{m}}dx = \frac{\rho\, P}{\dot{m}}\int_0^x h_m\, dx$$

$$\ell n \frac{\rho_{A,s} - \rho_{A,m}}{\rho_{A,s} - \rho_{A,m,i}} = -\frac{\rho\, P x \bar{h}_m}{\dot{m}} \quad \text{or} \quad \frac{\rho_{A,s} - \rho_{A,m}(x)}{\rho_{A,s} - \rho_{A,m,i}} = \exp\left(-\frac{\rho\, P\, x}{\dot{m}}\bar{h}_m\right). \qquad (1)$$ <

(b) With $\Delta\rho_A \equiv \rho_{A,s} - \rho_{A,m}$,

$$n_A = (\dot{m}/\rho)\left(\rho_{A,m,o} - \rho_{A,m,i}\right) = -(\dot{m}/\rho)\left(\Delta\rho_{A,o} - \Delta\rho_{A,i}\right)$$

and from Eq. (1) with

$$-\frac{\dot{m}}{\rho} = P\, L\, \bar{h}_m / \ell n \frac{\Delta\rho_{A,o}}{\Delta\rho_{A,i}}$$

it follows that

$$n_A = \bar{h}_m\, P\, L\, \frac{\Delta\rho_{A,o} - \Delta\rho_{A,i}}{\ell n\left(\Delta\rho_{A,o} / \Delta\rho_{A,i}\right)}.$$ <

COMMENTS: Due to the addition of vapor, $\dot{m}$ will actually increase with x. However, if the specific humidity of the saturated gas-vapor mixture is small (as is usually the case), the change in $\dot{m}$ will be small.

PROBLEM 8.108

KNOWN: Flow rate and temperature of air. Tube diameter and length. Presence of water film on tube inner surface.

FIND: (a) Vapor density at tube outlet, (b) Evaporation rate.

SCHEMATIC:

D=0.01m
Dry air, B
$\dot{m}=3\times10^{-4}$ kg/s
$T_m=25°C$
$p=1$ atm, $\rho_{A,m,i}=0$
$\rho_{A,m,o}$
L=1m
Water film (A)
$\rho_{A,s}=\rho_{A,sat}(T_s)$
x

ASSUMPTIONS: (1) Steady, incompressible flow, (2) Constant flow rate, (3) Isothermal system (water film maintained at 25°C), (4) Fully developed flow.

PROPERTIES: *Table A-4*, Air (1 atm, 298K): $\rho = 1.1707$ kg/m^3, $\mu = 183.6 \times 10^{-7}$ N·s/m^2, $\nu = 15.71 \times 10^{-6}$ m^2/s; *Table A-6*, Water vapor (298K): $\rho_{A,sat} = 1/v_g = (1/44.25\ \text{m}^3/\text{kg}) = 0.0226$ kg/m^3; *Table A-8*, Air-vapor (298K): $D_{AB} = 26 \times 10^{-6}$ m^2/s; $Sc = \nu/D_{AB} = 0.60$.

ANALYSIS: (a) From Equation 8.81,

$$\rho_{A,m,o} = \rho_{A,s} - \left(\rho_{A,s} - \rho_{A,m,i}\right)\exp\left(-\frac{\pi DL}{\dot{m}}\bar{h}_m\right)$$

$$Re_D = \frac{4\dot{m}}{\pi D\mu} = \frac{4\times3\times10^{-4}\,\text{kg/s}}{\pi(0.01\text{ m})183.6\times10^{-7}\,\text{N}\cdot\text{s/m}^2} = 2080.$$

Flow is laminar and from the mass transfer analogy to Eq. 8.57,

$$\overline{Sh}_D = 1.86\left(\frac{Re_D Sc}{L/D}\right)^{1/3} = 1.86\left(\frac{2080\times0.60}{100}\right)^{1/3} = 4.31$$

$$\bar{h}_m = \frac{\overline{Sh}_D\, D_{AB}}{D} = \frac{4.31\times26\times10^{-6}\text{m}^2/\text{s}}{0.01\text{ m}} = 0.0112\text{ m/s}$$

$$\rho_{A,m,o} = 0.0226\text{ kg/m}^3$$

$$-0.0226\text{ kg/m}^3\exp\left(-\frac{\pi\times0.01\text{ m}\times1\text{ m}\times1.17\text{ kg/m}^3\times0.0112\text{ m/s}}{3\times10^{-4}\text{ kg/s}}\right) = 0.0169\text{ kg/m}^3 \quad <$$

(b) The evaporation rate is

$$n_A = u_m A_c\left(\rho_{A,m,o} - \rho_{A,m,i}\right) = \frac{\dot{m}}{\rho}\left(\rho_{A,m,o}\right) = \frac{3\times10^{-4}\text{ kg/s}}{1.1707\text{ kg/m}^3}0.0169\frac{\text{kg}}{\text{m}^3} = 4.33\times10^{-6}\text{ kg/s.} \quad <$$

COMMENTS: With

$$\Delta\rho_{A,o} = \Delta\rho_{A,i}\exp\left(-\frac{\pi DL\rho}{\dot{m}}\bar{h}_m\right) = 0.0226\exp\left(-\frac{\pi\times0.01\times1\times1.17}{3\times10^{-4}\text{ kg/s}}0.0112\frac{\text{m}}{\text{s}}\right) = 5.73\times10^{-3}\text{ kg/m}^3$$

the evaporation rate is

$$n_A = \bar{h}_m\pi DL\frac{\Delta\rho_{A,o} - \Delta\rho_{A,i}}{\ell n\left(\Delta\rho_{A,o}/\Delta\rho_{A,i}\right)} = 0.0112\frac{\text{m}}{\text{s}}\pi(0.01\text{ m})1\text{m}\frac{(0.00573 - 0.0226)\text{kg/m}^3}{\ell n(0.00573/0.0226)} = 4.33\times10^{-6}\text{kg/s}$$

which agrees with the result of part (b).

PROBLEM 8.109

KNOWN: Flow rate and temperature of air in circular tube of prescribed diameter. Inner tube surface is wetted. Flow is fully developed and inlet air is dry.

FIND: Tube length required to reach 99% of saturation.

SCHEMATIC:

ASSUMPTIONS: (1) Steady, incompressible flow, (2) Constant flow rate, (3) Water film is also at 25°C.

PROPERTIES: *Table A-4*, Air (298K, 1 atm): $\rho = 1.17\ \text{kg/m}^3$, $\mu = 183.6 \times 10^{-7}\ \text{N·s/m}^2$, $\nu = 15.71 \times 10^{-6}\ \text{m}^2/\text{s}$; *Table A-6*, Water vapor (298K): $\rho_{A,sat} = 1/v_g = (1/44.25\ \text{m}^3/\text{kg}) = 0.0226\ \text{kg/m}^3$; *Table A-8*, Air-vapor (298K): $D_{AB} = 26 \times 10^{-6}\ \text{m}^2/\text{s}$, $Sc = \nu/D_{AB} = 0.60$.

ANALYSIS: If $\rho_{A,m,o} = 0.99\ \rho_{A,s}$, it follows from Problem 8.81 that

$$\frac{\rho_{A,s} - 0.99\ \rho_{A,s}}{\rho_{A,s}} = 0.01 = \exp\left(-\frac{\pi\ DL\rho}{\dot{m}}\overline{h}_m\right).$$

With

$$Re_D = \frac{4\ \dot{m}}{\pi\ D\mu} = \frac{4 \times 10^{-3}\text{kg/s}}{\pi\ (0.01\ \text{m})\ 183.6 \times 10^{-7}\text{N} \cdot \text{s/m}^2} = 6935$$

the flow is turbulent and the mass transfer version of the Colburn equation is

$$Sh_D = 0.023\ Re_D^{4/5}\ Sc^{1/3} = 0.023(6935)^{4/5}(0.60)^{1/3} = 22.9$$

$$h_m = \frac{Sh_D\ D_{AB}}{D} = \frac{22.9 \times 26 \times 10^{-6}\text{m}^2/\text{s}}{0.01\ \text{m}} = 0.0595\ \text{m/s}.$$

Hence

$$0.01 = \exp\left(-\frac{\pi \times 0.01\ \text{m} \times L \times 1.17\ \text{kg/m}^3}{10^{-3}\text{kg/s}} 0.0595\ \text{m/s}\right)$$

$$0.01 = \exp(-2.188\ L)$$

$$L = 2.1\ \text{m}.$$

<

PROBLEM 8.110

KNOWN: Flow rate and temperature of atmospheric air in circular tube of prescribed diameter. Flow is fully developed, and air is dry. Inner tube surface is wetted.

FIND: (a) Tube length required to reach 99% saturation, (b) Heat rate needed to maintain tube surface at air temperature.

SCHEMATIC:

ASSUMPTIONS: (1) Steady, incompressible flow, (2) Constant flow rate.

PROPERTIES: *Table A-4*, Air (298K, 1 atm): $\rho = 1.17\ \text{kg/m}^3$, $\mu = 183.6 \times 10^{-7}\ \text{N·s/m}^2$, $\nu = 15.71 \times 10^{-6}\ \text{m}^2\text{/s}$; *Table A-6*, Water vapor (298K): $v_g = 44.25\ \text{m}^3\text{/kg}$, $\rho_{A,sat} = 1/v_g = 0.0226\ \text{kg/m}^3$, $h_{fg} = 2443\ \text{kJ/kg}$; *Table A-8*, Air-vapor (298K): $D_{AB} = 26 \times 10^{-6}\ \text{m}^2\text{/s}$, $Sc = \nu/D_{AB} = 0.60$.

ANALYSIS: (a) If $\rho_{A,m,o} = 0.99\ \rho_{A,s}$, it follows from Problem 8.107 that

$$\frac{\rho_{A,s} - 0.99\ \rho_{A,s}}{\rho_{A,s}} = 0.01 = \exp\left(-\frac{\pi\ DL\rho}{\dot{m}}\overline{h}_m\right).$$

With $$\text{Re}_D = \frac{4\ \dot{m}}{\pi\ D\mu} = \frac{4\times10^{-3}\ \text{kg/s}}{\pi\,(0.02\text{m})\ 183.6\times10^{-7}\text{N}\cdot\text{s/m}^2} = 3467,$$

The flow is turbulent (weakly) and the mass transfer analog to the Colburn equation is

$$\text{Sh}_D = 0.023\ \text{Re}_D^{4/5}\ \text{Sc}^{1/3} = 0.023(3467)^{4/5}(0.60)^{1/3} = 13.2$$

$$h_m = \frac{\text{Sh}_D\ D_{AB}}{D} = \frac{13.2\times26\times10^{-6}\text{m}^2/\text{s}}{0.02\ \text{m}} = 0.0172\ \text{m/s}.$$

Hence,

$$L = -\frac{\dot{m}}{\pi\ D\rho\overline{h}_m}\ell\text{n}(0.01) = -\frac{10^{-3}\text{kg/s}\times\ell\text{n}(0.01)}{\pi\,(0.02\text{m})1.17\ \text{kg/m}^3(0.0172\ \text{m/s})} = 3.64\text{m}. \qquad <$$

(b) The required heat rate is

$$q = n_A\ h_{fg} \qquad n_A = \overline{h}_m\pi\ DL\frac{\Delta\rho_{A,o} - \Delta\rho_{A,i}}{\ell\text{n}(\Delta\rho_{A,o}/\Delta\rho_{A,i})}$$

$$n_A = 0.0172\text{m/s}\times\pi\,(0.02\text{m})3.64\text{m}\frac{0.01\ \rho_{A,s} - \rho_{A,s}}{\ell\text{n}(0.01)}$$

$$n_A = -8.542\times10^{-4}\text{m}^3/\text{s}\left(-0.99\times0.0226\ \text{kg/m}^3\right) = 1.91\times10^{-5}\text{kg/s}$$

$$q = n_A\ h_{fg} = 1.91\times10^{-5}\text{kg/s}\times2.443\times10^6\ \text{J/kg} = 46.7\ \text{W}. \qquad <$$

COMMENTS: The evaporation rate is low; hence the heat requirement is small.

PROBLEM 8.111

KNOWN: Tube length, diameter and temperature. Air temperature and velocity. Saturation pressure of thin liquid film and properties of vapor.

FIND: (a) Partial pressure and mass fraction of vapor at tube exit, (b) Mass rate at which liquid is removed from the tube.

SCHEMATIC:

D = 0.05 m
Dry air (B)
u_m = 0.5 m/s
T_m = 300 K
$p_{A,o}$, $\rho_{A,m,o}$
L = 5 m
Volatile liquid (A)
$p_{A,sat}$ = 15 mm Hg, $p_{A,i}$ = 0

ASSUMPTIONS: (1) System is isothermal at 300K, (2) Steady, incompressible flow, (3) Perfect gas behavior, (4) Mass flow rate is independent of x.

PROPERTIES: *Table A-4*, Air (300K, 1 atm): ρ = 1.16 kg/m^3, $\nu = 15.9 \times 10^{-6}$ m^2/s. Prescribed, Vapor (300K): $p_{A,sat}$ = 15 mm Hg, $\mathcal{M}_A$ = 70 kg/kmol, $D_{AB} = 10^{-5}$ m^2/s.

ANALYSIS: (a) With the vapor assumed to behave as an ideal gas, $p_A = C_A \Re T = \rho_A(\Re/\mathcal{M}_A)T$, and isothermal conditions, the vapor pressure at the outlet may be obtained from the expression

$$\frac{p_{A,sat} - p_{A,o}}{p_{A,sat} - p_{A,i}} = \frac{\rho_{A,s} - \rho_{A,m,o}}{\rho_{A,s} - \rho_{A,m,i}} = \exp\left(-\frac{\rho \pi D L \bar{h}_m}{\dot{m}}\right)$$

where $\dot{m} = \rho u_m A_c = 1.16\,\text{kg/m}^3 \times 0.5\,\text{m/s} \times \pi\,(0.05\text{m})^2/4 = 1.14\times10^{-3}\,\text{kg/s}$. With $Re_D = u_m D/\nu$ = 0.5 m/s × 0.05m/15.9 × 10^{-6} m^2/s = 1570, the flow is laminar and $\bar{h}_m$ may be determined from the mass transfer analog to Eq. 8.57. With Sc = ν/D_A = 1.59 and $[Re_D\,Sc/(L/D)]^{1/3}$ = 2.92 > 2

$$\bar{h}_m = \frac{\overline{Sh}_D D_{AB}}{D} = 1.86\left(\frac{Re\,Sc}{L/D}\right)^{1/3}\frac{D_{AB}}{D} = 1.86 \times 2.92 \times \frac{10^{-5}\text{m}^2/\text{s}}{0.05\text{m}} = 1.09\times10^{-3}\,\text{m/s}$$

Hence, with $p_{A,i} = 0$

$$p_{A,o} = p_{A,sat}\left[1 - \exp\left(-\frac{\rho\pi D L \bar{h}_m}{\dot{m}}\right)\right] = 15\,\text{mm Hg}\,[1 -$$

$$\left.\exp\left(-\frac{1.16\,\text{kg/m}^3 \times \pi \times 0.05\text{m} \times 5\text{m} \times 1.09\times10^{-3}\text{m/s}}{1.14\times10^{-3}\,\text{kg/s}}\right)\right] = 8.7\,\text{mm Hg} \qquad <$$

The corresponding mass density of the vapor is

$$\rho_{A,m,o} = \frac{p_{A,o}\mathcal{M}_A}{\Re T} = \frac{8.7\,\text{mm Hg} \times 70\,\text{kg/kmol}}{(760\,\text{mm Hg/atm})(0.082\,\text{m}^3\cdot\text{atm/kmol}\cdot\text{K})300\text{K}} = 0.0326\,\text{kg/m}^3 \qquad <$$

(b) The evaporation rate is

$$n_A = u_m A_c\left(\rho_{A,m,o} - \rho_{A,m,i}\right) = 0.5\text{m/s} \times 1.96\times10^{-3}\text{m}^2 \times 0.0326\,\text{kg/m}^3 = 3.20\times10^{-5}\,\text{kg/s} \qquad <$$

COMMENTS: (1) Since the evaporation rate ($n_A = 3.2 \times 10^{-5}$ kg/s) is much less than the air flow rate ($\dot{m} = 1.14 \times 10^{-3}$ kg/s), the assumption of a fixed flow rate is reasonable. (2) The evaporation rate is also given by $n_A = \bar{h}_m\,\pi D L\,\Delta\rho_{A,lm} = -\bar{h}_m\,\pi D L\,\rho_{A,m,o}/\ln[(p_{A,sat} - p_{A,o})/p_{A,sat}] = 3.22 \times 10^{-5}$ kg/s, which agrees with the calculation of part (b).

PROBLEM 8.112

KNOWN: Air flow rate through trachea of diameter D and length L.

FIND: (a) Average mass transfer convection coefficient, $\overline{h}_m$, and (b) Rate of water loss per day (liter/day).

SCHEMATIC:

Dry air (B)
$T_m = 310$ K
$\dot{\forall}_B = 10$ liter/min
$\rho_{A,m,i} = 0$
Trachea
D = 20 mm
L = 125 mm
$\rho_{A,m,o}$
Water (A)
$\rho_{A,s}$ (T_s)
$T_s = 37^\circ$C

ASSUMPTIONS: (1) Trachea can be approximated as a smooth tube with uniform surface temperature, (2) Laminar, fully developed flow, (3) Trachea inner surface is saturated with water at body temperature, $T_s = 37°C$, (4) Negligible water vapor in air at 310 K during inhalation, and (5) Heat-mass analogy is applicable.

PROPERTIES: *Table A-4*, Air (310 K, 1 atm): $\rho_B = 1.128$ kg/m^3, $\mu = 1.893 \times 10^{-5}$ N·s/m^2; *Table A-6*, Water ($T_s = 37°C = 310$ K): $\rho_{A,f} = 993$ kg/m^3, $\rho_{A,g} = 0.04361$ kg/m^3; *Table A-8*, Water-vapor air (310 K, 1 atm): $D_{AB} = 0.26 \times 10^{-4}\,(310/298)^{3/2} = 2.76 \times 10^{-5}$ m^2/s.

ANALYSIS: (a) Begin by characterizing the air (B) flow in the trachea modeled as a smooth tube,

$$Re_D = \frac{4\dot{m}}{\pi D\mu} = \frac{4\dot{\forall}\rho_B}{\pi D\mu}$$

$$Re_D = \frac{4\times 10\ \text{liter/min}\times 10^{-3}\text{m}^3/\text{liter}\times 1\text{min}/60\text{s}\times 1.128\,\text{kg/m}^3}{\pi\times 0.020\ \text{m}\times 1.893\times 10^{-5}\text{N}\cdot\text{s/m}^2} = 632$$

Hence, the flow is laminar, and for fully developed conditions and invoking the heat-mass analogy

$$Nu_D = Sh_D = 3.66 \qquad Sh = \overline{h}_m D/D_{AB}$$

$$\overline{h}_m = 3.66\ D_{AB}/D = 3.66\times 2.76\times 10^{-5}\text{m}^2/\text{s}/0.020\ \text{m} = 0.0050\ \text{m/s}$$ <

(b) The species (A) transfer rate equation, Eq. 8.75, has the form

$$n_A = \overline{h}_m A_s\,\Delta\rho_{A,\ell m}$$

$$\Delta\rho_{A,\ell m} = \frac{\left(\rho_{A,s}-\rho_{A,m,o}\right)-\left(\rho_{A,s}-\rho_{A,m,i}\right)}{\ell m\left[\left(\rho_{A,s}-\rho_{A,m,o}\right)/\left(\rho_{A,s}-\rho_{A,m,i}\right)\right]}$$

where the mean outlet species density, $\rho_{A,m,o}$, can be determined from Eq. 8.78

$$\frac{\rho_{A,s}-\rho_{A,m,o}}{\rho_{A,s}-\rho_{A,m,i}} = \exp\left(-\frac{\overline{h}_m\rho P}{\dot{m}}\right)$$

where $\dot{m}/\rho = u_m A_c = \forall_B$. Substituting numerical values with $P = \pi D$, find

$$\rho_{A,m,o} = 0.009233 \qquad n_A = 1.54\times 10^{-6}\,\text{kg/s}$$

The volumetric rate of water loss on a daily basis, assuming a 12 hour inhalation period, is

$$\dot{\forall}_A = \left(1.54\times 10^{-6}\text{kg/s}/993\ \text{kg/m}^3\right)\times 10^3\text{liter/m}^3\times\left(3600\,\text{s/h}\times 12\ \text{h/day}\right)$$

$$\dot{V}_A = 0.067\ \text{liter/day}$$

PROBLEM 8.113

KNOWN: Air (species B) is in fully developed, laminar flow as it enters a circular tube wetted with liquid A (water). Tube length and diameter. Flow rate of air and system temperature.

FIND: (a) Governing differential equation for species transfer, (b) Heat transfer analog and an expression for $\overline{Sh}_D$, (c) General expression for $\rho_{A,m,o}$, (d) Value of $\rho_{A,m,o}$ for prescribed conditions.

SCHEMATIC:

ASSUMPTIONS: (1) Steady, incompressible flow, (2) Flow rate is independent of x, (3) Laminar, fully developed flow (hydrodynamically), (4) Isothermal conditions, (5) Dry air at inlet.

PROPERTIES: *Table A.4*, Air (298 K, 1 atm): $\rho = 1.1707$ kg/m^3, $\mu = 183.6 \times 10^{-7}$ N·s/m^2, $\nu = 15.71 \times 10^{-6}$ m^2/s; *Table A.6*, Water vapor (298 K): $\rho_{A,sat} = 1/v_g = 0.0266$ kg/m^3; *Table A.8*, Air-vapor (298 K): $D_{AB} = 26 \times 10^{-6}$ m^2/s, $Sc = \nu/D_{AB} = 0.60$.

ANALYSIS: (a) The governing differential equation may be inferred by analogy to Eq. 8.48. In this case, the dependent variable is the vapor mass density, ρ_A (x,r), and the diffusivity is D_{AB}. With v = 0 for fully-developed flow, it follows that

$$u\frac{\partial \rho_A}{\partial x} = \frac{D_{AB}}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \rho_A}{\partial r}\right)$$ <

The entrance condition is

$$\rho_A(0,r) = 0$$ <

and the boundary conditions are

$$\rho_A(r_o, x) = \rho_{A,s} \qquad \partial\rho_A/\partial x\big|_{r=0} = 0$$ <

(b) The foregoing conditions are analogous to those of the thermal entry length condition associated with Eq. 8.56. Invoking this analogy the average Sherwood number for laminar, fully developed flow is

$$\overline{Sh}_D = 3.66 \frac{0.0668(D/L)Re_D\,Sc}{1+0.04\left[(D/L)Re_D\,Sc\right]^{2/3}}$$ <

(c) Applying conservation of species to the differential control volume,

$$\rho_{A,m}u_m A_c + dn_A = \left(\rho_{A,m} + \frac{d\rho_{A,m}}{dx}dx\right)u_m A_c$$

or, with $u_m A_c = \dot{m}/\rho$ and $dn_A = h_m \pi D dx\left(\rho_{A,s} - \rho_{A,m}\right)$

$$\frac{\dot{m}}{\rho}\frac{d\rho_{A,m}}{dx}dx = h_m \pi D dx\left(\rho_{A,s} - \rho_{A,m}\right)$$

Continued...

PROBLEM 8.113 (Cont.)

$$\int_{\rho_{A,m,i}}^{\rho_{A,m}} \frac{d\rho_{A,m}}{\rho_{A,s}-\rho_{A,m}} = \int_o^x \frac{\rho\pi D h_m}{\dot{m}} dx$$

or

$$\frac{\rho_{A,s}-\rho_{A,m}(x)}{\rho_{A,s}-\rho_{A,m,i}} = \exp\left(-\frac{\rho\pi D x \bar{h}_m(x)}{\dot{m}}\right)$$

at x = L,

$$\frac{\rho_{A,s}-\rho_{A,m,o}}{\rho_{A,s}-\rho_{A,m,i}} = \exp\left(-\frac{\rho\pi D L \bar{h}_m}{\dot{m}}\right)$$ <

(d) For the prescribed conditions, $Re_D = 4\dot{m}/\pi D\mu = 4\left(2.5\times10^{-4}\text{ kg/s}\right)/\pi(0.01\text{m})183.6\times10^{-7}\text{ N}\cdot\text{s/m}^2 =$ 1734 and $(D/L)Re_DSc$ = (0.01 m/1 m)1734(0.6) = 10.4. Hence,

$$\overline{Sh}_D = 3.66 + \frac{0.0668(10.4)}{1+0.04(10.4)^{2/3}} = 4.24$$

$$\bar{h}_m = \overline{Sh}_D\left(D_{AB}/D\right) = 4.24\left(26\times10^{-6}\text{ m}^2/\text{s}/0.01\text{m}\right) = 0.011\text{m/s}$$

Hence,

$$\frac{\rho_{A,s}-\rho_{A,m,o}}{\rho_{A,s}-\rho_{A,m,i}} = \exp\left(-\frac{1.1707\text{ kg/m}^3\times\pi\times0.01\text{m}\times1\text{m}\times0.011\text{m/s}}{2.5\times10^{-4}\text{ kg/s}}\right) = 0.198$$

$$\rho_{A,m,o} = \rho_{A,s} - 0.198\left(\rho_{A,s}-\rho_{A,m,i}\right) = 0.0226\text{ kg/m}^3(1-0.198) = 0.0181\text{kg/m}^3$$ <

COMMENTS: Due to evaporation, $\dot{m}$ actually increases with increasing x. However, the increase is small, and the assumption of fixed $\dot{m}$ is good.

PROBLEM 9.1

KNOWN: Tabulated values of density for water and definition of the volumetric thermal expansion coefficient, β.

FIND: Value of the volumetric expansion coefficient at 300K; compare with tabulated values.

PROPERTIES: *Table A-6*, Water (300K): $\rho = 1/v_f = 1/1.003 \times 10^{-3}\ m^3/kg = 997.0\ kg/m^3$, $\beta = 276.1 \times 10^{-6}\ K^{-1}$; (295K): $\rho = 1/v_f = 1/1.002 \times 10^{-3}\ m^3/kg = 998.0\ kg/m^3$; (305K): $\rho = 1/v_f = 1/1.005 \times 10^{-3}\ m^3/kg = 995.0\ kg/m^3$.

ANALYSIS: The volumetric expansion coefficient is defined by Eq. 9.4 as

$$\beta = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_p .$$

The density change with temperature at constant pressure can be estimated as

$$\left(\frac{\partial \rho}{\partial T}\right)_p \approx \left(\frac{\rho_1 - \rho_2}{T_1 - T_2}\right)_p$$

where the subscripts (1,2) denote the property values just above and below, respectively, the condition for T = 300K denoted by the subscript (o). That is,

$$\beta_o \approx -\frac{1}{\rho_o}\left(\frac{\rho_1 - \rho_2}{T_1 - T_2}\right)_p .$$

Substituting numerical values, find

$$\beta_o \approx \frac{-1}{997.0\ kg/m^3}\ \frac{(995.0 - 998.0)\ kg/m^3}{(305 - 295)\,K} = 300.9 \times 10^{-6}\ K^{-1}.$$ <

Compare this value with the tabulation, $\beta = 276.1 \times 10^{-6}\ K^{-1}$, to find our estimate is 8.7% high.

COMMENTS: (1) The poor agreement between our estimate and the tabulated value is due to the poor precision with which the density change with temperature is estimated. The tabulated values of β were determined from accurate equation of state data.

(2) Note that β is negative for T < 275K. Why? What is the implication for free convection?

PROBLEM 9.2

KNOWN: Object with specified characteristic length and temperature difference above ambient fluid.

FIND: Grashof number for air, hydrogen, water, ethylene glycol for a pressure of 1 atm.

SCHEMATIC:

$T_s - T_\infty = \Delta T = 25°C$

Quiescent fluid
T_∞, 1atm

$L = 0.25m$

ASSUMPTIONS: (1) Thermophysical properties evaluated at T_f = 350K, (2) Perfect gas behavior, ($\beta = 1/T_f$).

PROPERTIES: Evaluate at 1 atm, T_f = 350K:

Table A-4, Air: $\nu = 20.92 \times 10^{-6}$ m^2/s; Hydrogen: $\nu = 143 \times 10^{-6}$ m^2/s
Table A-6, Water (Sat. liquid): $\nu = \mu_f v_f = 37.5 \times 10^{-6}$ m^2/s, $\beta_f = 0.624 \times 10^{-3}$ K^{-1}
Table A-5, Ethylene glycol: $\nu = 3.17 \times 10^{-6}$ m^2/s, $\beta = 0.65 \times 10^{-3}$ K^{-1}.

ANALYSIS: The Grashof number is given by Eq. 9.12,

$$Gr_L = \frac{g\beta\,(T_s - T_\infty)\,L^3}{\nu^2}.$$

Substituting numerical values for *air* with $\beta = 1/T_f$, find

$$Gr_{L,air} = \frac{9.8\text{m/s}^2 \times (1/350\text{K})\ (25\text{K})\ (0.25\text{m})^3}{\left(20.92\times10^{-6}\ \text{m}^2/\text{s}\right)^2}$$

$$Gr_{L,air} = 2.50\times10^7.$$ <

Performing similar calculations for the other fluids, find

$$Gr_{L,hyd} = 5.35\times10^5$$ <

$$Gr_{L,water} = 1.70\times10^6$$ <

$$Gr_{L,eth} = 2.48\times10^8.$$ <

COMMENTS: Higher values of Gr_L imply increased free convection flows. However, other properties affect the value of the heat transfer coefficient.

PROBLEM 9.3

KNOWN: Relation for the Rayleigh number.

FIND: Rayleigh number for four fluids for prescribed conditions.

SCHEMATIC:

ASSUMPTIONS: (1) Perfect gas behavior for specified gases.

PROPERTIES: *Table A-4*, Air (400K, 1 atm): $\nu = 26.41 \times 10^{-6}$ m^2/s, $\alpha = 38.3 \times 10^{-6}$ m^2/s, $\beta = 1/T = 1/400K = 2.50 \times 10^{-3}$ K^{-1}; *Table A-4,* Helium (400K, 1 atm): $\nu = 199 \times 10^{-6}$ m^2/s, $\alpha = 295 \times 10^{-6}$ m^2/s, $\beta = 1/T = 2.50 \times 10^{-3}$ K^{-1}; *Table A-5*, Glycerin (12°C = 285K): $\nu = 2830 \times 10^{-6}$ m^2/s, $\alpha = 0.964 \times 10^{-7}$ m^2/s, $\beta = 0.475 \times 10^{-3}$ K^{-1}; *Table A-6*, Water (37°C = 310K, sat. liq.): $\nu = \mu_f v_f = 695 \times 10^{-6}$ $N \cdot s/m^2 \times 1.007 \times 10^{-3}$ $m^3/kg = 0.700 \times 10^{-6}$ m^2/s, $\alpha = k_f v_f/c_{p,f} = 0.628$ $W/m \cdot K \times 1.007 \times 10^{-3}$ $m^3/kg/4178$ $J/kg \cdot K = 0.151 \times 10^{-6}$ m^2/s, $\beta_f = 361.9 \times 10^{-6}$ K^{-1}.

ANALYSIS: The Rayleigh number, a dimensionless parameter used in free convection analysis, is defined as the product of the Grashof and Prandtl numbers.

$$Ra_L \equiv Gr \cdot Pr = \frac{g b \Delta T L^3}{n^2} \frac{m c_p}{k} = \frac{g b \Delta T L^3}{n^2} \cdot \frac{(n r) c_p}{k} = \frac{g b \Delta T L^3}{n a}$$

where $\alpha = k/\rho c_p$ and $\nu = \mu/\rho$. The numerical values for the four fluids follow:

Air (400K, 1 atm)

$$Ra_L = 9.8 m/s^2 \ (1/400K) \ 30K (0.01m)^3 / 26.41 \times 10^{-6} \ m^2/s \times 38.3 \times 10^{-6} \ m^2/s = 727 \quad <$$

Helium (400K, 1 atm)

$$Ra_L = 9.8 m/s^2 (1/400K) \ 30K (0.01m)^3 / 199 \times 10^{-6} m^2/s \times 295 \times 10^{-6} m^2/s = 12.5 \quad <$$

Glycerin (285K)

$$Ra_L = 9.8 m/s^2 \left(0.475 \times 10^{-3} K^{-1}\right) 30K \ (0.01m)^3 / 2830 \times 10^{-6} m^2/s \times 0.964 \times 10^{-7} m^2/s = 512 \quad <$$

Water (310K)

$$Ra_L = 9.8 m/s^2 \left(0.362 \times 10^{-3} K^{-1}\right) 30K \ (0.01m)^3 / 0.700 \times 10^{-6} m^2/s \times 0.151 \times 10^{-6} m^2/s = 9.35 \times 10^5. \quad <$$

COMMENTS: (1) Note the wide variation in values of Ra for the four fluids. A large value of Ra implies enhanced free convection, however, other properties affect the value of the heat transfer coefficient.

PROBLEM 9.4

KNOWN: Form of the Nusselt number correlation for natural convection and fluid properties.

FIND: Expression for figure of merit F_N and values for air, water and a dielectric liquid.

PROPERTIES: Prescribed. Air: k = 0.026 W/m·K, b = 0.0035 K^{-1}, ν = 1.5 × 10^{-5} m^2/s, Pr = 0.70. Water: k = 0.600 W/m·K, b = 2.7 × 10^{-4} K^{-1}, ν = 10^{-6} m^2/s, Pr = 5.0. Dielectric liquid: k = 0.064 W/m·K, b = 0.0014 K^{-1}, ν = 10^{-6} m^2/s, Pr = 25

ANALYSIS: With $Nu_L \sim Ra^n$, the convection coefficient may be expressed as

$$h \sim \frac{k}{L}\left(\frac{g b \Delta T L^3}{a n}\right)^n \sim \frac{\left(g \Delta T L^3\right)^n}{L}\left(\frac{k\, b^n}{a^n n^n}\right)$$

The figure of merit is therefore

$$F_N = \frac{k\, b^n}{a^n n^n}$$ <

and for the three fluids, with n = 0.33 and $a = n/\mathrm{Pr}$,

$F_N \left(W \cdot s^{2/3}/m^{7/3} \cdot K^{4/3}\right)$	Air	Water	Dielectric
	5.8	663	209

<

Water is clearly the superior heat transfer fluid, while air is the least effective.

COMMENTS: The figure of merit indicates that heat transfer is enhanced by fluids of large k, large b and small values of a and ν.

PROBLEM 9.5

KNOWN: Heat transfer rate by convection from a vertical surface, 1m high by 0.6m wide, to quiescent air that is 20K cooler.

FIND: Ratio of the heat transfer rate for the above case to that for a vertical surface that is 0.6m high by 1m wide with quiescent air that is 20K warmer.

SCHEMATIC:

ASSUMPTIONS: (1) Thermophysical properties independent of temperature; evaluate at 300K; (2) Negligible radiation exchange with surroundings, (3) Quiescent ambient air.

PROPERTIES: *Table A-4*, Air (300K, 1 atm): $\nu = 15.89 \times 10^{-6}\ m^2/s$, $\alpha = 22.5 \times 10^{-6}\ m^2/s$.

ANALYSIS: The rate equation for convection between the plates and quiescent air is

$$q = \bar{h}_L\ A_s \Delta T \tag{1}$$

where ΔT is either $(T_s - T_\infty)$ or $(T_\infty - T_s)$; for both cases, $A_s = Lw$. The desired heat transfer ratio is then

$$\frac{q_1}{q_2} = \frac{\bar{h}_{L1}}{\bar{h}_{L2}}. \tag{2}$$

To determine the dependence of $\bar{h}_L$ on geometry, first calculate the Rayleigh number,

$$Ra_L = g\ b\ \Delta T L^3 / n a \tag{3}$$

and substituting property values at 300K, find,

Case 1: $Ra_{L1} = 9.8\ m/s^2\ (1/300K)\ 20K\ (1m)^3 / 15.89 \times 10^{-6}\ m^2/s \times 22.5 \times 10^{-6}\ m^2/s = 1.82 \times 10^9$

Case 2: $Ra_{L2} = Ra_{L1}\ (L_2/L_1)^3 = 1.82 \times 10^4\ (0.6m/1.0m)^3 = 3.94 \times 10^8$.

Hence, Case 1 is turbulent and Case 2 is laminar. Using the correlation of Eq. 9.24,

$$\overline{Nu}_L = \frac{\bar{h}_L L}{k} = C\left(Ra_L\right)^n \qquad \bar{h}_L = \frac{k}{L} C\ Ra_L^n \tag{4}$$

where for *Case 1*: $C_1 = 0.10$, $n_1 = 1/3$ and for *Case 2*: $C_2 = 0.59$, $n_2 = 1/4$. Substituting Eq. (4) into the ratio of Eq. (2) with numerical values, find

$$\frac{q_1}{q_2} = \frac{\left(C_1 / L_1\right) Ra_{L1}^{n_1}}{\left(C_2 / L_2\right) Ra_{L2}^{n_2}} = \frac{(0.10/1m)\left(1.82\times10^9\right)^{1/3}}{(0.59/0.6m)\left(3.94\times10^8\right)^{1/4}} = 0.881 \qquad <$$

COMMENTS: Is this result to be expected? How do you explain this effect of plate orientation on the heat rates?

PROBLEM 9.6

KNOWN: Large vertical plate with uniform surface temperature of 130°C suspended in quiescent air at 25°C and atmospheric pressure.

FIND: (a) Boundary layer thickness at 0.25 m from lower edge, (b) Maximum velocity in boundary layer at this location and position of maximum, (c) Heat transfer coefficient at this location, (d) Location where boundary layer becomes turbulent.

SCHEMATIC:

ASSUMPTIONS: (1) Isothermal, vertical surface in an extensive, quiescent medium, (2) Boundary layer assumptions valid.

PROPERTIES: *Table A-4*, Air $\left(T_f = \left(T_s + T_\infty\right)/2 = 350\text{K, 1 atm}\right)$: $\nu = 20.92 \times 10^{-6}$ m^2/s, k = 0.030 W/m·K, Pr = 0.700.

ANALYSIS: (a) From the similarity solution results, Fig. 9.4 (see above right), the boundary layer thickness corresponds to a value of $\eta \approx 5$. From Eqs. 9.13 and 9.12,

$$y = \eta x \left(Gr_x / 4\right)^{-1/4} \tag{1}$$

$$Gr_x = g\beta\left(T_s - T_\infty\right)x^3/\nu^2 = 9.8\frac{m}{s^2}\times\frac{1}{350K}\left(130-25\right)K\,x^3/\left(20.92\times10^{-6}m^2/s\right)^2 = 6.718\times10^9\,x^3 \tag{2}$$

$$y \approx 5\left(0.25m\right)\left(6.718\times10^9\left(0.25\right)^3/4\right)^{-1/4} = 1.746\times10^{-2}m = 17.5\text{ mm.} \tag{3}$$ <

(b) From the similarity solution shown above, the maximum velocity occurs at $\eta \approx 1$ with $f'(\eta) = 0.275$. From Eq.9.15, find

$$u = \frac{2\nu}{x}Gr_x^{1/2}\,f'(\eta) = \frac{2\times20.92\times10^{-6}m^2/s}{0.25m}\left(6.718\times10^9\left(0.25\right)^3\right)^{1/2}\times0.275 = 0.47\text{ m/s.}$$ <

The maximum velocity occurs at a value of $\eta = 1$; using Eq. (3), it follows that this corresponds to a position in the boundary layer given as

$$y_{max} = 1/5\ \left(17.5\text{ mm}\right) = 3.5\text{ mm.}$$ <

(c) From Eq. 9.19, the local heat transfer coefficient at x = 0.25 m is

$$Nu_x = h_x x/k = \left(Gr_x/4\right)^{1/4}\ g\left(Pr\right) = \left(6.718\times10^9\left(0.25\right)^3/4\right)^{1/4}0.586 = 41.9$$

$$h_x = Nu_x k/x = 41.9\times0.030\text{ W/m}\cdot\text{K}/0.25\text{ m} = 5.0\text{ W/m}^2\cdot\text{K.}$$ <

The value for g(Pr) is determined from Eq. 9.20 with Pr = 0.700.

(d) According to Eq. 9.23, the boundary layer becomes turbulent at x_c given as

$$Ra_{x,c} = Gr_{x,c}Pr \approx 10^9 \qquad x_c \approx \left[10^9/6.718\times10^9\left(0.700\right)\right]^{1/3} = 0.60\text{ m.}$$ <

COMMENTS: Note that $\beta = 1/T_f$ is a suitable approximation for air.

PROBLEM 9.7

KNOWN: Thin, vertical plates of length 0.15m at 54°C being cooled in a water bath at 20°C.

FIND: Minimum spacing between plates such that no interference will occur between free-convection boundary layers.

SCHEMATIC:

ASSUMPTIONS: (a) Water in bath is quiescent, (b) Plates are at uniform temperature.

PROPERTIES: *Table A-6*, Water ($T_f = (T_s + T_\infty)/2 = (54 + 20)°C/2 = 310K$): $\rho = 1/v_f = 993.05$ kg/m^3, $\mu = 695 \times 10^{-6}$ N·s/m^2, $\nu = \mu/\rho = 6.998 \times 10^{-7}$ m^2/s, Pr = 4.62, $\beta = 361.9 \times 10^{-6}$ K^{-1}.

ANALYSIS: The minimum separation distance will be twice the thickness of the boundary layer at the trailing edge where x = 0.15m. Assuming laminar, free convection boundary layer conditions, the similarity parameter, η, given by Eq. 9.13, is

$$h = \frac{y}{x}\left(Gr_x / 4\right)^{1/4}$$

where y is measured normal to the plate (see Fig. 9.3). According to Fig. 9.4, the boundary layer thickness occurs at a value $\eta \approx 5$.
It follows then that,

$$y_{bl} = h\ x\ \left(Gr_x / 4\right)^{-1/4}$$

where $$Gr_x = \frac{g\, b\ \left(T_s - T_\infty\right) x^3}{n^2}$$

$$Gr_x = 9.8\ \text{m/s}^2 \times 361.9\times10^{-6}\ \text{K}^{-1}\left(54-20\right)\text{K}\times\left(0.15\text{m}\right)^3 / \left(6.998\times10^{-7}\ \text{m}^2/\text{s}\right)^2 = 8.310\times10^8.$$ <

Hence, $$y_{bl} = 5\times0.15\text{m}\left(8.310\times10^8/4\right)^{-1/4} = 6.247\times10^{-3}\text{m} = 6.3\ \text{mm}$$

and the minimum separation is

$$d = 2\ y_{bl} = 2\times6.3\ \text{mm} = 12.6\ \text{mm}.$$ <

COMMENTS: According to Eq. 9.23, the critical Grashof number for the onset of turbulent conditions in the boundary layer is $Gr_{x,c}\,Pr \approx 10^9$. For the conditions above, $Gr_x\,Pr = 8.31 \times 10^8 \times 4.62 = 3.8 \times 10^9$. We conclude that the boundary layer is indeed turbulent at x = 0.15m and our calculation is only an estimate which is likely to be low. Therefore, the plate separation should be greater than 12.6 mm.

PROBLEM 9.8

KNOWN: Square aluminum plate at 15°C suspended in quiescent air at 40°C.

FIND: Average heat transfer coefficient by two methods – using results of boundary layer similarity and results from an empirical correlation.

SCHEMATIC:

Quiescent air $T_\infty = 40°C$; $T_s = 15°C$; Plate, 200mm square, 5mm thickness; L

ASSUMPTIONS: (1) Uniform plate surface temperature, (2) Quiescent room air, (3) Surface radiation exchange with surroundings negligible, (4) Perfect gas behavior for air, $\beta = 1/T_f$.

PROPERTIES: *Table A-4*, Air ($T_f = (T_s + T_\infty)/2 = (40 + 15)°C/2 = 300K$, 1 atm): $\nu = 15.89 \times 10^{-6}$ m^2/s, k = 0.0263 W/m·K, $\alpha = 22.5 \times 10^{-6}$ m^2/s, Pr = 0.707.

ANALYSIS: Calculate the Rayleigh number to determine the boundary layer flow conditions,

$$Ra_L = g\, b\; \Delta T\, L^3 / n\; a$$

$$Ra_L = 9.8\ m/s^2\,(1/300K)\;(40-15)°C\,(0.2m)^3 / \left(15.89\times10^{-6} m^2/s\right)\;\left(22.5\times10^{-6} m^2/s\right) = 1.827\times10^7$$

where $\beta = 1/T_f$ and $\Delta T = T_\infty - T_s$. Since $Ra_L < 10^9$, the flow is laminar and the *similarity solution* of Section 9.4 is applicable. From Eqs. 9.21 and 9.20,

$$\overline{Nu}_L = \frac{\bar{h}_L\, L}{k} = \frac{4}{3}\left(Gr_L/4\right)^{1/4}\; g(Pr)$$

$$g(Pr) = \frac{0.75\, Pr^{1/2}}{\left[0.609 + 1.221\, Pr^{1/2} + 1.238\, Pr\right]^{1/4}}$$

and substituting numerical values with $Gr_L = Ra_L/Pr$, find

$$g(Pr) = 0.75(0.707)^{1/2} / \left[0.609 + 1.22(0.707)^{1/2} + 1.238\times0.707\right]^{1/4} = 0.501$$

$$\bar{h}_L = \left(\frac{0.0263\ W/m\cdot K}{0.20m}\right)\times\frac{4}{3}\left(\frac{1.827\times10^7/0.707}{4}\right)^{1/4}\times0.501 = 4.42\ W/m^2\cdot K. \quad <$$

The appropriate empirical correlation for estimating $\bar{h}_L$ is given by Eq. 9.27,

$$\overline{Nu}_L = \frac{\bar{h}_L\, L}{k} = 0.68 + \frac{0.670\, Ra_L^{1/4}}{\left[1+(0.492/Pr)^{9/16}\right]^{4/9}}$$

$$\bar{h}_L = (0.0263\ W/m\cdot K/0.20m)\;\left[0.68 + 0.670\left(1.827\times10^7\right)^{1/4} / \left[1+(0.492/0.707)^{9/16}\right]^{4/9}\right]$$

$$\bar{h}_L = 4.42\ W/m^2 \cdot K. \quad <$$

COMMENTS: The agreement of $\bar{h}_L$ calculated by these two methods is excellent. Using the Churchill-Chu correlation, Eq. 9.26, find $\bar{h}_L = 4.87\ W/m\cdot K$. This relation is not the most accurate for the laminar regime, but is suitable for both laminar and turbulent regions.

PROBLEM 9.9

KNOWN: Dimensions of vertical rectangular fins. Temperature of fins and quiescent air.

FIND: (a) Optimum fin spacing, (b) Rate of heat transfer from an array of fins at the optimal spacing.

SCHEMATIC:

ASSUMPTIONS: (1) Fins are isothermal, (2) Radiation effects are negligible, (3) Air is quiescent.

PROPERTIES: *Table A-4*, Air (T_f = 325K, 1 atm): $\nu = 18.41 \times 10^{-6}$ m^2/s, k = 0.0282 W/m·K, Pr = 0.703.

ANALYSIS: (a) If fins are too close, boundary layers on adjoining surfaces will coalesce and heat transfer will decrease. If fins are too far apart, the surface area becomes too small and heat transfer decreases. $S_{op} \approx \delta_{x=H}$. From Fig. 9.4, the edge of boundary layer corresponds to

$$h = (d/\mathrm{H})\ (\mathrm{Gr_H}/4)^{1/4} \approx 5.$$

Hence, $$\mathrm{Gr_H} = \frac{g b\,(\mathrm{T_s} - \mathrm{T_\infty})\,\mathrm{H}^3}{n\,\mathrm{H2}} = \frac{9.8\ \mathrm{m/s^2}\,(1/325\mathrm{K})\ 50\mathrm{K}\ (0.15\mathrm{m})^3}{\left(18.41\times10^{-6}\mathrm{m^2/s}\right)^2} = 1.5\times10^7$$

$$d\,(\mathrm{H}) = 5(0.15\mathrm{m})/\left(1.5\times10^7/4\right)^{1/4} = 0.017\mathrm{m} = 17\mathrm{mm} \qquad \mathrm{S_{op}} \approx 34\mathrm{mm}.$$ <

(b) The number of fins N can be found as

$$\mathrm{N} = \mathrm{W}/\left(\mathrm{S_{op}} + \mathrm{t}\right) = 355/35.5 = 10$$

and the rate is $\mathrm{q} = 2\ \mathrm{N}\ \bar{\mathrm{h}}(\mathrm{H}\cdot\mathrm{L})\ (\mathrm{T_s} - \mathrm{T_\infty})$.

For laminar flow conditions

$$\overline{\mathrm{Nu}}_\mathrm{H} = 0.68 + 0.67\ \mathrm{Ra_L^{1/4}}/\left[1 + (0.492/\mathrm{Pr})^{9/16}\right]^{4/9}$$

$$\overline{\mathrm{Nu}}_\mathrm{H} = 0.68 + 0.67\left(1.5\times10^7 \times 0.703\right)^{1/4}/\left[1 + (0.492/0.703)^{9/16}\right]^{4/9} = 30$$

$$\bar{\mathrm{h}} = \mathrm{k\ Nu_H}/\mathrm{H} = 0.0282\ \mathrm{W/m\cdot K}(30)/0.15\ \mathrm{m} = 5.6\ \mathrm{W/m^2\cdot K}$$

$$\mathrm{q} = 2(10)5.6\ \mathrm{W/m^2\cdot K}\,(0.15\mathrm{m}\times0.02\mathrm{m})\ (350 - 300)\mathrm{K} = 16.8\ \mathrm{W}.$$ <

COMMENTS: Part (a) result is a conservative estimate of the optimum spacing. The increase in area resulting from a further reduction in S would more than compensate for the effect of fluid entrapment due to boundary layer merger. From a more rigorous treatment (see Section 9.7.1), $S_{op} \approx$ 10 mm is obtained for the prescribed conditions.

PROBLEM 9.10

KNOWN: Interior air and wall temperatures; wall height.

FIND: (a) Average heat transfer coefficient when $T_\infty = 20°C$ and $T_s = 10°C$, (b) Average heat transfer coefficient when $T_\infty = 27°C$ and $T_s = 37°C$.

SCHEMATIC:

ASSUMPTIONS: (a) Wall is at a uniform temperature, (b) Room air is quiescent.

PROPERTIES: *Table A-4*, Air (T_f = 298K, 1 atm): $\beta = 1/T_f = 3.472 \times 10^{-3}\ K^{-1}$, $\nu = 14.82 \times 10^{-6}\ m^2/s$, k = 0.0253 W/m·K, $\alpha = 20.9 \times 10^{-6}\ m^2/s$, Pr = 0.710; ($T_f$ = 305K, 1 atm): $\beta = 1/T_f = 3.279 \times 10^{-3}\ K^{-1}$, $\nu = 16.39 \times 10^{-6}\ m^2/s$, k = 0.0267 W/m·K, $\alpha = 23.2 \times 10^{-6}\ m^2/s$, Pr = 0.706.

ANALYSIS: The appropriate correlation for the average heat transfer coefficient for free convection on a vertical wall is Eq. 9.26.

$$\overline{Nu}_L = \frac{\bar{h}L}{k} = \left\{0.825 + \frac{0.387\,Ra_L^{0.1667}}{\left[1+\left(0.492/Pr\right)^{0.563}\right]^{0.296}}\right\}^2$$

where $Ra_L = g\,\beta\,\Delta T\,L^3/\nu\alpha$, Eq. 9.25, with $\Delta T = T_s - T_\infty$ or $T_\infty - T_s$.

(a) Substituting numerical values typical of *winter* conditions gives

$$Ra_L = \frac{9.8\ m/s^2 \times 3.472\times10^{-3}\ K^{-1}\left(20-10\right)\ K\ \left(2.5m\right)^3}{14.82\times10^{-6} m^2/s \times 20.96\times10^{-6} m^2/s} = 1.711\times10^{10}$$

$$\overline{Nu}_L = \left\{0.825 + \frac{0.387\left(1.711\times10^{10}\right)^{0.1667}}{\left[1+\left(0.492/0.710\right)^{0.563}\right]^{0.296}}\right\}^2 = 299.6.$$

Hence, $\bar{h} = \overline{Nu}_L\ k/L = 299.6\left(0.0253\ W/m\cdot K\right)/2.5m = 3.03\ W/m^2\cdot K.$ <

(b) Substituting numerical values typical of *summer* conditions gives

$$Ra_L = \frac{9.8\ m/s^2 \times 3.279\times10^{-3}\ K^{-1}\left(37-27\right)\ K\ \left(2.5\ m\right)^3}{23.2\times10^{-6} m^2/s \times 16.39\times10^{-6} m^2/s} = 1.320\times10^{10}$$

$$\overline{Nu}_L = \left\{0.825 + \frac{0.387\left(1.320\times10^{10}\right)^{0.1667}}{\left[1+\left(0.492/0.706\right)^{0.563}\right]^{0.296}}\right\}^2 = 275.8.$$

Hence, $\bar{h} = \overline{Nu}_L\ k/L = 275.8\times0.0267\ W/m\cdot K/2.5m = 2.94\ W/m^2\cdot K.$ <

COMMENTS: There is a small influence due to T_f on $\bar{h}$ for these conditions. We should expect radiation effects to be important with such low values of $\bar{h}$.

PROBLEM 9.11

KNOWN: Vertical plate experiencing free convection with quiescent air at atmospheric pressure and film temperature 400 K.

FIND: Form of correlation for average heat transfer coefficient in terms of ΔT and characteristic length.

SCHEMATIC:

T_s

L

Quiescent air, T_∞

$T_f = 400K$

$\Delta T = T_s - T_\infty$

ASSUMPTIONS: (1) Air is extensive, quiescent medium, (2) Perfect gas behavior.

PROPERTIES: *Table A-6*, Air (T_f = 400K, 1 atm): $\nu = 26.41 \times 10^{-6}\ m^2/s$, k = 0.0338 W/m·K, $\alpha = 38.3 \times 10^{-6}\ m^2/s$.

ANALYSIS: Consider the correlation having the form of Eq. 9.24 with Ra_L defined by Eq. 9.25.

$$\overline{Nu}_L = \bar{h}_L L/k = C Ra_L^n \qquad (1)$$

where

$$Ra_L = \frac{g b \left(T_s - T_\infty\right) L^3}{n a} = \frac{9.8\ m/s^2 \left(1/400\ K\right) \Delta T \cdot L^3}{26.41 \times 10^{-6} m^2/s \times 38.3 \times 10^{-6} m^2/s} = 2.422 \times 10^7 \Delta T \cdot L^3. \qquad (2)$$

Combining Eqs. (1) and (2),

$$\bar{h}_L = \left(k/L\right) C Ra_L^n = \frac{0.0338\ W/m \cdot K}{L} C \left(2.422 \times 10^7 \Delta T L^3\right)^n. \qquad (3)$$

From Fig. 9.6, note that for laminar boundary layer conditions, $10^4 < Ra_L < 10^9$, C = 0.59 and n = 1/4. Using Eq. (3),

$$\bar{h} = 1.40 \left[L^{-1} \left(\Delta T \cdot L^3\right)^{1/4}\right] = 1.40 \left(\frac{\Delta T}{L}\right)^{1/4}$$ <

For turbulent conditions in the range $10^9 < Ra_L < 10^{13}$, C = 0.10 and n = 1/3. Using Eq. (3),

$$\bar{h}_L = 0.98 \left[L^{-1} \left(\Delta T \cdot L^3\right)^{1/3}\right] = 0.98 \Delta T^{1/3}.$$ <

COMMENTS: Note the dependence of the average heat transfer coefficient on ΔT and L for laminar and turbulent conditions. The characteristic length L does not influence $\bar{h}_L$ for turbulent conditions.

PROBLEM 9.12

KNOWN: Temperature dependence of free convection coefficient, $\bar{h} = C\Delta T^{1/4}$, for a solid suddenly submerged in a quiescent fluid.

FIND: (a) Expression for cooling time, t_f, (b) Considering a plate of prescribed geometry and thermal conditions, the time required to reach 80°C using the appropriate correlation from Problem 9.10 and (c) Plot the temperature-time history obtained from part (b) and compare with results using a constant $\bar{h}_o$ from an appropriate correlation based upon an average surface temperature $\bar{T} = (T_i + T_f)/2$.

SCHEMATIC:

Quiescent fluid, T_∞

$T(0)=T_i$
$T(t_f)=T_f$

$\dot{E}_{st}$

$q_{conv} = \bar{h}A_s(T-T_\infty)$
$\bar{h} = C(T-T_\infty)^{1/4}$

Al 2024 plate,
150 x 150 mm,
5 mm thickness,
$T(0) = 225\ ^oC$

q

Air
$T_\infty = 25\ ^oC$

ASSUMPTIONS: (1) Lumped capacitance approximation is valid, (2) Negligible radiation, (3) Constant properties.

PROPERTIES: *Table A.1*, Aluminum alloy 2024 $\left(\bar{T} = (T_i + T_f)/2 \approx 400\,K\right)$: ρ = 2770 kg/m^3, c_p = 925 J/kg·K, k = 186 W/m·K; *Table A.4*, Air ($\bar{T}_{film}$ = 362 K): ν = 2.221 × 10^{-5} m^2/s, k = 0.03069 W/m·K, α = 3.187 × 10^{-5} m^2/s, Pr = 0.6976, β = 1/$\bar{T}_{film}$.

ANALYSIS: (a) Apply an energy balance to a control surface about the object, $-\dot{E}_{out} = \dot{E}_{st}$, and substitute the convection rate equation, with $\bar{h} = C\Delta T^{1/4}$, to find

$$-CA_s(T - T_\infty)^{5/4} = d/dt(\rho VcT). \qquad (1)$$

Separating variables and integrating, find

$$dT/dt = -(CA_s/\rho Vc)(T - T_\infty)^{5/4}$$

$$\int_{T_i}^{T_f} \frac{dT}{(T - T_\infty)^{5/4}} = -\left(\frac{CA_s}{\rho Vc}\right)\int_0^{t_f} dt \qquad -4(T - T_\infty)^{-1/4}\Big|_{T_i}^{T_f} = -\frac{CA_s}{\rho Vc}t_f$$

$$t_f = \frac{4\rho Vc}{CA_s}\left[(T_f - T_\infty)^{-1/4} - (T_i - T_\infty)^{-1/4}\right] = \frac{4\rho Vc}{CA_s(T_i - T_\infty)^{1/4}}\left[\left(\frac{T_i - T_\infty}{T_f - T_\infty}\right)^{1/4} - 1\right]. \qquad (2) <$$

(b) Considering the aluminum plate, initially at T(0) = 225°C, and suddenly exposed to ambient air at $T_\infty = 25^\circ C$, from Problem 9.10 the convection coefficient has the form

$$\bar{h}_i = 1.40\left(\frac{\Delta t}{L}\right)^{1/4} \qquad \bar{h}_i = C\Delta T^{1/4}$$

where C = 1.40/L$^{1/4}$ = 1.40/(0.150)$^{1/4}$ = 2. 2496 $W/m^2 \cdot K^{3/4}$. Using Eq. (2), find

Continued...

PROBLEM 9.12 (Cont.)

$$t_f = \frac{4 \times 2770\,\text{kg}/\text{m}^3 \left(0.150^2 \times 0.005\right)\text{m}^3 \times 925\,\text{J/kg}\cdot\text{K}}{2.2496\,\text{W}/\text{m}^2\cdot\text{K}^{3/4} \times 2 \times (0.150\text{m})^2\,(225-25)^{1/4}\,\text{K}^{1/4}} \left[\left(\frac{225-25}{80-25}\right)^{1/4} - 1\right] = 1154\text{s}$$

(c) For the vertical plate, Eq. 9.27 is an appropriate correlation. Evaluating properties at

$$\overline{T}_{film} = \left(\overline{T}_s + T_\infty\right)/2 = \left(\left(T_i + T_f\right)/2 + T_\infty\right)/2 = 362\ \text{K}$$

where $\overline{T}_s = 426\text{K}$, the average plate temperature, find

$$Ra_L = \frac{g\beta\left(\overline{T}_s - T_\infty\right)L^3}{\nu\alpha} = \frac{9.8\,\text{m/s}^2\,(1/362\text{K})(426-298)\text{K}\,(0.150\text{m})^3}{2.221\times10^{-5}\,\text{m}^2/\text{s} \times 3.187\times10^{-5}\,\text{m}^2/\text{s}} = 1.652\times10^7$$

$$\overline{Nu}_L = 0.68 + \frac{0.670 Ra_L^{1/4}}{\left[1+\left(0.492/Pr\right)^{9/16}\right]^{4/9}} = 0.68 + \frac{0.670\left(1.652\times10^7\right)^{1/4}}{\left[1+\left(0.492/0.6976\right)^{9/16}\right]^{4/9}} = 33.4$$

$$\overline{h}_o = \frac{k}{L}\overline{Nu}_L = \frac{0.03069\,\text{W/m}\cdot\text{K}}{0.150\text{m}} \times 33.4 = 6.83\,\text{W}/\text{m}^2\cdot\text{K}$$

From Eq. 5.6, the temperature-time history with a constant convection coefficient is

$$T(t) = T_\infty + \left(T_i - T_\infty\right)\exp\left[-\left(\overline{h}_o A_s/\rho V c\right)t\right] \qquad (3)$$

where $A_s/V = 2L^2/(L\times L\times w) = 2/w = 400\text{m}^{-1}$. The temperature-time histories for the $h = C\Delta T^{1/4}$ and $\overline{h}_o$ analyses are shown in plot below.

COMMENTS: (1) The times to reach $T(t_o) = 80°\text{C}$ were 1154 and 1212s for the variable and constant coefficient analysis, respectively, a difference of 5%. For convenience, it is reasonable to evaluate the convection coefficient as described in part (b).

(2) Note that $Ra_L < 10^9$ so indeed the expression selected from Problem 9.10 was the appropriate one.

(3) Recognize that if the emissivity of the plate were unity, the average linearized radiation coefficient using Eq. (1.9) is $\overline{h}_{rad} = 11.0\,\text{W}/\text{m}^2\cdot\text{K}$ and radiative exchange becomes an important process.

PROBLEM 9.13

KNOWN: Oven door with average surface temperature of 32°C in a room with ambient air at 22°C.

FIND: Heat loss to the room. Also, find effect on heat loss if emissivity of door is unity and the surroundings are at 22°C.

SCHEMATIC:

Air $T_\infty = 22°C$

$L = 0.5m$ 0.7m

$T_s = 32°C$

A_s

ASSUMPTIONS: (1) Ambient air is quiescent, (2) Surface radiation effects are negligible.

PROPERTIES: *Table A-4*, Air (T_f = 300K, 1 atm): $\nu = 15.89 \times 10^{-6}\ m^2/s$, k = 0.0263 W/m·K, $\alpha = 22.5 \times 10^{-6}\ m^2/s$, Pr = 0.707, $\beta = 1/T_f = 3.33 \times 10^{-3}\ K^{-1}$.

ANALYSIS: The heat rate from the oven door surface by convection to the ambient air is

$$q = \bar{h} A_s \left(T_s - T_\infty \right) \qquad (1)$$

where $\bar{h}$ can be estimated from the free-convection correlation for a vertical plate, Eq. 9.26,

$$\overline{Nu}_L = \frac{\bar{h} L}{k} = \left\{ 0.825 + \frac{0.387 Ra_L^{1/6}}{\left[1 + \left(0.492/Pr\right)^{9/16}\right]^{8/27}} \right\}^2 . \qquad (2)$$

The Rayleigh number, Eq. 9.25, is

$$Ra_L = \frac{g\, b \left(T_s - T_\infty\right) L^3}{n a} = \frac{9.8\,m/s^2 \left(1/300K\right)\left(32-22\right)K \times 0.5^3 m^3}{15.89 \times 10^{-6} m^2/s \times 22.5 \times 10^{-6} m^2/s} = 1.142 \times 10^8.$$

Substituting numerical values into Eq. (2), find

$$\overline{Nu}_L = \left\{ 0.825 + \frac{0.387 \left(1.142 \times 10^8\right)^{1/6}}{\left[1 + \left(0.492/0.707\right)^{9/16}\right]^{8/27}} \right\}^2 = 63.5$$

$$\bar{h}_L = \frac{k}{L} \overline{Nu}_L = \frac{0.0263\,W/m \cdot K}{0.5m} \times 63.5 = 3.34\,W/m^2 \cdot K.$$

The heat rate using Eq. (1) is

$$q = 3.34\,W/m^2 \cdot K \times \left(0.5 \times 0.7\right) m^2 \left(32 - 22\right) K = 11.7W. \qquad <$$

Heat loss by radiation, assuming ε = 1, is

$$q_{rad} = e A_s s \left(T_s^4 - T_{sur}^4 \right)$$

$$q_{rad} = 1\left(0.5 \times 0.7\right) m^2 \times 5.67 \times 10^{-8} W/m^2 \cdot K^4 \left[\left(273+32\right)^4 - \left(273+22\right)^4 \right] K^4 = 21.4W. \qquad <$$

Note that heat loss by radiation is nearly double that by free convection. Using Eq. (1.9), the radiation heat transfer coefficient is $h_{rad} = 6.4\ W/m^2 \cdot K$, which is twice the coefficient for the free convection process.

PROBLEM 9.14

KNOWN: Aluminum plate (alloy 2024) at an initial uniform temperature of 227°C is suspended in a room where the ambient air and surroundings are at 27°C.

FIND: (a) Expression for time rate of change of the plate, (b) Initial rate of cooling (K/s) when plate temperature is 227°C, (c) Validity of assuming a uniform plate temperature, (d) Decay of plate temperature and the convection and radiation rates during cooldown.

SCHEMATIC:

Quiescent air, $T_\infty = 27\ ^oC$

L

$T_{sur} = T_\infty = 27\ ^oC$

Plate, 0.3 x 0.3 m,
15 mm thick,
Aluminum 2024,
plate area, A_s,
ε = 0.25

$T_i = 227\ ^oC$

$t = 0.015$ m

ASSUMPTIONS: (1) Plate temperature is uniform, (2) Ambient air is quiescent and extensive, (3) Surroundings are large compared to plate.

PROPERTIES: *Table A.1*, Aluminum alloy 2024 (T = 500 K): $\rho = 2770\ kg/m^3$, k = 186 W/m·K, c = 983 J/kg·K; *Table A.4*, Air (T_f = 400 K, 1 atm): $\nu = 26.41 \times 10^{-6}\ m^2/s$, k = 0.0388 W/m·K, $\alpha = 38.3 \times 10^{-6}\ m^2/s$, Pr = 0.690.

ANALYSIS: (a) From an energy balance on the plate with free convection and radiation exchange, $-\dot{E}_{out} = \dot{E}_{st}$, we obtain

$$-\bar{h}_L 2A_s (T_s - T_\infty) - \varepsilon 2A_s \sigma \left(T_s^4 - T_{sur}^4\right) = \rho A_s tc \frac{dT}{dt} \quad \text{or} \quad \frac{dT}{dt} = \frac{-2}{\rho tc}\left[\bar{h}_L (T_s - T_\infty) + \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right)\right]$$ <

where T_s, the plate temperature, is assumed to be uniform at any time.

(b) To evaluate (dT/dt), estimate $\bar{h}_L$. First, find the Rayleigh number,

$$Ra_L = g\beta (T_s - T_\infty) L^3 / \nu\alpha = \frac{9.8\,m/s^2\,(1/400\,K)(227-27)K \times (0.3\,m)^3}{26.41\times10^{-6}\,m^2/s \times 38.3\times10^{-6}\,m^2/s} = 1.308\times10^8 .$$

Eq. 9.27 is appropriate; substituting numerical values, find

$$\overline{Nu}_L = 0.68 + \frac{0.670 Ra_L^{1/4}}{\left[1+(0.492/Pr)^{9/16}\right]^{4/9}} = 0.68 + \frac{0.670\left(1.308\times10^8\right)^{1/4}}{\left[1+(0.492/0.690)^{9/16}\right]^{4/9}} = 55.5$$

$$\bar{h}_L = \overline{Nu}_L k/L = 55.5 \times 0.0338\,W/m\cdot K/0.3\,m = 6.25\,W/m^2\cdot K$$

Continued...

$$\frac{dT}{dt} = \frac{-2}{2770\,\text{kg/m}^3 \times 0.015\,\text{m} \times 983\,\text{J/kg}\cdot\text{K}} \times$$

$$\left[6.25\,\text{W/m}^2\cdot\text{K}\,(227-27)\,\text{K} + 0.25\left(5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\right)\left(500^4 - 300^4\right)\text{K}^4\right] = -0.099\,\text{K/s}.$$ <

(c) The uniform temperature assumption is justified if the Biot number criterion is satisfied. With $L_c \equiv (V/2A_s) = (A_s \cdot t/2A_s) = (t/2)$ and $\overline{h}_{tot} = \overline{h}_{conv} + \overline{h}_{rad}$, Bi = $\overline{h}_{tot}(t/2)/k \leq 0.1$. Using the linearized radiation coefficient relation, find

$$\overline{h}_{rad} = \varepsilon\sigma\left(T_s + T_{sur}\right)\left(T_s^2 + T_{sur}^2\right) = 0.25\left(5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\right)(500+300)\left(500^2 + 300^2\right)\text{K}^3 = 3.86\,\text{W/m}^2\cdot\text{K}$$

Hence, Bi = (6.25 + 3.86) $\text{W/m}^2\cdot\text{K}$(0.015 m/2)/186 W/m·K = 4.07×10^{-4}. Since Bi << 0.1, the assumption is appropriate.

(d) The temperature history of the plate was computed by combining the *Lumped Capacitance Model* of IHT with the appropriate *Correlations* and *Properties* Toolpads.

Temperature, T(C) vs. Time, t(s)

Heat rate, q(W) vs. Time, t(s)

Convection heat rate, qconv(W)
Radiation heat rate, qrad(W)

Due to the small values of $\overline{h}_L$ and $\overline{h}_{rad}$, the plate cools slowly and does not reach 30°C until t ≈ 14000s = 3.89h. The convection and radiation rates decrease rapidly with increasing t (decreasing T), thereby decelerating the cooling process.

COMMENTS: The reduction in the convection rate with increasing time is due to a reduction in the thermal conductivity of air, as well as the values of $\overline{h}_L$ and T.

PROBLEM 9.15

KNOWN: Instantaneous temperature and time rate of temperature change of a vertical plate cooling in a room.

FIND: Average free convection coefficient for the prescribed conditions; compare with standard empirical correlation.

SCHEMATIC:

Quiescent air, $T_\infty = 27°C$
Surroundings, $T_{sur} = T_\infty = 27°C$
$T_s = 127°C$, $\frac{dT}{dt} = -0.0465\ K/s$
Plate, aluminum alloy 2024, 0.3m square, $\varepsilon = 0.25$
$L = 0.3m$
$\ell = 15mm$

ASSUMPTIONS: (1) Uniform plate temperature, (2) Quiescent room air, (3) Large surroundings.

PROPERTIES: *Table A-1*, Aluminum alloy 2024 (T_s = 127°C = 400K): ρ = 2770 kg/m^3, c_p = 925 J/kg·K; *Table A-4*, Air ($T_f = (T_s + T_\infty)/2$ = 350K, 1 atm): $\nu = 20.92 \times 10^{-6}$ m^2/s, k = 0.020 W/m·K, $\alpha = 29.9 \times 10^{-6}$ m^2/s, Pr = 0.700.

ANALYSIS: From an energy balance on the plate considering free convection *and* radiation exchange,

T_s, $\dot{E}_{st}$, q_{conv}, q_{rad}

$$\dot{E}_{in} - \dot{E}_{out} = \dot{E}_{st}$$

$$-\bar{h}_L (2A_s)(T_s - T_\infty) - \varepsilon (2A_s) \sigma \left(T_s^4 - T_{sur}^4\right) = \rho A_s \ell c_p \frac{dT}{dt}.$$

Noting that the plate area is $2A_s$, solving for $\bar{h}_L$, and substituting numerical values, find

$$\bar{h}_L = \left[-\rho \ell c_p \frac{dT}{dt} - 2\varepsilon\sigma \left(T_s^4 - T_{sur}^4\right)\right] / 2(T_s - T_\infty)$$

$$\bar{h}_L = \left[-2770\text{kg/m}^3 \times 0.3\text{m} \times 925\text{J/kg}\cdot\text{K}\,(-0.0465\text{K/s}) - 2 \times 0.25 \times 5.67 \times 10^{-8}\text{W/m}^2\cdot\text{K}^4 \left(400^4 - 300^4\right)\text{K}^4\right]$$
$$/\ 2(127 - 27)^\circ\text{C} = (8.936 - 2.455)\ \text{W/m}^2\cdot\text{K}$$

$$\bar{h}_L = 6.5\text{W/m}^2\cdot\text{K}.$$ <

To select an appropriate empirical correlation, first evaluate the Rayleigh number,

$$Ra_L = g\beta\Delta T L^3/\nu\alpha$$

$$Ra_L = 9.8\text{m/s}^2 (1/350\text{K})(127 - 27)\text{K}\,(0.3\text{m})^3 / \left(20.92 \times 10^{-6}\text{m}^2/\text{s}\right)\left(29.9 \times 10^{-6}\text{m}^2/\text{s}\right) = 1.21 \times 10^8.$$

Since $Ra_L < 10^9$, the flow is laminar and Eq. 9.27 is applicable,

$$\overline{Nu}_L = \frac{\bar{h}_L L}{k} = 0.68 + \frac{0.670 Ra_L^{1/4}}{\left[1 + (0.492/Pr)^{9/16}\right]^{4/9}}$$

$$\bar{h}_L = \left(\frac{0.030\text{W/m}\cdot\text{K}}{0.3\text{m}}\right)\left\{0.68 + 0.670\left(1.21 \times 10^8\right)^{1/4} / \left[1 + (0.492/0.700)^{9/16}\right]^{4/9}\right\} = 5.5\text{W/m}^2\cdot\text{K}.$$ <

COMMENTS: (1) The correlation estimate is 15% lower than the experimental result. (2) This transient method, useful for obtaining an average free convection coefficient for spacewise isothermal objects, requires Bi ≤ 0.1.

PROBLEM 9.16

KNOWN: Person, approximated as a cylinder, experiencing heat loss in water or air at 10°C.

FIND: Whether heat loss from body in water is 30 times that in air.

ASSUMPTIONS: (1) Person can be approximated as a vertical cylinder of diameter D = 0.3 m and length L = 1.8 m, at 25°C, (2) Loss is only from the lateral surface.

PROPERTIES: *Table A-4*, Air $\left(\bar{T} = (25+10)^\circ C/2 = 290K, 1atm\right)$: k = 0.0293 W/m·K, ν = 19.91×10^{-6} m^2/s, $\alpha = 28.4 \times 10^{-6}$ m^2/s; *Table A-6*, Water (290K): k = 0.598 W/m·K, $\nu = \mu v_f$ = 1.081×10^{-6} m^2/s, $\alpha = k v_f / c_p = 1.431 \times 10^{-7}$ m^2/s, $\beta_f = 174 \times 10^{-6}$ K^{-1}.

ANALYSIS: In both water (wa) and air (a), the heat loss from the lateral surface of the cylinder approximating the body is

$$q = \bar{h} p\, DL \left(T_s - T_\infty\right)$$

where T_s and T_∞ are the same for both situations. Hence,

$$\frac{q_{wa}}{q_a} = \frac{\bar{h}_{wa}}{\bar{h}_a}$$

Vertical cylinder in air:

$$Ra_L = \frac{g b \Delta T L^3}{n a} = \frac{9.8\,m/s^2 \times (1/290K)(25-10)K\,(1.8m)^3}{19.91\times10^{-6} m^2/s \times 28.4\times10^{-6} m^2/s} = 5.228\times10^9$$

Using Eq. 9.24 with C = 0.1 and n = 1/3,

$$\overline{Nu}_L = \frac{\bar{h}_L L}{k} = C Ra_L^n = 0.1\left(5.228\times10^9\right)^{1/3} = 173.4 \qquad \bar{h}_L = 2.82\ W/m^2\cdot K.$$

Vertical cylinder in water:

$$Ra_L = \frac{9.8\,m/s^2 \times 174\times10^{-6} K^{-1}(25-10)K\,(1.8m)^3}{1.081\times10^{-6} m^2/s \times 1.431\times10^{-7} m^2/s} = 9.643\times10^{11}$$

Using Eq. 9.24 with C = 0.1 and n = 1/3,

$$\overline{Nu}_L = \frac{\bar{h}L}{k} = C Ra_L^n = 0.1\left(9.643\times10^{11}\right)^{1/3} = 978.9 \qquad \bar{h}_L = 328\ W/m^2\cdot K.$$

Hence, from this analysis we find

$$\frac{q_{wa}}{q_a} = \frac{328\ W/m^2\cdot K}{2.8\ W/m^2\cdot K} = 117$$

which compares poorly with the claim of 30.

PROBLEM 9.17

KNOWN: Dimensions of window pane with frost formation on inner surface. Temperature of room air and walls.

FIND: Heat loss through window.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state, (2) Surface of frost is isothermal with $T_s \approx 0°C$, (3) Radiation exchange is between a small surface (window) and a large enclosure (walls of room), (4) Room air is quiescent.

PROPERTIES: *Table A-4*, air (T_f = 9°C = 282 K): k = 0.0249 W/m·K, $\nu = 14.3 \times 10^{-6}$ m²/s, $\alpha = 20.1 \times 10^{-6}$ m²/s, Pr = 0.712, $\beta = 3.55 \times 10^{-3}$ K^{-1}.

ANALYSIS: Under steady-state conditions, the heat loss through the window corresponds to the rate of heat transfer to the frost by convection and radiaiton.

$$q = q_{conv} + q_{rad} = W \times L\left[\bar{h}\left(T_\infty - T_s\right) + \varepsilon\sigma\left(T_{sur}^4 - T_s^4\right)\right]$$

With $Ra_L = g\beta\left(T_\infty - T_s\right)L^3/\alpha\nu = 9.8\,m/s^2 \times 0.00355\,K^{-1} \times 18\,K\,(1m)^3/\left(14.3 \times 20.1 \times 10^{-12}\,m^4/s^2\right)$

$= 2.18 \times 10^9$, Eq. (9.26) yields

$$\overline{Nu}_L = \left\{0.825 + \frac{0.387\,Ra_L^{1/6}}{\left[1 + (0.492/Pr)^{9/16}\right]^{8/27}}\right\}^2 = 156.5$$

$$\bar{h} = Nu_L \frac{k}{L} = 156.5\left(\frac{0.0249\,W/m \cdot K}{1m}\right) = 3.9\,W/m^2 \cdot K$$

$$q = 1m^2\left[3.9\,W/m^2 \cdot K\,(18K) + 0.90 \times 5.67 \times 10^{-8}\,W/m^2 \cdot K^4\left(291^4 - 273^4\right)\right]$$

$$= 70.2\,W + 82.5\,W = 152.7\,W$$ <

COMMENTS: (1) The thickness of the frost layer does not affect the heat loss, since the inner surface of the layer remains at $T_s \approx 0°C$. However, the temperature of the glass/frost interface decreases with increasing thickness, from a value of 0°C for negligible thickness. (2) Since the thermal boundary layer thickness is zero at the top of the window and has its maximum value at the bottom, the temperature of the glass will actually be largest and smallest at the top and bottom, respectively. Hence, frost will first begin to form at the bottom.

PROBLEM 9.18

KNOWN: During a winter day, the window of a patio door with a height of 1.8 m and width of 1.0 m shows a frost line near its base.

FIND: (a) Explain why the window would show a frost layer at the base of the window, rather than at the top, and (b) Estimate the heat loss through the window due to free convection and radiation. If the room has electric baseboard heating, estimate the daily cost of the window heat loss for this condition based upon the utility rate of 0.08 \$/kW·h.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Window has a uniform temperature, (3) Ambient air is quiescent, and (4) Room walls are isothermal and large compared to the window.

PROPERTIES: *Table A-4*, Air ($T_f = (T_s + T_\infty)/2 = 280$ K, 1 atm): $\nu = 14.11 \times 10^{-6}$ m^2/s, k = 0.0247 W/m·K, $\alpha = 1.986 \times 10^{-5}$ m^2/s.

ANALYSIS: (a) For these winter conditions, a frost line could appear and it would be at the bottom of the window. The boundary layer is thinnest at the top of the window, and hence the heat flux from the warmer room is greater than compared to that at the bottom portion of the window where the boundary layer is thicker. Also, the air in the room may be stratified and cooler near the floor compared to near the ceiling.

(b) The heat loss from the room to the window having a uniform temperature $T_s = 0°C$ by convection and radiation is

$$q_{loss} = q_{cv} + q_{rad} \tag{1}$$

$$q_{loss} = A_s\left[\bar{h}_L\left(T_\infty - T_s\right) + \varepsilon\sigma\left(T_{sur}^4 - T_s^4\right)\right] \tag{2}$$

The average convection coefficient is estimated from the Churchill-Chu correlation, Eq. 9.26, using properties evaluated at $T_f = (T_s + T_\infty)/2$.

$$\overline{Nu}_L = \frac{\bar{h}_L L}{k} = \left\{0.825 + \frac{0.387\, Ra_L^{1/6}}{\left[1+\left(0.492/Pr\right)^{9/16}\right]^{8/27}}\right\}^2 \tag{3}$$

$$Ra_L = g\beta T\left(T_\infty - T_s\right)L^3/\nu\alpha \tag{4}$$

Substituting numerical values in the correlation expressions, find

$$Ra_L = 1.084\times10^{10} \qquad \overline{Nu}_L = 258.9 \qquad \bar{h}_L = 3.6\ W/m^2\cdot K$$

Continued …..

Using Eq. (2), the heat loss with $\sigma = 5.67 \times 10^{-8}$ W/m$^2 \cdot$K^4 is

$$q_{loss} = (1 \times 1.8) m^2 \left[3.6 \text{ W/m}^2 \cdot \text{K} (15 \text{ K}) + 0.940 \sigma \left(288^4 - 273^4 \right) \text{K}^4 \right]$$

$$q_{loss} = (96.1 + 127.1) \text{W} = 223 \text{ W}$$

The daily cost of the window heat loss for the given utility rate is

$$\text{cost} = q_{loss} \times (\text{utility rate}) \times 24 \text{ hours}$$

$$\text{cost} = 223 \text{ W} \times (10^{-3} \text{ kW/W}) \times 0.08 \text{ \$/kW} - \text{h} \times 24 \text{ h}$$

$$\text{cost} = 0.43 \text{ \$/day}$$ <

COMMENTS: Note that the heat loss by radiation is 30% larger than by free convection.

PROBLEM 9.19

KNOWN: Room and ambient air conditions for window glass.

FIND: Temperature of the glass and rate of heat loss.

SCHEMATIC:

Window glass (1 m x 1 m, T, ε = 1)
L = 1 m
Air
$T_{\infty,i}$ = 20 °C
$q''_{conv,i}$
$q''_{conv,o}$
Air
$T_{\infty,o}$ = -20 °C
$q''_{rad,i}$
$q''_{rad,o}$
$T_{sur,i}$ = 20 °C
x
$T_{sur,o}$ = -20 °C

ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible temperature gradients in the glass, (3) Inner and outer surfaces exposed to large surroundings.

PROPERTIES: *Table A.4*, air ($T_{f,i}$ and $T_{f,o}$): Obtained from the IHT *Properties* Tool Pad.

ANALYSIS: Performing an energy balance on the window pane, it follows that $\dot{E}_{in} = \dot{E}_{out}$, or

$$\varepsilon\sigma\left(T_{sur,i}^4 - T^4\right) + \bar{h}_i\left(T_{\infty,i} - T\right) = \varepsilon\sigma\left(T^4 - T_{sur}^4\right) + \bar{h}_o\left(T - T_{\infty,o}\right)$$

where $\bar{h}_i$ and $\bar{h}_o$ may be evaluated from Eq. 9.26.

$$\overline{Nu}_L = \left\{0.825 + \frac{0.387 Ra_L^{1/6}}{\left[1 + \left(0.492/Pr\right)^{9/16}\right]^{8/27}}\right\}^2$$

Using the *First Law* Model for an *Isothermal Plane Wall* and the *Correlations* and *Properties* Tool Pads of IHT, the energy balance equation was formulated and solved to obtain

T = 273.8 K <

The heat rate is then $q_i = q_o$, or

$$q_i = L^2\left[\varepsilon\sigma\left(T_{sur,i}^4 - T^4\right) + \bar{h}_i\left(T_\infty - T\right)\right] = 174.8\,W$$ <

COMMENTS: The radiative and convective contributions to heat transfer at the inner and outer surfaces are $q_{rad,i}$ = 99.04 W, $q_{conv,i}$ = 75.73 W, $q_{rad,o}$ = 86.54 W, and $q_{conv,o}$ = 88.23 W, with corresponding convection coefficients of $\bar{h}_i$ = 3.95 W/m^2·K and $\bar{h}_o$ = 4.23 W/m^2·K. The heat loss could be reduced significantly by installing a double pane window.

PROBLEM 9.20

KNOWN: Room and ambient air conditions for window glass. Thickness and thermal conductivity of glass.

FIND: Inner and outer surface temperatures and heat loss.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in the glass, (3) Inner and outer surfaces exposed to large surroundings.

PROPERTIES: *Table A.4*, air ($T_{f,i}$ and $T_{f,o}$): Obtained from the IHT *Properties* Tool Pad.

ANALYSIS: Performing energy balances at the inner and outer surfaces, we obtain, respectively,

$$\varepsilon\sigma\left(T_{sur,i}^4 - T_{s,i}^4\right) + \bar{h}_i\left(T_{\infty,i} - T_{s,i}\right) = \left(kg/tg\right)\left(T_{s,i} - T_{s,o}\right) \qquad (1)$$

$$\left(kg/tg\right)\left(T_{s,i} - T_{s,o}\right) = \varepsilon\sigma\left(T_{s,o}^4 - T_{sur,o}^4\right) + \bar{h}_o\left(T_{s,o} - T_{\infty,o}\right) \qquad (2)$$

where Eq. 9.26 may be used to evaluate $\bar{h}_i$ and $\bar{h}_o$

$$\overline{Nu}_L = \left\{0.825 + \frac{0.387 Ra_L^{1/6}}{\left[1 + \left(0.492/Pr\right)^{9/16}\right]^{8/27}}\right\}^2$$

Using the *First Law* Model for *One-dimensional Conduction* in a *Plane Wall* and the *Correlations* and *Properties* Tool Pads of IHT, the energy balance equations were formulated and solved to obtain

$T_{s,i} = 274.4$ K $\qquad$ $T_{s,o} = 273.2$ K <

from which the heat loss is

$$q = \frac{k_g L^2}{t_g}\left(T_{s,i} - T_{s,o}\right) = 168.8\,W$$ <

COMMENTS: By accounting for the thermal resistance of the glass, the heat loss is smaller (168.8 W) than that determined in the preceding problem (174.8 W) by assuming an isothermal pane.

PROBLEM 9.21

KNOWN: Plate dimensions, initial temperature, and final temperature. Air temperature.

FIND: (a) Initial cooling rate, (b) Time to reach prescribed final temperature.

SCHEMATIC:

L = 1 m
δ = 0.006 m
AISI 1010 steel
$T_i = 300^oC$
$T_f = 100^oC$
L = 1 m
Quiescent air
$T_\infty = 20^oC$
p = 1 atm
q
q

ASSUMPTIONS: (1) Plate is spacewise isothermal as it cools (lumped capacitance approximation), (2) Negligible heat transfer from minor sides of plate, (3) Thermal boundary layer development corresponds to that for an isolated plate (negligible interference between adjoining boundary layers). (4) Negligible radiation. (5) Constant properties.

PROPERTIES: *Table A-1*, AISI 1010 steel $(\bar{T} = 473\,K)$: $\rho = 7854$ kg/m^3, c = 513 J/kg·K. *Table A-4*, air ($T_{f,i}$ = 433 K): $\nu = 30.4 \times 10^{-6}$ m^2/s, k = 0.0361 W/m·K, $\alpha = 44.2 \times 10^{-6}$ m^2/s, Pr = 0.687, β = 0.0023 K^{-1}.

ANALYSIS: (a) The initial rate of heat transfer is $q_i = \bar{h} A_s (T_i - T_\infty)$, where $A_s \approx 2L^2 = 2\,m^2$. With $Ra_{L,i} = g\beta (T_i - T_\infty)L^3/\alpha\nu$ = 9.8 m/s^2 $\times$ 0.0021 (280)1m^3/44.2 $\times 10^{-6}$ m^2/s $\times$ 30.4 $\times 10^{-6}$ m^2/s = 4.72 $\times 10^9$, Eq. 9.26 yields

$$\bar{h} = \frac{0.0361\,W/m \cdot K}{1m}\left\{0.825 + \frac{0.387\left(4.72\times10^9\right)^{1/6}}{\left[1+\left(0.492/0.687\right)^{9/16}\right]^{8/27}}\right\}^2 = 7.16\,W/m^2 \cdot K$$

Hence,

$$q_i = 7.16\,W/m^2 \cdot K \times 2m^2 \times 280°C = 4010\,W$$ <

(b) From an energy balance at an instant of time for a control surface about the plate, $-q = \dot{E}_{st} = \rho L^2 \delta c\, dT/dt$, the rate of change of the plate temperature is

$$\frac{dT}{dt} = -\frac{\bar{h}\,2L^2\left(T - T_\infty\right)}{\rho L^2 \delta c} = -\frac{2\bar{h}}{\rho\,\delta c}\left(T - T_\infty\right)$$

where the Rayleigh number, and hence $\bar{h}$, changes with time due to the change in the temperature of the plate. Integrating the foregoing equation with the DER function of IHT, the following results are obtained for the temperature history of the plate.

Continued …..

Plate temperature, T(C)
300
260
220
180
140
100
0 500 1000 1500 2000 2500
Time, t(s)

The time for the plate to cool to 100°C is

$$t \approx 2365\,\text{s}$$ <

COMMENTS: (1) Although the plate temperature is comparatively large and radiation emission is significant relative to convection, much of the radiation leaving one plate is intercepted by the adjoining plate if the spacing between plates is small relative to their width. The net effect of radiation on the plate temperature would then be small. (2) Because of the increase in β and reductions in ν and α with increasing t, the Rayleigh number decreases only slightly as the plate cools from 300°C to 100°C (from 4.72×10^9 to 4.48×10^9), despite the significant reduction in $(T - T_\infty)$. The reduction in $\overline{h}$ from 7.2 to 5.6 $W/m^2 \cdot K$ is principally due to a reduction in the thermal conductivity.

PROBLEM 9.22

KNOWN: Thin-walled container with hot process fluid at 50°C placed in a quiescent, cold water bath at 10°C.

FIND: (a) Overall heat transfer coefficient, U, between the hot and cold fluids, and (b) Compute and plot U as a function of the hot process fluid temperature for the range $20 \le T_{\infty,h} \le 50°C$.

SCHEMATIC:

Hot process
$T_{\infty,h} = 50\ ^oC$
Thin-walled container
$L = 200$ mm
Cold water bath
$T_{\infty,c} = 10\ ^oC$

ASSUMPTIONS: (1) Steady-state conditions, (2) Heat transfer at the surfaces approximated by free convection from a vertical plate, (3) Fluids are extensive and quiescent, (4) Hot process fluid thermophysical properties approximated as those of water, and (5) Negligible container wall thermal resistance.

PROPERTIES: *Table A.6*, Water (assume $T_{f,h}$ = 310 K): $\rho_h = 1/1.007 \times 10^{-3} = 993$ kg/m^3, $c_{p,h}$ = 4178 J/kg·K, $\nu_h = \mu_h/\rho_h = 695 \times 10^{-6}$ N·s/m^2/993 kg/m^3 = 6.999×10^{-7} m^2/s, k_h = 0.628 W/m·K, Pr_h = 4.62, $\alpha_h = k_h/\rho_h c_{p,h} = 1.514 \times 10^{-7}$ m^2/s, $\beta_h = 361.9 \times 10^{-6}$ K^{-1} ; *Table A.6*, Water (assume $T_{f,c}$ = 295 K): $\rho_c = 1/1.002 \times 10^{-3} = 998$ kg/m^3, $c_{p,c}$ = 4181 J/kg·K, $\nu_c = \mu_c/\rho_c = 959 \times 10^{-6}$ N·s/m^2/998 kg/m^3 = 9.609×10^{-7} m^2/s, k_c = 0.606 W/m·K, Pr_c = 6.62, $\alpha_c = k_c/\rho_c c_{p,c} = 1.452\times 10^{-7}$ m^2/s, $\beta_c = 227.5 \times 10^{-6}$ K^{-1}.

ANALYSIS: (a) The overall heat transfer coefficient between the hot process fluid, $T_{\infty,h}$, and the cold water bath fluid, $T_{\infty,c}$, is

$$U = \left(1/\bar{h}_h + 1/\bar{h}_c\right)^{-1} \qquad (1)$$

where the average free convection coeffieicnts can be estimated from the vertical plate correlation Eq. 9.26, with the Rayleigh number, Eq. 9.25,

$$\overline{Nu}_L = \left\{0.825 + \frac{0.387 Ra_L^{1/6}}{\left[1 + (0.492/Pr)^{9/16}\right]^{8/27}}\right\}^2 \qquad Ra_L = \frac{g\beta\Delta T L^3}{\nu\alpha} \qquad (2,3)$$

To affect a solution, assume $T_s = \left(T_{\infty,h} - T_{\infty,i}\right)/2 = 30^\circ C = 303\,K$, so that the hot and cold fluid film temperatures are $T_{f,h}$ = 313 K ≈ 310 K and $T_{f,c}$ = 293 K ≈ 295 K. From an energy balance across the container walls,

$$\bar{h}_h\left(T_{\infty,h} - T_s\right) = \bar{h}_c\left(T_s - T_{\infty,c}\right) \qquad (4)$$

the surface temperature T_s can be determined. Evaluating the correlation parameters, find:

Hot process fluid:

$$Ra_{L,h} = \frac{9.8\,m/s^2 \times 361.9\times10^{-6}\,K^{-1}\,(50-30)\,K\,(0.200\,m)^3}{6.999\times10^{-7}\ m^2/s \times 1.514\times10^{-7}\ m^2/s} = 5.357\times10^9$$

Continued...

PROBLEM 9.22 (Cont.)

$$\overline{Nu}_{L,h} = \left\{0.825 + \frac{0.387\left(5.357\times10^{9}\right)^{1/6}}{\left[1+\left(0.492/4.62\right)^{9/16}\right]^{8/27}}\right\}^{2} = 251.5$$

$$\bar{h}_h = \overline{Nu}_{L,h}\,\frac{h_h}{L} = 251.5\times0.628\,W/m^2\cdot K/0.200m = 790\,W/m^2\cdot K$$

Cold water bath:

$$Ra_{L,c} = \frac{9.8\dot{m}/s^2\times227.5\times10^{-6}K^{-1}(30-10)K\,(0.200m)^3}{9.609\times10^{-7}\,m^2/s\times1.452\times10^{-7}\,m^2/s} = 2.557\times10^{9}$$

$$\overline{Nu}_{L,c} = \left\{0.825 + \frac{0.387\left(2.557\times10^{9}\right)^{1/6}}{\left[1+\left(0.492/6.62\right)^{9/16}\right]^{8/27}}\right\}^{2} = 203.9$$

$$\bar{h}_c = 203.9\times0.606\,W/m\cdot K/0.200\,m = 618\,W/m^2\cdot K$$

From Eq. (1) find

$$U = \left(1/790 + 1/618\right)^{-1}\,W/m^2\cdot K = 347\,W/m^2\cdot K \qquad <$$

Using Eq.(4), find the resulting surface temperature

$$790\,W/m^2\cdot K\,(50-T_s)K = 618\,W/m^2\cdot K\,(T_s-30)K \qquad T_s = 32.4^\circ C$$

Which compares favorably with our assumed value of 30°C.

(b) Using the *IHT Correlations Tool, Free Convection, Vertical Plate* and following the foregoing approach, the overall coefficient was computed as a function of the hot fluid temperature and is plotted below. Note that U increases almost linearly with $T_{\infty,h}$.

COMMENTS: For the conditions of part (a), using the IHT model of part (b) with thermophysical properties evaluated at the proper film temperatures, find U = 352 W/m·K with T_s = 32.4°C. Our approximate solution was a good one.

(2) Because the set of equations for part (b) is quite stiff, when using the IHT model you should follow the suggestions in the IHT Example 9.2 including use of the intrinsic function Tfluid_avg (T1,T2).

PROBLEM 9.23

KNOWN: Height, width, emissivity and temperature of heating panel. Room air and wall temperature.

FIND: Net rate of heat transfer from panel to room.

SCHEMATIC:

ASSUMPTIONS: (1) Quiescent air, (2) Walls of room form a large enclosure, (3) Negligible heat loss from back of panel.

PROPERTIES: *Table A-4*, Air (T_f = 350K, 1 atm): $\nu = 20.9 \times 10^{-6}\ m^2/s$, k = 0.03 W/m·K, $\alpha = 29.9 \times 10^{-6}\ m^2/s$, Pr = 0.700.

ANALYSIS: The heat loss from the panel by convection and radiation exchange is

$$q = \bar{h}A(T_s - T_\infty) + es\,A\left(T_s^4 - T_{sur}^4\right).$$

With

$$Ra_L = \frac{gb\,(T_s - T_\infty)L^3}{an} = \frac{9.8\,m/s^2(1/350K)(100K)(1m)^3}{(20.9)(29.9)\times 10^{-12}\,m^4/s^2} = 4.48\times 10^9$$

and using the Churchill-Chu correlation for free convection from a vertical plate,

$$\overline{Nu}_L = \frac{\bar{h}L}{k} = \left\{0.825 + \frac{0.387 Ra_L^{1/6}}{\left[1+(0.492/Pr)^{9/16}\right]^{8/27}}\right\}^2 = 196$$

$$\bar{h} = 196k/L = 196\times 0.03\,W/m\cdot K/1m = 5.87\,W/m^2\cdot K.$$

Hence,

$$q = 5.86\,W/m^2\cdot K\left(0.5m^2\right)100K$$

$$+0.9\times 5.67\times 10^{-8}\,W/m^2\cdot K^4\left(0.5m^2\right)\left[(400)^4 - (300)^4\right]K$$

$$q = 293\,W + 447\ W = 740\,W.$$ <

COMMENTS: As is typical of free convection in gases, heat transfer by surface radiation is comparable to, if not larger than, the convection rate. The *relative* contribution of free convection would increase with decreasing L and T_s.

PROBLEM 9.24

KNOWN: Initial temperature and dimensions of an aluminum plate. Condition of the plate surroundings. Plate emissivity.

FIND: (a) Initial cooling rate, (b) Validity of assuming negligible temperature gradients in the plate during the cooling process.

SCHEMATIC:

ASSUMPTIONS: (1) Plate temperature is uniform, (2) Chamber air is quiescent, (3) Chamber surface is much larger than that of plate, (4) Negligible heat transfer from edges.

PROPERTIES: *Table A-1*, Aluminum (573K): k = 232 W/m·K, c_p = 1022 J/kg·K, ρ = 2702 kg/m^3; *Table A-4*, Air (T_f = 436K, 1 atm): ν = 30.72 × 10^{-6} m^2/s, α = 44.7 × 10^{-6} m^2/s, k = 0.0363 W/m·K, Pr = 0.687, β = 0.00229 K^{-1}.

ANALYSIS: (a) Performing an energy balance on the plate,

$$-q=-2A_s\left[\bar{h}\left(T-T_\infty\right)+es\left(T^4-T_{sur}^4\right)\right]=\dot{E}_{st}=rVc_p\left[dT/dt\right]$$

$$dT/dt=-2\left[\bar{h}\left(T-T_\infty\right)+es\left(T^4-T_{sur}^4\right)\right]/\,rwc_p$$

Using the correlation of Eq. 9.27, with

$$Ra_L=\frac{gb\left(T_i-T_\infty\right)L^3}{na}=\frac{9.8\text{m/s}^2\times 0.00229\text{K}^{-1}\left(300-27\right)\text{K}\left(0.5\text{m}\right)^3}{30.72\times10^{-6}\text{m}^2/\text{s}\times 44.7\times10^{-6}\text{m}^2/\text{s}}=5.58\times10^8$$

$$\bar{h}=\frac{k}{L}\left\{0.68+\frac{0.670Ra_L^{1/4}}{\left[1+\left(0.492/\text{Pr}\right)^{9/16}\right]^{4/9}}\right\}=\frac{0.0363}{0.5}\left\{0.68+\frac{0.670\left(5.58\times10^8\right)^{1/4}}{\left[1+\left(0.492/0.687\right)^{9/16}\right]^{4/9}}\right\}$$

$$\bar{h}=5.8\text{W/m}^2\cdot\text{K}.$$ <

Hence the initial cooling rate is

$$\frac{dT}{dt}=-\frac{2\left(5.8\text{W/m}^2\cdot\text{K}\left(300-27\right)^\circ\text{C}+0.25\times5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4\left[\left(573\text{K}\right)^4-\left(300\text{K}\right)^4\right]\right)}{2702\text{kg/m}^3\left(0.016\text{m}\right)1022\text{J/kg}\cdot\text{K}}$$

$$\frac{dT}{dt}=-0.136\text{K/s}.$$ <

(b) To check the validity of neglecting temperature gradients across the plate thickness, calculate Bi = h_{eff} (w/2)/k where h_{eff} = q''_{tot} /(T_i - T_∞) = (1583 + 1413) W/m^2/273 K = 11.0 W/m^2·K. Hence

$$Bi=\left(11\text{W/m}^2\cdot\text{K}\right)\left(0.008\text{m}\right)/232\text{W/m}\cdot\text{K}=3.8\times10^{-4}$$ <

and the assumption is excellent.

COMMENTS: (1) Longitudinal (x) temperature gradients are likely to be more severe than those associated with the plate thickness due to the variation of h with x. (2) Initially $q''_{conv}\approx q''_{rad}$.

PROBLEM 9.25

KNOWN: Boundary conditions associated with a rear window experiencing uniform volumetric heating.

FIND: (a) Volumetric heating rate $\dot{q}$ needed to maintain inner surface temperature at $T_{s,i}$ = 15°C, (b) Effects of $T_{\infty,o}$, u_∞, and $T_{\infty,i}$ on $\dot{q}$ and $T_{s,o}$.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state, one-dimensional conditions, (2) Constant properties, (3) Uniform volumetric heating in window, (4) Convection heat transfer from interior surface of window to interior air may be approximated as free convection from a vertical plate, (5) Heat transfer from outer surface is due to forced convection over a flat plate in parallel flow.

PROPERTIES: *Table A.3*, Glass (300 K): k = 1.4 W/m·K: *Table A.4*, Air ($T_{f,i}$ = 12.5°C, 1 atm): $\nu = 14.6 \times 10^{-6}$ m²/s, k = 0.0251 W/m·K, $\alpha = 20.59 \times 10^{-6}$ m²/s, $\beta = (1/285.5) = 3.503 \times 10^{-3}$ K^{-1}, Pr = 0.711; ($T_{f,o} \approx 0$°C): $\nu = 13.49 \times 10^{-6}$ m²/s, k = 0.0241 W/m·K, Pr = 0.714.

ANALYSIS: (a) The temperature distribution in the glass is governed by the appropriate form of the heat equation, Eq. 3.39, whose general solution is given by Eq. 3.40.

$$T(x) = -(\dot{q}/2k)x^2 + C_1 x + C_2 .$$

The constants of integration may be evaluated by applying appropriate boundary conditions at x = 0. In particular, with T(0) = $T_{s,i}$, $C_2 = T_{s,i}$. Applying an energy balance to the inner surface, $q''_{cond} = q''_{conv,i}$

$$-k\frac{dT}{dx}\bigg|_{x=0} = \bar{h}_i\left(T_{\infty,i} - T_{s,i}\right) \qquad -k\left(-\frac{\dot{q}}{k}x + C_1\right)\bigg|_{x=0} = \bar{h}_i\left(T_{\infty,i} - T_{s,i}\right)$$

$$C_1 = -\left(\bar{h}_i/k\right)\left(T_{\infty,i} - T_{s,i}\right)$$

$$T(x) = -(\dot{q}/2k)x^2 - \frac{\bar{h}_i\left(T_{\infty,i} - T_{s,i}\right)}{k}x + T_{s,i} \qquad (1)$$

The required generation may then be obtained by formulating an energy balance at the outer surface, where $q''_{cond} = q''_{conv,o}$. Using Eq. (1),

$$-k\frac{dT}{dx}\bigg|_{x=L} = \bar{h}_o\left(T_{s,o} - T_{\infty,o}\right) \qquad (2)$$

Continued...

$$-k\left.\frac{dT}{dx}\right|_{x=L} = -k\left(-\frac{\dot{q}L}{k}\right) + \bar{h}_i\left(T_{\infty,i} - T_{s,i}\right) = \dot{q}L + \bar{h}_i\left(T_{\infty,i} - T_{s,i}\right) \quad (3)$$

Substituting Eq. (3) into Eq. (2), the energy balance becomes

$$\dot{q}L = \bar{h}_o\left(T_{s,o} - T_{\infty,o}\right) + \bar{h}_i\left(T_{s,i} - T_{\infty,i}\right) \quad (4)$$

where $T_{s,o}$ may be evaluated by applying Eq. (1) at x = L.

$$T_{s,o} = -\frac{\dot{q}L^2}{2k} - \frac{\bar{h}_i\left(T_{\infty,i} - T_{s,i}\right)}{k}L + T_{s,i}\,. \quad (5)$$

The *inside* convection coefficient may be obtained from Eq. 9.26. With

$$Ra_H = \frac{g\beta\left(T_{s,i} - T_{\infty,i}\right)H^3}{\nu\alpha} = \frac{9.8\,\text{m/s}^2\left(3.503\times10^{-3}\,\text{K}^{-1}\right)(15-10)\,\text{K}\,(0.5\,\text{m})^3}{14.60\times10^{-6}\,\text{m}^2/\text{s}\times20.59\times10^{-6}\,\text{m}^2/\text{s}} = 7.137\times10^7\,,$$

$$\overline{Nu}_H = \left[0.825 + \frac{0.387Ra_H^{1/6}}{\left[1+(0.492/Pr)^{9/16}\right]^{8/27}}\right]^2 = \left[0.825 + \frac{0.387\left(7.137\times10^7\right)^{1/6}}{\left[1+(0.492/0.711)^{9/16}\right]^{8/27}}\right]^2 = 56$$

$$\bar{h}_i = \overline{Nu}_H\frac{k}{H} = \frac{56\times0.0251\,\text{W/m}\cdot\text{K}}{0.5\,\text{m}} = 2.81\,\text{W/m}^2\cdot\text{K}$$

The *outside* convection coefficient may be obtained by first evaluating the Reynolds number. With

$$Re_H = \frac{u_\infty H}{\nu} = \frac{20\,\text{m/s}\times0.5\,\text{m}}{13.49\times10^{-6}\,\text{m}^2/\text{s}} = 7.413\times10^5$$

and with $Re_{x,c} = 5\times10^5$, mixed boundary layer conditions exist. Hence,

$$\overline{Nu}_H = \left(0.037\,Re_H^{4/5} - 871\right)Pr^{1/3} = \left[0.037\left(7.413\times10^5\right)^{4/5} - 871\right](0.714)^{1/3} = 864$$

$$\bar{h}_o = \overline{Nu}_H\,(k/H) = \left(864\times0.0241\,\text{W/m}\cdot\text{K}\right)/0.5\,\text{m} = 41.6\,\text{W/m}^2\cdot\text{K}\,.$$

Eq. (5) may now be expressed as

$$T_{s,o} = -\frac{\dot{q}\,(0.008\,\text{m})^2}{2\,(1.4\,\text{W/m}\cdot\text{K})} - \frac{2.81\,\text{W/m}^2\cdot\text{K}\,(10-15)\,\text{K}}{1.4\,\text{W/m}\cdot\text{K}}\times0.008\,\text{m} + 288\,\text{K} = -2.286\times10^{-5}\dot{q} + 288.1\,\text{K}$$

or, solving for $\dot{q}$,

$$\dot{q} = -43,745\left(T_{s,o} - 288.1\right) \quad (6)$$

and substituting into Eq. (4),

$$-43,745\left(T_{s,o} - 288.1\right)(0.008\,\text{m}) = 41.6\,\text{W/m}^2\cdot\text{K}\left(T_{s,o} - 263\,\text{K}\right) + 2.81\,\text{W/m}^2\cdot\text{K}\,(288\,\text{K} - 283\,\text{K})\,.$$

It follows that $T_{s,o}$ = 285.4 K in which case, from Eq. (6)

$$\dot{q} = 118\,\text{kW/m}^3\,.$$ <

(b) The parametric calculations were performed using the *One-Dimensional, Steady-state Conduction* Model of IHT with the appropriate *Correlations* and *Properties* Tool Pads, and the results are as follows.

Continued...

PROBLEM 9.25 (Cont.)

For fixed $T_{s,i}$ and $T_{\infty,i}$, $T_{s,o}$ and $\dot{q}$ are strongly influenced by $T_{\infty,o}$ and u_∞, increasing and decreasing, respectively, with increasing $T_{\infty,o}$ and decreasing and increasing, respectively with increasing u_∞. For fixed $T_{s,i}$ and u_∞, $T_{s,o}$ and $\dot{q}$ are independent of $T_{\infty,i}$, but increase and decrease, respectively, with increasing $T_{\infty,o}$.

COMMENTS: In lieu of performing a surface energy balance at x = L, Eq. (4) may also be obtained by applying an energy balance to a control volume about the entire window.

PROBLEM 9.26

KNOWN: Vertical panel with uniform heat flux exposed to ambient air.

FIND: Allowable heat flux if maximum temperature is not to exceed a specified value, T_{max}.

SCHEMATIC:

$T_L = T_{max} = 37°C$; $L = 1m$; q''_s; Quiescent air, $T_\infty = 25°C$; x

ASSUMPTIONS: (1) Constant properties, (2) Radiative exchange with surroundings negligible.

PROPERTIES: *Table A-4*, Air ($T_f = (T_{L/2} + T_\infty)/2 = (35.4 + 25)°C/2 = 30.2°C = 303K$, 1 atm): $\nu = 16.19 \times 10^{-6}\ m^2/s$, $k = 26.5 \times 10^{-3}\ W/m\cdot K$, $\alpha = 22.9 \times 10^{-6}\ m^2/s$, $Pr = 0.707$.

ANALYSIS: Following the treatment of Section 9.6.1 for a vertical plate with uniform heat flux (constant q''_s), the heat flux can be evaluated as

$$q''_s = \bar{h}\Delta T_{L/2} \qquad \text{where} \qquad \Delta T_{L/2} = T_s(L/2) - T_\infty \qquad (1,2)$$

and $\bar{h}$ is evaluated using an appropriate correlation for a constant temperature vertical plate. From Eq. 9.28,

$$\Delta T_x \equiv T_x - T_\infty = 1.15(x/L)^{1/5}\Delta T_{L/2} \qquad (3)$$

and recognizing that the maximum temperature will occur at the top edge, x = L, use Eq. (3) to find

$$\Delta T_{L/2} = (37-25)°C/1.15(1/1)^{1/5} = 10.4°C \qquad \text{or} \qquad T_{L/2} = 35.4°C.$$

Calculate now the Rayleigh number based upon $\Delta T_{L/2}$, with $T_f = (T_{L/2} + T_\infty)/2 = 303K$,

$$Ra_L = \frac{g\beta\Delta T L^3}{\nu\alpha} \qquad \text{where} \qquad \Delta T = \Delta T_{L/2} \qquad (4)$$

$$Ra_L = 9.8m/s^2(1/303K)\times 10.4K(1m)^3/16.19\times 10^{-6}m^2/s \times 22.9\times 10^{-6}m^2/s = 9.07\times 10^8.$$

Since $Ra_L < 10^9$, the boundary layer flow is laminar; hence the correlation of Eq. 9.27 is appropriate,

$$\overline{Nu}_L = \frac{\bar{h}L}{k} = 0.68 + \frac{0.670 Ra_L^{1/4}}{\left[1+(0.492/Pr)^{9/16}\right]^{4/9}} \qquad (5)$$

$$\bar{h} = \left[\frac{0.0265 W/m\cdot K}{1m}\right]\left\{0.68 + 0.670\left(9.07\times 10^8\right)^{1/4}/\left[1+(0.492/0.707)^{9/16}\right]^{4/9}\right\} = 2.38 W/m^2\cdot K.$$

From Eqs. (1) and (2) with numerical values for $\bar{h}$ and $\Delta T_{L/2}$, find

$$q''_s = 2.38 W/m^2\cdot K \times 10.4°C = 24.8 W/m^2. \qquad <$$

COMMENTS: Recognize that radiation exchange with the environment will be significant. Assuming $\bar{T}_s = T_{L/2}$, $T_{sur} = T_\infty$ and $\varepsilon = 1$, find $q''_{rad} = \sigma\left(\bar{T}_s^4 - T_{sur}^4\right) = 66\ W/m^2$.

PROBLEM 9.27

KNOWN: Vertical circuit board dissipating 5W to ambient air.

FIND: (a) Maximum temperature of the board assuming uniform surface heat flux and (b) Temperature of the board for an isothermal surface condition.

SCHEMATIC:

ASSUMPTIONS: (1) Either uniform q_s'' or T_s on the board, (2) Quiescent room air.

PROPERTIES: *Table A-4*, Air ($T_f = (T_{L/2} + T_\infty)/2$ or $(T_s + T_\infty)/2$, 1 atm), values used in iterations:

Iteration	T_f(K)	$\nu \cdot 10^6 (m^2/s)$	$k \cdot 10^3$(W/m·K)	$\alpha \cdot 10^6 (m^2/s)$	Pr
1	312	17.10	27.2	24.3	0.705
2	323	18.20	28.0	25.9	0.704
3	318	17.70	27.6	25.2	0.704
4	320	17.90	27.8	25.4	0.704

ANALYSIS: (a) For the uniform heat flux case (see Section 9.6.1), the heat flux is

$$q_s'' = \bar{h} \Delta T_{L/2} \qquad \text{where} \qquad \Delta T_{L/2} = T_{L/2} - T_\infty \qquad (1,2)$$

and $\quad q_s'' = q / A_s = 5\,W / (0.150 m)^2 = 222\,W/m^2.$

The maximum temperature on the board will occur at x = L and from Eq. 9.28 is

$$\Delta T_x = 1.15 (x/L)^{1/5} \Delta T_{L/2} \qquad (3)$$

$$T_L = T_{max} = T_\infty + 1.15 \Delta T_{L/2}.$$

The average heat transfer coefficient $\bar{h}$ is estimated from a vertical (uniform T_s) plate correlation based upon the temperature difference $\Delta T_{L/2}$. Recognize that an iterative procedure is required: (i) assume a value of $T_{L/2}$, use Eq. (2) to find $\Delta T_{L/2}$; (ii) evaluate the Rayleigh number

$$Ra_L = g\, b \Delta T_{L/2} L^3 / na \qquad (4)$$

and select the appropriate correlation (either Eq. 9.26 or 9.27) to estimate $\bar{h}$; (iii) use Eq. (1) with values of $\bar{h}$ and $\Delta T_{L/2}$ to find the calculated value of q_s''; and (iv) repeat this procedure until the calculated value for q_s'' is close to q_s'' = 222 W/m^2, the required heat flux.

Continued …..

To evaluate properties for the correlation, use the film temperature,

$$T_f = \left(T_{L/2} + T_\infty\right)/2. \tag{5}$$

Iteration #1: Assume $T_{L/2}$ = 50°C and from Eqs. (2) and (5) find

$$\Delta T_{L/2} = (50-27)^\circ C = 23^\circ C \qquad T_f = (50+27)^\circ C/2 = 312K.$$

From Eq. (4), with $\beta = 1/T_f$, the Rayleigh number is

$$Ra_L = 9.8 m/s^2 (1/312K) \times 23^\circ C (0.150m)^3 / \left(17.10\times10^{-6} m^2/s\right) \times \left(24.3\times10^{-6} m^2/s\right) = 5.868\times10^6.$$

Since $Ra_L < 10^9$, the flow is laminar and Eq. 9.27 is appropriate

$$\overline{Nu}_L = \frac{\bar{h}L}{k} = 0.68 + \frac{0.670 Ra_L^{1/4}}{\left[1+(0.492/Pr)^{9/16}\right]^{4/9}}$$

$$\bar{h}_L = \frac{0.0272 W/m\cdot K}{0.150m}\left\{0.68 + 0.670\left(5.868\times10^6\right)^{1/4} / \left[1+(0.492/0.705)^{9/16}\right]^{4/9}\right\} = 4.71 W/m^2\cdot K.$$

Using Eq. (1), the calculated heat flux is

$$q_s'' = 4.71 W/m^2\cdot K \times 23^\circ C = 108 W/m^2.$$

Since q_s'' < 222 W/m^2, the required value, another iteration with an increased estimate for $T_{L/2}$ is warranted. Further iteration results are tabulated.

Iteration	$T_{L/2}$(°C)	$\Delta T_{L/2}$(°C)	T_f(K)	Ra_L	$\bar{h}\left(W/m^2\cdot K\right)$	$q_s''\left(W/m^2\right)$
2	75	48	323	1.044×10^7	5.58	267
3	65	38	318	8.861×10^6	5.28	200
4	68	41	320	9.321×10^6	5.39	221

After Iteration 4, close agreement between the calculated and required q_s'' is achieved with $T_{L/2}$ = 68°C. From Eq. (3), the maximum board temperature is

$$T_L = T_{max} = 27^\circ C + 1.15(41)^\circ C = 74^\circ C. \qquad <$$

(b) For the uniform temperature case, the procedure for estimation of the average heat transfer coefficient is the same. Hence,

$$T_s = T_{L/2}\Big|_{q_s''} = 68^\circ C. \qquad <$$

COMMENTS: In both cases, q = 5W and $\bar{h} = 5.38 W/m^2$. However, the temperature distributions for the two cases are quite different as shown on the sketch. For q_s'' = constant, $\Delta T_x \sim x^{1/5}$ according to Eq. 9.28.

x/L
1
0.5
0
ΔT_L
q_s'' = const.
$\Delta T_{L/2}$
T_s = const.
$\Delta T_x = T_x - T_\infty$

PROBLEM 9.28

KNOWN: Coolant flow rate and inlet and outlet temperatures. Dimensions and emissivity of channel side walls. Temperature of surroundings. Power dissipation.

FIND: (a) Temperature of sidewalls for $\varepsilon_s = 0.15$, (b) Temperature of sidewalls for $\varepsilon_s = 0.90$, (c) Sidewall temperatures with loss of coolant for $\varepsilon_s = 0.15$ and $\varepsilon_s = 0.90$.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state, (2) Negligible heat transfer from top and bottom surfaces of duct, (3) Isothermal side walls, (4) Large surroundings, (5) Negligible changes in flow work and potential and kinetic energies of coolant, (6) Constant properties.

PROPERTIES: *Table A-4*, air $\left(\overline{T}_m = 298\,K\right)$: $c_p = 1007$ J/kg·K. Air properties required for the free convection calculations depend on T_s and were evaluated as part of the iterative solution obtained using the IHT software.

ANALYSIS: (a) The heat dissipated by the components is transferred by forced convection to the coolant (q_c), as well as by natural convection (q_{conv}) and radiation (q_{rad}) to the ambient air and the surroundings. Hence,

$$q = q_c + q_{conv} + q_{rad} = 200\,W \tag{1}$$

$$q_c = \dot{m} c_p \left(T_{m,o} - T_{m,i}\right) = 0.015\,kg/s \times 1007\,J/kg\cdot K \times 10°C = 151\,W \tag{2}$$

$$q_{conv} = 2\overline{h}\,A_s \left(T_s - T_\infty\right) \tag{3}$$

where $A_s = H \times L = 0.32\,m^2$ and $\overline{h}$ is obtained from Eq. 9.26, with $Ra_H = g\beta\left(T_s - T_\infty\right)H^3/\alpha\nu$.

$$\overline{h} = \frac{k}{H}\left\{0.825 + \frac{0.387\,Ra_H^{1/6}}{\left[1 + \left(0.492/Pr\right)^{9/16}\right]^{8/27}}\right\}^2 \tag{3a}$$

$$q_{rad} = 2\,A_s\,\varepsilon_s\,\sigma\left(T_s^4 - T_{sur}^4\right) \tag{4}$$

Substituting Eqs. (2) – (4) into (1) and solving using the IHT software with $\varepsilon_s = 0.15$, we obtain

$$T_s = 308.8\,K = 35.8°C$$ <

The corresponding heat rates are $q_{conv} = 39.6$ W and $q_{rad} = 9.4$ W.

(b) For $\varepsilon_s = 0.90$ and $q_c = 151$ W, the solution to Eqs. (1) – (4) yields

Continued …..

PROBLEM 9.28 (Cont.)

$$T_s = 301.8\,\text{K} = 28.8°\text{C}$$ <

with q_{conv} = 18.7 W and q_{rad} = 30.3 W. Hence, enhanced emission from the surface yields a lower operating temperature and heat transfer by radiation now exceeds that due to conduction.

(c) With loss of coolant flow, we can expect all of the heat to be dissipated from the sidewalls (q_c = 0). Solving Eqs. (1), (3) and (4), we obtain

$\varepsilon_s = 0.15:$ $\quad T_s = 341.8\,\text{K} = 68.8°\text{C}$ <

$\quad q_{conv} = 165.9\,\text{W}, \qquad q_{rad} = 34.1\,\text{W}$

$\varepsilon_s = 0.90:$ $\quad T_s = 322.5\,\text{K} = 49.5°\text{C}$ <

$\quad q_{conv} = 87.6\,\text{W}, \qquad q_{rad} = 112.4\,\text{W}$

Since the temperature of the electronic components exceeds that of the sidewalls, the value of T_s = 68.8°C corresponding to ε_s = 0.15 may be unacceptable, in which case the high emissivity coating should be applied to the walls.

COMMENTS: For the foregoing cases the convection coefficient is in the range $3.31 \le \overline{h} \le 5.31$ $W/m^2 \cdot K$, with the smallest value corresponding to (q_c = 151 W, ε_s = 0.90) and the largest value to (q_c = 0, ε_s = 0.15). The radiation coefficient is in the range $0.93 \le h_{rad} \le 5.96$ $W/m^2 \cdot K$, with the smallest value corresponding to (q_c = 151 W, ε_s = 0.15) and the largest value to (q_c = 0, ε_s = 0.90).

PROBLEM 9.29

KNOWN: Dimensions, interior surface temperature, and exterior surface emissivity of a refrigerator door. Temperature of ambient air and surroundings.

FIND: (a) Heat gain with no insulation, (b) Heat gain as a function of thickness for polystyrene insulation.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible thermal resistance of steel and polypropylene sheets, (3) Negligible contact resistance between sheets and insulation, (4) One-dimensional conduction in insulation, (5) Quiescent air.

PROPERTIES: *Table A.4,* air (T_f = 288 K): $\nu = 14.82 \times 10^{-6}$ m^2/s, $\alpha = 20.92 \times 10^{-6}$ m^2/s, k = 0.0253 W/m·K, Pr = 0.71, $\beta = 0.00347$ K^{-1}.

ANALYSIS: (a) Without insulation, $T_{s,o} = T_{s,i}$ = 278 K and the heat gain is

$$q_{wo} = \bar{h} A_s \left(T_\infty - T_{s,i}\right) + \varepsilon\sigma A_s \left(T_{sur}^4 - T_{s,i}^4\right)$$

where A_s = HW = 0.65 m^2. With a Rayleigh number of $Ra_H = g\beta\left(T_\infty - T_{s,i}\right)H^3 / \alpha\nu$ = 9.8 m/s^2(0.00347 K^{-1})(20 K)(1)^3/(20.92 × 10^{-6} m^2/s)(14.82 × 10^{-6} m^2/s) = 2.19 × 10^9, Eq. 9.26 yields

$$\overline{Nu}_H = \left\{0.825 + \frac{0.387\left(2.19\times10^9\right)^{1/6}}{\left[1+\left(0.492/0.71\right)^{9/16}\right]^{8/27}}\right\}^2 = 156.6$$

$$\bar{h} = \overline{Nu}_H\left(k/H\right) = 156.6\left(0.0253\,W/m\cdot K/1\,m\right) = 4.0\,W/m^2\cdot K$$

$$q_{wo} = 4.0\,W/m^2\cdot K\left(0.65\,m^2\right)\left(20\,K\right) + 0.6\left(5.67\times10^{-8}\,W/m^2\cdot K^4\right)\left(0.65\,m^2\right)\left(298^4 - 278^4\right)K^4$$

$$q_{wo} = \left(52.00 + 42.3\right)W = 94.3\,W$$

<

(b) With the insulation, $T_{s,o}$ may be determined by performing an energy balance at the outer surface, where $q''_{conv} + q''_{rad} = q''_{cond}$, or

$$\bar{h}\left(T_\infty - T_{s,o}\right) + \varepsilon\sigma\left(T_{sur}^4 - T_{s,o}^4\right) = \frac{k_i}{L}\left(T_{s,o} - T_{s,i}\right)$$

Using the *IHT First Law* Model for a *Nonisothermal Plane Wall* with the appropriate *Correlations* and *Properties* Tool Pads and evaluating the heat gain from

Continued...

PROBLEM 9.29 (Cont.)

$$q_w = \frac{k_i A_s}{L}\left(T_{s,o} - T_{s,i}\right)$$

the following results are obtained for the effect of L on $T_{s,o}$ and q_w.

The outer surface temperature increases with increasing L, causing a reduction in the rate of heat transfer to the refrigerator compartment. For L = 0.025 m, $\overline{h}$ = 2.29 W/m^2·K, h_{rad} = 3.54 W/m^2·K, q_{conv} = 5.16 W, q_{rad} = 7.99 W, q_w = 13.15 W, and $T_{s,o}$ = 21.5°C.

COMMENTS: The insulation is extremely effective in reducing the heat load, and there would be little value to increasing L beyond 25 mm.

PROBLEM 9.30

KNOWN: Air receiving tank of height 2.5 m and diameter 0.75 m; inside air is at 3 atm and 100°C while outside ambient air is 25°C.

FIND: (a) Receiver wall temperature and heat transfer to the ambient air; assume receiver wall is T_s = 60°C to facilitate use of the free convection correlations; (b) Whether film temperatures $T_{f,i}$ and $T_{f,o}$ were reasonable; if not, use an iteration procedure to find consistent values; and (c) Receiver wall temperatures, $T_{s,i}$ and $T_{s,o}$, considering radiation exchange from the exterior surface ($\varepsilon_{s,o}$ = 0.85) and thermal resistance of the wall (20 mm thick, k = 0.25W/m·K); represent the system by a thermal circuit.

SCHEMATIC:

ASSUMPTIONS: (1) Surface radiation effects are negligible, parts (a,b), (2) Losses from top and bottom of receiver are negligible, (3) Thermal resistance of receiver wall is negligible compared to free convection resistance, parts (a,b), (4) Interior and exterior air is quiescent and extensive.

PROPERTIES: *Table A-4,* Air (assume $T_{f,o}$ = 315 K, 1 atm): $\nu = 1.74 \times 10^{-5}$ m²/s, k = 0.02741 W/m·K, $\alpha = 2.472 \times 10^{-5}$ m²/s, Pr =0.7049; *Table A-4,* Air (assume $T_{f,i}$ = 350 K, 3 atm): $\nu = 2.092 \times 10^{-5}$ m²/s/3= 6.973×10^{-6} m²/s, k = 0.030 W/m·K, $\alpha = 2.990 \times 10^{-5}$ m²/s/3 = 9.967×10^{-6} m²/s, Pr = 0.700. Note that the pressure effect is present for ν and α since ρ(1 atm) = 1/3ρ(3 atm); other properties (c_p, k, μ) are assumed independent of pressure.

ANALYSIS: The heat transfer rate from the receiver follows from the thermal circuit,

$$q = \frac{\Delta T}{R_t} = \frac{T_{\infty,i} - T_{\infty,o}}{1/\bar{h}_o A_s + 1/\bar{h}_i A_s} = \frac{A_s\left(T_{\infty,i} - T_{\infty,o}\right)}{1/\bar{h}_o + 1/\bar{h}_i} \quad (1)$$

where $\bar{h}_o$ and $\bar{h}_i$ must be estimated from free convection correlations. We must assume a value of T_s in order to obtain first estimates for $\Delta T_o = T_s - T_{\infty,o}$ and $\Delta T_i = T_{\infty,o} - T_s$ as well as $T_{f,o}$ and $T_{f,i}$. Assume that T_s = 60°C, then ΔT_o = 60 - 25 = 35°C, $T_{f,o}$ = 315 K and ΔT_i = 100 - 60 = 40°C, and $T_{f,i}$ = 350 K.

$$Ra_{L,o} = \frac{g\beta\Delta T L^3}{\nu\alpha} = \frac{9.8\,m/s^2\,(1/315\,K)\times 35\,K\,(2.5\,m)^3}{1.74\times10^{-5}\,m^2/s\times 2.472\times10^{-5}\,m^2/s} = 3.952\times10^{10}$$

$$Ra_{L,i} = \frac{9.8\,m/s^2\,(1/350\,K)\times 40\,K\,(2.5\,m)^3}{6.973\times10^{-6}\,m^2/s\times 9.967\times10^{-6}\,m^2/s} = 2.518\times10^{11}$$

Approximating the receiver wall as a vertical plate, Eq. 9.26 yields

Continued...

PROBLEM 9.30 (Cont.)

$$\overline{Nu}_{L,o} = \left[0.825 + \frac{0.387 Ra_{L,o}^{1/6}}{\left[1+(0.492/Pr)^{9/16}\right]^{8/27}}\right]^2 = \left[0.825 + \frac{0.387\left(3.952\times10^{10}\right)^{1/6}}{\left[1+(0.492/0.7049)^{9/16}\right]^{8/27}}\right]^2 = 390.0$$

$$\overline{Nu}_{L,i} = \frac{\overline{h}_{L,i}L}{k} = \left[0.825 + \frac{0.387\left(2.518\times10^{11}\right)^{1/6}}{\left[1+(0.492/0.700)^{9/16}\right]^{8/27}}\right]^2 = 706.4$$

$$\overline{h}_{L,o} = \frac{0.02741\,W/m\cdot K}{2.5m}\times 390.0 = 4.27\,W/m^2\cdot K \qquad \overline{h}_{L,i} = \frac{0.030\,W/m\cdot K}{2.5m}\times 706.4 = 8.48\,W/m^2\cdot K$$

From Eq. (1),

$$q = \pi\times0.75m\times2.5m(100-25)K\Big/\left[\frac{1}{4.27}+\frac{1}{8.48}\right]m^2/K\cdot W = 1225W$$ <

Also,

$$T_s = T_{\infty,i} - q/\overline{h}_i\,A_s = 100^\circ C - 1255\,W\Big/\left(8.48\,W/m^2\cdot K\times\pi\times0.75\,m\times2.5\,m\right) = 74.9^\circ C$$ <

(b) From the above result for T_s, the computed film temperatures are

$$T_{f,o} = 323\,K \qquad T_{f,i} = 360\,K$$

as compared to assumed values of 315 and 350 K, respectively. Using *IHT Correlation Tools* for the *Free Convection*, *Vertical Plate*, and the thermal circuit representing Eq. (1) to find T_s, rather than using as assumed value,

$$\frac{T_{\infty,o} - T_s}{1/\overline{h}_o} = \frac{T_s - T_{\infty,o}}{1/\overline{h}_o}$$

we found

$q = 1262$ W $\qquad$ $T_s = 71.4°C$ <

with $T_{f,o}$ = 321K and 359 K. The iteration only influenced the heat rate slightly.

(c) Considering effects due to thermal resistance of the tank wall and radiation exchange, the thermal resistance network representing the system is shown below.

$T_{\infty,i}$ $T_{\infty,o}$ $T_{s,i}$ $T_{s,o}$ k = 0.25 W/m·K L = 20 mm D_i = 750 mm

q $T_{\infty,i}$ $R_{cv,i}$ $T_{s,i}$ R_{wall} $T_{s,o}$ $R_{cv,o}$ $T_{\infty,o}$ $R_{rad,o}$ T_{sur}

Continued …..

Using the *IHT Model*, *Thermal Network*, with the *Correlation Tool for Free Convection*, *Vertical Plate*, and *Properties Tool* for *Air*, a model was developed which incorporates all the foregoing equations of parts (a,b), but includes the thermal resistance of the wall, Table 3.3,

$$R_{wall} = \frac{\ell n\left(D_i / D_o\right)}{2\pi Lk} \qquad\qquad D_o = D_i + 2 \times t$$

Continued...

The results of the analyses are tabulated below showing for comparison those from parts (a) and (b):

Part	$R_{cv,i}$ (K/W)	R_w (K/W)	$R_{cv,o}$ (K/W)	R_{rad} (K/W)	$T_{s,i}$ (°C)	$T_{s,o}$ (°C)	q W
(a)	0.0200	0	0.0398	∞	74.9*	74.9*	1255
(b)	0.0227	0	0.0367	∞	71.4	71.4	1262
(c)	0.0219	0.0132	0.0419	0.0280	68.4	49.3	1445

*Recall we assumed T_s = 60°C in order to simplify the correlation calculation with fixed values of ΔT_i, ΔT_o as well as $T_{f,o}$, $T_{f,i}$.

COMMENTS: (1) In the table note the slight difference between results using assumed values for T_f and ΔT in the correlations (part (a)) and the exact solution (part (b)).

(2) In the part (c) results, considering thermal resistance of the wall and the radiation exchange process, the net effect was to reduce the overall thermal resistance of the system and, hence, the heat rate increased.

(3) In the part (c) analysis, the *IHT Thermal Resistance Network* model was used to create the thermal circuit and generate the required energy balances. The convection resistances were determined from appropriate *Convection Correlation Tools*. The code was developed in two steps: (1) Solve the energy balance relations from the *Network* with assigned values for h_i and h_o to demonstrate that the energy relations were correct and then (2) Call in the *Convection Correlations* and solve with variable coefficients. Because this equation set is very stiff, we used the intrinsic heat transfer function *Tfluid_avg* and followed these steps in the solution: Step (1): Assign constant values to the film temperatures, T_{fi} and T_{fo}, and to the temperature differences in the convection correlations, ΔT_i and ΔT_o; and in the *Initial Guesses* table, restrain all thermal resistances to be positive (minimum value = 1e-20); *Solve*; Step (2): Allow the film temperatures to be unknowns but keep assigned variables for the temperature differences; use the *Load* option and *Solve*. Step (3): Repeat the previous step but allowing the temperature differences to be unknowns. Even though you get a "successful solve" message, repeat the *Load-Solve* sequence until you see no changes in key variables so that you are assured that the Solver has fully converged on the solution.

PROBLEM 9.31

KNOWN: Dimensions and emissivity of cylindrical solar receiver. Incident solar flux. Temperature of ambient air.

FIND: (a) Heat loss and collection efficiency for a prescribed receiver temperature, (b) Effect of receiver temperature on heat losses and collector efficiency.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state, (2) Ambient air is quiescent, (3) Incident solar flux is uniformly distributed over receiver surface, (4) All of the incident solar flux is absorbed by the receiver, (5) Negligible irradiation from the surroundings, (6) Uniform receiver surface temperature, (7) Curvature of cylinder has a negligible effect on boundary layer development, (8) Constant properties.

PROPERTIES: *Table A-4*, air (T_f = 550 K): k = 0.0439 W/m·K, $\nu = 45.6 \times 10^{-6}$ m^2/s, $\alpha = 66.7 \times 10^{-6}$ m^2/s, Pr = 0.683, $\beta = 1.82 \times 10^{-3}$ K^{-1}.

ANALYSIS: (a) The total heat loss is

$$q = q_{rad} + q_{conv} = A_s \varepsilon \sigma T_s^4 + \bar{h} A_s (T_s - T_\infty)$$

With $Ra_L = g\beta(T_s - T_\infty)L^3/\nu\alpha = 9.8\ \text{m/s}^2\ (1.82 \times 10^{-3}\ \text{K}^{-1})\ 500\text{K}\ (12\text{m})^3/(45.6 \times 66.7 \times 10^{-12}\ \text{m}^4/\text{s}^2) = 5.07 \times 10^{12}$, Eq. 9.26 yields

$$\bar{h} = \frac{k}{L}\left\{0.825 + \frac{0.387\, Ra_L^{1/6}}{\left[1 + (0.492/Pr)^{9/16}\right]^{8/27}}\right\}^2 = \frac{0.0439\ \text{W/m}\cdot\text{K}}{12\text{m}}\{0.825 + 42.4\}^2 = 6.83\ \text{W/m}^2\cdot\text{K}$$

Hence, with $A_s = \pi DL = 264$ m^2

$$q = 264\,\text{m}^2 \times 0.2 \times 5.67 \times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4 (800\,\text{K})^4 + 264\,\text{m}^2 \times 6.83\,\text{W/m}^2\cdot\text{K}(500\,\text{K})$$

$$q = q_{rad} + q_{conv} = 1.23 \times 10^6\,\text{W} + 9.01 \times 10^5\,\text{W} = 2.13 \times 10^6\,\text{W}$$ <

With $A_s q_s'' = 2.64 \times 10^7$ W, the collector efficiency is

$$\eta = \left(\frac{A_s q_s'' - q}{A_s q_s''}\right)100 = \frac{\left(2.64 \times 10^7 - 2.13 \times 10^6\right)\text{W}}{2.64 \times 10^7\ \text{W}}(100) = 91.9\%$$ <

Continued …..

PROBLEM 9.31 (Cont.)

(b) As shown below, because of its dependence on temperature to the fourth power, q_{rad} increases more significantly with increasing T_s than does q_{conv}, and the effect on the efficiency is pronounced.

COMMENTS: The collector efficiency is also reduced by the inability to have a perfectly absorbing receiver. Partial reflection of the incident solar flux will reduce the efficiency by at least several percent.

PROBLEM 9.32

KNOWN: An experimental apparatus for measuring the local convection coefficient and the boundary layer temperature distribution for a heated vertical plate immersed in an extensive, quiescent fluid.

FIND: (a) An expression for estimating the radiation heat flux from the sensor as a function of the surface emissivity, surroundings temperature, and the quantity (T_s - T_∞); (b) Using this expression, apply the correction to the measured total heat flux, q''_{tot}, (see Table 1 below for data) to obtain the onvection heat flux, q''_{cv}, and calculate the convection coefficient; (c) Calculate and plot the local convection coefficient, $h_x(x)$, as a function the x-coordinate using the similarity solution, Eqs. 9.19 and 9.20; on the same graph, plot the experimental points; comment on the comparison between the experimental and analytical results; and (d) Compare the experimental boundary-layer air temperature measurements (see Table 2 below for data) with results from the similarity solution, Fig. 9.4(b). Summarize the results of your analysis using the similarity parameter, η, and the dimensionless temperature, T^*. Comment on the comparison between the experimental and analytical results.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Test plate at a uniform temperature, (3) Ambient air is quiescent, (4) Room walls are isothermal and at the same temperature as the plate.

PROPERTIES: *Table A-4*, Air (T_f =(T_s + T_∞)/2 = 303 K, 1 atm): $\nu = 16.19 \times 10^{-6}$ m^2/s, Pr = 0.707, $\beta = 1/T_f$.

ANALYSIS: (a) The radiation heat flux from the sensor as a function of the surface emissivity, surroundings temperature, and the quantity (T_s - T_∞) follows from Eqs. (1.8) and (1.9)

$$q''_{rad} = \bar{h}_{rad}\left(T_s - T_\infty\right) \qquad \bar{h}_{rad} = \varepsilon\sigma\left(T_s + T_\infty\right)\left(T_s^2 + T_\infty^2\right) \qquad (1,2)$$

where $T_{sur} = T_\infty$. Since $T_s \approx T_\infty$, $\bar{h}_{rad} \approx 4\varepsilon\sigma\bar{T}^3$ where $\bar{T} = \left(T_s + T_\infty\right)/2$.

(b) Using the above expression, the radiation heat flux, q''_{rad}, is calculated. This correction is applied to the measured total heat flux, q''_{tot}, to obtain the convection heat flux, q''_{cv}, from which the local convection coefficient, $h_{x,exp}$ is calculated.

$$q''_{cv} = q''_{tot} - q''_{rad} \qquad (3)$$

$$h''_{x,exp} = q''_{cv} / \left(T_s - T_\infty\right) \qquad (4)$$

Continued …..

The heat flux sensor data are given in the first row of the table below, and the subsequent rows labeled (b) are calculated using Eqs. (1, 3, 4).

Table 1
Heat flux sensor data and convection coefficient calculation results

		$T_s - T_\infty = 7.7$ K					
	x (mm)	25	75	175	275	375	475
Data	q''_{tot} (W/m^2)	41.4	27.2	22.0	20.1	18.3	17.2
(b)	q''_{rad} (W/m^2)	2.28	2.28	2.28	2.28	2.28	2.28
(b)	q''_{cv} (W/m^2)	39.12	24.92	19.72	17.82	16.02	14.92
(b)	$h_{x,exp}$ (W/m$^2\cdot$K)	5.08	3.24	2.56	2.31	2.08	1.94
(c)	$h_{x,ss}$ (W/m$^2\cdot$K)	4.44	3.37	2.73	2.44	2.26	2.13

(c) The similarity solution for the vertical surface, Section 9.4, provides the expression for the local Nusselt number in terms of the dimensionless parameters T^* and η. Using Eqs. (9.19) and (9.20),

$$Nu_x = \frac{h_{x,ss}x}{k} = \left(Gr_x/4\right)^{1/4} g\left(Pr\right) \tag{5}$$

$$g\left(Pr\right) = \frac{0.75\ Pr^{1/2}}{\left(0.609 + 1.221\ Pr^{1/2} + 1.238\ Pr\right)^{1/4}} \tag{6}$$

where the local Grashof number is

$$Gr_x = g\beta\left(T_s - T_\infty\right)x^3/\nu^2 \tag{7}$$

and the thermophysical properties are evaluated at the film temperature, $T_f = (T_s + T_\infty)/2$. Using the above relations in the *IHT* workspace along with the properties library for air, the convection coefficient $h_{x,ss}$ is calculated for selected values of x. The results are shown in Table 1 above and the graph below compared to the experimental results.

Continued …..

PROBLEM 9.32 (Cont.)

The experimental results and the calculated similarity solution coefficients are in good agreement. Except near the leading edge, the experimental results are systematically lower than those from the similarity solution.

(d) The experimental boundary-layer air temperature measurements for three discrete y-locations at two x-locations are shown in the first two rows of the table below. From Eq. 9.13, the similarity parameter is

$$\eta = \frac{y}{x}\left(\frac{Gr_x}{4}\right)^{1/4}$$

and the dimensionless temperature for the experimental data are

$$T^*_{exp} = \frac{T - T_\infty}{T_s - T_\infty}$$

Figure 9.4(b) is used to obtain the dimensionless temperature from the similarity solution, T^*_{ss}, for the required values of η and are tabulated below.

Table 2

Boundary-layer air temperature data and similarity solution results

	$T_s - T_\infty = 7.3$ K					
	$x = 200$ mm, $Gr_x = 8.9\times10^6$			$x = 400$ mm, $Gr_x = 7.2\times10^7$		
y (mm)	2.5	5.0	10.0	2.5	5.0	10.0
$T(x,y) - T_\infty$ (K)	5.5	3.8	1.6	5.9	4.5	2.0
T^*_{exp}	0.753	0.521	0.219	0.808	0.616	0.274
η	0.48	0.97	1.93	0.41	0.81	1.63
T^*_{ss}	0.77	0.55	0.22	0.79	0.62	0.28

The experimentally determined dimensionless temperatures, T^*_{exp}, are systematically lower than those from the similarity solution T^*_{ss}. The agreement is excellent at the x = 400 mm location, ranging from less than 1% near the wall to 2% far from the wall. For the x = 200 mm location, nearer to the leading edge, where the boundary layer is thinner and the boundary layer temperature gradient is higher, the agreement is good, but near the wall the differences are larger. Note that for both x-locations far from the wall, T^*_{exp} and T^*_{ss} are in excellent agreement. Would you have expected that behavior?

PROBLEM 9.33

KNOWN: Transformer which dissipates 1000 W whose surface is to be maintained at 47°C in quiescent air and surroundings at 27°C.

FIND: Power removal (a) by free convection and radiation from lateral and upper horizontal surfaces and (b) with 30 vertical fins attached to lateral surface.

SCHEMATIC:

Quiescent air, $T_\infty = 27°C$
$T_{sur} = T_\infty$
Transformer, D = 0.3m, H = 0.5m, 1000W dissipation
$T_s = 47°C$, $\varepsilon_s = 0.8$
Upper surface of transformer
Width, 75mm
Fin, thickness 5mm, 500mm length, $\varepsilon_s = 0.8$

ASSUMPTIONS: (1) Fins are isothermal at lateral surface temperature, T_s, (2) Vertical fins and lateral surface behave as vertical plate, (3) Transformer has isothermal surfaces and loses heat only on top and side.

PROPERTIES: *Table A-4*, Air (T_f = (27+47)°C/2=310K, 1 atm): $\nu = 16.90 \times 10^{-6}\ m^2/s$, $k = 27.0 \times 10^{-3}$ W/m·K, $\alpha = 23.98 \times 10^{-6}\ m^2/s$, Pr = 0.706, $\beta = 1/T_f$.

ANALYSIS: (a) For the vertical lateral (lat) and top horizontal (top) surfaces, the heat loss by radiation and convection is

$$q = q_{lat} + q_{top} = \left(\bar{h}_{lat} + h_r\right) p\, DL\left(T_s - T_\infty\right) + \left(\bar{h}_{top} + h_r\right)\left(p\ ^2D/4\right)\left(T_s - T_\infty\right)$$

where, from Eq. 1.9, the linearized radiation coefficient is

$$h_r = es\ \left(T_s + T_\infty\right)\left(T_s^2 + T_\infty^2\right)$$

$$h_r = 0.8 \times 5.67 \times 10^{-8} W/m^2 \cdot K^4 (320 + 300) K\left(320^2 + 300^2\right) K^2 = 5.41 W/m^2 \cdot K.$$

The free convection coefficient for the lateral and top surfaces is:

Lateral-vertical plate: Using Eq. 9.26 with

$$Ra_L = \frac{gb\ \left(T_s - T_\infty\right) H^3}{na} = \frac{9.8 m/s^2 (1/310K)(47 - 27) K (0.5m)^3}{16.90 \times 10^{-6} m^2/s \times 23.98 \times 10^{-6} m^2/s} = 1.950 \times 10^8$$

$$\overline{Nu}_L = \left\{0.825 + \frac{0.387 Ra_L^{1/6}}{\left[1 + (0.492/Pr)^{9/16}\right]^{8/27}}\right\}^2$$

$$\overline{Nu}_L = \left\{0.825 + \frac{0.387\left(1.950 \times 10^8\right)^{1/6}}{\left[1 + (0.492/0.706)^{9/16}\right]^{8/27}}\right\}^2 = 74.5$$

$$\bar{h}_{lat} = \overline{Nu}_L \cdot k/H = 74.5 \times 0.027 W/m \cdot K/0.5 m = 4.02 W/m^2 \cdot K.$$

Continued …..

PROBLEM 9.33 (Cont.)

Top-horizontal plate: Using Eq. 9.30 with

$$L_c = A_s / P = \frac{\pi D^2/4}{\pi D} = D/4 = 0.075\text{m}$$

$$Ra_L = \frac{g\beta\left(T_s - T_\infty\right)L_c^3}{\nu\alpha} = \frac{9.8\,\text{m/s}^2\left(1/310\,\text{K}\right)\left(47-27\right)\text{K}\left(0.075\text{m}\right)^3}{16.90\times10^{-6}\,\text{m}^2/\text{s}\times23.98\times10^{-6}\,\text{m}^2/\text{s}} = 6.598\times10^5$$

$$\overline{Nu}_L = 0.54 Ra_L^{1/4} = 0.54\left(6.598\times10^5\right)^{1/4} = 15.39$$

$$\bar{h}_{top} = \overline{Nu}_L \cdot k/L_c = 15.39\times0.027\,\text{W/m}\cdot\text{K}/0.075\text{m} = 5.54\,\text{W/m}^2\cdot\text{K}.$$

Hence, the heat loss by convection and radiation is

$$q = \left(4.02+5.41\right)\text{W/m}^2\cdot\text{K}\left(\pi\times0.30\text{m}\times0.50\text{m}\right)\left(47-20\right)\text{K}$$
$$+\left(5.54+5.41\right)\text{W/m}^2\cdot\text{K}\left(\pi\times0.30^2\,\text{m}^2/4\right)\left(47-20\right)\text{K}$$

$$q = \left(88.9+15.5\right)\text{W} = 104\text{W}.$$ <

(b) The effect of adding the vertical fins is to increase the area of the lateral surface to

$$A_{wf} = \left[\pi DH - 30\left(t\cdot H\right)\right] + 30\times2\left(w\cdot H\right)$$

$$A_{wf} = \left[\pi\,0.30\text{m}\times0.50\text{m} - 30\left(0.005\times0.500\right)\text{m}^2\right] + 30\times2\left(0.075\times0.500\right)\text{m}^2$$

$$A_{wf} = \left[0.471-0.075\right]\text{m}^2 + 2.25\text{m}^2 = 2.646\text{m}^2.$$

where t and w are the thickness and width of the fins, respectively. Hence, the heat loss is now

$$q = q_{lat} + q_{top} = \left(\bar{h}_{lat} + h_r\right)A_{wf}\left(T_s - T_\infty\right) + q_{top}$$

$$q = \left(4.02+5.41\right)\text{W/m}^2\times2.646\text{m}^2\times20\text{K} + 15.5\text{W} = 515\text{W}.$$ <

Adding the fins to the lateral surface increases the heat loss by a factor of five.

COMMENTS: Since the fins are not likely to have 100% efficiency, our estimate is optimistic. Further, since the fins see one another, as well as the lateral surface, the radiative heat loss is over predicted.

PROBLEM 9.34

KNOWN: Surface temperature of a long duct and ambient air temperature.

FIND: Heat gain to the duct per unit length of the duct.

SCHEMATIC:

Air, $T_\infty = 35°C$; ℓ; $W = 0.2\,m$; $H = 0.2\,m$; $T_s = 10°C$

s → sides of duct
t → top
b → bottom

ASSUMPTIONS: (1) Surface radiation effects are negligible, (2) Ambient air is quiescent.

PROPERTIES: *Table A-4*, Air ($T_f = (T_\infty + T_s)/2 \approx 300K$, 1 atm): $\nu = 15.89 \times 10^{-6}\ m^2/s$, k = 0.0263 W/m·K, $\alpha = 22.5 \times 10^{-6}\ m^2/s$, Pr = 0.707, $\beta = 1/T_f$.

ANALYSIS: The heat gain to the duct can be expressed as

$$q' = 2q'_s + q'_t + q'_b = \left(2\bar{h}_s \cdot H + \bar{h}_t \cdot W + \bar{h}_b \cdot W\right)\left(T_\infty - T_s\right). \quad (1)$$

Consider now correlations to estimate $\bar{h}_s$, $\bar{h}_t$, and $\bar{h}_b$. From Eq. 9.25, for the sides with L ≡ H,

$$Ra_L = \frac{g\beta\left(T_\infty - T_s\right)L^3}{\nu\alpha} = \frac{9.8\,m/s^2\,(1/300K)(35-10)K \times (0.2m)^3}{15.89\times10^{-6} m^2/s \times 22.5\times10^{-6} m^2/s} = 1.827\times10^7. \quad (2)$$

Eq. 9.27 is appropriate to estimate $\bar{h}_s$,

$$\overline{Nu}_L = 0.68 + \frac{0.670 Ra_L^{1/4}}{\left[1+\left(0.492/Pr\right)^{9/16}\right]^{4/9}} = 0.68 + \frac{0.670\left(1.827\times10^7\right)^{1/4}}{\left[1+\left(0.492/0.707\right)^{9/16}\right]^{4/9}} = 34.29$$

$$\bar{h}_s = \overline{Nu}_L \cdot k/L = 34.29 \times 0.0263\,W/m\cdot K/0.2m = 4.51\,W/m^2\cdot K.$$

(3)

For the top and bottom portions of the duct, $L \equiv A_s/P \approx W/2$, (see Eq. 9.29), find the Rayleigh number from Eq. (2) with L = 0.1 m, $Ra_L = 2.284 \times 10^6$. From the correlations, Eqs. 9.31 and 9.32 for the top and bottom surfaces, respectively, find

$$\bar{h}_t = \frac{k}{(W/2)} \times 0.15 Ra_L^{1/3} = \frac{0.0263\,W/m\cdot K}{0.1m} \times 0.15\left(2.284\times10^6\right)^{1/3} = 5.17\,W/m^2\cdot K. \quad (4)$$

$$\bar{h}_b = \frac{k}{(W/2)} \times \frac{0.0263\,W/m\cdot K}{0.1m} \times 0.27\left(2.284\times10^6\right)^{1/4} = 2.76\,W/m^2\cdot K. \quad (5)$$

The heat rate, Eq. (1), can now be evaluated using the heat transfer coefficients estimated from Eqs. (3), (4), and (5).

$$q' = \left(2\times4.51\,W/m^2\cdot K \times 0.2m + 5.17\,W/m^2\cdot K \times 0.2m + 2.76\,W/m^2\cdot K \times 0.2m\right)(35-10)K$$

$$q' = 84.8\,W/m.$$ <

COMMENTS: Radiation surface effects will be significant in this situation. With knowledge of the duct emissivity and surroundings temperature, the radiation heat exchange could be estimated.

PROBLEM 9.35

KNOWN: Inner surface temperature and dimensions of rectangular duct. Thermal conductivity, thickness and emissivity of insulation.

FIND: (a) Outer surface temperatures and heat losses from the walls, (b) Effect of insulation thickness on outer surface temperatures and heat losses.

SCHEMATIC:

ASSUMPTIONS: (1) Ambient air is quiescent, (2) One-dimensional conduction, (3) Steady-state.

PROPERTIES: *Table A.4*, air (obtained from *Properties* Tool Pad of IHT).

ANALYSIS: (a) The analysis follows that of Example 9.3, except the surface energy balance must now include the effect of radiation. Hence, $q''_{cond} = q''_{conv} + q''_{rad}$, in which case

$$\left(k_i/t\right)\left(T_{s,1} - T_{s,2}\right) = \bar{h}\left(T_{s,2} - T_\infty\right) + h_r\left(T_{s,2} - T_{sur}\right)$$

where $h_r = \varepsilon\sigma\left(T_{s,2} + T_{sur}\right)\left(T_{s,2}^2 + T_{sur}^2\right)$. Applying this expression to each of the top, bottom and side walls, with the appropriate correlation obtained from the *Correlations* Tool Pad of IHT, the following results are determined for t = 25 mm.

Sides: $T_{s,2} = 19.3°C$, $\bar{h} = 2.82\ W/m^2\cdot K$, $h_{rad} = 5.54\ W/m^2\cdot K$

Top: $T_{s,2} = 19.3°C$, $\bar{h} = 2.94\ W/m^2\cdot K$, $h_{rad} = 5.54\ W/m^2\cdot K$ <

Bottom: $T_{s,2} = 20.1°C$, $\bar{h} = 1.34\ W/m^2\cdot K$, $h_{rad} = 5.56\ W/m^2\cdot K$

With $q'' = q''_{cond}$, the surface heat losses may also be evaluated, and we obtain

Sides: $q' = 2H\,q'' = 21.6$ W/m; *Top:* $q' = w\,q'' = 27.0$ W/m; *Bottom:* $q' = w\,q'' = 26.2$ W/m <

(b) For the top surface, the following results are obtained from the parametric calculations

Continued...

PROBLEM 9.35 (Cont.)

COMMENTS: Contrasting the heat rates of part (a) with those predicted in Comment 1 of Example 9.3, it is evident that radiation is significant and increases the total heat loss from 57.6 W/m to 74.8 W/m. As shown in part (b), reductions in $T_{s,o}$ and q' may be effected by increasing the insulation thickness above 0.025 W/m·K, although attendant benefits diminish with increasing t.

PROBLEM 9.36

KNOWN: Electric heater at bottom of tank of 400mm diameter maintains surface at 70°C with engine oil at 5°C.

FIND: Power required to maintain 70°C surface temperature.

SCHEMATIC:

Tank, D = 400mm

Oil, $T_\infty = 5°C$

$T_s = 70°C$

ASSUMPTIONS: (1) Oil is quiescent, (2) Quasi-steady state conditions exist.

PROPERTIES: *Table A-5*, Engine Oil ($T_f = (T_\infty + T_s)/2 = 310K$): $\nu = 288 \times 10^{-6}\ m^2/s$, k = 0.145 W/m·K, $\alpha = 0.847 \times 10^{-7}\ m^2/s$, $\beta = 0.70 \times 10^{-3}\ K^{-1}$.

ANALYSIS: The heat rate from the bottom heater surface to the oil is

$$q = \bar{h} A_s \left(T_s - T_\infty\right)$$

where $\bar{h}$ is estimated from the appropriate correlation depending upon the Rayleigh number Ra_L, from Eq. 9.25, using the characteristic length, L, from Eq. 9.29,

$$L = \frac{A_s}{P} = \frac{\pi D^2/4}{\pi D} = \frac{D}{4} = \frac{0.4m}{4} = 0.1m.$$

The Rayleigh number is

$$Ra_L = \frac{g\beta\left(T_s - T_\infty\right)L^3}{\nu\alpha}$$

$$Ra_L = \frac{9.8m/s^2 \times 0.70 \times 10^{-3} K^{-1}\left(70-5\right)K \times 0.1^3 m^3}{288 \times 10^{-6} m^2/s \times 0.847 \times 10^{-7} m^2/s} = 1.828 \times 10^7.$$

The appropriate correlation is Eq. 9.31 giving

$$\overline{Nu}_L = \frac{\bar{h}L}{k} = 0.15 Ra_L^{1/3} = 0.15\left(1.828 \times 10^7\right)^{1/3} = 39.5$$

$$\bar{h} = \frac{k}{L}\overline{Nu}_L = \frac{0.145 W/m \cdot K}{0.1m} \times 39.5 = 57.3 W/m^2 \cdot K.$$

The heat rate is then

$$q = 57.3 W/m^2 \cdot K\left(\pi/4\right)\left(0.4m\right)^2\left(70-5\right)K = 468W.$$ <

COMMENTS: Note that the characteristic length is D/4 and not D; however, A_s is based upon D. Recognize that if the oil is being continuously heated by the plate, T_∞ could change. Hence, here we have analyzed a quasi-steady state condition.

PROBLEM 9.37

KNOWN: Horizontal, straight fin fabricated from plain carbon steel with thickness 6 mm and length 100 mm; base temperature is 150°C and air temperature is 26°C.

FIND: (a) Fin heat rate per unit width, q'_f, assuming an average fin surface temperature $\overline{T}_s = 125^\circ C$ for estimating free convection and linearized radiation coefficient; how sensitive is q'_f to the assumed value for $\overline{T}_s$?; (b) Compute and plot the heat rate, q'_f as a function of emissivity $0.05 \le \varepsilon \le 0.95$; show also the fraction of the total heat ratio due to radiation exchange.

SCHEMATIC:

q'_f
Air
T_∞ = 25 °C
l
t = 6mm
T_b = 150°C
L = 100mm
x

ASSUMPTIONS: (1) Air is quiescent medium, (2) Surface radiation effects are negligible, (3) One dimensional conduction in fin, (4) Characteristic length, $L_c = A_s/P = \ell L(2\ell + 2L) \approx L/2$.

PROPERTIES: Plain carbon steel, Given $\left(\overline{T}_{fin} \approx 125^\circ C \approx 400\,K\right)$: $k = 57\,W/m\cdot K$, $\varepsilon = 0.5$; *Table A-4*, Air ($T_f = \left(\overline{T}_{fin} + T_\infty\right)/2 = \left(125 + 25\right)^\circ C/2 \approx$ 350 K, 1 atm): $\nu = 20.92 \times 10^{-6}$ m²/s, $\alpha = 29.9 \times 10^{-6}$ m²/s, k = 0.030 W/m·K, Pr = 0.70, $\beta = 1/T_f$.

ANALYSIS: (a) We estimate $\overline{h}$ as the average of the values for a heated plate facing upward and a heated plate facing downward. See Table 9.2, Case 3(a) and (b). Begin by evaluating the Rayleigh number, using Eq. 9.29 for L_c.

$$Ra_L = \frac{g\beta\left(\overline{T}_{fin} - T_\infty\right)L_c^3}{\nu\alpha} = \frac{9.8\,m/s^2\,(1/350K)(125-25)K \times (0.1\,m/2)^3}{20.92\times10^{-6}\,m^2/s \times 29.9\times10^{-6}\,m^2/s} = 5.595\times10^5$$

An average fin temperature of $\overline{T}_{fin} \approx 125^\circ C$ has been assumed in evaluating properties and Ra_L. According to Table 9.2, Eqs. 9.30 and 9.32 are appropriate. For the *upper* fin surface, Eq. 9.30,

$$\overline{Nu}_L = \overline{h}\,L_c/k = 0.54 Ra_L^{1/4} = 0.54\left(5.595\times10^5\right)^{1/4} = 14.77$$

$$\overline{h}_{upper} = \overline{Nu}_L\,k/L_c = 14.77\times0.030\,W/m\cdot K/0.05\,m = 8.86\,W/m^2\cdot K.$$

For the *lower* fin surface, Eq. 9.32,

$$\overline{Nu}_L = \overline{h}\,L/k = 0.27 Ra_L^{1/4} = 0.27\left(5.595\times10^5\right)^{1/4} = 7.384$$

$$\overline{h}_{lower} = \overline{Nu}_L\,k/L = 7.384\times0.030\,W/m\cdot K/0.05\,m = 4.43\,W/m^2\cdot K.$$

The linearized radiation coefficient follows from Eq. 1.9

$$\overline{h}_r = \varepsilon\sigma\left(\overline{T}_{fin} + T_{sur}\right)\left(\overline{T}_{fin}^2 + T_{sur}^2\right)$$

$$\overline{h}_r = 0.5\times5.67\times10^{-8}\,W/m^2\cdot K^4\,(398+298)\left(398^2+298^2\right)K^3 = 4.88\,W/m^2\cdot K$$

Continued …..

Hence, the average heat transfer coefficient for the fin is

$$\bar{h} = \left(\bar{h}_{upper} + \bar{h}_{lower}\right)/2 + \bar{h}_r = \left[(8.86 + 4.43)/2 + 4.88\right] \mathrm{W/m^2 \cdot K} = 11.53 \mathrm{W/m^2 \cdot K}$$

Assuming the fin experiences convection at the tip, from Eq. 3.72,

$$q_f = M \tanh(mL)$$

$$M = \left(\bar{h}PkA_c\right)^{1/2} \theta_b = \left(11.53 \mathrm{W/m^2 \cdot K} \times 2\ell \times 57 \mathrm{W/m \cdot K}\left(6\times10^{-3} \mathrm{m} \times \ell\right)\right)^{1/2} (150 - 25)\mathrm{K} = 352.1 \mathrm{W}$$

$$m = \left(\bar{h} P/k A_c\right)^{1/2} = \left(11.53 \mathrm{W/m^2 \cdot K} \times 2\ell / 57 \mathrm{W/m \cdot K}\left(6\times10^{-3} \mathrm{m} \times \ell\right)\right)^{1/2} = 8.236 \mathrm{m^{-1}}$$

$$mL = 8.236 \mathrm{m^{-1}} \times 0.1 \mathrm{m} = 0.824$$

$$q_f' = q_f/\ell = 352.1 \mathrm{W/m} \times \tanh(0.824) = 238 \mathrm{W/m}.$$ <

To determine how sensitive the estimate for $\bar{h}$ is to the choice of the average fin surface temperature, the foregoing calculations were repeated using the *IHT Correlations Tool and Extended Surface Model* and the results are tabulated below; coefficients have units $\mathrm{W/m^2 \cdot K}$,

$\bar{T}_{fin}$ (°C)	125	135	145
$\bar{h}_{upper}$	4.43	4.54	4.64
$\bar{h}_{lower}$	8.86	9.08	9.28
$\bar{h}_r$	4.88	5.11	5.35
$\bar{h}$	11.5	11.9	12.3
q' (W/m)	238	245	252

The temperature distribution for the $\bar{T}_{fin} = 125^\circ\mathrm{C}$ case is shown above. With $\bar{T}_{fin} = 145^\circ\mathrm{C}$, the tip temperature is about $2^\circ\mathrm{C}$ higher. It appears that $\bar{T}_{fin} = 125^\circ\mathrm{C}$ was a reasonable choice. Note $\bar{T}_{fin}$ is the value at the mid length.

(b) Using the IHT code developed for part (a), the fin heat rate, q_f, was plotted as a function of the emissivity. In this analysis, the convection and radiation coefficients were evaluated for an average fin temperature $\bar{T}_{fin}$ evaluated at L/2. On the same plot we have also shown rad (%) = $\left(\bar{h}_r/\bar{h}\right)\times100$, which is the portion of the total heat rate due to radiation exchange.

PROBLEM 9.38

KNOWN: Width and thickness of sample material. Rate of heat dissipation at bottom surface of sample and temperatures of top and bottom surfaces. Temperature of quiescent air and surroundings.

FIND: Thermal conductivity and emissivity of the sample.

SCHEMATIC:

Sample, k
$T_{sur} = 25°C$
q_{rad}
q_{conv}
$T_\infty = 25°C$
Quiescent air
$T_2 = 100°C, \varepsilon$
L = 25 mm
q_{cond}
$T_1 = 150°C$
W = 250 mm
P_{elec}

ASSUMPTIONS: (1) Steady-state, (2) One-dimensional conduction in sample, (3) Quiescent air, (4) Sample is small relative to surroundings, (5) All of the heater power dissipation is transferred through the sample, (6) Constant properties.

PROPERTIES: *Table A-4*, air (T_f = 335.5K): $\nu = 19.5 \times 10^{-6}$ m²/s, k = 0.0289 W/m·K, $\alpha = 27.8 \times 10^{-6}$ m²/s, Pr = 0.703, $\beta = 0.00298$ K⁻¹.

ANALYSIS: The thermal conductivity is readily obtained by applying Fourier's law to the sample. Hence, with q = P_{elec},

$$k = \frac{P_{elec}/W^2}{(T_1 - T_2)/L} = \frac{70\,W/(0.250m)^2}{50°C/0.025m} = 0.560\,W/m\cdot K \qquad <$$

The surface emissivity may be obtained by applying an energy balance to a control surface about the sample, in which case

$$P_{elec} = q_{conv} + q_{rad} = \left[\bar{h}(T_2 - T_\infty) + \varepsilon\sigma\left(T_2^4 - T_{sur}^4\right)\right]W^2$$

$$\varepsilon = \frac{\left(P_{elec}/W^2\right) - \bar{h}(T_2 - T_\infty)}{\sigma\left(T_2^4 - T_{sur}^4\right)}$$

With L = A_s/P = $W^2/4W$ = W/4 = 0.0625m, $Ra_L = g\beta(T_2 - T_\infty)L^3/\nu\alpha = 9.86 \times 10^5$ and Eq. 9.30 yields

$$\bar{h} = \frac{\overline{Nu}_L\,k}{L} = \frac{k}{L}0.54\,Ra_L^{1/4} = \frac{0.0289\,W/m\cdot K}{0.0625m}0.54\left(9.86\times10^5\right)^{1/4} = 7.87\,W/m^2\cdot K \qquad <$$

Hence,

$$\varepsilon = \frac{70\,W/(0.250m)^2 - 7.87\,W/m^2\cdot K\,(75°C)}{5.67\times10^{-8}\,W/m^2\cdot K^4\left(373^4 - 298^4\right)K^4} = 0.815 \qquad <$$

COMMENTS: The uncertainty in the determination of ε is strongly influenced by uncertainties associated with using Eq. 9.30. If, for example, $\bar{h}$ is overestimated by 10%, the actual value of ε would be 0.905.

PROBLEM 9.39

KNOWN: Diameter, power dissipation, emissivity and temperature of gage(s). Air temperature (Cases A and B) and temperature of surroundings (Case A).

FIND: (a) Convection heat transfer coefficient (Case A), (b) Convection coefficient and temperature of surroundings (Case B).

SCHEMATIC:

T_{sur} = 25°C; Air T_∞ = 27°C; T = 67°C, ε = 0.8; q = 10.8 W; D = 0.160 m

T_{sur}; Air T_∞ = 17°C; q_1 = 9.70 W; q_2 = 5.67 W; T = 77°C, ε_1 = 0.8; T = 77°C, ε_2 = 0.1; D = 0.160 m

ASSUMPTIONS: (1) Steady-state, (2) Quiescent air, (3) Net radiation exchange from surface of gage approximates that of a small surface in large surroundings, (4) All of the electrical power is dissipated by convection and radiation heat transfer from the surface(s) of the gage, (5) Negligible thickness of strip separating semi-circular disks of Part B, (6) Constant properties.

PROPERTIES: *Table A-4*, air (T_f = 320K): $\nu = 17.9 \times 10^{-6}$ m²/s, $\alpha = 25.5 \times 10^{-6}$ m²/s, k = 0.0278 W/m·K, Pr = 0.704, $\beta = 0.00313$ K^{-1}.

ANALYSIS: (a) With $q = q_{conv} + q_{rad} = P_{elec}$ and $A_s = \pi D^2/4 = 0.0201$ m²,

$$\bar{h}_{meas} = \frac{P_{elec} - \varepsilon\sigma A_s\left(T^4 - T_{sur}^4\right)}{A_s\left(T - T_\infty\right)} = \frac{10.8\,\text{W} - 0.8\times 5.67\times 10^{-8}\,\text{W/m}^2\cdot\text{K}^4 \times 0.0201\text{m}^2\left(340^4 - 300^4\right)\text{K}^4}{0.0201\,\text{m}^2\,(40\,\text{K})} = 7.46\,\text{W/m}^2\cdot\text{K} \quad <$$

With $L = A_s/P = D/4 = 0.04$ m and $Ra_L = g\beta(T - T_\infty)L^3/\nu\alpha = 1.72 \times 10^5$, Eq. 9.30 yields

$$\bar{h} = \frac{k}{L}0.54\,Ra_L^{1/4} = \frac{0.0278\,\text{W/m}\cdot\text{K}\times 0.54\left(1.72\times 10^5\right)^{1/4}}{0.04\text{m}} = 7.64\,\text{W/m}^2\cdot\text{K} \quad <$$

Agreement between the two values of $\bar{h}$ is well within the uncertainty of the measurements.

(b) Since the semi-circular disks have the same temperature, each is characterized by the same convection coefficient and $q_{conv,1} = q_{conv,2}$. Hence, with

$$P_{elec,1} = q_{conv,1} + \varepsilon_1\sigma\left(A_s/2\right)\left(T^4 - T_{sur}^4\right) \quad (1)$$

$$P_{elec,2} = q_{conv,2} + \varepsilon_2\sigma\left(A_s/2\right)\left(T^4 - T_{sur}^4\right) \quad (2)$$

$$T_{sur} = \left[T^4 - \frac{P_{elec,1} - P_{elec,2}}{(\varepsilon_1 - \varepsilon_2)\sigma\left(A_s/2\right)}\right]^{1/4} = \left[(350)^4 - \frac{4.03\,\text{W}}{0.7\times 5.67\times 10^{-8}\,\text{W/m}^2\cdot\text{K}^4\times 0.01\,\text{m}^2}\right]^{1/4}$$

$$T_{sur} = 264\,\text{K} \quad <$$

From Eq. (1), the convection coefficient is then

$$\bar{h}_{meas} = \frac{P_{elec,1} - \varepsilon_1\sigma\left(A_s/2\right)\left(T^4 - T_{sur}^4\right)}{\left(A_s/2\right)\left(T - T_\infty\right)} = \frac{9.70\,\text{W} - 4.60\,\text{W}}{(0.01\times 60)\text{m}^2\cdot\text{K}} = 8.49\,\text{W/m}^2\cdot\text{K} \quad <$$

With $Ra_L = 2.58 \times 10^5$, Eq. 9.30 yields

$$\bar{h} = \frac{k}{L}0.054\,Ra_L^{1/4} = \frac{0.0278\,\text{W/m}\cdot\text{K}}{0.04\text{m}}0.54\left(2.58\times 10^5\right)^{1/4} = 8.46\,\text{W/m}^2\cdot\text{K} \quad <$$

Again, agreement between the two values of $\bar{h}$ is well within the experimental uncertainty of the measurements.

COMMENTS: Because the semi-circular disks are at the same temperature, the characteristic length corresponds to that of the circular disk, L = D/4.

PROBLEM 9.40

KNOWN: Horizontal, circular grill of 0.2m diameter with emissivity 0.9 is maintained at a uniform surface temperature of 130°C when ambient air and surroundings are at 24°C.

FIND: Electrical power required to maintain grill at prescribed surface temperature.

SCHEMATIC:

ASSUMPTIONS: (1) Room air is quiescent, (2) Surroundings are large compared to grill surface.

PROPERTIES: *Table A-4*, Air ($T_f = (T_\infty + T_s)/2 = (24 + 130)°C/2 = 350K$, 1 atm): $\nu = 20.92 \times 10^{-6}$ m^2/s, k = 0.030 W/m·K, $\alpha = 29.9 \times 10^{-6}$ m^2/s, $\beta = 1/T_f$.

ANALYSIS: The heat loss from the grill is due to free convection with the ambient air and to radiation exchange with the surroundings.

$$q = A_S \left[\bar{h}\left(T_S - T_\infty\right) + es\left(T_S^4 - T_{sur}^4\right)\right]. \quad (1)$$

Calculate Ra_L from Eq. 9.25,

$$Ra_L = gb\left(T_S - T_\infty\right)L_c^3/na$$

where for a horizontal disc from Eq. 9.29, $L_c = A_s/P = (\pi D^2/4)/\pi D = D/4$. Substituting numerical values, find

$$Ra_L = \frac{9.8\,m/s^2\,(1/350K)(130-24)K(0.25m/4)^3}{20.92\times10^{-6}m^2/s\times29.9\times10^{-6}m^2/s} = 1.158\times10^6.$$

Since the grill is an *upper surface heated*, Eq. 9.30 is the appropriate correlation,

$$\overline{Nu}_L = \bar{h}_L L_c/k = 0.54 Ra_L^{1/4} = 0.54\left(1.158\times10^6\right)^{1/4} = 17.72$$

$$\bar{h}_L = \overline{Nu}_L\, k/L_c = 17.72\times0.030\,W/m\cdot K/\left(0.25m/4\right) = 8.50\,W/m^2\cdot K. \quad (2)$$

Substituting from Eq. (2) for $\bar{h}$ into Eq. (1), the heat loss or required electrical power, q_{elec}, is

$$q = \frac{p}{4}(0.25m)^2\left[8.50\frac{W}{m^2\cdot K}(130-24)K + 0.9\times5.67\times10^{-8}\frac{W}{m^2\cdot K^4}\left((130+273)^4 - (24+273)^4\right)K^4\right]$$

$$q = 44.2W + 46.0W = 90.2W.$$ <

COMMENTS: Note that for this situation, free convection and radiation modes are of equal importance. If the grill were highly polished such that $\varepsilon \approx 0.1$, the required power would be reduced by nearly 50%.

PROBLEM 9.41

KNOWN: Plate dimensions and maximum allowable temperature. Freestream temperature.
FIND: Maximum allowable power dissipation.

SCHEMATIC:

Air $T_\infty = 300K$
$q = P_{elec}$
$T_s = 350K$
$A_s = 1.44 m^2$
$L = 1.2 m$

ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Negligible heat loss from sides and bottom, (4) Isothermal plate.

PROPERTIES: *Table A-4*, Air (T_f = 325K, 1 atm): $\nu = 18.4 \times 10^{-6}$ m²/s, k = 0.028 W/m·K, $\alpha = 26.2 \times 10^{-6}$ m²/s.

ANALYSIS: The power dissipated by convection is

$$P_{elec} = q = \bar{h} A_s \left(T_s - T_\infty \right).$$

With $\quad L = A_s / P = (1.2m)^2 / 4(1.2m) = 0.3\,m$

$$Ra_L = \frac{g b \left(T_s - T_\infty\right) L^3}{n a} = \frac{9.8 m/s^2 (325K)^{-1} (50K)(0.3m)^3}{\left(18.4 \times 10^{-6} m^2/s\right)\left(26.2 \times 10^{-6} m^2/s\right)}$$

$$Ra_L = 8.44 \times 10^7.$$

With the upper surface heated, Eq. 9.31 yields

$$\overline{Nu}_L = \frac{\bar{h} L}{k} = 0.15 Ra_L^{1/3} = 65.8$$

$$\bar{h} = 65.8 \frac{0.028 W/m \cdot K}{0.3m} = 6.14 W/m^2 \cdot K$$

and the power dissipated is

$$q = 6.14 W/m^2 \cdot K (1.2m)^2 (50K)$$

$$P_{elec} = q = 442W. \qquad <$$

COMMENTS: This result corresponds to an average surface heat flux of 442 W/1.44 m² = 307 W/m² = 0.03 W/cm², which is extremely small. Heat dissipation by free convection in this manner is a poor option compared to the heat flux with forced convection (u_∞ = 15 m/s) of 0.15 W/cm².

PROBLEM 9.42

KNOWN: Material properties, inner surface temperature and dimensions of roof of refrigerated truck compartment. Solar irradiation and ambient temperature.

FIND: Outer surface temperature of roof and rate of heat transfer to compartment.

SCHEMATIC:

ASSUMPTIONS: (1) Negligible irradiation from the sky, (2) $T_{s,o} > T_\infty$ (hot surface facing upward) and $Ra_L > 10^7$, (3) Constant properties.

PROPERTIES: *Table A-4*, air (p = 1 atm, $T_f \approx 310K$): $\nu = 16.9 \times 10^{-6}$ m²/s, k = 0.0270 W/m·K, Pr = 0.706, $\alpha = \nu/Pr = 23.9 \times 10^{-6}$ m²/s, $\beta = 0.00323$ K⁻¹.

ANALYSIS: From an energy balance for the outer surface,

$$\alpha_S G_S - q''_{conv} - E = q''_{cond} = \frac{T_{s,o} - T_{s,i}}{R''_{tot}}$$

$$\alpha_S G_S - \overline{h}\left(T_{s,o} - T_\infty\right) - \varepsilon\sigma T_{s,o}^4 = \frac{T_{s,o} - T_{s,i}}{2R''_p + R''_i}$$

where $R''_p = \left(t_1 / k_p\right) = 2.78 \times 10^{-5}\,m^2 \cdot K/W$ and $R''_i = \left(t_2 / k_i\right) = 1.923\,m^2 \cdot K/W$. For a hot surface facing upward and $Ra_L = g\beta\left(T_{s,o} - T_\infty\right)L^3/\alpha\nu > 10^7$, $\overline{h}$ is obtained from Eq. 9.31. Hence, with cancellation of L,

$$\overline{h} = \frac{k}{L} 0.15\,Ra_L^{1/3} = 0.15 \times 0.0270\,W/m\cdot K\left(\frac{9.8\,m/s^2 \times 0.00323\,K^{-1}}{16.9 \times 23.9 \times 10^{-12}\,m^4/s^2}\right)^{1/3}\left(T_{s,o} - T_\infty\right)^{1/3}$$

$$= 1.73\,W/m^2 \cdot K^{4/3}\left(T_{s,o} - 305\,K\right)^{4/3}$$

Hence,

$$0.5\left(750\,W/m^2 \cdot K\right) - 1.73\,W/m^2 \cdot K^{4/3}\left(T_{s,o} - 305\right)^{4/3} - 0.5 \times 5.67 \times 10^{-8}\,W/m^2 \cdot K^4\,T_{s,o}^4 = \frac{T_{s,o} - 263K}{\left(5.56 \times 10^{-5} + 1.923\right)m^2 \cdot K/W}$$

Solving, we obtain $$T_{s,o} = 318.3K = 45.3°C \quad <$$

Hence, the heat load is $$q = \left(W \cdot L_t\right)q''_{cond} = \left(3.5m \times 10m\right)\frac{\left(45.3 + 10\right)°C}{1.923\,m^2 \cdot K/W} = 1007\,W \quad <$$

COMMENTS: (1) The thermal resistance of the aluminum panels is negligible compared to that of the insulation. (2) The value of the convection coefficient is $\overline{h} = 1.73\left(T_{s,o} - T_\infty\right)^{1/3} = 4.10\,W/m^2 \cdot K$.

PROBLEM 9.43

KNOWN: Inner surface temperature and composition of a furnace roof. Emissivity of outer surface and temperature of surroundings.

FIND: (a) Heat loss through roof with no insulation, (b) Heat loss with insulation and inner surface temperature of insulation, and (c) Thickness of fire clay brick which would reduce the insulation temperature, $T_{ins,i}$, to 1350 K.

SCHEMATIC:

Air $T_\infty = 25\ ^oC$; $T_{sur} = 25^oC$; $T_{s,o}$; Roof (4m x 4m); $L_3 = 0.005m$; $L_2 = 0.02m$; $L_1 = 0.08m$; Steel (1010), ε = 0.3; Alumina silica (64kg / m²); Fireclay brick; $T_{ins,i}$; $T_{s,i} = 1700K$

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction through the composite wall, (3) Negligible contact resistance, (4) Constant properties.

PROPERTIES: *Table A-4*, Air ($T_f \approx 400$ K, 1 atm): k = 0.0338 W/m·K, $\nu = 26.4 \times 10^{-6}$ m²/s, $\alpha = 38.3 \times 10^{-6}$ m²/s, Pr = 0.69, $\beta = (400\text{ K})^{-1} = 0.0025\text{ K}^{-1}$; *Table A-1*, Steel 1010 (600 K): k = 48.8 W/m·K; *Table A-3* Alumina-Silica blanket (64 kg/m³, 750 K): k = 0.125 W/m·K; *Table A-3,* Fire clay brick (1478 K): k = 1.8 W/m·K.

ANALYSIS: (a) *Without the insulation*, the thermal circuit is

R_{rad}; $T_{s,i}$; $T_{s,o}$; $T_\infty = T_{sur}$; q_{cond}; $1/hA$; L_1/k_1A; L_3/k_3A

Performing an energy balance at the outer surface, it follows that

$$q_{cond} = q_{conv} + q_{rad} \qquad \frac{T_{s,i} - T_{s,o}}{L_1/k_1A + L_3/k_3A} = hA\left(T_{s,o} - T_\infty\right) + \varepsilon\sigma A\left(T_{s,o}^4 - T_{sur}^4\right) \quad (1,2)$$

where the radiation term is evaluated from Eq. 1.7. The characteristic length associated with free convection from the roof is, from Eq. 9.29 $L = A_s/P = 16\text{m}^2/16\text{ m} = 1\text{ m}$. From Eq. 9.25, with an assumed value for the film temperature, $T_f = 400$ K,

$$Ra_L = \frac{g\beta\left(T_{s,o} - T_\infty\right)L^3}{\nu\alpha} = \frac{9.8\text{ m/s}^2\left(0.0025\text{ K}^{-1}\right)\left(T_{s,o} - T_\infty\right)(1\text{ m})^3}{26.4\times10^{-6}\text{ m}^2/\text{s}\times38.3\times10^{-6}\text{ m}^2/\text{s}} = 2.42\times10^7\left(T_{s,o} - T_\infty\right)$$

Hence, from Eq. 9.31

$$h = \frac{k}{L}0.15Ra_L^{1/3} = \frac{0.0338\text{ W/m}\cdot\text{K}}{1\text{ m}}0.15\left(2.42\times10^7\right)^{1/3}\left(T_{s,o} - T_\infty\right)^{1/3} = 1.47\left(T_{s,o} - T_\infty\right)^{1/3}\text{ W/m}^2\cdot\text{K}.(3)$$

Continued...

PROBLEM 9.43 (Cont.)

The energy balance can now be written

$$\frac{(1700 - T_{s,o})\,\text{K}}{(0.08\text{m}/1.8\,\text{W/m}\cdot\text{K} + 0.005\text{m}/48.8\,\text{W/m}\cdot\text{K})} = 1.47\left(T_{s,o} - 298\,\text{K}\right)^{4/3}$$

$$+0.3\times5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left[T_{s,o}^4 - (298\,\text{K})^4\right]$$

and from iteration, find $T_{s,o} \approx 895$ K. Hence,

$$q = 16\text{m}^2\left\{1.47(895-298)^{4/3}\,\text{W/m}^2 + 0.3\times5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left[(895\,\text{K})^4 - (298\,\text{K})^4\right]\right\}$$

$$q = 16\text{m}^2\{7,389 + 10,780\}\,\text{W/m}^2 = 2.91\times10^5\,\text{W}. \qquad <$$

(b) *With the insulation*, an additional conduction resistance is provided and the energy balance at the outer surface becomes

$$\frac{T_{s,i} - T_{s,o}}{L_1/k_1A + L_2/k_2A + L_3/k_3A} = hA\left(T_{s,o} - T_\infty\right) + \varepsilon\sigma A\left(T_{s,o}^4 - T_{sur}^4\right) \qquad (4)$$

$$\frac{(1700 - T_{s,o})\,\text{K}}{(0.08\text{m}/1.8 + 0.02/0.125 + 0.005/48.8)\,\text{m}^2\cdot\text{K/W}} = 1.47\left(T_{s,o} - 298\text{K}\right)^{4/3}$$

$$+0.3\times5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left[T_{s,o}^4 - (298\,\text{K})^4\right].$$

From an iterative solution, it follows that $T_{s,o} \approx 610$ K. Hence,

$$q = 16\text{m}^2\left\{1.47(610-298)^{4/3}\,\text{W/m}^2 + 0.3\times5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left[(610\,\text{K})^4 - (298\,\text{K})^4\right]\right\}$$

$$q = 16\text{m}^2\{3111 + 2221\}\,\text{W/m}^2 = 8.53\times10^4\,\text{W}. \qquad <$$

The insulation inner surface temperature is given by

$$q = \frac{T_{s,i} - T_{ins,i}}{L_1/k_1A}.$$

Hence

$$T_{ins,i} = -q\frac{L_1}{k_1A} + T_{s,i} = -8.53\times10^4\,\text{W}\frac{0.08\,\text{m}}{1.8\,\text{W/m}\cdot\text{K}\times16\text{m}^2} + 1700\,\text{K} = 1463\,\text{K}. \qquad <$$

(c) To determine the required thickness L_1 of the fire clay brick to reduce $T_{ins,i}$ = 1350 K, we keyboarded Eq. (4) into the IHT Workspace and found

L_1 = 0.13 m. $\qquad <$

COMMENTS: (1) The accuracy of the calculations could be improved by re-evaluating thermophysical properties at more appropriate temperatures.

(2) Convection and radiation heat losses from the roof are comparable. The relative contribution of radiation increases with increasing $T_{s,o}$, and hence decreases with the addition of insulation.

(3) Note that with the insulation, $T_{ins,i}$ = 1463 K exceeds the melting point of aluminum (933 K). Hence, molten aluminum, which can seep through the refractory, would penetrate, and thereby degrade the insulation, under the specified conditions.

PROBLEM 9.44

KNOWN: Dimensions and emissivity of top surface of amplifier. Temperature of ambient air and large surroundings.

FIND: Effect of surface temperature on convection, radiation and total heat transfer from the surface.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state, (2) Quiescent air.

PROPERTIES: *Table A.4,* air (Obtained from *Properties* Tool Pad of IHT).

ANALYSIS: The total heat rate from the surface is $q = q_{conv} + q_{rad}$. Hence,

$$q = \overline{h} A_s \left(T_s - T_\infty\right) + \varepsilon \sigma A_s \left(T_s^4 - T_{sur}^4\right)$$

where $A_s = L^2 = 0.25\ m^2$. Using the *Correlations* and *Properties* Tool Pads of IHT to evaluate the average convection coefficient for the upper surface of a heated, horizontal plate, the following results are obtained.

Over the prescribed temperature range, the radiation and convection heat rates are virtually identical and the heat rate increases from approximately 66 to 153 W.

COMMENTS: A surface temperature above 50°C would be excessive and would accelerate electronic failure mechanisms. If operation involves large power dissipation (> 100 W), the receiver should be vented.

PROBLEM 9.45

KNOWN: Diameter, thickness, emissivity and initial temperature of silicon wafer. Temperature of air and surrounding.

FIND: (a) Initial cooling rate, (b) Time required to achieve prescribed final temperature.

SCHEMATIC:

$T_{sur} = 298$ K

Ambient air
$T_\infty = 298$ K

$q''_{cnv,t}$, q''_{rad}, $q''_{cnv,b}$, q''_{rad}

Silicon wafer
$T_i = 598$ K, $T_f = 323$ K
D = 150 mm, ε = 0.65
δ = 1 mm

ASSUMPTIONS: (1) Negligible heat transfer from side of wafer, (2) Large surroundings, (3) Wafer may be treated as a lumped capacitance, (4) Constant properties, (5) Quiescent air.

PROPERTIES: *Table A-1*, Silicon ($\overline{T}$ = 187°C = 460K): $\rho = 2330$ kg/m^3, $c_p = 813$ J/kg·K, k = 87.8 W/m·K. *Table A-4*, Air ($T_{f,i}$ = 175°C = 448K): $\nu = 32.15 \times 10^{-6}$ m^2/s, k = 0.0372 W/m·K, $\alpha = 46.8 \times 10^{-6}$ m^2/s, Pr = 0.686, $\beta = 0.00223$ K^{-1}.

SOLUTION: (a) Heat transfer is by natural convection and net radiation exchange from top and bottom surfaces. Hence, with $A_s = \pi D^2/4 = 0.0177$ m^2,

$$q = A_s\left[\left(\overline{h}_t + \overline{h}_b\right)\left(T_i - T_\infty\right) + 2\varepsilon\sigma\left(T_i^4 - T_{sur}^4\right)\right]$$

where the radiation flux is obtained from Eq. 1.7, and with $L = A_s/P = 0.0375$m and $Ra_L = g\beta\,(T_i - T_\infty)\,L^3/\alpha\nu = 2.30 \times 10^5$, the convection coefficients are obtained from Eqs. 9.30 and 9.32. Hence,

$$\overline{h}_t = \frac{k}{L}\left(0.54\,Ra_L^{1/4}\right) = \frac{0.0372\,\text{W/m}\cdot\text{K}\times 11.8}{0.0375\text{m}} = 11.7\,\text{W/m}^2\cdot\text{K}$$

$$\overline{h}_b = \frac{k}{L}\left(0.27\,Ra_L^{1/4}\right) = \frac{0.0372\,\text{W/m}\cdot\text{K}\times 5.9}{0.0375\text{m}} = 5.9\,\text{W/m}^2\cdot\text{K}$$

$$q = 0.0177\,\text{m}^2\left[\left(11.7+5.9\right)\text{W/m}^2\cdot\text{K}\left(300\text{K}\right) + 2\times 0.65\times 5.67\times 10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left(598^4 - 298^4\right)\text{K}^4\right]$$

$$q = 0.0177\,\text{m}^2\left[\left(5280 + 8845\right)\text{W/m}^2\right] = 250\,\text{W}$$ <

(b) From the generalized lumped capacitance model, Eq. 5.15,

$$\rho c A_s \delta\,\frac{dT}{dt} = -\left[\left(\overline{h}_t + \overline{h}_b\right)\left(T - T_\infty\right) + 2\varepsilon\sigma\left(T^4 - T_{sur}^4\right)\right]A_s$$

$$\int_{Ti}^{T} dT = -\int_0^t \left[\frac{\left(\overline{h}_t + \overline{h}_b\right)\left(T - T_\infty\right) + 2\varepsilon\sigma\left(T^4 - T_{sur}^4\right)}{\rho c\delta}\right]dt$$

Continued …..

PROBLEM 9.45 (Cont.)

Using the DER function of IHT to perform the integration, thereby accounting for variations in $\overline{h}_t$ and $\overline{h}_b$ with T, the time t_f to reach a wafer temperature of 50°C is found to be

$$t_f\left(T = 320\text{K}\right) = 181\text{s}$$ <

As shown above, the rate at which the wafer temperature decays with increasing time decreases due to reductions in the convection and radiation heat fluxes. Initially, the surface radiative flux (top or bottom) exceeds the heat flux due to natural convection from the top surface, which is twice the flux due to natural convection from the bottom surface. However, because q''_{rad} and q''_{cnv} decay approximately as T^4 and $T^{5/4}$, respectively, the reduction in q''_{rad} with decreasing T is more pronounced, and at t = 181s, q''_{rad} is well below $q''_{cnv,t}$ and only slightly larger than $q''_{cnv,b}$.

COMMENTS: With $\overline{h}_{r,i} = \varepsilon\sigma\left(T_i + T_{sur}\right)\left(T_i^2 + T_{sur}^2\right) = 14.7\ \text{W/m}^2\cdot\text{K}$, the largest cumulative coefficient of $\overline{h}_{tot} = \overline{h}_{r,i} + \overline{h}_{t,i} = 26.4\ \text{W/m}^2\cdot\text{K}$ corresponds to the top surface. If this coefficient is used to estimate a Biot number, it follows that $Bi = \overline{h}_{tot}\left(\delta/2\right)/k = 1.5\times10^{-4} \ll 1$ and the lumped capacitance approximation is excellent.

PROBLEM 9.46

KNOWN: Pyrex tile, initially at a uniform temperature $T_i = 140°C$, experiences cooling by convection with ambient air and radiation exchange with surroundings.

FIND: (a) Time required for tile to reach the safe-to-touch temperature of $T_f = 40°C$ with free convection and radiation exchange; use $\overline{T} = (T_i + T_f)/2$ to estimate the average free convection and linearized radiation coefficients; comment on how sensitive result is to this estimate, and (b) Time-to-cool if ambient air is blown in parallel flow over the tile with a velocity of 10 m/s.

SCHEMATIC:

T_{sur} = 25 °C
Air
T_∞ = 25 °C
Tile, $T_s(t)$, $T_s(0) = T_i$ = 140 °C, ε = 0.80
200 x 200 x 10 mm, Pyrex

ASSUMPTIONS: (1) Tile behaves as spacewise isothermal object, (2) Backside of tile is perfectly insulated, (3) Surroundings are large compared to the tile, (4) For forced convection situation, part (b), assume flow is fully turbulent.

PROPERTIES: *Table A.3*, Pyrex (300 K): ρ = 2225 kg/m^3, c_p = 835 J/kg·K, k = 1.4 W/m·K, ε = 0.80 (given); *Table A.4*, Air $\left(T_f = \left(\overline{T}_s + T_\infty\right)/2 = 330.5\,\text{K}, 1\,\text{atm}\right)$: ν = 18.96 × 10^{-6} m^2/s, k = 0.0286 W/m·K, α = 27.01 × 10^{-6} m^2/s, Pr = 0.7027, β = 1/T_f.

ANALYSIS: (a) For the lumped capacitance system with a constant coefficient, from Eq. 5.6,

$$\frac{T_s(t) - T_\infty}{T_i - T_\infty} = \exp\left[-\left(\frac{\overline{h}A_s}{\rho Vc}\right)t\right] \quad (1)$$

where $\overline{h}$ is the combined coefficient for the convection and radiation processes,

$$\overline{h} = \overline{h}_{cv} + \overline{h}_{rad} \quad (2)$$

and $\quad A_s = L^2 \qquad V = L^2 d \quad (3,4)$

The linearized radiation coefficient based upon the average temperature, $\overline{T}_s$, is

$$\overline{T}_s = \left(T_i + T_f\right)/2 = (140 + 40)^\circ C/2 = 90^\circ C = 363\,K \quad (5)$$

$$\overline{h}_{rad} = \varepsilon\sigma\left(\overline{T}_s + T_{sur}\right)\left(\overline{T}_s^2 + T_{sur}^2\right) \quad (6)$$

$$\overline{h}_{rad} = 0.8 \times 5.67 \times 10^{-8}\,W/m^2 \cdot K^4\,(363 + 298)\left(363^2 + 298^2\right)K^3 = 6.61\,W/m^2 \cdot K$$

The free convection coefficient can be estimated from the correlation for the flat plate, Eq. 9.30, with

$$Ra_L = \frac{g\beta\Delta T L^3}{\nu\alpha} \qquad L = A_s/P = L^2/4L = 0.25L \quad (7,8)$$

Continued...

PROBLEM 9.46 (Cont.)

$$Ra_L = \frac{9.8\,m/s^2\,(1/330\,K)(363-298)\,K\,(0.25\times 0.200\,m)^3}{18.96\times10^{-6}\,m^2/s\times 27.01\times10^{-6}\,m^2/s} = 4.712\times10^5$$

$$\overline{Nu}_L = 0.54 Ra_L^{1/4} = 0.54\left(4.712\times10^5\right)^{1/4} = 14.18$$

$$\bar{h}_{cv} = \overline{Nu}_L\,k/L = 14.18\times 0.0286\,W/m\cdot K/0.25\times 0.200\,m = 8.09\,W/m^2\cdot K$$

From Eq. (2), it follows

$$\bar{h} = (6.61+8.09)\,W/m^2\cdot K = 14.7\,W/m^2\cdot K$$

From Eq. (1), with $A_s/V = 1/d$, where d is the tile thickness, the time-to-cool is found as

$$\frac{40-25}{140-25} = \exp\left[-\frac{14.7\,W/m^2\cdot K\times t_f}{2225\,kg/m^3\times 0.010\,m\times 835\,J/kg\cdot K}\right]$$

$$t_f = 2574s = 42.9\,min$$ <

Using the *IHT Lumped Capacitance Model* with the *Correlations Tool, Free Convection, Flat Plate*, we can perform the analysis where both h_{cv} and h_{rad} are evaluated as a function of the tile temperature. The time-to-cool is

$$t_f = 2860s = 47.7\,min$$ <

which is 10% higher than the approximate value.

(b) Considering parallel flow with a velocity, $u_\infty = 10\,m/s$ over the tile, the Reynolds number is

$$Re_L = \frac{u_\infty L}{\nu} = \frac{10\,m/s\times 0.200m}{18.96\times10^{-6}\,m^2/s} = 1.055\times10^5$$

but, assuming the flow is turbulent at the upstream edge, use Eq. 7.41 to estimate $\bar{h}_{cv}$,

$$\overline{Nu}_L = 0.037\,Re_L^{4/5}\,Pr^{1/3} = 0.037\left(1.055\times10^5\right)^{4/5}(0.7027)^{1/3} = 343.3$$

$$\bar{h}_{cv} = \overline{Nu}_L k/L = 343.3\times 0.0286\,W/m\cdot K/0.200m = 49.1\,W/m^2\cdot K$$

Hence, using Eqs. (2) and (1), find

$$\bar{h} = 57.2\,W/m^2\cdot K \qquad t_f = 661s = 11.0\,min$$ <

COMMENTS: (1) For the conditions of part (a), Bi = hd/k= 14.7 $W/m^2\cdot K \times 0.01m$ / 1.4 W/m·K = 0.105. We conclude that the lumped capacitance analysis is marginally applicable. For the condition of part (b), Bi = 0.4 and, hence, we need to consider spatial effects as explained in Section 5.4. If we considered spatial effects, would our estimates for the time-to-cool be greater or less than those from the foregoing analysis?

(2) For the conditions of part (a), the convection and radiation coefficients are shown in the plot below as a function of cooling time. Can you use this information to explain the relative magnitudes of the t_f estimates?

Coefficients, (W/m^2.K)
20
15
10
5
0
0
1000
2000
3000
Elapsed cooling time, t (s)
Average coefficient, part (a)
Variable coefficient, hcv + hrad, part (b)
Convection coefficient, hcv
Radiation coefficient, hrad

PROBLEM 9.47

KNOWN: Stacked IC boards within a duct dissipating 500 W with prescribed air flow inlet temperature, flow rate, and internal convection coefficient. Outer surface has emissivity of 0.5 and is exposed to ambient air and large surroundings at 25°C.

FIND: Develop a model to estimate outlet temperature of the air, $T_{m,o}$, and the average surface temperature of the duct, $\overline{T}_s$, following these steps: (a) Estimate the average free convection for the outer surface, $\overline{h}_o$, assuming an average surface temperature of 37°C; (b) Estimate the average (linearized) radiation coefficient for the outer surface, $\overline{h}_{rad}$, assuming an average surface temperature of 37°C; (c) Perform an overall energy balance on the duct considering (i) advection of the air flow, (ii) dissipation of electrical power in the ICs, and (iii) heat transfer from the fluid to the ambient air and surroundings. Express the last process in terms of thermal resistances between the fluid and the mean fluid temperature, $\overline{T}_m$, and the outer temperatures T_∞ and T_{sur}; (d) Substituting numerical values into the expression of part (c), calculate $T_{m,o}$ and $\overline{T}_s$; comment on your results and the assumptions required to develop your model.

SCHEMATIC:

$T_{sur} = T_\infty$

Air $T_\infty = 25°C$

$\overline{T}_s$, $\overline{h}_o$, $\overline{h}_{rad}$, ε = 0.5

$\overline{h}_i$

Air

$T_{m,i} = 25°C$

$\dot{\forall} = 1.2\ m^3/min$

$T_{m,o}$

x

Square duct, H = w = 150 mm

$P_{elec} = 500$ W

L = 0.5 m

ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible potential energy, kinetic energy and flow work changes for the internal flow, (3) Constant properties, (4) Power dissipated in IC boards nearly uniform in longitudinal direction, (5) Ambient air is quiescent, and (5) Surroundings are isothermal and large relative to the duct.

PROPERTIES: *Table A-4*, Air ($T_f = (\overline{T}_s + T_\infty)/2 = 304$ K): $\nu = 1.629 \times 10^{-5}\ m^2/s$, $\alpha = 2.309 \times 10^{-5}\ m^2/s$, k = 0.0266 W/m·K, $\beta = 0.003289\ K^{-1}$, Pr = 0.706.

ANALYSIS: (a) *Average, free-convection coefficient over the duct*. Heat loss by free convection occurs on the vertical sides and horizontal top and bottom. The methodology for estimating the average coefficient assuming the average duct surface temperature $\overline{T}_s$ = 37°C follows that of Example 9.3. For the *vertical sides*, from Eq. 9.25 with L = H, find

$$Ra_L = \frac{g\beta\left(\overline{T}_s - T_\infty\right)H^3}{\nu\alpha}$$

$$Ra_L = \frac{9.8\ m/s^2 \times 0.003289\ K^{-1}(37-25)K \times (0.150\ m)^3}{1.629\times10^{-5}\ m^2/s \times 2.309\times10^{-5} m^2/s} = 3.47\times10^6$$

The free convection is laminar, and from Eq. 9.27,

$$\overline{Nu}_L = 0.68 + \frac{0.670\ Ra_L^{1/4}}{\left[1+(0.492/Pr)^{9/16}\right]^{4/9}}$$

Continued …..

$$\overline{Nu}_L = \frac{\bar{h}_v H}{k} = 0.68 + \frac{0.670 \times \left(3.47 \times 10^6\right)^{1/4}}{\left[1 + \left(0.492/0.706\right)^{9/16}\right]^{4/9}} = 23.2$$

$$\bar{h}_v = 4.11 \text{ W/m}^2 \cdot \text{K}$$

For the top and bottom surfaces, $L_c = (A_s/P) = (w \times L)/(2w + 2L) = 0.0577$ m, hence, $Ra_L = 1.974 \times 10^5$ and with Eqs. 9.30 and 9.32, respectively,

Top surface: $\overline{Nu}_L = \dfrac{\bar{h}_t L_c}{k} = 0.54\ Ra_L^{1/4}$; $\bar{h}_t = 5.25 \text{ W/m}^2 \cdot \text{K}$

Bottom surface: $\overline{Nu}_L = \dfrac{\bar{h}_b L_c}{k} = 0.27\ Ra_L^{1/4}$; $\bar{h}_b = 2.62 \text{ W/m}^2 \cdot \text{K}$

The average coefficient for the entire duct is

$$\bar{h}_{cv,o} = \left(2\bar{h}_v + \bar{h}_t + \bar{h}_b\right)/2 = \left(2 \times 4.11 + 5.25 + 2.62\right) \text{W/m}^2 \cdot \text{K} = 4.02 \text{ W/m}^2 \cdot \text{K} \quad <$$

(b) *Average (linearized) radiation coefficient over the duct*. Heat loss by radiation exchange between the duct outer surface and the surroundings on the vertical sides and horizontal top and bottom. With $\bar{T}_s$ = 37°C, from Eq. 1.9,

$$\bar{h}_{rad} = \varepsilon\sigma\left(\bar{T}_s + T_{sur}\right)\left(\bar{T}_s^2 + T_{sur}^2\right)$$

$$\bar{h}_{rad} = 0.5 \times 5.67 \times 10^{-8} \text{W/m}^2 \cdot \text{K}^4 \left(310 + 298\right)\left(310^2 + 298^2\right)\text{K}^3 = 3.2 \text{ W/m}^2 \cdot \text{K} \quad <$$

(c) *Overall energy balance on the fluid in the duct.* The control volume is shown in the schematic below and the energy balance is

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = 0$$

$$-q_{adv} + P_{elec} - q_{out} = 0 \tag{1}$$

The advection term has the form, with $\dot{m} = \dot{\forall}\rho$,

$$q_{adv} = \dot{m}\, c_p \left(T_{m,o} - T_{m,i}\right) \tag{2}$$

and the heat rate q_{out} is represented by the thermal circuit shown below and has the form, with $T_{sur} = T_\infty$,

$$q_{out} = \frac{\bar{T}_m - T_\infty}{R_{cv,i} + \left(1/R_{cv,o} + 1/R_{rad}\right)^{-1}} \tag{3}$$

where $\bar{T}_m$ is the average mean temperature of the fluid, $(T_{m,i} + T_{m,o})/2$. The thermal resistances are evaluated with $A_s = 2(w + H)\,L$ as

$$R_{cv,i} = 1/\bar{h}_i A_s \tag{4}$$

$$R_{cv,o} = 1/\bar{h}_{cv,o} A_s \tag{5}$$

$$R_{rad} = 1/\bar{h}_{rad} A_s \tag{6}$$

Continued …..

PROBLEM 9.47 (Cont.)

Using this energy balance, the outlet temperature of the air can be calculated. From the thermal circuit, the average surface temperature can be calculated from the relation

$$q_{out} = \left(\overline{T}_m - \overline{T}_s\right) / R_{cv,i} \tag{7}$$

(d) *Calculating $T_{m,o}$ and* $\overline{T}_s$. Substituting numerical values into the expressions of Part (c), find

$$T_{m,o} = 45.7°C \qquad \overline{T}_s = 34.0°C$$ <

The heat rates and thermal resistance results are

$$q_{adv} = 480.5\ W \qquad q_{out} = 19.5\ W$$

$$R_{cv,i} = 0.020\ K/W \qquad R_{cv,o} = 0.250\ K/W \qquad R_{rad} = 0.313\ K/W$$

COMMENTS: (1) We assumed $\overline{T}_s$ = 37°C for estimating $\overline{h}_{cv,o}$ and $\overline{h}_{rad}$, whereas from the energy balance we found the value was 34.0°C. Performing an interative solution, with different assumed $\overline{T}_s$ we would find that the results are not sensitive to the $\overline{T}_s$ value, and that the foregoing results are satisfactory.

(2) From the results of Part (d) for the heat rates, note that about 4% of the electrical power is transferred from the duct outer surface. The present arrangement does not provide a practical means to cool the IC boards.

(3) Note that $T_{m,i} < T_s < T_{m,o}$. As such, we can't utilize the usual log-mean temperature (LMTD) expression, Eq. 8.45, in the rate equation for the internal flow analysis. It is for this reason we used the overall coefficient approach representing the heat transfer by the thermal circuit. The average surface temperature of the duct, $\overline{T}_s$, is only used for the purposes of estimating $\overline{h}_{cv,o}$ and $\overline{h}_{rad}$. We represented the effective temperature difference between the fluid and the ambient/surroundings as $\overline{T}_m - T_\infty$. Because the fluid temperature rise is not very large, this assumption is a reasonable one.

PROBLEM 9.48

KNOWN: Parallel flow of air over a highly polished aluminum plate flat plate maintained at a uniform temperature T_s = 47°C by a series of segmented heaters.

FIND: (a) Electrical power required to maintain the heater segment covering the section between x_1 = 0.2 m and x_2 = 0.3m and (b) Temperature that the surface would reach if the air blower malfunctions and heat transfer occurs by free, rather than forced, convection.

SCHEMATIC:

ASSUMPTIONS : (l) Steady-state conditions, (2) Backside of plate is perfectly insulated, (3) Flow is turbulent over the entire length of plate, part (a), (4) Ambient air is extensive, quiescent at 23°C for part (b).

PROPERTIES: *Table A.4*, Air ($T_f = (T_s + T_\infty)/2$ = 308K): $\upsilon = 16.69 \times 10^{-6}$ m^2/s, k = 0.02689 W/m·K, $\alpha = 23.68 \times 10^{-6}$ m^2/s, Pr = 0.7059, $\beta = 1/T_f$; *Table A.12*, Aluminum, highly polished: ε = 0.03.

ANALYSIS: (a) The power required to maintain the segmented heater (x_1 - x_2) is

$$P_e = \bar{h}_{x1-x2}\left(x_2 - x_1\right)w\left(T_s - T_\infty\right) \qquad (1)$$

where $\bar{h}_{x1-x2}$ the average coefficient for the section between x_1 and x_2 can be evaluated as the average of the local values at x_1 and x_2,

$$\bar{h}_{x1-x2} = \left(h\left(x_1\right) + h\left(x_2\right)\right)/2 \qquad (2)$$

Using Eq. 7.37 appropriate for fully turbulent flow, with $Re_x = u_\infty x / k$,

$$Nu_{x1} = 0.0296\,Re_x^{4/5}\,Pr^{1/3}$$

$$Nu_{X1} = 0.0296\left(\frac{10\,\text{m/s} \times 0.2\,\text{m}}{16.69 \times 10^{-6}\,\text{m}^2/\text{s}}\right)^{4/5}\left(0.7059\right)^{1/3} = 304.6$$

$$h_{x1} = Nu_{x1}\,k/x_1 = 304.6 \times 0.02689\,\text{W/m}\cdot\text{K}/0.2\text{m} = 40.9\,\text{W/m}^2\cdot\text{K}$$

$$Nu_{x2} = 421.3 \qquad h_{x2} = 37.8\,\text{W/m}^2\cdot\text{K}$$

Hence, from Eq (2) to obtain $\bar{h}_{x1-x2}$ and Eq. (1) to obtain P_e,

$$\bar{h}_{x1-x2} = \left(40.9 + 37.8\right)\text{W/m}^2\cdot\text{K}/2 = 39.4\,\text{W/m}^2\cdot\text{K}$$

$$P_e = 39.4\,\text{W/m}^2\cdot\text{K}\left(0.3 - 0.2\right)\text{m} \times 0.2\text{m}\left(47 - 23\right)^\circ\text{C} = 18.9\,\text{W}$$ <

Continued...

(b) Without the airstream flow, the heater segment experiences free convection and radiation exchange with the surroundings,

$$P_e = \left[\bar{h}_{cv}\left(T_s - T_\infty\right) + \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right)\right]\left(x_2 - x_1\right)w \tag{3}$$

We will assume that the free convection coefficient, $\bar{h}_{cv}$, for the segment is the same as that for the entire plate. Using the correlation for a flat plate, Eq. 9.30, with

$$Ra_L = \frac{g\beta\Delta T L_c^3}{\nu\alpha} \qquad L_c = \frac{A_s}{P} = \frac{0.2\times0.5\,m^2}{2\left(0.2+0.5\right)m} = 0.0714\,m$$

and evaluating properties at T_f = 308 K,

$$Ra_L = \frac{9.8\,m/s^2\,(1/308K)(47-23)(0.0714m)^3}{16.69\times10^{-6}\,m/s^2\times23.68\times10^{-6}\,m^2/s} = 7.033\times10^5$$

$$\overline{Nu}_L = 0.54 Ra_L^{1/4} = 0.54\left(7.033\times10^5\right)^{1/4} = 15.64$$

$$\bar{h}_{cv} = \overline{Nu}_L\,k/L_c = 15.64\times0.02689\,W/m\cdot K/0.0714m = 5.89\,W/m^2\cdot K$$

Substituting numerical values into Eq. (3),

$$18.9W = \left[5.89\,W/m^2\cdot K\left(T_s - 296\right) + 0.03\times5.67\times10^{-8}\,W/m^2\cdot K^4\left(T_s^4 - 296^4\right)\right]\left(0.3-0.2\right)m\times0.2m$$

$$T_s = 447\,K = 174^\circ C$$ <

COMMENTS: Recognize that in part (b), the assumed value for T_f = 308 K is a poor approximation. Using the above relations in the IHT work space with the *Properties Tool*, find that T_s = 406 K = 133 °C using the properly evaluated film temperature (T_f) and temperature difference (ΔT) in the correlation. From this analysis, $\bar{h}_{cv} = 8.29\,W/m^2\cdot K$ and h_{rad} = 0.3 W/m$^2\cdot$K. Because of the low emissivity of the plate, the radiation exchange process is not significant.

PROBLEM 9.49

KNOWN: Correlation for estimating the average free convection coefficient for the exterior surface of a long horizontal rectangular cylinder (duct) exposed to a quiescent fluid. Consider a horizontal 0.15 m-square duct with a surface temperature of 35°C passing through ambient air at 15°C.

FIND: (a) Calculate the average convection coefficient and the heat rate per unit length using the H-D correlation, (b) Calculate the average convection coefficient and the heat rate per unit length considering the duct as formed by vertical plates (sides) and horizontal plates (top and bottom), and (c) Using an appropriate correlation, calculate the average convection coefficient and the heat rate per unit length for a duct of circular cross-section having a diameter equal to the wetted perimeter of the rectangular duct of part (a). Do you expect the estimates for parts (b) and (c) to be lower or higher than those obtained with the H-D correlation? Explain the differences, if any.

SCHEMATIC:

Air
$T_\infty = 15^oC$
w
H = w = 0.15 m
$T_s = 35^oC$

ASSUMPTIONS: (1) Steady-state conditions, (2) Ambient air is quiescent, (3) Duct surface has uniform temperature.

PROPERTIES: *Table A-4*, air ($T_f = (T_s + T_\infty)/2$ = 298 K, 1 atm): $\nu = 1.571 \times 10^{-5}$ m²/s, k = 0.0261 W/m·K, $\alpha = 2.22 \times 10^{-5}$ m²/s, Pr = 0.708.

ANALYSIS: (a) The Hahn-Didion (H-D) correlation [ASHRAE Proceedings, Part 1, pp 262-67, 1972] has the form

$$\overline{Nu}_p = 0.55\, Ra_p^{1/4} \left(\frac{H}{p}\right)^{1/8} \qquad Ra_p \le 10^7$$

where the characteristic length is the half-perimeter, p = (w + H), and w and H are the horizontal width and vertical height, respectively, of the duct. The thermophysical properties are evaluated at the film temperature. Using *IHT*, with the correlation and thermophysical properties, the following results were obtained.

Ra_p	$\overline{Nu}_D$	$\bar{h}_p\left(W/m^2\cdot K\right)$	$q'_p\,(W/m)$	
5.08×10^7	42.6	3.71	44.5	<

where the heat rate per unit length of the duct is

$$q'_p = \bar{h}_p\, 2\left(H + w\right)\left(T_s - T_\infty\right).$$

(b) Treating the duct as a combination of horizontal (*top*: hot-side up and *bottom*: hot-side down) and two vertical plates (v) as considered in Example 9.3, the following results were obtained

$\bar{h}_t$ (W/m²·K)	$\bar{h}_b$ (W/m²·K)	$\bar{h}_v$ (W/m²·K)	$\bar{h}_{hv}$ (W/m²·K)	q'_{hv} (W/m)	
5.62	2.81	4.78	4.50	54.0	<

Continued

PROBLEM 9.49 (Cont.)

where the average coefficient and heat rate per unit length for the horizontal-vertical plate duct are

$$\bar{h}_{hv} = \left(\bar{h}_t + \bar{h}_b + 2\bar{h}_v\right)/4$$

$$q'_{hv} = \bar{h}_{hv} 2(H+w)(T_s - T_\infty).$$

(c) Consider a circular duct having a wetted perimeter equal to that of the rectangular duct, for which the diameter is

$$\pi D = 2(H+w) \qquad\qquad D = 0.191 \text{ m}$$

Using the Churchill-Chu correlation, Eq. 9.34, the following results are obtained.

Ra_D	$\overline{Nu}_D$	$\bar{h}_D\left(W/m^2\cdot K\right)$	$q'_D\,(W/m)$	
1.31×10^7	30.6	4.19	50.3	<

where the heat rate per unit length for the circular duct is

$$q'_D = \pi D \bar{h}_D (T_s - T_\infty).$$

COMMENTS: (1) The H-D correlation, based upon experimental measurements, provided the lowest estimate for $\bar{h}$ and q'. The circular duct analysis results are in closer agreement than are those for the horizontal-vertical plate duct.

(2) An explanation for the relative difference in $\bar{h}$ and q' values can be drawn from consideration of the boundary layers and induced flows around the surfaces. Viewing the cross-section of the square duct, recognize that flow induced by the bottom surface flows around the vertical sides, joining the vertical plume formed on the top surface. The flow over the vertical sides is quite different than would occur if the vertical surface were modeled as an *isolated* vertical surface. Also, flow from the top surface is likewise modified by flow rising from the sides, and doesn't behave as an *isolated* horizontal surface. It follows that treating the duct as a combination of horizontal-vertical plates (hv results), each considered as *isolated*, would over estimate the average coefficient and heat rate.

(3) It follows that flow over the horizontal cylinder more closely approximates the situation of the square duct. However, the flow is more streamlined; thinnest along the bottom, and of increasing thickness as the flow rises and eventually breaks away from the upper surface. The edges of the duct disrupt the rising flow, lowering the convection coefficient. As such, we expect the horizontal cylinder results to be systematically higher than for the H-D correlation that accounts for the edges.

PROBLEM 9.50

KNOWN: Straight, rectangular cross-sectioned fin with prescribed geometry, base temperature, and environmental conditions.

FIND: (a) Effectiveness considering only free convection with average coefficient, (b) Effectiveness considering also radiative exchange, (c) Finite-difference equations suitable for considering local, rather than average, values.

SCHEMATIC:

A_c t = 2mm

Quiescent air
T_∞ = 25 °C

Θ_b, T_b = 45°C w

L = 100mm T_{sur} = 25 °C

x

ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) One-dimensional conduction in fin, (4) Width of fin much larger than length, w >> L, (5) Uniform heat transfer coefficient over length for Parts (a) and (b).

PROPERTIES: *Table A-1*, Aluminum alloy 2024-T6 (T ≈ (45 + 25) / 2 = 35 ° C ≈ 300 K), k = 177 W/m·K; *Table A-11*, Aluminum alloy 2024-T6 (Given), ε = 0.82; *Table A-4*. Air ($T_f \approx 300$ K), $\nu = 15.89 \times 10^{-6}$ m²/s, k = 26.3×10^{-3} W/m·K, $\alpha = 22.5 \times 10^{-6}$ m²/s, $\beta = 1/T_f = 33.3 \times 10^{-3}$ K⁻¹.

ANALYSIS: (a) The effectiveness of a fin is determined from Eq. 3.81

$$\varepsilon = q_f \big/ \bar{h} A_{c,b} \theta_b \tag{1}$$

where $\bar{h}$ is the average heat transfer coefficient. The fin heat transfer follows from Eq. 3.72

$$q_f = M \frac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL} \tag{2}$$

where

$$M = \left(hPkA_c\right)^{1/2}\theta_b \qquad \text{and} \qquad m = \left(hP/kA_c\right)^{1/2}. \tag{3,4}$$

Horizontal, flat plate correlations assuming $T_f = (T_b + T_\infty)/2 \approx 300$ K may be used to estimate $\bar{h}$, Eqs. 9.30 to 9.32. Calculate first the Rayleigh number

$$Ra_{L_c} = \frac{g\beta\left(\bar{T}_s - T_\infty\right)L_c^3}{\nu\alpha} \tag{5}$$

where $\bar{T}_s$ is the average temperature of the fin surface and L_c is the characteristic length from Eq. 9.29,

$$L_c \equiv \frac{A_s}{P} = \frac{L \times w}{2L + 2w} \approx \frac{L}{2}. \tag{6}$$

Substituting numerical values,

$$Ra_{L_c} = \frac{9.8\,m/s^2 \times 1/300K \times (310-298)K\left(100\times10^{-3}/2\right)^3 m^3}{22.5\times10^{-6}\,m^2/s \times 15.89\times10^{-6}\,m^2/s} = 1.37\times10^5 \tag{7}$$

Continued...

where $\overline{T}_s \approx (T_b + T_f)/2 = 310\text{K}$. Recognize the importance of this assumption which must be justified for a precise result. Using Eq. 9.30 and 9.32 for the upper and lower surfaces, respectively,

$$\overline{Nu}_{L_c} = 0.54\left(1.37\times10^5\right)^{1/4} = 10.4, \qquad \overline{h}_u = \overline{Nu}_{L_c}\times\frac{k}{L_c} = \frac{0.0263\,\text{W/m}\cdot\text{K}}{\left(100\times10^{-8}/2\right)\text{m}} = 5.47\,\text{W/m}^2\cdot\text{K}$$

$$\overline{Nu}_{L_c} = 0.27\left(1.37\times10^5\right)^{1/4} = 5.20, \qquad \overline{h}_\ell = 2.73\,\text{W/m}^2\cdot\text{K}$$

The average value is estimated as $\overline{h}_c = \left(\overline{h}_u + \overline{h}_\ell\right)/2 = 4.10\,\text{W/m}^2\cdot\text{K}$. Using this value in Eqs. (3) and (4), find

$$M = \left[4.10\,\text{W/m}^2\cdot\text{K}\,(2w)\,\text{m}\times177\,\text{W/m}\cdot\text{K}\left(w\times2\times10^{-3}\right)\text{m}^2\right]^{1/2}(45-25)^\circ\text{C} = 34.1w\text{ W}$$

$$m = \left(\overline{h}_c P/kA_c\right)^{1/2} = \left[4.1\,\text{W/m}^2\cdot\text{K}\,(2w)\,\text{m}\Big/177\,\text{W/m}\cdot\text{K}\left(w\times2\times10^{-3}\right)\text{m}^2\right]^{1/2} = 4.81\,\text{m}^{-1}.$$

Substituting these values into Eq. (2), with mL = 0.481 and $q_f/w = q'_f$.

$$q'_f = 34.1\,\text{W/m}\times\frac{\sinh 0.481 + \left(4.1\,\text{W/m}^2\cdot\text{K}/4.81\,\text{m}^{-1}\times177\,\text{W/m}\cdot\text{K}\right)\cosh 0.481}{\cosh 0.481 + \left(4.86\times10^{-3}\right)\sinh 0.481} = 15.2\,\text{W/m}$$

and then from Eq. (1), the effectiveness is

$$\varepsilon = 15.2\,\text{W/m}\times w\Big/4.1\,\text{W/m}^2\cdot\text{K}\left(w\times2\times10^{-3}\text{m}\right)(45-25)^\circ\text{C} = 92.7.$$ <

(b) If radiation exchange with the surroundings is considered, use Eq. 1.9 to determine

$$\overline{h}_r = \varepsilon\sigma\left(\overline{T}_s + T_{sur}\right)\left(\overline{T}_s^2 + T_{sur}^2\right) = 0.82\times5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\,(310+298)(310^2+298^2)\text{K}^3 = 5.23\,\text{W/m}^2\cdot\text{K}.$$

This assumes the fin surface is gray-diffuse and small compared to the surroundings. Using $\overline{h} = \overline{h}_c + \overline{h}_r$ where $\overline{h}_c$ is the convection parameter from part (a), find $\overline{h} = (4.10+5.23)\,\text{W/m}^2\cdot\text{K} = 9.33\,\text{W/m}^2\cdot\text{K}$, $M = 51.4w\text{ W},\ m = 7.26\text{m}^{-1},\ q'_f = 31.8\,\text{W/m}$ giving

$$\varepsilon = 85.2$$ <

(c) To perform the numerical method, we used the *IHT Finite Difference Equation Tool* for *1-D, SS*, extended surfaces. The convection coefficient for each node was expressed as

$$h_{tot,m} = \overline{h}_u(T_m) + \overline{h}_\ell(T_m)/2 + \overline{h}_r(T_m)$$

The effectiveness was calculated from Eq. (1) where the fin heat rate is determined from an energy balance on the base node.

$$q_f = q_{cond} + q_{cv} + q_{rad}$$

$$q_b = q_{cond} = kA_c\left(T_b - T_1\right)/\Delta x$$

$$q_a = q_{cv} + q_{rad} = \overline{h}_{tot,b}\left(P\cdot\Delta w/2\right)\left(T_b - T_{inf}\right)$$

$$\overline{h}_{tot,b} = \left(\overline{h}_u(T_b) + \overline{h}_\ell(T_b)\right)/2 + \overline{h}_r(T_b)$$

Continued...

The results of the analysis (15 nodes, $\Delta x = L/15$)

$$q_f = 33.6\,W/m \qquad \varepsilon = 83.2$$ <

COMMENTS: (1) From the analytical treatments, parts (a) and (b), considering radiation exchange nearly doubles the fin heat rate (31.8 vs. 15.2 W/m) and reduces the effectiveness from 92.7 to 85.2. The numerical method, part (c) considering local variations for h_c and h_{rad}, provides results for q_f' and ε which are in close agreement with the analytical method, part (b).

(2) The *IHT Finite Difference Equation Tool* provides a powerful approach to solving a problem as tedious as this one. Portions of the work space are copied below to illustrate the general logic of the analysis.

```
// Method of Solution:  The Finite-Difference Equation tool for One-Dimensional, Steady-State
Conditions for an extended surface was used to write 15 nodal equations.  The convection and linearized
radiation  coefficient for each node was separately calculated by a User-Defined Function.  */

// User-Defined Function - Upper surface convection coefficients:
/*  FUNCTION h_up ( Ts )
h_up = 0.0263 / 0.05 * NuLcu
NuLcu = 0.54 * (11,421 * (Ts - 298) )^0.25
RETURN h_up
END   */

// User-Defined Function - Linearized radiation coefficients:
/* FUNCTION h_rad ( Ts )
h_rad = 0.82 * sigma * (Ts + 298 ) * (Ts^2 + 298^2 )
sigma = 5.67e-8
RETURN h_rad
END */

/* Node 1: extended surface interior node;e and w labeled 2 and b. */
0.0 = fd_1d_xsur_i(T1,T2,Tb,k,qdot,Ac,P,deltax,Tinf,htot1,q''a)
q''a = 0         // Applied heat flux, W/m^2; zero flux shown
qdot = 0
htot1 = ( h_up(T1) + h_do(T1) ) / 2   +  h_rad(T1)

/* Node 2: extended surface interior node;e and w labeled 2 and b. */
0.0 = fd_1d_xsur_i(T2,T3,T1,k,qdot,Ac,P,deltax,Tinf,htot2,q''a)
htot2 = ( h_up(T2) + h_do(T2) ) / 2   +  h_rad(T2)

/* Node 15: extended surface end node, e-orientation; w labeled inf. */
0.0 = fd_1d_xend_e(T15,T14,k,qdot,Ac,P,deltax,Tinf,htot15,q''a,Tinf,htot15,q''a)
htot15 = ( h_up(T15) + h_do(T15) ) / 2   +  h_rad(T15)

// Assigned Variables:
Tb = 45 + 273             // Base temperature, K
Tinf = 25 + 273           // Ambient temperature, K
Tsur = 25 + 273           // Surroundings temperature, K
L  = 0.1                  // Length of fin, m
deltax = L / 15           // Space increment, m
k = 177                   // Thermal conductivity, W/m.K; fin material
Ac = t * w                // Cross-sectional area, m^2
t = 0.002                 // Fin thickness, m
w = 1                     // Fin width, m; unity value selected
P = 2 * w                 // Perimeter, m
Lc = L / 2                // Characteristic length, convection correlation, m

// Fin heat rate and effectiveness
qf = qcond + qcvrad                                  // Heat rate from the fin base, W
qcond = k * Ac * (Tb - T1 ) / deltax                 // Heat rate, conduction, W
qcvrad =  htotb * P * deltax / 2 * ( Tb - Tinf )     // Heat rate, combined radiation convection, W
htotb = ( h_up(Tb) + h_do(Tb) ) / 2   +  h_rad(Tb)   // Total heat transfer coefficient, W/m^2.K
eff = qf / ( htotb * Ac * (Tb - Tinf) )              // Effectivenss
```

PROBLEM 9.51

KNOWN: Dimensions, emissivity and operating temperatures of a wood burning stove. Temperature of ambient air and surroundings.

FIND: Rate of heat transfer.

SCHEMATIC:

$D_p = 0.25$ m, $L_p = 2$ m, $T_{s,p} = 400$ K, ε = 0.8; Ambient air $T_\infty = 300$ K; $T_{sur} = 300$ K; $L_s = 1$ m, $T_{s,s} = 500$ K, ε = 0.8; $T_{s,s} = 500$ K, ε = 0.8, $L_s = 1$ m; Pipe (p); Side wall (x4)(s); Top wall (t)

ASSUMPTIONS: (1) Steady-state, (2) Quiescent air, (3) Negligible heat transfer from pipe elbow, (4) Free convection from pipe corresponds to that from a vertical plate.

PROPERTIES: *Table A.4*, air ($T_f = 400$ K): $\nu = 26.41 \times 10^{-6}$ m²/s, k = 0.0338 W/m·K, $\alpha = 38.3 \times 10^{-6}$ m²/s, $\beta = 0.0025$ K⁻¹, Pr = 0.69. *Table A.4*, air ($T_f = 350$ K): $\nu = 20.92 \times 10^{-6}$ m²/s, k = 0.030 W/m·K, $\alpha = 29.9 \times 10^{-6}$ m²/s, Pr = 0.70, $\beta = 0.00286$ K⁻¹.

ANALYSIS: Three distinct contributions to the heat rate are made by the 4 side walls, the top surface, and the pipe surface. Hence $q_t = 4q_s + q_t + q_p$, where each contribution includes transport due to convection and radiation.

$$q_s = \bar{h}_s L_s^2 \left(T_{s,s} - T_\infty\right) + h_{rad,s} L_s^2 \left(T_{s,s} - T_{sur}\right)$$

$$q_t = \bar{h}_t L_s^2 \left(T_{s,s} - T_\infty\right) + h_{rad,s} L_s^2 \left(T_{s,s} - T_{sur}\right)$$

$$q_p = \bar{h}_p \left(\pi D_p L_p\right)\left(T_{s,p} - T_\infty\right) + h_{rad,p} \left(\pi D_p L_p\right)\left(T_{s,p} - T_{sur}\right)$$

The radiation coefficients are

$$h_{rad,s} = \varepsilon\sigma\left(T_{s,s} + T_{sur}\right)\left(T_{s,s}^2 + T_{sur}^2\right) = 12.3\,\text{W/m}^2\cdot\text{K}$$

$$h_{rad,p} = \varepsilon\sigma\left(T_{s,p} + T_{sur}\right)\left(T_{s,p}^2 + T_{sur}^2\right) = 7.9\,\text{W/m}^2\cdot\text{K}$$

For the stove side walls, $Ra_{L,s} = g\beta\left(T_{s,s} - T_\infty\right)L_s^3/\alpha\nu = 4.84 \times 10^9$. Similarly, with $(A_s/P) = L_s^2/4L_s = 0.25$ m, $Ra_{L,t} = 7.57 \times 10^7$ for the top surface, and with $L_p = 2$ m, $Ra_{L,p} = 3.59 \times 10^{10}$ for the stove pipe.

For the side walls and the pipe, the average convection coefficient may be determined from Eq. 9.26,

$$\overline{Nu}_L = \left\{0.825 + \frac{0.387 Ra_L^{1/6}}{\left[1 + \left(0.492/Pr\right)^{9/16}\right]^{8/27}}\right\}^2$$

Continued...

which yields $\overline{Nu}_{L,s}$ = 199.9 and $\overline{Nu}_{L,p}$ = 377.6. For the top surface, the average coefficient may be obtained from Eq. 9.31,

$$\overline{Nu}_L = 0.15 Ra_L^{1/3}$$

which yields $\overline{Nu}_{L,t}$ = 63.5. With $\bar{h} = \overline{Nu}(k/L)$, the convection coefficients are

$$\bar{h}_s = 6.8\,W/m^2\cdot K, \quad \bar{h}_t = 8.6\,W/m^2\cdot K, \quad \bar{h}_p = 5.7\,W/m^2\cdot K$$

Hence,

$$q_s = \left(\bar{h}_s + h_{rad,s}\right)L_s^2\left(T_{s,s} - 300\,K\right) = 19.1\,W/m^2\cdot K\left(1\,m^2\right)(200\,K) = 3820\,W$$

$$q_t = \left(\bar{h}_t + h_{rad,s}\right)L_s^2\left(T_{s,s} - 300\,K\right) = 20.9\,W/m^2\cdot K\left(1\,m^2\right)(200\,K) = 4180\,W$$

$$q_p = \left(\bar{h}_p + h_{rad,p}\right)\left(\pi D_p L_p\right)\left(T_{s,p} - 300\,K\right) = 13.6\,W/m^2\cdot K\left(\pi\times 0.25\,m\times 2\,m\right)(100\,K) = 2140\,W$$

and the total heat rate is

$$q_{tot} = 4q_s + q_t + q_p = 21,600\,W$$ <

COMMENTS: The amount of heat transfer is significant, and the stove would be capable of maintaining comfortable conditions in a large, living space under harsh (cold) environmental conditions.

PROBLEM 9.52

KNOWN: Plate, 1m × 1m, inclined at 45° from the vertical is exposed to a net radiation heat flux of 300 W/m^2; backside of plate is insulated and ambient air is at 0°C.

FIND: Temperature plate reaches for the prescribed conditions.

SCHEMATIC:

45°
$T_s, A_s = 1m \times 1m$
Quiescent air, $T_\infty = 0°C$
$q''_{rad} = 300 W/m^2$

ASSUMPTIONS: (1) Net radiation heat flux (300 W/m^2) includes exchange with surroundings, (2) Ambient air is quiescent, (3) No heat losses from backside of plate, (4) Steady-state conditions.

PROPERTIES: *Table A-4*, Air (assuming T_s = 84°C, $T_f = (T_s + T_\infty)/2$ = (84 + 0)°C/2 = 315K, 1 atm): $\nu = 17.40 \times 10^{-6}$ m^2/s, k = 0.0274 W/m·K, $\alpha = 24.7 \times 10^{-6}$ m^2/s, Pr = 0.705, $\beta = 1/T_f$.

ANALYSIS: From an energy balance on the plate, it follows that $q''_{rad} = q''_{conv}$. That is, the net radiation heat flux into the plate is equal to the free convection heat flux to the ambient air. The temperature of the surface can be expressed as

$$T_s = T_\infty + q''_{rad} / \overline{h}_L \qquad (1)$$

where $\overline{h}_L$ must be evaluated from an appropriate correlation. Since this is the *bottom surface of a heated inclined plate*, "*g*" may be replaced by "*g cos q*"; hence using Eq. 9.25, find

$$Ra_L = \frac{g\cos q b\,(T_s - T_\infty) L^3}{na} = \frac{9.8\,m/s^2 \times \cos 45°(1/315K)(84-0)\,K\,(1m)^3}{17.40\times10^{-6}\,m^2/s \times 24.7\times10^{-6}\,m^2/s} = 4.30\times10^9.$$

Since $Ra_L > 10^9$, conditions are turbulent and Eq. 9.26 is appropriate for estimating $\overline{Nu}_L$

$$\overline{Nu}_L = \left\{0.825 + \frac{0.387 Ra_L^{1/6}}{\left[1+(0.492/Pr)^{9/16}\right]^{8/27}}\right\}^2 \qquad (2)$$

$$\overline{Nu}_L = \left\{0.825 + \frac{0.387\left(4.30\times10^9\right)^{1/6}}{\left[1+(0.492/0.705)^{9/16}\right]^{8/27}}\right\}^2 = 193.2$$

$$\overline{h}_L = \overline{Nu}_L\, k/L = 193.2 \times 0.0274\,W/m\cdot K/1m = 5.29\,W/m^2\cdot K. \qquad (3)$$

Substituting $\overline{h}_L$ from Eq. (3) into Eq. (1), the plate temperature is

$$T_s = 0°C + 300\,W/m^2 / 5.29\,W/m^2\cdot K = 57°C.$$

<

COMMENTS: Note that the resulting value of $T_s \approx 57°C$ is substantially lower than the assumed value of 84°C. The calculation should be repeated with a new estimate of T_s, say, 60°C. An alternate approach is to write Eq. (2) in terms of T_s, the unknown surface temperature and then combine with Eq. (1) to obtain an expression which can be solved, by trial-and-error, for T_s.

PROBLEM 9.53

KNOWN: Horizontal rod immersed in water maintained at a prescribed temperature.

FIND: Free convection heat transfer rate per unit length of the rod, q'_{conv}

SCHEMATIC:

$T_s = 56°C$

Quiescent water, $T_\infty = 18°C$

q_{conv}

$D = 5 mm$

ASSUMPTIONS: (1) Water is extensive, quiescent medium.

PROPERTIES: *Table A-6*, Water ($T_f = (T_s + T_\infty)/2 = 310K$): $\rho = 1/v_f = 993.0\ kg/m^3$, $\nu = \mu/\rho = 695 \times 10^{-6}\ N \cdot s/m^2/993.0\ kg/m^3 = 6.999 \times 10^{-7}\ m^2/s$, $\alpha = k/\rho c = 0.628\ W/m \cdot K/993.0\ kg/m^3 \times 4178\ J/kg \cdot K = 1.514 \times 10^{-7}\ m^2/s$, $Pr = 4.62$, $\beta = 361.9 \times 10^{-6}\ K^{-1}$.

ANALYSIS: The heat rate per unit length by free convection is given as

$$q'_{conv} = \overline{h}_D \cdot p D \left(T_s - T_\infty\right). \qquad (1)$$

To estimate $\overline{h}_D$, first find the Rayleigh number, Eq. 9.25,

$$Ra_D = \frac{g\, b \left(T_s - T_\infty\right) D^3}{na} = \frac{9.8 m/s^2 \left(361.9 \times 10^{-6} K^{-1}\right)(56-18) K (0.005m)^3}{6.999 \times 10^{-7} m^2/s \times 1.514 \times 10^{-7} m^2/s} = 1.587 \times 10^5$$

and use Eq. 9.34 for a horizontal cylinder,

$$\overline{Nu}_D = \left\{ 0.60 + \frac{0.387 Ra_D^{1/6}}{\left[1 + (0.599/Pr)^{9/16}\right]^{8/27}} \right\}^2$$

$$\overline{Nu}_D = \left\{ 0.60 + \frac{0.387 \left(1.587 \times 10^5\right)^{1/6}}{\left[1 + (0.599/4.62)^{9/16}\right]^{8/27}} \right\}^2 = 10.40$$

$$\overline{h}_D = \overline{Nu}_D\, k/D = 10.40 \times 0.628 W/m \cdot K/0.005m = 1306 W/m^2 \cdot K. \qquad (2)$$

Substituting for $\overline{h}_D$ from Eq. (2) into Eq. (1),

$$q'_{conv} = 1306 W/m^2 \cdot K \times p\ (0.005m)(56-18) K = 780 W/m.$$

<

COMMENTS: (1) Note the relatively large value of $\overline{h}_D$; if the rod were immersed in air, the heat transfer coefficient would be close to 5 $W/m^2 \cdot K$.

(2) Eq. 9.33 with appropriate values of C and n from Table 9.1 could also be used to estimate $\overline{h}_D$. Find

$$\overline{Nu}_D = C Ra_D^n = 0.48 \left(1.587 \times 10^5\right)^{0.25} = 9.58$$

$$\overline{h}_D = \overline{Nu}_D\, k/D = 9.58 \times 0.628 W/m \cdot K/0.005m = 1203 W/m^2 \cdot K.$$

By comparison with the result of Eq. (2), the disparity of the estimates is ~8%.

PROBLEM 9.54

KNOWN: Horizontal, uninsulated steam pipe passing through a room.

FIND: Heat loss per unit length from the pipe.

SCHEMATIC:

ASSUMPTIONS: (1) Pipe surface is at uniform temperature, (2) Air is quiescent medium, (3) Surroundings are large compared to pipe.

PROPERTIES: *Table A-4*, Air ($T_f = (T_s + T_\infty)/2 = 350K$, 1 atm): $\nu = 20.92 \times 10^{-6}\ m^2/s$, $k = 0.030\ W/m\cdot K$, $\alpha = 29.9 \times 10^{-6}\ m^2/s$, $Pr = 0.700$, $\beta = 1/T_f = 2.857 \times 10^{-3}\ K^{-1}$.

ANALYSIS: Recognizing that the heat loss from the pipe will be by free convection to the air and by radiation exchange with the surroundings, we can write

$$q' = q'_{conv} + q'_{rad} = p\,D\left[\overline{h}_D\left(T_s - T_\infty\right) + e s\left(T_s^4 - T_{sur}^4\right)\right]. \qquad (1)$$

To estimate $\overline{h}_D$, first find Ra_L, Eq. 9.25, and then use the correlation for a horizontal cylinder, Eq. 9.34,

$$Ra_L = \frac{g\,b\left(T_s - T_\infty\right)D^3}{n a} = \frac{9.8 m/s^2\left(1/350K\right)\left(400-300\right)K\left(0.150m\right)^3}{20.92\times10^{-6} m^2/s \times 29.9\times10^{-6} m^2/s} = 1.511\times10^7$$

$$\overline{Nu}_D = \left\{0.60 + \frac{0.387 Ra_L^{1/6}}{\left[1+\left(0.559/Pr\right)^{9/16}\right]^{8/27}}\right\}^2$$

$$\overline{Nu}_D = \left\{0.60 + \frac{0.387\left(1.511\times10^7\right)^{1/6}}{\left[1+\left(0.559/0.700\right)^{9/16}\right]^{8/27}}\right\}^2 = 31.88$$

$$\overline{h}_D = \overline{Nu}_D \cdot k/D = 31.88\times0.030 W/m\cdot K/0.15m = 6.38 W/m^2\cdot K. \qquad (2)$$

Substituting for $\overline{h}_D$ from Eq. (2) into Eq. (1), find

$$q' = p\left(0.150m\right)\left[6.38 W/m^2\cdot K\left(400-300\right)K + 0.85\times5.67\times10^{-8} W/m^2\cdot K^4\left(400^4 - 300^4\right)K^4\right]$$

$$q' = 301 W/m + 397 W/m = 698 W/m.$$ <

COMMENTS: (1) Note that for this situation, heat transfer by radiation and free convection are of equal importance.

(2) Using Eq. 9.33 with constants C,n from Table 9.1, the estimate for $\overline{h}_D$ is

$$\overline{Nu}_D = C Ra_L^n = 0.125\left(1.511\times10^7\right)^{0.333} = 30.73$$

$$\overline{h}_D = \overline{Nu}_D k/D = 30.73\times0.030 W/m\cdot K/0.150m = 6.15 W/m^2\cdot K.$$

The agreement is within 4% of the Eq. 9.34 result.

PROBLEM 9.55

KNOWN: Diameter and outer surface temperature of steam pipe. Diameter, thermal conductivity, and emissivity of insulation. Temperature of ambient air and surroundings.

FIND: Effect of insulation thickness and emissivity on outer surface temperature of insulation and heat loss.

SCHEMATIC: See Example 9.4, Comment 2.

ASSUMPTIONS: (1) Pipe surface is small compared to surroundings, (2) Room air is quiescent.

PROPERTIES: *Table A.4*, air (evaluated using *Properties* Tool Pad of IHT).

ANALYSIS: The appropriate model is provided in Comment 2 of Example 9.4 and includes use of the following energy balance to evaluate $T_{s,2}$,

$$q'_{cond} = q'_{conv} + q'_{rad} \equiv q'$$

$$\frac{2\pi k_i \left(T_{s,1} - T_{s,2}\right)}{\ln\left(r_2/r_1\right)} = \bar{h} 2\pi r_2 \left(T_{s,2} - T_\infty\right) + \varepsilon 2\pi r_2 \sigma \left(T_{s,2}^4 - T_{sur}^4\right)$$

from which the total heat rate q' can then be determined. Using the IHT *Correlations* and *Properties* Tool Pads, the following results are obtained for the effect of the insulation thickness, with $\varepsilon = 0.85$.

The insulation significantly reduces $T_{s,2}$ and q', and little additional benefits are derived by increasing t above 25 mm. For t = 25 mm, the effect of the emissivity is as follows.

Continued...

PROBLEM 9.55 (Cont.)

Although the surface temperature decreases with increasing emissivity, the heat loss increases due to an increase in net radiation to the surroundings.

PROBLEM 9.56

KNOWN: Dimensions and temperature of beer can in refrigerator compartment.

FIND: Orientation which maximizes cooling rate.

SCHEMATIC:

D=0.06m
Air, $T_\infty = 4°C$, $p = 1 atm$
$L = 150 mm$
$T_s = 27°C$
Horizontal, (h)
Vertical, (v)

ASSUMPTIONS: (1) End effects are negligible, (2) Compartment air is quiescent, (3) Constant properties.

PROPERTIES: *Table A-4*, Air (T_f = 288.5K, 1 atm): $\nu = 14.87 \times 10^{-6}$ m^2/s, k =0.0254 W/m·K, α = 21.0×10^{-6} m^2/s, Pr = 0.71, $\beta = 1/T_f = 3.47 \times 10^{-3}$ K^{-1}.

ANALYSIS: The ratio of cooling rates may be expressed as

$$\frac{q_v}{q_h} = \frac{\bar{h}_v}{\bar{h}_h}\frac{p\,DL}{p\,DL}\frac{(T_s - T_\infty)}{(T_s - T_\infty)} = \frac{\bar{h}_v}{\bar{h}_h}.$$

For the *vertical* surface, find

$$Ra_L = \frac{g b\,(T_s - T_\infty)}{n a}L^3 = \frac{9.8\,m/s^2 \times 3.47\times10^{-3}\,K^{-1}\,(23°C)}{(14.87\times10^{-6} m^2/s)(21\times10^{-6} m^2/s)}L^3 = 2.5\times10^9 L^3$$

$$Ra_L = 2.5\times10^9 (0.15)^3 = 8.44\times10^6,$$

and using the correlation of Eq. 9.26,

$$\overline{Nu}_L = \left\{0.825 + \frac{0.387\left(8.44\times10^6\right)^{1/6}}{\left[1+(0.492/0.71)^{9/16}\right]^{8/27}}\right\}^2 = 29.7.$$

Hence

$$\bar{h}_L = \bar{h}_v = \overline{Nu}_L\,\frac{k}{L} = 29.7\frac{0.0254\,W/m\cdot K}{0.15\,m} = 5.03\,W/m^2\cdot K.$$

For the *horizontal* surface, find $Ra_D = \frac{g b\,(T_s - T_\infty)}{n a}D^3 = 2.5\times10^9 (0.06)^3 = 5.4\times10^5$

and using the correlation of Eq. 9.34,

$$\overline{Nu}_D = \left\{0.60 + \frac{0.387\left(5.4\times10^5\right)^{1/6}}{\left[1+(0.559/0.71)^{9/16}\right]^{8/27}}\right\}^2 = 12.24$$

$$\bar{h}_D = \bar{h}_h = \overline{Nu}_D\,\frac{k}{D} = 12.24\frac{0.0254\,W/m\cdot K}{0.06\,m} = 5.18\,W/m^2\cdot K.$$

Hence

$$\frac{q_v}{q_h} = \frac{5.03}{5.18} = 0.97.$$ <

COMMENTS: In view of the uncertainties associated with Eqs. 9.26 and 9.34 and the neglect of end effects, the above result is inconclusive. The cooling rates are approximately the same.

PROBLEM 9.57

KNOWN: Length and diameter of tube submerged in paraffin of prescribed dimensions. Properties of paraffin. Inlet temperature, flow rate and properties of water in the tube.

FIND: (a) Water outlet temperature, (b) Heat rate, (c) Time for complete melting.

SCHEMATIC:

ASSUMPTIONS: (1) Negligible k.e. and p.e. changes for water, (2) Constant properties for water and paraffin, (3) Negligible tube wall conduction resistance, (4) Free convection at outer surface associated with horizontal cylinder in an infinite quiescent medium, (5) Negligible heat loss to surroundings, (6) Fully developed flow in tube.

PROPERTIES: Water (given): c_p = 4185 J/kg·K, k = 0.653 W/m·K, $\mu = 467 \times 10^{-6}$ kg/s·m, Pr = 2.99; Paraffin (given): T_{mp} = 27.4°C, h_{sf} = 244 kJ/kg, k = 0.15 W/m·K, $\beta = 8 \times 10^{-4}\ K^{-1}$, ρ = 770 kg/m^3, $\nu = 5 \times 10^{-6}\ m^2/s$, $\alpha = 8.85 \times 10^{-8}\ m^2/s$.

ANALYSIS: (a) The overall heat transfer coefficient is

$$\frac{1}{U} = \frac{1}{h_i} + \frac{1}{h_o}.$$

To estimate h_i, find $\quad Re_D = \frac{4\dot{m}}{\pi D \mu} = \frac{4 \times 0.1\,kg/s}{\pi \times 0.025m \times 467 \times 10^{-6}\,kg/s \cdot m} = 10{,}906$

and noting the flow is turbulent, use the Dittus-Boelter correlation

$$Nu_D = 0.023 Re_D^{4/5} Pr^{0.3} = 0.023(10{,}906)^{4/5}(2.99)^{0.3} = 54.3$$

$$h_i = \frac{Nu_D\,k}{D} = \frac{54.3 \times 0.653\,W/m \cdot K}{0.025m} = 1418\,W/m^2 \cdot K.$$

To estimate h_o, find

$$Ra_D = \frac{g\beta\left(T_s - T_\infty\right)D^3}{\nu\alpha} = \frac{\left(9.8\,m/s^2\right)8 \times 10^{-4}K^{-1}(55 - 27.4)K\,(0.025m)^3}{5 \times 10^{-6}m^2/s \times 8.85 \times 10^{-8}m^2/s}$$

$$Ra_D = 7.64 \times 10^6$$

and using the correlation of Eq. 9.34, $\quad \overline{Nu}_D = \left\{0.60 + \frac{0.387 Ra_D^{1/6}}{\left[1 + \left(0.559/Pr\right)^{9/16}\right]^{8/27}}\right\}^2 = 35.0$

$$h_o = \overline{Nu}_D \frac{k}{D} = 35.0 \frac{0.15\,W/m \cdot K}{0.025m} = 210\,W/m^2 \cdot K.$$

Alternatively, using the correlation of Eq. 9.33,

Continued …..

PROBLEM 9.57 (Cont.)

$$Nu_D = CRa_D^n \text{ with } C = 0.48,\ n = 0.25 \qquad Nu_D = 25.2$$

$$h_o = 25.2\frac{0.15\,W/m\cdot K}{0.025m} = 151\,W/m^2\cdot K.$$

The significant difference in h_o values for the two correlations may be due to difficulties associated with high Pr applications of one or both correlations. Continuing with the result from Eq. 9.34,

$$\frac{1}{\overline{U}} = \frac{1}{h_i} + \frac{1}{\overline{h}_o} = \frac{1}{1418} + \frac{1}{210} = 5.467\times10^{-3}\,m^2\cdot K/W$$

$$\overline{U} = 183\,W/m^2\cdot K.$$

Using Eq. 8.46, find

$$\frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = \exp\left(-\frac{p\,DL}{\dot{m}c_p}\overline{U}\right) = \exp\left(-\frac{p\times0.025m\times3m}{0.1kg/s\times4185J/kg\cdot K}183\frac{W}{m^2\cdot K}\right)$$

$$T_{m,o} = T_\infty - \left(T_\infty - T_{m,i}\right)0.902 = \left[27.4 - \left(27.4 - 60\right)0.902\right]°C$$

$$T_{m,o} = 56.8°C. \qquad <$$

(b) From an energy balance, the heat rate is

$$q = \dot{m}c_p\left(T_{m,i} - T_{m,o}\right) = 0.1kg/s\times4185J/kg\cdot K\left(60 - 56.8\right)K = 1335W \qquad <$$

or using the rate equation,

$$q = \overline{U}\,A\,\Delta T_{\ell m} = 183\,W/m^2\cdot K\,p\left(0.025m\right)3m\frac{\left(60-27.4\right)K - \left(56.8-27.4\right)K}{\ell n\dfrac{60-27.4}{56.8-27.4}}$$

$$q = 1335W.$$

(c) Applying an energy balance to a control volume about the paraffin,

$$E_{in} = \Delta E_{st}$$

$$q\cdot t = r\,V\,h_{sf} = r\,L\left[WH - pD^2/4\right]h_{sf}$$

$$t = \frac{770kg/m^3\times3m}{1335W}\left[\left(0.25m\right)^2 - \frac{p}{4}\left(0.025m\right)^2\right]2.44\times10^5 J/kg$$

$$t = 2.618\times10^4 s = 7.27h. \qquad <$$

COMMENTS: (1) The value of $\overline{h}_o$ is overestimated by assuming an infinite quiescent medium. The fact that the paraffin is enclosed will increase the resistance due to free convection and hence decrease q and increase t.

(2) Using $\overline{h}_o = 151\,W/m^2\cdot K$ results in $\overline{U} = 136\,W/m^2\cdot K, T_{m,o} = 57.6°C$, q = 1009 W and t = 9.62 h.

PROBLEM 9.58

KNOWN: A long uninsulated steam line with a diameter of 89 mm and surface emissivity of 0.8 transports steam at 200°C and is exposed to atmospheric air and large surroundings at an equivalent temperature of 20°C.

FIND: (a) The heat loss per unit length for a calm day when the ambient air temperature is 20°C; (b) The heat loss on a breezy day when the wind speed is 8 m/s; and (c) For the conditions of part (a), calculate the heat loss with 20-mm thickness of insulation (k = 0.08 W/m·K). Would the heat loss change significantly with an appreciable wind speed?

SCHEMATIC:

$T_{s,b}$ = 200°C
Steam line (bare)
D_b = 89 mm
ε = 0.8
$T_{sur} = T_\infty$
Air
T_∞ = 20°C
V = 0 (calm day)
or 8 m/s
$T_{s,b}$
$T_{s,o}$
Insulation
D_o = 129 mm
t = 20 mm thick
k = 0.08 W/m-K
Part (c) with insulation

ASSUMPTIONS: (1) Steady-state conditions, (2) Calm day corresponds to quiescent ambient conditions, (3) Breeze is in crossflow over the steam line, (4) Atmospheric air and large surroundings are at the same temperature; and (5) Emissivity of the insulation surface is 0.8.

PROPERTIES: *Table A-4*, Air ($T_f = (T_s + T_\infty)/2$ = 383 K, 1 atm): $\nu = 2.454 \times 10^{-5}$ m²/s, k = 0.03251 W/m·K, $\alpha = 3.544 \times 10^{-5}$ m²/s, Pr = 0.693.

ANALYSIS: (a) The heat loss per unit length from the pipe by convection and radiation exchange with the surroundings is

$$q'_b = q'_{cv} + q'_{rad}$$

$$q'_b = \bar{h}_D P_b \left(T_{s,b} - T_\infty\right) + \varepsilon P_b \sigma \left(T_{s,b}^4 - T_\infty^4\right) \qquad P_b = \pi D_b \qquad (1,2)$$

where D_b is the diameter of the bare pipe. Using the Churchill-Chu correlation, Eq. 9.34, for *free convection* from a horizontal cylinder, estimate $\bar{h}_D$

$$\overline{Nu}_D = \frac{\bar{h} D_b}{k} = \left\{ 0.60 + \frac{0.387\ Ra_D^{1/6}}{\left[1 + \left(0.559/Pr\right)^{9/16}\right]^{8/27}} \right\}^2 \qquad (3)$$

where properties are evaluated at the film temperature, $T_f = (T_s + T_\infty)/2$ and

$$Ra_D = \frac{g\beta\left(T_s - T_\infty\right)D_b^3}{\nu\alpha} \qquad (4)$$

Substituting numerical values, find for the *bare* steam line

Ra_D	$\overline{Nu}_D$	$\bar{h}_D$ (W/m²·K)	q'_{cv} (W/m)	q'_{rad} (W/m)	q'_b (W/m)
3.73×10^6	21.1	7.71	388	541	929

<

Continued …..

PROBLEM 9.58 (Cont.)

(b) For forced convection conditions with V = 8 m/s, use the Churchill-Bernstein correlation, Eq. 7.56,

$$\overline{Nu}_D = \frac{\bar{h}_D D_b}{k} = 0.3 + \frac{0.62\, Re_D^{1/2}\, Pr^{1/3}}{\left[1+(0.4/Pr)^{2/3}\right]^{1/4}}\left[1+\left(\frac{Re_D}{282,000}\right)^{5/8}\right]^{4/5}$$

where $Re_D = VD/\nu$. Substituting numerical values, find

Re_D	$\overline{Nu}_D$	$\bar{h}_{D,b}$ (W/m^2·K)	q'_{cv} (W/m)	q'_{rad} (W/m)	q'_b (W/m)
2.17×10^4	82.5	30.1	1517	541	2058

<

(c) With 20-mm thickness insulation, and for the calm-day condition, the heat loss per unit length is

$$q'_{ins} = \left(T_{s,o} - T_\infty\right)/R'_{tot} \qquad (1)$$

$$R'_t = R'_{ins} + \left[1/R'_{cv} + 1/R'_{rad}\right]^{-1} \qquad (2)$$

where the thermal resistance of the insulation from Eq. 3.28 is

$$R'_{ins} = \ell n\left(D_o/D_b\right)/\left[2\pi k\right] \qquad (3)$$

and the convection and radiation thermal resistances are

$$R'_{cv} = 1/\left(\bar{h}_{D,o}\pi D_o\right) \qquad (4)$$

$$R'_{rad} = 1/\left(\bar{h}_{rad}\,\pi D_o\right) \qquad \bar{h}_{rad,o} = \varepsilon\sigma\left(T_{s,o}+T_\infty\right)\left(T_{s,o}^2+T_\infty^2\right) \qquad (5,6)$$

The outer surface temperature on the insulation, $T_{s,o}$, can be determined by an energy balance on the *surface node of* the thermal circuit.

R'rad
Tsur = T∞
q'
Ts,b
Ts,o
R'ins
T∞
R'cvi

$$\frac{T_{s,b} - T_{s,o}}{R'_{ins}} = \frac{T_{s,o} - T_\infty}{\left[1/R'_{cv} + 1/R'_{rad}\right]^{-1}}$$

Substituting numerical values with $D_{b,o}$ = 129 mm, find the following results.

$R'_{ins} = 0.7384\ m\cdot K/W$ $\qquad \bar{h}_{D,o} = 5.30\ W/m^2\cdot K$

$R'_{cv} = 0.4655\ K/W$ $\qquad \bar{h}_{rad} = 5.65\ W/m^2\cdot K$

$R'_{rad} = 0.4371\ K/W$ $\qquad q'_{ins} = 187\ W/m$ <

$T_{s,o} = 62.1°C$

Continued …..

COMMENTS: (1) For the calm-day conditions, the heat loss by radiation exchange is 58% of the total loss. Using a reflective shield (say, $\varepsilon = 0.1$) on the outer surface could reduce the heat loss by 50%.

(2) The effect of a 8-m/s breeze over the steam line is to increase the heat loss by more than a factor of two above that for a calm day. The heat loss by radiation exchange is approximately 25% of the total loss.

(3) The effect of the 20-mm thickness insulation is to reduce the heat loss to 20% the rate by free convection or to 9% the rate on the breezy day. From the results of part (c), note that the insulation resistance is nearly 3 times that due to the combination of the convection and radiation process thermal resistances. The effect of increased wind speed is to reduce R'_{cv}, but since R'_{ins} is the dominant resistance, the effect will not be very significant.

(4) Comparing the free convection coefficients for part (a), $D_b = 89$ mm with $T_{s,b} = 200°C$, and part (b), $D_{b,o} = 129$ mm with $T_{s,o} = 62.1°C$, it follows that $\overline{h}_{D,o}$ is less than $\overline{h}_{D,b}$ since, for the former, the steam line diameter is larger and the diameter smaller.

(5) The convection correlation models in *IHT* are especially useful for applications such as the present one to eliminate the tediousness of evaluating properties and performing the calculations. However, it is essential that you have experiences in hand calculations with the correlations before using the software.

PROBLEM 9.59

KNOWN: Horizontal tube, 12.5mm diameter, with surface temperature 240°C located in room with an air temperature 20°C.

FIND: Heat transfer rate per unit length of tube due to convection.

SCHEMATIC:

Air, T_∞=20°C

D=12.5mm

T_s=240°C

ASSUMPTIONS: (1) Ambient air is quiescent, (2) Surface radiation effects are not considered.

PROPERTIES: *Table A-4*, Air (T_f = 400K, 1 atm): $\nu = 26.41 \times 10^{-6}\ m^2/s$, k= 0.0338 W/m·K, $\alpha = 38.3 \times 10^{-6}\ m^2/s$, Pr = 0.690, $\beta = 1/T_f = 2.5 \times 10^{-3}\ K^{-1}$.

ANALYSIS: The heat rate from the tube, per unit length of the tube, is

$$q' = \overline{h}\,p\,D\left(T_s - T_\infty\right)$$

where $\overline{h}$ can be estimated from the correlation, Eq. 9.34,

$$\overline{Nu}_D = \left\{0.60 + \frac{0.387 Ra_D^{1/6}}{\left[1+\left(0.559/Pr\right)^{9/16}\right]^{8/27}}\right\}^2 .$$

From Eq. 9.25,

$$Ra_D = \frac{g b\left(T_s - T_\infty\right)D^3}{na} = \frac{9.8 m/s^2 \times 2.5\times10^{-3} K^{-1}\left(240-20\right)K\times\left(12.5\times10^{-3} m\right)^3}{26.41\times10^{-6} m^2/s \times 38.3\times10^{-6} m^2/s} = 10,410.$$

Hence, $$\overline{Nu}_D = \left\{0.60 + \frac{0.387\left(10,410\right)^{1/6}}{\left[1+\left(0.559/0.690\right)^{9/16}\right]^{8/27}}\right\}^2 = 4.40$$

$$\overline{h} = \frac{k}{D}\overline{Nu}_D = \frac{0.0338 W/m\cdot K}{12.5\times10^{-3} m}\times 4.40 = 11.9 W/m^2\cdot K.$$

The heat rate is

$$q' = 11.9 W/m^2\cdot K \times p\left(12.5\times10^{-3} m\right)\left(240-20\right)K = 103 W/m.$$ <

COMMENTS: Heat loss rate by radiation, assuming an emissivity of 1.0 for the surface, is

$$q'_{rad} = e P s\left(T_s^4 - T_\infty^4\right) = 1\times p\left(12.5\times10^{-3} m\right)\times 5.67\times10^{-8}\frac{W}{m^2\cdot K^4}\left[\left(240+273\right)^4 - \left(20+273\right)^4\right]K^4$$

$$q'_{rad} = 138 W/m.$$

Note that P = π D. Note also this estimate assumes the surroundings are at ambient air temperature. In this instance, $q'_{rad} > q'_{conv}$.

PROBLEM 9.60

KNOWN: Insulated steam tube exposed to atmospheric air and surroundings at 25°C.

FIND: (a) Heat transfer rate by free convection to the room, per unit length of the tube; effect on quality, x, at outlet of 30 m length of tube; (b) Effect of radiation on heat transfer and quality of outlet flow; (c) Effect of emissivity and insulation thickness on heat rate.

SCHEMATIC:

ASSUMPTIONS: (1) Ambient air is quiescent, (2) Negligible surface radiation (part a), (3) Tube wall resistance negligible.

PROPERTIES: Steam tables, steam (sat., 4 bar): h_f = 566 kJ/kg, T_{sat} = 416 K, h_g = 2727 kJ/kg, h_{fg} = 2160 kJ/kg, $v_g = 0.476 \times 10^3$ m^3/kg; *Table A.3*, magnesia, 85% (310 K): k_m = 0.051 W/m·K; *Table A.4*, air (assume T_s = 60°C, T_f = (60 + 25)°C/2 = 315 K, 1 atm): $\nu = 17.4 \times 10^{-6}$ m^2/s, k = 0.0274 W/m·K, $\alpha = 24.7 \times 10^{-6}$ m^2/s, Pr = 0.705, T_f = 1/315 K = 3.17×10^{-3} K^{-1}.

ANALYSIS: (a) The heat rate per unit length of the tube (see sketch) is given as,

$$q' = \frac{T_i - T_\infty}{R'_t} \qquad \text{where} \qquad \frac{1}{R'_t} = \left[\frac{1}{\bar{h}_o \pi D_3} + \frac{1}{2\pi k_m}\ln\frac{D_3}{D_2} + \frac{1}{\bar{h}_i \pi D_1}\right]^{-1} \qquad (1,2)$$

To estimate $\bar{h}_o$, we have assumed $T_s \approx 60°C$ in order to calculate Ra_L from Eq. 9.25,

$$Ra_D = \frac{g\beta(T_s - T_\infty)D_3^3}{\nu\alpha} = \frac{9.8\,m/s^2 \times 3.17\times10^{-3}K^{-1}(60-25)K(0.115\,m)^3}{17.4\times10^{-6}\,m^2/s \times 24.7\times10^{-6}\,m^2/s} = 3.85\times10^6 .$$

The appropriate correlation is Eq. 9.34; find

$$\overline{Nu}_D = \left\{0.60 + \frac{0.387(Ra_D)^{1/6}}{\left[1+(0.559/Pr)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.60 + \frac{0.387\left(3.85\times10^6\right)^{1/6}}{\left[1+(0.559/0.705)^{9/16}\right]^{8/27}}\right\}^2 = 21.4$$

$$\bar{h}_o = \frac{k}{D_3}\overline{Nu}_D = \frac{0.0274\,W/m\cdot K}{0.115\,m} \times 21.4 = 5.09\,W/m^2\cdot K .$$

Substituting numerical values into Eq. (2), find

$$\frac{1}{R'_t} = \left[\frac{1}{5.09\,W/m^2\cdot K\times\pi 0.115\,m} + \frac{1}{2\pi\times0.051\,W/m\cdot K}\ln\frac{115}{65} + \frac{1}{11,000\,W/m^2\cdot K\pi\times0.055\,m}\right]^{-1} = 0.430\,W/m\cdot K$$

and from Eq. (1), $\qquad q' = 0.430\,W/m\cdot K(416-298)K = 50.8\,W/m$ <

Continued...

We need to verify that the assumption of $T_s = 60°C$ is reasonable. From the thermal circuit,

$$T_s = T_\infty + q'/\bar{h}_o \pi D_3 = 25^\circ C + 50.8\,W/m \Big/ \left(5.09\,W/m^2 \cdot K \times \pi \times 0.115\,m\right) = 53^\circ C.$$

Another calculation using $T_s = 53°C$ would be appropriate for a more precise result.

Assuming q' is constant, the enthalpy of the steam at the outlet (L = 30 m), h_2, is

$$h_2 = h_1 - q' \cdot L/\dot{m} = 2727\,kJ/kg - 50.8\,W/m \times 30\,m/14.97\,kg/s = 2625\,kJ/kg$$

where $\dot{m} = \rho_g A_c u_m$ with $\rho_g = 1/v_g$ and $A_c = \pi D_1^2/4$. For negligible pressure drop,

$$x = (h_2 - h_f)/h_{fg} = (2625 - 566)\,kJ/kg/(2160\,kJ/kg) = 0.953.$$ <

(b) With radiation, we first determine T_s by performing an energy balance at the outer surface, where

$$q'_i = q'_{conv,o} + q'_{rad}$$

$$\frac{T_i - T_s}{R'_i} = \bar{h}_o \pi D_3 (T_s - T_\infty) + \pi D_3 \varepsilon \sigma \left(T_s^4 - T_{sur}^4\right)$$

and

$$R'_i = \frac{1}{\bar{h}_i \pi D_1} + \frac{1}{2\pi k_m} \ln \frac{D_3}{D_2}$$

From knowledge of T_s, $q'_i = (T_i - T_s)/R'_i$ may then be determined. Using the *Correlations* and *Properties* Tool Pads of IHT to determine $\bar{h}_o$ and the properties of air evaluated at $T_f = (T_s + T_\infty)/2$, the following results are obtained.

Condition	T_s (°C)	q'_i (W/m)
ε = 0.8, D_3 = 115 mm	41.8	56.9
ε = 0.8, D_3 = 165 mm	33.7	37.6
ε = 0.2, D_3 = 115 mm	49.4	52.6
ε = 0.2, D_3 = 165 mm	38.7	35.9

COMMENTS: Clearly, a significant reduction in heat loss may be realized by increasing the insulation thickness. Although T_s, and hence $q'_{conv,o}$, increases with decreasing ε, the reduction in q'_{rad} is more than sufficient to reduce the heat loss.

PROBLEM 9.61

KNOWN: Dissipation rate of an electrical cable suspended in air.

FIND: Surface temperature of the cable, T_s.

SCHEMATIC:

ASSUMPTIONS: (1) Quiescent air, (2) Cable in horizontal position, (3) Negligible radiation exchange.

PROPERTIES: *Table A-4*, Air ($T_f = (T_s + T_\infty)/2 = 325K$, based upon initial estimate for T_s, 1 atm): $\nu = 18.41 \times 10^{-6}\ m^2/s$, $k = 0.0282\ W/m{\cdot}K$, $\alpha = 26.2 \times 10^{-6}\ m^2/s$, $Pr = 0.704$.

ANALYSIS: From the rate equation on a unit length basis, the surface temperature is

$$T_s = T_\infty + q'/\pi D\bar{h}$$

where $\bar{h}$ is estimated by an appropriate correlation. Since such a calculation requires knowledge of T_s, an iteration procedure is required. Begin by assuming $T_s = 77°C$ and calculated Ra_D,

$$Ra_D = g\beta\Delta T D^3/\nu\alpha \quad \text{where} \quad \Delta T = T_s - T_\infty \quad \text{and} \quad T_f = (T_s + T_\infty)/2 \qquad (1,2,3)$$

For air, $\beta = 1/T_f$, and substituting numerical values,

$$Ra_D = 9.8\frac{m}{s^2}(1/325K)(77-27)K(0.025m)^3/18.41\times10^{-6}\frac{m^2}{s}\times26.2\times10^{-6}\frac{m^2}{s} = 4.884\times10^4.$$

Using the Churchill-Chu relation, find $\bar{h}$.

$$\overline{Nu}_D = \frac{\bar{h}D}{k} = \left\{0.60 + \frac{0.387Ra_D^{1/6}}{\left[1+(0.559/Pr)^{9/16}\right]^{8/27}}\right\}^2 \qquad (4)$$

$$\bar{h} = \frac{0.0282W/m\cdot K}{0.025m}\left\{0.60 + \frac{0.387\left(4.884\times10^4\right)^{1/6}}{\left[1+(0.559/0.704)^{9/16}\right]^{8/27}}\right\}^2 = 7.28W/m^2\cdot K.$$

Substituting numerical values into Eq. (1), the calculated value for T_s is

$$T_s = 27°C + (30W/m)/\pi\times0.025m\times7.28W/m^2\cdot K = 79.5°C.$$

This value is very close to the assumed value (77°C), but an iteration with a new value of 79°C is warranted. Using the same property values, find for this iteration:

$$Ra_D = 5.08\times10^4 \qquad \bar{h} = 7.35W/m^2\cdot K \qquad T_s = 79°C. \qquad <$$

We conclude that $T_s = 79°C$ is a good estimate for the surface temperature.

COMMENTS: Recognize that radiative exchange is likely to be significant and would have the effect of reducing the estimate for T_s.

PROBLEM 9.62

KNOWN: Dissipation rate of an immersion heater in a large tank of water.

FIND: Surface temperature in water and, if accidentally operated, in air.

SCHEMATIC:

ASSUMPTIONS: (1) Quiescent ambient fluid, (2) Negligible radiative exchange.

PROPERTIES: *Table A-6*, Water and *Table A-4*, Air:

	T(K)	$k \cdot 10^3$(W/m·K)	$\nu \cdot 10^7$(μ/ρ,m^2/s)	$\alpha \cdot 10^7$($k/\rho c_p$,m^2/s)	Pr	$\beta \cdot 10^6$(K^{-1})
Water	315	634	6.25	1.531	4.16	400.4
Air	1500	100	2400	3500	0685	666.7

ANALYSIS: From the rate equation, the surface temperature, T_s, is

$$T_s = T_\infty + q/\left(p D L \bar{h}\right) \tag{1}$$

where $\bar{h}$ is estimated by an appropriate correlation. Since such a calculation requires knowledge of T_s, an iteration procedure is required. Begin by assuming for *water* that T_s = 64°C such that T_f = 315K. Calculate the Rayleigh number,

$$Ra_D = \frac{g b \Delta T D^3}{na} = \frac{9.8\,m/s^2 \times 400.4\times10^{-6}\,K^{-1}(64-20)K(0.010m)^3}{6.25\times10^{-7}\,m^2/s \times 1.531\times10^{-7}\,m^2/s} = 1.804\times10^6. \tag{2}$$

Using the Churchill-Chu relation, find

$$\overline{Nu}_D = \frac{\bar{h}D}{k} = \left\{0.60 + \frac{0.387 Ra_D^{1/6}}{\left[1+(0.559/Pr)^{9/16}\right]^{8/27}}\right\}^2 \tag{3}$$

$$\bar{h} = \frac{0.634\,W/m\cdot K}{0.01m}\left\{0.60 + \frac{0.387\left(1.804\times10^6\right)^{1/6}}{\left[1+(0.559/4.16)^{9/16}\right]^{8/27}}\right\}^2 = 1301\,W/m^2\cdot K.$$

Substituting numerical values into Eq. (1), the calculated value for T_s in *water* is

$$T_s = 20°C + 550\,W/p \times 0.010m \times 0.30m \times 1301\,W/m^2\cdot K = 64.8°C.$$ <

Continued …..

PROBLEM 9.62 (Cont.)

Our initial assumption of T_s = 64°C is in excellent agreement with the calculated value.

With accidental operation in *air*, the heat transfer coefficient will be nearly a factor of 100 less. Suppose $\overline{h} \approx 25\,W/m^2 \cdot K$, then from Eq. (1), $T_s \approx 2360°C$. Very likely the heater will burn out. Using air properties at $T_f \approx 1500K$ and Eq. (2), find $Ra_D = 1.815 \times 10^2$. Using Eq. 9.33, $Nu_D = CRa_D^n$ with C= 0.85 and n = 0.188 from Table 9.1, find $\overline{h} = 22.6\,W/m^2 \cdot K$. Hence, our first estimate for the surface temperature in *air* was reasonable,

$$T_s \approx 2300°C.$$ <

However, radiation exchange will be the dominant mode, and would reduce the estimate for T_s. Generally such heaters could not withstand operating temperatures above 1000°C and safe operation in air is not possible.

PROBLEM 9.63

KNOWN: Motor shaft of 20-mm diameter operating in ambient air at T_∞ = 27°C with surface temperature $T_s \le 87°C$.

FIND: Convection coefficients and/or heat removal rates for different heat transfer processes: (a) For a rotating horizontal cylinder as a function of rotational speed 5000 to 15,000 rpm using the recommended correlation, (b) For free convection from a horizontal stationary shaft; investigate whether mixed free and forced convection effects for the range of rotational speeds in part (a) are significant using the recommended criterion; (c) For radiation exchange between the shaft having an emissivity of 0.8 and the surroundings also at ambient temperature, $T_{sur} = T_\infty$; and (d) For cross flow of ambient air over the stationary shaft, required air velocities to remove the heat rates determined in part (a).

SCHEMATIC:

Air T_∞ = 27 °C

$T_s \le$ 87 °C

Shaft, D = 20 mm, ε = 0.80

Ω(rad/s)

ASSUMPTIONS: (1) Steady-state conditions, (2) Shaft is horizontal with isothermal surface.

PROPERTIES: *Table A.4*, Air ($T_f = (T_s + T_\infty)/2$ = 330 K, 1 atm): $\nu = 18.91 \times 10^{-6}\ m^2/s$, k = 0.02852 W/m·K, $\alpha = 26.94 \times 10^{-6}\ m^2/s$, Pr = 0.7028, $\beta = 1/T_f$.

ANALYSIS: (a) The recommended correlation for the a horizontal rotating shaft is

$$\overline{Nu}_D = 0.133 Re_D^{2/3} Pr^{1/3} \qquad Re_D < 4.3\times10^5 \qquad 0.7 < Pr < 670$$

where the Reynolds number is

$$Re_D = \Omega D^2 / \nu$$

and $\Omega\ (\text{rad/s})$ is the rotational velocity. Evaluating properties at $T_f = (T_s + T_\infty)/2$, find for ω = 5000 rpm,

$$Re_D = (5000\text{rpm}\times 2\pi\ \text{rad/rev}/60\text{s/min})(0.020\text{m})^2 / 18.91\times10^{-6}\ \text{m}^2/\text{s} = 11,076$$

$$\overline{Nu}_D = 0.133(11,076)^{2/3}(0.7028)^{1/3} = 58.75$$

$$\bar{h}_{D,rot} = \overline{Nu}_D\, k/D = 58.75\times0.02852\ \text{W/m}\cdot\text{K}/0.020\text{m} = 83.8\ \text{W/m}^2\cdot\text{K}$$ <

The heat rate per unit shaft length is

$$q'_{rot} = \bar{h}_{D,rot}(\pi D)(T_s - T_\infty) = 83.8\ \text{W/m}^2\cdot\text{K}(\pi\times0.020\text{m})(87-27)^\circ\text{C} = 316\ \text{W/m}$$ <

The convection coefficient and heat rate as a function of rotational speed are shown in a plot below.

(b) For the stationary shaft condition, the free convection coefficient can be estimated from the Churchill-Chu correlation, Eq. (9.34) with

Continued...

PROBLEM 9.63 (Cont.)

$$\mathrm{Ra_D} = \frac{g\beta\Delta T D^3}{\nu\alpha}$$

$$\mathrm{Ra_D} = \frac{9.8\,\mathrm{m/s^2}\,(1/330\mathrm{K})(87-27)\mathrm{K}\,(0.020\mathrm{m})^3}{18.91\times10^{-6}\,\mathrm{m^2/s}\times 26.94\times10^{-6}\,\mathrm{m^2/s}} = 27,981$$

$$\overline{\mathrm{Nu}}_D = \left\{0.60 + \frac{0.387\mathrm{Ra}_D^{1/6}}{\left[1+(0.559/\mathrm{Pr})^{9/16}\right]^{8/27}}\right\}^2$$

$$\overline{\mathrm{Nu}}_D = \left\{0.60 + \frac{0.387\,(27,981)^{1/6}}{\left[1+(0.559/0.7028)^{9/16}\right]^{8/27}}\right\}^2 = 5.61$$

$$\bar{h}_{D,fc} = \overline{\mathrm{Nu}}_D\, k/D = 5.61\times0.02852\,\mathrm{W/m\cdot K}/0.020\mathrm{m} = 8.00\,\mathrm{W/m^2\cdot K}$$

$$q'_{fc} = 8.00\,\mathrm{W/m^2\cdot K}\,(\pi\times0.020\mathrm{m})(87-27)^\circ\mathrm{C} = 30.2\,\mathrm{W/m} \qquad <$$

Mixed free and forced convection effects may be significant if

$$\mathrm{Re_D} < 4.7\left(\mathrm{Gr}_D^3/\mathrm{Pr}\right)^{0.137}$$

where $Gr_D = Ra_D/Pr$, find using results from above and in part (a) for ω = 5000 rpm,

$$11,076 \;\; ?<? \;\; 4.7\left[(27,981/0.7028)^3/0.7018\right]^{0.137} = 383$$

We conclude that free convection effects are not significant for rotational speeds above 5000 rpm.

(c) Considering radiation exchange between the shaft and the surroundings,

$$h_{rad} = \varepsilon\sigma\,(T_s + T_{sur})\left(T_s^2 + T_{sur}^4\right)$$

$$h_{rad} = 0.8\times5.67\times10^{-8}\,\mathrm{W/m^2\cdot K}\,(360+300)\left(360^2+300^2\right)\mathrm{K}^3 = 6.57\,\mathrm{W/m^2\cdot K} \qquad <$$

and the heat rate by radiation exchange is

$$q'_{rad} = h_{rad}\,(\pi D)(T_s - T_{sur})$$

$$q'_{rad} = 6.57\,\mathrm{W/m^2\cdot K}\,(\pi\times0.020\mathrm{m})(87-27)\mathrm{K} = 24.8\,\mathrm{W/m} \qquad <$$

(d) For cross flow of ambient air at a velocity V over the shaft, the convection coefficient can be estimated using the Churchill-Bernstein correlation, Eq. 7.57, with

$$\mathrm{Re}_{D,cf} = \frac{VD}{\nu}$$

$$\overline{\mathrm{Nu}}_{D,cf} = \bar{h}_{D,cf}\,D/k = 0.3 + \frac{0.62\,\mathrm{Re}_{D,cf}^{1/2}\,\mathrm{Pr}^{1/3}}{\left[1+(0.4/\mathrm{Pr})^{2/3}\right]^{1/4}}\left[1+\left(\frac{\mathrm{Re}_{D,cf}}{282,000}\right)^{5/8}\right]^{4/5}$$

Continued...

PROBLEM 9.63 (Cont.)

From the plot below (left) for the rotating shaft condition of part (a), $\overline{h}_{D,rot}$ vs. rpm, note that the convection coefficient varies from approximately 75 to 175 $W/m^2 \cdot K$. Using the *IHT Correlations Tool, Forced Convection, Cylinder*, which is based upon the above relations, the range of air velocities V required to achieve $\overline{h}_{D,cf}$ in the range 75 to 175 $W/m^2 \cdot K$ was computed and is plotted below (right).

Note that the air cross-flow velocities are quite substantial in order to remove similar heat rates for the rotating shaft condition.

COMMENTS: We conclude for the rotational speed and surface temperature conditions, free convection effects are not significant. Further, radiation exchange, part (c) result, is less than 10% of the convection heat loss for the lowest rotational speed condition.

PROBLEM 9.64

KNOWN: Horizontal pin fin of 6-mm diameter and 60-mm length fabricated from plain carbon steel ($k = 57$ W/m·K, $\varepsilon = 0.5$). Fin base maintained at $T_b = 150°C$. Ambient air and surroundings at 25°C.

FIND: Fin heat rate, q_f, by two methods: (a) Analytical solution using average fin surface temperature of $\overline{T}_s = 125°C$ to estimate the free convection and linearized radiation coefficients; comment on sensitivity of fin heat rate to choice of $\overline{T}_s$; and, (b) Finite-difference method when coefficients are based upon local temperatures, rather than an average fin surface temperature; compare result of the two solution methods.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in the pin fin, (3) Ambient air is quiescent and extensive, (4) Surroundings are large compared to the pin fin, and (5) Fin tip is adiabatic.

PROPERTIES: *Table A.4*, Air ($T_f = \left(\overline{T}_s + T_\infty\right)/2$ = 348 K): $\nu = 20.72 \times 10^{-6}$ m^2/s, k = 0.02985 W/m·K, $\alpha = 29.60 \times 10^{-6}$ m^2/s, Pr = 0.7003, $\beta = 1/T_f$.

ANALYSIS: (a) The heat rate for the pin fin with an adiabatic tip condition is, Eq. 3.76,

$$q_f = M \tanh(mL) \tag{1}$$

$$M = \left(\overline{h}_{tot} P k A_c\right)^{1/2} \theta_b \qquad m = \left(hP/kA_c\right)^{1/2} \tag{2,3}$$

$$P = \pi D \qquad A_c = \pi D^2/4 \qquad \theta_b = T_b - T_\infty \tag{4-6}$$

and the average coefficient is the sum of the convection and linearized radiation processes, respectively,

$$\overline{h}_{tot} = \overline{h}_{fc} + \overline{h}_{rad} \tag{7}$$

evaluated at $\overline{T}_s = 125°C$ with $\overline{T}_f = \left(\overline{T}_s + T_\infty\right)/2 = 75°C = 348\,K$.

Estimating $\overline{h}_{fc}$*:* For the horizontal cylinder, Eq. 9.34, with

$$Ra_D = \frac{g\beta\Delta T D^3}{\nu\alpha}$$

Continued …..

$$Ra_D = \frac{9.8\,m/s^2\,(1/348K)(125-25)(0.006m)^3}{20.72\times10^{-6}\,m^2/s\times29.60\times10^{-6}\,m^2/s} = 991.79$$

$$\overline{Nu}_D = \left\{0.60 + \frac{0.387\,Ra_D^{1/6}}{\left[1+(0.559/Pr)^{9/16}\right]^{8/27}}\right\}^2$$

$$\overline{Nu}_D = \left\{0.60 + \frac{0.387\,(991.79)^{1/6}}{\left[1+(0.559/0.7003)^{9/16}\right]^{8/27}}\right\}^2 = 2.603$$

$$\bar{h}_{fc} = \overline{Nu}_D\,k/D = 2.603\times0.02985\,W/m\cdot K/0.006m = 12.95\,W/m^2\cdot K$$

Calculating $\bar{h}_{rad}$ **:** The linearized radiation coefficient is

$$\bar{h}_{rad} = \varepsilon\sigma\left(\bar{T}_s + T_{sur}\right)\left(\bar{T}_s^2 + T_{sur}^2\right) \qquad (8)$$

$$\bar{h}_{rad} = 0.5\times5.67\times10^{-8}\,W/m^2\cdot K^4\,(398+298)\left(398^2+298^2\right)K^3 = 4.88\,W/m^2\cdot K$$

Substituting numerical values into Eqs. (1-7) , find

$$q_{fin} = 2.04W \qquad <$$

with $\theta_b = 125\,K$, $A_c = 2.827\times10^{-5} m^2$, $P = 0.01885\,m$, $m = 2.603\,m^{-1}$, $M = 2.909\,W$, and $\bar{h}_{tot} = 17.83\,W/m^2\cdot K$.

Using the *IHT Model, Extended Surfaces, Rectangular Pin Fin,* with the *Correlations Tool* for *Free Convection* and the *Properties Tool* for *Air,* the above analysis was repeated to obtain the following results.

$\bar{T}_s\,(^\circ C)$	115	120	125	130	135
$q_f\,(W)$	1.989	2.012	2.035	2.057	2.079
$(q_f - q_{f,o})/q_{fo}$ (%)	-2.3	-1.1	0	+1.1	+2.2

The fin heat rate is not very sensitive to the choice of $\bar{T}_s$ for the range T_s = 125 ± 10 °C. For the base case condition, the fin tip temperature is T(L) = 114 °C so that $\bar{T}_s \approx (T(L) + T_b)/2$ = 132°C would be consistent assumed value.

Continued …..

PROBLEM 9.64 (Cont.)

(b) Using the *IHT Tool, Finite-Difference Equation, Steady- State, Extended Surfaces,* the temperature distribution was determined for a 15-node system from which the fin heat rate was determined. The local free convection and linearized radiation coefficients $h_{tot} = h_{fc} + h_{rad,}$ were evaluated at local temperatures, T_m, using IHT with the *Correlations Tool, Free Convection, Horizontal Cylinder,* and the *Properties Tool* for *Air*, and Eq. (8). The local coefficient h_{tot} vs. T_s is nearly a linear function for the range $114 \le T_s \le 150°C$ so that it was reasonable to represent h_{tot} (T_s) as a *Lookup Table Function.* The fin heat rate follows from an energy balance on the base node, (see schematic next page)

$$q_f = q_a + q_b = (0.08949 + 1.879)\,W = 1.97\,W$$

<

$$q_a = h_b\,(P\Delta x/2)(T_b - T_\infty)$$

$$q_b = kA_c\,(T_b - T_1)/\Delta x$$

where T_b = 150°C, T_1 = 418.3 K = 145.3°C, and $h_b = h_{tot}$ (T_b) = l8.99 $W/m^2 \cdot K$.

Considering variable coefficients, the fin heat rate is -3.3% lower than for the analytical solution with the assumed $\overline{T}_s$ = 125°C.

COMMENTS: (1) To validate the FDE model for part (b), we compared the temperature distribution and fin heat rate using a constant h_{tot} with the analytical solution ($\overline{T}_s$ = 125°C). The results were identical indicating that the 15-node mesh is sufficiently fine.

(2) The fin temperature distribution (K) for the IHT finite-difference model of part (b) is

Tb	T01	T02	T03	T04	T05	T06	T07
423	418.3`	414.1	410.3	406.8	403.7	401	398.6

T08	T09	T10	T11	T12	T13	T14	T15
396.6	394.9	393.5	392.4	391.7	391.2	391	390.9

PROBLEM 9.65

KNOWN: Diameter, thickness, emissivity and thermal conductivity of steel pipe. Temperature of water flow in pipe. Cost of producing hot water.

FIND: Cost of daily heat loss from an uninsulated pipe.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state, (2) Negligible convection resistance for water flow, (3) Negligible radiation from pipe surroundings, (4) Quiescent air, (5) Constant properties.

PROPERTIES: *Table A-4*, air (p = 1 atm, $T_f \approx 295K$): $k_a = 0.0259$ W/m·K. $\nu = 15.45 \times 10^{-6}$ m^2/s, $\alpha = 21.8 \times 10^{-6}$ m^2/s, Pr = 0.708, $\beta = 3.39 \times 10^{-3}$ K^{-1}.

ANALYSIS: Performing an energy balance for a control surface about the outer surface, $q'_{cond} = q'_{conv} + q'_{rad}$, it follows that

$$\frac{T - T_{s,o}}{R'_{cond}} = \overline{h}\pi D_o \left(T_{s,o} - T_\infty\right) + \varepsilon_p \pi D_o \sigma T_{s,o}^4 \qquad (1)$$

where $R'_{cond} = \ell n\left(D_o / D_i\right) / 2\pi k_p = \ell n\left(100/84\right)/2\pi\left(60\,W/m\cdot K\right) = 4.62\times10^{-4}\,m\cdot K/W$. The convection coefficient may be obtained from the Churchill and Chu correlation. Hence, with $Ra_D = g\beta\,(T_{s,o} - T_\infty)\,D_o^3/\alpha\nu = 9.8\,m/s^2 \times 3.39\times10^{-3}K^{-1}\left(0.1m\right)^3\left(T_{s,o} - 268K\right)/\left(21.8\times15.45\times10^{-12}m^4/s^2\right)$
$= 98,637\left(T_{s,o} - 268\right)$,

$$\overline{Nu}_D = \left\{0.60 + \frac{0.387\,Ra_D^{1/6}}{\left[1 + \left(0.559/Pr\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.60 + 2.182\left(T_{s,o} - 268\right)^{1/6}\right\}^2$$

$$\overline{h} = \frac{k_a}{D_o}\overline{Nu}_D = 0.259\,W/m^2\cdot K\left\{0.60 + 2.182\left(T_{s,o} - 268\right)^{1/6}\right\}^2$$

Substituting the foregoing expression for $\overline{h}$, as well as values of R'_{cond}, D_o, ε_p and σ into Eq. (1), an iterative solution yields $\quad T_{s,o} = 322.9\,K = 49.9°C$

It follows that $\overline{h} = 6.10\,W/m^2\cdot K$, and the heat loss per unit length of pipe is

$$q' = q'_{conv} + q'_{rad} = 6.10\,W/m^2\cdot K\left(\pi\times0.1m\right)54.9K + 0.6\left(\pi\times0.1m\right)5.67\times10^{-8}\,W/m^2\cdot K^4\left(322.9K\right)^4$$
$$= \left(105.2 + 116.2\right)W/m = 221.4\,W/m$$

The corresponding daily energy loss is $\quad Q' = 0.221\,kW/m\times24\,h/d = 5.3\,kW\cdot h/m\cdot d$

and the associated cost is $\quad C' = \left(5.3\,kW\cdot h/m\cdot d\right)\left(\$0.05/kW\cdot h\right) = \$0.265/m\cdot d$ <

COMMENTS: (1) The heat loss is significant, and the pipe should be insulated. (2) The conduction resistance of the pipe wall is negligible relative to the combined convection and radiation resistance at the outer surface. Hence, the temperature of the outer surface is only slightly less than that of the water.

PROBLEM 9.66

KNOWN: Insulated, horizontal pipe with aluminum foil having emissivity which varies from 0.12 to 0.36 during service. Pipe diameter is 300 mm and its surface temperature is 90°C.

FIND: Effect of emissivity degradation on heat loss with ambient air at 25°C and (a) quiescent conditions and (b) cross-wind velocity of 5 m/s.

SCHEMATIC:

$T_s = 90°C$, $\varepsilon = 0.12$ to 0.36

Quiescent air or $V = 10$ m/s, $T_\infty = 25°C$

$T_{sur} = 25°C$

$D = 300$mm

ASSUMPTIONS: (1) Steady-state conditions, (2) Surroundings are large compared to pipe, (3) Pipe has uniform temperature.

PROPERTIES: *Table A-4*, Air (T_f = (90 + 25)°C/2 = 330K, 1 atm): $\nu = 18.9 \times 10^{-6}$ m^2/s, k = 28.5×10^{-3} W/m·K, $\alpha = 26.9 \times 10^{-6}$ m^2/s, Pr = 0.703.

ANALYSIS: The heat loss per unit length from the pipe is

$$q' = \bar{h}P\left(T_s - T_\infty\right) + e s\, P\left(T_s^4 - T_{sur}^4\right)$$

where P = πD and $\bar{h}$ needs to be evaluated for the two ambient air conditions.

(a) *Quiescent air*. Treating the pipe as a horizontal cylinder, find

$$Ra_D = \frac{g b\left(T_s - T_\infty\right)D^3}{n a} = \frac{9.8\,\text{m/s}^2\,(1/330\text{K})(90-25)\,\text{K}\,(0.30\text{m})^3}{18.9\times10^{-6}\text{m}^2/\text{s}\times26.9\times10^{-6}\text{m}^2/\text{s}} = 1.025\times10^8$$

and using the Churchill-Chu correlation for $10^{-5} < Re_D < 10^{12}$.

$$\overline{Nu}_D = \left\{0.60 + \frac{0.387 Ra_D^{1/6}}{\left[1+\left(0.559/Pr\right)^{9/16}\right]^{8/27}}\right\}^2$$

$$\overline{Nu}_D = \left\{0.60 + \frac{0.387\left(1.025\times10^8\right)^{1/6}}{\left[1+\left(0.559/0.703\right)^{9/16}\right]^{8/27}}\right\}^2 = 56.93$$

$$\bar{h}_D = \overline{Nu}_D\, k/D = 56.93\times0.0285\ \text{W/m}\cdot\text{K}/0.300\text{m} = 5.4\ \text{W/m}^2\cdot\text{K}.$$

Continued …..

Hence, the heat loss is

$$q' = 5.4\,\mathrm{W/m^2\cdot K}(p\,0.30\mathrm{m})(90-25)\mathrm{K} + e \times 5.67\times10^{-8}\mathrm{W/m^2\cdot K}(p\,0.300\mathrm{m})\left(363^4 - 298^4\right)\mathrm{K}^4$$

$$q' = 331 + 506e \begin{cases} e = 0.12 \rightarrow q' = (331+61) = 392\,\mathrm{W/m} & < \\ e = 0.36 \rightarrow q' = (331+182) = 513\,\mathrm{W/m} & < \end{cases}$$

The radiation effect accounts for 16 and 35%, respectively, of the heat rate.

(b) *Cross-wind condition*. With a cross-wind, find

$$Re_D = \frac{VD}{n} = \frac{10\,\mathrm{m/s}\times0.30\mathrm{m}}{18.9\times10^{-6}\mathrm{m^2/s}} = 1.587\times10^5$$

and using the Hilpert correlation where C = 0.027 and m = 0.805 from Table 7.2,

$$\overline{Nu}_D = C\,Re_D^m\,Pr^{1/3} = 0.027\left(1.587\times10^5\right)^{0.805}(0.703)^{1/3} = 368.9$$

$$\overline{h}_D = Nu_D \cdot k/D = 368.9\times0.0285\,\mathrm{W/m\cdot K}/0.300\mathrm{m} = 35\,\mathrm{W/m^2\cdot K}.$$

Recognizing that *combined* free and forced convection conditions may exist, from Eq. 9.64 with n = 3,

$$Nu_m^3 = Nu_F^3 + Nu_N^3 \qquad \overline{h}_m = \left(5.4^3 + 35^3\right)^{1/3} = 35\,\mathrm{W/m^2\cdot K}$$

we find forced convection dominates. Hence, the heat loss is

$$q' = 35\,\mathrm{W/m^2\cdot K}(p\,0.300\mathrm{m})(90-25)\mathrm{K} + e\times5.67\times10^{-8}\mathrm{W/m^2\cdot K}(p\,0.300\mathrm{m})\left(393^4 - 298^4\right)\mathrm{K}^4$$

$$q' = 2144 + 853e \begin{cases} e = 0.12 \rightarrow q' = 2144 + 102 = 2246\,\mathrm{W/m} & < \\ e = 0.36 \rightarrow q' = 2144 + 307 = 2451\,\mathrm{W/m} & < \end{cases}$$

The radiation effect accounts for 5 and 13%, respectively, of the heat rate.

COMMENTS: (1) For high velocity wind conditions, radiation losses are quite low and the degradation of the foil is not important. However, for low velocity and quiescent air conditions, radiation effects are significant and the degradation of the foil can account for a nearly 25% change in heat loss.

(2) The radiation coefficient is in the range 0.83 to 2.48 $\mathrm{W/m^2\cdot K}$ for ε = 0.12 and 0.36, respectively. Compare these values with those for convection.

PROBLEM 9.67

KNOWN: Diameter, emissivity, and power dissipation of cylindrical heater. Temperature of ambient air and surroundings.

FIND: Steady-state temperature of heater and time required to come within 10°C of this temperature.

SCHEMATIC:

Quiescent air,
T_∞ = 300 K
T_{sur} = 300 K
q'_{rad}
D = 0.01 m
q'_{conv}
Electrical heater
T, q' = 1000 W/m, ε = 0.8
k = 240 W/m·K,
ρ = 2700 kg/m^3, c_p = 900 J/kg·K

ASSUMPTIONS: (1) Air is quiescent, (2) Duct wall forms large surroundings about heater, (3) Heater may be approximated as a lumped capacitance.

PROPERTIES: *Table A.4*, air (Obtained from *Properties* Tool Pad of IHT).

ANALYSIS: Performing an energy balance on the heater, the final (steady-state) temperature may be obtained from the requirement that $q' = q'_{conv} + q'_{rad}$, or

$$q' = \bar{h}(\pi D)(T - T_\infty) + h_r(\pi D)(T - T_{sur})$$

where $\bar{h}$ is obtained from Eq. 9.34 and $h_r = \varepsilon\sigma(T + T_{sur})(T^2 + T_{sur}^2)$. Using the *Correlations* Tool Pad of IHT to evaluate $\bar{h}$, this expression may be solved to obtain

T = 854 K = 581°C <

Under transient conditions, the energy balance is of the form, $\dot{E}'_{st} = q' - q'_{conv} - q'_{rad}$, or

$$\rho c_p \left(\pi D^2/4\right) dT/dt = q' - \bar{h}(\pi D)(T - T_\infty) - h_r(\pi D)(T - T_{sur})$$

Using the IHT *Lumped Capacitance* model with the *Correlations* Tool Pad, the above expression is integrated from t = 0, for which T_i = 562.4 K, to the time for which T = 844 K. The integration yields

t = 183s <

The value of T_i = 562.4 K corresponds to the steady-state temperature for which the power dissipation is balanced by convection and radiation (see solution to Problem 7.44).

COMMENTS: The forced convection coefficient (Problems 7.43 and 7.44) of 105 W/m^2·K is much larger than that associated with free convection for the steady-state conditions of this problem (14.6 W/m^2·K). However, because of the correspondingly larger heater temperature, the radiation coefficient with free convection (42.9 W/m^2·K) is much larger than that associated with forced convection (15.9 W/m^2·K).

PROBLEM 9.68

KNOWN: Cylindrical sensor of 12.5 mm diameter positioned horizontally in quiescent air at 27°C.

FIND: An expression for the free convection coefficient as a function of only $\Delta T = T_s - T_\infty$ where T_s is the sensor temperature.

ASSUMPTIONS: (1) Steady-state conditions, (2) Uniform temperature over cylindrically shaped sensor, (3) Ambient air extensive and quiescent.

PROPERTIES: *Table A-4*, Air (T_f, 1 atm): $\beta = 1/T_f$ and

T_s (°C)	T_f (K)	$\nu \times 10^6$ m²/s	$\alpha \times 10^6$ m²/s	$k \times 10^3$ W/m·K	Pr
30	302	16.09	22.8	26.5	0.707
55	314	17.30	24.6	27.3	0.705
80	327	18.61	26.5	28.3	0.703

ANALYSIS: For the cylindrical sensor, using Eqs. 9.25 and 9.34,

$$Ra_D = \frac{g b \Delta T D^3}{n a} \qquad \overline{Nu}_D = \frac{\bar{h}_D D}{k} = \left\{ 0.60 + \frac{0.387 Ra_D^{1/6}}{\left[1 + (0.559/Pr)^{9/16}\right]^{8/17}} \right\}^2 \tag{1,2}$$

where properties are evaluated at ($T_f = T_s + T_\infty$)/2. With $30 \le T_s \le 80$°C and $T_\infty = 27$°C, $302 \le T_f \le 326$ K. Using properties evaluated at the mid-range of T_f, $\bar{T}_f = 314\text{K}$, find

$$Ra_D = \frac{9.8\,\text{m/s}^2 (1/314\text{K}) \Delta T (0.0125\text{m})^3}{17.30 \times 10^{-6} \text{m}^2/\text{s} \times 24.6 \times 10^{-6} \text{m}^2/\text{s}} = 143.2 \Delta T$$

$$\bar{h}_D = \frac{0.0273\,\text{W/m} \cdot \text{K}}{0.0125\text{m}} \left\{ 0.60 + \frac{0.387 (143 \Delta T)^{1/6}}{\left[1 + (0.559/0.705)^{9/16}\right]^{8/27}} \right\}^2$$

$$\bar{h}_D = 2.184 \left\{ 0.60 + 0.734 \Delta T^{1/6} \right\}^2 . \tag{3}$$ <

COMMENTS: (1) The effect of using a fixed film temperature, $\bar{T}_f = 314\text{K} = 41$°C, for the full range $30 \le T_s \le 80$°C can be seen by comparing results from the approximate Eq. (3) and the correlation, Eq. (2), with the proper film temperature. The results are summarized in the table.

		Correlation			Eq. (3)
T_s (°C)	$\Delta T = T_s - T_\infty$ (°C)	Ra_D	$\overline{Nu}_D$	$\bar{h}_D\ (\text{W/m}^2 \cdot \text{K})$	$\bar{h}_D\ (\text{W/m}^2 \cdot \text{K})$
30	3	518	2.281	4.83	4.80
55	28	4011	3.534	7.72	7.71

The approximate expression for $\bar{h}_D$ is in excellent agreement with the correlation.

(2) In calculating heat rates it may be important to consider radiation exchange with the surroundings.

PROBLEM 9.69

KNOWN: Thin-walled tube mounted horizontally in quiescent air and wrapped with an electrical tape passing hot fluid in an experimental loop.

FIND: (a) Heat flux q''_e from the heating tape required to prevent heat loss from the hot fluid when (a) neglecting and (b) including radiation exchange with the surroundings, (c) Effect of insulation on q''_e and convection/radiation rates.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Ambient air is quiescent and extensive, (3) Surroundings are large compared to the tube.

PROPERTIES: *Table A.4*, Air ($T_f = (T_s + T_\infty)/2 = (45 + 15)°C/2 = 303$ K, 1 atm): $\nu = 16.19 \times 10^{-6}$ m^2/s, $\alpha = 22.9 \times 10^{-6}$ m^2/s, $k = 26.5 \times 10^{-3}$ W/m·K, Pr = 0.707, $\beta = 1/T_f$.

ANALYSIS: (a,b) To prevent heat losses from the hot fluid, the heating tape temperature must be maintained at T_m; hence $T_{s,i} = T_m$. From a surface energy balance,

$$q''_e = q''_{conv} + q''_{rad} = \left(\bar{h}D_i + h_r\right)\left(T_{s,i} - T_\infty\right)$$

where the linearized radiation coefficient, Eq. 1.9, is $h_r = \varepsilon\sigma\left(T_{s,i} + T_\infty\right)\left(T_{s,i}^2 + T_\infty^2\right)$, or

$$h_r = 0.95 \times 5.67 \times 10^{-8}\ \mathrm{W/m^2 \cdot K^4}\,(318 + 288)\left(318^2 + 288^2\right)\mathrm{K^3} = 6.01\,\mathrm{W/m^2 \cdot K}\,.$$

Neglecting radiation: For the horizontal cylinder, Eq. 9.34 yields

$$Ra_D = \frac{g\beta\left(T_{s,i} - T_\infty\right)D_i^3}{\nu\alpha} = \frac{9.8\,\mathrm{m/s^2}\,(1/303\,\mathrm{K})(45 - 15)\,\mathrm{K}\,(0.020\,\mathrm{m})^3}{16.19 \times 10^{-6}\,\mathrm{m^2/s} \times 22.9 \times 10^{-6}\,\mathrm{m^2/s}} = 20{,}900$$

$$\overline{Nu}_D = \frac{\bar{h}_{D_i} D_i}{k} = \left\{0.60 + \frac{0.386 Ra_D^{1/6}}{\left[1 + (0.559/Pr)^{9/16}\right]^{8/27}}\right\}^2$$

Continued ….

PROBLEM 9.69 (Cont.)

$$\overline{h}_{D_i} = \frac{0.0265\,W/m\cdot K}{0.020\,m}\left\{0.60 + \frac{0.386(20,900)^{1/6}}{\left[1+(0.559/0.707)^{9/16}\right]^{8/27}}\right\}^2 = 6.90\,W/m^2\cdot K$$

Hence, neglecting radiation, the required heat flux is

$$q''_e = 6.90\,W/m^2\cdot K(45-15)K = 207\,W/m^2\cdot K$$ <

Considering radiation: The required heat flux considering radiation is

$$q''_e = (6.90+6.01)W/m^2\cdot K(45-15)K = 387\,W/m^2\cdot K$$ <

(c) With insulation, the surface energy balance must be modified to account for an increase in the outer diameter from D_i to $D_o = D_i + 2t$ and for the attendant thermal resistance associated with conduction across the insulation. From an energy balance at the inner surface of the insulation,

$$q''_e(\pi D_i) = q'_{cond} = \frac{2\pi k_i\left(T_m - T_{s,o}\right)}{\ln(D_o/D_i)}$$

and from an energy balance at the outer surface,

$$q'_{cond} = q'_{conv} + q'_{rad} = \pi D_o\left(\overline{h}_{D_o} + h_r\right)\left(T_{s,o} - T_\infty\right)$$

The foregoing expressions may be used to determine $T_{s,o}$ and q''_e as a function of t, with the IHT *Correlations* and *Properties* Tool Pads used to evaluate $\overline{h}_{D_o}$. The desired results are plotted as follows.

By adding 20 mm of insulation, the required power dissipation is reduced by a factor of approximately 3. Convection and radiation heat rates at the outer surface are comparable.

COMMENTS: Over the range of insulation thickness, $T_{s,o}$ decreases from 45°C to 20°C, while $\overline{h}_{D_o}$ and h_r decrease from 6.9 to 3.5 $W/m^2\cdot K$ and from 3.8 to 3.3 $W/m^2\cdot K$, respectively.

PROBLEM 9.70

KNOWN: A billet of stainless steel AISI 316 with a diameter of 150 mm and length 500 mm emerges from a heat treatment process at 200°C and is placed into an unstirred oil bath maintained at 20°C.

FIND: (a) Determine whether it is advisable to position the billet in the bath with its centerline horizontal or vertical in order decrease the cooling time, and (b) Estimate the time for the billet to cool to 30°C for the better positioning arrangement.

SCHEMATIC:

Billet
D = 150 mm
L = 500 mm,
$T_s = 200^oC$
Unstirred oil bath
$T_\infty = 20^oC$
$T_s = 200^oC$
Vertical position
Horizontal position

ASSUMPTIONS: (1) Steady-state conditions for part (a), (2) Oil bath approximates a quiescent fluid, (3) Consider only convection from the lateral surface of the cylindrical billet; and (4) For part (b), the billet has a uniform initial temperature.

PROPERTIES: *Table A-5*, Engine oil ($T_f = (T_s + T_\infty)/2$): see Comment 1. *Table A-1*, AISI 316 (400 K): $\rho = 8238$ kg/m^3, $c_p = 468$ J/kg·K, k = 15 W/m·K.

ANALYSIS: (a) For the purpose of determining whether the horizontal or vertical position is preferred for faster cooling, consider only free convection from the lateral surface. The heat loss from the lateral surface follows from the rate equation

$$q = \bar{h} A_s \left(T_s - T_\infty\right)$$

Vertical position. The lateral surface of the cylindrical billet can be considered as a vertical surface of height L, width P = πD, and area A_s = PL. The Churchill-Chu correlation, Eq. 9.26, is appropriate to estimate $\bar{h}_L$,

$$\overline{Nu}_L = \frac{\bar{h}_L L}{k} = \left\{0.825 + \frac{0.387\ Ra_L^{1/6}}{\left[1+\left(0.492/Pr\right)^{9/16}\right]^{8/27}}\right\}^2$$

$$Ra_L = \frac{g\beta\left(T_s - T_\infty\right)L^3}{\nu\alpha}$$

with properties evaluated at $T_f = (T_s + T_\infty)/2$.

Horizontal position. In this position, the billet is considered as a long horizontal cylinder of diameter D for which the Churchill-Chu correlation of Eq. 9.34 is appropriate to estimate $\bar{h}_D$,

$$\overline{Nu}_L = \frac{\bar{h}_D D}{k} = \left\{0.60 + \frac{0.387\ Ra_D^{1/6}}{\left[1+\left(0.55/Pr\right)^{9/16}\right]^{8/27}}\right\}^2$$

Continued …..

$$Ra_D = \frac{g\beta(T_s - T_\infty)D^3}{\nu\alpha}$$

with properties evaluated at T_f. The heat transfer area is also A_s = PL.

Using the foregoing relations in *IHT* with the thermophysical properties library as shown in Comment 1, the analysis results are tabulated below.

$Ra_L = 1.36 \times 10^{11}$	$\overline{Nu}_L = 801$	$\bar{h}_L = 218\ W/m^2 \cdot K$	(vertical)
$Ra_D = 3.67 \times 10^9$	$\overline{Nu}_D = 245$	$\bar{h}_D = 221\ W/m^2 \cdot K$	(horizontal)

Recognize that the orientation has a small effect on the convection coefficient for these conditions, but we'll select the horizontal orientation as the preferred one.

(b) Evaluate first the Biot number to determine if the lumped capacitance method is valid.

$$Bi = \frac{\bar{h}_D(D_o/2)}{k} = \frac{221\ W/m^2 \cdot K(0.150\ m/2)}{15\ W/m \cdot K} = 1.1$$

Since Bi >> 0.1, the spatial effects are important and we should use the one-term series approximation for the infinite cylinder, Eq. 5.49. Since $\bar{h}_D$ will decrease as the billet cools, we need to estimate an average value for the cooling process from 200°C to 30°C. Based upon the analysis summarized in Comment 1, use $\bar{h}_D = 119\ W/m^2 \cdot K$. Using the transient model for the infinite cylinder in *IHT*, (see Comment 2) find for $T(r_o, t_o)$ = 30°C,

$$t_o = 3845\ s = 1.1\ h$$ <

COMMENTS: (1) The *IHT* code using the convection correlation functions to estimate the coefficients is shown below. This same code was used to calculate $\bar{h}_D$ for the range $30 \le T_s \le 200$°C and determine that an average value for the cooling period of part (b) is 119 W/m$^2\cdot$K.

```
/* Results - convection coefficients, Ts = 200 C
hDbar  hLbar   D      L      Tinf_C  Ts_C
221.4  217.5   0.15   0.5    20      200     */

/* Results - correlation parameters, Ts = 200 C
NuDbar         NuLbar  Pr     RaD      RaL
244.7          801.3   219.2  3.665E9  1.357E11 */

/* Results - properties, Ts = 200 C; Tf = 383 K
Pr     alpha     beta    deltaT  k       nu        Tf
219.2  7.188E-8  0.0007  180     0.1357  1.582E-5  383

/*  Correlation description: Free convection (FC), long horizontal cylinder (HC),
10^-5<=RaD<=10^12, Churchill-Chu correlation, Eqs 9.25 and 9.34 . See Table 9.2 .  */
NuDbar = NuD_bar_FC_HC(RaD,Pr)              // Eq 9.34
NuDbar = hDbar * D / k
RaD = g * beta * deltaT * D^3 / (nu * alpha)       //Eq 9.25
deltaT = abs(Ts - Tinf)
g = 9.8     // gravitational constant, m/s^2
// Evaluate properties at the film temperature, Tf.
Tf = Tfluid_avg(Tinf,Ts)
```

Continued …..

PROBLEM 9.70 (Cont.)

```
/*  Correlation description: Free convection (FC) for a vertical plate (VP),  Eqs 9.25 and 9.26 .
See Table 9.2 .  */
NuLbar = NuL_bar_FC_VP(RaL,Pr)                  // Eq 9.26
NuLbar = hLbar * L / k
RaL = g * beta * deltaT * L^3 / (nu * alpha)        //Eq 9.25

// Input variables
D = 0.15
L = 0.5
Tinf_C = 20
Ts_C = 200

// Engine Oil property functions : From Table A.5
// Units: T(K)
nu = nu_T("Engine Oil",Tf)              // Kinematic viscosity, m^2/s
k = k_T("Engine Oil",Tf)                // Thermal conductivity, W/m·K
alpha = alpha_T("Engine Oil",Tf)        // Thermal diffusivity, m^2/s
Pr = Pr_T("Engine Oil",Tf)              // Prandtl number
beta = beta_T("Engine Oil",Tf)          // Volumetric coefficient of expansion, K^(-1)

// Conversions
Tinf_C = Tinf - 273
Ts_C = Ts - 273
```

(2) The portion of the *IHT* code used for the transient analysis is shown below. Recognize that we have not considered heat losses from the billet end surfaces, also, we should consider the billet as a three-dimensional object rather than as a long cylinder.

```
/* Results - time to cool to 30 C, center and surface temperatures
D       T_xt_C    Ti_C    Tinf_C    r        h       t
0.15    30.01     200     20        0.075    119     3845    */
0.15    33.19     200     20        0        119     3845

// Transient conduction model, cylinder (series solution)
// The temperature distribution T(r,t) is
T_xt = T_xt_trans("Cylinder",rstar,Fo,Bi,Ti,Tinf)    // Eq 5.47
// The dimensionless parameters are
rstar = r / ro
Bi = h * ro / k
Fo= alpha * t / ro^2
alpha = k/ (rho * cp)
```

PROBLEM 9.71

KNOWN: Diameter, initial temperature and emissivity of long steel rod. Temperature of air and surroundings.

FIND: (a) Average surface convection coefficient, (b) Effective radiation coefficient, (c,d) Maximum allowable conveyor time.

SCHEMATIC:

T_{sur} = 300K

Steel
ε = 0.4, k = 25 W/m·K
α = 5.2 x 10^{-6} m^2/s,
T_i = 1000 K

D = 50 mm

Air T_∞ = 300K

ASSUMPTIONS: (1) Negligible effect of forced convection, (2) Constant properties, (3) Large surroundings, (4) Quiescent air.

PROPERTIES: Stainless steel (given): k = 25 W/m·K, $\alpha = 5.2 \times 10^{-6}$ m^2/s; *Table A.4,* Air (T_f = 650 K, 1 atm): $\nu = 6.02 \times 10^{-5}$ m^2/s, $\alpha = 8.73 \times 10^{-5}$ m^2/s, k = 0.0497 W/m·K, Pr = 0.69.

ANALYSIS: (a) For free convection from a horizontal cylinder,

$$Ra_D = \frac{g\beta(T_s - T_\infty)D^3}{\alpha\nu} = \frac{9.8\,m/s^2\,(1/650\,K)(0.05\,m)^3}{6.02\times 8.73\times 10^{-10}\,m^4/s^2} = 2.51\times 10^5$$

The Churchill and Chu correlation yields

$$\overline{Nu}_D = \left\{0.60 + \frac{0.387 Ra_D^{1/6}}{\left[1+(0.559/Pr)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.60 + \frac{0.387\left(2.51\times 10^5\right)^{1/6}}{\left[1+(0.559/0.69)^{9/16}\right]^{8/27}}\right\}^2 = 9.9$$

$$\bar{h} = \overline{Nu}_D\,k/D = 9.9(0.0497\,W/m\cdot K)/0.05\,m = 9.84\,W/m^2\cdot K$$ <

(b) The radiation heat transfer coefficient is

$$h_r = \varepsilon\sigma(T_s + T_{sur})\left(T_s^2 + T_{sur}^2\right) = 0.4\times 5.67\times 10^{-8}\,W/m^2\cdot K^4\,(1000$$

$$+300)K\left[(1000)^2 + (300)^2\right]K^2 = 32.1\,W/m^2\cdot K$$ <

(c) For the long stainless steel rod and the initial values of $\bar{h}$ and h_r,

$$Bi = \left(\bar{h} + h_r\right)(r_o/2)/k = 42.0\,W/m^2\cdot K\times 0.0125\,m/25\,W/m\cdot K = 0.021.$$

Hence, the lumped capacitance method can be used.

$$\frac{T - T_\infty}{T_i - T_\infty} = \frac{600\,K}{700\,K} = \exp(-Bi\cdot Fo) = \exp(-0.021 Fo)$$

Continued...

$$\text{Fo} = 7.34 = \alpha t \Big/ \left(r_o / 2 \right)^2 = 0.0333t$$

$t = 221$ s. <

(d) Using the IHT *Lumped Capacitance* Model with the *Correlations* and *Properties* Tool Pads, a more accurate estimate of the maximum allowable transit time may be obtained by evaluating the numerical integration,

$$\int_o^t dt = -\frac{\rho c_p D}{4} \int_{1000\,K}^{900\,K} \frac{dT}{\left(\overline{h} + h_r \right)\left(T - T_\infty \right)}$$

where $\rho c_p = k/\alpha = 4.81 \times 10^6 \ J\big/K \cdot m^3$. The integration yields

$t = 245$ s <

At this time, the convection and radiation coefficients are $\overline{h} = 9.75$ and $h_r = 24.5$ W/m^2·K, respectively.

COMMENTS: Since $\overline{h}$ and h_r decrease with increasing time, the maximum allowable conveyor time is underestimated by the result of part (c).

PROBLEM 9.72

KNOWN: Velocity and temperature of air flowing through a duct of prescribed diameter. Temperature of duct surroundings. Thickness, thermal conductivity and emissivity of applied insulation.

FIND: (a) Duct surface temperature and heat loss per unit length with no insulation, (b) Surface temperatures and heat loss with insulation.

SCHEMATIC:

T_{sur} = 0 °C
Air
T_∞ = 0 °C, $\bar{h}_o$
Insulation, $T_{s,o}$
D_o = 0.20 m, ε_o = 0.60
k_s = 0.050 W/m•K
Air
u_m = 3 m/s,
T_m = 70 °C, $\bar{h}_i$
Duct wall, $T_{s,i}$
D_i = 0.15 m, ε_i = 0.5

T_m, $T_{s,i}$, $T_{s,o}$, $q'_{conv,o}$, T_∞, $q'_{conv,i}$, q'_{cond}, T_{sur}, $q'_{rad,o}$

ASSUMPTIONS: (1) Steady-state conditions, (2) Fully-developed internal flow, (3) Negligible duct wall resistance, (4) Duct outer surface is diffuse-gray, (5) Outside air is quiescent, (6) Pressure of inside and outside air is atmospheric.

PROPERTIES: *Table A.4*, Air (T_m = 70°C): $\nu = 20.22 \times 10^{-6}$ m²/s, Pr = 0.70, k = 0.0295 W/m·K; *Table A.4*, Air ($T_f \approx 27$°C): $\nu = 15.89 \times 10^{-6}$ m²/s, Pr = 0.707, k = 0.0263 W/m·K, $\alpha = 22.5 \times 10^{-6}$ m²/s, β = 0.00333 K^{-1}.

ANALYSIS: (a) Performing an energy balance on the duct wall with no insulation ($T_{s,i} = T_{s,o}$),

$$q'_{conv,i} = q'_{conv,o} + q'_{rad,o} \qquad h_i(\pi D_i)(T_m - T_{s,i}) = h_o(\pi D_i)(T_{s,i} - T_\infty) + \varepsilon_i \sigma (\pi D_i)\left(T_{s,i}^4 - T_{sur}^4\right)$$

with $Re_{D,i} = u_m D_i/\nu$ = 3 m/s × 0.15 m/(20.22 × 10^{-6} m²/s) = 2.23 × 10^4, the internal flow is turbulent, and from the Dittus-Boelter correlation,

$$h_i = \frac{k}{D_i} 0.023 Re_{D,i}^{4/5} Pr^{0.3} = \frac{0.0295\,W/m\cdot K}{0.15\,m} 0.023\left(2.23\times10^4\right)^{4/5}(0.7)^{0.3} = 12.2\,W/m^2\cdot K.$$

For free convection, the Rayleigh number is

$$Ra_{D,i} = \frac{g\beta(T_{s,i} - T_\infty)D_i^3}{\nu\alpha} = \frac{9.8\,m/s^2\,(0.0033)(T_{s,i} - 273)(0.15)^3\,m^3}{15.89\times10^{-6}\,m^2/s\times22.5\times10^{-6}\,m^2/s} = 3.08\times10^5\left(T_{s,i} - T_\infty\right)$$

and from Eq. 9.34,

$$\bar{h}_o = \frac{k}{D_i}\left[0.60 + \frac{0.387 Ra_{D,i}^{1/6}}{\left[1 + (0.559/Pr)^{9/16}\right]^{8/27}}\right]^2 = \frac{0.0263}{0.15}\left[0.60 + \frac{0.387\left[3.08\times10^5\left(T_{s,i} - T_\infty\right)\right]^{1/6}}{\left[1 + (0.559/0.707)^{9/16}\right]^{8/27}}\right]^2$$

Continued...

$$\bar{h}_o = 0.175\left[0.60 + 2.64\left(T_{s,i} - T_\infty\right)^{1/6}\right]^2$$

Hence

$$12.2\left(343 - T_{s,i}\right) = 0.175\left\{0.60 + 2.64\left(T_{s,i} - 273\right)^{1/6}\right\}^2\left(T_{s,i} - 273\right) + 0.5 \times 5.67 \times 10^{-8}\left[T_{s,i}^4 - (273)^4\right]$$

A trial-and-error solution gives $T_{s,i} \approx 314.7\,K \approx 41.7^\circ C$ <

The heat loss per unit length is then

$$q' = q'_{conv,i} \approx 12.2(\pi \times 0.15)(70 - 42) \approx 163\,W/m\,.$$ <

(b) Performing energy balances at the inner and outer surfaces, we obtain, respectively,

$$q'_{conv,i} = q'_{cond}$$

or,

$$\bar{h}_i(\pi D_i)\left(T_m - T_{s,i}\right) = \frac{2\pi k_s\left(T_{s,i} - T_{s,o}\right)}{\ln\left(D_o/D_i\right)}$$

and,

$$q'_{cond} = q'_{conv,o} + q'_{rad,o}$$

or,

$$\frac{2\pi k_s\left(T_{s,i} - T_{s,o}\right)}{\ln\left(D_o/D_i\right)} = \bar{h}_o(\pi D_o)\left(T_s - T_\infty\right) + \varepsilon_o\sigma(\pi D_o)\left(T_{s,o}^4 - T_{sur}^4\right)$$

Using the IHT workspace with the *Correlations* and *Properties* Tool Pads to solve the energy balances for the unknown surface temperatures, we obtain

$T_{s,i} = 60.8^\circ C$ $T_{s,o} = 12.5°C$ <

With the heat loss per unit length again evaluated from the inside convection process, we obtain

$$q' = q'_{conv,i} = 52.8\,W/m$$ <

COMMENTS: For part (a), the outside convection coefficient is $\bar{h}_o$ = 5.4 W/m^2·K < h_i. The outside heat transfer rates are $q'_{conv,o}$ ≈ 106 W/m and $q'_{rad,o}$ ≈ 57 W/m. For part (b), $\bar{h}_o$ = 3.74 W/m^2·K, $q'_{conv,o}$ = 29.4 W/m, and $q'_{rad,o}$ = 23.3 W/m. Although $T_{s,i}$ increases with addition of the insulation, there is a substantial reduction in $T_{s,o}$ and hence the heat loss.

PROBLEM 9.73

KNOWN: Biological fluid with prescribed flow rate and inlet temperature flowing through a coiled, thin-walled, 5-mm diameter tube submerged in a large water bath maintained at 50°C.

FIND: (a) Length of tube and number of coils required to provide an exit temperature of $T_{m,o}$ = 38°C, and (b) Variations expected in $T_{m,o}$ for a ±10 % change in the mass flow rate for the tube length determined in part (a) .

SCHEMATIC:

Water bath T_∞ = 50 °C

Tube, D = 5mm; coil diameter, D_c = 200 mm

Biological fluid

$\dot{m}$ = 0.02 kg/s

L

$T_{m,i}$ = 25 °C $\quad$ $T_{m,o}$ = 38 °C

ASSUMPTIONS: (1) Steady-state conditions, (2) Coiled tube approximates a horizontal tube experiencing free convection in a quiescent, extensive medium (water bath), (3) Biological fluid has thermophysical properties of water, and (4) Negligible tube wall thermal resistance.

PROPERTIES: *Table A.4* Water - cold side ($T_{m,c} = (T_{m,i} + T_{m,o}) / 2$ = 304.5 K) : $c_{p,c}$ = 4178 J/kg·K, μ_c = 777.6×10^{-6} N·s/m^2 , k_c = 0.6193 W/m·K , Pr_c = 5.263 ; *Table A.4,* Water - hot side ($\overline{T}_f = (T_s + T_\infty)/2$ = 320.1 K, see comment 1) : ρ_h = 989.1 kg/m^3 , $c_{p,h}$ = 4180 J/kg·K, $\mu_h = 575.6 \times 10^{-6}$ N·s/m^2, k_h = 0.6401 W/m·K , Pr_h = 3.76, $\nu_h = \mu_h/\rho_h = 5.827 \times 10^{-7}$ m^2/s , $\alpha_h = k_h / \rho_h c_{ph}$, = 15.48×10^{-8} m^2/s.

ANALYSIS: (a) Following the treatment of Section 8.3.3, the coil experiences internal flow of the cold biological fluid (c) and free convection with the external hot fluid (h). From Eq. 8.46a,

$$\frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = \exp\left(-\frac{\overline{U}PL}{\dot{m}c_{p,c}}\right) \qquad \overline{U} = \left(1/\overline{h}_c + 1/\overline{h}_h\right)^{-1} \tag{1}$$

with P = πD and for the overall coefficient $\overline{U}$, $\overline{h}_c$ and $\overline{h}_h$ are the average convection coefficients for internal flow and external free convection, respectively. These coefficients are estimated as follows.

Internal flow, $\overline{h}_c$*:* To characterize the flow, calculate the Reynolds number,

$$Re_{D,c} = \frac{4\dot{m}}{\pi D \mu_c} = \frac{4 \times 0.02\,\text{kg/s}}{\pi \times 0.005\text{m} \times 777.6 \times 10^{-6}\,\text{N}\cdot\text{s/m}^2} = 6550 \tag{3}$$

evaluating properties at $\overline{T}_m = \left(T_{m,i} + T_{m,o}\right)/2 = (25+38)^\circ\text{C}/2 = 31.5^\circ\text{C} = 304.5\text{K}$. Note that $Rc_{D,c}$ is between the laminar upper limit (2300) and the turbulent lower limit (10,000). To provide a conservative estimate, we choose to consider the flow as laminar and anticipate that the flow will be fully developed. From Eq. 8.55, $Nu_{D,c}$ = 3.66,

$$\overline{h}_c = Nu_{D,c}\, k_c / D\; 3.66 \times 0.6193\,\text{W/m}\cdot\text{K} / 0.005\text{m} = 453\,\text{W/m}^2\cdot\text{K} \tag{4}$$

External free convection , $\overline{h}_h$: For the horizontal tube, Eq. 9.34, with

$$Ra_{D,h} = \frac{g\beta_h \Delta T D^3}{\nu_h \alpha_h} \qquad \Delta T = \overline{T}_s - T_\infty \tag{5,6}$$

Continued...

where $\overline{T}_s$ is the average tube wall temperature determined from the thermal circuit for which

$$\overline{h}_c\left(\overline{T}_m-\overline{T}_s\right)=\overline{h}_h\left(\overline{T}_s-T_\infty\right) \qquad (7)$$

and the average film temperature at which to evaluate properties is

$$\overline{T}_{f,c}=\left(\overline{T}_s+T_\infty\right)/2 \qquad (8)$$

T_∞
$1/\overline{h}_h$
$\overline{T}_s$
$1/\overline{h}_c$
$\overline{T}_m$

We need to guess a value for $\overline{T}_s$ and iterate the solution of the system of equations until all the equations are satisfied. See Comments 1 and 2.

Results of the analysis: Using the foregoing relations in IHT (see Comment 2) the following results were obtained

$$\overline{U}=313.4\,W/m^2\cdot K, \qquad \overline{h}_c=453\,W/m^2\cdot K \qquad \overline{h}_h=1015\,W/m^2\cdot K$$

$$\overline{T}_{m,c}=304.5\,K, \qquad \overline{T}_{f,h}=320.1\,K, \qquad \overline{T}_s=317.0\,K \qquad L=12.46\,m \quad <$$

From knowledge of the tube length with the diameter of the coil D_c = 200 mm, the number of coils required is

$$N=\frac{L}{\pi D_c}=\frac{12.46\,m}{\pi\times 0.200\,m}=19.8\approx 20 \quad <$$

(b) With the length fixed at L = 12.46 m, we can backsolve the foregoing IHT workspace model to find what effect a ±10% change in the mass flow rate has on the outlet temperature, $T_{m,o}$. The results of the analysis are tabulated below.

$\dot{m}\,(kg/s)$	0.018	0.02	0.022
$T_{m,o}\,(^\circ C)$	38.95	38.00	37.17

That is, a ±10 % change in the flow rate causes a ±1°C change in the outlet temperature. While this change seems quite small, the effect on biological processes can be significant.

COMMENTS: (1) For the hot fluid, the Properties section shows the relevant thermophysical properties evaluated at the proper average (rather than a guess value for the film temperature).

(2) For the tube L/D = 12.46m/0.005m = 2492 which is substantially greater than the entrance length criterion, $0.05Re_D = 0.05\times 6550 = 328$. Hence, the assumption of fully developed internal flow is justified.

(3) The IHT model for the system can be constructed beginning with the *Rate Equation Tools, Tube Flow, Constant Surface Temperature* along with the *Correlation Tools* for *Free Convection, Horizontal Cylinder* and *Internal Flow, Laminar, Fully Developed Flow* and the *Properties Tool* for the hot and cold fluids (water). The full set of equations is extensive and very stiff. Review of the IHT Example 8.5 would be helpful in understanding how to organize the complete model.

PROBLEM 9.74

KNOWN: Volume, thermophysical properties, and initial and final temperatures of a pharmaceutical. Diameter and length of submerged tubing. Pressure of saturated steam flowing through the tubing.

FIND: (a) Initial rate of heat transfer to the pharmaceutical, (b) Time required to heat the pharmaceutical to 70°C and the amount of steam condensed during the process.

SCHEMATIC:

ASSUMPTIONS: (1) Pharmaceutical may be approximated as an infinite, quiescent fluid of uniform, but time-varying temperature, (2) Free convection heat transfer from the coil may be approximated as that from a heated, horizontal cylinder, (3) Negligible thermal resistance of condensing steam and tube wall, (4) Negligible heat transfer from tank to surroundings, (5) Constant properties.

PROPERTIES: *Table A-4*, Saturated water (2.455 bars): T_{sat} = 400K = 127°C, $h_{fg} = 2.183 \times 10^6$ J/kg. Pharmaceutical: See schematic.

ANALYSIS: (a) The initial rate of heat transfer is $q = \bar{h} A_s (T_s - T_i)$, where $A_s = \pi DL = 0.707\ m^2$ and $\bar{h}$ is obtained from Eq. 9.34. With $\alpha = \nu/Pr = 4.0 \times 10^{-7}\ m^2/s$ and $Ra_D = g\beta (T_s - T_i) D^3/\alpha\nu$ = $9.8\ m/s^2\ (0.002\ K^{-1})\ (102K)\ (0.015m)^3/16 \times 10^{-13}\ m^4/s^2 = 4.22 \times 10^6$,

$$\overline{Nu}_D = \left\{0.60 + \frac{0.387\, Ra_D^{1/6}}{\left[1+(0.559/Pr)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.60 + \frac{0.387\left(4.22\times10^6\right)^{1/6}}{\left[1+(0.559/10)^{9/16}\right]^{8/27}}\right\}^2 = 27.7$$

Hence,

$$\bar{h} = Nu_D\, k/D = 27.7 \times 0.250\, W/m\cdot K / 0.015m = 462\, W/m^2\cdot K$$

and

$$q = \bar{h} A_s (T_s - T_i) = 462\, W/m^2\cdot K \times 0.707\, m^2\, (102°C) = 33{,}300\, W \quad <$$

(b) Performing an energy balance at an instant of time for a control surface about the liquid,

$$\frac{d(\rho\forall c T)}{dt} = q(t) = \bar{h}(t) A_s (T_s - T(t))$$

where the Rayleigh number, and hence $\bar{h}$, changes with time due to the change in the temperature of the liquid. Integrating the foregoing equation using the DER function of IHT, the following results are obtained for the variation of T and $\bar{h}$ with t.

Continued …..

PROBLEM 9.74 (Cont.)

The time at which the liquid reaches 70°C is

$$t_f \approx 855\,\text{s}$$ <

The rate at which T increases decreases with increasing time due to the corresponding reduction in (T_s – T), and hence reductions in Ra_D, $\overline{h}$ and q. The Rayleigh number decreases from 4.22×10^6 to 2.16×10^6, while the heat rate decreases from 33,300 to 14,000 W. The convection coefficient decreases approximately as $(T_s - T)^{1/3}$, while q ~ $(T_s - T)^{4/3}$. The latent energy released by the condensed steam corresponds to the increase in thermal energy of the pharmaceutical. Hence, $m_c h_{fg} = \rho \forall c\left(T_f - T_i\right)$, and

$$m_c = \frac{\rho \forall c\left(T_f - T_i\right)}{h_{fg}} = \frac{1100\,\text{kg/m}^3 \times 0.2\,\text{m}^3 \times 2000\,\text{J/kg}\cdot\text{K} \times 45°\text{C}}{2.183 \times 10^6\,\text{J/kg}} = 9.07\,\text{kg}$$ <

COMMENTS: (1) Over such a large temperature range, the fluid properties are likely to vary significantly, particularly ν and Pr. A more accurate solution could therefore be performed if the temperature dependence of the properties were known. (2) Condensation of the steam is a significant process expense, which is linked to the equipment (capital) and energy (operating) costs associated with steam production.

PROBLEM 9.75

KNOWN: Fin of uniform cross section subjected to prescribed conditions.

FIND: Tip temperature and fin effectiveness based upon (a) *average* values for free convection and radiation coefficients and (b) *local* values using a numerical method of solution.

SCHEMATIC:

T_{sur} = 27 °C
Quiescent air
T_∞ = 27°C
T_b = 125°C
Fin, AISI 316,
D = 6mm, ε = 0.6
L = 50mm
x

ASSUMPTIONS: (1) Steady-state conditions, (2) Surroundings are isothermal and large compared to the fin, (3) One-dimensional conduction in fin, (4) Constant fin properties, (5) Tip of fin is insulated, (6) Fin surface is diffuse-gray.

PROPERTIES: *Table A-4,* Air (T_f = 325 K, 1 atm): $\nu = 18.41 \times 10^{-6}\ m^2/s$, k = 0.0282 W/m·K, $\alpha = 26.2 \times 10^{-6}\ m^2/s$, Pr = 0.704, $\beta = 1/T_f = 3.077 \times 10^{-3}\ K^{-1}$; *Table A-1,* Steel AISI 316 $\left(\bar{T}_s = 350\,K\right)$: k = 14.3 W/m·K.

ANALYSIS: (a) *Average value* $\bar{h}_c$ *and* $\bar{h}_r$: From Table 3.4 for a fin of constant cross section with an insulated tip and constant heat transfer coefficient $\bar{h}$, the tip temperature (x = L) is given by Eq. 3.75,

$$\theta_L = \theta_b \frac{\cosh m(L-x)}{\cosh mL} = \theta_b / \cosh(mL) \qquad m = \left(\bar{h}P/kA_c\right)^{1/2} \qquad (1,2)$$

where $\theta_L = T_L - T_\infty$ and $\theta_b = T_b - T_\infty$. For this situation, the average heat transfer coefficient is

$$\bar{h} = \bar{h}_c + \bar{h}_r \qquad (3)$$

and is evaluated at the average temperature of the fin. The fin effectiveness ε_f follows from Eqs. 3.81 and 3.76

$$\varepsilon_f \equiv q_f / \bar{h}A_{c,b}\theta_b, \qquad q_f = M \cdot \tanh(mL), \qquad M = \left(\bar{h}PkA_c\right)^{1/2}\theta_b. \qquad (4,5,6)$$

To estimate the coefficients, assume a value of $\bar{T}_s$; the lowest $\bar{T}_s$ occurs when the tip reaches T_∞. That is,

$$\bar{T}_s = \left(\bar{T}_\infty + T_b\right)/2 = (27+125)^\circ C/2 = 76^\circ \approx 350\,K \qquad T_f = \left(\bar{T}_s + T_\infty\right)/2 = 325\ K.$$

The free convection coefficient can be estimated from Eq. 9.33,

$$\overline{Nu}_D = \frac{\bar{h}_c D}{k} = C Ra_D^n \qquad (7)$$

$$Ra_D = \frac{g\beta\Delta T D^3}{\nu\alpha} = \frac{9.8\,m/s^2 \times 3.077\times10^{-3} K^{-1}(350-300)K(0.006\,m)^3}{18.41\times10^{-6}\ m^2/s \times 26.2\times10^{-6}\ m^2/s} = 675$$

and from Table 9.1 with $10^2 < Ra_L < 10^4$, C = 0.850 and n = 0.188. Hence

Continued...

PROBLEM 9.75 (Cont.)

$$\bar{h}_c = \frac{0.0282\ \mathrm{W/m\cdot K}}{0.006\mathrm{m}} \times 0.850(675)^{0.188} = 13.6\ \mathrm{W/m^2\cdot K}. \qquad (8)$$

The radiation coefficient is estimated from Eq. 1.9,

$$\bar{h}_r = \varepsilon\sigma\left(\bar{T}_s + T_{sur}\right)\left(\bar{T}_s^2 + T_{sur}^2\right)$$

$$\bar{h}_r = 0.6\times 5.67\times 10^{-8}\ \mathrm{W/m^2\cdot K^4}\,(350+300)\left(350^2+300^2\right)\mathrm{K^3} = 4.7\ \mathrm{W/m^2\cdot K} \qquad (9)$$

Hence, the average coefficient, Eq. (3), is

$$\bar{h} = (13.6+4.7)\,\mathrm{W/m^2\cdot K} = 18.3\,\mathrm{W/m^2\cdot K}.$$

Evaluate the fin parameters, Eq. (2) and (6) with

$$P = \pi D = \pi\times 0.006\mathrm{m} = 1.885\times 10^{-2}\,\mathrm{m} \qquad A_c = \pi D^2/4 = \pi\,(0.006\mathrm{m})^2/4 = 2.827\times 10^{-5}\,\mathrm{m^2}$$

$$m = \left(18.3\,\mathrm{W/m^2\cdot K}\times 1.885\times 10^{2}\,\mathrm{m}/14.3\,\mathrm{W/m\cdot K}\times 2.827\times 10^{-5}\right)^{1/2} = 29.21\mathrm{m}^{-1}$$

$$M = \left(18.3\ \mathrm{W/m^2\cdot K}\times 1.885\times 10^{-2}\mathrm{m}\times 14\cdot 3\,\mathrm{W/m\cdot K}\times 2.827\times 10^{-5}\mathrm{m^2}\right)^{1/2}(125-27)\,\mathrm{K} = 1.157\ \mathrm{W}.$$

From Eq. (1), the tip temperature is

$$\theta_L = T_L - T_b = (125-27)\,\mathrm{K}\Big/\cosh\left(29.21\mathrm{m}^{-1}\times 0.050\mathrm{m}\right) = 43.2\mathrm{K} \qquad T_L = 70.2^\circ\mathrm{C} = 343\,\mathrm{K}. \quad <$$

Note this value of T_L provides for $\bar{T}_s \approx 370$ K; so we underestimated $\bar{T}_s$. For best results, an iteration is warranted. The fin effectiveness, Eqs. (4) and (5), is

$$q_f = 1.157\,\mathrm{W}\tanh\left(29.21\,\mathrm{m}^{-1}\times 0.050\,\mathrm{m}\right) = 1.039\,\mathrm{W}$$

$$\varepsilon_f = 1.039\,\mathrm{W}/18.3/\,\mathrm{W/m^2\cdot K}\times 2.827\times 10^{-5}\mathrm{m^2}\,(125-27)\,\mathrm{K} = 20.5. \qquad <$$

(b) *Local values* h_c *and* h_r: Consider the nodal arrangement for using a numerical method to find the tip temperature T_L, the heat rate q_f, and the fin effectiveness ε.

From an energy balance on a control volume about node m, the finite-difference equation is of the form

$$T_m = \left[T_{m+1} + T_{m-1} + (h_c + h_r)\left(4\Delta x^2/kD\right)T_\infty\right]\Big/\left[2 + (h_r + h_c)\left(4\Delta x^2/kD\right)\right]. \qquad (10)$$

The local coefficient h_c follows from Eq. (3), with Eq. 9.33, yielding

$$h_c = \frac{k}{D}C\,Ra_D^n$$

$$h_c = \frac{0.0282\,\mathrm{W/m\cdot K}}{0.006\,\mathrm{m}}\times 0.850\left(675\left[\Delta T/(350-300)\right]\right)^{0.188} = 6.517\,(T_m - 300)^{0.188}. \qquad (11)$$

The local coefficient h_r follows from Eq. (9),

$$h_r = 0.6\times 5.67\times 10^{-8}\,\mathrm{W/m^2\cdot K^4}\,(T_m + 300)\left(T_m^2 + 300^2\right)$$

Continued...

$$h_r = 3.402\times10^{-8}\left(T_m + 300\right)\left(T_m^2 + 300^2\right). \tag{12}$$

The 20-node system of finite-difference equations based upon Eq. (10) with the variable coefficients h_c and h_r prescribed Eqs. (11) and (12), respectively, can be solved simultaneously using IHT or another approach. The temperature distribution is

Node	T_m(K)	Node	T_m(K)	Node	T_m(K)	Node	T_m(K)
1	391.70	6	367.61	11	353.02	16	345.49
2	385.95	7	364.03	12	351.00	17	344.70
3	380.70	8	360.81	13	349.25	18	344.15
4	375.92	9	357.91	14	347.75	19	343.82
5	371.56	10	355.32	15	346.50	20	343.71

From these results the tip temperature is

$$T_L = T_{fd} = 343.7\,\text{K} = 70.7^\circ\text{C}.$$ <

The fin heat rate follows from an energy balance for the control surface about node b.

$$q_f = q_{conv} + q_{cond}$$

$$q_f = h_b P \frac{\Delta x}{2}\left(T_b - T_\infty\right) + kA_c \frac{T_b - T_1}{\Delta x}$$

where h_b follows from Eqs. (11) and (12), with $T_b = 125^\circ\text{C} = 398$ K,

$$h_b = 6.517\left(398 - 300\right)^{0.188} + 3.402\times10^{-8}\left(398+300\right)\left(398^2 + 300^2\right) = 21.33\,\text{W/m}^2\cdot\text{K}$$

$$q_f = 21.33\,\text{W/m}^2\cdot\text{K}\times1.855\times10^{-2}\,\text{m}\left(0.0025\,\text{m}/2\right)\left(398-300\right)\text{K}$$

$$+14.3\,\text{W/m}\cdot\text{K}\times2.827\times10^{-5}\,\text{m}^2\,\frac{\left(398-391.70\right)\text{K}}{0.0025\text{m}} = \left(0.049 + 1.018\right)\text{W} = 1.067\,\text{W}.$$

The effectiveness follows from Eq. (4)

$$\varepsilon_f = 1.067\,\text{W}/21.33\,\text{W/m}^2\cdot\text{K}\times2.827\times10^{-5}\text{m}^2\left(125-27\right)\text{K} = 18.1$$

COMMENTS: (1) The results by the two methods of solution compare as follows:

Coefficients	T(L),K	q_f(W)	ε_f
average	343.1	1.039	20.5
local	343.7	1.067	18.1

The temperature predictions are in excellent agreement and the heat rates very close, within 4%.

(2) To obtain the finite-different equation for node n = 20, use Eq. (10) but consider the adiabatic surface as a symmetry plane.

PROBLEM 9.76

KNOWN: Horizontal tubes of different shapes each of the same cross-sectional area transporting a hot fluid in quiescent air. Lienhard correlation for immersed bodies.

FIND: Tube shape which has the minimum heat loss to the ambient air by free convection.

SCHEMATIC:

ASSUMPTIONS: (1) Ambient air is quiescent, (2) Negligible heat loss by radiation, (3) All shapes have the same cross-sectional area and uniform surface temperature.

PROPERTIES: *Table A-4*, Air ($T_f \approx$ 300K, 1 atm): $\nu = 15.89 \times 10^{-6}$ m^2/s, $\alpha = 22.5 \times 10^{-6}$ m^2/s, k = 0.0263 W/m·K, Pr = 0.707, $\beta = 1/T_f$.

ANALYSIS: The Lienhard correlation approximates the laminar convection coefficient for an immersed body on which the boundary layer does not separate from the surface by $\overline{\mathrm{Nu}}_\ell = \left(\bar{h}\ell\right)/k = 0.52\mathrm{Ra}_\ell^{1/4}$, where the characteristic length, ℓ, is the length of travel of the fluid in the boundary layer across the shape surface. The heat loss per unit length from any shape is $q' = \bar{h}P\left(T_s - T_\infty\right)$. For the shapes,

$$\mathrm{Ra}_\ell = \frac{gb\Delta T\ell^3}{na} = \frac{9.8\,\mathrm{m/s^2}\,(1/300\mathrm{K})(35-25)\mathrm{K}\,\ell^3\mathrm{m}^3}{15.89\times10^{-6}\mathrm{m}^2/\mathrm{s}\times22.5\times10^{-6}\mathrm{m}^2/\mathrm{s}} = 9.137\times10^8\,\ell^3$$

$$\bar{h}_\ell = \left(0.0263\,\mathrm{W/m\cdot K}/\ell\right)0.52\left(9.137\times10^8\ell^3\right)^{1/4} = 2.378\ell^{-1/4}.$$

For the shapes, ℓ is half the total wetted perimeter P. Evaluating $\bar{h}_\ell$ and q', find

Shape	P (mm)	ℓ (mm)	$\bar{h}_\ell\left(\mathrm{W/m^2\cdot K}\right)$	$q'(\mathrm{W/m})$
1	2 × 40 + 2 × 10 = 100	50	5.03	5.03
2	4 × 20 = 80	40	5.32	4.26
3	4 × 20 = 80	40	5.32	4.26
4	π × 22.56 = 70.9	35.4	5.48	3.89

Hence, it follows that shape 4 has the minimum heat loss. <

COMMENTS: Using the Lienhard correlation for a sphere of D = 22.56 mm, find $\ell = 35.4\mathrm{mm}$, the same as for a cylinder, namely, h_4 = 5.48 W/m^2·K. Using the Churchill correlation, Eq. 9.35, find $\bar{h} = 7.69\,\mathrm{W/m^2\cdot K}$. Hence, the approximation for the sphere is 29% low. For a cylinder, using Eq. 9.34, find $\bar{h} = 5.15\,\mathrm{W/m^2\cdot K}$. The approximation for the cylinder is 6% high.

PROBLEM 9.77

KNOWN: Sphere of 2-mm diameter immersed in a fluid at 300 K.

FIND: (a) The conduction limit of heat transfer from the sphere to the quiescent, extensive fluid, $Nu_{D,cond} = 2$; (b) Considering free convection, surface temperature at which the Nusselt number is twice that of the conduction limit for the fluids air and water; and (c) Considering forced convection, fluid velocity at which the Nusselt number is twice that of the conduction limit for the fluids air and water.

SCHEMATIC:

ASSUMPTIONS: (1) Sphere is isothermal, (2) For part (a), fluid is stationary, and (3) For part (b), fluid is quiescent, extensive.

ANALYSIS: (a) Following the hint provided in the problem statement, the thermal resistance of a hollow sphere, Eq. 3.36 of inner and outer radii, r_1 and r_2 , respectively, and thermal conductivity k, is

$$R_{t,cond} = \frac{1}{4\pi k}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \qquad (1)$$

and as $r_2 \to \infty$, that is the medium is extensive

$$R_{t,cond} = \frac{1}{4\pi k r_1} = \frac{1}{2\pi k D} \qquad (2)$$

The Nusselt number can be expressed as

$$Nu = \frac{hD}{k} \qquad (3)$$

and the conduction resistance in terms of a convection coefficient is

$$R_{t,cond} = \frac{1}{hA_s} = \frac{1}{h\pi D^2} \qquad (4)$$

Combining Eqs. (3) and (4)

$$Nu_{D,cond} = \frac{\left(1/R_{t,cond}\pi D^2\right)D}{k} = \frac{\left[1/(1/2\pi kD)\left(\pi D^2\right)\right]D}{k} = 2$$

<

(b) For free convection, the recommended correlation, Eq. 9.35, is

$$\overline{Nu}_D = 2 + \frac{0.589 Ra_D^{1/4}}{\left[1 + (0.469/Pr)^{9/16}\right]^{4/9}}$$

Continued...

$$Ra_D = \frac{g\beta\Delta T D^3}{\nu\alpha} \qquad \Delta T = T_s - T_\infty$$

where properties are evaluated at $T_f = (T_s + T_\infty) / 2$. What value of T_s is required for $\overline{Nu}_D = 4$ for the fluids air and water? Using the *IHT Correlations Tool*, *Free Convection, Sphere* and the *Properties Tool* for *Air* and *Water*, find

Air: $\overline{Nu} \le 3.1$ for all T_s <

Water: $T_s = 301.1K$ <

(c) For forced convection, the recommended correlation, Eq. 7.59, is

$$\overline{Nu}_D = 2 + \left(0.4\,Re_D^{1/2} + 0.06\,Re_D^{2/3}\right)Pr^{0.4}\left(\mu/\mu_s\right)^{1/4}$$

$$Re_D = VD/\nu$$

where properties are evaluated at T_∞, except for μ_s evaluated at T_s. What value of V is required for $\overline{Nu}_D = 4$ if the fluids are air and water? Using the *IHT Correlations Tool*, *Forced Convection, Sphere* and the *Properties Tool* for *Air* and *Water*, find

Air: V = 0.17 m/s *Water*: V = 0.00185 m/s <

COMMENTS: (1) For water, $\overline{Nu}_D = 2\times\overline{Nu}_{D,cond}$ can be achieved by $\Delta T \approx 1$ for free convection and with very low velocity, V< 0.002 m/s, for forced convection.

(2) For air, $\overline{Nu}_D = 2\times\overline{Nu}_{D,cond}$ can be achieved in forced convection with low velocities, V< 0.2 m/s. In free convection, $\overline{Nu}_D$ increases with increasing T_s and reaches a maximum, $\overline{Nu}_{D,max} = 3.1$, around 450 K. Why is this so? Hint: Plot Ra_D as a function of T_s and examine the role of β and ΔT as a function of T_s.

PROBLEM 9.78

KNOWN: Sphere with embedded electrical heater is maintained at a uniform surface temperature when suspended in various media.

FIND: Required electrical power for these media: (a) atmospheric air, (b) water, (c) ethylene glycol.

SCHEMATIC:

ASSUMPTIONS: (1) Negligible surface radiation effects, (2) Extensive and quiescent media.

PROPERTIES: Evaluated at $T_f = (T_s + T_\infty)/2 = 330K$:

	$\nu \cdot 10^6$, m^2/s	$k \cdot 10^3$, W/m·K	$\alpha \cdot 10^6$, m^2/s	Pr	$\beta \cdot 10^3$, K^{-1}
Table A-4, Air (1 atm)	18.91	28.5	26.9	0.711	3.03
Table A-6, Water	0.497	650	0.158	3.15	0.504
Table A-5, Ethylene glycol	5.15	260	0.0936	55.0	0.65

ANALYSIS: The electrical power (P_e) required to offset convection heat transfer is

$$q_{conv} = \bar{h} A_s \left(T_s - T_\infty\right) = p\, \bar{h} D^2 \left(T_s - T_\infty\right). \quad (1)$$

The free convection heat transfer coefficient for the sphere can be estimated from Eq. 9.35 using Eq. 9.25 to evaluate Ra_D.

$$\overline{Nu}_D = 2 + \frac{0.589 Ra_D^{1/4}}{\left[1 + (0.469/Pr)^{9/16}\right]^{4/9}} \begin{cases} Pr \geq 0.7 \\ Ra_D \leq 10^{11} \end{cases} \qquad Ra_D = \frac{g\, b\, \Delta T\, D^3}{n a}. \quad (2,3)$$

(a) For air

$$Ra_D = \frac{9.8 m/s^2 \left(3.03 \times 10^{-3} K^{-1}\right)(94-20) K \,(0.025 m)^3}{18.91 \times 10^{-6} m^2/s \times 26.9 \times 10^{-6} m^2/s} = 6.750 \times 10^4$$

$$\bar{h}_D = \frac{k}{D} \overline{Nu}_D = \frac{0.02285 W/m \cdot K}{0.025 m} \left\{ 2 + \frac{0.589 \left(6.750 \times 10^4\right)^{1/4}}{\left[1 + (0.469/0.711)^{9/16}\right]^{4/9}} \right\} = 10.6 W/m^2 \cdot K$$

$$q_{conv} = p \times 10.6 W/m^2 \cdot K (0.025 m)^2 (94 - 20) K = 1.55 W.$$

Continued …..

(b,c) Summary of the calculations above and for water and ethylene glycol:

Fluid	Ra_D	$\bar{h}_D \left(W/m^2 \cdot K\right)$	q(W)	
Air	6.750×10^4	10.6	1.55	<
Water	7.273×10^7	1299	187	<
Ethylene glycol	15.82×10^6	393	57.0	<

COMMENTS: Note large differences in the coefficients and heat rates for the fluids.

PROBLEM 9.79

KNOWN: Surface temperature and emissivity of a 20W light bulb (spherical) operating in room air

FIND: Heat loss from bulb surface.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Quiescent room air, (3) Surroundings much larger than bulb.

PROPERTIES: *Table A-4*, Air ($T_f = (T_s + T_\infty)/2 = 348K$, 1 atm): $\nu = 20.72 \times 10^{-6}$ m^2/s, k = 0.0298 W/m·K, $\alpha = 29.6 \times 10^{-6}$ m^2/s, Pr = 0.700, $\beta = 1/T_f$.

ANALYSIS: Heat loss from the surface of the bulb is by free convection and radiation. The rate equations are

$$q = q_{conv} + q_{rad} = \bar{h} A_s (T_s - T_\infty) + \varepsilon A_s \sigma \left(T_s^4 - T_{sur}^4\right)$$

where $A_s = \pi D^2$. To estimate $\bar{h}$ for free convection, first evaluate the Rayleigh number.

$$Ra_D = \frac{g\beta \Delta T D^3}{\nu\alpha} = \frac{9.8\text{m/s}^2 (1/348\text{K})(125-25)\text{K}(0.040\text{m})^3}{20.72\times10^{-6}\text{m}^2/\text{s}\times29.6\times10^{-6}\text{m}^2/\text{s}} = 2.93\times10^5.$$

Since Pr ≥ 0.7 and $Ra_D < 10^{11}$, the Churchill relation, Eq. 9.35, is appropriate.

$$\overline{Nu}_D = 2 + \frac{0.589 Ra_D^{1/4}}{\left[1+(0.469/Pr)^{9/16}\right]^{4/9}} = 2 + \frac{0.589\left(2.93\times10^5\right)^{1/4}}{\left[1+(0.469/0.700)^{9/16}\right]^{4/9}} = 12.55$$

$$\bar{h} = \overline{Nu}_D\, k/D = 12.55(0.0298\text{W/m}\cdot\text{K})/0.040\text{m} = 9.36\text{W/m}^2\cdot\text{K}.$$

Substituting numerical values, the heat loss from the bulb is,

$$q = \pi (0.040\text{m})^2 \left[9.36\frac{\text{W}}{\text{m}^2\cdot\text{K}}(125-25)\text{K} + 0.80\times5.67\times10^{-8}\frac{\text{W}}{\text{m}^2\cdot\text{K}^4}\left[(125+273)^4 - (25+273)^4\right]\text{K}^4\right]$$

$$q = (4.70 + 3.92)\text{W} = 8.62\text{W}.$$ <

COMMENTS: (1) The contributions of convection and radiation to the surface heat loss are comparable.

(2) The remaining heat loss (20 – 8.62 = 11.4 W) is due to the transmission of radiant energy (light) through the bulb and heat conduction through the base.

PROBLEM 9.80

KNOWN: A copper sphere with a diameter of 25 mm is removed from an oven at a uniform temperature of 85°C and allowed to cool in a quiescent fluid maintained at 25°C.

FIND: (a) Convection coefficients for the initial condition for the cases when the fluid is air and water, and (b) Time for the sphere to reach 30°C when the cooling fluid is air and water using two different approaches, average coefficient and numerically integrated energy balance.

SCHEMATIC:

Quiescent fluid
Water or air
$T_\infty = 25°C$
$T_s(0) = 85°C$
Copper sphere
D = 25 mm

ASSUMPTIONS: (1) Steady-state conditions for part (a); (2) Low emissivity coating makes radiation exchange negligible for the in-air condition; (3) Fluids are quiescent, and (4) Constant properties.

PROPERTIES: *Table A-4*, Air (T_f = (25 + 85)°C/2 = 328 K, 1 atm): $\nu = 1.871 \times 10^{-5}$ m²/s, k = 0.0284 W/m·K, $\alpha = 2.66 \times 10^{-5}$ m²/s, Pr = 0.703, $\beta = 1/T_f$; *Table A-6*, Water (T_f = 328 K): $\nu = 5.121 \times 10^{-7}$ m²/s, k = 0.648 W/m·K, $\alpha = 1.57 \times 10^{-7}$ m²/s, Pr = 3.26, $\beta = 4.909 \times 10^{-4}$ K^{-1}; *Table A-1*, Copper, pure ($\overline{T}$ = (25 + 85)°C/2 = 328 K): ρ = 8933 kg/m³, c = 382 J/kg·K, k = 399 W/m·K.

ANALYSIS: (a) For the initial condition, the average convection coefficient can be estimated from the Churchill-Chu correlation, Eq. 9.35,

$$\overline{Nu}_D = \frac{\overline{h}_D D}{k} = 2 + \frac{0.589\ Ra_D^{1/4}}{\left[1 + (0.469/Pr)^{9/16}\right]^{4/9}} \qquad (1)$$

$$Ra_D = \frac{g\beta\left(T_s - T_\infty\right)D^3}{\nu\alpha} \qquad (2)$$

with properties evaluated at $T_f = (T_s + T_\infty)/2$ = 328 K. Substituting numerical values find these results:

Fluid	T_s(°C)	T_f(K)	Ra_D	Nu_D	$\overline{h}_D$ (W/m²·K)	
Air	85	328	5.62×10^4	8.99	10.2	<
Water	85	328	5.61×10^7	46.8	1213	<

(b) To establish the validity of the lumped capacitance (LC) method, calculate the Biot number for the worst condition (air).

$$Bi = \frac{\overline{h}_D\left(D/3\right)}{k} = 10.2\ W/m^2\cdot K\left(0.025\ m/3\right)/399\ W/m\cdot K = 2.1\times10^{-4}$$

Since Bi << 0.1, the sphere can be represented by this energy balance for the cooling process

$$\dot{E}_{in} - \dot{E}_{out} = \dot{E}_{st} \qquad -q_{cv} = Mc\frac{dT_s}{dt}$$

$$-\overline{h}_D A_s\left(T_s - T_\infty\right) = \rho Vc\frac{dT_s}{dt} \qquad (3)$$

where $A_s = \pi D^2$ and $V = \pi D^3/6$. Two approaches are considered for evaluating appropriate values for $\overline{h}_D$.

Average coefficient. Evaluate the convection coefficient corresponding to the average temperature of the sphere, $\overline{T}_s$ = (30 + 85)°C/2 = 57.5°C, for which the film temperature is $T_f = (\overline{T}_s + T_\infty)/2$. Using the foregoing analyses of part (a), find these results.

Continued …..

Fluid	$\overline{T}_s$ (°C)	T_f(K)	Ra_D	Nu_D	$\overline{h}_D$ (W/m^2·K)
Air	57.5	314	3.72×10^4	8.31	9.09
Water	57.5	314	1.99×10^7	37.1	940

Numerical integration of the energy balance equation. The more accurate approach is to numerically integrate the energy balance equation, Eq. (3), with $\overline{h}_D$ evaluated as a function of T_s using Eqs. (1) and (2). The properties in the correlation parameters would likewise be evaluated at T_f, which varies with T_s. The integration is performed in the *IHT* workspace; see Comment 3.

Results of the lumped-capacitance analysis. The results of the LC analyses using the two approaches are tabulated below, where t_o is the time to cool from 85°C to 30°C:

	t_o (s)	
Approach	Air	Water
Average coefficient	3940	39
Numerical coefficient	4600	49

COMMENTS: (1) For these condition, the convection coefficient for the water is nearly two orders of magnitude higher than for air.

(2) Using the average-coefficient approach, the time-to-cool, t_o, values for both fluids is 15-20% faster than the more accurate numerical integration approach. Evaluating the average coefficient at $\overline{T}_s$ results in systematically over estimating the coefficient.

(3) The *IHT* code used for numerical integration of the energy balance equation and the correlations is shown below for the fluid water.

```
//  LCM energy balance
- hDbar * As * (Ts - Tinf) = M * cps * der(Ts,t)
As = pi * D^2
M = rhos * Vs
Vs = pi * D^3 / 6

// Input variables
D = 0.025
// Ts = 85 + 273                // Initial condition, Ts
Tinf_C = 25
rhos = 8933                     // Table A.1, copper, pure
cps = 382
ks =  399

/*  Correlation description: Free convection (FC), sphere (S),  RaD<=10^11, Pr >=0.7, Churchill
correlation, Eqs 9.25 and 9.35 . See Table 9.2 .  */
NuDbar = NuD_bar_FC_S(RaD,Pr)                  // Eq 9.35
NuDbar = hDbar * D / k
RaD = g * beta * deltaT * D^3 / (nu * alpha)   //Eq 9.25
deltaT = abs(Ts - Tinf)
g = 9.8     // gravitational constant, m/s^2
// Evaluate properties at the film temperature, Tf.
Tf = Tfluid_avg(Tinf,Ts)

// Water property functions :T dependence, From Table A.6
// Units: T(K), p(bars);
x = 0                          // Quality (0=sat liquid or 1=sat vapor)
nu = nu_Tx("Water",Tf,x)       // Kinematic viscosity, m^2/s
k = k_Tx("Water",Tf,x)         // Thermal conductivity, W/m·K
Pr = Pr_Tx("Water",Tf,x)       // Prandtl number
beta = beta_T("Water",Tf)      // Volumetric coefficient of expansion, K^(-1) (f, liquid, x = 0)
alpha = k / (rho * cp)         // Thermal diffusivity, m^2/s

// Conversions
Ts_C = Ts - 273
Tinf_C = Tinf - 273
```

PROBLEM 9.81

KNOWN: Temperatures and spacing of vertical, isothermal plates.

FIND: (a) Shape of velocity distribution, (b) Forms of mass, momentum and energy equations for laminar flow, (c) Expression for the temperature distribution, (d) Vertical pressure gradient, (e) Expression for the velocity distribution.

SCHEMATIC:

ASSUMPTIONS: (1) Laminar, incompressible, fully-developed flow, (2) Constant properties, (3) Negligible viscous dissipation, (4) Boussinesq approximation.

ANALYSIS: (a) For the prescribed conditions, there must be buoyancy driven ascending and descending flows along the surfaces corresponding to $T_{s,1}$ and $T_{s,2}$, respectively (see schematic). However, conservation of mass dictates equivalent rates of *upflow* and *downflow* and, assuming constant properties, inverse symmetry of the velocity distribution about the midplane.

(b) For fully-developed flow, which is achieved for *long* plates, $v_x = 0$ and the continuity equation yields

$$\partial v_z / \partial z = 0$$ <

Hence, there is no net transfer of momentum or energy by advection, and the corresponding equations are, respectively,

$$0 = -(dp/dz) + \mu\left(d^2 v_z / dx^2\right) - \rho\,(g/g_c)$$ <

$$0 = (dT^2/dx^2)$$ <

(c) Integrating the energy equation twice, we obtain

$$T = C_1 x + C_2$$

and applying the boundary conditions, $T(-L) = T_{s,1}$ and $T(L) = T_{s,2}$, it follows that $C_1 = -(T_{s,1} - T_{s,2})/2L$ and $C_2 = (T_{s,1} + T_{s,2})/2 \equiv T_m$, in which case,

$$\frac{T - T_m}{T_{s,1} - T_{s,2}} = -\frac{x}{2L}$$ <

Continued...

PROBLEM 9.81 (Cont.)

(d) From hydrostatic considerations and the assumption of a constant density ρ_m, the balance between the gravitational and net pressure forces may be expressed as $dp/dz = -\rho_m(g/g_c)$. The momentum equation is then of the form

$$0 = \mu\left(d^2v_z/dx^2\right) - (\rho - \rho_m)(g/g_c)$$

or, invoking the Boussinesq approximation, $\rho - \rho_m \approx -\beta\rho_m(T - T_m)$,

$$d^2v_z/dx^2 = -(\beta\rho_m/\mu)(g/g_c)(T - T_m)$$

or, from the known temperature distribution,

$$d^2v_z/dx^2 = (\beta\rho_m/2\mu)(g/g_c)\left(T_{s,1} - T_{s,2}\right)(x/L)$$

<

(e) Integrating the foregiong expression, we obtain

$$dv_z/dx = (\beta\rho_m/4\mu)(g/g_c)\left(T_{s,1} - T_{s,2}\right)\left(x^2/L\right) + C_1$$

$$v_z = (\beta\rho_m/12\mu)(g/g_c)\left(T_{s,1} - T_{s,2}\right)\left(x^3/L\right) + C_1x + C_2$$

Applying the boundary conditions $v_z(-L) = v_z(L) = 0$, it follows that $C_1 = -(\beta\rho_m/12\mu)(g/g_c)\left(T_{s,1} - T_{s,2}\right)L$ and $C_2 = 0$. Hence,

$$v_z = \left(\beta\rho_m L^2/12\mu\right)(g/g_c)\left(T_{s,1} - T_{s,2}\right)\left[\left(x^3/L^3\right) - (x/L)\right]$$

<

COMMENTS: The validity of assuming fully-developed conditions improves with increasing plate length and would be satisfied precisely for infinite plates.

PROBLEM 9.82

KNOWN: Dimensions of vertical rectangular fins. Temperature of fins and quiescent air.

FIND: Optimum fin spacing and corresponding fin heat transfer rate.

SCHEMATIC:

ASSUMPTIONS: (1) Isothermal fins, (2) Negligible radiation, (3) Quiescent air, (4) Negligible heat transfer from fin tips, (5) Negligible radiation.

PROPERTIES: *Table A-4*, Air (T_f = 325K, 1 atm): $\nu = 18.41 \times 10^{-6}\ m^2/s$, k = 0.0282 W/m·K, $\alpha = 26.1 \times 10^{-6}\ m^2/s$, Pr = 0.703.

ANALYSIS: From Table 9.3

$$S_{opt} = 2.71\left(Ra_S / S^3 H\right)^{-1/4} = 2.71\left[\frac{g b\,(T_s - T_\infty)}{an\,H}\right]^{-1/4}$$

$$S_{opt} = 2.71\left[\frac{9.8\,m/s^2\,(325K)^{-1}(50K)}{26.1\times10^{-6} m^2/s \times 18.4\times10^{-6} m^2/s \times 0.15m}\right]^{-1/4} = 7.12mm \qquad <$$

From Eq. 9.45 and Table 9.3

$$\overline{Nu}_s = \left[\frac{576}{(Ra_S S/L)^2} + \frac{2.87}{(Ra_S S/L)^{1/2}}\right]^{-1/2}$$

$$Ra_S(S/L) = \frac{g b\,(T_s - T_\infty) S^4}{an\,H} = \frac{9.8\,m/s^2\,(325K)^{-1}(50K)\left(7.12\times10^{-3} m\right)^4}{25.4\times10^{-6} m^2/s \times 18.4\times10^{-6} m^2/s \times 0.15m}$$

$$Ra_S(S/L) = 53.2$$

$$\overline{Nu}_S = \left[\frac{576}{(53.2)^2} + \frac{2.87}{(53.2)^{1/2}}\right]^{-1/2} = [0.204 + 0.393]^{-1/2} = 1.29$$

$$\bar{h} = \overline{Nu}_S\ k/S = 1.29(0.0282 W/m\cdot K / 0.00712m) = 5.13 W/m^2\cdot K.$$

With N = W/(t + S) = (355 mm)/(8.62×10^{-3} m) = 41.2 ≈ 41,

$$q = 2N\bar{h}(L\times H)(T_s - T_\infty) = 82\left(5.13 W/m^2\cdot K\right)(0.02m\times0.15m)50K$$

$$q = 63.1W. \qquad <$$

COMMENTS: S_{opt} = 7.12 mm is considerably less than the value of 34 mm predicted from previous considerations. Hence, the corresponding value of q = 63.1 W is considerably larger than that of the previous predication.

PROBLEM 9.83

KNOWN: Length, width and spacing of vertical circuit boards. Maximum allowable board temperature.

FIND: Maximum allowable power dissipation per board.

SCHEMATIC:

ASSUMPTIONS: (1) Circuit boards are flat with uniform heat flux at each surface, (2) Negligible radiation.

PROPERTIES: *Table A-4*, Air $\left(\overline{T} = 320K, 1atm\right)$: $\nu = 17.9 \times 10^{-6}\ m^2/s$, $k = 0.0278\ W/m\cdot K$, $\alpha = 25.5 \times 10^{-6}\ m^2/s$.

ANALYSIS: From Eqs. 9.41 and 9.46 and Table 9.3,

$$\frac{q_s''}{T_{s,L} - T_\infty}\frac{S}{k} = \left[\frac{48}{Ra_S^* S/L} + \frac{2.51}{\left(Ra_S^* S/L\right)^{2/5}}\right]^{-1/2}$$

where $$Ra_S^* \frac{S}{L} = \frac{g\beta q_s'' S^5}{k\alpha\nu L} = \frac{9.8 m/s^2 (320K)^{-1} (0.025m)^5 q_s''}{0.0278 W/m\cdot K\left(25.5\times10^{-6} m^2/s\right)\left(17.9\times10^{-6} m^2/s\right)0.4m}$$

$$Ra_S^* \frac{S}{L} = 58.9 q_s''$$

and $$\frac{q_s''}{T_{s,L} - T_\infty}\frac{S}{k} = \frac{0.025m \cdot q_s''}{(60\ K)0.0278 W/m\cdot K} = 0.015 q_s''.$$

Hence, $$0.015 q_s'' = \left[\frac{0.815}{q_s''} + \frac{0.492}{(q_s'')^{0.4}}\right]^{-1/2}.$$

A trial-and-error solution yields

$$q_s'' = 287\ W/m^2.$$

Hence, $$q = 2A_s q_s'' = 2(0.4m)^2\left(287 W/m^2\right) = 91.8\ W.$$ <

COMMENTS: Larger heat rates may be achieved by using a fan to superimpose a forced flow on the buoyancy driven flow.

PROBLEM 9.84

KNOWN: Array of isothermal vertical fins attached to heat sink at 42°C with ambient air temperature at 27°C.

FIND: (a) Heat removal rate for 24 pairs of fins and (b) Optimum fin spacing for maximizing heat removal rate if overall size of sink is to remain unchanged.

SCHEMATIC:

S = 6mm
Vertical plates, L=150mm, 24 pairs
w = 25mm
Air, T_∞=27°C
T_s = 42°C
Sink
Circuit board

ASSUMPTIONS: (1) Fins form vertical, symmetrically heated, isothermal plates, (2) Negligible radiation effects, (3) Ambient air is quiescent.

PROPERTIES: *Table A-4*, Air $\left(T_f = (T_s + T_\infty)/2 = (42+27)/2°C = 308K, 1atm\right)$:

$n = 16.69 \times 10^{-6} m^2/s$, $a = 23.5 \times 10^{-6} m^2/s$, $k = 26.9 \times 10^{-3}$ W/m·K.

ANALYSIS: Considering the fins as vertical isothermal plates, the heat rate can be determined from Eq. 9.37 with the Elenbaas correlation, Eq. 9.36,

$$Ra_S = \frac{gb\,(T_s - T_\infty)S^3}{na} = \frac{9.8 m/s^2 (1/308\ K)(42-27)K(0.006\ m)^3}{16.69 \times 10^{-6} m^2/s \times 23.5 \times 10^{-6} m^2/s} = 262.9$$

$$\overline{Nu}_S = \left(\frac{q/A_s}{T_s - T_\infty}\right)\frac{S}{k} = \frac{1}{24} Ra_S (S/L)\left\{1 - \exp\left[-\frac{35}{Ra_S (S/L)}\right]\right\}^{3/4}$$

$$\overline{Nu}_S = \frac{1}{24}(262.9)\left(\frac{0.006m}{0.150m}\right)\left\{1 - \exp\left[-\frac{35}{262.9(0.006m/0.150m)}\right]\right\}^{3/4} = 0.4263$$

find the heat rate as

$$q_s = A_s\,(T_s - T_\infty)\frac{k}{S}\overline{Nu}_S$$

$$q_s = 2 \times 2 \times 24 (0.025 \times 0.150) m^2 (42-27) K \frac{0.0269 W/m \cdot K}{0.006\ m} 0.4263$$

$$q_s = 10.4\ W.$$

<

(b) For symmetric isothermal plates, from Table 9.3, the optimum spacing to maximize the heat rate with L = 0.150 m is

$$S_{opt} = 2.71\left(Ra_S / S^3 L\right)^{-1/4} = 2.71\left[\frac{gb\,(T_s - T_\infty)}{na}\frac{1}{L}\right]^{-1/4} = 9.03\ mm$$

Continued …..

and using Eq. 9.45 with values of C_1 and C_2 from Table 9.3,

$$Ra_S = \frac{g b \left(T_s - T_\infty\right) S_{opt}^3}{n a} = 896$$

$$\overline{Nu}_S = \left[\frac{q/A_s}{T_s - T_\infty}\right]\frac{S}{k} = \left[\frac{C_1}{\left(Ra_S S/L\right)^2} + \frac{C_2}{\left(Ra_S S/L\right)^{1/2}}\right]^{-1/2}$$

and solving for the heat rate,

$$q = 0.240 m^2 \left(42 - 27\right) K \frac{0.0269\ W/m \cdot K}{0.00903\ m}\left[\frac{576}{\left(896 \times 9/150\right)^2} + \frac{2.87}{\left(896 \times 9/150\right)^{1/2}}\right]^{-1/2}$$

$$q = 14.0\ W$$ <

where with spacing S_{opt} = 9.03 mm, rather than 6 mm, the available space for the fins has decreased by (9.03 – 6)/6 = 50%; hence only 16 pairs are possible and $A_s = 4 \times 16(0.025 \times 0.150)m^2$ = 0.240 m^2.

COMMENTS: Note that with S_{opt} = 9 mm, the convection coefficient is increased by (3.88 – 1.91)1.91 = 103%. However, the increased spacing reduces the number of surfaces possible within the given space constraint.

PROBLEM 9.85

KNOWN: Vertical air vent in front door of dishwasher with prescribed width and height. Spacing between isothermal and insulated surface of 20 mm.

FIND: (a) Heat loss from the tub surface and (b) Effect on heat rate of changing spacing by ± 10 mm.

SCHEMATIC:

L = 500 mm
$T_s = 52°C$
Insulated surface, width w = 580 mm
Ambient air, $T_\infty = 27°C$
S = 20 mm

ASSUMPTIONS: (1) Steady-state conditions, (2) Vent forms vertical parallel isothermal/adiabatic plates, (3) Ambient air is quiescent.

PROPERTIES: *Table A-4*, ($T_f = (T_s + T_\infty)/2 = 312.5K$, 1 atm): $\nu = 17.15 \times 10^{-6}\ m^2/s$, $\alpha = 24.4 \times 10^{-6}\ m^2/s$, $k = 27.2 \times 10^{-3}$ W/m·K, $\beta = 1/T_f$.

ANALYSIS: The vent arrangement forms two vertical plates, one is isothermal, T_s, and the other is adiabatic $(q'' = 0)$. The heat loss can be estimated from Eq. 9.37 with the correlation of Eq. 9.45 using $C_1 = 144$ and $C_2 = 2.87$ from Table 9.3:

$$Ra_S = \frac{gb\,(T_s - T_\infty)S^3}{na} = \frac{9.8\,m/s^2\,(1/312.5\ K)(52-27)K\,(0.020m)^3}{17.15\times10^{-6} m^2/s \times 24.4\times10^{-6} m^2/s} = 14{,}988$$

$$q = A_s\,(T_s - T_\infty)\frac{k}{S}\left[\frac{C_1}{(Ra_S S/L)^2} + \frac{C_2}{(Ra_S S/L)^{1/2}}\right]^{-1/2} = (0.500\times0.580)m^2 \times$$

$$(52-27)K\,\frac{0.0272 W/m\cdot K}{0.020m}\left[\frac{C_1}{(Ra_S S/L)^2} + \frac{C_2}{(Ra_S S/L)^{1/2}}\right]^{-1/2} = 28.8W. <$$

(b) To determine the effect of the spacing at S = 30 and 10 mm, we need only repeat the above calculations with these results

S (mm)	Ra_S	q (W)	
10	1874	26.1	<
30	50,585	28.8	<

Since it would be desirable to minimize heat losses from the tub, based upon these calculations, you would recommend a decrease in the spacing.

COMMENTS: For this situation, according to Table 9.3, the spacing corresponding to the maximum heat transfer rate is $S_{max} = (S_{max}/S_{opt}) \times 2.15(Ra_S/S^3L)^{-1/4} = 14.5$ mm. Find $q_{max} = 28.5$ W. Note that the heat rate is not very sensitive to spacing for these conditions.

PROBLEM 9.86

KNOWN: Dimensions, spacing and temperature of plates in a vertical array. Ambient air temperature. Total width of the array.

FIND: Optimal plate spacing for maximum heat transfer from the array and corresponding number of plates and heat transfer.

SCHEMATIC:

W_{ar} = 150 mm
1 2 ··· N
Heated plate
T_s = 75°C, W = L
L = 0.3m
T_∞ = 20°C
S_{op}
Ambient air

ASSUMPTIONS: (1) Steady-state, (2) Negligible plate thickness, (3) Constant properties.

PROPERTIES: *Table A-4*, air $\left(p = 1\,\text{atm},\ \overline{T} = 320\text{K}\right)$: $\nu = 17.9 \times 10^{-6}$ m²/s, k = 0.0278 W/m·K, $\alpha = 25.5 \times 10^{-6}$ m²/s, Pr = 0.704, $\beta = 0.00313\ \text{K}^{-1}$.

ANALYSIS: With $Ra_S/S^3L = g\beta(T_s - T_\infty)/\alpha\nu L = (9.8\ \text{m/s}^2 \times 0.00313\ \text{K}^{-1} \times 55°\text{C})/(25.5 \times 17.9 \times 10^{-12}\ \text{m}^4/\text{s}^2 \times 0.3\text{m}) = 1.232 \times 10^{10}\ \text{m}^4$, from Table 9.3, the spacing which maximizes heat transfer for the array is

$$S_{opt} = \frac{2.71}{\left(Ra_S/S^3L\right)^{1/4}} = \frac{2.71}{\left(1.232\times10^{10}\,\text{m}^{-4}\right)^{1/4}} = 8.13\times10^{-3}\,\text{m} = 8.13\,\text{mm} \qquad <$$

With the requirement that $(N - 1)\, S_{opt} \le W_{ar}$, it follows that $N \le 1 + 150\ \text{mm}/8.13\ \text{mm} = 19.4$, in which case

$$N = 19 \qquad <$$

The corresponding heat rate is $q = N(2WL)\overline{h}(T_s - T_\infty)$, where, from Eq. 9.45 and Table 9.3,

$$\overline{h} = \frac{k}{S}\overline{Nu}_S = \frac{k}{S}\left[\frac{576}{(Ra_S S/L)^2} + \frac{2.87}{(Ra_S S/L)^{1/2}}\right]^{1/2}$$

With $Ra_S\, S/L = (Ra_S/S^3L)S^4 = 1.232 \times 10^{10}\ \text{m}^{-4} \times (0.00813\text{m})^4 = 53.7$,

$$\overline{h} = \frac{0.0278\,\text{W/m}\cdot\text{K}}{0.00457\text{m}}\left[\frac{576}{(53.7)^2} + \frac{2.87}{(53.7)^{1/2}}\right] = 6.08(0.200 + 0.392) = 3.60\,\text{W/m}^2\cdot\text{K}$$

$$q = 19(2\times0.3\text{m}\times0.3\text{m})3.60\,\text{W/m}^2\cdot\text{K}\times55°\text{C} = 677\,\text{W} \qquad <$$

COMMENTS: It would be difficult to fabricate heater plates of thickness $\delta << S_{opt}$. Hence, subject to the constraint imposed on W_{ar}, N would be reduced, where $N \le 1 + W_{ar}/(S_{opt} + \delta)$.

PROBLEM 9.87

KNOWN: A bank of drying ovens is mounted on a rack in a room with an ambient temperature of 27°C; the cubical ovens are 500 mm to a side and the spacing between the ovens is 15 mm.

FIND: (a) Estimate the heat loss from the facing side of an oven when its surface temperature is 47°C, and (b) Explore the effect of the spacing dimension on the heat loss. At what spacing is the heat loss a maximum? Describe the boundary layer behavior for this condition. Can this condition be analyzed by treating the oven side-surface as an isolated vertical plate?

SCHEMATIC:

T_s = 47°C
Drying oven
500 mm sides
L
Ambient air
T_∞ = 27°C
Spacing
S = 15 mm

ASSUMPTIONS: (1) Steady-state conditions, (2) Adjacent oven sides form a vertical channel with symmetrically heated plates, (3) Room air is quiescent, and channel sides are open to the room air, and (4) Constant properties.

PROPERTIES: *Table A-4*, Air ($T_f = (T_s + T_\infty)/2$ = 310 K, 1 atm): $\nu = 1.69 \times 10^{-5}$ m^2/s, k = 0.0270 W/m·K, $\alpha = 2.40 \times 10^{-5}$ m^2/s, Pr = 0.706, $\beta = 1/T_f$.

ANALYSIS: (a) For the isothermal plate channel, Eq. 9.45 with Eqs. 9.37 and 9.38, allow for calculation of the heat transfer from a plate to the ambient air.

$$\overline{Nu}_S = \left[\frac{C_1}{Ra_S S/L} + \frac{C_2}{(Ra_S S/L)^{1/2}}\right]^{-1/2} \tag{1}$$

$$\overline{Nu}_S = \frac{q/A}{T_s - T_\infty}\frac{S}{k} \tag{2}$$

$$Ra_S = \frac{g\beta(T_s - T_\infty)S^3}{\alpha\nu} \tag{3}$$

where, from Table 9.3, for the *symmetrical isothermal plates*, C_1 = 576 and C_2 = 2.87. Properties are evaluated at the film temperature T_f. Substituting numerical values, evaluate the correlation parameters and the heat rate.

$$Ra_S = \frac{9.8\ \text{m/s}^2(1/310\ \text{K})(47-27)\text{K}(0.015\ \text{m})^3}{2.40\times10^{-5}\ \text{m}^2/\text{s}\times1.69\times10^{-5}\ \text{m}^2/\text{s}} = 5267$$

$$\overline{Nu}_S = \left[\frac{576}{5267\times0.015\ \text{m}/0.50\ \text{m}} + \frac{2.87}{(5267\times0.015\ \text{m}/0.050\ \text{m})^{1/2}}\right]^{-1/2} = 1.994$$

$$1.994 = \frac{q/(0.50\times0.50)\text{m}^2}{(47-27)\text{K}}\frac{0.015\ \text{m}}{0.0274\ \text{W/m}\cdot\text{K}} \qquad q = 18.0\ \text{W}$$ <

Continued …..

(b) Using the foregoing relations in *IHT*, the heat rate is calculated for a range of spacing S.

Note that the heat rate increases with increasing spacing up to about S = 20 mm. This implies that for S > 20 mm, the side wall of the oven behaves as an *isolated vertical plate*. From the treatment of the vertical channel, Section 9.7.1, the spacing to provide maximum heat rate from a plate occurs at S_{max} which, from Table 9.3, is evaluated by

$$S_{max} = 1.71\, S_{opt} = 0.01964 \text{ m} = 19.6 \text{ mm}$$

$$S_{opt} = 2.71\left(Ra_S / S^3 L\right)^{-1/4} = 0.01147 \text{ m}$$

For the condition $S = S_{max}$, the spacing is sufficient that the boundary layers on the plates do not overlap.

COMMENTS: Using the Churchill-Chu correlation, Eq. 9.26, for the isolated vertical plate, where the characteristic dimension is the height L, find q = 20.2 W ($Ra_L = 1.951 \times 10^8$ and $\overline{h}_L = 4.03$ $W/m^2 \cdot K$). This value is slightly larger than that from the channel correlation when $S > S_{max}$, but a good approximation.

PROBLEM 9.88

KNOWN: Inclination angle of parallel plate solar collector. Plate spacing. Absorber plate and inlet temperature.

FIND: Rate of heat transfer to collector fluid.

SCHEMATIC:

ASSUMPTIONS: (1) Flow in collector corresponds to buoyancy driven flow between parallel plates with quiescent fluids at the inlet and outlet, (2) Constant properties.

PROPERTIES: *Table A-6*, Water $(\overline{T} = 320K)$: ρ = 989 kg/m^3, c_p = 4180 J/kg·K, μ = 577 × 10^{-6} kg/s·m, k = 0.640 W/m·K, β = 436.7 × 10^{-6} K^{-1}.

ANALYSIS: With

$$a = \frac{k}{r c_p} = \frac{0.640 W/m \cdot K}{989 kg/m^3 (4180 J/kg \cdot K)} = 1.55 \times 10^{-7} m^2/s$$

$$n = (m/r) = \left(577 \times 10^{-6} kg/s \cdot m\right)/989 kg/m^3 = 5.83 \times 10^{-7} m^2/s$$

find

$$Ra_S \frac{S}{L} = \frac{gb(T_s - T_\infty)S^4}{anL} = \frac{9.8 m/s^2 \left(436.7 \times 10^{-6} K^{-1}\right)(40K)(0.015m)^4}{\left(1.55 \times 10^{-7} m^2/s\right)\left(5.83 \times 10^{-7} m^2/s\right)1.5m}$$

$$Ra_S \frac{S}{L} = 6.39 \times 10^4.$$

Since $Ra_S(S/L) > 200$, Eq. 9.47 may be used,

$$\overline{Nu}_S = 0.645\left[Ra_S(S/L)\right]^{1/4} = 0.645\left(6.39 \times 10^4\right)^{1/4} = 10.3$$

$$\overline{h} = \overline{Nu}_S \frac{k}{S} = 10.3(0.64 W/m \cdot K/0.015m) = 438 W/m^2 \cdot K.$$

Hence the heat rate is

$$q = \overline{h}A(T_s - T_\infty) = 438 W/m^2 \cdot K(1.5m)(67 - 27)K = 26{,}300 W/m. \quad <$$

COMMENTS: Such a large heat rate would necessitate use of a concentrating solar collector for which the normal solar flux would be significantly amplified.

PROBLEM 9.89

KNOWN: Critical Rayleigh number for onset of convection in vertical cavity filled with atmospheric air. Temperatures of opposing surfaces.

FIND: Maximum allowable spacing for heat transfer by conduction across the air. Effect of surface temperature and air pressure.

SCHEMATIC:

ASSUMPTIONS: (1) Critical Rayleigh number is $Ra_{L,c} = 2000$, (2) Constant properties.

PROPERTIES: *Table A-4*, air [$T = (T_1 + T_2)/2 = 1°C = 274K$]: $\nu = 13.6 \times 10^{-6}$ m^2/s, k = 0.0242 W/m·K, $\alpha = 19.1 \times 10^{-6}$ m^2/s, $\beta = 0.00365$ K^{-1}.

ANALYSIS: With $Ra_{L,c} = g\,\beta\,(T_1 - T_2)\,L_c^3/\alpha\nu$,

$$L_c = \left[\frac{\alpha\nu\,Ra_{L,c}}{g\beta\left(T_1 - T_2\right)}\right]^{1/3} = \left[\frac{19.1\times 13.6\times 10^{-12}\,m^4/s^2\times 2000}{9.8\,m/s^2\times 0.00365\,K^{-1}\times 42°C}\right]^{1/3} = 0.007m = 7\,mm$$ <

The critical value of the spacing, and hence the corresponding thermal resistance of the air space, increases with a decreasing temperature difference, $T_1 - T_2$, and decreasing air pressure. With $\nu = \mu/\rho$ and $\alpha \equiv k/\rho c_p$, both quantities increase with decreasing p, since ρ decreases while μ, k and c_p are approximately unchanged.

COMMENTS: (1) For the prescribed conditions and L_c = 7 mm, the conduction heat flux across the air space is $q'' = k\left(T_1 - T_2\right)/L_c = 0.0242\,W/m\cdot K\times 42°C/0.007m = 145\,W/m^2$, (2) With triple pane construction, the conduction heat loss could be reduced by a factor of approximately two, (3) Heat loss is also associated with radiation exchange between the panes.

PROBLEM 9.90

KNOWN: Temperatures and dimensions of a window-storm window combination.

FIND: Rate of heat loss by free convection.

SCHEMATIC:

$T_2 = -10°C$
Air, 1atm
$L = 0.06m$
$W = 0.8m$
$H = 1.2m$
$T_1 = 20°C$
Vertical cavity

ASSUMPTIONS: (1) Both glass plates are of uniform temperature with insulated interconnecting walls and (2) Negligible radiation exchange.

PROPERTIES: *Table A-4*, Air (278K, 1 atm): $\nu = 13.93 \times 10^{-6}$ m^2/s, k = 0.0245 W/m·K, α = 19.6×10^{-6} m^2/s, Pr = 0.71, β = 0.00360 K^{-1}.

ANALYSIS: For the vertical cavity,

$$Ra_L = \frac{g\, b\, (T_1 - T_2) L^3}{an} = \frac{9.8\,m/s^2 \left(0.00360 K^{-1}\right)(30°C)(0.06m)^3}{19.6\times10^{-6} m^2/s \times 13.93\times10^{-6} m^2/s}$$

$$Ra_L = 8.37 \times 10^5.$$

With (H/L) = 20, Eq. 9.52 may be used as a first approximation for Pr = 0.71,

$$\overline{Nu}_L = 0.42 Ra_L^{1/4} Pr^{0.012} (H/L)^{-0.3} = 0.42\left(8.37\times10^5\right)^{1/4} (0.71)^{0.012} (20)^{-0.3}$$

$$\overline{Nu}_L = 5.2$$

$$\bar{h} = \overline{Nu}_L \frac{k}{L} = 5.2 \frac{0.0245 W/m\cdot K}{0.06m} = 2.1 W/m^2 \cdot K.$$

The heat loss by free convection is then

$$q = \bar{h} A (T_1 - T_2)$$

$$q = 2.1 W/m^2 \cdot K (1.2m \times 0.8m)(30°C) = 61W.$$ <

COMMENTS: In such an application, radiation losses should also be considered, and infiltration effects could render heat loss by free convection significant.

PROBLEM 9.91

KNOWN: Absorber plate and cover plate temperatures and geometry for a flat plate solar collector.

FIND: Heat flux due to free convection.

SCHEMATIC:

Cover plate, $T_2 = 35°C$
$L = 0.05m$
H
Air, 1atm
Absorber plate, $T_1 = 70°C$
$\tau = 60°$

ASSUMPTIONS: (1) Aspect ratio, H/L, is greater than 12.

PROPERTIES: *Table A-4*, Air (325K, 1 atm): $\nu = 18.4 \times 10^{-6}$ m^2/s, k = 0.028 W/m·K, $\alpha = 26.2 \times 10^{-6}$ m^2/s, Pr = 0.703, $\beta = 3.08 \times 10^{-3}$ K^{-1}.

ANALYSIS: For the inclined enclosure,

$$\mathrm{Ra}_L = \frac{g\, b\,(T_1 - T_2)L^3}{an} = \frac{9.8\,\mathrm{m/s^2}\left(3.08\times10^{-3}\mathrm{K}^{-1}\right)(70-35)°\mathrm{C}\left(0.05\mathrm{m}\right)^3}{\left(26.2\times10^{-6}\mathrm{m^2/s}\right)\left(18.4\times10^{-6}\mathrm{m^2/s}\right)}$$

$$\mathrm{Ra}_L = 2.74\times10^5.$$

With $t < t^* = 70°$, Table 9.4,

$$\overline{\mathrm{Nu}}_L = 1 + 1.44\left[1 - \frac{1708}{\mathrm{Ra}_L \cos t}\right]^{\bullet}\left[1 - \frac{1708\left(\sin 1.8t\right)^{1.6}}{\mathrm{Ra}_L \cos t}\right] + \left[\left(\frac{\mathrm{Ra}_L \cos t}{5830}\right)^{1/3} - 1\right]^{\bullet}$$

$$\overline{\mathrm{Nu}}_L = 1 + 1.44(0.99)(0.99) + 1.86 = 4.28$$

$$\bar{h} = \overline{\mathrm{Nu}}_L \frac{k}{L} = 4.28 \frac{0.028\,\mathrm{W/m\cdot K}}{0.05\mathrm{m}} = 2.4\,\mathrm{W/m^2\cdot K}.$$

Hence, the heat flux is

$$q'' = h\left(T_1 - T_2\right) = 2.4\,\mathrm{W/m^2\cdot K}\left(70-35\right)°\mathrm{C}$$

$$q'' = 84\,\mathrm{W/m^2}.$$ <

COMMENTS: Radiation exchange between the absorber and cover plates will also contribute to heat loss from the collector.

PROBLEM 9.92

KNOWN: Horizontal solar collector cover and absorber plates.

FIND: Heat loss from absorber to cover plate for spacings (a) 60mm and (b) 10mm.

SCHEMATIC:

$T_2 = 29°C$; $L = 60, 10mm$; Glass cover plate, $3m^2$

$T_1 = 50°C$; Absorber plate, $3m^2$

ASSUMPTIONS: (1) Both plates are of uniform temperature with insulated interconnecting walls, (2) Surface radiation effects are negligible.

PROPERTIES: *Table A-4*, Air ($T_f = (T_1 + T_2)/2 = 312K$, 1 atm): $\nu = 17.15 \times 10^{-6}$ m^2/s, k = 0.0272 W/m·K, $\alpha = 24.35 \times 10^{-6}$ m^2/s, Pr = 0.705, $\beta = 1/T_f = 3.21 \times 10^{-3}$ K^{-1}.

ANALYSIS: The heat rate between the plates is

$$q = q'' \cdot A_s = \bar{h} A_s (T_1 - T_2) \tag{1}$$

where h can be estimated by an appropriate correlation depending upon the magnitude of the Rayleigh number, Eq. 9.25,

$$Ra_L = g\,b\,(T_1 - T_2)L^3/na. \tag{2}$$

(a) For separation distance L = 60mm,

$$Ra_L = \frac{9.8\text{m/s}^2 \times 3.21 \times 10^{-3}(50-29)\text{K}(0.060\text{m})^3}{17.15 \times 10^{-6}\text{m}^2/\text{s} \times 24.35 \times 10^{-6}\text{m}^2/\text{s}} = 3.417 \times 10^5. \tag{3}$$

As a first approximation, Eq. 9.49 is appropriate (note $Ra_L > 3 \times 10^5$),

$$\bar{h} = \overline{Nu}_L \cdot \frac{k}{L} = \frac{k}{L}\left[0.069 Ra_L^{1/3} Pr^{0.074}\right] \tag{4}$$

$$\bar{h} = \frac{0.0272\text{W/m}\cdot\text{K}}{0.060\text{m}}\left[0.069\left(3.417 \times 10^5\right)^{1/3}(0.705)^{0.074}\right] = 2.13\text{W/m}^2\cdot\text{K}$$

$$q = 2.13\text{W/m}^2\cdot\text{K} \times 3\text{m}^2(50-29)\text{K} = 134\text{W}. \quad <$$

(b) For separation distance L = 10mm, from Eq. (3) it follows that $Ra_L = (10/60)^3 \times 3.417 \times 10^5 = 1582$. Since $Ra_L < 1700$, heat transfer occurs by conduction only, such that

$$\overline{Nu}_L = \frac{\bar{h}L}{k} = 1 \qquad \text{or} \qquad \bar{h} = 1 \times \frac{0.0272\text{W/m}\cdot\text{K}}{0.010\text{m}} = 2.72\text{W/m}^2\cdot\text{K}$$

$$q = 2.72\text{W/m}^2\cdot\text{K} \times 3\text{m}^2(50-29)\text{K} = 171\text{W}. \quad <$$

COMMENTS: Note that as L increases, from 10mm to 60mm, the heat rate decreases. However, For L ≥ 60mm, the heat rate will not change. This follows from Eq. 4 which, for $Ra > 3 \times 10^5$, $\bar{h}$ is independent of separation distance L.

PROBLEM 9.93

KNOWN: Rectangular cavity of two parallel, 0.5m square plates with insulated inter-connecting sides and with prescribed separation distance and surface temperatures.

FIND: Heat flux between surfaces for three orientations of the cavity: (a) Vertical τ = 90°C, (b) Horizontal with τ = 0°, and (c) Horizontal with τ = 180°.

SCHEMATIC:

H=0.5m
T_2 = 275K
T_1 = 325K
Air
L=50mm
τ = 0, 90, or 180°

ASSUMPTIONS: (1) Radiation exchange is negligible, (2) Air is at atmospheric pressure.

PROPERTIES: *Table A-4*, Air ($T_f = (T_1 + T_2)/2$ = 300K, 1 atm): $\nu = 15.89 \times 10^{-6}\ m^2/s$, k = 0.0263 W/m·K, $\alpha = 22.5 \times 10^{-6}\ m^2/s$, Pr = 0.707, $\beta = 1/T_f = 3.333 \times 10^{-3}\ K^{-1}$.

ANALYSIS: The convective heat flux between the two cavity plates is

$$q''_{conv} = \overline{h}\left(T_1 - T_2\right)$$

where $\overline{h}$ is estimated from the appropriate enclosure correlation which will depend upon the Rayleigh number. From Eq. 9.25, find

$$Ra_L = \frac{g\,b\,(T_1 - T_2)L^3}{na} = \frac{9.8\,m/s^2 \times 3.333\times10^{-3}K^{-1}(325-275)K(0.05m)^3}{15.89\times10^{-6}m^2/s \times 22.5\times10^{-6}m^2/s} = 5.710\times10^5.$$

Note that H/L = 0.5/0.05 = 10, a factor which is important in selecting correlations.

(a) With τ = 90°, for a vertical cavity, Eq. 9.50, is appropriate,

$$\overline{Nu}_L = 0.22\left(\frac{Pr}{0.22+Pr}Ra_L\right)^{0.28}\left(\frac{H}{L}\right)^{-1/4} = 0.22\left(\frac{0.707}{0.22+0.707}\times5.71\times10^5\right)^{0.28}(10)^{-1/4} = 4.692$$

$$\overline{h}_L = \frac{k}{L}\overline{Nu}_L = \frac{0.0263\,W/m\cdot K}{0.05m}\times4.692 = 2.47\,W/m^2\cdot K$$

$$q''_{conv} = 2.47\,W/m^2\cdot K(325-275)K = 123\,W/m^2. \qquad <$$

(b) With τ = 0* for a horizontal cavity heated from below, Eq. 9.49 is appropriate.

$$\overline{h} = \frac{k}{L}\overline{Nu}_L = 0.069\frac{k}{L}Ra_L^{1/3}Pr^{0.074} = 0.069\frac{0.0263\,W/m\cdot K}{0.05m}\left(5.710\times10^5\right)^{1/3}(0.707)^{0.074}$$

$$\overline{h} = 2.92\,W/m^2\cdot K$$

$$q''_{conv} = 2.92\,W/m^2\cdot K(325-275)K = 146\,W/m^2. \qquad <$$

(c) For τ = 180° corresponding to the horizontal orientation with the heated plate on the top, heat transfer will be by conduction. That is,

$$\overline{Nu}_L = 1 \qquad \text{or} \qquad \overline{h}_L = \overline{Nu}_L\cdot\frac{k}{L} = 1\times0.0263\,W/m\cdot K/(0.05m) = 0.526\,W/m^2\cdot K.$$

$$q''_{conv} = 0.526\,W/m^2\cdot K(325-275)K = 26.3\,W/m^2. \qquad <$$

COMMENTS: Compare the heat fluxes for the various orientations and explain physically their relative magnitudes.

PROBLEM 9.94

KNOWN: Horizontal flat roof and vertical wall sections of same dimensions exposed to identical temperature differences.

FIND: (a) Ratio of convection heat rate for horizontal section to that of the vertical section and (b) Effect of inserting a baffle at the mid-height of the vertical wall section on the convection heat rate.

SCHEMATIC:

ASSUMPTIONS: (1) Ends of sections and baffle adiabatic, (2) Steady-state conditions.

PROPERTIES: *Table A-4*, Air $\left(\overline{T} = (T_1 + T_2)/2 = 277\text{K}, 1\text{atm}\right)$: $\nu = 13.84 \times 10^{-6}$ m^2/s, k = 0.0245 W/m·K, $\alpha = 19.5 \times 10^{-6}$ m^2/s, Pr = 0.713.

ANALYSIS: (a) The ratio of the convection heat rates is

$$\frac{q_{hor}}{q_{vert}} = \frac{\overline{h}_{hor} A_s \Delta T}{\overline{h}_{vert} A_s \Delta T} = \frac{\overline{h}_{hor}}{\overline{h}_{vert}}. \qquad (1)$$

To estimate coefficients, recognizing both sections have the same characteristics length, L = 0.1m, with $Ra_L = g\beta\Delta T L^3/\nu\alpha$ find

$$Ra_L = \frac{9.8\,\text{m/s}^2 \times (1/277\text{K})(18-(-10))\text{K}\,(0.1\text{m})^3}{13.84\times10^{-6}\text{m}^2/\text{s} \times 19.5\times10^{-6}\text{m}^2/\text{s}} = 3.67\times10^6.$$

The appropriate correlations for the sections are Eqs. 9.49 and 9.52 (with H/L = 30),

$$\overline{Nu}_L\Big|_{hor} = 0.069 Ra_L^{1/3}\, Pr^{0.074} \qquad \overline{Nu}_L\Big|_{vert} = 0.42 Ra_L^{1/4}\, Pr^{0.012}\,(H/L)^{-0.3}. \qquad (3,4)$$

Using Eqs. (3) and (4), the ratio of Eq. (1) becomes,

$$\frac{q_{hor}}{q_{vert}} = \frac{0.069 Ra_L^{1/3}\, Pr^{0.074}}{0.42 Ra_L^{1/4}\, Pr^{0.012}\,(H/L)^{-0.3}} = \frac{0.069\left(3.67\times10^6\right)^{1/3}(0.713)^{0.074}}{0.42\left(3.67\times10^6\right)^{1/4}(0.713)^{0.012}(30)^{-0.3}} = 1.57.$$ <

(b) The effect of the baffle in the vertical wall section is to reduce H/L from 30 to 15. Using Eq. 9.52, it follows,

$$\frac{q_{baf}}{q} = \frac{\overline{h}_{baf}}{\overline{h}} = \frac{(H/L)_{baf}^{-0.3}}{(H/L)^{-0.3}} = \left(\frac{15}{30}\right)^{-0.3} = 1.23.$$ <

That is, the effect of the baffle is to increase the convection heat rate.

COMMENTS: (1) Note that the heat rate for the horizontal section is 57% larger than that for the vertical section for the same $(T_1 - T_2)$. This indicates the importance of heat losses from the ceiling or roofs in house construction. (2) Recognize that for Eq. 9.52, the Pr > 1 requirement is not completely satisfied. (3) What is the physical explanation for the result of part (b)?

PROBLEM 9.95

KNOWN: Double-glazed window of variable spacing L between panes filled with either air or carbon dioxide.

FIND: Heat transfer across window for variable spacing when filled with either gas. Consider these conditions (outside, T_1; inside, T_2): winter (-10, 20°C) and summer (35°C, 25°C).

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Radiation exchange is negligible, (3) Gases are at atmospheric pressure, (4) Perfect gas behavior.

PROPERTIES: *Table A-4*: Winter, $\overline{T} = (-10+20)°C/2 = 288K$, Summer, $\overline{T} = (35+25)°C/2 = 303K$:

Gas (1 atm)	$\overline{T}$ (K)	α ($m^2/s \times 10^6$)	ν ($m^2/s \times 10^6$)	$k \times 10^3$ (W/m·K)
Air	288	20.5	14.82	24.9
Air	303	22.9	16.19	26.5
CO_2	288	10.2	7.78	15.74
CO_2	303	11.2	8.55	16.78

ANALYSIS: The heat flux by convection across the window is

$$q'' = h(T_1 - T_2)$$

where the convection coefficient is estimated from the correlation of Eq. 9.53 for large aspect ratios 10< H/L < 40,

$$\overline{Nu}_L = \overline{h}L/k = 0.046Ra_L^{1/4}.$$

Substituting numerical values for winter (w) and summer (s) conditions,

$$Ra_{L,w,air} = \frac{9.8m/s^2(1/288K)(20-(-10))KL^3}{20.5\times10^{-6}m^2/s\times14.82\times10^{-6}m^2/s} = 3.360\times10^9L^3$$

$$Ra_{L,s,air} = 8.724\times10^8L^3 \qquad Ra_{L,w,CO_2} = 1.286\times10^{10}L^3 \qquad Ra_{L,s,CO_2} = 3.378\times10^9L^3$$

the heat transfer coefficients are

$$\overline{h}_{w,air} = (0.0249W/m\cdot K/L)\times0.046\left(3.360\times10^9L^3\right)^{1/4} = 0.276L^{-1/4}$$

$$h_{s,air} = 0.209L^{-1/4} \quad h_{w,CO_2} = 0.244L^{-1/4} \quad h_{s,CO_2} = 0.186L^{-1/4}.$$

For a separation distance such that H/L = 40, the maximum aspect ratio for the correlation, with H = 1.5 m, L = 37.5 mm find

$$q''_{w,air} = 18.8W/m^2 \quad q''_{s,air} = 4.7W/m^2 \quad q''_{w,CO_2} = 16.6W/m^2 \quad q''_{s,CO_2} = 4.2W/m^2.$$

Using CO_2 rather than air reduces the heat loss/gain by approximately 12%. Note the winter heat rate for this window is nearly 4 times that for summer.

PROBLEM 9.96

KNOWN: Dimensions of double pane window. Thickness of air gap. Temperatures of room and ambient air.

FIND: (a) Temperatures of glass panes and heat rate through window, (b) Resistance of glass pane relative to smallest convection resistance.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state, (2) Negligible glass pane thermal resistance, (3) Constant properties.

PROPERTIES: *Table A-3*, Plate glass: $k_p = 1.4$ W/m·K. *Table A-4*, Air (p = 1 atm). $T_{f,i} = 287.6$K: $\nu_i = 14.8 \times 10^{-6}$ m^2/s, $k_i = 0.0253$ W/m·K, $\alpha_i = 20.9 \times 10^{-6}$ m^2/s, $Pr_i = 0.710$, $\beta_i = 0.00348$ K^{-1}. $\overline{T} = (T_{s,i} + T_{s,o})/2 = 272.8$K: $\nu = 13.49 \times 10^{-6}$ m^2/s, k = 0.0241 W/m·K, $\alpha = 18.9 \times 10^{-6}$ m^2/s, Pr = 0.714, $\beta = 0.00367$ K^{-1}. $T_{f,o} = 258.2$K: $\nu_o = 12.2 \times 10^{-6}$ m^2/s, $k_o = 0.0230$W/m·K, $\alpha = 17.0 \times 10^{-6}$ m^2/s, Pr = 0.718, $\beta_o = 0.00387$ K^{-1}.

ANALYSIS: (a) The heat rate may be expressed as

$$q = q_o = \overline{h}_o H^2 \left(T_{s,o} - T_{\infty,o}\right) \tag{1}$$

$$q = q_g = \overline{h}_g H^2 \left(T_{s,i} - T_{s,o}\right) \tag{2}$$

$$q = q_i = \overline{h}_i H^2 \left(T_{\infty,i} - T_{s,i}\right) \tag{3}$$

where $\overline{h}_o$ and $\overline{h}_i$ may be obtained from Eq. (9.26),

$$\overline{Nu}_H = \left\{0.825 + \frac{0.387\,Ra_H^{1/6}}{\left[1 + \left(0.492/Pr\right)^{9/16}\right]^{8/27}}\right\}^2$$

with $Ra_H = g\beta_o\left(T_{s,o} - T_{\infty,o}\right)H^3/\alpha_o\nu_o$ and $Ra_H = g\beta_i\left(T_{\infty,i} - T_{s,i}\right)H^3/\alpha_i\nu_i$, respectively. Assuming $10^4 < Ra_L < 10^7$, $\overline{h}_g$ is obtained from

$$\overline{Nu}_L = 0.42\,Ra_L^{1/4}\,Pr^{0.012}\left(H/L\right)^{-0.3}$$

where $Ra_L = g\beta\left(T_{s,i} - T_{s,o}\right)L^3/\alpha\nu$. A simultaneous solution to Eqs. (1) – (3) for the three unknowns yields

Continued …..

$$T_{s,i} = 9.1°C, \quad T_{s,o} = -9.6°C, q = 35.7\ W$$ <

where $\overline{h}_i = 3.29\ W/m^2 \cdot K$, $\overline{h}_o = 3.45\ W/m^2 \cdot K$ and $\overline{h}_g = 1.90\ W/m^2 \cdot K$.

(b) The unit conduction resistance of a glass pane is $R''_{cond} = L_p / k_p = 0.00429\ m^2 \cdot K/W$, and the smallest convection resistance is $R''_{conv,o} = (1/\overline{h}_o) = 0.290\ m^2 \cdot K/W$. Hence,

$$R''_{cond} << R''_{conv,min}$$ <

and it is reasonable to neglect the thermal resistance of the glass.

COMMENTS: (1) Assuming a heat flux of 35.7 W/m^2 through a glass pane, the corresponding temperature difference across the pane is $\Delta T = q''(L_p / k_p) = 0.15°C$. Hence, the assumption of an isothermal pane is good. (2) Equations (1) – (3) were solved using the IHT workspace and the temperature-dependent air properties provided by the software. The property values provided in the PROPERTIES section of this solution were obtained from the software.

PROBLEM 9.97

KNOWN: Top surface of an oven maintained at 60°C.

FIND: (a) Reduction in heat transfer from the surface by installation of a cover plate with specified air gap; temperature of the cover plate, (b) Effect of cover plate spacing.

SCHEMATIC:

Air, $T_\infty = 23\ ^oC$
0.5 m x 0.5 m
q_s
$Ts = 60\ ^oC$
Exposed surface

Air, $T_\infty = 23\ ^oC$
q_{cp}
$5 \le L \le 50$ mm
$H = 0.5$ m
T_2
Air gap q_g
$T_1 = T_s = 60\ ^oC$
With cover plate

ASSUMPTIONS: (1) Steady-state conditions, (2) Oven surface at $T_1 = T_s$ for both cases, (3) Negligible radiative exchange with surroundings and across air gap.

PROPERTIES: *Table A.4,* Air ($T_f = (T_s + T_\infty)/2 = 315$ K, 1 atm): $\nu = 17.40 \times 10^{-6}$ m²/s, k = 0.0274 W/m·K, $\alpha = 24.7 \times 10^{-6}$ m²/s; *Table A.4*, Air ($\overline{T} = (T_1 + T_2)/2$ and $T_{f2} = (T_2 + T_\infty)/2$): Properties obtained form *Correlations Toolpad* of IHT.

ANALYSIS: (a) The convective heat loss from the exposed top surface of the oven is $q_s = \overline{h}A_s(T_s - T_\infty)$. With $L = A_s/P = (0.5\ m)^2/(4 \times 0.5\ m) = 0.125$ m,

$$Ra_L = \frac{g\beta\Delta TL^3}{\nu\alpha} = \frac{9.8\,m/s^2\,(1/315\,K)(60-23)^\circ C\,(0.125\,m)^3}{17.40\times10^{-6}\,m/s^2 \times 24.7\times10^{-6}\,m/s^2} = 5.231\times10^6.$$

The appropriate correlation for a heated plate facing upwards, Eq. 9.30, is

$$\overline{Nu}_L = \frac{\overline{h}L}{k} = 0.54Ra_L^{1/4} \qquad 10^4 \le Ra_L \le 10^7$$

$$\overline{h} = \left(\frac{0.0274\,W/m\cdot K}{0.125\,m}\right)\times 0.54\left(5.231\times10^6\right)^{1/4} = 5.66\,W/m^2\cdot K$$

Hence, the heat rate for the exposed surface is

$$q_s = 5.66\,W/m^2\cdot K\,(0.5\,m)^2\,(60-23)^\circ C = 52.4\,W.$$ <

With the cover plate, the surface temperature ($T_s = T_2$) is unknown and must be obtained by performing an energy balance at the top surface.

$T_\infty = 23^oC$
R_{fc}
T_2
R_{enc}
$T_1 = T_s = 60^oC$
q_{cp}

Continued…..

PROBLEM 9.97 (Cont.)

Equating heat flow across the gap to that from the top surface, $q_g = q_{cp}$. Hence, for a unit surface area,

$$\bar{h}_g \left(T_1 - T_2\right) = \bar{h}_{cp}\left(T_2 - T_\infty\right)$$

where $\bar{h}_{cp}$ is obtained from Eq. 9.30 and $\bar{h}_g$ is evaluated from Eq. 9.49.

$$\overline{Nu}_L = \frac{\bar{h}_g L}{k} = 0.069 Ra_L^{1/3} Pr^{0.074}$$

Entering this expression from the keyboard and Eq. 9.30 from the *Correlations* Toolpad, with the *Properties* Toolpad used to evaluate air properties at $\bar{T}$ and T_{fs}, IHT was used with L = 0.05 m to obtain

$T_2 = 35.4°C$ $\qquad\qquad$ $q_{cp} = 13.5$ W $\qquad$ <

where $\bar{h}_g = 2.2\,W/m^2\cdot K$ and $\bar{h}_{cp} = 4.4\,W/m^2\cdot K$. Hence, the effect of installing the cover plate creating the enclosure is to reduce the heat loss by

$$\frac{q_s - q_{cp}}{q_s}\times 100 = \frac{52.4 - 13.5}{52.4}\times 100 = 74\%.$$ <

Note, however, that for L = 0.05 m, $Ra_L = 2.05 \times 10^5$ is slightly less than the lower limit of applicability for Eq. 9.49.

(b) If we use the foregoing model to evaluate T_2 and q_{cp} for $0.005 \le L \le 0.05$ m, we find that there is no effect. This seemingly unusual result is a consequence of the fact that, in Eq. 9.49, $\overline{Nu}_L \propto Ra_L^{1/3}$, in which case $\bar{h}_g$ is independent of L. However, Ra_L and Nu_L do decrease with decreasing L, eventually approaching conditions for which transport across the airspace is determined by conduction and not convection. If transport is by conduction, the heat rate must be determined from Fourier's law, for which $q_g'' = (k/L)(T_1 - T_2)$ and the equivalent, *pseudo*, Nusselt number is $\overline{Nu}_L = \bar{h}L/k = 1$. If this expression is used to determine $\bar{h}_g$ in the energy balance, q_{cp} increases with decreasing L. The results would only apply if there is negligible advection in the airspace and hence for Rayleigh numbers less than 1708, which corresponds to $L \approx 10.5$ mm. For this value of L, $q_{cp} = 15.4$ W exceeds that previously determined for L = 50 mm. Hence, there is little variation in q_{cp} over the range $10.5 < L < 50$ mm. However, q_{cp} increases with decreasing L below 10.5 mm, achieving a value of 24.2 W for L = 5 mm. Hence, a value of L slightly larger than 10.5 mm could be considered an optimum.

COMMENTS: Radiation exchange across the cavity and with the surroundings is likely to be significant and should be considered in a more detailed analysis.

PROBLEM 9.98

KNOWN: The sample compartment of an optical instrument in the form of a rectangular cavity; one face in contact with instrument cover maintained at 27°C; other face having 10-mm thick insulation pad in contact with oven wall maintained at 227°C.

FIND: Heat gain to the instrument, and average air temperature in the compartment.

SCHEMATIC:

Sample compartment
15 x 15 x 10 cm
Insulating pad
k = 0.05 W/m-K
t = 10 mm
15 cm
Instrument cover
$T_c = 27^oC$
Oven wall
$T_o = 227^oC$
10 cm

T_c T_i T_o
q
R_{cav} R_{ins}
Thermal circuit

ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible heat losses from the edges of the rectangular-cavity shaped compartment, and (3) Constant properties.

PROPERTIES: *Table A-4*, air ($T_f = \overline{T}_{air} = (T_i + T_c)/2 = 354$ K, 1 atm): $\nu = 2.132 \times 10^{-5}$ m^2/s, k = 0.0303 W/m·K, $\alpha = 3.051 \times 10^{-5}$ m^2/s, Pr = 0.699, $\beta = 1/T_f$.

ANALYSIS: The system comprised of the rectangular cavity and the insulating pad can be represented by the thermal circuit shown above. The heat gain to the instrument and the hot-side temperature of the cavity, T_i, can be expressed as

$$q = \frac{T_o - T_c}{R_{ins} + R_{cav}} \qquad \frac{T_o - T_i}{R_{ins}} = \frac{T_i - T_c}{R_{cav}} \qquad (1,2)$$

The thermal resistance of the insulating pad is

$$R_{ins} = \frac{L_i}{k_i A_s} = \frac{0.010 \text{ m}}{0.05 \text{ W/m}\cdot\text{K}(0.15 \times 0.15)\text{m}^2} = 8.89 \text{ K/W} \qquad (3)$$

The thermal resistance of the rectangular cavity is

$$R_{cav} = 1/\left(\overline{h} A_s\right) \qquad (4)$$

where L_c is the cavity width and the average convection coefficient follows from Eq. 9.51 (since H/L_c = 15/10 = 1.5),

$$\overline{Nu}_L = \frac{\overline{h} L_c}{k} = 0.18\left(\frac{Pr}{0.2 + Pr} Ra_{Lc}\right)^{0.29} \qquad (5)$$

$$Ra_{Lc} = g\beta\left(T_i - T_c\right)L_c^3/\alpha\nu \qquad (6)$$

where the properties are evaluated at the average cavity air temperature

$$T_f = \overline{T}_{air} = \left(T_i + T_c\right)/2 \qquad (7)$$

Recognize that the system of equations needs to be solved iteratively by initially guessing values of T_i or solved simultaneously using equation-solving software with a properties library. The results are:

$$q = 10.4 \text{ W} \qquad \overline{T}_{air} = 134°\text{C}$$ <

COMMENTS: Other parameters resulting from the analyses are:

$\overline{Nu}_{Lc} = 14.3$ $\quad \overline{Ra}_L = 4.57 \times 10^6$ $\quad \overline{h} = 4.33 \text{ W/m}^2\cdot\text{K}$ $\quad R_{cav} = 10.28 \text{ K/W}$

PROBLEM 9.99

KNOWN: Rectangular cavity of two parallel, 0.5m square plates with insulated boundaries and with prescribed separation distance and surface temperatures.

FIND: Convective heat flux between surfaces for tilt angles of (a) 45° and (b) 75°.

SCHEMATIC:

ASSUMPTIONS: (1) Radiation exchange is negligible, (2) Cavity air is 1 atm.

PROPERTIES: *Table A-4*, Air ($T_f = (T_1 + T_2)/2 = 300K$, 1 atm): k = 0.0263 W/m·K, ν = 15.89 × 10^{-6} m^2/s, α = 22.5 × 10^{-6} m^2/s, Pr = 0.707, β = $1/T_f$ = 3.333 × 10^{-3} K^{-1}.

ANALYSIS: (a) The convective heat flux between the plates is $q'' = \overline{h}(T_1 - T_2)$ where $\overline{h}$ is estimated from the appropriate correlation with

$$Ra_L = \frac{g\,b\,(T_1 - T_2)L^3}{na} = \frac{9.8\,m/s^2 \times 3.333\times10^{-3}K^{-1}(325-275)K(0.05m)^3}{22.5\times10^{-6}m^2/s\times15.89\times10^{-6}m^2/s} = 5.710\times10^5.$$

For H/L = 0.5m/0.05m = 10 and τ < τ* (τ* ≈ 64° from Table 9.4), Eq. 9.55 is suitable,

$$\overline{Nu}_L = \overline{Nu}_L(t=0)\left[\frac{\overline{Nu}_L(t=90)}{\overline{Nu}_L(t=0)}\right]^{t/t^*}(\sin t^*)^{t/4t^*}. \qquad (1)$$

For $\overline{Nu}_L(t=90°)$, Eq. 9.50 is appropriate,

$$\overline{Nu}_L(t=90°) = 0.22\left(\frac{Pr}{0.2+Pr}Ra_L\right)^{0.28}\left(\frac{H}{L}\right)^{-1/4} = 0.22\left(\frac{0.707}{0.2+0.707}5.71\times10^5\right)^{0.28}(10)^{-1/4} = 4.72.$$

For $\overline{Nu}_L(t=0°)$, Eq. 9.49 is appropriate,

$$\overline{Nu}_L(t=0°) = 0.069Ra_L^{1/3}Pr^{0.074} = 0.069\times\left(5.71\times10^5\right)^{1/3}(0.707)^{0.074} = 5.58.$$

Substituting numerical values into Eq. (1) with τ = 45°,

$$\overline{Nu}_L = 5.58[4.72/5.58]^{45/64}(\sin64)^{45/4\times64} = 4.86$$

$$\overline{h} = \overline{Nu}_L\;k/L = 4.86\times0.0263\,W/m\cdot K/0.05m = 2.56\,W/m^2\cdot K.$$

$$q'' = 2.56\,W/m^2\cdot K(325-275)K = 128\,W/m^2. \qquad <$$

(b) For τ = 75°, τ > τ*, the critical tilt angle, Eq. 9.56 is appropriate for estimating $\overline{h}$.

$$\overline{Nu}_L = \overline{Nu}_L(t=90)\cdot(\sin t)^{1/4} = 4.72(\sin75°)^{1/4} = 4.68$$

$$\overline{h} = \overline{Nu}_L\;k/L = 4.68\times0.0263\,W/m\cdot K/0.05m = 2.46\,W/m^2\cdot K.$$

$$q'' = 2.46\,W/m^2\cdot K(325-275)K = 123\,W/m^2. \qquad <$$

COMMENTS: Note that $\overline{Nu}_L(t=0) > \overline{Nu}_L(t=90°)$. For the cavity conditions there is little change in $\overline{h}$ for tilt angles, τ, from 45° to 90°.

PROBLEM 9.100

KNOWN: Dimensions and surface temperatures of a flat-plate solar collector.

FIND: (a) Heat loss across collector cavity, (b) Effect of plate spacing on the heat loss.

SCHEMATIC:

5 < L < 50 mm
Solar collector
A_s = 4 m^2
T_1 = 70 ^{o}C
T_2 = 30 ^{o}C
Air gap
τ = 60^o

ASSUMPTIONS: Negligible radiation.

PROPERTIES: *Table A.4*, Air ($\overline{T} = (T_1 + T_2)/2 = 323$ K): $\nu = 18.2 \times 10^{-6}$ m^2/s, k = 0.028 W/m·K, α = 25.9×10^{-6} m^2/s, $\beta = 0.0031$ K^{-1}.

ANALYSIS: (a) Since H/L = 2 m/0.03 m = 66.7 > 12, $\tau < \tau^*$ and Eq. 9.54 may be used to evaluate the convection coefficient associated with the air space. Hence, $q = \overline{h} A_s(T_1 - T_2)$, where $\overline{h} = (k/L)\overline{Nu}_L$ and

$$\overline{Nu}_L = 1 + 1.44\left[1 - \frac{1708}{Ra_L \cos\tau}\right]^{\bullet}\left[1 - \frac{1708(\sin 1.8\tau)^{1.6}}{Ra_L \cos\tau}\right] + \left[\left(\frac{Ra_L \cos\tau}{5830}\right)^{1/3} - 1\right]^{\bullet}$$

For L = 30 mm, the Rayleigh number is

$$Ra_L = \frac{g\beta(T_1 - T_2)L^3}{\alpha\nu} = \frac{9.8\,\text{m/s}^2\left(0.0031\,\text{K}^{-1}\right)\left(40^\circ\text{C}\right)(0.03\,\text{m})^3}{25.9\times10^{-6}\,\text{m}^2/\text{s}\times18.2\times10^{-6}\,\text{m}^2/\text{s}} = 6.96\times10^4$$

and $Ra_L \cos\tau = 3.48 \times 10^4$. It follows that $\overline{Nu}_L = 3.12$ and $\overline{h}$ = (0.028 W/m·K/0.03 m)3.12 = 2.91 W/m^2·K. Hence,

$$q = 2.91\,\text{W/m}^2\cdot\text{K}\left(4\,\text{m}^2\right)\left(40^\circ\text{C}\right) = 466\,\text{W}$$ <

(b) The foregoing model was entered into the workspace of IHT, and results of the calculations are plotted as follows.

Heat loss, q(W) vs. Plate spacing, L(m) (0.017 – 0.05)

Heat loss, q(W) vs. Plate spacing, L(m) (0.011 – 0.016)

Continued...

PROBLEM 9.100 (Cont.)

The plots are influenced by the fact that the third and second terms on the right-hand side of the correlation are set to zero at $L \approx 0.017$ m and $L \approx 0.011$ m, respectively. For the range of conditions, minima in the heat loss of $q \approx 410$ W and $q = 397$ W are achieved at $L \approx 0.012$ m and $L = 0.05$ m, respectively. Operation at $L \approx 0.02$ m corresponds to a maximum and is clearly undesirable, as is operation at $L < 0.011$ m, for which conditions are conduction dominated.

COMMENTS: Because the convection coefficient is low, radiation effects would be significant.

PROBLEM 9.101

KNOWN: Cylindrical 120-mm diameter radiation shield of Example 9.5 installed concentric with a 100-mm diameter tube carrying steam; spacing provides for an air gap of L = 10 mm.

FIND: (a) Heat loss per unit length of the tube by convection when a second shield of diameter 140 m is installed; compare the result to that for the single shield calculation of the example; and (b) The heat loss per unit length if the gap dimension is made L = 15 mm (rather than 10 mm). Do you expect the heat loss to increase or decrease?

SCHEMATIC:

1st shield
T_1, D_1 = 120 or 130 mm
2nd shield
T_2 = 35°C
D_2 = 140 or 160 mm
Air gaps
L = 10 or 15 mm
Tube
D_i = 100 mm
T_i = 120°C
r
q'
T_i, T_1, T_2
R_{g1}, R_{g2}
Thermal circuit

ASSUMPTIONS: (1) Steady-state conditions, and (b) Constant properties.

PROPERTIES: *Table A-4*, Air ($T_f = (T_s + T_\infty)/2$ = 350 K, 1 atm): $\nu = 20.92 \times 10^{-6}$ m^2/s, k = 0.030 W/m·K, Pr = 0.700.

ANALYSIS: (a) The thermal circuit representing the tube with two concentric cylindrical radiation shields having gap spacings L = 10 mm is shown above. The heat loss per unit length by convection is

$$q' = \frac{T_i - T_2}{R'_{g1} + R'_{g2}} \tag{1}$$

where the R'_g represents the thermal resistance of the annular gap (spacing). From Eq. 9.58, 59 and 60, find

$$R'_g = \frac{\ell n\left(D_o / D_i\right)}{2\pi k_{eff}} \tag{2}$$

$$\frac{k_{eff}}{k} = 0.386\left(\frac{Pr}{0.861 + Pr}\right)^{1/4}\left(Ra_c^*\right)^{1/4} \tag{3}$$

$$Ra_c^* = \frac{\left[\ell n\left(D_o / D_i\right)\right]^4}{L^3\left(D_i^{-3/5} + D_o^{-3/5}\right)^5} Ra_L \tag{4}$$

$$Ra_L = g\beta\left(T_o - T_i\right)L^3 / \alpha\nu \tag{5}$$

where the properties are evaluated at the average temperature of the bounding surfaces, $T_f = (T_i + T_o)/2$. Recognize that the above system of equations needs to be solved iteratively by initial guess values of T_1, or solved simultaneously using equation-solving software with a properties library. The results are tabulated below.

Continued …..

PROBLEM 9.101 (Cont.)

(b) Using the foregoing relations, the analyses can be repeated with L = 15 mm, so that D_i = 130 mm and D_2 = 160 mm. The results are tabulated below along with those from Example 9.5 for the single-shield configuration.

Shields	L(mm)	R'_{g1} (m·K/W)	R'_{g2} (m·K/W)	R'_{tot} (m·K/W)	T_1(°C)	q' (W/m)
1	10	0.7658	---	0.76	---	100
2	10	1.008	0.8855	1.89	74.8	44.9
2	15	0.9773	0.8396	1.82	74.3	46.8

COMMENTS: (1) The effect of adding the second shield is to more than double the thermal resistance of the shields to convection heat transfer.

(2) The effect of gap increase from 10 to 15 mm for the two-shield configuration is slight. Increasing L allows for greater circulation in the annular space, thereby reducing the thermal resistance.

(3) Note the difference in thermal resistances for the annular spaces R'_{g1} of the one-and two-shield configurations with L = 10 mm. Why are they so different (0.7658 vs. 1.008 m·K/W, respectively)?

(4) See Example 9.5 for details on how to evaluate the properties for use with the correlation.

PROBLEM 9.102

KNOWN: Operating conditions of a concentric tube solar collector.

FIND: Convection heat transfer per unit length across air space between tubes.

SCHEMATIC:

$D_o = 0.15m$
$T_o = 30°C$
$D_i = 0.10m$
$T_i = 70°C$
Annular region between tubes is air filled (p=1atm)
$L = (D_o - D_i)/2 = 0.025m$

ASSUMPTIONS: (1) Steady-state conditions, (2) Long tubes.

PROPERTIES: *Table A-4*, Air (T = 50°C, 1 atm): $\nu = 18.2 \times 10^{-6}$ m^2/s, k = 0.028 W/m·K, $\alpha = 25.9 \times 10^{-6}$ m^2/s, Pr = 0.71, $\beta = 0.0031$ K^{-1}.

ANALYSIS: For the annular region

$$Ra_L = \frac{g\, b\, (T_s - T_\infty) L^3}{na} = \frac{\left(9.8 m/s^2\right)\left(0.0031 K^{-1}\right)(70-30)\,°C\,(0.025m)^3}{\left(18.2\times10^{-6} m^2/s\right)\left(25.9\times10^{-6} m^2/s\right)}$$

$$Ra_L = 4.03\times10^4.$$

Hence, from Eq. 9.60,

$$Ra_c^* = \frac{\left[\ln(0.15/0.10)\right]^4}{(0.025m)^3\left[(0.10)^{-3/5}+(0.15)^{-3/5}\right]^5}\times4.03\times10^4 = 3857.$$

Accordingly, Eq. 9.59 may be used, in which case

$$k_{eff} = 0.386k\left(\frac{Pr}{0.861+Pr}\right)^{1/4}\left(Ra_c^*\right)^{1/4}$$

$$k_{eff} = 0.386(0.028 W/m\cdot K)\left(\frac{0.71}{0.861+0.71}\right)^{1/4}(3857)^{1/4} = 0.07 W/m\cdot K.$$

From Eq. 9.58, it then follows that

$$q' = \frac{2p\, k_{eff}}{\ln(D_o/D_i)}(T_i - T_o) = \frac{2p\,(0.07 W/m\cdot K)}{\ln(0.15/0.10)}(70-30)\,°C = 43.4 W/m. \qquad <$$

COMMENTS: An additional heat loss is related to thermal radiation exchange between the inner and outer surfaces.

PROBLEM 9.103

KNOWN: Annulus formed by two concentric, horizontal tubes with prescribed diameters and surface temperatures is filled with water.

FIND: Convective heat transfer rate per unit length of the tubes.

SCHEMATIC:

$T_i = 300K, D_i = 50mm$

$L = \dfrac{D_o - D_i}{2} = 12.5mm$ $\quad T_o = 350K, D_o = 75mm$

Water

ASSUMPTIONS: Steady-state conditions.

PROPERTIES: *Table A-6*, Water (T_f = 325K): $\rho = (1/1.013 \times 10^{-3}\ m^3/kg)$, c_p = 4182 J/kg·K, $\mu = 528 \times 10^{-6}\ N \cdot s/m^2$, k = 0.645 W/m·K, Pr = 3.42, $\beta = 471.2 \times 10^{-6}\ K^{-1}$.

ANALYSIS: From Eqs. 9.58 and 9.59,

$$q' = \frac{2p\, k_{eff}}{\ell n\left(D_o / D_i\right)}\left(T_i - T_o\right) \qquad \frac{k_{eff}}{k} = 0.386\left(\frac{Pr}{0.861 + Pr}\right)^{1/4}\left(Ra_c^*\right)^{1/4}. \qquad (1,2)$$

From Eq. 9.60,

$$Ra_c^* = \left[\ell n\left(D_o / D_i\right)\right]^4 \cdot Ra_L / L^3\left(D_i^{-3/5} + D_o^{-3/5}\right)^5. \qquad (3)$$

The Rayleigh number follows from Eq. 9.25 using $\nu = \mu/\rho$ and $\alpha = k/\rho\ c_p$,

$$Ra_L = \frac{g\, b\left(T_o - T_i\right) L^3}{na} = \frac{9.8 m/s^2 \times 471.2 \times 10^{-6} K^{-1}\left(350 - 300\right) K \times \left(12.5 \times 10^{-3} m\right)^3}{\left(528 \times 10^{-6} N \cdot s/m^2 \times 1.013 \times 10^{-3} \dfrac{m^3}{kg}\right) \times \left(\dfrac{0.645 W/m \cdot K \times 1.013 \times 10^{-3} m^3/kg}{4182 J/kg \cdot K}\right)}$$

$$Ra_L = 5.396 \times 10^5.$$

Using this value in Eq. (3), find

$$Ra_c^* = \left[\ell n\left(\frac{75}{50}\right)\right]^4 \times 5.396 \times 10^5 / \left(12.5 \times 10^{-3} m\right)^3\left(\left[50 \times 10^{-3} m\right]^{-3/5} + \left[75 \times 10^{-3} m\right]^{-3/5}\right)^5 = 5.164 \times 10^4$$

and then evaluating Eq. (2), find

$$\frac{k_{eff}}{k} = 0.386\left(\frac{3.42}{0.861 + 3.42}\right)^{1/4}\left(5.164 \times 10^4\right)^{1/4} = 5.50$$

$$k_{eff} = 5.50 \times 0.645 W/m \cdot K = 3.55 W/m \cdot K.$$

The heat rate from Eq. (1) is then,

$$q' = \frac{2p \times 3.55 W/m \cdot K}{\ell n\left(75/50\right)}\left(350 - 300\right) K = 2.75 kW/m. \qquad <$$

COMMENTS: Note that the Ra_c^* value is within prescribed limits for Eq. 9.50 or Eq. (3). Note also the characteristic length in Ra_L is $L = (D_o - D_i)/2$, the annulus gap.

PROBLEM 9.104

KNOWN: Annulus formed by two concentric, horizontal tubes with prescribed diameters and surface temperatures is filled with nitrogen at 5 atm.

FIND: Convective heat transfer rate per unit length of the tubes.

SCHEMATIC:

$T_i = 300K,\ D_i = 200\,mm$

$L = 25\,mm$

$T_o = 400K,\ D_o = 250\,mm$

Nitrogen, 5atm

ASSUMPTIONS: (1) Thermophysical properties k, μ, and Pr, are independent of pressure, (2) Density is proportional to pressure, (3) Perfect gas behavior.

PROPERTIES: *Table A-4*, Nitrogen $\left(\overline{T} = \left(T_i + T_o\right)/2 = 350K,\ 5\ atm\right)$: k = 0.0293 W/m·K, μ = 200×10^{-7} N·s/m^2, ρ(5 atm) = 5 ρ (1 atm) = 5 × 0.9625 kg/m^3 = 4.813 kg/m^3, Pr = 0.711, ν = μ/ρ = 4.155×10^{-6} m^2/s, α = k/ρc = 0.0293 W/m·K/(4.813 kg/m^3 × 1042 J/kg·K) = 5.842×10^{-6} m^2/s.

ANALYSIS: From Eqs. 9.58 and 9.59

$$q' = \frac{2p\ k_{eff}}{\ell n\left(D_o/D_i\right)}\left(T_o - T_i\right) \qquad \frac{k_{eff}}{k} = 0.386\left(\frac{Pr}{0.861 + Pr}\right)^{1/4}\left(Ra_c^*\right)^{1/4}. \qquad (1,2)$$

From Eq. 9.60,

$$Ra_c^* = \left[\ell n\left(D_o/D_i\right)\right]^4 Ra_L / L^3\left(D_i^{-3/5} + D_o^{-3/5}\right)^5.$$

(3)

The Rayleigh number, Ra_L, follows from Eq. 9.25, and Ra_c^* from Eq. (3),

$$Ra_L = \frac{gb\left(T_o - T_i\right)L^3}{an} = \frac{9.8\,m/s^2\left(1/350K\right)\left(400 - 300\right)K\left(0.025m\right)^3}{5.842\times10^{-6}m^2/s \times 4.155\times10^{-6}m^2/s} = 1.802\times10^6.$$

$$Ra_c^* = \left[\ell n\frac{250}{200}\right]^4 \times 1.802\times10^6 / \left(0.025m\right)^3\left(0.20^{-3/5} + 0.25^{-3/5}\right)^5 m^3 = 98{,}791$$

and then evaluating Eq. (2),

$$\frac{k_{eff}}{k} = 0.386\left(\frac{0.711}{0.861 + 0.711}\right)^{1/4}(98{,}791)^{1/4} = 5.61.$$

Hence, the heat rate, Eq. (1), becomes

$$q' = \frac{2p \times 5.61 \times 0.0293\,W/m\cdot K}{\ell n\left(250/200\right)}\left(400 - 300\right)K = 463\,W/m. \qquad <$$

COMMENTS: Note that the heat loss by convection is nearly six times that for conduction. Radiation transfer is likely to be important for this situation. The effect of nitrogen pressure is to decrease ν which in turn increases Ra_L; that is, free convection heat transfer will increase with increase in pressure.

PROBLEM 9.105

KNOWN: Diameters and temperatures of concentric spheres.

FIND: Rate at which stored nitrogen is vented.

SCHEMATIC:

ASSUMPTIONS: (1) Negligible radiation.

PROPERTIES: Liquid nitrogen (given): $h_{fg} = 2 \times 10^5$ J/kg; *Table A-4*, Helium ($\overline{T} = (T_i + T_o)/2 =$ 180K, 1 atm): $\nu = 51.3 \times 10^{-6}$ m^2/s, k = 0.107 W/m·K, $\alpha = 76.2$ m^2/s, Pr = 0.673, $\beta = 0.00556$ K^{-1}.

ANALYSIS: Performing an energy balance for a control surface about the liquid nitrogen, it follows that

$$q = q_{conv} = \dot{m}h_{fg}.$$

From the Raithby and Hollands expressions for free convection between concentric spheres,

$$q_{conv} = k_{eff}p\left(D_iD_o / L\right)\left(T_o - T_i\right)$$

$$k_{eff} = 0.74k\left[Pr/\left(0.861+Pr\right)\right]^{1/4}\left(Ra_s^*\right)^{1/4}$$

where

$$Ra_s^* = \left[\frac{L}{\left(D_oD_i\right)^4}\frac{Ra_L}{\left(D_i^{-7/5}+D_o^{-7/5}\right)^5}\right]$$

$$Ra_L = \frac{gb\left(T_o - T_i\right)L^3}{na} = \frac{9.8\,m/s^2\left(0.00556K^{-1}\right)\left(206K\right)\left(0.05m\right)^3}{\left(51.3\times10^{-6}m^2/s\right)\left(76.2\times10^{-6}m^2/s\right)} = 3.59\times10^5$$

$$Ra_s^* = \frac{0.05m}{\left(1.10m^2\right)^4}\frac{3.59\times10^5}{\left[1+\left(1.1\right)^{-7/5}\right]^5 m^{-7}} = 529$$

$$k_{eff} = 0.74\left(0.107W/m\cdot K\right)\left[0.673/\left(0.861+0.673\right)\right]^{1/4}\left(529\right)^{1/4} = 0.309W/m\cdot K.$$

Hence,

$$q_{conv} = \left(0.309W/m\cdot K\right)p\left(1.10m^2/0.05m\right)206\text{ K} = 4399\text{ W}.$$

The rate at which nitrogen is lost from the system is therefore

$$\dot{m} = q_{conv}/h_{fg} = 4399W/2\times10^5 J/kg = 0.022\text{ kg/s}. \quad <$$

COMMENTS: The heat gain and mass loss are large. Helium should be replaced by a noncondensing gas of smaller k, or the cavity should be evacuated.

PROBLEM 9.106

KNOWN: Concentric spheres with prescribed surface temperatures.

FIND: Convection heat transfer rate.

SCHEMATIC:

$T_i = 325K, D_i = 75mm$

$L = 12.5mm$ $T_o = 275K, D_o = 100mm$

Air, 3atm

ASSUMPTIONS: (1) Quiescent air in void space, (2) Density ~ pressure; other properties independent, (3) Perfect gas behavior.

PROPERTIES: *Table A-4*, Air (T_f = 300K, 3 atm): $\beta = 3.33 \times 10^{-3}\ K^{-1}$, $\nu = 1/3 \times 15.89 \times 10^{-6}\ m^2/s$, k = 0.263 W/m·K, $\alpha = 1/3 \times 22.5 \times 10^{-6}\ m^2/s$, Pr = 0.707.

ANALYSIS: The heat transfer rate due to free convection is

$$q = k_{eff}\, \pi \left(D_i D_o / L\right)\left(T_i - T_o\right) \tag{9.61}$$

where

$$\frac{k_{eff}}{k} = 0.74\left(\frac{Pr}{0.861 + Pr}\right)^{1/4}\left(Ra_s^*\right)^{1/4} \tag{9.62}$$

$$Ra_s^* = \left[\frac{L}{\left(D_o D_i\right)^4}\,\frac{Ra_L}{\left(D_i^{-7/5} + D_o^{-7/5}\right)^5}\right] \tag{9.63}$$

$$Ra_L = \frac{g\,\beta\left(T_i - T_o\right)L^3}{\nu\alpha}. \tag{9.25}$$

Substituting numerical values in the above expressions, find that

$$Ra_L = \frac{9.8\,m/s^2 \times 3.333 \times 10^{-3} K^{-1}(325 - 375)K\left(12.5 \times 10^{-3}\right)^3 m^3}{(1/3)15.89 \times 10^{-6} m^2/s\,(1/3)\,22.5 \times 10^{-6} m^2/s} = 80{,}928$$

$$Ra_s^* = \left[\frac{\left(12.5 \times 10^{-3}\right)m}{\left(100 \times 10^{-3} \times 75 \times 10^{-3}\right)^4 m^4} \cdot \frac{80{,}928}{\left(\left(75 \times 10^{-3} m\right)^{-7/5} + \left(100 \times 10^{-3} m\right)^{-7/5}\right)^5}\right] = 330.1$$

$$\frac{k_{eff}}{k} = 0.74\left(\frac{0.707}{0.861 + 0.707}\right)^{1/4}(330.1)^{1/4} = 2.58.$$

Hence, the heat rate becomes

$$q = \left(2.58 \times 0.263 \frac{W}{m \cdot K}\right)\pi\left(\frac{75 \times 10^{-3} m \times 100 \times 10^{-3} m}{12.5 \times 10^{-3} m}\right)(325 - 275)K = 64.0W. \quad <$$

COMMENTS: Note the manner in which the thermophysical properties vary with pressure. Assuming perfect gas behavior, $\rho \sim p$. Also, k, μ and c_p are independent of pressure. Hence, Pr is independent of pressure, but $\nu = \mu/\rho \sim p^{-1}$ and $\alpha = k/\rho c \sim p^{-1}$.

PROBLEM 9.107

KNOWN: Cross flow over a cylinder with prescribed surface temperature and free stream conditions.

FIND: Whether free convection will be significant if the fluid is water or air.

SCHEMATIC:

Water or air, 1atm
V=0.05m/s
T_∞=35°C
D=50mm, Cylinder
T_s=20°C

ASSUMPTIONS: (1) Constant properties, (2) Combined free and forced heat transfer.

PROPERTIES: *Table A-6*, Water ($T_f = (T_\infty + T_s)/2 = 300K$): $\nu = \mu\, v_f = 855 \times 10^{-6}$ N·s/m^2 $\times$ 1.003 $\times 10^{-3}$ m^3/kg = 8.576 $\times 10^{-7}$ m^2/s, $\beta = 276.1 \times 10^{-6}$ K^{-1}; *Table A-4*, Air (300K, 1 atm): $\nu = 15.89 \times 10^{-6}$ m^2/s, $\beta = 1/T_f = 3.333 \times 10^{-3}$ K^{-1}.

ANALYSIS: Following the discussion of Section 9.9, the general criterion for delineating the relative significance of free and forced convection depends upon the value of Gr/Re^2. If free convection is significant.

$$Gr_D / Re_D^2 \geq 1 \qquad (1)$$

where $$Gr_D = g\, b\, (T_\infty - T_s) D^3 / n^2 \quad \text{and} \quad Re_D = VD/n. \qquad (2,3)$$

(a) When the surrounding fluid is *water*, find

$$Gr_D = 9.8\,\text{m/s}^2 \times 276.1 \times 10^{-6}\,\text{K}^{-1} (35-20)\,\text{K}\,(0.05\text{m})^3 / \left(8.576 \times 10^{-7}\,\text{m}^2/\text{s}\right)^2 = 68,980$$

$$Re_D = 0.05\,\text{m/s} \times 0.05\text{m} / 8.576 \times 10^{-7}\,\text{m}^2/\text{s} = 2915$$

$$Gr_D / Re_D^2 = 68{,}980/2915^2 = 0.00812.$$ <

We conclude that since $Gr_D / Re_D^2 << 1$, free convection is not significant. It is apparent that forced convection dominates the heat transfer process.

(b) When the surrounding fluid is *air*, find

$$Gr_D = 9.8\,\text{m/s}^2 \times 3.333 \times 10^{-3}\,\text{K}^{-1} (35-20)\,\text{K}\,(0.05\text{m})^3 / \left(15.89 \times 10^{-6}\,\text{m}^2/\text{s}\right)^2 = 242{,}558$$

$$Re_D = 0.05\,\text{m/s} \times 0.05\text{m} / 15.89 \times 10^{-6}\,\text{m}^2/\text{s} = 157$$

$$Gr_D / Re_D^2 = 242{,}558/157^2 = 9.8.$$ <

We conclude that, since $Gr_D / Re_D^2 >> 1$, free convection dominates the heat transfer process.

COMMENTS: Note also that for the air flow situation, surface radiation exchange is likely to be significant.

PROBLEM 9.108

KNOWN: Parallel air flow over a uniform temperature, heated vertical plate; the effect of free convection on the heat transfer coefficient will be 5% when $Gr_L / Re_L^2 = 0.08$.

FIND: Minimum vertical velocity required of air flow such that free convection effects will be less than 5% of the heat rate.

SCHEMATIC:

$T_s = 60°C$

$L = 0.3m$

Air
$T_\infty = 25°C$

$T_\infty = 25°C$
u_∞

ASSUMPTIONS: (1) Steady-state conditions, (2) Criterion for combined free-forced convection determined from experimental results.

PROPERTIES: *Table A-4*, Air ($T_f = (T_s + T_\infty)/2 = 315K$, 1 atm): $\nu = 17.40 \times 10^{-6}\ m^2/s$, $\beta = 1/T_f$.

ANALYSIS: To delineate flow regimes, according to Section 9.9, the general criterion for predominately forced convection is that

$$Gr_L / Re_L^2 << 1. \tag{1}$$

From experimental results, when $Gr_L / Re_L^2 \approx 0.08$, free convection will be equal to 5% of the total heat rate.

For the vertical plate using Eq. 9.12,

$$Gr_L = \frac{g\,b\,(T_1 - T_2)\,L^3}{n^2} = \frac{9.8\,m/s^2 \times 1/315K \times (60 - 25)K \times (0.3m)^3}{\left(17.40 \times 10^{-6} m^2/s\right)^2} = 9.711 \times 10^7.$$

(2)

For the vertical plate with forced convection,

$$Re_L = \frac{u_\infty L}{n} = \frac{u_\infty (0.3m)}{17.4 \times 10^{-6} m^2/s} = 1.724 \times 10^4\ u_\infty. \tag{3}$$

By combining Eqs. (2) and (3),

$$\frac{Gr_L}{Re_L^2} = \frac{9.711 \times 10^7}{\left[1.724 \times 10^4\ u_\infty\right]^2} = 0.08$$

find that

$$u_\infty = 2.02 m/s.$$ <

That is, when $u_\infty \geq 2.02$ m/s, free convection effects will not exceed 5% of the total heat rate.

PROBLEM 9.109

KNOWN: Vertical array of circuit boards 0.15m high with maximum allowable uniform surface temperature for prescribed ambient air temperature.

FIND: Allowable electrical power dissipation per board, $q'[W/m]$, for these cooling arrangements: (a) Free convection only, (b) Air flow downward at 0.6 m/s, (c) Air flow upward at 0.3 m/s, and (d) Air flow upward or downward at 5 m/s.

SCHEMATIC:

ASSUMPTIONS: (1) Uniform surface temperature, (2) Board horizontal spacing sufficient that boundary layers don't interfere, (3) Ambient air behaves as quiescent medium, (4) Perfect gas behavior.

PROPERTIES: *Table A-4*, Air ($T_f = (T_s + T_\infty)/2 \approx 315K$, 1 atm): $\nu = 17.40 \times 10^{-6}$ m^2/s, k = 0.0274 W/m·K, $\alpha = 24.7 \times 10^{-6}$ m^2/s, Pr = 0.705, $\beta = 1/T_f$.

ANALYSIS: (a) For *free convection* only, the allowable electrical power dissipation rate is

$$q' = \overline{h}_L (2L)(T_s - T_\infty) \tag{1}$$

where $\overline{h}_L$ is estimated using the appropriate correlation for free convection from a vertical plate. Find the Rayleigh number,

$$Ra_L = \frac{g\,b\,\Delta T\,L^3}{na} = \frac{9.8\text{m/s}^2 (1/315\text{K})(60-25)\text{K}(0.150\text{m})^3}{17.4\times10^{-6}\text{m}^2/\text{s}\times24.7\times10^{-6}\text{m}^2/\text{s}} = 8.551\times10^6. \tag{2}$$

Since $Ra_L < 10^9$, the flow is laminar. With Eq. 9.27 find

$$\overline{Nu}_L = \frac{\overline{h}L}{k} = 0.68 + \frac{0.670 Ra_L^{1/4}}{\left[1+(0.492/Pr)^{9/16}\right]^{4/9}} = 0.68 + \frac{\left(0.670\left[8.551\times10^6\right]^{1/4}\right)}{\left[1+(0.492/0.705)^{9/16}\right]^{4/9}} = 28.47 \tag{3}$$

$$\overline{h}_L = (0.0274\text{W/m}\cdot\text{K}/0.150\text{m})\times28.47 = 5.20\text{W/m}^2\cdot\text{K}.$$

Hence, the allowable electrical power dissipation rate is,

$$q' = 5.20\text{W/m}^2\cdot\text{K}(2\times0.150\text{m})(60-25)°\text{C} = 54.6\text{W/m}.$$ <

(b) With *downward velocity* V = 0.6 m/s, the possibility of mixed forced-free convection must be considered. With $Re_L = VL/\nu$, find

$$\left(Gr_L/Re_L^2\right) = \left(\frac{Ra_L}{Pr}/Re_L^2\right) \tag{4}$$

$$\left(Gr_L/Re_L^2\right) = \left(8.551\times10^6/0.705\right)/\left(0.6\text{m/s}\times0.150\text{m}/17.40\times10^{-6}\text{m}^2/\text{s}\right)^2 = 0.453.$$

Continued …..

PROBLEM 9.109 (Cont.)

Since $\left(Gr_L / Re_L^2\right) \sim 1$, flow is mixed and the average heat transfer coefficient may be found from a correlating equation of the form

$$\overline{Nu}^n = Nu_F^n \pm Nu_N^n \qquad (5)$$

where n = 3 for the vertical plate geometry and the minus sign is appropriate since the natural convection (N) flow opposes the forced convection (F) flow. For the forced convection flow, Re_L = 5172 and the flow is laminar; using Eq. 7.31,

$$\overline{Nu}_F = 0.664\ Re_L^{1/2}\ Pr^{1/3} = 0.664(5172)^{1/2}(0.705)^{1/3} = 42.50. \qquad (6)$$

Using $\overline{Nu}_N = 28.47$ from Eq. (3), Eq. (5) now becomes

$$\overline{Nu}^3 = \left(\frac{\bar{h}L}{k}\right)^3 = (42.50)^3 - (28.47)^3 \qquad \overline{Nu} = 37.72$$

$$\bar{h} = \left(\frac{0.0274\,W/m\cdot K}{0.150m}\right) \times 37.72 = 6.89\,W/m^2\cdot K.$$

Substituting for $\bar{h}$ into the rate equation, Eq. (1), the allowable power dissipation with a downward velocity of 0.6 m/s is

$$q' = 6.89\,W/m^2\cdot K\,(2\times 0.150m)\,(60-25)°C = 72.3\,W/m. \qquad <$$

(c) With an *upward velocity* V = 0.3 m/s, the positive sign of Eq. (5) applies since the N-flow is assisting the F-flow. For forced convection, find

$$Re_L = VL/\nu = 0.3m/s \times 0.150m/\left(17.40\times 10^{-6}m^2/s\right) = 2586.$$

The flow is again laminar, hence Eq. (6) is appropriate.

$$\overline{Nu}_F = 0.664(2586)^{1/2}(0.705)^{1/3} = 30.05.$$

From Eq. (5), with the positive sign, and $\overline{Nu}_N$ from Eq. (4),

$$\overline{Nu}^3 = (30.05)^3 + (28.47)^3 \quad \text{or} \quad \overline{Nu} = 36.88 \quad \text{and} \quad \bar{h} = 6.74\,W/m^2\cdot K.$$

From Eq. (1), the allowable power dissipation with an upward velocity of 0.3 m/s is

$$q' = 6.74\,W/m^2\cdot K\,(2\times 0.150m)\,(60-25)°C = 70.7\,W/m. \qquad <$$

(d) With a *forced convection* velocity V = 5 m/s, very likely forced convection will dominate. Check by evaluating whether $\left(Gr_L\ Re_L^2\right) << 1$ where $Re_L = VL/\nu$ = 5 m/s × 0.150m/(17.40 × 10^{-6} m^2/s) = 43,103. Hence,

$$\left(Gr_L / Re_L^2\right) = \left(\frac{Ra_L}{Pr}/Re_L^2\right) = \left(8.551\times 10^6/0.705\right)/43{,}103^2 = 0.007.$$

The flow is not mixed, but pure forced convection. Using Eq. (6), find

$$\bar{h} = (0.0274\,W/m\cdot K/0.150m)\,0.664(43{,}103)^{1/2}(0.705)^{1/3} = 22.4\,W/m^2\cdot K$$

and the allowable dissipation rate is

$$q' = 22.4\,W/m^2\cdot K\,(2\times 0.150m)\,(60-25)°C = 235\,W/m. \qquad <$$

COMMENTS: Be sure to compare dissipation rates to see relative importance of mixed flow conditions.

PROBLEM 9.110

KNOWN: Horizontal pipe passing hot oil used to heat water.

FIND: Effect of water flow direction on the heat rate.

SCHEMATIC:

Water, $T_\infty = 37°C$; (b) ↓; (c) ↑; $V = 0.5$ m/s (a) →; $T_s = 85°C$; Oil; Pipe, $D = 100$ mm

ASSUMPTIONS: (1) Uniform pipe surface temperature, (2) Constant properties.

PROPERTIES: *Table A-6*, Water ($T_f = (T_s + T_\infty)/2 \approx 335$K): $\nu = \mu_f\, v_f = 4.625 \times 10^{-7}$ m^2/s, k = 0.656 W/m·K, $\alpha = k\, v_f/c_p = 1.595 \times 10^{-7}$ m^2/s, Pr = 2.88, $\beta = 535.5 \times 10^{-6}$ K^{-1}; *Table A-6*, Water (T_∞ = 310K): $\nu = \mu_f\, v_f = 6.999 \times 10^{-7}$ m^2/s, k = 0.028 W/m·K, Pr = 4.62; *Table A-6,* Water (T_s = 358K): Pr = 2.07

ANALYSIS: The rate equation for the flow situations is of the form

$$q' = \overline{h}\,(\pi D)\,(T_s - T_\infty).$$

To determine whether mixed flow conditions are present, evaluate $\left(Gr_D / Re_D^2\right)$.

$$Gr_D = \frac{g\,\beta \Delta T\, D^3}{\nu^2} = \frac{9.8\,\text{m/s}^2 \times 535.5\times10^{-6}\,\text{K}^{-1}(85-37)\,\text{K}\,(0.100\,\text{m})^3}{\left(4.625\times10^{-7}\,\text{m}^2/\text{s}\right)^2} = 1.178\times10^9$$

$$Re_D = VD/\nu = 0.5\,\text{m/s}\times0.100\,\text{m}/6.999\times10^{-7}\,\text{m}^2/\text{s} = 7.144\times10^4.$$

It follows that $\left(Gr_D / Re_D^2\right) = 0.231$; since this ratio is of order unity, the flow condition is mixed. Using

Eq. 9.64, $\overline{Nu}^n = \overline{Nu}_F^n \pm \overline{Nu}_N^n$ and for the three flow arrangements,

(a) Transverse flow: $\overline{Nu}^4 = \overline{Nu}_F^4 + \overline{Nu}_N^4$

(b) Opposing flow: $\overline{Nu}^3 = \overline{Nu}_F^3 - \overline{Nu}_N^3$

(c) Assisting flow: $\overline{Nu}^3 = \overline{Nu}_F^3 + \overline{Nu}_N^3$

For *natural convection* from the cylinder, use Eq. 9.34 with Ra = Gr·Pr.

$$\overline{Nu}_N = \left\{0.60 + \frac{0.387 Ra_D^{1/6}}{\left[1+(0.559/Pr)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.60 + \frac{0.387\left(1.178\times10^9\times2.88\right)^{1/6}}{\left[1+(0.559/2.88)^{9/16}\right]^{8/27}}\right\}^2 = 201.2$$

For *forced convection* in cross flow over the cylinder, from Table 7-4 use

$$\overline{Nu}_F = C\, Re_D^m\, Pr^n\, (Pr/Pr_s)^{1/4}$$

$$\overline{Nu}_F = 0.26\left(7.144\times10^4\right)^{0.6}(4.62)^{0.37}(4.62/2.07)^{1/4} = 457.5$$

Continued …..

PROBLEM 9.110 (Cont.)

where n = 0.37 since Pr ≤ 10. The results of the calculations are tabulated.

Flow	$\overline{Nu}$	$\bar{h}\left(W/m^2 \cdot K\right)$	$q' \times 10^{-4}\ (W/m)$
(a) Transverse	461.7	3029	4.57
(b) Opposing	444.1	2913	4.39
(c) Assisting	470.1	3083	4.65

COMMENTS: Note that the flow direction has a minor effect (<6%) for these conditions.

PROBLEM 9.111

KNOWN: Diameter and surface temperature of long tube housing heat dissipating electronic components. Temperature of cooling water.

FIND: (a) Heat dissipation per unit length to quiescent water, (b) Percent enhancement for imposed cross flow.

SCHEMATIC:

ASSUMPTIONS: Constant properties evaluated at T_f.

PROPERTIES: *Table A-6*, Water (325K): ρ = 987 kg/m^3, $\mu = 528 \times 10^{-6}$ kg/s·m, $\nu = \mu/\rho = 0.535 \times 10^{-6}$ m^2/s, k = 0.645 W/m·K, Pr = 3.42, $\beta = 471 \times 10^{-6}$ K^{-1}.

ANALYSIS: (a) With

$$Ra_D = \frac{\beta g \Delta T D^3}{\nu^2} Pr = \frac{9.8\,m/s^2 \times 471\times10^{-6} K^{-1}(50\ K)(0.1m)^3\, 3.42}{\left(0.535\times10^{-6} m^2/s\right)^2} = 2.76\times10^9$$

use the Churchill and Chu correlation

$$\overline{Nu}_D = \left\{0.60 + \frac{0.387 Ra_D^{1/6}}{\left[1+(0.559/Pr)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.60 + \frac{0.387\left(2.76\times10^9\right)^{1/6}}{\left[1+(0.559/3.42)^{9/16}\right]^{8/27}}\right\}^2 = 191$$

$$\bar{h} = \overline{Nu}_D\,(k/D) = 191(0.645 W/m\cdot K/0.1m) = 1232 W/m^2\cdot K.$$

Hence, $$q' = \pi D \bar{h}\,(T_s - T_\infty) = \pi\,(0.1m) 1232 W/m^2\cdot K\,(350 - 300) K = 19.4 kW/m.$$ <

(b) Using the Hilpert correlation (for which properties are evaluated at T_f), it follows that, for pure forced convection,

$$Re_D = \frac{VD}{\nu} = \frac{1\,m/s\times0.1m}{0.535\times10^{-6} m^2/s} = 1.87\times10^5.$$

Hence, using the Hilpert correlation,

$$\overline{Nu}_{D,F} = 0.027 Re_D^{0.805} Pr^{1/3} = 0.027\left(1.87\times10^5\right)^{0.805}(3.42)^{1/3} = 713.$$

For mixed convection with n = 4,

$$\overline{Nu}^n = \overline{Nu}_F^n + \overline{Nu}_N^n = (713)^4 + (191)^4 = 2.59\times10^{11} \quad \overline{Nu} = 714$$

$$\bar{h} = \overline{Nu}(k/D) = 714(0.645 W/m\cdot K/0.1m) = 4605 W/m^2\cdot K.$$

Hence, $$q' = \bar{h}\pi D\,(T_s - T_\infty) = 4605 W/m^2\cdot K\times\pi\,(0.1m)(350-300)K = 72.3 kW/m.$$ <

The cross flow enhances the heat rate by a factor of 72.3/19.4 = 3.7.

COMMENTS: (1) With V = 1 m/s, heat transfer is dominated by forced convection.

PROBLEM 9.112

KNOWN: Horizontal square panel removed from an oven and cooled in quiescent or moving air.

FIND: Initial convection heat rates for both methods of cooling.

SCHEMATIC:

Quiescent air, $T_\infty = 29°C$
$T_s = 125°C$
Highly reflective square plate, 250mm
Batch method-stationary plate

Air $u_\infty = 0.5 m/s$, $T_\infty = 29°C$
Quiescent air, $T_\infty = 29°C$
$T_s = 125°C$
Conveyor method-moving air

ASSUMPTIONS: (1) Quasi-steady state conditions, (2) Backside of plates insulated, (3) Air flow is in the length-wise (not diagonal) direction, (4) Constant properties, (5) Radiative exchange negligible.

PROPERTIES: *Table A-4*, Air ($T_f = (T_\infty + T_s)/2 = 350K$, 1 atm): $\nu = 20.92 \times 10^{-6}$ m^2/s, k = 0.030 W/m·K, $\alpha = 29.9 \times 10^{-6}$ m^2/s, Pr = 0.700, $\beta = 1/T_f$.

ANALYSIS: The initial heat transfer rate from the plates by convection is given by the rate equation $q = \bar{h} A_s (T_s - T_\infty)$. Test for the existence of combined free-forced convection by calculation of the ratio Gr_L / Re_L^2. Use the same characteristic length in both parameters, L = 250mm, the side length.

$$Gr_L = \frac{g\, b\, \Delta T\, L^3}{n^2} = \frac{9.8 m/s^2 (1/350K)(125-29) K (0.250m)^3}{\left(20.92\times10^{-6} m/s^2\right)^2} = 9.597\times10^7$$

$$Re_L = u_\infty L / n = 0.5 m/s \times 0.250 m / \left(20.92\times10^{-6} m^2/s\right) = 5.975\times10^3.$$

Since $Gr_L / Re_L^2 = 2.69$ flow is mixed. For the *stationary plate*, $Ra_L = Gr_L \cdot Pr = 6.718 \times 10^7$ and Eq. 9.31 is the appropriate correlation,

$$\overline{Nu}_N = \frac{\bar{h}L}{k} = 0.15 Ra_L^{1/3} = 0.15\left(6.718\times10^7\right)^{1/3} = 60.9$$

$$\bar{h} = (0.030 W/m\cdot K / 0.250m) \times 60.9 = 7.31 W/m^2\cdot K.$$

$$q = 7.31 W/m^2\cdot K \times (0.250m)^2 (125-29) K = 43.9W. \qquad <$$

For the *plate with moving air*, $Re_L = 5.975 \times 10^3$ and the flow is laminar.

$$\overline{Nu}_F = 0.664\, Re_L^{1/2}\, Pr^{1/3} = 0.664\left(5.975\times10^3\right)^{1/2} (0.700)^{1/3} = 45.6.$$

For combined free-forced convection, use the correlating equation with n = 7/2.

$$\overline{Nu}^{7/2} = \overline{Nu}_F^{7/2} + \overline{Nu}_N^{7/2} = (45.6)^{7/2} + (60.9)^{7/2} \qquad \overline{Nu} = 66.5.$$

$$\bar{h} = \overline{Nu}\, k/L = 66.5(0.030 W/m\cdot K/0.25m) = 7.99 W/m^2\cdot K$$

$$q = 7.99 W/m^2\cdot K (0.250m)^2 (125-29) K = 47.9W. \qquad <$$

COMMENTS: (1) The conveyor method provides only slight enhancement of heat transfer.

PROBLEM 9.113

KNOWN: Wet garment at 25°C hanging in a room with still, dry air at 40°C.

FIND: Drying rate per unit width of garment.

SCHEMATIC:

ASSUMPTIONS: (1) Analogy between heat and mass transfer applies, (2) Water vapor at garment surface is saturated at T_s, (3) Perfect gas behavior of vapor and air.

PROPERTIES: *Table A-4*, Air ($T_f \approx (T_s + T_\infty)/2 = 305K$, 1 atm): $\nu = 16.39 \times 10^{-6}$ m^2/s; *Table A-6*, Water vapor (T_s = 298K, 1 atm): $p_{A,s} = 0.0317$ bar, $\rho_{A,s} = 1/v_f = 0.02660$ kg/m^3; *Table A-8*, Air-water vapor (305 K): $D_{AB} = 0.27 \times 10^{-4}$ m^2/s, Sc = ν/DAB = 0.607.

ANALYSIS: The drying rate per unit width of the garment is

$$\dot{m}'_A = \overline{h}_m \cdot L\left(\rho_{A,s} - \rho_{A,\infty}\right)$$

where $\overline{h}_m$ is the mass transfer coefficient associated with a vertical surface that models the garment. From the heat and mass transfer analogy, Eq. 9.24 and Fig. 9.6 yield

$$\overline{Sh}_L = 0.59\left(Gr_L\ Sc\right)^{1/4}$$

where $GrL = g\Delta\rho L^3/\rho\nu^2$ and $\Delta\rho = \rho_s - \rho_\infty$. Since the still air is dry, $\rho_\infty = \rho_{B,\infty} = p_{B,\infty}/R_B T_\infty$, where $R_B = \Re/\mathcal{M}_B = 8.314 \times 10^{-2}$ m^3·bar/kmol·K/29 kg/kmol = 0.00287 m^3·bar/kg·K. With $p_{B,\infty}$ = 1 atm = 1.0133 bar,

$$\rho_\infty = \frac{1.0133 \text{ bar}}{0.00287 \text{ m}^3 \cdot \text{bar/kg} \cdot \text{K} \times 313 \text{ K}} = 1.1280 \text{ kg/m}^3$$

The density of the air/vapor mixture at the surface is $\rho_s = \rho_{A,s} + \rho_{B,s}$. With $p_{B,s}$ = 1 atm – $p_{A,s}$ = 1.0133 bar – 0.0317 bar = 0.9816 bar,

$$\rho_{B,s} = \frac{p_{B,s}}{R_B T_s} = \frac{0.9816 \text{ bar}}{0.00287\left(\text{m}^3 \cdot \text{bar/kg} \cdot \text{K}\right) \times 298 \text{ K}} = 1.1477 \text{ kg/m}^3$$

Hence, ρ_s = (0.0266 + 1.1477) kg/m^3 = 1.1743 kg/m^3 and $\rho = (\rho_s + \rho_\infty)/2 = 1.512$ kg/m^3. The Grashof number is then

$$Gr_L = \frac{9.8 \text{ m/s}^2 \times (1.1743 - 1.1280)\text{ kg/m}^3 (1 \text{ m})^3}{1.1512 \text{ kg/m}^3 \times \left(16.39 \times 10^{-6} \text{m}^2/\text{s}\right)^2} = 1.467 \times 10^9$$

and (Gr_L Sc) = 8.905 × 10^8. The convection coefficient is then

$$\overline{h}_m = \frac{D_{AB}}{L}\overline{Sh}_L = \frac{0.27 \times 10^{-4} \text{m}^2/\text{s}}{1 \text{ m}} \times 0.59\left(8.905 \times 10^8\right)^{1/4} = 0.00275 \text{ m/s}$$

The drying rate is then

$$\dot{m}'_A = 2.750 \times 10^{-3} \text{m/s} \times 1.0\text{m}\left(0.0226 - 0\right)\text{kg/m}^3 = 6.21 \times 10^{-5} \text{kg/s} \cdot \text{m}. \quad <$$

COMMENTS: Since $\rho_s > \rho_\infty$, the buoyancy driven flow *descends* along the garment.

PROBLEM 9.114

KNOWN: Circular pan of water at 37°C exposed to dry, still air at 17°C.

FIND: Evaporation rate and total heat transfer rate from the pan.

SCHEMATIC:

Quiescent air, $T_\infty = 17°C$

$q_{evap} = \dot{m}_A h_{fg}$

q_{conv}

Water (A), $T_s = 37°C$, $\rho_{A,s} = \rho_{A,sat}(T_s)$

Pan, D=225mm

q_{elec}

ASSUMPTIONS: (1) Dry room air, (2) Negligible radiation, (3) Water vapor and air behave as perfect gases.

PROPERTIES: *TableA-4*, Air ($T_f = (T_s + T_\infty)/2$=300K, 1 atm): $\nu = 15.89 \times 10^{-6}$ m^2/s, k = 0.0263 W/m·K, Pr = 0.707, $\beta = 1/T_f$; *Table A-6*, Water (T_s = 310K): $\rho_{A,s} = \rho_{A,sat} = 1/v_g$ = 0.04361 kg/m^3, $p_{A,s}$ = 0.0622 bar, h_{fg} = 2414 kJ/kg; *Table A-8*, Air-water vapor (1 atm, T_f = 300K): $D_{AB} \approx 0.26 \times 10^{-4}$ m^2/s, Sc = ν/D_{AB} = 0.611

ANALYSIS: The evaporation rate and total heat transfer rate from the pan are

$$\dot{m}_A = \bar{h}_m A_s \left(r_{A,s} - r_{A,\infty} \right) = \bar{h}_m A_s\, r_{A,sat}(T_s) \qquad q = q_{conv} + q_{evap} = \bar{h} A_s (T_s - T_\infty) + \dot{m}_A h_{fg} \qquad (1,2)$$

where $A_s = \pi D^2/4$. The convection coefficients can be estimated from the free convection correlation for a horizontal, circular plate with $L = A_s/P = D/4 = 0.0563$ m.

To determine the appropriate convection correlation, we must first determine ρ_∞ and ρ_s. Since $\phi_\infty = 0$ and the gas constant for air is $R_B = \Re/M_B$ =8.314 × 10-2 m^3·bar/kmol·K/29 kg/kmol = 0.00287 m^3·bar/kg·K,

$$r_\infty = \frac{p_{B,\infty}}{R_B T_\infty} = \frac{1.0133\text{bar}}{0.00287\ \text{m}^3 \cdot \text{bar/kg} \cdot \text{K} \times 290\ \text{K}} = 1.2175\ \text{kg/m}^3$$

At the surface, $\rho_s = \rho_{A,s} + \rho_{B,s}$. With $p_{B,s}$ = 1.0133 bar – $p_{A,s}$ = 0.9511 bar, $\rho_{B,s} = p_{B,s}/R_B T_s$ = 1.0690 kg/m3 and

$$r_s = r_{A,s} + r_{B,s} = (0.0436 + 1.0690)\text{kg/m}^3 = 1.1126\ \text{kg/m}^3$$

From Eq. 9.65, with $\rho = (\rho_s + \rho_\infty)/2$ = 1.1651 kg/m^3, the Grashof number is

$$Gr_L = \frac{g(r_\infty - r_s)L^3}{rn^2} = \frac{9.8\ \text{m/s}^2 \left(0.0524\ \text{kg/m}^3\right)(0.0563\ \text{m})^3}{1.1651\text{kg/m}^3 \left(15.89 \times 10^{-6}\text{m}^2/\text{s}\right)^2} = 3.12 \times 10^5$$

in which case $Ra_L = Gr_L Pr = 2.21 \times 10^5$ and $Gr_L Sc = 1.91 \times 10^5$. From Eq. 9.30,

$$h = \left(\frac{k}{L}\right) 0.54\ Ra_L^{1/4} = \left(\frac{0.0263\ \text{W/m} \cdot \text{K}}{0.0563\ \text{m}}\right) 0.54 \left(2.21 \times 10^5\right)^{1/4} = 5.47\ \text{W/m}^2 \cdot \text{K}$$

Continued …..

and from its mass transfer analog,

$$h_m = \frac{D_{AB}}{L} 0.54 (Gr_L \, Sc)^{1/4} = \left(\frac{0.26 \times 10^{-4} \, m^2/s}{0.0563 \, m} \right) 0.54 \left(1.91 \times 10^5\right)^{1/4} = 0.00521 \, m/s$$

Substituting numerical values into the rate equations, Eqs. (1) and (2), find

$$\dot{m}_A = 5.21 \times 10^{-3} \, m/s \left(\pi (0.225m)^2 / 4\right) \times 0.04361 \, kg/m^3 = 9.034 \times 10^{-6} \, kg/s = 32.5 \, g/h \quad <$$

$$q = 5.47 \, W/m^2 \cdot K \left(\pi (0.225m)^2 / 4\right)(37 - 17) K + 9.034 \times 10^{-6} \, kg/s \times 2414 \times 10^3 \, J/kg$$

$$q = (4.4 + 21.8) \, W = 26.2 \; W. \quad <$$

COMMENTS: As expected, the heat loss is more strongly influenced by the loss of latent energy.

PROBLEM 9.115

KNOWN: A water bath maintained at a uniform temperature of 37°C with top surface exposed to draft-free air and uniform temperature walls in a laboratory.

FIND: (a) The heat loss from the surface of the bath by radiation exchange with the surroundings; (b) Calculate the Grashof number using Eq. 9.65 with a characteristic length L that is appropriate for the exposed surface of the water bath; (c) Estimate the free convection heat transfer coefficient using the result for Gr_L obtained in part (b); (d) Invoke the heat-mass analogy and use an appropriate correlation to estimate the mass transfer coefficient using Gr_L; calculate the water evaporation rate on a daily basis and the heat loss by evaporation; and (e) Calculate the total heat loss from the surface and compare relative contributions of the sensible, latent and radiative effects. Review assumptions made in your analysis, especially those relating to the heat-mass analogy.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Laboratory air is quiescent, (3) Laboratory walls are isothermal and large compared to water bath exposed surface, (4) Emissivity of the water surface is 0.96, (5) Heat-mass analogy is applicable, and (6) Constant properties.

PROPERTIES: *Table A-6*, Water vapor (T_∞ = 293 K): $\rho_{A,\infty,sat}$ = 0.01693 kg/m^3; (T_s = 310 K): $\rho_{A,s}$ = 0.04361 kg/m^3, $h_{fg} = 2.414 \times 10^6$ J/kg; *Table A-4*, Air (T_∞ = 293 K, 1 atm): $\rho_{B,\infty}$ = 1.194 kg/m^3; (T_s = 310 K, 1 atm): $\rho_{B,s}$ = 1.128 kg/m^3; ($T_f = (T_s + T_\infty)/2$ = 302 K, 1 atm): $\nu_B = 1.604 \times 10^{-5}$ m^2/s, k = 0.0270 W/m·K, Pr = 0.706; *Table A-8*, Water vapor-air (T_f = 302 K, 1 atm): $D_{AB} = 0.24 \times 10^{-4}$ m^2/s $(302/298)^{3/2} = 2.65 \times 10^{-5}$ m^2/s.

ANALYSIS: (a) Using the linearized form of the radiation exchange rate equation, the heat rate and radiation coefficient can be estimated.

$$h_{rad} = \varepsilon\sigma\left(T_s + T_{sur}\right)\left(T_s^2 + T_{sur}^2\right) \qquad (1)$$

$$h_{rad} = 0.96\sigma\left(310 + 298\right)\left(310^2 + 298^2\right)K^3 = 6.12\ W/m^2 \cdot K \qquad <$$

$$q_{rad} = h_{rad} A_s \left(T_s - T_{sur}\right) \qquad (2)$$

$$q_{rad} = 6.12\ W/m^2 \cdot K \times \left(0.25 \times 0.50\right) m^2 \times \left(37 - 25\right) K = 9.18\ W$$

(b) The general form of the Grashof number, Eq. 9.65, applied to natural convection flows driven by concentration gradients

$$Gr_L = g\left(\rho_\infty - \rho_s\right) L^3 / \rho\nu^2 \qquad (3)$$

where L is the characteristic length defined in Eq. 9.29 as $L = A_s/P$, where A_s and P are the exposed surface area and perimeter, respectively; ρ_s and ρ_∞ are the density of the mixture at the surface and in the quiescent fluid, respectively; and, ρ is the mean boundary layer density, $(\rho_\infty + \rho_s)/2$, and ν is the kinematic viscosity of fluid B, evaluated at the film temperature $T_f = (T_s + T_\infty)/2$. Using the property values from above,

Continued …..

$$\rho_s = \rho_{A,s} + \rho_{B,s} = (0.04361 + 1.128)\,\text{kg/m}^3 = 1.1716\ \text{kg/m}^3$$

$$\rho_\infty = \rho_{A,\infty} + \rho_{B,\infty} = \phi_\infty \rho_{A,\infty,sat} + \rho_{B,\infty}$$

$$\rho_\infty = (0.6 \times 0.01693 + 1.194)\,\text{kg/m}^3 = 1.2042\ \text{kg/m}^3$$

$$\rho = (\rho_s + \rho_\infty)/2 = 1.4601\ \text{kg/m}^3$$

Substituting numerical values in Eq. (3), find the Grashof number.

$$Gr_L = \frac{9.8\ \text{m/s}^2 (1.2042 - 1.1716)\,\text{kg/m}^3 \times (0.0833\ \text{m})^3}{1.4601\ \text{kg/m}^3 \left(1.604 \times 10^{-5}\text{m}^2/\text{s}\right)^2}$$

$$Gr_L = 4.916 \times 10^5 \qquad <$$

where the characteristic length is defined by Eq. 9.29,

$$L = A_s / P = (0.25 \times 0.5)\text{m}^2 / 2(0.25 + 0.50)\text{m} = 0.0833\ \text{m}$$

(c) The free convection heat transfer coefficient for the horizontal surface, Eq. 9.30, for *upper surface of heated plate*, is estimated as follows:

$$Ra_L = Gr_L\ Pr_L = 4.916 \times 10^5 \times 0.706 = 3.471 \times 10^5$$

$$\overline{Nu}_L = \frac{\bar{h}L}{k} = 0.54\ Ra_L^{1/4} = 13.11$$

$$\bar{h} = 13.11 \times 0.0270\ \text{W/m}\cdot\text{K} / 0.0833\ \text{m} = 4.25\ \text{W/m}^2\cdot\text{K} \qquad <$$

(d) Invoking the heat-mass analogy, the mass transfer coefficient is estimated as follows,

$$Ra_{L,m} = Gr_L Sc = 4.916 \times 10^5 \times 0.605 = 2.975 \times 10^5$$

where the Schmidt number is given as

$$Sc = \nu / D_{AB} = 1.604 \times 10^{-5}\text{m}^2/\text{s} / 2.65 \times 10^{-5}\text{m}^2/\text{s} = 0.605$$

The correlation has the form

$$\overline{Sh}_L = \frac{\bar{h}_m L}{D_{AB}} = 0.54\ Ra_{L,m}^{1/4} = 12.61$$

$$\bar{h}_m = 12.61 \times 2.65 \times 10^{-5}\text{m}^2/\text{s} / 0.0833\ \text{m} = 0.00401\ \text{m/s} \qquad <$$

The water evaporation rate on a daily basis is

$$n_A = \bar{h}_m A_s \left(\rho_{A,sat} - \rho_{A,\infty}\right)$$

$$n_A = 0.00401\ \text{m/s}(0.25 \times 0.50)\text{m}^2 (0.04361 - 0.6 \times 0.01693)\,\text{kg/m}^3$$

Continued …..

$$nA = 1.677 \times 10^{-5}\, \text{kg/s} = 1.45\ \text{kg/day} \qquad <$$

and the *heat loss by evaporation* is

$$q_{evap} = n_A\, h_{fg} = 1.677 \times 10^{-5}\, \text{kg/s} \times 2.414 \times 10^{6}\, \text{J/kg} = 40.5\ \text{W} \qquad <$$

(e) The *convective heat loss* is that of free convection,

$$q_{cv} = \overline{h} A_s \left(T_s - T_\infty \right)$$

$$q_{cv} = 4.25\ \text{W/m}^2 \times (0.25 \times 0.50)\, \text{m}^2\, (37 - 20)\, \text{K} = 9.02\ \text{W} \qquad <$$

In summary, the *total heat loss* from the surface of the bath, which must be supplied as electrical power to the bath heaters, is

$$q_{tot} = q_{rad} + q_{cv} + q_{evap}$$

$$q_{tot} = (9.18 + 9.02 + 40.5)\, \text{W} = 59\ \text{W} \qquad <$$

The *sensible heat losses* are by convection (q_{rad} + q_{cv}), which represent 31% of the total; the balance is the *latent loss* by evaporation, 69%.

PROBLEM 10.1

KNOWN: Water at 1 atm with $T_s - T_{sat} = 10°C$.

FIND: Show that the Jakob number is much less than unity; what is the physical significance of the result; does result apply to other fluids?

ASSUMPTIONS: (1) Boiling situation, $T_s > T_{sat}$.

PROPERTIES: *Table A-5* and *Table A-6*, (1 atm):

	h_{fg} (kJ/kg)	$c_{p,v}$ (J/kg·K)	T_{sat}(K)
Water	2257	2029	373
Ethylene glycol	812	2742*	470
Mercury	301	135.5*	630
R-12	165	1015*	243

* Estimated based upon value at highest temperature cited in Table A-5.

ANALYSIS: The Jakob number is the ratio of the maximum sensible energy absorbed by the vapor to the latent energy absorbed by the vapor during boiling. That is,

$$Ja = \left(c_p \Delta T\right)_v / h_{fg} = c_{p,v}\,\Delta T_e / h_{fg}$$

For *water* with an excess temperature $\Delta T_s = T_e - T_\infty = 10°C$, find

$$Ja = \left(2029\,J/kg \cdot K \times 10K\right)/2257 \times 10^3\,J/kg$$

$$Ja = 0.0090.$$

Since Ja << 1, the implication is that the sensible energy absorbed by the vapor is much less than the latent energy absorbed during the boiling phase change. Using the appropriate thermophysical properties for three other fluids, the Jakob numbers are:

Ethylene glycol: $Ja = \left(2742\,J/kg \cdot K \times 10K\right)/812 \times 10^3\,J/kg = 0.0338$ <

Mercury: $Ja = \left(135.5\,J/kg \cdot K \times 10K\right)/301 \times 10^3\,J/kg = 0.0045$ <

Refrigerant, R-12: $Ja = \left(1015\,J/kg \cdot K \times 10K\right)/165 \times 10^3\,J/kg = 0.0615$ <

For ethylene glycol and R-12, the Jakob number is larger than the value for water, but still much less than unity. Based upon these example fluids, we conclude that generally we'd expect Ja to be much less than unity.

COMMENTS: We would expect the same low value of Ja for the condensation process since $c_{p,g}$ and $c_{p,f}$ are of the same order of magnitude.

PROBLEM 10.2

KNOWN: Horizontal 20 mm diameter cylinder with $\Delta T_e = T_s - T_{sat} = 5°C$ in saturated water, 1 atm.

FIND: Heat flux based upon free convection correlation; compare with boiling curve. Estimate maximum value of the heat transfer coefficient from the boiling curve.

SCHEMATIC:

$\Delta T_e = 5°C$
$T_s = 105°C$
D=20mm
Water, 1atm

ASSUMPTIONS: (1) Horizontal cylinder, (2) Free convection, no bubble information.

PROPERTIES: *Table A-6*, Water (Saturated liquid, $T_f = (T_{sat} + T_s)/2 = 102.5$ °C ≈ 375K): $r_\ell = 956.9$ kg/m^3, $c_{p,\ell}$ = 4220 J/kg·K, $m_\ell = 274 \times 10^{-6}$ N·s/m^2, $k_\ell = 0.681$ W/m·K, Pr = 1.70, $\beta = 761 \times 10^{-6}$ K^{-1}.

ANALYSIS: To estimate the free convection heat transfer coefficient, use the Churchill-Chu correlation,

$$\overline{Nu}_D = \frac{\bar{h}D}{k} = \left\{0.60 + \frac{0.387 Ra_D^{1/6}}{\left[1+(0.559/Pr)^{9/16}\right]^{8/27}}\right\}^2 .$$

Substituting numerical values, with $\Delta T = \Delta T_e = 5°C$, find

$$Ra_D = \frac{g\, b\, \Delta T\, D^3}{n a} = \frac{9.8\,m/s^2 \times 761\times10^{-6} K^{-1} \times 5°C(0.020m)^3}{\left[274\times10^{-6} N\cdot s/m^2 /956.9\ kg/m^3\right]\times1.686\times10^{-7} m^2/s} = 6.178\times10^6$$

where $\alpha = k/\rho\, c_p$ = (0.681 W/m·K/956.9 kg/m^3 × 4220 J/kg·K) = 1.686×10^{-7} m^2/s. Note that Ra_D is within the prescribed limits of the correlation. Hence,

$$\overline{Nu}_D = \left\{0.60 + \frac{0.387\left(6.178\times10^6\right)^{1/6}}{\left[1+(0.559/1.70)^{9/16}\right]^{8/27}}\right\}^2 = 27.22$$

$$\bar{h}_{fc} = Nu_D \frac{k}{D} = \frac{27.22\times0.681 W/m\cdot K}{0.020m} = 928 W/m^2\cdot K.$$ <

Hence, $q_s'' = h_{fc}\Delta T_e = 4640\ W/m^2$

From the typical boiling curve for water at 1 atm, Fig. 10.4, find at $\Delta T_e = 5°C$ that

$$q_s'' \approx 8.5\times10^3 W/m^2$$ <

The free convection correlation underpredicts (by 1.8) the boiling curve. The maximum value of h_{bc} can be estimated as

$$h_{max} \approx q_{max}''/\Delta T_e = 1.2\times10^6 MW/m^2/30°C = 40{,}000 W/m^2\cdot K.$$ <

COMMENTS: (1) Note the large increase in h with a slight change in ΔT_e.

(2) The maximum value of h occurs at point P on the boiling curve.

PROBLEM 10.3

KNOWN: Spherical bubble of pure saturated vapor in mechanical and thermal equilibrium with its liquid.

FIND: (a) Expression for the bubble radius, (b) Bubble vapor and liquid states on a p-v diagram; how changes in these conditions cause bubble to collapse or grow, and (c) Bubble size for specified conditions.

SCHEMATIC:

Saturated vapor, v
$T_{sat,v}, p_{sat,v}$
Interface ℓ-v
r_b
Liquid, ℓ
T_ℓ, p_ℓ
Conditions:
Vapor saturated at 101°C
liquid pressure corresponding
to saturation temperature 100°C
Surface tension force
F_{st}
Interface bubble
Liquid
$\Delta p = p_i - p_o$
p_i
Saturated vapor
F_{st}

ASSUMPTIONS: (1) Liquid-vapor medium, (2) Thermal and mechanical equilibrium.

PROPERTIES: *Table A-6*, Water (T_{sat} = 101°C = 374.15K): p_{sat} = 1.0502 bar; (T_{sat} = 100°C = 373.15K): p_{sat} = 1.0133 bar, $\sigma = 58.9 \times 10^{-3}$ N/m.

ANALYSIS: (a) For mechanical equilibrium, the difference in pressure between the vapor inside the bubble and the liquid outside the bubble will be offset by the surface tension of the liquid-vapor interface. The force balance follows from the free-body diagram shown above (right),

$$F_{st} = \left(\pi r_b^2\right)\Delta p = \left(p_i - p_o\right)\left(\pi r_b^2\right)$$

$$\left(2\pi r_b\right)\sigma = \left(\pi r_b^2\right)\left(p_i - p_o\right)$$

$$r_b = 2\sigma / \left(p_i - p_o\right) \qquad (1)$$

Thermal equilibrium requires that the temperatures of the vapor and liquid be equal. Since the vapor inside the bubble is saturated, $p_i = p_{sat,v}$ (T). Since $p_o < p_i$, it follows that the liquid outside the bubble must be superheated; hence, $p_o = p_\ell$ (T), the pressure of superheated liquid at T. Hence, we can write,

$$r_b = 2\sigma / \left(p_{sat,v} - p_\ell\right) \qquad (2)$$ <

(b) The vapor [1] and liquid [2] states are represented on the following p-v diagram. Thermal equilibrium requires both the vapor and liquid to be at the same temperature [3]. But mechanical equilibrium requires that the outside liquid pressure be less than the inside vapor pressure [4]. Hence the liquid must be in a superheated state. That is, its saturation temperature, $T_{sat}(p_o)$ [5] is less than $T_{sat}(p_i)$; $T_\ell = T_{sat}(p_o)$ and $p_o = p_\ell$.

Continued …..

PROBLEM 10.3 (Cont.)

<

The equilibrium condition for the bubble is unstable. Consider situations for which the pressure of the surrounding liquid is greater or less than the equilibrium value. These situations are presented on portions of the p-v diagram

When $p_o' < p_o$, $T_\ell' < T_{sat,v}$ and heat must be transferred out of the bubble and vapor condenses. Hence, the bubble collapses.

A similar argument for the condition $p_o' > p_o$ leads to $T_\ell' > T_{sat,v}$ and heat is transferred into the bubble causing evaporation with the formation of vapor. Hence, the bubble begins to grow.

(c) Consider the specific conditions

$$T_{sat,v} = 101°C \qquad \text{and} \qquad T_\ell = T_{sat}(p_o) = 100°C$$

and calculate the radius of the bubble using the appropriate properties in Eq. (2).

$$r_b = 2 \times 58.9 \times 10^{-3} \frac{N}{m} / (1.0502 - 1.0133) \text{bar} \times \left(10^5 \frac{N}{m^2} / \text{bar} \right)$$

$$r_b = 0.032 \text{mm}.$$

<

Note the small bubble size. This implies that nucleation sites of the same magnitude formed by pits and crevices are important in promoting the boiling process.

PROBLEM 10.4

KNOWN: Long wire, 1 mm diameter, reaches a surface temperature of 126°C in water at 1 atm while dissipating 3150 W/m.

FIND: (a) Boiling heat transfer coefficient and (b) Correlation coefficient, $C_{s,f}$, if nucleate boiling occurs.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Nucleate boiling.

PROPERTIES: *Table A-6*, Water (saturated, 1 atm): T_s = 100°C, $r_\ell = 1/v_f = 957.9\ kg/m^3$, $\rho_f = 1/v_g = 0.5955\ kg/m^3$, $c_{p,\ell} = 4217$ J/kg·K, $m_\ell = 279 \times 10^{-6}\ N{\cdot}s/m^2$, $Pr_\ell = 1.76$, $h_{fg} = 2257$ kJ/kg, $\sigma = 58.9\times 10^{-3}$ N/m.

ANALYSIS: (a) For the boiling process, the rate equation can be rewritten as

$$\bar{h} = q_s''/\left(T_s - T_{sat}\right) = \frac{q_s'}{p\,D}/\left(T_s - T_{sat}\right)$$

$$\bar{h} = \frac{3150\,W/m}{p \times 0.001m}/\left(126-100\right)°C = 1.00\times10^6\,\frac{W}{m^2}/26°C = 38{,}600\,W/m^2\cdot K. \qquad <$$

Note the heat flux is very close to q''_{max}, and nucleate boiling does exist.

(b) For nucleate boiling, the Rohsenow correlation may be solved for $C_{s,f}$ to give

$$C_{s,f} = \left\{\frac{m_\ell\, h_{fg}}{q_s''}\right\}^{1/3}\left[\frac{g\left(r_\ell - r_v\right)}{s}\right]^{1/6}\left(\frac{c_{p,\ell}\,\Delta T_e}{h_{fg}\, Pr_\ell^n}\right).$$

Assuming the liquid-surface combination is such that n = 1 and substituting numerical values with $\Delta T_e = T_s - T_{sat}$, find

$$C_{s,f} = \left\{\frac{279\times10^{-6}\,N\cdot s/m^2 \times 2257\times10^3\,J/kg}{1.00\times10^6\,W/m^2}\right\}^{1/3}\left[\frac{9.8\frac{m}{s^2}\left(957.9-0.5955\right)\frac{kg}{m^3}}{58.9\times10^{-3}\,N/m}\right]^{1/6}\times$$

$$\left(\frac{4217\,J/kg\cdot K\times26K}{2257\times10^3\,J/kg\times1.76}\right)$$

$$C_{s,f} = 0.017. \qquad <$$

COMMENTS: By comparison with the values of $C_{s,f}$ for other water-surface combinations of Table 10.1, the $C_{s,f}$ value for the wire is large, suggesting that its surface must be highly polished. Note that the value of the boiling heat transfer coefficient is much larger than values common to single-phase convection.

PROBLEM 10.5

KNOWN: Nucleate pool boiling on a 10 mm-diameter tube maintained at $\Delta T_e = 10°C$ in water at 1 atm; tube is platinum-plated.

FIND: Heat transfer coefficient.

SCHEMATIC:

Water, 1 atm

Platinum-coated tube, $T_s - T_{sat} = \Delta T_e = 10°C$

ASSUMPTIONS: (1) Steady-state conditions, (2) Nucleate pool boiling.

PROPERTIES: *Table A-6*, Water (saturated, 1 atm): $T_s = 100°C$, $r_\ell = 1/v_f = 957.9\ kg/m^3$, $\rho_v = 1/v_g = 0.5955\ kg/m^3$, $c_{p,\ell} = 4217\ J/kg{\cdot}K$, $m_\ell = 279 \times 10^{-6}\ N{\cdot}s/m^2$, $Pr_\ell = 1.76$, $h_{fg} = 2257\ kJ/kg$, $\sigma = 58.9 \times 10^{-3}\ N/m$.

ANALYSIS: The heat transfer coefficient can be estimated using the Rohsenow nucleate-boiling correlation and the rate equation

$$h = \frac{q''_s}{\Delta T_e} = \frac{m_\ell h_{fg}}{\Delta T_e}\left[\frac{g(r_\ell - r_v)}{s}\right]^{1/2}\left(\frac{c_{p,\ell}\Delta T_e}{C_{s,f} h_{fg} Pr_\ell^n}\right)^3 .$$

From Table 10.1, find $C_{s,f} = 0.013$ and $n = 1$ for the water-platinum surface combination. Substituting numerical values,

$$h = \frac{279\times10^{-6}\,N\cdot s/m^2 \times 2257\times10^3\,J/kg}{10K}\left[\frac{9.8\,m/s^2\,(957.9-0.5955)\,kg/m^3}{58.9\times10^{-3}\,N/m}\right]^{1/2}\times$$

$$\left(\frac{4217\,J/kg\cdot K\times10K}{0.013\times2257\times10^3\,J/kg\times1.76}\right)^3$$

$$h = 13{,}690\ W/m^2\cdot K.$$ <

COMMENTS: For this liquid-surface combination, $q''_s = 0.137\,MW/m^2$, which is in general agreement with the *typical* boiling curve of Fig. 10.4. To a first approximation, the effect of the tube diameter is negligible.

PROBLEM 10.6

KNOWN: Water boiling on a mechanically polished stainless steel surface maintained at an excess temperature of 15°C; water is at 1 atm.

FIND: Boiling heat transfer coefficient.

SCHEMATIC:

Water, 1atm

$\Delta T_e = 15°C$

Mechanically polished stainless steel

ASSUMPTIONS: (1) Steady-state conditions, (2) Nucleate pool boiling occurs.

PROPERTIES: *Table A-6*, Saturated water (1 atm): T_{sat} = 100°C, r_ℓ = 957.9 kg/m^3, ρ_v = 0.596 kg/m^3, $c_{p,\ell}$ = 4217 J/kg·K, m_ℓ = 279 × 10^{-6} N·s/m^2, Pr_ℓ = 1.76, σ = 58.9 × 10^{-3} N/m, h_{fg} = 2257 kJ/kg.

ANALYSIS: The heat transfer coefficient can be expressed as

$$h = q_s'' / \Delta T_e$$

where the nucleate pool boiling heat flux can be estimated using the Rohsenow correlation.

$$q_s'' = m_\ell h_{fg} \left[\frac{g(r_\ell - r_v)}{s}\right]^{1/2} \left(\frac{c_{p,\ell} \Delta T_e}{C_{s,f} h_{fg} Pr_\ell^n}\right)^3 .$$

From Table 10.1, find for this liquid-surface combination, $C_{s,f}$ = 0.013 and n = 1, and substituting numerical values,

$$q_s'' = 279\times10^{-6} N\cdot s/m^2 \times 2257\times10^3 J/kg \left[\frac{9.8 m/s^2 (957.9 - 0.596) kg/m^3}{58.9\times10^{-3} N/m}\right]^{1/2} \times \left(\frac{4217 J/kg\cdot K \times 15°C}{0.013\times 2257 kJ/kg \times 1.76}\right)^3$$

$$q_s'' = 461.9 kW/m^2.$$

Hence, the heat transfer coefficient is

$$h = 461.9\times10^3 W/m^2 / 15°C = 30{,}790\ W/m^2\cdot K.$$ <

COMMENTS: Note that this value of q_s'' for ΔT_e = 15°C is consistent with the typical boiling curve, Fig. 10.4.

PROBLEM 10.7

KNOWN: Simple expression to account for the effect of pressure on the nucleate boiling convection coefficient in water.

FIND: Compare predictions of this expression with the Rohsenow correlation for specified ΔT_e and pressures (2 and 5 bar) applied to a horizontal plate.

ASSUMPTIONS: (1) Steady-state conditions, (2) Nucleate pool boiling, (3) $C_{s,f} = 0.013$, n = 1.

PROPERTIES: *Table A-6*, Saturated water (2 bar): $\rho_\ell = 942.7\ \text{kg/m}^3$, $c_{p,\ell} = 4244.3$ J/kg·K, $\mu_\ell = 230.7 \times 10^{-6}\ \text{N·s/m}^2$, $Pr_\ell = 1.43$, $h_{fg} = 2203$ kJ/kg, $\sigma = 54.97 \times 10^{-3}$ N/m, $\rho_v = 1.1082\ \text{kg/m}^3$; Saturated water (5 bar): $\rho_\ell = 914.7\ \text{kg/m}^3$, $c_{p,\ell} = 4316$ J/kg·K, $\mu_\ell = 179 \times 10^{-6}\ \text{N·s/m}^2$, $Pr_\ell = 1.13$, $h_{fg} = 2107.8$ kJ/kg, $\sigma = 48.4 \times 10^{-3}$ N/m, $\rho_v = 2.629\ \text{kg/m}^3$.

ANALYSIS: The simple expression by Jakob [51] accounting for pressure effects is

$$h = C(\Delta T_e)^n (p/p_a)^{0.4} \tag{1}$$

where p and p_a are the system and standard atmospheric pressures. For a horizontal plate, C = 5.56 and n = 3 for the range $15 < q_s'' < 235\ \text{kW/m}^2$. For $\Delta T_e = 10°C$,

p = 2 bar $\quad h = 5.56(10)^3(2\text{bar}/1.0133\text{bar})^{0.4} = 7{,}298\ \text{W/m}^2\cdot\text{K}, \quad q_s'' = 73\ \text{kW/m}^2$ <

p = 5 bar $\quad h = 5.56(10)^3(5\text{bar}/1.0133\text{bar})^{0.4} = 10{,}529\ \text{W/m}^2\cdot\text{K}, \quad q_s'' = 105\ \text{kW/m}^2$ <

where $q_s'' = h\Delta T_e$. The Rohsenow correlation, Eq. 10.5, with $C_{s,f} = 0.013$ and n = 1, is of the form

$$q_s'' = \mu_\ell h_{fg}\left[\frac{g(\rho_\ell - \rho_v)}{\sigma}\right]^{1/2}\left[\frac{c_{p,\ell}\Delta T_e}{C_{s,f}h_{fg}Pr_\ell^n}\right]^3. \tag{2}$$

p = 2 bar: $$q_s' = 230.7\times10^{-6}\frac{\text{N}\cdot\text{s}}{\text{m}^2}\times2203\times10^3\frac{\text{J}}{\text{kg}}\left[\frac{9.8\frac{\text{m}}{\text{s}^2}(942.7-1.1082)\frac{\text{kg}}{\text{m}^3}}{54.97\times10^{-3}\ \text{N/m}}\right]^{1/2}\times\left[\frac{4244.3\text{J/kg}\cdot\text{K}\times10\text{K}}{0.013\times2203\times10^3\frac{\text{J}}{\text{kg}}\times1.43^1}\right]^3$$

$$q_s'' = 232\ \text{kW/m}^2$$ <

p = 4 bar: $$q_s'' = 439\ \text{kW/m}^2.$$ <

COMMENTS: For ease of comparison, the results with $p_a = 1.0133$ bar are:

Correlation/p (bar)	$q_s''\ (\text{kW/m}^2)$ 1	2	4
Simple	56	73	105
Rohsenow	135	232	439

Note that the range of q_s'' is within the limits of the Simple correlation. The comparison is poor and therefore the correlation is not to be recommended. By manipulation of the Rohsenow results, find that the $(p/p_o)^m$ dependence provides m ≈ 0.75, compared to the exponent of 0.4 in the Simple correlation.

PROBLEM 10.8

KNOWN: Diameter of copper pan. Initial temperature of water and saturation temperature of boiling water. Range of heat rates ($1 \le q \le 100$ kW).

FIND: (a) Variation of pan temperature with heat rate for boiling water, (b) Pan temperature shortly after start of heating with q = 8 kW.

SCHEMATIC:

Water
$T_i = 20^oC$
$T_{sat} = 100^oC$
q
Copper, T_s
D = 0.30 m

ASSUMPTIONS: (1) Conditions of part (a) correspond to steady nucleate boiling, (2) Surface of pan corresponds to polished copper, (3) Conditions of part (b) correspond to natural convection from a heated plate to an infinite quiescent medium, (4) Negligible heat loss to surroundings.

PROPERTIES: *Table A-6*, saturated water (T_{sat} = 100°C): $\rho_\ell = 957.9\ \text{kg/m}^3$, $\rho_v = 0.60\,\text{kg/m}^3$, $c_{p,\ell} = 4217\,\text{J/kg}\cdot\text{K}$, $\mu_\ell = 279\times10^{-6}\ \text{N}\cdot\text{s/m}^2$, $\text{Pr}_\ell = 1.76$, $h_{fg} = 2.257\times10^6\ \text{J/kg}$, $\sigma = 0.0589\,\text{N/M}$.
Table A-6, saturated water (assume T_s = 100°C, T_f = 60°C = 333 K): $\rho = 983$ kg/m^3, $\mu = 467 \times 10^{-6}$ N·s/m^2, k = 0.654 W/m·K, Pr = 2.99, $\beta = 523 \times 10^{-6}$ K^{-1}. Hence, $\nu = 0.475 \times 10^{-6}$ m^2/s, $\alpha = 0.159 \times 10^{-6}$ m^2/s.

ANALYSIS: (a) From Eq. (10.5),

$$\Delta T_e = T_s - T_{sat} = \frac{C_{s,f}\ h_{fg}\ \text{Pr}_\ell^n}{c_{p,\ell}} \times \left\{ \frac{q_s / \mu_\ell\, h_{fg}\, A_s}{\left[g\left(\rho_\ell - \rho_v\right)/\sigma\right]^{1/2}} \right\}^{1/3}$$

For n = 1.0, $C_{s,f}$ = 0.013 and $A_s = \pi D^2/4 = 0.0707$ m^2, the following variation of T_s with q_s is obtained.

Surface temperature (C)
125
120
115
110
105
100
0 20000 40000 60000 80000 100000
Heat rate (W)

<

As indicated by the correlation, the surface temperature increases as the cube root of the heat rate, permitting large increases in q for modest changes in T_s. For q = 1 kW, Ts = 104.7°C, which is barely sufficient to sustain boiling.

(b) Assuming $10^7 < Ra_L < 10^{11}$, the convection coefficient may be obtained from Eq. (9.31). Hence, with $L = A_s/P = D/4 = 0.075$m,

Continued …..

$$\bar{h} = \left(\frac{k}{L}\right)0.15\,Ra_L^{1/3} = \left(\frac{0.654\,W/m\cdot K}{0.075m}\right)0.15\left[\frac{9.8\,m/s^2 \times 523\times10^{-6}\,K^{-1}(T_s - T_i)(0.075m)^3}{0.475\times0.159\times10^{-12}\,m^4/s^2}\right]^{1/3}$$

$$= 1.308\left(2.86\times10^7\right)^{1/3}(T_s - T_i)^{1/3} = 400(T_s - T_i)^{1/3}$$

With As = $\pi D^2/4$ = 0.0707 m^2, the heat rate is then

$$q = \bar{h}A_s(T_s - T_i) = \left(400\,W/m^2\cdot K^{4/3}\right)0.0707\,m^2(T_s - T_i)^{4/3}$$

With q = 8000 W,

$$T_s = T_i + 69°C = 89°C$$ <

COMMENTS: (1) With $(T_s – T_i)$ = 69°C, $Ra_L = 1.97 \times 10^9$, which is within the assumed Rayleigh number range. (2) The surface temperature increases as the temperature of the water increases, and bubbles may nucleate when it exceeds 100°C. However, while the water temperature remains below the saturation temperature, the bubbles will collapse in the subcooled liquid.

PROBLEM 10.9

KNOWN: Fluids at 1 atm: mercury, ethanol, R-12.

FIND: Critical heat flux; compare with value for water also at 1 atm.

ASSUMPTIONS: (1) Steady-state conditions, (2) Nucleate pool boiling.

PROPERTIES: *Table A-5* and *Table A-6* at 1 atm,

	h_{fg} (kJ/kg)	ρ_v	r_ℓ (kg/m^3)	$\sigma \times 10^3$ (N/m)	T_{sat} (K)
Mercury	301	3.90	12,740	417	630
Ethanol	846	1.44	757	17.7	351
R-12	165	6.32	1,488	15.8	243
Water	2257	0.596	957.9	58.9	373

ANALYSIS: The critical heat flux can be estimated by the modified Zuber-Kutateladze correlation, Eq. 10.7,

$$q''_{max} = 0.149\ h_{fg}\ r_v \left[\frac{s\ g(r_\ell - r_v)}{r_v^2}\right]^{1/4}.$$

To illustrate the calculation procedure, consider numerical values for *mercury*.

$$q''_{max} = 0.149 \times 301 \times 10^3 \text{J/kg} \times 3.90\text{kg/m}^3 \times \left[\frac{417 \times 10^{-3}\text{N/m} \times 9.8\text{m/s}^2 (12{,}740 - 3.90)\text{kg/m}^3}{\left(3.90\text{kg/m}^3\right)^2}\right]^{1/4}$$

$$q''_{max} = 1.34\ \text{MW/m}^2.$$

For the other fluids, the results are tabulated along with the ratio of the critical heat fluxes to that for water.

Flluid	$q''_{max}\left(\text{MW/m}^2\right)$	$q''_{max} / q''_{max,water}$
Mercury	1.34	1.06
Ethanol	0.512	0.41
R-12	0.241	0.19
Water	1.26	1.00

<

COMMENTS: Note that, despite the large difference between mercury and water properties, their critical heat fluxes are similar.

PROBLEM 10.10

KNOWN: Copper pan, 150 mm diameter and filled with water at 1 atm, is maintained at 115°C.

FIND: Power required to boil water and the evaporation rate; ratio of heat flux to critical heat flux; pan temperature required to achieve critical heat flux.

SCHEMATIC:

ASSUMPTIONS: (1) Nucleate pool boiling, (2) Copper pan is polished surface.

PROPERTIES: *Table A-6*, Water (1 atm): T_{sat} = 100°C, ρ_ℓ = 957.9 kg/m^3, ρ_v = 0.5955 kg/m^3, $c_{p,\ell}$ = 4217 J/kg·K, μ_ℓ = 279 × 10^{-6} N·s/m^2, Pr_ℓ = 1.76, h_{fg} = 2257 kJ/kg, σ = 58.9 × 10^{-3} N/m.

ANALYSIS: The power requirement for boiling and the evaporation rate can be expressed as follows,

$$q_{boil} = q_s'' \cdot A_s \qquad \dot{m} = q_{boil} / h_{fg}.$$

The heat flux for nucleate pool boiling can be estimated using the Rohsenow correlation.

$$q_s'' = \mu_\ell h_{fg} \left[\frac{g(\rho_\ell - \rho_v)}{\sigma}\right]^{1/2} \left(\frac{c_{p,\ell}\Delta T_e}{C_{s,f}\, h_{fg}\, Pr_\ell^n}\right)^3.$$

Selecting $C_{s,f}$ = 0.013 and n = 1 from Table 10.1 for the polished copper finish, find

$$q_s'' = 279\times10^{-6}\,\frac{N\cdot s}{m^2}\times2257\times10^3\,\frac{J}{kg}\left[\frac{9.8\,\frac{m}{s^2}(957.9-0.5955)\frac{kg}{m^3}}{589\times10^{-3}\,N/m}\right]^{1/2}\left(\frac{4217\,\frac{J}{kg\cdot K}\times15°C}{0.013\times2257\times10^3\,\frac{J}{kg}\times1.76}\right)^3$$

$$q_s'' = 4.619\times10^5\,W/m^2.$$

The power and evaporation rate are

$$q_{boil} = 4.619\times10^5\,W/m^2\times\frac{\pi}{4}(0.150m)^2 = 8.16kW \qquad <$$

$$\dot{m}_{boil} = 8.16kW/2257\times10^3 J/kg = 3.62\times10^{-3} kg/s = 13 kg/h. \qquad <$$

The maximum or critical heat flux was found in Example 10.1 as

$$q_{max}'' = 1.26 MW/m^2.$$

Hence, the ratio of the operating to maximum heat flux is

$$\frac{q_s''}{q_{max}''} = 4.619\times10^5\,W/m^2/1.26MW/m^2 = 0.367. \qquad <$$

From the boiling curve, Fig. 10.4, $\Delta T_e \approx$ 30°C will provide the maximum heat flux. <

PROBLEM 10.11

KNOWN: Nickel-coated heater element exposed to saturated water at atmospheric pressure; thermocouple attached to the insulated, backside surface indicates a temperature $T_o = 266.4°C$ when the electrical power dissipation in the heater element is 6.950×10^7 W/m^3.

FIND: (a) From the foregoing data, calculate the surface temperature, T_s, and the heat flux at the exposed surface, and (b) Using an appropriate boiling correlation, estimate the surface temperature based upon the surface heat flux determined in part (a).

SCHEMATIC:

Saturated water
$T_{sat} = 100°C$
Nickel-coated surface, T_s
Current flow
Heater element
$k = 50$ W/m-K
$\dot{q} = 6.95 \times 10^7$ W/m^3
L = 15 mm
$T_o = 266.4°C$

ASSUMPTIONS: (1) Steady-state conditions, (2) Water exposed to standard atmospheric pressure and uniform temperature, T_{sat}, and (3) Nucleate pool boiling occurs on exposed surface, (4) Uniform volumetric generation in element, and (5) Backside of heater is perfectly insulated.

PROPERTIES: *Table A-6*, Saturated water, liquid (100°C): $\rho_\ell = 1/v_f = 957.9 \text{ kg/m}^3$, $c_{p,\ell} = c_{p,f} = 4.217 \text{ kJ/kg}\cdot\text{K}$, $\mu_\ell = \mu_f = 279\times10^{-6} \text{ N}\cdot\text{s/m}^2$, $Pr_\ell = Pr_f = 1.76$, $h_{fg} = 2257$ kJ/kg, $\sigma = 58.9 \times 10^{-3}$ N/m; Saturated water, vapor (100°C): $\rho_v = 1/v_g = 0.5955$ kg/m^3.

ANALYSIS: (a) From Eq. 3.43, the temperature at the exposed surface, T_s, is

$$T_s = T_o - \frac{\dot{q}L^2}{2k} = 266.4°C - \frac{6.95\times10^7 \text{ W/m}^3 (0.015 \text{ m})^2}{2\times50 \text{ W/m}\cdot\text{K}}$$

$$T_s = 110.0°C$$ <

The heat flux at the exposed surface is

$$q''_s = \dot{q}/L = 6.95\times10^7 \text{ W/m}^3 / 0.015 \text{ m} = 4.63\times10^9 \text{ W/m}^2$$ <

(b) Since $\Delta T_e = T_s - T_{sat} = (110 - 100)°C = 10°C$, nucleate pool boiling occurs and the Rohsenow correlation, Eq. 10.5, with q''_s from part (a) can be used to estimate the surface temperature, $T_{s,c}$,

$$q''_s = \mu_\ell h_{fg} \left[\frac{g(\rho_\ell - \rho_v)}{\sigma}\right]^{1/2} \left(\frac{c_{p,\ell}\,\Delta T_{e,c}}{C_{s,f}\, h_{fg}\, Pr_\ell^n}\right)^3$$

From Table 10.1, for the water-nickel surface-fluid combination, $C_{s,f} = 0.006$ and $n = 1.0$. Substituting numerical values, find $\Delta T_{e,c}$ and $T_{s,c}$.

Continued …..

$$4.63\times10^{9}\ \mathrm{W/m^2} = 279\times10^{-6}\,\mathrm{N\cdot s/m^2}\times2257\times10^{3}\,\mathrm{J/kg}$$

$$\times\left[\frac{9.8\ \mathrm{m/s^2}\,(957.9-0.5955)\,\mathrm{kg/m^3}}{58.9\times10^{-3}\,\mathrm{N/m}}\right]^{1/2}$$

$$\times\left(\frac{4.217\times10^{3}\,\mathrm{J/kg\cdot K}\times\Delta T_{e,c}}{0.006\times2257\times10^{3}\,\mathrm{J/kg}\times1.76}\right)^{3}$$

$$\Delta T_{e,c} = T_{s,c} - T_{sat} = 9.1^\circ\mathrm{C} \qquad T_{s,c} = 109.1^\circ\mathrm{C}$$ $<$

COMMENTS: From the experimental data, part (a), the surface temperature is determined from the conduction analyses as T_s = 110.0°C. Using the traditional nucleate boiling correlation with the experiential value for the heat flux, the surface temperature is estimated as $T_{s,c}$ = 109.1°C. The two approaches provide excess temperatures that are 10.0 vs. 9.1°C, which amounts to nearly a 10% difference.

PROBLEM 10.12

KNOWN: Chips on a ceramic substrate operating at power levels corresponding to 50% of the critical heat flux.

FIND: (a) Chip power level and temperature rise of the chip surface, and (b) Compute and plot the chip temperature T_s as a function of heat flux for the range $0.25 \le q''_s / q''_{max} \le 0.90$.

SCHEMATIC:

ASSUMPTIONS: (1) Nucleate boiling, (2) Fluid-surface with $C_{s,f}$ = 0.004, n = 1.7 for Rohsenow correlation, (3) Backside of substrate insulated.

PROPERTIES: *Table A-5,* Refrigerant R-113 (1 atm): T_{sat} = 321 K = 48°C, ρ_ℓ = 1511 kg/m^3, ρ_v = 7.38 kg/m^3, h_{fg} = 147 kJ/kg, $\sigma = 15.9 \times 10^{-3}$ N/m; R-113, sat. liquid (given, 321 K): $c_{p,\ell}$ = 983.8 J/kg·K, $\mu_\ell = 5.147 \times 10^{-4}$ N·s/m^2, Pr_ℓ = 7.183.

ANALYSIS: (a) The operating power level (flux) is $0.50\, q''_{max}$, where the critical heat flux is estimated from Eq. l0.7 for nucleate pool boiling,

$$q''_{max} = 0.149 h_{fg} \rho_v \left[\sigma g \left(\rho_\ell - \rho_v \right) / \rho_v^2 \right]^{1/4}$$

$$q''_{max} = 0.149 \times 147 \times 10^3 \frac{J}{kg} \times 7.38 \frac{kg}{m^3} \left[15.9 \times 10^{-3} \frac{N}{m} \times 9.8 \frac{m}{s^2} (1511 - 7.38) \frac{kg}{m^3} / \left(7.38 \frac{kg}{m^3} \right)^2 \right]^{1/4}$$

$$q''_{max} = 233\, kW/m^2 .$$

Hence, the heat flux on a chip is 0.5×233 kW/m^2 = 116 kW/m^2 and the power level is

$$q_{chip} = q''_s \times A_s = 116 \times 10^3\, W/m^2 \times 25\, mm^2 \left(10^{-3}\, m/mm \right)^2 = 2.9\, W. \qquad <$$

To determine the chip surface temperature for this condition, use the Rohsenow equation to find $\Delta T_e = T_s - T_{sat}$ with $q''_s = 116 \times 10^3$ W/m^2. The correlation, Eq. 10.5, solved for ΔT_e is

$$\Delta T_e = \frac{C_{s,f} h_{fg} Pr_\ell^n}{c_{p,\ell}} \left(\frac{q''_s}{\mu_\ell h_{fg}} \right)^{1/3} \left[\frac{\sigma}{g(\rho_\ell - \rho_v)} \right]^{1/6} = \frac{0.004 \times 147 \times 10^3\, J/kg\, (7.18)^{1.7}}{983.8\, J/kg \cdot K} \times$$

$$\left(\frac{116 \times 10^3\, W/m^2 \cdot}{5.147 \times 10^{-4} \frac{N \cdot s}{m^2} \times 147 \times 10^3 \frac{J}{kg}} \right)^{1/3} \left[\frac{15.9 \times 10^{-3}\, N/m}{9.8 \frac{m}{s^2} (1511 - 7.38) \frac{kg}{m^3}} \right]^{1/6} = 19.9^\circ C.$$

Hence, the chip surface temperature is

Continued...

PROBLEM 10.12 (Cont.)

$$T_s = T_{sat} + \Delta T_e = 48^\circ C + 19.9^\circ C \approx 68^\circ C. \qquad <$$

(b) Using the *IHT Correlations Tools, Boiling, Nucleate Pool Boiling -- Heat flux* and *Maximum heat flux,* the chip surface temperature, T_s, was calculated as a function of the ratio q''_s / q''_{max}. The required thermophysical properties as provided in the problem statement were entered directly into the IHT workspace. The results are plotted below.

COMMENTS: (1) Refrigerant R-113 is attractive for electronic cooling since its boiling point is slightly above room temperature. The reliability of electronic devices is highly dependent upon operating temperature.

(2) A copy of the *IHT Workspace* model used to generate the above plot is shown below.

```
// Correlations Tool - Boiling, Nucleate pool boiling, Critical heat flux
q"max = qmax_dprime_NPB(rhol,rhov,hfg,sigma,g)          // Eq 10.7
g = 9.8                         // Gravitational constant, m/s^2
/*  Correlation description: Critical (maximum) heat flux for nucleate pool boiling (NPB). Eq 10.7, Table
10.1 . See boiling curve, Fig 10.4 . */
// Correlations Tool - Boiling, Nucleate pool boiling, Heat flux
qs" = qs_dprime_NPB(Csf,n,rhol,rhov,hfg,cpl,mul,Prl,sigma,deltaTe,g)    // Eq 10.5
//g = 9.8                       // Gravitational constant, m/s^2
deltaTe = Ts - Tsat             // Excess temperature, K
Ts =  Ts_C + 273                // Surface temperature, K
Ts_C = 68                       // Surface temperature, C
//Tsat =                        // Saturation temperature, K
/* Evaluate liquid(l) and vapor(v) properties at Tsat. From Table 10.1.  */
// Fluid-surface combination:
Csf =   0.004                   // Given
n =   1.7                       // Given
/*  Correlation description: Heat flux for nucleate pool boiling (NPB), water-surface combination (Cf,n), Eq
10.5, Table 10.1 . See boiling curve, Fig 10.4 . */
// Heat rates:
qsqm = qs" / q"max              // Ratio, heat flux over critical heat flux
qsqm = 0.5
// Thermophysical Properties (Given):
Tsat = 321                      // Saturation temperature, K
Tsat_C = Tsat - 273             // Saturation temperature, C
rhol = 1511                     // Density, liquid, kg/m^3
rhov = 7.38                     // Density, vapor, kg/m^3
hfg = 147000                    // Heat of vaporization, J/kg
sigma = 15.9e-3                 // Surface tension/ N/m
cpl = 983.3                     // Specific heat, saturated liquid, J/kg.K
mul  = 5.147e-4                 // Viscosity, saturated liquid, N.s/m^2
Prl = 7.183                     // Prandtl number, saturated liquid
```

PROBLEM 10.13

KNOWN: Saturated ethylene glycol at 1 atm heated by a chromium-plated heater of 200 mm diameter and maintained at 480K.

FIND: Heater power, rate of evaporation, and ratio of required power to maximum power for critical heat flux.

SCHEMATIC:

Ethylene glycol
1atm
q_s''
Chromium plated heater,
T_s=480K, D=200mm

ASSUMPTIONS: (1) Nucleate pool boiling, (2) Fluid-surface, $C_{s,f}$ = 0.010 and n = 1.

PROPERTIES: *Table A-5*, Saturated ethylene glycol (1atm): T_{sat} = 470K, h_{fg} = 812 kJ/kg, ρ_f = 1111 kg/m^3, $\sigma = 32.7 \times 10^{-3}$ N/m; Saturated ethylene glycol (given, 470K): ρ_v = 1.66kg/m^3, $m_\ell = 0.38 \times 10^{-3}$ N·s/m^2, $c_{p,\ell}$ = 3280 J/kg·K, Pr_ℓ = 8.7, k_ℓ = 0.15 W/m·K.

ANALYSIS: The power requirement for boiling and the evaporation rate are $q_{boil} = q_s'' \cdot A_s$ and $\dot{m} = q_{boil}/h_{fg}$. Using the Rohsenow correlation,

$$q_s'' = m_\ell\, h_{fg}\left[\frac{g(r_\ell - r_v)}{s}\right]^{1/2}\left(\frac{c_{p,\ell}\,\Delta T_e}{C_{s,f}\,h_{fg}\,Pr_\ell^n}\right)^3$$

$$q_s'' = 0.38\times10^{-3}\,\frac{N\cdot s}{m^2}\times 812\times10^3\,\frac{J}{kg}\left[\frac{9.8\,m/s^2\,(1111-1.66)\,kg/m^3}{32.7\times10^{-3}\,N/m}\right]^{1/2}\left(\frac{3280\,J/kg\cdot K(480-470)K}{0.01\times812\times10^3\,\frac{J}{kg}(8.7)^1}\right)^3$$

$$q_s'' = 1.78\times10^4\,W/m^2 \qquad q_{boil} = 1.78\times10^4\,W/m^2 \times p/4(0.200m)^2 = 559\ W$$ <

$$\dot{m} = 559\,W/812\times10^3\ J/kg = 6.89\times10^{-4}\,kg/s.$$ <

For this fluid, the critical heat flux is estimated from Eq. 10.7,

$$q_{max}'' = 0.149\ h_{fg}\ r_v\left[s\ g(r_\ell - r_v)/r_v^2\right]^{1/4}$$

$$q_{max}'' = 0.149\times812\times10^3\,\frac{J}{kg}\times1.66\,\frac{kg}{m^3}\left[\frac{32.7\times10^{-3}\,N/m\times9.8\,m/s^2\,(1111-1.66)\,kg/m^3}{\left(1.66\,kg/m^3\right)^2}\right]^{1/4}$$

$$q_{max}'' = 6.77\times10^5\,W/m^2.$$

Hence, the ratio of the operating heat flux to the critical heat flux is,

$$\frac{q_s''}{q_{max}''} = \frac{1.78\times10^4\,W/m^2}{6.77\times10^5\,W/m^2} \approx 0.026 \qquad \text{or} \qquad 2.6\%.$$ <

COMMENTS: Recognize that the results are crude approximations since the values for $C_{s,f}$ and n are just estimates. This fluid is not normally used for boiling processes since it decomposes at higher temperatures.

PROBLEM 10.14

KNOWN: Copper tubes, 25 mm diameter × 0.75 m long, used to boil saturated water at 1 atm operating at 75% of the critical heat flux.

FIND: (a) Number of tubes, N, required to evaporate at a rate of 750 kg/h; tube surface temperature, T_s, for these conditions, and (b) Compute and plot T_s and N required to provide the prescribed vapor production as a function of the heat flux ratio, $0.25 \le q''_s/q''_{max} \le 0.90$.

SCHEMATIC:

Water, 1 atm

q''_s

Copper tube, D = 25 mm, T_s

Conditions:
heat flux 75% of q''_{max},
evaporation rate 750 kg/h

L = 0.75 m

ASSUMPTIONS: (1) Nucleate pool boiling, (2) Polished copper tube surfaces.

PROPERTIES: *Table A-6*, Saturated water (100°C): ρ_ℓ = 957.9 kg/m^3, $c_{p,\ell}$ = 4217 J/kg·K, μ_ℓ = 279 × 10^{-6} N·s/m^2, Pr_ℓ = 1.76, h_{fg} = 2257 kJ/kg, σ = 58.9 × 10^{-3} N/m, ρ_v = 0.5955 kg/m^3.

ANALYSIS: (a) The total number of tubes, N, can be evaluated from the rate equations

$$q = q''_s A_s = q''_s N\pi DL \qquad q = \dot{m}h_{fg} \qquad N = \dot{m}h_{fg}/q''_s \pi DL. \qquad (1,2,3)$$

The tubes are operated at 75% of the critical flux (1.26 MW/m^2, see Example 10.1). Hence, the heat flux is

$$q''_s = 0.75\, q''_{max} = 0.75\times 1.26\ \text{MW/m}^2 = 9.45\times 10^5\ \text{W/m}^2.$$

Substituting numerical values into Eq. (3), find

$$N = 750\,\text{kg/h}\,(1\text{h}/3600\text{s})\times 2257\times 10^3\ \text{J/kg}\Big/\left(9.45\times 10^5\ \text{W/m}^2\times\pi\times 0.025\,\text{m}\times 0.75\,\text{m}\right) = 8.5 \approx 9. \quad <$$

To determine the tube surface temperature, use the Rohsenow correlation,

$$\Delta T_e = \frac{C_{s,f}\, h_{fg}\, Pr_\ell^n}{c_{p,\ell}}\left(\frac{q''_s}{\mu_\ell h_{fg}}\right)^{1/3}\left[\frac{\sigma}{g(\rho_\ell - \rho_v)}\right]^{1/6}.$$

From Table 10.1 for the polished copper-water combination, $C_{s,f}$ = 0.013 and n = 1.0.

$$\Delta T_e = \frac{0.013\times 2257\times 10^3\ \text{J/kg}\,(1.76)^1}{4217\ \text{J/kg}\cdot\text{K}}\left(\frac{9.45\times 10^5\ \text{W/m}^2}{279\times 10^{-6}\ \text{N}\cdot\text{s/m}^2\times 2257\times 10^3\ \text{J/kg}}\right)^{1/3}\times$$

$$\left[\frac{58.9\times 10^{-3}\ \text{N/m}}{9.8\,\text{m/s}^2\,(957.9 - 0.5955)\,\text{kg/m}^3}\right]^{1/6} = 19.0^\circ\text{C}.$$

Hence,

$$T_s = T_{sat} + \Delta T_e = (100 + 19)^\circ\text{C} = 119^\circ\text{C}. \quad <$$

Continued...

PROBLEM 10.14 (Cont.)

(b) Using the *IHT Correlations Tool*, *Boiling*, *Nucleate Pool Boiling*, *Heat flux* and the *Properties Tool* for *Water*, combined with Eqs. (1,2,3) above, the surface temperature T_s and N can be computed as a function of q''_s/q''_{max}. The results are plotted below.

Ts (C) and N

300
200
100
0

0.2 0.4 0.6 0.8 1

Ratio q"s/q"max

Tube surface temperature, Ts (C)
Number of tubes, N*10

Note that the tube surface temperature increases only slightly (113 to 120°C) as the ratio q''_s/q''_{max} increases. The number of tubes required to provide the prescribed evaporation rate decreases markedly as q''_s/q''_{max} increases.

COMMENTS: (1) The critical heat flux, q''_{max} =1.26 MW/m^2, for saturated water at 1 atm is calculated in Example 10.1 using the Zuber-Kutateladze relation, Eq. 10.7. The *IHT Correlation Tool, Boiling*, *Nucleate pool boiling*, *Maximum heat flux*, with the *Properties Tool* for *Water* could also be used to determine q''_{max}.

PROBLEM 10.15

KNOWN: Diameter and length of tube submerged in pressurized water. Flowrate and inlet temperature of gas flow through the tube.

FIND: Tube wall and gas outlet temperatures.

SCHEMATIC:

Tube
L = 8 m
D = 0.025 m
N = 5
T_s
$T_{m,o}$
Boiling water
T_{sat} = 420 K
Combustion gas
$T_{m,i}$ = 973 K
$\dot{m}$ = 0.08 kg/s
q_{cnv}
q_{nb}

ASSUMPTIONS: (1) Steady-state, (2) Uniform tube wall temperature, (3) Nucleate boiling at outer surface of tube, (4) Fully developed flow in tube, (5) Negligible flow work and potential and kinetic energy changes for tube flow, (7) Constant properties.

PROPERTIES: *Table A-6*, saturated water (p_{sat} = 4.37 bars): T_{sat} = 420 K, $h_{fg} = 2.123 \times 10^6$ J/kg, $\rho_\ell = 919\,\text{kg/m}^3$, $\rho_v = 2.4\,\text{kg/m}^3$, $\mu_\ell = 185\times10^{-6}\,\text{N}\cdot\text{s/m}^2$, $c_{p,\ell} = 4302\,\text{J/kg}\cdot\text{K}$, $Pr_\ell = 2.123\times 10^6\,\text{J/kg}\cdot\text{K}$, $\sigma = 0.0494$ N/m. *Table A-4*, air $(p = 1\,\text{atm}, \overline{T}_m \approx 700\,\text{K})$: c_p = 1075 J/kg·K, $\mu = 339 \times 10^{-7}$ N·s/m^2, k = 0.0524 W/m·K, Pr = 0.695.

ANALYSIS: From an energy balance performed for a control surface that bounds the tube, we know that the rate of heat transfer by convection from the gas to the inner surface must equal the rate of heat transfer due to boiling at the outer surface. Hence, from Eqs. (8.44), (8.45) and (10.5), the energy balance for a single tube is of the form

$$\overline{h}A_s\left[\frac{\Delta T_o - \Delta T_i}{\ln\left(\Delta T_o/\Delta T_i\right)}\right] = A_s\mu_\ell h_{fg}\left[\frac{g\left(\rho_\ell - \rho_v\right)}{\sigma}\right]^{1/2}\left(\frac{c_{p,\ell}\Delta T_e}{C_{s,f}\,h_{fg}\,Pr_\ell^n}\right)^3 \quad (1)$$

where $\overline{U} = \overline{h}$ and $C_{s,f} = 0.013$ and n = 1.0 from Table 10.1. The corresponding unknowns are the wall temperature T_s and gas outlet temperature, $T_{m,o}$, which are also related through Eq. (8.43).

$$\frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp\left(-\frac{\pi DL}{\dot{m}c_p}\overline{h}\right) \quad (2)$$

For $Re_D = 4\dot{m}/\pi D\mu = 119,600$, the flow is turbulent, and with n = 0.3, Eq. (8.60) yields,

$$\overline{h} = h_{fd} = \left(\frac{k}{D}\right)0.023\,Re_D^{4/5}\,Pr^{0.3} = \left(\frac{0.0524\,\text{W/m}\cdot\text{K}}{0.025\text{m}}\right)0.023\left(119,600\right)^{4/5}\left(0.695\right)^{0.3} = 502\,\text{W/m}^2\cdot\text{K}$$

Solving Eqs. (1) and (2), we obtain

$$T_s = 152.6°\text{C}, \qquad T_{m,o} = 166.7°\text{C}$$ <

COMMENTS: (1) The heat rate per tube is $q = \dot{m}c_p\,(T_{m,i} - T_{m,o})$ = 45,930 W, and the total heat rate is Nq = 229,600 W, in which case the rate of steam production is $\dot{m}_{steam} = q/h_{fg} = 0.108\,\text{kg/s}$.

(2) It would not be possible to maintain isothermal tube walls without a large wall thickness, and T_s, as well as the intensity of boiling, would decrease with increasing distance from the tube entrance. However the foregoing analysis suffices as a first approximation.

PROBLEM 10.16

KNOWN: Nickel wire passing current while submerged in water at atmospheric pressure.

FIND: Current at which wire burns out.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Pool boiling.

ANALYSIS: The burnout condition will occur when electrical power dissipation creates a surface heat flux exceeding the critical heat flux, q''_{max}. This burn out condition is illustrated on the boiling curve to the right and in Figure 10.6.

The criterion for burnout can be Expressed as

$$q''_{max} \cdot \pi D = q'_{elec} \qquad q'_{elec} = I^2 R'_e. \tag{1,2}$$

That is,

$$I = [q''_{max} \pi D / R'_e]^{1/2}. \tag{3}$$

For pool boiling of water at 1 atm, we found in Example 10.1 that

$$q''_{max} = 1.26 \text{MW/m}^2.$$

Substituting numerical values into Eq. (3), find

$$I = \left[1.26\times10^6 \text{W/m}^2 (\pi \times 0.001\text{m})/0.129\Omega/\text{m}\right]^{1/2} = 175\text{A}. \qquad <$$

COMMENTS: The magnitude of the current required to burn out the 1 mm diameter wire is very large. What current would burn out the wire in air?

PROBLEM 10.17

KNOWN: Saturated water boiling on a brass plate maintained at 115°C.

FIND: Power required (W/m^2) for pressures of 1 and 10 atm; fraction of critical heat flux at which plate is operating.

SCHEMATIC:

Water, 1 or 10atm

$\Delta T_e = 15°C$,
Brass plate

ASSUMPTIONS: (1) Nucleate pool boiling, (2) ΔT_e = 15°C for both pressure levels.

PROPERTIES: *Table A-6*, Saturated water, liquid (1 atm, T_{sat} = 100°C): r_ℓ = 957.9 kg/m^3, $c_{p,\ell}$ = 4217 J/kg·K, m_ℓ =279 × 10^{-6} N·s/m^2, Pr = 1.76, h_{fg} = 2257 kJ/kg, σ = 58.9 × 10^{-3} N/m; *Table A-6*, Saturated water, vapor (1 atm): ρ_v = 0.596 kg/m^3; *Table A-6*, Saturated water, liquid (10 atm = 10.133 bar, T_{sat} = 453.4 K = 180.4°C): r_ℓ = 886.7 kg/m^3, $c_{p,\ell}$ = 4410 J/kg·K, m_ℓ = 149 × 10^{-6} N·s/m^2, Pr_ℓ = 0.98, h_{fg} = 2012 kJ/kg, σ = 42.2 × 10^{-3} N/m; *Table A-6*, Water, vapor (10.133 bar): ρ_v = 5.155 kg/m^3.

ANALYSIS: With ΔT_e = 15°C, we expect nucleate pool boiling. The Rohsenow correlation with $C_{s,f}$ = 0.006 and n = 1.0 for the brass-water combination gives

$$q''_s = m_\ell h_{fg}\left[\frac{g(r_\ell - r_v)}{s}\right]^{1/2}\left(\frac{c_{p,\ell}\Delta T_e}{C_{s,f}\, h_{fg}\, Pr_\ell^n}\right)^3$$

1 atm:
$$q''_s = 279\times10^{-6}\,N\cdot s/m^2 \times 2257\times10^3\,J/kg\left[\frac{9.8\,m/s^2\,(957.9-0.596)\,kg/m^3}{58.9\times10^{-3}\,N/m}\right]^{1/2}\times$$

$$\left(\frac{4217\,J/kg\cdot K\times15K}{0.006\times2257\times10^3\,J/kg\times1.76^1}\right)^3 = 4.70\,MW/m^2$$

10 atm: $\quad q''_s = 23.8\,MW/m^2$

From Example 10.1, $q''_{max}(1atm) = 1.26\,MW/m^2$. To find the critical heat flux at 10 atm, use the correlation of Eq. 10.7,

$$q''_{max} = 0.149\, h_{fg}\, r_v\left[s\, g(r_\ell - r_v)/r_v^2\right]^{1/4}.$$

$$q''_{max}(10atm) = 0.149\times2012\times10^3\,J/kg\times5.155\,kg/m^3\times$$

$$\left[\frac{42.2\times10^{-3}\,N/m\times9.8\,m/s^2\,(886.7-5.16)\,kg/m^3}{\left(5.155\,kg/m^3\right)^2}\right]^{1/4} = 2.97\,MW/m^2.$$

For both conditions, the Rohsenow correlation predicts a heat flux that exceeds the maximum heat flux, q''_{max}. We conclude that the boiling condition with ΔT_e = 15°C for the brass-water combination is beyond the inflection point (P, see Fig. 10.4) where the boiling heat flux is no longer proportional to ΔT_e^3.

$$q''_s \approx q''_{max}(1atm) \le 1.26\,MW/m^2 \qquad\qquad q''_s \approx q''_{max}(10\ atm) \le 2.97\,MW/m^2.$$ <

PROBLEM 10.18

KNOWN: Zuber-Kutateladze correlation for critical heat flux, q''_{max}.

FIND: Pressure dependence of q''_{max} for water; demonstrate maximum value occurs at approximately 1/3 p_{crit}; suggest coordinates for a universal curve to represent other fluids.

ASSUMPTIONS: Nucleate pool boiling conditions.

PROPERTIES*:* *Table A-6*, Water, saturated at various pressures; see below.

ANALYSIS: The Z-K correlation for estimating the critical heat flux, has the form

$$q''_{max} = 0.149\ r_v h_{fg} \left[\frac{g\, s\, (r_\ell - r_v)}{r_v^2} \right]^{1/4}$$

where the properties for saturation conditions are a function of pressure. The properties (Table A-6) and the values for q''_{max} are as follows:

p (bar)	p/p_c	r_ℓ (kg/m^3)	r_v (kg/m^3)	h (kJ/kg)	$\sigma \times 10^3$ (N/m)	q''_{max} (MW/m^2)
1.01	0.0045	957.9	0.5955	2257	58.9	1.258
11.71	0.053	879.5	5.988	1989	40.7	3.138
26.40	0.120	831.3	13.05	1825	31.6	3.935
44.58	0.202	788.1	22.47	1679	24.5	4.398
61.19	0.277	755.9	31.55	1564	19.7	4.549
82.16	0.372	718.4	43.86	1429	15.0	4.520
123.5	0.557	648.9	72.99	1176	8.4	4.047
169.1	0.765	562.4	117.6	858	3.5	2.905
221.2	1.000	315.5	315.5	0	0	0

The q''_{max} values are plotted as a function of p/p_c, where p_c is the critical pressure. Note the rapid decrease of hfg and σ with increasing pressure. The universal curve coordinates would be $q''_{max} / q''_{max}\ (1/3\, p_{crit})$ vs. p / p_c.

PROBLEM 10.19

KNOWN: Kutateladze's dimensional analysis and the bubble diameter parameter.

FIND: (a) Verify the dimensional consistency of the critical heat flux expression, and (b) Estimate heater size with water at 1 atm required such that the Bond number will exceed 3, i.e., Bo ≥ 3.

ASSUMPTIONS: Nucleate pool boiling.

ANALYSIS: (a) Kutateladze postulated that the critical flux was dependent upon four parameters,

$$q''_{max} = q''_{max}\left(h_{fg}, r_v, s\ ,\ D_b\right)$$

where D_b is the bubble diameter parameter having the form

$$D_b = \left[s\ /\ g\left(r_\ell - r_v\right)\right]^{1/2}. \tag{1}$$

The form of the critical heat flux expression was presumed to be

$$q''_{max} = C\ h_{fg}\ \ r_v^{1/2}\ D_b^{-1/2}\ s^{1/2} \tag{2}$$

where C is a constant. It is not possible to derive this equation from a dimensional (Pi) analysis. We can only determine that the equation is dimensionally consistent. Using SI units, check Eq. (1) for D_b,

$$D_b \Rightarrow \left[\left(Nm^{-1}\right)\left(m^{-1}s^2\right)\left(kg^{-1}m^3\right)\right]^{1/2} \Rightarrow \left[N\left(\frac{s^2}{kg\cdot m^2}\right)m^3\right]^{1/2} \Rightarrow [m]$$

and in Eq. (2) for q''_{max},

$$q''_{max} \Rightarrow \left[\left(Jkg^{-1}\right)\left(kg^{1/2}m^{-3/2}\right)\left(m^{-1/2}\right)\left(N^{1/2}m^{-1/2}\right)\right] \Rightarrow \left[\frac{J}{s}\cdot\left(\frac{N\cdot s^2}{kg\cdot m}\right)^{1/2} m^{-2}\right] \Rightarrow \left[\frac{W}{m^2}\right].$$

Hence, the equations are dimensionally consistent.

(b) The Bond number, Bo, is defined as the ratio of the characteristic length L (width or diameter) of the heater surface to the bubble diameter parameter, D_b. That is, Bo ≡ L/D_B. The number squared is also indicative of the ratio of the buoyant to capillary forces. For water at 1 atm (see Example 10.1 for properties listing), Eq. (1) yields

$$D_b = \left[58.9\times10^{-3}\frac{N}{m}/9.8\frac{m}{s^2}(957.9 - 0.5955)\frac{kg}{m^3}\right]^{1/2} = 0.0025m = 2.5mm.$$

Eq. 10.7 for the critical heat flux is appropriate for an "infinite" heater (Bo ≥ 3). To meet this requirement, the heater dimension must be

$$L \geq Bo \cdot D_b = 3\times2.5mm = 7.5mm. \qquad <$$

COMMENTS: As the heater size decreases (Bo decreasing), the boiling curve no longer exhibits the characteristic q''_{max} and q''_{min} features. The very small heater, such as a wire, is enveloped with vapor at small ΔT_e and film boiling occurs.

PROBLEM 10.20

KNOWN: Lienhard-Dhir critical heat flux correlation for small horizontal cylinders.

FIND: Critical heat flux for 1 mm and 3 mm diameter horizontal cylinders in water at 1 atm.

SCHEMATIC:

Water, 1 atm

1 or 3mm diameter cylinder

ASSUMPTIONS: Nucleate pool boiling.

PROPERTIES: *Table A-6*, Water (1 atm): $\rho_\ell = 957.9$ kg/m^3, $\rho_v = 0.5955$ kg/m^3, $\sigma = 58.9 \times 10^{-3}$ N/m.

ANALYSIS: The Lienhard-Dhir correlation for small horizontal cylinders is

$$q''_{max} = q''_{max,Z}\left[0.94\,(Bo)^{-1/4}\right] \qquad 0.15 \le Bo \le 1.2 \qquad (1)$$

where $q''_{max,Z}$ is the critical heat flux predicted by the Zuber-Kutateladze correlation for the infinite heater (Eq. 10.6) and the Bond number is

$$Bo = \frac{r}{D_b} = r/\left[\sigma / g\left(\rho_\ell - \rho_v\right)\right]^{1/2}. \qquad (2)$$

Note the characteristic length is the cylinder radius. From Example 10.1, using Eq. 10.6,

$$q''_{max,Z} = 1.11\,MW/m^2$$

and substituting property values for water at 1 atm into Eq. (2),

$$D_b = \left[58.9\times10^{-3}\,\frac{N}{m}/9.8\frac{m}{s^2}(957.9-0.5955)\frac{kg}{m^3}\right]^{1/2} = 2.51 mm.$$

Substituting appropriate values into Eqs. (1) and (2),

1 mm dia cylinder Bo = 0.5 mm/2.51 mm = 0.20

$$q''_{max} = 1.11\,MW/m^2\left[0.94(0.20)^{-1/4}\right] = 1.56\,MW/m^2. \qquad <$$

3 mm dia cylinder Bo = 1.5 mm/2.51 mm = 0.60

$$q''_{max} = 1.11\,MW/m^2\left[0.94(0.60)^{-1/4}\right] = 1.19\ MW/m^2. \qquad <$$

Note that for the 3 mm diameter cylinder, the critical heat flux is 1.19/1.11 = 1.07 times larger than the value for a very large horizontal cylinder.

COMMENTS: For practical purposes a horizontal cylinder of diameter greater than 3 mm can be considered as a very large one. The critical heat flux for a 1 mm diameter cylinder is 40% larger than that for the large cylinder.

PROBLEM 10.21

KNOWN: Boiling water at 1 atm pressure on moon where the gravitational field is 1/6 that of the earth.

FIND: Critical heat flux.

ASSUMPTIONS: Nucleate pool boiling conditions.

PROPERTIES: *Table A-6*, Water (1 atm): T_{sat} = 100°C, ρ_ℓ = 957.9 kg/m^3, ρ_v = 0.5955 kg/m^3, h_{fg} = 2257 kJ/kg, $\sigma = 58.9 \times 10^{-3}$ N/m.

ANALYSIS: The modified Zuber-Kutateladze correlation for the critical heat flux is Eq. 10.7.

$$q''_{max} = 0.149\, \rho_v^{1/2} h_{fg} \left[\sigma\, g \left(\rho_\ell - \rho_v \right) \right]^{1/4}.$$

The relation predicts the critical flux dependency on the gravitational acceleration as

$$q''_{max} \sim g^{1/4}.$$

It follows that if $g_{moon} = (1/6)\, g_{earth}$ and recognizing $q''_{max,e}$ = 1.26 MW/m^2 for earth acceleration (see Example 10.1),

$$q''_{max,moon} = q''_{max,earth} \left(g_{moon} / g_{earth} \right)^{1/4}$$

$$q''_{max,moon} = 1.26 \frac{\text{MW}}{\text{m}^2} \left(\frac{1}{6} \right)^{1/4} = 0.81\,\text{MW/m}^2.$$ <

COMMENTS: Note from the discussion in Section 10.4.5 that the g1/4 dependence on the critical heat flux has been experimentally confirmed. In the nucleate pool boiling regime, the heat flux is nearly independent of the gravitational field.

PROBLEM 10.22

KNOWN: Concentric stainless steel tubes packed with dense boron nitride powder. Inner tube has heat generation while outer tube surface is exposed to boiling heat flux, $q_s'' = C\left(T_s - T_{sat}\right)^3$. Saturation temperature of boiling liquid and temperature of the outer tube surface.

FIND: Expressions for the maximum temperature in the stainless steel (ss) tubes and in the boron nitride (bn).

SCHEMATIC:

Boron nitride (bn), k_{bn}
Tube, stainless steel (ss), $\dot{q}$, k_{ss}
Tube, ss
r_1, r_2, r_3, r_4
T_s
Boiling liquid, T_{sat}
$q''_{boil} = q''_s = C(T_s - T_{sat})^3$
q'_{gen} → T_2 — R'_{23} — T_3 — R'_{34} — $T_4 = T_s$ → q'_{boil}

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional (cylindrical) steady-state heat transfer in tubes and boron nitride.

ANALYSIS: Construct the thermal circuit shown above where R'_{23} and R'_{34} represent the resistances due to the boron nitride between r_2 and r_3 and to the outer stainless steel tube, respectively. From an overall energy balance,

$$q'_{gen} = q'_{boil},$$

$$\dot{q}\pi\left(r_2^2 - r_1^2\right) = \left(2\pi r_4\right) C\left(T_s - T_{sat}\right)^3.$$

With prescribed values for T_{sat}, T_s and C, the required volumetric heating of the inner stainless steel tube is

$$\dot{q} = \frac{2r_4}{\left(r_2^2 - r_1^2\right)} C\left(T_s - T_{sat}\right)^3.$$

Using the thermal circuit, we can write expressions for the *maximum* temperature of the stainless steel (ss) and boron nitride (bn).

Stainless steel: $T_{ss,max}$ occurs at r_1. Using the results of Section 3.4.2, the temperature distribution in a radial tube of inner and outer radii r_1 and r_2 is

$$T(r) = -\frac{\dot{q}}{2k_{ss}} r^2 + C_1 \ln r + C_2$$

for which the boundary conditions are

BC#1: $\quad r = r_1 \quad \frac{dT}{dr} = 0 \quad 0 = -\frac{\dot{q}}{2k_{ss}} 2r_1 + \frac{C_1}{r_1} + 0 \rightarrow C_1 = +\frac{\dot{q}r_1^2}{k_{ss}}$

Continued …..

PROBLEM 10.22 (Cont.)

BC#2: $\quad r = r_2 \quad T(r_2) = T_2$

$$T_2 = -\frac{\dot{q}}{2k_{ss}} r_2^2 + \frac{\dot{q} r_1^2}{k_{ss}} \ln r_2 + C_2$$

$$C_2 = T_2 + \frac{\dot{q}}{2k_{ss}} r_2^2 - \frac{\dot{q} r_1^2}{k_{ss}} \ln r_2$$

Hence,

$$T(r) = -\frac{\dot{q}}{2k_{ss}}\left(r^2 - r_2^2\right) + \frac{\dot{q} r_1^2}{k_{ss}} \ln\left(r / r_2\right) + T_2.$$

Using the thermal circuit, find T_2 in terms of known parameters T_s, T_{sat} and C.

$$\frac{T_2 - T_s}{R'_{23} + R'_{34}} = \left(2\pi r_4\right) C \left(T_s - T_{sat}\right)^3.$$

Hence, the maximum temperature in the inner stainless steel tube ($r = r_1$) is

$$T_{ss,max} = T(r_1) = -\frac{\dot{q}}{2k_{ss}}\left(r_1^2 - r_2^2\right) + \frac{\dot{q} r_1^2}{k_{ss}} \ln\left(r_1 / r_2\right) + T_s$$

$$+ \left(R'_{23} + R'_{34}\right)\left(2\pi r_4\right) C \left(T_s - T_{sat}\right)^3$$ <

where from Eq. 3.27

$$R'_{23} = \frac{\ln\left(r_3 / r_2\right)}{2\pi k_{bn}} \qquad R'_{34} = \frac{\ln\left(r_4 / r_3\right)}{2\pi k_{ss}}.$$

Boron nitride: $T_{bn,max}$ occurs at r_1. Hence

$$T_{bn,max} = T(r_1)$$ <

as derived above for the inner stainless steel tube.

PROBLEM 10.23

KNOWN: Thickness and thermal conductivity of a silicon chip. Properties of saturated fluorocarbon liquid.

FIND: (a) Temperature at bottom surface of chip for a prescribed heat flux and 90% of CHF, (b) Effect of heat flux on chip surface temperatures; maximum allowable heat flux.

SCHEMATIC:

Sat. FC, T_{sat} = 57 °C

q''_s, T_s

Silicon, k_s = 135 W/m•K

q''_{cond}

L = 2.5 mm

q''_o = 5 x 10^4 W/m^2 or $C_1\, q''_{max}$

N.B. Constants: $C_{s,f}$ = 0.005, n = 1.7

T_o

q''_o

ASSUMPTIONS: (1) Steady-state conditions, (2) Uniform heat flux and adiabatic sides, hence one-dimensional conduction in chip, (3) Constant properties, (4) Nucleate boiling in liquid.

PROPERTIES: Saturated fluorocarbon (given): $c_{p,\ell}$ = 1100 J/kg·K, h_{fg} = 84,400 J/kg, ρ_ℓ = 1619.2 kg/m^3, ρ_v = 13.4 kg/m^3, $\sigma = 8.1 \times 10^{-3}$ kg/s^2, $\mu_\ell = 440 \times 10^{-6}$ kg/m·s, Pr_ℓ = 9.01.

ANALYSIS: (a) Energy balances at the top and bottom surfaces yield $q''_o = q''_{cond} = k_s\left(T_o - T_s\right)/L$ = q''_s; where T_s and q''_s are related by the Rohsenow correlation,

$$T_s - T_{sat} = \frac{C_{s,f}\, h_{fg}\, Pr_\ell^n}{c_{p,\ell}}\left(\frac{q''_s}{\mu_\ell h_{fg}}\right)^{1/3}\left[\frac{\sigma}{g\left(\rho_\ell - \rho_v\right)}\right]^{1/6}$$

Hence, for $q''_s = 5 \times 10^4$ W/m^2,

$$T_s - T_{sat} = \frac{0.005\left(84,400\,\text{J/kg}\right)9.01^{1.7}}{1100\,\text{J/kg}\cdot\text{K}}\left(\frac{5\times10^4\,\text{W/m}^2}{440\times10^{-6}\,\text{kg/m}\cdot\text{s}\times84,400\,\text{J/kg}}\right)^{1/3}$$

$$\times\left[\frac{8.1\times10^{-3}\,\text{kg/s}^2}{9.807\,\text{m/s}^2\left(1619.2-13.4\right)\text{kg/m}^3}\right]^{1/6} = 15.9^\circ\text{C}$$

$$T_s = \left(15.9+57\right)^\circ\text{C} = 72.9^\circ\text{C}.$$

From the rate equation,

$$T_o = T_s + \frac{q''_o L}{k_s} = 72.9^\circ\text{C} + \frac{5\times10^4\,\text{W/m}^2\times0.0025\,\text{m}}{135\,\text{W/m}\cdot\text{K}} = 73.8^\circ\text{C} \qquad <$$

For a heat flux which is 90% of the critical heat flux (C_1 = 0.9), it follows that

$$q''_o = 0.9q''_{max} = 0.9\times0.149 h_{fg}\rho_v\left[\frac{\sigma g\left(\rho_\ell-\rho_v\right)}{\rho_v^2}\right]^{1/4} = 0.9\times0.149\times84,400\,\text{J/kg}\times13.4\,\text{kg/m}^3$$

Continued...

PROBLEM 10.23 (Cont.)

$$\times\left[\frac{8.1\times10^{-3}\,\text{kg}/\text{s}^2\times9.807\,\text{m}/\text{s}^2\,(1619.2-13.4)\,\text{kg}/\text{m}^3}{\left(13.4\,\text{kg}/\text{m}^3\right)^2}\right]^{1/4}$$

$$q''_o = 0.9\times15.5\times10^4\,\text{W}/\text{m}^2 = 13.9\times10^4\,\text{W}/\text{m}^2$$

From the results of the previous calculation and the Rohsenow correlation, it follows that

$$\Delta T_e = 15.9^\circ\text{C}\left(q''_o/5\times10^4\,\text{W}/\text{m}^2\right)^{1/3} = 15.9^\circ\text{C}\,(13.9/5)^{1/3} = 22.4^\circ\text{C}$$

Hence, $T_s = 79.4°C$ and

$$T_o = 79.4^\circ\text{C} + \frac{13.9\times10^4\,\text{W}/\text{m}^2\times0.0025\,\text{m}}{135\,\text{W}/\text{m}\cdot\text{K}} = 82^\circ\text{C}$$ <

(b) Using the energy balance equations with the *Correlations* Toolpad of IHT to perform the parametric calculations for $0.2 \le C_1 \le 0.9$, the following results are obtained.

The chip surface temperatures, as well as the difference between temperatures, increase with increasing heat flux. The maximum chip temperature is associated with the bottom surface, and $T_o = 80°C$ corresponds to

$$q''_{o,max} = 11.3\times10^4\,\text{W}/\text{m}^2$$ <

which is 73% of CHF (q''_{max} = 15.5×10^4 W/m^2).

COMMENTS: Many of today's VLSI chip designs involve heat fluxes well in excess of 15 W/cm^2, in which case pool boiling in a fluorocarbon would not be an appropriate means of heat dissipation.

PROBLEM 10.24

KNOWN: Operating conditions of apparatus used to determine surface boiling characteristics.

FIND: (a) Nucleate boiling coefficient for special coating, (b) Surface temperature as a function of heat flux; apparatus temperatures for a prescribed heat flux; applicability of nucleate boiling correlation for a specified heat flux.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional, steady-state conduction in the bar, (2) Water is saturated at 1 atm, (3) Applicability of Rohsenow correlation with n = 1.

PROPERTIES: *Table A.6*, saturated water (100°C): ρ_ℓ = 957.9 kg/m^3, $c_{p,\ell}$ = 4217 J/kg·K, μ_ℓ = 279×10^{-6} N·s/m^2, Pr_ℓ = 1.76, $h_{fg} = 2.257 \times 10^6$ J/kg, σ = 0.0589 N/m, ρ_v = 0.5955 kg/m^3.

ANALYSIS: (a) The coefficient $C_{s,f}$ associated with Eq. 10.5 may be determined if q''_s and T_s are known. Applying Fourier's law between x_1 and x_2,

$$q''_s = q''_{cond} = k\frac{T_2 - T_1}{x_2 - x_1} = 400\,\text{W/m}\cdot\text{K} \times \frac{(158.6 - 133.7)^\circ\text{C}}{0.015\,\text{m}} = 6.64 \times 10^5\,\text{W/m}^2$$

Since the temperature distribution in the bar is linear, $T_s = T_1 - (dT/dx)x_1 = T_1 - [(T_2 - T_1)/(x_2 - x_1)]x_1$. Hence,

$$T_s = 133.7^\circ\text{C} - \left[24.9^\circ\text{C}/0.015\,\text{m}\right]0.01\,\text{m} = 117.1^\circ\text{C}$$

From Eq. 10.5, with n = 1,

$$C_{s,f} = \frac{c_{p,\ell}\Delta T_e}{h_{fg}\,Pr_\ell}\left(\frac{\mu_\ell h_{fg}}{q''_s}\right)^{1/3}\left[\frac{g(\rho_\ell - \rho_v)}{\sigma}\right]^{1/6}$$

$$C_{s,f} = \frac{4217\,\text{J/kg}\cdot\text{K}\left(17.1^\circ\text{C}\right)}{2.257 \times 10^6\,\text{J/kg}\,(1.76)}\left(\frac{279 \times 10^{-6}\,\text{kg/s}\cdot\text{m} \times 2.257 \times 10^6\,\text{J/kg}}{6.64 \times 10^5\,\text{W/m}^2}\right)^{1/3}\left[\frac{9.8\,\text{m/s}^2 \times 957.3\,\text{kg/m}^3}{0.0589\,\text{kg/s}^2}\right]^{1/6}$$

$C_{s,f}$ = 0.0131 <

(b) Using the appropriate IHT *Correlations* and *Properties* Toolpads, the following portion of the nucleate boiling regime was computed.

Continued...

PROBLEM 10.24 (Cont.)

For q''_s = 10^6 W/m^2 = q''_{cond}, T_s = 119.6°C and

T_1 = 144.6°C and T_2 = 182.1°C <

With the critical heat flux given by Eq. 10.7,

$$q''_{max} = 0.149 h_{fg} \rho_v \left[\frac{\sigma g (\rho_\ell - \rho_v)}{\rho_v^2} \right]^{1/4}$$

$$q''_{max} = 0.149 \left(2.257 \times 10^6 \text{ J/kg} \right) 0.5955 \text{ kg/m}^3 \left[\frac{0.0589 \text{ kg/s}^2 \times 9.8 \text{ m/s}^2 \times 957.3 \text{ kg/m}^3}{\left(0.5955 \text{ kg/m}^3 \right)^2} \right]^{1/4}$$

$$q''_{max} = 1.25 \times 10^6 \text{ W/m}^2$$

Since q''_s = 1.5×10^6 W/m^2 > q''_{max}, the heat flux exceeds that associated with nucleate boiling and the foregoing results can not be used.

COMMENTS: For $q''_s > q''_{max}$, conditions correspond to film boiling, for which T_s may exceed acceptable operating conditions.

PROBLEM 10.25

KNOWN: Small copper sphere, initially at a uniform temperature, T_i, greater than that corresponding to the Leidenfrost point, T_D, suddenly immersed in a large fluid bath maintained at T_{sat}.

FIND: (a) Sketch the temperature-time history, T(t), during the quenching process; indicate temperature corresponding to T_i , T_D, and T_{sat} , identify regimes of film, transition and nucleate boiling and the single-phase convection regime; identify key features; and (b) Identify times(s) in this quenching process when you expect the surface temperature of the sphere to deviate most from its center temperature.

SCHEMATIC:

ANALYSIS: (a) In the right-hand schematic above, the quench process is shown on the "boiling curve" similar to Figure 10.4. Beginning at an initial temperature, $T_i > T_D$, the process proceeds as indicated by the arrows: film regime from i to D, transition regime from D to C, nucleate regime from C to A, and single-phase (free convection) from A to the condition when $\Delta T_e = T_s - T_{sat} = 0$. The quench process is shown on the temperature-time plot below and the boiling regimes and key temperatures are labeled..

The highest temperature-time change should occur in the nucleate pool boiling regime, especially near the critical flux condition, T_c . The lowest temperature-time change will occur in the single-phase, free convection regime.

(b) The difference between the center and surface temperature will occur when $Bi = hr_o/3k \geq 0.1$. This could occur in regimes with the highest convection coefficients. For example, $h = 10{,}000\ W/m^2 \cdot K$ which might be the case for water in the nucleate boiling regime, C-A, $Bi \approx 10{,}000\ W/m^2\ (0.010m)/3\times400\ W/m\cdot K = 0.08$. For a sphere of larger dimension, in the nucleate and film pool boiling regimes, we could expect temperature differences between the center and surface temperatures since Bi might be greater than 0.1.

PROBLEM 10.26

KNOWN: A sphere (aluminum alloy 2024) with a uniform temperature of 500°C and emissivity of 0.25 is suddenly immersed in a saturated water bath maintained at atmospheric pressure.

FIND: (a) The total heat transfer coefficient for the initial condition; fraction of the total coefficient contributed by radiation; and (b) Estimate the temperature of the sphere 30 s after it has been immersed in the bath.

SCHEMATIC:

Saturated water
$T_{sat} = 100^oC$

Sphere, Al 2024 alloy, T(t)
D = 20 mm
ε = 0.25

$T_i = T_s(0) = 500^oC$

ASSUMPTIONS: (1) Water exposed to standard atmospheric pressure and uniform temperature, T_{sat}, and (2) Lumped capacitance method is valid.

PROPERTIES: See Comment 2; properties obtained with *IHT* code.

ANALYSIS: (a) For the initial condition with T_s = 500°C, *film boiling* will occur and the coefficients due to convection and radiation are estimated using Eqs. 10.9 and 10.11, respectively,

$$\overline{Nu}_D = \frac{\bar{h}_{conv} D}{k_v} = C\left[\frac{g\left(\rho_\ell - \rho_v\right) h'_{fg} D^3}{\eta_v k_v \left(T_s - T_{sat}\right)}\right]^{1/4} \qquad (1)$$

$$\bar{h}_{rad} = \frac{\varepsilon\sigma\left(T_s^4 - T_{sat}^4\right)}{T_s - T_{sat}} \qquad (2)$$

where C = 0.67 for spheres and σ = 5.67×10^{-8} $W/m^2 \cdot K^4$. The corrected latent heat is

$$h'_{fg} = h_{fg} + 0.8\, c_{p,v}\left(T_s - T_{sat}\right) \qquad (3)$$

The total heat transfer coefficient is given by Eq. 10.10a as

$$\bar{h}^{4/3} = \bar{h}_{conv}^{4/3} + \bar{h}_{rad} \cdot \bar{h}^{1/3} \qquad (4)$$

The vapor properties are evaluated at the film temperature,

$$T_f = \left(T_s + T_{sat}\right)/2 \qquad (5)$$

while the liquid properties are evaluated at the saturation temperature. Using the foregoing relations in *IHT* (see Comments), the following results are obtained.

$\overline{Nu}_D$	$\bar{h}_{cnv}\left(W/m^2 \cdot K\right)$	$\bar{h}_{rad}\left(W/m^2 \cdot K\right)$	$\bar{h}\left(W/m^2 \cdot K\right)$	
226	867	12.0	876	<

The radiation process contribution is 1.4% that of the total heat rate.

(b) For the lumped-capacitance method, from Section 5.3, the energy balance is

$$-\bar{h}A_s\left(T_s - T_{sat}\right) = \rho_s V c_s \frac{dT_s}{dt} \qquad (6)$$

where ρ_s and c_s are properties of the sphere. To determine $T_s(t)$, it is necessary to evaluate $\bar{h}$ as a function of T_s. Using the foregoing relations in *IHT* (see Comments), the sphere temperature after 30s is

$$T_s\left(30s\right) = 333°C. \qquad \text{<}$$

Continued …..

COMMENTS: (1) The Biot number associated with the aluminum alloy sphere cooling process for the initial condition is Bi = 0.09. Hence, the lumped-capacitance method is valid.

(2) The *IHT* code to solve this application uses the film-boiling correlation function, the water properties function, and the lumped capacitance energy balance, Eq. (6). The results for part (a), including the properties required of the correlation, are shown at the outset of the code.

```
/* Results, Part (a): Initial condition, Ts = 500C
NuDbar          hbar     hcvbar   hradbar    F
226             875.5    866.5    11.97      0.01367    */

/* Properties: Initial condition, Ts = 500 C, Tf = 573 K
Pr      cpv     h'fg      hfg       kv       nuv       rhol     rhov
1.617   5889    3.291E6   1.406E6   0.0767   4.33E-7   712.1    45.98 */

/* Results: with initial condition, Ts = 500 C; after 30 s
Bi              F         Ts_C      hbar     t
0.09414         0.01367   500       875.5    0
0.04767         0.01587   333.2     443.3    30

// LCM analysis, energy balance
- hbar * As * (Ts - Tsat) = rhos * Vol * cps * der(Ts,t)
As = pi * D^2 / 4
Vol = pi * D^3 / 6

/*  Correlation description: coefficients for film pool boiling (FPB).  Eqs. 10.9, 10.10 and 10.11 .
See boiling curve, Fig 10.4 . */
NuDbar = NuD_bar_FPB(C,rhol,rhov,h'fg,nuv,kv,deltaTe,D,g)        // Eq 10.9
NuDbar = hcvbar * D / kv
g = 9.8                              // gravitational constant, m/s^2
deltaTe = Ts - Tsat                  // excess temperature, K
// Ts_C = 500                        // surface temperature, K
Tsat = 373                                   // saturation temperature, K
// The vapor properties are evaluated at the film temperature,Tf,
Tf = Tfluid_avg(Ts,Tsat)
// The correlation constant is 0.62 or 0.67 for cylinders or spheres,
C = 0.67
// The corrected latent heat is
h'fg = hfg + 0.80*cpv*(Ts - Tsat)
// The radiation coefficient is
hradbar = eps * sigma * (Ts^4 - Tsat^4) / (Ts - Tsat)     // Eq 10.11
sigma = 5.67E-8                      // Stefan-Boltzmann constant, W/m^2·K^4
eps =    0.25                        // surface emissivity
// The total heat transfer coefficient is
hbar^(4/3) = hcvbar^(4/3) + hradbar * hbar^(1/3)     // Eq 10.10a
F = hradbar / hbar                   // fraction contribution of radiation

// Input variables
D = 0.020
rhos = 2702                          // Sphere properties, aluminum alloy 2024
cps = 875
ks = 186
Bi = hbar * D / ks                   // Biot number

// Water property functions :T dependence, From Table A.6
// Units: T(K), p(bars);
x = 1                                // Quality (0=sat liquid or 1=sat vapor)
xx = 0
rhov = rho_Tx("Water",Tf,x)          // Density, kg/m^3
rhol = rho_Tx("Water",Tf,xx)                 // Density, kg/m^3
hfg = hfg_T("Water",Tf)              // Heat of vaporization, J/kg
cpv = cp_Tx("Water",Tf,x)            // Specific heat, J/kg·K
nuv = nu_Tx("Water",Tf,x)            // Kinematic viscosity, m^2/s
kv = k_Tx("Water",Tf,x)              // Thermal conductivity, W/m·K
Pr = Pr_Tx("Water",Tf,x)             // Prandtl number

// Conversions
Ts_C = Ts - 273
```

PROBLEM 10.27

KNOWN: Steel bar upon removal from a furnace immersed in water bath.

FIND: Initial heat transfer rate from bar.

SCHEMATIC:

D=20mm
q_s
Water 1atm
T_s = 455°C, ε = 0.9
L=200mm

ASSUMPTIONS: (1) Uniform bar surface temperature, (2) Film pool boiling conditions.

PROPERTIES: *Table A-6*, Water, liquid (1 atm, T_{sat} = 100°C): ρ_ℓ = 957.9 kg/m^3, h_{fg} = 2257 kJ/kg; *Table A-6*, Water, vapor ($T_f = (T_s + T_{sat})/2$ = 550K): ρ_v = 31.55 kg/m^3, $c_{p,v}$ = 4640 J/kg·K, μ_v = 18.6 × 10^{-6} N·s/m^2, k_v = 0.0583 W/m·K.

ANALYSIS: The total heat transfer rate from the bar at the instant of time it is removed from the furnace and immersed in the water is

$$q_s = \bar{h} A_s (T_s - T_{sat}) = \bar{h} A_s \Delta T_e \quad (1)$$

where ΔT_e = 455 – 100 = 355K. According to the boiling curve of Figure 10.4, with such a high ΔT_e, film pool boiling will occur. From Eq. 10.10,

$$\bar{h}^{4/3} = \bar{h}_{conv}^{4/3} + \bar{h}_{rad} \cdot \bar{h}^{1/3} \quad \text{or} \quad \bar{h} = \bar{h}_{conv} + \frac{3}{4}\bar{h}_{rad} \ \left(\text{if } h_{conv} > h_{rad}\right). \quad (2)$$

To estimate the convection coefficient, use Eq. 10.9,

$$\overline{Nu}_D = \frac{\bar{h}_{conv} D}{k_v} = C\left[\frac{g(\rho_\ell - \rho_v) h'_{fg} D^3}{\nu_v k_v \Delta T_e}\right]^{1/4} \quad (3)$$

where C = 0.62 for the horizontal cylinder and $h'_{fg} = h_{fg} + 0.8\, c_{p,v}(T_s - T_{sat})$. Find

$$\bar{h}_{conv} = \frac{0.0583\,\text{W/m}\cdot\text{K}}{0.020\ \text{m}} 0.62\left[\frac{9.8\,\text{m/s}^2\,(957.9 - 31.55)\,\text{kg/m}^3\left[2257\times10^3 + 0.8\times4640\times355\right]\text{J/kg}\,(0.020\text{m})^3}{\left(18.6\times10^{-6}/31.55\right)\text{m}^2/\text{s}\times0.0583\,\text{W/m}\cdot\text{K}\times355\text{K}}\right]^{1/4}$$

$$\bar{h}_{conv} = 690\ \text{W/m}^2\cdot\text{K}.$$

To estimate the radiation coefficient, use Eq. 10.11,

$$\bar{h}_{rad} = \frac{\varepsilon\sigma\left(T_s^4 - T_{sat}^4\right)}{T_s - T_{sat}} = \frac{0.9\times5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left(728^4 - 373^4\right)\text{K}^4}{355\text{K}} = 37.6\,\text{W/m}^2\cdot\text{K}.$$

Substituting numerical values into the simpler form of Eq. (2), find

$$\bar{h} = \left(690 + (3/4)37.6\right)\text{W/m}^2\cdot\text{K} = 718\,\text{W/m}^2\cdot\text{K}.$$

Using Eq. (1), the heat rate, with $A_s = \pi D L$, is

$$q_s = 718\ \text{W/m}^2\cdot\text{K}\left(\pi\times0.020\text{m}\times0.200\text{m}\right)\times355\text{K} = 3.20\text{kW}. \quad <$$

COMMENTS: For these conditions, the convection process dominates.

PROBLEM 10.28

KNOWN: Electrical conductor with prescribed surface temperature immersed in water.

FIND: (a) Power dissipation per unit length, q'_s and (b) Compute and plot q'_s as a function of surface temperature $250 \le T_s \le 650°C$ for conductor diameters of 1.5, 2.0, and 2.5 mm; separately plot the percentage contribution of radiation as a function of T_s .

SCHEMATIC:

Water

T_s = 555 °C, ε = 0.50, D = 2 mm

q'_s

Current

ASSUMPTIONS: (1) Steady-state conditions, (2) Water saturated at l atm, (3) Film pool boiling.

PROPERTIES: *Table A-6*, Water, liquid (1 atm, T_{sat} = 100°C): ρ_ℓ = 957.9 kg/m^3 , h_{fg} = 2257 kJ/kg; *Table A-6*, Water, vapor ($T_f = (T_s + T_{sat}) / 2$ = 600 K): ρ_v = 72.99 kg/m^3, $c_{p,v}$ = 8750 J/kg·K, μ_v = 22.7 × 10^{-6} N·s/m^2 , k_v = 0.0929 W/m·K.

ANALYSIS: (a) The heat rate per unit length due to electrical power dissipation is

$$q'_s = \frac{q_s}{\ell} = \bar{h}\frac{A_s}{\ell}(T_s - T_{sat}) = \bar{h}\pi D\Delta T_e$$

where ΔT_e = (555 - 100)°C = 455°C. According to the boiling curve of Figure 10.4, with such a high ΔT_e, film pool boiling will occur. From Eq 10.10,

$$\bar{h}^{4/3} = \bar{h}_{conv}^{4/3} + \bar{h}_{rad}\cdot\bar{h}^{1/3} \qquad \text{or} \qquad \bar{h} = \bar{h}_{conv} + \frac{3}{4}\bar{h}_{rad} \qquad \left(\text{if } \bar{h}_{conv} > \bar{h}_{rad}\right).$$

To estimate the convection coefficient, use Eq. 10.9,

$$\overline{Nu}_D = \frac{\bar{h}_{conv}D}{k_v} = C\left[\frac{g(\rho_\ell - \rho_v)h'_{fg}D^3}{\nu_v k_v \Delta T_e}\right]^{1/4}$$

where C = 0.62 for the horizontal cylinder and $h'_{fg} = h_{fg} + 0.8c_{p,v}(T_s - T_{sat})$. Find

$$\bar{h}_{conv} = \frac{0.0929\,W/m\cdot K}{0.002\,m}\times 0.62\left[\frac{9.8\,m/s^2\,(957.9-72.99)\,kg/m^3\left[2257\times10^3 + 0.8\times8750\times455\right]J/kg\,(0.002m)^3}{\left(22.7\times10^{-6}/72.99\right)m^2/s\times0.0929\,W/m\cdot K\times455\,K}\right]^{1/4}$$

$$\bar{h}_{conv} = 2108\,W/m^2\cdot K.$$

To estimate the radiation coefficient, use Eq. 10.11.

$$\bar{h}_{rad} = \frac{\varepsilon\sigma\left(T_s^4 - T_{sat}^4\right)}{T_s - T_{sat}} = \frac{0.5\times5.67\times10^{-8}\,W/m^2\cdot K^4\left(828^4 - 373^4\right)K^4}{455\,K} = 28\,W/m^2\cdot K.$$

Since $h_{conv} > h_{rad}$, the simpler form of Eq. 10.10b is appropriate. Find,

$$\bar{h} = \left(2108 + (3/4)\times28\right)W/m^2\cdot K = 2129\,W/m^2\cdot K$$

Continued...

PROBLEM 10.28 (Cont.)

The heat rate is

$$q' = 2129\,\mathrm{W/m^2 \cdot K} \times \pi\,(0.002\mathrm{m}) \times 455\,\mathrm{K} = 6.09\,\mathrm{kW/m}. \qquad <$$

(b) Using the *IHT Correlations Tool, Boiling, Film Pool Boiling*, combined with the *Properties Tool* for *Water*, the heat rate, q', was calculated as a function of the surface temperature, T_s, for conductor diameters of 1.5, 2.0 and 2.5 mm. Also, plotted below is the ratio (%) of q'_{rad}/q' as a function of surface temperature.

Heat rate, q' (W/m)
Surface temperature, Ts (C)
D = 1.5 mm
D = 2.0 mm
D = 2.5 mm

q'rad/q' (%)
Surface temperature, Ts (C)
D = 1.5 mm
D = 2.0 mm
D = 2.5 mm

From the q' vs. T_s plot, note that the heat rate increases markedly with increasing surface temperatures, and, as expected the heat rate increases with increasing diameter. The discontinuity near T_s = 650°C is caused by the significant changes in the thermophysical properties as the film temperature, T_f, approaches the critical temperature, 647.3 K. From the q'_{rad}/q' vs. T_s plot, the maximum contribution by radiation is 2% and surprisingly doesn't occur at the maximum surface temperature. By examining a plot of q'_{rad} vs. T_s, we'd see that indeed q'_{rad} increases markedly with increasing T_s; but q'_{conv} increases even more markedly so the relative contribution of the radiation mode actually decreases with increasing temperature for $T_s > 350$°C.

PROBLEM 10.29

KNOWN: Horizontal, stainless steel bar submerged in water at 25°C.

FIND: Heat rate per unit length of the bar.

SCHEMATIC:

Water, 25°C

T_s=250°C

q'_s

Polished stainless steel rod, D=50mm, ε = 0.10

ASSUMPTIONS: (1) Steady-state conditions, (2) Film pool boiling, (3) Water at 1 atm.

PROPERTIES: *Table A-6*, Water, liquid (1 atm, Tsat = 100°C): r_ℓ = 957.9 kg/m3, hfg = 2257 kJ/kg; *Table A-6*, Water, vapor, ($T_f = (T_s + T_{sat})/2 \approx 450$K): $\rho_v = 4.81$ kg/m^3, $c_{p,v} = 2560$ J/kg·K, $\mu_v = 14.85 \times 10^{-6}$ N·s/m^2, $k_v = 0.0331$ W/m·K.

ANALYSIS: The heat rate per unit length is

$$q'_s = q_s/\ell = q'' p\, D = \bar{h} p D\left(T_s - T_{sat}\right) = \bar{h} p\, D\, \Delta T_e$$

where ΔT_e = (250-100)°C = 150°C. Note from the boiling curve of Figure 10.4, that film boiling will occur. From Eq. 10.10,

$$\bar{h}^{4/3} = \bar{h}_{conv}^{4/3} + \bar{h}_{rad}\bar{h}^{1/3} \qquad \text{or} \qquad \bar{h} = \bar{h}_{conv} + \frac{3}{4}\bar{h}_{rad} \qquad \left(\text{if } \bar{h}_{conv} > \bar{h}_{rad}\right).$$

To estimate the convection coefficient, use Eq. 10.9,

$$\overline{Nu}_D = \frac{\bar{h}_{conv} D}{k_v} = C\left[\frac{g\left(r_\ell - r_v\right)h'_{fg}D^3}{n_v\, k_v\, \Delta T_e}\right]^{1/4}$$

where C = 0.62 for the horizontal cylinder and $h'_{fg} = h_{fg} + 0.8 c_{p,v}\left(T_s - T_{sat}\right)$. Find

$$\bar{h}_{conv} = \frac{0.0331\,\text{W/m}\cdot\text{K}}{0.050\text{m}} 0.62\left[\frac{9.8\,\text{m/s}^2\,(957.9 - 4.81)\,\text{kg/m}^3\left[2257\times10^3 + 0.8\times2560\,\text{J/kg}\cdot\text{K}\times150\,\text{K}\right](0.050\text{m})^3}{\left(14.85\times10^{-6}/4.81\right)\text{m}^2/\text{s}\times0.0331\,\text{W/m}\cdot\text{K}\times150\text{K}}\right]^{1/4}$$

$$\bar{h}_{conv} = 273\,\text{W/m}^2\cdot\text{K}.$$

To estimate the radiation coefficient, use Eq. 10.11,

$$\bar{h}_{rad} = \frac{e\, s\left(T_s^4 - T_{sat}^4\right)}{T_s - T_{sat}} = \frac{0.50\times5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left(523^4 - 373^4\right)\text{K}^4}{150\text{K}} = 11\,\text{W/m}^2\cdot\text{K}.$$

Since $h_{conv} > h_{rad}$, the simpler form of Eq. 10.10 is appropriate. Find,

$$\bar{h} = \left[273 + (3/4)\times11\right]\,\text{W/m}^2\cdot\text{K} = 281\,\text{W/m}^2\cdot\text{K}.$$

Using the rate equation, find

$$q'_s = 281\,\text{W/m}^2\cdot\text{K}\times p\times(0.050\text{m})\times150\text{K} = 6.62\,\text{kW/m}. \qquad <$$

COMMENTS: The effect of the water being subcooled (T = 25°C < T_{sat}) is considered to be negligible.

PROBLEM 10.30

KNOWN: Heater element of 5-mm diameter maintained at a surface temperature of 350°C when immersed in water under atmospheric pressure; element sheath is stainless steel with a mechanically polished finish having an emissivity of 0.25.

FIND: (a) The electrical power dissipation and the rate of evaporation per unit length; (b) If the heater element were operated at the same power dissipation rate in the nucleate boiling regime, what temperature would the surface achieve? Calculate the rate of evaporation per unit length for this operating condition; and (c) Make a sketch of the boiling curve and represent the two operating conditions of parts (a) and (b). Compare the results of your analysis. If the heater element is operated in the power-controlled mode, explain how you would achieve these two operating conditions beginning with a cold element.

SCHEMATIC:

Heater element (SS mechanically polished), D = 5 mm, ε = 0.25; $T_s = 350^oC$; Saturated water $T_{sat} = 100^oC$

ASSUMPTIONS: (1) Steady-state conditions, and (2) Water exposed to standard atmospheric pressure and uniform temperature, T_{sat}.

PROPERTIES: *Table A-6*, Saturated water, liquid (100°C): $\rho_\ell = 957.9\ \text{kg/m}^3$, $c_{p,\ell} = 4217$ J/kg·K, $\mu_\ell = 279 \times 10^{-6}\ \text{N}\cdot\text{s/m}^2$, $Pr_\ell = 1.76$, h_{fg} = 2257 kJ/kg, $h'_{fg} = h_{fg} + 0.80\ c_{p,v}(T_s - T_{sat})$ = 2905 kJ/kg, $\sigma = 58.9 \times 10^{-3}$ N/m; Saturated water, vapor (100°C): $\rho_v = 0.5955\ \text{kg/m}^3$; Water vapor ($T_f$ = 498 K): $\rho_v = 1/v_v = 12.54\ \text{kg/m}^3$, $c_{p,v}$ = 3236 J/kg·K, k_v = 0.04186 W/m·K, $\eta_v = 1.317 \times 10^{-6}\ \text{m}^2/\text{s}$.

ANALYSIS: (a) Since $\Delta T_e > 120°C$, the element is operating in the *film-boiling* (FB) regime. The electrical power dissipation per unit length is

$$q'_s = \bar{h}(\pi D)(T_s - T_{sat}) \tag{1}$$

where the total heat transfer coefficient is

$$\bar{h}^{4/3} = \bar{h}_{conv}^{4/3} + \bar{h}_{rad}\,\bar{h}^{1/3} \tag{2}$$

The convection coefficient is given by the correlation, Eq. 10.9, with C = 0.62,

$$\frac{\bar{h}_{conv}D}{k_v} = C\left[\frac{g(\rho_\ell - \rho_v)h'_{fg}D^3}{\eta_v k_v (T_s - T_{sat})}\right]^{1/4} \tag{3}$$

$$\bar{h}_{conv} = 0.62\left[\frac{9.8\ \text{m/s}^2(833.9 - 12.54)\text{kg/m}^3 \times 2.905\times10^6\,\text{J/kg}\cdot\text{K}(0.005\ \text{m})^3}{1.31\times10^{-6}\text{m}^2/\text{s}\times0.04186\ \text{W/m}\cdot\text{K}(350-100)\text{K}}\right]^{1/4}$$

$$\bar{h}_{conv} = 626\ \text{W/m}^2\cdot\text{K}$$

The radiation coefficient, Eq. (10.11), with $\sigma = 5.67 \times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4$, is

Continued …..

PROBLEM 10.30 (Cont.)

$$\bar{h}_{rad} = \frac{\varepsilon\sigma\left(T_s^4 - T_{sat}^4\right)}{\left(T_s - T_{sat}\right)}$$

$$\bar{h}_{rad} = \frac{0.25\sigma\left(623^4 - 373^4\right)K^4}{\left(350 - 100\right)K} = 4.5\ W/m^2 \cdot K$$

Substituting numerical values into Eq. (2) for $\bar{h}$, and into Eq. (1) for q_s', find

$$\bar{h} = 630\ W/m^2 \cdot K$$

$$q_s' = 630\ W/m^2 \cdot K\left(\pi \times 0.005\ m\right)\left(350 - 100\right)K = 2473\ W/m$$ <

$$q_s'' = q_s'/\pi D = 0.157\ MW/m^2$$

The evaporation rate per unit length is

$$\dot{m}_b' = q_s'/h_{fg} = 3.94\ kg/h \cdot m$$ <

(b) For the same heat flux, $q_s'' = 0.157\ MW/m^2$, using the Rohsenow correlation for the *nucleate boiling* (NB) regime, find ΔT_e, and hence T_s.

$$q_s'' = \mu_\ell h_{fg}\left[\frac{g\left(\rho_\ell - \rho_v\right)}{\sigma}\right]^{1/2}\left(\frac{c_{p,\ell}\,\Delta T_e}{C_{s,f}\,h_{fg}\,Pr_\ell^n}\right)^3$$

where, from Table 10.1, for stainless steel mechanically polished finish with water, $C_{s,f}$ = 0.013 and n = 1.0.

$$0.157 \times 10^6 W/m^2 = 279 \times 10^{-6} N \cdot s/m^2 \times 2.257 \times 10^6 J/kg$$

$$\times\left[\frac{9.8\ m/s^2\left(957.9 - 0.5955\right)kg/m^3}{58.9 \times 10^{-3} N/m}\right]^{1/2}$$

$$\times\left(\frac{4217\ J/kg \cdot K \times \Delta T_e}{0.013 \times 2.257 \times 10^6 J/kg \times 1.76}\right)^3$$

$$\Delta T_e = T_s - T_{sat} = 10.5\ K \qquad T_s = 110.5°C$$ <

The evaporation rate per unit length is

$$\dot{m}_b' = q_s''\left(\pi D\right)h_{fg} = 3.94\ kg/h \cdot m$$ <

Continued …..

PROBLEM 10.30 (Cont.)

(c) The two operating conditions are shown on the boiling curve, which is fashioned after Figure 10.4. For FB the surface temperature is T_s = 350°C (ΔT_e = 250°C). The element can be operated at NB with the same heat flux, q_s'' = 0.157 MW/m^2, with a surface temperature of T_s = 110°C (ΔT_e = 10°C). Since the heat fluxes are the same for both conditions, the evaporation rates are the same.

If the element is cold, and operated in a power-controlled mode, the element would be brought to the NB condition following the arrow shown next to the boiling curve near ΔT_e = 0. If the power is increased beyond that for the NB point, the element will approach the critical heat flux (CHF) condition. If q_s'' is increased beyond q_{max}'', the temperature of the element will increase abruptly, and the burnout condition will likely occur. If burnout does not occur, reducing the heat flux would allow the element to reach the FB point.

PROBLEM 10.31

KNOWN: Inner and outer diameters, outer surface temperature and thermal conductivity of a tube. Saturation pressure of surrounding water and convection coefficient associated with gas flow through the tube.

FIND: (a) Heat rate per unit tube length, (b) Mean temperature of gas flow through tube.

SCHEMATIC:

Water p_{sat} = 1.523 bar
Vapor
Gas
T_m, h_i = 500 W/m²-K
Tube wall
D_i = 0.026 m
D_o = 0.030 m
k_s = 15 W/m-K
ε = 0.30
$T_{s,o}$ = 575 K

q' → T_m — R'_{conv} — R'_{cond} — $T_{s,o}$ — R'_{fb} — T_{sat}

ASSUMPTIONS: (1) Steady-state, (2) Uniform surface temperature, (3) Water is at saturation temperature, (4) Tube is horizontal.

PROPERTIES: *Table A-6*, saturated water, liquid (p = 1.523 bars): T_{sat} = 385 K, $\rho_\ell = 950\,\text{kg/m}^3$, h_{fg} = 2225 kJ/kg. *Table A-6*, saturated water, vapor (T_f = 480 K): $\rho_v = 9.01\,\text{kg/m}^3$, $c_{p,v}$ = 2940 J/kg·K. $\mu_v = 15.9 \times 10^{-6}$ N·s/m², k_v = 0.0381 W/m·K, $\nu_v = 1.77\times10^{-6}\,\text{m}^2/\text{s}$.

ANALYSIS: (a) The heat rate per unit length is $q' = h_o \pi D_o (T_{s,o} - T_{sat})$, where h_o includes contributions due to convection and radiation in film boiling. With C = 0.62 and $h'_{fg} = h_{fg} + 0.80\, c_{p,v}(T_{s,o} - T_{sat}) = 2.67\times10^6\,\text{J/kg}$, Eq. 10.9 yields

$$\overline{h}_{conv,o} = \left(\frac{0.0381\,\text{W/m}\cdot\text{K}}{0.030\text{m}}\right)0.62\left[\frac{9.8\,\text{m/s}^2\,(950-9)\,\text{kg/m}^3 \times 2.67\times10^6\,\text{J/kg}\,(0.03\text{m})^3}{1.77\times10^{-6}\,\text{m}^2/\text{s}\times 0.0381\,\text{W/m}\cdot\text{K}\times190\,\text{K}}\right]^{1/4} = 376\,\text{W/m}^2\cdot\text{K}$$

From Eq. 10.11, the radiation coefficient is

$$\overline{h}_{rad,o} = \frac{0.30\times5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left(575^4 - 385^4\right)\text{K}^4}{(575-385)\,\text{K}} = 7.8\,\text{W/m}^2\cdot\text{K}$$

From Eq. 10.10b, it follows that

$$\overline{h}_o = \overline{h}_{conv,o} + 0.75\,\overline{h}_{rad,o} = 382\,\text{W/m}^2\cdot\text{K}$$

and the heat rate is

$$q' = \overline{h}_o \pi D_o (T_{s,o} - T_{sat}) = 382\,\text{W/m}^2\cdot\text{K}\,(\pi\times0.03\text{m})190\,\text{K} = 6840\,\text{W}$$ <

(b) From the thermal circuit, with $R'_{conv,i} = (h_i \pi D_i)^{-1} = \left(500\,\text{W/m}^2\cdot\text{K}\times\pi\times0.026\text{m}\right)^{-1} = 0.0245\,\text{m}\cdot\text{K/W}$ and $R'_{cond} = \ell n(\text{Do/Di})/2\pi k_s = \ell n(0.030/0.026)/2\pi(15\,\text{W/m}\cdot\text{K}) = 0.00152\,\text{m}\cdot\text{K/W}$,

$$q' = \frac{T_m - T_{s,o}}{R_{conv,i} + R'_{cond}} = \frac{T_m - 575\,\text{K}}{(0.0245 + 0.00152)\,\text{m}\cdot\text{K/W}}$$

$$T_m = 575\,\text{K} + 6840\,\text{W/m}\,(0.0260\,\text{m}\cdot\text{K/W}) = 753\,\text{K}$$ <

COMMENTS: Despite the large temperature of the gas, the rate of heat transfer is limited by the large thermal resistances associated with convection from the gas and film boiling. The resistance due to film boiling is $R'_{fb} = (\pi D_o \overline{h}_o)^{-1} = 0.0278\,\text{m}\cdot\text{K/W}$.

PROBLEM 10.32

KNOWN: Cylinder of 120 mm diameter at 1000K quenched in saturated water at 1 atm

FIND: Describe the quenching process and estimate the maximum heat removal rate per unit length during cooling.

SCHEMATIC:

ASSUMPTIONS: Water exposed to 1 atm pressure, T_{sat} = 100°C.

ANALYSIS: At the start of the quenching process, the surface temperature is $T_s(0)$ = 1000K. Hence, $\Delta T_e = T_s - T_{sat}$ = 1000K – 373K = 627K, and from the typical boiling curve of Figure 10.4, film boiling occurs.

As the cylinder temperature decreases, ΔT_e decreases, and the cooling process follows the boiling curve sketched above. The cylinder boiling process passes through the Leidenfrost point D, into the transition or unstable boiling regime (D $\rightarrow$ C).

At point C, the boiling heat flux has reached a maximum, q''_{max} = 1.26 MW/m^2 (see Example 10.1). Hence, the heat rate per unit length of the cylinder is

$$q'_s = q'_{max} = q''_{max}(\pi D) = 1.26\,\mathrm{MW/m^2}\left[\pi\,(0.120\mathrm{m})\right] = 0.475\,\mathrm{MW/m}. \qquad <$$

As the cylinder cools further, nucleate boiling occurs (C $\rightarrow$ A) and the heat rate drops rapidly. Finally, at point A, boiling no longer is present and the cylinder is cooled by free convection.

COMMENTS: Why doesn't the quenching process follow the cooling curve of Figure 10.3?

PROBLEM 10.33

KNOWN: Horizontal platinum wire of diameter of 1 mm, emissivity of 0.25, and surface temperature of 800 K in saturated water at 1 atm pressure.

FIND: (a) Surface heat flux, q''_s, when the surface temperature is T_s = 800 K and (b) Compute and plot on log-log coordinates the heat flux as a function of the excess temperature, $\Delta T_e = T_s - T_{sat}$, for the range $150 \leq \Delta T_e \leq 550$ K for emissivities of 0.1, 0.25, and 0.95; separately plot the percentage contribution of radiation as a function of ΔT_e.

SCHEMATIC:

Water; q''_s; T_s = 800 K, ε = 0.25, D = 1 mm; Current

ASSUMPTIONS: (1) Steady-state conditions, (2) Film pool boiling.

PROPERTIES: *Table A.6*, Saturated water, liquid (T_{sat} = 100°C, 1 atm): ρ_ℓ = 957.9 kg/m^3, h_{fg} = 2257 kJ/kg; *Table A.6*, Water, vapor ($T_f = (T_s + T_{sat})/2$ = (800 + 373)K/2 = 587 K): ρ_v = 58.14 kg/m^3, $c_{p,v}$ = 7065 J/kg·K, $\mu_v = 21.1 \times 10^{-6}$ N·s/m^2, $k_v = 81.9 \times 10^{-3}$ W/m·K.

ANALYSIS: (a) The heat flux is

$$q''_s = \bar{h}\left(T_s - T_{sat}\right) = \bar{h}\Delta T_e$$

where ΔT_e = (800 - 373)K = 427 indicative of film boiling. From 10.10,

$$\bar{h}^{4/3} = \bar{h}_{conv}^{4/3} + \bar{h}_{rad}\bar{h}^{-1/3} \qquad \text{or} \qquad \bar{h} = \bar{h}_{conv} + (3/4)\bar{h}_{rad}$$

if $h_{rad} < h_{conv}$. Use Eq. 10.9 with C = 0.62 for a horizontal cylinder,

$$\overline{Nu}_D = \frac{\bar{h}_{conv}D}{k_v} = C\left[\frac{g\left(\rho_\ell - \rho_v\right)h'_{fg}D^3}{\nu_v k_v\left(T_s - T_{sat}\right)}\right]^{1/4}$$

$$\frac{\bar{h}_{conv} \times 0.001\,\text{m}}{81.9\times10^{-3}\,\text{W/m}\cdot\text{K}} = 0.62\left[\frac{9.8\,\text{m/s}^2\left(957.9 - 58.14\right)\text{kg/m}^3 \times 4670\,\text{kJ/kg}\left(0.001\,\text{m}\right)^3}{\left(21.1\times10^{-6}\,\text{N}\cdot\text{s/m}^2/58.14\,\text{kg/m}^3\right)\times 0.0819\,\text{W/m}\cdot\text{K}\left(800-373\right)\text{K}}\right]^{1/4}$$

$$\bar{h}_{conv} = 2155\ \text{W/m}^2\cdot\text{K}$$

where $h'_{fg} = h_{fg} + 0.8c_{p,v}\left(T_s - T_{sat}\right) = 2257\,\text{kJ/kg} + 0.8\times7065\,\text{J/kg}\cdot\text{K}\left(800-373\right)\text{K} = 4670\,\text{kJ/kg}$. To estimate the radiation coefficient, use Eq. 10.11,

$$\bar{h}_{rad} = \frac{\varepsilon\sigma\left(T_s^4 - T_{sat}^4\right)}{T_s - T_{sat}} = \frac{0.25\sigma\left(800^4 - 373^4\right)\text{K}^4}{\left(800-373\right)\text{K}} = 13.0\,\text{W/m}^2\cdot\text{K}.$$

Since $\bar{h}_{rad} < \bar{h}_{conv}$, use the simpler expression,

$$\bar{h} = 2155\,\text{W/m}^2\cdot\text{K} + (3/4)13.0\,\text{W/m}^2\cdot\text{K} = 2165\,\text{W/m}^2\cdot\text{K}.$$

Using the rate equation, find

Continued...

$$q''_s = 2165\,\text{W/m}^2\cdot\text{K}\,(800-373)\,\text{K} = 0.924\,\text{MW/m}^2.$$ <

(b) Using the *IHT Correlations Tool, Boiling, Film Pool Boiling*, combined with the *Properties Tool* for *Water*, the heat flux, q''_s, was calculated as a function of the excess temperature, ΔT_e for emissivities of 0.1, 0.25 and 0.95. Also plotted is the ratio (%) of q''_{rad}/q'' as a function of ΔT_e.

Heat flux, q'' (W/m^2)
8E6
4E6
1E6
600000
200000
80000
40000
10000
100
400
800
Temperature excess, deltaTe (K)
eps = 0.1
eps = 0.25
eps = 0.95
Critical heat flux, q''max (W/m^2)
Minimum heat flux, q''min (W/m^2)

q''rad/q''s (%)
3
2
1
0
100 200 300 400 500 600
Temperature excess, deltaTe (K)
eps = 0.1
eps = 0.25
eps = 0.95

From the q''_s vs. ΔT_e plot, note that the heat rate increases markedly with increasing excess temperature. On the plot scale, the curves for the three emissivity values, 0.1, 0.25 and 0.95, overlap indicating that the overall effect of emissivity change on the total heat flux is slight. Also shown on the plot are the critical heat flux, q''_{max} = 1.26 MW/m^2, and the minimum heat flux, q''_{min} = 18.9kW/m^2, at the Leidenfrost point. These values are computed in Example 10.1. Note that only for the extreme value of ΔT_e is the heat flux in film pool boiling in excess of the critical heat flux. The relative contribution of the radiation mode is evident from the q''_{rad}/q''_s vs. ΔT_e plot. The maximum contribution by radiation is less than 3% and surprisingly doesn't occur at the maximum excess temperature. By examining a plot of q''_{rad} vs. ΔT_e, we'd see that indeed q''_{rad} increases markedly with increasing ΔT_e; however, q''_{conv} increases even more markedly so that the relative contribution of the radiation mode actually decreases with increasing temperature for ΔT_e > 250 K. Note that, as expected, the radiation heat flux, q''_{rad}, is proportional to the emissivity.

COMMENTS: Since $q''_s < q''_{max} = 1.26\,\text{MW/m}^2$, the prescribed condition can only be achieved in power-controlled heating by first exceeding q''_{max} and then decreasing the flux to 0.924 MW/m^2.

PROBLEM 10.34

KNOWN: Surface temperature and emissivity of strip steel.

FIND: Heat flux across vapor blanket.

SCHEMATIC:

Wall jet

q''_s

Water vapor

Steel strip, $T_s = 907K$, $\varepsilon = 0.35$

ASSUMPTIONS: (1) Steady-state conditions, (2) Vapor/jet interface is at T_{sat} for p = 1 atm, (3) Negligible effect of jet and strip motion.

PROPERTIES: *Table A-6*, Saturated water (100°C): r_ℓ = 957.9 kg/m^3, h_{fg} = 2257 kJ/kg; Saturated water vapor (T_f = 640K): ρ_v = 175.4 kg/m^3, $c_{p,v}$ = 42 kJ/kg·K, $\mu_v = 32 \times 10^{-6}$ N·s/m^2, k = 0.155 W/m·K, $\nu_v = 0.182 \times 10^{-6}$ m^2/s.

ANALYSIS: The heat flux is

$$q''_s = \bar{h}\Delta T_e$$

where $\Delta T_e = 907\ \text{K} - 373\ \text{K} = 534\ \text{K}$

and $\bar{h}^{4/3} = \bar{h}_{conv}^{4/3} + \bar{h}_{rad}\bar{h}^{1/3}$ or $\bar{h} = \bar{h}_{conv} + (3/4)\bar{h}_{rad}$.

With $h'_{fg} = h_{fg} + 0.80 c_{p,v}(T_s - T_{sat}) = 2.02\times10^7\ \text{J/kg}$

Equation 10.9 yields

$$\overline{Nu}_D = 0.62\left[\frac{9.8\,\text{m/s}^2(957.9-175.4)\text{kg/m}^3\left(2.02\times10^7\ \text{J/kg}\right)(1\ \text{m})^3}{0.182\times10^{-6}\,\text{m}^2/\text{s}(0.155\ \text{W/m}\cdot\text{K})(907-373)\text{K}}\right]^{1/4} = 6243.$$

Hence,

$$\bar{h}_{conv} = \overline{Nu}_D k_v/D = 6243\,\text{W/m}^2\cdot\text{K}(0.155\ \text{W/m}\cdot\text{K}/1\ \text{m}) = 968\ \text{W/m}^2\cdot\text{K}$$

$$\bar{h}_{rad} = \frac{e\, s\left(T_s^4 - T_{sat}^4\right)}{T_s - T_{sat}} = \frac{0.35\times5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\left(907^4-373^4\right)\text{K}^4}{(907-373)\text{K}}$$

$$\bar{h}_{rad} = 24\ \text{W/m}^2\cdot\text{K}$$

Hence, $\bar{h} = 968\ \text{W/m}^2\cdot\text{K} + (3/4)\left(24\ \text{W/m}^2\cdot\text{K}\right) = 986\ \text{W/m}^2\cdot\text{K}$

And $q''_s = 986\ \text{W/m}^2\cdot\text{K}(907-373)\text{K} = 5.265\times10^5\ \text{W/m}^2.$ <

COMMENTS: The foregoing analysis is a very rough approximation to a complex problem. A more rigorous treatment is provided by Zumbrunnen et al. In ASME Paper 87-WA/HT-5.

PROBLEM 10.35

KNOWN: Copper sphere, 10 mm diameter, initially at a prescribed elevated temperature is quenched in a saturated (1 atm) water bath.

FIND: The time for the sphere to cool (a) from T_i = 130 to 110°C and (b) from T_i = 550°C to 220°C .

SCHEMATIC:

Sphere, D = 10 mm, copper
Quenching processes:
(a) T_i = 130 to 110 °C
(b) T_i = 550 to 220 °C
Water
T_{sat} = 100 °C
q''_s

ASSUMPTIONS: (1) Sphere approximates lumped capacitance, (2) Water saturated at 1 atm.

PROPERTIES: *Table A-1*, Copper: ρ = 8933 kg/m^3; *Table A.11*, Copper (polished) : ε = 0.04, typical value; *Table A.4, Water*: as required for the pool boiling correlations.

ANALYSIS: Treating the sphere as a lumped capacitance and performing an energy balance, see Eq. 5.14,

$$\dot{E}_{in} - \dot{E}_{out} = \dot{E}_{st} \qquad -q''_s \cdot A_s = \rho c \forall \frac{dT}{dt} \qquad (1,2)$$

For the sphere, $V = \pi D^3 / 6$ and $A_s = \pi D^2$. Using the *IHT Lumped Capacitance Model* to solve this differential equation, we need to specify (1) the specific heat of the copper sphere as a function sphere temperature; use *IHT Properties Tool, Copper;* and (2) the heat flux, q''_s, associated with the pool boiling processes; use *IHT Correlations Tool, Boiling:*

(a) *Cooling from T_i =130° to 110°: Nucleate pool boiling*, Rohsenhow correlation, Eq. 10.5,

(b) *Cooling from T_i = 550 to 220°C : Film Pool Boiling*, Eq. 10.9 with C = 0.67 (sphere).

The thermophysical properties for water required of the correlations are provided by the *IHT Tool, Properties-Water*. The specific heat of copper as a function of sphere temperature is provided by the *IHT Tool, Properties-Copper.* The temperature-time histories for each of the cooling processes are plotted below.

Nucleate pool boiling quench process

Sphere temperature, Ts (C)
130
120
110
100
0 1 2 3 4 5
Elapsed time, t (s)

Film pool boiling quench process

Sphere temperature, T (s)
500
400
300
200
100
0 10 20 30
Elapsed time, t (s)

Continued...

Using the *Explore* feature in the *IHT Plot Window*, the elapsed times for the quench process were found as:

Quench process	T_i - T_f (°C)	$\Delta t(s)$
Nucleate pool boiling	130-110	0.76
Film pool boiling	550-220	13.5

COMMENTS: (1) Comparing the elapsed times for the two processes, the nucleate pool boiling process cools 20°C in 0.76s (26.3°C/s) vs. 330°C in 13.5s (24.4°C/s) for the film pool boiling process.

(2) The IHT Workspace used to generate the temperature-time history for the nucleate pool boiling process is shown below.

```
// Correlations Tool - Boiling, Nucleate Pool Boiling, Heat flux
qs'' = qs_dprime_NPB(Csf,n,rhol,rhov,hfg,cpl,mul,Prl,sigma,deltaTe,g)    // Eq 10.5
g = 9.8                          // Gravitational constant, m/s^2
deltaTe = Ts - Tsat              // Excess temperature, K
Ts =  Ts_C + 273                 // Surface temperature, K
//Ts_C = 130
Tsat =   100 + 273               // Saturation temperature, K
/* Evaluate liquid(l) and vapor(v) properties at Tsat. From Table 10.1 (Fill in as required), */
// fluid-surface combination:
Csf =   0.013                    // Polished copper-water combination, Table 10.1
n =   1.0
/*  Correlation description: Heat flux for nucleate pool boiling (NPB), water-surface combination (Cf,n), Eq 10.5,
Table 10.1 . See boiling curve, Fig 10.4 . */

// Properties Tool- Water:
// Water property functions :T dependence, From Table A.6
// Units: T(K), p(bars);
xv =  1                                  // Quality (0=sat liquid or 1=sat vapor)
rhov = rho_Tx("Water",Tsat,xv)           // Density, kg/m^3
hfg = hfg_T("Water",Tsat)                // Heat of vaporization, J/kg
sigma = sigma_T("Water",Tsat)            // Surface tension, N/m (liquid-vapor)
// Water property functions :T dependence, From Table A.6
// Units: T(K), p(bars);
xl = 0                                   // Quality (0=sat liquid or 1=sat vapor)
rhol = rho_Tx("Water",Tsat,xl)           // Density, kg/m^3
cpl = cp_Tx("Water",Tsat,xl)             // Specific heat, J/kg·K
mul = mu_Tx("Water",Tsat,xl)             // Viscosity, N·s/m^2
Prl = Pr_Tx("Water",Tsat,xl)             // Prandtl number

// Lumped Capacitance Model:
/* Conservation of energy requirement on the control volume, CV.  */
Edotin - Edotout = Edotst
Edotin = 0
Edotout = As * ( + qs'' )
Edotst = rho * vol * cp * Der(Ts,t)
/* The independent variables for this system and their assigned numerical values are */
As =  pi * D^2 / 4      // surface area, m^2
vol =  pi * D^3 / 6      // volume, m^3
D = 0.01
rho =  8933             // density, kg/m^3

// Properties Tool - Copper
// Copper (pure) property functions : From Table A.1
// Units: T(K)
cp = cp_T("Copper",Ts)          // Specific heat, J/kg·K
```

PROBLEM 10.36

KNOWN: Saturated water at 1 atm is heated in cross flow with velocities 0 – 2 m/s over a 2 mm-diameter tube.

FIND: Plot the critical heat flux as a function of water velocity; identify the pool boiling and transition regions between the low and high velocity ranges.

SCHEMATIC:

Water, 1atm

V=0-2m/s

D=2mm

$T_s > T_{sat}$

ASSUMPTIONS: Nucleate boiling in the presence of external forced convection.

PROPERTIES: *Table A-6*, Water (1 atm): T_{sat} = 100°C, r_ℓ = 957.9 kg/m^3, ρ_v = 0.5955 kg/m^3, h_{fg} = 2257 kJ/kg, $\sigma = 58.9 \times 10^{-3}$ N/m.

ANALYSIS: The Lienhard-Eichhorn correlations for forced convection boiling with cross flow over a cylinder are appropriate for estimating q''_{max}, Eqs. 10.12 and 10.13.

Low Velocity

$$q''_{max} = \frac{r_v h_{fg}}{p}\left[1 + \left(\frac{4s}{r_v V^2 D}\right)^{1/3}\right] V$$

$$q''_{max} = \frac{1}{p} 0.5955 \frac{\text{kg}}{\text{m}^3} \times 2257 \times 10^3 \frac{\text{J}}{\text{kg}}\left[1 + \left(\frac{4 \times 58.9 \times 10^{-3}\,\text{N/m}}{0.5955\,\text{kg/m}^3\,V^2\,0.002\text{m}}\right)^{1/3}\right] V$$

$$q''_{max} = 4.2782 \times 10^5 V + 2.921 \times 10^6 V^{1/3}.$$

High Velocity

$$q''_{max} = \frac{r_v h_{fg}}{p}\left[\frac{1}{169}\left(\frac{r_\ell}{r_v}\right)^{3/4} + \frac{1}{19.2}\left(\frac{r_\ell}{r_v}\right)^{1/2}\left(\frac{s}{r_v V^2 D}\right)^{1/3}\right] V$$

$$q''_{max} = \frac{1}{p} 0.5955 \frac{\text{kg}}{\text{m}^3} \times 2257 \times 10^3 \frac{\text{J}}{\text{kg}}\left[\frac{1}{169}\left(\frac{957.9}{0.5955}\right)^{3/4} + \right.$$

$$\left. \frac{1}{19.2}\left(\frac{957.9}{0.5955}\right)^{1/2}\left(\frac{58.9 \times 10^{-3}\,\text{N/m}}{0.5955\,\text{kg/m}^3\,V^2\,0.002\text{m}}\right)^{1/3}\right] V$$

$$q''_{max} = 6.4299 \times 10^5 V + 3.280 \times 10^6 V^{1/3}$$

Continued …..

PROBLEM 10.36 (Cont.)

The *transition* between the low and high velocity regions occurs when

$$q''_{max} = r_v h_{fg}\ V\left[\frac{0.275}{p}\left(\frac{r_\ell}{r_v}\right)^{1/2} + 1\right]$$

$$q''_{max} = 0.5955\frac{kg}{m^3}\times 2257\times 10^3\frac{J}{kg}V\left[\frac{0.275}{p}\left(\frac{957.9}{0.5955}\right)^{1/2} + 1\right] = 6.0627\times 10^6 V. \qquad (3)$$

For pool boiling conditions when the velocity is zero, the critical heat flux must be estimated according to the correlation for the small horizontal cylinder as introduced in Problem 10.22. If the cylinder were "large," the critical heat flux would be 1.26 MW/m^2 as given by the Zuber-Kutateladze correlation, Eq. 10.7. Following the analysis of Problem 10.22, find Bo = 0.40 and the critical heat flux for the "small" 2 mm cylinder is

$$q''_{max})_{pool} = 1.18\times 1.26\ MW/m^2 = 1.49\ W/m^2.$$

The graph below identifies four regions: pool boiling where $q''_{max} = 1.49\ MW/m^2$ from V = 0 to 0.15 m/s and the low velocity, transition and high velocity regimes.

PROBLEM 10.37

KNOWN: Saturated water at 1 atm and velocity 2 m/s in cross flow over a heater element of 5 mm diameter.

FIND: Maximum heating rate, $q'[W/m]$.

SCHEMATIC:

ASSUMPTIONS: Nucleate boiling in the presence of external forced convection.

PROPERTIES: *Table A-6*, Water (1 atm): T_{sat} = 100°C, ρ_ℓ = 957.9 kg/m^3, ρ_v = 0.5955 kg/m^3, h_{fg} = 2257 kJ/kg, $\sigma = 58.9 \times 10^{-3}$ N/m.

ANALYSIS: The Lienhard-Eichhorn correlation for forced convection with cross flow over a cylinder is appropriate for estimating q''_{max}. Assuming high-velocity region flow, Eq. 10.13 with Eq. 10.14 can be written as

$$q''_{max} = \frac{\rho_v h_{fg} V}{\pi}\left[\frac{1}{169}\left(\frac{\rho_\ell}{\rho_v}\right)^{3/4} + \frac{1}{19.2}\left(\frac{\rho_\ell}{\rho_v}\right)^{1/2}\left(\frac{\sigma}{\rho_v V^2 D}\right)^{1/3}\right].$$

Substituting numerical values, find

$$q''_{max} = \frac{1}{\pi} 0.5955\,\text{kg/m}^3 \times 2257\times10^3\,\text{J/kg}\times 2\,\text{m/s}\left[\frac{1}{169}\left(\frac{957.9}{0.5955}\right)^{3/4} + \right.$$

$$\left.\frac{1}{19.2}\left(\frac{957.9}{0.5955}\right)^{1/2}\left(\frac{58.9\times10^{-3}\,\text{N/m}}{0.5955\,\text{kg/m}^3\,(2\,\text{m/s})^2\,0.005\text{m}}\right)^{1/3}\right]$$

$$q''_{max} = 4.331\,\text{MW/m}^2.$$

The high-velocity region assumption is satisfied if

$$\frac{q''_{max}}{\rho_v h_{fg} V} \overset{?}{<} \frac{0.275}{\pi}\left(\frac{\rho_\ell}{\rho_v}\right)^{1/2} + 1$$

$$\frac{4.331\times10^6\,\text{W/m}^2}{0.5955\,\text{kg/m}^3 \times 2257\times10^3\,\text{J/kg}\times 2\,\text{m/s}} = 1.61 \overset{?}{<} \frac{0.275}{\pi}\left(\frac{957.9}{0.5955}\right)^{1/2} + 1 = 4.51.$$

The inequality is satisfied. Using the q''_{max} estimate, the maximum heating rate is

$$q'_{max} = q''_{max}\cdot\pi D = 4.331\,\text{MW/m}^2 \times \pi\,(0.005\text{m}) = 68.0\,\text{kW/m}. \qquad <$$

COMMENTS: Note that the effect of the forced convection is to increase the critical heat flux by 4.33/1.26 = 3.4 over the pool boiling case.

PROBLEM 10.38

KNOWN: Correlation for forced-convection local boiling inside a vertical tube.

FIND: Boiling heat transfer rate per unit length of the tube.

SCHEMATIC:

$D = 50mm$

$T_s - T_{sat} = 15°C$

T_s

Water, 4 bar

ASSUMPTIONS: (1) Steady-state conditions, (2) Local boiling occurs when tube wall is 15°C above the saturation temperature.

ANALYSIS: From experimental results, the heat transfer coefficient can be estimated by the correlation

$$h = 2.54\left(\Delta T_e\right)^3 \exp\left(\frac{p}{15.3}\right) \qquad \left[W/m^2 \cdot K\right]$$

where ΔT_e is the excess temperature, $T_s - T_{sat}$ $[K]$, and p is the pressure [bar]. The heat transfer rate per unit length is

$$q' = p\, D\, h\, \Delta T_e.$$

Evaluating the heat transfer coefficient, find

$$h = 2.54(15K)^3 \exp(4\ \text{bar}/15.3) = 11{,}134\ W/m^2 \cdot K.$$

The heat rate is then.

$$q' = p\,(0.050m) \times 11{,}134 W/m^2 \cdot K \times 15K = 26.2\ kW/m. \qquad <$$

COMMENTS: The saturation temperature at 4 bar is T_{sat} = 406.5K according to Table A-6.

PROBLEM 10.39

KNOWN: Forced convection and boiling processes occur in a smooth tube with prescribed water velocity and surface temperature.

FIND: Heat transfer rate per unit length of the tube.

SCHEMATIC:

$T_s = 110°C$

Water

$u_m = 1.5 m/s$
$T_m = 95°C$

Brass tube, D = 15mm

ASSUMPTIONS: (1) Fully-developed flow, (2) Nucleate boiling conditions occur on inner wall of tube, (3) Forced convection and boiling effects can be separately estimated.

PROPERTIES: *Table A-6*, Water (T_m = 95°C = 368K): $\rho_\ell = 1/v_f = 962$ kg/m^3, $\rho_v = 1/v_g = 0.500$ kg/m^3, $h_{fg} = 2270$ kJ/kg, $c_{p,\ell} = 4212$ J/kg·K, $\mu_\ell = 296 \times 10^{-6}$ N·s/m^2, $k_\ell = 0.678$ W/m·K, $Pr_\ell = 1.86$, $\sigma = 60 \times 10^{-3}$ N/m, $\nu_\ell = 3.08 \times 10^{-7}$ m^2/s.

ANALYSIS: Experimentation has indicated that the heat transfer rate can be estimated as the sum of the separate effects due to forced convection and boiling. On a per unit length basis,

$$q' = q'_{fc} + q'_{boil}.$$

For *forced convection*, $Re_D = u_m D/\nu_\ell = 1.5 m/s \times 0.015 m/3.08 \times 10^{-7} m^2/s = 73{,}052$. Since Re > 2300, flow is turbulent and since fully developed, use the Dittus-Boelter correlation but with the 0.023 coefficient replaced by 0.019 and n = 0.4,

$$Nu_D = hD/k = 0.019 Re_D^{4/5} Pr^n$$

$$h = \frac{k}{D} Nu_D = \frac{0.678 W/m\cdot K}{0.015m} \times 0.019(73{,}052)^{4/5}(1.86)^{0.4} = 8563 W/m^2\cdot K.$$

$$q'_{fc} = h\pi D(T_s - T_m) = 8{,}563 W/m^2\cdot K \cdot \pi(0.015m)(110-95)°C = 6052 W/m.$$

For *boiling*, ΔT_e = (110 – 100)°C = 10°C and hence nucleate boiling occurs. From the Rohsenow equation, with C_{sf} = 0.006 and n = 1.0,

$$q''_{boil} = \mu_\ell h_{fg}\left[\frac{g(\rho_\ell - \rho_v)}{\sigma}\right]^{1/2}\left[\frac{c_{p,\ell}\Delta T_e}{C_{sf} h_{fg} Pr_\ell^n}\right]^3$$

$$q'_{boil} = 296\times10^{-6}\frac{N\cdot s}{m^2}\times 2270\times10^3\frac{J}{kg}\left[\frac{9.8 m/s^2 (962-0.5) kg/m^3}{60\times10^{-3} N/m}\right]^{1/2}\left[\frac{4212 J/kg\cdot K\times 10K}{0.006\times2270\times10^3\frac{J}{kg}\times(1.86)^{1.0}}\right]^3$$

$$q''_{boil} = 1.22\times10^6 W/m^2 \qquad q'_{boil} = q''_{boil}(\pi D) = 1.22\times10^6 W/m^2(\pi\times0.015m) = 57{,}670 W/m.$$

The total heat rate for both processes is

$$q' = (6052 + 57{,}670) W/m = 6.37\times10^4 W/m.$$ <

COMMENTS: Recognize that this method provides only an estimate since the processes are surely coupled.

PROBLEM 10.40

KNOWN: Saturated steam condensing on the outside of a brass tube and water flowing on the inside of the tube; convection coefficients are prescribed.

FIND: Steam condensation rate per unit length of the tube.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions.

PROPERTIES: *Table A-6*, Water, vapor (0.1 bar): $T_{sat} \approx 320K$, $h_{fg} = 2390 \times 10^3$ J/kg; *Table A-1*, Brass $\left(\overline{T} = (T_m + T_{sat})/2 \approx 300K\right)$: $k = 110\ W/m \cdot K$

ANALYSIS: The condensation rate per unit length follows from Eq. 10.33 written as

$$\dot{m}' = q'/h'_{fg} \tag{1}$$

where the heat rate follows from Eq. 10.32 using an overall heat transfer coefficient

$$q' = U_o \cdot \pi D_o (T_{sat} - T_m) \tag{2}$$

and from Eq. 3.31,

$$U_o = \left[\frac{1}{h_o} + \frac{D_o/2}{k} \ell n \frac{D_o}{D_i} + \frac{D_o}{D_i}\frac{1}{h_i}\right]^{-1} \tag{3}$$

$$U_o = \left[\frac{1}{6800 W/m^2 \cdot K} + \frac{0.0095m}{110 W/m \cdot K} \ell n \frac{19}{16.5} + \frac{19}{16.5}\frac{1}{5200 W/m^2 \cdot K}\right]^{-1}$$

$$U_o = \left[147.1 \times 10^{-6} + 12.18 \times 10^{-6} + 192.3 \times 10^{-6}\right]^{-1} W/m^2 \cdot K = 2627 W/m^2 \cdot K.$$

Combining Eqs. (1) and (2) and substituting numerical values (see below for h'_{fg}), find

$$\dot{m}' = U_o \pi D_o (T_{sat} - T_m)/h'_{fg}$$

$$\dot{m}' = 2627 W/m^2 \cdot K \pi\ (0.019m)(320 - 303) K / 2410 \times 10^3 J/kg = 1.11 \times 10^{-3} kg/s. \quad <$$

COMMENTS: (1) Note from evaluation of Eq. (3) that the thermal resistance of the brass tube is not negligible. (2) From Eq. 10.26, with Ja = $c_{p,\ell}(T_{sat} - T_s)/h_{fg}$, $h'_{fg} = h_{fg}[1 + 0.68 Ja]$. Note from expression for U_o, that the internal resistance is the largest. Hence, estimate $T_{s,o} \approx T_o - (R_o/\Sigma R)(T_o - T_m) \approx 313K$. Hence,

$$h'_{fg} \approx 2390 \times 10^3 J/kg \left[1 + 0.68 \times 4179 J/kg \cdot K (320 - 313) K / 2390 \times 10^3 J/kg\right]$$

$$h'_{fg} = 2410 kJ/kg$$

where $c_{p,\ell}$ for water (liquid) is evaluated at $T_f = (T_{s,o} + T_o)/2 \approx 317K$.

PROBLEM 10.41

KNOWN: Insulated container having cold bottom surface and exposed to saturated vapor.

FIND: Expression for growth rate of liquid layer, δ(t); thickness formed for prescribed conditions; compare with vertical plate condensate for same conditions.

SCHEMATIC:

ASSUMPTIONS: (1) Side wall effects are negligible and, (2) Vapor-liquid interface is at T_{sat}, (3) Temperature distribution in liquid is linear, (4) Constant properties.

PROPERTIES: *Table A-6*, Saturated vapor (p = 1.0133 bar): T_{sat} = 100°C, ρ_v = 0.596 kg/m^3, h_{fg} = 2257 kJ/kg; *Table A-6*, Saturated liquid (T_f = 90°C = 363K): r_ℓ = 1000 kg/m^3, m_ℓ = 313 × 10^{-6} N·s/m^2, k_ℓ = 0.676 W/m·K, $c_{p,\ell}$ = 4207 J/kg·K.

ANALYSIS: Perform a surface energy balance on the interface (see above) recognizing that $\dot{m}_\ell / A = r_\ell \, d d / dt$ from an overall mass rate balance on the liquid to obtain

$$\dot{E}''_{in} - \dot{E}''_{out} = q''_{conds} - q''_{cond} = \frac{\dot{m}}{A} h_{fg} - k_\ell \frac{T_{sat} - T_s}{d} = r_\ell \frac{dd}{dt} h_{fg} - k_\ell \frac{T_{sat} - T_s}{d} = 0 \quad (1)$$

where q''_{conds} is the condensation heat flux and q''_{cond} is the conduction heat flux into the liquid layer of thickness δ with linear temperature distribution. Eq. (1) can be rewritten as

$$r_\ell h_{fg} \frac{dd}{dt} = k_\ell \frac{T_{sat} - T_s}{d}.$$

Separate variables and integrate with limits shown to obtain the liquid layer growth rate,

$$\int_0^d d \, dd = \int_0^t \frac{k_\ell \left(T_{sat} - T_s\right)}{r_\ell h_{fg}} dt \qquad \text{or} \qquad d = \left[\frac{2k_\ell \left(T_{sat} - T_s\right)}{r_\ell h_{fg}} t\right]^{1/2}. \qquad (2) \quad <$$

For the prescribed conditions, the liquid layer thickness and condensate formed in one hour are

$$d\,(1\text{hr}) = \left[2 \times 0.676 \frac{\text{W}}{\text{m} \cdot \text{K}} (100 - 80)^\circ\text{C} \times 3600\text{s} / 1000 \frac{\text{kg}}{\text{m}^3} \times 2257 \times 10^3 \frac{\text{J}}{\text{kg}}\right]^{1/2} = 6.57\text{mm} \quad <$$

$$M\,(1\text{hr}) = r_\ell A d = 1000\text{kg/m}^3 \times 200 \times 10^{-6} \text{m}^2 \times 6.57 \times 10^{-3} \text{m} = 1.314 \times 10^{-3} \text{kg}. \quad <$$

Continued …..

The condensate formed on a vertical plate with the same conditions follows from Eq. 10.33,

$$M_{vp} = \dot{m} \cdot t = \overline{h}_L A \left(T_{sat} - T_s\right) \cdot t / h'_{fg}$$

where h'_{fg} and $\overline{h}_L$ follow from Eqs. 10.26 and 10.30, respectively.

$$h'_{fg} = h_{fg}\left(1 + 0.68\text{Ja}\right) = h_{fg}\left(1 + 0.68 c_{p,\ell}\,\Delta T / h_{fg}\right)$$

$$h'_{fg} = 2257\times10^3\,\text{J/kg}\left(1 + 0.68\times4207\,\frac{\text{J}}{\text{kg}\cdot\text{K}}\left(100-80\right)^\circ\text{C}/2257\times10^3\,\text{J/kg}\right) = 2314\,\text{kJ/kg}$$

$$\overline{h}_L = 0.943\left[g\,\rho_\ell\left(\rho_\ell - \rho_v\right)k_\ell^3\,h'_{fg} / \mu_\ell\left(T_{sat} - T_s\right)L\right]^{1/4}$$

$$\overline{h}_L = 0.943\Big[9.8\,\text{m/s}^2\times1000\,\text{kg/m}^3\left(1000 - 0.596\right)\text{kg/m}^3\left(0.676\,\text{W/m}\cdot\text{K}\right)^3$$

$$\times2314\times10^3\,\text{J/kg}/313\times10^{-6}\,\text{N}\cdot\text{s/m}^2\left(100-80\right)^\circ\text{C}\times0.2\,\text{m}\Big]^{1/4}$$

$$\overline{h}_L = 8155\,\text{W/m}^2\cdot\text{K}.$$

Hence,

$$M_{vp} = 8155\,\text{W/m}^2\cdot\text{K}\times200\times10^{-6}\,\text{m}^2\left(100-80\right)^\circ\text{C}\times3600\,\text{s}/2314\times10^3\,\text{J/kg}$$

$$M_{vp} = 5.08\times10^{-2}\,\text{kg}.$$ <

COMMENTS: (1) Note that the condensate formed by the vertical plate is an order of magnitude larger. For the vertical plate the rate of condensate formation is constant. For the container bottom surface, the rate decreases with increasing time since the conduction resistance increases as the liquid layer thickness increases.

(2) For the vertical plate, assumed to be square 14.1 × 14.1 mm, the Reynolds number, Eq. 10.35 and 10.33, is

$$\text{Re}_\delta = \frac{4\dot{m}}{\mu_\ell b} = \frac{4}{\mu_\ell b}\frac{\overline{h}_L A\left(T_{sat} - T_s\right)}{h'_{fg}}$$

$$\text{Re}_\delta = \frac{4}{313\times10^{-6}\,\text{N}\cdot\text{s/m}^2\times14.1\times10^{-3}\,\text{m}}\frac{8155\,\text{W/m}^2\cdot\text{K}\left(200\times10^{-6}\,\text{m}^2\right)\left(100-80\right)^\circ\text{C}}{2314\,\text{kJ/kg}}$$

$$\text{Re}_\delta = 12.8.$$

Hence, using Eq. 10.30 to estimate $\overline{h}_L$ is correct since, in fact, the film is laminar.

PROBLEM 10.42

KNOWN: Vertical tube experiencing condensation of steam on its outer surface.

FIND: Heat transfer and condensation rates.

SCHEMATIC:

Saturated steam, 1atm

$L = 1m$

$D=100mm$

$T_s = 94°C$

Condensate

ASSUMPTIONS: (1) Film condensation, (2) Negligible non-condensibles, (3) D/2 >> δ, vertical plate behavior.

PROPERTIES: *Table A-6*, Water, vapor (1.0133 bar): $T_{sat} = 100°C$, $\rho_v = 0.596\ kg/m^3$, $h_{fg} = 2257$ kJ/kg; *Table A-6*, Water, liquid ($T_f = 97°C$): $r_\ell = 960.6\ kg/m^3$, $m_\ell = 289 \times 10^{-6}\ N{\cdot}s/m^2$, $c_{p,\ell} = 4214$ J/kg·K, $k_\ell = 0.679$ W/m·K.

ANALYSIS: The heat transfer and condensation rates are

$$q = \overline{h}_L (p\ D L)(T_{sat} - T_s) \qquad\qquad \dot{m} = q / h'_{fg}$$

where $h'_{fg} = h_{fg}(1 + 0.68Ja)$ and $Ja = c_{p,\ell}(T_{sat} - T_s)/h_{fg}$. Hence Ja = 4214 J/kg·K (100 - 94)K/2257 × 10^3 J/kg = 0.0112 and h'_{fg} = 2274 kJ/kg. Assume laminar film condensation and use Eq. 10.31 to estimate $\overline{h}_L$,

$$\overline{Nu}_L = \frac{\overline{h}_L L}{k_\ell} = 0.943\left[\frac{r_\ell g(r_\ell - r_v)h'_{fg}L^3}{m_\ell k_\ell (T_{sat} - T_s)}\right]^{1/4}$$

$$\overline{h}_L = \frac{0.679 W/m\cdot K}{1.0m}\times 0.943\left[\frac{960.6kg/m^3 \times 9.8m/s^2(960.6 - 0.596)kg/m^3 \times 2274\times 10^3 J/kg \times (1m)^3}{289\times 10^{-6} N\cdot s/m^2 \times 0.679 W/m\cdot K(100-94)K}\right]^{1/4} = 7360 W/m^2\cdot K.$$

Hence,
$$q = 7360 W/m^2\cdot K(p \times 0.100m \times 1m)(100-94)K = 13.87kW.$$

$$\dot{m} = 13.9\times 10^3 W/2274\times 10^3 J/kg = 0.00610kg/s.$$

Check the laminar film assumption: $Re_d = 4\dot{m}/m_\ell b = 4 \times 0.00610$ kg/s/289 × 10^{-6} N·s/m² × (π × 0.100m) = 269. Since 30 < Re_δ < 1800, the flow is wavy, not laminar. By combining Eqs. 10.33 and 10.35 with 10.38 (see Example 10.3), find Re_δ,

$$\frac{Re_d\ m_\ell\ b\ h'_{fg}}{4A_s(T_{sat} - T_s)} = \frac{Re_d}{1.08Re_d^{1.22} - 5.2}\cdot\frac{k_\ell}{\left(n_\ell^2/g\right)^{1/3}}$$

Continued …..

$$\frac{289\times10^{-6}\,\mathrm{N\cdot s/m^2}\left(p\,0.10\mathrm{m}\right)\times2274\times10^{3}\mathrm{J/kg}}{4\left(p\times0.10\mathrm{m}\times1\mathrm{m}\right)\left(100-94\right)\mathrm{K}}=\frac{1}{1.08\mathrm{Re}_d^{1.22}-5.2}\cdot\frac{0.679\,\mathrm{W/m\cdot K}}{\left[\left(289\times10^{-6}/960.6\right)^2\mathrm{m^4/s^2}/9.8\mathrm{m/s^2}\right]^{1/3}}$$

Solving, we obtain $\mathrm{Re}_\delta = 311$. Using Eq. 10.38, find

$$\frac{\overline{h}_L\left(\overline{n}_\ell^{2}/g\right)^{1/3}}{k_\ell}=\frac{\mathrm{Re}_d}{1.08\mathrm{Re}_d^{1.22}-5.2}\qquad \overline{h}_L=8507\,\mathrm{W/m^2\cdot K}$$

$$q=8507\,\mathrm{W/m^2\cdot K}\left(p\times0.10\mathrm{m}\times1\mathrm{m}\right)\left(100-94\right)\mathrm{K}=16.0\,\mathrm{kW}\qquad <$$

$$\dot{m}=16.0\times10^{3}\,\mathrm{W}/2274\times10^{3}\,\mathrm{J/kg}=7.05\times10^{-3}\,\mathrm{kg/s}.\qquad <$$

COMMENTS: To determine whether the assumption D/2 >> δ is satisfied, use Eq. 10.25 to estimate δ(L) ≈ 0.12mm. Despite the laminar film assumption, clearly the assumption is justified and the vertical plate correlation is applicable.

PROBLEM 10.43

KNOWN: Vertical tube experiencing condensation of steam on its outer surface.

FIND: Heat transfer and condensation rates.

SCHEMATIC:

ASSUMPTIONS: (1) Film condensation, (2) Negligible condensibles in steam, (3) D/2 >> δ, vertical plate behavior.

PROPERTIES: *Table A-6*, Water, vapor (1.5 bar): $T_{sat} \approx 385K$, $\rho_v = 0.876\ kg/m^3$, $h_{fg} = 2225$ kJ/kg; *Table A-6*, Water, (liquid $T_f = 376K$): $r_\ell = 956.2\ kg/m^3$, $c_{p,\ell} = 4220\ J/kg\cdot K$, $m_\ell = 271 \times 10^{-6}$ $N\cdot s/m^2$, $k_\ell = 0.681\ W/m\cdot K$.

ANALYSIS: The heat transfer and condensation rates are

$$q = \bar{h}_L (p\,DL)(T_{sat} - T_s) \qquad\qquad \dot{m} = q/h'_{fg}$$

where $h'_{fg} = h_{fg}(1 + 0.68Ja)$ and $Ja = c_{p,\ell}(T_{sat} - T_s)/h_{fg}$. Hence, Ja = 4220 J/kg·K (385 – 367)K/2225 × 10^3 J/kg = 0.0171 and h'_{fg} = 2277 kJ/kg. Assume the flow is wavy. Combine Eqs. 10.33 and 10.35 with 10.38, find Re_δ.

$$\frac{Re_d\, m_\ell\, b h'_{fg}}{4A_s(T_{sat} - T_s)} = \frac{Re_d}{1.08Re_d^{1.22} - 5.2} \cdot \frac{k_\ell}{\left(n^2/g\right)^{1/3}}$$

$$\frac{271\times10^{-6} N\cdot s/m^2 (p \times 0.10m) \times 2277\times10^3 J/kg}{4\times(p\times0.10m\times1m)(385-367)K}$$

$$= \frac{1}{1.08Re_d^{1.22} - 5.2} \cdot \frac{0.681 W/m\cdot K}{\left[\left(271\times10^{-6}/956.2\right)^2 m^4/s^2/9.8m/s^2\right]^{1/3}}$$

$$Re_d = 832.$$

Using Eq. 10.38, find $\dfrac{\bar{h}_L\left(n_\ell^2/g\right)^{1/3}}{k_\ell} = \dfrac{Re_d}{1.08Re_d^{1.22} - 5.2}$ $\qquad \bar{h}_L = 7{,}127 W/m^2\cdot K.$

$$q = 7127 W/m^2\cdot K(p\times0.1m\times1m)(385-367)K = 40.3kW \qquad <$$

$$\dot{m} = 40.3\times10^3 W/2277\times10^3 J/kg = 0.0177kg/s. \qquad <$$

Continued …..

COMMENTS: Since $30 < Re_\delta < 1800$, the wavy flow film assumption is justified. By comparing these results with those of Problem 10.42, the effect of increased pressure on condensation can be seen.

p (bar)	T_{sat}(K)	T_{sat}-T_s(K)	$\overline{h}_L \left(W/m^2 \cdot K\right)$	q (kW)	$\dot{m} \cdot 10^3 (kg/s)$
1.01	373	6	8507	16.0	7.05
1.5	385	18	7127	40.3	17.7

The effect of increasing the pressure from 1.01 to 1.5 bar is to increase the excess temperature three-fold, to decrease $\overline{h}_L$ by 16%, and to increase the rates by a factor of 2.5.

PROBLEM 10.44

KNOWN: Saturated steam at one atmosphere condenses on the outer surface of a vertical tube; water flow within tube experiences 4°C temperature rise.

FIND: Required flow rate to maintain tube wall at 94°C.

SCHEMATIC:

ASSUMPTIONS: (1) Laminar wavy film condensation on a vertical surface, (2) Negligible concentration of non-condensible gases in the stream, (3) Thermal resistance of tube wall is negligible, (4) Water flow is fully developed, (5) Tube wall surface is at uniform temperature T_s.

PROPERTIES: *Table A-6,* Water (Assume $\overline{T}_m \approx 300K$): $c_p = 4179$ J/kg·K, $\mu = 855 \times 10^{-6}$ N·s/m^2, k = 0.613 W/m·K, Pr = 5.83.

ANALYSIS: From the results of Problem 10.42, the heat rate for laminar wavy condensation on the outside surface of the tube was found to be q = 16.0 kW. From an energy balance on the water flowing within the tube, the flow rate is

$$\dot{m} = q / c_p \left(T_{m,o} - T_{m,i}\right) = 16.0 \times 10^3 \text{W} / 4179 \text{J/kg} \cdot \text{K} \times 4\text{K} = 0.957 \text{kg/s}. \qquad (1) \quad <$$

To determine the inlet temperature of the water, the rate equation is required.

$$q = U A \Delta T_{lm} \qquad U = \frac{1}{1/h_i + 1/h_o} \qquad \Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln\left(\Delta T_1 / \Delta T_2\right)}. \qquad (2,3,4)$$

From Problem 10.42, $h_o = 8507$ W/m^2·K. Evaluate Re for the water flow using Eq. 8.6

$$Re = 4\dot{m}/\pi D \mu = 4 \times 0.957 \text{kg/s}/\pi \times 0.092\text{m} \times 855 \times 10^{-6} \text{N} \cdot \text{s/m}^2 = 15{,}493.$$

The flow is turbulent and since fully-developed, Eq. 8.60 is an appropriate correlation.

$$Nu = h_i D_i / k = 0.023 Re_D^{4/5} Pr^{0.4} = 0.023(15{,}493)^{4/5}(5.83)^{0.4} = 104.7$$

$$h_i = Nu \cdot k / D_i = 104.7 \times 0.613 \text{W/m} \cdot \text{K} / 0.092\text{m} = 698 \text{W/m}^2 \cdot \text{K}.$$

Hence, U = 1/[1/698 + 1/8507] = 645 W/m^2·K. Substituting numerical values into the rate equation, Eq. (2), with $A = \pi D_i L$, find

$$\Delta T_{lm} = q/UA = 16.0 \times 10^3 \text{W} / 645 \text{W/m}^2 \cdot \text{K} \times (\pi\, 0.092\text{m} \times 1\text{m}) = 85.6\text{K}.$$

Recalling now Eq. (4), note that $\Delta T_1 - \Delta T_2 = 4K$ and that $T_{m,o} - T_{m,i} = 4K$, hence,

$$85.6\text{K} = 4\text{K} / \ln \frac{94 - T_{m,i}}{94 - \left(T_{m,i} + 4\right)} \qquad \text{giving} \qquad T_{m,i} = 6.3°\text{C}.$$

COMMENTS: Note that the $\overline{T}_m = 300K$ assumption is not reasonable and an iteration should be made. Also, it is likely that the thermal resistance of the tube wall is not negligible.

PROBLEM 10.45

KNOWN: Cooled vertical plate 500-mm high and 200-mm wide condensing saturated steam at 1 atm.

FIND: (a) Surface temperature, T_s, required to achieve a condensation rate of $\dot{m}$ = 25 kg/h, (b) Compute and plot T_s as a function of the condensation rate for the range $15 \le \dot{m} \le 50\ \text{kg/h}$, and (c) Compute and plot T_s for the same range of $\dot{m}$, but if the plate is 200 mm high and 500 mm wide (vs. 500 mm high and 200 mm wide for parts (a) and (b)).

SCHEMATIC:

Plate, T_s
b = 0.2 m
L = 0.5 m
Steam, 1 atm
$\dot{m}$ = 25 kg/h

ASSUMPTIONS: (1) Film condensation, (2) Negligible non-condensables in steam.

PROPERTIES: *Table A-6*, Water, vapor (1.0133 bar): T_{sat} = 100°C, $\rho_v = 0.5963\ \text{kg/m}^3$, h_{fg} = 2257 kJ/kg; *Table A-6*, Water, liquid ($T_f \approx (74 + 100)$°C/2 ≈ 360 K): ρ_ℓ = 967.1 kg/m^3, $c_{p,\ell}$ = 4203 J/kg·K, $\mu_\ell = 324 \times 10^{-6}$ N·s/m^2, k_ℓ = 0.674 W/m·K.

ANALYSIS: (a) The surface temperature can be determined from the rate equation, Eq. 10.32, written as

$$T_s = T_{sat} - q/\bar{h}_L A_s = T_{sat} - \dot{m}h'_{fg}/\bar{h}_L A_s$$

where $h'_{fg} = h_{fg}(1 + 0.68\,Ja)$ and $Ja = c_{p,\ell}(T_{sat} - T_s)/h_{fg}$. To evaluate T_s, we need values of h_L and h'_{fg}, both of which require knowledge of T_s. Hence, we need to assume a value of T_s and iterate the solution until good agreement with calculated T_s value is achieved. *Assume* T_s = 74°C and evaluate h'_{fg} and Re_δ.

$$h'_{fg} = 2257\,\text{kJ/kg}\left(1+0.68\left[4203\,\text{J/kg}\cdot\text{K}\,(100-74)\,\text{K}/2257\times10^3\,\text{J/kg}\right]\right) = 2331\,\text{kJ/kg}$$

$$Re_\delta = 4\dot{m}/\mu_\ell b = 4\times(25/3600)\,\text{kg/s}/324\times10^{-6}\,\text{N}\cdot\text{s/m}^2\times0.2\text{m} = 429.$$

Since $30 < Re_\delta < 1800$, the flow is wavy-laminar and Eq. 10.38 is appropriate,

$$\bar{h}_L = \frac{Re_\delta}{1.08\,Re_\delta^{1.22} - 5.2}\cdot\frac{k_\ell}{\left(\nu_\ell^2/g\right)^{1/3}}$$

$$\bar{h}_L = \frac{429}{108(429)^{1.22} - 5.2}\cdot\frac{0.674\,\text{W/m}\cdot\text{K}}{\left[\left(324\times10^{-6}/967.1\right)^2 \text{m}^4/\text{s}^2/9.8\,\text{m/s}^2\right]^{1/3}} = 7312\,\text{W/m}^2\cdot\text{K}$$

Hence, $T_s = 100^\circ\text{C} - (25/3600)\dfrac{\text{kg}}{\text{s}}\times2331\times10^3\dfrac{\text{J}}{\text{kg}}\Big/\left[7312\dfrac{\text{W}}{\text{m}^2\cdot\text{K}}\times(0.2\times0.5)\,\text{m}^2\right] = 78^\circ\text{C}.$ <

This value is to be compared to the assumed value of 74°C. See comment 1.

(b,c) Using the *IHT Correlations Tool*, *Film Condensation*, *Vertical Plate* for *laminar*, *wavy-laminar* and *turbulent regions*, combined with the *Properties Tool* for *Water*, the surface temperature T_s was

Continued...

calculated as a function of the condensation rate, $\dot{m}$, considering the two plate configurations as indicated in the plot below.

As expected the condensation rate increases with decreasing surface temperature. The plate with the shorter height (L = 200 mm vs 500 mm) will have the thinner boundary layer and, hence, the higher average convection coefficient. Since both plate configurations have the same total surface area, the 200-mm height plate will have the larger heat transfer and condensation rates. For the range of conditions examined, the condensate flow is in the wavy-laminar region.

COMMENTS: (1) With the IHT model developed for parts (b) and (c), the result for the part (a) conditions with $\dot{m}$ = 25 kg/h is T_s = 78.2°C (Re_δ = 439 and $\bar{h}_L = 7403\ W/m^2 \cdot K$). Hence, the assumed value (Ts = 74°C) required to initiate the analysis was a good one.

(2) A copy of the IHT Workspace model used to generate the above plot is shown below.

```
/* Correlations Tool
- Film Condensation, Vertical Plate, Laminar, wavy-laminar and turbulent regions: */
NuLbar = NuL_bar_FCO_VP(Redelta,Prl)     // Eq 10.37, 38, 39
NuLbar = hLbar * (nul^2 / g)^(1/3) / kl
g = 9.8                                   // Gravitational constant, m/s^2
Ts =  Ts_C + 273                          // Surface temperature, K
Ts_C = 78                                 // Initial guess value used to solve the model
Tsat =  100 + 273                         // Saturation temperature, K
// The liquid properties are evaluated at the film temperature, Tf,
Tf = Tfluid_avg(Ts,Tsat)
// The condensation and heat rates are
q = hLbar * As * (Tsat - Ts)      // Eq 10.32
As = L * b                                // Surface Area, m^2
mdot = q / h'fg                           // Eq 10.33
h'fg = hfg + 0.68 * cpl * (Tsat - Ts)     // Eq 10.26
// The Reynolds number based upon film thickness is
Redelta = 4 * mdot / (mul * b)            // Eq 10.35
// Assigned Variables:
L = 0.5                        // Vertical height, m
b = 0.2                        // Width, m
mdot_h = mdot * 3600           // Condensation rate, kg/h
//mdot_h = 25                  // Design value, part (a)
// Properties Tool - Water:
// Water property functions :T dependence, From Table A.6
// Units: T(K), p(bars);
xl =  0                        // Quality (0=sat liquid or 1=sat vapor)
rhol = rho_Tx("Water",Tf,xl)   // Density, kg/m^3
hfg = hfg_T("Water",Tsat)      // Heat of vaporization, J/kg
cpl = cp_Tx("Water",Tf,xl)     // Specific heat, J/kg·K
mul = mu_Tx("Water",Tf,xl)     // Viscosity, N·s/m^2
nul = nu_Tx("Water",Tf,xl)     // Kinematic viscosity, m^2/s
kl = k_Tx("Water",Tf,xl)       // Thermal conductivity, W/m·K
Prl = Pr_Tx("Water",Tf,xl)     // Prandtl number
```

PROBLEM 10.46

KNOWN: Plate dimensions, temperature and inclination. Pressure of saturated steam.

FIND: (a) Heat transfer and condensation rates for vertical plate, (b) Heat transfer and condensation rates for inclined plate.

SCHEMATIC:

b = 2 m
L = 2 m
T_s = 50°C
Sat steam p = 1 atm
(a)

b
θ = 45°
T_s
L
Sat steam p = 1 atm
(b)

ASSUMPTIONS: (1) Conditions correspond to the turbulent film region, (2) Constant properties.

PROPERTIES: *Table A-6*, saturated vapor (p=1.0133 bars): T_{sat} = 100°C, ρ_v = 0.596 kg/m^3, h_{fg} = 2257 kJ/kg. *Table A-6*, saturated liquid (T_f = 75°C): $\rho_\ell = 975\,kg/m^3$, $\mu_\ell = 375\times10^{-6}\,N\cdot s/m^2$, $k_\ell = 0.668\,W/m\cdot K$, $c_{p,\ell} = 4193\,J/kg\cdot K$.

ANALYSIS: (a) Expressing $\overline{h}_L$ in terms of Re_δ by combining Eqs. (10.33) and (10.35) and substituting into Eq. (10.38), it follows that

$$\frac{Re_\delta\,\mu_\ell h'_{fg}}{4L\left(T_{sat}-T_s\right)} = \frac{Re_\delta}{1.08\,Re_\delta^{1.22}-5.2}\cdot\frac{k_\ell}{\left(\nu_\ell^2/g\right)^{1/3}} \tag{1}$$

where, with Ja = $c_{p,\ell}\left(T_{sat}-T_s\right)/h_{fg} = 0.0929$, $h'_{fg} = h_{fg}\left(1+0.68\,Ja\right) = 2400\,kJ/kg$. From an iterative solution to Eq. (1), we obtain Re_δ = 2370, and the assumption of a turbulent film is justified. From Eqs. (10.35) and (10.33) the condensation and heat rates are then

$$\dot{m} = \frac{\mu_\ell b\,Re_\delta}{4} = 0.444\,kg/s$$ <

$$q = \dot{m}\,h'_{fg} = 0.444\,kg/s\times2.4\times10^6\,J/kg = 1.065\times10^6\,W$$ <

From Eq. (10.32), we also obtain $\overline{h}_L = q/\left[\left(bL\right)\left(T_{sat}-T_s\right)\right] = 5325\,W/m^2\cdot K$.

(b) With $\overline{h}_{L(incl)} \approx \left(\cos\theta\right)^{1/4}\overline{h}_L$, we obtain $\overline{h}_{L(incl)} \approx 0.917\times5325\,W/m^2\cdot K = 4880\,W/m^2\cdot K$. If the inclination reduces $\overline{h}_L$ by 8.73%, the heat and condensation rates are reduced by equivalent amounts. Hence,

$$\dot{m} = 0.407\,kg/s, \qquad q = 0.977\times10^6\ W$$ <

COMMENTS: The initial guess of a turbulent film region was motivated by the value of L = 2m, which was believed to be large enough for transition to turbulence. Note that the solution could also have been obtained by accessing the Film Condensation correlations of IHT, implementation of which does not require an assumption of flow conditions.

PROBLEM 10.47

KNOWN: Saturated ethylene glycol (1 atm) condensing on a vertical plate at 420K.

FIND: Heat transfer rate to the plate and condensation rate.

SCHEMATIC:

Plate, b=0.10m
$L=0.3m$
$T_s=420K$
Ethylene glycol
1 atm, $T_{sat}=470K$

ASSUMPTIONS: (1) Film condensation, (2) Negligible non-condensible gases in vapor.

PROPERTIES: *Table A-5*, Ethylene glycol vapor (1 atm): $T_{sat} = 470K$, $\rho_v \approx 0$ kg/m^3, $h_{fg} = 812$ kJ/kg; *Table A-5*, Ethylene glycol, liquid ($T_f = (T_s + T_{sat})/2 \approx 445K$; use properties at upper limit of table 373K): $r_\ell = 1058.5$ kg/m^3, $c_{p,\ell} = 2742$ J/kg·K, $m_\ell = 0.215 \times 10^{-2}$ N·s/m^2, $k_\ell = 0.263$, W/m·K.

ANALYSIS: The heat transfer and condensation rates are given by Eqs. 10.32 and 10.33.

$$q = \overline{h}_L A_s (T_{sat} - T_s) \qquad \dot{m} = q / h'_{fg},$$

where $h'_{fg} = h_{fg}(1 + 0.68\,Ja)$ and $Ja = c_{p,\ell}(T_{sat} - T_s)/h_{fg}$. Substituting property values at $T_f = (T_s + T_{sat})/2$, find $h'_{fg} = 812$ kJ/kg $(1 + 0.68\,[2742$ J/kg·K $(470 - 420)$K$/812 \times 10^3$ J/kg$]) = 905$ kJ/kg. Assuming the flow is laminar, use Eq. 10.30 to evaluate $\overline{h}_L$.

$$\overline{h}_L = 0.943\left[\frac{g\, r_\ell (r_\ell - r_v) k_\ell^3 h'_{fg}}{m_\ell (T_{sat} - T_s) L}\right]^{1/4} \left[\frac{9.8\,\text{m/s}^2 \times 1058.5\,\text{kg/m}^3 (1058.5 - 0)\,\text{kg/m}^3 (0.263\,\text{W/m}\cdot\text{K})^3 \times 905 \times 10^3\,\text{J/kg}}{0.215 \times 10^{-2}\,\text{N}\cdot\text{s/m}^2 (470 - 420)\,\text{K} \times 0.3\text{m}}\right]^{1/4}$$

find $\overline{h}_L = 1451$ W/m^2·K. Using the rate equations, find

$$q = 1451\ \text{W/m}^2\cdot\text{K}\,(0.3 \times 0.1)\,\text{m}^2\,(470 - 420)\,\text{K} = 2.18\text{kW}$$

$$\dot{m} = 2.18 \times 10^3\,\text{W}/905 \times 10^3\,\text{J/kg} = 0.002405\,\text{kg/s} = 8.66\,\text{kg/h}.$$

Determine whether the flow is indeed laminar: $Re_d = 4\dot{m}/m_\ell b = 4 \times 0.002405$ kg/s$/0.215 \times 10^{-2}$ N·s/m$^2 \times$ 0.1m = 44.7. Since $30 < Re_\delta < 1800$, the flow is in the wavy-laminar region. Hence, the correlation of Eq. 10.38 is more appropriate. Combining Eq. 10.33 and 10.35 with 10.38 (see Example 10.3),

$$\frac{Re_d\, m_\ell\, b\, h'_{fg}}{4 A_s (T_{sat} - T_s)} = \frac{Re_d}{1.08 Re_d^{1.22} - 5.2} \cdot \frac{k_\ell}{(n_\ell^2/g)^{1/3}}$$

$$\frac{0.215 \times 10^{-2}\,\text{N}\cdot\text{s/m}^2 \times 0.1\text{m} \times 905 \times 10^3\,\text{J/kg}}{4 \times (0.3 \times 0.1)\,\text{m}^2\,(470 - 420)\,\text{K}} = \frac{1}{1.08 Re_d^{1.22} - 5.2} \frac{0.263\,\text{W/m}\cdot\text{K}}{\left[\left(0.215 \times 10^{-2}/1058.5\right)\text{m}^4/\text{s}^2/9.8\,\text{m/s}^2\right]^{1/3}}$$

find $Re_\delta = 45$ and using Eq. 10.38, find

$$\overline{h}_L = \frac{Re_d}{1.08 Re_d^{1.22} - 5.2} \cdot \frac{k_\ell}{(n_\ell^2/g)^{1/3}} = 1470\,\text{W/m}^2\cdot\text{K}.$$

Hence, $\qquad q = 2.21\text{kW} \qquad \dot{m} = 2.44 \times 10^{-3}\,\text{kg/s}.$ $\quad$ <

COMMENTS: Note the wavy-laminar value of Re_δ is within 1.3% of the laminar value.

PROBLEM 10.48

KNOWN: Vertical plate 2.5 m high at a surface temperature $T_s = 54°C$ exposed to steam at atmospheric pressure.

FIND: (a) Condensation and heat transfer rates, (b) Whether turbulent flow would still exist if the height were halved, and (c) Compute and plot the condensation rates for the two plate heights (2.5 m and 1.25 m) as a function of surface temperature for the range, $54 \le T_s \le 90°C$.

SCHEMATIC:

Plate, unit width (b)
$T_s = 54°C$
Steam, 1atm
$L = 2.5$ m
q'_s
$\dot{m}$

ASSUMPTIONS: (1) Film condensation, (2) Negligible non-condensables in steam.

PROPERTIES: *Table A-6*, Water, vapor (1 atm): $T_{sat} = 100°C$, $\rho_v = 0.596$ kg/m^3, $h_{fg} = 2257$ kJ/kg; *Table A-6*, Water, liquid ($T_f = (100 + 54)°C/2 = 350$ K): $\rho_\ell = 973.7$ kg/m^3, $k_\ell = 0.668$ W/m·K, $\mu_\ell = 365 \times 10^{-6}$ N·s/m^2, $c_{p,\ell} = 4195$ J/kg·K, $Pr_\ell = 2.29$.

ANALYSIS: (a) The heat transfer and condensation rates are given by Eqs. 10.32 and 10.33,

$$q' = \bar{h}_L L\left(T_{sat} - T_s\right) \qquad \dot{m}' = q'/h'_{fg} \qquad (1,2)$$

where, from Eq. 10.26, with Ja = $c_{p,\ell}\,(T_{sat} - T_s)/h_{fg}$,

$$h'_{fg} = h_{fg}\left(1 + 0.68\left[c_{p,\ell}\left(T_{sat} - T_s\right)/h_{fg}\right]\right)$$

$$h'_{fg} = 2257\frac{\text{kJ}}{\text{kg}}\left(1 + 0.68\left[\frac{4195\,\text{J/kg}\cdot\text{K}\,(100-54)\text{K}}{2257\times10^3\,\text{J/kg}}\right]\right) = 2388\,\text{kJ/kg}\,.$$

Assuming turbulent flow conditions, Eq. 10.39 is the appropriate correlation,

$$\frac{\bar{h}L\left(\nu_\ell^2/g\right)^{1/3}}{k_\ell} = \frac{Re_\delta}{8750 + 58\,Pr^{-0.5}\left(Re_\delta^{0.75} - 253\right)} \qquad Re_\delta > 1800 \qquad (3)$$

Not knowing Re_δ or $\bar{h}_L$, another relation is required. Combine Eq. 10.33 and 10.35,

$$\bar{h}_L = \frac{\dot{m}h'_{fg}}{A\left(T_{sat} - T\right)} = \left(\frac{Re_\delta\,\mu_\ell b}{4}\right)\frac{h'_{fg}}{A\left(T_{sat} - T\right)}\,. \qquad (4)$$

Substitute Eq. (4) for $\bar{h}_L$ into Eq. (3), with A = bL,

$$\frac{Re_\delta\,\mu_\ell b h'_{fg}}{4(bL)\left(T_{sat} - T\right)} = \frac{Re_\delta}{8750 + 58\,Pr_\ell^{-0.5}\left(Re_\delta^{0.75} - 253\right)}\cdot\frac{k_\ell}{\left(\nu_\ell^2/g\right)^{1/3}}\,. \qquad (5)$$

Using appropriate properties with L = 2.5 m, find

Continued...

$$\frac{365\times 10^{-6}\,\mathrm{N\cdot s/m^2}\times 2388\times 10^{3}\,\mathrm{J/kg}}{4\times 2.5\,\mathrm{m}\,(100-54)\,\mathrm{K}} \tag{6}$$

$$=\frac{1}{8750+58(2.29)^{-0.5}\left(\mathrm{Re}_{\delta}^{0.75}-253\right)}\cdot\frac{0.668\,\mathrm{W/m\cdot K}}{\left[\left(365\times 10^{-6}/973.7\right)^{2}\mathrm{m^4/s^2}/9.8\,\mathrm{m/s^2}\right]^{1/3}}$$

$$\mathrm{Re}_{\delta}=2979\,.$$

Note that $\mathrm{Re}_{\delta} > 1800$, so indeed the flow is turbulent, and using Eq. (4) or (3), find

$$\bar{h}_L = 5645\,\mathrm{W/m^2\cdot K}\,.$$

From the rate equations (1) and (2), the heat transfer and condensation rates are

$$q' = 5645\,\mathrm{W/m^2\cdot K}\times 2.5\,\mathrm{m}\,(100-54)\,\mathrm{K} = 649\,\mathrm{kW/m}$$ <

$$\dot{m}' = 649\times 10^{3}\,\mathrm{W/m}/2388\times 10^{3}\,\mathrm{J/kg} = 0.272\,\mathrm{kg/s\cdot m}\,.$$ <

(b) If the height of the plate were halved, L = 1.25 m, Eq. (6) would only need to be modified for this new value. Using the calculated values for the LHS and the last term on the RHS, Eq. (6) becomes,

$$3.78960=\frac{1}{8750+58(2.29)^{-0.5}\left(\mathrm{Re}_{\delta}^{0.75}-253\right)}\times 27,493 \tag{7}$$

and after some manipulation , find

$$\mathrm{Re}_{\delta}=1280\,.$$

Since $1800 > \mathrm{Re}_{\delta}$, the flow is not turbulent, but wavy-laminar. Now the procedure follows that of Example 10.3. For L = 1.25 m with wavy-laminar flow, Eq. 10.38 is the appropriate correlation. The calculations yield these results:

$$\mathrm{Re}_{\delta}=1372 \qquad \bar{h}_L = 5199\,\mathrm{W/m^2\cdot K}$$

$$q' = 299\,\mathrm{kW/m} \qquad \dot{m}' = 0.125\,\mathrm{kg/s\cdot m}\,.$$ <

Note that the height was decreased by a factor of 2 while the rates decreased by a factor of 2.2! Would you have expected this result?

(c) Using the *IHT Correlation Tool*, *Film Condensation*, *Vertical Plate* for *laminar, wavy-laminar*, and *turbulent regions*, combined with the *Properties Tool* for *Water*, the condensation rates were calculated as a function of the surface temperature considering the two plate heights indicated.

Condensation rate, mdot' (kg/s.m)

Plate temperature, Ts (C)

2.5 m high plate
1.25 m high plate

Continued...

PROBLEM 10.48 (Cont.)

The condensation rate decreases nearly linearly with increasing surface temperature. The inflection in the upper curve (L = 2.5 m) corresponds to the flow transition at $Re_\delta = 1800$ between wavy-laminar and turbulent. For surface temperature lower than 76°C, the flow is turbulent over the 2.5 m plate. The flow over the 1.25 m plate is always in the wavy-laminar region. The fact that the 2.5 m plate experiences turbulent flow explains the height-rate relationship mentioned in the closing sentences of part (b).

COMMENTS: A copy of the IHT model used to generate the above plot is shown below.

```
/* Correlations Tool
- Film Condensation, Vertical Plate, Laminar, wavy-laminar and turbulent regions: */
NuLbar = NuL_bar_FCO_VP(Redelta,Prl)     // Eq 10.37, 38, 39
NuLbar = hLbar * (nul^2 / g)^(1/3) / kl
g = 9.8                                  // Gravitational constant, m/s^2
Ts =  Ts_C + 273                         // Surface temperature, K
Ts_C = 54                                // Part (a) design condition
Tsat =  100 + 273                        // Saturation temperature, K
// The liquid properties are evaluated at the film temperature, Tf,
Tf = Tfluid_avg(Ts,Tsat)
// The condensation and heat rates are
q = hLbar * As * (Tsat - Ts)             // Eq 10.32
As = L * b                               // Surface Area, m^2
mdot = q / h'fg                          // Eq 10.33
h'fg = hfg + 0.68 * cpl * (Tsat - Ts)    // Eq 10.26
// The Reynolds number based upon film thickness is
Redelta = 4 * mdot / (mul * b)           // Eq 10.35

// Assigned Variables:
L = 1.25          // Height, m
b = 1             // Width, m

// Properties Tool - Water:
// Water property functions :T dependence, From Table A.6
// Units: T(K), p(bars);
xl =  0                          // Quality (0=sat liquid or 1=sat vapor)
rhol = rho_Tx("Water",Tf,xl)     // Density, kg/m^3
hfg = hfg_T("Water",Tsat)        // Heat of vaporization, J/kg
cpl = cp_Tx("Water",Tf,xl)       // Specific heat, J/kg·K
mul = mu_Tx("Water",Tf,xl)       // Viscosity, N·s/m^2
nul = nu_Tx("Water",Tf,xl)       // Kinematic viscosity, m^2/s
kl = k_Tx("Water",Tf,xl)         // Thermal conductivity, W/m·K
Prl = Pr_Tx("Water",Tf,xl)       // Prandtl number
```

PROBLEM 10.49

KNOWN: Two vertical plate configurations maintained at 90°C for condensing saturated steam at 1 atm: single plate L × w and two plates each L/2 × w where L and w are the vertical and horizontal dimensions, respectively.

FIND: Which case will provide the larger heat transfer or condensation rate.

SCHEMATIC:

ASSUMPTIONS: (1) Negligible concentration of non-condensible gases in the steam.

PROPERTIES: *Table A-6*, Saturated water vapor (1 atm): T_{sat} = 100°C, $\rho_v = (1/v_g) = 0.596\ kg/m^3$, h_{fg} = 2257 kJ/kg; Saturated water ($T_f = (T_s + T_{sat})/2$ = (90 + 100)°C/2 = 95°C = 368K): $r_\ell = (1/v_f)$ = 962 kg/m^3, $m_\ell = 296 \times 10^{-6}\ N{\cdot}s/m^2$, k_ℓ = 0.678 W/m·K, $c_{p,\ell}$ = 4212 J/kg·K.

ANALYSIS: The heat transfer and condensation rates are

$$q = \bar{h}_L A_s \left(T_{sat} - T_s\right) \qquad \dot{m} = q / h'_{fg}$$

where, for the two cases,

$$\bar{h}_{L,1} A_{s,1} = \bar{h}_{L,1}(L)[L \times w] \qquad \bar{h}_{L,2} A_{s,2} = \bar{h}_{L,2}(L/2)\left[2(L/2 \times w)\right]$$

and the average convection coefficients are evaluated at L and L/2, respectively. Hence,

$$\frac{q_1}{q_2} = \frac{\dot{m}_1}{\dot{m}_2} = \frac{\bar{h}_{L,1}(L)[L \times w]}{\bar{h}_{L,2}(L/2)\left[2(L/2 \times w)\right]} = \frac{\bar{h}_{L,1}(L)}{\bar{h}_{L,2}(L/2)}.$$

For *laminar film* condensation on both plates, using the correlation of Eq. 10.30, with $\bar{h}_L \;\alpha\; L^{-1/4}$,

$$q_1 / q_2 = \left(L / [L/2]\right)^{-1/4} = 0.84.$$

Hence, case 2 is preferred and provides 16% more heat transfer. <

When Re_δ = 30 for case 1 with the given conditions, find from Eq. 10.37

$$\frac{\bar{h}_L \left(n_\ell^2 / g\right)^{1/3}}{k_\ell} = \frac{\bar{h}_L \left[\left(296 \times 10^{-6} N \cdot s/m^2 / 962 kg/m^3\right)^2 / 9.8 m/s^2\right]^{1/3}}{0.678 W/m \cdot K}$$

Continued …..

$$\frac{\overline{h}_L\left(\nu_\ell^2/g\right)^{1/3}}{k_\ell}=1.47\,Re_\delta^{-1/3}=1.47(30)^{-1/3}$$

$$\overline{h}_L=15{,}061\,W/m^2\cdot K$$

and then from Eq. 10.30,

$$\overline{h}_L=0.943\left[\frac{g\,\rho_\ell(\rho_\ell-\rho_v)k_\ell^3 h'_{fg}}{\mu_\ell(T_{sat}-T_s)L}\right]^{1/4}$$

where

$$h'_{fg}=h_{fg}+0.68c_{p,\ell}(T_{sat}-T_s)$$

$$h'_{fg}=2257\,kJ/kg+0.68\times4212\,J/kg\cdot K(100-90)K=2286\,kJ/kg,$$

$$15{,}061\,W/m^2\cdot K=$$

$$0.943\left[\frac{9.8\,m/s^2\times962\,kg/m^3(962-0.596)\,kg/m^3(0.678\,W/m\cdot K)^3}{296\times10^{-6}\,N\cdot s/m^2(100-90)K\,L}2286\,kJ/kg\right]^{1/4}$$

$$L=34\ mm.$$

We can anticipate for other, larger values of L that the comparison of $\overline{h}_L$ values cannot be so easily made. However, according to Figure 10.15, we expect the same behavior of $\overline{h}_L$ in the *wavy* region and anticipate that indeed case 2 will provide the greater condensation rate. Note that in the turbulent region with the increase in $\overline{h}_L$ with Re_δ, we cannot conclude with certainty which case is preferred.

COMMENTS: In dealing with single-phase, forced or free convection, we associate thin thermal boundary layers with higher heat transfer rates. For vertical plates, we would expect the shorter plate to have the higher convection heat transfer coefficient. The results of this problem suggest the same is true for condensation on the vertical plate.

PROBLEM 10.50

KNOWN: Number, diameter and wall temperature of condenser tubes in a square array. Pressure of saturated steam around tubes.

FIND: Rates of heat transfer and condensation per unit length of the array.

SCHEMATIC:

Sat steam
p = 0.105 bar

Tube, D = 0.025 m
T_s = 17°C
N = 25

ASSUMPTIONS: (1) Laminar film condensation on tubes, (2) Negligible concentration of noncondensable gases in steam.

PROPERTIES: *Table A-6*, saturated vapor (p_{sat} = 0.105 bar): T_{sat} = 320 K = 47°C, ρ_v = 0.0715 kg/m^3, h_{fg} = 2390 kJ/kg. *Table A-6*, saturated liquid (T_f = 32°C = 305 K): $\rho_\ell = 995\,\text{kg/m}^3$, $\mu_\ell = 769\times10^{-6}\,\text{N}\cdot\text{s/m}^2$, $k_\ell = 0.620\,\text{W/m}\cdot\text{K}$, $c_{p,\ell} = 4178\,\text{J/kg}\cdot\text{K}$.

ANALYSIS: The average heat rate per unit length for a single tube is $q_1' = \overline{h}_{D,N}(\pi D)(T_{sat} - T_s)$, where $\overline{h}_{D,N}$ is obtained from Eq. 10.41. With $Ja = c_{p,\ell}(T_{sat} - T_s)/h_{fg} = 0.052$ and $h_{fg}' = h_{fg}$ (l + 0.68 Ja) = 1.04 (2.390 × 10^6 J/kg) = 2.48 × 10^6 J/kg,

$$\overline{h}_{D,N} = 0.729\left[\frac{g\,\rho_\ell(\rho_\ell - \rho_v)k_\ell^3 h_{fg}'}{N\,\mu_\ell(T_{sat} - T_s)D}\right]^{1/4}$$

$$\overline{h}_{D,N} = 0.729\left[\frac{9.8\,\text{m/s}^2 \times 995\,\text{kg/m}^3 (995 - 0.0715)\text{kg/m}^3 (0.62\,\text{W/m}\cdot\text{K})^3\, 2.48\times10^6\,\text{J/kg}}{25\times769\times10^{-6}\,\text{N}\cdot\text{s/m}^2\,(30°\text{C})\,0.025\text{m}}\right]^{1/4} = 3260\,\text{W/m}^2\cdot\text{K}$$

The heat rate per unit length of the array is $q' = N^2 q_1'$. Hence,

$$q' = N^2\overline{h}_{D,N}(\pi D)(T_{sat} - T_s) = 625\times3260\,\text{W/m}^2\cdot\text{K}(\pi\times0.025\text{m})30°\text{C} = 4.79\times10^6\,\text{W/m}$$ <

The corresponding condensation rate is

$$\dot{m}' = \frac{q'}{h_{fg}'} = \frac{4.79\times10^6\,\text{W/m}}{2.48\times10^6\,\text{J/kg}} = 1.93\,\text{kg/s}\cdot\text{m}$$ <

COMMENTS: Because of turbulence generation due to *splashing* from one tube to another in a vertical column, the foregoing value of $\overline{h}_{D,N}$ is expected to underestimate the actual value of $\overline{h}_{D,N}$ and hence to underpredict the heat and condensation rates.

PROBLEM 10.51

KNOWN: Tube wall diameters and thermal conductivity. Mean temperature and flow rate of water flow through tube. Pressure of saturated steam around tube.

FIND: (a) Rates of heat transfer and condensation per unit length, (b) Effect of flow rate on heat transfer.

SCHEMATIC:

Tube wall
$D_i = 0.026$ m
$D_o = 0.030$ m
$k_s = 15$ W/m-K
$T_{s,o}$
Condensate
Sat steam
p = 0.135 bar
Water
$T_m = 290$ K
$\dot{m} = 0.25$ kg/s

T_{sat}, $T_{s,o}$, T_m, q', R'_{fc}, R'_{cond}, R'_{conv}

ASSUMPTIONS: (1) Negligible concentration of noncondensible gases in the steam, (2) Uniform tube surface temperatures, (3) Laminar film condensation, (4) Fully-developed internal flow, (5) Constant properties.

PROPERTIES: *Table A-6*, water (T_m = 290 K): $\mu = 0.00108\ \text{N·s/m}^2$, k = 0.598 W/m·K, Pr = 7.56. *Table A-6*, saturated vapor (p = 0.135 bar): T_{sat} = 325 K = 52°C, $\rho_v = 0.0904\ \text{kg/m}^3$, h_{fg} = 2378 kJ/kg. *Table A-6*, saturated liquid ($T_f \approx T_{sat}$): $\rho_\ell = 987\ \text{kg/m}^3$, $c_{p,\ell} = 4182\ \text{J/kg}\cdot\text{K}$, $\mu_\ell = 528\times10^{-6}\ \text{N}\cdot\text{s/m}^2$, $k_\ell = 0.645\ \text{W/m}\cdot\text{K}$.

ANALYSIS: (a) From the thermal circuit, the heat rate may be expressed as

$$q' = \frac{T_{sat} - T_m}{R'_{fc} + R'_{cond} + R'_{conv}} \tag{1}$$

where, $$R'_{cond} = \ell n\left(D_o / D_i\right)/2\pi k_s = 0.00152\ \text{m}\cdot\text{K/W}$$

The convection resistance is $R'_{conv} = \left(\pi D_i h_i\right)^{-1}$. With $Re_D = 4\dot{m}/\pi D_i \mu = 11,336$, the flow is turbulent and the Dittus-Boelter correlation yields

$$h_i = \left(\frac{k}{D_i}\right)0.023 Re_D^{4/5} Pr^{0.4} = \left(\frac{0.598\ \text{W/m}\cdot\text{K}}{0.026\text{m}}\right)0.023(11,336)^{4/5}(7.56)^{0.4} = 2082\ \text{W/m}^2\cdot\text{K}$$

The convection resistance is then

$$R'_{conv} = \left(\pi D_i h_i\right)^{-1} = \left(\pi\times0.026\text{m}\times2082\ \text{W/m}^2\cdot\text{K}\right)^{-1} = 0.00588\ \text{m}\cdot\text{K/W}$$

The resistance associated with the condensate film is $R'_{fc} = \left(\pi D_o \bar{h}_o\right)$, where $\bar{h}_o$ is given by Eq. 10.40. With C = 0.729,

$$\bar{h}_o = C\left[\frac{g\rho_\ell\left(\rho_\ell - \rho_v\right)k_\ell^3 h'_{fg}}{\mu_\ell\left(T_{sat} - T_{s,o}\right)D_o}\right]^{1/4} = 0.729\left[\frac{9.8\ \text{m/s}^2 \times 987\left(987 - 0.09\right)\text{kg}^2/\text{m}^6\left(0.645\ \text{W/m}\cdot\text{K}\right)^3 h'_{fg}}{528\times10^{-6}\ \text{N}\cdot\text{s/m}^2\left(325 - T_{s,o}\right)\times0.030\text{m}}\right]^{1/4}$$

$$\bar{h}_o = 462\left(\frac{\text{W}^3\cdot\text{kg}}{\text{m}^8\cdot\text{K}^3\cdot\text{s}}\right)^{1/4}\left(\frac{h'_{fg}}{325 - T_{s,o}}\right)^{1/4}$$

where $h'_{fg} = h_{fg} + 0.68 c_{p,\ell}\left(T_{sat} - T_{s,o}\right) = 2.38\times10^6\ \text{J/kg} + 2844\ \text{J/kg}\cdot\text{K}\left(325 - T_{s,o}\right)$

The unknown surface temperature may be determined from an additional rate equation, such as

Continued …..

$$q' = \frac{T_{s,o} - T_m}{R'_{cond} + R'_{conv}} \tag{2}$$

Substituting the thermal resistances into Eqs. (1) and (2), an iterative solution yields

$$T_{s,o} = 321.6\,K = 48.6°C \qquad q' = 4270\,W/m$$ <

The condensation rate is then

$$\dot{m}'_{cond} = \frac{q'}{h'_{fg}} = \frac{4270\,W/m}{2.39 \times 10^6\,J/kg} = 0.00179\,kg/s \cdot m$$ <

The corresponding values of the condensate convection coefficient and resistance are

$$\bar{h}_o = 13{,}380\,W/m^2 \cdot K$$

and $$R'_{fc} = 0.000793\,m \cdot K/W$$

Because R'_{conv} is much larger than R'_{cond} and R'_{fc}, attention should be paid to reducing the convection resistance in order to increase the heat rate. The resistance to heat flow by convection is the *limiting factor*.

(b) The effects of varying the flow rate are shown below

The effect of increasing $\dot{m}$ on q' is significant and is accompanied by a reduction in $T_{s,o}$.

COMMENTS: (1) Use of the IHT convection and condensation correlations, as well as its temperature-dependent properties of water facilitated the numerical solution. (2) Evaluation of the film properties at T_{sat} is reasonable for part (a), since $T_f = (T_{s,o} + T_{sat})/2 = 50.3°C \approx T_{sat}$. However, with increasing $\dot{m}$ and hence decreasing $T_{s,o}$, the approximation would become inappropriate.

PROBLEM 10.52

KNOWN: Inner surface of a vertical thin-walled container of length L and diameter D experiences condensation of a saturated vapor. Container wall maintained at a uniform surface temperature by flowing cold water across its outer surface.

FIND: Expression for the time, t_f , required to fill the container with condensate assuming the condensate film is laminar. Express your result in terms of D, L, $(T_{sat} - T_s)$, g and appropriate fluid properties.

SCHEMATIC:

ASSUMPTIONS: (1) Laminar film condensation on a vertical surface, (2) Uniform temperature container wall surface, and (3) Mass of liquid condensate in the laminar film negligible compared to liquid mass on bottom of container.

ANALYSIS: From an instantaneous mass balance on the container,

$$\dot{m}(t) = \frac{dM}{dt} \tag{1}$$

Where $\dot{m}(t)$ is the condensate rate and the liquid mass in the container, M, is

$$M = \rho_\ell \left(\pi D^2/4\right)(L - x) \tag{2}$$

The condensate rate from Eq. 10.33 can be expressed as

$$\dot{m}(t) = \frac{q}{h'_{fg}} = \frac{\bar{h}_s A_s (T_{sat} - T_s)}{h'_{fg}} \tag{3}$$

where the average film coefficient over the height 0 to x from Eq. 10.30 is,

$$\bar{h}_s = 0.943 \left[\frac{g\rho_\ell (\rho_\ell - \rho_v) k_\ell^3 h'_{fg}}{\mu_\ell (T_{sat} - T_s) x} \right]^{1/4} \tag{4}$$

and the surface area over which condensation occurs is

$$A_s = \pi D x \tag{5}$$

Continued...

PROBLEM 10.52 (Cont.)

Substituting Eqs (2-5) into Eq. (1),

$$0.943\left[\frac{g\rho_\ell(\rho_\ell-\rho_v)k_\ell^3 h'_{fg}}{\mu_\ell(T_{sat}-T_s)L}\right]^{1/4}\frac{L^{1/4}}{x^{1/4}}(\pi Dx)\Big/h'_{fg}=-\rho_\ell\left(\pi D^2/4\right)\frac{dx}{dt} \qquad (6)$$

Separate variables and identify the limits of integration,

$$\left\{0.943\left[\frac{g\rho_\ell(\rho_\ell-\rho_v)k_\ell^3 h'_{fg}}{\mu_\ell(T_{sat}-T_s)L}\right]^{1/4}L^{1/4}(\pi D)\Big/\left[h'_{fg}\rho_\ell\left(\pi D^2/4\right)\right]\right\}\int_0^{t_f}dt=-\int_{x=L}^{0}x^{-3/4}dx \qquad (7)$$

The RHS integrates to

$$-\left[x^{1/4}\Big/(1/4)\right]_L^0=4L^{1/4} \qquad (8)$$

and solving for t_f,

$$t_f=4\left[\frac{\rho_\ell\left(\pi D^2/4\right)Lh'_{fg}}{0.943\left[\frac{g\rho_\ell(\rho_\ell-\rho_v)k_\ell^3 h'_{fg}}{\mu_\ell(T_{sat}-T_s)L}\right]^{1/4}(\pi DL)(T_{sat}-T_s)}\right]$$

<

COMMENTS: The numerator and denominator in the bracketed expression are of special significance. The numerator is product of the mass in the filled container and the latent heat of vaporization; that is, the total energy removed by the cold water. What is physical significance of the denominator? Can you interpret the time-to-fill, t_f , expression in light of these terms?

PROBLEM 10.53

KNOWN: Tube of Problem 10.42 in horizontal position experiences condensation on its outer surface.

FIND: Heat transfer and condensation rates.

SCHEMATIC:

Saturated steam, 1atm

$T_s = 94°C$

Horizontal tube, $D = 0.1\,m$, $L = 1\,m$

ASSUMPTIONS: (1) Laminar film condensation, (2) End effects negligible, (3) Negligible concentration of non-condensible gases in steam.

PROPERTIES: *Table A-6*, Water, vapor (1 atm): T_{sat} = 100°C, ρ_v = 0.596 kg/m^3, h_{fg} = 2257 kJ/kg; *Table A-6*, Water, liquid ($T_f = (T_s + T_{sat})/2$ = 370K): r_ℓ = 960.6 kg/m^3, $c_{p,\ell}$ = 4214 J/kg·K, m_ℓ = 289×10^{-6} N·s/m^2, k_ℓ =0.679 W/m·K.

ANALYSIS: From Eq. 10.32 with A = π D L and Eq. 10.33, the heat transfer and condensation rates are

$$q = \overline{h}_L\left(p\,D\,L\right)\left(T_{sat} - T_s\right) \qquad\qquad \dot{m} = q / h'_{fg}$$

where from Eq. 10.26 with $Ja = c_{p,\ell}\left(T_{sat} - T_s\right)/h_{fg}$, find

$$h'_{fg} = h_{fg}\left[1 + 0.68Ja\right] = 2257\,kJ/kg\left[1 + 0.68\left[4214\,J/kg\cdot K\left(100 - 94\right)K / 2257 \times 10^3\ J/kg\right]\right] = 2274\frac{kJ}{kg}.$$

For laminar film condensation, Eq. 10.40 is the appropriate correlation for a cylinder with C = 0.729,

$$\overline{h}_D = 0.729\left[\frac{g\,r_\ell\left(r_\ell - r_v\right)k_\ell^3\,h'_{fg}}{m_\ell\left(T_{sat} - T_s\right)D}\right]^{1/4}.$$

$$\overline{h}_D = 0.729\left[\frac{9.8\,m/s^2 \times 960.6\,kg/m^3\left(960.6 - 0.596\right)kg/m^3\left(0.679\,W/m\cdot K\right)^3 \times 2274 \times 10^3 J/kg}{289 \times 10^{-6}\ N\cdot s/m^2\left(100 - 94\right)K \times 0.1m}\right]^{1/4}$$

$$\overline{h}_D = 10{,}120\ W/m^2\cdot K.$$

Hence, the heat transfer and condensation rates are

$$q = 10{,}120\ W/m^2\cdot K\left(p \times 0.1m \times 1m\right)\left(100 - 94\right)K = 19.1kW \qquad <$$

$$\dot{m} = 19.1 \times 10^3 W / 2274 \times 10^3\ J/kg = 8.39 \times 10^{-3} kg/s. \qquad <$$

COMMENTS: A comparison of the above results for the horizontal tube with those for a vertical tube (Problem 10.42) follows:

Position	$\overline{h}\left(W/m^2\cdot K\right)$	q(kW)	$\dot{m}\cdot 10^3\left(kg/s\right)$
Vertical	8,507	16.0	7.05
Horizontal	10,120	19.1	8.39

The rates are higher for the horizontal case. Why?

PROBLEM 10.54

KNOWN: Horizontal pipe passing through an air space with prescribed temperature and relative humidity.

FIND: Water condensation rate per unit length of the pipe.

SCHEMATIC:

Ambient air, $T_\infty = 37°C$, $\phi = 0.75$

Pipe, D=25mm

$T_s = 15°C$

$\dot{m}$

ASSUMPTIONS: (1) Film condensation occurs on horizontal tube.

PROPERTIES: *Table A-6*, Water, vapor (T_∞ = 37°C = 310K): $p_{A,sat}$ = 0.06221 bar; *Table A-6*, Water, vapor ($p_A = \phi \cdot p_{A,sat}$ = 0.04666 bar): $T_{A,sat} \approx$ 305K, ρ_v = 0.04 kg/m^3, h_{fg} = 2426 kJ/kg; *Table A-6*, Water, liquid ($T_f = (T_s + T_{A,sat})/2$ = 297K): ρ_ℓ = 997.2 kg/m^3, $c_{p,\ell}$ = 4180 J/kg·K, $\mu_\ell = 917 \times 10^{-6}$ N·s/m^2, k_ℓ = 0.609 W/m·K.

ANALYSIS: From Eq. 10.33, the condensate rate per unit length is

$$\dot{m}' = \frac{q'}{h'_{fg}} = \frac{\bar{h}_L (\pi D)(T_{sat} - T_s)}{h'_{fg}}$$

where, from Eq. 10.26, with $Ja = c_{p,\ell}(T_{sat} - T_s)/h_{fg}$,

$$h'_{fg} = h_{fg}\left[1 + 0.68 c_{p,\ell}(T_{sat} - T_s)/h_{fg}\right] = 2426\frac{kJ}{kg}\left[1 + 0.68 \times 4180\frac{J}{kg \cdot K}(305 - 288)K/2426 \times 10^3 \frac{J}{kg}\right]$$

$$h'_{fg} = 2474 \text{ kJ/kg}.$$

Note that $T_{sat} = T_{A,sat}$ is the saturation temperature of the water vapor in air at 37°C having a relative humidity ϕ = 0.75. That is, T_{sat} = 305K while T_s = 15°C = 288K. Assuming laminar film condensation on the horizontal pipe, it follows from Eq. 10.40 that,

$$\bar{h}_D = 0.729\left[\frac{g\,\rho_\ell(\rho_\ell - \rho_v)k_\ell^3 h'_{fg}}{\mu_\ell(T_{sat} - T_s)D}\right]^{1/4}$$

$$\bar{h}_D = 0.729\left[\frac{9.8\,m/s^2 \times 997.2\,kg/m^3 (997.2 - 0.04)kg/m^3 (0.609\,W/m \cdot K)^3 \times 2474 \times 10^3 J/kg}{917 \times 10^{-6} N \cdot s/m^2 (305 - 288) K \times 0.025m}\right]^{1/4}$$

$$\bar{h}_D = 7925 \text{ W/m}^2 \cdot \text{K}.$$

Hence, the condensate rate is,

$$\dot{m}' = 7925 \text{ W/m}^2 \cdot K(\pi \times 0.025m)(305 - 288)K/2474 \times 10^3 J/kg$$

$$\dot{m}' = 4.28 \times 10^{-3} \text{kg/s} \cdot \text{m}.$$ <

COMMENTS: The actual dropwise condensation rate exceeds the foregoing estimate.

PROBLEM 10.55

KNOWN: Horizontal tube, 50mm diameter, with surface temperature of 34°C is exposed to steam at 0.2 bar.

FIND: Estimate the heat transfer and condensation rates per unit length of the tube.

SCHEMATIC:

ASSUMPTIONS: (1) Laminar film condensation, (2) Negligible non-condensibles in steam.

PROPERTIES: *Table A-6*, Saturated steam (0.2 bar): T_{sat} = 333K, ρ_v = 0.129 kg/m^3, h_{fg} = 2358 kJ/kg; *Table A-6*, Water, liquid ($T_f = (T_s + T_{sat})/2$ = 320K): r_ℓ = 989.1 kg/m^3, $c_{p,\ell}$ = 4180 J/kg·K, m_ℓ = 577 × 10^{-6} N·s/m^2, k_ℓ = 0.640 W/m·K.

ANALYSIS: From Eqs. 10.32 and 10.33, the heat transfer and condensate rates per unit length of the tube are

$$q' = \bar{h}_D (p D)(T_{sat} - T_s) \qquad\qquad \dot{m}' = q'/h'_{fg}$$

where from Eq. 10.26 with $Ja = c_{p,\ell}(T_{sat} - T_s)/h_{fg}$,

$$h'_{fg} = h_{fg}[1 + 0.68\ Ja] = 2358\frac{kJ}{kg}\left[1 + 0.68 \times 4180\,J/kg\cdot K(333 - 307)K/2358 \times 10^3\,J/kg\right]$$

$$h'_{fg} = 2432\ kJ/kg.$$

For laminar film condensation, Eq. 10.40 is appropriate for estimating $\bar{h}_D$ with C = 0.729,

$$\bar{h}_D = 0.729\left[\frac{g\, r_\ell (r_\ell - r_v) k_\ell^3 h'_{fg}}{m_\ell (T_{sat} - T_s) D}\right]^{1/4}$$

$$\bar{h}_D = 0.729\left[\frac{9.8\,m/s^2 \times 989.1\,kg/m^3 (989.1 - 0.129)kg/m^3 (0.640\,W/m\cdot K)^3 \times 2432 \times 10^3\,J/kg}{577 \times 10^{-6}\,N\cdot s/m^2\,(333 - 307)K \times 0.050m}\right]$$

$$\bar{h}_D = 6926\ W/m^2\cdot K.$$

Hence, the heat transfer and condensation rates are

$$q' = 6926\ W/m^2\cdot K(p \times 0.050m)(333 - 307)K = 28.3kW/m \qquad <$$

$$\dot{m}' = 28.3 \times 10^3\,W/m / 2432 \times 10^3\,J/kg = 1.16 \times 10^{-2}\,kg/s\cdot m. \qquad <$$

PROBLEM 10.56

KNOWN: Horizontal tube 1m long with surface temperature of 70°C used to condense steam at 1 bar.

FIND: Diameter required for condensation rate of 125 kg/h.

SCHEMATIC:

T_s = 70°C = 343K

Tube, L = 1m, D

Steam, 1atm

$\dot{m}$ = 125kg/h

ASSUMPTIONS: (1) Laminar film condensation, (2) Negligible non-condensibles in steam.

PROPERTIES: *Table A-6*, Water, vapor (1 atm): T_{sat} = 100°C, ρ_v = 0.596 kg/m^3, h_{fg} = 2257 kJ/kg; *Table A-6*, Water, liquid ($T_f = (T_s + T_{sat})/2$ = 358K): ρ_ℓ = 968.6 kg/m^3, $c_{p,\ell}$ = 4201 J/kg·K, μ_ℓ = 332×10^{-6} N·s/m^2, k_ℓ = 0.673 W/m·K.

ANALYSIS: From the rate equation, Eq. 10.33, with A = π D L, the required diameter is

$$D = \dot{m}\, h'_{fg} / \pi\, L\, \bar{h}_D \left(T_{sat} - T_s\right) \tag{1}$$

where from Eq. 10.26 with $Ja = c_{p,\ell}\left(T_{sat} - T_s\right)/h_{fg}$,

$$h'_{fg} = h_{fg}\left(1 + 0.68 Ja\right) = 2257 \frac{kJ}{kg}\left(1 + 0.68 \frac{4201 J/kg \cdot K \times (100 - 70) K}{2257 \times 10^3 J/kg}\right) = 2343 kJ/kg. \tag{2}$$

Substituting numerical values, Eq. (1) becomes

$$D = \frac{125}{3600}\frac{kg}{s} \times 2343 \times 10^3 \frac{J}{kg} / \pi \times 1m \times \bar{h}_D\left(100 - 70\right) K = 863.2 \bar{h}_D^{-1}. \tag{3}$$

The appropriate correlation for $\bar{h}_D$ is Eq. 10.40 with C = 0.729,

$$\bar{h}_D = 0.729 \left[\frac{g\, \rho_\ell \left(\rho_\ell - \rho_v\right) k_\ell^3\, h'_{fg}}{\mu_\ell \left(T_{sat} - T_s\right) D}\right]^{1/4}. \tag{4}$$

Substitute Eq. (4) for $\bar{h}_D$ into Eq. (3) and use numerical values,

$$863.2\ D^{-1} = 0.729 \times$$

$$\left[\frac{9.8 m/s^2 \times 968.6 kg/m^3 \left(968.6 - 0.596\right) kg/m^3 \left(0.673 W/m \cdot K\right)^3 \times 2343 \times 10^3 J/kg}{332 \times 10^{-6} N \cdot s/m^2 \left(100 - 70\right) K \times D}\right]^{1/4}$$

$$863.2\ D^{-1} = 3693.4\ D^{-1/4}$$

$$D = 0.144 m = 144 mm.$$ <

COMMENTS: Note for this situation Ja = 0.06.

PROBLEM 10.57

KNOWN: Saturated R-12 vapor at 1 atm condensing on the outside of a horizontal tube.

FIND: Tube surface temperature required for condensation rate of 50 kg/h.

SCHEMATIC:

ASSUMPTIONS: (1) Laminar film condensation, (2) Negligible non-condensibles in vapor.

PROPERTIES: *Table A-5*, R-12 Saturated vapor (1 atm): T_{sat} = 243K, ρ_v = 6.32 kg/m^3, h_{fg} = 165 kJ/kg; *Table A-5*, R-12 Saturated liquid ($T_f \approx$ 240K): r_ℓ = 1498 kg/m^3, $c_{p,\ell}$ = 892.3 J/kg·K, m_ℓ = 0.0385×10^{-2} N·s/m^2, k_ℓ = 0.069 W/m·K.

ANALYSIS: The surface temperature or temperature difference can be written as follows from Eq. 10.33,

$$\Delta T = T_{sat} - T_s = \dot{m}\, h'_{fg} / \bar{h}_D\, p\, D\, L \qquad (1)$$

where A = π D L. To evaluate h'_{fg} and $\bar{h}_D$, we require knowledge of T_s or ΔT. Assume a ΔT = 10°C, then T_s = 233K and $T_f = (T_s + T_{sat})/2$ = 240K. From Eq. 10.26 with $Ja = c_{p,\ell}\,\Delta T / h_{fg}$, find

$$h'_{fg} = h_{fg}\left(1 + 0.68 Ja\right) = 165\frac{kJ}{kg}\left[1 + 0.68 \times 892.3 \frac{J}{kg \cdot K} \times 10K / 165 \times 10^3 \frac{J}{kg}\right] = 171 kJ/kg. \qquad (2)$$

The appropriate correlation for $\bar{h}_D$ is Eq. 10.40 with C = 0.729; substitute properties and find $\bar{h}_D$ in terms of ΔT.

$$\bar{h}_D = 0.729\left[\frac{g\, r_\ell \left(r_\ell - r_v\right) k_\ell^3 h'_{fg}}{m_\ell \left(T_{sat} - T_s\right) D}\right]^{1/4}$$

$$\bar{h}_D = 0.729\left[\frac{9.8 m/s^2 \times 1498 kg/m^3 \left(1498 - 6.32\right) kg/m^3 \left(0.069 W/m \cdot K\right)^3 \times 171 \times 10^3 J/kg}{0.0385 \times 10^{-2} N \cdot s/m^2 \times \Delta T \times 0.010m}\right]^{1/4}$$

$$\bar{h}_D = 3082 \Delta T^{1/4}. \qquad (3)$$

Substitute Eq. (3) into Eq. (1) for $\bar{h}_D$, and solve for ΔT,

$$\Delta T = \frac{50}{3600} kg/s \times 171 \times 10^3 J/kg / \left(3082\, \Delta T^{1/4}\right) p \left(0.010m\right) \times 1m$$

$$\Delta T = 12.9K \qquad \text{or} \qquad T_s = 230K.$$

<

COMMENTS: We used the assumed value of T_s or ΔT only to evaluate properties. Our estimate for T_f = 240K is to be compared to the calculated value of $T_f \approx$ 236K. An iteration is probably not necessary.

PROBLEM 10.58

KNOWN: Saturation temperature and inlet flow rate of R-12. Diameter, length and temperature of tube.

FIND: Rate of condensation and outlet flow rate.

SCHEMATIC:

ASSUMPTIONS: (1) Negligible concentration of noncondensables in vapor.

PROPERTIES: Given, R-12, saturated vapor: $\rho_v = 6\ kg/m^3$, $h_{fg} = 160\ kJ/kg$, $\mu_v = 150 \times 10^{-7}\ N{\cdot}s/m^2$. *Table A-5*, R-12, saturated liquid (T_f = 300 K): $\rho_\ell = 1306\ kg/m^3$, $c_{p,\ell} = 978\ J/kg\cdot K$, $\mu_\ell = 0.0254\ N\cdot s/m^2$, $k_\ell = 0.072\ W/m\cdot K$.

ANALYSIS: The Reynolds number associated with the inlet vapor flow is $Re_{v,i} = 4\dot{m}_{v,i}/\pi D\mu_v$ $= 0.04\ kg/s/\pi \times 0.025m \times 150 \times 10^{-7}\ N\cdot s/m^2 = 33,950 < 35,000$. Hence, the average convection coefficient may be obtained from Eq. 10.42, where $h'_{fg} = h_{fg} + 0.375\, c_{p,\ell}\left(T_{sat} - T_s\right) = (1.6 \times 10^5 + 0.375 \times 978 \times 20)\ J/kg = 1.67 \times 10^5\ J/kg$.

$$\overline{h}_D = 0.555\left[\frac{g\rho_\ell\left(\rho_\ell - \rho_v\right)k_\ell^3\, h'_{fg}}{\mu_\ell\left(T_{sat} - T_s\right)D}\right]^{1/4} \approx 0.555\left[\frac{9.8\ m/s^2\left(1306\ kg/m^3\right)^2\left(0.072\ W/m\cdot K\right)^3 1.67\times10^5\ J/kg}{0.0254\ N\cdot s/m^2 \times 20\ K \times 0.025m}\right]^{1/4}$$

$$\overline{h}_D = 297\ W/m^2\cdot K$$

The heat rate is then

$$q = \pi DL\,\overline{h}_D\left(T_{sat} - T_s\right) = \pi \times 0.025m \times 2m \times 297\ W/m^2\cdot K \times 20\ K = 933\ W$$

and the condensation rate is

$$\dot{m}_{cond} = \frac{q}{h'_{fg}} = \frac{933\ W}{1.67\times10^5} = 0.0056\ kg/s \qquad <$$

The flow rate of vapor leaving the tube is then

$$\dot{m}_{v,o} = \dot{m}_{v,i} - \dot{m}_{cond} = \left(0.0100 - 0.0056\right)kg/s = 0.0044\ kg/s \qquad <$$

PROBLEM 10.59

KNOWN: Array of condenser tubes exposed to saturated steam at 0.1 bar.

FIND: (a) Condensation rate per unit length of square array, (b) Options for increasing the condensation rate.

SCHEMATIC:

Saturated steam, 0.1 bar

T_s

D = 8 mm, N_t = 100 tubes

ASSUMPTIONS: (1) Film condensation on tubes, (2) Negligible non-condensable gases in steam.

PROPERTIES: *Table A.6*, Saturated water vapor (0.1 bar): $T_{sat} \approx 320$ K, $\rho_v = 0.072$ kg/m^3, $h_{fg} = 2390$ kJ/kg; *Table A.6*, Water, liquid ($T_f = (T_s + T_{sat})/2 = 310$ K): $\rho_\ell = 993.1$ kg/m^3, $c_{p,\ell} = 4178$ J/kg·K, $\mu_\ell = 695 \times 10^{-6}$ N·s/m^2, $k_\ell = 0.628$ W/m·K.

ANALYSIS: (a) From Eq. 10.33, the condensation rate for a N ×N square array is

$$\dot{m}' = \dot{m}/L = \bar{h}_{D,N} \cdot N_t (\pi D)(T_{sat} - T_s)/h'_{fg}$$

where $\bar{h}_{D,N}$ is the average coefficient for the tubes in a vertical array of N tubes. With $Ja = c_{p,\ell}\Delta T/h_{fg} = 4178$ J/kg·K × (320 - 300)K/2390 × 10^3 J/kg = 0.035, Eq. 10.26 yields $h'_{fg} = h_{fg}(1 + 0.68\,Ja) = 2390$ kJ/kg(1 + 0.68 × 0.035) = 2447 kJ/kg.

For a vertical tier of N = 10 horizontal tubes, the average coefficient is given by Eq. 10.41,

$$\bar{h}_{D,N} = 0.729\left[\frac{g\rho_\ell(\rho_\ell - \rho_v)k_\ell^3 h'_{fg}}{N\mu_\ell(T_{sat} - T_s)D}\right]^{1/4}$$

$$\bar{h}_{D,N} = 0.729\left[\frac{9.8\,\text{m/s}^2 \times 993.1\,\text{kg/m}^3\,(993.1 - 0.072)\,\text{kg/m}^3\,(0.628\,\text{W/m}\cdot\text{K})^3 \times 2447\times10^3\,\text{J/kg}}{10\times695\times10^{-6}\,\text{N}\cdot\text{s/m}^2\,(320-300)\,\text{K}\times0.008\,\text{m}}\right]^{1/4}$$

$$\bar{h}_{D,N} = 6210\,\text{W/m}^2\cdot\text{K}.$$

Hence, the condensation rate for the entire array per unit tube length is

$$\dot{m}' = 6210\,\text{W/m}^2\cdot\text{K}(100)\pi\times0.008\,\text{m}(320-300)\text{K}/2447\times10^3\,\text{J/kg}$$

$$\dot{m}' = 0.128\,\text{kg/s}\cdot\text{m} = 459\,\text{kg/h}\cdot\text{m}.$$ <

(b) Options for increasing the condensation rate include reducing the surface temperature and/or the number of tubes in a vertical tier. By varying the temperature of cold water flowing through the tubes, it is feasible to maintain surface temperatures in the range $280 \le T_s \le 300$ K. Using the *Correlations* and *Properties* Toolpads of IHT, the following results were obtained for N = 10, 5 and 2, with $N_t = 100$ in each case. The results are based on properties evaluated at p = 0.1 bar, for which the Properties Toolpad yielded $T_{sat} = 318.9$ K.

Continued...

PROBLEM 10.59 (Cont.)

Condensation rate, mdot'(kg/s.m)

0.3
0.2
0.1

280 290 300

Surface temperature, Ts(K)

N = 10
N = 5
N = 2

Clearly, there are significant benefits associated with reducing both T_s and N.

COMMENTS: Note that, since $\overline{h}_{D,N} \propto N^{-1/4}$, the average coefficient decreases with increasing N due to a corresponding increase in the condensate film thickness. From the result of part (a), the coefficient for the topmost tube is $\overline{h}_D = 6210\ W/m^2 \cdot K(10)^{1/4} = 11{,}043\ W/m^2 \cdot K$.

PROBLEM 10.60

KNOWN: Thin-walled concentric tube arrangement for heating deionized water by condensation of steam.

FIND: Estimates for convection coefficients on both sides of the inner tube. Inner tube wall outlet temperature. Whether condensation provides fairly uniform inner tube wall temperature approximately equal to the steam saturation temperature.

SCHEMATIC:

$D_o = 60$ mm, $D_i = 30$ mm, $L = 0.19$ m, T_s, Sat. steam, 1 atm, Deionized water, $T_{m,i} = 40°C$, $\dot{m} = 5$ kg/s, $T_{m,o} = 60°C$

ASSUMPTIONS: (1) Negligible thermal resistance of inner tube wall, (2) Internal flow is fully developed.

PROPERTIES: Deionized water (given): $\rho = 982.3$ kg/m^3, $c_p = 4181$ J/kg·K, k = 0.643 W/m·K, $\mu = 548 \times 10^{-6}$ N·s/m^2, Pr = 3.56; *Table A-6*, Saturated vapor (1 atm): $T_{sat} = 100°C$, $\rho_v = (1/v_g) = 0.596$ kg/m^3, $h_{fg} = 2265$ kJ/kg; *Table A-6*, Saturated water (assume $T_s \approx 75°C$, $T_f = (75 + 100)°C/2 = 360K$): $r_\ell = (1/v_f) = 967$ kg/m^3, $m_\ell = 324 \times 10^{-6}$ N·s/m^2, $k_\ell = 0.674$ W/m·K, $c_{p,\ell} = 4203$ J/kg·K.

ANALYSIS: From an energy balance on the inner tube assuming a constant wall temperature,

$$\bar{h}_c \left(T_{sat} - T_{s,o}\right) = h_i \left(T_{s,o} - T_{m,o}\right)$$

where $\bar{h}_c$ and h_i are, respectively, the heat transfer coefficients for condensation (c) on a horizontal cylinder and internal (i) flow in a tube.

Condensation. From Eq. 10.40, for the horizontal tube,

$$\bar{h}_c = 0.729\left[\frac{g\, r_\ell (r_\ell - r_v) k_\ell^3 h'_{fg}}{m_\ell (T_{sat} - T_s) D}\right]^{1/4}$$

where $h'_{fg} = h_{fg}\left\{1 + 0.68 c_{p,\ell}\left(T_{sat} - T_s\right)/h_{fg}\right\}$

$$h'_{fg} = 2265 \text{ kJ/kg}\left\{1 + 0.68 \times 4203 \text{ J/kg}\cdot\text{K}\left(100 - T_s\right)/2265 \times 10^3 \text{ J/kg}\right\}$$

$$h'_{fg} = 2265 \text{ kJ/kg}\left\{1 + 1.262 \times 10^{-3}\left(100 - T_s\right)\right\}$$

$$\bar{h}_c = 0.729\Big[9.8 \text{m/s}^2 \times 967 \text{kg/m}^3 (067 - 0.596) \text{kg/m}^3 (0.674 \text{W/m}\cdot\text{K})^3 \times$$

$$2265\left\{1 + 1.262 \times 10^{-3}\left(100 - T_s\right)\right\} \text{kJ/kg} / 324 \times 10^{-6} \text{N}\cdot\text{s/m}^2 \left(100 - T_s\right) 0.030 \text{m}\Big]^{1/4}$$

Continued …..

$$\overline{h}_c = 2.843\times10^4\left[\frac{1+1.262\times10^{-3}(100-T_s)}{100-T_s}\right]^{1/4}.$$

Internal flow. From Eq. 8.6, evaluating properties at $\overline{T}_m$, find

$$Re_D = \frac{4\dot{m}}{p m D} = \frac{4\times5\ kg/s}{p\times548\times10^{-6}\,N\cdot s/m^2\times0.030\ m} = 3.872\times10^5$$

and for turbulent flow use the Colburn equation,

$$Nu_D = \frac{h_i D}{k} = 0.023 Re_D^{0.8} Pr^{1/3}$$

$$h_i = \frac{0.023\times0.643\ W/m\cdot K}{0.03\ m}\left(3.872\times10^5\right)^{0.8}(3.56)^{1/3} = 2.22\times10^4\ W/m^2\cdot K.$$ <

Substituting numerical values into the energy balance relation,

$$2.843\times10^4\left[\frac{1+1.262\times10^{-3}\left(100-T_{s,o}\right)}{100-T_{s,o}}\right]^{1/4}\left(100-T_{s,o}\right)K$$

$$= 2.22\times10^4\ W/m^2\cdot K\left(T_{s,o}-60\right)K$$

and by trial-and-error, find

$$T_{s,o} \approx 75°C.$$

With this value of T_s, find that

$$\overline{h}_c = 1.29\times10^4\ W/m^2\cdot K$$ <

which is approximately half that for the internal flow. Hence, the tube wall cannot be at a uniform temperature. This could only be achieved if $\overline{h}_c \gg h_i$.

PROBLEM 10.61

KNOWN: Heat dissipation from multichip module to saturated liquid of prescribed temperature and properties. Diameter and inlet and outlet water temperatures for a condenser coil.

FIND: (a) Condensation and water flow rates. (b) Tube surface inlet and outlet temperatures. (c) Coil length.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions since rate of heat transfer from the module is balanced by rate of heat transfer to coil, (2) Fully developed flow in tube, (3) Negligible changes in potential and kinetic energy for tube flow.

PROPERTIES: Saturated fluorocarbon (T_{sat} = 57°C, given): k_ℓ = 0.0537 W/m·K, $c_{p,\ell}$ = 1100 J/kg·K, $h'_{fg} \approx h_{fg}$ = 84,400 J/kg. r_ℓ = 1619.2 kg/m^3, ρ_v = 13.4 kg/m^3, $\sigma = 8.1 \times 10^{-3}$ kg/s^2, m_ℓ = 440 $\times 10^{-6}$ kg/m·s, Pr_ℓ = 9; *Table A-6*, Water, sat. liquid $(\overline{T}_m = 300K)$: ρ = 997 kg/m^3, c_p = 4179 J/kg·K, $\mu = 855 \times 10^{-6}$ N·s/m^2, k = 0.613 W/m·K, Pr = 5.83.

ANALYSIS: (a) With

$$q = (q'' \times A)_{module} = 10^5\ W/m^2 (0.1 m)^2 = 10^3\ W$$

the condensation rate is

$$\dot{m}_{con} = \frac{q}{h'_{fg}} = \frac{10^3\ W}{84,400 J/kg} = 0.0118\ kg/s$$ <

and the required water flow rate is

$$\dot{m} = \frac{q}{c_p (T_{m,o} - T_{m,i})} = \frac{1000\ W}{4179 J/kg \cdot K (30K)} = 7.98 \times 10^{-3} kg/s.$$ <

(b) The Reynolds number for flow through the tube is

$$Re_D = \frac{4\ \dot{m}}{p D m} = \frac{4 \times 7.98 \times 10^{-3} kg/s}{p\ (0.01m) 855 \times 10^{-6} N \cdot s/m^2} = 1188.$$

Hence, the flow is laminar. Assuming a uniform wall temperature,

$$h_i = Nu_D\ k/D = 3.66(0.613\ W/m \cdot K/0.01m) = 224\ W/m^2 \cdot K.$$

Continued …..

PROBLEM 10.61 (Cont.)

For film condensation on the outer surface, Eq. 10.40 yields

$$h_o = 0.729\left[\frac{9.8\,\mathrm{m/s^2}\left(1619.2\ \mathrm{kg/m^3}\right)\left(1605.8\,\mathrm{kg/m^3}\right)\left(0.0537\ \mathrm{W/m\cdot K}\right)^3 84{,}400\,\mathrm{J/kg}}{440\times10^{-6}\,\mathrm{kg/m\cdot s}\times 0.01\,\mathrm{m}\left(T_{sat}-T_s\right)}\right]^{1/4}$$

$$h_o = 2150\left(57 - T_s\right)^{-1/4}.$$

From an energy balance on a portion of the tube surface,

$$h_o\left(T_{sat}-T_s\right) = h_i\left(T_s - T_m\right)$$

or

$$2150\left(57-T_s\right)^{3/4} = 224\left(T_s - T_m\right)$$

At the entrance where $\left(T_{m,i} = 285\mathrm{K}\right)$, trial-and-error yields:

$$T_{s,i} = 50.6°\mathrm{C}$$ <

and at the exit where $\left(T_{m,o} = 315\mathrm{K}\right)$,

$$T_{s,o} = 55.4°\mathrm{C}$$ <

(c) From Eqs. 8.44 and 8.45,

$$L = \frac{q}{h_i \pi D \Delta T_{\ell m}}$$

where

$$\Delta T_{\ell m} = \frac{\left(T_s - T_{m,i}\right)-\left(T_s - T_{m,o}\right)}{\ln\left[\left(T_s - T_{m,i}\right)/\left(T_s - T_{m,o}\right)\right]} = \frac{41-11}{\ln\left(41/11\right)} = 22.8°\mathrm{C}$$

$$L = \frac{1000\ \mathrm{W}}{\left(224\ \mathrm{W/m^2\cdot K}\right)\pi\left(0.01\mathrm{m}\right)22.8°\mathrm{C}} = 6.23\mathrm{m}.$$ <

COMMENTS: Some control over system performance may be exercised by adjusting the water flow rate. By increasing $\dot{m}$, $\left(T_{m,o} - T_{m,i}\right)$ is reduced for a prescribed q. The value of h_i is increased substantially if the internal flow is turbulent.

PROBLEM 10.62

KNOWN: Saturated ethylene glycol vapor at 1 atm condensing on a sphere of 100 mm diameter having surface temperature of 150°C.

FIND: Condensation rate.

SCHEMATIC:

ASSUMPTIONS: (1) Laminar film condensation, (2) Negligible non-condensibles in vapor.

PROPERTIES: *Table A-5*, Saturated ethylene glycol, vapor (1 atm): T_{sat} = 470K, $\rho_v \approx 0$ kg/m^3, h_{fg} = 812 kJ/kg; *Table A-5*, Ethylene glycol, liquid (T_f = 423K, but use values at 373K, limit of data in table): r_ℓ = 1058.5 kg/m^3, $c_{p,\ell}$ = 2742 J/kg·K, m_ℓ = 0.215 × 10^{-2} N·s/m^2, k_ℓ = 0.263 W/m·K.

ANALYSIS: The condensation rate is given by Eq. 10.33 as

$$\dot{m} = \frac{q}{h'_{fg}} = \frac{\bar{h}_L \left(p\, D^2\right)\left(T_{sat} - T_s\right)}{h'_{fg}}$$

where A = π D^2 for the sphere and h'_{fg}, with $Ja = c_{p,\ell}\,\Delta T / h_{fg}$, is given by Eq. 10.26 as

$$h'_{fg} = h_{fg}\left(1 + 0.68 Ja\right) = 812\frac{kJ}{kg}\left(1 + 0.68 \times 2742\frac{J}{kg\cdot K}\left(470 - 423\right)K/812 \times 10^3\ J/kg\right) = 900\,kJ/kg.$$

The average heat transfer coefficient for the sphere follows from Eq. 10.40with C = 0.815,

$$\bar{h}_D = 0.815\left[\frac{g\, r_\ell\left(r_\ell - r_v\right)k_\ell^3 h'_{fg}}{m_\ell\left(T_{sat} - T_s\right)D}\right]^{1/4}$$

$$\bar{h}_D = 0.815\left[\frac{9.8\,m/s^2 \times 1058.5\,kg/m^3\left(1058.5 - 0\right)kg/m^3\left(0.263\,W/m\cdot K\right)^3 \times 900 \times 10^3 J/kg}{0.215 \times 10^{-2}\,N\cdot s/m^2\left(470 - 423\right)K \times 0.100m}\right]^{1/4}$$

$$\bar{h}_D = 1674\ W/m^2\cdot K.$$

Hence, the condensation rate is

$$\dot{m} = 1674\,W/m^2\cdot K \times p\left(0.100m\right)^2\left(470 - 423\right)K/900 \times 10^3\,J/kg$$

$$\dot{m} = 2.75 \times 10^{-3} kg/s. \qquad <$$

COMMENTS: Recognize this estimate is likely to be a poor one since properties were not evaluated at the proper T_f which was beyond the limit of the table.

PROBLEM 10.63

KNOWN: Copper sphere of 10 mm diameter, initially at 50°C, is placed in a large container filled with saturated steam at 1 atm.

FIND: Time required for sphere to reach equilibrium and the condensate formed during this period.

SCHEMATIC:

Saturated steam, 1atm

Copper sphere, $D = 10mm$
$T_s(0) = T_i = 50°C$

$\dot{m}(t)$

$\dot{E}_{st}$

$\dot{E}_{in} = \dot{m} h'_{fg}$

ASSUMPTIONS: (1) Laminar film condensation, (2) Negligible non-condensibles in vapor, (3) Sphere is spacewise isothermal, (4) Sphere experiences heat gain by condensation only.

PROPERTIES: *Table A-6*, Saturated water vapor (1 atm): T_{sat} = 100°C, ρ_v = 0.596 kg/m^3, h_{fg} = 2257 kJ/kg; *Table A-6*, Water, liquid ($T_f \approx$ (75 + 100)°C/2 = 360K): r_ℓ = 967.1 kg/m^3, $c_{p,\ell}$ = 4203 J/kg·K, m_ℓ = 324 × 10^{-6} N·s/m^2, k_ℓ = 0.674 W/m·K; *Table A-1*, Copper, pure $(\overline{T} = 75° C)$: ρ_{sp} = 8933 kg/m^3, $c_{p,sp}$ = 389 J/kg·K.

ANALYSIS: Using the lumped capacitance approach, an energy balance on the sphere provides,

$$\dot{E}_{in} - \dot{E}_{out} = \dot{E}_{st}$$

$$\dot{m} h'_{fg} = \overline{h}_D A_s (T_{sat} - T_s) = r_{sp} c_{p,sp} V_s \frac{dT_s}{dt}. \quad (1)$$

Properties of the sphere, ρ_{sp} and $c_{p,sp}$,. Will be evaluated at $\overline{T}_s = (50 + 100)°C/2 = 75°C$, while water (liquid) properties will be evaluated at $\overline{T}_f = (\overline{T}_s + T_{sat})/2 = 87.5°C \approx 360K$. From Eq. 10.26 with Ja = $c_{p,\ell} \Delta T / h_{fg}$ where ΔT = Tsat - $\overline{T}_s$, find

$$h'_{fg} = h_{fg}(1 + 0.68Ja) = 2257 \frac{kJ}{kg}\left(1 + 0.68\left[4203 \frac{J}{kg \cdot K} \times (100 - 75)K / 2257 \times 10^3 J/kg\right]\right) = 2328 \frac{kJ}{kg}. \quad (2)$$

To estimate the time required to reach equilibrium, we need to integrate Eq. (1) with appropriate limits. However, to perform the integration, an appropriate relation for the temperature dependence of $\overline{h}_D$ needs to be found. Using Eq. 10.40 with C = 0.815,

$$\overline{h}_D = 0.815 \left[\frac{g\, r_\ell (r_\ell - r_v) k_\ell^3 h'_{fg}}{m_\ell (T_{sat} - T_s) D}\right]^{1/4}.$$

Substitute numerical values and find,

$$\overline{h}_D = 0.815 \left[\frac{9.8 m/s^2 \times 967.1 kg/m^3 (967.1 - 0.596) kg/m^3 (0.674 W/m \cdot K)^3 \times 2328 \times 10^3 J/kg}{324 \times 10^{-6} N \cdot s/m^2 (T_{sat} - T_s) \times 0.010m}\right]^{1/4}$$

$$\overline{h}_D = B(T_{sat} - T_s)^{-1/4} \qquad \text{where} \qquad B = 30{,}707 W/m^2 \cdot (K)^{3/4}. \quad (3)$$

Continued …..

PROBLEM 10.63 (Cont.)

Substitute Eq. (3) into Eq. (1) for $\overline{h}_D$ and recognize $V_s / A_s = \frac{1}{6}\pi D^3 / \pi D^2 = D/6$,

$$B\left(T_{sat} - T_s\right)^{-1/4}\left(T_{sat} - T_s\right) = \rho_{sp}\, c_{p,sp}\left(D/6\right)\frac{dT_s}{dt}. \tag{4}$$

Note that $d(T_s) = - d(T_{sat} - T_s)$; letting $\Delta T \equiv T_{sat} - T_s$ and separating variables, the energy balance relation has the form

$$\int_0^t dt = -\frac{\rho_{sp}\, c_{p,sp}\left(D/6\right)}{B}\int_{\Delta T_o}^{\Delta T}\frac{d\left(\Delta T\right)}{\Delta T^{3/4}} \tag{5}$$

where the limits of integration have been identified, with $\Delta T_o = T_{sat} - T_i$ and $T_i = T_s(0)$. Performing the integration, find

$$t = -\frac{\rho_{sp}\, c_{p,sp}\left(D/6\right)}{B}\cdot\frac{1}{1-3/4}\left[\Delta T^{1/4} - \Delta T_o^{1/4}\right].$$

Substituting numerical values with the limits, $\Delta T = 0$ and $\Delta T_o = 100\text{-}50 = 50°C$,

$$t = -\frac{8933\,\text{kg/m}^3 \times 389\,\text{J/kg}\cdot\text{K}\left(0.010\,\text{m}/6\right)}{30{,}707\ \text{W/m}^2\cdot\text{K}^{3/4}}\times 4\left[0^{1/4} - 50^{1/4}\right]\text{K}^{1/4}$$

$$t = 2.0\text{s}.$$ <

To determine the total amount of condensate formed during this period, perform an energy balance on a time interval basis,

$$E_{in} - E_{out} = \Delta E = E_{final} - E_{initial}$$

$$E_{in} = \rho_{sp}\, c_{p,sp}\, V\left(T_{final} - T_{initial}\right] \tag{6}$$

where $T_{final} = T_{sat}$ and $T_{initial} = T_i = T_s(0)$. Recognize that

$$E_{in} = M\, h'_{fg} \tag{7}$$

where M is the total mass of vapor that condenses. Combining Eqs. (6) and (7),

$$M = \frac{\rho_{sp}\, c_{p,sp}\, V}{h'_{fg}}\left[T_{sat} - T_i\right]$$

$$M = \frac{8933\,\text{kg/m}^3 \times 389\,\text{J/kg}\cdot\text{K}\left(\pi/6\right)\left(0.010\,\text{m}\right)^3}{2328\times 10^3\ \text{J/kg}}\left[100 - 50\right]\text{K}$$

$$M = 3.91\times 10^{-5}\,\text{kg}.$$ <

COMMENTS: The total amount of condensate could have been evaluated from the integral,

$$M = \int_0^t \dot{m}\, dt = \int_0^t \frac{q}{h'_{fg}}\, dt = \int_0^t \frac{\overline{h}_D A_s\left(T_{sat} - T_s\right)dt}{h'_{fg}}$$

giving the same result, but with more effort.

PROBLEM 10.64

KNOWN: Saturated steam condensing on the inside of a horizontal pipe.

FIND: Heat transfer coefficient and the condensation rate per unit length of the pipe.

SCHEMATIC:

ASSUMPTIONS: (1) Film condensation with low vapor velocities.

PROPERTIES: *Table A-6*, Saturated water vapor (1.5 bar): $T_{sat} \approx 385K$, $\rho_v = 0.88\ kg/m^3$, $h_{fg} = 2225\ kJ/kg$; *Table A-6*, Saturated water ($T_f = (T_{sat} + T_s)/2 \approx 380K$): $r_\ell = 953.3\ kg/m^3$, $c_{p,\ell} = 4226\ J/kg{\cdot}K$, $m_\ell = 260 \times 10^{-6}\ N{\cdot}s/m^2$, $k_\ell = 0.683\ W/m{\cdot}K$.

ANALYSIS: The condensation rate per unit length follows from Eq. 10.33 with A = π D L and has the form

$$\dot{m}' = \frac{\dot{m}}{L} = \bar{h}_D (p D)(T_{sat} - T_s)/h'_{fg}$$

where $\bar{h}_D$ is estimated from the correlation of Eq. 10.42 with Eq. 10.43,

$$\bar{h}_D = 0.555\left[\frac{g\, r_\ell (r_\ell - r_v) k_\ell^3 h'_{fg}}{m_\ell (T_{sat} - T_s) D}\right]^{1/4}$$

where

$$h'_{fg} = h_{fg} + \frac{3}{8} c_{p,\ell}(T_{sat} - T_s) = 2225\times10^3 \frac{J}{kg} + \frac{3}{8}\times 4226 \frac{J}{kg\cdot K}(385 - 373)K$$

$$h'_{fg} = 2244 kJ/kg.$$

Hence,

$$\bar{h}_D = 0.555\left[\frac{9.8 m/s^2 \times 953.3 \frac{kg}{m^3}(953.3 - 0.88)\frac{kg}{m^3}(0.683 W/m\cdot K)^3\, 2244\times10^3 J/kg}{260\times10^{-6} N\cdot s/m^2 (385 - 373) K \times 0.075 m}\right]^{1/4}$$

$$\bar{h}_D = 7127 W/m^2\cdot K.$$

It follows that the condensate rate per unit length of the tube is

$$\dot{m}' = 7127 W/m^2\cdot K\,(p\times 0.075 m)(385 - 373)K\ /\ 2225\times10^3 J/kg = 9.06\times10^{-3} kg/s\cdot m.$$ <

PROBLEM 10.65

KNOWN: Horizontal pipe passing through an air space with prescribed temperature and relative humidity.

FIND: Water condensation rate per unit length of pipe.

SCHEMATIC:

Ambient air, $T_\infty = 37°C$, $\phi = 0.75$

$T_s = 15°C = 288K$

Pipe, D=25mm

ASSUMPTIONS: (1) Drop-wise condensation, (2) Copper tube approximates well promoted surface.

PROPERTIES: *Table A-6*, Water vapor (T_∞ = 37°C = 310K): $p_{A,sat}$ = 0.06221 bar; *Table A-6*, Water vapor ($p_A = \phi \cdot p_{a,sat}$ = 0.04666 bar): T_{sat} = 305K = 32°C, h_{fg} = 2426 kJ/kg; *Table A-6*, Water, liquid ($T_f = (T_s + T_{sat})/2$ = 297K): $c_{p,\ell} = 4180\ \text{J/kg}\cdot\text{K}$.

ANALYSIS: From Eq. 10.33, the condensate rate per unit length is

$$\dot{m}' = \frac{q'}{h'_{fg}} = \frac{h_L(p\,D)(T_{sat} - T_s)}{h'_{fg}}$$

where from Eq. 10.26, with $Ja = c_{p,\ell}(T_{sat} - T_s)/h_{fg}$,

$$h'_{fg} = h_{fg}[1 + 0.68Ja] = 2426\frac{kJ}{kg}[1 + 0.68 \times 4180 J/kg \cdot K(305 - 288)K/2426 kJ/kg]$$

$$h'_{fg} = 2474 kJ/kg.$$

Note that T_{sat} is the saturation temperature of the water vapor in air at 37°C having a relative humidity, ϕ = 0.75. That is, T_{sat} = 305K and T_s = 15°C + 288K. For *drop-wise condensation*, the correlation of Eq. 10.44 yields

$$\bar{h}_{dc} = 51{,}104 + 2044T_{sat} \qquad 22°C < T_{sat} < 100°C$$

where the units of $\bar{h}_{dc}$ and T_{sat} are W/m^2·K and °C.

$$\bar{h}_{dc} = 51{,}104 + 2044(32°C) = 116{,}510\ W/m^2 \cdot K.$$

Hence, the condensation rate is

$$\dot{m}' = 116{,}510\ W/m^2 \cdot K(p \times 0.025m)(305 - 288)K/2474 \times 10^3 J/kg$$

$$\dot{m}' = 6.288 \times 10^{-2}\ kg/s \cdot m$$

<

COMMENTS: From the result of Problem 10.54 assuming laminar film condensation, the condensation rate was $\dot{m}'_{film} = 4.28 \times 10^{-3}\ kg/s \cdot m$ which is an order of magnitude less than for the rate assuming drop-wise condensation.

PROBLEM 10.66

KNOWN: Beverage can at 5°C is placed in a room with ambient air temperature of 32°C and relative humidity of 75%.

FIND: The condensate rate for (a) drop-wise and (b) film condensation.

SCHEMATIC:

Air
$T_\infty = 32^oC$
$\phi_\infty = 75\%$

Beverage can
$T_s = 5^oC$
D = 65 mm
L = 120 mm

Drop-wise or film condensation

ASSUMPTIONS: (1) Condensation on top and bottom surface of can neglected, (2) Negligible non-condensibles in water vapor-air, and (b) For film condensation, film thickness is small compared to diameter of can.

PROPERTIES: *Table A-6*, Water vapor (T_∞ = 32°C = 305 K): $p_{A,sat}$ = 0.04712 bar; Water vapor ($p_A = \phi \cdot p_{A,sat}$ = 0.03534 bar): $T_{sat} \approx$ 300 K = 27°C, h_{fg} = 2438 kJ/kg; Water, liquid ($T_f = (T_s + T_{sat})/2$ = 289 K): $c_{p,\ell}$ = 4185 J/kg·K.

ANALYSIS: From Eq. 10.33, the condensate rate is

$$\dot{m} = \frac{q}{h'_{fg}} = \frac{\bar{h}(\pi DL)(T_{sat} - T_s)}{h'_{fg}}$$

where from Eq. 10.26, with Ja = $c_{p,\ell}(T_{sat} - T_s)/h_{fg}$,

$$h'_{fg} = h_{fg}[1 + 0.68 \text{ Ja}]$$

$$h'_{fg} = 2438 \text{ kJ/kg}\left[1 + 0.68 \times 4185 \text{ J/kg} \cdot \text{K}(300 - 278)\text{K}/2438 \text{ kJ/kg}\right]$$

$$h'_{fg} = 2501 \text{ kJ/kg}$$

Note that T_{sat} is the saturation temperature of the water vapor in air at 32°C having a relative humidity of ϕ_∞ = 0.75.

(a) For drop-wise condensation, the correlation of Eq. 10.44 with T_{sat} = 300 K = 27°C yields

$$\bar{h} = \bar{h}_{dc} = 51{,}104 + 2044\, T_{sat} \qquad 22^\circ C < T_{sat} \le 100^\circ C$$

where the units of $\bar{h}_{dc}$ are W/m^2·K and T_{sat} are °C,

$$\bar{h}_{dc} = 51{,}104 + 2044 \times 27 = 106{,}292 \text{ W/m}^2 \cdot \text{K}$$

Hence, the condensation rate is

$$\dot{m} = 1.063 \times 10^5 \text{W/m}^2 \cdot \text{K}(\pi \times 0.065 \text{ m} \times 0.125 \text{ m})(27 - 5)\text{K}/2501 \text{ kJ/kg}$$

$$\dot{m} = 0.0229 \text{ kg/s}$$ <

Continued …..

PROBLEM 10.66 (Cont.)

(b) For film condensation, we used the *IHT* tool *Correlations, Film Condensation*, which is based upon Eqs. 10.37, 10.38 or 10.39 depending upon the flow regime. The code is shown in the Comments section, and the results are

$$\mathrm{Re}_\delta = 24, \text{ flow is laminar} \qquad \dot{m} = 0.00136 \text{ kg/s}$$ <

Note that the film condensation rate estimate is nearly 20 times less than for drop-wise condensation.

COMMENTS: The *IHT* code identified in part (b) follows:

```
/* Results,        Part (b)  - input variables   and rate parameters
NuLbar             Redelta  hLbar   mdot      D      L      Ts     Tsat
0.5093             24.05    6063    0.001362  0.065  0.125  278    300    */

/* Thermophysical properties evaluated at Tf; hfg at Tsat
Prl    Tf     cpl     h'fg     hfg      kl      mul       nul
7.81   289    4185    2.501E6  2.438E6  0.5964  0.001109  1.11E-6*/

// Other input variables required in the correlation
L = 0.125
b = pi * D
D = 0.065

/*  Correlation description: Film condensation (FCO) on a vertical plate (VP). If Redelta<29,
laminar region,  Eq 10.37 . If 31<Redelta<1750, wavy-laminar region,  Eq 10.38 . If Redelta>=1850,
turbulent region,  Eq 10.26, 10.32, 10.33, 10.35, 10.39 . In laminar-wavy and wavy-turbulent transition
regimes, function interpolates between laminar and wavy, and wavy and turbulent correlations.  See
Fig 10.15 .  */
NuLbar = NuL_bar_FCO_VP(Redelta,Prl)        // Eq 10.37, 38, 39
NuLbar = hLbar * (nul^2 / g)^(1/3) / kl
g = 9.8                             // gravitational constant, m/s^2
Ts =  5 + 273                       // surface temperature, K
Tsat = 300                          // saturation temperature, K
// The liquid properties are evaluated at the film temperature, Tf,
Tf= (Ts + Tsat) / 2
// The condensation and heat rates are
q = hLbar * As * (Tsat - Ts)        // Eq 10.32
As = L * b                          // surface Area, m^2
mdot = q / h'fg                               // Eq 10.33
h'fg = hfg + 0.68 * cpl * (Tsat - Ts)     // Eq 10.26; hfg evaluated at Tsat
// The Reynolds number based upon film thickness is
Redelta = 4 * mdot / (mul * b)      // Eq 10.35

// Water property functions :T dependence, From Table A.6
// Units: T(K), p(bars);
x = 0                               // Quality (0=sat liquid or 1=sat vapor)
hfg = hfg_T("Water",Tsat)           // Heat of vaporization, J/kg; evaluated at Tsat
cpl = cp_Tx("Water",Tf,x)           // Specific heat, J/kg·K
mul = mu_Tx("Water",Tf,x)           // Viscosity, N·s/m^2
nul = nu_Tx("Water",Tf,x)           // Kinematic viscosity, m^2/s
kl = k_Tx("Water",Tf,x)             // Thermal conductivity, W/m·K
Prl = Pr_Tx("Water",Tf,x)           // Prandtl number
```

PROBLEM 10.67

KNOWN: Surface temperature and area of integrated circuits submerged in a dielectric fluid of prescribed properties. Height and temperature of condenser plates.

FIND: (a) Heat dissipation by an integrated circuit, (b) Condenser surface area needed to balance heat load.

SCHEMATIC:

ASSUMPTIONS: (1) Nucleate pool boiling in liquid, (2) Laminar film condensation of vapor, (3) Negligible heat loss to surroundings.

PROPERTIES: Dielectric fluid (given, T_{sat} = 50°C): r_ℓ = 1700kg/m^3, $c_{p,\ell}$ = 1005J/kg·K, m_ℓ = 6.80 × 10^{-4} kg/s·m, k_ℓ = 0.062W/m·K, Pr_ℓ = 11, σ = 0.013 kg/s^2, h_{fg} = 1.05 × 10^5 J/kg, $C_{s,f}$ = 0.004, n = 1.7.

ANALYSIS: (a) For nucleate pool boiling,

$$q_s'' = m_\ell h_{fg}\left[\frac{g(r_\ell - r_v)}{s}\right]^{1/2}\left(\frac{c_{p,\ell}\Delta T_e}{C_{s,f}h_{fg}Pr_\ell^n}\right)^3 \approx 6.8\times10^{-4}\,kg/s\cdot m\left(1.05\times10^5\,J/kg\right)$$

$$\times\left[\frac{9.8\,m/s^2\times1700\,kg/m^3}{0.013\,kg/s^2}\right]^{1/2}\left(\frac{1005\,J/kg\cdot K\times25K}{0.004\times1.05\times10^5\,J/kg\times11^{1.7}}\right)^3 = 84,530\,W/m^2$$

$$q_s = A_s q_s'' = 84,530\,W/m^2\times25\times10^{-6}\,m^2 = 2.11\,W.$$ <

(b) For laminar film condensation on a vertical surface, $\overline{Nu}_L = 0.943\left[\frac{g r_\ell(r_\ell - r_v)h_{fg}'L^3}{m_\ell k_\ell(T_{sat}-T_s)}\right]^{1/4}$

$$h_{fg}' = h_{fg}\left(1+0.68\frac{c_{p,\ell}\Delta T}{h_{fg}}\right) = 1.05\times10^5 + 0.68\left(1005\,J/kg\cdot K\times35K\right) = 1.29\times10^5\,J/kg$$

$$\overline{Nu}_L \approx 0.943\left[\frac{9.8\,m/s^2\left(1700\,kg/m^3\right)^2 1.29\times10^5\,J/kg\left(0.05\,m\right)^3}{6.8\times10^{-4}\,kg/s\cdot m\left(0.062\,W/m\cdot K\right)\left(35K\right)}\right]^{1/4} = 703$$

$$\overline{h}_L = \left(k_\ell/L\right)\overline{Nu}_L = \left(0.062\,W/m\cdot K/0.05m\right)703 = 872\,W/m^2\cdot K$$

$$q_c = \overline{h}_L A_c\left(T_{sat}-T_c\right) = 872\,W/m^2\cdot K\left(35K\right)A_c = 30,500A_c\left(m^2\right)$$

To balance the heat load, $q_c = Nq_s$. Hence

$$A_c = \frac{500\times2.11\,W}{30,500\,W/m^2} = 0.0346\,m^2$$ <

COMMENTS: (1) With A_c = 0.0346m^2 and H = 0.05m, the total condenser width is W = A_c/H = 692mm. (2) With $\dot{m}_c/b = \Gamma = q_c/h_{fg}'W = 1055W/1.29\times10^5 J/kg\times0.692m = 0.0118 kg/s\cdot m$, $Re_d = 4\Gamma/m_\ell$ = 4(0.0118kg/s·m)/6.8 × 10^{-4} kg/s·m = 69.4. Hence condensate film is in the laminar-wavy regime, and a more accurate estimate of A_c would require iteration.

PROBLEM 10.68

KNOWN: Thin-walled thermosyphon. Absorbs heat by boiling saturated water at atmospheric pressure on boiling section L_b. Rejects heat by condensing vapor into a thick film which falls length of condensation section L_c back into boiling section.

FIND: (a) Mean surface temperature, Ts,b, of the boiling surface if nucleate boiling flux is 30% critical flux, (b) Mean surface temperature, Ts,c of condensation section, and (c) Total condensation flow rate, $\dot{m}$, in thermosyphon. Explain how to determine whether film is laminar, wavy-laminar or turbulent.

SCHEMATIC:

$T_{s,c}$ — Saturated steam — Condensation film — $L_c = 40\,mm$ Condensation section — g — $L_i = 40\,mm$ Insulation — Saturated water — Electric heater — $T_{s,b}$ — $L_b = 20\,mm$ Boiling section — $D = 20\,mm$

ASSUMPTIONS: (1) Laminar film condensation occurs in condensation section which approximates a vertical plate, (2) Boiling and condensing section are separated by insulated length L_i, (3) Top surface of condensation section is insulated, (4) For condensation, liquid properties evaluated at $T_f = 90°C$.

PROPERTIES: *Table A-6*, Saturated water (100°C): $r_\ell = 1/v_f = 957.9\ kg/m^3$, $c_{p,\ell} = 4217$ J/kg·K, $m_\ell = 279 \times 10^{-6}\ N \cdot s/m^2$, $Pr_\ell = 1.76$, $h_{fg} = 2257$ kJ/kg, $\sigma = 58.9 \times 10^{-3}$ N/m; Saturated vapor (100°C): $\rho_v = 1/v_g = 0.5955\ kg/m^3$; Saturated water (90°C): $r_\ell = 1/v_f = 964.9 kg/m^3$, $c_{p,\ell} = 4207$ J/kg·K, $m_\ell = 313 \times 10^{-6}$ N·s/m^2, $k_\ell = 0.676\ W/m\cdot K$.

ANALYSIS: (a) The heat flux for the boiling section is 30% the critical heat flux which at atmospheric pressure is

$$q''_{s,b} = 0.30 q''_{max} = 0.30 \times 1.26 \times 10^6\ W/m^2 = 3.78 \times 10^5\ W/m^2.$$

Using the Rohsenow correlation for nucleate boiling with $T_{sat} = 100°C$ and typical values for the surface of $C_{s,f} = 0.0130$ and n = 1.0, find

$$q''_{s,b} = m_\ell h_{fg} \left[\frac{g(r_\ell - r_v)}{s} \right]^{1/2} \left(\frac{c_{p,\ell}\left(T_{s,b} - T_{sat}\right)}{C_{s,f} h_{fg} Pr_\ell^n} \right)^3$$

$$3.78 \times 10^5\ W/m^2 = 279 \times 10^{-6} N \cdot s/m^2 \times 2257 \times 10^3\ J/kg \times$$

$$\left[\frac{9.8 m/s^2 (957.9 - 0.5955) kg/m^3}{58.9 \times 10^{-3}\ N/m} \right]^{1/2} \left(\frac{4217\ J/kg \cdot K \left(T_{s,b} - 100\right)}{0.013 \times 2257 \times 10^3\ J/kg 1.76^{1.0}} \right)^3$$

Continued …..

$$T_{s,b} = 114.0°C. \qquad <$$

(b) The heat transferred into the boiling section must be rejected by film condensation,

$$q_c = q_b = q''_{s,b}\left[\pi D^2/4 + \pi D L_b\right]$$

$$q_c = 3.78\times10^5\ \text{W/m}^2\left[\pi\,(0.020\text{m})^2/4 + \pi\,(0.020\text{m})\times0.020\text{m}\right] = 592\ \text{W}.$$

The mean surface temperature can be determined from the rate equation

$$q_c = \overline{h}_{Lc}\left(\pi D L_c\right)\left(T_{sat} - T_{s,c}\right)$$

where the convection coefficient is determined from Eq. 10.30,

$$\overline{h}_{Lc} = 0.943\left[\frac{g\rho_\ell\left(\rho_\ell - \rho_v\right)k_\ell^3 h'_{fg}}{\mu_\ell\left(T_{sat} - T_{s,c}\right)L_c}\right]^{1/4}$$

$$\overline{h}_{Lc} = 0.943\left[\frac{9.8\,\text{m/s}^2\times964.9\,\text{kg/m}^3\,(964.9-0.5955)\,\text{kg/m}^3\,(0.676\,\text{W/m}\cdot\text{K})^3\,2257\times10^3\ \text{J/kg}}{313\times10^{-6}\ \text{N}\cdot\text{s/m}^2\left(100-T_{s,c}\right)0.040\ \text{m}}\right]^{1/4}$$

where $h'_{fg} = h_{fg}\left\{1 + 0.68 c_{p,\ell}\left(T_{sat} - T_{s,c}\right)/h_{fg}\right\}$

$$h'_{fg} = 2257\times10^3\ \text{J/kg}\left\{1 + 0.68\times4207\ \text{J/kg}\cdot\text{K}\left(100 - T_{s,c}\right)/2257\times10^3\ \text{J/kg}\right\} \approx 2257\times10^3\ \text{J/kg}.$$

Hence, $\qquad \overline{h}_{Lc} = 2.517\times10^4\left(100 - T_{s,c}\right)^{-1/4}.$

Using the rate equation, now find $T_{s,c}$ by trial-and-error,

$$592\ \text{W} = 2.517\times10^4\left(100 - T_{s,c}\right)^{-1/4}\left(\pi\times0.020\text{m}\times0.040\text{m}\right)\left(100 - T_{s,c}\right)\text{K}$$

$$9.358 = \left(100 - T_{s,c}\right)^{0.75}$$

$$T_{s,c} = 80.3°C. \qquad <$$

(c) The condensation rate in the condenser section is

$$\dot{m} = q_c/h'_{fg} = 592\,\text{W}/\left(2257\times10^3\ \text{J/kg}\right) = 2.623\times10^{-4}\ \text{kg/s} \qquad <$$

and from Eq. 10.35,

$$Re_\delta = \frac{4\dot{m}}{\mu_\ell b} = \frac{4\dot{m}}{\mu_\ell\left(\pi D\right)} = \frac{4\times2.623\times10^{-4}\ \text{kg/s}}{313\times10^{-6}\ \text{N}\cdot\text{s/m}^2\left(\pi\times0.020\text{m}\right)} = 53.3.$$

Since $30 < Re_\delta < 1800$, we conclude the film is laminar-wavy.

PROBLEM 10.69

KNOWN: Thermosyphon configuration for cooling a computer chip of prescribed size.

FIND: (a) Chip temperature and total power dissipation when chip operates at 90% of critical heat flux, (b) Required condenser length.

SCHEMATIC:

$D = 30$ mm

$T_s = 25\ ^oC$

L

Chip

$L_c = 20$ mm

$T,\ q'' = 0.9\ q''_{max}$

$C_{s,f} = 0.005,\ n = 1.7$

ASSUMPTIONS: (1) Steady-state, (2) Saturated liquid/vapor conditions, (3) Negligible heat transfer from bottom of chip.

PROPERTIES: Fluorocarbon (prescribed): $T_{sat} = 57°C$, $c_{p,\ell} = 1100$ J/kg·K, $h_{fg} = 84{,}400$ J/kg, $\rho_\ell = 1619.2$ kg/m^3, $\rho_v = 13.4$ kg/m^3, $\sigma = 8.1 \times 10^{-3}$ kg/s^2, $\mu_\ell = 440 \times 10^{-6}$ kg/m·s, $Pr_\ell = 9.01$, $k_\ell = 0.054$ W/m·K, $\nu_\ell = \mu_\ell / \rho_\ell = 0.272 \times 10^{-6}$ m^2/s.

ANALYSIS: (a) With $q'' = 0.9\ q''_{max}$ and the critical heat flux given by Eq. 10.7, the chip power dissipation is

$$q = 0.9 L_c^2 \times 0.149 h_{fg} \rho_v \left[\frac{\sigma g(\rho_\ell - \rho_v)}{\rho_v^2}\right]^{1/4}$$

$$q = 0.9(0.02\,\text{m})^2 \times 0.149(84,400\,\text{J/kg})13.4\,\text{kg/m}^3 \left[\frac{0.0081\,\text{kg/s}^2 (9.8\,\text{m/s}^2)(1605.8\,\text{kg/m}^3)}{(13.4\,\text{kg/m}^3)^2}\right]^{1/4}$$

$$q_c = 0.9\left(4\times10^{-4}\,\text{m}^2\right)1.55\times10^5\,\text{W/m}^2 = 55.7\,\text{W}$$ <

With operation at $q'' = 1.40 \times 10^5$ W/m^2 in the nucleate boiling region, Eq. 10.5 yields

$$T = T_{sat} + \frac{C_{s,f} h_{fg} Pr_\ell^n}{c_{p,\ell}} \left(\frac{q''}{\mu_\ell h_{fg}}\right)^{1/3} \left[\frac{\sigma}{g(\rho_\ell - \rho_v)}\right]^{1/6}$$

$$T = 57^\circ C + \frac{0.005(84,400\,\text{J/kg})(9.01)^{1.7}}{1100\,\text{J/kg}\cdot\text{K}} \left(\frac{1.40\times10^5\,\text{W/m}^2}{4.4\times10^{-4}\,\text{kg/m}\cdot\text{s}\times84,400\,\text{J/kg}}\right)^{1/3} \left[\frac{0.0081\,\text{kg/s}^2}{9.8\,\text{m/s}^2(1605.8\,\text{kg/m}^3)}\right]^{1/6}$$

Continued...

$$T = 57^\circ C + 22.4^\circ C = 79.4^\circ C$$ <

(b) The power dissipated by the chip must be balanced by the rate of heat transfer from the condensing section. Hence, with A = πDL, Eq. 10.32 yields the requirement that

$$\overline{h}_L L = \frac{q}{\pi D\left(T_{sat} - T_s\right)} = \frac{55.7\,W}{\pi(0.03\,m)\left(32^\circ C\right)} = 18.5\,W/m\cdot K$$

To determine $\overline{h}_L$, we combine Eqs. 10.33 and 10.35 to obtain $Re_\delta = 4q/\mu_\ell b h'_{fg}$, where b = πD = 0.0942 m and $h'_{fg} = h_{fg} + 0.68c_{p,1}\left(T_{sat} - T_s\right) = 84{,}400\,J/kg + 0.68(1100\,J/kg\cdot K)32^\circ C$ = 108,300 J/kg. Hence, Re_δ = 4(55.7 W)/4.4 × 10^{-4} kg/m·s(0.0942 m)108,300 J/kg = 49.6 and the condensate film is in the laminar-wavy region. Hence, from Eq. 10.38

$$\overline{h}_L = \frac{k_\ell}{\left(\nu_\ell^2/g\right)^{1/3}} \frac{Re_\delta}{1.08\,Re_\delta^{1.22} - 5.2} = \frac{0.054\,W/m\cdot K \times 0.409}{\left[\left(0.272\times10^{-6}\,m^2/s\right)^2 / 9.8\,m/s^2\right]^{1/3}} = 1130\,W/m^2\cdot K$$

in which case,

$$L = \frac{18.5\,W/m\cdot K}{1130\,W/m^2\cdot K} = 0.0164\,m = 16.4\,mm$$ <

COMMENTS: The chip operating temperature (T = 79.4°C) is not excessive, and the proposed scheme provides a compact means of cooling high performance chips.

PROBLEM 10.70

KNOWN: Copper plate, 2m × 2m, in a condenser-boiler section maintained at $T_s = 100°C$ separates condensing saturated steam and nucleate-pool boiling of saturated liquid X.

FIND: (a) Rates of evaporation and condensation (kg/s) for the two fluids and (b) Saturation temperature T_{sat} and pressure p for the steam, assuming that film condensation occurs.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Isothermal copper plate.

PROPERTIES: *Fluid-X* (Given, 1 atm): $T_{sat} = 80°C$, $h_{fg} = 700$ kJ/kg, portion of boiling curve shown above for operating condition, $\Delta T_e = T_s - T_{sat} = (100 - 80)°C = 20°C$, $q''_s = 5 \times 10^4\ W/m^2$; *Table A.4*, Water (saturated, 1 atm): $T_{sat} = 100°C$, $h_{fg} = 2.25 \times 10^6$ J/kg; Water (saturated, T_{sat}): as required in part (b); Water (saturated, $T_f = (T_{sat} + T_s)/2$): as required in part (b).

ANALYSIS: (a) For fluid-X, with $\Delta T_e = T_s - T_{sat} = (100 - 80)°C = 20$ K, the heat flux from the boiling curve is

$$q''_s = 50,000\,W/m^2$$

and the heat rate from the copper plate section into liquid-X is

$$q_s = q''_s \times A_s = 50,000\,W/m^2 \times (2 \times 2)\,m^2 = 200,000\,W$$

From an energy balance around liquid-X, the evaporation rate for fluid-X is

$$\dot{m}_X = q_s / h_{fg,X} = 200,000\,W / 700,000\,J/kg = 0.286\,kg/s$$ <

The heat rate into the copper plate section from the steam is $q_s = 200,000$ W, and from an energy balance around the condensate film, the condensation rate for steam (w)

$$\dot{m}_w = q_s / h'_{fg,w} = 200,000\,W / 2.25 \times 10^6\,J/kg = 0.0889\,kg/s$$

where we are assuming that $T_{sat,w}$ is only a few degrees above T_s so that $h'_{fg'} \approx h_{fg}$.

(b) The condensation heat rate, Eq. 10.32 can be expressed as

$$q_s = \bar{h}_L A_s \left(T_{sat} - T_s\right)$$

and assuming laminar film condensation, Eq. 10.30,

$$\bar{h}_L = 0.943 \left[\frac{g \rho_\ell \left(\rho_\ell - \rho_v\right) k_\ell^3 h'_{fg}}{\mu_\ell k_\ell \left(T_{sat} - T_s\right)} \right]^{1/4}$$

Continued...

PROBLEM 10.70 (Cont.)

Recognize that with q_s , A_s and T_s known, this relation can be used to determine T_{sat} , and from the steam table, the corresponding p_{sat} can be found. The vapor properties (v) are evaluated at T_{sat} while the liquid properties (ℓ) are evaluated at the film temperature $T_f = (T_{sat} + T_s)/2$. An iterative solution is required, beginning by assuming a value for T_{sat} , evaluate properties and check whether the rate equation returns the assumed value for T_{sat} . Using the *IHT Correlations Tool, Film Condensation, Vertical Plate* for the *laminar* region, the results are

$$T_{sat} = 381.7\,\text{K} \qquad p_{sat} = 1.367\,\text{bar}$$

for which $Re_\delta = 661$, so that the flow is wavy-laminar, not laminar. Repeating the analysis but with the *IHT Tool* for the *laminar*, *wavy-laminar*, *turbulent* regions, the results with $Re_\delta = 652$ are

$$T_{sat} = 379.6\,\text{K} \qquad P_{sat} = 1.27\,\text{bar}$$ <

COMMENTS: A copy of the IHT model for determining T_{sat} and p_{sat} for part (b) is shown below.

```
// Correlations Tool -
//Film Condensation, Vertical Plate, laminar, wavy-laminar, turbulent regions
NuLbar = NuL_bar_FCO_VP(Redelta,Prl)        // Eq 10.37, 38, 39
NuLbar = hLbar * (nul^2 / g)^(1/3) / kl
g = 9.8                                     // Gravitational constant, m/s^2
Ts = 100 + 273                              // Surface temperature, K
Tsat =    380                               // Saturation temperature, K; explore over range to match q
// The liquid properties are evaluated at the film temperature, Tf,
Tf = Tfluid_avg(Ts,Tsat)
// The condensation and heat rates are
q = hLbar * As * (Tsat - Ts)                // Eq 10.32
As = L * b    // Surface Area, m^2
mdot = q / h'fg                             // Eq 10.33
h'fg = hfg + 0.68 * cpl * (Tsat - Ts)       // Eq 10.26
// The Reynolds number based upon film thickness is
Redelta = 4 * mdot / (mul * b)              // Eq 10.35
/*  Correlation description: Film condensation (FCO) on a vertical plate (VP). If Redelta<29, laminar
region,  Eq 10.37 . If 31<Redelta<1750, wavy-laminar region,  Eq 10.38 . If Redelta>=1850, turbulent
region,  Eq 10.22, 10.32, 10.33, 10.35, 10.39 . In laminar-wavy and wavy-turbulent transition regimes,
function interpolates between laminar and wavy, and wavy and turbulent correlations.  See Fig 10.15 .  */
// Assigned Variables:
L = 2                                       // Plate height, m
b = 2                                       // Plate width, m
//q = 200000                                // Heat rate, W; required heat rate for suitable Tsat
// Properties Tool - Water:
// Water property functions :T dependence, From Table A.6
// Units: T(K), p(bars);
xl =  0                                     // Quality (0=sat liquid or 1=sat vapor)
pl = psat_T("Water", Tf)                    // Saturation pressure, bar
vl = v_Tx("Water",Tf,xl)                    // Specific volume, m^3/kg
rhol = rho_Tx("Water",Tf,xl)                // Density, kg/m^3
cpl = cp_Tx("Water",Tf,xl)                  // Specific heat, J/kg·K
mul = mu_Tx("Water",Tf,xl)                  // Viscosity, N·s/m^2
nul = nu_Tx("Water",Tf,xl)                  // Kinematic viscosity, m^2/s
kl = k_Tx("Water",Tf,xl)                    // Thermal conductivity, W/m·K
Prl = Pr_Tx("Water",Tf,xl)                  // Prandtl number
// Water property functions :T dependence, From Table A.6
// Units: T(K), p(bars);
xv =  1                                     // Quality (0=sat liquid or 1=sat vapor)
pv = psat_T("Water", Tsat)                  // Saturation pressure, bar
vv = v_Tx("Water",Tsat,xv)                  // Specific volume, m^3/kg
rhov = rho_Tx("Water",Tsat,xv)              // Density, kg/m^3
hfg = hfg_T("Water",Tsat)                   // Heat of vaporization, J/kg
cpv = cp_Tx("Water",Tsat,xv)                // Specific heat, J/kg·K
muv = mu_Tx("Water",Tsat,xv)                // Viscosity, N·s/m^2
nuv = nu_Tx("Water",Tsat,xv)                // Kinematic viscosity, m^2/s
kv = k_Tx("Water",Tsat,xv)                  // Thermal conductivity, W/m·K
Prv = Pr_Tx("Water",Tsat,xv)                // Prandtl number
```

PROBLEM 10.71

KNOWN: Thin-walled container filled with a low boiling point liquid (A) at $T_{sat,A}$. Outer surface of container experiences laminar-film condensation with the vapor of a high-boiling point fluid (B). Laminar film extends from the location of the liquid-A free surface. The heat flux for nucleate pool boiling in liquid-A along the container wall is given as $q''_{npb} = C(T_s - T_{sat})^3$, where C is a known empirical constant.

FIND: (a) Expression for the average temperature of the container wall, T_s; assume that the properties of fluids A and B are known; (b) Heat rate supplied to liquid-A, and (c) Time required to evaporate all the liquid-A in the container, assuming that initially the container is filled, y = L.

SCHEMATIC:

ASSUMPTIONS: (1) Nucleate pool boiling occurs on the inner surface of the container with liquid-A, (2) Laminar film condensation occurs on the outer surface of the container with fluid-B over the liquid-A free surface, y, and (3) Negligible wall thermal resistance.

ANALYSIS: (a) Perform an energy balance on the control surface about the container wall along locations experiencing boiling (A) and condensation (B) as shown in the schematic above.

$$\dot{E}''_{in} - \dot{E}''_{out} = 0 \quad (1)$$

$$q''_{cond} - q''_{npb} = 0 \quad (2)$$

$$\bar{h}_y (\pi D y)\left(T_{sat,B} - T_s\right) - (\pi D y) C\left(T_s^3 - T_{sat,A}\right) = 0$$

$$\bar{h}_y \left(T_{sat,B} - T_s\right) = C\left(T_s - T_{sat,A}\right)^3 \quad (3) \quad <$$

where $\bar{h}_y$ is the average convection coefficient for laminar film condensation over the surface length 0 to y. From Eq. 10.30 and 10.26,

Continued...

PROBLEM 10.71 (Cont.)

$$\bar{h}_y = 0.943\left[\frac{g\rho_\ell(\rho_\ell - \rho_v)k_\ell^3 h'_{fg}}{\mu_\ell(T_{sat} - T_s)y}\right]_B^{1/4} \quad (3)$$

$$h'_{fg} = h_{fg,B} + 0.68c_{p,B}(T_{sat,B} - T_s) \quad (4)$$

where the properties are for fluid-B.

(b) The heat flux supplied to liquid-A is, from Eq. (2), $q''_{cond} = q''_{npb}$. Since $\bar{h}_y$ is a function of y, T_s and, hence, the heat fluxes will be functions of y, the height of liquid A in the container.

(c) To determine the dry-out time, t_f, begin with an energy balance on the inside of the container (fluid-A). The heat transfer supplied to liquid-A results in an evaporation rate of liquid-A,

$$q''_{npb}(\pi D y) - \frac{dM}{dt}h_{fg} = 0 \quad (4)$$

where M is the mass of liquid-A in the container,

$$M = \rho_{\ell,A}\left(\pi D^2/4\right)y \quad (5)$$

Substituting Eq. (5) into (4), separating variables and identifying integration limits, find

$$C\left(T_s - T_{sat,A}\right)^3(\pi D y) = \frac{d}{dt}\left[\rho_{\ell,A}\left(\pi D^2/4\right)y\right]h_{fg}$$

$$\int_0^{t_f} dt = t_f = \frac{\rho_{\ell,A}\left(\pi D^2/4\right)h_{fg}}{C\pi D}\int_L^0 \frac{dy}{\left(T_s - T_{sat,A}\right)^3 y} \quad (6)$$

The definite integral could be numerically evaluated using values for $T_s(y)$ obtained by solving Eq. (3).

PROBLEM 11.1

KNOWN: Initial overall heat transfer coefficient of a fire-tube boiler. Fouling factors following one year's application.

FIND: Whether cleaning should be scheduled.

SCHEMATIC:

Boiling water

Products of combustion

$R''_{f,i} = 0.0015\ m^2 \cdot K/W$

$R''_{f,o} = 0.0005\ m^2 \cdot K/W$

$U(t=0) = 400\ W/m^2 \cdot K$

ASSUMPTIONS: (1) Negligible tube wall conduction resistance, (2) Negligible changes in h_c and h_h.

ANALYSIS: From Equation 11.1, the overall heat transfer coefficient after one year is

$$\frac{1}{U} = \frac{1}{h_i} + \frac{1}{h_o} + R''_{f,i} + R''_{f,o}.$$

Since the first two terms on the right-hand side correspond to the reciprocal of the initial overall coefficient,

$$\frac{1}{U} = \frac{1}{400\ W/m^2 \cdot K} + (0.0015 + 0.0005)\ m^2 \cdot K/W = 0.0045\ m^2 \cdot K/W$$

$$U = 222\ W/m^2 \cdot K.$$

COMMENTS: Periodic cleaning of the tube inner surfaces is essential to maintaining efficient fire-tube boiler operations.

PROBLEM 11.2

KNOWN: Type-302 stainless tube with prescribed inner and outer diameters used in a cross-flow heat exchanger. Prescribed fouling factors and internal water flow conditions.

FIND: (a) Overall coefficient based upon the outer surface, U_o, with air at T_o =15°C and velocity V_o = 20 m/s in cross-flow; compare thermal resistances due to convection, tube wall conduction and fouling; (b) Overall coefficient, U_o, with water (rather than air) at T_o = 15°C and velocity V_o = 1 m/s in cross-flow; compare thermal resistances due to convection, tube wall conduction and fouling; (c) For the water-air conditions of part (a), compute and plot U_o as a function of the air cross-flow velocity for $5 \le V_o \le 30$ m/s for water mean velocities of $u_{m,i}$ = 0.2, 0.5 and 1.0 m/s; and (d) For the water-water conditions of part (b), compute and plot U_o as a function of the water mean velocity for $0.5 \le u_{m,i} \le 2.5$ m/s for air cross-flow velocities of V_o = 1, 3 and 8 m/s.

SCHEMATIC:

Fouling factors
$R''_{f,o}$ = 0.0004 m²·K/W
$R''_{f,i}$ = 0.0002 m²·K/W
Tube, SS302, k_w
Water
$T_{m,i}$ = 75 °C
$u_{m,i}$ = 0.5 m/s
Air
V_o = 20 m/s
T_o = 15 °C
D_o = 27 mm
D_i = 22 mm

ASSUMPTIONS: (1) Steady-state conditions, (2) Fully developed internal flow,

PROPERTIES: *Table A.1*, Stainless steel, AISI 302 (300 K): k_w = 15.1 W/m·K; *Table A.6*, Water ($\bar{T}_{m,i}$ = 348 K): ρ_i = 974.8 kg/m³, $\mu_i = 3.746 \times 10^{-4}$ N·s/m², k_i = 0.668 W/m·K, Pr_i = 2.354; *Table A.4,* Air (assume $\bar{T}_{f,o}$ = 315K, 1 atm): k_o = 0.02737 W/m·K, $\nu_o = 17.35 \times 10^{-6}$ m²/s, Pr_o = 0.705.

ANALYSIS: (a) For the water-air condition, the overall coefficient, Eq. 11.1, based upon the outer area can be expressed as the sum of the thermal resistances due to convection (cv), tube wall conduction (w) and fouling (f):

$$1/U_o A_o = R_{tot} = R_{cv,i} + R_{f,i} + R_w + R_{f,o} + R_{cv,o}$$

$$R_{cv,i} = 1/\bar{h}_i A_i \qquad R_{cv,o} = 1/\bar{h}_o A_o$$

$$R_{f,i} = R''_{f,i}/A_i \qquad R_{f,o} = R''_{f,o}/A_o$$

and from Eq. 3.28,

$$R_w = \ln(D_o/D_i)/(2\pi L k_w)$$

The convection coefficients can be estimated from appropriate correlations.

Continued...

PROBLEM 11.2 (Cont.)

Estimating $\overline{h}_i$: For internal flow, characterize the flow evaluating thermophysical properties at $T_{m,i}$ with

$$Re_{D,i} = \frac{u_{m,i} D_i}{\nu_i} = \frac{0.5\,\text{m/s} \times 0.022\text{m}}{3.746 \times 10^{-4}\,\text{N}\cdot\text{s/m}^2 / 974.8\,\text{kg/m}^3} = 28,625$$

For the turbulent flow, use the Dittus-Boelter correlation, Eq. 8.60,

$$Nu_{D,i} = 0.023\, Re_{D,i}^{0.8}\, Pr_i^{0.4}$$

$$Nu_{D,i} = 0.023(28,625)^{0.8}(2.354)^{0.4} = 119.1$$

$$\overline{h}_i = Nu_{D,i}\, k_i / D_i = 119.1 \times 0.668\,\text{W/m}^2\cdot\text{K} / 0.022\text{m} = 3616\,\text{W/m}^2\cdot\text{K}$$

Estimating $\overline{h}_o$: For external flow, characterize the flow with

$$Re_{D,o} = \frac{V_o D_o}{\nu_o} = \frac{20\,\text{m/s} \times 0.027\text{m}}{17.35 \times 10^{-6}\,\text{m}^2/\text{s}} = 31,124$$

evaluating thermophysical properties at $T_{f,o} = (T_{s,o} + T_o)/2$ when the surface temperature is determined from the thermal circuit analysis result,

$$(T_{m,i} - T_o)/R_{tot} = (T_{s,o} - T_o)/R_{cv,o}$$

$T_{m,i}$
$R_{cv,i}$
$R_{f,i}$
R_w
$R_{f,o}$
$T_{s,o}$
$R_{cv,o}$
T_o
q'

Assume $T_{f,o}$ = 315 K, and check later. Using the Churchill-Bernstein correlation, Eq. 7.57, find

$$\overline{Nu}_{D,o} = 0.3 + \frac{0.62\, Re_{D,o}^{1/2}\, Pr_o^{1/3}}{\left[1 + (0.4/Pr_o)^{2/3}\right]^{1/4}} \left[1 + \left(\frac{Re_{D,o}}{282,000}\right)^{5/8}\right]^{4/5}$$

$$\overline{Nu}_{D,o} = 0.3 + \frac{0.62(31,124)^{1/2}(0.705)^{1/3}}{\left[1 + (0.4/0.705)^{2/3}\right]^{1/4}} \left[1 + \left(\frac{31,124}{282,000}\right)^{5/8}\right]^{4/5}$$

$$\overline{Nu}_{D,o} = 102.6$$

$$\overline{h}_o = \overline{Nu}_{D,o}\, k_o / D_o = 102.6 \times 0.02737\,\text{W/m}\cdot\text{K} / 0.027\text{m} = 104.0\,\text{W/m}\cdot\text{K}$$

Using the above values for $\overline{h}_i$, and $\overline{h}_o$, and other prescribed values, the thermal resistances and overall coefficient can be evaluated and are tabulated below.

$R_{cv,i}$ (K/W)	$R_{f,i}$ (K/W)	R_w (K/W)	$R_{f,o}$ (K/W)	$R_{cv,o}$ (K/W)	U_o (W/m^2·K)	R_{tot} (K/W)
0.00436	0.00578	0.00216	0.00236	0.1134	92.1	0.128

The major thermal resistance is due to outside (air) convection, accounting for 89% of the total resistance. The other thermal resistances are of similar magnitude, nearly 50 times smaller than $R_{cv,o}$.

(b) For the water-water condition, the method of analysis follows that of part (a). For the internal flow, the estimated convection coefficient is the same as part (a). For an assumed outer film coefficient, $\overline{T}_{f,o}$ = 292 K, the convection correlation for the outer water flow condition V_o = 1 m/s and T_o = 15°C, find

PROBLEM 11.2 (Cont.)

$$Re_{D,o} = 26,260 \qquad Nu_{D,o} = 220.6 \qquad \overline{h}_o = 4914\ W/m^2 \cdot K$$

The thermal resistances and overall coefficient are tabulated below.

$R_{cv,i}$ (K/W)	$R_{f,i}$ (K/W)	R_w (K/W)	$R_{f,o}$ (K/W)	$R_{cv,o}$ (K/W)	R_{tot} (K/W)	U_o ($W/m^2 \cdot K$)
0.00436	0.00579	0.00216	0.00236	0.00240	0.0171	691

Note that the thermal resistances are of similar magnitude. In contrast with the results for the water-air condition of part (a), the thermal resistance of the outside convection process, $R_{cv,o}$, is nearly 50 times smaller. The overall coefficient for the water-water condition is 7.5 times greater than that for the water-air condition.

(c) For the water-air condition, using the IHT workspace with the analysis of part (a), U_o was calculated as a function of the air cross-flow velocity for selected mean water velocities.

Water (i) - air (o) condition

Uo (W/m^2.K)
120
100
80
60
40
5 10 15 20 25 30
Air velocity, Vo (m/s)

Water mean velocity, umi = 0.2 m/s
umi = 0.5 m/s
umi = 1.0 m/s

The effect of increasing the cross-flow air velocity is to increase U_o since the $R_{cv,o}$ is the dominant thermal resistance for the system. While increasing the water mean velocity will increase $\overline{h}_i$, because $R_{cv,i} << R_{cv,o}$, this increase has only a small effect on U_o.

(d) For the water-water condition, using the IHT workplace with the analysis of part (b), U_o was calculated as a function of the mean water velocity for selected air cross-flow velocities.

Water (i) - water (o) condition

Uo (W/m^2.K)
1000
900
800
700
600
0.5 1 1.5 2 2.5
Water mean velocity, umi (m/s)

Air velocity, Vo = 1 m/s
Vo = 3 m/s
Vo = 8 m/s

Because the thermal resistances for the convection processes, $R_{cv,i}$ and $R_{cv,o}$, are of similar magnitude according to the results of part (b), we expect to see U_o significantly increase with increasing water mean velocity and air cross-flow velocity.

PROBLEM 11.3

KNOWN: Copper tube with prescribed inner and outer diameters used in a shell-and-tube heat exchanger. Conditions prescribed for internal water flow and steam condensation on external surface.

FIND: (a) Overall heat transfer coefficient based upon the outer surface area, U_o; compare thermal resistances due to convection, tube wall conduction and condensation, and (b) Compute and plot U_o, water-side convection coefficient, h_i, and steam-side convection coefficient, h_o, as a function of the water flow rate for the range $0.2 \le \dot{m}_i \le 0.8$ kg/s.

SCHEMATIC:

Saturated steam, 1 atm

Tube, Copper, k_w

Water

$T_{m,i}$ = 25 °C

$\dot{m}_i$ = 0.2 kg/s

D_o = 18 mm

D_i = 13 mm

ASSUMPTIONS: (1) Steady-state conditions, (2) Fully developed internal flow.

PROPERTIES: *Table A.1*, Copper, pure (300 K): k_w = 401 W/m·K; *Table A.6*, Water ($T_{m,i}$ = 298 K): $\mu_i = 8.966 \times 10^{-4}$ N·s/m^2, k_i = 0.6102 W/m·K, Pr_i = 6.146. *Table A.6*, Water, (assume $T_{s,o}$ = 351 K, $T_{f,o}$ = 362 K): ρ_ℓ = 965.7 kg/m^3, $c_{p,\ell}$ = 4205 J/kg·K, $\mu_\ell = 3.172 \times 10^{-4}$ N·s/m^2, k_ℓ = 0.6751 W/m·K; *Table A.6* Water (T_{sat} = 373 K, 1 atm): ρ_v = 0.5909 kg/m^3, h_{fg} = 2257 kJ/kg.

ANALYSIS: (a) The overall coefficient, Eq 11.1, based upon the outer surface area can be expressed as the sum of the thermal resistances due to convection (cv), tube wall conduction (w, see Eq. 3.28) and condensation (cnd):

$$1/U_oA_o = R_{tot} = R_{cv} + R_w + R_{cnd}$$

$$R_{cv} = 1/h_i A_i \qquad R_w = \ell n(D_o/D_i)/(2\pi L k_w) \qquad R_{cnd} = 1/h_o A_o$$

The convection coefficients can be estimated from appropriate correlations.

Estimating h_i: For internal flow, characterize the flow using thermophysical properties evaluated at $T_{m,i}$ with

$$Re_{D,i} = \frac{4\dot{m}_i}{\pi D_i \mu_i} = \frac{4 \times 0.2\,\text{kg/s}}{\pi \times 0.013\text{m} \times 8.966 \times 10^{-4}\ \text{N/s}\cdot\text{m}^2} = 21{,}847$$

For turbulent flow, use the Dittus-Boelter correlation, Eq. 8.60,

$$Nu_{D,i} = 0.023 Re_{D,i}^{0.8} Pr_i^{0.4} = 0.023(21{,}847)^{0.8}(6.146)^{0.4} = 140.8$$

$$h_i = Nu_{D,i}\, k_i / D_i = 140.8 \times 0.6102\,\text{W/m}\cdot\text{K}/0.013\text{m} = 6610\,\text{W/m}^2\cdot\text{K}$$

Continued...

PROBLEM 11.3 (Cont.)

Estimating h_o : For the horizontal tube, average convection coefficient for film condensation, Eq. 10.40, is

$$h_o = 0.729\left[\frac{g\rho_\ell(\rho_\ell - \rho_v)k_\ell^3 h'_{fg}}{\mu_\ell(T_{sat} - T_{s,o})D_o}\right]^{1/4}$$

$$h'_{fg} = h_{fg} + 0.68c_{p,\ell}(T_{sat} - T_{s,o})$$

q'

T_{sat} — R_{cnd} — $T_{s,o}$ — R_w — R_{cv} — $T_{m,i}$

The vapor (v) properties and h_{fg} are evaluated at T_{sat} , while the liquid properties (ℓ) are evaluated at the film temperature $T_{f,o} = (T_{s,o} - T_{sat})$ where the surface temperature is determined from the thermal circuit analysis result,

$$(T_{m,i} - T_{sat})/R_{tot} = (T_{s,o} - T_{sat})/R_{cnd}$$

Assume $T_{s,o}$ = 351 K so that $T_{f,o}$ = 362 K, and check later. Hence,

$$h_o = 0.729\left[\frac{9.8\,m/s^2 \times 965.7\,kg/m^3 \times (965.7 - 0.5909)\,kg/m^3 \times (0.6751\,W/m\cdot K)^3 \times 2321\,kJ/kg}{3.172\times10^{-4}\,N\cdot s/m^2\,(373-351)K \times 0.018\,m}\right]^{1/4}$$

$$h_o = 11,005\ W/m^2\cdot K$$

Using the above values for h_i, h_o and other prescribed values, the thermal resistances and overall coefficient can be evaluated and are tabulated below.

$R_{cv}\times10^3$ (K/W)	$R_w\times10^3$ (K/W)	$R_{cnd}\times10^3$ (K/W)	$R_{tot}\times10^3$ (K/W)	U_o ($W/m^2\cdot K$)
3.704	1.292	1.610	5.444	3249

The largest resistance is that due to convection on the water-side. Interestingly, the wall thermal resistance for the pure copper, while the smallest for all the process, is still significant relative to that for the condensation process.

(b) The foregoing relations were entered into the IHT workspace along with the *Correlations Tools* for *Forced Convection, Internal Flow, Turbulent Flow* and for *Film Condensation, Horizontal Cylinder* with the appropriate *Properties Tools* for *Air* and *Water.* The coefficients U_o, h_i and h_o were computed and plotted as a function of the water flow rate.

Note that the overall coefficient increases nearly 50% over the range of the water flow rate. The water-side coefficient increases markedly, by nearly a factor of 4, with increasing flow rate. The steam-side coefficient, h_o , is larger than h_i by a factor of 2 at the lowest flow rate. However, h_o decreases with increasing water flow rate since the tube wall temperature, $T_{s,o}$, decreases causing the water film thickness to increase with the net effect of reducing h_o.

25000
20000
15000
10000
5000
0
hi, ho, Uo (W/m^2.K)
0.2 0.3 0.4 0.5 0.6 0.7 0.8
Water flow rate, mdoti (kg/s)
Water-side, hi (W/m^2.K)
Steam-side, ho (W/m^2.K)
Overall coefficient, Uo (W/m^2.K)

PROBLEM 11.4

KNOWN: Dimensions of heat exchanger tube with or without fins. Cold and hot side convection coefficients.

FIND: Cold side overall heat transfer coefficient without and with fins.

SCHEMATIC:

ASSUMPTIONS: (1) Negligible fouling, (2) Negligible contact resistance between fins and tube wall, (3) h_h is not affected by fins, (4) One-dimensional conduction in fins, (5) Adiabatic fin tip.

ANALYSIS: From Eq. 11.1,

$$\frac{1}{U_c} = \frac{1}{(h_o h)_c} + \frac{D_i \ln(D_o / D_i)}{2k} + \frac{A_c}{(h_o hA)_h}$$

Without fins: $h_{o,c} = h_{o,h} = 1$

$$\frac{1}{U_c} = \frac{1}{8000\ W/m^2 \cdot K} + \frac{(0.02m)\ln(26/20)}{100\ W/m \cdot K} + \frac{1}{200\ W/m^2 \cdot K}\frac{20}{26}$$

$$1/U_c = \left(1.25\times10^{-4} + 5.25\times10^{-5} + 3.85\times10^{-3}\right) m^2 \cdot K/W = 4.02\times10^{-3}\ m^2 \cdot K/W$$

$$U_c = 249\ W/m^2 \cdot K.$$ <

With fins: $h_{o,c} = 1,\ h_{o,h} = 1-(A_f/A)(1-h_f)$ Per unit length along the tube axis,

$$A_f = N(2L_f + t) = 16(30+2)mm = 512\ mm$$

$$A_h = A_f + (p\, D_o - 16t) = (512 + 81.7 - 32)mm = 561.7\ mm$$

With $m = (2h/kt)^{1/2} = \left(400\ W/m^2 \cdot K / 50\ W/m \cdot K \times 0.002m\right)^{1/2} = 63.3m^{-1}$

$$mL_f = \left(63.3m^{-1}\right)(0.015m) = 0.95$$

and Eq. 11.4 yields

$$h_f = \tanh(mL_f)/mL_f = 0.739/0.95 = 0.778.$$

The overall surface efficiency is then

$$h_o = 1-(A_f/A_h)(1-h_f) = 1-(512/561.7)(1-0.778) = 0.798.$$

Hence

$$\frac{1}{U_c} = \left(1.25\times10^{-4} + 5.25\times10^{-5} + \frac{p(20)}{0.798(200)561.7}\right) m^2 \cdot K/W = 8.78\times10^{-4} m^2 \cdot K/W$$

$$U_c = 1138\ W/m^2 \cdot K.$$ <

PROBLEM 11.5

KNOWN: Geometry of finned, annular heat exchanger. Gas-side temperature and convection coefficient. Water-side flowrate and temperature.

FIND: Heat rate per unit length.

SCHEMATIC:

$h = 100\,W/m^2\cdot K$
Gas, $T_{m,h} = 800K$
Water, $\dot{m}_c = 0.161\,kg/s$, $T_{m,c} = 300K$
$k = 50\,W/m\cdot K$

$D_o = 60$ mm
$D_{i,1} = 24$ mm
$D_{i,2} = 30$ mm
t = 3 mm = 0.003m
L = (60-30)/2 mm = 0.015m

ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) One-dimensional conduction in strut, (4) Adiabatic outer surface conditions, (5) Negligible gas-side radiation, (6) Fully-developed internal flow, (7) Negligible fouling.

PROPERTIES: *Table A-6*, Water (300 K): k = 0.613 W/m·K, Pr = 5.83, $\mu = 855 \times 10^{-6}$ $N\cdot s/m^2$.

ANALYSIS: The heat rate is

$$q = (UA)_c\left(T_{m,h} - T_{m,c}\right)$$

where

$$1/(UA)_c = 1/(hA)_c + R_w + 1/(\eta_o hA)_h$$

$$R_w = \frac{\ln\left(D_{i,2}/D_{i,1}\right)}{2\pi kL} = \frac{\ln(30/24)}{2\pi\,(50\ W/m\ K)1m} = 7.10\times10^{-4}\,K/W.$$

With

$$Re_D = \frac{4\dot{m}}{\pi D_{i,1}\mu} = \frac{4\times0.161\,kg/s}{\pi\,(0.024m)855\times10^{-6}\,N\cdot s/m^2} = 9990$$

internal flow is turbulent and the Dittus-Boelter correlation gives

$$h_c = \left(k/D_{i,1}\right)0.023Re_D^{4/5}\,Pr^{0.4} = \left(\frac{0.613\,W/m\cdot K}{0.024m}\right)0.023(9990)^{4/5}(5.83)^{0.4} = 1883\ W/m^2\cdot K$$

$$(hA)_c^{-1} = \left(1883\ W/m^2\cdot K\times\pi\times0.024m\right)^{-1} = 7.043\times10^{-3}\ K/W.$$

Find the fin efficiency as

$$\eta_o = 1 - \left(A_f/A\right)\left(1-\eta_f\right)$$

$$A_f = 8\times2\,(L\cdot w) = 8\times2(0.015m\times1m) = 0.24m^2$$

$$A = A_f + \left(\pi D_{i,2} - 8t\right)w = 0.24m^2 + (\pi\times0.03m - 8\times0.003m) = 0.31m^2.$$

PROBLEM 11.5 (Cont.)

From Eq. 11.4,

$$h_f = \frac{\tanh(mL)}{mL}$$

where

$$m = [2h/kt]^{1/2} = \left[2 \times 100\ W/m^2 \cdot K / 50\ W/m \cdot K\,(0.003m)^{1/2}\right] = 36.5m^{-1}$$

$$mL = (2h/kt)^{1/2} L = 36.5m^{-1} \times 0.015m = 0.55$$

$$\tanh\left[(2h/kt)^{1/2} L\right] = 0.499.$$

Hence

$$h_f = 0.800/1.10 = 0.907$$

$$h_o = 1 - (A_f / A)(1 - h_f) = 1 - (0.24/0.31)(1 - 0.907) = 0.928$$

$$(h_o hA)_h^{-1} = \left(0.928 \times 100\ W/m^2 \cdot K \times 0.31m^2\right)^{-1} = 0.0347\ K/W.$$

Hence

$$(UA)_c^{-1} = \left(7.043 \times 10^{-3} + 7.1 \times 10^{-4} + 0.0347\right) K/W$$

$$(UA)_c = 23.6\ W/K$$

and

$$q = 23.6\ W/K\,(800 - 300)\,K = 11{,}800\ W \qquad <$$

for a 1m long section.

COMMENTS: (1) The gas-side resistance is substantially decreased by using the fins ($A_f >> pD_{i,2}$) and q is increased.

(2) Heat transfer enhancement by the fins could be increased further by using a material of larger k, but material selection would be limited by the large $T_{m,h}$.

PROBLEM 11.6

KNOWN: Condenser arrangement of tube with six longitudinal fins (k = 200 W/m·K). Condensing refrigerant temperature at 45°C flows axially through inner tube while water flows at 0.012 kg/s and 15°C through the six channels formed by the splines.

FIND: Heat removal rate per unit length of the exchanger.

SCHEMATIC:

$D_1 = 10$ mm
$D_2 = 14$ mm
$D_3 = 50$ mm
$t = 2$ mm

ASSUMPTIONS: (1) No heat loss/gain to the surroundings, (2) Negligible kinetic and potential energy changes, (3) Negligible thermal resistance on condensing refrigerant side, $h_i \to \infty$, (4) Water flow is fully developed, (5) Negligible thermal contact between splines and inner tube, (6) Heat transfer from outer tube negligible.

PROPERTIES: *Table A-6*, Water ($\overline{T}_c$ = 15°C = 288 K): ρ = 1000 kg/m^3, k = 0.595 W/m·K, $\nu = \mu/\rho$ = 1138 × 10^{-6} N·s/m^2/1000 kg/m^3 = 1.138 × 10^{-6} m^2/s, Pr = 8.06; Tube fins (given): k = 200 W/m·K.

ANALYSIS: Following the discussion of Section 11.2,

$$q' = UA'(T_h - T_c)$$

$$\frac{1}{UA'} = R'_h + R'_w + R'_c = R'_w + \frac{1}{(\eta_o hA')_c}$$

where $R'_h = 0$, due to the negligible thermal resistance on the refrigerant side (h), and

$$R'_w = \frac{\ln(D_2/D_1)}{2\pi k} = \frac{\ln(14/10)}{2\pi(200\ \text{W/m}\cdot\text{K})} = 2.678\times10^{-4}\,\text{m}\cdot\text{K/W}.$$

To estimate the thermal resistance on the water side (c), first evaluate the convection coefficient. The hydraulic diameter for a passage, where A_c is the cross-sectional area of the passage is

$$D_{h,c} = \frac{4A_c}{P} = \frac{4\left[\pi\left(D_3^2 - D_2^2\right)/4 - 6(D_3 - D_2)t/2\right]/6}{(\pi D_2 - 6t)/6 + (\pi D_3 - 6t)/6 + 2(D_3 - D_2)/2}$$

$$D_{h,c} = \frac{4\left[\pi\left(50^2 - 14^2\right)/4 - 6(50-14)\right]\times10^{-6}\,\text{m}^2/6}{\left[(14\pi - 6\times2)/6 + (50\pi - 6\times2)/6 + (50-14)\right]\times10^{-3}\,\text{m}}$$

$$D_{h,c} = \frac{4\times2.656\times10^{-4}\,\text{m}^2}{6.551\times10^{-2}\,\text{m}} = 0.01622\,\text{m}.$$

Hence the Reynolds number is

Continued …..

PROBLEM 11.6 (Cont.)

$$Re_{D,c} = \frac{\left[(0.012\ \text{kg/s}/6)/\left(1000\ \text{kg/m}^3 \times 2.656\times10^{-4}\text{m}^2\right)\right]\times 0.01622\text{m}}{1.138\times10^{-6}\text{m}^2/\text{s}} = 107$$

and assuming the flow is fully developed,

$$Nu_{D,c} = \frac{h_c D_{h,c}}{k} = 3.66$$

$$h_c = 3.66\times0.595\ \text{W/m}\cdot\text{K}/0.01622 = 134\ \text{W/m}^2\cdot\text{K}.$$

The temperature effectiveness of the splines (fins) on the cold side is

$$\eta_o = 1 - \frac{A_{f,c}}{A_c}(1-\eta_f)$$

where $A_{f,c}$ and A_c are, respectively, the finned and total (fin plus prime) surface areas, while

$$\eta_f = \frac{\tanh(mL)}{mL}$$

$$m = (2h_c/kt)^{1/2} = \left[\left(2\times134\ \text{W/m}^2\cdot\text{K}\right)/\left(200\ \text{W/m}\cdot\text{K}\times0.002\text{m}\right)\right]^{1/2} = 25.88\,\text{m}^{-1}$$

$$\eta_f = \frac{\tanh\left(25.88\,\text{m}^{-1}\times0.018\text{m}\right)}{25.88\,\text{m}^{-1}\times0.018\text{m}} = \frac{0.4348}{0.4658} = 0.934.$$

Hence

$$\eta_o = 1 - \frac{6(D_3-D_2)}{6(D_3-D_2)+(\pi D_2 - 6t)}[1-\eta_f]$$

$$\eta_o = 1 - \frac{6(50-14)}{6(50-14)+(14\pi-6\times2)}(1-0.934) = 0.943$$

$$\frac{1}{\eta_o h A_c'} = \frac{1}{0.943\times134\ \text{W/m}^2\cdot\text{K}\left[6(50-14)+(14\pi-6\times2)\right]\times10^{-3}\text{m}} = 3.22\times10^{-2}\ \text{m}\cdot\text{K/W}$$

and the heat rate is

$$q' = \frac{T_h - T_c}{R_w' + 1/(\eta_o h A')_c}$$

$$q' = \frac{(45-15)\text{K}}{2.678\times10^{-4}\text{m}\cdot\text{K/W} + 3.22\times10^{-2}\text{m}\cdot\text{K/W}} = 924\ \text{W/m}.$$

$<$

COMMENTS: (1) The effective length of the fin representing the splines was conservatively estimated. The heat transfer by conduction through the splines to the outer tube and then by convection to the water was ignored.

(2) Without the splines, find $D_h = (D_3 - D_2) = 36$ mm so that $h_c = 60.5$ W/m$^2\cdot$K. The heat rate with $A_c' = \pi D_2$ is

$$q' = (hA_c')(T_h - T_c) = 60.5\ \text{W/m}^2\cdot\text{K}(0.014\pi\ \text{m})(45-15)\text{K} = 79\ \text{W/m}.$$

The splines enhance the heat transfer rate by a factor of 924/79 = 11.7.

PROBLEM 11.7

KNOWN: Number, inner-and outer diameters, and thermal conductivity of condenser tubes. Convection coefficient at outer surface. Overall flow rate, inlet temperature and properties of water flow through the tubes. Flow rate and pressure of condensing steam. Fouling factor for inner surface.

FIND: (a) Overall coefficient based on outer surface area, U_o, without fouling, (b) Overall coefficient with fouling, (c) Temperature of water leaving the condenser.

SCHEMATIC:

ASSUMPTIONS: (1) Negligible flow work and kinetic and potential energy changes for water flow, (2) Fully-developed flow in tubes, (3) Negligible effect of fouling on D_i.

PROPERTIES: Water (Given): c_p = 4180 J/kg·K, $\mu = 9.6 \times 10^{-4}$ N·s/m^2, k = 0.60 W/m·K, Pr = 6.6. *Table A-6*, Water, saturated vapor (p = 0.0622 bars): T_{sat} = 310 K, $h_{fg} = 2.414 \times 10^6$ J/kg.

ANALYSIS: (a) Without fouling, Eq. 11.5 yields

$$\frac{1}{U_o} = \frac{1}{h_i}\left(\frac{D_o}{D_i}\right) + \frac{D_o \ln(Do/Di)}{2k_t} + \frac{1}{h_o}$$

With $Re_{D_i} = 4\dot{m}_1/\pi D_i \mu = 1.60\,kg/s/\left(\pi \times 0.025m \times 9.6 \times 10^{-4}\,N \cdot s/m^2\right) = 21,220$, flow in the tubes is turbulent, and from Eq. 8.60

$$h_i = \left(\frac{k}{D_i}\right) 0.023 Re_{D_i}^{4/5} Pr^{0.4} = \left(\frac{0.60\,W/m \cdot K}{0.025m}\right) 0.023(21,200)^{4/5}(6.6)^{0.4} = 3400\,W/m^2 \cdot K$$

$$U_o = \left[\frac{1}{3400}\left(\frac{28}{25}\right) + \frac{0.028\ln(28/25)}{2 \times 110} + \frac{1}{10,000}\right]^{-1} W/m^2 \cdot K =$$

$$\left(3.29 \times 10^{-4} + 1.44 \times 10^{-5} + 10^{-4}\right)^{-1} W/m^2 \cdot K = 2255\,W/m^2 \cdot K \qquad <$$

(b) With fouling, Eq. 11.5 yields

$$U_o = \left[4.43 \times 10^{-4} + (Do/Di)R''_{f,i}\right]^{-1} = \left(5.55 \times 10^{-4}\right)^{-1} = 1800\,W/m^2 \cdot K \qquad <$$

(c) The rate at which energy is extracted from the steam equals the rate of heat transfer to the water, $\dot{m}_h h_{fg} = \dot{m}_c c_p \left(T_{m,o} - T_{m,i}\right)$, in which case

$$T_{m,o} = T_{m,i} + \frac{\dot{m}_h h_{fg}}{\dot{m}_c c_p} = 15°C + \frac{10\,kg/s \times 2.414 \times 10^6\,J/kg}{400\,kg/s \times 4180\,J/kg \cdot K} = 29.4°C \qquad <$$

COMMENTS: (1) The largest contribution to the thermal resistance is due to convection at the interior of the tube. To increase U_o, hi could be increased by increasing $\dot{m}_1$, either by increasing $\dot{m}_c$ or decreasing N. (2) Note that $T_{m,o}$ = 302.4 K < T_{sat} = 310 K, as must be the case.

PROBLEM 11.8

KNOWN: Diameter and inner and outer convection coefficients of a condenser tube. Thickness, outer diameter, and pitch of aluminum fins.

FIND: (a) Overall heat transfer coefficient without fins, (b) Effect of fin thickness and pitch on overall heat transfer coefficient with fins.

SCHEMATIC:

ASSUMPTIONS: (1) Negligible tube wall conduction resistance, (2) Negligible fouling and fin contact resistance, (3) One-dimensional conduction in fin.

PROPERTIES: *Table A.1*, Aluminum (T = 300 K): k = 237 W/m·K.

ANALYSIS: (a) With no fins, Eq. 11.1 yields

$$U = \left(h_i^{-1} + h_o^{-1}\right)^{-1} = \left(2\times10^{-4} + 0.01\right)^{-1} \text{W/m}^2\cdot\text{K} = 98.0\,\text{W/m}^2\cdot\text{K}$$ <

(b) With fins and a unit tube length, Eqs. 11.1 and 11.3 yield

$$\frac{1}{U_i \pi D_i} = \frac{1}{h_i \pi D_i} + \frac{1}{\eta_o h_o A'_o}$$

and $\eta_o = 1 - \left(A'_f / A'_o\right)\left(1 - \eta_f\right)$. The total fin surface area per unit length is $A'_f = N' 2\pi \left(r_{oc}^2 - r_i^2\right)$, where the number of fins per unit length is $N' = 1\text{m}/S(\text{m})$. The total outside surface area per unit length is $A'_o = A'_f + (1 - N't)\pi D_i$, and the fin efficiency is given by Eq. 3.91 or Fig. 3.19.

For t = 0.0015 m and S = 0.0035 m, $r_{oc} = (D_o/2) + (t/2) = 0.01075$ m, $N' \approx 286$, $A'_f = 0.163\ \text{m}^2/\text{m}$, and $A'_o = (0.163 + 0.018)\ \text{m}^2/\text{m} = 0.181\ \text{m}^2/\text{m}$. With $r_{oc}/r_i = 2.15$, $L_c = 0.00575$ m, $A_p = 8.625 \times 10^{-6}\ \text{m}^2$, and $L_c^{3/2}\left(h_o/kA_p\right)^{1/2} = 0.0964$, Fig. 3.19 yields $\eta_f \approx 0.99$. Hence, $\eta_o \approx 1 - (0.163/0.181)(0.01) = 0.99$ and

$$U_i = \left[(1/h_i) + \left(\pi D_i / \eta_o h_o A'_o\right)\right]^{-1}$$

$$U_i = \left[2\times10^{-4}\,\text{m}^2\cdot\text{K/W} + \pi\times0.01\,\text{m} / 0.99\times100\,\text{W/m}^2\cdot\text{K}\times0.181\,\text{m}^2/\text{m}\right]^{-1} = 512\,\text{W/m}^2\cdot\text{K}$$ <

We may use the IHT *Extended Surface* Model (*Performance Calculations* for a *Circular Rectangular Fin Array*) to consider the effect of varying t and S. To maximize N', the minimum allowable value of

Continued...

PROBLEM 11.8 (Cont.)

S - t = 1.5 mm should be selected. It is then a matter of choosing between a large number of thin fins or a smaller number of thicker fins. Calculations were performed for the following options.

t (mm)	S (mm)	N'	U_i ($W/m^2 \cdot K$)
1	2.5	400	640
2	3.5	286	512
3	4.5	222	460
4	5.5	182	420

Since heat transfer increases with U_i, the best configuration corresponds to t = 1 mm and S = 2.5 mm, which provides the largest airside surface area.

COMMENTS: The best performance is always associated with a large number of closely spaced fins, so long as the flow between adjoining fins is sufficient to maintain the convection coefficient.

PROBLEM 11.9

KNOWN: Operating conditions and surface area of a finned-tube, cross-flow exchanger.

FIND: Overall heat transfer coefficient.

SCHEMATIC:

ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible kinetic and potential energy changes, (3) Constant properties, (4) Exhaust gas properties are those of air.

PROPERTIES: *Table A-6*, Water $\left(\overline{T}_m = 87°C\right)$: $\bar{c}_p = 4203\,J/kg\cdot K$; *Table A-4*, Air $\left(T_m \approx 275°C\right)$: $\bar{c}_p = 1040\,J/kg\cdot K$.

ANALYSIS: From the energy balance equations

$$q = \dot{m}_c c_{p,c}\left(T_{c,o} - T_{c,i}\right) = 0.5\,kg/s \times 4203\,J/kg\cdot K\,(150-25)\,°C = 2.63\times10^5\,W$$

$$T_{h,o} = T_{h,i} - \frac{q}{\dot{m}_h c_{p,h}} = 325°C - \frac{2.63\times10^5\,W}{2\,kg/s\times1040\,J/kg\cdot K} = 198.6°C.$$

Hence

$$U = q/A\Delta T_{\ell m} \qquad \text{where} \qquad \Delta T_{\ell m} = F\Delta T_{\ell m,CF}.$$

From Fig. 11.12, with

$$P = \frac{t_o - t_i}{T_i - t_i} = \frac{150-25}{325-25} = 0.42,\quad R = \frac{T_i - T_o}{t_o - t_i} = \frac{325-198.6}{150-25} = 1.01,\quad F = 0.94$$

$$\Delta T_{\ell m,CF} = \frac{(325-150)-(198.6-25)}{\ell n\dfrac{325-150}{198.6-25}} = 174.3°C.$$

Hence

$$U = \frac{q}{AF\Delta T_{\ell m,CF}} = \frac{2.63\times10^5\,W}{10\,m^2\times0.94\times174.3°C} = 160\,W/m^2\cdot K.$$ <

COMMENTS: From the *e* - NTU method, $C_c = 2102$ W/K, $C_h = 2080$ W/K, $(C_{min}/C_{max}) \approx 1$, $q_{max} = 6.24\times10^5$ W and $e = 0.42$. Hence, from Fig. 11.18, NTU ≈ 0.75 and $U \approx 156\,W/m^2\cdot K$.

PROBLEM 11.10

KNOWN: Heat exchanger with two shell passes and eight tube passes having an area $925 m^2$; 45,500 kg/h water is heated from 80°C to 150°C; hot exhaust gases enter at 350°C and exit at 175°C.

FIND: Overall heat transfer coefficient.

SCHEMATIC:

ASSUMPTIONS: (1) Negligible losses to surroundings, (2) Negligible kinetic and potential energy changes, (3) Constant properties, (4) Exhaust gas properties are approximated as those of atmospheric air.

PROPERTIES: *Table A-6*, Water $\left(\overline{T}_c = (80+150)°C/2 = 388K\right)$: $c = c_{p,f} = 4236$ J/kg·K.

ANALYSIS: The overall heat transfer coefficient follows from Eqs. 11.9 and 11.18 written in the form

$$U = q / A F \Delta T_{\ell m,CF}$$

where F is the correction factor for the HXer configuration, Fig. 11.11, and $\Delta T_{\ell m,CF}$ is the log mean temperature difference (CF), Eqs. 11.15 and 11.16. From Fig. 11.11, find

$$R = \frac{T_{h,i} - T_{h,o}}{T_{c,o} - T_{c,i}} = \frac{(350-175)°C}{(150-80)°C} = 2.5 \quad P = \frac{T_{c,o} - T_{c,i}}{T_{h,i} - T_{c,i}} = \frac{(150-80)°C}{(350-80)°C} = 0.26$$

find $F \approx 0.97$. The log-mean temperature difference, Eqs. 11.15 and 11.17, is

$$\Delta T_{\ell m,CF} = \frac{\Delta T_1 - \Delta T_2}{\ell n(\Delta T_1 / \Delta T_2)} = \frac{(350-150)°C - (175-80)°C}{\ell n\left[(350-150)/(175-80)\right]} = 141.1°C.$$

From an overall energy balance on the cold fluid (water), the heat rate is

$$q = \dot{m}_c c_c \left(T_{c,o} - T_{c,i}\right)$$

$$q = 45{,}500 kg/h \times 1h/3600s \times 4236 J/kg \cdot K (150-80)°C = 3.748 \times 10^6 W.$$

Substituting values with $A = 925 m^2$, find

$$U = 3.748 \times 10^6 W / 925 m^2 \times 0.97 \times 141.1 K = 29.6 W/m^2 \cdot K.$$ <

COMMENTS: Compare the above result with representative values for air-water exchangers, as given in Table 11.2. Note that in this exchanger, two shells with eight tube passes, the correction factor effect is very small, since F = 0.97.

PROBLEM 11.11

KNOWN: Dimensions and thermal conductivity of tubes with or without annular fins. Convection coefficients associated with condensation and natural convection at the inner and outer surfaces, respectively.

FIND: (a) Overall heat transfer coefficient U_i for aluminum and copper tubes without fins, (b) Value of U_i associated with adding aluminum fins.

SCHEMATIC:

Aluminum fins
k = 180 W/m-K
N' = 300
t = 1 mm
$h_i = 5000\ W/m^2\text{-}K$
$D_f = 25$ mm
Tube
Aluminum or copper
k = 180 or 400 W/m-K
$D_o = 13$ mm
$D_i = 11$ mm
$h_o = 10\ W/m^2\text{-}K$

ASSUMPTIONS: (1) Negligible fouling and fin contact resistances, (2) One-dimensional conduction in fins.

ANALYSIS: (a) For unfinned, *aluminum* tubes of unit length, Eq. 11.5 yields

$$\frac{1}{U_i} = \frac{1}{h_i} + \frac{D_i \ln(Do/Di)}{2k} + \frac{1}{h_o}\left(\frac{D_i}{D_o}\right)$$

$$U_i = \left[\frac{1}{5000} + \frac{0.011 \ln(13/11)}{2 \times 180} + \frac{1}{10}\left(\frac{11}{13}\right)\right]^{-1} = \left(2 \times 10^{-4} + 5.1 \times 10^{-6} + 846 \times 10^{-4}\right)^{-1} = 11.8\ W/m^2 \cdot K \quad <$$

For *copper* the tube conduction resistance is reduced from $5.1 \times 10^{-6}\ m^2 \cdot K/W$ to 2.3×10^{-6}, but U_i is essentially unchanged.

$$U_i = 11.8\ W/m^2 \cdot K \quad <$$

(b) With fins and a unit tube length, Eqs. 11.1 and 11.3 yield

$$\frac{1}{U_i} = \frac{1}{h_i} + \frac{D_i \ln(Do/Di)}{2k} + \frac{\pi D_i}{\eta_o h_o A'_o}$$

and $\eta_o = 1 - (A'_f / A'_o)(1 - \eta_f)$. The fin surface area is $A'_f = N' 2\pi \left(r_{fc}^2 - r_o^2\right)$ and the total outer surface area is $A'_o = A'_f + (1 - N't)\pi D_o$. With t = 0.001m, $r_{fc} = r_f + t/2 = (0.0125 + 0.0005)m = 0.0130m$ and

$A'_f = 300\ m^{-1}(2\pi)\left(0.0130^2 - 0.0065^2\right)m^2 = 0.239m$ and $A'_o = 0.239m + (1 - 0.300)\pi(0.013m)$

$=0.268m$. With $r_{2c} = r_f + t/2 = 0.013m$, $L_c = (r_f - r_o) + t/2 = 0.0065m$, $r_{2c}/r_o = 2$, $A_p = L_c t = 3.25 \times 10^{-6}\ m^2$, and $L_c^{3/2}\left(h_o / k A_p\right)^{1/2} = 0.0685$, Fig. 3.19 yields $\eta_f \approx 0.97$. Hence, $\eta_o = 1 - (0.239/0.268)(0.03) \approx 0.973$, and

$$U_i = \left[\frac{1}{5000} + \frac{0.011 \ln(13/11)}{360} + \frac{\pi \times 0.011}{0.973 \times 10 \times 0.244}\right]^{-1}$$

$$= \left(2 \times 10^{-4} + 5.1 \times 10^{-6} + 145 \times 10^{-4}\right)^{-1} = 68.0\ W/m^2 \cdot K \quad <$$

COMMENTS: There is significant advantage to installing fins on the outer surface, which has a much smaller convection coefficient. The thermal resistance at the outer surface has been reduced from 0.0846 to 0.0145 $m^2 \cdot K/W$ and could be reduced further by increasing D_f and/or N'. However, the spacing between adjoining fins must not be so small as to restrict buoyancy driven flow in the associated air space.

PROBLEM 11.12

KNOWN: Properties and flow rates for the hot and cold fluid to a heat exchanger.

FIND: Which fluid limits the heat transfer rate of the exchanger?

ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, and (3) Negligible losses to the surroundings and kinetic and potential energy changes.

ANALYSIS: The properties and flow rates for the hot and cold fluid to the heat exchanger are tabulated below.

	Cold fluid	*Hot fluid*
Density, kg/m^3	997	1247
Specific heat, J/kg·K	4179	2564
Thermal conductivity, W/m·K	0.613	0.287
Viscosity, $N \cdot s/m^2$	8.55×10^{-4}	1.68×10^{-4}
Flow rate, m^3/h	14	16

The fluid which limits the heat transfer rate of the exchanger is the minimum fluid, $C_{min} = \left(\dot{m} \cdot c\right)_{min}$. For the hot and cold fluids, find

$$C_h = \dot{m}_h\, c_h = 16\ m^3/h \times 1247\ kg/m^3 \times 2564\ J/kg \cdot K \times \left(1h/3600s\right) = 14.21\ kW/K$$

$$C_c = \dot{m}_c\, c_c = 14\ m^3/h \times 997\ kg/m^3 \times 4179\ J/kg \cdot K \times \left(1h/3600s\right) = 16.20\ kW/K$$

Hence, the hot fluid is the minimum fluid,

$$C_{min} = C_h$$ <

For any exchanger, the heat rate is $q = \varepsilon\, q_{max}$, where ε depends upon the exchanger type. The maximum heat rate is $q_{max} = C_{min}\,(T_{h,i} - T_{c,i})$. Hence, it is the conditions for the minimum fluid that limit the performance of the exchanger.

PROBLEM 11.13

KNOWN: Process (hot) fluid having a specific heat of 3500 J/kg·K and flowing at 2 kg/s is to be cooled from 80°C to 50°C with chilled-water (cold fluid) supplied at 2.5 ks/g and 15°C assuming an overall heat transfer coefficient of 2000 $W/m^2 \cdot K$.

FIND: The required heat transfer areas for the following heat exchanger configurations; (a) Concentric tube (CT) - parallel flow, (b) CT - counterflow, (c) Shell and tube, one-shell pass and 2 tube passes; (d) Cross flow, single pass, both fluids unmixed. Use the *IHT Tools | Heat Exchanger* models as your solution tool.

SCHEMATIC:

(a) Parallel flow hxer

(b) Counterflow hxer

ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible losses to the surroundings and kinetic and potential energy changes, (3) Overall heat transfer coefficient remains constant with different configurations, and (4) Constant properties.

ANALYSIS: The *IHT Tools | Heat Exchanger* models are based upon the effectiveness-NTU method and suited for design-type problems. The table below summarizes the results of our analysis using the IHT models including model equations, figures, and the required heat transfer area. The cold fluid outlet temperature for all configurations is $T_{c,o}$ = 35.1°C. The IHT code for the concentric tube, parallel flow heat exchanger is provided in the Comments.

Heat exchanger type	*Eqs.*	*Figs*	*A(m^2)*
(a) CT -Parallel flow	11.29b	11.14	3.09
(b) CT -Counterflow	11.30b	11.15	2.64
(c) Shell and tube (1 - sp, 2 - tp)	11.31b	11.10, 16	2.83
(d) Crossflow (1 - p, unmixed)	11.33	11.12, 18	2.84

COMMENTS: (1) Referring to the tabulated results, note that for the concentric tube exchangers, the area required for parallel flow is 17% larger than for counterflow. Under what circumstances would you choose to use the PF arrangement if the area has to be significantly larger?

(2) The shell-tube and crossflow exchangers require nearly the same heat transfer area. What are other factors that might influence your decision to select one type over the other for an application?

(3) Based upon area considerations only, the CF arrangement requires the smallest heat transfer area. What practical issues need to be considered in making a CF heat exchanger with a 2.6 m^2 area?

Continued …..

PROBLEM 11.13 (Cont.)

(4) The *IHT* code used for the concentric tube, parallel flow heat exchanger is shown below. Note the use of the water property function, *cp_Tx*, and the intrinsic function, *Tfluidavg*, to provide the specific heat at the mean water (cold fluid) temperature.

```
/" Results - energy balance only
Cc          Ch          Tco     cc      q       Tci     Thi     Tho     ch
1.045E4     7000        35.1    4180    2.1E5   15      80      50      3500*/

/" Results of sizing
A       CR      NTU     eps
3.87    0.6699  0.882   0.4615  */

// Design conditions
Thi = 80
Tho = 50
mdoth = 2
ch = 3500
mdotc = 2.5
Tci = 15
U = 2000

// For the parallel-flow, concentric-tube heat exchanger,
// For the parallel-flow, concentric-tube heat exchanger,
NTU = -ln(1 - eps * (1 + Cr))/(1 + Cr)          // Eq 11.29b
// where the heat-capacity ratio is
Cr = Cmin/Cmax
// and the number of transfer units, NTU, is
NTU = U * A/Cmin                                // Eq 11.25
// The effectiveness is defined as
eps = q/qmax
qmax = Cmin * (Thi - Tci)                       // Eq 11.20
// See Tables 11.3 and 11.4 and Fig 11.14

// Energy balances
q = Cc * (Tco - Tci)
q = Ch * (Thi - Tho)
Cc = mdotc * cc
Ch = mdoth * ch
Cmin = Ch
Cmax = Cc

// Water property functions: T dependence, From Table A.6
// Units: T(K), p(bars):
xc = 0                                          // Quality (0=sat liquid or 1-sat vapor)
cc = cp_Tx("Water", Tcm,xc)                     // Specific heat, J/kg·K
Tcm = Tfluid_avg(Tci, Tco)                      // Mean temperature; K; intrinsic function
```

PROBLEM 11.14

KNOWN: A shell and tube Hxer (two shells, four tube passes) heats 10,000 kg/h of pressurized water from 35°C to 120°C with 5,000 kg/h water entering at 300°C.

FIND: Required heat transfer area, A_s.

SCHEMATIC:

ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible kinetic and potential energy changes, (3) Constant properties.

PROPERTIES: *Table A-6,* Water $(\overline{T}_c = 350\text{K})$: c_p = 4195 J/kg·K; *Table A-6,* Water (Assume $T_{h,o} \approx 150°C$, $\overline{T}_h \approx 500$ K): c_p = 4660 J/kg·K.

ANALYSIS: The rate equation, Eq. 11.14, can be written in the form

$$A_s = q / U\Delta T_{\ell m} \qquad (1)$$

and from Eq. 11.18,

$$\Delta T_{\ell m} = F\Delta T_{\ell m,CF} \qquad \text{where} \qquad \Delta T_{\ell m,CF} = \frac{\Delta T_1 - \Delta T_2}{\ell n(\Delta T_1 / \Delta T_2)}. \qquad (2,3)$$

From an energy balance on the cold fluid, the heat rate is

$$q = \dot{m}_c c_{p,c}(T_{c,o} - T_{c,i}) = \frac{10{,}000 \text{ kg/h}}{3600 \text{s/h}} \times 4195 \frac{\text{J}}{\text{kg} \cdot \text{K}}(120 - 35)\text{K} = 9.905 \times 10^5 \text{ W}.$$

From an energy balance on the hot fluid, the outlet temperature is

$$T_{h,o} = T_{h,i} - q / \dot{m}_h c_{p,h} = 300°\text{C} - 9.905 \times 10^5 \text{ W} / \frac{5000}{3600} \frac{\text{kg}}{\text{s}} \times 4660 \frac{\text{J}}{\text{kg} \cdot \text{K}} = 147°\text{C}.$$

From Fig. 11.11, determine F from values of P and R, where P = (120 – 35)°C/(300 – 35)°C = 0.32, R = (300 – 147)°C/(120-35)°C = 1.8, and F ≈ 0.97. The log-mean temperature difference based upon a CF arrangement follows from Eq. (3); find

$$\Delta T_{\ell m} = [(300 - 120) - (147 - 35)]\text{K} / \ell n \frac{(300 - 120)}{(147 - 35)} = 143.3\text{K}. \qquad <$$

$$A_s = 9.905 \times 10^5 \text{W} / 1500 \text{W/m}^2 \cdot \text{K} \times 0.97 \times 143.3\text{K} = 4.75\text{m}^2 \qquad <$$

COMMENTS: (1) Check $\overline{T}_h \approx 500$ K used in property determination; $\overline{T}_h$ = (300 + 147)°C/2 = 497 K.

(2) Using the NTU-*e* method, determine first the capacity rate ratio, C_{min}/C_{max} = 0.56. Then

$$e \equiv \frac{q}{q_{max}} = \frac{C_{max}(T_{c,o} - T_{c,i})}{C_{min}(T_{h,i} - T_{c,i})} = \frac{1}{0.56} \times \frac{(120 - 35)°\text{C}}{(300 - 35)°\text{C}} = 0.57.$$

From Fig. 11.17, find that NTU = $AU/C_{min} \approx 1.1$ giving A_s = 4.7 m^2.

PROBLEM 11.15

KNOWN: The shell and tube Hxer (two shells, four tube passes) of Problem 11.14, known to have an area $4.75 m^2$, provides 95°C water at the cold outlet (rather than 120°C) after several years of operation. Flow rates and inlet temperatures of the fluids remain the same.

FIND: The fouling factor, R_f.

SCHEMATIC:

T; $T_{h,i} = 300°C$; $T_{c,o} = 95°C$; ΔT_1; $T_{h,o}$; ΔT_2; $T_{c,i} = 35°C$; $A_s = 4.75 m^2$; $\dot{m}_h = 5000 kg/h$; $\dot{m}_c = 10,000 kg/h$; 1; 2

ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible kinetic and potential energy changes, (3) Constant properties, (4) Thermal resistance for the clean condition is R''_t = (1500 $W/m^2 \cdot K)^{-1}$.

PROPERTIES: *Table A-6*, Water ($\bar{T}_c \approx 338$ K): c_p = 4187 J/kg·K; *Table A-6*, Water (Assume $T_{h,o} \approx 190°C$, $\bar{T}_h \approx 520$ K): c_p = 4840 J/kg·K.

ANALYSIS: The overall heat transfer coefficient can be expresses as

$$U = 1/\left(R''_t + R''_f\right) \qquad \text{or} \qquad R''_f = 1/U - R''_t \qquad (1)$$

where R''_t is the thermal resistance for the clean condition and R''_f, the fouling factor, represents the additional resistance due to fouling of the surface. The rate equation, Eq. 11.14 with Eq. 11.18, has the form,

$$U = q / A_s F \Delta T_{\ell m,CF} \qquad \Delta T_{\ell m,CF} = \left(\Delta T_1 - \Delta T_2\right)/\ell n\left(\Delta T_1/\Delta T_2\right). \qquad (2)$$

From energy balances on the cold and hot fluids, find

$$q = \dot{m}_c c_{p,c}\left(T_{c,o} - T_{c,i}\right) = (10{,}000/3600\ kg/s)4187\ J/kg\cdot K(95-35)K = 6.978\times10^5\ W$$

$$T_{h,o} = T_{h,i} - q/\dot{m}_h c_{p,h} = 300°C - 6.978\times10^5 W/\left(5000/3600 kg/s \times 4840 J/kg\cdot K\right) = 196.2°C.$$

The factor, F, follows from values of P and R as given by Fig. 11.11 with

$$P = (95-35)/(300-35) = 0.23 \qquad R = (300-196)/(120-35) = 1.22$$

giving F ≈ 1. Based upon CF arrangement,

$$\Delta T_{\ell m,CF} = \left[(300-95)-(196-35)\right]°C/\ell n\left[(300-95)/(196.2-35)\right] = 182K.$$

Using Eq. (2), find now the overall heat transfer coefficient as

$$U = 6.978\times10^5 W/4.75 m^2 \times 1 \times 182K = 806\ W/m^2\cdot K.$$

From Eq. (1), the fouling factor is

$$R''_f = \frac{1}{806\ W/m^2\cdot K} - \frac{1}{1500\ W/m^2\cdot K} = 5.74\times10^{-4} m^2\cdot K/W. \qquad <$$

COMMENTS: Note that the effect of fouling is to nearly double (U_{clean}/U_{fouled} = 1500/806 ≈ 1.9) the resistance to heat transfer. Note also the assumption for Th,o used for property evaluation is satisfactory.

PROBLEM 11.16

KNOWN: Inner tube diameter (D = 0.02 m) and fluid inlet and outlet temperatures corresponding to design conditions for a concentric tube heat exchanger. Overall heat transfer coefficient (U = 500 $W/m^2 \cdot K$) and desired heat rate (q = 3000 W). Cold fluid outlet temperature after three years of operation.

FIND: (a) Required heat exchanger length, (b) Heat rate, hot fluid outlet temperature, overall heat transfer coefficient, and fouling factor after three years.

SCHEMATIC:

$T_{h,i}$ = 160 °C
Oil
T
$T_{h,o}$ = 140 °C
$T_{c,o}$ = 80 °C or 65 °C
Water
$T_{c,i}$ = 20 °C
O
x
L

ASSUMPTIONS: (1) Negligible heat loss to the surroundings and kinetic and potential energy changes, (2) Negligible tube wall conduction resistance, (3) Constant properties.

ANALYSIS: (a) The tube length needed to achieve the prescribed conditions may be obtained from Eqs. 11.14 and 11.15 where $\Delta T_1 = T_{h,i} - T_{c,o} = 80°C$ and $\Delta T_2 = T_{h,o} - T_{c,i} = 120°C$. Hence, $\Delta T_{1m} = (120 - 80)°C/\ln(120/80) = 98.7°C$ and

$$L = \frac{q}{(\pi D) U \Delta T_{lm}} = \frac{3000\,W}{(\pi \times 0.02\,m) 500\,W/m^2 \cdot K \times 98.7°C} = 0.968\,m$$ <

(b) With $q = C_c(T_{c,o} - T_{c,i})$, the following ratio may be formed in terms of the design and 3 year conditions.

$$\frac{q}{q_3} = \frac{C_c\left(T_{c,o} - T_{c,i}\right)}{C_c\left(T_{c,o} - T_{c,i}\right)_3} = \frac{60°C}{45°C} = 1.333$$

Hence,

$$q_3 = q/1.33 = 3000\,W/1.333 = 2250\,W$$ <

Having determined the ratio of heat rates, it follows that

$$\frac{q}{q_3} = \frac{C_h\left(T_{h,i} - T_{h,o}\right)}{C_h\left(T_{h,i} - T_{h,o}\right)_3} = \frac{20°C}{160°C - T_{h,o(3)}} = 1.333$$

Hence,

$$T_{h,o(3)} = 160°C - 20°C/1.333 = 145°C$$ <

With $\Delta T_{lm,3} = (125 - 95)/\ln(125/95) = 109.3°C$,

$$U_3 = \frac{q_3}{(\pi D L)\Delta T_{lm,3}} = \frac{2250\,W}{\pi(0.02\,m)0.968\,m\left(109.3°C\right)} = 338\,W/m^2 \cdot K$$ <

Continued...

PROBLEM 11.16 (Cont.)

With $U = \left[(1/h_i) + (1/h_o)\right]^{-1}$ and $U_3 = \left[(1/h_i) + (1/h_o) + R''_{f,c}\right]^{-1}$,

$$R''_{f,c} = \frac{1}{U_3} - \frac{1}{U} = \left(\frac{1}{338} - \frac{1}{500}\right) m^2 \cdot K/W = 9.59 \times 10^{-4} m^2 \cdot K/W$$

$<$

COMMENTS: Over time fouling will always contribute to a degradation of heat exchanger performance. In practice it is desirable to remove fluid contaminants and to implement a regular maintenance (cleaning) procedure.

PROBLEM 11.17

KNOWN: Counterflow, concentric tube heat exchanger of Example 11.1; maintaining the outlet oil temperature of 60°C, but with variable rate of cooling water, all other conditions remaining the same.

FIND: (a) Calculate and plot the required exchanger tube length L and water outlet temperature $T_{c,o}$ for the cooling water flow rate in the range 0.15 to 0.3 kg/s, and (b) Calculate U as a function of the water flow rate assuming the water properties are independent of temperature; justify using a constant value of U for the part (a) calculations.

SCHEMATIC:

$T_{h,i} = 100^oC$ $\dot{m}_h = 0.1$ kg/s   Oil $T_{h,o} = 60^oC$

$T_{c,o}$ Concentric tube, CF hxer

$D_i = 25$ mm, L, $U = 37.8\ W/m^2$-K

Water $\dot{m}_c = 0.15$-0.30 kg/s $T_{c,i} = 30^oC$

ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible losses to the surroundings and kinetic and potential energy changes, (3) Overall heat transfer coefficient independent of water flow rate for this range, and (4) Constant properties.

PROPERTIES: *Table A-6*, Water $\left(\overline{T}_c = 35^\circ C = 308\ K\right)$: $c_p = 4178\ J/kg\cdot K$, $\mu = 725\times10^{-6}$ $N\cdot s/m^2$, $k = 0.625\ W/m\cdot K$, $Pr = 4.85$, *Table A-4*, Unused engine oil $\left(\overline{T}_h = 353\ K\right)$: $c_p = 2131\ J/kg\cdot K$.

ANALYSIS: (a) The NTU-ε method will be used to calculate the tube length L and water outlet temperature $T_{c,o}$ using this system of equations in the *IHT* workspace:

NTU relation, CF hxer, Eq. 11.30b

$$NTU = \frac{1}{C_r - 1}\ell n\frac{(\varepsilon - 1)}{(\varepsilon C_r - 1)} \qquad C_r = C_{max}/C_{min} \qquad (1, 2)$$

$$NTU = U\cdot A/C_{min} \qquad (3)$$

$$A = \pi D_i \cdot L \qquad (4)$$

Capacity rates, find minimum fluid

$$C_h = \dot{m}_h c_h = 0.1\ kg/s\times2131\ J/kg\cdot K = 213.1\ W/K$$

$$C_c = \dot{m}_c c_c = (0.15\ \text{to}\ 0.30)kg/s\times4178\ J/kg\cdot K = 626.7 - 1253\ W/K \qquad (5)$$

$$C_{min} = C_h \qquad (6)$$

Effectiveness and maximum heat rate, Eqs. 11.19 and 11.20

$$\varepsilon = q/q_{max} \qquad (7)$$

$$q_{max} = C_{min}\left(T_{h,i} - T_{c,i}\right) = C_c\left(T_{h,i} - T_{c,i}\right) \qquad (8)$$

Continued …..

PROBLEM 11.17 (Cont.)

$$q = C_h\left(T_{h,i} - T_{h,o}\right) \tag{9}$$

With the foregoing equations and the parameters specified in the schematic, the results are plotted in the graphs below.

The effect of water flow rate on outlet temperature

Temperature, Tco (C)

Water flow rate, modtc (kg/s)

The effect of water flow rate on required exchanger length

Exchanger length, L (m)

Water flow rate, mdotc (kg/s)

(b) The overall coefficient can be written in terms of the inner (cold) and outer (hot) side convection coefficients,

$$U = 1/\left(1/h_i + 1/h_o\right) \tag{10}$$

From Example 11.1, h_o = 38.4 W/m^2·K, and h_i will vary with the flow rate from Eq. 8.60 as

$$h_i = h_{i,b}\left(\dot{m}_i / \dot{m}_{i,b}\right)^{0.8} \tag{11}$$

where the subscript *b* denotes the base case when $\dot{m}_i = 0.2\ \text{kg/s}$. From these equations, the results are tabulated.

$\dot{m}_c(\text{kg/s})$	$h_i\left(\text{W/m}^2\cdot\text{K}\right)$	$h_o\left(\text{W/m}^2\cdot\text{K}\right)$	$U\left(\text{W/m}^2\cdot\text{K}\right)$
015	1787	38.4	37.6
0.20	2250	38.4	37.8
0.25	2690	38.4	37.9
0.30	3112	38.4	37.9

Note that while h_i varies nearly 50%, there is a negligible effect on the value of U.

COMMENTS: Note from the graphical results, that by doubling the flow rate (from 0.15 to 0.30 kg/s), the required length of the exchanger can be decreased by approximately 6%. Increasing the flow rate is not a good strategy for reducing the length of the exchanger. However, any increase in the hot-side (oil) convection coefficient would provide a proportional decrease in the length.

PROBLEM 11.18

KNOWN: Concentric tube heat exchanger with area of 50 m^2 with operating conditions as shown on the schematic.

FIND: (a) Outlet temperature of the hot fluid; (b) Whether the exchanger is operating in counterflow or parallel flow; or can't tell from information provided; (c) Overall heat transfer coefficient; (d) Effectiveness of the exchanger; and (e) Effectiveness of the exchanger if its length is made very long

SCHEMATIC:

Operating conditions
Concentric tube HXer, A = 50 m^2

	Hot fluid (h)	Cold fluid (c)
Capacity rate, kW/K	6	3
Inlet temperature, °C	60	30
Outlet temperature, °C	--	54

$T_{h,i}$ = 60°C
ΔT_1
$T_{c,o}$ = 54°C
C_h
CF
A, U
$T_{h,o}$
ΔT_2
$T_{c,i}$ = 30°C
C_c

ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible kinetic and potential energy changes, and (3) Constant properties.

ANALYSIS: From overall energy balances on the hot and cold fluids, find the hot fluid outlet temperature

$$q = C_c\left(T_{c,o} - T_{c,i}\right) = C_h\left(T_{h,i} - T_{h,o}\right) \tag{1}$$

$$3000\ \text{W/K}\ (54 - 30)\text{K} = 6000\left(60 - T_{h,o}\right) \qquad T_{h,o} = 48°\text{C}$$ <

(b) HXer must be operating in counterflow (CF) since $T_{h,o} < T_{c,o}$. See schematic for temperature distribution.

(c) From the rate equation with A = 50 m^2, with Eq. (1) for q,

$$q = C_c\left(T_{c,o} - T_{c,i}\right) = UA\Delta T_{\ell m} \tag{2}$$

$$\Delta T_{\ell m} = \frac{\Delta T_1 - \Delta T_2}{\ell m\left(\Delta T_1 / \Delta T_2\right)} = \frac{(60 - 54)\text{K} - (48 - 30)\text{K}}{\ell n\left(6/18\right)} = 10.9°\text{C} \tag{3}$$

$$3000\ \text{W/K}(54 - 30)\text{K} = U \times 50\ \text{m}^2 \times 10.9\,\text{K}$$

$$U = 132\ \text{W/m}^2 \cdot \text{K}$$ <

(d) The effectiveness, from Eq. 11.20, with the cold fluid as the minimum fluid, $C_c = C_{min}$,

$$\varepsilon = \frac{q}{q_{max}} = \frac{C_c\left(T_{c,o} - T_{c,i}\right)}{C_{min}\left(T_{h,i} - T_{c,i}\right)} = \frac{(54 - 30)\text{K}}{(60 - 30)\text{K}} = 0.8$$ <

(e) For a very long CF HXer, the outlet of the minimum fluid, $C_{min} = C_c$, will approach $T_{h,i}$. That is,

$$q \rightarrow C_{min}\left(T_{c,o} - T_{c,i}\right) \rightarrow q_{max} \qquad \varepsilon = 1$$ <

PROBLEM 11.19

KNOWN: Specifications for a water-to-water heat exchanger as shown in the schematic including the flow rate, and inlet and outlet temperatures.

FIND: (a) Design a heat exchanger to meet the specifications; that is, size the heat exchanger, and (b) Evaluate your design by identifying what features and configurations could be explored with your customer in order to develop more complete, detailed specifications.

SCHEMATIC:

$T_{h,i} = 90°C$ $\dot{m}_h = 28$ kg/s CF

$T_{c,o} = 60°C$ $T_{h,o}$

A = ? U = ?

$\dot{m}_c = 27$ kg/s $T_{c,i} = 34°C$

D_i D_o $\dot{m}_c$ $\dot{m}_h$ L

Concentric N-tube, CF HXer

ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible kinetic and potential energy changes, (3) Tube walls have negligible thermal resistance, (4) Flow is fully developed, and (5) Constant properties.

ANALYSIS: (a) Referring to the schematic above and using the rate equation, we can determine the value of the UA product required to satisfy the design requirements. Sizing the heat exchanger involves determining the heat transfer area, A (tube diameter, length and number), and the associated overall convection coefficient, U, such that U × A satisfies the required UA product. Our approach has five steps: (1) *Calculate the UA product:* Select a configuration and calculate the required UA product; (2) *Estimate the area, A:* Assume a range for the overall coefficient, calculate the area and consider suitable tube diameter(s); (3) *Estimate the overall coefficient, U:* For selected tube diameter(s), use correlations to estimate hot- and cold-side convection coefficients and the overall coefficient; (4) *Evaluate first-pass design:* Check whether the A and U values (U × A) from Steps 2 and 3 satisfy the required UA product; if not, then (5) *Repeat the analysis:* Iterate on different values for area parameters until a satisfactory match is made, (U × A) = UA.

To perform the analysis, *IHT* models and tools will be used for the effectiveness-NTU method relations, internal flow convection correlations, and thermophysical properties. See the Comments section for details.

Step 1 Calculate the required UA. For the initial design, select a concentric tube, counterflow heat exchanger. Calculate UA using the following set of equations, Eqs. 11.30a,

$$\varepsilon = \frac{1-\exp\left[-NTU\left(1-C_r\right)\right]}{1-C_r\exp\left[-NTU\left(1-C_r\right)\right]} \quad (1)$$

$$NTU = UA/C_{min} \qquad C_r = C_{min}/C_{max} \quad (2,3)$$

$$\varepsilon = q/q_{max} \qquad q_{max} = C_{min}\left(T_{h,i} - T_{c,i}\right) \quad (4,5)$$

where $C = \dot{m}\,c_p$, and c_p is evaluated at the average mean temperature of the fluid, $\overline{T}_m = (T_{m,i} + T_{m,o})/2$. Substituting numerical values, find

$$\varepsilon = 0.464 \qquad NTU = 0.8523 \quad q = 2.934\times10^6\,W \qquad T_{h,o} = 65.0°C$$

Continued …..

$$UA = 9.62 \times 10^{4}\, W/K$$ <

Step 2 Estimate the area, A. From Table 11.2, the typical range of U for water-to-water exchangers is 850 – 1700 $W/m^2 \cdot K$. With UA = 9.619 × 10^4 W/K, the range for A is 57 – 113 m^2, where

$$A = \pi D_i LN \tag{6}$$

where L and N are the length and number of tubes, respectively. Consider these values of D_i with L = 10 m to describe the exchanger:

Case	D_i (mm)	L (m)	N	A (m^2)
1	25	10	73-146	57-113
2	50	10	36-72	57-113
3	75	10	24-48	57-113

<

Step 3 Estimate the overall coefficient, U. With the inner (hot) and outer (cold) fluids in the concentric tube arrangement, the overall coefficient is

$$1/U = 1/\bar{h}_i + 1/\bar{h}_o \tag{7}$$

and the $\bar{h}$ are estimated using the Dittus-Boelter correlation assuming fully developed turbulent flow.

Coefficient, hot side, $\bar{h}_i$. For flow in the inner tube,

$$Re_{Di} = \frac{4\,\dot{m}_{h,i}}{\pi D_i \mu_h} \qquad \dot{m}_h = \dot{m}_{hi} \cdot N \tag{8,9}$$

and the correlation, Eq. 8.60 with n = 0.3, is

$$\overline{Nu}_D = \frac{\bar{h}_i D_i}{k} = 0.037\, Re_{Di}^{4/5}\, Pr^{0.3} \tag{10}$$

where properties are evaluated at the average mean temperature, $\bar{T}_h = (T_{hi} + T_{ho})/2$.

Coefficient, cold side, $\bar{h}_o$. For flow in the annular space, $D_o - D_i$, the above relations apply where the characteristic dimension is the hydraulic diameter,

$$D_{h,o} = 4A_{c,o}/P_o \qquad A_{c,o} = \pi\left(D_o^2 - D_i^2\right)/4 \qquad P_o = \pi\left(D_o + D_i\right) \tag{11-13}$$

To determine the outer diameter D_o, require that the inner and outer fluid flow areas are the same, that is,

$$A_{c,i} = A_{c,o} \qquad \pi D_i^2/4 = \pi\left(D_o^2 - D_i^2\right)/4 \tag{14,15}$$

Summary of the convection coefficient calculations. The results of the analysis with L = 10 m are summarized below.

Continued …..

PROBLEM 11.19 (Cont.)

Case	D_i (mm)	N	A (m^2)	$\bar{h}_i$ ($W/m^2 \cdot K$)	$\bar{h}_o$ ($W/m^2 \cdot K$)	U ($W/m^2 \cdot K$)	U × A W/K
1a	25	73	57	4795	4877	2418	1.39×10^5
2a	50	36	57	2424	2465	1222	6.91×10^4
3a	75	24	57	1616	1644	814	4.61×10^4

For all these cases, the Reynolds numbers are above 10,000 and turbulent flow occurs.

Step 4 Evaluate first-pass design. The required UA product value determined in step 1 is UA = 9.62×10^4 W/K. By comparison with the results in the above table, note that the U × A values for cases 1a and 2a are, respectively, larger and smaller than that required. In this first-pass design trial we have identified the range of D_i and N (with L = 10 m) that could satisfy the exchanger specifications. A strategy can now be developed in *Step 5* to iterate the analysis on values for D_i and N, as well as with different L, to identify a combination that will meet specifications.

(b) What information could have been provided by the customer to simplify the analysis for design of the exchanger? Looking back at the analysis, recognize that we had to assume the exchanger configuration (type) and overall length. Will knowledge of the customer's installation provide any insight? While no consideration was given in our analysis to pumping power limitations, that would affect the flow velocities, and hence selection of tube diameter.

COMMENTS: The *IHT* workspace with the relations for step 3 analysis is shown below, including summary of key correlation parameters. The set of equations is quite stiff so that good initial guesses are required to make the initial solve.

```
/* Results, Step 3 - Di = 25 mm, N = 73, L = 10 m
A       Do        U         UA        Di      L     N
57.33   0.03536   2418      1.386E5   0.025   10    73
ReDi              ReDo      hDi       hDo
5.384E4           1.352E4   4795      4877    */

/* Results, Step 3 - Di = 50 mm, N = 36, L = 10 m
A       Do        U         UA        Di      L     N
56.55   0.07071   1222      6.912E4   0.05    10    36
ReDi              ReDo      hDi       hDo
5.459E4           1.371E4   2424      2465     */

/* Results, Step 3 - Di = 75 mm, N = 24, L = 10 m
A       Do        U         UA        Di      L     N
56.55   0.1061    814.8     4.608E4   0.075   10    24
ReDi              ReDo      hDi       hDo
5.459E4           1.371E4   1616      1644    */

// Input variables
//Di = 0.050
Di = 0.025
//Di = 0.075
//N = 36
N = 73
//N = 24
L = 10
mdoth = 28
Thi_C = 90
Tho_C = 65.0          // From Step 1
mdotc = 27
Tci_C = 34
Tco_C = 60
```

Continued …..

PROBLEM 11.19 (Cont.)

```
// Flow rate and number of tubes, inside parameters (hot)
mdoth = N * umi * rhoi * Aci
Aci = pi * Di^2 /4
1 / U = 1 / hDi + 1/ hDo
UA = U * A
A = pi * Di * L * N

// Flow rate, outside parameters (cold)
mdotc = rhoo * Aco * umo * N
Aco = Aci                          // Make cross-sectional areas of equal size
Aco = pi * (Do^2 - Di^2) / 4
Dho = 4 * Aco / P                  // hydraulic diameter
P = pi * ( Di + Do)                // wetted perimeter of the annular space

// Inside coefficient, hot fluid
NuDi = NuD_bar_IF_T_FD(ReDi,Pri,n)     // Eq 8.60
n = 0.3   // n = 0.4 or 0.3 for Tsi>Tmi or Tsi<Tmi
NuDi = hDi * Di / ki
ReDi = umi * Di / nui
/* Evaluate properties at the fluid average mean temperature, Tmi. */
Tmi = Tfluid_avg(Thi,Tho)
//Tmi = 310

// Outside coefficient, cold fluid
NuDo = NuD_bar_IF_T_FD(ReDo,Pro,nn)     // Eq 8.60
nn = 0.4   // n = 0.4 or 0.3 for Tsi>Tmi or Tsi<Tmi
NuDo= hDo * Dho / ko
ReDo= umo * Dho/ nuo
/* Evaluate properties at the fluid average mean temperature, Tmo. */
Tmo = Tfluid_avg(Tci,Tco)
//Tmo = 310

// Water property functions :T dependence, From Table A.6
// Units: T(K), p(bars);
x = 0                              // Quality (0=sat liquid or 1=sat vapor)
rhoi = rho_Tx("Water",Tmi,x)       // Density, kg/m^3
nui = nu_Tx("Water",Tmi,x)         // Kinematic viscosity, m^2/s
ki = k_Tx("Water",Tmi,x)           // Thermal conductivity, W/m·K
Pri = Pr_Tx("Water",Tmi,x)         // Prandtl number
rhoo = rho_Tx("Water",Tmo,x)       // Density, kg/m^3
nuo = nu_Tx("Water",Tmo,x)         // Kinematic viscosity, m^2/s
ko = k_Tx("Water",Tmo,x)           // Thermal conductivity, W/m·K
Pro = Pr_Tx("Water",Tmo,x)                 //Prandtl number

// Conversions
Thi_C = Thi - 273
Tho_C = Tho - 273
Tci_C = Tci - 273
Tco_C = Tco - 273
```

PROBLEM 11.20

KNOWN: Counterflow concentric tube heat exchanger.

FIND: (a) Total heat transfer rate and outlet temperature of the water and (b) Required length.

SCHEMATIC:

$T_{h,i}=100°C$, $\dot{m}_h=0.1 kg/s$ Oil, h

$T_{h,o}=60°C$, ΔT_2, ΔT_1, $T_{c,o}$

$\dot{m}_c=0.1 kg/s$ Water, c

$T_{c,i}=30°C$

Concentric tube: $U=60 W/m^2 \cdot K$, $D=25 mm$

ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible kinetic and potential energy changes, (3) Negligible thermal resistance due to tube wall thickness.

PROPERTIES: (given):

	ρ (kg/m^3)	c_p (J/kg·K)	ν (m^2/s)	k (W/m·K)	Pr
Water	1000	4200	7×10^{-7}	0.64	4.7
Oil	800	1900	1×10^{-5}	0.134	140

ANALYSIS: (a) With the outlet temperature, $T_{c,o}$ = 60°C, from an overall energy balance on the hot (oil) fluid, find

$$q = \dot{m}_h c_h \left(T_{h,i} - T_{h,o}\right) = 0.1 kg/s \times 1900 J/kg \cdot K (100 - 60)°C = 7600 \text{ W}. \qquad <$$

From an energy balance on the cold (water) fluid, find

$$T_{c,o} = T_{c,i} + q/\dot{m}_c c_c = 30°C + 7600 W/0.1 kg/s \times 4200 J/kg \cdot K = 48.1°C. \qquad <$$

(b) Using the LMTD method, the length of the CF heat exchanger follows from

$$q = UA\Delta T_{lm,CF} = U(p DL)\Delta T_{lm,CF} \qquad L = q/U(p D)\Delta T_{lm,CF}$$

where

$$\Delta T_{lm,CF} = \frac{\Delta T_1 - \Delta T_2}{\ln\left(\Delta T_1/\Delta T_2\right)} = \frac{(60-30)°C - (100 - 48.1)°C}{\ln(30/51.9)} = 40.0°C$$

$$L = 7600 W / 60 W/m^2 \cdot K (p \times 0.025 m) \times 40.0°C = 40.3 m. \qquad <$$

COMMENTS: Using the ε-NTU method, find $C_{min} = C_h$ = 190 W/K and $C_{max} = C_c$ = 420 W/K. Hence

$$q_{max} = C_{min}\left(T_{h,i} - T_{c,i}\right) = 190 \text{ W/K}(100-30)\text{K} = 13{,}300 \text{ W}$$

and $\varepsilon = q/q_{max}$ = 0.571. With $C_r = C_{min}/C_{max}$ = 0.452 and using Eq. 11.30b,

$$NTU = \frac{UA}{C_{min}} = \frac{1}{C_r - 1}\ln\left(\frac{e-1}{eC_r - 1}\right) = \frac{1}{0.452-1}\ln\left(\frac{0.571-1}{0.571 \times 0.452 - 1}\right) = 1.00$$

so that with $A = \pi DL$, find L = 40.3 m.

PROBLEM 11.21

KNOWN: Counterflow, concentric tube heat exchanger undergoing test after service for an extended period of time; surface area of 5 m^2 and design value for the overall heat transfer coefficient of $U_d = 38\ W/m^2 \cdot K$.

FIND: Fouling factor, if any, based upon the test results of engine oil flowing at 0.1 kg/s cooled from 110°C to 66°C by water supplied at 25°C and a flow rate of 0.2 kg/s.

SCHEMATIC:

$T_{h,i} = 110^oC$
$\dot{m}_h = 0.1$ kg/s
$T_{h,o} = 66^oC$
$T_{c,o}$
$A = 5\ m^2$
$U_d = 38\ W/m^2$-K
$\dot{m}_c = 0.2$ kg/s
$T_{c,i} = 25^oC$

ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible losses to the surroundings and kinetic and potential energy changes, (3) Constant properties.

PROPERTIES: *Table A-5*, Engine oil ($\overline{T}_h$ = 361 K): c = 2166 J/kg·K; *Table A-6*, Water $\left(\overline{T}_c = 304\text{ K, assuming } T_{c,o} = 36^\circ C\right)$: c = 4178 J/kg·K.

ANALYSIS: For the CF conditions shown in the Schematic, find the heat rate, q, from an energy balance on the hot fluid (oil); the cold fluid outlet temperature, $T_{c,o}$, from an energy balance on the cold fluid (water); the overall coefficient U from the rate equation; and a fouling factor, R, by comparison with the design value, U_d.

Energy balance on hot fluid

$$q = \dot{m}_h c_h \left(T_{h,i} - T_{h,o}\right) = 0.1\ kg/s \times 2166\ J/kg \cdot K\ (110 - 66)K = 9530\ W$$

Energy balance on the cold fluid

$$q = \dot{m}_c c_c \left(T_{c,o} - T_{c,i}\right), \qquad \text{find} \quad T_{c,o} = 36.4^\circ C$$

Rate equation

$$q = UA\Delta T_{\ell n,CF}$$

$$\Delta T_{\ell n,CF} = \frac{\left(T_{h,i} - T_{c,o}\right) - \left(T_{h,o} - T_{c,i}\right)}{\ell n\left[\left(T_{h,i} - T_{c,o}\right)/\left(T_{h,o} - T_{c,i}\right)\right]} = \frac{(110 - 36.4)^\circ C - (66 - 25)^\circ C}{\ell n[73.6/41.0]} = 55.7^\circ C$$

$$9530\ W = U \times 5\ m^2 \times 55.7^\circ C$$

$$U = 34.2\ W/m^2 \cdot K$$

Overall resistance including fouling factor

$$U = 1/\left[1/U_d + R''_f\right]$$

$$34.2\ W/m^2 \cdot K = 1/\left[1/38\ W/m^2 \cdot K + R''_f\right]$$

$$R''_f = 0.0029\ m^2 \cdot K/W$$ <

PROBLEM 11.22

KNOWN: Prescribed flow rates and inlet temperatures for hot and cold water; UA value for a shell-and-tube heat exchanger (one shell and two tube passes).

FIND: Outlet temperature of the hot water.

SCHEMATIC:

$T_{h,i} = 80°C$, $T_{c,i} = 20°C$, $\dot{m}_c = 5000$ kg/h, $UA = 11{,}600$ W/K, $\dot{m}_h = 10{,}000$ kg/h; $T_{h,i} = 80°C$, $T_{h,o}$, $T_{c,o}$, $T_{c,i} = 20°C$, $\dot{m}_c$, T, x, CF

ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Constant properties, (3) Negligible kinetic and potential energy changes.

PROPERTIES: *Table A-6*, Water ($\overline{T}_c$ = (20 + 60)/2 = 40°C ≈ 310 K): $c_c = c_{p,f}$ = 4178 J/kg·K; Water ($\overline{T}_h$ = (80 +60)/2=70°C≈340 K): $c_h = c_{p,f}$ = 4188 J/kg·K.

ANALYSIS: From an energy balance on the hot fluid, the outlet temperature is

$$T_{h,o} = T_{h,i} - q / \dot{m}_h c_h. \tag{1}$$

The heat rate can be written in terms of the effectiveness and q_{max} as

$$q = \varepsilon q_{max} = \varepsilon C_{min}\left(T_{h,i} - T_{c,i}\right) \tag{2}$$

where for this HXer, the cold fluid is the minimum fluid giving

$$q_{max} = (\dot{m}c)_c\left(T_{h,i} - T_{c,i}\right)$$

$$q_{max} = (5000/3600)\,\text{kg/s} \times 4178\,\text{J/kg}\cdot\text{K}\,(80-20)\,°\text{C} = 348.2\ \text{kW}.$$

The effectiveness can be determined from Figure 11.16 with

$$NTU = \frac{UA}{C_{min}} = \frac{11{,}600\ \text{W/K}}{(5000/3600)\,\text{kg/s} \times 4178\,\text{J/kg}\cdot\text{K}} = 2.0$$

giving, ε = 0.7 for $C_r = C_{min}/C_{max}$ = (5,000 × 4178)/(10,000 × 4188) = 0.499. Combining Eqs. (1) and (2), find

$$T_{h,o} = 80°\text{C} - \left(0.7 \times 348.2 \times 10^3\,\text{W}\right)/\,(10{,}000/3600)\,\text{kg/s} \times 4188\,\text{J/kg}\cdot\text{K}$$

$$T_{h,o} = (80 - 21.0)\,°\text{C} = 59°\text{C}. \qquad <$$

COMMENTS: (1) From an energy balance on the cold fluid, $q = (\dot{m}c)_c\,(T_{c,o} - T_{c,i})$, find that $T_{c,o}$ = 62°C. For evaluating properties at average mean temperatures, we should use $\overline{T}_h$ = (59 + 80)/2 = 70°C = 343 K and $\overline{T}_c$ = (20 + 62)/2 = 41°C = 314 K. Note from above that we have indeed assumed reasonable temperatures at which to obtain specific heats.

(2) We could have also used Eq. 11.31a to evaluate ε using C_r = 0.5 and NTU = 2 to obtain ε = 0.693.

PROBLEM 11.23

KNOWN: Flow rates and inlet temperatures for automobile radiator configured as a cross-flow heat exchanger with both fluids unmixed. Overall heat transfer coefficient.

FIND: (a) Area required to achieve hot fluid (water) outlet temperature, $T_{m,o}$ = 330 K, and (b) Outlet temperatures, $T_{h,o}$ and $T_{c,o}$, as a function of the overall coefficient for the range, $200 \le U \le 400\ W/m^2 \cdot K$ with the surface area A found in part (a) with all other heat transfer conditions remaining the same as for part (a).

SCHEMATIC:

Water $T_{h,i}$ = 400 K
$\dot{m}_h$ = 0.05 kg/s
Air
$T_{c,i}$ = 300 K
$\dot{m}_c$ = 0.75 kg/s
$T_{c,o}$
$U = 200\ W/m^2 \cdot K$
$T_{h,o}$ = 330 K

ASSUMPTIONS: (1) Negligible heat loss to surroundings and kinetic and potential energy changes, (2) Constant properties.

PROPERTIES: *Table A.6*, Water ($\overline{T}_h$ = 365 K): $c_{p,h}$ = 4209 J/kg·K; *Table A.4*, Air $\left(\overline{T}_c \approx 310\,K\right)$: $c_{p,c}$ = 1007 J/kg·K.

ANALYSIS: (a) The required heat transfer rate is

$$q = \dot{m}_h c_{p,h}\left(T_{h,i} - T_{h,o}\right) = 0.05\,kg/s\left(4209\,J/kg \cdot K\right)70\,K = 14,732\,W\,.$$

Using the ε -NTU method,

$$C_{min} = C_h = 210.45\,W/K \qquad C_{max} = C_c = 755.25\,W/K\,.$$

Hence, C_{min}/C_{max} = 0.279 and

$$q_{max} = C_{min}\left(T_{h,i} - T_{c,i}\right) = 210.45\ W/K\,(100\ K) = 21,045\,W$$

$$\varepsilon = q/q_{max} = 14,732\,W/21,045\,W = 0.700\,.$$

Figure 11.18 yields NTU ≈ 1.5, hence,

$$A = NTU\left(C_{min}/U\right) = 1.5 \times 210.45\,W/K\Big/\left(200\,W/m^2 \cdot K\right) = 1.58\,m^2.$$ <

(b) Using the *IHT Heat Exchanger Tool* for *Cross-flow with both fluids unmixed* arrangement and the *Properties Tool* for *Air* and *Water*, a model was generated to solve part (a) evaluating the efficiency using Eq. 11.33. The following results were obtained:

$$A = 1.516\,m^2 \qquad NTU = 1.441 \qquad T_{c,o} = 319.5\,K$$

Using the model but assigning A = 1.516 m^2, the outlet temperature $T_{h,o}$ and $T_{c,o}$ were calculated as a function of U and the results plotted below.

Continued...

With a higher U, the outlet temperature of the hot fluid (water) decreases. A benefit is enhanced heat removal from the engine block and a cooler operating temperature. If it is desired to cool the engine with water at 330 K, the heat exchanger surface area and, hence its volume in the engine component could be reduced.

COMMENTS: (1) For the results of part (a), the air outlet temperature is

$$T_{c,o} = T_{c,i} + q/C_c \;= 300\,\text{K} + (14,732\,\text{W}/755.25\,\text{W/K}) = 319.5\,\text{K}.$$

(2) For the conditions of part (a), using the LMTD approach, ΔT_{lm} = 51.2 K, R = 0.279 and P = 0.7. Hence, Fig. 11.12 yields F ≈ 0.95 and

$$A = q/FU\Delta T_{lm} = (14,732\text{W})\Big/\Big[0.95\big(200\,\text{W/m}^2\cdot\text{K}\big)51.2\text{K}\Big] = 1.51\text{m}^2.$$

(3) The IHT workspace with the model to generate the above plot is shown below. Note that it is necessary to enter the overall energy balances on the fluids from the keyboard.

```
// Heat Exchanger Tool - Cross-flow with both fluids unmixed:
// For the cross-flow, single-pass heat exchanger with both fluids unmixed,
eps = 1 - exp((1 / Cr) * (NTU^0.22) * (exp(-Cr * NTU^0.78) - 1))      // Eq 11.33
// where the heat-capacity ratio is
Cr = Cmin / Cmax
// and the number of transfer units, NTU, is
NTU = U * A / Cmin                          // Eq 11.25
// The effectiveness is defined as
eps = q / qmax
qmax = Cmin * (Thi - Tci)                   // Eq 11.20
// See Tables 11.3 and 11.4 and Fig 11.18
// Overall Energy Balances on Fluids:
q = mdoth * cph * (Thi - Tho)
q = mdotc * cpc * (Tco - Tci)
// Assigned Variables:
Cmin = Ch                   // Capacity rate, minimum fluid, W/K
Ch = mdoth * cph            // Capacity rate, hot fluid, W/K
mdoth = 0.05                // Flow rate, hot fluid, kg/s
Thi = 400                   // Inlet temperature, hot fluid, K
Tho = 330                   // Outlet temperature, hot fluid, K; specified for part (a)
Cmax = Cc                   // Capacity rate, maximum fluid, W/K
Cc = mdotc * cpc            // Capacity rate, cold fluid, W/K
mdotc = 0.75                // Flow rate, cold fluid, kg/s
Tci = 300                   // Inlet temperature, cold fluid, K
U = 200                     // Overall coefficient, W/m^2.K
// Properties Tool - Water (h)
// Water property functions :T dependence, From Table A.6
// Units: T(K), p(bars);
xh =  0                             // Quality (0=sat liquid or 1=sat vapor)
rhoh = rho_Tx("Water",Tmh,xh)       // Density, kg/m^3
cph = cp_Tx("Water",Tmh,xh)         // Specific heat, J/kg·K
Tmh =  Tfluid_avg(Thi,Tho )
// Properties Tool - Air(c)
// Air property functions : From Table A.4
// Units: T(K); 1 atm pressure
rhoc = rho_T("Air",Tmc)             // Density, kg/m^3
cpc = cp_T("Air",Tmc)               // Specific heat, J/kg·K
Tmc = Tfluid_avg(Tci,Tco)
```

PROBLEM 11.24

KNOWN: Flowrates and inlet temperatures of a cross-flow heat exchanger with both fluids unmixed. Total surface area and overall heat transfer coefficient for clean surfaces. Fouling resistance associated with extended operation.

FIND: (a) Fluid outlet temperatures, (b) Effect of fouling, (c) Effect of UA on air outlet temperature.

SCHEMATIC:

$\dot{m}_c = 5$ kg/s
$T_{c,i} = 300$ K
$\dot{m}_h = 1$ kg/s
$T_{h,i} = 800$ K
$T_{h,o}$
$T_{c,o}$
$U = 35$ W/m^2•K
$A = 25$ m^2
$R''_f = 0.004$ m^2•K/W

ASSUMPTIONS: (1) Negligible heat loss to surroundings and negligible kinetic and potential energy changes, (2) Constant properties, (3) Negligible tube wall resistance.

PROPERTIES: Air and gas (given): $c_p = 1040$ J/kg·K.

ANALYSIS: (a) With $C_{min} = C_h = 1$ kg/s × 1040 J/kg·K = 1040 W/K and $C_{max} = C_c = 5$ kg/s × 1040 J/kg·K = 5200 W/K, $C_{min}/C_{max} = 0.2$. Hence, NTU = UA/C_{min} = 35 W/m^2·K(25 m^2)/1040 W/K = 0.841 and Fig. 11.18 yields ε ≈ 0.57. With $C_{min}(T_{h,i} - T_{c,i})$ = 1040 W/K(500 K) = 520,000 W = q_{max}, Eqs. (11.21) and (11.22) yield

$$T_{h,o} = T_{h,i} - \varepsilon q_{max}/C_h = 800\,\text{K} - 0.56(520,000\,\text{W})/1040\,\text{W/K} = 520\,\text{K}$$ <

$$T_{c,o} = T_{c,i} + \varepsilon q_{max}/C_c = 300\,\text{K} + 0.56(520,000\,\text{W})/5200\,\text{W/K} = 356\,\text{K}$$ <

(b) With fouling, the overall heat transfer coefficient is reduced to

$$U_f = \left(U^{-1} + R''_f\right)^{-1} = \left[(0.029 + 0.004)\,\text{m}^2 \cdot \text{K/W}\right]^{-1} = 30.3\,\text{W/m}^2 \cdot \text{K}$$

This 13.4% reduction in performance is large enough to justify cleaning of the tubes. <

(c) Using the *Heat Exchangers* option from the IHT Toolpad to explore the effect of UA, we obtain the following result.

Air outlet temperature, Tco(K)
400
380
360
340
320
300
500
900
1300
1700
2100
2500
Heat transfer parameter, UA(W/K)

The heat rate, and hence the air outlet temperature, increases with increasing UA, with $T_{c,o}$ approaching a maximum outlet temperature of 400 K as UA → ∞ and ε → 1.

COMMENTS: Note that, for conditions of part (a), Eq. 11.33 yields a value of ε = 0.538, which reveals the level of approximation associated with reading ε from Fig. 11.18.

PROBLEM 11.25

KNOWN: Cooling milk from a dairy operation to a safe-to-store temperature, $T_{h,o} \le 13°C$, using ground water in a counterflow concentric tube heat exchanger with a 50-mm diameter inner pipe and overall heat transfer coefficient of 1000 $W/m^2 \cdot K$.

FIND: (a) The UA product required for the chilling process and the length L of the exchanger, (b) The outlet temperature of the ground water, and (c) the milk outlet temperatures for the cases when the water flow rate is halved and doubled, using the UA product found in part (a)

SCHEMATIC:

T
$T_{h,i}$ = 38.6°C
$\dot{\forall}_h$ = 250 liter/h
U = 1000 W/m²-K
D = 50 mm
$T_{c,o}$
$T_{h,o}$ ≤ 13°C
$T_{c,i}$ = 10°C
$\dot{\forall}_c$ = 0.72 m³/h

ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible heat loss to surroundings and kinetic and potential energy changes, and (3) Constant properties.

PROPERTIES: *Table A-6*, Water $\left(\overline{T}_c = 287\text{ K, assume } T_{c,o} = 18°C\right)$: $\rho = 1000\text{ kg/m}^3$, $c_p = 4187\text{ J/kg}\cdot\text{K}$; Milk (given): $\rho = 1030\text{ kg/m}^3$, $c_p = 3860\text{ J/kg}\cdot\text{K}$.

ANALYSIS: (a) Using the effectiveness-NTU method, determine the capacity rates and the minimum fluid.

Hot fluid, milk:

$$\dot{m}_h = \rho_h \dot{\forall}_h = 1030\text{ kg/m}^3 \times 250\text{ liter/h} \times 10^{-3}\text{m}^3\text{/liter} \times 1\text{ h/3600 s} = 0.0715\text{ kg/s}$$

$$C_h = \dot{m}_h c_h = 0.0715\text{ kg/s} \times 3860\text{ J/kg}\cdot\text{K} = 276\text{ W/K}$$

Cold fluid, water:

$$C_c = \dot{m}_c c_c = 1000\text{ kg/m}^3 \times \left(0.72/3600\text{ m}^3\text{/s}\right) \times 4187\text{ J/kg}\cdot\text{K} = 837\text{ W/K}$$

It follows that $C_{min} = C_h$. The effectiveness of the exchanger from Eqs. 11.19 and 11.21 is

$$\varepsilon = \frac{q}{q_{max}} = \frac{C_h\left(T_{h,i} - T_{h,o}\right)}{C_{min}\left(T_{h,i} - T_{c,i}\right)} = \frac{(38.6 - 13)\text{K}}{(38.6 - 10)\text{K}} = 0.895 \qquad (1)$$

The NTU can be calculated from Eq. 10.30b, where $C_r = C_{min}/C_{max} = 0.330$,

$$\text{NTU} = \frac{1}{C_r - 1}\ell n\left(\frac{\varepsilon - 1}{\varepsilon C_r - 1}\right) \qquad (2)$$

$$\text{NTU} = \frac{1}{0.330 - 1}\ell n\left(\frac{0.895 - 1}{0.895 \times 0.330 - 1}\right) = 2.842$$

Continued …..

PROBLEM 11.25 (Cont.)

From Eq. 11.25, find UA

$$[UA] = NTU \cdot C_{min} = 2.842 \times 276\ W/K = 785\ W/K$$ <

and the exchanger tube length with A = π DL is

$$L = [UA]/\pi DU\ =\ 785\ W/K/\pi\, 0.050\ m \times 1000\ W/m^2 \cdot K = 5.0\ m$$ <

(b) The water outlet temperature, $T_{c,o}$, can be calculated from the heat rates,

$$C_h\left(T_{h,i} - T_{h,o}\right) = C_c\left(T_{c,o} - T_{c,i}\right) \quad (3)$$

$$276\ W/K\,(38.6 - 13)K = 837\ W/K\,\left(T_{c,o} - 10\right)K$$

$$T_{c,o} = 18.4^\circ C$$ <

(c) Using the foregoing Eqs. (1 - 3) in the *IHT* workspace, the hot fluid (milk) outlet temperatures are evaluated with UA = 785 W/K for different water flow rates. The results, including the hot fluid outlet temperatures, are compared to the base case, part (a).

Case	C_c *(W/K)*	$T_{c,o}$ *(°C)*	$T_{h,o}$ *(°C)*
1, halved flow rate	419	14.9	25.6
Base, part (a)	837	13	18.4
2, doubled flow rate	1675	12.3	14.3

COMMENTS: (1) From the results table in part (c), note that if the water flow rate is halved, the milk will not be properly chilled, since $T_{c,o}$ = 14.9°C > 13°C. Doubling the water flow rate reduces the outlet milk temperature by less than 1°C.

(2) From the results table, note that the water outlet temperature changes are substantially larger than those of the milk with changes in the water flow rate. Why is this so? What operational advantage is achieved using the heat exchanger under the present conditions?

(3) The water thermophysical properties were evaluated at the average cold fluid temperature, $\overline{T}_c = \left(T_{c,i} + T_{c,o}\right)/2$. We assumed an outlet temperature of 18°C, which as the results show, was a good choice. Because the water properties are not highly temperature dependent, it was acceptable to use the same values for the calculations of part (c). You could, of course, use the properties function in *IHT* that will automatically use the appropriate values.

PROBLEM 11.26

KNOWN: Flow rate, inlet temperatures and overall heat transfer coefficient for a regenerator. Desired regenerator effectiveness. Cost of natural gas.

FIND: (a) Heat transfer area required for regenerator and corresponding heat recovery rate and outlet temperatures, (b) Annual energy and fuel cost savings.

SCHEMATIC:

$T_{h,i} = 70^oC$

$T_{c,o}$

$T_{c,i} = 5^oC$

$U = 2000$ W/m^2-K

$\varepsilon = 0.50$

$T_{h,o}$

70^{o}C, $T_{c,o}$, $\dot{m}_h$, $T_{h,o}$, $\dot{m}_c$, 5^{o}C

$\dot{m}_c = \dot{m}_h = 5$ kg/s

ASSUMPTIONS: (a) Negligible heat loss to surroundings, (b) Constant properties.

PROPERTIES: *Table A-6*, water $(\overline{T}_m \approx 310\text{K})$: $c_p = 4178\ \text{J/kg}\cdot\text{K}$.

ANALYSIS: (a) With $C_r = 1$ and $\varepsilon = 0.50$ for one shell and two tube passes, Eq. 11.31c yields E = 1.414. With $C_{min} = 5$ kg/s $\times$ 4178 J/kg·K = 20,890 W/K, Eq. 11.31b then yields

$$A = -\frac{C_{min}}{U}\frac{\ln\left[(E-1)/(E+1)\right]}{\left(1+C_r^2\right)^{1/2}} = -\frac{20,890\ \text{W/K}}{2000\ \text{W/m}^2\cdot\text{K}}\frac{\ln(0.171)}{1.414} = 13.05\ \text{m}^2$$ <

With $\varepsilon = 0.50$, the heat recovery rate is then

$$q = \varepsilon C_{min}\left(T_{h,i} - T_{c,i}\right) = 679,000\ \text{W}$$ <

and the outlet temperatures are

$$T_{c,o} = T_{c,i} + \frac{q}{C_c} = 5°\text{C} + \frac{679,000\ \text{W}}{20,890\ \text{W/K}} = 37.5°\text{C}$$ <

$$T_{h,o} = T_{h,i} - \frac{q}{C_h} = 70°\text{C} - \frac{679,000\ \text{W}}{20,890\ \text{W/K}} = 37.5°\text{C}$$ <

(b) The amount of energy recovered for continuous operation over 365 days is

$$\Delta E = 679,000\ \text{W}\times 365\ \text{d/yr}\times 24\ \text{h/d}\times 3600\ \text{s/h} = 2.14\times 10^{13}\ \text{J/yr}$$

The annual fuel savings S_A is then

$$S_A = \frac{\Delta E\times C_{ng}}{\eta} = \frac{2.14\times 10^7\ \text{MJ/yr}\times \$0.0075/\text{MJ}}{0.9} = \$178,000/\text{yr}$$ <

COMMENTS: (1) With $C_c = C_h$, the temperature changes are the same for the two fluids, (2) A larger effectiveness and hence a smaller value of A can be achieved with a counterflow exchanger (compare Figs. 11.15 and 11.16 for $C_r = 1$), (c) The savings are significant and well worth the cost of the heat exchanger. An additional benefit is that, with $T_{h,o}$ reduced from 70 to 37.5°C, less energy is consumed by the refrigeration system used to restore it to 5°C.

PROBLEM 11.27

KNOWN: Heat exchanger in car operating between warm radiator fluid and cooler outside air. Effectiveness of heater is $\varepsilon \sim \dot{m}_{air}^{-0.2}$ since water flow rate is large compared to that of the air. For low-speed fan condition, heat warms outdoor air from 0°C to 30°C.

FIND: (a) Increase in heat added to car for high-speed fan condition causing $\dot{m}_{air}$ to be doubled while inlet temperatures remain the same, and (b) Air outlet temperature for medium-speed fan condition where air flow rate increases 50% and heat transfer increases 20%.

SCHEMATIC:

ASSUMPTIONS: (1) Negligible heat losses from heat exchanger to surroundings, (2) $T_{h,i}$ and $T_{c,i}$ remain fixed for all fan-speed conditions, (3) Water flow rate is much larger than that of air.

ANALYSIS: (a) Assuming the flow rate of the water is much larger than that of air,

$$C_{min} = C_c = \dot{m}_{air}\, c_{p,c}.$$

Hence, the heat rate can be written as

$$q = \varepsilon q_{max} = \varepsilon C_{min}\left(T_{h,i} - T_{c,i}\right) = \varepsilon \cdot \dot{m}_{air} c_{p,air}\left(T_{h,i} - T_{c,i}\right).$$

Taking the ratio of the heat rates for the high and low speed fan conditions, find

$$\frac{q_{hi}}{q_{lo}} = \frac{\left(\varepsilon \dot{m}_{air}\right)_{hi}}{\left(\varepsilon \dot{m}_{air}\right)_{lo}} = \frac{\left(\dot{m}_{air}^{0.8}\right)_{hi}}{\left(\dot{m}_{air}^{0.8}\right)_{lo}} = 2^{0.8} = 1.74$$ <

where we have used $\varepsilon \sim \dot{m}_{air}^{-0.2}$ and recognized that for the high speed fan condition, the air flow rate is doubled. Hence the heat rate is increased by 74%.

(b) Considering the medium and low speed conditions, it was observed that,

$$\frac{q_{med}}{q_{lo}} = 1.2 \qquad \frac{\left(\dot{m}_{air}\right)_{med}}{\left(\dot{m}_{air}\right)_{lo}} = 1.5.$$

To find the outlet air temperature for the medium speed condition,

$$\frac{q_{med}}{q_{lo}} = \frac{\left[\dot{m}_{air} c_{p,c}\left(T_{c,o} - T_{c,i}\right)\right]_{med}}{\left[\dot{m}_{air} c_{p,c}\left(T_{c,o} - T_{c,i}\right)\right]_{lo}}$$

$$1.2 = \frac{1.5\ \dot{m}_{air} c_{p,c}\left(T_{c,o} - 0°C\right)}{\dot{m}_{air} c_{p,c}\left(30 - 0°C\right)} \qquad T_{c,o} = 24°C.$$ <

PROBLEM 11.28

KNOWN: Counterflow heat exchanger formed by two brazed tubes with prescribed hot and cold fluid inlet temperatures and flow rates.

FIND: Outlet temperature of the air.

SCHEMATIC:

D_1=10mm
Water
Nickel tubes, t=2mm, L=40m
K_t'=100W/m·K
Air
D_2=30mm
$T_{h,i}$=85°C
Water
$\dot{m}_h$= 0.04kg/s
T
x
$\dot{m}_c$= 0.12kg/s
Air
$T_{c,i}$=23°C

ASSUMPTIONS: (1) Negligible loss/gain from tubes to surroundings, (2) Negligible changes in kinetic and potential energy, (3) Flow in tubes is fully developed since L/D_h = 40 m/0.030m = 1333.

PROPERTIES: *Table A-6*, Water ($\overline{T}_h$ = 335 K): $c_h = c_{p,h}$ = 4186 J/kg·K, $\mu = 453 \times 10^{-6}$ N·s/m^2, k = 0.656 W/m·K, Pr = 2.88; *Table A-4*, Air (300 K): $c_c = c_{p,c}$ = 1007 J/kg·K, $\mu = 184.6 \times 10^{-7}$ N·s/m^2, k = 0.0263 W/m·K, Pr = 0.707; *Table A-1*, Nickel ($\overline{T}$ = (23 + 85)°C/2 = 327 K): k = 88 W/m·K.

ANALYSIS: Using the NTU - ε method, from Eq. 11.30a,

$$e = \frac{1-\exp\left[-NTU\left(1-C_r\right)\right]}{1-C_r\exp\left[-NTU\left(1-C_r\right)\right]} \qquad NTU = UA/C_{min} \qquad C_r = C_{min}/C_{max}. \qquad (1,2,3)$$

Estimate UA from a model of the tubes and flows, and determine the outlet temperature from the expression

$$e = C_c\left(T_{c,o} - T_{c,i}\right)/C_{min}\left(T_{h,i} - T_{c,i}\right). \qquad (4)$$

Water-side: $$Re_D = \frac{4\dot{m}_h}{p D m} = \frac{4 \times 0.04\ kg/s}{p \times 0.010m \times 453 \times 10^{-6}\ N\cdot s/m^2} = 11,243.$$

The flow is turbulent and since fully developed, use the Dittus-Boelter correlation,

$$\overline{Nu}_h = \overline{h}_h D/k = 0.023 Re_D^{0.8} Pr^{0.3} = 0.023(11,243)^{0.8}(2.88)^{0.3} = 54.99$$

$$\overline{h}_h = 54.99 \times 0.656\ W/m{\cdot}K / 0.01m = 3,607\ W/m^2\cdot K.$$

Air-side: $$Re_D = \frac{4\dot{m}_c}{p D m} = \frac{4 \times 0.120\ kg/s}{p \times 0.030m \times 184.6 \times 10^{-7}\ N\cdot s/m^2} = 275,890.$$

The flow is turbulent and since fully developed, again use the correlation

$$\overline{Nu}_c = \overline{h}_c D/K = 0.023 Re_D^{0.8} Pr^{0.4} = 0.023(275,890)^{0.8}(0.707)^{0.4} = 450.9$$

$$\overline{h}_c = 450.9 \times 0.0263\ W/m\cdot K/0.030m = 395.3\ W/m^2\cdot K.$$

Overall coefficient: From Eq. 11.1, considering the temperature effectiveness of the tube walls and the thermal conductance across the brazed region,

Continued …..

PROBLEM 11.28 (Cont.)

$$\frac{1}{UA} = \frac{1}{(h_o hA)_h} + \frac{1}{K'_t L} + \frac{1}{(h_o hA)_c} \qquad (5)$$

where η_o needs to be evaluated for each of the tubes.

Water-side temperature effectiveness: $A_h = p D_h L = p\,(0.010\text{m})\,40\text{m} = 1.257\text{m}^2$

$$h_{o,h} = h_{f,h} = \tanh(mL_h)/mL_h \qquad m = \left(\bar{h}_h P/kA\right)^{1/2} = \left(h_h/kt\right)^{1/2}$$

$$m = \left(3607\ \text{W/m}^2\cdot\text{K}/88\ \text{W/m}\cdot\text{K}\times 0.002\text{m}\right)^{1/2} = 143.2\text{m}^{-1}$$

and with $L_h = 0.5\ \pi D_h$, $\eta_{o,h} = \tanh(143.2\ \text{m}^{-1} \times 0.5\ \pi \times 0.010\text{m})/143.2\ \text{m}^{-1} \times 0.5\ \pi \times 0.010\ \text{m} = 0.435$.

Air-side temperature effectiveness: $A_c = \pi D_c L = \pi(0.030\text{m})40\text{m} = 3.770\ \text{m}^2$

$$h_{o,c} = h_{f,c} = \tanh(mL_c)/mL_c \quad m = \left(395.3\ \text{W/m}^2\cdot\text{K}/88\ \text{W/m}\cdot\text{K}\times 0.002\text{m}\right)^{1/2} = 47.39\text{m}^{-1}$$

and with $L_c = 0.5\pi D_c$, $\eta_{o,c} = \tanh(47.39\ \text{m}^{-1} \times 0.5\ \pi \times 0.030\text{m})/47.39\ \text{m}^{-1} \times 0.5\ \pi \times 0.030\text{m} = 0.438$.

Hence, the *overall heat transfer coefficient* using Eq. (5) is

$$\frac{1}{UA} = \frac{1}{0.435\times 3607\ \text{W/m}^2\cdot\text{K}\times 1.257\text{m}^2} + \frac{1}{100\ \text{W/m K}(40\text{m})} + \frac{1}{0.438\times 395.3\ \text{W/m}^2\cdot\text{K}\times 3.770\text{m}^2}$$

$$UA = \left[5.070\times 10^{-4} + 2.50\times 10^{-4} + 1.533\times 10^{-3}\right]^{-1}\ \text{W/K} = 437\ \text{W/K}.$$

Evaluating now the *heat exchanger effectiveness* from Eq. (1) with

$$\left.\begin{array}{l} C_h = \dot{m}_h c_h = 0.040\text{kg/s}\times 4186\ \text{J/kg}\cdot\text{K} = 167.4\ \text{W/K} \leftarrow C_{max} \\ C_c = \dot{m}_c c_c = 0.120\text{kg/s}\times 1007\ \text{J/kg}\cdot\text{K} = 120.8\ \text{W/K} \leftarrow C_{min} \end{array}\right\} C_r = C_{min}/C_{max} = 0.722$$

$$NTU = \frac{UA}{C_{min}} = \frac{437\ \text{W/K}}{120.8\ \text{W/K}} = 3.62 \quad e = \frac{1-\exp[-3.62(1-0.722)]}{1-0.722\ \exp[-3.62(1-0.722)]} = 0.862$$

and finally from Eq. (4) with $C_{min} = C_c$,

$$0.862 = \frac{C_c\left(T_{c,o} - 23°\text{C}\right)}{C_c(85-23)°\text{C}} \qquad T_{c,o} = 76.4°\text{C} \qquad <$$

COMMENTS: (1) Using overall energy balances, the water outlet temperature is

$$T_{h,o} = T_{h,i} + (C_c/C_h)\left(T_{c,o} - T_{c,i}\right) = 85°\text{C} - 0.722(76.4-23)°\text{C} = 46.4°\text{C}.$$

(2) To initially evaluate the properties, we assumed that $\bar{T}_h \approx 335$ K and $\bar{T}_c \approx 300$ K. From the calculated values of $T_{h,o}$ and $T_{c,o}$, more appropriate estimates of $\bar{T}_h$ and $\bar{T}_c$ are 338 K and 322 K, respectively. We conclude that proper thermophysical properties were used for water but that the estimates could be improved for air.

PROBLEM 11.29

KNOWN: Twin-tube counterflow heat exchanger with balanced flow rates, $\dot{m} = 0.003$ kg/s. Cold airstream enters at 280 K and must be heated to 340 K. Maximum allowable pressure drop of cold airstream is 10 kPa.

FIND: (a) Tube diameter D and length L which satisfies the heat transfer and pressure drop requirements, and (b) Compute and plot the cold stream outlet temperature $T_{c,o}$, the heat rate q, and pressure drop Δp as a function of the balanced flow rate from 0.002 to 0.004 kg/s.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible heat loss to surroundings, (3) Average pressure of the airstreams is 1 atm, (4) Tube walls act as fins with 100% efficiency, (4) Fully developed flow.

PROPERTIES: *Table A.4*, Air ($\overline{T}_m$ = 310 K, 1 atm) : ρ = 1.128 kg/m^3, c_p = 1007 J/kg·K, $\mu = 18.93 \times 10^{-6}$ m^2 / s, k = 0.0270 W/m·K, Pr = 0.7056.

ANALYSIS: (a) The heat exchanger diameter D and length L can be specified through two analyses: (1) heat transfer based upon the effectiveness-NTU method to meet the cold air heating requirement and (2) pressure drop calculation to meet the requirement of 10 kPa. The *heat transfer analysis* begins by determining the effectiveness from Eq. 11.22, since $C_{min} = C_{max}$ and $C_r = 1$,

$$\varepsilon = \frac{q}{q_{max}} = \frac{C\left(T_{c,o} - T_{c,i}\right)}{C\left(T_{h,i} - T_{c,i}\right)} = \frac{(340-280)\text{K}}{(360-280)\text{K}} = 0.750 \qquad (1)$$

From Table 11.4, Eq. 11.29b for $C_r = 1$,

$$\text{NTU} = \frac{\varepsilon}{1-\varepsilon} = \frac{0.750}{1-0.750} = 3 \qquad (2)$$

where NTU, following its definition, Eq. 11.25, is

$$\text{NTU} = \frac{\overline{U}A}{C_{min}} \qquad (3)$$

with

$$C_{min} = \dot{m}c_p = 0.003\,\text{kg/s} \times 1007\,\text{J/kg}\cdot\text{K} = 3.021\,\text{K/W} \qquad (4)$$

Continued...

and $1/\bar{U}A$ represents the thermal resistance between the two fluids at $T_{m,h}$ and $T_{m,c}$ as illustrated in the above-right schematic. Since the tube walls are isothermal, it follows that

$$1/UA = 1/\bar{h}_c A + 1/\bar{h}_h A \quad (5)$$

and since the flow conditions are nearly identical $\bar{h}_c = \bar{h}_h$ so that

$$U = 0.5\bar{h} \quad (6)$$

where the heat transfer area is

$$A = \pi DL \quad (7)$$

Hence, Eq. (3) can now be expressed as

$$3 = \frac{0.5\bar{h}(\pi DL)}{3.021\,K/W}$$

$$\bar{h}DL = 5.7697 \quad (8)$$

Assuming an average mean temperature $\bar{T}_{m,c} = 310\,K$, characterize the flow with

$$Re_D = \frac{4\dot{m}}{\pi D\mu} = \frac{4\times 0.003\,kg/s}{\pi\times D\times 18.93\times 10^{-6}\,m^2/s} = \frac{201.78}{D} \quad (9)$$

and assuming the flow is both turbulent and fully developed using the Dittus-Boelter correlation, Eq. 8.57, with n = 0.4,

$$\overline{Nu}_D = \frac{\bar{h}D}{k} = 0.023\,Re_D^{0.8}\,Pr^{0.4}$$

$$\bar{h}D = 0.023\times 0.0270\,W/m\cdot K\,(201.78/D)^{0.8}\,(0.7056)^{0.4}$$

$$\bar{h}D^{1.8} = 0.0377 \quad (10)$$

The *pressure drop* for fully developed flow, Eq. 8.22a, is

$$\Delta p = f\,\frac{\rho u_m^2}{2D}\,L \quad (11)$$

where the mean velocity is $u_m = \dot{m}/(\rho\pi D^2/4)$ so that

$$\Delta p = f\,\frac{4\rho\left(\dot{m}/\rho\pi D^2\right)L}{2D} = \frac{2}{\pi^2}\,f\,\frac{m^2\rho^2 L}{D^5}$$

$$\Delta p = \frac{2}{\pi^2}\,f\,\frac{(0.003\,kg/s)^2\left(1.128\,kg/m^3\right)L}{D^5} = 2.3206\times 10^{-6}\,f\,L\,D^{-5} \quad (12)$$

Recall that the pressure drop requirement is $\Delta p = 10\ kPa = 10^4\ N/m^2$, so that Eq. (12) can be rewritten as

$$fLD^{-5} = 4.3092\times 10^{10} \quad (13)$$

Continued...

PROBLEM 11.29 (Cont.)

For the Reynolds number range, $3000 \le Re_D \le 5 \times 10^6$, Eq. 8.21 provides an estimate for the friction factor,

$$f = \left[\left(0.790\ell n\left(Re_D\right) - 1.64\right)\right]^{-2}$$

$$f = \left[\left(0.790\ell n\left(201.78/D\right) - 1.46\right)\right]^{-2} \qquad (14)$$

In the foregoing analysis, there are 4 unknowns (D, L, f, $\overline{h}$) and 4 equations (8, 10, 13, 14). Using the IHT workspace, find

$$D = 8.96\,\text{mm} \quad L = 3.52\,\text{m} \quad f = 0.02538 \quad \overline{h} = 182.9\,\text{W/m}^2 \cdot \text{K}$$

For this configuration, Re_D = 22,520 so the flow is turbulent and since L/D = 3.52/0.00896 = 390 >> 10, the fully developed assumption is reasonable.

(b) The foregoing analysis entered into the IHT workspace was used to determine $T_{c,o}$, q and Δp as a function of the balanced flow rate, $\dot{m}$.

The outlet temperature of the cold air, $T_{c,o}$, is nearly insensitive to the flow rate. It follows that the heat rate, q, must be nearly proportional to the flow rate as can be seen in the q vs. $\dot{m}$ plot above. The pressure drop varies with the mean velocity squared.

PROBLEM 11.30

KNOWN: Dimensions and thermal conductivity of twin-tube, counterflow heat exchanger. Contact resistance between tubes. Air inlet conditions for one tube and pressure of saturated steam in other tube.

FIND: Air outlet temperature and condensation rate.

SCHEMATIC:

ASSUMPTIONS: (1) Negligible heat exchange with surroundings, (2) Negligible kinetic and potential energy and flow work changes, (3) Fully developed air flow, (4) Negligible fouling, (5) Constant properties.

PROPERTIES: *Table A-4*, air $\left(\bar{T}_c \approx 325\,K,\ p = 5\,atm\right)$: c_p = 1008 J/kg·K, $\mu = 196.4 \times 10^{-7}$ N·s/m^2, k = 0.0281 W/m·K, Pr = 0.703. *Table A-6*, sat. steam (p = 2.455 bar): $T_{h,i} = T_{h,o}$ = 400 K, h_{fg} = 2183 kJ/kg.

ANALYSIS: With $C_{max} \to \infty$, $C_r = 0$ and Eqs. 11.22 and 11.36a yield

$$\varepsilon = \frac{T_{c,o} - T_{c,i}}{T_{h,i} - T_{c,i}} = 1 - \exp(-NTU) \tag{1}$$

From Eq. 11.1,

$$\frac{1}{UA} = \frac{1}{(\eta_o hA)_c} + \frac{R'_t}{L} + \frac{1}{(\eta_o hA)_h} \tag{2}$$

With $Re_D = 4\dot{m}/\pi D_i \mu = 0.12\ kg/s/\pi(0.05m)196.4\times10^{-7}\ N\cdot s/m^2 = 38,900,$ the air flow is turbulent and the Dittus-Boelter correlation yields

$$h_c \approx h_{fD} = \left(\frac{k}{D_i}\right) 0.023\ Re_D^{4/5}\ Pr^{0.4} = \left(\frac{0.0281\ W/m\cdot K}{0.05m}\right) 0.023(38,900)^{4/5}(0.703)^{0.4} = 52.7\ W/m^2\cdot K$$

As shown on the inset, each tube wall may be modelled as two fins, each of length $L_f \approx \pi D_i/2$ = 0.0785m. The total surface area for heat transfer is $A_t = \pi D_i L$ = 0.785 m^2 = A_c, which is equivalent to the surface area of the fins. With $NA_f = A_t$ from Eq. 3.102, $\eta_o = \eta_f$. Because the outer surface of the tube is insulated, a wall thickness of 2t must be used in evaluating η_f. With $m = (2h/k \times 2t)^{1/2} = (h/kt)^{1/2} = [52.7\ W/m^2\cdot K/(60\ W/m\cdot K \times 0.004m)]^{1/2} = 14.8\ m^{-1}$, $L_c = L_f$ for an adiabatic tip, and mL_f = 1.163, Eq. 3.89 yields

$$\eta_f = \frac{\tanh\ mL_f}{mL_f} = \frac{0.821}{1.163} = 0.706 = \eta_{o,c}$$

Continued …..

Similarly, for the steam tube, $m = (h/kt)^{1/2} = [5,000\ W/m^2 \cdot K/(60\ W/m \cdot K \times 0.004m)]^{1/2} = 144.3\ m^{-1}$ and $mL_f = 11.33$. Hence,

$$\eta_f = \frac{\tanh\ mL_f}{mL_f} = \frac{1.00}{11.33} = 0.088 = \eta_{o,h}$$

Substituting into Eq. (2),

$$UA = \left[\frac{1}{0.706 \times 52.7 \times 0.785} + \frac{0.01}{5} + \frac{1}{0.088 \times 5000 \times 0.785}\right]^{-1} \frac{W}{K} = 25.6 \frac{W}{K}$$

Hence, with $C_{min} = \left(\dot{m}\, c_p\right)_c = 0.03\ kg/s \times 1008\ J/kg \cdot K = 30.2\ W/K$, $NTU = UA/C_{min} = 0.847$ and $\varepsilon = 1 - \exp(-NTU) = 0.571$. From Eq. (1), the air outlet temperature is then

$$T_{c,o} = T_{c,i} + \varepsilon\left(T_{h,i} - T_{c,i}\right) = 17°C + 0.571(127 - 17)°C = 79.8°C \qquad <$$

The rate of heat transfer to the air is

$$q = \dot{m}\, c_p\left(T_{c,o} - T_{c,i}\right) = 0.03\,kg/s \times 1008\,J/kg \cdot K \times 62.8°C = 1900\,W$$

and the rate of condensation is

$$\dot{m}_{cond} = \frac{q}{h_{fg}} = \frac{1900\,W}{2.183 \times 10^6\,J/kg} = 8.70 \times 10^{-4}\ kg/s \qquad <$$

COMMENTS: (1) With $\overline{T}_c = 321.4\,K$, the initial estimate of 325K is reasonable and iteration on the property values is not necessary, (2) The major contribution to the total thermal resistance is due to air-side convection, (3) The foregoing results are independent of air pressure.

PROBLEM 11.31

KNOWN: Tube inner and outer diameters and longitudinal and transverse pitches for a cross-flow heat exchanger. Number of tubes in transverse plane. Water and gas flow rates and inlet temperatures. Water outlet temperature.

FIND: (a) Gas outlet temperature and number of longitudinal tube rows, (b) Effect of gas flowrate and inlet temperature on fluid outlet temperatures.

SCHEMATIC:

$T_{c,o}$ = 350K (Part a)
W = 2m
$T_{h,o}$
Gases
$20 \le \dot{m}_h \le 40$ kg/s
$500 \le T_{h,i} \le 700$ K
H = 1.2m, N_T = 30
D_i = 15 mm, D_o = 20 mm
k = 60 W/m·K
$S_T = S_L$ = 40 mm
Water
$\dot{m}_c$ = 50 kg/s
$T_{c,i}$ = 290 K
h_c = 3000 W/m²·K

ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible kinetic and potential energy changes, (3) Constant properties, (4) Negligible fouling.

PROPERTIES: *Table A.6*, Water ($\bar{T}_c$ = 320 K: c_p = 4180 J/kg·K, $\mu = 577 \times 10^{-6}$ N·s/m², k_f = 0.640 W/m·K, Pr = 3.77; *Table A.4*, Air ($\bar{T}_h \approx 550$ K): c_p = 1040 J/kg·K, $\mu = 288.4 \times 10^{-7}$ N·s/m², k = 0.0439 W/m·K, Pr = 0.683, ρ = 0.633 kg/m³.

ANALYSIS: (a) The required heat transfer rate is

$$q = \dot{m}_c c_{p,c}\left(T_{c,o} - T_{c,i}\right) = 50\,\text{kg/s}\left(4180\,\text{J/kg}\cdot\text{K}\right)60\,\text{K} = 1.254\times10^7\,\text{W}.$$

Hence, with $T_{h,o} = T_{h,i} - q/\dot{m}_h c_{p,h}$,

$$T_{h,o} = 700\,\text{K} - 1.254\times10^7\,\text{W}/\left(40\,\text{kg/s}\times1040\,\text{J/kg}\cdot\text{K}\right) = 398.6\,\text{K}$$ <

Use the ε - NTU method to compute the hot side HX surface area, A_H. To calculate U_h, we must find h_h. For the tube bank, S_D = 44.7 mm > $(S_T + D)/2$ = 30 mm. Hence, with $\rho V_{max} = \left[S_T/(S_T - D_o)\right]\rho V = \left[S_T/(S_T - D_o)\right]\left(\dot{m}_h/WH\right)$,

$$\rho V_{max} = (40/20)\left[40\,\text{kg/s}/(2\times1.2)\text{m}^2\right] = 33.3\,\text{kg/s}\cdot\text{m}^2$$

$$Re_{D,max} = \left(\rho V_{max} D_o\right)/\mu = \left[33.3\,\text{kg/s}\cdot\text{m}^2\,(0.02\,\text{m})\right]/288.4\times10^{-7}\,\text{N}\cdot\text{s/m}^2 = 23{,}116.$$

From the Zhukauskas correlation, with $(Pr/Pr_s) \approx 1$, and Table 7.7,

$$\overline{Nu}_D = 0.35\,Re_D^{0.6}\,Pr^{0.36} = 0.35(23{,}116)^{0.6}(0.683)^{0.36} = 127$$

where it is assumed that $N_L > 20$. Hence,

$$h_h = \overline{Nu}_D\left(k/D_o\right) = 127\left(0.0439\,\text{W/m}\cdot\text{K}/0.02\,\text{m}\right) = 279\,\text{W/m}^2\cdot\text{K}.$$

From Eq. 11.1,

Continued...

$$\frac{1}{U_h}=\frac{1}{h_c}\frac{D_o}{D_i}+\frac{D_o\ln(D_o/D_i)}{2k}+\frac{1}{h_h}=\frac{1}{3000\,W/m^2\cdot K}\frac{20}{15}+\frac{0.02\,m\ln(20/15)}{60\,W/m\cdot K}+\frac{1}{279\,W/m^2\cdot K}$$

$$\frac{1}{U_h}=\left(4.44\times10^{-4}+9.59\times10^{-5}+3.58\times10^{-3}\right)m^2\cdot K/W=4.12\times10^{-3}m^2\cdot K/W$$

$$U_h=243\,W/m^2\cdot K.$$

With $C_h = C_{min} = 4.160 \times 10^4$ W/K and $C_c = C_{max} = 2.09 \times 10^5$ W/K, $C_{min}/C_{max} = 0.199$ and $q_{max} = C_{min}(T_{h,i} - T_{c,i}) = 4.16 \times 10^4$ W/K(410 K) = 1.71×10^7 W. Hence, $\varepsilon = (q/q_{max}) = (1.254 \times 10^7\ W/1.71 \times 10^7\ W) = 0.735$. With C_{min} mixed and C_{max} unmixed, Eq. 11.35b gives NTU = 1.54 and

$$A_h = NTU\left(C_{min}/U_h\right)=1.54\left(4.160\times10^4\ W/K\big/243\,W/m^2\cdot K\right)=264\,m^2.$$

Hence, $N_L=\dfrac{A_h}{(\pi D_o W)N_T}=\dfrac{264\,m^2}{\pi(0.02)2(30)m^2}=70$ <

(b) Using the IHT *Correlations*, *Heat Exchangers* and *Properties* Toolpads to perform the parametric calculations, we obtain the following results for $N_L = 90$.

Water outlet temperature, Tco(K)
Gas flowrate, mdoth (kg/s)
Thi = 700 K
Thi = 600 K
Thi = 500 K

Gas outlet temperature, Tho(K)
Gas flowrate, mdoth (kg/s)
Thi = 700 K
Thi = 600 K
Thi = 500 K

Since h_h, and hence U_h, increases with $\dot{m}_h$, q, and hence, $T_{c,o}$, increases with increasing $\dot{m}_h$, as well as with increasing $T_{h,i}$. Although q increases with $\dot{m}_h$, the proportionality is not linear ($q \propto \dot{m}_h^a$, where a < 1) and $(T_{h,i} - T_{h,o})$ must decrease with increasing $\dot{m}_h$, in which case $T_{h,o}$ must increase. From the above results, it is clear that operation is restricted to $\dot{m}_h \geq 40$ kg/s and $T_{h,i} \geq 700$ K, if corrosion of the heat exchanger surfaces is to be avoided.

COMMENTS: To check the presumed value of $h_c = 3000\ W/m^2\cdot K$, compute

$$Re_D=\frac{4(\dot{m}_c/N)}{\pi D_i\mu}=\frac{4(50\,kg/s)/70\times30}{\pi(0.015\,m)577\times10^{-6}\,N\cdot s/m^2}=3500.$$

Hence, $Nu_D=0.023Re_D^{4/5}Pr^{0.4}=0.023(3500)^{4/5}(3.77)^{0.4}=26.8$

$$h_c=(k/D)Nu_D=(0.640\,W/m\cdot K/0.015\,m)26.8=1142\,W/m^2\cdot K.$$

Hence, the cold side convection coefficient has been overestimated and the calculations should be repeated using the smaller value of h_c.

PROBLEM 11.32

KNOWN: Single pass, cross-flow heat exchanger with hot exhaust gases (mixed) to heat water (unmixed)

FIND: Required surface area.

SCHEMATIC:

ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible kinetic and potential energy changes, (3) Exhaust gas properties assumed to be those of air.

PROPERTIES: *Table A-6*, Water ($\overline{T}_c$ = (80 + 30)°C/2 = 328 K): c_p = 4184 J/kg·K; *Table A-4,* Air (1 atm, $\overline{T}_h$ = (100 + 225)°C/2 = 436 K): c_p = 1019 J/kg·K.

ANALYSIS: The rate equation for the heat exchanger follows from Eqs. 11.14 and 11.18. The area is given as

$$A = q/U\Delta T_{\ell m} = q/UF\Delta T_{\ell m,CF} \quad (1)$$

where F is determined from Fig. 11.13 using

$$P = \frac{80-30}{225-30} = 0.26 \text{ and } R = \frac{225-100}{80-30} = 2.50 \text{ giving } F \approx 0.92. \quad (2)$$

From an energy balance on the cold fluid, find

$$q = \dot{m}_c c_c \left(T_{c,o} - T_{c,i}\right) = 3\frac{kg}{s} \times 4184\frac{J}{kg \cdot K}(80-30)K = 627{,}600 \text{ W}. \quad (3)$$

From Eq. 11.15, the LMTD for counter-flow conditions is

$$\Delta T_{\ell m,CF} = \frac{\Delta T_1 - \Delta T_2}{\ell n\left(\Delta T_1/\Delta T_2\right)} = \frac{(225-80)-(100-30)}{\ell n(145/70)}°C = 103.0°C. \quad (4)$$

Substituting numerical values resulting from Eqs. (2-4) into Eq. (1), find the required surface area to be

$$A = 627{,}600\,W/200\,W/m^2 \cdot K \times 0.92 \times 103.0K = 33.1m^2.$$ <

COMMENTS: Note that the properties of the exhaust gases were not needed in this method of analysis. If the ε-NTU method were used, find first C_h/C_c = 0.40 with $C_{min} = C_h$ = 5021 W/K. From Eqs. 11.19 and 11.20, with $C_h = C_{min}$, $\varepsilon = q/q_{max} = (T_{h,i} - T_{h,o})/(T_{h,i} - T_{c,i})$ = (225 – 100)/(225 – 30) = 0.64. Using Fig. 11.19 with C_{min}/C_{max} = 0.4 and ε = 0.64, find NTU = UA/C_{min} ≈ 1.4. Hence,

$$A = NTU \cdot C_{min}/U \approx 1.4 \times 5021 W/K/200\,W/m^2 \cdot K = 35.2m^2.$$

Note agreement with above result.

PROBLEM 11.33

KNOWN: Concentric tube heat exchanger operating in parallel flow (PF) conditions with a thin-walled separator tube of 100-mm diameter; fluid conditions as specified.

FIND: (a) Required length for the exchanger; (b) Convection coefficient for water flow, assumed to be fully developed; (c) Compute and plot the heat transfer rate, q, and fluid inlet temperatures, $T_{h,o}$ and $T_{c,o}$, as a function of the tube length for $60 \le L \le 400$ m with the PF arrangement and overall coefficient $\left(U = 200\,W/m^2 \cdot K\right)$, inlet temperatures ($T_{h,i}$ = 225°C and $T_{c,i}$ = 30°C), and fluid flow rates from Problem 11.23; (d) Reduction in required length relative to the value found in part (a) if the exchanger were operated in the counterflow (CF) arrangement; and (e) Compute and plot the effectiveness and fluid outlet temperatures as a function of tube length for $60 \le L \le 400$ m for the CF arrangement of part (c).

SCHEMATIC:

ASSUMPTIONS: (1) No losses to surroundings, (2) Negligible kinetic and potential energy changes, (3) Separation tube has negligible thermal resistance, (4) Water flow is fully developed, (5) Constant properties, (6) Exhaust gas properties are those of atmospheric air.

PROPERTIES: *Table A-4*, Hot fluid, Air (1 atm, $\overline{T}$ = (225 +100)°C /2 = 436 K): c_p = 1019 J/kg·K; *Table A-6*, Cold fluid, Water $\overline{T}$ = (30 + 80)°C /2 ≈ 328 K): $\rho = 1/v_f$ = 985.4 kg/m^3, c_p = 4183 J/kg·K, k = 0.648 W/m·K, $\mu = 505 \times 10^{-6}$ N·s/m^2, Pr = 3.58.

ANALYSIS: (a) From the rate equation, Eq. 11.14, with A = πDL, the length of the exchanger is

$$L = q/U \cdot \pi D \cdot \Delta T_{\ell n,PF} \,. \qquad (1)$$

The heat rate follows from an energy balance on the cold fluid, using Eq. 11.7, find

$$q = \dot{m}_c c_c \left(T_{c,o} - T_{c,i}\right) = 3\,kg/s \times 4183\,J/kg \cdot K\,(80 - 30)\,K = 627.5 \times 10^3\,W \,.$$

Using an energy balance on the hot fluid, find $\dot{m}_h$ for later use.

$$\dot{m}_h = q/c_h \left(T_{h,i} - T_{h,o}\right) = 627.5 \times 10^3\,W/1019\,J/kg \cdot K\,(225 - 100)\,K = 4.93\,kg/s \qquad (2)$$

For parallel flow, Eqs. 11.15 and 11.16,

$$\Delta T_{\ell m,PF} = \frac{\Delta T_1 - \Delta T_2}{\ell n \Delta T_1/\Delta T_2} = \frac{(225-30)^\circ C - (100-80)^\circ C}{\ell n\,(225-30)/(100-80)} = 76.8^\circ C \,.$$

Substituting numerical values into Eq. (1), find

$$L = 627.5 \times 10^3\,W/200\,W/m^2 \cdot K\,(\pi \times 0.1m)\,76.8K = 130m \,.$$

<

Continued...

PROBLEM 11.33 (Cont.)

(b) Considering the water flow within the separator tube, from Eq. 8.6,

$$\mathrm{Re_D} = 4\dot{m}/\pi D\mu = 4\times 3\,\mathrm{kg/s}\Big/\left(\pi\times 0.1\mathrm{m}\times 505\times 10^{-6}\ \mathrm{N/s\cdot m^2}\right) = 75,638\,.$$

Since $R_{eD} > 2300$, the flow is turbulent and since flow is assumed to be fully developed, use the Dittus-Boelter correlation with n = 0.4 for heating,

$$\mathrm{Nu_D} = 0.023\,\mathrm{Re_D^{0.8}}\,\mathrm{Pr^{0.4}} = 0.023(75,638)^{0.8}(3.58)^{0.4} = 306.4$$

$$h = \mathrm{Nu_D}\frac{k}{D} = 306.4\times 0.648\,\mathrm{W/m\cdot K}\big/(0.1\mathrm{m}) = 1985\,\mathrm{W\big/m^2\cdot K}\,.$$ <

(c) Using the *IHT Heat Exchanger Tool*, *Concentric Tube*, *Parallel Flow*, *Effectiveness relation*, and the *Properties Tool* for *Water* and *Air*, a model was developed for the PF arrangement. With U = 200 $W/m^2\cdot K$ and prescribed inlet temperatures, $T_{h,i}$ = 225°C and $T_{c,i}$ = 30°C, the outlet temperatures, $T_{h,o}$ and $T_{c,o}$ and heat rate, q, were computed as a function of tube length L.

Parallel flow arrangement

Temperature (C) or Heat rate (W*10^-4)

140
115
90
65
40

60 120 180 240 300 360

Tube length, L (m)

Cold outlet temperature, Tco (C)
Hot outlet temperature, Tho (C)
Heat rate, q*10^-4 (W)

As the tube length increases, the outlet temperatures approach one another and eventually reach $T_{h,o} = T_{c,o}$ = 85.6°C.

(d) If the exchanger as for part (a) is operated in counterflow (rather than parallel flow), the log mean temperature difference is

T
CF
$T_{h,i}$ = 225 °C
$T_{h,o}$ = 100 °C
ΔT_1
$T_{c,o}$ = 80 °C
ΔT_2
$T_{c,i}$ = 30 °C
1
2

$$\Delta T_{\ell m,CF} = \frac{\Delta T_i - \Delta T_2}{\ell n\,\Delta T_1/\Delta T_2}$$

$$\Delta T_{\ell m,CF} = \frac{(225-80)-(100-30)}{\ell n\,(225-80)\big/100-30} = 103.0^\circ\mathrm{C}\,.$$

Using Eq. (1), the required length is

$$L = 627.5\times 10^3\ \mathrm{W}/200\,\mathrm{W\big/m^2\cdot K}\times\pi\times 0.1\mathrm{m}\times 103.0\,\mathrm{K} = 97\,\mathrm{m}\,.$$

The reduction in required length of CF relative to PF operation is

Continued...

PROBLEM 11.33 (Cont.)

$$\Delta L = \left(L_{PF} - L_{CF}\right)/L_{PF} = (103 - 97)/103 = 5.8\%$$ <

(e) Using the *IHT Heat Exchanger Tool*, *Concentric Tube*, *Counterflow*, *Effectiveness relation*, and the *Properties Tool* for *Water* and *Air*, a model was developed for the CF arrangement. For the same conditions as part (c), but CF rather than PF, the effectiveness and fluid outlet temperatures were computed as a function of tube length L.

Counterflow arrangement

Tho, Tco (C) or eps*100

140
120
100
80
60
40
20

60 120 180 240 300 360

Tube length, L (m)

Cold outlet temperature, Tco (C)
Hot outlet temperature, Tho (C)
Effectiveness, eps*100

Note that as the length increases, the effectiveness tends toward unity, and the hot fluid outlet temperature tends toward $T_{c,i}$ = 30°C. Remember the heat rate for an infinitely long CF heat exchanger is q_{max} and the minimum fluid (hot in our case) experiences the temperature change, $T_{h,i} - T_{c,i}$.

COMMENTS: (1) As anticipated, the required length for CF operations was less than for PF operation.

(2) Note that U is substantially less than h_i implying that the gas-side coefficient must be the controlling thermal resistance.

PROBLEM 11.34

KNOWN: Cross-flow heat exchanger (both fluids unmixed) cools blood to induce body hypothermia using ice-water as the coolant.

FIND: (a) Heat transfer rate from the blood, (b) Water flow rate, $\dot{\forall}_c$ (liter/min), (c) Surface area of the exchanger, and (d) Calculate and plot the blood and water outlet temperatures as a function of the water flow rate for the range, $2 \leq \dot{\forall} \leq 4$ liter/min, assuming all other parameters remain unchanged.

SCHEMATIC:

$T_{h,i} = 37^oC$ $\quad \dot{\forall}_h = 5$ liter/min $\quad T_{h,o} = 25^oC$

$T_{c,o} = 15^oC$ $\quad \dot{\forall}_c$ $\quad T_{c,i} = 0^oC$

Cross-flow hxer
(both fluids unmixed)
$U = 750\ W/m^2\text{-}K$

ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible losses to the surroundings and kinetic and potential energy changes, (3) Overall heat transfer coefficient remains constant with water flow rate changes, and (4) Constant properties.

PROPERTIES: *Table A-6*, Water $\left(\overline{T}_c = 280\text{K}\right)$, $\rho = 1000\ \text{kg/m}^3$, $c = 4198\ \text{J/kg}\cdot\text{K}$. Blood (given): $\rho = 1050\ \text{kg/m}^3$, $c = 3740\ \text{J/kg}\cdot\text{K}$.

ANALYSIS: (a) The heat transfer rate from the blood is calculated from an energy balance on the hot fluid,

$$\dot{m}_h = \rho_h \dot{\forall}_h = 1050\ \text{kg/m}^3 \times \left(5\ \text{liter/min} \times 1\ \text{min}/60\ \text{s}\right) \times 10^{-3}\text{m}^3/\text{liter} = 0.0875\ \text{kg/s}$$

$$q = \dot{m}_h c_h \left(T_{h,i} - T_{h,o}\right) = 0.0875\ \text{kg/s} \times 3740\ \text{J/kg}\cdot\text{K}\left(37-25\right)\text{K} = 3927\ \text{W} \qquad < \quad (1)$$

(b) From an energy balance on the cold fluid, find the coolant water flow rate,

$$q = \dot{m}_c\, c_c \left(T_{c,o} - T_{c,i}\right) \qquad (2)$$

$$3927\ \text{W} = \dot{m}_c \times 4198\ \text{J/kg}\cdot\text{K}\left(15-0\right)\text{K} \qquad \dot{m}_c = 0.0624\ \text{kg/s}$$

$$\dot{\forall}_c = \dot{m}_c/\rho_c = 0.0624\ \text{kg/s}/1000\ \text{kg/m}^3 \times 10^3\text{liter/m}^3 \times 60\ \text{s/min} = 3.74\ \text{liter/min} \qquad <$$

(c) The surface area can be determined using the effectiveness-NTU method. The capacity rates for the exchanger are

$$C_h = \dot{m}_h c_h = 327\ \text{W/K} \qquad C_c = \dot{m}_c\, c_c = 262\ \text{W/K} \qquad C_{min} = C_c \qquad (3, 4, 5)$$

From Eq. 11.19 and 11.20, the maximum heat rate and effectiveness are

Continued …..

PROBLEM 11.34 (Cont.)

$$q_{max} = C_{min}\left(T_{h,i} - T_{c,i}\right) = 262\ \text{W/K}\ (37 - 0)\text{K} = 9694\ \text{W} \tag{6}$$

$$\varepsilon = q / q_{max} = 3927 / 9694 = 0.405 \tag{7}$$

For the cross flow exchanger, with both fluids unmixed, substitute numerical values into Eq. 11.33 to find the number of transfer units, NTU, where $C_r = C_{min} / C_{max}$.

$$\varepsilon = 1 - \exp\left[(1/C_r)\text{NTU}^{0.22}\left\{\exp\left[-C_r \text{NTU}^{0.78}\right] - 1\right\}\right] \tag{8}$$

$$\text{NTU} = 0.691$$

From Eq. 11.25, find the surface area, A.

$$\text{NTU} = UA / C_{min}$$

$$A = 0.691 \times 262\ \text{W/K} / 750\ \text{W/m}^2 \cdot \text{K} = 0.241\ \text{m}^2$$ <

(d) Using the foregoing equations in the *IHT* workspace, the blood and water outlet temperatures, $T_{h,o}$ and $T_{c,o}$, respectively, are calculated and plotted as a function of the water flow rate, all other parameters remaining unchanged.

Outlet temperatures for blood flow rate 5 liter/min

Temperature, Tco and Tho (C)

28
26
24
22
20
18
16
14

2
2.5
3
3.5
4

Water flow rate, Qc (liter/min)

Water outlet temperature, Tco
Blood outlet temperature, Tho

From the graph, note that with increasing water flow rate, both the blood and water outlet temperatures decrease. However, the effect of the water flow rate is greater on the water outlet temperature. This is an advantage for this application, since it is desirable to have the blood outlet temperature relatively insensitive to changes in the water flow rate. That is, if there are pressure changes on the water supply line or a slight miss-setting of the water flow rate controller, the outlet blood temperature will not change markedly.

$$h_i = \frac{k_f}{D_i} Nu_D = \frac{0.620\,W/m \cdot K}{13.4 \times 10^{-3}\,m} \times 130.9 = 6057\,W/m^2 \cdot K.$$

Substituting numerical values into Eq. (1), the overall heat transfer coefficient is

$$U_o = \left[\frac{1}{13,500\,W/m^2 \cdot K} + \frac{\left(15.9 \times 10^{-3}\,m\right)/2}{115\,W/m \cdot K} \ell n \frac{15.9}{13.4} + \frac{15.9}{13.4} \times \frac{1}{6057\,W/m^2 \cdot K} \right]^{-1}$$

$$U_o = \left[7.407 \times 10^{-5} + 1.183 \times 10^{-5} + 19.590 \times 10^{-5} \right]^{-1} W/m^2 \cdot K = 3549\,W/m^2 \cdot K. \quad <$$

To find the outlet temperature of the water, we'll employ the ε – NTU method. From an energy balance on the cold fluid,

$$T_{c,o} = T_{c,i} + q/C_c \tag{3}$$

where the heat rate can be expressed as

$$q = \varepsilon q_{max} \qquad q_{max} = C_{min}\left(T_{h,i} - T_{h,o}\right). \tag{4,5}$$

The minimum capacity rate is that of the cold water since $C_h \rightarrow \infty$. Evaluating, find

$$C_{min} = C_c = \left(\dot{m}c_p\right)_c = 22.8\,kg/s \times 4178\,J/kg \cdot K = 95,270\,W/K.$$

where

$$\dot{m}_c = \left(\rho A u_m\right) N = 995.0\,kg/m^3 \times \pi/4\left(0.0134 m\right)^2 \times 1.25\,m/s \times 130 = 22.8\,kg/s$$

To determine ε, use Fig. 11.16 (one shell and any multiple of tube passes) with

$$NTU = \frac{U_o A_o}{C_{min}} = \frac{3549\,W/m^2 \cdot K\left(\pi 0.0159 m \times 2 m \times 130 \times 2\right)}{95,270\,W/K} = 0.968$$

where 130 and 2 represent the number of tubes and passes, respectively, to find ε ≈ 0.62. Combining Eqs. (4) and (5) into Eq. (3), find

$$T_{c,o} = T_{c,i} + \varepsilon C_{min}\left(T_{h,i} - T_{c,i}\right)/C_c = 20^\circ C + 0.62\left(327 - 293\right)K = 41.1^\circ C. \quad <$$

The condensation rate of the steam is given by

$$\dot{m}_h = q/h_{fg} \tag{6}$$

where the heat rate can be determined from Eq. (3) with $T_{c,o}$,

$$\dot{m}_h = C_c\left(T_{c,o} - T_{c,i}\right)/h_{fg} = 95,270\,W/K\left(41.1 - 20.0\right)K/2373 \times 10^3\,J/kg \cdot K = 0.85\,kg/s. \quad <$$

(b) Using the *IHT Heat Exchanger Tool*, *All Exchangers*, $C_r = 0$, and the *Properties Tool* for *Water*, a model was developed and the cold outlet temperature and condensation rate were computed and plotted.

Continued...

PROBLEM 11.35 (Cont.)

With increasing cold flow rate, the cold outlet temperature decreases as expected. The condensation rate increases with increasing cold flow rate. Note that $T_{c,o}$ and $\dot{m}_h$ are nearly linear with the cold flow rate.

COMMENTS: For part (a) analysis, note that the assumption $T_{c,o} \approx 44°C$ used for evaluation of the cold fluid properties was reasonable. Using the IHT model of part (b), we found $T_{c,o} = 40.2°C$ and $\dot{m}_h$ = 0.812 kg /s.

PROBLEM 11.36

KNOWN: Shell-and-tube (one shell, two tube passes) heat exchanger design. Water flow rate and inlet temperature. Steam pressure and convection coefficient.

FIND: (a) Water outlet temperature, $T_{c,o}$; (b) $T_{c,o}$ as a function of flow rate, $\dot{m}_c$, for the range, $5 \le \dot{m}_c \le 20$ kg/s, with all other conditions remaining the same, but accounting for changes in the overall coefficient, U; and (c) Plot $T_{c,o}$ on the same graph considering fouling factors of $R''_f = 0.0002$ and $0.0005\ m^2\cdot K/W$

SCHEMATIC:

ASSUMPTIONS: (1) Negligible heat loss to surroundings and kinetic and potential energy changes, (2) Negligible wall conduction and fouling resistances, (3) Constant properties.

PROPERTIES: *Table A-6*, Sat. water (p = 1.0133 bar): $T_{sat} = T = 373.1$ K; ($\bar{T}_c \approx 320$ K): $c_p = 4180$ J/kg·K, $\mu = 577 \times 10^{-6}\ N\cdot s/m^2$, k = 0.640 W/m·K, Pr = 3.77.

ANALYSIS: Using the NTU-effectiveness method, calculate for U by finding h_i. With

$$Re_D = 4\dot{m}/\pi D\mu = \left[4(10\,kg/s)/100\right]\Big/\left[\pi(0.02m)\left(577\times10^{-6}\ N\cdot s/m^2\right)\right] = 11,033 \qquad (1)$$

and using the Dittus-Boelter correlation,

$$Nu_D = 0.023 Re_D^{4/5} Pr^{0.4} = 0.023(11,033)^{4/5}(3.77)^{0.4} = 67.05 \qquad (2)$$

$$h_i = (k/D)Nu_D = (0.640\,W/m\cdot K/0.02m)67.05 = 2146\,W/m^2\cdot K.$$

From Eq. 11.5

$$1/U = 1/h_i + 1/h_o = \left[(1/10,000) + (1/2146)\right]m^2\cdot K/W = 5.66\times10^{-4}\,m^2\cdot K/W \qquad (3)$$

$$U = 1766\,W/m^2\cdot K.$$

The heat transfer surface area, capacity rates and NTU are

$$A = N(\pi D)2L = 100(\pi 0.02m)2\times 2m = 25.1m^2$$

$$C_{min} = C_c = 10\,kg/s(4180\,J/kg\cdot K) = 41,800\,W/K$$

$$NTU = UA/C_{min} = 1766\,W/m^2\cdot K\times 25.1\,m^2/41,800\,W/K = 1.06$$

From Eq. 11.36a

Continued...

PROBLEM 11.36 (Cont.)

$$\varepsilon = 1 - \exp(-NTU) = 1 - \exp(-1.06) = 0.654 \, . \qquad (4)$$

With

$$q_{max} = C_{min}\left(T_{h,i} - T_{c,i}\right) = 41{,}800 \, W/K \left(373.15 - 290\right) K = 3.48 \times 10^{6} \, W \qquad (5)$$

$$q = \varepsilon q_{max} = 0.654\left(3.48 \times 10^{6} \, W\right) = 2.27 \times 10^{6} \, W$$

find

$$T_{c,o} = T_{c,i} + \left(q/C_c\right) = 290 \, K + \left(2.27 \times 10^{6} \, W/41{,}800 W/K\right) = 344.4 K \, . \qquad (6)$$ <

(b,c) Using the *IHT Heat Exchanger Tool, All Exchangers,* $C_r = 0$, the *Properties Tool* for *Water* and the *Correlation Tool, Forced Convection, Internal Flow,* for *Turbulent, fully developed conditions,* a model was developed following the foregoing analysis to compute and plot the outlet temperature $T_{c,o}$ as a function of the cold fluid flow rate, $\dot{m}_c$. The expression for the overall coefficient, Eq.(1), was modified to include the fouling factor,

$$1/U = 1/h_i + R_f'' + 1/h_o \, .$$

Outlet temperature, Tco (K)
360
350
340
330
320
310
5 8 11 14 17 20
Water flow rate, mdotc (kg/s)
Fouling factor, R"f = 0
R"f = 0.0002 m^2.K/W
R"f = 0.0005 m^2.K/W

The effect of increasing the cold flow rate is to decrease the outlet temperature. The effect of the fouling resistance is to decrease the outlet temperature as well.

COMMENTS: (1) For the part (a) analysis, $\overline{T}_c = 317$ K and the initial guess of 320 K was reasonably good.

(2) In the anlysis of parts (b,c), $Re_{D,c}$ is as low as 4880, below the turbulent range (10,000) and above the laminar range (2300). We chose to treat the flow as turbulent.

PROBLEM 11.37

KNOWN: Saturated steam at 110°C condensing in a shell and tube heat exchanger (one shell pass, 2, 4, tube passes) with a UA value of 2.5 kW/K; cooling water enters at 40°C.

FIND: Cooling water flow rate required to maintain a heat rate of 150 kW; and (b) Calculate and plot the water flow rate required to provide heat rates over the range 130 to 160 kW, assuming that UA is independent of flow rate. Comment on the validity of the assumption.

SCHEMATIC:

Sat. steam
$T_{h,i}$ = 110°C
Cooling water
$\dot{m}_c$
$T_{c,i}$ = 40°C
One shell pass
2, 4, ... tube passes
UA = 2.5 kW/K
q = 150 kW

ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible kinetic and potential energy changes, (3) UA independent of flow rate, and (4) Constant properties.

PROPERTIES: *Table A-6*, Water ($T_{m,c} = (T_{c,i} + T_{c,o})/2$ = 49.5°C = 322.5 K): $c_{p,c}$ = 4181 J/kg·K.

ANALYSIS: (a) For the shell-tube heat exchanger with any multiple of two-tube passes, from Eq. 11.36a with $C_r = 0$, using Eqs. 11.20 and 11.23,

$$\varepsilon = 1 - \exp(-NTU) \qquad NTU = UA/C_{min} \tag{1,2}$$

$$\varepsilon = q/q_{max} \qquad q_{max} = C_c\left(T_{h,i} - T_{c,i}\right) \tag{3,4}$$

By combining the equations with $C_{min} = C_c = \dot{m}_c\, c_{p,c}$,

$$\frac{q}{\dot{m}_c\, c_{p,c}\left(T_{h,i} - T_{c,i}\right)} = 1 - \exp\left(-\frac{UA}{\dot{m}_c\, c_{p,c}}\right) \tag{5}$$

Substituting numerical values, and solving using *IHT* find

$$\dot{m}_c = 1.89\ \text{kg/s}$$ <

The specific heat of the cold fluid, $c_{p,c}$, is evaluated at the average of the mean inlet and outlet temperatures, $T_{m,c} = (T_{c,i} + T_{c,o})/2$, with $T_{c,o}$ determined from the energy balance equation,

$$q = \dot{m}\, c_{p,c}\left(T_{c,o} - T_{c,i}\right). \tag{6}$$

(b) Solving the above system of equations in the *IHT* workspace, the graph below illustrates the water flow rate required to provide a range of heat rates.

Continued …..

PROBLEM 11.37 (Cont.)

Water flow rate required for specified heat rate

Water flow rate, mdotc (kg/s)

Heat rate, q (kW)

COMMENTS: (1) The assumption that UA is constant with flow rate is a poor one. Because the heat transfer coefficient for condensation is so high, the overall coefficient is controlled by the water-side coefficient. Presuming the flow is turbulent, from the Dittus-Boelter correlation, we'd expect $U \propto m_c^{0.8}$. Over the range of the graph above, U will vary by approximately a factor of $(3.5/1)^{0.8}$ = 2.7.

(2) If we considered UA to vary with the cold water flow rate as just described, make a sketch of $\dot{m}_c$ vs. q and compare it to the graph above.

PROBLEM 11.38

KNOWN: Temperature, convection coefficient and condensation rate of saturated steam. Tube diameter for shell-and-tube heat exchanger with one shell pass and two tube passes. Velocity and inlet and maximum allowable exit temperatures of cooling water.

FIND: (a) Minimum number of tubes and tube length per pass, (b) Effect of tube-side heat transfer enhancement on tube length.

SCHEMATIC:

Saturated steam
$\dot{m}_{cond}$ = 2.73 kg/s, T_h = 340 K
h_o = 10,000 W/m^2-K
$T_{c,o} \le 30^oC$
L
Water
N tubes
D = 0.019m
$T_{c,i} = 15^oC$
u_m = 0.5 m/s

ASSUMPTIONS: (1) Negligible heat exchange with surroundings, (2) Negligible flow work and kinetic and potential energy changes, (3) Negligible tube wall conduction and fouling resistance, (4) Constant properties, (5) Fully developed internal flow throughout.

PROPERTIES: *Table A-6*, Sat. water (340 K): $h_{fg} = 2.342 \times 10^6$ J/kg; Sat. water $\left(\overline{T}_c = 22.5°C \approx 295\ K\right)$: ρ=998 kg/m^3, c_p = 4181 J/kg·K, $\mu = 959 \times 10^{-6}$ N·s/m^2, k = 0.606 W/m·K, Pr = 6.62.

ANALYSIS: (a) The required heat rate and the maximum allowable temperature rise of the water determine the minimum allowable flow rate. That is, with

$$q = q_{cond} = \dot{m}_{cond} h_{fg} = 2.73\ kg/s \times 2.342 \times 10^6\ J/kg = 6.39 \times 10^6\ W$$

$$\dot{m}_{c,min} = \frac{q}{c_{p,c}\left(T_{c,o} - T_{c,i}\right)} = \frac{6.39 \times 10^6\ W}{4181\ J/kg \cdot K\left(15°C\right)} = 101.9\ kg/s$$

With a specified flow rate per tube of $\dot{m}_{c,1} = \rho u_m \pi D^2/4$ = 998 kg/m^3 × 0.5 m/s × π (0.019m)^2/4 = 0.141 kg/s, the minimum number of tubes is

$$N_{min} = \frac{\dot{m}_{c,min}}{\dot{m}_{c,1}} = \frac{101.9\ kg/s}{0.141\ kg/s} = 720$$ <

To determine the corresponding tube length, we must first find the required heat transfer surface area. With $Re_D = \rho u_m D/\mu$ = 998 kg/m^3 (0.5 m/s) 0.019m/959 × 10^{-6} N·s/m^2 = 9,886, the Dittus-Boelter equation yields

$$\overline{h}_i = (k/D) 0.023\ Re_D^{4/5}\ Pr^{0.4} = \left(0.606\ W/m \cdot K/0.019m\right) 0.023 \left(9886\right)^{4/5} \left(6.62\right)^{0.4} = 2454\ W/m^2 \cdot K$$

Continued …..

PROBLEM 11.38 (Cont.)

Hence, $U = \left[\bar{h}_i^{-1} + h_o^{-1}\right]^{-1} = 1970\ W/m^2 \cdot K$

With $C_r = 0$, $C_{min} = \dot{m}_c\ c_{p,c}$ = 101.9 kg/s × 4181 J/kg·K = 4.26 × 10^5 W/K, $q_{max} = C_{min}\ (T_{h,i} - T_{c,i})$ = 4.26 × 10^5 W/K (340 – 288) K = 2.215 × 10^7 W and $\varepsilon = q/q_{max}$ = 0.289, Eq. 11.36b yields NTU = – ln (1 – ε) = – ln (1 – 0.289) = 0.341. Hence the tube length per pass is

$$L = \frac{A}{2N\pi D} = \frac{NTU \times C_{min}}{2N\pi DU} = \frac{0.341 \times 4.26 \times 10^5\ W/K}{2 \times 720 \times \pi\,(0.019m)1970\ W/m^2 \cdot K} = 0.858m \qquad <$$

(b) If the tube-side convection coefficient is doubled, $\bar{h}_i = 4908\ W/m^2 \cdot K$ and U = 3292 W/m^2·K. Since q, C_r, C_{min}, q_{max} and hence ε are unchanged, the number of transfer units is still NTU = 0.341. Hence, the tube length per pass is now

$$L = \frac{NTU \times C_{min}}{2\,N\pi DU} = \frac{0.341 \times 4.26 \times 10^5\ W/K}{2 \times 720 \times \pi\,(0.019m)3292\ W/m^2 \cdot K} = 0.513m \qquad <$$

COMMENTS: Heat transfer enhancement for the flow with the smallest convection coefficient significantly reduces the size of the heat exchanger.

PROBLEM 11.39

KNOWN: Pressure and initial flow rate of water vapor. Water inlet and outlet temperatures. Initial and final overall heat transfer coefficients.

FIND: (a) Surface area for initial U and water flow rate, (b) Vapor flow rate for final U.

SCHEMATIC:

ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible wall conduction resistance.

PROPERTIES: *Table A-6*, Sat. water ($\overline{T}_c$ = 310 K): $c_{p,c}$ = 4178 J/kg·K; (p = 0.51 bars): T_{sat} = 355 K, h_{fg} = 2304 kJ/kg.

ANALYSIS: (a) The required heat transfer rate is

$$q = \dot{m}_h h_{fg} = 1.5 \text{ kg/s}\left(2.304\times10^6 \text{J/kg}\right) = 3.46\times10^6 \text{W}$$

and the corresponding heat capacity rate for the water is

$$C_c = C_{min} = q/\left(T_{c,o} - T_{c,i}\right) = 3.46\times10^6 \text{ W}/40 \text{ K} = 86{,}400 \text{ W/K}.$$

Hence, $$e = q/\left(C_{min}\left[T_{h,i} - T_{c,i}\right]\right) = 3.46\times10^6 \text{ W}/86{,}400 \text{ W/K}\left(65 \text{ K}\right) = 0.62.$$

Since $C_{min}/C_{max} = 0$, Eq. 11.36b yields

$$NTU = -\ln\left(1-e\right) = -\ln\left(1-0.62\right) = 0.97$$

and $$A = NTU\left(C_{min}/U\right) = 0.97\left(86{,}400 \text{ W/K}/2000 \text{ W/m}^2\cdot\text{K}\right) = 41.9\text{m}^2$$ <

$$\dot{m}_c = C_c/c_{p,c} = 86{,}400 \text{ W/K}/4178 \text{ J/kg}\cdot\text{K} = 20.7 \text{ kg/s}.$$ <

(b) Using the final overall heat transfer coefficient, find

$$NTU = UA/C_{min} = 1000 \text{ W/m}^2\cdot\text{K}\left(41.9\text{m}^2\right)/86{,}400 \text{ W/K} = 0.485.$$

Since $C_{min}/C_{max} = 0$, Eq. 11.36a yields

$$e = 1 - \exp\left(-NTU\right) = 1 - \exp\left(-0.485\right) = 0.384.$$

Hence, $$q = eC_{min}\left(T_{h,i} - T_{c,i}\right) = 0.384\left(86{,}400 \text{ W/K}\right)65 \text{ K} = 2.16\times10^6 \text{ W}$$

$$\dot{m}_h = q/h_{fg} = 2.16\times10^6 \text{ W}/2.304\times10^6 \text{ J/kg} = 0.936 \text{ kg/s}.$$ <

COMMENTS: The significant reduction (38%) in $\dot{m}_h$ represents a significant loss in turbine power. Periodic cleaning of condenser surfaces should be employed to minimize the adverse effects of fouling.

PROBLEM 11.40

KNOWN: Two-fluid heat exchanger with prescribed inlet and outlet temperatures of the two fluids.

FIND: (a) Whether exchanger is operating in parallel or counter flow, (b) Effectiveness of the exchanger when $C_c = C_{min}$.

SCHEMATIC:

ASSUMPTIONS: (1) Negligible heat loss to the surroundings, (2) Negligible kinetic and potential energy changes, (3) Cold fluid is minimum fluid.

ANALYSIS: (a) To determine whether operation is PF or CF, consider the temperature distributions. From the distributions we note that PF or CF operation is possible.

(b) The effectiveness of the exchanger follows from Eq. 11.20,

$$\varepsilon = q / q_{max} \tag{1}$$

where from Eq. 11.19,

$$q_{max} = C_{min}\left(T_{h,i} - T_{c,i}\right). \tag{2}$$

Since $C_{min} = C_c$ and performing an energy balance on the cold fluid, Eq. (1) with Eq. (2) becomes

$$\varepsilon = C_c\left(T_{c,o} - T_{c,i}\right)/C_{min}\left(T_{h,i} - T_{c,i}\right) = \left(T_{c,o} - T_{c,i}\right)/\left(T_{h,i} - T_{c,i}\right)$$

$$\varepsilon = (30 - 15)°C/(65 - 15)°C = 0.30.$$ <

COMMENTS: If $T_{c,o}$ were greater than $T_{h,o}$, parallel-flow operation would not be possible.

PROBLEM 11.41

KNOWN: Concentric tube heat exchanger.

FIND: Length of the exchanger.

SCHEMATIC:

ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible kinetic and potential energy changes, (3) Constant properties.

PROPERTIES: *Table A-6*, Water ($\bar{T}_c$ = (35 + 95)°C/2 = 338 K): $c_{p,c}$ = 4188 J/kg·K

ANALYSIS: From the rate equation, Eq. 11.14, with $A_o = \pi D_o L$,

$$L = q / U_o \pi D_o \Delta T_{\ell m}.$$

The heat rate, q, can be evaluated from an energy balance on the cold fluid,

$$q = \dot{m}_c c_c \left(T_{c,o} - T_{c,i}\right) = \frac{225 \text{ kg/h}}{3600 \text{ s/h}} \times 4188 \text{ J/kg}\cdot\text{K}\left(95-35\right)\text{K} = 15{,}705 \text{ W}.$$

In order to evaluate $\Delta T_{\ell m}$, we need to know whether the exchanger is operating in CF or PF. From an energy balance on the hot fluid, find

$$T_{h,o} = T_{h,i} - q/\dot{m}_h c_h = 210°\text{C} - 15{,}705 \text{ W} / \frac{225 \text{ kg/h}}{3600 \text{s/h}} \times 2095 \frac{\text{J}}{\text{kg}\cdot\text{K}} = 90.1°\text{C}.$$

Since $T_{h,o} < T_{c,o}$ it follows that Hxer operation must be CF. From Eq. 11.15,

$$\Delta T_{\ell m,CF} = \frac{\Delta T_1 - \Delta T_2}{\ell n\left(\Delta T_1 / \Delta T_2\right)} = \frac{\left(210-95\right)-\left(90.1-35\right)}{\ell n\left(115/55.1\right)}°\text{C} = 81.4°\text{C}.$$

Substituting numerical values, the HXer length is

$$L = 15{,}705 \text{ W} / 550 \text{ W/m}^2\cdot\text{K}\,\pi\left(0.10\text{m}\right)\times 81.4\text{K} = 1.12\text{m}.$$ <

COMMENTS: The ε-NTU method could also be used. It would be necessary to perform the hot fluid energy balance to determine if CF operation existed. The capacity rate ratio is C_{min}/C_{max} = 0.50. From Eqs. 11.19 and 11.20 with q evaluated from an energy balance on the hot fluid,

$$\varepsilon = \frac{T_{h,i} - T_{h,o}}{T_{h,i} - T_{c,i}} = \frac{210-90.1}{210-35} = 0.69.$$

From Fig. 11.15, find NTU ≈ 1.5 giving

$$L = \text{NTU}\cdot C_{min} / U_o \pi D_o \approx 1.5 \times 130.94 \frac{\text{W}}{\text{K}} / 550 \frac{\text{W}}{\text{m}^2\cdot\text{K}} \cdot \pi\left(0.10\text{m}\right) \approx 1.14\text{m}.$$

Note the good agreement in both methods.

PROBLEM 11.42

KNOWN: A *very long*, concentric tube heat exchanger having hot and cold water inlet temperatures, 85°C and 15°C, respectively; flow rate of hot water is twice that of the cold water.

FIND: Outlet temperatures for counterflow and parallel flow operation.

SCHEMATIC:

$T_{h,i} = 85°C$, $T_{c,i} = 15°C$, C_c, $C_h = 2C_c$

ASSUMPTIONS: (1) Equivalent hot and cold water specific heats, (2) Negligible kinetic and potential energy changes, (3) No heat loss to surroundings.

ANALYSIS: The heat rate for a concentric tube heat exchanger with very large surface area operating in the *counterflow* mode is

CF: $T(x)$, $T_{h,i}=85°C$, $C_h=2C_c$, $T_{c,i}=15°C$, C_c, x

$$q = q_{max} = C_{min}\left(T_{h,i} - T_{c,i}\right)$$

where $C_{min} = C_c$. From an energy balance on the hot fluid,

$$q = C_h\left(T_{h,i} - T_{h,o}\right).$$

Combining the above relations and rearranging, find

$$T_{h,o} = -\frac{C_{min}}{C_h}\left(T_{h,i} - T_{c,i}\right) + T_{h,i} = -\frac{C_c}{C_h}\left(T_{h,i} - T_{c,i}\right) + T_{h,i}.$$

Substituting numerical values,

$$T_{h,o} = -\frac{1}{2}(85-15)°C + 85°C = 50°C.$$ <

For *parallel flow* operation, the hot and cold outlet temperatures will be equal; that is, $T_{c,o} = T_{h,o}$. Hence,

PF: $T(x)$, $T_{h,i}=85°C$, $C_h=2C_c$, $T_{c,o}=T_{h,o}$, C_c, $T_{c,i}=15°C$, x

$$C_c\left(T_{c,o} - T_{c,i}\right) = C_h\left(T_{h,i} - T_{h,o}\right).$$

Setting $T_{c,o} = T_{h,o}$ and rearranging,

$$T_{h,o} = \left(T_{h,i} + \frac{C_c}{C_h}T_{c,i}\right) \Big/ \left(1 + \frac{C_c}{C_h}\right)$$

$$T_{h,o} = \left(85 + \frac{1}{2}\times 15\right)°C \Big/ \left(1 + \frac{1}{2}\right) = 61.7°C.$$ <

COMMENTS: Note that while $\varepsilon = 1$ for CF operation, for PF operation find $\varepsilon = q/q_{max} = 0.67$.

PROBLEM 11.43

KNOWN: Saturation temperature and condensation rate of refrigerant. Frontal area of condenser and dependence of overall coefficient on inlet velocity. Operational range of the air inlet temperature.

FIND: (a) Required heat exchanger area and air outlet temperature for prescribed air inlet velocity and temperature, (b) Variation in air velocity needed to achieve prescribed condensation rate.

SCHEMATIC:

T

ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Constant properties.

PROPERTIES: Given (R-12): $h_{fg} = 1.35 \times 10^5$ J/kg. *Table A-4*, air ($T_{c,i}$ = 303 K): $\rho_c = 1.17$ kg/m^3, $c_{p,c}$ = 1007 J/kg·K.

ANALYSIS: (a) With $\dot{m}_c = \rho_c V A_{fr} = 1.17\,kg/m^3 \times 2\,m/s \times 0.25\,m^2 = 0.585\,kg/s$, $C_{min} = \dot{m}_c c_{p,c} = 589\,W/K$. Hence, from Eq. (11.19), with $T_{h,i} = T_{sat}$,

$$q_{max} = C_{min}\left(T_{h,i} - T_{c,i}\right) = 589\,W/K\left(45-30\right)K = 8,836\,W$$

and with $q = \dot{m}_h\, h_{fg} = 0.015\,kg/s \times 1.35 \times 10^5\,J/kg = 2025\,W$

$$\varepsilon = \frac{q}{q_{max}} = \frac{2025}{8836} = 0.229$$

From Eq. 11.36b we then obtain (for $C_r = 0$),

$$A = \frac{C_{min}}{U}NTU = -\frac{C_{min}}{U}\ln\left(1-\varepsilon\right) = -\frac{589\,W/K}{50\,W/m^2\cdot K}\ln\left(0.771\right) = 3.067\,m^2 \quad <$$

With $q = C_{min}(T_{c,o} - T_{c,i})$, the outlet temperature is

$$T_{c,o} = T_{c,i} + \frac{q}{C_{min}} = 30°C + \frac{2025\,W}{589\,W/K} = 33.4°C \quad <$$

(b) With q = 2025 W, A = 3.06 m^2 and U = 50 W/m^2·K $(V/2)^{0.7}$, the foregoing equations may be solved to obtain V as a function of $T_{c,i}$.

Continued…..

PROBLEM 11.43 (Cont.)

With increasing $T_{c,i}$, the driving potential for heat transfer, $T_{h,i} - T_{c,i}$, decreases and a larger value of U, and hence V, is needed to maintain the required heat rate. For $27 \le T_{c,i} \le 38°C$, $1.56 \le V \le 5.66$ m/s and $42.1 \le U \le 103.6\ W/m^2 \cdot K$.

COMMENTS: The variation of V with $T_{c,i}$ is nonlinear, and, in principle, $V \to \infty$ as $T_{c,i} \to T_{sat}$.

PROBLEM 11.44

KNOWN: Conditions of oil and water for heat exchanger, one shell with 4 tube passes.

FIND: Length of exchanger tubes per pass, L; and (b) Compute and plot the effectiveness, ε, fluid outlet temperatures, $T_{h,o}$ and $T_{c,o}$, and water-side convection coefficient, h_c, as a function of the water flow rate for $5000 \le \dot{m}_c \le 15{,}000$ kg / h for the tube length found in part (a) with all other conditions remaining the same.

SCHEMATIC:

$T_{h,i}$ = 160 °C
Oil
Brass tubes, L
$T_{c,i}$ = 16 °C
$\dot{m}_c$ = 10,000 kg/h
Water
11 tubes, D_i = 22.9 mm
D_o = 25.4 mm, 4 passes,
h_o = 400 W/m²•K
$T_{c,o}$ = 84 °C
Hxer: single shell, 4 tube passes
$T_{h,o}$ = 94 °C

ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible kinetic and potential energy changes, (3) Constant properties, (4) Fully-developed flow in tubes.

PROPERTIES: *Table A-1*, Brass (400 K): k = 137 W/m·K; *Table A-5*, Water (323 K): ρ = 998.1 kg/m³, k = 0.643 W/m·K, c_p = 4182 J/kg·K, μ = 548 × 10⁻⁶ N·s/m², Pr = 3.56.

ANALYSIS: (a) From an energy balance on the water, the heat rate required is

$$q = \dot{m}_c c_c \left(T_{c,o} - T_{c,i}\right) = 10{,}000/3600\,\text{kg/s} \times 4182\,\text{J/kg}\cdot\text{K}\,(84-16)^\circ\text{C} = 789{,}933\,\text{W}. \qquad (1)$$

The required tube length may be obtained from Eqs. 11.14 and 11.15,

$$q = U_o A_o F \Delta T_{\ell m,CF} \qquad (2)$$

$$\Delta T_{\ell m,CF} = \left[(160-84)^\circ\text{C} - (94-16)^\circ\text{C}\right] / \ell n\left(160-84/94-16\right) = 77.0^\circ\text{C}.$$

From Fig. 11.10, F = 0.86 using P = (84 - 16)/(160 - 16) = 0.47 and R = (160 - 94)/(84 - 16) = 0.97. From Eq. 11.5,

$$U_o = \left[\frac{1}{h_o} + \frac{r_o}{k}\ell n\frac{r_o}{r_i} + \frac{r_o}{r_i}\frac{1}{h_i}\right]^{-1}$$

where h_i must be estimated from the appropriate correlation. With N = 11, the number of tubes,

$$Re_D = \frac{4\dot{m}/N}{\pi D \mu} = \frac{4 \times (10{,}000/3600)\,\text{kg/s}/(11)}{\pi \times 22.9 \times 10^{-3}\,\text{m} \times 548 \times 10^{-6}\,\text{N}\cdot\text{s/m}^2} = 25{,}621.$$

For fully developed turbulent flow, the Dittus-Boelter correlation with n = 0.4 yields

$$Nu_D = h_i D/k = 0.023 Re_D^{0.8} Pr^{0.4} = 0.023(25{,}621)^{0.8}(3.56)^{0.4} = 128.6$$

$$h_i = Nu_D (k/D) = 128.6 \times 0.643\,\text{W/m}\cdot\text{K}/\left(22.9 \times 10^{-3}\,\text{m}\right) = 3610\,\text{W/m}^2\cdot\text{K}.$$

Continued...

PROBLEM 11.44 (Cont.)

$$U_o = \left[\frac{1}{400\,\text{W/m}^2\cdot\text{K}} + \frac{25.4\times10^{-3}\,\text{m}}{2\times137\,\text{W/m}\cdot\text{K}} \ell n \frac{25.4}{22.9} + \frac{25.4}{22.9} \times \frac{1}{3610\,\text{W/m}^2\cdot\text{K}} \right]^{-1} = 355\,\text{W/m}^2\cdot\text{K}\,.$$

Returning now to Eq. (2), find A_o, then the length,

$$A_o = \pi D_o L \times \text{No. of Passes} \times \text{No. of Tubes} = \pi \times 25.4\times10^{-3}\,\text{m}\times 4\times 11\,L = 3.511\ L$$

$$L = 789{,}933\,\text{W} \big/ 3.511\ \text{m}\times 355\,\text{W/m}^2\cdot\text{K}\times 0.86\times 77.0^\circ\,\text{C} = 9.6\,\text{m}$$ <

(b) Using the *IHT Heat Exchanger Tool, Shell and Tube, One-shell pass* and *N tube passes,* the *Correlation Tool, Forced Convection, Internal Flow* for *Turbulent, fully developed condition,* and the *Properties Tool* for *Water,* a model was developed using the effectiveness - NTU method to compute and plot $T_{c,o}$, $T_{h,o}$, ε, and h_i as a function of $\dot{m}_c$.

In order to avoid a boiling condition in the cold fluid, the cold flow rate should not be less than 8000 kg/h. As expected, $T_{c,o}$ and $T_{h,o}$ decrease and the internal convection coefficient increases nearly linearly with increasing flow rate. The effectiveness increases with increasing flow rate since the overall convection coefficient is increasing.

COMMENTS: (1) The thermal resistance of the brass tubes is negligible. Since $L/D_i = 400$, fully-developed conditions are reasonable.

(2) In the analysis of part (b), you have to specify the capacity rate for the hot fluid in order to solve the model. From the analysis of part (a) using the model, we found L = 9.56 m and C_h = 11,974 W/K.

PROBLEM 11.45

KNOWN: Properties and flow rate of computer coolant. Diameter and number of heat exchanger tubes. Heat exchanger transfer rate and inlet temperature of computer coolant. Flow rate, specific heat, inlet temperature, and average convection coefficient of water.

FIND: (a) Tube flow convection coefficient, $\bar{h}_i$, (b) Tube length/pass required to achieve prescribed fluid outlet temperature, (c) Compute and plot the dielectric fluid outlet temperature, $T_{f,o}$, as a function of its flow rate $\dot{m}_f$ for the range $4 \le \dot{m}_f \le 6$ kg/s based upon the length/pass found in part (c), (d) the effect of ±10% change in the water flow rate, $\dot{m}_w$, on $T_{f,o}$ and (e) the effect of ±3°C change in inlet water temperature, $T_{w,i}$, on $T_{f,o}$. For parts (c, d, e), account for any changes in the overall convection coefficient, while all other conditions remain the same.

SCHEMATIC:

$\dot{m}_w$ = 2.5 kg/s, $T_{w,i}$ = 5 °C

$T_{f,i}$ = 25 °C
$T_{f,o}$ = 15 °C
$\dot{m}_f$ = 4.81 kg/s

q

$\bar{h}_o = 10^4$ W/m²·K
N = 72 tubes, D = 0.01 m

$T_{w,o}$

ASSUMPTIONS: (1) Negligible heat loss to surroundings, kinetic and potential energy changes, fouling and tube wall resistance; (2) Constant properties; (3) Fully developed flow, (4) Convection coefficient on shell side, $\bar{h}_o$, remains constant for all operating conditions.

PROPERTIES: Coolant (given): c_p = 1040 J/kg·K, $\mu = 7.65 \times 10^{-4}$ kg/s·m, k = 0.058 W/m·K, Pr = 14; Water (given): c_p = 4200 J/kg·K.

ANALYSIS: (a) For flow through a single tube,

$$Re_D = \frac{4\dot{m}_{f,t}}{\pi D \mu} = \frac{4(4.81\,kg/s)/72}{\pi(0.01m)7.65\times10^{-4}\,kg/s\cdot m} = 11{,}120$$

and using the Dittus-Boelter correlation, find

$$h_i = (k/D)0.023\,Re_D^{4/5}\,Pr^{0.3} = 0.023\frac{0.058\,W/m\cdot K}{0.01m}(11{,}120)^{4/5}(14)^{0.3} = 508\,W/m^2\cdot K\,.$$ <

(b) Find the capacity ratio

$$C_f = \dot{m}_f c_{p,f} = 4.81\,kg/s(1040\,J/kg\cdot K) = 5002\,W/K = C_{min}$$

$$C_w = \dot{m}_w c_{p,w} = 2.5\,kg/s(4200\,J/kg\cdot K) = 10{,}500\,W/K = C_{max}$$

hence, $C_r = C_{min}/C_{max} = 0.476$ and

$$\varepsilon = \frac{q}{q_{max}} = \frac{C_f\left(T_{f,i} - T_{f,o}\right)}{C_f\left(T_{f,i} - T_{w,i}\right)} = \frac{(25-15)^\circ C}{(25-5)^\circ C} = 0.500\,.$$

Using Fig. 11.16 with NTU = $(UA/C_{min}) = (UN\pi D2L/C_{min}) \approx 0.85$,

Continued...

$$U = \left(h_i^{-1} + h_o^{-1}\right)^{-1} = \left[(508)^{-1} + \left(10^4\right)^{-1}\right]^{-1} \text{W/m}^2 \cdot \text{K} = 483\,\text{W/m}^2 \cdot \text{K}$$

$$L = 0.85\left(5002\ \text{W/K}\right) \Big/ 144\pi\left(483\,\text{W/m}^2 \cdot \text{K}\right)0.01\text{m} = 1.95\text{m}\ .$$ <

(c) Using the *IHT Heat Exchanger Tool, Shell and Tube, One-shell pass and N-tube passes,* and the *Correlation Tool, Forced Convection, Internal Flow* for *Turbulent, fully developed conditions*, a model was developed using the effectiveness-NTU method employed above to compute and plot $T_{f,o}$ as a function of $\dot{m}_f$.

Outlet temperature, Tfo (C)
16
15
14
4
5
6
Dielectric flow rate, mdotf (kg/s)

A change in the dielectric fluid flow rate of ±1 kg/s causes approximately ±0.5°C change in its outlet temperature.

(d) Using the above IHT model with the base conditions for part (c), the effect of a ±10% change in the water flow rate from its design value, $\dot{m}_w$ = 2.5 kg/s (2.25 ≤ $\dot{m}_w$ ≤ 2.75 kg/s) causes the dielectric fluid outlet temperature to change as

$$T_{f,o} = 15 \pm 0.14^\circ\text{C}$$ <

(e) Using the IHT model of part (c) with the base case conditions for part (c), the effect of a ±3°C in the water inlet temperature from its design value, $T_{c,i}$ = 5°C (2 ≤ $T_{c,i}$ ≤ 8°C) cause the dielectric fluid outlet temperature to change as

$$T_{f,o} = 15 \pm 1.5^\circ\text{C}$$ <

COMMENTS: (1) For the analyses of part (a), Eq. 11.31 yields NTU = 0.85 and q = 50 kW and $T_{w,o}$ = 9.76°C.

(2) The results of the analyses provide operating performance information on the effect of changes due to dielectric fluid flow rate (±1 kg/s of $\dot{m}_f$), water fluid flow rate (≤ 10% of $\dot{m}_w$) and water inlet temperature (±3°C of $T_{w,i}$) on the dielectric fluid outlet temperature, $T_{f,o}$, supplied to the computer. The greatest effect on $T_{f,o}$, is that by the input water temperature.

PROBLEM 11.46

KNOWN: Shell and tube heat exchanger with 135 tubes (one shell, double pass) of inner diameter 12.5 mm and surface area 47.5 m^2.

FIND: (a) Exchanger gas and water outlet temperatures, (b) Tube heat transfer coefficient, $\bar{h}_i$, assuming fully developed flow, (c) Compute and plot the effectiveness and fluid outlet temperatures, $T_{c,o}$ and $T_{h,o}$ for the water flow rate range $6 \le \dot{m}_c \le 12$ kg/s with all other conditions remaining the same, and (d) Hot gas inlet temperature, $T_{h,i}$, required to supply 10 kg/s of hot water with an outlet temperature of 42°C with all other conditions the same; determine also the effectiveness.

SCHEMATIC:

$T_{h,i}$ = 200 °C
$\dot{m}_h$ = 5 kg/s
Exhaust gases, 1 atm
Shell and tube hxer
A = 47.5 m^2
D_i = 12.5 mm
N = 135
Tubes
$T_{h,o}$
U = 200 $W/m^2 \cdot K$
Water
$\dot{m}_c$ = 6.5 kg/s
$T_{c,i}$ = 15 °C

ASSUMPTIONS: (1) Negligible heat lost to surroundings, (2) Negligible kinetic and potential energy changes, (3) Fully-developed conditions for internal flow of water in tubes, (4) Exhaust gas properties are those of air, and (5) The overall coefficient remains unchanged for the operating conditions examined.

PROPERTIES: *Table A-6*, Water ($\bar{T}_c \approx 300$ K): ρ = 997 kg/m^3, c = 4179 J/kg·K, k = 0.613 W/m·K, $\mu = 855 \times 10^{-6}$ N·s/m^2, Pr = 5.83; *Table A-4*, Air (1 atm, $\bar{T}_h \approx 400$ K): ρ = 0.8711 kg/m^3, c = 1014 J/kg·K.

ANALYSIS: (a) Using the ε-NTU method, first find the capacity rates, $C = \dot{m}c$,

$$C_c = 6.5\,\mathrm{kg/s} \times 4179\,\mathrm{J/kg \cdot K} = 27{,}164\,\mathrm{W/K} \qquad C_h = 5.0\,\mathrm{kg/s} \times 1014\,\mathrm{J/kg \cdot K} = 5{,}070\,\mathrm{W/K}\,.$$

Recognize that $C_h = C_{min}$ and determine

$$\frac{C_{min}}{C_{max}} = \frac{C_h}{C_c} = \frac{5{,}070}{27{,}164} = 0.19 \qquad NTU = \frac{AU}{C_{min}} = \frac{47.5\mathrm{m}^2 \times 200\,\mathrm{W/m^2 \cdot K}}{5{,}070\,\mathrm{W/K}} = 1.87\,.$$

From Fig. 11.16 for the shell and tube exchanger, find with NTU = 1.87 and $C_{min}/C_{max} = 0.19$ that $\varepsilon \approx 0.78$. From the definition of effectiveness,

$$\varepsilon = \frac{q}{q_{max}} = \frac{C_h\left(T_{h,i} - T_{h,o}\right)}{C_{min}\left(T_{h,i} - T_{c,i}\right)} = \frac{200 - T_{h,o}}{200 - 15} = 0.78 \quad \text{or} \quad T_{h,o} = 55.7^\circ\mathrm{C}\,.$$

<

From energy balances on the two fluids, $C_h (T_{h,i} - T_{h,o}) = C_c (T_{c,o} - T_{c,i})$, find

$$T_{c,o} = T_{c,i} + \left(C_h/C_c\right)\left(T_{h,i} - T_{h,o}\right) = 15^\circ\mathrm{C} + 0.19\left(200 - 55.7\right)^\circ\mathrm{C} = 42.4^\circ\mathrm{C}\,.$$

<

(b) To estimate $\bar{h}_i$ for the water, find first the Reynolds number. From Eq. 8.6,

Continued...

PROBLEM 11.46 (Cont.)

$$Re_{D_i} = \frac{4\dot{m}}{\pi D_i \mu} = \frac{4\dot{m}_c/N}{\pi D_i \mu} = \frac{4 \times 6.5\,kg/s/135}{\pi\, 12.5 \times 10^{-3} m \times 855 \times 10^{-6}\, N/s \cdot m^2} = 5736$$

While the flow is fully developed and turbulent, Re_D = 10,000 such that Dittus-Boelter correlation is not strictly applicable. However, its use allows a first estimate.

$$\overline{Nu}_{D_i} = \bar{h}\, D_i/k = 0.023\, Re_D^{4/5}\, Pr^{0.4} = 0.023(5736)^{4/5}(5.83)^{0.4} = 47.3$$

$$\bar{h}_i = \overline{Nu}_D\, k/D_i = 47.3 \times 0.613\, W/m^2 \cdot K / 12.5 \times 10^{-3} m = 2320\, W/m^2 \cdot K\,.$$ <

(c) Using the *IHT Heat Exchanger Tool*, *Shell and Tube*, *One-shell pass and N-tube passes*, and the prescribed properties, a model was developed following the analysis of part (a) to compute and plot ε, $T_{c,o}$, and $T_{h,o}$ for a function of $\dot{m}_c$.

100
80
60
40
20
Tco, Tho (C), eps*100
6
8
10
12
Cold flow rate, mdotc (kg/s)
Cold outlet temperature, Tco (C)
Hot outlet temperature, Tho (C)
Effectiveness, eps * 100

The outlet temperatures decrease nearly linearly with increasing cold fluid flow rate; the decrease in the cold outlet temperature is nearly twice that of the hot fluid. The change in the effectiveness with increasing flow rate is only slightly increased.

(d) Using the above IHT model, the hot inlet temperature $T_{h,i}$, required to provide $\dot{m}_c$ = 10 kg/s with $T_{c,o}$ = 42°C and the effectiveness for this operating condition are

$$T_{h,i} = 74.4^\circ C \qquad \varepsilon = 0.55$$ <

COMMENTS: (1) Check that assumptions for $\bar{T}_h$ and $\bar{T}_c$ used in part (a) for evaluation of the fluid properties are satisfactory as $\bar{T}_h = 400.7$ K and $\bar{T}_c = 301.5$ K.

(2) From part (b), with $\bar{h}_i$ = 2320 W/m^2·K and U = 200 W/m^2·K, the shell-side convection coefficient is $\bar{h}_o$ = 219 W/m^2·K. As such, U is controlled by shell-side conditions. Assuming U as a constant in part (c) with changes in $\dot{m}_c$ is therefore reasonable. However, for part (d) with $\dot{m}_h$ doubling, we should expect U to increase.

PROBLEM 11.47

KNOWN: Power output and efficiency of an ocean energy conversion system. Temperatures and overall heat transfer coefficient of shell-and-tube evaporator.

FIND: (a) Evaporator area, (b) Water flow rate.

SCHEMATIC:

One shell and two tube passes

$T_{h,i}$, $T_{c,o}$, $T_{c,i}$, $T_{h,o}$, $U = 1200\ W/m^2 \cdot K$

$T_{h,i} = 300K$, $T_{h,o} = 292K$, $T_{c,i} = T_{c,o} = 290K$

ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible kinetic and potential energy changes, (3) Constant properties.

PROPERTIES: *Table A-6*, Water ($\overline{T}_m$ = 296 K): c_p = 4181 J/kg·K.

ANALYSIS: (a) The efficiency is

$$h = \frac{\dot{W}}{q} = \frac{2MW}{q} = 0.03.$$

Hence the required heat transfer rate is

$$q = \frac{2MW}{0.03} = 66.7MW.$$

Also

$$\Delta T_{\ell m,CF} = \frac{(300-290)-(292-290)°C}{\ell n \dfrac{300-290}{292-290}} = 5°C$$

and, with P = 0 and R = ∞, from Fig. 11.10 it follows that F = 1. Hence

$$A = \frac{q}{U F \Delta T_{\ell m,CF}} = \frac{6.67 \times 10^7\ W}{1200\ W/m^2 \cdot K \times 1 \times 5°C}$$

$$A = 11{,}100 m^2.$$ <

(b) The water flow rate through the evaporator is

$$\dot{m}_h = \frac{q}{c_{p,h}\left(T_{h,i} - T_{h,o}\right)} = \frac{6.67 \times 10^7\ W}{4181 J/kg \cdot K (300 - 292)}$$

$$\dot{m}_h = 1994\ kg/s.$$ <

COMMENTS: (1) From the ε-NTU method, $(C_{min}/C_{max}) = 0$, $q_{max} = 8.34 \times 10^7$ W, ε = 0.80 and from Fig. 11.16, NTU ≈ 1.65, giving A = 11,500 m^2. (2) The required heat exchanger size is enormous due to the small temperature differences involved.

PROBLEM 11.48

KNOWN: Length and tube diameter for a shell-and-tube (one shell pass, multiple tube passes) heat exchanger. Flow rate and temperature of saturated steam. Condensation convection coefficient. Velocity and inlet and outlet temperatures of cooling water.

FIND: (a) Required number of tubes and if the heat exchanger length is not to exceed 1.5 m, the number of tube passes; (b) Compute and plot water outlet temperature $T_{c,o}$, and condensation rate, $\dot{m}_h$ as a function of the mean velocity for the range $0.5 \le u_m \le 3$ m/s, for the heat transfer area found from part (a), accounting for changes in the overall coefficient, but all other conditions remaining the same; and (c) Repeat the analysis of part (b) for tube diameters of 15 and 25 mm.

SCHEMATIC:

$\dot{m}_h$ = 2.73 kg/s, $T_{h,i}$ = 340 K

h_o = 12,500 W/m^2·K

D = 0.019 m

L = 1.5 m

$T_{c,o}$ = 303 K

$T_{c,i}$ = 291 K

u_m = 1.5 m/s

$T_{h,o}$ = 340 K

ASSUMPTIONS: (1) Negligible heat loss to surroundings and changes in kinetic and potential energy, (2) Negligible tube wall conduction and fouling resistances, (3) Constant properties, and (4) The shell-side coefficient $\overline{h}_o$ remains unchanged for the operating conditions examined.

PROPERTIES: *Table A.6*, Sat. water (340 K): $h_{fg} = 2.342 \times 10^6$ J/kg; Sat. water ($\overline{T}_c$ = 297 K): ρ = 998 kg/m^3, c_p = 4180 J/kg·K, $\mu = 917 \times 10^{-6}$ kg/s·m, k = 0.609 W/m·K, Pr = 6.3.

ANALYSIS: (a) The required heat rate is

$$q = \dot{m}_h h_{fg} = 2.73\,\text{kg/s}\left(2.342\times10^6\ \text{J/kg}\right) = 6.39\times10^6\,\text{W}\ .$$

Hence, from conservation of energy,

$$\dot{m}_c = q\Big/\left[c_{p,c}\left(T_{c,o} - T_{c,i}\right)\right] = 6.39\times10^6\ \text{W}\big/4180\,\text{J/kg}\cdot\text{K}\,(12\text{K}) = 127.5\,\text{kg/s}\ .$$

Hence the number of tubes is

$$N = \dot{m}_c/\dot{m}_{c,t} = \dot{m}_c\Big/\left(\pi D^2/4\right)\rho u_m = 4\times127.5\,\text{kg/s}/\pi\,(0.019\text{m})^2\ 998\,\text{kg/m}^3\,(1.5\,\text{m/s}) = 300\ .$$ <

To determine the heat transfer surface area A, use the ε - NTU method. Find first

$$Re_D = \rho u_m D/\mu = 998\,\text{kg/m}^3\,(1.5\,\text{m/s})(0.019\text{m})\big/917\times10^{-6}\,\text{kg/s}\cdot\text{m} = 31,017$$

and using the Dittus-Boelter equation,

$$h_i = (k/D)\,0.023\,Re_D^{4/5}\,Pr^{0.4} = (0.609\,\text{W/m}\cdot\text{K}/0.019\text{m})\,0.023\,(31,017)^{4/5}\,(6.3)^{0.4} = 6034\,\text{W/m}^2\cdot\text{K}$$

$$U = \left[1/h_i + 1/h_o\right]^{-1} = \left[(1/6034) + (1/12,500)\right]^{-1}\,\text{W/m}^2\cdot\text{K} = 4070\,\text{W/m}^2\cdot\text{K}\ .$$

Continued...

PROBLEM 11.48 (Cont.)

With $C_{min} = \dot{m}_c c_{p,c}$ = 127.5 kg/s(5180 J/kg·K) = 5.33×10^5 W/K

$$\varepsilon = q/q_{max} = q/C_{min}\left(T_{h,i} - T_{c,i}\right) = 6.39\times10^6 \text{ W}\Big/5.33\times10^5 \text{ W/K}\left(49\text{K}\right) = 0.245 .$$

Hence, with $C_r = 0$, Eq. 11.36b yields NTU = -ln(1 - ε) = -ln(1 - 0.245) = 0.281,

$$A = NTU\left(C_{min}/U\right) = 0.281\left(5.33\times10^5 \text{ W/K}\Big/4070 \text{ W/m}^2\cdot\text{K}\right) = 36.8\text{m}^2$$

The tube length, L, in terms of the number of tubes, N, and passes, P, is

$$L = A/N\cdot P\pi D$$

and if P = 2,

$$L = 36.8\text{m}^2\Big/300\times2\times\pi\times0.019\text{m} = 1.03\,\text{m}$$ <

which is less than the maximum length 1.5 m.

(b) Using the *IHT Heat Exchanger Tool, All Exchangers, $C_r = 0$*, the *Properties Tool* for *Water*, and the *Correlations Tool*, *Forced Convection, Internal Flow*, for *Turbulent fully developed conditions*, a model was developed following the foregoing analysis to compute $T_{c,o}$ and $\dot{m}_h$ as a function of u_m with A = 36.8 m^2 as determined from part (a). The plot is shown below with the results for part (c).

(c) The IHT model was used to compute and plot $T_{c,o}$ and $\dot{m}_h$ as a function of u_m for tube diameters of 15, 19, and 25 mm.

The effect of tube diameter on $T_{c,o}$ as a function of the water flow rate is significant. As D increases at any flow rate, the outlet temperatures decreases. The effect of tube diameter on the condensation rate is slight. However, the condensation rate increases markedly as the water flow rate increases.

PROBLEM 11.49

KNOWN: Shell(1)-and-tube (two passes, p = 2) heat exchanger for condensing saturated steam at 1 atm. Inlet cooling water temperature and mean velocity. Thin-walled tube diameter and length prescribed, as well as, convective heat transfer coefficient on outer tube surface, h_o.

FIND: (a) Number of tubes/pass, N, required to condense 2.3 kg/s of steam, (b) Outlet water temperature, $T_{c,o}$, (c) Maximum condensation rate possible for same water flowrate and inlet temperature, and (d) Compute and plot $T_{c,o}$ and the condensation rate, $\dot{m}_h$, for water mean velocity, u_m, in the range $1 \le u_m \le 5$ ms/, using the heat transfer surface area found in part (a) assuming the shell-side convection coefficient remains unchanged.

SCHEMATIC:

T
x
$C_h \longrightarrow \infty$
h_o = 21,800 W/m²•K
Saturated steam, 1 atm, $\dot{m}_h$ = 2.3 kg/s
$T_{c,o}$
Water, u_m = 3.5 m/s
C_c
Tubes: D = 14 mm, L = 0.5 m
$T_{c,i}$ = 15 °C

ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible kinetic and potential energy changes, (3) Negligible thermal resistance due to the tube walls.

PROPERTIES: *Table A.6*, Saturated steam (1 atm): T_{sat} = 100°C, h_{fg} = 2257 kJ/kg; Water (assume $T_{c,o}$ ≈ 25°C, $\overline{T}_m = (T_h + T_c)/2 \approx 295$ K): $\rho = 1/v_f = 998$ kg/m³, $c_c = c_{p,h} = 4181$ J/kg·K, $\mu = \mu_f = 959 \times 10^{-6}$ N·s/m², $k = k_f = 0.606$ W/m·K, $Pr = Pr_\ell = 6.62$.

ANALYSIS: (a) The heat transfer rate for the heat exchanger is

$$q = \dot{m}_h h_{fg} = 2.3\,\text{kg/s} \times 2257 \times 10^3\,\text{J/kg} = 5.191 \times 10^6\,\text{W} \tag{1}$$

Using the ε-NTU method, evaluate the following parameters:

Water-side heat transfer coefficient:

$$Re_D = \frac{u_m D}{\mu/\rho} = \frac{3.5\,\text{m/s} \times 0.014\,\text{m}}{959 \times 10^{-6}\,\text{N}\cdot\text{s/m}^2 / 998\,\text{kg/m}^3} = 50,993 \tag{2}$$

$$h_i = \frac{k}{D} Nu_D = \frac{k}{D} 0.023 Re_D^{0.8} Pr^{1/3} = \frac{0.606\,\text{W/m}\cdot\text{K}}{0.014\,\text{m}} \times 0.023 (50,993)^{0.8} (6.62)^{1/3} = 10,906\,\text{W/m}^2\cdot\text{K} \tag{3}$$

using the Colburn equation for fully developed turbulent conditions.

Overall coefficient:

$$\overline{U} = (1/h_i + 1/h_o)^{-1} = (1/10,906 + 1/21,800)^{-1} = 7269\,\text{W/m}^2\cdot\text{K} \tag{4}$$

Effectiveness relations: With $C_{min} = C_c$ and $\dot{m}_c = \rho(\pi D^2/4)u_m N$,

$$q = \varepsilon q_{max} = \varepsilon C_{min}\left(T_{h,i} - T_{c,i}\right) \tag{5}$$

$$C_{min} = \dot{m}_c c_c = 998\,\text{kg/m}^3 \left(\pi \times 0.014^2\,\text{m}^2/4\right) \times 3.5\,\text{m/s} \times N \times 4181\,\text{J/kg}\cdot\text{K} = 2248\,N \tag{6}$$

Continued...

PROBLEM 11.49 (Cont.)

$$5.191\times10^{6}\,\text{W} = \varepsilon\times2248\,\text{N}\,(100-15)\,\text{K}$$

$$\varepsilon N = 27.17 \qquad (7)$$

From Eq. 11.36a with $C_r = 0$, the effectiveness is

$$\varepsilon = 1-\exp(-\text{NTU}) = 1-\exp(-0.142) = 0.132 \qquad (8)$$

where, using $A_s = \pi DLNP$, NTU is evaluated as,

$$\text{NTU} = \frac{\overline{U}A_s}{C_{min}} = \frac{7269\,\text{W/m}^2\cdot\text{K}\,(\pi\times0.014\,\text{m}\times0.5\,\text{m})\,N\times2}{2248\,\text{N}} = 0.142$$

Hence, using Eq. (7), the required number of tubes is

$$N = 27.17/\varepsilon = 205.8 \approx 206$$ <

and the total surface area is

$$A_s = \pi DLNP = \pi\times0.014\,\text{m}\times0.5\,\text{m}\times206\times2 = 9.06\,\text{m}^2 .$$

(b) The water outlet temperature with $C_{min} = 2248\,N = 463{,}090$ W/K,

$$T_{c,o} = T_{c,i} + q/C_{min} = 15^\circ\text{C} + 5.191\times10^{6}\,\text{W}/463{,}090\,\text{W/K} = 26.1^\circ\text{C}$$ <

(c) The maximum condensation rate will occur when $q = q_{max}$. Hence

$$\dot{m}_{h,max} = \frac{q_{max}}{h_{fg}} = \frac{C_{min}(T_{h,i}-T_{c,i})}{h_{fg}} = \frac{463{,}090\,\text{W/K}\,(100-15)\,\text{K}}{2257\times10^{3}\,\text{J/kg}} = 17.44\,\text{kg/s} .$$ <

(d) Using the *IHT Heat Exchanger Tool*, *All Exchangers*, *$C_r = 0$*, along with the *Properties Tool* for *Water*, the foregoing analysis was performed to obtain $T_{h,o}$ and $\dot{m}_h$ using the heat transfer surface area $A_s = 9.06$ m^2 (part a) as a function of u_m.

Tco (C), mdoth*10 (kg/s)
Water mean velocity, um (m/s)
Outlet temperature, Tco (C)
Condensation rate, mdoth*10 (kg/s)

Note that the condensation rate increases nearly linearly with the water mean velocity. The cold water outlet temperature decreases nearly linearly with u_m. We should expect this behavior from energy balance considerations. Since h_h is nearly two times greater than h_c, $\overline{U}$ is controlled by the water side coefficient. Hence $\overline{U}$ will increase with increasing u_m.

COMMENTS: Note that the assumed value for $\overline{T}_m$ to evaluate water properties in part (a) was a good choice.

PROBLEM 11.50

KNOWN: Feed water heater (single shell, two tube passes) with inlet temperature 20°C supplies 10,000 kg/h of water at 65°C by condensing steam at 1.30 bar. Overall heat transfer coefficient is 2000 $W/m^2 \cdot K$.

FIND: (a) Required area using LMTD and NTU approaches, (b) Steam condensation rate.

SCHEMATIC:

ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible kinetic and potential energy changes, (3) Constant properties.

PROPERTIES: *Table A-6*, Steam (1.3 bar, saturated): $T_h = 380.3$ K, $h_{fg} = 2238 \times 10^3$ J/kg·K; *Table A-6*, Water ($\overline{T}_c = 316$ K): $c_p = 4179$ J/kg·K.

ANALYSIS: (a) Using the *LMTD approach*, from Eqs. 11.14 and 11.18,

$$A = q / U F \Delta T_{\ell m,CF} \qquad \Delta T_{\ell m,CF} = [\Delta T_1 - \Delta T_2] / \ell n (\Delta T_1 / \Delta T_2) \qquad (1,2)$$

$$\Delta T_{\ell m,CF} = [(380.3 - 338) - (380.3 - 293)] K / \ell n \frac{(380.3 - 338)}{(380.3 - 293)} = 62.1 K.$$

Since T_h is uniform throughout the HXer, F = 1. From an energy balance on the cold fluid,

$$q = \dot{m}_c c_{p,c} (T_{c,o} - T_{c,i}) = \frac{10{,}000}{3600} \frac{kg}{s} \times 4179 \frac{J}{kg \cdot K} (338 - 293) K = 5.224 \times 10^5 W.$$

Substituting numerical values into Eq. (1) find that

$$A = 5.224 \times 10^5 W / 2000 W/m^2 \cdot K \times 1 \times 62.1 K = 4.21 m^2.$$ <

Using the *NTU approach*, recognize that $C_{min} = C_c$ and $C_{max} = C_h \to \infty$ so that $C_{min}/C_{max} = 0$. The effectiveness, defined by Eq. 11.20, is

$$e \equiv \frac{q}{q_{max}} = \frac{C_c (T_{c,o} - T_{c,i})}{C_{min} (T_{h,i} - T_{c,i})} = \frac{(338 - 293) K}{(380.3 - 293) K} = 0.515.$$

From Fig. 11.16, with $\varepsilon = 0.52$ and $C_{min}/C_{max} = 0$, find NTU ≈ 0.70. Hence,

$$A = C_{min} NTU / U = \frac{(10{,}000)/(3600) kg/s \times 4179 J/kg \cdot K \times 0.70}{2000 W/m^2 \cdot K} = 4.1 m^2.$$ <

(b) The condensation rate of steam is

$$\dot{m}_h = q / h_{fg} = 5.224 \times 10^5 W / (2238 \times 10^3 J/kg) = 0.233 \ kg/s = 840 \ kg/h.$$ <

COMMENTS: Note both methods of solution given the same result. Eq. 11.31 could have been used to obtain a more precise NTU value.

PROBLEM 11.51

KNOWN: Shell-and-tube HXer with one shell and one tube pass.

FIND: (a) Oil outlet temperature for prescribed conditions, (b) Effect of fouling and water flowrate on oil outlet temperature.

SCHEMATIC:

N_T = 100 copper tubes, L = 0.5 m, D_o = 8 mm, D_i = 6 mm, $R''_{f,o}$ = 0.0003 m²·K/W (Part b), h_o = 500 W/m² K, Water, Oil

$T_{h,i}$ = 140 °C, $\dot{m}_h$ = 1 kg/s, Oil, $T_{h,o}$, $T_{c,o}$, $\dot{m}_c$ = 2 kg/s, $T_{c,i}$ = 15 °C, Water, 0, 0.5, $x(m)$

ASSUMPTIONS: (1) Constant properties, (2) Negligible changes in kinetic and potential energies, (3) Negligible fouling and losses to surroundings, (4) Uniform tube outer surface temperature.

PROPERTIES: *Table A.5*, Engine oil ($\overline{T}_h \approx 350$ K): $\rho_h = 854$ kg/m^3, $c_{p,h} = 2118$ J/kg·K, $\mu_h = 0.0356$ N·s/m^2, $k_h = 0.318$ W/m·K, $Pr_h = 546$; ($\overline{T}_s \approx 330$ K): $\mu_s = 0.0836$ N·s/m^2; *Table A.6*, Water ($\overline{T}_c \approx 320$ K): $c_{p,c} = 4180$ J/kg·K; *Table A.1*, Copper ($\overline{T} \approx 320$ K): k = 399 W/m·K.

ANALYSIS: (a) To determine the outlet temperature of the oil, we will need to know the overall heat transfer coefficient. From Eq. 11.5,

$$\frac{1}{UA} = \frac{1}{h_i A_i} + \frac{\ln(D_o/D_i)}{2\pi k L_t} + \frac{R''_{f,o}}{A_o} + \frac{1}{h_o A_o} \tag{1}$$

where $h_o = 500$ W/m^2·K (water-side) and h_i (oil-side) must be estimated from an appropriate correlation. Using properties evaluated at an estimated average mean temperature $\overline{T}_h \approx 350$ K, find

$$Re_{D,h} = \frac{4\dot{m}_{h,1}}{\pi D_i \mu_h} = \frac{4\times(1\,\text{kg/s}/100)}{\pi(0.006\,\text{m})\times 0.0356\,\text{N}\cdot\text{s/m}^2} = 59.6\,. \tag{2}$$

Since $Re_D < 2300$, the flow is laminar. To assess flow conditions, evaluate

$$Gz^{-1} = \frac{L/D_i}{Re_{D,h}\,Pr_h} = \frac{0.5\,\text{m}/0.006\,\text{m}}{59.6\times 546} = 0.00256 \tag{3}$$

Since $Gz^{-1} < 0.05$, the flow is characterized by combined entry length conditions (Fig. 8.9), and

$$\overline{Nu}_D = 1.86\left[\left(\frac{Re_D\,Pr}{L/D}\right)^{1/3}\left(\frac{\mu}{\mu_s}\right)^{0.14}\right] \tag{4}$$

where [ ] ≥ 2. To evaluate μ_s, assume $\overline{T}_s \approx 330$ K. Hence,

$$\overline{Nu}_D = 1.86\left[(0.00256)^{-1/3}\left(\frac{0.0356}{0.0836}\right)^{0.14}\right] = 1.86\times 6.48 = 12.1$$

Note that [ ] = 6.48 > 2 as required. Hence,

Continued...

$$\bar{h}_i = \overline{Nu}_D \frac{k}{D} = 12.1 \times 0.138\ \mathrm{W/m \cdot K} / (0.006\ \mathrm{m}) = 277\ \mathrm{W/m^2 \cdot K}.$$

With $R''_{f,o} = 0$ and $L_t = N_t L$, Eq. 1 yields

$$\frac{1}{UA} = \frac{1}{\pi N_t L}\left(\frac{1}{h_i D_i} + \frac{\ln(D_o/D_i)}{2k} + \frac{1}{h_o D_o}\right) \qquad (5)$$

$$\frac{1}{UA} = \frac{1}{\pi \times 100 \times 0.5\ \mathrm{m}}\left[1\Big/\left(500\ \mathrm{W/m^2 \cdot K} \times 0.008\ \mathrm{m}\right) + \ln(8/6)/(2 \times 399\ \mathrm{W/m \cdot K}) + 1\Big/\left(277\ \mathrm{W/m^2 \cdot K} \times 0.006\ \mathrm{m}\right)\right]$$

$$\frac{1}{UA} = 6.366 \times 10^{-3}\left[0.2500 + 0.0003 + 0.6017\right] = 5.424 \times 10^{-3}\ \mathrm{K/W}$$

$$UA = 184\ \mathrm{W/K}$$

With knowledge of UA, we can now use the ε - NTU method to obtain the oil outlet temperature, $T_{h,o}$. Find the capacity rates, $C = \dot{m}c_p$,

$$C_c = \dot{m}_c c_{p,c} = 2\ \mathrm{kg/s} \times 4180\ \mathrm{J/kg \cdot K} = 8360\ \mathrm{W/K} = C_{max}$$

$$C_h = \dot{m}_h c_{p,h} = 1\ \mathrm{kg/s} \times 2118\ \mathrm{J/kg \cdot K} = 2118\ \mathrm{W/K} = C_{min}$$

$$C_r = C_{min}/C_{max} = 2118/8360 = 0.253$$

From Eq. 11.25, find

$$NTU = UA/C_{min} = 184\ \mathrm{W/K}/(2118\ \mathrm{W/K}) = 0.0869. \qquad (6)$$

For this exchanger - one shell and one pass - there are no figures (11.14-19) or relations (Table 11.3) that can be directly used to evaluate ε. However, the HXer approximates a CF concentric tube HXer; hence, use Eq. 11.30a.

$$\varepsilon = \frac{1 - \exp[-NTU(1 - C_r)]}{1 - C_r \exp[-NTU(1 - C_r)]} = \frac{1 - \exp[-0.0869(1 - 0.253)]}{1 - 0.253 \exp[-0.0869(1 - 0.253)]} = 0.0824 \qquad (7)$$

From the definition of effectiveness,

$$\varepsilon = \frac{q}{q_{max}} = \frac{C_h(T_{h,i} - T_{h,o})}{C_{min}(T_{h,i} - T_{c,i})}$$

$$T_{h,o} = T_{h,i} - \varepsilon(T_{h,i} - T_{c,i}) = 140^\circ C - 0.0824(140 - 15)^\circ C = 129.7^\circ C \qquad <$$

The foregoing result indicates that $\bar{T}_h \approx 408$ K, which is much larger than the assumed value of 350 K. Since the properties of oil depend strongly on temperature, they should be re-evaluated and the foregoing calculations repeated until convergence is achieved. Using the *Correlations*, *Properties* and *Heat Exchangers* Toolpads of IHT, we obtain $h_i = 226\ \mathrm{W/m^2 \cdot K}$, UA = 159 W/K, ε = 0.064, and $T_{h,o} = 132°C$.

Continued …..

PROBLEM 11.51 (Cont.)

(b) If the foregoing calculations are repeated with $R''_{f,o}$ = 0.0003 $m^2 \cdot K/W$, there is only a slight increase in the oil outlet temperature to $T_{h,o}$ = 132.3°C. The effect is small because the fouling resistance is approximately an order of magnitude smaller than the convection resistances. As shown below,

132.5
132.3
132.1
131.9
Oil outlet temperature, Tho(C)
1 1.2 1.4 1.6 1.8 2
Water flowrate, mdotc(kg/s)
R"fo = 0
R"fo = 0.0003 m^2.K/W

the effect of the water flowrate is also small, because, even for $\dot{m}_c$ = 1 kg/s, $T_{c,o}$ is only approximately 4.5°C larger than $T_{c,i}$. Although the effect of $\dot{m}_c$ on h_o has not been considered, it would also be small since the water-side convection resistance is substantially larger than the oil side resistance.

COMMENTS: In Part (a), note that the Nusselt number for the oil entrance region flow is 12.1/3.66 ≈ 3.3 times that for fully developed flow.

PROBLEM 11.52

KNOWN: Shell-and-tube heat exchanger with one shell pass and 20 tube passes.

FIND: Average convection coefficient for the outer tube surface.

SCHEMATIC:

ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible changes in kinetic and potential energies, (3) Constant properties, (4) Type of oil not specified, (5) Thermal resistance of tubes negligible; no fouling.

PROPERTIES: *Table A-6*, Water, liquid ($\overline{T}_h$ = 330 K): c_p = 4184 J/kg·K, k = 0.650 W/m·K, μ = 489 $\times 10^{-6}$ N·s/m^2, Pr = 3.15.

ANALYSIS: To find the average coefficient for the outer tube surface, h_o, we need to evaluate h_i for the internal tube flow and U, the overall coefficient. From Eq. 11.5,

$$\frac{1}{UA} = \frac{1}{h_i A_i} + \frac{1}{h_o A_o} = \frac{1}{N_t \pi L}\left[\frac{1}{h_i D_i} + \frac{1}{h_o D_o}\right]$$

where N_t is the total number of tubes. Solving for h_o,

$$h_o = D_o^{-1}\left[(UA)^{-1} N_t \pi L - 1/h_i D_i\right]^{-1}. \qquad (1)$$

Evaluate h_i from an appropriate correlation; begin by calculating the Reynolds number.

$$Re_{D,i} = \frac{4\,\dot{m}_h}{\pi D_i \mu} = \frac{4 \times 0.2\ \text{kg/s}}{\pi\,(0.020\text{m})\,489 \times 10^{-6}\ \text{N}\cdot\text{s/m}^2} = 26{,}038.$$

Hence, flow is turbulent and since L >> D_i, the flow is likely to be fully developed. Use the Dittus-Boelter correlation with n = 0.3 since $T_s < T_m$, $Nu_D = 0.023\ Re_D^{4/5}\ Pr^{0.3}$

$$h_i = \frac{k}{D} Nu_D = \frac{0.650\ \text{W/m}\cdot\text{K}}{0.020\text{m}} \times 0.023(26{,}038)^{4/5}(3.15)^{0.3} = 3594\ \text{W/m}^2\cdot\text{K}. \qquad (2)$$

To evaluate UA, we need to employ the rate equation, written as

$$UA = q / F\,\Delta T_{\ell n,CF} \qquad (3)$$

where q = $\dot{m}_h\ c_{p,h}\ (T_{h,i} - T_{h,o})$ = 0.2 kg/s × 4184 J/kg·K) (87-27)°C = 50,208 W and $\Delta T_{\ell n,CF}$ = $[\Delta T_1 - \Delta T_2]/\ell n\ (\Delta T_1/\Delta T_2)$ = [(87 – 37) – (27 – 7)]°C/ ℓn (87 – 37/27 – 7) = 32.7°C. Find F ≈ 0.5 using Fig. 11.10 with P = (27 – 87)/(7 – 87) = 0.75 and R = (7 – 37)/(27 – 87) = 0.50. Substituting numerical values in Eqs. (3) and (1), find

$$UA = 50{,}208\,\text{W}/0.5 \times 32.7°\text{C} = 3071\,\text{W/K} \qquad (4)$$

$$h_o = (0.024\text{m})^{-1}\left[(3071\ \text{W/K})^{-1} \times 20 \times \pi \times 3\text{m} - 1/3594\ \text{W/m}^2\cdot\text{K} \times 0.020\text{m}\right]^{-1} = 878\ \text{W/m}^2\cdot\text{K}. \quad <$$

COMMENTS: Using the ε-NTU method: find C_h and C_c to obtain C_r = 0.5 and ε = 0.75. From Eq. 11.31b,c find NTU = 3.59 and UA = 3003 W/K.

PROBLEM 11.53

KNOWN: Engine oil cooled by air in a cross-flow heat exchanger with both fluids unmixed.

FIND: (a) Heat transfer coefficient on oil side of exchanger assuming fully-developed conditions and constant wall heat flux, (b) Effectiveness, and (c) Outlet temperature of the oil.

SCHEMATIC:

ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible KE and PE changes, (3) Constant properties, (4) Oil flow and thermal conditions are fully developed, (5) Oil cooling process approximates constant wall flux conditions.

PROPERTIES: *Table A-5*, Engine oil (assume $T_{h,o} \approx 45°C$, $\overline{T}_h$ = (45 + 75)°C/2 = 333 K): c_h = 2047 J/kg·K, $\mu = 7.45 \times 10^{-2}$ N·s/m^2, k = 0.140 W/m·K; *Table A-4*, Air (assume $T_{c,o} \approx 40°C$, $\overline{T}_c$ = (30 + 40)°C/2 = 308 K, 1 atm): c_c = 1007 J/kg·K.

ANALYSIS: (a) For the oil side, using Eq. 8.6, find,

$$Re_D = 4\,\dot{m}/p\,D\,m = 4(0.026\ \text{kg/s})/\left(p\,(0.01\text{m})\,7.45\times10^{-2}\,\text{N}\cdot\text{s/m}^2\right) = 44.4$$

Since $Re_D < 2000$ the flow is laminar. For the fully-developed conditions with constant wall flux,

$$Nu_D = \frac{h_i D}{k} = 4.36, \qquad h_i = 4.36\frac{k}{D} = 4.36\frac{0.140\ \text{W/m}\cdot\text{K}}{0.01\text{m}} = 61.0\ \text{W/m}^2\cdot\text{K}. \qquad <$$

(b) The effectiveness can be determined by the ε-NTU method.

$$C_h = \dot{m}_h c_h = 0.026\ \text{kg/s}\times 2047\ \text{J/kg}\cdot\text{K} = 53.22\ \text{W/K} \qquad C_{min} = C_h$$

$$C_c = \dot{m}_c c_c = 0.53\ \text{kg/s}\times 1007\ \text{J/kg}\cdot\text{K} = 533.7\ \text{W/K} \qquad C_{min}/C_{max} = 0.10$$

$$NTU = UA/C_{min} = 53\ \text{W/m}^2\cdot\text{K}\times 1\text{m}^2/53.22\ \text{W/K} = 1.00.$$

Using Fig. 11.18, with C_{min}/C_{max} = 0.1 and NTU = 1, find ε ≈ 0.64. <

(c) From Eqs. 11.20 and 11.19,

$$e = \frac{q}{q_{max}} = \frac{C_h\left(T_{h,i} - T_{h,o}\right)}{C_{min}\left(T_{h,i} - T_{c,i}\right)} = \frac{T_{h,i} - T_{h,o}}{T_{h,i} - T_{c,i}}.$$

Solving for $T_{h,o}$ and substituting numerical values, find

$$T_{h,o} = T_{h,i} - e\left(T_{h,i} - T_{c,i}\right) = 75°C - 0.64(75 - 30)°C = 46.2°C. \qquad <$$

COMMENTS: Note that the $\overline{T}_h$ value at which the oil properties were evaluated is reasonable.

PROBLEM 11.54

KNOWN: Shell-tube heat exchanger with one shell and single tube pass; Tube side: exhaust gas with specified flow rate and temperature change; Shell side: supply of saturated water at 11.7 bar; Tube dimensions and thermal conductivity, and fouling resistance on gas side, $R''_{f,h}$, specified.

FIND: Number of tubes and their length if the gas velocity is not to exceed $u_{m,i}$ = 25 m/s.

SCHEMATIC:

Sat. water
p = 11.7 bar

Exhaust gas
$T_{h,i}$ = 400°C
m_h = 2 kg/s

$T_{h,o}$ = 215°C

One shell
Single pass, N tubes, length L
D_i = 50 mm, D_o = 58 mm
k = 40 W/m-K, $u_{m,i}$ < 25 m/s
$R''_{f,h}$ = 0.0015 m^2-K/W

ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible losses to the surroundings and kinetic and potential energy changes, (3) Negligible water-side thermal resistance, (4) Exhaust gas properties are those of atmospheric air, (5) Gas-side flow is fully developed, and (6) Constant properties.

PROPERTIES: *Table A-4*, Air $\left(\overline{T}_h = 581\text{ K}\right)$: $\rho = 0.600\text{ kg/m}^3$, $c = 1047\text{ J/kg}\cdot\text{K}$, $\nu = 4.991\times10^{-5}\text{ m}^2/\text{s}$, $k = 0.0457\text{ W/m}\cdot\text{K}$, $Pr = 0.684$. *Table A-6*, Water (11.7 bar, saturated): $T_{c,i} = 460\text{ K} = 187^\circ\text{C}$.

ANALYSIS: We'll employ the NTU-ε method to design the exchanger. Since C_r = 0, use Eq. 11.36b.

$$NTU = -\ell n(1-\varepsilon)$$

where the effectiveness can be evaluated from Eqs. 11.19 and 11.20.

$$C_{min} = C_h = \dot{m}_h c_h = 2\text{ kg/s}\times1047\text{ J/kg}\cdot\text{K} = 2094\text{ W/K}$$

$$\varepsilon = \frac{C_h\left(T_{h,i}-T_{h,o}\right)}{C_{min}\left(T_{h,i}-T_{c,i}\right)} = \frac{(400-215)^\circ\text{C}}{(400-187)^\circ\text{C}} = 0.868$$

$$NTU = -\ell n(1-0.868) = 2.029$$

From Eq. 11.25,

$$UA = C_{min}\cdot NTU = 2094\text{ W/K}\times2.029 = 4249\text{ W/K} \qquad (1)$$

Considering the gas-side flow rate and velocity criteria, find the number of tubes required as

$$\dot{m}_h = N\cdot\rho_h\cdot A_c\cdot u_{m,i} = N\cdot\rho_h\left(\pi D_i^2/4\right)u_{m,i}$$

Continued …..

PROBLEM 11.54 (Cont.)

$$2\ \text{kg/s} = N \times 0.6009\ \text{kg/m}^3 \times \pi (0.050\ \text{m})^2 / 4 \times 25\ \text{m/s}$$

$$N = 67.8\ \text{tubes, specify } 68$$ <

The overall coefficient, considering the convection process, fouling resistance and the tube thermal resistance, is evaluated as

$$U_i = 1 / \left[R''_{f,i} + R''_{cv,i} + R''_{cd,t} \right] = 56.4\ \text{W/m}^2 \cdot \text{K}$$

$$R''_{f,i} = 0.0015\ \text{m}^2 \cdot \text{K/W}$$

$$R''_{cv,i} = 1/h_i = 1/62\ \text{W/m}^2 \cdot \text{K} = 0.0161\ \text{m}^2 \cdot \text{K/W}$$

$$R''_{cd,t} = \frac{D_i\ \ell n\ (D_o / D_i)}{2\ k} = \frac{0.050\ \text{m}\ \ell n (58/50)}{2 \times 40\ \text{W/m} \cdot \text{K}} = 9.28 \times 10^{-5}\ \text{m}^2 \cdot \text{K/W}$$

where the gas-side convection coefficient estimate is explained in the Comments section. Substituting numerical values, determine the required tube length

$$[UA] = U_i \cdot A_i = U_i\ (N\ \pi\ D_i\ L)$$

$$4249\ \text{W/K} = 56.4\ \text{W/m}^2 \cdot \text{K} \times 68 \times \pi \times 0.050\ \text{m} \times L$$

$$L = 7.1\ \text{m}$$ <

COMMENTS: (1) Is the assumption of negligible water-side thermal resistance reasonable? Explain why.

(2) Knowing the tube gas-side velocity, the usual convection correlation calculation methodology is followed. The flow is turbulent, $Re_{Di} = 2.5 \times 10^4$, and assuming fully developed flow, use the Dittius-Boelter correlation, Eq. 8.60, to find $Nu_{Di} = 67.8$ and $h_i = 62.0\ \text{W/m}^2 \cdot \text{K}$.

PROBLEM 11.55

KNOWN: Hot and cold gas flow rates and inlet temperatures of a recuperator. Overall heat transfer coefficient. Desired cold gas outlet temperature.

FIND: (a) Required surface area, (b) Effect of surface area on cold-gas outlet temperature.

SCHEMATIC:

Combustion products

$\dot{m}_h = 6.5$ kg/s
$T_{h,i} = 700$ K

Air
$\dot{m}_c = 6.2$ kg/s
$T_{c,i} = 300$ K

$T_{c,o} = 500$ K

Cross-flow HX
(both fluids unmixed)
$U = 100\ W/m^2\text{-}K$

ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible kinetic and potential energy and flow work changes, (3) Constant properties.

PROPERTIES: Given: $c_{p,c} = c_{p,h} = 1040$ W/m·K.

ANALYSIS: (a) With $C_{min} = C_c = 6.2$ kg/s × 1040 J/kg·K = 6,448 W/K, $C_{max} = C_h = 6.5$ kg/s × 1040 J/kg·K = 6,760 W/K, $C_r = C_{min}/C_{max} = 0.954$, $q = C_c (T_{c,o} - T_{c,i}) = 6{,}448$ W/K (200 K) = 1.29×10^6 W, $q_{max} = C_{min} (T_{h,i} - T_{c,i}) = 6{,}448$ W/K (400 K) = 2.58×10^6 W, and $\varepsilon = q/q_{max} = 0.50$, Fig. 11.18 yields NTU ≈ 1.10. Hence

$$A = \frac{NTU \times C_{min}}{U} = \frac{1.10 \times 6,448\ W/K}{100\ W/m^2 \cdot K} = 70.9\ m^2$$ <

(b) Using the Heat Exchanger option of *IHT*, the following result was obtained

Air outlet temperature, K

600
550
500
450
400
350
300

0 50 100 150 200

Heat exchanger area, m^2

The air outlet temperature increases, of course, with increasing heat exchanger area, but the approach to the maximum possible outlet temperature, $T_{h,i}$, is slow and the heat exchanger size needed to achieve a large outlet temperature may be prohibitively expensive.

PROBLEM 11.56

KNOWN: Inlet temperature and flow rates for a concentric tube heat exchanger. Hot fluid outlet temperature.

FIND: (a) Maximum possible heat transfer rate and effectiveness, (b) Preferred mode of operation.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state operation, (2) Negligible KE and PE changes, (3) Negligible heat loss to surroundings, (4) Fixed overall heat transfer coefficient.

PROPERTIES: *Table A-5*, Ethylene glycol ($\overline{T}_m$ = 80°C): c_p = 2650 J/kg·K; *Table A-6*, Water ($\overline{T}_m$ ≈ 30°C): c_p = 4178 J/kg·K.

ANALYSIS: (a) Using the ε-NTU method, find

$$C_{min} = C_h = \dot{m}_h c_{p,h} = (0.5\,kg/s)(2650\,J/kg\cdot K) = 1325\,W/K.$$

Hence from Eqs. 11.19 and 11.6,

$$q_{max} = C_{min}(T_{h,i} - T_{c,i}) = (1325\,W/K)(100-15)°C = 1.13\times10^5\,W.$$

$$q = \dot{m}_h c_{p,h}(T_{h,i} - T_{h,o}) = 0.5\,kg/s(2650\,J/kg\cdot K)(100-60)°C = 0.53\times10^5\,W. \quad <$$

Hence from Eq. 11.20,

$$e = q/q_{max} = 0.53\times10^5/1.13\times10^5 = 0.47. \quad <$$

(b) From Eq. 11.7,

$$T_{c,o} = T_{c,i} + \frac{q}{\dot{m}_c c_{p,c}} = 15°C + \frac{0.53\times10^5}{0.5\,kg/s\times4178\,J/kg\cdot K} = 40.4°C.$$

Since $T_{c,o} < T_{h,o}$, a *parallel flow* mode of operation is possible. However, with $(C_{min}/C_{max}) = (\dot{m}_h c_{p,h}/\dot{m}_c c_{p,c}) = 0.63$,

Fig. 11.14 → $(NTU)_{PF} \approx 0.95$ Fig. 11.15 → $(NTU)_{CF} \approx 0.75$.

Hence from Eq. 11.15

$$(A_{CF}/A_{PF}) = (NTU)_{CF}/(NTU)_{PF} \approx (0.75/0.95) = 0.79.$$

Because of the reduced size requirement, and hence capital investment, the *counterflow* mode of operation is preferred.

PROBLEM 11.57

KNOWN: Single-pass, cross-flow heat exchanger with both fluids (water) unmixed; hot water enters at 90°C and at 10,000 kg/h while cold water enters at 10°C and at 20,000 kg/h; effectiveness is 60%.

FIND: Cold water exit temperature, $T_{c,o}$.

SCHEMATIC:

ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible kinetic and potential energy changes, (3) Constant properties.

PROPERTIES: *Table A-6*, Water ($\overline{T}_c \approx (10 + 40)°C/2 \approx 300$ K): $c_c = 4179$ J/kg·K; *Table A-6*, Water ($\overline{T}_h \approx (90 + 60)°C/2 \approx 350$ K): $c_h = 4195$ J/kg·K.

ANALYSIS: From an energy balance on the cold fluid, Eq. 11.7, the outlet temperature can be expressed as

$$T_{c,o} = T_{c,i} + q / \dot{m}_c C_c.$$

The heat rate can be written in terms of the effectiveness and q_{max}. Using Eqs. 11.20 and 11.19,

$$q = e\, q_{max} = e\, C_{min}\left(T_{h,i} - T_{c,i}\right).$$

By inspection, it can be noted that the hot fluid is the minimum capacity fluid. Substituting numerical values,

$$q = e\left(\dot{m}_h\, c_h\right)\left(T_{h,i} - T_{c,i}\right)$$

$$q = 0.60\left(10{,}000\,\text{kg/h}/3600\,\text{s/h}\right)4195\,\text{J/kg}\cdot\text{K}\left(90-10\right)°\text{C} = 559.3\times10^3\ \text{W}.$$

The exit temperature of the cold water is then

$$T_{c,o} = 10°\text{C} + 559.3\times10^3\ \text{W} / \frac{20{,}000}{3600}\text{kg/s}\times4179\,\text{J/kg}\cdot\text{K} = 34.1°\text{C}.$$ <

COMMENTS: (1) The properties of the cold fluid should be evaluated at $\overline{T} = (T_{c,o} + T_{c,i})/2 = (34.1 + 10)°C/2 = 295$ K. Note the analysis assumed $\overline{T}_c \approx 300$ K, hence little error is incurred. For best precision, one should check $\overline{T}_h$ and C_h.

(2) From Fig. 11.18, the value of NTU could be determined. First evaluate the term

$$C_{min} / C_{max} = \dot{m}_h\, C_h / \dot{m}_c\, C_c = \frac{10{,}000\times4195}{20{,}000\times4179} = 0.50$$

and with $\varepsilon = 0.60$, find NTU ≈ 1.2.

PROBLEM 11.58

KNOWN: Hxer consisting of 32 tubes in 0.6m square duct. Hot water enters tubes at 150°C with mean velocity 0.5 m/s. Atmospheric air at 10°C enters exchanger with volumetric flow rate of 1 m^3/s. Heat transfer coefficient on tube outer surfaces is 400 W/m^2·K.

FIND: Outlet temperatures of the fluids, $T_{c,o}$ and $T_{h,o}$.

SCHEMATIC:

ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible potential and kinetic energy changes, (3) Constant properties, (4) Hxer is a single-pass, cross-flow type with one fluid mixed (air) and the other unmixed (water), (5) Tube water flow is fully developed, (6) Negligible thermal resistance due to tube wall.

PROPERTIES: *Table A-4*, Air ($T_{c,i}$ = 10°C = 283 K, 1 atm): r = 1.2407 kg/m^3; *Table A-4*, Air (assume $T_{c,o} \approx 40°C$, $\overline{T}_c$ = (10 + 40)°C/2 = 298 K, 1 atm): c_p = 1007 J/kg·K; *Table A-6*, Water (assume $T_{h,o} \approx 140°C$, $\overline{T}_h$ = (140 + 150)°C/2 = 418 K): $r = 1/v_f = 1/1.0850 \times 10^{-3}$ m^3/kg, c_p = 4297 J/kg·K, $m_f = 188 \times 10^{-6}$ N·s/m^2, k_f = 0.688 W/m·K, Pr_f = 1.18.

ANALYSIS: Using the ε-NTU method, first find the capacity rates.

$$C_h = \dot{m}_h c_{p,h} = \left(r\, A_c\, u_m \right)_h N \cdot c_{p,h}$$

$$C_h = \frac{1}{1.0850 \times 10^{-3}\, m^3/kg} \times \frac{p}{4}\left(10.2 \times 10^{-3} m\right)^2 \times 0.5\frac{m}{s} \times 32 \times 4297\frac{J}{kg \cdot K} = 5178\frac{W}{K}$$

$$C_c = \dot{m}_c c_{p,c} = \left(r\, V \right)_c c_{p,c} = 1.2407\frac{kg}{m^3} \times 1m^3/s \times 1007 J/kg \cdot K = 1249\frac{W}{K}. \qquad (1,2)$$

Note that the cold fluid is the minimum fluid, $C_c = C_{min}$. The overall heat transfer coefficient follows from Eq. 11.5,

$$U_o A_o = \left[\frac{1}{h_i A_i} + \frac{1}{h_o A_o} \right]^{-1} \qquad (3)$$

where h_i must be estimated from an appropriate internal flow correlation. The Reynolds number for water flow is

$$Re_D = \frac{r\, u_m D_i}{m} = \frac{\left(1/1.0850 \times 10^{-3} m^3 kg\right) \times 0.5 m/s \times \left(10.2 \times 10^{-3} m\right)}{188 \times 10^{-6} N \cdot s/m^2} = 25{,}002. \qquad (4)$$

Continued …..

The flow is turbulent and since $L/D_i = 0.6\text{m}/10.2 \times 10^{-3}\text{m} = 59$, fully developed conditions may be assumed. The Dittus-Boelter correlation with n = 0.3 is appropriate.

$$Nu_D = \frac{h_i D_i}{k} = 0.023 Re_D^{0.8} Pr^{0.3} = 0.023(25{,}002)^{0.8}(1.18)^{0.3} = 79.7$$

$$h_i = \frac{k}{D_i} Nu_D = \frac{0.688\,\text{W/m}\cdot\text{K}}{10.2\times10^{-3}\,\text{m}} \times 79.7 = 5376\,\text{W/m}^2\cdot\text{K}.$$

Substituting numerical values into Eq. (3), find

$$U_o = \left[\left(\frac{12.5\text{mm}}{10.2\text{mm}}\right)\frac{1}{5376\,\text{W/m}^2\cdot\text{K}} + \frac{1}{400\,\text{W/m}^2\cdot\text{K}}\right]^{-1} = 366.6\,\text{W/m}^2\cdot\text{K}.$$

It follows from Eq. 11.25, with $A_o = N(\pi D_o L)$, that

$$NTU = \frac{U_o A_o}{C_{min}} = 366.6\frac{\text{W}}{\text{m}^2\cdot\text{K}} \times \left(32\times p \times 12.5\times10^{-3}\text{m}\times0.6\text{m}\right)/1249\frac{\text{W}}{\text{K}} = 0.22.$$

From Fig. 11.19, noting that $C_{min} = C_c$ is the mixed fluid (solid curves),

$$\frac{C_{mixed}}{C_{unmixed}} = \frac{C_{min}}{C_{max}} = \frac{C_c}{C_h} = \frac{1249\,\text{W/K}}{5178\,\text{W/K}} = 0.24$$

and with NTU = 0.22 find $\varepsilon \approx 0.19$. From the definition of effectiveness, Eq. 11.20,

$$e = \frac{q}{q_{max}} = \frac{C_c\left(T_{c,o} - T_{c,i}\right)}{C_{min}\left(T_{h,i} - T_{c,i}\right)}$$

$$T_{c,o} = T_{c,i} + e\left(T_{h,i} - T_{c,i}\right) = 10^\circ\text{C} + 0.19(150 - 10)^\circ\text{C} = 36.6^\circ\text{C}.$$ <

Equating the energy balances on both fluids,

$$C_c\left(T_{c,o} - T_{c,i}\right) = C_h\left(T_{h,i} - T_{h,o}\right)$$

or

$$T_{h,o} = T_{h,i} - \frac{C_c}{C_h}\left(T_{c,o} - T_{c,i}\right)$$

$$T_{h,o} = 150^\circ\text{C} - \frac{1249\,\text{W/K}}{5178\,\text{W/K}}(36.6 - 10)^\circ\text{C} = 143.5^\circ\text{C}.$$ <

COMMENTS: (1) Note that the assumptions of $T_{h,o}$ and $T_{c,o}$ used in evaluating properties are reasonable.

(2) Note that to calculate $\dot{m}_c$ from V, the density at 10°C is more appropriate than at $\overline{T}_c$.

PROBLEM 11.59

KNOWN: Flow rates and inlet temperatures of exhaust gases and combustion air used in a cross-flow (one fluid mixed) heat exchanger. Overall heat transfer coefficient. Desired air outlet temperature.

FIND: Required heat exchanger surface area.

SCHEMATIC:

Air $T_{c,i} = 300K$ $\dot{m}_c = 10 kg/s$

Exhaust gases $T_{h,i} = 1100K$ $\dot{m}_h = 15 kg/s$

$T_{h,o}$

Cross flow hxer, air mixed, $U = 100 W/m^2 \cdot K$

$T_{c,o} = 850K$

ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible heat loss to surroundings, (3) Negligible kinetic and potential energy changes, (4) Constant properties, (5) Gas properties are those of air.

PROPERTIES: *Table A-4*, Air ($\overline{T}_m \approx 700$ K, 1 atm): $c_p = 1075$ J/kg·K.

ANALYSIS: From Eqs. 11.6 and 11.7,

$$T_{h,o} = T_{h,i} - \frac{\dot{m}_c c_{p,c}}{\dot{m}_h c_{p,h}}\left(T_{c,o} - T_{c,i}\right) = 1100K - \frac{10 \text{ kg/s}}{15 \text{ kg/s}}(850 - 300)K = 733K.$$

From Eqs. 11.15, 11.17 and 11.18,

$$\Delta T_{\ell m} = F\frac{\left(T_{h,i} - T_{c,o}\right) - \left(T_{h,o} - T_{c,i}\right)}{\ell n\left[\left(T_{h,i} - T_{c,o}\right)/\left(T_{h,o} - T_{c,i}\right)\right]} = F\frac{250 - 433}{\ell n\left(250/433\right)} = F \times 333K.$$

From Fig. 11.13, with R = (300 – 850)/(733 – 1100) = 1.50 and P = (733 – 1100)/(300 – 1100) = 0.46, F ≈ 0.73. With

$$q = \dot{m}_h c_{p,h}\left(T_{h,i} - T_{h,o}\right) = 15 kg/s \times 1075 J/kg \cdot K\left(367K\right) = 5.92 \times 10^6 W$$

it follows from Eq. 11.14 that

$$A = \frac{5.92 \times 10^6 W}{100 W/m^2 \cdot K \times 0.73(333K)} = 243 m^2.$$

<

COMMENTS: Using the effectiveness-NTU method, from Eq. 11.22

$$e = \frac{T_{c,o} - T_{c,i}}{T_{h,i} - T_{c,i}} = \frac{(850 - 300)K}{(1100 - 300)K} = 0.688.$$

Hence, with $C_{mixed}/C_{unmixed} = C_c/C_h = 0.67$, Fig. 11.19 gives NTU ≈ 2.3. From Eq. 11.25,

$$A = NTU\frac{C_{min}}{U} \approx 2.3\frac{10 \text{ kg/s} \times 1075 \text{ J/kg} \cdot K}{100 \text{ W/m}^2 \cdot K} \approx 247 m^2.$$

PROBLEM 11.60

KNOWN: Dimensions, configuration and material of a single-pass, cross-flow heat exchanger. Inlet conditions of inner and outer flow. Fouling factor of inner surface.

FIND: (a) Percent fuel savings for prescribed conditions, (b) Effect of UA on air outlet temperature and fuel savings.

SCHEMATIC:

$\dot{m}_h = 1.05$ kg/s, $\dot{m}_c = 1.0$ kg/s, $T_{h,i} = 1400$ K, $T_{c,o}$, $T_{h,o}$, $T_{c,i} = 300$ K

Tube, $k = 20$ W/m·K, $D_o = 80$ mm, $D_i = 55$ mm, $V = 1$m/s, $R''_{f,i} = 2 \times 10^{-4}$ m²·K/W, $L = 1.4$ m, $N = 80$, $S_L = 100$ mm, $S_T = 120$ mm

ASSUMPTIONS: (1) Negligible heat loss to surroundings and potential and kinetic energy changes, (2) Air properties are those of atmospheric air at 300 K, (3) Gas properties are those of atmospheric air at 1400 K, (4) Tube wall temperature may be approximated as 800 K for treating variable property effects.

PROPERTIES: *Table A.4*, Air (1 atm, T = 300 K): $\nu = 15.89 \times 10^{-6}$ m²/s, $c_p = 1007$ J/kg·K, k = 0.0263 W/m·K, Pr = 0.707; (T = 1400 K): $\mu = 530 \times 10^{-7}$ kg/s·m, $c_p = 1207$ J/kg·K, k = 0.091 W/m·K, Pr = 0.703; (T = 800 K): $\mu = 370 \times 10^{-7}$ kg/s·m, Pr = 0.709.

ANALYSIS: (a) With capacity rates of $C_c = \dot{m}_c c_{p,c}$ = 1 kg/s × 1007 J/kg·K = 1007 W/K = C_{min} and $C_h = \dot{m}_h c_{p,h}$ = 1.05 kg/s × 1207 J/kg·K = 1267 W/K = C_{max}, $C_{min}/C_{max} = 0.795$. The overall coefficient is

$$\frac{1}{UA} = \frac{1}{h_i A_i} + \frac{R''_{f,i}}{A_i} + \frac{\ln(D_o/D_i)}{(2\pi kL)N} + \frac{1}{h_o A_o}.$$

For flow through a single tube,

$$Re_D = \frac{4\dot{m}_h}{N\pi D_i \mu} = \frac{4 \times 1.05\,\text{kg/s}}{80\pi(0.055\,\text{m})530\times10^{-7}\,\text{kg/s}\cdot\text{m}} = 5733.$$

Assuming fully developed turbulent flow throughout and using the Sieder-Tate correlation,

$$Nu_D = 0.027\,Re_D^{4/5}\,Pr^{1/3}(\mu/\mu_s)^{0.14} = 0.027(5733)^{4/5}(0.703)^{1/3}(530/370)^{0.14} = 25.6$$

$$h_i = Nu_D k/D_i = 25.6(0.091\,\text{W/m}\cdot\text{K})/0.055\,\text{m} = 42.4\,\text{W/m}^2\cdot\text{K}.$$

For flow over the tube bank,

$$V_{max} = [S_T/(S_T - D_o)]V = [0.12\,\text{m}/(0.12 - 0.08)\,\text{m}]1\,\text{m/s} = 3\,\text{m/s}$$

$$Re_{D,max} = \frac{V_{max} D_o}{\nu} = \frac{3\,\text{m/s}(0.08\,\text{m})}{15.89\times10^{-6}\,\text{m}^2/\text{s}} = 15{,}100$$

From the Zhukauskas correlation for a tube bank,

$$\overline{Nu}_D = 0.27(15{,}100)^{0.63}(0.707)^{0.36}(0.707/0.709)^{1/4} = 102.3$$

$$\bar{h}_o = \overline{Nu}_D(k/D_o) = 102.3(0.0263\,\text{W/m}\cdot\text{K})/0.08\,\text{m} = 33.6\,\text{W/m}^2\cdot\text{K}.$$

Hence, based on the inner surface, the *overall coefficient* is

Continued...

$$\frac{1}{U_i} = \frac{1}{h_i} + R''_{f,i} + \frac{D_i \ln(D_o/D_i)}{2k} + \frac{D_i}{D_o h_o}$$

$$\frac{1}{U_i} = \left(0.0236 + 0.0002 + \frac{0.055 \ln(0.08/0.055)}{40} + \frac{0.055}{0.08 \times 33.6}\right) m^2 \cdot K/W$$

$$U_i = \left[(0.0236 + 0.0002 + 0.0005 + 0.0246) m^2 \cdot K/W\right]^{-1} = 22.3\, W/m^2 \cdot K\,.$$

Hence, $(UA)_i = U_i N \pi D_i L = 22.3\, W/m^2 \cdot K \times 80\pi (0.055\, m) 1.4\, m = 432\, W/K$. The number of transfer units is then NTU = UA/C_{min} = 432 W/K/1007 W/K = 0.429, and with $C_{mixed}/C_{unmixed} = C_c/C_h = C_{min}/C_{max}$ = 0.795, Fig. 11.19 yields $\varepsilon \approx 0.3$ or, from Eq. 11.35 a,

$$\varepsilon = 1 - \exp\left(-C_r^{-1}\{1 - \exp[-C_r \cdot NTU]\}\right) = 0.305\,.$$

Hence, with

$$q_{max} = C_{min}(T_{h,i} - T_{c,i}) = 1007\, W/K (1100\, K) = 1.11 \times 10^6\, W$$

$$q = \varepsilon q_{max} = 0.305 \times 1.11 \times 10^6\, W = 337,800\, W$$

$$T_{c,o} = T_{c,i} + q/C_{min} = 300\, K + (337,800\, W/1007\, W/K) = 635\, K\,.$$

Hence,

$$\%\text{ fuel savings} \equiv FS = (\Delta T_c/10\, K) \times 1\% = (335\, K/10\, K) \times 1\% = 33.5\%$$ <

(b) Using the Heat Exchangers Toolpad of IHT to perform the parametric calculations, the following results are obtained.

Significant benefits are derived by increasing UA, with values of $T_{c,o}$ = 716 K and FS = 41.6% obtained for UA = 600 W/K. The major contributions to the total resistance are made by the inner and outer convection resistances. These contributions could be reduced by using extended surfaces on both the inner and outer surfaces.

COMMENTS: For part (a), properties of the flue gas should be evaluated at $(T_{h,i} + T_{h,o})/2$ and the calculations repeated. The Colburn equation yields

$$Nu_D = 0.023 Re_D^{4/5} Pr^{1/3} = 20.8$$

which is 19% less than the result of the Sieder-Tate correlation.

PROBLEM 11.61

KNOWN: Rate of thermal energy production in combustor and transfer to load in furnace. Cold air and flue gas flowrates and specific heats in recuperator. Recuperator cold air inlet temperature.

FIND: Recuperator hot gas inlet and outlet temperatures and air outlet temperature for a recuperator effectiveness of $\varepsilon = 0.3$. Value of ε needed to achieve a recuperator outlet temperature of 800 K.

SCHEMATIC:

ASSUMPTIONS: (1) Negligible flowwork and potential and kinetic energy changes, (2) Constant properties, (3) Negligible effect of fuel addition on flowrate.

PROPERTIES: Air and gas: $c_{p,c} = c_{p,h} = 1200$ J/kg·K.

ANALYSIS: With $C_c = C_h = C_{min}$, the effectiveness of the recuperator, $\varepsilon = q/q_{max}$, may be expressed as

$$\varepsilon = \frac{C_c\left(T_{c,o} - T_{c,i}\right)}{C_{min}\left(T_{h,i} - T_{c,i}\right)} = \frac{T_{c,o} - 300\,K}{T_{h,i} - 300\,K} = 0.3$$

The unknown temperatures, $T_{c,o}$ and $T_{h,i}$, are also related through an energy balance performed on the air entering the combustor and leaving the furnace. Specifically,

$$C\left(T_{h,i} - T_{c,o}\right) = q_{comb} - q_{load} = 0.6 \times 10^6\,W$$

where C = 1 kg/s × 1200 J/kg·K = 1200 W/K. Solving the foregoing equations, we obtain

$$T_{h,i} = 1014\,K \qquad T_{c,o} = 514\,K$$ <

Expressing the effectiveness as

$$\varepsilon = \frac{C_h\left(T_{h,i} - T_{h,o}\right)}{C_{min}\left(T_{h,i} - T_{c,i}\right)} = \frac{1014\,K - T_{h,o}}{714\,K}$$

we also obtain $T_{h,o} = 800$ K. <

For a combustor air inlet temperature of $T_{c,o} = 800$ K and $T_{h,i} = 1014$ K, the required effectiveness is

$$\varepsilon = \frac{T_{c,o} - T_{c,i}}{T_{h,i} - T_{c,i}} = \frac{(800 - 300)\,K}{(1014 - 300)\,K} = 0.70$$ <

COMMENTS: The effectiveness of the recuperator may be increased by increasing NTU and hence UA, as, for example, by increasing the number of tubes.

PROBLEM 11.62

KNOWN: Single-shell, two-tube pass heat exchanger with surface area 0.5 m^2 and overall heat transfer coefficient of 2000 $W/m^2 \cdot K$; saturated steam at 100°C condenses on one side while water at a flow rate of 0.5 kg/s enters at 15°C.

FIND: (a) Outlet temperature of the water, $T_{c,o}$, (b) Rate of condensation of steam, $\dot{m}_h$.

SCHEMATIC:

Steam, T_h; One shell, two tube pass; Water. $C_h \to \infty$, $T_h = 100°C$; $T_{c,i} = 15°C$, $T_{c,o}$; $\dot{m}_c = 0.5 kg/s$; $A = 0.5 m^2$, $U = 2000 W/m^2 \cdot K$

ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible, kinetic and potential energy changes, (3) Constant properties.

PROPERTIES: *Table A-6*, Steam (100°C, 1 atm): h_{fg} = 2257 kJ/kg; *Table A-6*, Water ($\overline{T}_c \approx$ (15 + 35)°C/2 ≈ 300 K): c_c = 4179 J/kg·K.

ANALYSIS: (a) Using the ε-NTU method of analysis, recognize that the minimum capacity fluid is the cold fluid since for the hot fluid, $C_h \to \infty$. See Fig. 11.9a. That is,

$$C_{min} = \dot{m}_c c_c = 0.5 \text{ kg/s} \times 4179 \text{ J/kg} \cdot \text{K} = 2090 \text{ W/K}.$$

It follows also that,

$$NTU = AU/C_{min} = 0.5 m^2 \times 2000 \text{ W/m}^2 \cdot \text{K}/2090 \text{ W/K} = 0.48.$$

Using NTU = 0.48 and $C_{min}/C_{max} = 0$, find from Fig. 11.16 that ε = 0.39. Since $\dot{m}_c$ is the minimum fluid, from Eq. 11.22

$$e = \left(T_{c,o} - T_{c,i}\right) / \left(T_{h,i} - T_{c,i}\right)$$

$$T_{c,o} = T_{c,i} + e\left(T_{h,i} - T_{c,i}\right) = 15°C + 0.39(100 - 15)°C = 48.2°C. \qquad <$$

(b) The rate of steam condensation can be expressed as

$$\dot{m}_h = q/h_{fg}.$$

From Eqs. 11.19 and 11.20

$$q = e q_{max} = e C_{min}\left(T_{h,i} - T_{c,i}\right)$$

$$q = 0.39 \times 2090 \text{W/K} (100 - 15) \text{K} = 69{,}284 \text{W}.$$

Hence, the condensation rate is

$$\dot{m}_h = 69{,}284 \text{W} / 2257 \times 10^3 \text{J/kg} = 0.031 \text{kg/s}. \qquad <$$

COMMENTS: (1) Be sure to recognize why $C_h \to \infty$. Note also that $\dot{m}_c \gg \dot{m}_h$.

(2) Note that $\overline{T}_c = (T_{c,i} + T_{c,o})/2 = (15 + 48.2)°C/2 \approx 305$ K. This compares favorably with the value of 300 K at which properties of the cold fluid were evaluated.

PROBLEM 11.63

KNOWN: Concentric tube heat exchanger with prescribed conditions.

FIND: (a) Maximum possible heat transfer, (b) Effectiveness, (c) Whether heat exchanger should be run in PF or CF to minimize size or weight; determine ratio of required areas for the two flow conditions.

SCHEMATIC:

$\dot{m}_c = 0.125\,kg/s$

$\dot{m}_h = 0.125\,kg/s$

PF: $T_{h,i} = 210°C$, $T_{h,o} = 100°C$, $T_{c,i} = 40°C$, $T_{c,o} = 95°C$, ΔT_1, ΔT_2

CF: $T_{h,i} = 210°C$, $T_{h,o} = 100°C$, $T_{c,o} = 95°C$, $T_{c,i} = 40°C$, ΔT_1, ΔT_2

ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible kinetic and potential energy changes, (3) Constant properties, (4) Overall heat transfer coefficient remains unchanged for PF or CF conditions.

PROPERTIES: Hot fluid (given): c = 2100 J/kg·K; Cold fluid (given): c = 4200 J/kg·K.

ANALYSIS: (a) The maximum possible heat transfer rate is given by Eq. 11.19.

$$q_{max} = C_{min}\left(T_{h,i} - T_{c,o}\right).$$

The minimum capacity fluid is the hot fluid with $C_{min} = \dot{m}_h\, c_h$, giving

$$q_{max} = \dot{m}_h\, c_h\left(T_{h,i} - T_{c,o}\right) = 0.125\frac{kg}{s}\times 2100\frac{J}{kg\cdot K}(210-40)K = 44{,}625W. \quad <$$

(b) The effectiveness is defined by Eq. 11.20 and the heat rate, q, can be determined from an energy balance on the cold fluid.

$$e = q/q_{max} = \dot{m}_c\, c_c\left(T_{c,o} - T_{c,i}\right)/q_{max}$$

$$e = 0.125\ kg/s\times 4200\,J/kg\cdot K\,(95-40)K/44{,}625W = 0.65. \quad <$$

(c) Operating the heat exchanger under CF conditions will require a smaller heat transfer area than for PF conditions. The ratio of the areas is

$$\frac{A_{CF}}{A_{PF}} = \frac{q/U\Delta T_{\ell m,CF}}{q/U\Delta T_{\ell m,PF}} = \frac{\Delta T_{\ell m,PF}}{\Delta T_{\ell m,CF}}.$$

To calculate the LMTD, first find $T_{h,o}$ from overall energy balances on the two fluids.

$$T_{h,o} = T_{h,i} - \frac{\dot{m}_c\, c_c}{\dot{m}_h\, c_h}\left(T_{c,o} - T_{c,i}\right) = 210°C - \frac{0.125\times 4200}{0.125\times 2100}(95-40)°C = 100°C.$$

Using Eq. 11.15 with ΔT_1 and ΔT_2 as shown below, find $\Delta T_{\ell m} = (\Delta T_1 - \Delta T_2)/\ell n\,(\Delta T_1/\Delta T_2)$. Substituting values, find

$$\frac{A_{CF}}{A_{PF}} = \frac{\left[(210-40)-(100-95)\right]/\ell n\,(170/5)}{\left[(210-95)-(100-40)\right]\ell n(115/60)} = \frac{46.8°C}{84.5°C} = 0.55. \quad <$$

COMMENTS: In solving part (c), it is also possible to use Figs. 11.15 and 11.16 to evaluate NTU values for corresponding ε and C_{min}/C_{max} values. With knowledge of NTU it is then possible to find A_{CF}/A_{PF}.

PROBLEM 11.64

KNOWN: Concentric tube HXer with prescribed inlet fluid temperatures, fluid flow rates and overall coefficient.

FIND: (a) Maximum heat transfer rate, q_{max}; (b) Outlet fluid temperatures when area is 0.33 m^2 with CF operation; (c) Compute and plot the effectiveness, ε, and fluid outlet temperatures, $T_{c,o}$ and $T_{h,o}$, as a function of UA for the range $50 \leq UA \leq 1000$ W/K for CF operation with all other conditions remaining the same; as UA becomes very large, find asymptotic value for $T_{h,o}$; (d) Largest heat transfer rate which could be achieved if HXer is very long with PF operation; effectiveness for this arrangement; and (e) Compute and plot ε, $T_{c,o}$ and $T_{h,o}$ as a function of UA for the range $50 \leq UA \leq 1000$ W/K for PF operation with all other conditions remaining the same; as UA becomes very large, find asymptotic value for $T_{c,o}$ and $T_{h,o}$.

SCHEMATIC:

T
U = 180 W/m²·K
CF
$T_{h,i}$ = 200 °C
$T_{h,o}$
$\dot{m}_h$ = 42 kg/h
$T_{c,o}$
$T_{c,i}$ = 35 °C
$\dot{m}_c$ = 84 kg/h

T
PF
$T_{h,i}$ = 200 °C
$\delta T_{h,max}$
$\delta T_{c,max}$
$T_{c,i}$ = 35 °C
Part (d,e)

ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible kinetic and potential energy changes, (3) Constant properties.

PROPERTIES: *Table A-6,* Water (Assume $T_{c,o} \approx 85°C$, $\bar{T}_c \approx$ 335 K): c_c = 4186 J/kg·K, (Assume $T_{h,o} \approx$ 100°C, $\bar{T}_h \approx 100°C$, $\bar{T}_h \approx$ 420 K): c_h = 4302 J/kg·K.

ANALYSIS: (a) With $C_{min} = C_h$, the maximum heat transfer rate from Eq. 11.19 is

$$q_{max} = C_{min}\left(T_{h,i} - T_{c,i}\right) = C_h\left(T_{h,i} - T_{c,i}\right) = \frac{42}{3600}\frac{kg}{s}\times 4302 \times \frac{J}{kg\cdot K}\left(200-35\right)K = 8281\,W\,.$$ <

(b) Using the ε - NTU method, find ε from values of C_{min}, C_{min}/C_{max}, and NTU.

$$C_{min} = 42/3600\,kg/s \times 4302\,J/kg\cdot K = 50.19\,W/K\,, \quad C_{min}/C_{max} = \frac{42\,kg/h \times 4302\,J/kg\cdot K}{84\,kg/h \times 4186\,J/kg\cdot K} = 0.514$$

$$NTU = UA/C_{min} = 180\,W/m^2\cdot K \times 0.33m^2 / 50.19\,W/K = 1.184\,.$$

Using Eq. 11.30 for counter flow operation, with $C_r = C_{min}/C_{max}$, find that

$$\varepsilon = \frac{1-\exp\left[-NTU\left(1-C_r\right)\right]}{1-C_r\exp\left[-NTU\left(1-C_r\right)\right]} = \frac{1-\exp\left[-1.18\left(1-0.514\right)\right]}{1-0.514\exp\left[-1.18\left(1-0.514\right)\right]} = 0.616\,.$$

From the definition of effectiveness, $\varepsilon = C_h\,(T_{h,i} - T_{h,o})/C_{min}\,(T_{h,i} - T_{c,i})$, it follows that

$$T_{h,o} = T_{h,i} - \varepsilon\left(T_{h,i} - T_{c,i}\right) = 200^\circ C - 0.62\left(200-35\right)^\circ C = 98.4^\circ C\,.$$ <

Equating the energy balances on both fluids, $C_h\,(T_{h,i} - T_{h,o}) = C_c\,(T_{c,o} - T_{c,i})$, find

$$T_{c,o} = \left(C_h/C_e\right)\left(T_{h,i} - T_{h,o}\right) + T_{c,i} = 0.514\left(200-98.4\right)^\circ C + 35^\circ C = 87.2^\circ C\,.$$ <

Continued...

PROBLEM 11.64 (Cont.)

(c) Using the *IHT Heat Exchanger Tool*, *Concentric Tube, counter flow operation* and the *Properties Tool* for *Water,* a model was developed using the effectiveness NTU method employed in the previous analysis to compute ε, $T_{c,o}$ and $T_{h,o}$ as a function of UA for CF operation. The results are plotted and discussed below.

(d) For PF with same prescribed inlet conditions, the temperature distributions appear as shown above when $A \rightarrow \infty$. At the outlet, $T_{c,o} = T_{h,o}$, and from the sketch $\delta T_{h,max} + \delta T_{c,max} = (200 - 35)°C = 165°C$. From the energy balance, find

$$C_h \delta T_{h,max} = C_c \delta T_{c,max}$$

and solving simultaneously, find

$$\delta T_{h,max} = 109.0^\circ C \qquad T_{h,o} = T_{h,i} - \delta T_{h,max} = 200 - 109.0 = 91.0^\circ C.$$

The heat rate and effectiveness are

$$q = C_h \cdot \delta T_{h,max} = 50.19\,W/K \times 109.0K = 5471W$$ <

$$\varepsilon = q/q_{max} = 5471\,W/8,281W = 0.661.$$ <

(e) Using the IHT model from part (c), but for PF operation, the effectiveness, $T_{c,o}$ and $T_{h,o}$ were computed and plotted as a function of UA.

Counterflow operation

Tco , Tho (C), eps*100

UA (W/K)

Tho (C), minumum fluid
Tco (C), maximum fluid
Effectiveness, eps*100

Parallel flow operation

Tco, Tho (C), eps*100

UA (W/K)

Tho (C), minumum fluid
Tco (C), maximum fluid
Effectiveness, eps*100

COMMENTS: (1) From the plot for CF operation as UA increases, the minimum (hot) fluid outlet temperature, $T_{h,o}$, decreases to the cold fluid temperature, $T_{c,i}$. That is when $UA \rightarrow \infty$, $T_{h,o} \rightarrow T_{c,i}$. As UA $\rightarrow \infty$, the effectiveness approaches unity as expected since a very large CF heat exchanger has a heat rate q_{max} and ε = 1.

(2) From the plot for PF operation, as UA increases, $T_{h,o}$ and $T_{c,o}$ approach an asymptotic value, 91.0°C. Also, as $UA \rightarrow \infty$, the effectiveness increases, approaching 0.661, rather than unity as would be the case for CF operation.

PROBLEM 11.65

KNOWN: Flow rates and inlet temperatures of water and glycol in counterflow heat exchanger. Desired glycol outlet temperature. Heat exchanger diameter and overall heat transfer coefficient without and with spherical inserts.

FIND: (a) Required length without spheres, (b) Required length with spheres, (c) Explanation for reduction in fouling and pump power associated with using spheres.

SCHEMATIC:

$T_{h,i}$ = 100°C

$T_{h,o}$ = 40°C
$\dot{m}_h$ = 0.5 kg/s
Ethylene glycol

$T_{c,i}$ = 15°C
$\dot{m}_c$ = 0.5 kg/s
Water

L

D_i = 0.075 m

ASSUMPTIONS: (1) Negligible kinetic energy, potential energy and flow work changes, (2) Negligible heat loss to surroundings, (3) Constant properties, (4) Negligible tube wall thickness.

PROPERTIES: *Table A-5*, Ethylene glycol $\left(\overline{T}_h = 70°C\right)$: $c_{p,h}$ = 2606 J/kg·K; *Table A-6*, Water $\left(\overline{T}_c \approx 35°C\right)$: $c_{p,c}$ = 4178 J/kg·K.

ANALYSIS: (a) With $C_h = C_{min}$ = 1303 W/K and $C_c = C_{max}$ = 2089 W/K, C_r = 0.624. With actual and maximum possible heat rates of

$$q = C_h\left(T_{h,i} - T_{h,o}\right) = 1303\ W/K\left(100-40\right)°C = 78{,}180\ W$$

$$q_{max} = C_{min}\left(T_{h,i} - T_{c,i}\right) = 1303\ W/K\left(100-15\right)°C = 110{,}755\ W$$

the effectiveness is $\varepsilon = q/q_{max}$ = 0.706. From Eq. 11.30b,

$$NTU = \frac{1}{C_r - 1}\ln\left(\frac{\varepsilon - 1}{\varepsilon C_r - 1}\right) = -2.66\ \ln\left(\frac{0.294}{0.559}\right) = 1.71$$

Hence, with $A = \pi DL$ and $NTU = UA/C_{min}$,

$$L = \frac{C_{min}\ NTU}{\pi D_i U} = \frac{1303\ W/K \times 1.71}{\pi\left(0.075m\right)1000\ W/m^2\cdot K} = 9.46m$$ <

(b) Since $\dot{m}_c$, $\dot{m}_h$, $T_{h,i}$, $T_{h,o}$ and $T_{c,i}$ are unchanged, C_r, ε and NTU are unchanged. Hence, with U = 2000 W/m^2·K,

$$L = 4.73m$$ <

(c) Because the spheres induce mixing of the flows, the potential for contaminant build-up on the surfaces, and hence fouling, is reduced. Although the obstruction to flow imposed by the spheres acts to increase the pressure drop, the reduction in the heat exchanger length reduces the pressure drop. The second effect may exceed that of the first, thereby reducing pump power requirements.

COMMENTS: The water outlet temperature is Tc,o = $T_{c,i} + q/C_c$ = 15°C + 78,180 W/2089 W/K = 52.4°C. The mean temperature $\left(\overline{T}_c = 33.7°C\right)$ is close to that used to evaluate the specific heat of water.

PROBLEM 11.66

KNOWN: Concentric tube, counter-flow heat exchanger.

FIND: Total heat transfer rate and outlet temperatures of both fluids.

SCHEMATIC:

A=23 m², U=1000 W/m²·K
Hot water, $\dot{m}_h$=2.5 kg/s
Cold water, $\dot{m}_c$=5.0 kg/s
$T_{h,i}$=100°C, $T_{c,o}$, $T_{h,o}$, $T_{c,i}$=20°C

ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible kinetic and potential energy changes, (3) Constant properties.

PROPERTIES: *Table A-6*, Water ($\overline{T}_h$ ≈ 68°C ≈ 340 K): c_h = 4188 J/kg·K; *Table A-6*, Water ($\overline{T}_c$ ≈ 37°C = 310 K): c_c = 4178 J/kg·K.

ANALYSIS: Using the ε-NTU method, begin by evaluating the capacity rates.

$$C_h = \dot{m}_h c_h = 2.5\,\text{kg/s} \times 4188\,\text{J/kg}\cdot\text{K} = 10{,}470\,\text{W/K}$$

$$C_c = \dot{m}_c c_c = 5.0\,\text{kg/s} \times 4178\,\text{J/kg}\cdot\text{K} = 20{,}890\,\text{W/K}$$

Hence, $C_{min} = C_h$ and $C_{min}/C_{max} = 0.50$

From the definition, Eq. 11.25,

$$NTU = UA/C_{min} = 1000\,\text{W/m}^2\cdot\text{K} \times 23\,\text{m}^2 / (10{,}470\,\text{W/K}) = 2.20.$$

Using values of NTU and C_{min}/C_{max}, find from Fig. 11.15, that

$$e \approx 0.80.$$

From the definition of ε, Eq. 11.20, it follows that

$$q = e\, q_{max} = e\, C_{min}\left(T_{h,i} - T_{c,i}\right) = 0.80 \times 10{,}470\,\text{W/K}\,(100-20)\,\text{K} = 670\,\text{kW}. \quad <$$

Performing energy balances on both fluids, find

$$T_{c,o} = T_{c,i} + q/C_c = 20°\text{C} + 670\,\text{kW}/20{,}890\,\text{W/K} = 52.1°\text{C} \quad <$$

$$T_{h,o} = T_{h,i} - q/C_h = 100°\text{C} - 670\,\text{kW}/10{,}470\,\text{W/K} = 36.0°\text{C}. \quad <$$

COMMENTS: (1) Note that $\overline{T}_c$ = (20 + 52.1)°C/2 ≈ 310 K and $\overline{T}_h$ = (100 + 36)°C/2 = 341 K and that these values agree well with those used to evaluate the properties.

(2) Eq. 11.30 could be used to evaluate ε; the result gives ε = 0.800.

PROBLEM 11.67

KNOWN: Shell and tube heat exchanger for cooling exhaust gases with water.

FIND: Required surface area using ε-NTU method.

SCHEMATIC:

Gases
$T_{h,i} = 200°C$
$T_{c,o} = 85°C$
$T_{c,i} = 35°C$
Water
$\dot{m}_c = 2.5 kg/s$
$T_{h,o} = 93°C$
$U = 180 W/m^2 \cdot K$

ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible changes in kinetic and potential energies, (3) Constant properties, (4) Gases have properties of air.

PROPERTIES: *Table A-6*, Water, liquid ($\overline{T}_c$ = (85 + 35)°C/2 = 333 K): c_p = 4185 J/kg·K.

ANALYSIS: Using the ε-NTU method, the area can be expressed as

$$A = NTU \cdot C_{min} / U \qquad (1)$$

where NTU must be found from knowledge of ε and $C_{min}/C_{max} = C_r$. The capacity rates are:

$$C_c = \dot{m}_c c_{p,c} = 2.5 kg/s \times 4185 J/kg \cdot K = 10,463 W/K$$

Equating the energy balance relation for each fluid,

$$C_h = C_c (T_{c,o} - T_{c,i})/(T_{h,i} - T_{h,o}) = 10,463 W/K (85-35)/(200-93) = 4889 W/K.$$

Hence,

$$C_r = C_{min}/C_{max} = C_h/C_c = 4889/10,463 = 0.467.$$

The effectiveness of the exchanger, with $q_{max} = C_{min}(T_{h,i} - T_{c,i})$ and $C_{min} = C_h$, is

$$e = q/q_{max} = C_h (T_{h,i} - T_{h,o}) / C_h (T_{h,i} - T_{c,i}) = (200-93)/(200-35) = 0.648.$$

Considering the HXer to be a single shell with 2,4….tube passes, Eqs. 11.31b,c are appropriate to evaluate NTU.

$$NTU = -\left(1 + C_r^2\right)^{-1/2} \ell n \frac{E-1}{E+1} \qquad E = \frac{2/e_1 - (1 + C_r)}{\left(1 + C_r^2\right)^{1/2}}.$$

Substituting numerical values,

$$E = \frac{2/0.648 - (1 + 0.467)}{\left(1 + 0.467^2\right)^{1/2}} = 1.467 \quad NTU = -\left(1 + (0.467)^2\right)^{-1/2} \ell n \frac{1.467 - 1}{1.467 + 1} = 1.51.$$

Using the appropriate numerical values in Eq. (1), the required area is

$$A = 1.51 \times 4889 W/K / 180 W/m^2 \cdot K = 40.9 m^2. \qquad <$$

COMMENTS: Figure 11.16 could also have been used with C_r and ε to find NTU.

PROBLEM 11.68

KNOWN: Dimensions, fluid flow rates, and fluid temperatures for a counterflow heat exchanger used to heat blood.

FIND: (a) Outlet temperature of the blood, (b) Effect of water flowrate and inlet temperature on heat rate and blood outlet temperature.

SCHEMATIC:

ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible kinetic and potential energy changes, (3) Constant properties.

PROPERTIES: *Table A.6*, Water ($\bar{T}_m \approx 55°C$): c_p = 4183 J/kg·K.

ANALYSIS: (a) Using the ε - NTU method, we first obtain $C_h = (\dot{m}_h c_{p,h}) = (0.10\text{ kg/s} \times 4183\text{ J/kg·K})$ = 418.3 W/K and $C_c = (\dot{m}_c c_{p,c}) = (0.05\text{ kg/s} \times 3500\text{ J/kg·K}) = 175\text{ W/K} = C_{min}$. Hence, (C_{min}/C_{max}) = 0.418 and

$$NTU = \frac{UA}{C_{min}} = \frac{\left(500\,W/m^2\cdot K\right)\pi\left(0.055\,m\right)\left(0.5\,m\right)}{175\,W/K} = 0.247\,.$$

From Eq. 11.30, ε = 0.21. Hence, from Eq. 11.23

$$q = \varepsilon C_{min}\left(T_{h,i} - T_{c,i}\right) = 0.21\left(175\,W/K\right)\left(60-18\right)^{\circ}C = 1544\,W\,.$$

From Eq. 11.7,

$$T_{c,o} = T_{c,i} + \frac{q}{C_c} = 18^{\circ}C + \frac{1544\,W}{175\,W/K} = 26.8^{\circ}C$$ <

(b) Because the variation of C_{min}/C_{max} with $\dot{m}_h$ does not have a significant effect on ε for the prescribed NTU, $T_{c,o}$ and q increase only slightly with increasing $\dot{m}_h$.

However, the water inlet temperature does have a significant effect, and accelerated heating is achieved with $T_{h,i}$ = 70°C.

COMMENTS: With $\dot{m}_h$ = 0.2 kg/s and $T_{h,i}$ = 70°C, the outlet temperature of the blood is still below the desired level of $T_{c,o} \approx 37°C$. This value of $T_{c,o}$ could be increased by increasing L or $T_{h,i}$.

PROBLEM 11.69

KNOWN: Inlet temperatures and flow rates of water (c) and ethylene glycol (h) in a shell-and-tube heat exchanger (one shell pass and two tube passes) of prescribed area and overall heat transfer coefficient.

FIND: (a) Heat transfer rate and fluid outlet temperatures and (b) Compute and plot the effectiveness, ε, and fluid outlet temperatures, $T_{c,o}$ and $T_{h,o}$ as a function of the flow rate of ethylene glycol, $\dot{m}_h$, for the range $0.5 \le \dot{m}_h \le 5$ kg/s.

SCHEMATIC:

$T_{h,i}$, $\dot{m}_h$ = 2 kg/s
One shell pass and two tube passes
$T_{c,o}$
$T_{c,i}$, $\dot{m}_c$ = 5 kg/s
$T_{h,o}$
A = 15 m^2, U = 800 W/m^2·K
T
$T_{h,i}$ = 60 °C
Ethylene glycol
$T_{c,i}$ = 10 °C, water

ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible kinetic and potential energy changes, (3) Constant properties, and (4) Overall coefficient remains unchanged.

PROPERTIES: *Table A-5*, Ethylene glycol ($\overline{T}_m \approx 40°C$): c_p = 2474 J/kg·K; *Table A-6*, Water ($\overline{T}_m \approx 15°C$): c_p = 4186 J/kg·K.

ANALYSIS: (a) Using the ε-NTU method we first obtain

$$C_h = \left(\dot{m}_h c_{p,h}\right) = \left(2\,\text{kg/s} \times 2474\,\text{J/kg}\cdot\text{K}\right) = 4948\,\text{W/K}$$

$$C_c = \left(\dot{m}_c c_{p,c}\right) = \left(5\,\text{kg/s} \times 4186\,\text{J/kg}\cdot\text{K}\right) = 20{,}930\,\text{W/K}.$$

Hence with $C_{min} = C_h$ = 4948 W/K and $C_r = C_{min}/C_{max}$ = 0.236,

$$NTU = \frac{UA}{C_{min}} = \frac{\left(800\,\text{W/m}^2\cdot\text{K}\right)15\,\text{m}^2}{4948\,\text{W/K}} = 2.43.$$

From Fig. 11.16, ε = 0.81 and from Eq. 11.23

$$q = \varepsilon C_{min}\left(T_{h,i} - T_{c,i}\right) = 0.81\left(4948\,\text{W/K}\right)\left(60-10\right)\text{K} = 2\times10^5\,\text{W}. \quad <$$

From Eqs. 11.6 and 11.7, energy balances on the fluids,

$$T_{h,o} = T_{h,i} - \frac{q}{C_h} = 60^\circ\text{C} - \frac{2\times10^5\,\text{W}}{4948\,\text{W/K}} = 19.6^\circ\text{C} \quad <$$

$$T_{c,o} = T_{c,i} + \frac{q}{C_c} = 10^\circ\text{C} + \frac{2\times10^5\,\text{W}}{20{,}930\,\text{W/K}} = 19.6^\circ\text{C}. \quad <$$

(b) Using the *IHT Heat Exchanger Tool*, *Shell and Tube*, and the *Properties Tool* for *Water* and *Ethylene Glycol*, $T_{c,o}$, $T_{h,o}$, and ε as a function of $\dot{m}_h$ were computed and plotted.

At very low C_{min}, (low $\dot{m}_h$) note that $\varepsilon \to 1$ while $T_{h,o} \to T_{c,i}$. As $\dot{m}_h$ increases, both fluid outlet temperatures increase and the effectiveness decreases.

Tco, Tho (C), eps*100
100
80
60
40
20
0
0 1 2 3 4 5
Hot fluid flow rate, mdoth (kg/s)
Tco (C), max fluid
Tho (C), min fluid
Effectiveness, eps*100

PROBLEM 11.70

KNOWN: Flow rate, specific heat and inlet temperature of gas in cross-flow heat exchanger. Flow rate and temperature of water which enters as saturated liquid and leaves as saturated vapor. Number of tubes, tube diameter and overall heat transfer coefficient.

FIND: Required tube length.

SCHEMATIC:

$\dot{m}_h = 10\,kg/s$, $T_{h,i} = 1400\,K$, $c_{p,h} = 1120\,J/kg\cdot K$

$\dot{m}_c = 3\,kg/s$
$T_{c,i} = 450\,K$

$N = 500$, $D_o = 0.025\,m$
$U_o = 50\,W/m^2\cdot K$

ASSUMPTIONS: (1) Negligible kinetic and potential energy changes, (2) Negligible heat loss to surroundings, (3) Constant gas specific heat.

PROPERTIES: *Table A-6*, Saturated Water, (T = 450 K): $h_{fg} = 2.024 \times 10^6$ J/kg.

ANALYSIS: Use effectiveness-NTU method

$$\varepsilon = \frac{q}{q_{max}} = \frac{q}{C_{min}\left(T_{h,i} - T_{c,i}\right)} = \frac{q}{\dot{m}_h\, c_{p,h}\left(T_{h,i} - T_{c,i}\right)}$$

$$q = \dot{m}_c\, h_{fg} = 3\,kg/s \times 2.024\times10^6\,J/kg = 6.072\times10^6\,W$$

$$\varepsilon = \frac{6.072\times10^6\,W}{10\,kg/s\times1120\,J/kg\cdot K\left(1400-450\right)K} = 0.571 \qquad C_{min}/C_{max} = 0.$$

From Fig. 11.19, find

$$NTU \approx 0.8 \approx U_o N \pi D_o L / C_{min}$$

$$L \approx \frac{0.8\times10\,kg/s\times1120\,J/kg\cdot K}{50\,W/m^2\cdot K\times500\pi\times0.025\,m} = 4.56\,m.$$

<

COMMENTS: (1) The gas outlet temperature is

$$T_{h,o} = T_{h,i} - q/\dot{m}_h\, c_{p,h} = 1400K - 6.072\times10^6\,W/10\,kg/s\times1120\,J/kg\cdot K = 857.9\,K.$$

(2) Using the LMTD method,

$$\Delta T_{\ell m,CF} = \left[\left(1400-450\right)-\left(858-450\right)\right]/\ell n\left[\left(1400-450\right)/\left(858-450\right)\right] = 641K.$$

From Fig. 11.13, find F = 1, so the area and length are

$$A_o = q/U_o F\,\Delta T_{\ell m,CF} = 6.072\times10^6\,W/\left(50\,W/m^2\cdot K\times1\times641K\right) = 189\,m^2$$

$$L = A/N\pi D_o = 189\,m^2/500\pi\left(0.025\,m\right) = 4.82\,m.$$

PROBLEM 11.71

KNOWN: Gas flow conditions upstream of a tube bank of prescribed geometry. Flow rate and inlet temperature of water passing through the tubes.

FIND: (a) Overall heat transfer coefficient, (b) Water and gas outlet temperatures, (c) Effect of water flow rate on heat recovery and outlet temperatures.

SCHEMATIC:

Exhaust gas
$T_{h,o}$
Tube bank: 10 x 10 square array of aligned tubes, $D = 0.025$ m, $S_T = 0.05$ m, $L = 4$ m, $0.02 \le \dot{m}_{c,1} \le 0.20$ kg/s
Water
$2 \le \dot{m}_c \le 20$ kg/s
$T_{c,i} = 300$ K
$T_{c,o}$
$\dot{m}_h = 2.25$ kg/s, $T_{h,i} = 800$ K, $V = 5.0$ m/s

ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Negligible heat loss to the surroundings and kinetic and potential energy changes, (4) Negligible tube fouling and wall thermal resistance, (5) Fully developed water flow, (6) Gas properties are those of air.

PROPERTIES: *Table A.6*, Water (Assume $\overline{T}_m \approx 340$ K): $c_p = 4188$ J/kg·K, $\mu = 420 \times 10^{-6}$ N·s/m², k = 0.660 W/m·K, Pr = 2.66; *Table A.4*, Air (Assume $\overline{T}_m \approx 600$ K): $c_p = 1051$ J/kg·K, $\nu = 52.7 \times 10^{-6}$ m²/s, k = 0.047 W/m·K, Pr = 0.69.

ANALYSIS: (a) For the prescribed conditions, $U = (1/h_i + 1/h_o)^{-1}$. For the *internal* flow, with $\dot{m}_{c,1}$ = 0.025 kg/s,

$$Re_D = \frac{4\dot{m}_{c,1}}{\pi D \mu} = \frac{4 \times 0.025\,\text{kg/s}}{\pi(0.025\,\text{m})420 \times 10^{-6}\,\text{N}\cdot\text{s/m}^2} = 3032\,.$$

Hence, assuming turbulent flow,

$$Nu_D = 0.023 Re_D^{4/5} Pr^{0.4} = 0.023(3032)^{4/5}(2.66)^{0.4} = 20.8$$

$$h_i = \frac{k}{D} Nu_D = \frac{0.660\,\text{W/m}\cdot\text{K}}{0.025\,\text{m}} 20.8 = 548\,.$$

For the external flow, $V_{max} = \dfrac{0.05\,\text{m}}{(0.05 - 0.025)\,\text{m}} 5.0\,\text{m/s} = 10.0\,\text{m/s}$. Hence

$$Re_{D,max} = \frac{V_{max} D}{\nu} = \frac{10\,\text{m/s} \times 0.025}{52.7 \times 10^{-6}\,\text{m}^2/\text{s}} = 4744$$

From the Zhukauskas correlation and Tables 7.7 and 7.8, $\overline{Nu}_D = (0.97)0.27 Re_{D,max}^{0.63} Pr^{0.36} (Pr/Pr_s)^{1/4}$. Neglecting the Prandtl number ratio,

$$\overline{Nu}_D = (0.97)0.27(4744)^{0.63}(0.69)^{0.36} = 47.4$$

$$\overline{h}_o = \frac{k}{D}\overline{Nu}_D = \frac{0.047\,\text{W/m}\cdot\text{K}}{0.025\,\text{m}} 47.4 = 89.1\,\text{W/m}^2\cdot\text{K}\,.$$

Continued...

PROBLEM 11.71 (Cont.)

Hence, $U = (1/548 + 1/89.1)^{-1} = 76.7\ W/m^2\cdot K$. <

(b) The fluid outlet temperatures may be determined from the ε-NTU method. With $\dot{m}_c$ = 2.5 kg/s, $C_c = \dot{m}_c c_{p,c}$ = 2.5 kg/s × 4188 J/kg·K = 10,470 W/K. With $C_h = \dot{m}_h c_{p,h}$ = 2.25 kg/s × 1051 J/kg·K = 2365 W/K, $C_{min}/C_{max} = C_{mixed}/C_{unmixed}$ = 2365/10,470 = 0.23. Hence, with A = NπDL = 100π × 0.025 m × 4 m = 31.4 m²,

$$NTU = \frac{UA}{C_{min}} = \frac{76.7\ W/m^2\cdot K\left(31.4\ m^2\right)}{2365\ W/K} = 0.95$$

From Fig. 11.19, ε ≈ 0.61. From Eq. 11.19, $q_{max} = C_{min}(T_{h,i} - T_{c,i})$ = 2365 W/K(800 - 300)K = 1.18 × 10⁶ W. Hence, $q = \varepsilon q_{max} = 0.72 \times 10^6$ W. From Eq. 11.6b,

$$\left(T_{h,i} - T_{h,o}\right) = \frac{q}{C_h} = \frac{0.72\times10^6\ W}{2365\ W/K} = 304\ K \qquad T_{h,o} = 496\ K$$ <

From Eq. 11.7b,

$$\left(T_{c,o} - T_{c,i}\right) = \frac{q}{C_c} = \frac{0.72\times10^6\ W}{10,470\ W/K} = 69\ K \qquad T_{c,o} = 369\ K$$ <

(c) Using the appropriate *Heat Exchangers, Correlations* and *Properties* Toolpads of IHT, the following results were obtained.

Outlet temperatures (K) vs. Water flow rate, mdotc(kg/s)

—●— Gas outlet temperature, Tho(K)
—▲— Water outlet temperature, Tco(K)

Heat rate, q(W) vs. Water flow rate, mdotc(kg/s)

With increasing $\dot{m}_c$ (and $\dot{m}_{c,1}$), h_i increases, thereby increasing U and q. However, because the total resistance is dominated by the gas-side condition, $\dot{m}_c$ = 20 kg/s only yields U = 83.9 W/m²·K, despite the fact that h_i = 2180 W/m²·K. Because the extent to which q increases with increasing $\dot{m}_c$ is much smaller than the increase in $\dot{m}_c$ itself, $T_{c,o}$ decreases with increasing $\dot{m}_c$. Hence, there is a trade-off between the amount of hot water and the temperature at which it is delivered. If, for example, the temperature must exceed 50°C ($T_{c,o}$ > 323 K), $\dot{m}_c$ cannot exceed 8 kg/s. To maintain an acceptable value of $T_{c,o}$, while increasing $\dot{m}_c$, $\dot{m}_h$ (and V) should be increased, thereby increasing h_o, and hence U and q.

COMMENTS: If the air and water property functions of IHT are used to evaluate properties at appropriate mean values of the inlet and outlet fluid temperatures, the following, more accurate, results would be obtained for Parts (a) and (b): ε = 0.582, q = 0.697 × 10⁶ W, $T_{c,o}$ = 366.6 K, $T_{h,o}$ = 508.8 K, h_i = 523 W/m²·K, h_o = 86.5 W/m²·K and U = 74.2 W/m²·K.

PROBLEM 11.72

KNOWN: Tube arrangement in steam-to-air, cross-flow heat exchanger. Flow rate $\dot{m}_c$ and inlet temperature of air. Condensing temperature of steam.

FIND: (a) Air outlet temperature for $\dot{m}_c$ = 12 kg/s, (b) Effect of $\dot{m}_c$ on air outlet temperature, heat rate and condensation rate.

SCHEMATIC:

$10 \le \dot{m}_c \le 50$ kg/s
$T_{c,i}$ = 300 K
p = 1 atm
D = 70 mm
L = 2 m
S_T = 140 mm
S_L = 140 mm
$T_{c,o}$
N = 1200
N_L = 40
N_T = 30
Condensing steam, T_s = 400 K

ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible kinetic and potential energy changes, (3) Negligible steam side convection and tube wall conduction resistance, (4) Mean air temperature is 350 K.

PROPERTIES: *Table A.4*, Air (Assume $\overline{T}_c \equiv \left(T_{c,i} + T_{c,o}\right)/2 \approx 350$ K, 1 atm): $\rho = 0.995$ kg/m^3, c_p = 1009 J/kg·K, $\nu = 20.92 \times 10^{-6}$ m^2/s, k = 0.030 W/m·K, Pr = 0.700; T_s = 400 K: Pr = 0.690.

ANALYSIS: (a) For a single-pass, cross-flow heat exchanger with one fluid mixed and the other unmixed, Fig. 11.19 can be used to obtain ε, where $C_{min}/C_{max} = C_{mixed}/C_{unmixed} = 0$ and $NTU = UA/C_{min} = U(\pi DL)N/\dot{m}_c c_p$. From Eq. 11.5, $U = \overline{h}_o$, and the Zhukauskas correlation may be used to estimate $\overline{h}_o$. The upstream velocity may be obtained from $\dot{m}_c = \rho VA \approx \rho V N_T L S_T$. Hence,

$$V = \frac{\dot{m}_c}{\rho N_T L S_T} = \frac{12\,\text{kg/s}}{0.995\,\text{kg/m}^3 \times 30 \times 2\,\text{m} \times 0.14\,\text{m}} = 1.44\,\text{m/s}.$$

For aligned tubes,

$$V_{max} = \frac{S_T}{S_T - D}V = \frac{0.14\,\text{m}}{(0.14 - 0.07)\text{m}} 1.44\,\text{m/s} = 2.88\,\text{m/s}$$

$$Re_{D,max} = \frac{V_{max}D}{\nu} = \frac{2.88\,\text{m/s} \times 0.07\,\text{m}}{20.92 \times 10^{-6}\,\text{m}^2/\text{s}} = 9637.$$

From Table 7.7, select values of C = 0.27 and m = 0.63. Hence,

$$\overline{Nu}_D = 0.27\,Re_{D,max}^{0.63}\,Pr^{0.36}\left(Pr/Pr_s\right)^{0.25}$$

$$\overline{Nu}_D = 0.27(9637)^{0.63}(0.70)^{0.36}(0.70/0.69)^{0.25} = 77.1$$

$$\overline{h}_o = \overline{Nu}_D\frac{k}{D} = 77.1\frac{0.030\,\text{W/m}\cdot\text{K}}{0.07\,\text{m}} = 33.0\,\text{W/m}^2\cdot\text{K}.$$

Hence,

$$NTU = \frac{\overline{h}_o \pi DLN}{\dot{m}_c c_p} = \frac{33.0\,\text{W/m}^2\cdot\text{K} \times \pi(0.07\,\text{m})2\,\text{m}(1200)}{12\,\text{kg/s} \times 1009\,\text{J/kg}\cdot\text{K}} = 1.44.$$

From Fig. 11.19, find $\varepsilon \approx 0.77$ and then determine

Continued...

PROBLEM 11.72 (Cont.)

$$\varepsilon = \frac{q}{q_{max}} = \frac{\dot{m}_c c_p \left(T_{c,o} - T_{c,i}\right)}{\dot{m}_c c_p \left(T_s - T_{c,i}\right)} = \frac{T_{c,o} - T_{c,i}}{T_s - T_{c,i}}$$

$$T_{c,o} = T_{c,i} + \varepsilon\left(T_s - T_{c,i}\right) = 300\,\text{K} + 0.77\left(400 - 300\right)\text{K} = 377\,\text{K} = 104^\circ\text{C}$$ <

(b) With $q = \varepsilon q_{max} = \varepsilon C_c(T_s - T_{c,i})$ and the condensation rate given by Eqs. 10.33 and 10.26,

$$\dot{m}_{cd} = \frac{q}{h'_{fg}} \approx \frac{q}{h_{fg}}$$

the foregoing model may be used with the Heat Exchangers, Correlations and Properties Toolpads of IHT to determine the effect of $\dot{m}_c$ on $T_{c,o}$, q and $\dot{m}_{cd}$.

Since $\overline{h}_o$ increases with increasing $\dot{m}_c$, q must also increase. However, since the increase in q is proportionally less than the increase in $\dot{m}_c$, $T_{c,o}$ decreases with increasing $\dot{m}_c$.

The condensation rate increases proportionally with the increase in q, and if the objective is to maximize the condensation rate, the largest value of $\dot{m}_c$ should be maintained.

COMMENTS: If the objective is to heat the air, there is obviously a trade-off between maintaining elevated values of the flowrate and outlet temperature.

PROBLEM 11.73

KNOWN: Heat exchanger operating in parallel-flow configuration.

FIND: Expression for R_{lm}/R_t which doesn't involve temperatures. Plot result.

SCHEMATIC:

$T_{h,i}$, $T_{c,i}$ → HXer → $T_{h,o}$, $T_{c,o}$; q → $T_{h,i}$ —R_t— $T_{c,i}$

ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible change in kinetic and potential energy.

ANALYSIS: (a) For the exchanger, the rate equation is

$$q = UA\Delta T_{lm}$$

and we can define thermal resistances as

$$R_t = \left(T_{h,i} - T_{c,i}\right)/q \qquad \text{or} \qquad R_{lm} = \left(\Delta T_{lm}\right)/q = 1/UA.$$

Using the rate equation and the definition of effectiveness, find the thermal resistance based upon the inlet temperatures of the hot and cold fluids as

$$R_t = C_{min}\left(T_{h,i} - T_{c,i}\right)/C_{min} \cdot q = 1/\varepsilon C_{min}.$$

The ratio of these resistances is

$$\frac{R_{lm}}{R_t} = \frac{1/UA}{1/\varepsilon C_{min}} = \frac{\varepsilon}{UA/C_{min}} = \frac{\varepsilon}{NTU}$$

and for the parallel flow, concentric tube configuration using Eq. 11.29a,

$$\frac{R_{lm}}{R_t} = \frac{1-\exp\left[-NTU\left(1+C_r\right)\right]}{NTU\left(1+C_r\right)} = \frac{1-\exp\left(-B\right)}{B} \qquad <$$

where $B = NTU(1 + C_r)$. Evaluating the ratio for various values of B, find <

B	R_{lm}/R_t
0.1	0.95
0.5	0.79
1.0	0.63
3.0	0.32
5.0	0.20
10.0	0.10

COMMENTS: (1) For $C_{max} \to \infty$, $C_r \to 0$; hence $B \to NTU$. (2) For $C_{max} \approx C_{min}$, $B \to 2NTU$ or $B \sim C_{min}^{-1}$. (3) For $B << 1$, $R_{lm}/R_t \to 1$. (4) For $B >> 1$, $R_{lm}/R_t \to B^{-1}$. (5) We conclude that care must be taken in representing heat exchangers with a thermal resistance, recognizing that the resistance will depend on flow rates for wide ranges of conditions.

PROBLEM 11.74

KNOWN: Heat exchanger condensing steam at 100°C with cooling water supplied at 15°C.

FIND: (a) Thermal resistance of the exchanger, (b) Change in thermal resistance if fouling is 0.0002 $m^2 \cdot K/W$ on each of the inner and outer tube surfaces, and (c) Plot the thermal resistance as a function of tube water inlet rate assuming all other conditions remain unchanged; comment on whether UA will remain constant if the flow rate changes.

SCHEMATIC:

$T_h = 100°C$, $C_h \to \infty$

$T_{c,o}$

$\dot{m}_c = 0.5\, kg/s$

$T_{c,i} = 15°C$

Shell (1)-2 tube passes
$A = 0.5 m^2$, $U = 2000 W/m^2 \cdot K$

ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible kinetic and potential energy changes.

PROPERTIES: *Table A-6*, Water ($\overline{T}_m$ = 42°C=315 K): c_p = 4179 J/kg·K.

ANALYSIS: (a) For an exchanger, using the rate equation,

$$q = UA\Delta T_{lm} = \left(T_{h,i} - T_{c,i}\right) / R_t,$$

the thermal resistance of the exchanger can be expressed as

$$R_t = \frac{T_{h,i} - T_{c,i}}{UA\Delta T_{lm}} = \frac{C_{min}\left(T_{h,i} - T_{c,i}\right)}{C_{min} \cdot q} = \frac{1}{C_{min}} \cdot \frac{q_{max}}{q} = \frac{1}{\varepsilon C_{min}}.$$

For the present exchanger with $C_r = 0$, use Eq. 11.36a with

$$C_{min} = \dot{m}_c c_{p,c} = 0.5 kg/s \times 4179 J/kg \cdot K = 2090 W/K$$

$$NTU = UA/C_{min} = 2000 W/m^2 \cdot K \times 0.5 m^2 / 2090 W/K = 0.478$$

$$\varepsilon = 1 - \exp(-NTU) = 0.380.$$

Hence, the thermal resistance is

$$R_t = 1/0.380 \times 2090 W/K = 1.258 \times 10^{-3} K/W.$$ <

(b) With fouling present, the overall heat transfer coefficient will decrease.

No fouling:
$$\frac{1}{U_o A} = \frac{1}{h_h A_h} + \frac{1}{h_c A_c}$$

With fouling:
$$\frac{1}{U_f A} = \frac{1}{U_o A} + \frac{R''_{f,c}}{A_c} + \frac{R''_{f,h}}{A_h} = \frac{1}{U_o A} + \frac{2R''_f}{A}$$

Continued …..

PROBLEM 11.74 (Cont.)

$$\frac{1}{U_f A} = \frac{1}{2000\ W/m^2 \cdot K \times 0.5m^2} + \frac{2 \times 0.0002\ m^2 \cdot K/W}{0.5m^2}$$

$$U_f A = 555.6\,W/K.$$

It follows that NTU = $U_f A/C_{min}$ = 0.266 and ε_f = 0.233 giving

$$R_{t,f} = 1/e_f C_{min} = 2.050 \times 10^{-3}\ K/W$$

and hence the increase in thermal resistance due to fouling is

$$\left(R_{t,f} - R_t\right)/R_t = \left(2.050 - 1.258\right)/1.258 = 63\%.$$

<

(c) With no fouling, the thermal resistance, when all other conditions ($U_o A$ = 1000 W/K) remain unchanged, depends on C_{min} only as NTU = $U_o A/C_{min}$,

$$R_t = \frac{1}{eC_{min}} = \frac{1}{C_{min}}\left[1 - \exp\left(-\frac{UA}{C_{min}}\right)\right]^{-1} = \frac{1}{C_{min}}\left[1 - \exp\left(-\frac{1000\ W/K}{C_{min}}\right)\right]^{-1}$$

C_{min} (W/K)	200	400	600	1000	1500	2000	3000
R_t (K/W × 10^3)	4.967	2.723	2.055	1.582	1.370	1.271	1.176

From the plot note that R_t is a weak function of C_{min} above C_{min} > 1000 W/K, from which we conclude that using a constant R_t would be reasonable.

Concerning the variability of UA with changing C_{min}: if most of the resistance is on the water side and the flow is turbulent, $h_c \approx Re_D^{0.8} \approx u_m^{0.8} \approx \dot{m}_c^{0.8}$. It follows that h_c will depend significantly on changes in C_{min}. However, if h_c and h_h are of similar magnitude, the effect of C_{min} on U may not be significant.

PROBLEM 11.75

KNOWN: Air conditioner modeled as a reversed Carnot heat engine, with refrigerant as the working fluid, operating between indoor and outdoor temperatures of 23 and 43°C, respectively, removing 5 kW from a building. Compressor and fan motor efficiency of 80%.

FIND: (a) Required motor power assuming negligible thermal resistances *between* the refrigerant in the condenser and the outside air and *between* the refrigerant in the evaporator and the inside air, and (b) Required power if thermal resistances are each 3×10^{-3} K/W.

SCHEMATIC:

$T_o = T_H = 43°C = 316K$

$T(°C)$, 43, 23, q_H, q_L, $T_H = T_o$, $T_L = T_i$, s, $\dot{w}$

$T_i = T_L = 23°C = 296K$

ASSUMPTIONS: (1) Ideal heat exchanger with no losses, (2) Air conditioner behaves as reversed Carnot engine.

ANALYSIS: (a) With negligible thermal resistances, the Carnot cycle and reversed heat engine can be represented as shown above. Hence,

$$\dot{w}_{ideal} = q_H - q_L = q_L\left[\left(T_H / T_L\right) - 1\right] = 5\text{kW}\left[\left(316\,\text{K}/296\,\text{K}\right) - 1\right] = 0.3378\ \text{kW}.$$

Considering the fan power requirement and the efficiency of the motor,

$$\dot{w}_{act} = \left(\dot{w}_{ideal} + \dot{w}_{fan}\right)/h_c = \left(0.3378 + 0.200\right)\text{kW}/0.8 = 0.672\ \text{kW}. \quad <$$

(b) Consider now thermal resistances of $R_t = 3 \times 10^{-3}$ K/W on the high temperature (condenser) and low temperature (evaporator) sides.

Low side: in order to remove heat from the room, $T_C < T_i$. That is

$$T_i - T_C = qR_t = 5\ \text{kW}\left(3\times10^{-3}\ \text{K/W}\right) = 15\ \text{K}$$

$$T_C = T_i - 15\,\text{K} = 23°\text{C} - 15\ \text{K} = 8°\text{C}.$$

$T(°C)$, 61, 43, 23, 8, T_H, T_o, T_i, T_C, s

High side: in order to reject heat from the condenser to the outside air, $T_H > T_o$,

$$T_H - T_o = q_H R_t = q_c\left(T_H / T_c\right) R_t$$

$$T_H - \left(43 + 273\right)\text{K} = 5\ \text{kW}\left[T_H / \left(8 + 273\right)\right] 3\times10^{-3}\ \text{K/W} \qquad T_H = 333.9\ \text{K} = 61°\text{C}.$$

The work required for this cycle is

$$\dot{w}_{ideal} = q_H - q_L = q_L\left[\left(T_H / T_L\right) - 1\right] = 5\ \text{kW}\left[\left(61 + 273\right)\text{K}\ /\left(8 + 273\right)\text{K} - 1\right] = 0.943\text{kW}$$

$$\dot{w}_{act} = \left(\dot{w}_{ideal} + \dot{w}_{fan}\right)/h_c = \left(0.943 + 0.2\right)\text{kW}/0.8 = 1.43\ \text{kW}. \quad <$$

The effect of finite thermal resistances in the evaporator and condenser is to increase the power by a factor of two.

PROBLEM 11.76

KNOWN: Flow rate and pressure of saturated vapor entering a condenser. Number and diameter of condenser tubes. Water flow rate and inlet temperature. Tube outside convection coefficient.

FIND: (a) Water outlet temperature, (b) Total tube length, (c) Effect of fouling on mass condensation, (d) Effect of water flow rate and inlet temperature on condenser performance.

SCHEMATIC:

Sat. vapor, p = 0.51 bars
$\dot{m}_h$ = 1.5 kg/s
N = 100 tubes, D = 10 mm
$T_{c,i}$ = 280 K
$\dot{m}_c$ = 15 kg/s
Parts (a) - (c)
Sat. liquid
$\bar{h}_o$ = 5000 W/m²·K

ASSUMPTIONS: (1) Negligible heat loss to surroundings and potential and kinetic energy changes, (2) Constant properties, (3) Negligible wall conduction resistance and fouling (initially).

PROPERTIES: Water (given): c_p = 4178 J/kg·K, $\mu = 700 \times 10^{-6}$ kg/s·m, k = 0.628 W/m·K, Pr = 4.6; *Table A.6*, Sat. steam (355 K): $h_{fg} = 2.304 \times 10^6$ J/kg; With fouling: $R''_f = 0.0003\ m^2 \cdot K/W$.

ANALYSIS: (a) From an energy balance, $q_h = \dot{m}_h \left(i_{h,i} - i_{h,o}\right) = \dot{m}_h h_{fg} = q_c = \dot{m}_c c_{p,c}\left(T_{c,o} - T_{c,i}\right)$, or

$$T_{c,o} = T_{c,i} + \frac{\dot{m}_h c_{c,p}}{\dot{m}_c c_{p,c}} = 280\,K + \frac{1.5\,kg/s \times 2.304 \times 10^6\ J/kg}{15\,kg/s \times 4178\,J/kg \cdot K} = 335.1\,K.$$

<

(b) Since $C_r = 0$, NTU = -ln(1 - ε), where

$$\varepsilon = \frac{q}{q_{max}} = \frac{\dot{m}_c c_{p,c}\left(T_{c,o} - T_{c,i}\right)}{\dot{m}_c c_{p,c}\left(T_{h,i} - T_{c,i}\right)} = \frac{(335.1 - 280)\,K}{(355 - 280)\,K} = 0.735$$

Hence, NTU = -ln(1 - 0.735) = 1.327 = UA/C_{min}. The overall heat transfer coefficient is given by $1/U = 1/\bar{h}_i + 1/\bar{h}_o$. For the internal tube flow,

$$Re_D = \frac{4\dot{m}_{c,1}}{\pi D \mu} = \frac{4 \times 15\,kg/s/100}{\pi(0.01\,m)700 \times 10^{-6}\ kg/s \cdot m} = 27{,}284$$

Hence, assuming fully developed flow with the Dittus-Boelter correlation,

$$Nu_D = 0.023 Re_D^{4/5} Pr^n = 0.023(27{,}284)^{4/5}(4.6)^{0.4} = 149.8$$

$$\bar{h}_i = (k/D)Nu_D = \frac{0.628\,W/m \cdot K}{0.01\,m} 149.8 = 9408\,W/m^2 \cdot K$$

and $U = \left[(1/9408) + (1/5000)\right]^{-1} W/m^2 \cdot K = 3265\,W/m^2 \cdot K$. Hence, the heat transfer area is

$$A = \dot{m}_c c_{p,c}(NTU/U) = 15\,kg/s(4178\,J/kg \cdot K)\left(1.327/3265\,W/m^2 \cdot K\right) = 25.5\,m^2$$

and the tube length is $L = A/N\pi D = 25.5\ m^2/100\pi(0.01\ m) = 8.11$ m.

<

(c) With fouling, the overall heat transfer coefficient is $1/U_w = 1/U_{wo} + R''_f$. Hence,

Continued...

PROBLEM 11.76 (Cont.)

$$1/U_w = \left(3.063\times10^{-4} + 3\times10^{-4}\right) m^2 \cdot K/W$$

U_w = 1649 W/m^2·K.

$$NTU = UA/C_{min} = \left(1649\,W/m^2\cdot K \times 25.5\,m^2\right) / \left(15\,kg/s \times 4178\,J/kg\cdot K\right) = 0.671$$

From Eq. 11.36a, ε = 1 - exp(-NTU) = 1 - exp(-0.671) = 0.489. Hence, $q = \varepsilon q_{max}$ = 0.489 × 15 kg/s × 4178 J/kg·K(355 - 280)K = 2.30 × 10^6 W. Without fouling the heat rate was

$$q = \dot{m}_h h_{fg} = 1.5\,kg/s \times 2.304\times10^6\,J/kg = 3.46\times10^6\,W.$$

Hence, $\dot{m}_{h,w}/\dot{m}_{h,wo} = 2.30\times10^6/3.46\times10^6 = 0.666$. <

The condensation rate with fouling is then $\dot{m}_{h,w} = 0.666\times1.5\,kg/s = 0.998\,kg/s$.

(d) The prescribed water inlet temperature of $T_{c,i}$ = 280 K is already at the lower limit of available sources, and it would not be feasible to consider smaller values. In addition, with $\bar{h}_i$ already quite large, an increase in $\dot{m}_c$ is not likely to provide a significant improvement in performance. Using the *Heat Exchanger* and *Correlations* Tools from IHT, the following results were obtained for 15 ≤ $\dot{m}_c$ ≤ 25 kg/s.

Condensation rate, mdoth(kg/s)

Water mass flow rate, mdotc(kg/s)

Over the specified range of $\dot{m}_c$, there is approximately an 18% increase in the heat rate, and hence in the condensation rate. This increase is, in part, due to the increase in $\bar{h}_i$ from 9408 to 14,160 W/m^2·K, which increases U from 1649 to 1752 W/m^2·K, as well as to a reduction in $T_{c,o}$ from 316.6 to 306.0 K, which increases the mean driving potential for heat transfer.

COMMENTS: There is a significant reduction in performance due to fouling, which can not be restored by increasing $\dot{m}_c$. The desired performance could be achieved by oversizing the condenser, that is, by increasing the number of tubes and/or the tube length.

PROBLEM 11.77

KNOWN: Rankine cycle with saturated steam leaving the boiler at 2 MPa and a condenser pressure of 10 kPa. Net reversible work of 0.5 MW.

FIND: (a) Thermal efficiency of ideal Rankine cycle, (b) Required cooling water flow rate to condenser at 15°C with allowable temperature rise of 10°C, and (c) Design of a shell and tube heat exchanger (one shell and multiple tube passes) to satisfy condenser flow rate and temperature rise.

SCHEMATIC:

ASSUMPTIONS: (1) Negligible loss from condenser to surroundings, (2) Negligible kinetic and potential energy changes in heat exchanger, (3) Ideal Rankine cycle, and (4) Negligible thermal resistance on condensate side of exchanger tubes.

PROPERTIES: *Steam Tables*, (Wark, 4th Edition): (1) $p_1 = p_4 = 10$ kPa = 0.10 bar, $T_{sat} = 45.8°C$ = 319 K, $v_f = 1.0102 \times 10^{-3}$ m^3/kg, h_f = 191.83 kJ/kg; (3) $p_2 = p_3 = 2$ Mpa = 20 bar, h_g = 2799.5 kJ/kg, s_g = 6.3409 kJ/kg·K; (4) $s_4 = s_3$ = 6.3409 kJ/kg·K, p_4 = 0.10 bar, s_f = 0.6493 kJ/kg·K, s_g = 8.1502 kJ/kg·K, h_f = 191.83 kJ/kg·K, h_{fg} = 2392.8 kJ/kg; *Table A-6*, Water (T_{sat} = 293 K): $c_{p,c}$ = 4182 J/kg·K, $\mu = 1007 \times 10^{-6}$ N·s/m^2, k = 0.603 W/m·K, Pr = 7.0. Note: 1 bar = 10^5 N/m^2 = 10^5 Pa.

ANALYSIS: (a) Referring to your thermodynamics text, find that

$$h = \frac{w_{net}}{Q_H} = \frac{w_t - w_p}{Q_H} = \frac{(h_3 - h_4) - v_1(p_2 - p_1)}{h_3 - h_2}$$

where the net work is the turbine minus the pump work. Assuming the liquid in the pump is incompressible,

$$w_p = v_1(p_2 - p_1) = 1.0102 \times 10^{-3}\,\text{m}^3/\text{kg}\left(2\times10^6 - 10\times10^3\right)\text{N/m}^2 = 2.01\,\text{kJ/kg}.$$

To find the enthalpies at states 2, 3, and 4, consider the individual processes. For the *pump*,

$$h_2 = h_1 + w_p = (191.83 + 2.01)\,\text{kJ/kg} = 193.84\ \text{kJ/kg}.$$

Since the exit state of the boiler is saturated at p_3 = 2 Mpa,

$$h_3 = h_g = 2799.5\ \text{kJ/kg}.$$

$$Q_H = h_3 - h_2 = (2799.5 - 193.84)\,\text{kJ/kg} = 2605.7\ \text{kJ/kg}.$$

Since the process from 3 to 4 is isentropic, $s_4 – s_3$, hence

$$x_4 = (s_4 - s_f)/(s_g - s_f) = (6.3409 - 0.6493)/(8.1502 - 0.6493) = 0.759$$

$$h_4 = h_f + xh_{fg} = [191.83 + 0.759(2392.8)]\,\text{kJ/kg} = 2007.5\,\text{kJ/kg}.$$

Continued …..

PROBLEM 11.77 (Cont.)

$$w_t = h_3 - h_4 = (2799.5 - 2007.5)\,\text{kJ/kg} = 792.0\ \text{kJ/kg}.$$

Substituting appropriate values, the thermal efficiency is

$$h = \frac{(792.0 - 2.01)\,\text{kJ/kg}}{2605.7\ \text{kJ/kg}} = 0.303 = 30.3\%.$$ <

(b) From an overall balance on the cycle, the heat rejected to the condenser is

$$Q_c = Q_H - w_{net} = \left[2605.7 - (792.0 - 2.01)\right]\text{kJ/kg} = 1815.7\ \text{kJ/kg}.$$

Since the net reversible power is 0.5 MW, the required steam rate (h) is

$$\dot{m}_h = \dot{W}_{net} / w_{net} = 0.5 \times 10^6\,\text{W} / (792.0 - 2.01)\,\text{kJ/kg} = 0.6329\ \text{kg/s}.$$

Hence, the heat rate to be removed by the cold water passing through the condenser is

$$q_c = Q_c \cdot \dot{m}_h = \dot{m}_c\, c_{p,c}\left(T_{c,out} - T_{c,in}\right)$$

$$1815.7\ \text{kJ/kg} \times 0.6329\ \text{kg/s} = 1.149 \times 10^6\ \text{W} = \dot{m}_c \times 4182\ \text{J/kg}\cdot\text{K}\,(25 - 15)\,\text{K}$$

$$\dot{m}_c = 27.47\ \text{kg/s}$$ <

where $c_{p,c} = c_{p,f}$ is evaluated at T_2, $T_{c,in}$ = 15°C and $T_{c,out} - T_{c,in}$ = 10°C, the specified allowable rise.

(c) To design the heat exchanger we need to evaluate UA. Considering the shell-tube configuration and since $C_r = C_{min}/C_{max} = 0$,

$$e = 1 - \exp(-NTU) = 1 - \exp\left[-\left(UA/C_{min}\right)\right]$$

$$e = \frac{q}{q_{max}} = \frac{q_c}{\dot{m}_c\, c_{p,c}\left(T_h - T_{c,i}\right)}$$

$$e = \frac{1.149 \times 10^6\ \text{W}}{27.47\,\text{kg/s} \times 4182\ \text{J/kg}\cdot\text{K}\,(45.7 - 15)\,\text{K}} = 0.326$$

$$0.326 = 1 - \exp\left(-\frac{UA}{27.47\ \text{kg/s} \times 4182\ \text{J/kg}\cdot\text{K}}\right)$$

$$UA_s = 45{,}372\ \text{W/K}$$

where $C_{min} = \dot{m}_c\, c_{p,c}$. Our design process will involve the following steps: select tube diameter, D = 15 mm; set u_m = 2 m/s in each tube and find number of tubes; perform internal flow calculation to estimate $\overline{h}_c$ and then determine the length.

$$\dot{m}_c = r A_c N u_m = \left(1.010 \times 10^{-3}\,\text{m}^3/\text{kg}\right)^{-1}\left(p\,(0.015\text{m})^2/4\right) 2\,\text{m/s} \times N = 27.47\ \text{kg/s}$$

$$N = 78.5 \approx 79.$$

Continued …..

PROBLEM 11.77 (Cont.)

For flow in a single tube,

$$\mathrm{Re}_D = \frac{4\dot{m}_t}{\pi D\mu} = \frac{4(27.47\ \mathrm{kg/s}/79)}{\pi\,(0.015\mathrm{m})1007\times10^{-6}\ \mathrm{N\cdot s/m^2}} = 29{,}310.$$

Assuming the flow is fully developed and using the Dittus-Boelter correlation,

$$\mathrm{Nu} = \frac{hD}{k} = 0.023\mathrm{Re}_D^{0.8}\,\mathrm{Pr}^{0.4} = 0.023(29{,}310)^{0.8}(7.00)^{0.4} = 187.7$$

$$h = 0.603\,\mathrm{W/m\cdot K}\times 187.7/0.015\mathrm{m} = 7544\ \mathrm{W/m^2\cdot K}.$$

Hence, the tube length is

$$UA_s = h(\pi DL)\,N = 45{,}372\ \mathrm{W/K}$$

$$L = 45{,}372\ \mathrm{W/K}/7544\ \mathrm{W/m^2\cdot K}\times\pi\,(0.015\mathrm{m})79 = 1.6\mathrm{m}$$

and our design has the following parameters:

$N = 79$ tubes $\qquad L = 1.6\mathrm{m} \qquad D = 15$ mm. <

COMMENTS: (1) The selection of the tube diameter and water velocity values (15 mm, 2 m/s) was based upon prior experience; they seemed reasonable. We could, however, establish other requirements which would influence these choices such as allowable pressure drop and standard tube sizes.

PROBLEM 11.78

KNOWN: Rankine cycle with saturated steam leaving the boiler at 2 Mpa and a condenser pressure of 10 kPa. Heat rejected to the condenser of 2.3 MW. Condenser supplied with cooling water at rate of 70 kg/s at 15°C.

FIND: (a) Size of the condenser as determined by the parameter, UA, and (b) Reduction in thermal efficiency of the cycle if U decreases by 10% due to fouling assuming water flow rate and inlet temperature and the condenser steam pressure remain fixed.

SCHEMATIC:

ASSUMPTIONS: (1) Negligible loss from condenser to surroundings, (2) Negligible kinetic and potential energy changes in exchanger, (3) Ideal Rankine cycle, (4) For fouled operating condition, $\dot{m}_c$, $T_{c,i}$ and p_4 remain the same.

PROPERTIES: *Steam Tables* (Wark, 4th Edition): See previous problem for calculations to obtain cycle enthalpies; h_1 = 191.83 kJ/kg, h_4 = 2007.5 kJ/kg.

ANALYSIS: (a) For the condenser, recognize that $C_{min} = C_c$, and $C_r = C_{min}/C_{max} = 0$,

$$e = \frac{q}{q_{max}} = 1 - \exp(-NTU) = 1 - \exp(-UA/C_{min})$$

$$C_{min} = \dot{m}_c c_{p,c} = 70 \text{ kg/s} \times 4182 \text{ J/kg}\cdot\text{K} = 292{,}740 \text{ W/K}$$

$$q_{max} = C_{min}(T_h - T_{c,i}) = 292{,}740 \text{ W/K}(45.7 - 15)\text{K} = 8.987 \times 10^6 \text{ W.}$$

$$q = q_h = 2.30 \times 10^6 \text{ W}$$

$$\frac{2.30 \times 10^6 \text{ W}}{8.987 \times 10^6 \text{ W}} = 0.256 = 1 - \exp\left(-\frac{UA}{292{,}740 \text{ W/K}}\right)$$

$$UA = 86{,}538 \text{ W/K.}$$ <

(b) In the fouled condition, U is reduced 10%, hence

$$U_f A = 0.9UA = 77{,}884 \text{ W/K}$$

and

$$NTU_f = \frac{U_f A}{C_{min}} = \frac{77{,}884 \text{ W/K}}{292{,}740 \text{ W/K}} = 0.266$$

$$e_f = 1 - \exp(-NTU_f) = 1 - \exp(-0.266) = 0.234.$$

Continued …..

If we operate the cycle at the same back pressure p_4 = 10 kPa so that T_h = 45.7°C, the heat removal rate must decrease,

$$q_h = \varepsilon q_{max} = 0.234 \times 8.987 \times 10^6 \text{ W} = 2.103 \times 10^6 \text{ W}$$

since $q_{max} = C_{min} (T_h - T_{c,i})$ remains the same. From the previous problem, we found the heat rejected as

$$h_4 - h_1 = (2007.5 - 191.83) \text{kJ/kg} = 1815.7 \text{ kJ/kg}$$

and hence the cycle steam rate through the *fouled* condenser is

$$\dot{m}_{h,f} = q_h / (h_4 - h_1) = 2.103 \times 10^6 \text{ W}/1815.7 \text{ kJ/kg} = 1.158 \text{ kg/s}.$$

For the *unfouled* condenser of part (a), the steam rate was

$$\dot{m}_h = 2.3\text{MW}/1815.7 \text{ kJ/kg} = 1.267 \text{ kg/s}.$$

Hence, we see that fouling reduces the steam rate by 8.5% when U is decreased 10%. Since p_4 remains the same, the thermal efficiency remains unchanged,

$$\eta = 30.3\% \qquad <$$

as calculated in the previous problem. However, the net work of the cycle will decrease 8.5%.

COMMENTS: Fouling of the condenser heat exchanger has no effect on the thermal efficiency of the cycle since the back pressure at the condenser is maintained constant. The effect is, however, to reduce the heat rejection rate while maintaining exchanger flow rate and inlet temperature fixed. Comparing the conditions:

Parameter	Clean	Fouled	Change (%)
UA, W/K	86,538	77,884	-10.0
ε	0.256	0.234	-8.6
q_h, MW	2.300	2.103	-8.6
$\dot{w}_{net}$	--	--	-8.6

PROBLEM 11.79

KNOWN: Compact heat exchanger (see Example 11.6) after extended use has prescribed fouling factors on water and gas sides.

FIND: Gas-side overall heat transfer coefficient.

SCHEMATIC:

Water, c
$h_c = 1500\ W/m^2\cdot K$
Gas, h
Gas
$h_h = 183\ W/m^2\cdot K,\ \eta_{o,h} = 0.91$
$A_h R_w = 3.51\times10^{-5}\ m^2\cdot K/W$
$A_c/A_h = 0.143$
Fouling factors: $R''_{f,c} = 0.0005\ m^2\cdot K/W$
$R''_{f,h} = 0.001\ m^2\cdot K/W$

ASSUMPTIONS: (1) Heat transfer coefficients on the inside and outside (cold- and hot-sides) are the same as for the unfouled condition, (2) Temperature effectiveness of the finned hot side surface is the same as for the unfouled condition.

ANALYSIS: The overall heat transfer coefficient follows from Eq. 11.1 as

$$\frac{1}{U_h A_h} = \frac{1}{(\eta_o h A)_c} + \frac{R''_{f,c}}{(\eta_o A)_c} + R_w + \frac{R''_{f,h}}{(\eta_o A)_h} + \frac{1}{(\eta_o h A)_h}$$

where R_w and R''_f are the wall resistance and fouling factors, respectively. Multiply both sides by A_h and recognizing that $\eta_{o,c} = 1$, obtain

$$\frac{1}{U_h} = \frac{1}{h_c(A_c/A_h)} + \frac{R''_{f,c}}{(A_c/A_h)} + A_h R_w + \frac{R''_{f,h}}{\eta_{o,h}} + \frac{1}{\eta_o h_h}.$$

Substitute numerical values from Example 11.6 results (h_h, $\eta_{o,h}$, $A_h R_w$, A_c/A_h) and those from the problem statement $\left(R''_{f,h}, R''_{f,c}, h_c\right)$ to find,

$$\frac{1}{U_h} = \frac{1}{1500\ W/m^2\cdot K(0.143)}$$

$$+\frac{0.0005 m^2\cdot K/W}{(0.143)} + 3.51\times10^{-5} m^2\cdot K/W + \frac{0.001 m^2\cdot K/W}{0.91} + \frac{1}{0.91\times183\ W/m^2\cdot K}$$

$$\frac{1}{U_h} = \left(4.662\times10^{-3} + 6.993\times10^{-3} + 3.51\times10^{-5} + 5.495\times10^{-4} + 6.005\times10^{-3}\right) m^2\cdot K/W$$

$$U_h = 65.4\ W/m^2\cdot K. \qquad <$$

COMMENTS: For the unfouled condition, we found $U_h = 93.4\ W/m^2\cdot K$ from Example 11.6. Note that the thermal resistance of the tube-fin material is negligible and that fouling has a significant effect, reducing U_h by 41%.

PROBLEM 11.80

KNOWN: Compact heat exchanger with prescribed core geometry and operating parameters.

FIND: Required heat exchanger volume; number of tubes in the longitudinal and transverse directions, N_L and N_T; required tube length.

SCHEMATIC:

Longitudinal direction → N_L
Transverse direction (single pass) → N_T
Gases, h
$\dot{m}_h = 1.25$ kg/s
$T_h = 700$ K
$A_{f,r} = 0.20$ m²
34.3 mm = ℓ
$D_i = 13.8$ mm, $V_i = 100$ mm/s
$T_{c,i} = 300$ K
Water, c
$\dot{m}_c = 2$ kg/s
$T_{c,o} = 350$ K
ℓ_L
ℓ_T
Other parameters: $U_h = 93.4$ W/m²·K, $\alpha = 269$ m²/m³, $D_f = 28.5$ mm, $D_o = 16.4$ mm, $A_f/A_h = 0.830$

ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible KE and PE changes, (3) Single pass operation, (4) Gas properties are those of air.

PROPERTIES: *Table A-6*, Water ($\overline{T}_c$ = 325 K): ρ = 987.2 kg/m³, c_p = 4182 J/kg·K; *Table A-4*, Air (Assume $T_{h,o} \approx 400$ K, $\overline{T}_h \approx 550$ K, 1 atm): c_p = 1040 J/kg·K.

ANALYSIS: To find the Hxer volume, first find A_h using the ε-NTU method. By definition,

$$V = A_h / a \qquad \text{and} \qquad A_h = NTU \cdot C_{min} / U_h. \qquad (1,2)$$

Find the capacity rates, q, q_{max} and ε:

$$C_c = \dot{m}_c c_{p,c} = 2\ \text{kg/s} \times 4182\ \text{J/kg}\cdot\text{K} = 8364\ \text{W/K}$$

$$C_h = \dot{m}_h c_{p,h} = 1.25\ \text{kg/s} \times 1040\ \text{J/kg}\cdot\text{K} = 1300\ \text{W/K} \leftarrow C_{min}$$

Hence,

$$C_r = \frac{C_{min}}{C_{max}} = 0.155.$$

It follows that

$$e = \frac{q}{q_{max}} = \frac{C_c\left(T_{c,o} - T_{c,i}\right)}{C_{min}\left(T_{h,i} - T_{c,i}\right)} = \frac{8364\ \text{W/K}(350-300)\text{K}}{1300\ \text{W/K}(700-300)\text{K}} = 0.804.$$

With ε = 0.804 and C_r = 0.155, find NTU ≈ 1.7 from Fig. 11.18 for a single-pass, cross flow Hxer with both fluids unmixed. Using Eqs. (2) and (1), find

$$A_h = 1.7 \times 1300\ \text{W/K} / 93.4\ \text{W/m}^2\cdot\text{K} = 23.7\text{m}^2$$

$$V = 23.7\text{m}^2 / 269\text{m}^2/\text{m}^3 = 0.0880\text{m}^3.$$

Continued …..

PROBLEM 11.80 (Cont.)

To determine the number of tubes in the longitudinal direction, consider the tubular arrangement in the sketch. The Hxer volume can be written as

$$V = A_{fr} \times \ell_L \qquad (3)$$

where

$$\ell_L = (N_L - 1)\ell + D_f \qquad (4)$$

and N_L is the number of tubes in the longitudinal direction. Combining Eqs. (3) and (4) and substituting numerical values, find

$$N_L = (V / A_{fr} - D_f)/\ell + 1 \qquad (5)$$

where D_f is the overall diameter of the finned tube, and

$$N_L = \left(0.0880\text{m}^3/0.20\ \text{m}^2 - 0.0285\text{m}\right)/0.0343 + 1 = 13.0 \approx 13.$$ <

To determine the number of tubes in the transverse direction, compare the overall water flow rate $\dot{m}_c$ with that for a single tube, $\dot{m}_t$. That is,

$$\dot{m}_t = \rho_c A_t V_i \qquad (6)$$

where A_t is the tube inner cross-sectional area $\left(\pi D_i^2/4\right)$ and V_i the internal velocity. Hence,

$$N = \dot{m}_c/\dot{m}_t = (2\ \text{kg/s})/987.2\ \text{kg/m}^3 \times \frac{\pi}{4}(0.0138\text{m})^2 \times 0.100\ \text{m/s} = 135.4 \approx 135.$$

The total number of tubes required, N, is 135; the number in the transverse direction is

$$N_T = N/N_L = 135/13 = 10.4 \approx 11.$$ <

To determine the water tube length, recognize that the total area (A_h), less that of the finned surfaces (A_f), will be that of the water tube surface area. That is,

$$A_h - A_f = \pi D_o \ell_T \cdot N.$$

From specification of the core geometry, we know $A_f/A_h = 0.830$; solve for ℓ_T to obtain

$$\ell_T = A_h(1 - A_f/A_h)/\pi D_o \cdot N \qquad (7)$$

$$\ell_T = 23.7\text{m}^2(1 - 0.830)/\pi(0.0164\text{m}) \times 135 = 0.58\text{m}.$$ <

COMMENTS: In summary we find that

Total number of tubes, N ($N_T \times N_L$)	143
Tubes in longitudinal direction, N_L	13
Tubes in transverse direction, N_T	11

with a total surface area of 27.3 m^2. The length of the exchanger is

Length in longitudinal direction, ℓ_L	0.44 m
Length in transverse direction, ℓ_T	0.58 m.

PROBLEM 11.81

KNOWN: Compact heat exchanger geometry, gas-side flow rate and inlet temperature, water-side convection coefficient, water flow rate, and water inlet and outlet temperatures.

FIND: Gas-side overall heat transfer coefficient. Required heat exchanger volume.

SCHEMATIC:

Hypothetical circular fin
Water
$h_c = 1500\ W/m^2\text{-}K$
$T_{c,i} = 290\ K$
$T_{c,o} = 370\ K$
Combustion gas
$\dot{m}_h = 1.25\ kg/s$
$T_{h,i} = 825\ K$
$A_{fr} = 0.20\ m^2$
$D_i = 8.2\ mm$
$D_o = 10.2\ mm$
$r_2 = D_o/2$
$r_2 = 15.8\ mm$
Continuous fin
(surface 8.0-3/8T)

ASSUMPTIONS: (1) Gas has properties of atmospheric air at an assumed mean temperature of 700 K, (2) Negligible fouling, (3) Negligible heat exchange with the surroundings and negligible kinetic and potential energy and flow work changes.

PROPERTIES: *Table A-1*, aluminum (T ≈ 300 K): k = 237 W/m·K. *Table A-4*, air (p = 1 atm, $\bar{T}$ = 700 K): c_p = 1075 J/kg·K, $\mu = 338.8 \times 10^{-7}\ N{\cdot}s/m^2$, Pr = 0.695. *Table A-6*, water ($\bar{T}$ = 330 K): c_p = 4184 J/kg·K.

ANALYSIS: For the prescribed heat exchanger core,

$$\frac{1}{U_h} = \frac{1}{h_c\left(A_c / A_h\right)} + A_h R_w + \frac{1}{\eta_{o,h} h_h}$$

where

$$\frac{A_c}{A_h} \approx \frac{D_i}{D_o}\left(1 - \frac{A_{f,h}}{A_h}\right) = \frac{8.2}{10.2}(1 - 0.913) = 0.070$$

The product of A_h and the wall conduction resistance is

$$A_h R_w = \frac{\ln\left(D_o / D_i\right)}{2\pi k L / A_h} = \frac{D_i \ln\left(D_o / D_i\right)}{2k\left(A_c / A_h\right)} = \frac{0.0082\text{m} \times \ln(10.2/8.2)}{2 \times 237\ \text{W/m}\cdot\text{K}\,(0.070)} = 5.39 \times 10^{-4}\ \text{m}^2 \cdot \text{K/W}$$

With a gas-side mass velocity of $G = \dot{m}_h / \sigma A_{fr}$ = 1.25 kg/s/0.534 × 0.20 m^2 = 11.7 kg/s·m^2,

$$Re = \frac{G D_h}{\mu} = \frac{11.7\ \text{kg/s}\cdot\text{m}^2 \times 0.00363\text{m}}{338.8 \times 10^{-7}\ \text{N}\cdot\text{s/m}^2} = 1254$$

and Fig. 11.21 yields $j_H \approx 0.0096$. Hence,

$$h_h \approx \frac{0.0096\ G c_p}{Pr^{2/3}} = \frac{0.0096\left(11.7\ \text{kg/s}\cdot\text{m}^2\right)(1075\ \text{J/kg}\cdot\text{K})}{(0.695)^{2/3}} = 154\ \text{W/m}^2\cdot\text{K}$$

Continued …..

With $r_{2c} = r_2 + t/2 = 15.8$ mm + 0.330 mm/2 = 15.97 mm, $r_{2c}/r_1 = 15.97/5.1 = 3.13$, $L = r_2 - r_1 = 10.7$ mm, $L_c = L + t/2 = 10.87$ mm = 0.0109m, $A_p = L_c t = 3.59 \times 10^{-6}$ m^2, and $L_c^{3/2}\,(h_h/kA_p)^{1/2} = 0.484$, Fig. 3.19 yields $\eta_f \approx 0.77$. Hence,

$$\eta_{o,h} = 1 - \frac{A_f}{A}(1 - \eta_f) = 1 - 0.913(1 - 0.77) = 0.790$$

$$U_h^{-1} = \left(1500\ \text{W/m}^2 \cdot \text{K} \times 0.07\right)^{-1} + 5.39 \times 10^{-4}\,\text{m}^2 \cdot \text{K/W} + \left(0.79 \times 154\ \text{W/m}^2 \cdot \text{K}\right)^{-1} = 0.0183\,\text{m}^2 \cdot \text{K/W}$$

$$U_h = 54.7\ \text{W/m}^2 \cdot \text{K}$$ <

With $q = C_c\,(T_{c,o} - T_{c,i}) = 4184$ W/K × 80 K = 3.35×10^5 W, $q_{max} = C_{min}\,(T_{h,i} - T_{c,i}) = 1344$ W/K × 535 K = 7.19×10^5 W, $\varepsilon = 0.466$ and $C_r = 0.321$. From Figure 11.18, we then obtain NTU ≈ 0.65. The required gas-side surface area is then

$$A_h = \frac{NTU \times C_{min}}{U_h} = \frac{0.65 \times 1344\ \text{W/K}}{54.7\ \text{W/m}^2 \cdot \text{K}} = 16.0\ \text{m}^2$$

With $\alpha = 587$ m^2/m^3, the required volume is

$$V = \frac{A_h}{\alpha} = \frac{16\,\text{m}^2}{587\,\text{m}^2/\text{m}^3} = 0.0273\,\text{m}^3$$ <

COMMENTS: (1) Although U_h is small and A_h larger for the continuous fins than for the circular fins of Example 11.6, the much larger value of α, renders the volume requirement smaller.

(2) The heat exchanger length is $L = V/A_{fr} = 0.137$ m, and the number of tube rows is

$$N_L \approx \frac{L}{S_L} + 1 = 7.23 \approx 7.$$

(3) The hypothetical fin radius ($r_2 = 15.8$ mm) was estimated to be the arithmetic mean of one-half the center-to-center spacing between one tube and its six neighbors.

PROBLEM 11.82

KNOWN: Cooling coil geometry. Air flow rate and inlet and outlet temperatures. Freon pressure and convection coefficient.

FIND: Required number of tube rows.

SCHEMATIC:

Air
$\dot{m}_h = 1.5\ kg/s$
$T_{h,i} = 310K$
$A_{fr} = 0.16m^2$
L
R-12
$p = 1atm$
$h_c = 5000W/m^2 \cdot K$
$T_{h,o} = 290K$
Surface CF-7.0-5/8 J,
$D_i = 13.8mm$

ASSUMPTIONS: (1) Negligible fouling, (2) Constant properties, (3) Negligible heat loss to surroundings.

PROPERTIES: *Table A-4*, Air ($\overline{T}_h$ = 300 K, 1 atm): c_p = 1007 J/kg·K, $\mu = 184.6 \times 10^{-7}$ N·s/m^2, k = 0.0263 W/m·K, Pr = 0.707; *Table A-5*, Sat. R-12 (1 atm): $T_{sat} = T_c$ = 243 K, h_{fg} = 165 kJ/kg.

ANALYSIS: The required number of tube rows is

$$N_L = (L - D_f)/S_L + 1$$

where

$$L = V/A_{fr} \qquad V = A_h/a \qquad A_h = NTU(C_{min}/U_h)$$

$$1/U_h = 1/h_c(A_c/A_h) + A_h R_w + 1/h_{o,h} h_h.$$

From Ex. 11.6, $(A_c/A_h) = 0.143$ and $A_h R_w = 3.51 \times 10^{-5}$ m^2·K/W. With

$$G = \frac{\dot{m}_h}{s\,A_{fr}} = \frac{1.50\ kg/s}{0.449 \times 0.16m^2} = 20.9\ kg/s \cdot m^2$$

$$Re = \frac{GD_h}{m} = \frac{20.9\ kg/s \cdot m^2 \times 6.68 \times 10^{-3} m}{184.6 \times 10^{-7}\ N \cdot s/m^2} = 7563$$

and Fig. 11.20 gives $j_H \approx 0.0068$. Hence,

$$h_h = j_h \frac{Gc_p}{Pr^{2/3}} = 0.0068 \frac{20.9\ kg/s \cdot m^2 \times 1007\ J/kg \cdot K}{(0.707)^{2/3}} = 180\ W/m^2 \cdot K.$$

With L_c = 6.18 mm and $A_p = 1.57 \times 10^{-6}$ m^2 from Ex. 11.6, $L_c^{3/2}(h_h/kA_p)^{1/2} = 0.338$ and, from Fig. 3.19, $\eta_f \approx 0.89$ for $r_{2c}/r_1 = 1.75$. Hence, as in Ex. 11.6, $\eta_{o,h} = 0.91$ and

$$1/U_h = 1/\left(5000\ W/m^2 \cdot K\right) 0.143 + 3.51 \times 10^{-5} m^2 \cdot K/W + 1/\left(0.91 \times 180\ W/m^2 \cdot K\right)$$

$$U_h = 133\ W/m^2 \cdot K.$$

Continued …..

With $C_{min}/C_{max} = 0$ and $C_{min} = \dot{m}_h c_{p,h} = 1511$ W/K,

$$e = \frac{q}{q_{max}} = \frac{C_h\left(T_{h,i} - T_{h,o}\right)}{C_h\left(T_{h,i} - T_{c,i}\right)} = \frac{20\ \text{K}}{67\ \text{K}} = 0.30$$

$$\text{NTU} = -\ln(1 - e) = 0.355$$

and

$$A_h = \text{NTU}\frac{C_{min}}{U_h} = 0.355\frac{1511\,\text{W/K}}{133\ \text{W/m}^2\cdot\text{K}} = 4.03\text{m}^2.$$

Hence,

$$L = \frac{A_h}{a A_{fr}} = \frac{4.03\text{m}^2}{\left(269\ \text{m}^2/\text{m}^3\right)0.16\text{m}^2} = 0.0937\text{m}$$

and

$$N_L = \frac{L - D_f}{S_L} + 1 = \frac{0.0652}{0.0343\text{m}} + 1 = 2.9.$$

Hence, three or more rows must be used. <

COMMENTS: For the prescribed operating conditions, the heat rate would be

$$q = C_h\left(T_{h,i} - T_{h,o}\right) = 1511\,\text{W/K}\left(20\ \text{K}\right) = 30{,}220\ \text{W}.$$

If R-12 enters the tubes as saturated liquid, a flow rate of at least

$$\dot{m}_c = \frac{q}{h_{fg}} = \frac{30{,}220\ \text{W}}{165{,}000\,\text{J/kg}} = 0.183\ \text{kg/s}$$

would be needed to maintain saturated conditions in the tubes.

PROBLEM 11.83

KNOWN: Cooling coil geometry. Air flow rate and inlet temperature. Freon pressure and convection coefficient.

FIND: Air outlet temperature.

SCHEMATIC:

Air
$\dot{m}_h = 1.5\,kg/s$
$T_{h,i} = 310\,K$
$A_{fr} = 0.16\,m^2$

L

R-12
$p = 1\,atm$
$h_c = 5000\,W/m^2 \cdot K$

$T_{h,o}$

Surface CF-7.0-5/8 J,
$D_i = 13.8\,mm$, $N_L = 4$

ASSUMPTIONS: (1) Negligible fouling, (2) Constant properties, (3) Negligible heat loss to surroundings.

PROPERTIES: *Table A-4*, Air ($\overline{T}_h \approx 300$ K, 1 atm): $c_p = 1007$ J/kg·K, $\mu = 184.6 \times 10^{-7}$ N·s/m^2, k = 0.0263 W/m·K, Pr = 0.707; *Table A-5*, Sat. R-12 (1 atm): $T_{sat} = T_c = 243$ K, $h_{fg} = 165$ kJ/kg.

ANALYSIS: To obtain the air outlet temperature, we must first obtain the heat rate from the ε-NTU method. To find A_h, first find the heat exchanger length,

$$L \approx (N_L - 1)S_L + D_f = 3(0.0343\text{m}) + 0.0285\text{m} = 0.131\text{m}.$$

Hence,

$$V = A_{fr}L = 0.16\text{m}^2(0.131\text{m}) = 0.021\text{m}^3$$

$$A_h = aV = \left(269\text{m}^2/\text{m}^3\right)0.021\text{m}^3 = 5.65\text{m}^2.$$

The overall coefficient is

$$\frac{1}{U_h} = \frac{1}{h_c\,(A_c/A_h)} + A_h R_w + \frac{1}{h_{o,h} h_h}$$

where Ex. 11.6 yields $(A_c/A_h) = 0.143$ and $A_h R_w = 3.51 \times 10^{-5}$ m^2·K/W. With

$$G = \frac{\dot{m}_h}{s\,A_{fr}} = \frac{1.50\ \text{kg/s}}{0.449 \times 0.16\ \text{m}^2} = 20.9\ \text{kg/s}\cdot\text{m}^2$$

$$Re = \frac{GD_h}{m} = \frac{20.9\ \text{kg/s}\cdot\text{m}^2 \times 6.68 \times 10^{-3}\text{m}}{184.6 \times 10^{-7}\ \text{N}\cdot\text{s/m}^2} = 7563.$$

Fig. 11.20 gives $j_H \approx 0.0068$. Hence,

$$h_h = j_h \frac{Gc_p}{Pr^{2/3}} = 0.0068 \frac{20.9\ \text{kg/s}\cdot\text{m}^2 \times 1007\ \text{J/kg}\cdot\text{K}}{(0.707)^{2/3}}$$

$$h_h = 180\ \text{W/m}^2\cdot\text{K}.$$

Continued …..

With L_c = 6.18 mm and $A_p = 1.57 \times 10^{-6}$ m^2 from Ex. 11.6, $L_c^{3/2}\left(h_h / kA_p\right)^{1/2}$ = 0.338 and, from Fig. 3.19, $\eta_f \approx 0.89$ for r_{2c}/r_1 = 1.75. Hence, as in Ex. 11.6, $\eta_{o,h}$ = 0.91 and

$$\frac{1}{U_h} = \frac{1}{\left(5000\text{ W/m}^2\cdot\text{K}\right)0.143} + 3.51\times10^{-5}\text{ m}^2\cdot\text{K/W} + \frac{1}{0.91\left(180\text{ W/m}^2\cdot\text{K}\right)}$$

$$U_h = 133\text{ W/m}^2\cdot\text{K}.$$

With

$$C_{min} = C_h = \dot{m}_h c_{p,h} = 1.5\text{ kg/s}\left(1007\text{ J/kg}\cdot\text{K}\right) = 1511\text{ W/K}$$

$$NTU = \frac{U_h A_h}{C_{min}} = \frac{133\text{ W/m}^2\cdot\text{K}\times 5.65\text{m}^2}{1511\text{ W/K}} = 0.497.$$

With $C_{min}/C_{max} = 0$, Eq. 11.36a yields

$$\varepsilon = 1 - \exp(-NTU) = 1 - \exp(-0.497) = 0.392.$$

Hence,

$$q = \varepsilon q_{max} = \varepsilon C_{min}\left(T_{h,i} - T_{c,i}\right) = 0.392\left(1511\text{ W/K}\right)67\text{ K}$$

$$q = 39{,}685\text{ W}.$$

The air outlet temperature is

$$T_{h,o} = T_{h,i} - \frac{q}{C_h} = 310\text{ K} - \frac{39{,}685\text{ W}}{1511\text{ W/K}} = 283.7\text{ K}. \quad <$$

COMMENTS: If R-12 enters the tubes as saturated liquid, a flow rate of at least

$$\dot{m}_c = \frac{q}{h_{fg}} = \frac{39{,}685\text{ W}}{165{,}000\text{ J/kg}} = 0.241\text{ kg/s}$$

would be needed to maintain saturated conditions in the tubes.

PROBLEM 11.84

KNOWN: Cooling coil geometry. Gas flow rate and inlet temperature. Water pressure, flow rate and convection coefficient.

FIND: Required number of tube rows.

SCHEMATIC:

Gas
$\dot{m}_h = 3\,kg/s$
$T_{h,i} = 900\,K$
$A_{fr} = 0.36\,m^2$

Sat. water
$p = 2.455$ bars,
$\dot{m}_c = 0.5\,kg/s$
$h_c = 10^4\,W/m^2\cdot K$

L

Surface CF-7.0-5/8 J,
$D_i = 13.8\,mm$

ASSUMPTIONS: (1) Negligible fouling, (2) Constant properties, (3) Negligible heat loss to surroundings.

PROPERTIES: *Table A-4*, Air ($\bar{T}_h \approx 725$ K, 1 atm): $c_p = 1081$ J/kg·K, $\mu = 346.7 \times 10^{-7}$ N·s/m^2, k = 0.0536 W/m·K, Pr = 0.698; *Table A-6*, Sat. water (2.455 bar): $T_{sat} = T_c = 400$ K, h_{fg} = 2183 kJ/kg.

ANALYSIS: The required number of tube rows is

$$N_L = \frac{L - D_f}{S_L} + 1$$

where

$$L = \frac{V}{A_{fr}} \qquad V = \frac{A_h}{a} \qquad A_h = NTU\frac{C_{min}}{U_h}$$

$$\frac{1}{U_h} = \frac{1}{h_c\left(A_c/A_h\right)} + A_h R_w + \frac{1}{h_{o,h} h_h}.$$

From Ex. 11.6, $(A_c/A_h) \approx 0.143$ and

$$A_h R_w = \frac{D_i \ln\left(D_o/D_i\right)}{2k\left(A_c/A_h\right)} = \frac{(0.0138\text{m})\ln(16.4/13.8)}{2(15\text{ W/m}\cdot\text{K})(0.143)} = 5.55\times10^{-4}\text{ m}^2\cdot\text{K/W}.$$

With

$$G = \frac{\dot{m}_h}{s\,A_{fr}} = \frac{3.0\text{ kg/s}}{0.449\times0.36\text{m}^2} = 18.6\text{ kg/s}\cdot\text{m}^2$$

$$Re = \frac{GD_h}{m} = \frac{18.6\text{ kg/s}\cdot\text{m}^2\times6.68\times10^{-3}\text{ m}}{346.7\times10^{-7}\text{ N}\cdot\text{s/m}^2} = 3576$$

and Fig. 11.20 gives $j_h \approx 0.009$. Hence,

$$h_h = j_h\frac{Gc_p}{Pr^{2/3}} = 0.009\frac{18.6\text{ kg/s}\cdot\text{m}^2\times1081\text{J/kg}\cdot\text{K}}{(0.698)^{2/3}} = 230\text{ W/m}^2\cdot\text{K}.$$

Continued …..

With $r_{2c}/r_1 = 1.75$, $L_c = 6.18$ mm and $A_p = 1.57 \times 10^{-6}$ m^2 from Ex. 11.6, $L_c^{3/2}\left(h_h/kA_p\right)^{1/2} = 1.52$ and Fig. 3.19 gives $\eta_f \approx 0.40$. Hence,

$$h_{o,h} = 1 - \frac{A_f}{A}\left(1 - h_f\right) = 1 - 0.83(1 - 0.4) = 0.50.$$

Hence,

$$\frac{1}{U_h} = \frac{1}{\left(10^4\ \text{W/m}^2\cdot\text{K}\right)0.143} + 5.55\times10^{-4}\ \text{m}^2\cdot\text{K/W} + \frac{1}{0.50\left(230\ \text{W/m}^2\cdot\text{K}\right)}$$

$$U_h = 100.5\ \text{W/m}^2\cdot\text{K}.$$

With

$$q = \dot{m}_c h_{fg} = 0.5\ \text{kg/s}\left(2.183\times10^6\ \text{J/kg}\right) = 1.092\times10^6\ \text{W}$$

$$C_{min} = C_h = 3.0\ \text{kg/s}\left(1081\,\text{J/kg}\cdot\text{K}\right) = 3243\ \text{W/K}$$

$$q_{max} = C_{min}\left(T_{h,i} - T_{c,i}\right) = 3243\,\text{W/K}\left(500\ \text{K}\right) = 1.622\times10^6\ \text{W}$$

find

$$e = \frac{q}{q_{max}} = \frac{1.092\times10^6\ \text{W}}{1.622\times10^6\ \text{W}} = 0.674.$$

From Eq. 11.36b

$$NTU = -\ln(1 - e) = -\ln(1 - 0.674) = 1.121.$$

Hence,

$$A_h = NTU\frac{C_{min}}{U_h} = 1.121\frac{3243\ \text{W/K}}{100.5\ \text{W/m}^2\cdot\text{K}} = 36.17\text{m}^2$$

$$L = \frac{A_h}{A_{fr}a} = \frac{36.17\text{m}^2}{0.36\text{m}^2\left(269\text{m}^2/\text{m}^3\right)} = 0.373\text{m}$$

$$N_L = \frac{L - D_f}{S_L} + 1 = \frac{373 - 28.5}{34.3} + 1 = 11.06 \approx 11.$$ <

COMMENTS: The gas outlet temperature is

$$T_{h,o} = T_{h,i} - \frac{q}{C_{min}} = 900\ \text{K} - \frac{1.092\times10^6\ \text{W}}{3243\ \text{W/K}} = 564\ \text{K}.$$

Hence $\overline{T}_h$ = (900 K + 564 K)/2 = 732 K is in good agreement with the assumed value.

PROBLEM 11.85

KNOWN: Cooling coil geometry. Gas flow rate and inlet temperature. Water pressure and convection coefficient.

FIND: Gas outlet temperature.

SCHEMATIC:

Gas
$\dot{m}_h = 3 kg/s$
$T_{h,i} = 900K$
$A_{fr} = 0.36m^2$
L
Sat. water
$p = 2.455$ bars,
$h_c = 10^4 W/m^2 \cdot K$
Surface CF-7.0-5/8J,
$D_i = 13.8mm$, $N_L = 11$

ASSUMPTIONS: (1) Negligible fouling, (2) Constant properties, (3) Negligible heat loss to surroundings.

PROPERTIES: *Table A-4*, Air ($\overline{T}_h \approx 725$ K, 1 atm): c_p = 1081 J/kg·K, $\mu = 346.7 \times 10^{-7}$ N·s/m^2, k = 0.0536 W/m·K, Pr = 0.698; *Table A-6*, Sat. water (2.455 bar): $T_{sat} = T_c$ = 400 K, h_{fg} = 2183 kJ/kg.

ANALYSIS: To obtain $T_{h,o}$, first obtain q from the ε-NTU method. To determine NTU, Ah must be found from knowledge of L.

$$L \approx (N_L - 1)S_L + D_f = 10(0.0343m) + 0.0285m = 0.372m.$$

Hence,

$$V = A_{fr}L = 0.36m^2(0.372m) = 0.134m^3$$

$$A_h = aV = \left(269m^2/m^3\right)0.134m^3 = 36.05m^2.$$

The overall coefficient is

$$\frac{1}{U_h} = \frac{1}{h_c(A_c/A_h)} + A_h R_w + \frac{1}{h_{o,h} h_h}.$$

From Ex. 11.6, $(A_c/A_h) \approx 0.143$ and

$$A_h R_w = \frac{D_i \ln(D_o/D_i)}{2k(A_c/A_h)} = \frac{(0.0138m)\ln(16.4/13.8)}{2(15\ W/m \cdot K)(0.143)} = 5.55 \times 10^{-4} m^2 \cdot K/W.$$

With

$$G = \frac{\dot{m}_h}{s\, A_{fr}} = \frac{3.0\ kg/s}{0.449 \times 0.36m^2} = 18.6\ kg/s \cdot m^2$$

$$Re = \frac{GD_h}{m} = \frac{18.6\ kg/s \cdot m^2 \times 6.68 \times 10^{-3}\ m}{346.7 \times 10^{-7}\ N \cdot s/m^2} = 3576$$

and Fig. 11.20 gives $j_H \approx 0.009$. Hence,

Continued …..

$$h_h = j_h \frac{Gc_p}{Pr^{2/3}} = 0.009 \frac{18.6\ kg/s \cdot m^2 \times 1081\,J/kg \cdot K}{(0.698)^{2/3}}$$

$$h_h = 230\ W/m^2 \cdot K.$$

With r_{2c}/r_1 = 1.75, L_c = 6.18 mm and $A_p = 1.57 \times 10^{-6}\ m^2$ from Ex. 11.6, $L_c^{3/2}\left(h_h/kA_p\right)^{1/2}$ = 1.52 and Fig. 3.19 gives $\eta_f \approx 0.40$. Hence,

$$h_{o,h} = 1 - \frac{A_f}{A}(1 - h_f) = 1 - 0.83(1 - 0.4) = 0.50.$$

Hence,

$$\frac{1}{U_h} = \frac{1}{\left(10^4\ W/m^2 \cdot K\right)0.143} + 5.55 \times 10^{-4}\,m^2 \cdot K/W + \frac{1}{0.50\left(230\ W/m^2 \cdot K\right)}$$

$$U_h = 100.5\ W/m^2 \cdot K.$$

With

$$C_{min} = C_h = 3\,kg/s\left(1081\,J/kg \cdot K\right) = 3243\ W/K$$

$$NTU = \frac{U_h A_h}{C_{min}} = \frac{100.5\ W/m^2 \cdot K\left(36.05 m^2\right)}{3243\ W/K} = 1.117.$$

Since $C_{min}/C_{max} = 0$, Eq. 11.36a gives

$$e = 1 - \exp(-NTU) = 1 - \exp(-1.117) = 0.673.$$

Hence,

$$q = eC_{min}\left(T_{h,i} - T_{c,i}\right) = 0.673\left(3243\ W/K\right)\left(500\ K\right) = 1.091 \times 10^6\ W$$

and

$$T_{h,o} = T_{h,i} - \frac{q}{C_{min}} = 900\ K - \frac{1.091 \times 10^6\ W}{3243\ W/K} = 564\ K.$$ <

COMMENTS: (1) The assumption of $\overline{T}_h$ = 725 K is good.

(2) If water enters the tubes as saturated liquid, a flow rate of at least

$$\dot{m}_c = \frac{q}{h_{fg}} = \frac{1.091 \times 10^6\ W}{2.183 \times 10^6\ J/kg} = 0.50\ kg/s$$

would be need to maintain saturated conditions in the tubes.

PROBLEM 12.1

KNOWN: Rate at which radiation is intercepted by each of three surfaces (see (Example 12.1).

FIND: Irradiation, G[W/m^2], at each of the three surfaces.

SCHEMATIC:

$A_1 = A_2 = A_3 = A_4 = 10^{-3}\ m^2$

$q_{1-3} = 28.0\ mW$, $q_{1-2} = 12.1\ mW$, $q_{1-4} = 19.8\ mW$, 30°, 45°, 60°

Intensity from A_1, $I = 7000\ W/m^2 \cdot sr$, diffuse emitter

ANALYSIS: The irradiation at a surface is the rate at which radiation is incident on a surface per unit area of the surface. The irradiation at surface j due to emission from surface 1 is

$$G_j = \frac{q_{1-j}}{A_j}.$$

With $A_1 = A_2 = A_3 = A_4 = 10^{-3}\ m^2$ and the incident radiation rates q_{1-j} from the results of Example 12.1, find

$$G_2 = \frac{12.1 \times 10^{-3}\ W}{10^{-3}\ m^2} = 12.1\ W/m^2 \qquad <$$

$$G_3 = \frac{28.0 \times 10^{-3}\ W}{10^{-3}\ m^2} = 28.0\ W/m^2 \qquad <$$

$$G_4 = \frac{19.8 \times 10^{-3}\ W}{10^{-3}\ m^2} = 19.8\ W/m^2. \qquad <$$

COMMENTS: The irradiation could also be computed from Eq. 12.15, which, for the present situation, takes the form

$$G_j = I_1 \cos\theta_j\, \omega_{1-j}$$

where $I_1 = I = 7000\ W/m^2 \cdot sr$ and ω_{1-j} is the solid angle subtended by surface 1 with respect to j. For example,

$$G_2 = I_1 \cos\theta_2\, \omega_{1-2}$$

$$G_2 = 7000\ W/m^2 \cdot sr \times$$

$$\cos 30° \frac{10^{-3}\ m^2 \times \cos 60°}{(0.5m)^2}$$

$$G_2 = 12.1\ W/m^2.$$

Note that, since A_1 is a diffuse radiator, the intensity I is independent of direction.

PROBLEM 12.2

KNOWN: A diffuse surface of area $A_1 = 10^{-4} m^2$ emits diffusely with total emissive power $E = 5 \times 10^4$ W/m^2 .

FIND: (a) Rate this emission is intercepted by small surface of area $A_2 = 5 \times 10^{-4}$ m^2 at a prescribed location and orientation, (b) Irradiation G_2 on A_2, and (c) Compute and plot G_2 as a function of the separation distance r_2 for the range $0.25 \le r_2 \le 1.0$ m for zenith angles $\theta_2 = 0$, 30 and 60°.

SCHEMATIC:

ASSUMPTIONS: (1) Surface A_1 emits diffusely, (2) A_1 may be approximated as a differential surface area and that $A_2/r_2^2 << 1$.

ANALYSIS: (a) The rate at which emission from A_1 is intercepted by A_2 follows from Eq. 12.5 written on a total rather than spectral basis.

$$q_{1\to 2} = I_{e,1}(\theta,\phi) A_1 \cos\theta_1 d\omega_{2-1}. \quad (1)$$

Since the surface A_1 is diffuse, it follows from Eq. 12.13 that

$$I_{e,1}(\theta,\phi) = I_{e,1} = E_1/\pi \ . \quad (2)$$

The solid angle subtended by A_2 with respect to A_1 is

$$d\omega_{2-1} \approx A_2 \cdot \cos\theta_2 / r_2^2 \ . \quad (3)$$

Substituting Eqs. (2) and (3) into Eq. (1) with numerical values gives

$$q_{1\to 2} = \frac{E_1}{\pi} \cdot A_1 \cos\theta_1 \cdot \frac{A_2 \cos\theta_2}{r_2^2} = \frac{5\times 10^4 \ W/m^2}{\pi \ sr} \times \left(10^{-4} m^2 \times \cos 60^\circ\right) \times \left[\frac{5\times 10^{-4} m^2 \times \cos 30^\circ}{(0.5m)^2}\right] sr \quad (4)$$

$$q_{1\to 2} = 15{,}915 \ W/m^2 sr \times \left(5\times 10^{-5} m^2\right) \times 1.732\times 10^{-3} sr = 1.378\times 10^{-3} W. \quad <$$

(b) From section 12, 2.3, the irradiation is the rate at which radiation is incident upon the surface per unit surface area,

$$G_2 = \frac{q_{1\to 2}}{A_2} = \frac{1.378\times 10^{-3} W}{5\times 10^{-4} m^2} = 2.76 \ W/m^2 \quad (5) \ <$$

(c) Using the IHT workspace with the foregoing equations, the G_2 was computed as a function of the separation distance for selected zenith angles. The results are plotted below.

Continued...

PROBLEM 12.2 (Cont.)

For all zenith angles, G_2 decreases with increasing separation distance r_2 . From Eq. (3), note that $d\omega_{2-1}$ and, hence G_2, vary inversely as the square of the separation distance. For any fixed separation distance, G_2 is a maximum when $\theta_2 = 0°$ and decreases with increasing θ_2, proportional to $\cos\theta_2$.

COMMENTS: (1) For a diffuse surface, the intensity, I_e, is independent of direction and related to the emissive power as $I_e = E/\pi$. Note that π has the units of $[\mathrm{sr}]$ in this relation.

(2) Note that Eq. 12.5 is an important relation for determining the radiant power leaving a surface in a prescribed manner. It has been used here on a total rather than spectral basis.

(3) Returning to part (b) and referring to Figure 12.10, the irradiation on A2 may be expressed as

$$G_2 = I_{i,2} \cos\theta_2 \frac{A_1 \cos\theta_1}{r_2^2}$$

Show that the result is $G_2 = 2.76\ \mathrm{W/m^2}$. Explain how this expression follows from Eq. (12.15).

PROBLEM 12.003

KNOWN: Intensity and area of a diffuse emitter. Area and rotational frequency of a second surface, as well as its distance from and orientation relative to the diffuse emitter.

FIND: Energy intercepted by the second surface during a complete rotation.

SCHEMATIC:

$\dot{\theta}_2 = 2$ rad/s

$I_e = 100$ W/m^2-sr

$r = 0.5$ m

$A_1 = 10^{-4}$ m^2

$A_2 = 10^{-4}$ m^2

ASSUMPTIONS: (1) A_1 and A_2 may be approximated as differentially small surfaces, (2) A_1 is a diffuse emitter.

ANALYSIS: From Eq. 12.5, the rate at which radiation emitted by A_1 is intercepted by A_2 is

$$q_{1-2} = I_e\, A_1 \cos\theta_1\, \omega_{2-1} = I_e\, A_1\left(A_2 \cos\theta_2 / r^2\right)$$

where $\theta_1 = 0$ and θ_2 changes continuously with time. The amount of energy intercepted by both sides of A_2 during one rotation, ΔE, may be grouped into four equivalent parcels, each corresponding to rotation over an angular domain of $0 \le \theta_2 < \pi/2$. Hence, with $dt = d\theta_2/\dot{\theta}_2$, the radiant energy intercepted over the period T of one revolution is

$$\Delta E = \int_0^T q dt = \frac{4 I_e\, A_1}{\dot{\theta}_2}\left(\frac{A_2}{r^2}\right)\int_0^{\pi/2} \cos\theta_2 d\theta_2 = \frac{4 I_e\, A_1}{\dot{\theta}_2}\left(\frac{A_2}{r^2}\right)\sin\theta_2\Big|_0^{\pi/2}$$

$$\Delta E = \frac{4\times 100\ \text{W}/\text{m}^2\cdot \text{sr}\times 10^{-4}\text{m}^2}{2\ \text{rad}/\text{s}}\left[\frac{10^{-4}\ \text{m}^2}{(0.50\text{m})^2}\right]\text{sr} = 8\times 10^{-6}\ \text{J}$$

COMMENTS: The maximum rate at which A_2 intercepts radiation corresponds to $\theta_2 = 0$ and is $q_{max} = I_e\, A_1\, A_2/r^2 = 4 \times 10^{-6}$ W. The period of rotation is $T = 2\pi/\dot{\theta}_2 = 3.14$ s.

PROBLEM 12.004

KNOWN: Furnace with prescribed aperture and emissive power.

FIND: (a) Position of gauge such that irradiation is $G = 1000\ W/m^2$, (b) Irradiation when gauge is tilted $\theta_d = 20^o$, and (c) Compute and plot the gage irradiation, G, as a function of the separation distance, L, for the range $100 \le L \le 300$ mm and tilt angles of $\theta_d = 0$, 20, and 60^o.

SCHEMATIC:

ASSUMPTIONS: (1) Furnace aperture emits diffusely, (2) $A_d << L^2$.

ANALYSIS: (a) The irradiation on the detector area is defined as the power incident on the surface per unit area of the surface. That is

$$G = q_{f\rightarrow d}/A_d \qquad q_{f\rightarrow d} = I_e A_f \cos\theta_f\ \omega_{d-f} \qquad (1,2)$$

where $q_{f\rightarrow d}$ is the radiant power which leaves A_f and is intercepted by A_d. From Eqs. 12.2 and 12.5, ω_{d-f} is the solid angle subtended by surface A_d with respect to A_f,

$$\omega_{d-f} = A_d \cos\theta_d / L^2\ . \qquad (3)$$

Noting that since the aperture emits diffusely, $I_e = E/\pi$ (see Eq. 12.14), and hence

$$G = (E/\pi) A_f \cos\theta_f \left(A_d \cos\theta_d / L^2\right) / A_d \qquad (4)$$

Solving for L^2 and substituting for the condition $\theta_f = 0^o$ and $\theta_d = 0^o$,

$$L^2 = E \cos\theta_f \cos\theta_d\ A_f / \pi G\ . \qquad (5)$$

$$L = \left[3.72\times10^5\ W/m^2 \times \frac{\pi}{4}(20\times10^{-3})^2\ m^2 / \pi\times1000\ W/m^2\right]^{1/2} = 193\ mm\ . \qquad <$$

(b) When $\theta_d = 20^o$, $q_{f\rightarrow d}$ will be reduced by a factor of $\cos\theta_d$ since ω_{d-f} is reduced by a factor $\cos\theta_d$. Hence,

$$G = 1000\ W/m^2 \times \cos\theta_d = 1000 W/m^2 \times \cos 20^o = 940\ W/m^2\ . \qquad <$$

(c) Using the IHT workspace with Eq. (4), G is computed and plotted as a function of L for selected θ_d. Note that G decreases inversely as L^2. As expected, G decreases with increasing θ_d and in the limit, approaches zero as θ_d approaches 90^o.

PROBLEM 12.005

KNOWN: Radiation from a diffuse radiant source A_1 with intensity $I_1 = 1.2 \times 10^5$ W/m^2·sr is incident on a mirror A_m, which reflects radiation onto the radiation detector A_2.

FIND: (a) Radiant power incident on A_m due to emission from the source, A_1, $q_{1\rightarrow m}$ (mW), (b) Intensity of radiant power leaving the perfectly reflecting, diffuse mirror A_m, I_m (W/m^2·sr), and (c) Radiant power incident on the detector A_2 due to the reflected radiation leaving A_m, $q_{m\rightarrow 2}$ (μW), (d) Plot the radiant power $q_{m\rightarrow 2}$ as a function of the lateral separation distance y_o for the range $0 \le y_o \le 0.2$ m; explain features of the resulting curve.

SCHEMATIC:

ASSUMPTIONS: (1) Surface A_1 emits diffusely, (2) Surface A_m does not emit, but reflects perfectly and diffusely, and (3) Surface areas are much smaller than the square of their separation distances.

ANALYSIS: (a) The radiant power leaving A_1 that is incident on A_m is

$$q_{1\rightarrow m} = I_1 \cdot A_1 \cdot \cos\theta_1 \cdot \Delta\omega_{m\text{-}1}$$

where $\omega_{m\text{-}1}$ is the solid angle A_m subtends with respect to A_1, Eq. 12.2,

$$\Delta\omega_{m\text{-}1} \equiv \frac{dA_n}{r^2} = \frac{A_m \cos\theta_m}{x_o^2 + y_o^2} = \frac{2\times 10^{-4}\ \text{m}^2 \cdot \cos 45°}{\left[0.1^2 + 0.1^2\right]\text{m}^2} = 7.07\times 10^{-3}\ \text{sr}$$

with $\theta_m = 90° - \theta_1$ and $\theta_1 = 45°$,

$$q_{1\rightarrow m} = 1.2\times 10^5\ \text{W/m}^2\cdot\text{sr} \times 1\times 10^{-4}\ \text{m}^2 \times \cos 45° \times 7.07\times 10^{-3}\ \text{sr} = 60\ \text{mW}$$ <

(b) The intensity of radiation leaving A_m, after perfect and diffuse reflection, is

$$I_m = \left(q_{1\rightarrow m}/A_m\right)/\pi = \frac{60\times 10^{-3}\ \text{W}}{\pi\times 2\times 10^{-4}\ \text{m}^2} = 95.5\ \text{W/m}^2\cdot\text{sr}$$

(c) The radiant power leaving A_m due to reflected radiation leaving A_m is

$$q_{m\rightarrow 2} = q_2 = I_m \cdot A_m \cdot \cos\theta_m \cdot \Delta\omega_{2-m}$$

where $\Delta\omega_{2\text{-}m}$ is the solid angle that A_2 subtends with respect to A_m, Eq. 12.2,

Continued …..

PROBLEM 12.005 (Cont.)

$$\Delta\omega_{2-m} \equiv \frac{dA_n}{r^2} = \frac{A_2 \cos\theta_2}{(L_o - x_o)^2 + y_o^2} = \frac{1\times10^{-4}\ m^2 \times \cos 45°}{\left[0.1^2 + 0.1^2\right] m^2} = 3.54\times10^{-3}\ sr$$

with $\theta_2 = 90° - \theta_m$

$$q_{m\to2} = q_2 = 95.5\ W/m^2\cdot sr \times 2\times10^4\ m^2 \times \cos 45° \times 3.54\times10^{-3}\ sr = 47.8\ \mu W$$ <

(d) Using the foregoing equations in the *IHT* workspace, q_2 is calculated and plotted as a function of y_o for the range $0 \le y_o \le 0.2$ m.

Emitted power from A1 reflected from Am onto A2

q2 (uW)

yo (m)

From the relations, note that q_2 is dependent upon the geometric arrangement of the surfaces in the following manner. For small values of y_o, that is, when $\theta_1 \approx 0°$, the cos θ_1 term is at a maximum, near unity. But, the solid angles $\Delta\omega_{m-1}$ and $\Delta\omega_{2-m}$ are very small. As y_o increases, the cos θ_1 term doesn't diminish as much as the solid angles increase, causing q_2 to increase. A maximum in the power is reached as the cos θ_1 term decreases and the solid angles increase. The maximum radiant power occurs when $y_o = 0.058$ m which corresponds to $\theta_1 = 30°$.

PROBLEM 12.6

KNOWN: Flux and intensity of direct and diffuse components, respectively, of solar irradiation.

FIND: Total irradiation.

SCHEMATIC:

ANALYSIS: Since the irradiation is based on the actual surface area, the contribution due to the direct solar radiation is

$$G_{dir} = q''_{dir} \cdot \cos\theta .$$

From Eq. 12.19 the contribution due to the diffuse radiation is

$$G_{dif} = \pi I_{dif} .$$

Hence

$$G = G_{dir} + G_{dif} = q''_{dir} \cdot \cos\theta + \pi I_{dif}$$

or

$$G = 1000 \, W/m^2 \times 0.866 + \pi \, sr \times 70 \, W/m^2 \cdot sr$$

$$G = (866 + 220) \, W/m^2$$

or

$$G = 1086 \, W/m^2. \qquad <$$

COMMENTS: Although a diffuse approximation is often made for the non-direct component of solar radiation, the actual directional distribution deviates from this condition, providing larger intensities at angles close to the direct beam.

PROBLEM 12.007

KNOWN: Daytime solar radiation conditions with direct solar intensity $I_{dir} = 2.10 \times 10^7$ W/m^2·sr within the solid angle subtended with respect to the earth, $\Delta\omega_S = 6.74 \times 10^{-5}$ sr, and diffuse intensity $I_{dif} = 70$ W/m^2·sr.

FIND: (a) Total solar irradiation at the earth's surface when the direct radiation is incident at 30°, and (b) Verify the prescribed value of $\Delta\omega_S$ recognizing that the diameter of the earth is $D_S = 1.39 \times 10^9$ m, and the distance between the sun and the earth is $r_{e\text{-}S} = 1.496 \times 10^{11}$ m (1 astronomical unity).

SCHEMATIC:

I_{diir} = 2.10x10^7 W/m^2-sr

30°

ω_S = 6.74x10^{-5} sr

I_{dif} = 70 W/m^2-sr

ANALYSIS: (a) The total solar irradiation is the sum of the diffuse and direct components,

$$G_S = G_{dif} + G_{dir} = (220 + 1226) W/m^2 = 1446 \text{ W}/m^2 \qquad <$$

From Eq. 12.19 the diffuse irradiation is

$$G_{dif} = \pi I_{dif} = \pi \text{ sr} \times 70 \text{ W}/m^2 \cdot sr = 220 \text{ W}/m^2$$

The direct irradiation follows from Eq. 12.15, expressed in terms of the solid angle

$$G_{dir} = I_{dir} \cos\theta \, \Delta\omega_S$$

$$G_{dir} = 2.10 \times 10^7 \text{ W}/m^2 \cdot sr \times \cos 30° \times 6.74 \times 10^{-5} \text{ sr} = 1226 \text{ W}/m^2$$

(b) The solid angle the sun subtends with respect to the earth is calculated from Eq. 12.2,

$$\Delta\omega_S = \frac{dA_n}{r^2} = \frac{\pi D_S^2/4}{r_{e\text{-}S}^2} = \frac{\pi \left(1.39 \times 10^9 \text{ m}\right)^2 / 4}{\left(1.496 \times 10^{11} \text{ m}\right)^2} = 6.74 \times 10^{-5} \text{ sr} \qquad <$$

where dA_n is the projected area of the sun and $r_{e\text{-}S}$, the distance between the earth and sun. We are assuming that $r_{e\text{-}S}^2 >> D_S^2$.

COMMENTS: Can you verify that the direct solar intensity, I_{dir}, is a reasonable value, assuming that the solar disk emits as a black body at 5800 K? $\left(I_{b,S} = \sigma T_S^4 / \pi = \sigma (5800 \text{ K})^4 / \pi = 2.04 \times 10^7 \text{ W}/m^2 \cdot sr\right)$. Because of local cloud formations, it is possible to have an appreciable diffuse component. But it is not likely to have such a high direct component as given in the problem statement.

PROBLEM 12.8

KNOWN: Directional distribution of solar radiation intensity incident at earth's surface on an overcast day.

FIND: Solar irradiation at earth's surface.

SCHEMATIC:

n

$I_i = I_n \cos\theta$

$I_n = 80\ \text{W/m}^2\cdot\text{sr}$

ASSUMPTIONS: (1) Intensity is independent of azimuthal angle θ.

ANALYSIS: Applying Eq. 12.17 to the total intensity

$$G = \int_0^{2\pi}\int_0^{\pi/2} I_i(\theta)\cos\theta\,\sin\theta\,d\theta\,d\phi$$

$$G = 2\pi\, I_n \int_0^{\pi/2} \cos^2\theta\,\sin\theta\,d\theta$$

$$G = (2\pi\ \text{sr})\times 80\,\text{W/m}^2\cdot\text{sr}\left(-\frac{1}{3}\cos^3\theta\right)\Bigg|_0^{\pi/2}$$

$$G = -167.6\,\text{W/m}^2\cdot\text{sr}\left(\cos^3\frac{\pi}{2} - \cos^3 0\right)$$

$$G = 167.6\,\text{W/m}^2.$$

<

PROBLEM 12.9

KNOWN: Emissive power of a diffuse surface.

FIND: Fraction of emissive power that leaves surface in the directions $\pi/4 \le \theta \le \pi/2$ and $0 \le \phi \le \pi$.

SCHEMATIC:

ASSUMPTIONS: (1) Diffuse emitting surface.

ANALYSIS: According to Eq. 12.12, the total, hemispherical emissive power is

$$E = \int_0^\infty \int_0^{2\pi} \int_0^{\pi/2} I_{\lambda,e}(\lambda,\theta,\phi)\cos\theta \sin\theta \, d\theta \, d\phi \, d\lambda .$$

For a diffuse surface $I_{\lambda,e}$ (λ, θ, ϕ) is independent of direction, and as given by Eq. 12.14,

$$E = \pi I_e .$$

The emissive power, which has directions prescribed by the limits on θ and ϕ, is

$$\Delta E = \int_0^\infty I_{\lambda,e}(\lambda) d\lambda \left[\int_0^\pi d\phi\right]\left[\int_{\pi/4}^{\pi/2} \cos\theta \sin\theta \, d\theta\right]$$

$$\Delta E = I_e [\phi]_0^\pi \times \left[\frac{\sin^2\theta}{2}\right]_{\pi/4}^{\pi/2} = I_e[\pi]\left[\frac{1}{2}\left(1 - 0.707^2\right)\right]$$

$$\Delta E = 0.25\pi I_e .$$

It follows that

$$\frac{\Delta E}{E} = \frac{0.25\pi I_e}{\pi I_e} = 0.25.$$ <

COMMENTS: The diffuse surface is an important concept in radiation heat transfer, and the directional independence of the intensity should be noted.

PROBLEM 12.10

KNOWN: Spectral distribution of E_λ for a diffuse surface.

FIND: (a) Total emissive power E, (b) Total intensity associated with directions $\theta = 0^o$ and $\theta = 30^o$, and (c) Fraction of emissive power leaving the surface in directions $\pi/4 \le \theta \le \pi/2$.

SCHEMATIC:

E_λ (W/m^2•μm)
200
100
0 5 10 15 20 λ(μm)

ASSUMPTIONS: (1) Diffuse emission.

ANALYSIS: (a) From Eq. 12.11 it follows that

$$E = \int_0^\infty E_\lambda(\lambda)\,d\lambda = \int_0^5 (0)\,d\lambda + \int_5^{10} (100)\,d\lambda + \int_{10}^{15} (200)\,d\lambda + \int_{15}^{20} (100)\,d\lambda + \int_{20}^{\infty} (0)\,d\lambda$$

$$E = 100\ \text{W/m}^2 \cdot \mu\text{m}\ (10-5)\ \mu\text{m} + 200\text{W/m}^2 \cdot \mu\text{m}\ (15-10)\ \mu\text{m} + 100\ \text{W/m}^2 \cdot \mu\text{m}\ (20-15)\ \mu\text{m}$$

$$E = 2000\ \text{W/m}^2$$ <

(b) For a diffuse emitter, I_e is *independent* of θ and Eq. 12.14 gives

$$I_e = \frac{E}{\pi} = \frac{2000\ \text{W}\big/\text{m}^2}{\pi\ \text{sr}}$$

$$I_e = 637\ \text{W}\big/\text{m}^2 \cdot \text{sr}$$ <

(c) Since the surface is diffuse, use Eqs. 12.10 and 12.14,

$$\frac{E(\pi/4 \to \pi/2)}{E} = \frac{\int_0^{2\pi} \int_{\pi/4}^{\pi/2} I_e \cos\theta \sin\theta\, d\theta\, d\phi}{\pi I_e}$$

$$\frac{E(\pi/4 \to \pi/2)}{E} = \frac{\int_{\pi/4}^{\pi/2} \cos\theta \sin\theta\, d\theta \int_0^{2\pi} d\phi}{\pi} = \frac{1}{\pi}\left[\frac{\sin^2\theta}{2}\right]_{\pi/4}^{\pi/2} \phi\Big|_0^{2\pi}$$

$$\frac{E(\pi/4 \to \pi/2)}{E} = \frac{1}{\pi}\left[\frac{1}{2}(1^2 - 0.707^2)(2\pi - 0)\right] = 0.50$$ <

COMMENTS: (1) Note how a spectral integration may be performed in parts.

(2) In performing the integration of part (c), recognize the significance of the diffuse emission assumption for which the intensity is uniform in all directions.

PROBLEM 12.11

KNOWN: Diffuse surface ΔA_o, 5-mm square, with total emissive power E_o = 4000 W/m^2.

FIND: (a) Rate at which radiant energy is emitted by ΔA_o, q_{emit}; (b) Intensity $I_{o,e}$ of the radiation field emitted from the surface ΔA_o; (c) Expression for q_{emit} presuming knowledge of the intensity $I_{o,e}$ beginning with Eq. 12.10; (d) Rate at which radiant energy is incident on the hemispherical surface, r = R_1 = 0.5 m, due to emission from ΔA_o; (e) Rate at which radiant energy leaving ΔA_o is intercepted by the small area ΔA_2 located in the direction (40°, ϕ) on the hemispherical surface using Eq. 12.5; also determine the irradiation on ΔA_2; (f) Repeat part (e), for the location (0°, ϕ); are the irradiations at the two locations equal? and (g) Irradiation G_1 on the hemispherical surface at r = R_1 using Eq. 12.5.

SCHEMATIC:

ASSUMPTIONS: (1) Diffuse surface, ΔA_o, (2) Medium above ΔA_o is also non-participating, (3) $R_1^2 >> \Delta A_o, \Delta A_2$.

ANALYSIS: (a) The radiant power leaving ΔA_o by emission is

$$q_{emit} = E_o \cdot \Delta A_o = 4000 \text{ W/m}^2 (0.005 \text{ m} \times 0.005 \text{ m}) = 0.10 \text{ W}$$ <

(b) The emitted intensity is $I_{o,e}$ and is independent of direction since ΔA_o is a diffuser emitter,

$$I_{o,e} = E_o / \pi = 1273 \text{ W/m}^2 \cdot \text{sr}$$ <

The intensities at points P_1 and P_2 are also $I_{o,e}$ and the intensity in the directions shown in the schematic above will remain constant no matter how far the point is from the surface ΔA_o since the space is non-participating.

(c) From knowledge of $I_{o,e}$, the radiant power leaving ΔA_o from Eq. 12.10 is,

$$q_{emit} = \int_h I_{o,e} \Delta A_o \cos\theta \sin\theta d\theta d\phi = I_{o,e} \Delta A_o \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} \cos\theta \sin\theta d\theta d\phi = \pi I_{o,e} \Delta A_o = 0.10 \text{ W}$$ <

(d) Defining control surfaces above ΔA_o and on A_1, the radiant power leaving ΔA_o must pass through A_1. That is,

$$q_{1,inc} = E_o \Delta A_o = 0.10 \text{ W}$$ <

Recognize that the average irradiation on the hemisphere, A_1, where $A_1 = 2\pi R_1^2$, based upon the definition, Section 12.2.3,

$$\bar{G}_1 = q_{1,inc} / A_1 = E_o \Delta A_o / 2\pi R_1^2 = 63.7 \text{ mW/m}^2$$

where $q_{1,inc}$ is the radiant power incident on surface A_1.

Continued...

PROBLEM 12.11 (Cont.)

(e) The radiant power leaving ΔA_o intercepted by ΔA_2, where $\Delta A_2 = 4\times10^{-6}$ m^2, located at ($\theta = 45^o$, ϕ) as per the schematic, follows from Eq. 12.5,

$$q_{\Delta A_o \to \Delta A_2} = I_{o,e} \Delta A_o \cos\theta_o \Delta\omega_{2-o}$$

where $\theta_o = 45^o$ and the solid angle ΔA_2 subtends with respect to ΔA_o is

$$\Delta\omega_{2-o} = \Delta A_2 \cos\theta_2 \Big/ R_1^2 = 4\times10^{-6} m^2 \cdot 1\big/(0.5m)^2 = 1.60\times10^{-5} sr$$

where $\theta_2 = 0^o$, the direction normal to ΔA_2,

$$q_{\Delta A_o \to \Delta A_2} = 1273\, W\big/m^2 \cdot sr \times 25\times10^{-6} m^2 \cos 45^o \times 1.60\times10^{-5} sr = 3.60\times10^{-7}\ W \qquad <$$

From the definition of irradiation, Section 12.2.3,

$$G_2 = q_{\Delta A_o \to \Delta A_2} \Big/ \Delta A_2 = 90\, mW\big/m^2$$

(f) With ΔA_2, located at ($\theta = 0^o$, ϕ), where $\cos\theta_o = 1$, $\cos\theta_2 = 1$, find

$$\Delta\omega_{2-o} = 1.60\times10^{-5} sr \qquad q_{\Delta A_o \to \Delta A_2} = 5.09\times10^{-7}\ W \qquad G_2 = 127\, mW\big/m^2 \qquad <$$

Note that the irradiation on ΔA_2 when it is located at (0^o, ϕ) is larger than when ΔA_2 is located at (45^o, ϕ); that is, 127 mW/m^2 > 90 W/m^2. Is this intuitively satisfying?

(g) Using Eq. 12.15, based upon Figure 12.10, find

$$\bar{G}_1 = \int_h I_{1,i} dA_1 \cdot d\omega_{0-1} / A_1 = \pi I_{o,e}\, \Delta A_o / \Delta A_1 = 63.7\, mW\big/m^2 \qquad <$$

where the elemental area on the hemispherical surface A_1 and the solid angle ΔA_o subtends with respect to ΔA_1 are, respectively,

$$dA_1 = R_1^2 \sin\theta\, d\theta\, d\phi \qquad\qquad d\omega_{o-1} = \Delta A_o \cos\theta \big/ R_1^2$$

From this calculation you found that the *average* irradiation on the hemisphere surface, $r = R_1$, is $\bar{G}_1 = 63.7\, mW\big/m^2$. From parts (e) and (f), you found irradiations, G_2 on ΔA_2 at (0^o, ϕ) and (45^o, ϕ) as 127 mW/m^2 and 90 mW/m^2, respectively. Did you expect $\bar{G}_1$ to be less than either value for G_2? How do you explain this?

COMMENTS: (1) Note that from Parts (e) and (f) that the irradiation on A_1 is not uniform. Parts (d) and (g) give an average value.

(2) What conclusions would you reach regarding G_1 if ΔA_o were a sphere?

PROBLEM 12.12

KNOWN: Hemispherical and spherical arrangements for radiant heat treatment of a thin-film material. Heater emits diffusely with intensity $I_{e,h} = 169{,}000\ \text{W/m}^2\cdot\text{sr}$ and has an area $0.0052\ \text{m}^2$.

FIND: (a) Expressions for the irradiation on the film as a function of the zenith angle, θ, and (b) Identify arrangement which provides the more uniform irradiation, and hence better quality control for the treatment process.

SCHEMATIC:

ASSUMPTIONS: (1) Heater emits diffusely, (2) All radiation leaving the heater is absorbed by the thin film.

ANALYSIS: (a) The irradiation on any differential area, dA_s, due to emission from the heater, A_h , follows from its definition, Section 12.2.3,

$$G = \frac{q_{h\to s}}{dA_s} \qquad (1)$$

Where $q_{h\to s}$ is the radiant heat rate leaving A_h and intercepted by dA_s. From Eq. 12.5,

$$q_{h\to s} = I_{e,h} \cdot dA_h \cos\theta_h \cdot \omega_{s-h} \qquad (2)$$

where ω_{s-h} is the solid angle dA_s subtends with respect to any point on A_h. From the definition, Eq. 12.2,

$$\omega = \frac{dA_n}{r^2} \qquad (3)$$

where dA_n is normal to the viewing direction and r is the separation distance.

For the hemisphere: Referring to the schematic above, the solid angle is

$$\omega_{s-h} = \frac{dA_s}{R^2}$$

and the irradiation distribution on the hemispheric surface as a function of θ_h is

$$G = I_{e,h} A_h \cos\theta_h / R^2 \qquad (1)$$ <

For the sphere: From the schematic, the solid angle is

$$\omega_{s,h} = \frac{dA_s \cos\theta_s}{R_o^2} = \frac{dA_s}{4R^2 \cos\theta_h}$$

where R_o, from the geometry of sphere cord and radii with $\theta_s = \theta_h$, is

Continued...

PROBLEM 12.12 (Cont.)

$$R_o = 2R\cos\theta_h$$

and the irradiation distribution on the spherical surface as a function of θ_h is

$$G = I_{e,h}\, d\,A_h \big/ 4R^2 \qquad (2) \quad <$$

(b) The spherical shape provides more uniform irradiation as can be seen by comparing Eqs. (1) and (2). In fact, for the spherical shape, the irradiation on the thin film is uniform and therefore provides for better quality control for the treatment process. Substituting numerical values, the irradiations are:

$$G_{hem} = 169{,}000\ \mathrm{W/m^2 \cdot sr} \times 0.0052\mathrm{m^2}\cos\theta_h \big/ (2\mathrm{m})^2 = 219.7\cos\theta_h\ \mathrm{W/m^2} \qquad (3)$$

$$G_{sph} = 169{,}000\ \mathrm{W/m^2 \cdot sr} \times 0.0052\,\mathrm{m^2} \big/ 4(2\mathrm{m})^2 = 54.9\,\mathrm{W/m^2} \qquad (4)$$

COMMENTS: (1) The radiant heat rate leaving the diffuse heater surface by emission is

$$q_{tot} = \pi I_{e,h} A_h = 276.1\mathrm{W}$$

The average irradiation on the *spherical surface*, $A_{sph} = 4\pi R^2$,

$$\bar{G}_{sph} = q_{tot} \big/ A_{sph} = 276.1\mathrm{W} \big/ 4\pi\,(2\mathrm{m})^2 = 54.9\,\mathrm{W/m^2}$$

while the average irradiation on the *hemispherical surface*, $A_{hem} = 2\pi R^2$ is

$$\bar{G}_{hem} = 276.1\mathrm{W} \big/ 2\pi\,(2\mathrm{m})^2 = 109.9\,\mathrm{W/m^2}$$

(2) Note from the foregoing analyses for the *sphere* that the result for $\bar{G}_{sph}$ is identical to that found as Eq. (4). That follows since the irradiation is uniform.

(3) Note that $\bar{G}_{hem} > \bar{G}_{sph}$ since the surface area of the hemisphere is half that of the sphere. Recognize that for the hemisphere thin film arrangement, the distribution of the irradiation is quite variable with a maximum at $\theta = 0°$ (top) and half the maximum value at $\theta = 30°$.

N: Hot part, ΔA_p ... n s
e L_d = 1 m.

(a) Location x_1 at
n the sensor, S_o, and
he range $0.2 \le S/S_o$
$S/S_o = 0.75$.

EMATIC:

PROBLEM 12.13 (Cont.)

When the part is directly under the sensor, x = 0, $S/S_o = 1$ for all values of L_d. With increasing x, S/S_o decreases most rapidly with the smallest L_d. From the IHT model we found the part position x corresponding to $S/S_o = 0.75$ as follows.

S/S_o	L_d (m)	x_1 (m)
0.75	0.8	0.315
0.75	1.0	0.393
0.75	1.2	0.472

If the sensor system is set so that when S/S_o reaches 0.75 a process is initiated, the technician can use the above plot and table to determine at what position the part will begin to experience the treatment process.

PROBLEM 12.14

KNOWN: Diameter and temperature of burner. Temperature of ambient air. Burner efficiency.

FIND: (a) Radiation and convection heat rates, and wavelength corresponding to maximum spectral emission. Rate of electric energy consumption. (b) Effect of burner temperature on convection and radiation rates.

SCHEMATIC:

q_{rad} q_{conv} Air $T_\infty = 20°C$

D = 0.20 m $100°C \le T_s \le 350°C$

P_{elec}

ASSUMPTIONS: (1) Burner emits as a blackbody, (2) Negligible irradiation of burner from surrounding, (3) Ambient air is quiescent, (4) Constant properties.

PROPERTIES: *Table A-4*, air (T_f = 408 K): k = 0.0344 W/m·K, $\nu = 27.4 \times 10^{-6}$ m^2/s, $\alpha = 39.7 \times 10^{-6}$ m^2/s, Pr = 0.70, $\beta = 0.00245$ K^{-1}.

ANALYSIS: (a) For emission from a black body

$$q_{rad} = A_s\ E_b = \left(\pi D^2/4\right)\sigma T^4 = \left[\pi\,(0.2\text{m})^2/4\right]5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\,(523\,\text{K})^4 = 133\ \text{W}$$ <

With $L = A_s/P = D/4 = 0.05$m and $Ra_L = g\beta(T_s - T_\infty)\,L^3/\alpha\nu = 9.8\ \text{m/s}^2 \times 0.00245\ \text{K}^{-1}\ (230\ \text{K})\ (0.05\text{m})^3/(27.4 \times 39.7 \times 10^{-12}\ \text{m}^4/\text{s}^2) = 6.35 \times 10^5$, Eq. (9.30) yields

$$\bar{h} = \frac{k}{L}\overline{Nu}_L = \left(\frac{k}{L}\right)0.54\ Ra_L^{1/4} = \left(\frac{0.0344\ \text{W/m}\cdot\text{K}}{0.05\text{m}}\right)0.54\left(6.35\times10^5\right)^{1/4} = 10.5\ \text{W/m}^2\cdot\text{K}$$

$$q_{conv} = \bar{h}A_s\,(T_s - T_\infty) = 19.4\ \text{W/m}^2\cdot\text{K}\left[\pi\,(0.2\text{m})^2/4\right]230\,\text{K} = 75.7\ \text{W}$$ <

The electric power requirement is then

$$P_{elec} = \frac{q_{rad} + q_{conv}}{\eta} = \frac{(133 + 75.7)\,\text{W}}{0.9} = 232\ \text{W}$$ <

The wavelength corresponding to peak emission is obtained from Wien's law, Eq. (12.27)

$$\lambda_{max} = 2898\mu\text{m}\cdot\text{K}/523\text{K} = 5.54\mu\text{m}$$ <

(b) As shown below, and as expected, the radiation rate increases more rapidly with temperature than the convection rate due to its stronger temperature dependence $\left(T_s^4\ \text{vs.}\ T_s^{5/4}\right)$.

Continued …..

PROBLEM 12.14(Cont.)

500
400
300
200
100
0
Heat rate (W)
100 150 200 250 300 350
Surface temperature (C)
qconv
qrad
Pelec

COMMENTS: If the surroundings are treated as a large enclosure with isothermal walls at $T_{sur} = T_\infty$ = 293 K, irradiation of the burner would be $G = \sigma T_{sur}^4 = 418\ W/m^2$ and the corresponding heat rate would be $A_s\,G = 13\ W$. This input is much smaller than the energy outflows due to convection and radiation and is justifiably neglected.

PROBLEM 12.15

KNOWN: Evacuated, aluminum sphere (D = 2m) serving as a radiation test chamber.

FIND: Irradiation on a small test object when the inner surface is lined with carbon black and at 600K. What effect will surface coating have?

SCHEMATIC:

Aluminum sphere, D=2m
Small test surface, A_1
Carbon black coating,
T_s=600K

ASSUMPTIONS: (1) Sphere walls are isothermal, (2) Test surface area is small compared to the enclosure surface.

ANALYSIS: It follows from the discussion of Section 13.3 that this isothermal sphere is an enclosure behaving as a blackbody. For such a condition, see Fig. 12.12(c), the irradiation on a small surface within the enclosure is equal to the blackbody emissive power at the temperature of the enclosure. That is

$$G_1 = E_b(T_s) = s\ T_s^4$$

$$G_1 = 5.67\times10^{-8}\text{W/m}^2\cdot\text{K}^4(600\text{K})^4 = 7348\text{W/m}^2.$$ <

The irradiation is independent of the nature of the enclosure surface coating properties.

COMMENTS: (1) The irradiation depends only upon the enclosure surface temperature and is independent of the enclosure surface properties.

(2) Note that the test surface area must be small compared to the enclosure surface area. This allows for inter-reflections to occur such that the radiation field, within the enclosure will be uniform (diffuse) or isotropic.

(3) The irradiation level would be the same if the enclosure were not evacuated since, in general, air would be a non-participating medium.

PROBLEM 12.16

KNOWN: Isothermal enclosure of surface area, A_s, and small opening, A_o, through which 70W emerges.

FIND: (a) Temperature of the interior enclosure wall if the surface is black, (b) Temperature of the wall surface having ε = 0.15.

SCHEMATIC:

ASSUMPTIONS: (1) Enclosure is isothermal, (2) $A_o << A_s$.

ANALYSIS: A characteristic of an isothermal enclosure, according to Section 12.3, is that the radiant power emerging through a small aperture will correspond to blackbody conditions. Hence

$$q_{rad} = A_o E_b(T_s) = A_o s T_s^4$$

where q_{rad} is the radiant power leaving the enclosure opening. That is,

$$T_s = \left(\frac{q_{rad}}{A_o s}\right)^{1/4} = \left(\frac{70W}{0.02m^2 \times 5.670 \times 10^{-8} W/m^2 \cdot K^4}\right)^{1/4} = 498K.$$ <

Recognize that the radiated power will be independent of the emissivity of the wall surface. As long as $A_o << A_s$ *and* the enclosure is isothermal, then the radiant power will depend only upon the temperature.

COMMENTS: It is important to recognize the unique characteristics of isothermal enclosures. See Fig. 12.12 to identify them.

PROBLEM 12.17

KNOWN: Sun has equivalent blackbody temperature of 5800 K. Diameters of sun and earth as well as separation distance are prescribed.

FIND: Temperature of the earth assuming the earth is black.

SCHEMATIC:

ASSUMPTIONS: (1) Sun and earth emit as blackbodies, (2) No attenuation of solar irradiation enroute to earth, and (3) Earth atmosphere has no effect on earth energy balance.

ANALYSIS: Performing an energy balance on the earth,

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$A_{e,p} \cdot G_S = A_{e,s} \cdot E_b\left(T_e\right)$$

$$\left(\pi D_e^2 / 4\right) G_S = \pi D_e^2 \sigma T_e^4$$

$$T_e = \left(G_S / 4\sigma\right)^{1/4}$$

where $A_{e,p}$ and $A_{e,s}$ are the projected area and total surface area of the earth, respectively. To determine the irradiation G_S at the earth's surface, equate the rate of emission from the sun to the rate at which this radiation passes through a spherical surface of radius $R_{S,e} - D_e/2$.

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$\pi D_S^2 \cdot \sigma T_S^4 = 4\pi \left[R_{S,e} - D_e/2\right]^2 G_S$$

$$\pi \left(1.39\times10^9\ \text{m}\right)^2 \times 5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4 \left(5800\ \text{K}\right)^4$$

$$= 4\pi \left[1.5\times10^{11} - 1.29\times10^7/2\right]^2\ \text{m}^2 \times G_S$$

$$G_S = 1377.5\ \text{W/m}^2.$$

Substituting numerical values, find

$$T_e = \left(1377.5\ \text{W/m}^2 / 4\times5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)^{1/4} = 279\ \text{K}.$$ <

COMMENTS: (1) The average earth's temperature is greater than 279 K since the effect of the atmosphere is to reduce the heat loss by radiation.

(2) Note carefully the different areas used in the earth energy balance. Emission occurs from the total spherical area, while solar irradiation is absorbed by the projected spherical area.

PROBLEM 12.18

KNOWN: Solar flux at outer edge of earth's atmosphere, 1353 W/m^2.

FIND: (a) Emissive power of sun, (b) Surface temperature of sun, (c) Wavelength of maximum solar emission, (d) Earth equilibrium temperature.

SCHEMATIC:

ASSUMPTIONS: (1) Sun and earth emit as blackbodies, (2) No attenuation of solar radiation enroute to earth, (3) Earth atmosphere has no effect on earth energy balance.

ANALYSIS: (a) Applying conservation of energy to the solar energy crossing two concentric spheres, one having the radius of the sun and the other having the radial distance from the edge of the earth's atmosphere to the center of the sun

$$E_s\left(\pi D_s^2\right) = 4\pi\left(R_{s-e} - \frac{D_e}{2}\right)^2 q''_s.$$

Hence

$$E_s = \frac{4\left(1.5\times10^{11}\,m - 0.65\times10^7\,m\right)^2 \times 1353\,W/m^2}{\left(1.39\times10^9\,m\right)^2} = 6.302\times10^7\,W/m^2.$$ <

(b) From Eq. 12.28, the temperature of the sun is

$$T_s = \left(\frac{E_s}{\sigma}\right)^{1/4} = \left(\frac{6.302\times10^7\,W/m^2}{5.67\times10^{-8}\,W/m^2\cdot K^4}\right)^{1/4} = 5774\,K.$$ <

(c) From Wien's law, Eq. 12.27, the wavelength of maximum emission is

$$\lambda_{max} = \frac{C_3}{T} = \frac{2897.6\,\mu m\cdot K}{5774\,K} = 0.50\,\mu m.$$

(d) From an energy balance on the earth's surface

$$E_e\left(\pi D_e^2\right) = q''_S\left(\pi D_e^2/4\right).$$

Hence, from Eq. 12.28,

$$T_e = \left(\frac{q''_S}{4\sigma}\right)^{1/4} = \left(\frac{1353\,W/m^2}{4\times5.67\times10^{-8}\,W/m^2\cdot K^4}\right)^{1/4} = 278K.$$ <

COMMENTS: The average earth temperature is higher than 278 K due to the shielding effect of the earth's atmosphere (transparent to solar radiation but not to longer wavelength earth emission).

PROBLEM 12.19

KNOWN: Small flat plate positioned just beyond the earth's atmosphere oriented such that its normal passes through the center of the sun. Pertinent earth-sun dimensions from Problem 12.18.

FIND: (a) Solid angle subtended by the sun about a point on the surface of the plate, (b) Incident intensity, I_i, on the plate using the known value of the solar irradiation about the earth's atmosphere, $G_S = 1353$ W/m^2, and (c) Sketch of the incident intensity as a function of the zenith angle θ, where θ is measured from the normal to the plate.

SCHEMATIC:

Sun, $T_S = 5800$ K, $D_S = 1.39 \times 10^9$ m

Earth, $D_e = 1.29 \times 10^7$ m

$R_{S,e} = 1.5 \times 10^{11}$ m

(a)

ω_{s-p} Plate

$L_{S,p} = R_{S,e} + D_e/2$

(b)

ASSUMPTIONS: (1) Plate oriented normal to centerline between sun and earth, (2) Height of earth's atmosphere negligible compared to distance from the sun to the plate, (3) Dimensions of the plate are very small compared to sun-earth dimensions.

ANALYSIS: (a) The pertinent sun-earth dimensions are shown in the schematic (a) above while the position of the plate relative to the sun and the earth is shown in (b). The solid angle subtended by the sun with respect to any point on the plate follows from Eq. 12.2,

$$\omega_{S-p} = \frac{A_S \cos\theta_p}{L_{S,p}^2} = \frac{\left(\pi D_S^2/4\right)\cos\theta_p}{\left(R_{S,e}+D_{e/2}\right)^2} = \frac{\pi\left(1.39\times10^9\,\text{m}\right)^2 \big/ 4\times1}{\left(1.5\times10^{11}\text{m}+1.29\times10^7\,\text{m}/2\right)^2} = 6.74\times10^{-5}\,\text{sr} \quad (1) \quad <$$

where A_S is the projected area of the sun (the solar disk), θ_p is the zenith angle measured between the plate normal and the centerline between the sun and earth, and $L_{S,p}$ is the separation distance between the plate at the sun's center.

(b) The plate is irradiated by solar flux in the normal direction only (not diffusely). Using Eq. (12.7), the radiant power incident on the plate can be expressed as

$$G_S \Delta A_p = I_i \cdot \Delta A_p \cos\theta_p \cdot \omega_{S-p} \quad (2)$$

and the intensity I_i due to the solar irradiation G_S with $\cos\theta_p = 1$,

$$I_i = G_S/\omega_{S-p} = 1353\,\text{W/m}^2 \big/ 6.74\times10^{-5}\,\text{sr} = 2.01\times10^7\,\text{W/m}^2\cdot\text{sr} \quad <$$

(c) As illustrated in the schematic to the right, the intensity I_i will be constant for the zenith angle range $0 \le \theta_p \le \theta_{p,o}$ where

$\theta_{p,o}$, I_i

$$\theta_{p,o} = \frac{D_S/2}{L_{S,p}} = \frac{1.39\times10^9\,\text{m}/2}{\left(1.5\times10^{11}\text{m}+1.29\times10^7\,\text{m}/2\right)}$$

$$\theta_{p,o} = 4.633\times10^{-3}\,\text{rad} \approx 0.27^\circ$$

For the range $\theta_p > \theta_{p,o}$, the intensity will be zero. Hence the I_i as a function of θ_p will appear as shown to the right.

I_i, $I(\theta)$, 0, 0, 0.27, 90, θ_p(°)

PROBLEM 12.20

KNOWN: Various surface temperatures.

FIND: (a) Wavelength corresponding to maximum emission for each surface, (b) Fraction of solar emission in UV, VIS and IR portions of the spectrum.

ASSUMPTIONS: (1) Spectral distribution of emission from each surface is approximately that of a blackbody, (2) The sun emits as a blackbody at 5800 K.

ANALYSIS: (a) From Wien's law, Eq. 12.27, the wavelength of maximum emission for blackbody radiation is

$$l_{max} = \frac{C_3}{T} = \frac{2897.6\ mm \cdot K}{T}.$$

For the prescribed surfaces

Surface	Sun (5800K)	Tungsten (2500K)	Hot metal (1500K)	Skin (305K)	Cool metal (60K)	
$\lambda_{max}(\mu m)$	0.50	1.16	1.93	9.50	48.3	<

(b) From Fig. 12.3, the spectral regions associated with each portion of the spectrum are

Spectrum	**Wavelength limits, mm**
UV	**0.0 – 0.4**
VIS	**0.4 – 0.7**
IR	**0.7 - 100**

For T = 5800K and each of the wavelength limits, from Table 12.1 find:

$\lambda(\mu m)$	10^{-2}	0.4	0.7	10^2
$\lambda T(\mu m \cdot K)$	58	2320	4060	5.8×10^5
$F_{(0 \to \lambda)}$	0	0.125	0.491	1

Hence, the fraction of the solar emission in each portion of the spectrum is:

$F_{UV} = 0.125 - 0 = 0.125$ <

$F_{VIS} = 0.491 - 0.125 = 0.366$ <

$F_{IR} = 1 - 0.491 = 0.509.$ <

COMMENTS: (1) Spectral concentration of surface radiation depends strongly on surface temperature.

(2) Much of the UV solar radiation is absorbed in the earth's atmosphere.

PROBLEM 12.21

KNOWN: Visible spectral region 0.47 μm (blue) to 0.65 μm (red). Daylight and incandescent lighting corresponding to blackbody spectral distributions from the solar disk at 5800 K and a lamp bulb at 2900 K, respectively.

FIND: (a) Band emission fractions for the visible region for these two lighting sources, and (b) wavelengths corresponding to the maximum spectral intensity for each of the light sources. Comment on the results of your calculations considering the rendering of true colors under these lighting conditions.

ASSUMPTIONS: Spectral distributions of radiation from the sources approximates those of blackbodies at their respective temperatures.

ANALYSIS: (a) From Eqs. 12.30 and 12.31, the band-emission fraction in the spectral range λ_1 to λ_2 at a blackbody temperature T is

$$F_{(\lambda 1-\lambda 2,\, T)} = F_{(0\to \lambda 2,\, T)} - F_{(0\to \lambda 1,\, T)}$$

where the $F_{(0\to \lambda\ T)}$ values can be read from Table 12.1 (or, more accurately calculated using the *IHT Radiation | Band Emission* tool)

Daylight source (T = 5800 K)

$$F_{(\lambda 1-\lambda 2,\, T)} = 0.4374 - 0.2113 = 0.2261$$ <

where at $\lambda_2 \cdot T$ = 0.65 μm × 5800 K = 3770 μm·K, find $F_{(0\text{ - }\lambda T)}$ = 0.4374, and at $\lambda_1 \cdot T$ = 0.47 μm × 5800 K = 2726 μm·K, find $F_{(0\text{ - }\lambda T)}$ = 0.2113.

Incandescent source (T = 2900 K)

$$F_{(\lambda 1-\lambda 2,\, T)} = 0.05098 - 0.00674 = 0.0442$$ <

(b) The wavelengths corresponding to the peak spectral intensity of these blackbody sources can be found using Wien's law, Eq. 12.27.

$$\lambda_{max} = C_3 / T = 2898\ \mu m \cdot K$$

For the daylight (d) and incandescent (i) sources, find

$$\lambda_{max,\ d} = 2898\ \mu m \cdot K / 5800\ K = 0.50\ \mu m$$ <

$$\lambda_{max,\ i} = 2898\ \mu m \cdot K / 2800\ K = 1.0\ \mu m$$ <

COMMENTS: (1) From the band-emission fraction calculation, part (a), note the substantial difference between the fractions for the daylight and incandescent sources. The fractions are a measure of the relative amount of total radiant power that is useful for lighting (visual illumination).

(2) For the daylight source, the peak of the spectral distribution is at 0.5 μm within the visible spectral region. In contrast, the peak for the incandescent source at 1 μm is considerably outside the visible region. For the daylight source, the spectral distribution is "flatter" (around the peak) than that for the incandescent source for which the spectral distribution is decreasing markedly with decreasing wavelength (on the short-wavelength side of the blackbody curve). The eye has a bell-shaped relative spectral response within the visible, and will therefore interpret colors differently under illumination by the two sources. In daylight lighting, the colors will be more "true," whereas with incandescent lighting, the shorter wavelength colors (blue) will appear less bright than the longer wavelength colors (red)

PROBLEM 12.22

KNOWN: Lamp with prescribed filament area and temperature radiates like a blackbody at 2900 K when consuming 100 W.

FIND: (a) Efficiency of the lamp for providing visible radiation, and (b) Efficiency as a function of filament temperature for the range 1300 to 3300 K.

SCHEMATIC:

Glass Envelope

q_{vis}

Filament, 2 mm x 5 mm, T_s = 2900 K

q_{elec} = 100 W

ASSUMPTIONS: (1) Filament behaves as a blackbody, (2) Glass envelope transmits all visible radiation incident upon it.

ANALYSIS: (a) We define the efficiency of the lamp as the ratio of the radiant power within the visible spectrum (0.4 - 0.7 μm) to the electrical power required to operate the lamp at the prescribed temperature.

$$\eta = q_{vis}/q_{elec} \;.$$

The radiant power for a blackbody within the visible spectrum is given as

$$q_{vis} = F(0.4\mu m \rightarrow 0.7\mu m) A_s \sigma T_s^4 = \left[F_{(0 \rightarrow 0.7\mu m)} - F_{(0 \rightarrow 0.4\mu m)} \right] A_s \sigma T_s^4$$

using Eq. 12.31 to relate the band emission factors. From Table 12.1, find

$$\lambda_2 T_s = 0.7\ \mu m \times 2900\ K = 2030\ \mu m \cdot K, \qquad F_{(0 \rightarrow 0.7\mu m)} = 0.0719$$

$$\lambda_1 T_s = 0.4\ \mu m \times 2900\ K = 1600\ \mu m \cdot K, \qquad F_{(0 \rightarrow 0.4\mu m)} = 0.0018$$

The efficiency is then

$$\eta = [0.0719 - 0.0018] \times 2(2 \times 10^{-3} m \times 5 \times 10^{-3} m) 5.67 \times 10^{-8}\ W/m^2 \cdot K^4 (2900\ K)^4\ 100\ W$$

$$\eta = 5.62\ W/100\ W = 5.6\%$$ <

(b) Using the *IHT Radiation Exchange Tool*, *Blackbody Emission Factor*, and Eqs. (1) and (2) above, a model was developed to compute and plot η as a function of T_s.

Efficiency, eta (%)

Filament temperature, Ts (K)

Continued...

PROBLEM 12.22 (Cont.)

Note that the efficiency decreases markedly with reduced filament temperature. At 2900 K, η = 5.6% while at 2345 K, the efficiency decreases by more than a factor of five to η = 1%.

COMMENTS: (1) Based upon this analysis, less than 6% of the energy consumed by the lamp operating at 2900 K is converted to visible light. The transmission of the glass envelope will be less than unity, so the efficiency will be less than the calculated value.

(2) Most of the energy is absorbed by the glass envelope and then lost to the surroundings by convection and radiation. Also, a significant amount of power is conducted to the lamp base and into the lamp base socket.

(3) The IHT workspace used to generate the above plot is shown below.

```
// Radiation Exchange Tool - Blackbody Band Emission Factor:
/* The blackbody band emission factor, Figure 12.14 and Table 12.1, is */
FL1Ts = F_lambda_T(lambda1,Ts)              // Eq 12.30
// where units are lambda (micrometers, mum) and T (K)
/* The blackbody band emission factor, Figure 12.14 and Table 12.1, is */
FL2Ts = F_lambda_T(lambda2,Ts)              // Eq 12.30

// Efficiency and rate expressions:
eta = qvis / qelec                          // Eq. (1)
eta_pc = eta * 100                          // Efficiency, %
qelec = 100                                 // Electrical power, W
qvis = (FL2Ts - FL1Ts) * As * sigma * Ts^4  // Eq (2)
sigma = 5.67e-8                             // Stefan-Boltzmann constant, W/m^2.K

//Assigned Variables:
Ts = 2900                   // Filament temperature, K
As = 0.005 * 0.005          // Filament area, m^2
lambda1 = 0.4               // Wavelength, mum; lower limit of visible spectrum
lambda2 = 0.7               // Wavelength, mum; upper limit of visible spectrum

/*Data Browser Results - Part (a):
FL1Ts       FL2Ts    eta      eta_pc   qvis     As       Ts       lambda1
lambda2     qelec    sigma
0.001771    0.07185  0.07026  7.026    7.026    2.5E-5   2900     0.4
0.7         100      5.67E-8  */
```

PROBLEM 12.23

KNOWN: Solar disc behaves as a blackbody at 5800 K.

FIND: (a) Fraction of total radiation emitted by the sun that is in the visible spectral region, (b) Plot the percentage of solar emission that is at wavelengths less than λ as a function of λ, and (c) Plot on the same coordinates the percentage of emission from a blackbody at 300 K that is at wavelengths less than λ as a function of λ; compare the plotted results with the upper abscissa scale of Figure 12.23.

ASSUMPTIONS: (1) Visible spectral region has limits $\lambda_1 = 0.40$ μm and $\lambda_2 = 0.70$ μm.

ANALYSIS: (a) Using the blackbody functions of Table 12.1, find $F_{(0\rightarrow\lambda)}$, the fraction of radiant flux leaving a black surface in the spectral interval 0→λ as a function of the product λT. From the tabulated values for $F_{(0\rightarrow\lambda)}$ with T = 5800 K,

$$\lambda_2 = 0.70\ \mu m \qquad \lambda_2 T = 4060\ \mu m\cdot K \qquad F_{(0\rightarrow l_1)} = 0.4914$$

$$\lambda_1 = 0.40\ \mu m \qquad \lambda_1 T = 2320\ \mu m\cdot K \qquad F_{(0\rightarrow l_2)} = 0.1245$$

Hence, for the visible spectral region, the fraction of total emitted solar flux is

$$F_{(l_1\rightarrow l_2)} = F_{(0\rightarrow l_2)} - F_{(0\rightarrow l_1)} = 0.4914 - 0.1245 = 0.3669 \quad \text{or} \quad 37\% \qquad <$$

(b,c) Using the *IHT Radiation Tool, Band Emission Factor*, $F_{(0\text{-}\lambda T)}$ are evaluated for the solar spectrum (T = 5800 K) and that for a blackbody temperature (T = 300 K) as a function of wavelength and are plotted below.

F(0 -> lambda) (%)

Wavelength, lambda (mum)

Solar spectrum, TbS = 5800 K (left)
Blackbody, Tbs = 300 K (right)

The left-hand curve in the plot represents the percentage of solar flux approximated as the 5800 K-blackbody spectrum in the spectral region less than λ. The right-hand curve represents the percentage of 300 K-blackbody flux in the spectral region less than λ. Referring to upper abscissa scale of Figure 12.23, for the solar flux, 75% of the solar flux is at wavelengths shorter than 1 μm. For the blackbody flux (300 K), 75% of the blackbody flux is at wavelengths shorter than 20 μm. These values are in agreement with points on the solar and 300K-blackbody curves, respectively, in the above plot.

PROBLEM 12.24

KNOWN: Thermal imagers operating in the spectral regions 3 to 5 μm and 8 to 14 μm.

FIND: (a) Band-emission factors for each of the spectral regions, 3 to 5 μm and 8 to 14 μm, for temperatures of 300 and 900 K, (b) Calculate and plot the band-emission factors for each of the spectral regions for the temperature range 300 to 1000 K; identify the maxima, and draw conclusions concerning the choice of an imager for an application; and (c) Considering imagers operating at the maximum-fraction temperatures found from the graph of part (b), determine the sensitivity (%) required of the radiation detector to provide a noise-equivalent temperature (NET) of 5°C.

ASSUMPTIONS: The sensitivity of the imager's radiation detector within the operating spectral region is uniform.

ANALYSIS: (a) From Eqs. 12.30 and 12.31, the band-emission fraction $F(\lambda 1 \rightarrow \lambda 2, T)$ for blackbody emission in the spectral range λ_1 to λ_2 for a temperature T is

$$F_{(l1 \rightarrow l2, T)} = F_{(0 \rightarrow l2, T)} - F_{(0 \rightarrow l1, T)}$$

Using the *IHT Radiation | Band Emission* tool (or Table 12.1), evaluate $F_{(0-\lambda T)}$ at appropriate $\lambda \cdot T$ products:

3 to 5 mm region

$$F_{(l1-l2, 300\ K)} = 0.1375 - 0.00017 = 0.01359$$ <

$$F_{(l1-l2, 900\ K)} = 0.5640 - 0.2055 = 0.3585$$ <

8 to 14mm region

$$F_{(l1-l2, 300\ K)} = 0.5160 - 0.1403 = 0.3758$$ <

$$F_{(l1-l2, 900\ K)} = 0.9511 - 0.8192 = 0.1319$$ <

(b) Using the *IHT Radiation | Band Emission* tool, the band-emission fractions for each of the spectral regions is calculated and plotted below as a function of temperature.

Band fractions for thermal imaging spectral regions

Band fraction for range: 0, 0.1, 0.2, 0.3, 0.4

Temperature, T (K): 300, 400, 500, 600, 700, 800, 900, 1000

3 to 5 um region
8 to 14 um region

Continued …..

PROBLEM 12.24 (Cont.)

For the 3 to 5 μm imager, the band-emission factor increases with increasing temperature. For low temperature applications, not only is the radiant power $\left(s\,T^4,\ T \approx 300\ K\right)$ low, but the band fraction is small. However, for high temperature applications, the imager operating conditions are more favorable with a large band-emission factor, as well as much higher radiant power $\left(s\,T^4,\ T \rightarrow 900\ K\right)$.

For the 8 to 14 μm imager, the band-emission factor decreases with increasing temperature. This is a more favorable instrumentation feature, since the band-emission factor (proportionally more power) becomes larger as the radiant power decreases. This imager would be preferred over the 3 to 5 μm imager at lower temperatures since the band-emission factor is 8 to 10 times higher.

Recognizing that from Wien's law, the peaks of the blackbody curves for 300 and 900 K are approximately 10 and 3.3 μm, respectively, it follows that the imagers will receive the most radiant power when the peak target spectral distributions are close to the operating spectral region. It is good application practice to chose an imager having a spectral operating range close to the peak of the blackbody curve (or shorter than, if possible) corresponding to the target temperature.

The maxima band fractions for the 3 to 5 μm and 8 to 14 μm spectral regions correspond to temperatures of 960 and 355 K, respectively. Other application factors not considered (like smoke, water vapor, etc), the former imager is better suited with higher temperature scenes, and the latter with lower temperature scenes.

(c) Consider the 3 to 5 μm and 8 to 14 μm imagers operating at their band-emission peak temperatures, 355 and 960 K, respectively. The sensitivity S (% units) of the imager to resolve an NET of 5°C can be expressed as

$$S(\%) = \frac{F(l1 - l2, T1) - F(l1 - l2, T2)}{F(l1 - l2, T1)} \times 100$$

where T_1 = 355 or 960 K and T_2 = 360 or 965 K, respectively. Using this relation in the *IHT* workspace, find

$$S_{3-5} = 0.035\% \qquad\qquad S_{8-14} = 0.023\%$$ <

That is, we require the radiation detector (with its signal-processing system) to resolve the output signal with the foregoing precision in order to indicate a 5°C change in the scene temperature.

PROBLEM 12.25

KNOWN: Tube furnace maintained at T_f = 2000 K used to calibrate a heat flux gage of sensitive area 5 mm^2 mounted coaxial with the furnace centerline, and positioned 60 mm from the opening of the furnace.

FIND: (a) Heat flux (kW/m^2) on the gage, (b) Radiant flux in the spectral region 0.4 to 2.5 μm, the sensitive spectral region of a solid-state (photoconductive type) heat-flux gage, and (c) Calculate and plot the heat fluxes for each of the gages as a function of the furnace temperature for the range 2000 ≤ T_f ≤ 3000 K. Compare the values for the two types of gages; explain why the solid-state gage will always indicate systematically low values; does the solid-state gage performance improve, or become worse as the source temperature increases?

SCHEMATIC:

Furnace, T_f = 2000 K
D = 12.5 mm
Cold extension tube
$\Delta\omega_{f-g}$
Heat flux gage, A_g = 5 mm^2
L = 60 mm

ASSUMPTIONS: (1) Steady-state conditions, (2) Graphite tube furnace behaves as a blackbody, (3) Areas of gage and furnace opening are small relative to separation distance squared, and (4) Extension tube is cold relative to the furnace.

ANALYSIS: (a) The heat flux to the gage is equal to the irradiation, G_g, on the gage and can be expressed as (see Section 12.2.3)

$$G_g = I_f \cdot \cos\theta_g \cdot \Delta\omega_{f-g}$$

where $\Delta\omega_{f-g}$ is the solid angle that the furnace opening subtends relative to the gage. From Eq. 12.2, with $\theta_g = 0°$

$$\Delta\omega_{f-g} \equiv \frac{dA_n}{r^2} = \frac{A_f \cos\theta_g}{L^2} = \frac{\pi(0.0125\text{ m})^2/4\times 1}{(0.060\text{ m})^2} = 3.409\times 10^{-2}\text{ sr}$$

The intensity of the radiation from the furnace is

$$I_f = E_{b,f}(T_f)/\pi = \sigma T_f^4/\pi = 5.67\times 10^{-8}\text{ W/m}^2\cdot\text{K}^4\,(2000\text{ K})^4/\pi = 2.888\times 10^5\text{ W/m}^2\cdot\text{sr}$$

Substituting numerical values,

$$G_g = 2.888\times 10^5\text{ W/m}^2\cdot\text{sr}\times 1\times 3.409\times 10^{-2}\text{ sr} = 9.84\text{ kW/m}^2$$ <

(b) The solid-state detector gage, sensitive only in the spectral region λ_1 = 0.4 μm to λ_2 = 2.5 μm, will receive the band irradiation.

$$G_{g,\lambda 1-\lambda 2} = F_{(\lambda 1\to\lambda 2,\,Tf)}\cdot G_{g,b} = \left[F_{(0\to\lambda 2,\,Tf)} - F_{(0\to\lambda 1,\,Tf)}\right] G_{g,b}$$

Continued …..

where for $\lambda_1 T_f = 0.4\ \mu m \times 2000\ K = 800\ \mu m \cdot K$, $F_{(0-\lambda 1)} = 0.0000$ and for $\lambda_2 \cdot T_f = 2.5\ \mu m \times 2000\ K = 5000\ \mu m \cdot K$, $F_{(0-\lambda 2)} = 0.6337$. Hence,

$$G_{g,\lambda 1-\lambda 2} = [0.6337 - 0.0000] \times 9.84\ kW/m^2 = 6.24\ kW/m^2 \qquad <$$

(c) Using the foregoing equation in the *IHT* workspace, the heat fluxes for each of the gage types are calculated and plotted as a function of the furnace temperature.

Heat heat flux gage calibrations

Heat flux, Gg (kW/m^2)

Furnace temperature, Tf (K)

Black heat flux gage

Solid-state gage, 0.4 to 2.5 um

For the black gage, the irradiation received by the gage, G_g, increases as the fourth power of the furnace temperature. For the solid-state gage, the irradiation increases slightly greater than the fourth power of the furnace temperature since the band-emission factor for the spectral region, $F_{(\lambda 1-\lambda 2, Tf)}$, increases with increasing temperature. The solid-state gage will always indicate systematic low readings since its band-emission factor never approaches unity. However, the error will decrease with increasing temperature as a consequence of the corresponding increase in the band-emission factor.

COMMENTS: For this furnace-gage geometrical arrangement, evaluating the solid angle, $\Delta\omega_{f-g}$, and the areas on a differential basis leads to results that are systematically high by 1%. Using the view factor concept introduced in Chapter 13 and Eq. 13.8, the results for the black and solid-state gages are 9.74 and 6.17 kW/m^2, respectively.

PROBLEM 12.26

KNOWN: Geometry and temperature of a ring-shaped radiator. Area of irradiated part and distance from radiator.

FIND: Rate at which radiant energy is incident on the part.

SCHEMATIC:

ASSUMPTIONS: (1) Heater emits as a blackbody.

ANALYSIS: Expressing Eq. 12.5 on the basis of the total radiation, $dq = I_e \, dA_h \cos\theta \, d\omega$, the rate at which radiation is incident on the part is

$$q_{h-p} = \int dq = I_e \int\int \cos\theta d\omega_{p-h} dA_h \approx I_e \cos\theta \cdot \omega_{p-h} \cdot A_h$$

Since radiation leaving the heater in the direction of the part is oriented normal to the heater surface, $\theta = 0$ and $\cos\theta = 1$. The solid angle subtended by the part with respect to the heater is $\omega_{p-h} = A_p \cos\theta_1/L^2$, while the area of the heater is $A_h \approx 2\pi r_h W = 2\pi(L \sin\theta_1)W$. Hence, with $I_e = E_b/\pi = \sigma T_h^4/\pi$,

$$q_{h-p} \approx \frac{5.67\times10^{-8}\,\mathrm{W/m^2\cdot K^4}\,(3000\,\mathrm{K})^4}{\pi} \times \frac{0.007\,\mathrm{m}^2\,(\cos 30^\circ)}{(3\,\mathrm{m})^2} \times 2\pi\,(1.5\,\mathrm{m})\,0.03\,\mathrm{m}$$

$$q_{h-p} \approx 278.4\,\mathrm{W}$$ <

COMMENTS: The foregoing representation for the double integral is an excellent approximation since $W << L$ and $A_p << L^2$.

PROBLEM 12.27

KNOWN: Spectral distribution of the emissive power given by Planck's law.

FIND: Approximations to the Planck distribution for the extreme cases when (a) $C_2/\lambda T >> 1$, Wien's law and (b) $C_2/\lambda T << 1$, Rayleigh-Jeans law.

ANALYSIS: Planck's law provides the spectral, hemispherical emissive power of a blackbody as a function of wavelength and temperature, Eq. 12.26,

$$E_{\lambda,b}(\lambda,T) = C_1/\lambda^5\left[\exp(C_2/\lambda T)-1\right].$$

We now consider the extreme cases of $C_2/\lambda T >> 1$ and $C_2/\lambda T << 1$.

(a) When $C_2/\lambda T >> 1$ (or $\lambda T << C_2$), it follows $\exp(C_2/\lambda T) >> 1$. Hence, the –1 term in the denominator of the Planck law is insignificant, giving

$$E_{\lambda,b}(\lambda,T) \approx \left(C_1/\lambda^5\right)\exp(-C_2/\lambda T). \qquad <$$

This approximate relation is known as *Wien's law*. The ratio of the emissive power by Wien's law to that by the Planck law is,

$$\frac{E_{\lambda,b,\text{Wien}}}{E_{\lambda,b,\text{Planck}}} = \frac{1/\exp(C_2/\lambda T)}{1/\left[\exp(C_2/\lambda T)-1\right]}.$$

For the condition $\lambda T = \lambda_{max} T = 2898\ \mu m\cdot K$, $C_2/\lambda T = \dfrac{14388\ \mu m\cdot K}{2898\ \mu m\cdot K} = 4.966$ and

$$\frac{E_{\lambda,b}\big|_{\text{Wien}}}{E_{\lambda,b}\big|_{\text{Planck}}} = \frac{1/\exp(4.966)}{1/\left[\exp(4.966)-1\right]} = 0.9930. \qquad <$$

That is, for $\lambda T \le 2898\ \mu m\cdot K$, Wien's law is a good approximation to the Planck distribution.

(b) If $C_2/\lambda T << 1$ (or $\lambda T >> C_2$), the exponential term may be expressed as a series that can be approximated by the first two terms. That is,

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots \approx 1 + x \qquad \text{when} \qquad x << 1.$$

The *Rayleigh-Jeans* approximation is then

$$E_{\lambda,b}(\lambda,T) \approx C_1/\lambda^5\left[1+(C_2/\lambda T)-1\right] = C_1 T/C_2\lambda^4.$$

For the condition $\lambda T = 100{,}000\ \mu m\cdot K$, $C_2/\lambda T = 0.1439$

$$\frac{E_{\lambda,b,R-J}}{E_{\lambda,b,\text{Planck}}} = \frac{C_1 T/C_2\lambda^4}{C_1/\lambda^5}\left[\exp(C_2/\lambda T)-1\right]^{-1} = (\lambda T/C_2)\left[\exp(C_2/\lambda T)-1\right] = 1.0754. \qquad <$$

That is, for $\lambda T \ge 100{,}000\ \mu m\cdot K$, the Rayleigh-Jeans law is a good approximation (better than 10%) to the Planck distribution.

COMMENTS: The Wien law is used extensively in optical pyrometry for values of λ near 0.65 μm and temperatures above 700 K. The Rayleigh-Jeans law is of limited use in heat transfer but of utility for far infrared applications.

PROBLEM 12.28

KNOWN: Aperture of an isothermal furnace emits as a blackbody.

FIND: (a) An expression for the ratio of the fractional change in the spectral intensity to the fractional change in temperature of the furnace aperture, (b) Allowable variation in temperature of a furnace operating at 2000 K such that the spectral intensity at 0.65μm will not vary by more than 1/2%. Allowable variation for 10μm.

SCHEMATIC:

$I_{\lambda,b}$ T=2000K
0.65 λ_{max} 10 $\lambda(\mu m)$

ASSUMPTIONS: (1) Furnace is isothermal and aperture radiates as a blackbody.

ANALYSIS: (a) The Planck spectral distribution, Eq. 12.26, is

$$I_\lambda(\lambda,T) = C_1/\pi\lambda^5\left[\exp(C_2/\lambda T)-1\right].$$

Taking natural logarithms of both sides, find $\ell n I_\lambda = \ell n\left[C_1/\pi\lambda^5\right] - \ell n\left[\exp(C_2/\lambda T)-1\right]$. Take the total derivative of both sides, but consider the λ variable as a constant.

$$\frac{dI_\lambda}{I_\lambda} = -\frac{d\left[\exp(C_2/\lambda T)-1\right]}{\left[\exp(C_2/\lambda T)-1\right]} = -\frac{\left\{\exp(C_2/\lambda T)\right\}(C_2/\lambda)\left(-1/T^2\right)dT}{\left[\exp(C_2/\lambda T)-1\right]}$$

$$\frac{dI_\lambda}{I_\lambda} = \frac{C_2}{\lambda T}\cdot\frac{\exp(C_2/\lambda T)}{\left[\exp(C_2/\lambda T)-1\right]}\cdot\frac{dT}{T} \quad \text{or} \quad \frac{dI_\lambda/I_\lambda}{dT/T} = \frac{C_2}{\lambda T}\cdot\frac{1}{1-\exp(-C_2/\lambda T)}.$$ <

(b) If the furnace operates at 2000 K and the desirable fractional change of the spectral intensity is 0.5% at 0.65 μm, the allowable temperature variation is

$$\frac{dT}{T} = \frac{dI_\lambda}{I_\lambda}\Big/\left\{\frac{C_2}{\lambda T}\,\frac{1}{\left[1-\exp(-C_2/\lambda T)\right]}\right\}$$

$$\frac{dT}{T} = 0.005\Big/\left\{\frac{14{,}388\,\mu m\cdot K}{0.65\,\mu m\times 2000K}\Big/\left[1-\exp\left(\frac{-14{,}388\,\mu m\cdot K}{0.65\,\mu m\times 2000K}\right)\right]\right\} = 4.517\times10^{-4}.$$

That is, the allowable fractional variation in temperature is 0.045%; at 2000 K, the allowable temperature variation is

$$\Delta T \approx 4.517\times10^{-4}T = 4.517\times10^{-4}\times 2000K = 0.90K.$$ <

Substituting with T = 2000 K and λ = 10 μm, find that

$$\frac{dT}{T} = 3.565\times10^{-3} \quad \text{and} \quad \Delta T \approx 3.565\times10^{-3}T = 7.1K.$$ <

COMMENTS: Note that the power control requirements to satisfy the spectral intensity variation for 0.65 μm and 10 μm conditions are quite different. The peak of the blackbody curve for 2000 K is λ_{max} = 2898 μm·K/2000 K=1.45 μm.

PROBLEM 12.29

KNOWN: Spectral emissivity, dimensions and initial temperature of a tungsten filament.

FIND: (a) Total hemispherical emissivity, ε, when filament temperature is T_s = 2900 K; (b) Initial rate of cooling, dT_s/dt, assuming the surroundings are at T_{sur} = 300 K when the current is switched off; (c) Compute and plot ε as a function of T_s for the range $1300 \le T_s \le 2900$ K; and (d) Time required for the filament to cool from 2900 to 1300 K.

SCHEMATIC:

0.45
ε_λ
0.10
0
2
4
λ(μm)
Evacuated bulb
q_{rad}
Tungsten filament,
D = 0.8 mm,
L = 20 mm,
T_s = 2900 K,
T_f = 1300 K

ASSUMPTIONS: (1) Filament temperature is uniform at any time (lumped capacitance), (2) Negligible heat loss by conduction through the support posts, (3) Surroundings large compared to the filament, (4) Spectral emissivity, density and specific heat constant over the temperature range, (5) Negligible convection.

PROPERTIES: *Table A-1*, Tungsten (2900 K); $\rho = 19,300\ \text{kg/m}^3$, $c_p \approx 185\ \text{J/kg}\cdot\text{K}$.

ANALYSIS: (a) The total emissivity at Ts = 2900 K follows from Eq. 12.38 using Table 12.1 for the band emission factors,

$$\varepsilon = \int_0^\infty \varepsilon_\lambda E_{\lambda,b}(T_s)d\lambda = \varepsilon_1 F_{(0\to 2\mu m)} + \varepsilon_2(1 - F_{0\to 2\mu m}) \quad (1)$$

$$\varepsilon = 0.45 \times 0.72 + 0.1\,(1 - 0.72) = 0.352$$ <

where $F_{(0\to 2\mu m)} = 0.72$ at $\lambda T = 2\mu m \times 2900\ K = 5800\ \mu m\cdot K$.

(b) Perform an energy balance on the filament at the instant of time at which the current is switched off,

$$\dot{E}_{in} - \dot{E}_{out} = Mc_p \frac{dT_s}{dt}$$

$$A_s(\alpha G_{sur} - E) = A_s\left(\alpha\sigma T_s^4 - \varepsilon\sigma T_s^4\right) = Mc_p\, dT_s/dt$$

and find the change in temperature with time where $A_s = \pi DL$, $M = \rho\forall$, and $\forall = (\pi D^2/4)L$,

$$\frac{dT_s}{dt} = -\frac{\pi DL\sigma(\varepsilon T_s^4 - \alpha T_{sur}^4)}{\rho\left(\pi D^2/4\right)Lc_p} = -\frac{4\sigma}{\rho c_p D}\left(\varepsilon T_s^4 - \alpha T_{sur}^4\right)$$

$$\frac{dT_s}{dt} = -\frac{4\times 5.67\times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4(0.352\times 2900^4 - 0.1\times 300^4)\text{K}^4}{19,300\ \text{kg/m}^2\times 185\ \text{J/kg}\cdot\text{K}\times 0.0008\text{m}} = -1977\ \text{K/s}$$ <

(c) Using the *IHT Tool*, *Radiation, Band Emission Factor*, and Eq. (1), a model was developed to calculate and plot ε as a function of T_s. See plot below.

Continued...

PROBLEM 12.29 (Cont.)

(d) Using the IHT *Lumped Capacitance Model* along with the IHT workspace for part (c) to determine ε as a function of T_s, a model was developed to predict T_s as a function of cooling time. The results are shown below for the variable emissivity case (ε vs. T_s as per the plot below left) and the case where the emissivity is fixed at ε(2900 K) = 0.352. For the variable and fixed emissivity cases, the times to reach T_s = 1300 K are

t_{var} = 8.3 s $\qquad$ t_{fix} = 5.1 s <

eps
0.4
0.3
0.2
0.1
1000 1500 2000 2500 3000
Filament temperature, Ts (K)

Ts (K)
3000
2000
1000
0 2 4 6 8 10
Elapsed time, t (s)
Variable emissivity
Fixed emissivity, eps = 0.35

COMMENTS: (1) From the ε vs. T_s plot, note that ε increases as T_s increases. Could you have surmised as much by looking at the spectral emissivity distribution, ε_λ vs. λ?

(2) How do you explain the result that $t_{var} > t_{fix}$?

PROBLEM 12.30

KNOWN: Spectral distribution of emissivity for zirconia and tungsten filaments. Filament temperature.

FIND: (a) Total emissivity of zirconia, (b) Total emissivity of tungsten and comparative power requirement, (c) Efficiency of the two filaments.

SCHEMATIC:

ASSUMPTIONS: (1) Negligible reflection of radiation from bulb back to filament, (2) Equivalent surface areas for the two filaments, (3) Negligible radiation emission from bulb to filament.

ANALYSIS: (a) From Eq. (12.38), the emissivity of the zirconia is

$$\varepsilon = \int_o^\infty \varepsilon_\lambda \left(E_\lambda / E_b\right) d_\lambda = \varepsilon_1 F_{(0\to 0.4\mu m)} + \varepsilon_2 F_{(0.4\to 0.7\mu m)} + \varepsilon_3 F_{(0.7\mu m\to\infty)}$$

$$\varepsilon = \varepsilon_1 F_{(0\to 0.4\mu m)} + \varepsilon_2 \left(F_{(0\to 0.7\mu m)} - F_{(0\to 0.4\mu m)}\right) + \varepsilon_3 \left(1 - F_{(0\to 0.7\mu m)}\right)$$

From Table 12.1, with T = 3000 K

$$\lambda T = 0.4\mu m \times 3000 \equiv 1200\mu m\cdot K: \quad F_{(0\to 0.4\mu m)} = 0.0021$$
$$\lambda T = 0.7\mu m \times 3000\ K = 2100\mu m\cdot K: F_{(0\to 0.7\mu m)} = 0.0838$$

$$\varepsilon = 0.2\times 0.0021 + 0.8(0.0838 - 0.0021) + 0.2\times(1 - 0.0838) = 0.249$$ <

(b) For the tungsten filament,

$$\varepsilon = \varepsilon_1 F_{(0\to 2\mu m)} + \varepsilon_2 \left(1 - F_{(0\to 2\mu m)}\right)$$

With $\lambda T = 6000\mu m\cdot K$, $F(0 \to 2\mu m) = 0.738$

$$\varepsilon = 0.45\times 0.738 + 0.1(1 - 0.738) = 0.358$$ <

Assuming, no reflection of radiation from the bulb back to the filament and with no losses due to natural convection, the power consumption per unit surface area of filament is $P''_{elec} = \varepsilon\sigma T^4$.

Continued …..

Zirconia: $P''_{elec} = 0.249 \times 5.67 \times 10^{-8}\ W/m^2 \cdot K^4 (3000\ K)^4 = 1.14 \times 10^6\ W/m^2$

Tungsten: $P''_{elec} = 0.358 \times 5.67 \times 10^{-8}\ W/m^2 \cdot K^4 (3000\ K)^4 = 1.64 \times 10^6\ W/m^2$

Hence, for an equivalent surface area and temperature, the tungsten filament has the largest power consumption. $<$

(c) Efficiency with respect to the production of visible radiation may be defined as

$$\eta_{vis} = \frac{\int_{0.4}^{0.7} \varepsilon_\lambda E_{\lambda,b} d\lambda}{E} = \frac{\int_{0.4}^{0.7} \varepsilon_\lambda \left(E_{\lambda,b}/E_b\right)}{\varepsilon} = \frac{\varepsilon_{vis}}{\varepsilon} F_{(0.4 \to 0.7 \mu m)}$$

With $F_{(0.4 \to 0.7\ \mu m)} = 0.0817$ for T = 3000 K,

Zirconia: $\eta_{vis} = (0.8/0.249)0.0817 = 0.263$

Tungsten: $\eta_{vis} = (0.45/0.358)0.0817 = 0.103$

Hence, the zirconia filament is the more efficient. $<$

COMMENTS: The production of visible radiation per unit filament surface area is $E_{vis} = \eta_{vis}$ P''_{elec}. Hence,

Zirconia: $E_{vis} = 0.263 \times 1.14 \times 10^6\ W/m^2 = 3.00 \times 10^5\ W/m^2$

Tungsten: $E_{vis} = 0.103 \times 1.64 \times 10^6\ W/m^2 = 1.69 \times 10^5\ W/m^2$

Hence, not only is the zirconia filament more efficient, but it also produces more visible radiation with less power consumption. This problem illustrates the benefits associated with carefully considering spectral surface characteristics in radiative applications.

PROBLEM 12.31

KNOWN: Variation of spectral, hemispherical emissivity with wavelength for two materials.

FIND: Nature of the variation with temperature of the total, hemispherical emissivity.

SCHEMATIC:

ASSUMPTIONS: (1) ε_λ is independent of temperature.

ANALYSIS: The total, hemispherical emissivity may be obtained from knowledge of the spectral, hemispherical emissivity by using Eq. 12.38

$$\varepsilon(T) = \frac{\int_0^\infty \varepsilon_\lambda(\lambda) E_{\lambda,b}(\lambda,T)\, d\lambda}{E_b(T)} = \int_0^\infty \varepsilon_\lambda(\lambda) \frac{E_{\lambda,b}(\lambda,T)}{E_b(T)} d\lambda.$$

We also know that the spectral emissive power of a blackbody becomes more concentrated at lower wavelengths with increasing temperature (Fig. 12.13). That is, the weighting factor, $E_{\lambda,b}(\lambda,T)/E_b(T)$ increases at lower wavelengths and decreases at longer wavelengths with increasing T. Accordingly,

Material A: $\varepsilon(T)$ increases with increasing T <

Material B: $\varepsilon(T)$ decreases with increasing T. <

PROBLEM 12.32

KNOWN: Metallic surface with prescribed spectral, directional emissivity at 2000 K and 1 μm (see Example 12.6) and additional measurements of the spectral, hemispherical emissivity.

FIND: (a) Total hemispherical emissivity, ε, and the emissive power, E, at 2000 K, (b) Effect of temperature on the emissivity.

SCHEMATIC:

ANALYSIS: (a) The total, hemispherical emissivity, ε, may be determined from knowledge of the spectral, hemispherical emissivity, ε_λ, using Eq. 12.38.

$$\varepsilon(T) = \int_0^\infty \varepsilon_\lambda(\lambda) E_{\lambda,b}(\lambda, T)\, d\lambda \big/ E_b(T) = \varepsilon_1 \int_0^{2\mu m} \frac{E_{\lambda,b}(\lambda, T) d\lambda}{E_b(T)} + \varepsilon_2 \int_{2\mu m}^{4\mu m} \frac{E_{\lambda,b}(\lambda, T) d\lambda}{E_b(T)}$$

or from Eqs. 12.28 and 12.30,

$$\varepsilon(T) = \varepsilon_1 F_{(0\to\lambda_1)} + \varepsilon_2 \left[F_{(0\to\lambda_2)} - F_{(0\to\lambda_1)} \right]$$

From Table 12.1,

$$\lambda_1 = 2\,\mu m, \quad T = 2000\text{ K}: \quad \lambda_1 T = 4000\,\mu m \cdot K, \quad F_{(0\to\lambda_1)} = 0.481$$

$$\lambda_2 = 4\,\mu m, \quad T = 2000\text{ K}: \quad \lambda_2 T = 8000\,\mu m \cdot K, \quad F_{(0\to\lambda_2)} = 0.856$$

Hence,

$$\varepsilon(T) = 0.36 \times 0.481 + 0.20(0.856 - 0.481) = 0.25$$ <

From Eqs. 12.28 and 12.37, the total emissive power at 2000 K is

$$E(2000\text{ K}) = \varepsilon(2000\text{ K}) \cdot E_b(2000\text{ K})$$

$$E(2000\text{ K}) = 0.25 \times 5.67 \times 10^{-8}\, W/m^2 \cdot K^4 \times (2000\text{ K})^4 = 2.27 \times 10^5\, W/m^2 .$$ <

(b) Using the *Radiation* Toolpad of IHT, the following result was generated.

Continued...

PROBLEM 12.32 (Cont.)

At T ≈ 500 K, most of the radiation is emitted in the far infrared region ($\lambda > 4$ μm), in which case $\varepsilon \approx 0$. With increasing T, emission is shifted to lower wavelengths, causing ε to increase. As $T \rightarrow \infty$, $\varepsilon \rightarrow 0.36$.

COMMENTS: Note that the value of ε_λ for $0 < \lambda \leq 2$ μm cannot be read directly from the ε_λ distribution provided in the problem statement. This value is calculated from knowledge of $\varepsilon_{\lambda,\theta}(\theta)$ in Example 12.6.

PROBLEM 12.33

KNOWN: Relationship for determining total, hemispherical emissivity, ε, by integration of the spectral emissivity distribution, ε_λ (Eq. 12.38).

FIND: Evaluate ε from ε_λ for the following cases: (a) Ex. 12.5, use the result to benchmark your code, (b) tungsten at 2800 K, and (c) aluminum oxide at 1400 K. Use the intrinsic function *INTEGRAL* of *IHT* as your solution tool.

SCHEMATIC:

ASSUMPTIONS: (1) Surfaces are diffuse emitters.

ANALYSIS: (a) Using *IHT* as the solution tool, Eq. 12.38 is entered into the workspace, and a look-up table created to represent the spectral emissivity distribution. See Comment 1 for the *IHT* annotated code. The result is $\varepsilon = 0.558$, in agreement with the analysis of Ex. 12.5 using the band-emission factors.

(b, c) Using the same code as for the benchmarking exercise in Part (a), but with new look-up table files (*.lut) representing the spectral distributions tabulated below, the total hemispherical emissivities for the tungsten at 2800 K and aluminum oxide at 1400 K are:

$$\varepsilon_W = 0.31 \qquad\qquad \varepsilon_{Al2O3} = 0.38$$ <

These results compare favorably with values of 0.29 and 0.41, respectively, from Fig. 12.19. See Comment 2.

Tungsten, 2800 K

λ (μm)	ε_λ	λ (μm)	ε_λ
0.3	0.47	2.0	0.26
0.4	0.48	4.0	0.17
0.5	0.47	6.0	0.05
0.6	0.44	8.0	0.03
1.0	0.38	10	0.03

Aluminum oxide, 1400 K

λ (μm)	ε_λ	λ (μm)	ε_λ
0.6	0.19	4.5	0.50
0.8	0.18	5	0.70
1.0	0.175	6	0.88
1.5	0.175	10	0.96
2	0.19	12.5	0.9
3	0.29	15	0.53
4	0.4	20	0.39

Continued …..

COMMENTS: (1) The *IHT* code to obtain ε from ε_λ for the case of Ex. 12.5 spectral distribution is shown below.

```
// Benchmarking use of INTEGRAL and LOOKUPVAL functions
// Calculating total emissivity from spectral distribution

/* Results:  integration from 0.05 to 15 by steps of 0.02, tabulated every 10
ILb       eps      eps_t     T        lambda   LLB
198.2     0.001    0.5579    1600     14.85    0.1982 */

// Emissivity integral, Eq. 12.38
eps_t = pi * INTEGRAL(IL,lambda) / (sigma * T^4)
sigma = 5.67 e-8

// Blackbody Spectral intensity, Tools | Radiation
/* From Planck's law, the blackbody spectral intensity is */
IL = eps *ILb
ILb = I_lambda_b(lambda, T, C1, C2)   // Eq. 12.25
// where units are ILb(W/m^2.sr.mum), lambda (mum) and T (K) with
C1 = 3.7420e8       // First radiation constant, W·mum^4/m^2
C2 = 1.4388e4       // Second radiation constant, mum·K
// and (mum) represents (micrometers).

// Emissivity function
eps = LOOKUPVAL(eps_L, 1, lambda, 2)
/* The table file name is eps_L.lut, with 2 columns and 6 rows.  See Help | Solver |
Lookup Tables | Lookupval
0.05      0.4
1.99      0.4
2         0.8
4.99      0.8
5         0.001
100       0.001      */

// Input variable
T = 1600
```

(2) For tungsten at 2800 K, the spectral limits for 98% of the blackbody spectrum are 0.51 and 8.3 μm. For aluminum at 1400 K, the spectral limits for 98% of the blackbody spectrum are 1.0 and 16.7 μm. For both cases, the foregoing tabulated spectral emissivity distributions are adequately represented for integration within the 98% limits.

PROBLEM 12.34

KNOWN: Spectral directional emissivity of a diffuse material at 2000K.

FIND: (a) Total, hemispherical emissivity, (b) Emissive power over the spectral range 0.8 to 2.5 μm and for directions $0 \le \theta \le \pi/6$.

SCHEMATIC:

ASSUMPTIONS: (1) Surface is diffuse emitter.

ANALYSIS: (a) Since the surface is diffuse, $\varepsilon_{\lambda,\theta}$ is independent of direction; from Eq. 12.36, $\varepsilon_{\lambda,\theta} = \varepsilon_\lambda$. Using Eq. 12.38,

$$\varepsilon(\mathrm{T}) = \int_0^\infty \varepsilon_\lambda(\lambda) \mathrm{E}_{\lambda,\mathrm{b}}(\lambda,\mathrm{T}) d\lambda / \mathrm{E}_\mathrm{b}(\mathrm{T})$$

$$\mathrm{E}(\mathrm{T}) = \int_0^{1.5} \varepsilon_1 \mathrm{E}_{\lambda,\mathrm{b}}(\lambda,2000) d\lambda / \mathrm{E}_\mathrm{b} + \int_{1.5}^\infty \varepsilon_2 \mathrm{E}_{\lambda,\mathrm{b}}(\lambda,2000)\, d\lambda / \mathrm{E}_\mathrm{b}.$$

Written now in terms of $F_{(0 \to \lambda)}$, with $F_{(0 \to 1.5)} = 0.2732$ at $\lambda T = 1.5 \times 2000 = 3000$ μm·K, (Table 12.1) find,

$$\varepsilon(2000\mathrm{K}) = \varepsilon_1 \times F_{(0\to1.5)} + \varepsilon_2\left[1 - F_{(0\to1.5)}\right] = 0.2 \times 0.2732 + 0.8[1 - 0.2732] = 0.636. \qquad <$$

(b) For the prescribed spectral and geometric limits, from Eq. 12.12,

$$\Delta \mathrm{E} = \int_{0.8}^{2.5} \int_0^{2\pi} \int_0^{\pi/6} \varepsilon_{\lambda,\theta} \mathrm{I}_{\lambda,\mathrm{b}}(\lambda,\mathrm{T}) \cos\theta \sin\theta \, d\theta \, d\phi \, d\lambda$$

where $I_{\lambda,e}(\lambda, \theta, \phi) = \varepsilon_{\lambda,\theta} I_{\lambda,b}(\lambda,T)$. Since the surface is diffuse, $\varepsilon_{\lambda,\theta} = \varepsilon_\lambda$, and noting $I_{\lambda,b}$ is independent of direction and equal to $E_{\lambda,b}/\pi$, we can write

$$\Delta \mathrm{E} = \left\{\int_0^{2\pi} \int_0^{\pi/6} \cos\theta \sin\theta \, d\theta \, d\phi\right\} \frac{\mathrm{E}_\mathrm{b}(\mathrm{T})}{\pi} \left\{\frac{\int_{0.8}^{1.5} \varepsilon_1 \mathrm{E}_{\lambda,\mathrm{b}}(\lambda,\mathrm{T}) d\lambda}{\mathrm{E}_\mathrm{b}(\mathrm{T})} + \frac{\int_{1.5}^{2.5} \varepsilon_2 \mathrm{E}_{\lambda,\mathrm{b}}(\lambda,\mathrm{T}) d\lambda}{\mathrm{E}_\mathrm{b}(\mathrm{T})}\right\}$$

or in terms $F_{(0 \to \lambda)}$ values,

$$\Delta \mathrm{E} = \left\{\phi\Big|_0^{2\pi} \times \frac{\sin^2\theta}{2}\Big|_0^{\pi/6}\right\} \frac{\sigma \mathrm{T}^4}{\pi} \left\{\varepsilon_1\left[F_{0\to1.5} - F_{0\to0.8}\right] + \varepsilon_2\left[F_{0\to2.5} - F_{0\to1.5}\right]\right\}.$$

From Table 12.1:	$\lambda T = 0.8 \times 2000 = 1600$ μm·K	$F_{(0 \to 0.8)} = 0.0197$
	$\lambda T = 2.5 \times 2000 = 5000$ μm·K	$F_{(0 \to 2.5)} = 0.6337$

$$\Delta \mathrm{E} = \left\{2\pi \times \frac{\sin^2 \pi/6}{2}\right\} \frac{5.67 \times 10^{-8} \times 2000^4}{\pi} \frac{\mathrm{W}}{\mathrm{m}^2} \cdot \left\{0.2[0.2732 - 0.0197] + 0.8[0.6337 - 0.2732]\right\}$$

$$\Delta \mathrm{E} = 0.25 \times \left(5.67 \times 10^{-8} \times 2000^4\right) \mathrm{W/m^2} \times 0.339 = 76.89\ \mathrm{kW/m^2}. \qquad <$$

PROBLEM 12.35

KNOWN: Directional emissivity, ε_θ, of a selective surface.

FIND: Ratio of the normal emissivity, ε_n, to the hemispherical emissivity, ε.

SCHEMATIC:

ASSUMPTIONS: Surface is isotropic in ϕ direction.

ANALYSIS: From Eq. 12.36 written on a total, rather than spectral, basis, the hemispherical emissivity is

$$\varepsilon = 2\int_0^{\pi/2} \varepsilon_\theta\,(\theta)\cos\theta\,\sin\theta\,d\,\theta.$$

Recognizing that the integral can be expressed in two parts, find

$$\varepsilon = 2\left[\int_0^{\pi/4} \varepsilon(\theta)\cos\theta\,\sin\theta\,d\theta + \int_{\pi/4}^{\pi/2} \varepsilon(\theta)\cos\theta\,\sin\theta\,d\theta\right]$$

$$\varepsilon = 2\left[0.8\int_0^{\pi/4} \cos\theta\,\sin\theta\,d\theta + 0.3\int_{\pi/4}^{\pi/2} \cos\theta\,\sin\theta\,d\theta\right]$$

$$\varepsilon = 2\left[0.8\frac{\sin^2\theta}{2}\Bigg|_0^{\pi/4} + 0.3\frac{\sin^2\theta}{2}\Bigg|_{\pi/4}^{\pi/2}\right]$$

$$\varepsilon = 2\left[0.8\frac{1}{2}(0.50-0) + 0.3\times\frac{1}{2}(1-0.50)\right] = 0.550.$$

The ratio of the normal emissivity (ε_n) to the hemispherical emissivity is

$$\frac{\varepsilon_n}{\varepsilon} = \frac{0.8}{0.550} = 1.45.$$ <

COMMENTS: Note that Eq. 12.36 assumes the directional emissivity is independent of the ϕ coordinate. If this is not the case, then Eq. 12.35 is appropriate.

PROBLEM 12.35

KNOWN: Directional emissivity, ε_θ, of a selective surface.

FIND: Ratio of the normal emissivity, ε_n, to the hemispherical emissivity, ε.

SCHEMATIC:

ASSUMPTIONS: Surface is isotropic in ϕ direction.

ANALYSIS: From Eq. 12.36 written on a total, rather than spectral, basis, the hemispherical emissivity is

$$\varepsilon = 2\int_0^{\pi/2} \varepsilon_\theta(\theta)\cos\theta\,\sin\theta\,d\theta.$$

Recognizing that the integral can be expressed in two parts, find

$$\varepsilon = 2\left[\int_0^{\pi/4} \varepsilon(\theta)\cos\theta\,\sin\theta\,d\theta + \int_{\pi/4}^{\pi/2} \varepsilon(\theta)\cos\theta\,\sin\theta\,d\theta\right]$$

$$\varepsilon = 2\left[0.8\int_0^{\pi/4} \cos\theta\,\sin\theta\,d\theta + 0.3\int_{\pi/4}^{\pi/2} \cos\theta\,\sin\theta\,d\theta\right]$$

$$\varepsilon = 2\left[0.8\frac{\sin^2\theta}{2}\bigg|_0^{\pi/4} + 0.3\frac{\sin^2\theta}{2}\bigg|_{\pi/4}^{\pi/2}\right]$$

$$\varepsilon = 2\left[0.8\frac{1}{2}(0.50-0) + 0.3\times\frac{1}{2}(1-0.50)\right] = 0.550.$$

The ratio of the normal emissivity (ε_n) to the hemispherical emissivity is

$$\frac{\varepsilon_n}{\varepsilon} = \frac{0.8}{0.550} = 1.45.$$ <

COMMENTS: Note that Eq. 12.36 assumes the directional emissivity is independent of the ϕ coordinate. If this is not the case, then Eq. 12.35 is appropriate.

PROBLEM 12.36

KNOWN: The total directional emissivity of non-metallic materials may be approximated as $\varepsilon_\theta = \varepsilon_n \cos\theta$ where ε_n is the total normal emissivity.

FIND: Show that for such materials, the total hemispherical emissivity, ε, is 2/3 the total normal emissivity.

SCHEMATIC:

ANALYSIS: From Eq. 12.36, written on a total rather than spectral basis, the hemispherical emissivity ε can be determined from the directional emissivity ε_θ as

$$\varepsilon = 2\int_0^{\pi/2} \varepsilon_\theta \cos\theta \sin\theta \, d\theta$$

With $\varepsilon_\theta = \varepsilon_n \cos\theta$, find

$$\varepsilon = 2\,\varepsilon_n \int_0^{\pi/2} \cos^2\theta \sin\theta \, d\theta$$

$$\varepsilon = -2\,\varepsilon_n \left(\cos^3\theta / 3\right)\Big|_0^{\pi/2} = 2/3\,\varepsilon_n$$ <

COMMENTS: (1) Refer to Fig. 12.17 illustrating on cartesian coordinates representative directional distributions of the total, directional emissivity for nonmetallic and metallic materials. In the schematic above, we've shown ε_θ vs. θ on a polar plot for both types of materials, in comparison with a diffuse surface.

(2) See Section 12.4 for discussion on other characteristics of emissivity for materials.

PROBLEM 12.37

KNOWN: Incandescent sphere suspended in air within a darkened room exhibiting these characteristics:

initially: brighter around the rim
after some time: brighter in the center

FIND: Plausible explanation for these observations.

ASSUMPTIONS: (1) The sphere is at a uniform surface temperature, T_s.

ANALYSIS: Recognize that in observing the sphere by eye, emission from the central region is in a nearly normal direction. Emission from the rim region, however, has a large angle from the normal to the surface.

Sphere
T_s

Note now the directional behavior, ε_θ, for conductors and non-conductors as represented in Fig. 12.17.

Assume that the sphere is fabricated from a *metallic* material. Then, the rim would appear brighter than the central region. This follows since ε_θ is higher at higher angles of emission.

If the metallic sphere oxidizes with time, then the ε_θ characteristics change. Then ε_θ at small angles of θ become larger than at higher angles. This would cause the sphere to appear brighter at the center portion of the sphere.

COMMENTS: Since the emissivity of non-conductors is generally larger than for metallic materials, you would also expect the oxidized sphere to appear brighter for the same surface temperature.

PROBLEM 12.38

KNOWN: Detector surface area. Area and temperature of heated surface. Radiant power measured by the detector for two orientations relative to the heated surface.

FIND: (a) Normal emissivity of heated surface, (b) Whether surface is a diffuse emitter.

SCHEMATIC:

ASSUMPTIONS: (1) Detector intercepts negligible radiation from surroundings, (2) A_1 and A_2 are differential surfaces.

ANALYSIS: The radiant power leaving the heated surface and intercepting the detector is

$$q_{12}(\theta) = I_1(\theta) A_1 \cos\theta \, \omega_{2-1}$$

$$I_1(\theta) = \varepsilon_1(\theta) I_{b,1} = \varepsilon_1(\theta) \sigma T_1^4 / \pi \qquad \omega_{2-1} = \frac{A_2 \cos\theta}{(L/\cos\theta)^2}.$$

Hence,

$$q_{12}(\theta) = \varepsilon_1(\theta) \frac{\sigma T_1^4}{\pi} A_1 \cos\theta \frac{A_2 \cos\theta}{(L/\cos\theta)^2} \qquad \varepsilon_1(\theta) = \frac{q_{12}(\theta)\pi}{\sigma T_1^4} \frac{L^2}{A_1 A_2 (\cos\theta)^4}$$

(a) For the normal condition, $\theta = 0$, $\cos\theta = 1$, and $\varepsilon_1(\theta) \equiv \varepsilon_{1,n}$ is

$$\varepsilon_{1,n} = \frac{1.155\times10^{-6}\,\mathrm{W}\,\pi}{5.67\times10^{-8}\,\mathrm{W/m^2\cdot K^4}(1000\mathrm{K})^4} \frac{(0.5\mathrm{m})^2}{5\times10^{-6}\,\mathrm{m^2}\times4\times10^{-6}\,\mathrm{m^2}}$$

$$\varepsilon_{1,n} = 0.80.$$ <

(b) For the orientation with $\theta = 60°$ and $\cos\theta = 0.5$, so that $\varepsilon_1(\theta = 60°)$ is

$$\varepsilon_1(60°) = \frac{5.415\times10^{-8}\,\mathrm{W}\,\pi}{5.67\times10^{-8}\,\mathrm{W/m^2\cdot K^4}(1000\mathrm{K})^4} \frac{(0.5\mathrm{m})^2}{5\times10^{-6}\,\mathrm{m^2}\times4\times10^{-6}\,\mathrm{m^2}(0.5)^4}$$

$$\varepsilon_1(60°) = 0.60.$$ <

Since $\varepsilon_{1,n} \neq \varepsilon_1(60°)$, the surface is not a diffuse emitter.

COMMENTS: Even if $\varepsilon_1(60°)$ were equal to $\varepsilon_{1,n}$, it could not be concluded there was diffuse emission until results were obtained for a wider range of θ.

PROBLEM 12.39

KNOWN: Radiation thermometer responding to radiant power within a prescribed spectral interval and calibrated to indicate the temperature of a blackbody.

FIND: (a) Whether radiation thermometer will indicate temperature greater than, less than, or equal to T_s when surface has $\varepsilon < 1$, (b) Expression for T_s in terms of spectral radiance temperature and spectral emissivity, (c) Indicated temperature for prescribed conditions of T_s and ε_λ.

SCHEMATIC:

ω_t
q_{th}
Surface
$T_s = 1000K$,
$\varepsilon_\lambda(0.65 \mu m) = 0.9$
Target viewed by thermometer, A_t

ASSUMPTIONS: (1) Surface is a diffuse emitter, (2) Thermometer responds to radiant flux over interval $d\lambda$ about λ.

ANALYSIS: (a) The radiant power which reaches the radiation thermometer is

$$q_\lambda = \varepsilon_\lambda I_{\lambda,b}(\lambda, T_s) \cdot A_t \cdot \omega_t \tag{1}$$

where A_t is the area of the surface viewed by the thermometer (referred to as the target) and ω_t the solid angle through which A_t is viewed. The thermometer responds as if it were viewing a blackbody at T_λ, the spectral radiance temperature,

$$q_\lambda = I_{\lambda,b}(\lambda, T_\lambda) \cdot A_t \cdot \omega_t . \tag{2}$$

By equating the two relations, Eqs. (1) and (2), find

$$I_{\lambda,b}(\lambda, T_\lambda) = \varepsilon_\lambda I_{\lambda,b}(\lambda, T_s). \tag{3}$$

Since $\varepsilon_\lambda < 1$, it follows that $I_{\lambda,b}(\lambda, T_\lambda) < I_{\lambda,b}(\lambda, T_s)$ or that $T_\lambda < T_s$. That is, the thermometer will always indicate a temperature lower than the true or actual temperature for a surface with $\varepsilon < 1$.

(b) Using Wien's law in Eq. (3), find

$$I_\lambda(\lambda, T) = \frac{1}{\pi} C_1 \lambda^{-5} \exp(-C_2 / \lambda T)$$

$$\frac{1}{\pi} C_1 \lambda^{-5} \exp(-C_2 / \lambda T_\lambda) = \varepsilon_\lambda \cdot \frac{1}{\pi} C_1 \lambda^{-5} \exp(-C_2 / \lambda T_s).$$

Canceling terms ($C_1 \lambda^{-5}/\pi$), taking natural logs of both sides of the equation and rearranging, the desired expression is

$$\frac{1}{T_s} = \frac{1}{T_\lambda} + \frac{\lambda}{C_2} \ell n \varepsilon_\lambda . \tag{4}$$ <

(c) For $T_s = 1000K$ and $\varepsilon = 0.9$, from Eq. (4), the indicated temperature is

$$\frac{1}{T_\lambda} = \frac{1}{T_s} - \frac{\lambda}{C_2} \ln \varepsilon_\lambda = \frac{1}{1000K} - \frac{0.65 \mu m}{14{,}388 \mu m \cdot K} \ell n (0.9) \qquad T_\lambda = 995.3K.$$ <

That is, the thermometer indicates 5K less than the true temperature.

PROBLEM 12.40

KNOWN: Spectral distribution of emission from a blackbody. Uncertainty in measurement of intensity.

FIND: Corresponding uncertainities in using the intensity measurement to determine (a) the surface temperature or (b) the emissivity.

ASSUMPTIONS: Diffuse surface behavior.

ANALYSIS: From Eq. 12.25, the spectral intensity associated with emission may be expressed as

$$I_{l,e} = e_l \, I_{l,b} = \frac{e_l \, C_1 / p}{l^5 \left[\exp\left(C_2 / l T\right) - 1\right]}$$

(a) To determine the effect of temperature on intensity, we evaluate the derivative,

$$\frac{\partial I_{l,e}}{\partial T} = -\frac{\left(e_l \, C_1/p\right) l^5 \exp\left(C_2 / l T\right)\left(-C_2 / l T^2\right)}{\left\{l^5 \left[\exp\left(C_2 / l T\right) - 1\right]\right\}^2}$$

$$\frac{\partial I_{l,e}}{\partial T} = \frac{\left(C_2 / l T^2\right) \exp\left(C_2 / l T\right)}{\exp\left(C_2 / l T\right) - 1} I_{l,e}$$

Hence,

$$\frac{dT}{T} = \frac{1 - \exp\left(-C_2 / l T\right)}{\left(C_2 / l T\right)} \frac{d I_{l,e}}{I_{l,e}}$$

With $\left(d I_{l,e} / I_{l,e}\right) = 0.1$, $C_2 = 1.439 \times 10^4 \, m\text{m} \cdot \text{K}$ and $l = 10 m\text{m}$,

$$\frac{dT}{T} = \left[\frac{1 - \exp\left(-1439\text{K}/T\right)}{1439\text{K}/T}\right] \times 0.1$$

T = 500 K: dT/T = 0.033 → 3.3% uncertainty <

T = 1000 K: dT/T = 0.053 → 5.5% uncertainty <

(b) To determine the effect of the emissivity on intensity, we evaluate

$$\frac{\partial I_{l,e}}{\partial e_l} = I_{l,b} = \frac{I_{l,e}}{e_l}$$

Hence, $\dfrac{d e_l}{e_l} = \dfrac{d I_{l,e}}{I_{l,e}} = 0.10 \rightarrow 10\%$ uncertainty <

COMMENTS: The uncertainty in the temperature is less than that of the intensity, but increases with increasing temperature (and wavelength). In the limit as $C_2 / l T \rightarrow 0$, exp $(-C_2 / l T) \rightarrow 1 - C_2 / l T$ and $dT/T \rightarrow d I_{l,e} / I_{l,e}$. The uncertainty in temperature then corresponds to that of the intensity measurement. The same is true for the uncertainty in the emissivity, irrespective of the value of T or l.

PROBLEM 12.41

KNOWN: Temperature, thickness and spectral emissivity of steel strip emerging from a hot roller. Temperature dependence of total, hemispherical emissivity.

FIND: (a) Initial total, hemispherical emissivity, (b) Initial cooling rate, (c) Time to cool to prescribed final temperature.

SCHEMATIC:

ASSUMPTIONS: (1) Negligible conduction (in longitudinal direction), convection and radiation from surroundings, (2) Negligible transverse temperature gradients.

PROPERTIES: Steel (given): ρ = 7900 kg/m^3, c = 640 J/kg·K, $\varepsilon = 1200\varepsilon_i/T$ (K).

ANALYSIS: (a) The initial total hemispherical emissivity is

$$\varepsilon_i = \int_0^\infty \varepsilon_\lambda \left[E_{\lambda b}(1200)/E_b(1200)\right]d\lambda$$

and integrating by parts using values from Table 12.1, find

$$\lambda T = 1200\mu m \cdot K \rightarrow F_{(0-1\mu m)} = 0.002;\ \lambda T = 7200\mu m \cdot K \rightarrow F_{(0-6\mu m)} = 0.819$$

$$\varepsilon_i = 0.6 \times 0.002 + 0.4(0.819 - 0.002) + 0.25(1 - 0.819) = 0.373.$$ <

(b) From an energy balance on a unit surface area of strip (top and bottom),

$$-\dot{E}_{out} = dE_{st}/dt \qquad -2\varepsilon\sigma T^4 = d(\rho\delta cT)/dt$$

$$\left.\frac{dT}{dt}\right)_i = -\frac{2\varepsilon_i\sigma T_i^4}{\rho\delta c} = \frac{-2(0.373)5.67\times 10^{-8}\ W/m^2\cdot K^4(1200\ K)^4}{7900\ kg/m^3(0.003\ m)(640\ J/kg\cdot K)} = -5.78\ K/s.$$ <

(c) From the energy balance,

$$\frac{dT}{dt} = -\frac{2\varepsilon_i(1200/T)\sigma T^4}{\rho\delta c}, \quad \int_{T_i}^{T_f}\frac{dT}{T^3} = -\frac{2400\varepsilon_i\sigma}{\rho\delta c}\int_0^t dt, \quad t = \frac{\rho\delta c}{4800\varepsilon_i\sigma}\left(\frac{1}{T_f^2} - \frac{1}{T_i^2}\right)$$

$$t = \frac{7900\ kg/m^3(0.003m)640\ J/kg\cdot K}{4800\ K \times 0.373 \times 5.67 \times 10^{-8}\ W/m^2\cdot K^4}\left(\frac{1}{600^2} - \frac{1}{1200^2}\right)K^{-2} = 311s.$$ <

COMMENTS: Initially, from Eq. 1.9, $h_r = \varepsilon_i\sigma T_i^3$ = 36.6 W/m^2·K. Assuming a plate width of W = 1m, the Rayleigh number may be evaluated from $Ra_L = g\beta(T_i - T_\infty)(W/2)^3/\nu\alpha$. Assuming T_∞ = 300 K and evaluating properties at T_f = 750 K, $Ra_L = 2.4 \times 10^8$. From Eq. 9.31, Nu_L = 93, giving $\bar{h}$ = 10 W/m^2·K. Hence heat loss by radiation exceeds that associated with free convection. To check the validity of neglecting transverse temperature gradients, compute Bi = h(δ/2)/k. With h = 36.6 W/m^2·K and k = 28 W/m·K, Bi = 0.002 << 1. Hence the assumption is valid.

PROBLEM 12.42

KNOWN: Large body of nonluminous gas at 1200 K has emission bands between 2.5 – 3.5 μm and between 5 – 8 μm with effective emissivities of 0.8 and 0.6, respectively.

FIND: Emissive power of the gas.

SCHEMATIC:

ASSUMPTIONS: (1) Gas radiates only in specified bands, (2) Emitted radiation is diffuse.

ANALYSIS: The emissive power of the gas is

$$E_g = \varepsilon E_b\left(T_g\right) = \int_0^\infty \varepsilon_\lambda E_{\lambda,b}\left(T_g\right) d\lambda$$

$$E_g = \int_{2.5}^{3.5} \varepsilon_{\lambda,1} E_{\lambda,b}\left(T_g\right) d\lambda + \int_5^8 \varepsilon_{\lambda,2} E_{\lambda,b}\left(T_g\right) d\lambda$$

$$E_g = \left[\varepsilon_1 F_{(2.5-3.5\,\mu m)} + \varepsilon_2 F_{(5-8\,\mu m)}\right] \sigma T_g^4.$$

Using the blackbody function $F_{(0-\lambda T)}$ from Table 12.1 with T_g = 1200 K,

λT(μm·K)	2.5 × 1200 3000	3.5 × 1200 4200	5 × 1200 6000	8 × 1200 9600
$F_{(0-\lambda T)}$	0.273	0.516	0.738	0.905

so that

$$F_{(2.5-3.5\,\mu m)} = F_{(0-3.5\,\mu m)} - F_{(0-2.5\,\mu m)} = 0.516 - 0.273 = 0.243$$

$$F_{(5-8\,\mu m)} = F_{(0-8\,\mu m)} - F_{(0-5\,\mu m)} = 0.905 - 0.738 = 0.167.$$

Hence the emissive power is

$$E_g = [0.8 \times 0.243 + 0.6 \times 0.167]\, 5.67 \times 10^{-8}\, W/m^2 \cdot K^4 (1200\ K)^4$$

$$E_g = 0.295 \times 117{,}573\ W/m^2 = 34{,}684\ W/m^2.$$ <

COMMENTS: Note that the effective emissivity for the gas is 0.295. This seems surprising since emission occurs only at the discrete bands. Since λ_{max} = 2.4 μm, all of the spectral emissive power is at wavelengths beyond the peak of blackbody radiation at 1200 K.

PROBLEM 12.43

KNOWN: An opaque surface with prescribed spectral, hemispherical reflectivity distribution is subjected to a prescribed spectral irradiation.

FIND: (a) The spectral, hemispherical absorptivity, (b) Total irradiation, (c) The absorbed radiant flux, and (d) Total, hemispherical absorptivity.

SCHEMATIC:

ASSUMPTIONS: (1) Surface is opaque.

ANALYSIS: (a) The spectral, hemispherical absorptivity, α_λ, for an opaque surface is given by Eq. 12.58,

$$\alpha_\lambda = 1 - \rho_\lambda$$ <

which is shown as a dashed line on the ρ_λ distribution axes.

(b) The total irradiation, G, follows from Eq. 12.16 which can be integrated by parts,

$$G = \int_0^\infty G_\lambda d\lambda = \int_0^{5\,\mu m} G_\lambda d\lambda + \int_{5\,\mu m}^{10\,\mu m} G_\lambda d\lambda + \int_{10\,\mu m}^{20\,\mu m} G_\lambda d\lambda$$

$$G = \frac{1}{2} \times 600 \frac{W}{m^2 \cdot \mu m}(5-0)\mu m + 600 \frac{W}{m^2 \cdot \mu m}(10-5)\mu m + \frac{1}{2} \times 600 \frac{W}{m^2 \cdot \mu m} \times (20-10)\mu m$$

$$G = 7500\ W/m^2.$$ <

(c) The absorbed irradiation follows from Eqs. 12.45 and 12.46 with the form

$$G_{abs} = \int_0^\infty \alpha_\lambda G_\lambda d\lambda = \alpha_1 \int_0^{5\,\mu m} G_\lambda d\lambda + G_{\lambda,2} \int_{5\,\mu m}^{10\,\mu m} \alpha_\lambda d\lambda + \alpha_3 \int_{10\,\mu m}^{20\,\mu m} G_\lambda d\lambda.$$

Noting that $\alpha_1 = 1.0$ for $\lambda = 0 \to 5\ \mu m$, $G_{\lambda,2} = 600\ W/m^2 \cdot \mu m$ for $\lambda = 5 \to 10\ \mu m$ and $\alpha_3 = 0$ for $\lambda > 10\ \mu m$, find that

$$G_{abs} = 1.0\left(0.5 \times 600\ W/m^2 \cdot \mu m\right)(5-0)\,\mu m + 600\ W/m^2 \cdot \mu m\,(0.5 \times 0.5)(10-5)\mu m + 0$$

$$G_{abs} = 2250\ W/m^2.$$ <

(d) The total, hemispherical absorptivity is defined as the fraction of the total irradiation that is absorbed. From Eq. 12.45,

$$\alpha = \frac{G_{abs}}{G} = \frac{2250\ W/m^2}{7500\ W/m^2} = 0.30.$$ <

COMMENTS: Recognize that the total, hemispherical absorptivity, $\alpha = 0.3$, is for the given spectral irradiation. For a different G_λ, one would then expect a different value for α.

PROBLEM 12.44

KNOWN: Temperature and spectral emissivity of small object suspended in large furnace of prescribed temperature and total emissivity.

FIND: (a) Total surface emissivity and absorptivity, (b) Reflected radiative flux and net radiative flux to surface, (c) Spectral emissive power at $\lambda = 2\ \mu m$, (d) Wavelength $\lambda_{1/2}$ for which one-half of total emissive power is in spectral region $\lambda \geq \lambda_{1/2}$.

SCHEMATIC:

ASSUMPTIONS: (1) Surface is opaque and diffuse, (2) Walls of furnace are much larger than object.

ANALYSIS: (a) The emissivity of the object may be obtained from Eq. 12.38, which is expressed as

$$\varepsilon(T_s) = \frac{\int_o^\infty \varepsilon_\lambda(\lambda) E_{\lambda,b}(\lambda, T_s)\, d\lambda}{E_b(T)} = \varepsilon_1\left[F_{(0\to 3\mu m)} - F_{(0\to 1\mu m)}\right] + \varepsilon_2\left[1 - F_{(0\to 3\mu m)}\right]$$

where, with $\lambda_1 T_s = 400\ \mu m\cdot K$ and $\lambda_2 T_s = 1200\ \mu m\cdot K$, $F_{(0\to 1\mu m)} = 0$ and $F_{(0\to 3\mu m)} = 0.002$. Hence,

$$\varepsilon(T_s) = 0.7(0.002) + 0.5(0.998) = 0.500$$ <

The absorptivity of the surface is determined by Eq. 12.46,

$$\alpha = \frac{\int_o^\infty \alpha_\lambda(\lambda) G_\lambda(\lambda)\, d\lambda}{\int_o^\infty G_\lambda(\lambda)\, d\lambda} = \frac{\int_o^\infty \alpha_\lambda(\lambda) E_{\lambda,b}(\lambda, T_f)\, d\lambda}{E_b(T_f)}$$

Hence, with $\lambda_1 T_f = 2000\ \mu m\cdot K$ and $\lambda_2 T_f = 6000\ \mu m\cdot K$, $F_{(0\to 1\mu m)} = 0.067$ and $F_{(0\to 3\mu m)} = 0.738$. It follows that

$$\alpha = \alpha_1\left[F_{(0\to 3\mu m)} - F_{(0\to 1\mu m)}\right] + \alpha_2\left[1 - F_{(0\to 3\mu m)}\right] = 0.7\times 0.671 + 0.5\times 0.262 = 0.601$$ <

(b) The reflected radiative flux is

$$G_{ref} = \rho G = (1-\alpha) E_b(T_f) = 0.399 \times 5.67\times 10^{-8}\ W/m^2\cdot K^4\, (2000\,K)^4 = 3.620\times 10^5\ W/m^2$$ <

The net radiative flux to the surface is

$$q''_{rad} = G - \rho G - \varepsilon E_b(T_s) = \alpha E_b(T_f) - \varepsilon E_b(T_s)$$

$$q''_{rad} = 5.67\times 10^{-8}\ W/m^2\cdot K^4 \left[0.601(2000\,K)^4 - 0.500(400\,K)^4\right] = 5.438\times 10^5\ W/m^2$$ <

(c) At $\lambda = 2\ \mu m$, $\lambda T_s = 800$ K and, from Table 12.1, $I_{\lambda,b}(\lambda,T)/\sigma T^5 = 0.991 \times 10^{-7}\ (\mu m\cdot K\cdot sr)^{-1}$. Hence,

Continued...

$$I_{\lambda,b} = 0.991\times10^{-7}\times5.67\times10^{-8}\,\frac{\mathrm{W/m^2\cdot K^4}}{\mu\mathrm{m\cdot K\cdot sr}}\times(400\,\mathrm{K})^5 = 0.0575\,\frac{\mathrm{W}}{\mathrm{m^2\cdot\mu m\cdot sr}}$$

Hence, with $E_\lambda = \varepsilon_\lambda E_{\lambda,b} = \varepsilon_\lambda \pi I_{\lambda,b}$,

$$E_\lambda = 0.7(\pi\,\mathrm{sr})0.0575\,\mathrm{W/m^2\cdot\mu m\cdot sr} = 0.126\,\mathrm{W/m^2\cdot\mu m} \qquad <$$

(d) From Table 12.1, $F_{(0\rightarrow\lambda)} = 0.5$ corresponds to $\lambda T_s \approx 4100\ \mu\mathrm{m\cdot K}$, in which case,

$$\lambda_{1/2} \approx 4100\,\mu\mathrm{m\cdot K}/400\,\mathrm{K} \approx 10.3\,\mu\mathrm{m} \qquad <$$

COMMENTS: Because of the significant difference between T_f and T_s, $\alpha \neq \varepsilon$. With increasing $T_s \rightarrow T_f$, ε would increase and approach a value of 0.601.

PROBLEM 12.45

KNOWN: Small flat plate maintained at 400 K coated with white paint having spectral absorptivity distribution (Figure 12.23) approximated as a stairstep function. Enclosure surface maintained at 3000 K with prescribed spectral emissivity distribution.

FIND: (a) Total emissivity of the enclosure surface, ε_{es}, and (b) Total emissivity, ε, and absorptivity, α, of the surface.

SCHEMATIC:

Coated plate: α_λ vs. λ(μm); $\alpha_1 = 0.75$ for $0 < \lambda < 0.4$, $\alpha_2 = 0.15$ for $0.4 < \lambda < 3.0$, $\alpha_3 = 0.96$ for $\lambda > 3.0$.

Enclosed surface: ε_λ vs. λ(μm); $\varepsilon_1 = 0.2$ for $0 < \lambda < 2.0$, $\varepsilon_2 = 0.9$ for $\lambda > 2.0$.

ASSUMPTIONS: (1) Coated plate with white paint is diffuse and opaque, so that $\alpha_\lambda = \varepsilon_\lambda$, (2) Plate is small compared to the enclosure surface, and (3) Enclosure surface is isothermal, diffuse and opaque.

ANALYSIS: (a) The total emissivity of the enclosure surface at T_{es} = 3000 K follows from Eq. 12.38 which can be expressed in terms of the bond emission factor, $F_{(0-\lambda T)}$, Eq. 12.30,

$$\varepsilon_{e,s} = \varepsilon_1 F_{(0-\lambda_1 T_{es})} + \varepsilon_2\left[1 - F_{(0-\lambda_1 T_{es})}\right] = 0.2\times 0.738 + 0.9[1-0.738] = 0.383 \qquad <$$

where, from Table 12.1, with $\lambda_1 T_{es}$ = 2 μm × 3000 K = 6000 μm·K, $F_{(0-\lambda T)}$ = 0.738.

(b) The total emissivity of the coated plate at T = 400 K can be expressed as

$$\varepsilon = \alpha_1 F_{(0-\lambda_1 T_s)} + \alpha_2\left[F_{(0-\lambda_2 T_s)} - F_{(0-\lambda_1 T_s)}\right] + \alpha_3\left[1 - F_{(0-\lambda_2 T_s)}\right]$$

$$\varepsilon = 0.75\times 0 + 0.15[0.002134 - 0.000] + 0.96[1 - 0.002134] = 0.958 \qquad <$$

where, from Table 12.1, the band emission factors are: for $\lambda_1 T_s$ = 0.4 × 400 = 160 μm·K, find $F_{(0-\lambda_1 T_s)}$ = 0.000; for $\lambda_2 T_{es}$ = 3.0 × 400 = 1200 μm·K, find $F_{(0-\lambda_2 T_s)}$ = 0.002134. The total absorptivity for the irradiation due to the enclosure surface at T_{es} = 3000 K is

$$\alpha = \alpha_1 F_{(0-\lambda_1 T_{es})} + \alpha_2\left[F_{(0-\lambda_2 T_{es})} - F_{(0-\lambda_2 T_{es})}\right] + \alpha_3\left[1 - F_{(0-\lambda_2 T_{es})}\right]$$

$$\alpha = 0.75\times 0.002134 + 0.15[0.8900 - 0.002134] + 0.96[1 - 0.8900] = 0.240 \qquad <$$

where, from Table 12.1, the band emission factors are: for $\lambda_1 T_{es}$ = 0.4 × 3000 = 1200 μm·K, find $F_{(0-\lambda_1 T_{es})}$ = 0.002134; for $\lambda_2 T_{es}$ = 3.0 × 3000 = 9000 μm·K, find $F_{(0-\lambda_2 T_{es})}$ = 0.8900.

COMMENTS: (1) In evaluating the total emissivity and absorptivity, remember that $\varepsilon = \varepsilon(\varepsilon_\lambda, T_s)$ and $\alpha = \alpha(\alpha_\lambda, G_\lambda)$ where T_s is the temperature of the surface and G_λ is the spectral irradiation, which if the surroundings are large and isothermal, $G_\lambda = E_{b,\lambda}(T_{sur})$. Hence, $\alpha = \alpha(\alpha_\lambda, T_{sur})$. For the opaque, diffuse surface, $\alpha_\lambda = \varepsilon_\lambda$.

(2) Note that the coated plate (white paint) has an absorptivity for the 3000 K-enclosure surface irradiation of α = 0.240. You would expect it to be a low value since the coating appears visually "white".

(3) The emissivity of the coated plate is quite high, ε = 0.958. Would you have expected this of a "white paint"? Most paints are oxide systems (high absorptivity at long wavelengths) with pigmentation (controls the "color" and hence absorptivity in the visible and near infrared regions).

PROBLEM 12.46

KNOWN: Area, temperature, irradiation and spectral absorptivity of a surface.

FIND: Absorbed irradiation, emissive power, radiosity and net radiation transfer from the surface.

SCHEMATIC:

ASSUMPTIONS: (1) Opaque, diffuse surface behavior, (2) Spectral distribution of solar radiation corresponds to emission from a blackbody at 5800 K.

ANALYSIS: The absorptivity to solar irradiation is

$$\alpha_S = \frac{\int_0^\infty \alpha_\lambda G_\lambda d\lambda}{G} = \frac{\int_0^\infty \alpha_\lambda E_{\lambda b}(5800\ \text{K}) d\lambda}{E_b} = \alpha_1 F_{(0.5\to 1\,\mu m)} + \alpha_2 F_{(2\to\infty)}.$$

From Table 12.1, $\lambda T = 2900\ \mu m \cdot K$: $F_{(0\to 0.5\,\mu m)} = 0.250$

$\lambda T = 5800\ \mu m \cdot K$: $F_{(0\to 1\,\mu m)} = 0.720$

$\lambda T = 11{,}600\ \mu m \cdot K$: $F_{(0\to 2\,\mu m)} = 0.941$

$$\alpha_S = 0.8(0.720 - 0.250) + 0.9(1 - 0.941) = 0.429.$$

Hence, $$G_{abs} = \alpha_S G_S = 0.429\left(1200\ \text{W/m}^2\right) = 515\ \text{W/m}^2.$$ <

The emissivity is

$$\varepsilon = \int_0^\infty \varepsilon_\lambda E_{\lambda b}(400\ \text{K}) d\lambda / E_b = \varepsilon_1 F_{(0.5\to 1\,\mu m)} + \varepsilon_2 F_{(2\to\infty)}.$$

From Table 12.1, $\lambda T = 200\ \mu m \cdot K$: $F_{(0\to 0.5\,\mu m)} = 0$

$\lambda T = 400\ \mu m \cdot K$: $F_{(0\to 1\,\mu m)} = 0$

$\lambda T = 800\ \mu m \cdot K$ $F_{(0\to 2\,\mu m)} = 0.$

Hence, $\varepsilon = \varepsilon_2 = 0.9$,

$$E = \varepsilon\sigma T_s^4 = 0.9\times 5.67\times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4\ (400\ \text{K})^4 = 1306\ \text{W/m}^2.$$ <

The radiosity is

$$J = E + \rho_S G_S = E + (1-\alpha_S) G_S = [1306 + 0.571\times 1200]\ \text{W/m}^2 = 1991\ \text{W/m}^2.$$ <

The net radiation transfer from the surface is

$$q_{net} = (E - \alpha_S G_S) A_s = (1306 - 515)\ \text{W/m}^2 \times 4\ \text{m}^2 = 3164\ \text{W}.$$ <

COMMENTS: Unless 3164 W are supplied to the surface by other means (for example, by convection), the surface temperature will decrease with time.

PROBLEM 12.47

KNOWN: Temperature and spectral emissivity of a receiving surface. Direction and spectral distribution of incident flux. Distance and aperture of surface radiation detector.

FIND: Radiant power received by the detector.

SCHEMATIC:

$\theta_i = 30°$
$q'' = 1000\,W/m^2$
$A_d = 10^{-5} m^2$
$\Delta\omega_{d-s}$
$L = 1m$
$T_s = 700K$
$A_s = 10^{-4} m^2$
ε_λ
0.9
0.5
ε_1
ε_2
3
10
$\lambda(\mu m)$

ASSUMPTIONS: (1) Target surface is diffuse, (2) $A_d/L^2 << 1$.

ANALYSIS: The radiant power received by the detector depends on emission and reflection from the target.

$$q_d = I_{e+r} A_s \cos q_{d-s} \Delta w_{d-s}$$

$$q_d = \frac{e s T_s^4 + rG}{p} A_s \frac{A_d}{L^2}$$

$$e = \frac{\int_0^\infty e_l E_{lb}(700\ K)\,dl}{E_b(700\ K)} = e_1 F_{(3\to 10\,mm)} + e_2 F_{(10\to\infty)}.$$

From Table 12.1, $\lambda T = 2100\ \mu m{\cdot}K$: $F_{(0\to 3\,\mu m)} = 0.0838$

$\lambda T = 7000\ \mu m{\cdot}K$: $F_{(0\to 10\,\mu m)} = 0.8081$.

The emissivity can be expected as

$$e = 0.5(0.8081 - 0.0838) + 0.9(1 - 0.8081) = 0.535.$$

Also,

$$r = \frac{\int_0^\infty r_l G_l\,dl}{G} = \frac{\int_0^\infty (1 - e_l) q''_l\,dl}{q''} = 1\times F_{(0\to 3\,mm)} + 0.5\times F_{(3\to 6mm)}$$

$$r = 1\times 0.4 + 0.5\times 0.6 = 0.70.$$

Hence, with $G = q'' \cos q_i = 866\ W/m^2$,

$$q_d = \frac{0.535\times 5.67\times 10^{-8}\ W/m^2\cdot K^4 (700\ K)^4 + 0.7\times 866\ W/m^2}{p} 10^{-4} m^2 \frac{10^{-5} m^2}{(1m)^2}$$

$$q_d = 2.51\times 10^{-6}\ W.$$ <

COMMENTS: A total radiation detector cannot discriminate between emitted and reflected radiation from a surface.

PROBLEM 12.48

KNOWN: Small disk positioned at center of an isothermal, hemispherical enclosure with a small aperture.

FIND: Radiant power [μW] leaving the aperture.

SCHEMATIC:

R=100mm
θ_1=45°
q_{ap}
Aperture (A_2), D_2=2mm
Hemispherical enclosure (A_3), T_3=300K, ε_3=0.85
Disk (A_1), T_1=900K, D_1=5mm, ε_1=0.7

ASSUMPTIONS: (1) Disk is diffuse-gray, (2) Enclosure is isothermal and has area much larger than disk, (3) Aperture area is very small compared to enclosure area, (4) Areas of disk and aperture are small compared to radius squared of the enclosure.

ANALYSIS: The radiant power leaving the aperture is due to radiation leaving the disk and to irradiation on the aperture from the enclosure. That is,

$$q_{ap} = q_{1\to 2} + G_2 \cdot A_2. \qquad (1)$$

The radiation leaving the disk can be written in terms of the radiosity of the disk. For the diffuse disk,

$$q_{1\to 2} = \frac{1}{\pi} J_1 \cdot A_1 \cos\theta_1 \cdot \omega_{2-1}$$

(2)

and with ε = α for the gray behavior, the radiosity is

$$J_1 = \varepsilon_1 E_b(T_1) + \rho G_1 = \varepsilon_1 \sigma T_1^4 + (1-\varepsilon_1)\sigma T_3^4 \qquad (3)$$

where the irradiation G_1 is the emissive power of the black enclosure, E_b (T_3); $G_1 = G_2 = E_b$ (T_3). The solid angle ω_{2-1} follows from Eq. 12.2,

$$\omega_{2-1} = A_2 / R^2. \qquad (4)$$

Combining Eqs. (2), (3) and (4) into Eq. (1) with $G_2 = \sigma T_3^4$, the radiant power is

$$q_{ap} = \frac{1}{\pi}\sigma\left[\varepsilon_1 T_1^4 + (1-\varepsilon_1)T_3^4\right] A_1 \cos\theta_1 \cdot \frac{A_2}{R^2} + A_2 \sigma T_3^4$$

$$q_{ap} = \frac{1}{\pi} 5.67\times10^{-8} \frac{W}{m^2\cdot K^4}\left[0.7(900K)^4 + (1-0.7)(300K)^4\right]\frac{\pi}{4}(0.005m)^2 \cos 45°\times$$

$$\frac{\pi/4(0.002m)^2}{(0.100m)^2} + \frac{\pi}{4}(0.002m)^2\, 5.67\times10^{-8} W/m^2\cdot K^4 (300K)^4$$

$$q_{ap} = (36.2 + 0.19 + 1443)\mu W = 1479\,\mu W. \qquad <$$

COMMENTS: Note the relative magnitudes of the three radiation components. Also, recognize that the emissivity of the enclosure ε_3 doesn't enter into the analysis. Why?

PROBLEM 12.49

KNOWN: Spectral, hemispherical absorptivity of an opaque surface.

FIND: (a) Solar absorptivity, (b) Total, hemispherical emissivity for T_s = 340K.

SCHEMATIC:

ASSUMPTIONS: (1) Surface is opaque, (2) $\varepsilon_\lambda = \alpha_\lambda$, (3) Solar spectrum has $G_\lambda = G_{\lambda,S}$ proportional to $E_{\lambda,b}$ (λ, 5800K).

ANALYSIS: (a) The solar absorptivity follows from Eq. 12.47.

$$a_S = \int_0^\infty a_l(l) E_{l,b}(l, 5800K)\, dl \,/ \int_0^\infty E_{l,b}(l, 5800K)\, dl.$$

The integral can be written in three parts using $F_{(0 \to \lambda)}$ terms.

$$a_S = a_1 F_{(0\to0.3)} + a_2\left[F_{(0\to1.5)} - F_{(0\to0.3)}\right] + a_3\left[1 - F_{(0\to1.5)}\right].$$

From Table 12.1,

$\lambda T = 0.3 \times 5800 = 1740\ \mu m \cdot K$ $\quad F_{(0 \to 0.3\ \mu m)} = 0.0335$

$\lambda T = 1.5 \times 5800 = 8700\ \mu m \cdot K$ $\quad F_{(0 \to 1.5\ \mu m)} = 0.8805.$

Hence,

$$a_S = 0 \times 0.0355 + 0.9[0.8805 - 0.0335] + 0.1[1 - 0.8805] = 0.774. \qquad <$$

(b) The total, hemispherical emissivity for the surface at 340K will be

$$e = \int_0^\infty e_l(l) E_{l,b}(l, 340K)\, dl \,/ E_b(340K).$$

If $\varepsilon_\lambda = \alpha_\lambda$, then using the α_λ distribution above, the integral can be written in terms of $F_{(0 \to \lambda)}$ values. It is readily recognized that since

$F_{(0\to1.5 mm, 340K)} \approx 0.000$ $\quad$ at $\quad$ $lT = 1.5 \times 340 = 510\ mm \cdot K$

there is negligible spectral emissive power below 1.5 μm. It follows then that

$$e = e_l = a_l = 0.1 \qquad <$$

COMMENTS: The assumption $\varepsilon_\lambda = \alpha_\lambda$ can be satisfied if this surface were irradiated diffusely or if the surface itself were diffuse. Note that for this surface under the specified conditions of solar irradiation and surface temperature $\alpha_S \neq \varepsilon$. Such a surface is referred to as a spectrally selective surface.

PROBLEM 12.50

KNOWN: Spectral distribution of the absorptivity and irradiation of a surface at 1000 K.

FIND: (a) Total, hemispherical absorptivity, (b) Total, hemispherical emissivity, (c) Net radiant flux to the surface.

SCHEMATIC:

ASSUMPTIONS: (1) $\alpha_\lambda = \varepsilon_\lambda$.

ANALYSIS: (a) From Eq. 12.46,

$$a = \frac{\int_0^\infty a_l\, G_l\, d_l}{\int_0^\infty G_l\, d_l} = \frac{\int_0^{2mm} a_l\, G_l\, dl + \int_2^{4mm} a_l\, G_l\, dl + \int_4^{6mm} a_l\, G_l\, dl}{\int_0^{2mm} G_l\, dl + \int_2^{4mm} G_l\, dl + \int_4^{6mm} G_l\, dl}$$

$$a = \frac{0 \times 1/2(2-0)5000 + 0.6(4-2)5000 + 0.6 \times 1/2(6-4)5000}{1/2(2-0)5000 + (4-2)(5000) + 1/2(6-4)5000}$$

$$a = \frac{9000}{20{,}000} = 0.45.$$ <

(b) From Eq. 12.38,

$$e = \frac{\int_0^\infty e_l\, E_{l,b}\, dl}{E_b} = \frac{0\int_0^{2mm} E_{l,b}\, dl}{E_b} + \frac{0.6\int_2^\infty E_{l,b}\, dl}{E_b}$$

$$e = 0.6 F_{(2mm \to \infty)} = 0.6\left[1 - F_{(0 \to 2\ mm)}\right].$$

From Table 12.1, with $\lambda T = 2\ \mu m \times 1000K = 2000\ \mu m \cdot K$, find $F_{(0 \to 2\ \mu m)} = 0.0667$. Hence,

$$e = 0.6[1 - 0.0667] = 0.56.$$ <

(c) The net radiant heat flux *to* the surface is

$$q''_{rad,net} = aG - E = aG - e\, s\, T^4$$

$$q''_{rad,net} = 0.45\left(20{,}000 W/m^2\right) - 0.56 \times 5.67 \times 10^{-8} W/m^2 \cdot K^4 \times (1000K)^4$$

$$q''_{rad,net} = (9000 - 31{,}751)\, W/m^2 = -22{,}751 W/m^2.$$ <

PROBLEM 12.51

KNOWN: Spectral distribution of surface absorptivity and irradiation. Surface temperature.

FIND: (a) Total absorptivity, (b) Emissive power, (c) Nature of surface temperature change.

SCHEMATIC:

ASSUMPTIONS: (1) Opaque, diffuse surface behavior, (2) Convection effects are negligible.

ANALYSIS: (a) From Eqs. 12.45 and 12.46, the absorptivity is defined as

$$\alpha \equiv G_{abs} / G = \int_0^\infty \alpha_\lambda G_\lambda d\lambda \Big/ \int_0^\infty G_\lambda d\lambda.$$

The absorbed irradiation is,

$$G_{abs} = 0.4\left(5000\,\text{W/m}^2 \cdot \mu\text{m} \times 2\,\mu\text{m}\right)/2 + 0.8 \times 5000\,\text{W/m}^2 \cdot \mu\text{m}\,(5-2)\,\mu\text{m} + 0 = 14{,}000\ \text{W/m}^2.$$

The irradiation is,

$$G = \left(2\,\mu\text{m} \times 5000\ \text{W/m}^2 \cdot \mu\text{m}\right)/2 + (10-2)\,\mu\text{m} \times 5000\ \text{W/m}^2 \cdot \mu\text{m} = 45{,}000\ \text{W/m}^2.$$

Hence, $\alpha = 14{,}000\ \text{W/m}^2 / 45{,}000\ \text{W/m}^2 = 0.311.$ <

(b) From Eq. 12.38, the emissivity is

$$\varepsilon = \int_0^\infty \varepsilon_\lambda E_{\lambda,b}\, d\lambda \,/ E_b = 0.4 \int_0^2 E_{\lambda,b}\, d\lambda \,/ E_b + 0.8 \int_2^5 E_{\lambda,b}\, d\lambda \,/ E_b$$

From Table 12.1, $\lambda T = 2\ \mu\text{m} \times 1250\text{K} = 2500\text{K}$, $F_{(0-2)} = 0.162$

$\lambda T = 5\ \mu\text{m} \times 1250\text{K} = 6250\text{K}$, $F_{(0-5)} = 0.757.$

Hence, $\varepsilon = 0.4 \times 0.162 + 0.8(0.757 - 0.162) = 0.54.$

$$E = \varepsilon E_b = \varepsilon \sigma T^4 = 0.54 \times 5.67 \times 10^{-8}\,\text{W/m}^2 \cdot \text{K}^4 (1250\text{K})^4 = 74{,}751\,\text{W/m}^2. \quad <$$

(c) From an energy balance on the surface, the net heat flux *to* the surface is

$$q''_{net} = \alpha G - E = (14{,}000 - 74{,}751)\ \text{W/m}^2 = -60{,}751\,\text{W/m}^2.$$

Hence the temperature of the surface is *decreasing*. <

COMMENTS: Note that $\alpha \neq \varepsilon$. Hence the surface is not gray for the prescribed conditions.

PROBLEM 12.52

KNOWN: Power dissipation temperature and distribution of spectral emissivity for a tungsten filament. Distribution of spectral absorptivity for glass bulb. Temperature of ambient air and surroundings. Bulb diameter.

FIND: Bulb temperature.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state, (2) Uniform glass temperature, T_s, and uniform irradiation of inner surface, (3) Surface of glass is diffuse, (4) Negligible absorption of radiation by filament due to emission from inner surface of bulb, (5) Net radiation transfer from outer surface of bulb is due to exchange with large surroundings, (6) Bulb temperature is sufficiently low to provide negligible emission at $\lambda < 2\mu$m, (7) Ambient air is quiescent.

PROPERTIES: *Table A-4*, air (assume T_f = 323 K): $\nu = 1.82 \times 10^{-5}$ m^2/s, $\alpha = 2.59 \times 10^{-5}$ m^2/s,k = 0.028 W/m·K, $\beta = 0.0031$ K^{-1}, Pr = 0.704.

ANALYSIS: From an energy balance on the glass bulb,

$$q''_{rad,i} = q''_{rad,o} + q''_{conv} = \varepsilon_b \sigma \left(T_s^4 - T_{sur}^4 \right) + \bar{h} \left(T_s - T_\infty \right) \quad (1)$$

where $\varepsilon_b = \varepsilon_{\lambda > 2\mu m} = \alpha_{\lambda > 2\mu m} = 1$ and $\bar{h}$ is obtained from Eq. (9.35)

$$\overline{Nu}_D = 2 + \frac{0.589\ Ra_D^{1/4}}{\left[1 + (0.469/Pr)^{9/16}\right]^{4/9}} = \frac{\bar{h}D}{k} \quad (2)$$

with $Ra_D = g\beta \left(T_s - T_\infty \right) D^3 / \nu\alpha$. Radiation absorption at the inner surface of the bulb may be expressed as

$$q''_{rad,i} = \alpha G = \alpha \left(P_{elec} / \pi D^2 \right) \quad (3)$$

where, from Eq. (12.46),

$$\alpha = \alpha_1 \int_0^{0.4} (G_\lambda / G) d\lambda + \alpha_2 \int_{0.4}^{2.0} (G_\lambda / G) d\lambda + \alpha_3 \int_2^\infty (G_\lambda / G) d\lambda$$

Continued …..

PROBLEM 12.52 (Cont.)

The irradiation is due to emission from the filament, in which case $(G_\lambda/G) \sim (E_\lambda/E)_f = (\varepsilon_{f,\lambda}E_{\lambda,b}/\varepsilon_f E_b)$. Hence,

$$\alpha = (\alpha_1/\varepsilon_f)\int_0^{0.4}\varepsilon_{f,\lambda}(E_{\lambda,b}/E_b)d\lambda + (\alpha_2/\varepsilon_f)\int_{0.4}^{2.0}\varepsilon_{f,\lambda}(E_{\lambda,b}/E_b)d\lambda + (\alpha_3/\varepsilon_f)\int_2^{\infty}\varepsilon_{f,\lambda}(E_{\lambda,b}/E_b)d\lambda \quad (4)$$

where, from the spectral distribution of Problem 12.25, $\varepsilon_{f,\lambda} \equiv \varepsilon_1 = 0.45$ for $\lambda < 2\mu m$ and $\varepsilon_{f,\lambda} \equiv \varepsilon_2 = 0.10$ for $\lambda > 2\mu m$. From Eq. (12.38)

$$\varepsilon_f = \int_0^{\infty}\varepsilon_{f,\lambda}(E_{\lambda,b}/E_b)d\lambda = \varepsilon_1 F_{(0\to 2\mu m)} + \varepsilon_2\left(1 - F_{(0\to 2\mu m)}\right)$$

With $\lambda T_f = 2\mu m \times 3000\ K = 6000\ \mu m \cdot K$, $F_{(0\to 2\mu m)} = 0.738$ from Table 12.1. Hence,

$$\varepsilon_f = 0.45 \times 0.738 + 0.1(1 - 0.738) = 0.358$$

Equation (4) may now be expressed as

$$\alpha = (\alpha_1/\varepsilon_f)\varepsilon_1 F_{(0\to 0.4\mu m)} + (\alpha_2/\varepsilon_f)\varepsilon_1\left(F_{(0\to 2\mu m)} - F_{(0\to 0.4\mu m)}\right) + (\alpha_3/\varepsilon_f)\varepsilon_2\left(1 - F_{(0\to 2\mu m)}\right)$$

where, with $\lambda T = 0.4\mu m \times 3000\ K = 1200\ \mu m \cdot K$, $F_{(0\to 0.4\mu m)} = 0.0021$. Hence,

$$\alpha = (1/0.358)0.45 \times 0.0021 + (0.1/0.358)0.45 \times (0.738 - 0.0021) + (1/0.358)0.1(1 - 0.738) = 0.168$$

Substituting Eqs. (2) and (3) into Eq. (1), as well as values of $\varepsilon_b = 1$ and $\alpha = 0.168$, an iterative solution yields

$$T_s = 348.1\ K$$ <

COMMENTS: For the prescribed conditions, $q''_{rad,i} = 713\ W/m^2$, $q''_{rad,o} = 385.5\ W/m^2$ and $q''_{conv} = 327.5\ W/m^2$.

PROBLEM 12.53

KNOWN: Spectral emissivity of an opaque, diffuse surface.

FIND: (a) Total, hemispherical emissivity of the surface when maintained at 1000 K, (b) Total, hemispherical absorptivity when irradiated by large surroundings of emissivity 0.8 and temperature 1500 K, (c) Radiosity when maintained at 1000 K and irradiated as prescribed in part (b), (d) Net radiation flux into surface for conditions of part (c), and (e) Compute and plot each of the parameters of parts (a)-(c) as a function of the surface temperature T_s for the range $750 < T_s \leq 2000$ K.

SCHEMATIC:

ASSUMPTIONS: (1) Surface is opaque, diffuse, and (2) Surroundings are large compared to the surface.

ANALYSIS: (a) When the surface is maintained at 1000 K, the total, hemispherical emissivity is evaluated from Eq. 12.38 written as

$$\varepsilon = \int_0^\infty \varepsilon_\lambda E_{\lambda,b}(T)\,d\lambda \Big/ E_b(T) = \varepsilon_{\lambda,1}\int_0^{\lambda_1} E_{\lambda,b}(T)\,d\lambda \Big/ E_b(T) + \varepsilon_{\lambda,2}\int_{\lambda_1}^\infty E_{\lambda,b}(T)\,d\lambda \Big/ E_b(T)$$

$$\varepsilon = \varepsilon_{\lambda,1} F_{(0-\lambda_1 T)} + \varepsilon_{\lambda,2}\left(1 - F_{(0-\lambda_1 T)}\right)$$

where for $\lambda T = 6\mu m \times 1000\ K = 6000 \mu m\cdot K$, from Table 12.1, find $F_{0-\lambda T} = 0.738$. Hence,

$$\varepsilon = 0.8 \times 0.738 + 0.3(1 - 0.738) = 0.669.$$ <

(b) When the surface is irradiated by large surroundings at $T_{sur} = 1500$ K, $G = E_b(T_{sur})$.
From Eq. 12.46,

$$\alpha = \int_0^\infty \alpha_\lambda G_\lambda\,d\lambda \Big/ \int_0^\infty G_\lambda\,d\lambda = \int_0^\infty \varepsilon_\lambda E_{\lambda,b}(T_{sur})\,d\lambda \Big/ E_b(T_{sur})$$

$$\alpha = \varepsilon_{\lambda,1} F_{(0-\lambda_1 T_{sur})} + \varepsilon_{\lambda,2}\left(1 - F_{(0-\lambda_1 T_{sur})}\right)$$

where for $\lambda_1 T_{sur} = 6\ \mu m \times 1500\ K = 9000\ \mu m\cdot K$, from Table 12.1, find $F_{(0-\lambda T)} = 0.890$. Hence,

$$\alpha = 0.8 \times 0.890 + 0.3\,(1 - 0.890) = 0.745.$$ <

Note that $\alpha_\lambda = \varepsilon_\lambda$ for all conditions and the emissivity of the surroundings is irrelevant.

(c) The radiosity for the surface maintained at 1000 K and irradiated as in part (b) is

$$J = \varepsilon E_b(T) + \rho G = \varepsilon E_b(T) + (1 - \alpha) E_b(T_{sur})$$

$$J = 0.669 \times 5.67 \times 10^{-8}\ W/m^2\cdot K^4\,(1000\ K)^4 + (1 - 0.745)\,5.67 \times 10^{-8}\ W/m^2\cdot K^4\,(1500\ K)^4$$

$$J = (37{,}932 + 73{,}196)\ W/m^2 = 111{,}128\ W/m^2$$ <

Continued...

(d) The net radiation flux into the surface with $G = \sigma T_{sur}^4$ is

$$q''_{rad,in} = \alpha G - \varepsilon E_b(T) = G - J$$

$$q''_{rad,in} = 5.67 \times 10^{-8}\ \mathrm{W/m^2 \cdot K}\ (1500\ \mathrm{K})^4 - 111{,}128\ \mathrm{W/m^2}$$

$$q''_{rad,in} = 175{,}915\ \mathrm{W/m^2}.$$

<

(e) The foregoing equations were entered into the IHT workspace along with the *IHT Radiaton Tool, Band Emission Factor*, to evaluate $F_{(0-\lambda T)}$ values and the respective parameters for parts (a)-(d) were computed and are plotted below.

Note that the absorptivity, $\alpha = \alpha(\alpha_\lambda, T_{sur})$, remains constant as T_s changes since it is a function of α_λ (or ε_λ) and Tsur only. The emissivity $\varepsilon = \varepsilon(\varepsilon_\lambda, T_s)$ is a function of T_s and increases as T_s increases. Could you have surmised as much by looking at the spectral emissivity distribution? At what condition is $\varepsilon = \alpha$?

The radiosity, J_1 increases with increasing T_s since $E_b(T)$ increases markedly with temperature; the reflected irradiation, $(1 - \alpha)E_b(T_{sur})$ decreases only slightly as T_s increases compared to $E_b(T)$. Since G is independent of T_s, it follows that the variation of $q''_{rad,in}$ will be due to the radiosity change; note the sign difference.

COMMENTS: We didn't use the emissivity of the surroundings ($\varepsilon = 0.8$) to determine the irradiation onto the surface. Why?

PROBLEM 12.54

KNOWN: Furnace wall temperature and aperture diameter. Distance of detector from aperture and orientation of detector relative to aperture.

FIND: (a) Rate at which radiation from the furnace is intercepted by the detector, (b) Effect of aperture window of prescribed spectral transmissivity on the radiation interception rate.

SCHEMATIC:

$T_f = 1500\,K$; Aperture, $D = 0.02\,m$, $A_a = \pi D^2/4$; $\theta_1 = 30°$; $\theta_2 = 45°$; $r = 1m$; $A_s = 10^{-5}\,m^2$

τ_λ; 0.8; λ; $2\,\mu m$

ASSUMPTIONS: (1) Radiation emerging from aperture has characteristics of emission from a blackbody, (2) Cover material is diffuse, (3) Aperture and detector surface may be approximated as infinitesimally small.

ANALYSIS: (a) From Eq. 12.5, the heat rate leaving the furnace aperture and intercepted by the detector is

$$q = I_e A_a \cos\theta_1 \,\omega_{s-a}.$$

From Eqs. 12.14 and 12.28

$$I_e = \frac{E_b(T_f)}{\pi} = \frac{\sigma T_f^4}{\pi} = \frac{5.67\times10^{-8}(1500)^4}{\pi} = 9.14\times10^4\ W/m^2\cdot sr.$$

From Eq. 12.2,

$$\omega_{s-a} = \frac{A_n}{r^2} = \frac{A_s\cdot\cos\theta_2}{r^2} = \frac{10^{-5}\ m^2\times\cos45°}{(1m)^2} = 0.707\times10^{-5}\ sr.$$

Hence

$$q = 9.14\times10^4 W/m^2\cdot sr\left[\pi\,(0.02m)^2/4\right]\cos30°\times0.707\times10^{-5} sr = 1.76\times10^{-4} W. \quad <$$

(b) With the window, the heat rate is

$$q = \tau\left(I_e A_a \cos\theta_1 \,\omega_{s-a}\right)$$

where τ is the transmissivity of the window to radiation emitted by the furnace wall. From Eq. 12.55,

$$\tau = \frac{\int_0^\infty \tau_\lambda G_\lambda\, d\lambda}{\int_0^\infty G_\lambda\, d\lambda} = \frac{\int_0^\infty \tau_\lambda E_{\lambda,b}(T_f)\, d\lambda}{\int_0^\infty E_{\lambda,b}\, d\lambda} = 0.8\int_0^2 \left(E_{\lambda,b}/E_b\right) d\lambda = 0.8F_{(0\to2\,\mu m)}.$$

With $\lambda T = 2\ \mu m \times 1500K = 3000\ \mu m\cdot K$, Table 12.1 gives $F_{(0\to2\,\mu m)} = 0.273$. Hence, with $\tau = 0.273 \times 0.8 = 0.218$, find

$$q = 0.218\times1.76\times10^{-4} W = 0.384\times10^{-4} W. \quad <$$

PROBLEM 12.55

KNOWN: Thermocouple is irradiated by a blackbody furnace at 1500 K with 25 mm^2 aperture. Optical fiber of prescribed spectral transmissivity in sight path.

FIND: (a) Distance L from the furnace detector should be positioned such that its irradiation is G = 50 W/m^2 and, (b) Compute and plot irradiation, G, vs separation distance L for the range $100 \le L \le 400$ mm for blackbody furnace temperatures of T_f = 1000, 1500 and 2000 K.

SCHEMATIC:

ASSUMPTIONS: (1) Furnace aperture emits diffusely, (2) $A_d << L^2$.

ANALYSIS: (a) The irradiation on the detector due to emission from the furnace which passes through the filter is defined as

$$G_d = q_{f\to d}/A_d = 50\,W/m^2 \quad (1)$$

where the power leaving the furnace and intercepted at the detector is

$$q_{f\to d} = \left[I_f \cdot A_f \cos\theta_f \cdot \omega_{d-f}\right]\tau_{filter} = \left[\frac{\sigma T^4}{\pi}\cdot A_f \cos\theta_f \cdot \frac{A_d}{L^2}\right]\tau_{filter}\,. \quad (2)$$

The transmittance of the filter is

$$\tau_{filter} = \tau_{\lambda 1}F_{0-\lambda T} + \tau_{\lambda 2}(1-F_{0-\lambda T}) = 0\times 0.4036 + 0.8(1-0.4036) = 0.477 \quad (3)$$

where $F_{0-\lambda T} = 0.4036$ with $\lambda T = 2.4\times 1500 = 3600$ μm·K from Table 12.1. Combining Eqs. (1) and (2) and substituting numerical values,

$$G_d = (1/\pi)5.67\times10^{-8}\,W/m^2\cdot K^4\times(1500K)^4\,(25\times10^{-6}m^2\times1)(A_d/L^2)\times0.477/A_d = 50\,W/m^2$$

find

$$L = 147\text{ mm.}$$ <

(b) Using the foregoing equations in the IHT workspace along with the *IHT Radiation Tool*, *Band Emission Factor*, G was computed and plotted as a function of L for selected blackbody temperatures.

Continued...

PROBLEM 12.55 (Cont.)

The irradiation decreases with increasing separation distance x as the inverse square of the distance. At any fixed separation distance, the irradiation increases as T_f increases. In what manner will G depend upon T_f? Is $G \sim T_f^4$? Why not?

COMMENTS: The IHT workspace used to generate the above plot is shown below.

```
// Irradiation, Eq (2):
G = qfd / Ad
qfd = Ief * Af * omegadf * tauf
omegadf = Ad / L^2
Ief = Ebf / pi
Ebf = sigma * Tf^4
sigma = 5.67e-8
// Transmittance, Eq (3):
tauf = tau1 * FL1Tf  +  tau2 * ( 1 - FL1Tf )
/* The blackbody band emission factor, Figure 12.14 and Table 12.1, is */
FL1Tf = F_lambda_T(lambda1,Tf)    // Eq 12.30
// where units are lambda (micrometers, mum) and T (K)
// Assigned Variables:
G = 50                      // Irradiation on detector, W/m^2
Tf = 1000                   // Furnace temperature, K
//Tf = 1500
//Tf = 2000
Af = 25 * 1e-6              // Furnace aperture, m^2
Ad = 0.003 * 0.007          // Detector area, m^2
tau1 = 0                    // Spectral transmittance, <= lambda1
tau2 = 0.8                  // Spectral transmittance, >= lambda2
lambda1 = 2.4               // Wavelength, mum
//L = 0.194                 // Separation distance, m
L_mm = L * 1000             // Separation distance, mm
/* Data Browser Results - Part (a)
Ebf     FL1Tf   Ief       L       omegadf    qfd     tauf     Ad      Af      Df      G
        Tf      lambda1   sigma   tau1       tau2
2.87E5  0.4036  9.137E4   0.1476  0.0009634          0.00105  0.4771  2.1E-5  2.5E-5  0.025
        50      1500      2.4     5.67E-8    0       0.8   */
```

PROBLEM 12.56

KNOWN: Spectral transmissivity of a plain and tinted glass.

FIND: (a) Solar energy transmitted by each glass, (b) Visible radiant energy transmitted by each with solar irradiation.

SCHEMATIC:

ASSUMPTIONS: (1) Spectral distribution of solar irradiation is proportional to spectral emissive power of a blackbody at 5800K.

ANALYSIS: To compare the energy transmitted by the glasses, it is sufficient to calculate the transmissivity of each glass for the prescribed spectral range when the irradiation distribution is that of the solar spectrum. From Eq. 12.55,

$$\tau_S = \int_0^\infty \tau_\lambda \cdot G_{\lambda,S} d\lambda \Big/ \int_0^\infty G_{\lambda,S} d\lambda = \int_0^\infty \tau_\lambda \cdot E_{\lambda,b}(\lambda, 5800\text{K})\, d\lambda \,/\, E_b(5800\text{K}).$$

Recognizing that τ_λ will be constant for the range $\lambda_1 \to \lambda_2$, using Eq. 12.31, find

$$\tau_S = \tau_\lambda \cdot F_{(\lambda_1 \to \lambda_2)} = \tau_\lambda \left[F_{(0 \to \lambda_2)} - F_{(0 \to \lambda_1)} \right].$$

(a) For the two glasses, the solar transmissivity, using Table 12.1 for F, is then

Plain glass: $\lambda_2 = 2.5\ \mu\text{m}$ $\quad \lambda_2 T = 2.5\ \mu\text{m} \times 5800\text{K} = 14{,}500\ \mu\text{m}\cdot\text{K}$ $\quad F_{(0 \to \lambda 2)} = 0.966$

$\lambda_1 = 0.3\ \mu\text{m}$ $\quad \lambda_1 T = 0.3\ \mu\text{m} \times 5800\text{K} = 1{,}740\ \mu\text{m}\cdot\text{K}$ $\quad F_{(0 \to \lambda_1)} = 0.033$

$\tau_S = 0.9\ [0.966 - 0.033] = 0.839.$ <

Tinted glass: $\lambda_2 = 1.5\ \mu\text{m}$ $\quad \lambda_2 T = 1.5\ \mu\text{m} \times 5800\text{K} = 8{,}700\ \mu\text{m}\cdot\text{K}$ $\quad F_{(0 \to \lambda_2)} = 0.881$

$\lambda_1 = 0.5\ \mu\text{m}$ $\quad \lambda_1 T = 0.5\ \mu\text{m} \times 5800\text{K}=2{,}900\ \mu\text{m}\cdot\text{K}$ $\quad F_{(0 \to \lambda_1)} = 0.033$

$\tau_S = 0.9\ [0.886 - 0.250] = 0.568.$ <

(b) The limits of the visible spectrum are $\lambda_1 = 0.4$ and $\lambda_2 = 0.7\ \mu\text{m}$. For the tinted glass, $\lambda_1 = 0.5\ \mu\text{m}$ rather than 0.4 μm. From Table 12.1,

$\lambda_2 = 0.7\ \mu\text{m}$ $\quad \lambda_2 T = 0.7\ \mu\text{m} \times 5800\text{K} = 4{,}060\ \mu\text{m}\cdot\text{K}$ $\quad F_{(0 \to \lambda_2)} = 0.491$

$\lambda_1 = 0.5\ \mu\text{m}$ $\quad \lambda_1 T = 0.5\ \mu\text{m} \times 5800\text{K} = 2{,}900\ \mu\text{m}\cdot\text{K}$ $\quad F_{(0 \to \lambda_1)} = 0.250$

$\lambda_1 = 0.4\ \mu\text{m}$ $\quad \lambda_1 T = 0.4\ \mu\text{m} \times 5800\text{K}=2{,}320\ \mu\text{m}\cdot\text{K}$ $\quad F_{(0 \to \lambda_1)} = 0.125$

Plain glass: $\tau_{vis} = 0.9\ [0.491 - 0.125] = 0.329$ <

Tinted glass: $\tau_{vis} = 0.9\ [0.491 - 0.250] = 0.217$ <

COMMENTS: For solar energy, the transmissivities are 0.839 for the plain glass vs. 0.568 for the plain and tinted glasses. Within the visible region, τ_{vis} is 0.329 vs. 0.217. Tinting reduces solar flux by 32% and visible solar flux by 34%.

PROBLEM 12.57

KNOWN: Spectral transmissivity and reflectivity of light bulb coating. Dimensions, temperature and spectral emissivity of a tungsten filament.

FIND: (a) Advantages of the coating, (b) Filament electric power requirement for different coating spectral reflectivities.

SCHEMATIC:

τ_λ, ρ_λ vs. λ: $\tau_\lambda = 1$ for $0 < \lambda < \lambda_c$, 0 beyond; $\rho_\lambda = 0$ for $\lambda < \lambda_c$, 1 beyond.

ASSUMPTIONS: (1) All of the radiation reflected from the inner surface of bulb is absorbed by the filament.

ANALYSIS: (a) For λ_c = 0.7 μm, the coating has two important advantages: (i) It transmits all of the visible radiation emitted by the filament, thereby maximizing the lighting efficiency. (ii) It returns all of the infrared radiation to the filament, thereby reducing the electric power requirement and conserving energy. (b) The power requirement is simply the amount of radiation transmitted by the bulb, or

$$P_{elec} = A_f\, E_{(0\to\lambda_c)} = \pi\left(DL + D^2/2\right)\int_0^{\lambda_c} \varepsilon_\lambda\, E_{\lambda,b}\, d\lambda$$

From the spectral distribution of Problem 12.25, ε_λ = 0.45 for both values of λ_c. Hence,

$$P_{elec} = \left\{\pi\left[0.0008\times0.02 + (0.0008)^2/2\right]m^2\right\}0.45\, E_b \int_0^{\lambda_c}\left(E_{\lambda,b}/E_b\right)d\lambda$$

$$P_{elec} = 5.13\times10^{-5}\,m^2 \times 0.45\times5.67\times10^{-8}\ W/m^2\cdot K^4\,(3000\ K)^4\, F_{(0\to\lambda_c)}$$

$$P_{elec} = 106\ W\ F_{(0\to\lambda_c)}$$

For $\lambda_c = 0.7\mu m$, $\lambda_c T = 2100\,\mu m\cdot K$ and from Table 12.1, $F_{(0\to\lambda_c)} = 0.0838$. Hence,

$$\lambda_c = 0.7\,\mu m:\ P_{elec} = 106\ W\times0.0838 = 8.88\ W \qquad <$$

For $\lambda_c = 2\,\mu m$, $\lambda_c T = 6000\,\mu m\cdot K$ and $F_{(0\to\lambda_c)} = 0.738$. Hence,

$$\lambda_c = 2.0\,\mu m:\ P_{elec} = 106\ W\times0.738 = 78.2\ W \qquad <$$

COMMENTS: Clearly, significant energy conservation could be realized with a reflective coating and λ_c = 0.7 μm. Although a coating with the prescribed spectral characteristics is highly idealized and does not exist, there are coatings that may be used to reflect a portion of the infrared radiation from the filament and to thereby provide some energy savings.

PROBLEM 12.58

KNOWN: Spectral transmissivity of low iron glass (see Fig. 12.24).

FIND: Interpretation of the greenhouse effect.

SCHEMATIC:

ANALYSIS: The glass affects the net radiation transfer to the contents of the greenhouse. Since most of the solar radiation is in the spectral region $\lambda < 3$ μm, the glass will transmit a large fraction of this radiation. However, the contents of the greenhouse, being at a comparatively low temperature, emit most of their radiation in the medium to far infrared. This radiation is not transmitted by the glass. Hence the glass allows short wavelength solar radiation to enter the greenhouse, but does not permit long wavelength radiation to leave.

PROBLEM 12.59

KNOWN: Spectrally selective, diffuse surface exposed to solar irradiation.

FIND: (a) Spectral transmissivity, τ_λ, (b) Transmissivity, τ_S, reflectivity, ρ_S, and absorptivity, α_S, for solar irradiation, (c) Emissivity, ε, when surface is at T_s = 350K, (d) Net heat flux by radiation to the surface.

SCHEMATIC:

ASSUMPTIONS: (1) Surface is diffuse, (2) Spectral distribution of solar irradiation is proportional to $E_{\lambda,b}$ (λ, 5800K).

ANALYSIS: (a) Conservation of radiant energy requires, according to Eq. 12.56, that $\rho_\lambda + \alpha_\lambda + \tau_\lambda = 1$ or $\tau_\lambda = 1 - \rho_\lambda - \alpha_\lambda$. Hence, the spectral transmissivity appears as shown above (dashed line). Note that the surface is opaque for λ > 1.38 μm.

(b) The transmissivity to solar irradiation, G_S, follows from Eq. 12.55,

$$t_S = \int_0^\infty t_l\, G_{l,S}\, dl / G_S = \int_0^\infty t_l\, E_{l,b}(l, 5800K)\, dl / E_b(5800K)$$

$$t_S = t_{l,b} \int_0^{1.38} E_{l,b}(l, 5800K)\, dl / E_b(5800K) = t_{l,1} F_{(0 \to l_1)} = 0.7 \times 8.56 = 0.599 \qquad <$$

where $\lambda_1 T_S = 1.38 \times 5800 = 8000$ μm·K and from Table 12.1, $F_{(0 \to l_1)} = 0.856$. From Eqs. 12.52 and 12.57,

$$r_S = \int_0^\infty r_l\, G_{l,S}\, dl / G_S = r_{l,1} F_{(0 \to l_1)} = 0.1 \times 0.856 = 0.086 \qquad <$$

$$a_S = 1 - r_S - t_S = 1 - 0.086 - 0.599 = 0.315. \qquad <$$

(c) For the surface at T_s = 350K, the emissivity can be determined from Eq. 12.38. Since the surface is diffuse, according to Eq. 12.65, $\alpha_\lambda = \varepsilon_\lambda$, the expression has the form

$$e = \int_0^\infty e_l\, E_{l,b}(T_s)\, dl / E_b(T_s) = \int_0^\infty a_l\, E_{l,b}(350K)\, dl / E_b(350K)$$

$$e = a_{l,1} F_{(0-1.38mm)} + a_{l,2}\left[1 - F_{(0-1.38mm)}\right] = a_{l,2} = 1 \qquad <$$

where from Table 12.1 with $\lambda_1 T_S = 1.38 \times 350 = 483$ μm·K, $F_{(0-lT)} \approx 0$.

(d) The net heat flux by radiation *to* the surface is determined by a radiation balance

$$q''_{rad} = G_S - r_S G_S - t_S G_S - E$$

$$q''_{rad} = a_S G_S - E$$

$$q''_{rad} = 0.315 \times 750\ W/m^2 - 1.0 \times 5.67 \times 10^{-8}\ W/m^2 \cdot K^4 (350K)^4 = -615\ W/m^2. \qquad <$$

PROBLEM 12.60

KNOWN: Large furnace with diffuse, opaque walls (T_f, ε_f) and a small diffuse, spectrally selective object (T_o, τ_λ, ρ_λ).

FIND: For points on the furnace wall and the object, find ε, α, E, G and J.

SCHEMATIC:

T_f=3000K
Point A
B
T_o=300K, τ_λ, ρ_λ

1.0
ρ_λ
τ_λ
0
5
$\lambda(\mu m)$

ASSUMPTIONS: (1) Furnace walls are isothermal, diffuse, and gray, (2) Object is isothermal and diffuse.

ANALYSIS: Consider first the furnace wall (A). Since the wall material is diffuse and gray, it follows that

$$e_A = e_f = a_A = 0.85. \qquad <$$

The emissive power is

$$E_A = e_A E_b(T_f) = e_A s T_f = 0.85 \times 5.67 \times 10^{-8}\,W/m^2 \cdot K^4 (3000\,K)^4 = 3.904 \times 10^6\,W/m^2. \qquad <$$

Since the furnace is an isothermal enclosure, blackbody conditions exist such that

$$G_A = J_A = E_b(T_f) = s\,T_f^4 = 5.67 \times 10^{-8} W/m^2 \cdot K^4 (3000K)^4 = 4.593 \times 10^6 W/m^2. \qquad <$$

Considering now the semitransparent, diffuse, spectrally selective object at T_o = 300 K. From the radiation balance requirement, find

$$a_l = 1 - r_l - t_l \quad \text{or} \quad a_1 = 1 - 0.6 - 0.3 = 0.1 \quad \text{and} \quad a_2 = 1 - 0.7 - 0.0 = 0.3$$

$$a_B = \int_0^\infty a_l G_l\, dl \,/\, G = F_{0-lT} \cdot a_1 + (1 - F_{0-lT}) \cdot a_2 = 0.970 \times 0.1 + (1 - 0.970) \times 0.3 = 0.106 \qquad <$$

where $F_{0\text{ - }\lambda T}$ = 0.970 at λT = 5 μm × 3000 K = 15,000 μm·K since G = $E_b(T_f)$. Since the object is diffuse, $\varepsilon_\lambda = \alpha_\lambda$, hence

$$e_B = \int_0^\infty e_l E_{l,b}(T_o)\, dl \,/\, E_{b,o} = F_{0-lT} a_1 + (1 - F_{0-lT}) \cdot a_2 = 0.0138 \times 0.1 + (1 - 0.0138) \times 0.3 = 0.297 \qquad <$$

where $F_{0-\lambda T}$ = 0.0138 at λT = 5 μm × 300 K = 1500 μmK. The emissive power is

$$E_B = e_B E_{b,B}(T_o) = 0.297 \times 5.67 \times 10^{-8} W/m^2 \cdot K^4 (300\,K)^4 = 136.5\,W/m^2. \qquad <$$

The irradiation is that due to the large furnace for which blackbody conditions exist,

$$G_B = G_A = s\,T_f^4 = 4.593 \times 10^6\,W/m^2. \qquad <$$

The radiosity leaving point B is due to emission and reflected irradiation,

$$J_B = E_B + r_B G_B = 136.5\,W/m^2 + 0.3 \times 4.593 \times 10^6\,W/m^2 = 1.378 \times 10^6\,W/m^2. \qquad <$$

If we include transmitted irradiation, $J_B = E_B + (\rho_B + \tau_B)\,G_B = E_B + (1 - \alpha_B)\,G_B = 4.106 \times 10^6$ W/m^2.
In the first calculation, note how we set $\rho_B \approx \rho_\lambda$ (λ < 5 μm).

PROBLEM 12.61

KNOWN: Spectral characteristics of four diffuse surfaces exposed to solar radiation.

FIND: Surfaces which may be assumed to be gray.

SCHEMATIC:

ASSUMPTIONS: (1) Diffuse surface behavior.

ANALYSIS: A gray surface is one for which α_λ and ε_λ are constant over the spectral regions of the irradiation and the surface emission.

For $\lambda = 3\ \mu m$ and $T = 5800K$, $\lambda T = 17{,}400\ \mu m \cdot K$ and from Table 12.1, find $F_{(0 \to \lambda)} = 0.984$. Hence, 98.4% of the solar radiation is in the spectral region below 3 μm.

For $\lambda = 6\ \mu m$ and $T = 300K$, $\lambda T = 1800\ \mu m \cdot K$ and from Table 12.1, find $F_{(0 \to \lambda)} = 0.039$. Hence, 96.1% of the surface emission is in the spectral region above 6 μm.

Hence: Surface A is gray: $\alpha_S \approx \varepsilon = 0.8$ <

Surface B is not gray: $\alpha_S \approx 0.8,\ \varepsilon \approx 0.3$ <

Surface C is not gray: $\alpha_S \approx 0.3,\ \varepsilon \approx 0.7$ <

Surface D is gray: $\alpha_S \approx \varepsilon = 0.3.$ <

PROBLEM 12.62

KNOWN: A gray, but directionally selective, material with $\alpha\,(\theta, \phi) = 0.5(1 - \cos\phi)$.

FIND: (a) Hemispherical absorptivity when irradiated with collimated solar flux in the direction (θ = 45° and ϕ = 0°) and (b) Hemispherical emissivity of the material.

SCHEMATIC:

ASSUMPTIONS: (1) Gray surface behavior.

ANALYSIS: (a) The surface has the directional absorptivity given as

$$\alpha\left(\theta,\phi\right) = \alpha_{\lambda,\phi} = 0.5\left[1-\cos\phi\right].$$

When irradiated in the direction θ = 45° and ϕ = 0°, the directional absorptivity for this condition is

$$\alpha\left(45°, 0°\right) = 0.5\left[1-\cos\left(0°\right)\right] = 0.$$ <

That is, the surface is completely reflecting (or transmitting) for irradiation in this direction.

(b) From Kirchhoff's law,

$$\alpha_{\theta,\phi} = \varepsilon_{\theta,\phi}$$

so that

$$\varepsilon_{\theta,\phi} = \alpha_{\theta,\phi} = 0.5\left(1-\cos\phi\right).$$

Using Eq. 12.35 find

$$\varepsilon = \frac{\int_0^{2\pi}\int_0^{\pi/2} \varepsilon_{\theta,\phi,\lambda}\cos\theta\sin\theta\,d\theta\,d\phi}{\int_0^{2\pi}\int_0^{\pi/2}\cos\theta\sin\theta\,d\theta\,d\phi}$$

$$\varepsilon = \frac{\int_0^{2\pi} 0.5\left(1-\cos\phi\right)d\phi}{\int_0^{2\pi} d\phi} = \left.\frac{0.5\left(\phi-\sin\phi\right)}{2\pi}\right|_0^{2\pi} = 0.5.$$ <

PROBLEM 12.63

KNOWN: Area and temperature of an opaque surface. Rate of incident radiation, absorbed radiation and heat transfer by convection.

FIND: Surface irradiation, emissive power, radiosity, absorptivity, reflectivity and emissivity.

SCHEMATIC:

GA_s = 1300 W

E

q_{conv} = 300 W

A_s = 3 m^2

T_s = 500 K

G_{abs}

$G_{abs}A_s$ = 1000 W

ASSUMPTIONS: (1) Steady-state, (2) Adiabatic sides and bottom.

ANALYSIS: The irradiation, emissive power and radiosity are

$$G = 1300\,\text{W}\big/3\text{m}^2 = 433\,\text{W}\big/\text{m}^2 \qquad <$$

$$E = G_{abs} - q''_{conv} = (1000-300)\,\text{W}\big/3\text{m}^2 = 233\,\text{W}\big/\text{m}^2 \qquad <$$

$$J = E + G_{ref} = E + (G - G_{abs}) = \left[233 + (433-333)\right]\text{W}\big/\text{m}^2 = 333\,\text{W}\big/\text{m}^2 \qquad <$$

The absorptivity, reflectivity and emissivity are

$$\alpha = G_{abs}/G = \left(333\,\text{W}/\text{m}^2\right)\big/\left(433\,\text{W}/\text{m}^2\right) = 0.769 \qquad <$$

$$\rho = 1 - \alpha = 0.231 \qquad <$$

$$\varepsilon = E/E_b = E\big/\sigma T_s^4 = 233\,\text{W}/\text{m}^2\big/5.67\times10^{-8}\,\text{W}/\text{m}^2\cdot\text{K}^4\,(500\,\text{K})^4 = 0.066 \qquad <$$

COMMENTS: The expression for E follows from a surface energy balance for which the absorbed irradiation is balanced by emission and convection.

PROBLEM 12.64

KNOWN: Isothermal enclosure at a uniform temperature provides a known irradiation on two small surfaces whose absorption rates have been measured.

FIND: (a) Net heat transfer rates and temperatures of the two surfaces, (b) Absorptivity of the surfaces, (c) Emissive power of the surfaces, (d) Emissivity of the surfaces.

SCHEMATIC:

ASSUMPTIONS: (1) Enclosure is at a uniform temperature and large compared to surfaces A and B, (2) Surfaces A and B have been in the enclosure a long time, (3) Irradiation to both surfaces is the same.

ANALYSIS: (a) Since the surfaces A and B have been within the enclosure a long time, thermal equilibrium conditions exist. That is,

$$q_{A,net} = q_{B,net} = 0.$$

Furthermore, the surface temperatures are the same as the enclosure, $T_{s,A} = T_{s,B} = T_{enc}$. Since the enclosure is at a uniform temperature, it follows that blackbody radiation exists within the enclosure (see Fig. 12.12) and

$$G = E_b\left(T_{enc}\right) = \sigma\, T_{enc}^4$$

$$T_{enc} = \left(G/\sigma\right)^{1/4} = \left(6300\,\text{W/m}^2 / 5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\right)^{1/4} = 577.4\text{K}.$$ <

(b) From Eq. 12.45, the absorptivity is G_{abs}/G,

$$\alpha_A = \frac{5600\,\text{W/m}^2}{6300\,\text{W/m}^2} = 0.89 \qquad \alpha_B = \frac{630\,\text{W/m}^2}{6300\,\text{W/m}^2} = 0.10.$$ <

(c) Since the surfaces experience zero net heat transfer, the energy balance is $G_{abs} = E$. That is, the absorbed irradiation is equal to the emissive power,

$$E_A = 5600\,\text{W/m}^2 \qquad E_B = 630\,\text{W/m}^2.$$ <

(d) The emissive power, E(T), is written as

$$E = \varepsilon E_b(T) = \varepsilon\,\sigma\,T^4 \qquad \text{or} \qquad \varepsilon = E/\sigma\,T^4.$$

Since the temperature of the surfaces and the emissive powers are known,

$$\varepsilon_A = 5600\,\text{W/m}^2 \Big/ \left[5.67\times10^{-8}\,\frac{\text{W}}{\text{m}^2\cdot\text{K}^4}\left(577.4\text{K}\right)^4\right] = 0.89 \qquad \varepsilon_B = 0.10.$$ <

COMMENTS: Note for this equilibrium condition, $\varepsilon = \alpha$.

PROBLEM 12.65

KNOWN: Opaque, horizontal plate, well insulated on backside, is subjected to a prescribed irradiation. Also known are the reflected irradiation, emissive power, plate temperature and convection coefficient for known air temperature.

FIND: (a) Emissivity, absorptivity and radiosity and (b) Net heat transfer per unit area of the plate.

SCHEMATIC:

ASSUMPTIONS: (1) Plate is insulated on backside, (2) Plate is opaque.

ANALYSIS: (a) The total, hemispherical emissivity of the plate according to Eq. 12.37 is

$$\varepsilon = \frac{E}{E_b(T_s)} = \frac{E}{\sigma T_s^4} = \frac{1200\ \mathrm{W/m^2}}{5.67\times10^{-8}\ \mathrm{W/m^2\cdot K^4}\times(227+273)^4\ \mathrm{K^4}} = 0.34. \quad <$$

The total, hemispherical absorptivity is related to the reflectivity by Eq. 12.57 for an opaque surface. That is, $\alpha = 1 - \rho$. By definition, the reflectivity is the fraction of irradiation reflected, Eq. 12.51, such that

$$\alpha = 1 - G_{ref}/G = 1 - 500\ \mathrm{W/m^2}/\left(2500\ \mathrm{W/m^2}\right) = 1 - 0.20 = 0.80. \quad <$$

The radiosity, J, is defined as the radiant flux leaving the surface by emission and reflection per unit area of the surface (see Section 12.24).

$$J = \rho G + \varepsilon E_b = G_{ref} + E = 500\ \mathrm{W/m^2} + 1200\ \mathrm{W/m^2} = 1700\ \mathrm{W/m^2}. \quad <$$

(b) The net heat transfer is determined from an energy balance,

$$q''_{net} = q''_{in} - q''_{out} = G - G_{ref} - E - q''_{conv}$$

$$q''_{net} = (2500 - 500 - 1200)\ \mathrm{W/m^2} - 15\ \mathrm{W/m^2\cdot K}\,(227-127)\ \mathrm{K} = -700\ \mathrm{W/m^2}. \quad <$$

An alternate approach to the energy balance using the radiosity,

$$q''_{net} = G - J - q''_{conv}$$

$$q''_{net} = (2500 - 1700 - 1500)\ \mathrm{W/m^2}$$

$$q''_{net} = -700\ \mathrm{W/m^2}.$$

COMMENTS: (1) Since the net heat rate per unit area is negative, energy must be added to the plate in order to maintain it at $T_s = 227°C$. (2) Note that $\alpha \neq \varepsilon$. Hence, the plate is not a gray body. (3) Note the use of radiosity in performing energy balances. That is, considering only the radiation processes, $q''_{net} = G - J$.

PROBLEM 12.66

KNOWN: Horizontal, opaque surface at steady-state temperature of 77°C is exposed to a convection process; emissive power, irradiation and reflectivity are prescribed.

FIND: (a) Absorptivity of the surface, (b) Net radiation heat transfer rate for the surface; indicate direction, (c) Total heat transfer rate for the surface; indicate direction.

SCHEMATIC:

ASSUMPTIONS: (1) Surface is opaque, (2) Effect of surroundings included in the specified irradiation, (3) Steady-state conditions.

ANALYSIS: (a) From the definition of the thermal radiative properties and a radiation balance for an opaque surface on a total wavelength basis, according to Eq. 12.59,

$$\alpha = 1 - \rho = 1 - 0.4 = 0.6.$$ <

(b) The net radiation heat transfer rate to the surface follows from a surface energy balance considering only radiation processes. From the schematic,

$$q''_{net,rad} = \left(\dot{E}''_{in} - \dot{E}''_{out}\right)_{rad}$$

$$q''_{net,rad} = G - \rho G - E = (1380 - 0.4 \times 1380 - 628)\, W/m^2 = 200\, W/m^2.$$ <

Since $q''_{net,rad}$ is positive, the net radiation heat transfer rate is *to* the surface.

(c) Performing a surface energy balance considering all heat transfer processes, the local heat transfer rate is

$$q''_{tot} = \left(\dot{E}''_{in} - \dot{E}''_{out}\right)$$

$$q''_{tot} = q''_{net,rad} - q''_{conv}$$

$$q''_{tot} = 200\, W/m^2 - 28\, W/m^2 \cdot K\,(77 - 27)\,K = -1200\, W/m^2.$$ <

The total heat flux is shown as a negative value indicating the heat flux is *from* the surface.

COMMENTS: (1) Note that the surface radiation balance could also be expresses as

$$q''_{net,rad} = G - J \qquad \text{or} \qquad \alpha G - E.$$

Note the use of radiosity to express the radiation flux leaving the surface.

(2) From knowledge of the surface emissive power and T_s, find the emissivity as

$$\varepsilon \equiv E/\sigma T_s^4 = 628\, W/m^2 / \left(5.67 \times 10^{-8}\, W/m^2 \cdot K^4\right)(77 + 273)^4\, K^4 = 0.74.$$

Since ε ≠ α, we know the surface is not gray.

(c) Using the foregoing equations in the IHT workspace along with the *IHT Radiation Tool*, *Band Emission Factor*, $q''_{rad,in}$ was computed and plotted as a function of T_s.

250000
200000
150000
100000
50000
q"radin (W/m^2)
500 600 700 800 900 1000
Surface temperature, Ts (K)

Grey surface, eps = 0.4
Selective surface, eps
Grey surface, eps = 0.8

The net radiation heat rate, $q''_{rad,in}$ decreases with increasing surface temperature since E increases with T_s and the absorbed irradiation remains constant according to Eq. (1). The heat flux is largest for the gray surface with $\varepsilon = 0.4$ and the smallest for the gray surface with $\varepsilon = 0.8$. As expected, the heat flux for the selective surface is between the limits of the two gray surfaces.

(d) Using the IHT model of part (c), the emissivity and absorptivity of the surface are computed and plotted below.

0.8
0.7
0.6
0.5
0.4
eps or alpha
500 600 700 800 900 1000
Surface temperature, Ts (K)

Emissivity, eps
Absorptivity, alpha

The absorptivity, $\alpha = \alpha(\alpha_\lambda, T_{sur})$, remains constant as T_s changes since it is a function of α_λ (or ε_λ) and T_{sur} only. The emissivity, $\varepsilon = \varepsilon(\varepsilon_\lambda, T_s)$ is a function of T_s and decreases as T_s increases. Could you have surmised as much by looking at the spectral emissivity distribution? Under what condition would you expect $\alpha = \varepsilon$?

PROBLEM 12.68

KNOWN: Opaque, diffuse surface with prescribed spectral reflectivity and at a temperature of 750K is subjected to a prescribed spectral irradiation, G_λ.

FIND: (a) Total absorptivity, α, (b) Total emissivity, ε, (c) Net radiative heat flux to the surface.

SCHEMATIC:

ASSUMPTIONS: (1) Opaque and diffuse surface, (2) Backside insulated.

ANALYSIS: (a) The total absorptivity is determined from Eq. 12.46 and 12.56,

$$a_l = 1 - r_l \qquad \text{and} \qquad a = \int_0^\infty a_l \, G_l \, dl / G. \qquad (1,2)$$

Evaluating by separate integrals over various wavelength intervals.

$$a = \frac{(1-r_{l,1})\int_1^3 G_l\,dl + (1-r_{l,2})\int_3^6 G_l\,dl + (1-r_{l,2})\int_6^8 G_l\,dl}{\int_1^3 G_l\,dl + \int_3^6 G_l\,dl + \int_6^8 G_l\,dl} = \frac{G_{abs}}{G}$$

$$G_{abs} = (1-0.6)\left[0.5\times500\,W/m^2\cdot mm(3-1)mm\right] + (1-0.2)\left[500\,W/m^2\cdot mm(6-3)mm\right]$$

$$+(1-0.2)\left[0.5\times500\,W/m^2\cdot mm(8-6)mm\right]$$

$$G = 0.5\times500\,W/m^2\cdot mm\times(3-1)mm + 500\,W/m^2\cdot mm\,(6-3)mm + 0.5\times500\,W/m^2\cdot mm\,(8-6)\,mm$$

$$a = \frac{[200+1200+400]W/m^2}{[500+1500+500]W/m^2} = \frac{1800\,W/m^2}{2500\,W/m^2} = 0.720.$$ <

(b) The total emissivity of the surface is determined from Eq. 12.38 and 12.65,

$$e_l = a_l \qquad \text{and, hence} \qquad e_l = 1 - r_l. \qquad (3,4)$$

The total emissivity can then be expressed as

$$e = \int_0^\infty e_l\, E_{l,b}(l,T_s)\,dl / E_b(T_s) = \int_0^\infty (1-r_l)\,E_{l,b}(l,T_s)\,dl / E_b(T_s)$$

$$e = (1-r_{l,1})\int_0^3 E_{l,b}(l,T_s)\,dl / E_b(T_s) + (1-r_{l,2})\int_3^\infty E_{l,b}(l,T_s)\,dl / E_b(T_s)$$

$$e = (1-r_{l,1})F_{(0\to3\,mm)} + (1-r_{l,2})\left[1-F_{(0\to3\,mm)}\right]$$

$$e = (1-0.6)\times0.111 + (1-0.2)[1-0.111] = 0.756$$ <

where Table 12.1 is used to find $F_{(0-\lambda)} = 0.111$ for $\lambda_1 T_s = 3\times750 = 2250$ μm·K.

(c) The net radiative heat flux *to* the surface is

$$q''_{rad} = aG - e\,E_b(T_s) = aG - e\,s\,T_s^4$$

$$q''_{rad} = 0.720\times2500\,W/m^2$$

$$-0.756\times5.67\times10^{-8}\,W/m^2\cdot K^4\,(750K)^4 = -11{,}763\,W/m^2.$$ <

PROBLEM 12.69

KNOWN: Opaque, gray surface at 27°C with prescribed irradiation, reflected flux and convection process.

FIND: Net heat flux from the surface.

SCHEMATIC:

ASSUMPTIONS: (1) Surface is opaque and gray, (2) Surface is diffuse, (3) Effects of surroundings are included in specified irradiation.

ANALYSIS: From an energy balance on the surface, the net heat flux *from* the surface is

$$q''_{net} = \dot{E}''_{out} - \dot{E}''_{in}$$

$$q''_{net} = q''_{conv} + E + G_{ref} - G = h\left(T_s - T_\infty\right) + \varepsilon\sigma T_s^4 + G_{ref} - G. \qquad (1)$$

To determine ε, from Eq. 12.59 and Kirchoff's law for a diffuse-gray surface, Eq. 12.62,

$$\varepsilon = \alpha = 1 - \rho = 1 - \left(G_{ref}/G\right) = 1 - \left(800/1000\right) = 1 - 0.8 = 0.2 \qquad (2)$$

where from Eq. 12.51, $\rho = G_{ref}/G$. The net heat flux from the surface, Eq. (1), is

$$q''_{net} = 15\,W/m^2 \cdot K\left(27 - 17\right)K + 0.2 \times 5.67 \times 10^{-8}\,W/m^2 \cdot K^4 \left(27 + 273\right)^4 K^4$$
$$+ 800\,W/m^2 - 1000\,W/m^2$$

$$q''_{net} = \left(150 + 91.9 + 800 - 1000\right)W/m^2 = 42\,W/m^2. \qquad <$$

COMMENTS: (1) For this situation, the radiosity is

$$J = G_{ref} + E = \left(800 + 91.9\right)W/m^2 = 892\,W/m^2.$$

The energy balance can be written involving the radiosity (radiation leaving the surface) and the irradiation (radiation to the surface).

$$q''_{net} = J - G + q''_{conv} = \left(892 - 1000 + 150\right)W/m^2 = 42\,W/m^2.$$

(2) Note the need to assume the surface is diffuse, gray and opaque in order that Eq. (2) is applicable.

PROBLEM 12.70

KNOWN: Diffuse glass at $T_g = 750$ K with prescribed spectral radiative properties being heated in a large oven having walls with emissivity of 0.75 and 1800 K.

FIND: (a) Total transmissivity r, total reflectivity ρ, and total emissivity ε of the glass; Net radiative heat flux to the glass, (b) $q''_{rad,in}$; and (c) Compute and plot $q''_{rad,in}$ as a function of glass temperatures for the range $500 \le T_g \le 800$ K for oven wall temperatures of $T_w = 1500$, 1800 and 2000 K.

SCHEMATIC:

Oven wall, T_w = 1800 K, ε_w = 0.75

Glass, T_g = 750 K, τ_λ, ρ_λ

Oven wall, T_w, ε_w

τ_λ: 0.9, 0

ρ_λ: 0.5, 0.05

0.2, 1.6, λ(μm)

ASSUMPTIONS: (1) Glass is of uniform temperature, (2) Glass is diffuse, (3) Furnace walls large compared to the glass; ε_w plays no role, (4) Negligible convection.

ANALYSIS: (a) From knowledge of the spectral transmittance,τ_w, and spectral reflectivity, ρ_λ, the following radiation properties are evaluated:

Total transmissivity, r: For the irradiation from the furnace walls, $G_\lambda = E_{\lambda,b}(\lambda, T_w)$. Hence

$$\tau = \int_0^\infty \tau_\lambda E_{\lambda,b}(\lambda, T_w)\, d\lambda \Big/ \sigma T_w^4 \approx \tau_{\lambda 1} F_{(0-\lambda T)} = 0.9 \times 0.25 = 0.225 .$$ <

where $\lambda T = 1.6\ \mu m \times 1800\ K = 2880\ \mu m \cdot K \approx 2898\ \mu m \cdot K$ giving $F_{(0-\lambda T)} \approx 0.25$.

Total reflectivity, ρ: With $G_\lambda = E_{\lambda,b}(\lambda, T_w)$, $T_w = 1800$ K, and $F_{0-\lambda T} = 0.25$,

$$\rho \approx \rho_{\lambda 1} F_{(0-\lambda T)} + \rho_{\lambda 2}\left(1 - F_{(0-\lambda T)}\right) = 0.05 \times 0.25 + 0.5(1 - 0.25) = 0.388$$ <

Total absorptivity, α: To perform the energy balance later, we'll need α. Employ the conservation expression,

$$\alpha = 1 - \rho - \tau = 1 - 0.388 - 0.225 = 0.387 .$$

Emissivity, ε: Based upon surface temperature $T_g = 750$ K, for

$$\lambda T = 1.6\ \mu m \times 750 K = 1200\ \mu m \cdot K, \qquad F_{0-\lambda T} \approx 0.002 .$$

Hence for $\lambda > 1.6\ \mu m$, $\varepsilon \approx \varepsilon_\lambda \approx 0.5$. <

(b) Performing an energy balance on the glass, the net radiative heat flux by radiation into the glass is,

Continued...

$$q''_{net,in} = E''_{in} - E''_{out}$$

$$q''_{net,in} = 2\left(\alpha G - \varepsilon E_b\left(T_g\right)\right)$$

where $G = \sigma T_w^4$

$$q''_{net,in} = 2\left[0.387\sigma\left(1800K\right)^4 - 0.5\sigma\left(750K\right)^4\right]$$

$$q''_{net,in} = 442.8\,kW/m^2\ .$$

(b) Using the foregoing equations in the IHT Workspace along with the *IHT Radiation Tool, Band Emission Factor*, the net radiative heat flux, $q''_{rad,in}$, was computed and plotted as a function of T_g for selected wall temperatures T_w .

As the glass temperature increases, the rate of emission increases so we'd expect the net radiative heat rate into the glass to decrease. Note that the decrease is not very significant. The effect of increased wall temperature is to increase the irradiation and, hence the absorbed irradiation to the surface and the net radiative flux increase.

PROBLEM 12.71

KNOWN: Temperature, absorptivity, transmissivity, radiosity and convection conditions for a semitransparent plate.

FIND: Plate irradiation and total hemispherical emissivity.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Uniform surface conditions.

ANALYSIS: From an energy balance on the plate

$$\dot{E}_{in} = \dot{E}_{out}$$

$$2G = 2q''_{conv} + 2J.$$

Solving for the irradiation and substituting numerical values,

$$G = 40\ W/m^2 \cdot K(350 - 300)K + 5000\ W/m^2 = 7000\ W/m^2. \qquad <$$

From the definition of J,

$$J = E + rG + tG = E + (1 - a)G.$$

Solving for the emissivity and substituting numerical values,

$$e = \frac{J - (1 - a)G}{s\,T^4} = \frac{\left(5000\ W/m^2\right) - 0.6\left(7000\ W/m^2\right)}{5.67 \times 10^{-8}\ W/m^2 \cdot K^4 (350K)^4} = 0.94. \qquad <$$

Hence,

$$a \neq e$$

and the surface is not gray for the prescribed conditions.

COMMENTS: The emissivity may also be determined by expressing the plate energy balance as

$$2aG = 2q''_{conv} + 2E.$$

Hence

$$e\,s\,T^4 = aG - h(T - T_\infty)$$

$$e = \frac{0.4\left(7000\ W/m^2\right) - 40\ W/m^2 \cdot K(50\ K)}{5.67 \times 10^{-8}\ W/m^2 \cdot K^4 (350\ K)^4} = 0.94.$$

PROBLEM 12.72

KNOWN: Material with prescribed radiative properties covering the peep hole of a furnace and exposed to surroundings on the outer surface.

FIND: Steady-state temperature of the cover, T_s; heat loss from furnace.

SCHEMATIC:

ASSUMPTIONS: (1) Cover is isothermal, no gradient, (2) Surroundings of the outer surface are large compared to cover, (3) Cover is insulated from its mount on furnace wall, (4) Negligible convection on interior surface.

PROPERTIES: Cover material (given): For irradiation from the furnace interior: $\tau_f = 0.8$, $\rho_f = 0$; For room temperature emission: $\tau = 0$, $\varepsilon = 0.8$.

ANALYSIS: Perform an energy balance identifying the modes of heat transfer,

$$\dot{E}_{in} - \dot{E}_{out} = 0 \qquad \alpha_f G_f + \alpha_{sur} G_{sur} - 2\varepsilon E_b(T_s) - h(T_s - T_\infty) = 0. \qquad (1)$$

Recognize that $\qquad G_f = \sigma T_f^4 \qquad G_{sur} = \sigma T_{sur}^4. \qquad (2,3)$

From Eq. 12.57, it follows that $\qquad \alpha_f = 1 - \tau_f - \rho_f = 1 - 0.8 - 0.0 = 0.2. \qquad (4)$

Since the irradiation G_{sur} will have nearly the same spectral distribution as the emissive power of the cover, $E_b(T_s)$, and since G_{sur} is diffuse irradiation,

$$\alpha_{sur} = \varepsilon = 0.8. \qquad (5)$$

This reasoning follows from Eqs. 12.65 and 12.66. Substituting Eqs. (2-5) into Eq. (1) and using numerical values,

$$0.2 \times 5.67 \times 10^{-8}(450+273)^4\,W/m^2 + 0.8 \times 5.67 \times 10^{-8} \times 300^4\,W/m^2 \qquad (2\text{-}5)$$

$$-2 \times 0.8 \times 5.67 \times 10^{-8}\,T_s^4\,W/m^2 - 50\,W/m^2 \cdot K(T_s - 300)K = 0 \qquad <$$

$$9.072 \times 10^{-8}\,T_s^4 + 50T_s = 18{,}466 \qquad \text{or} \qquad T_s = 344K.$$

The heat loss from the furnace (see energy balance schematic) is

$$q_{f,loss} = A_s\left[\alpha_f G_f + \tau_f G_f - \varepsilon E_b(T_s)\right] = \frac{\pi D^2}{4}\left[(\alpha_f + \tau_f)G_f - \varepsilon E_b(T_s)\right]$$

$$q_{f,loss} = \pi(0.050m)^2/4\Big[(0.8+0.2)(723K)^4$$

$$-0.8(344K)^4\Big]5.67 \times 10^{-8}\,W/m^2 \cdot K^4 = 29.2\ W. \qquad <$$

PROBLEM 12.73

KNOWN: Window with prescribed τ_λ and ρ_λ mounted on cooled vacuum chamber passing radiation from a solar simulator.

FIND: (a) Solar transmissivity of the window material, (b) State-state temperature reached by window with simulator operating, (c) Net radiation heat transfer to chamber.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Diffuse behavior of window material, (3) Chamber and room surroundings large compared to window, (4) Solar simulator flux has spectral distribution of 5800K blackbody, (5) Window insulated from its mount, (6) Window is isothermal at T_w.

ANALYSIS: (a) Using Eq. 12.55 and recognizing that $G_{\lambda,S} \sim E_{b,\lambda}$ (λ, 5800K),

$$\tau_S = \tau_1 \int_{0.38}^{1.9} E_{\lambda,b}(\lambda, 5800\text{K})\, d\lambda / E_b(5800\text{K}) = \tau_1 \left[F_{(0\to 1.9\mu m)} - F_{(0\to 0.38\mu m)} \right].$$

From Table 12.1 at $\lambda T = 1.9 \times 5800 = 11{,}020\ \mu m\cdot K$, $F_{(0 \to \lambda)} = 0.932$; at $\lambda T = 0.38 \times 5800\ \mu m\cdot K = 2{,}204\ \mu m\cdot K$, $F_{(0 \to \lambda)} = 0.101$; hence

$$\tau_S = 0.90[0.932 - 0.101] = 0.748.$$ <

Recognizing that later we'll need α_S, use Eq. 12.52 to find ρ_S

$$\rho_S = \rho_1 F_{(0\to 0.38\mu m)} + \rho_2 \left[F_{(0\to 1.9\mu m)} - F_{(0\to 0.38\mu m)} \right] + \rho_3 \left[1 - F_{(0\to 1.9\mu m)} \right]$$

$$\rho_S = 0.15 \times 0.101 + 0.05[0.932 - 0.101] + 0.15[1 - 0.932] = 0.067$$

$$\alpha_S = 1 - \rho_S - \tau_S = 1 - 0.067 - 0.748 = 0.185.$$

(b) Perform an energy balance on the window.

$$\alpha_S G_S - q''_{w-c} - q''_{w-sur} - q''_{conv} = 0$$

$$\alpha_S G_S - \varepsilon\sigma\left(T_w^4 - T_c^4\right) - \varepsilon\sigma\left(T_w^4 - T_{sur}^4\right) - \bar{h}\left(T_w - T_\infty\right) = 0.$$

Recognize that ρ_λ (λ > 1.9) = 0.15 and that $\varepsilon \approx 1 - 0.15 = 0.85$ since T_w will be near 300K. Substituting numerical values, find by trial and error,

$$0.185 \times 3000\ W/m^2 - 0.85 \times \sigma \left[2T_w^4 - 298^4 - 77^4 \right] K^4 - 28\ W/m^2\cdot K\left(T_w - 298\right)K = 0$$

$$T_w = 302.6\text{K} = 29.6°\text{C}.$$ <

(c) The net radiation transfer per unit area of the window to the vacuum chamber, excluding the transmitted simulated solar flux is

$$q''_{w-c} = \varepsilon\sigma\left(T_w^4 - T_c^4\right) = 0.85 \times 5.67 \times 10^{-8}\ W/m^2\cdot K^4 \left[302.6^4 - 77^4 \right] K^4 = 402\ W/m^2.$$ <

PROBLEM 12.74

KNOWN: Reading and emissivity of a thermocouple (TC) located in a large duct to measure gas stream temperature. Duct wall temperature and emissivity; convection coefficient.

FIND: (a) Gas temperature, T_∞, (b) Effect of convection coefficient on measurement error.

SCHEMATIC:

Gas
T_∞, $\bar{h}$
T_s
$\varepsilon_s = 0.6$
Thermocouple (TC) support
Duct wall,
$\varepsilon_w = 0.85$, $T_w = 450\ ^oC$
q''_{conv}
q''_{w-s}
T_s

ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible heat loss from TC sensing junction to support, (3) Duct wall much larger than TC, (4) TC surface is diffuse-gray.

ANALYSIS: (a) Performing an energy balance on the thermocouple, it follows that

$$q''_{w-s} - q''_{conv} = 0 .$$

where radiation exchange between the duct wall and the TC is given by Eq. 1.7. Hence,

$$\varepsilon_s \sigma (T_w^4 - T_s^4) - \bar{h}(T_s - T_\infty) = 0 .$$

Solving for T_∞ with $T_s = 180^oC$,

$$T_\infty = T_s - \frac{\varepsilon_s \sigma}{\bar{h}} (T_w^4 - T_s^4)$$

$$T_\infty = (180 + 273)K - \frac{0.6(5.67 \times 10^{-8}\ W/m^2 \cdot K^4)}{125\ W/m^2 \cdot K} \left([450 + 273]^4 - [180 + 273]^4 \right) K^4$$

$$T_\infty = 453\ K - 62.9\ K = 390\ K = 117^\circ C .$$ <

(b) Using the IHT *First Law* model for an *Isothermal Solid Sphere* to solve the foregoing energy balance for T_s, with $T_\infty = 125^oC$, the measurement error, defined as $\Delta T = T_s - T_\infty$, was determined and is plotted as a function of $\bar{h}$.

Measurement error, delta(C)
0, 50, 100, 150, 200, 250, 300
0, 200, 400, 600, 800, 1000
Convection coefficient, hbar(W/m^2.K)

The measurement error is enormous ($\Delta T \approx 270^oC$) for $\bar{h} = 10\ W/m^2 \cdot K$, but decreases with increasing $\bar{h}$. However, even for $\bar{h} = 1000\ W/m^2 \cdot K$, the error ($\Delta T \approx 8^\circ C$) is not negligible. Such errors must always be considered when measuring a gas temperature in surroundings whose temperature differs significantly from that of the gas.

Continued...

PROBLEM 12.74 (Cont.)

COMMENTS: (1) Because the duct wall surface area is much larger than that of the thermocouple, its emissivity is not a factor. (2) For such a situation, a shield about the thermocouple would reduce the influence of the hot duct wall on the indicated TC temperature. A low emissivity thermocouple coating would also help.

PROBLEM 12.75

KNOWN: Diameter and emissivity of a horizontal thermocouple (TC) sheath located in a large room. Air and wall temperatures.

FIND: (a) Temperature indicated by the TC, (b) Effect of emissivity on measurement error.

SCHEMATIC:

ASSUMPTIONS: (1) Room walls approximate isothermal, large surroundings, (2) Room air is quiescent, (3) TC approximates horizontal cylinder, (4) No conduction losses, (5) TC surface is opaque, diffuse and gray.

PROPERTIES: *Table A-4*, Air (assume $T_s = 25\ ^\circ C$, $T_f = (T_s + T_\infty)/2 \approx 296$ K, 1 atm):
$\nu = 15.53\times10^{-6}\ m^2/s$, $k = 0.026\ W/m\cdot K$, $\alpha = 22.0\times10^{-6}\ m^2/s$, $Pr = 0.708$, $\beta = 1/T_f$.

ANALYSIS: (a) Perform an energy balance on the thermocouple considering convection and radiation processes. On a unit area basis, with $q''_{conv} = \bar{h}(T_s - T_\infty)$,

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$\alpha G - \varepsilon E_b(T_s) - \bar{h}(T_s - T_\infty) = 0 . \qquad (1)$$

Since the surroundings are isothermal and large compared to the thermocouple, $G = E_b(T_{sur})$. For the gray-diffuse surface, $\alpha = \varepsilon$. Using the Stefan-Boltzman law, $E_b = \sigma T^4$, Eq. (1) becomes

$$\varepsilon\sigma(T_{sur}^4 - T_s^4) - \bar{h}(T_s - T_\infty) = 0 . \qquad (2)$$

Using the Churchill-Chu correlation for a horizontal cylinder, estimate $\bar{h}$ due to free convection.

$$\overline{Nu}_D = \frac{\bar{h}D}{k} = \left\{0.60 + \frac{0.387 Ra_D^{1/6}}{\left[1+\left(0.559/Pr\right)^{9/16}\right]^{8/27}}\right\}^2 , \quad Ra_D = \frac{g\beta\Delta T D^3}{\nu\alpha} . \qquad (3,4)$$

To evaluate Ra_D and $\overline{Nu}_D$, assume $T_s = 25^\circ C$, giving

$$Ra_D = \frac{9.8\,m/s^2 (1/296\ K)(25-20)K(0.004m)^3}{15.53\times10^{-6}\ m^2/s \times 22.0\times10^{-6}\ m^2/s} = 31.0$$

$$\bar{h} = \frac{0.026\ W/m\cdot K}{0.004m}\left\{0.60 + \frac{0.387(31.0)^{1/6}}{\left[1+\left(0.559/0.708\right)^{9/16}\right]^{8/27}}\right\}^2 = 8.89\ W/m^2\cdot K . \qquad (5)$$

With $\varepsilon = 0.4$, the energy balance, Eq. (2), becomes

$$0.4\times5.67\times10^{-8}\ W/m^2\cdot K^4[(30+273)^4 - T_s^4]K^4 - 8.89\ W/m^2\cdot K[T_s - (20+273)]K = 0 \qquad (6)$$

where all temperatures are in kelvin units. By trial-and-error, find

$T_s \approx 22.2^\circ C$ <

Continued...

PROBLEM 12.75 (Cont.)

(b) The thermocouple measurement error is defined as $\Delta T = T_s - T_\infty$ and is a consequence of radiation exchange with the surroundings. Using the IHT *First Law* Model for an *Isothermal Solid Cylinder* with the appropriate *Correlations* and *Properties* Toolpads to solve the foregoing energy balance for T_s, the measurement error was determined as a function of the emissivity.

Measurement error, delta(C)

5
4
3
2
1
0

0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

Emissivity, eps

The measurement error decreases with decreasing ε, and hence a reduction in net radiation transfer from the surroundings. However, even for ε = 0.1, the error ($\Delta T \approx 1^\circ C$) is not negligible.

PROBLEM 12.76

KNOWN: Temperature sensor imbedded in a diffuse, gray tube of emissivity 0.8 positioned within a room with walls and ambient air at 30 and 20 °C, respectively. Convection coefficient is 5 $W/m^2 \cdot K$.

FIND: (a) Temperature of sensor for prescribed conditions, (b) Effect of surface emissivity and using a fan to induce air flow over the tube.

SCHEMATIC:

q''_{conv} Tube with sensor, T_t, ε_t = 0.2, 0.5 or 0.8

q''_{rad}

Walls T_w = 30 °C

Air T_∞ = 20 °C

$2 \le h \le 25$ W/m²·K

ASSUMPTIONS: (1) Room walls (surroundings) much larger than tube, (2) Tube is diffuse, gray surface, (3) No losses from tube by conduction, (4) Steady-state conditions, (5) Sensor measures temperature of tube surface.

ANALYSIS: (a) Performing an energy balance on the tube, $\dot{E}_{in} - \dot{E}_{out} = 0$. Hence, $q''_{rad} - q''_{conv} = 0$, or $\varepsilon_t \sigma (T_w^4 - T_t^4) - h(T_t - T_w) = 0$. With h = 5 $W/m^2 \cdot K$ and ε_t = 0.8, the energy balance becomes

$$0.8 \times 5.67 \times 10^{-8}\ W/m^2 \cdot K^4 \left[(30+273)^4 - T_t^4\right] K^4 = 5\ W/m^2 \cdot K \left[T_t - (20+273)\right] K$$

$$4.5360 \times 10^{-8} \left[303^4 - T_t^4\right] = 5\left[T_t - 293\right]$$

which yields T_t = 298 K = 25°C. <

(b) Using the IHT *First Law* Model, the following results were determined.

Sensor temperature, Tt(C)

Convection coefficient, h(W/m^2.K)

epst = 0.8
epst = 0.5
epst = 0.2

The sensor temperature exceeds the air temperature due to radiation absorption, which must be balanced by convection heat transfer. Hence, the excess temperature $T_t - T_\infty$, may be reduced by increasing h or by decreasing α_t, which equals ε_t for a diffuse-gray surface, and hence the absorbed radiation.

COMMENTS: A fan will increase the air velocity over the sensor and thereby increase the convection heat transfer coefficient. Hence, the sensor will indicate a temperature closer to T_∞

PROBLEM 12.77

KNOWN: Diffuse-gray sphere is placed in large oven with known wall temperature and experiences convection process.

FIND: (a) Net heat transfer rate to the sphere when its temperature is 300 K, (b) Steady-state temperature of the sphere, (c) Time required for the sphere, initially at 300 K, to come within 20 K of the steady-state temperature, and (d) Elapsed time of part (c) as a function of the convection coefficient for $10 \leq h \leq 25$ $W/m^2 \cdot K$ for emissivities 0.2, 0.4 and 0.8.

SCHEMATIC:

ASSUMPTIONS: (1) Sphere surface is diffuse-gray, (2) Sphere area is much smaller than the oven wall area, (3) Sphere surface is isothermal.

PROPERTIES: Sphere (Given) : $\alpha = 7.25 \times 10^{-5}\ m^2/s$, $k = 185\ W/m \cdot K$.

ANALYSIS: (a) From an energy balance on the sphere find

$$q_{net} = q_{in} - q_{out}$$

$$q_{net} = \alpha G A_s + q_{conv} - E A_s$$

$$q_{net} = \alpha\sigma T_o^4 A_s + h A_s \left(T_\infty - T_s\right) - \varepsilon\sigma T_s^4 A_s. \qquad (1)$$

Note that the irradiation to the sphere is the emissive power of a blackbody at the temperature of the oven walls. This follows since the oven walls are isothermal and have a much larger area than the sphere area. Substituting numerical values, noting that $\alpha = \varepsilon$ since the surface is diffuse-gray and that $A_s = \pi D^2$, find

$$q_{net} = \Big[0.8 \times 5.67 \times 10^{-8}\ W/m^2 \cdot K^4 (600K)^4 + 15\ W/m^2 \cdot K \times (400 - 300) K$$

$$- 0.8 \times 5.67 \times 10^{-8}\ W/m^2 \cdot K^4 (300K)^4 \Big] \pi\left(30 \times 10^{-3} m\right)^2$$

$$q_{net} = [16.6 + 4.2 - 1.0] W = 19.8\ W. \qquad (1)$$ <

(b) For steady-state conditions, q_{net} in the energy balance of Eq. (1) will be zero,

$$0 = \alpha\sigma T_o^4 A_s + h A_s \left(T_\infty - T_{ss}\right) - \varepsilon\sigma T_{ss}^4 A_s \qquad (2)$$

Substitute numerical values and find the steady-state temperature as

$$T_{ss} = 538.2K$$ <

Continued...

PROBLEM 12.77 (Cont.)

(c) Using the *IHT Lumped Capacitance Model* considering convection and radiation processes, the temperature- time history of the sphere, initially at $T_s(0) = T_i = 300$ K, can be determined. The elapsed time required to reach

$$T_s(t_o) = (538.2 - 20)\,K = 518.2\,K$$

was found as

$$t_o = 855\,s = 14.3\,min$$ <

(d) Using the IHT model of part (c), the elapsed time for the sphere to reach within 20 K of its steady-state temperature, t_f, as a function of the convection coefficient for selected emissivities is plotted below.

Time-to-reach within 20 K of steady-state temperature

Time, tf (s)

Convection coefficient, h (W/m^2.K)

eps = 0.2
eps = 0.4
eps = 0.8

For a fixed convection coefficient, t_f increases with decreasing ε since the radiant heat transfer into the sphere decreases with decreasing emissivity. For a given emissivity, the t_f decreases with increasing h since the convection heat rate increases with increasing h. However, the effect is much more significant with lower values of emissivity.

COMMENTS: (1) Why is t_f more strongly dependent on h for a lower sphere emissivity? Hint: Compare the relative heat rates by convection and radiation processes.

(2) The steady-state temperature, T_{ss}, as a function of the convection coefficient for selected emmissivities calculated using (2) is plotted below. Are these results consistent with the above plot of t_f vs h ?

Steady-state temperature, Tss (K)

Convection coefficient, h (W/m^2.K)

eps = 0.2
eps = 0.4
eps = 0.8

PROBLEM 12.78

KNOWN: Thermograph with spectral response in 9 to 12 μm region views a target of area 200mm^2 with solid angle 0.001 sr in a normal direction.

FIND: (a) For a black surface at 60°C, the emissive power in 9 – 12 μm spectral band, (b) Radiant power (W), received by thermograph when viewing black target at 60°C, (c) Radiant power (W) received by thermograph when viewing a gray, diffuse target having ε = 0.7 and considering the surroundings at T_{sur} = 23°C.

SCHEMATIC:

ASSUMPTIONS: (1) Wall is diffuse, (2) Surroundings are black with T_{sur} = 23°C.

ANALYSIS: (a) Emissive power in spectral range 9 to 12 μm for a 60°C black surface is

$$E_t \equiv E_b(9-12\,\mu m) = E_b\left[F(0 \rightarrow 12\,\mu m) - F(0-9\,\mu m)\right]$$

where $E_b(T_s) = \sigma T_s^4$. From Table 12.1:

$$\lambda_2 T_s = 12 \times (60+273) \approx 4000\,\mu m\ K, \qquad F(0-12\,\mu m) = 0.491$$

$$\lambda_1 T_s = 9 \times (60+273) \approx 3000\,\mu m\ K, \qquad F(0-9\,\mu m) = 0.273.$$

Hence

$$E_t = 5.667 \times 10^{-8}\,W/m^2 \cdot K^4 \times (60+273)^4\,K^4\,[0.491-0.273] = 144.9\,W/m^2. \qquad <$$

(b) The radiant power, q_b (w), received by the thermograph from a black target is determined as

$$q_b = \frac{E_t}{\pi} \cdot A_s \cos\theta_1 \cdot \omega$$

where E_t = emissive power in 9 – 12 μm spectral region, part (a) result
A_s = target area viewed by thermograph, 200mm^2 (2×10^{-4} m^2)
ω = solid angle thermograph aperture subtends when viewed from the target, 0.001 sr
θ = angle between target area normal and view direction, 0°.

Hence,

$$q_b = \frac{144.9\,W/m^2}{\pi\ sr} \times \left(2 \times 10^{-4}\,m^2\right) \times \cos 0° \times 0.001\,sr = 9.23\,\mu W. \qquad <$$

Continued …..

PROBLEM 12.78 (Cont.)

(c) When the target is a gray, diffuse emitter, $\varepsilon = 0.7$, the thermograph will receive emitted power from the target and reflected irradiation resulting from the surroundings at $T_{sur} = 23°C$. Schematically:

The power is expressed as

$$q = q_e + q_r = \varepsilon\, q_b + I_r \cdot A_s \cos\theta_1 \cdot \omega \left[F_{(0\to 12\,\mu m)} - F_{(0\to 9\,\mu m)} \right]$$

where

q_b = radiant power from black surface, part (b) result

$F_{(0-\lambda)}$ = band emission fraction for $T_{sur} = 23°C$; using Table 12.1

$\lambda_2 T_{sur} = 12 \times (23 + 273) = 3552\ \mu m \cdot K$, $F_{(0-\lambda_2)} = 0.394$

$\lambda_1 T_{sur} = 9 \times (23 + 273) = 2664\ \mu m \cdot K$, $F_{(0-\lambda_1)} = 0.197$

I_r = reflected intensity, which because of diffuse nature of surface

$$I_r = \rho \frac{G}{\pi} = (1-\varepsilon) \frac{E_b(T_{sur})}{\pi}.$$

Hence

$$q = 0.7 \times 9.23\,\mu W + (1 - 0.7) \frac{5.667 \times 10^{-8}\ W/m^2 \cdot K^4 \times (273 + 23)^4\ K}{\pi\ sr}$$

$$\times \left(2 \times 10^{-4}\ m^2 \right) \times \cos 0° \times 0.001\, sr\, [0.394 - 0.197]$$

$$q = 6.46\,\mu W + 1.64\,\mu W = 8.10\,\mu W.$$ <

COMMENTS: (1) Comparing the results of parts (a) and (b), note that the power to the thermograph is slightly less for the gray surface with $\varepsilon = 0.7$. From part (b) see that the effect of the irradiation is substantial; that is, 1.64/8.10 ≈ 20% of the power received by the thermograph is due to reflected irradiation. Ignoring such effects leads to misinterpretation of temperature measurements using thermography.

(2) Many thermography devices have a spectral response in the 3 to 5 μm wavelength region as well as 9 – 12 μm.

PROBLEM 12.79

KNOWN: Radiation thermometer (RT) viewing a steel billet being heated in a furnace.

FIND: Temperature of the billet when the RT indicates 1160K.

SCHEMATIC:

Billet, T, ε = 0.8
Furnace, T_{wall} = 1500K
Radiation thermometer, T_a = 1160K

ASSUMPTIONS: (1) Billet is diffuse-gray, (2) Billet is small object in large enclosure, (3) Furnace behaves as isothermal, large enclosure, (4) RT is a radiometer sensitive to total (rather than a prescribed spectral band) radiation and is calibrated to correctly indicate the temperature of a black body, (5) RT receives radiant power originating from the target area on the billet.

ANALYSIS: The radiant power reaching the radiation thermometer (RT) is proportional to the radiosity of the billet. For the diffuse-gray billet within the large enclosure (furnace), the radiosity is

$$J = e\, E_b(T) + r\, G = e\, E_b(T) + (1-e) E_b(T_w)$$

$$J = e\, s\, T^4 + (1-e)\, s\, T_w^4 \qquad (1)$$

where $\alpha = \varepsilon$, $G = E_b(T_w)$ and $E_b = \sigma T^4$. When viewing the billet, the RT indicates T_a = 1100K, referred to as the apparent temperature of the billet. That is, the RT *indicates* the billet is a blackbody at T_a for which the radiosity will be

$$E_b(T_a) = J_a = s\, T_a^4. \qquad (2)$$

Recognizing that $J_a = J$, set Eqs. (1) and (2) equal to one another and solve for T, the billet true temperature.

$$T = \left[\frac{1}{e} T_a^4 - \frac{1-e}{e} T_w^4 \right]^{1/4}.$$

Substituting numerical values, find

$$T = \left[\frac{1}{0.8} (1160\text{K})^4 - \frac{1-0.8}{0.8} (1500\text{K})^4 \right]^{1/4} = 999\text{K}. \qquad <$$

COMMENTS: (1) The effect of the reflected wall irradiation from the billet is to cause the RT to indicate a temperature higher than the true temperature.

(2) What temperature would the RT indicate when viewing the furnace wall assuming the wall emissivity were 0.85?

(3) What temperature would the RT indicate if the RT were sensitive to spectral radiation at 0.65 μm instead of total radiation? Hint: in Eqs. (1) and (2) replace the emissive power terms with spectral intensity. Answer: 1365K.

PROBLEM 12.80

KNOWN: Irradiation and temperature of a small surface.

FIND: Rate at which radiation is received by a detector due to emission and reflection from the surface.

SCHEMATIC:

$A_d = 10^{-6} m^2$
n
θ = 30°
r = 1m
G = 1500 W/m²
$A_s = 10^{-4} m^2$, $T_s = 500K$, ε = 0.7

ASSUMPTIONS: (1) Opaque, diffuse-gray surface behavior, (2) A_s and A_d may be approximated as differential areas.

ANALYSIS: Radiation intercepted by the detector is due to emission and reflection from the surface, and from the definition of the intensity, it may be expressed as

$$q_{s-d} = I_{e+r} A_s \cos\theta \, \Delta\omega.$$

The solid angle intercepted by A_d with respect to a point on A_s is

$$\Delta\omega = \frac{A_d}{r^2} = 10^{-6} \text{ sr}.$$

Since the surface is diffuse it follows from Eq. 12.24 that

$$I_{e+r} = \frac{J}{\pi}$$

where, since the surface is opaque and gray (ε = α = 1 - ρ),

$$J = E + \rho G = \varepsilon E_b + (1 - \varepsilon) G.$$

Substituting for E_b from Eq. 12.28

$$J = \varepsilon \sigma T_s^4 + (1 - \varepsilon) G = 0.7 \times 5.67 \times 10^{-8} \frac{W}{m^2 \cdot K^4} (500K)^4 + 0.3 \times 1500 \frac{W}{m^2}$$

or

$$J = (2481 + 450) W/m^2 = 2931 W/m^2.$$

Hence

$$I_{e+r} = \frac{2931 W/m^2}{\pi \text{ sr}} = 933 W/m^2 \cdot sr$$

and

$$q_{s-d} = 933 \text{ W/m}^2 \cdot \text{sr} \left(10^{-4} \text{ m}^2 \times 0.866\right) 10^{-6} \text{sr} = 8.08 \times 10^{-8} \text{ W}.$$ <

PROBLEM 12.81

KNOWN: Small, diffuse, gray block with ε = 0.92 at 35°C is located within a large oven whose walls are at 175°C with ε = 0.85.

FIND: Radiant power reaching detector when viewing (a) a deep hole in the block and (b) an area on the block's surface.

SCHEMATIC:

ASSUMPTIONS: (1) Block is isothermal, diffuse, gray and small compared to the enclosure, (2) Oven is isothermal enclosure.

ANALYSIS: (a) The small, deep hole in the isothermal block approximates a blackbody at T_s. The radiant power to the detector can be determined from Eq. 12.54 written in the form:

$$q = I_e \cdot A_t \cdot \omega_t = \frac{\sigma T_s^4}{\pi} \cdot A_t \cdot \omega_t$$

$$q = \frac{1}{\pi \text{ sr}}\left[5.67\times10^{-8}\times(35+273)^4\right]\frac{\text{W}}{\text{m}^2}\times\frac{\pi\left(3\times10^{-3}\right)^2 \text{m}^2}{4}\times0.001\text{sr} = 1.15\,\mu\text{W} \quad <$$

where $A_t = \pi D_t^2/4$. Note that the hole diameter must be greater than 3mm diameter.

(b) When the detector views an area on the surface of the block, the radiant power reaching the detector will be due to emission and reflected irradiation originating from the enclosure walls. In terms of the radiosity, Section 12.24, we can write using Eq. 12.24,

$$q = I_{e+r} \cdot A_t \cdot \omega_t = \frac{J}{\pi} \cdot A_t \cdot \omega_t.$$

Since the surface is diffuse and gray, the radiosity can be expressed as

$$J = \varepsilon E_b(T_s) + \rho G = \varepsilon E_b(T_s) + (1-\varepsilon)E_b(T_{sur})$$

recognizing that ρ = 1 - ε and $G = E_b(T_{sur})$. The radiant power is

$$q = \frac{1}{\pi}\left[\varepsilon E_b(T_s) + (1-\varepsilon)E_b(T_{sur})\right] \cdot A_t \cdot \omega_t$$

$$q = \frac{1}{\pi \text{ sr}}\left[0.92\times5.67\times10^{-8}(35+273)^4 + (1-0.92)\times5.67\times10^{-8}(175+273)^4\right]\text{W/m}^2\times$$

$$\frac{\pi\left(3\times10^{-3}\right)^2 \text{m}^2}{4}\times0.001\text{sr} = 1.47\,\mu\text{W}. \quad <$$

COMMENTS: The effect of reflected irradiation when ε < 1 is important for objects in enclosures. The practical application is one of measuring temperature by radiation from objects within furnaces.

PROBLEM 12.82

KNOWN: Diffuse, gray opaque disk (1) coaxial with a ring-shaped disk (2), both with prescribed temperatures and emissivities. Cooled detector disk (3), also coaxially positioned at a prescribed location.

FIND: Rate at which radiation is incident on the detector due to emission and reflection from A_1.

SCHEMATIC:

ASSUMPTIONS: (1) A_1 is diffuse-gray, (2) A_2 is black, (3) A_1 and $A_3 << R^2$, the distance of separation, (4) $\Delta r << r_i$, such that $A_2 \approx 2\pi r_i \Delta r$, and (5) Backside of A_2 is insulated.

ANALYSIS: The radiant power leaving A_1 intercepted by A_3 is of the form

$$q_{1\to3} = (J_1/\pi) A_1 \cos\theta_1 \cdot \omega_{3-1}$$

where for this configuration of A_1 and A_3,

$$\theta_1 = 0^\circ \qquad \omega_{3-1} = A_3 \cos\theta_3 / (L_A + L_B)^2 \qquad \theta_3 = 0^\circ.$$

Hence,

$$q_{1\to3} = (J_1/\pi) A_1 \cdot A_3 / (L_A + L_B)^2 \qquad J_1 = \rho G_1 + \varepsilon E_b(T_1) = \rho G_1 + \varepsilon\sigma T_1^4.$$

The irradiation on A_1 due to emission from A_2, G_1, is

$$G_1 = q_{2\to1}/A_1 = \left(I_2 \cdot A_2 \cos\theta_2' \cdot \omega_{1-2}\right)/A_1$$

where

$$\omega_{1-2} = A_1 \cos\theta_1' / R^2$$

is constant over the surface A_2. From geometry,

$$\theta_1' = \theta_2' = \tan^{-1}\left[(r_i + \Delta r/2)/L_A\right] = \tan^{-1}\left[(0.500 + 0.005)/1.000\right] = 26.8^\circ$$

$$R = L_A/\cos\theta_1' = 1\,m/\cos 26.8^\circ = 1.12\,m.$$

Hence,

$$G_1 = \left(\sigma T_2^4/\pi\right) A_2 \cos 26.8^\circ \cdot \left[A_1 \cos 26.8^\circ/(1.12\,m)^2\right]/A_1 = 360.2\ W/m^2$$

using $A_2 = 2\pi r_i \Delta r = 3.142 \times 10^{-2}\ m^2$ and

$$J_1 = (1-0.3) \times 360.2\ W/m^2 + 0.3 \times 5.67 \times 10^{-8}\ W/m^2 \cdot K^4 (400\ K)^4 = 687.7\ W/m^2.$$

Hence the radiant power is

$$q_{1\to3} = \left(687.7\ W/m^2/\pi\right)\left[\pi (0.010\ m)^2/4\right]^2/(1\ m + 1\ m)^2 = 337.6 \times 10^{-9}\ W. \quad <$$

PROBLEM 12.83

KNOWN: Area and emissivity of opaque sample in hemispherical enclosure. Area and position of detector which views sample through an aperture. Sample and enclosure temperatures.

FIND: (a) Detector irradiation, (b) Spectral distribution and maximum intensities.

SCHEMATIC:

ASSUMPTIONS: (1) Diffuse-gray surfaces, (2) Hemispherical enclosure forms a blackbody cavity about the sample, $A_h >> A_s$, (3) Detector field of view is limited to sample surface.

ANALYSIS: (a) The irradiation can be evaluated as $G_d = q_{s\text{-}d}/A_d$ and $q_{s\text{-}d} = I_{s(e+r)}\, A_s\, \omega_{d\text{-}s}$. Evaluating parameters: $\omega_{d\text{-}s} \approx A_d/L^2 = 2\ \text{mm}^2/(300\ \text{mm})^2 = 2.22 \times 10^{-5}$ sr, find

$$I_{s(e)} = \frac{E_s}{p} = \frac{e_s s\, T_s^4}{p} = \frac{0.1\left(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)(400\ \text{K})^4}{p\ \text{sr}} = 46.2\ \text{W/m}^2\cdot\text{sr}$$

$$I_{s(r)} = \frac{r_s G_s}{p} = \frac{(1-e_s)\, s\, T_h^4}{p} = \frac{0.9\left(5.67\times10^{-8}\text{W/m}^2\cdot\text{K}^4\right)(273\ \text{K})^4}{p\ \text{sr}} = 90.2\ \text{W/m}^2\cdot\text{sr}$$

$$q_{s-d} = (46.2+90.2)\,\text{W/m}^2\cdot\text{sr}\left(5\times10^{-6}\text{m}^2\times2.22\times10^{-5}\text{sr}\right) = 1.51\times10^{-8}\ \text{W}$$

$$G_d = 1.51\times10^{-8}\,\text{W}/2\times10^{-6}\ \text{m}^2 = 7.57\times10^{-3}\ \text{W/m}^2. \qquad <$$

(b) Since $\lambda_{max} T = 2898\ \mu\text{m·K}$, it follows that $\lambda_{max(e)} = 2898\ \mu\text{m·K}/400\ \text{K} = 7.25\ \mu\text{m}$ and $\lambda_{max(r)} = 2898\ \mu\text{m·K}/273\ \text{K} = 10.62\ \mu\text{m}$.

<

$\lambda = 7.25\ \mu\text{m}$: Table 12.1 $\rightarrow I_{\lambda,b}(400\ \text{K}) = 0.722 \times 10^{-4}\ (5.67 \times 10^{-8})(400)^5 = 41.9\ \text{W/m}^2\cdot\mu\text{m·sr}$

$I_{\lambda,b}(273\ \text{K}) = 0.48 \times 10^{-4}\ (5.67 \times 10^{-8})(273)^5 = 4.1\ \text{W/m}^2\cdot\mu\text{m·sr}$

$I_\lambda = I_{\lambda,e} + I_{\lambda,r} = \varepsilon_s I_{\lambda,b}(400\ \text{K}) + \rho I_{\lambda,b}(273\text{K}) = 0.1 \times 41.9 + 0.9 \times 4.1 = 7.90\ \text{W/m}^2\cdot\mu\text{m·sr}$ <

$\lambda = 10.62\ \mu\text{m}$: Table 12.1 $\rightarrow I_{\lambda,b}(400\ \text{K}) = 0.53 \times 10^{-4}\ (5.67 \times 10^{-8})(400)^5 = 30.9\ \text{W/m}^2\cdot\mu\text{m·sr}$

$I_{\lambda,b}(273\ \text{K}) = 0.722 \times 10^{-4}\ (5.67 \times 10^{-8})(273)^5 = 6.2\ \text{W/m}^2\cdot\mu\text{m·sr}$

$I_\lambda = 0.1 \times 30.9 + 0.9 \times 6.2 = 8.68\ \text{W/m}^2\cdot\mu\text{m·sr}$. <

COMMENTS: Although Th is substantially smaller than T_s, the high sample reflectivity renders the reflected component of J_s comparable to the emitted component.

PROBLEM 12.84

KNOWN: Sample at T_s = 700 K with ring-shaped cold shield viewed normally by a radiation detector.

FIND: (a) Shield temperature, T_{sh}, required so that its emitted radiation is 1% of the total radiant power received by the detector, and (b) Compute and plot T_{sh} as a function of the sample emissivity for the range $0.05 \le \varepsilon \le 0.35$ subject to the parametric constraint that the radiation emitted from the cold shield is 0.05, 1 or 1.5% of the total radiation received by the detector.

SCHEMATIC:

ASSUMPTIONS: (1) Sample is diffuse and gray, (2) Cold shield is black, and (3) $A_d, D_s^2, D_t^2 \ll L_t^2$.

ANALYSIS: (a) The radiant power intercepted by the detector from within the target area is

$$q_d = q_{s\to d} + q_{sh\to d}$$

The contribution from the sample is

$$q_s \to d = I_{s,e} A_s \cos\theta_s \,\Delta\omega_{d-s} \qquad \theta_s = 0^\circ$$

$$I_{s,e} = \varepsilon_s E_b/\pi = \varepsilon_s \sigma T_s^4/\pi$$

$$\Delta\omega_{d-s} = \frac{A_d \cos\theta_d}{L_t^2} = \frac{A_d}{L_t^2} \qquad \theta_d = 0^\circ$$

$$q_{s\to d} = \varepsilon_s \sigma T_s^4 A_s A_d / \pi L_t^2 \tag{1}$$

The contribution from the ring-shaped cold shield is

$$q_{sh\to d} = I_{sh,e} A_{sh} \cos\theta_{sh} \Delta\omega_{d-sh}$$

$$I_{sh,e} = E_b/\pi = \sigma T_{sh}^4/\pi$$

and, from the geometry of the shield -detector,

$$A_{sh} = \frac{\pi}{4}\left(D_t^2 - D_s^2\right)$$

$$\cos\theta_{sh} = L_t \Big/ \left[\left(\bar{D}/2\right)^2 = L_t^2\right]^{1/2}$$

Continued...

PROBLEM 12.84 (Cont.)

where $\bar{D} = (D_s + D_t)/2$

$$\Delta\omega_{d-sh} = \frac{A_d \cos\theta_d}{R^2} \qquad \cos\theta_d = \cos\theta_{sh}$$

where $R = \left[L_t^2 + \bar{D}^2\right]^{1/2}$

$$q_{sh\to d} = \frac{\sigma T_{sh}^4}{\pi} A_{sh} \left[\frac{L_t}{\left[((D_s + D_t)/4)^2 + L_t^2\right]^{1/2}}\right]^2 \frac{A_d}{\left[((D_s + D_t)/4)^2 + L_t^2\right]} \qquad (2)$$

The requirement that the emitted radiation from the cold shield is 1% of the total radiation intercepted by the detector is expressed as

$$\frac{q_{sh-d}}{q_{tot}} = \frac{q_{sh-d}}{q_{sh-d} + q_{s-d}} = 0.01 \qquad (3)$$

By evaluating Eq. (3) using Eqs. (1) and (3), find

$$T_{sh} = 134\,K$$ <

(b) Using the foregoing equations in the IHT workspace, the required shield temperature for q_{sh-d}/q_{tot} = 0.5, 1 or 1.5% was computed and plotted as a function of the sample emissivity.

As the shield emission-to-total radiant power ratio decreases (from 1.5 to 0.5%) , the required shield temperature decreases. The required shield temperature increases with increasing sample emissivity for a fixed ratio.

PROBLEM 12.84

KNOWN: Sample at T_s = 700 K with ring-shaped cold shield viewed normally by a radiation detector.

FIND: (a) Shield temperature, T_{sh}, required so that its emitted radiation is 1% of the total radiant power received by the detector, and (b) Compute and plot T_{sh} as a function of the sample emissivity for the range $0.05 \le \varepsilon \le 0.35$ subject to the parametric constraint that the radiation emitted from the cold shield is 0.05, 1 or 1.5% of the total radiation received by the detector.

SCHEMATIC:

Radiation detector, $A_d = 4 \times 10^{-6}\ m^2$

L_t = 1 m

Sample, T_s = 700 K, ε_s = 0.1

Cold shield, T_{sh}, ε = 1

D_s = 20 mm

D_t = 40 mm

ASSUMPTIONS: (1) Sample is diffuse and gray, (2) Cold shield is black, and (3) $A_d, D_s^2, D_t^2 << L_t^2$.

ANALYSIS: (a) The radiant power intercepted by the detector from within the target area is

$$q_d = q_{s\to d} + q_{sh\to d}$$

The contribution from the sample is

$$q_s \to d = I_{s,e} A_s \cos\theta_s\ \Delta\omega_{d-s} \qquad \theta_s = 0^\circ$$

$$I_{s,e} = \varepsilon_s E_b / \pi = \varepsilon_s \sigma T_s^4 / \pi$$

$$\Delta\omega_{d-s} = \frac{A_d \cos\theta_d}{L_t^2} = \frac{A_d}{L_t^2} \qquad \theta_d = 0^\circ$$

$$q_{s\to d} = \varepsilon_s \sigma T_s^4 A_s A_d / \pi L_t^2 \qquad (1)$$

The contribution from the ring-shaped cold shield is

θ_d, R, L_t, θ_{sh}, D_s, D_t

$$q_{sh\to d} = I_{sh,e} A_{sh} \cos\theta_{sh} \Delta\omega_{d-sh}$$

$$I_{sh,e} = E_b / \pi = \sigma T_{sh}^4 / \pi$$

and, from the geometry of the shield -detector,

$$A_{sh} = \frac{\pi}{4}\left(D_t^2 - D_s^2\right)$$

$$\cos\theta_{sh} = L_t \Big/ \left[\left(\bar{D}/2\right)^2 = L_t^2\right]^{1/2}$$

Continued...

PROBLEM 12.84 (Cont.)

where $\bar{D} = (D_s + D_t)/2$

$$\Delta\omega_{d-sh} = \frac{A_d \cos\theta_d}{R^2} \qquad \cos\theta_d = \cos\theta_{sh}$$

where $R = \left[L_t^2 + \bar{D}^2\right]^{1/2}$

$$q_{sh\to d} = \frac{\sigma T_{sh}^4}{\pi} A_{sh} \left[\frac{L_t}{\left[((D_s + D_t)/4)^2 + L_t^2\right]^{1/2}}\right]^2 \frac{A_d}{\left[((D_s + D_t)/4)^2 + L_t^2\right]} \qquad (2)$$

The requirement that the emitted radiation from the cold shield is 1% of the total radiation intercepted by the detector is expressed as

$$\frac{q_{sh-d}}{q_{tot}} = \frac{q_{sh-d}}{q_{sh-d} + q_{s-d}} = 0.01 \qquad (3)$$

By evaluating Eq. (3) using Eqs. (1) and (3), find

$$T_{sh} = 134\,K$$ <

(b) Using the foregoing equations in the IHT workspace, the required shield temperature for q_{sh-d}/q_{tot} = 0.5, 1 or 1.5% was computed and plotted as a function of the sample emissivity.

Shield temperature, Th (K)
Sample emissivity, epss
Shield /total radiant power = 0.5 %
1.0%
1.5 %

As the shield emission-to-total radiant power ratio decreases (from 1.5 to 0.5%) , the required shield temperature decreases. The required shield temperature increases with increasing sample emissivity for a fixed ratio.

PROBLEM 12.85

KNOWN: Infrared scanner (radiometer) with a 3- to 5-micrometer spectral bandpass views a metal plate maintained at T_s = 327°C having four diffuse, gray coatings of different emissivities. Surroundings at T_{sur} = 87°C.

FIND: (a) Expression for the scanner output signal, S_o, in terms of the responsivity, R ($\mu V \cdot m^2/W$), the black coating ($\varepsilon_o = 1$) emissive power and appropriate band emission fractions; assuming R = 1 $\mu V \cdot m^2/W$, evaluate $S_o(V)$; (b) Expression for the output signal, S_c, in terms of the responsivity R, the blackbody emissive power of the coating, the blackbody emissive power of the surroundings, the coating emissivity, ε_c, and appropriate band emission fractions; (c) Scanner signals, S_c (μV), when viewing with emissivities of 0.8, 0.5 and 0.2 assuming R = 1 $\mu V \cdot m^2/W$; and (d) Apparent temperatures which the scanner will indicate based upon the signals found in part (c) for each of the three coatings.

SCHEMATIC:

T_{sur} = 27 °C

$E_b(T_s)$

T_s= 327 °C, ε_o = 1

Black coating, ε_o

G

$\rho_c G_{sur}$

$\varepsilon_c E_b(T_s)$

T_s= 327 °C, ε_c = 1

Coating, $\varepsilon_c < 1$

ASSUMPTIONS: (1) Plate has uniform temperature, (2) Surroundings are isothermal and large compared to the plate, and (3) Coatings are diffuse and gray so that $\varepsilon = \alpha$ and $\rho = 1 - \varepsilon$.

ANALYSIS: (a) When viewing the black coating ($\varepsilon_o = 1$), the scanner output signal can be expressed as

$$S_o = R F_{(\lambda_1 - \lambda_2, T_s)} E_b(T_s) \qquad (1)$$

where R is the responsivity ($\mu V \cdot m^2/W$), $E_b(T_s)$ is the blackbody emissive power at T_s and $F_{(\lambda_1 - \lambda_2, T_s)}$ is the fraction of the spectral band between λ_1 and λ_2 in the spectrum for a blackbody at T_s,

$$F_{(\lambda_1 - \lambda_2, T_s)} = F_{(0 - \lambda_2, T_s)} - F_{(0 - \lambda_1, T_s)} \qquad (2)$$

where the band fractions Eq. 12.38 are evaluated using Table 12.1 with $\lambda_1 T_s = 3\ \mu m\ (327 + 273)K = 1800\ \mu m \cdot K$ and $\lambda_2 T_s = 5\ \mu m\ (327 + 273) = 3000\ \mu m \cdot K$. Substituting numerical values with R = 1 $\mu V \cdot m^2/W$, find

$$S_o = 1 \mu V \cdot m^2 / W [0.2732 - 0.0393] 5.67 \times 10^{-8}\ W/m^2 \cdot K^4\ (600 K)^4$$

$$S_o = 1718\ \mu V$$ <

(b) When viewing one of the coatings ($\varepsilon_c < \varepsilon_o = 1$), the scanner output signal as illustrated in the schematic above will be affected by the emission and reflected irradiation from the surroundings,

$$S_c = R \left\{ F_{(\lambda_1 - \lambda_2, T_s)} \varepsilon_c E_b(T_s) + F_{(\lambda_1 - \lambda_2, T_{sur})} \rho_c G_c \right\} \qquad (3)$$

where the reflected irradiation parameters are

Continued...

$$\rho_c = 1 - \varepsilon_c \qquad G_c = \sigma T_{sur}^4 \tag{4,5}$$

and the related band fractions are

$$F_{(\lambda_1 - \lambda_2, T_{sur})} = F_{(0-\lambda_2, T_{sur})} - F_{(0-\lambda_1, T_{sur})} \tag{6}$$

Combining Eqs. (2-6) above, the scanner output signal when viewing a coating is

$$S_c = R\left\{\left[F_{(0-\lambda_2 T_s)} - F_{(0-\lambda_1 T_s)}\right]\varepsilon_c \sigma T_s^4 + \left[F_{(0-\lambda_2 T_{sur})} - F_{(0-\lambda_2 T_{sur})}\right](1-\varepsilon_c)\sigma T_{sur}^4\right\} \tag{7}$$

(c) Substituting numerical values into Eq. (7), find

$$S_c = 1\,\mu V \cdot m^2/W\left\{[0.2732 - 0.0393]\varepsilon_c \sigma (600K)^4 + [0.0393 - 0.0010](1-\varepsilon_c)\sigma (360K)^4\right\}$$

where for $\lambda_2 T_{sur}$ = 5 μm × 360 K = 1800 μm·K, $F_{(0-\lambda_2 T_{sur})}$ = 0.0393 and $\lambda_1 T_{sur}$ = 3 μm × 360 K = 1080 μm·K, $F_{(0-\lambda_1 T_{sur})}$ = 0.0010. For ε_c = 0.80, find

$$S_c(\varepsilon_c = 0.8) = 1\,\mu V \cdot m^2/W\{1375 + 7.295\}W/m^2 = 1382\,\mu V$$ <

$$S_c(\varepsilon_c = 0.5) = 1\,\mu V \cdot m^2/W\{859.4 + 18.238\}W/m^2 = 878\,\mu V$$ <

$$S_c(\varepsilon_c = 0.2) = 1\,\mu V \cdot m^2/W\{343.8 + 29.180\}W/m^2 = 373\,\mu V$$ <

(d) The scanner calibrated against a black surface (ε_1 = 1) interprets the radiation reaching the detector by emission and reflected radiation from a coating target ($\varepsilon_c < \varepsilon_o$) as that from a blackbody at an apparent temperature T_a. That is,

$$S_c = RF_{(\lambda_1 - \lambda_2, T_a)} E_b(T_a) = R\left[F_{(0-\lambda_2 T_a)} - F_{(0-\lambda_1 T_a)}\right]\sigma T_a^4 \tag{8}$$

For each of the coatings in part (c), solving Eq. (8) using the IHT workspace with the *Radiation Tool*, *Band Emission Factor*, the following results were obtained,

ε_c	S_c (μV)	T_a (K)	$T_a - T_s$ (K)
0.8	1382	579.3	-20.7
0.5	878	539.2	-60.8
0.2	373	476.7	-123.3

COMMENTS: (1) From part (c) results for S_c, note that the contribution of the reflected irradiation becomes relatively more significant with lower values of ε_c.

(2) From part (d) results for the apparent temperature, note that the error, $(T - T_a)$, becomes larger with decreasing ε_c. By rewriting Eq. (8) to include the emissivity of the coating,

$$S_c' = R\left[F_{(0-\lambda_2 T_a)} - F_{(0-\lambda_1 T_a)}\right]\varepsilon_c \sigma T_a^4$$

The apparent temperature T_a' will be influenced only by the reflected irradiation. The results correcting only for the emissivity, ε_c, are

ε_c	0.8	0.5	0.2
$T_a'(K)$	600.5	602.2	608.5
$T_a' - T_s(K)$	+0.5	+2.2	+8.5

PROBLEM 12.86

KNOWN: Billet at T_t = 500 K which is diffuse, gray with emissivity ε_t = 0.9 heated within a large furnace having isothermal walls at T_f = 750 K with diffuse, gray surface of emissivity ε_f = 0.8. Radiation detector with sensitive area $A_d = 5.0 \times 10^{-4}$ m² positioned normal to and at a distance R = 0.5 m from the billet. Detector receives radiation from a billet target area $A_t = 3.0 \times 10^{-6}$ m².

FIND: (a) Symbolic expressions and numerical values for the following radiation parameters associated with the target surface (t): irradiation on the target, G_t; intensity of the reflected irradiation leaving the target, $I_{t,ref}$; emissive power of the target, E_t; intensity of the emitted radiation leaving the target, $I_{t,emit}$; and radiosity of the target J_t; and (b) Expression and numerical value for the radiation which leaves the target in the spectral region $\lambda \geq 4$ μm and is intercepted by the radiation detector, $q_{t \to d}$; write the expression in terms of the target reflected and emitted intensities $I_{t,ref}$ and $I_{t,emit}$, respectively, as well as other geometric and radiation parameters.

SCHEMATIC:

ASSUMPTIONS: (1) Furnace wall is isothermal and large compared to the billet, (2) Billet surface is diffuse gray, and (3) $A_t, A_d << R^2$.

ANALYSIS: (a) Expressions and numerical values for radiation parameters associated with the target are:

Irradiation, G_t: due to blackbody emission from the furnace walls which are isothermal and large relative to the billet target,

$$G_t = E_b(T_f) = \sigma T_f^4 = 5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4\ (750\ \text{K})^4 = 17,940\ \text{W/m}^2 \qquad <$$

Intensity of reflected irradiation, $I_{t,ref}$: since the billet is diffuse, $I = G_i/\pi$ from Eq. 12.19, and diffuse-gray, $\rho_t = 1 - \varepsilon_t$,

$$I_{t,ref} = \rho_t G_t / \pi = (1 - \varepsilon_t) G_t / \pi$$

$$I_{t,ref} = (1 - 0.9) \times 19,740\ \text{W/m}^2 / \pi = 571\ \text{W/m}^2 \cdot \text{sr} \qquad <$$

Emissive power, E_t: from the Stefan-Boltzmann law, Eq. 12.28, and the definition of the total emissivity, Eq. 12.37,

$$E_t = \varepsilon_t E_b(T_t) = \varepsilon_t \sigma T_t^4$$

$$E_t = 0.9 \times 5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4\ (500\ \text{K})^4 = 3189\ \text{W/m}^2 \qquad <$$

Continued...

PROBLEM 12.86 (Cont.)

Intensity of emitted radiation, $I_{t,emit}$: since the billet is diffuse, $I_e = E/\pi$ from Eq. 12.14,

$$I_{t,emit} = E_t/\pi = 3189\,\text{W/m}^2/\pi = 1015\,\text{W/m}^2\cdot\text{sr}$$ <

Radiosity, J_t: the radiosity accounts for the emitted radiation and reflected portion of the irradiation; for the diffuse surface, from Eq. 12.24,

$$J_t = \pi\left(I_{t,ref} + I_{t,emit}\right)$$

$$J_t = \pi\,\text{sr}\left(571+1015\right)\text{W/m}^2\cdot\text{sr} = 4983\,\text{W/m}^2$$ <

(b) The radiant power in the spectral region $\lambda \geq 4\ \mu$m leaving the target which is intercepted by the detector follows from Eq. 12.5,

$$q_{t\to d} = \left[F_{ref}I_{t,ref} + F_{emit}I_{t,emit}\right]A_t\cos\theta_t\omega_{d-t}$$

The F factors account for the fraction of the total spectral region for $\lambda \geq 4\ \mu$m,

$$F_{ref} = 1 - F\left(0 - \lambda T_f\right) = 1 - 0.2732 = 0.727$$

$$F_{emit} = 1 - F\left(0 - \lambda T_t\right) = 1 - 0.06673 = 0.933$$

where from Eq. 12.30 and Table 12.1, for $\lambda T_f = 4\ \mu\text{m} \times 750$ K = 3000 μm·K, $F(0 - \lambda T_f) = 0.2732$ and for $\lambda T_t = 4\ \mu\text{m} \times 500$ K = 2000 μm·K, $F(0 - \lambda T_t) = 0.06673$. Since the radiation detector is normal to the billet, $\cos\theta_t = 1$. The solid angle subtended by the detector area with respect to the target area is

$$\omega_{d-t} = \frac{A_d\cos\theta_d}{R^2} = \frac{5\times10^{-4}\,\text{m}^2\times1}{\left(0.5\,\text{m}\right)^2} = 2.00\times10^{-3}\text{sr}$$

Hence, the radiant power is

$$q_{t\to d} = \left(0.727\times571 + 0.933\times1015\right)\text{W/m}^2\cdot\text{sr}\times3.0\times10^{-6}\,\text{m}^2\times1\times2.00\times10^{-3}\text{sr}$$

$$q_{t\to d} = \left(2.491 + 5.682\right)\times10^{-6}\,\text{W} = 8.17\,\mu\text{W}$$ <

COMMENTS: (1) Why doesn't the emissivity of the furnace walls, ε_f, affect the target irradiation?

(2) Note the importance of the diffuse, gray assumption for the billet target surface. In what ways was the assumption used in the analysis?

(3) From the calculation of the radiant power to the detector, $q_{t\to d}$, note that the contribution of the reflected irradiation is nearly a third of the total.

PROBLEM 12.87

KNOWN: Painted plate located inside a large enclosure being heated by an infrared lamp bank.

FIND: (a) Lamp irradiation required to maintain plot at $T_s = 140^oC$ for the prescribed convection and enclosure irradiation conditions, (b) Compute and plot the lamp irradiation, G_{lamp}, required as a function of the plate temperature, T_s, for the range $100 \le T_s \le 300\ ^oC$ and for convection coefficients of h = 15, 20 and 30 $W/m^2 \cdot K$, and (c) Compute and plot the air stream temperature, T_∞, required to maintain the plate at 140^oC as a function of the convection coefficient h for the range $10 \le h \le 30\ W/m^2 \cdot K$ with a lamp irradiation $G_{lamp} = 3000\ W/m^2$.

SCHEMATIC:

Air
$T_\infty = 27\ ^oC$
$\bar{h} = 20\ W/m^2 \cdot K$
q''_{conv}
$G_{wall} = 450\ W/m^2$
G_{lamp}
E_s
Plate
$T_s = 140\ ^oC$
$\varepsilon_s = 0.8$
$\alpha_{wall} = 0.7$
$\alpha_{lamp} = 0.6$

ASSUMPTIONS: (1) Steady-state conditions, (2) No losses on backside of plate.

ANALYSIS: (a) Perform an energy balance on the plate, per unit area,

$$\dot{E}_{in} - \dot{E}_{out} = 0 \tag{1}$$

$$\alpha_{wall} \cdot G_{wall} + \alpha_{lamp} G_{lamp} - q''_{conv} - E_s = 0 \tag{2}$$

where the emissive power of the surface and convective fluxes are

$$E_s = \varepsilon_s E_b(T_s) = \varepsilon_s \cdot \sigma T_s^4 \qquad q''_{conv} = h(T_s - T_\infty) \tag{3,4}$$

Substituting values, find the lamp irradiation

$$0.7 \times 450\ W/m^2 + 0.6 \times G_{lamp} - 20\ W/m^2 \cdot K(413 - 300\ K$$
$$-0.8 \times 5.67 \times 10^{-8}\ W/m^2 \cdot K^4 (413\ K)^4 = 0 \tag{5}$$

$$G_{lamp} = 5441\ W/m^2$$ <

(b) Using the foregoing equations in the IHT workspace, the irradiation, G_{lamp}, required to maintain the plate temperature in the range $100 \le T_s \le 300\ ^oC$ for selected convection coefficients was computed. The results are plotted below.

Continued...

PROBLEM 12.87 (Cont.)

As expected, to maintain the plate at higher temperatures, the lamp irradiation must be increased. At any plate operating temperature condition, the lamp irradiation must be increased if the convection coefficient increases. With forced convection (say, $h \geq 20\ W/m^2 \cdot K$) of the airstream at 27^oC, excessive irradiation levels are required to maintain the plate above the cure temperature of 140^oC.

(c) Using the IHT model developed for part (b), the airstream temperature, T_∞, required to maintain the plate at $T_s = 140^oC$ as a function of the convection coefficient with $G_{lamp} = 3000\ W/m^2 \cdot K$ was computed and the results are plotted below.

Air temperature, Tinf (C)
120
100
80
60
10
20
30
Convection coefficient, h (W/m^2.K)

As the convection coefficient increases, for example by increasing the airstream velocity over the plate, the required air temperature must increase. Give a physical explanation for why this is so.

COMMENTS: (1) For a spectrally selective surface, we should expect the absorptivity to depend upon the spectral distribution of the source and $\alpha \neq \varepsilon$.

(2) Note the new terms used in this problem; use your Glossary, Section 12.9 to reinforce their meaning.

PROBLEM 12.88

KNOWN: Small sample of reflectivity, ρ_λ, and diameter, D, is irradiated with an isothermal enclosure at T_f.

FIND: (a) Absorptivity, α, of the sample with prescribed ρ_λ, (b) Emissivity, ε, of the sample, (c) Heat removed by coolant to the sample, (d) Explanation of why system provides a measure of ρ_λ.

SCHEMATIC:

ASSUMPTIONS: (1) Sample is diffuse and opaque, (2) Furnace is an isothermal enclosure with area much larger than the sample, (3) Aperture of furnace is small.

ANALYSIS: (a) The absorptivity, α, follows from Eq. 12.42, where the irradiation on the sample is G = $E_b(T_f)$ and $\alpha_\lambda = 1 - \rho_\lambda$.

$$a = \int_0^\infty a_l G_l \, dl / G = \int_0^\infty (1 - r_l) E_{l,b}(l, 1000\text{K}) \, dl / E_b(1000\text{K})$$

$$a = (1 - r_{l,1}) F_{(0\to l_1)} + (1 - r_{l,2}) \left[1 - F_{(0\to l_1)}\right].$$

Using Table 12.1 for $\lambda_1 T_f = 4 \times 1000 = 4000\ \mu\text{m}\cdot\text{K}$, $F_{(0-\lambda)} = 0.491$ giving

$$a = (1 - 0.2) \times 0.491 + (1 - 0.8) \times (1 - 0.491) = 0.49.$$ <

(b) The emissivity, ε, follows from Eq. 12.37 with $\varepsilon_\lambda = \alpha_\lambda = 1 - \rho_\lambda$ since the sample is diffuse.

$$e = E(T_s)/E_b(T_s) = \int_0^\infty e_l \, E_{l,b}(l, 300\text{K}) \, dl / E_b(300\text{K})$$

$$e = (1 - r_{l,1}) F_{(0-l_1)} + (1 - r_{l,2}) \left[1 - F_{(0\to l_1)}\right].$$

Using Table 12.1 for $\lambda_1 T_s = 4 \times 300 = 1200\ \mu\text{m}\cdot\text{K}$, $F_{(0-\lambda)} = 0.002$ giving

$$e = (1 - 0.2) \times 0.002 + (1 - 0.8) \times (1 - 0.002) = 0.20.$$

(c) Performing an energy balance on the sample, the heat removal rate by the cooling water is

$$q_{cool} = A_s \left[a G + q''_{conv} - e E_b(T_s) \right]$$

where $G = E_b(T_f) = E_b(1000\text{K})$

$$q''_{conv} = h(T_f - T_s) \qquad A_s = p D^2 / 4$$

$$q_{cool} = (p/4)(0.03\text{m})^2 \Big[0.49 \times 5.67 \times 10^{-8} \text{W/m}^2 \cdot \text{K}^4 \times (1000\text{K})^4$$

$$+ 10\,\text{W/m}^2 \cdot \text{K}(1000 - 300)\text{K} - 0.20 \times 5.67 \times 10^{-8} \text{W/m}^2 \cdot \text{K}^4 \times (300\text{K})^4 \Big] = 24.4\ \text{W}.$$ <

(d) Assume that reflection makes the dominant contribution to the radiosity of the sample. When viewing in the direction A, the spectral radiant power is proportional to $\rho_\lambda G_\lambda$. In direction B, the spectral radiant power is proportional to $E_{\lambda,b}(T_f)$. Noting that $G_\lambda = E_{\lambda,b}(T_f)$, the ratio gives ρ_λ.

PROBLEM 12.89

KNOWN: Small, opaque surface initially at 1200 K with prescribed α_λ distribution placed in a large enclosure at 2400 K.

FIND: (a) Total, hemispherical absorptivity of the sample surface, (b) Total, hemispherical emissivity, (c) α and ε after long time has elapsed, (d) Variation of sample temperature with time.

SCHEMATIC:

Enclosure, T_{sur} = 2400 K, ε = 0.2

Surface, α_λ
T_s = 1200 K (initial)
D = 10 mm

α_λ: 0.8, α_2; 0.1, α_1; 0; 2; λ (μm)

ASSUMPTIONS: (1) Surface is diffusely radiated, (2) Enclosure is much larger than surface and at a uniform temperature.

PROPERTIES: *Table A.1*, Tungsten (T ≈ 1800 K): ρ = 19,300 kg/m^3, c_p = 163 J/kg·K, k ≈ 102 W/m·K.

ANALYSIS: (a) The total, hemispherical absorptivity follows from Eq. 12.46, where $G_\lambda = E_{\lambda,b}(T_{sur})$. That is, the irradiation corresponds to the spectral emissive power of a blackbody at the enclosure temperature and is independent of the enclosure emissivity.

$$\alpha = \int_0^\infty \alpha_\lambda G_\lambda \, d\lambda \Big/ \int_0^\infty G_\lambda \, d\lambda = \int_0^\infty \alpha_\lambda E_{\lambda,b}(\lambda, T_{sur}) d\lambda / E_b(T_{sur})$$

$$\alpha = \alpha_1 \int_0^{2\mu m} E_{\lambda,b}(\lambda, T_{sur}) d\lambda / \sigma T_{sur}^4 + \alpha_2 \int_{2\mu m}^\infty E_{\lambda,b}(\lambda, T_{sur}) d\lambda \Big/ \sigma T_{sur}^4$$

$$\alpha = \alpha_1 F_{(0\to 2\mu m)} + \alpha_2 \left[1 - F_{(0\to 2\mu m)}\right] = 0.1 \times 0.6076 + 0.8[1 - 0.6076] = 0.375$$ <

where at $\lambda T = 2 \times 2400 = 4800\, \mu m \cdot K$, $F_{(0\to 2\mu m)} = 0.6076$ from Table 12.1.

(b) The total, hemispherical emissivity follows from Eq. 12.38,

$$\varepsilon = \int_0^\infty \varepsilon_\lambda E_{\lambda,b}(\lambda, T_s) d\lambda \Big/ \int_0^\infty E_{\lambda,b}(\lambda, T_s) d\lambda .$$

Since the surface is diffuse, $\varepsilon_\lambda = \alpha_\lambda$ and the integral can be expressed as

$$\varepsilon = \alpha_1 \int_0^{2\mu m} E_{\lambda,b}(\lambda, T_s) d\lambda \Big/ \sigma T_s^4 + \alpha_2 \int_{2\mu m}^\infty E_{\lambda,b}(\lambda, T_s) d\lambda \Big/ \sigma T_s^4$$

$$\varepsilon = \alpha_1 F_{(0\to 2\mu m)} + \alpha_2 \left[1 - F_{(0\to 2\mu m)}\right] = 0.1 \times 0.1403 + 0.8[1 - 0.1403] = 0.702$$ <

where at $\lambda T = 2 \times 1200 = 2400\, \mu m \cdot K$, find $F_{(0\to 2\mu m)} = 0.1403$ from Table 12.1.

(c) After a long period of time, the surface will be at the temperature of the enclosure. This condition of thermal equilibrium is described by Kirchoff's law, for which

$$\varepsilon = \alpha = 0.375.$$ <

Continued...

PROBLEM 12.89 (Cont.)

(d) Using the IHT *Lumped Capacitance* Model, the energy balance relation is of the form

$$\rho c_p \forall \frac{dT}{dt} = A_s[\alpha G - \varepsilon(T)E_b(T)]$$

where T = T_s, $\forall = \pi D^3/6$, $A_s = \pi D^2$ and $G = \sigma T_{sur}^4$. Integrating over time in increments of Δt = 0.5s and using the *Radiation* Toolpad to determine ε(t),
the following results are obtained.

The temperature of the specimen increases rapidly with time and achieves a value of 2399 K within t ≈ 47s. The emissivity decreases with increasing time, approaching the absorptivity as T approaches T_{sur}.

COMMENTS: (1) Recognize that α always depends upon the spectral irradiation distribution, which, in this case, corresponds to emission from a blackbody at the temperature of the enclosure.

(2) With $h_r = \varepsilon\sigma(T + T_{sur})(T^2 + T_{sur}^2) = 0.375\sigma 4T_{sur}^3 = 1176\, W/m^2 \cdot K$, $Bi = h_r\,(r_o/3)/k$
$= (1176\, W/m^2 \cdot K)\ \ .667 \times 10^{-3}\, m/102\, W/m \cdot K = 0.0192 << 1$, use of the lumped capacitance model is justified.

PROBLEM 12.90

KNOWN: Vertical plate of height L = 2 m suspended in quiescent air. Exposed surface with diffuse coating of prescribed spectral absorptivity distribution subjected to simulated solar irradiation, $G_{S,\lambda}$. Plate steady-state temperature T_s = 400 K.

FIND: (a) Plate emissivity, ε, plate absorptivity, α, plate irradiation, G, and using an appropriate correlation, the free convection coefficient, $\overline{h}$, and (b) Plate steady-state temperature if the irradiation found in part (a) were doubled.

SCHEMATIC:

L = 2 m
$G_{s,\lambda}$
Plate, α_λ
T_s = 400 K
Air
T_∞ = 300 K
0.9
0.1
α_1
α_2
α_λ
1
λ (μm)

ASSUMPTIONS: (1) Steady-state conditions, (2) Ambient air is extensive, quiescent, (3) Spectral distribution of the simulated solar irradiation, $G_{S,\lambda}$, proportional to that of a blackbody at 5800 K, (4) Coating is opaque, diffuse, and (5) Plate is perfectly insulated on the edges and the back side, and (6) Plate is isothermal.

PROPERTIES: *Table A.4*, Air (T_f = 350 K, 1 atm): $\nu = 20.92 \times 10^{-6}$ m²/s, k = 0.030 W/m·K, $\alpha = 29.90 \times 10^{-6}$ m²/s, Pr = 0.700.

ANALYSIS: (a) Perform an energy balance on the plate as shown in the schematic on a per unit plate width basis,

E
G
T_∞, $\overline{h}$
q'_{conv}
T_s = 400 K

$$\dot{E}_{in} - E_{out} = 0$$

$$\left[\alpha G - \varepsilon\sigma T_s^4 - \overline{h}\left(T_s - T_\infty\right)\right]L = 0 \qquad (1)$$

where α and ε are determined from knowledge of α_λ and $\overline{h}$ is estimated from an appropriate correlation.

Plate total emmissivity: From Eq. 12.38 written in terms of the band emission factor, $F_{(0-\lambda T)}$, Eq. 12.30,

$$\varepsilon = \alpha_1 F_{(0-\lambda_1 T_s)} + \alpha_2\left[1 - F_{(0-\lambda_1 T_s)}\right]$$

$$\varepsilon = 0.9\times 0 + 0.1[1-0] = 0.1 \qquad <$$

where, from Table 12.1, with λ, T_s = 1μm × 400 K = 400 μm·K, F(0-λT) = 0.000.

Plate absorptivity: With the spectral distribution of simulated solar irradiation proportional to E_b (T_s = 5800 K),

Continued...

$$\alpha = \alpha_1 F_{(0-\lambda_1 T_s)} + \alpha_2 \left[1 - F_{(0-\lambda_1 T_s)}\right]$$

$$\alpha = 0.9 \times 0.7202 + 0.1[1 - 0.7202] = 0.676$$ <

where, from Table 12.1, with $\lambda_1 T_s = 5800$ μm·K, $F_{(0-\lambda T)} = 0.7202$.

Estimating the free convection coefficient, $\bar{h}$*:* Using the Churchill-Chu correlation Eq. (9.26) with properties evaluated at $T_f = (T_s + T_\infty)/2 = 350$ K,

$$Ra_L = \frac{g\beta(T_s - T_\infty)L^3}{\nu\alpha}$$

$$Ra_L = \frac{9.8\,m/s^2\,(1/350\,K) \times 100\,K\,(2\,m)^3}{20.92 \times 10^{-6}\,m^2/s \times 29.90 \times 10^{-6}\,m^2/s} = 3.581 \times 10^{10}$$

$$\overline{Nu}_L = \left\{0.825 + \frac{0.387 Ra_L^{1/6}}{\left[1 + (0.492/Pr)^{9/16}\right]^{8/27}}\right\}^2$$

$$\overline{Nu}_L = \left\{0.825 + \frac{0.387 Ra_L^{1/6}}{\left[1 + (0.492/0.700)^{9/16}\right]^{8/27}}\right\}^2 = 377.6$$

$$\bar{h}_L = \overline{Nu}_L\, k/L = 377.6 \times 0.030\,W/m \cdot K/2\,m = 5.66\,W/m^2 \cdot K$$ <

Irradiation on the Plate: Substituting numerical values into Eq. (1), find G.

$$0.676G - 0.1 \times 5.67 \times 10^{-8}\,W/m^2 \cdot K^4\,(400\,K)^4 - 5.66\,W/m^2 \cdot K\,(400 - 300)\,K = 0$$

$$G = 1052\,W/m^2$$ <

(b) If the irradiation were doubled, $G = 2104$ W/m^2, the plate temperature T_s will increase, of course, requiring re-evaluation of ε and $\bar{h}$. Since α depends upon the irradiation distribution, and not the plate temperature, α will remain the same. As a first approximation, assume ε = 0.1 and $\bar{h}$ = 5.66 W/m^2·K and with G = 2104 W/m^2, use Eq. (1) to find

$$T_s \approx 492\,K$$ <

With $T_f = (T_s + T_\infty)/2 = (492 + 300)K/2 \approx 400$ K, use the correlation to reevaluate $\bar{h}$. For $T_s = 492$ K, ε = 0.1 is yet a good assumption. We used IHT with the foregoing equations in part (a) and found these results.

$$T_s = 477K \qquad T_f = 388.5 \qquad \bar{h} = 6.38\,W/m^2 \cdot K \qquad \varepsilon = 0.1$$ <

PROBLEM 12.91

KNOWN: Diameter and initial temperature of copper rod. Wall and gas temperature.

FIND: (a) Expression for initial rate of change of rod temperature, (b) Initial rate for prescribed conditions, (c) Transient response of rod temperature.

SCHEMATIC:

Nitrogen $T_\infty = 1500$ K; $D = 0.01$ m; $T_w = 1500$ K; E, G, $\dot{E}_{st}$, q''_{conv}; Thin film. Plot of ε_λ vs λ (μm): $\varepsilon_1 = 0.9$ for $\lambda < 2$, $\varepsilon_2 = 0.4$ for $\lambda > 2$.

ASSUMPTIONS: (1) Applicability of lumped capacitance approximation, (2) Furnace approximates a blackbody cavity, (3) Thin film is diffuse and has negligible thermal resistance, (4) Properties of nitrogen approximate those of air (Part c).

PROPERTIES: *Table A.1*, copper (T = 300 K): $c_p = 385$ J/kg·K, $\rho = 8933$ kg/m^3, k = 401 W/m·K. *Table A.4*, nitrogen (p = 1 atm, $T_f = 900$ K): $\nu = 100.3 \times 10^{-6}$ m^2/s, $\alpha = 139 \times 10^{-6}$ m^2/s, k = 0.0597 W/m·K, Pr = 0.721.

ANALYSIS: (a) Applying conservation of energy at an instant of time to a control surface about the cylinder, $\dot{E}_{in} - \dot{E}_{out} = \dot{E}_{st}$, where energy inflow is due to natural convection and radiation from the furnace wall and energy outflow is due to emission. Hence, for a unit cylinder length,

$$q_{conv} + q_{rad,net} = \frac{\rho \pi D^2}{4} c_p \frac{dT}{dt}$$

where

$$q_{conv} = \bar{h}(\pi D)(T_\infty - T)$$

$$q_{rad,net} = \pi D(\alpha G - \varepsilon E_b) = \pi D\left[\alpha E_b(T_w) - \varepsilon E_b(T)\right]$$

Hence, at t = 0 ($T = T_i$),

$$dT/dt)_i = \left(4/\rho c_p D\right)\left[\bar{h}(T_\infty - T_i) + \alpha E_b(T_w) - \varepsilon E_b(T_i)\right]$$

(b) With $Ra_D = \dfrac{g\beta(T_\infty - T_i)D^3}{\alpha \nu} = \dfrac{9.8\,\text{m/s}^2(1/900\,\text{K})(1200\,\text{K})(0.01\,\text{m})^3}{100.3 \times 139 \times 10^{-12}\,\text{m}^4/\text{s}^2} = 937$, Eq. (9.34) yields

$$\overline{Nu}_D = \left\{0.60 + \frac{0.387 Ra_D^{1/6}}{\left[1 + (0.559/Pr)^{9/16}\right]^{8/27}}\right\}^2 = 2.58$$

$$\bar{h} = k\frac{\overline{Nu}_D}{D} = \frac{(0.0597\,\text{W/m}\cdot\text{K})\,2.58}{0.01\,\text{m}} = 15.4\,\text{W/m}^2\cdot\text{K}$$

With $T = T_i = 300$ K, $\lambda T = 600$ μm·K yields $F_{(0\to\lambda)} = 0$, in which case $\varepsilon = \varepsilon_1 F_{(0\to\lambda)} + \varepsilon_2\left[1 - F_{(0\to\lambda)}\right] = 0.4$.

With $T = T_w = 1500$ K, $\lambda T = 3000$ K yields $F_{(0\to\lambda)} = 0.273$. Hence, with $\alpha_\lambda = \varepsilon_\lambda$, $\alpha = \varepsilon_1 F_{(0\to\lambda)} + \varepsilon_2[1 - F_{(0\to\lambda)}] = 0.9(0.273) + 0.4(1 - 0.273) = 0.537$. It follows that

Continued...

PROBLEM 12.91 (Cont.)

$$\left.\frac{dT}{dt}\right)_i = \frac{4}{8933\,\frac{kg}{m^3}\left(385\,\frac{J}{kg\cdot K}\right)0.01\,m}\left[15\,\frac{W}{m^2\cdot K}(1500-300)\,K\right.$$

$$\left.+0.537\times 5.67\times 10^{-4}\,\frac{W}{m^2\cdot K^4}(1500\,K)^4 - 0.4\times 5.67\times 10^{-8}\,\frac{W}{m^2\cdot K^4}(300\,K)^4\right]$$

$$dT/dt)_i = 1.163\times 10^{-4}\,m^2\cdot K/J\,[18,480+154,140-180]\,W/m^2 = 20\,K/s \qquad <$$

Defining a pseudo radiation coefficient as $h_r = (\alpha G - \varepsilon E_b)/(T_w - T_i) = (153,960\ W/m^2)/1200\ K = 128.3\ W/m^2\cdot K$, $Bi = (h + h_r)(D/4)/k = 143.7\ W/m^2\cdot K\ (0.0025\ m)/401\ W/m\cdot K = 0.0009$. Hence, the lumped capacitance approximation is appropriate.

(c) Using the IHT *Lumped Capacitance* Model with the *Correlations*, *Radiation* and *Properties* (copper and air) Toolpads, the transient response of the rod was computed for $300 \le T < 1200$ K, where the upper limit was determined by the temperature range of the copper property table.

The rate of change of the rod temperature, dT/dt, decreases with increasing temperature, in accordance with a reduction in the convective and *net* radiative heating rates. Note, however, that even at $T \approx 1200$ K, αG, which is fixed, is large relative to q''_{conv} and εE_b and dT/dt is still significant.

PROBLEM 12.92

KNOWN: Temperatures of furnace wall and top and bottom surfaces of a planar sample. Dimensions and emissivity of sample.

FIND: (a) Sample thermal conductivity, (b) Validity of assuming uniform bottom surface temperature.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in sample, (3) Constant k, (4) Diffuse-gray surface, (5) Irradiation equal to blackbody emission at 1400K.

PROPERTIES: *Table A-6,* Water coolant (300K): $c_{p,c} = 4179$ J/kg·K

ANALYSIS: (a) From energy balance at top surface,

$$\alpha G - E = q''_{cond} = k_s (T_s - T_c)/L$$

where $E = \varepsilon_s \sigma T_s^4, G = \sigma T_w^4, \alpha = \varepsilon_s$ giving

$$\varepsilon_s \sigma T_w^4 - \varepsilon_s \sigma T_s^4 = k_s (T_s - T_c)/L.$$

Solving for the thermal conductivity and substituting numerical values, find

$$k_s = \frac{\varepsilon_s L \sigma}{T_s - T_c}\left(T_w^4 - T_s^4\right)$$

$$k_s = \frac{0.85 \times 0.015\,\text{m} \times 5.67 \times 10^{-8}\,\text{W/m}^2 \cdot \text{K}^4}{(1000 - 300)\,\text{K}}\left[(1400\,\text{K})^4 - (1000\,\text{K})^4\right]$$

$$k_s = 2.93\ \text{W/m} \cdot \text{K}.$$ <

(b) Non-uniformity of bottom surface temperature depends on coolant temperature rise. From the energy balance

$$q = \dot{m}_c c_{p,c} \Delta T_c = (\alpha G - E) W^2$$

$$\Delta T_c = 0.85 \times 5.67 \times 10^{-8}\,\text{W/m}^2 \cdot \text{K}^4 \left[1400^4 - 1000^4\right] \text{K}^4 (0.10\,\text{m})^2 / 0.1\,\text{kg/s} \times 4179\,\text{J/kg} \cdot \text{K}$$

$$\Delta T_c = 3.3\text{K}.$$ <

The variation in T_c (~ 3K) is small compared to $(T_s - T_c) \approx 700$K. Hence it is not large enough to introduce significant error in the k determination.

PROBLEM 12.93

KNOWN: Thicknesses and thermal conductivities of a ceramic/metal composite. Emissivity of ceramic surface. Temperatures of vacuum chamber wall and substrate lower surface. Receiving area of radiation detector, distance of detector from sample, and sample surface area viewed by detector.

FIND: (a) Ceramic top surface temperature and heat flux, (b) Rate at which radiation emitted by the ceramic is intercepted by detector, (c) Effect of an interfacial (ceramic/substrate) contact resistance on sample top and bottom surface temperatures.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional, steady-state conduction in sample, (2) Constant properties, (3) Chamber forms a blackbody enclosure at T_w, (4) Ceramic surface is diffuse/gray, (5) Negligible interface contact resistance for part (a).

PROPERTIES: Ceramic: $k_c = 60$ W/m·K, $\varepsilon_c = 0.8$. Substrate: $k_s = 25$ W/m·K.

ANALYSIS: (a) From an energy balance at the exposed ceramic surface, $q''_{cond} = q''_{rad}$, or

$$\frac{T_1 - T_2}{(L_s/k_s) + (L_c/k_c)} = \varepsilon_c \sigma \left(T_2^4 - T_w^4\right)$$

$$\frac{1500\,\text{K} - T_2}{\dfrac{0.008\,\text{m}}{25\,\text{W/m}\cdot\text{K}} + \dfrac{0.005\,\text{m}}{60\,\text{W/m}\cdot\text{K}}} = 0.8 \times 5.67 \times 10^{-8}\,\text{W/m}^2\cdot\text{K}^4 \left(T_2^4 - 90^4\right)\text{K}^4$$

$$3.72 \times 10^6 - 2479 T_2 = 4.54 \times 10^{-8} T_2^4 - 2.98$$

$$4.54 \times 10^{-8} T_2^4 + 2479 T_2 = 3.72 \times 10^6$$

Solving, we obtain

$$T_2 = 1425\,\text{K}$$ <

$$q''_h = \frac{T_1 - T_2}{(L_s/k_s) + (L_c/k_c)} = \frac{(1500 - 1425)\,\text{K}}{4.033 \times 10^{-4}\,\text{m}^2 \cdot \text{K/W}} = 1.87 \times 10^5\,\text{W/m}^2$$ <

(b) Since the ceramic surface is diffuse, the total intensity of radiation emitted in all directions is $I_e = \varepsilon_c E_b(T_s)/\pi$. Hence, the rate at which *emitted* radiation is intercepted by the detector is

$$q_{c(em)-d} = I_e \Delta A_c \left(A_d / L_{s-d}^2\right)$$

$$q_{c(em)-d} = \frac{0.8 \times 5.67 \times 10^{-8}\,\text{W/m}^2\cdot\text{K}^4\,(1425\,\text{K})^4}{\pi\,\text{sr}} \times 10^{-4}\,\text{m}^2 \times 10^{-5}\,\text{sr} = 5.95 \times 10^{-5}\,\text{W}$$

Continued...

PROBLEM 12.93 (Cont.)

(c) With the development of an interfacial thermal contact resistance and fixed values of q''_h and T_w, (i) T_2 remains the same (its value is determined by the requirement that $q''_h = \varepsilon_c \sigma \left(T_2^4 - T_w^4\right)$, while (ii) T_1 increases (its value is determined by the requirement that $q''_h = \left(T_1 - T_2\right) / R''_{tot}$, where R''_{tot} = [(L_s/k_s) + $R''_{t,c}$ + (L_c/k_c)]; if q''_h and T_2 are fixed, T_1 must increase with increasing R''_{tot}).

COMMENTS: The detector will also see radiation which is reflected from the ceramic. The corresponding radiation rate is $q_{c(reflection)-d} = \rho_c G_c \Delta A_c \, A_d / L_{s-d}^2 = 0.2\,\sigma(90\text{ K})^4 \times 10^{-4}\text{ m}^2 \times (10^{-5}\text{ sr}) = 7.44 \times 10^{-10}$ W. Hence, reflection is negligible.

PROBLEM 12.94

KNOWN: Wafer heated by ion beam source within large process-gas chamber with walls at uniform temperature; radiometer views a 5 × 5 mm target on the wafer. Black panel mounted in place of wafer in a pre-production test of the equipment.

FIND: (a) Radiant power (μW) received by the radiometer when the black panel temperature is T_{bp} = 800 K and (b) Temperature of the wafer, T_w, when the ion beam source is adjusted so that the radiant power received by the radiometer is the same as that of part (a)

SCHEMATIC:

Objective, 25 mm dia
Window
Radiometer
500 mm
30°
Black panel, T_{bp} = 800 K
Wafer, ε = 0.7, T_w = ?
Ion beam
Target, 5x5 mm
Chamber, T_{ch} = 400 K

ASSUMPTIONS: (1) Steady-state conditions, (2) Chamber represents large, isothermal surroundings, (3) Wafer is opaque, diffuse-gray, and (4) Target area << square of distance between target and radiometer objective.

ANALYSIS: (a) The radiant power leaving the black-panel target and reaching the radiometer as illustrated in the schematic below is

$$q_{bp-rad} = \left[E_{b,bp}\left(T_{bp}\right)/\pi\right] A_t \cos\theta_t \cdot \Delta\omega_{rad-t} \qquad (1)$$

where $\theta_t = 0°$ and the solid angle the radiometer subtends with respect to the target follows from Eq. 12.2,

$$\Delta\omega_{rad-t} = \frac{dA_n}{r^2} = \frac{\left(\pi D_o^2/4\right)}{r^2} = \frac{\pi(0.025\text{ m})^2/4}{(0.500\text{ m})^2} = 1.964\times10^{-3}\text{ sr}$$

With $E_{b,bp} = \sigma T_{bp}^4$, find

$$q_{bp-rad} = \left[5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4(800\text{ K})^4/\pi\text{ sr}\right]$$

$$\times(0.005\text{ m})^2 \times \cos 30° \times 1.964\times10^{-3}\text{ sr}$$

$$q_{bp-rad} = 314\ \mu\text{W}$$ <

$E_{b,bp}(T_{bp})$
q_{bp-rad}
30°
$\Delta\omega_{rad-t}$
Target, 5x5 mm

(a) Black panel, T_{bp} = 800 K

J_w
q_{w-rad}
$G_w = E_b(T_{ch})$

(b) Wafer, ε_w = 0.7, T_w = ?

Continued …..

(b) With the wafer mounted, the ion beam source is adjusted until the radiometer receives the same radiant power as with part (a) for the black panel. The power reaching the radiometer is expressed in terms of the wafer radiosity,

$$q_{w-rad} = \left[J_w / \pi\right] A_t \cos\theta_t \cdot \Delta\omega_{rad-t} \qquad (2)$$

Since $q_{w-rad} = q_{bp-rad}$ (see Eq. (1)), recognize that

$$J_w = E_{b,bp}\left(T_{bp}\right) \qquad (3)$$

where the radiosity is

$$J_w = \varepsilon_w E_{b,w}\left(T_w\right) + \rho_w G_w = \varepsilon_w E_{b,w}\left(T_w\right) + \left(1 - \varepsilon_w\right) E_b\left(T_{ch}\right) \qquad (4)$$

and G_w is equal to the blackbody emissive power at T_{ch}. Using Eqs. (3) and (4) and substituting numerical values, find

$$\sigma T_{bp}^4 = \varepsilon_w \sigma T_w^4 + \left(1 - \varepsilon_w\right) \sigma T_{ch}^4$$

$$(800\ \text{K})^4 = 0.7\ T_w^4 + 0.3(400\ \text{K})^4$$

$$T_w = 871\ \text{K}$$ <

COMMENTS: (1) Explain why T_w is higher than 800 K, the temperature of the black panel, when the radiometer receives the same radiant power for both situations.

(2) If the chamber walls were cold relative to the wafer, say near liquid nitrogen temperature, T_{ch} = 80 K, and the test repeated with the same indicated radiometer power, is the wafer temperature higher or lower than 871 K?

(3) If the chamber walls were maintained at 800 K, and the test repeated with the same indicated radiometer power, what is the wafer temperature?

PROBLEM 12.95

KNOWN: Spectrally selective workpiece placed in an oven with walls at T_f = 1000 K experiencing convection with air at T_∞ = 600 K.

FIND: (a) Steady-state temperature, T_s, by performing an energy balance on the workpiece; show control surface identifying all relevant processes; (b) Compute and plot T_s as a function of the convection coefficient h, for the range $10 \le h \le 120\ \mathrm{W/m^2 \cdot K}$; on the same plot, show the behavior for diffuse surfaces of emissivity 0.2 and 0.8.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Convection coefficient constant, independent of temperature, (3) Workpiece is diffusely irradiated by oven wall which is large isothermal surroundings, (4) Spectral emissivity independent of workpiece temperature.

ANALYSIS: (a) Performing an energy balance on the workpiece,

$$\dot{E}_{in} - \dot{E}_{out} = \alpha G - \varepsilon E_b - q''_{conv} = 0 \qquad \text{and} \qquad G = E_b(T_f), \tag{1}$$

$$\alpha E_b(T_f) - \varepsilon E_b(T_s) - h(T_s - T_\infty) = 0 \tag{2}$$

<

where the total absorptivity is, with $\alpha_\lambda = \varepsilon_\lambda$,

$$\alpha = F_{(0\to 5\mu\mathrm{m},1000\ \mathrm{K})} \cdot \varepsilon_1 + \left(1 - F_{(0\to 5\mu\mathrm{m},1000\ \mathrm{K})}\right)\varepsilon_2 = 0.6337 \times 0.2 + (1 - 0.6337) \times 0.8 = 0.419$$

using Table 12.1 with $\lambda T = 5 \times 1000 = 5000\ \mu\mathrm{m \cdot K}$ for which $F_{(0-\lambda T)} = 0.6337$. The total emissivity is,

$$\varepsilon = F_{(0\to 5\mu\mathrm{m}\cdot T_s)}\varepsilon_1 + \left(1 - F_{(0\to 5\times T_s)}\right) \cdot \varepsilon_2 \tag{3}$$

which requires knowing T_s. Hence, an iterative solution is required, beginning by assuming a value of T_s to find ε using Eq. (3), and then using that value in Eq. (2) to find T_s. The result is,

T_s = 800 K <

for which ε = 0.512.

(b) Using the IHT workspace with the foregoing equations and the *Radiation Exchange Tool*, *Band Emission Factor*, a model was developed to calculate T_s as a function of the convection coefficient. Additionally, T_s was plotted for the cases when the workpiece is diffuse, gray with ε = 0.2 and 0.8

Continued...

For the workpiece with the selective surface (ε_λ as shown in the schematic), the temperature decreases with increasing convection coefficient. For the gray surface, $\varepsilon = 0.2$ or 0.8, the temperature is lower and higher, respectively, than that of the workpiece. Recall from part (c), at $h = 60\ \mathrm{W/m^2 \cdot K}$, $\varepsilon = 0.512$ and $\alpha = 0.419$, so that it is understandable why the curve for the workpiece is between that for the two gray surfaces.

COMMENTS: (1) For the conditions in part (b), make a sketch of the workpiece emissivity and the absorptivity as a function of its temperature.

(2) The IHT workspace model used to generate the plot is shown below. Note how we used this model to also calculate T_s vs h for the gray surfaces by adjusting λ_1.

```
// Energy Balance:
alpha * Gf  - eps * Ebs  -  h * (Ts - Tinf) = 0
Gf = Ebf                              // Irradiation from furnace, W/m^2
Ebf = sigma * Tf^4                    // Blackbody emissive power, W/m^2; furnace wall
Ebs = sigma * Ts^4                    // Blackbody emissive power, W/m^2; workpiece
sigma = 5.67e-8                       // Stefan-Boltzmann constant, W/m^2.K^4

// Radiation Tool - Band Emission Factor, Total emissivity and absorptivity
eps = FL1Ts * eps1 + (1 - FL1Ts) * eps2
/* The blackbody band emission factor, Figure 12.14 and Table 12.1, is */
FL1Ts = F_lambda_T(lambda1,Ts)                  // Eq 12.30
// where units are lambda (micrometers, mum) and T (K)
alpha = FL1Tf * eps1 + (1 - FL1Tf) * eps2
FL1Tf = F_lambda_T(lambda1,Tf)                  // Eq 12.30

// Assigned Variables:
Ts > 0                    // Workpiece temperature, K; assures positive value for Ts
Tf = 1000                 // Furnace wall temperature, K
Tinf = 600                // Air temperature, K
h = 60                    // Convection coefficient, W/m^2.K
eps1 = 0.2                // Spectral emissivity, 0 <= epsL <= 5 micrometers
eps2 = 0.8                // Spectral emissivity, 5 <= epsL <= infinity
//lambda1 = 5             // Wavelength, micrometers; spectrally selective workpiece
//lambda1 = 1e6           // Wavelength; gray surface for which eps = 0.2
lambda1 = 0.5             // Wavelength; gray surface for which eps = 0.8
```

PROBLEM 12.96

KNOWN: Laser-materials-processing apparatus. Spectrally selective sample heated to the operating temperature $T_s = 2000$ K by laser irradiation (0.5 μm), G_{laser}, experiences convection with an inert gas and radiation exchange with the enclosure.

FIND: (a) Total emissivity of the sample, ε ; (b) Total absorptivity of the sample, α, for irradiation from the enclosure; (c) Laser irradiation required to maintain the sample at $T_s = 2000$ K by performing an energy balance on the sample; (d) Sketch of the sample emissivity during the cool-down process when the laser and inert gas flow are deactivated; identify key features including the emissivity for the final condition ($t \rightarrow \infty$); and (e) Time-to-cool the sample from the operating condition at $T_s(0) = 2000$ K to a safe-to-touch temperature of $T_s(t) = 40°C$; use the lumped capacitance method and include the effects of convection with inert gas ($T_\infty = 300$ K , h = 50 W/ m^2·K) as well as radiation exchange $T_{enc} = T_\infty$.

SCHEMATIC:

ASSUMPTIONS: (1) Enclosure is isothermal and large compared to the sample, (2) Sample is opaque and diffuse, but spectrally selective, so that $\varepsilon_\lambda = \alpha_\lambda$, (3) Sample is isothermal.

PROPERTIES: Sample (Given) ρ = 3900 kg/m^3, c_p = 760 J/kg , k = 45 W/m·K.

ANALYSIS: (a) The total emissivity of the sample, ε, at $T_s = 2000$ K follows from Eq. 12.38 which can be expressed in terms of the band emission factor, $F_{(0\text{-}\lambda,T)}$ Eq. 12.30,

$$\varepsilon = \varepsilon_1 F_{(0-\lambda_1 T_s)} + \varepsilon_2\left[1 - F_{(0-\lambda_1 T_s)}\right] \quad (1)$$

$$\varepsilon = 0.8 \times 0.7378 + 0.2[1 - 0.7378] = 0.643$$ <

where from Table 12.1, with $\lambda_1 T_s$ = 3μm × 2000 K = 6000 μm·K, $F_{(0\text{-}\lambda T)}$ = 0.7378.

(b) The total absorptivity of the sample, α, for irradiation from the enclosure at T_{enc} = 300 K, is

$$\alpha = \varepsilon_1 F_{(0-\lambda_1 T_{enc})} + \varepsilon_2\left[1 - F_{(0-\lambda_1 T_{enc})}\right] \quad (2)$$

$$\alpha = 0.8 \times 0 + 0.2[1 - 0] = 0.200$$ <

where, from Table 12.1, with $\lambda_1 T_{enc}$ = 3 μm × 300 K = 900 μm·K, $F_{(0\text{-}\lambda T)}$ =0.

Continued...

(c) The energy balance on the sample, on a per unit area basis, as shown in the schematic at the right is

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$+\alpha_{las} G_{laser} + 2\alpha G - 2\varepsilon E_b (T_s) - q''_{cv} = 0$$

$$\alpha_{las} G_{laser} + 2\alpha\sigma T_{enc}^4 - 2\varepsilon\tau T_s^4 - 2h(T_s - T_\infty) = 0 \qquad (3)$$

Recognizing that $\alpha_{las}(0.5\ \mu m) = 0.8$, and substituting numerical values find,

$$0.8 \times G_{laser} + 2 \times 0.200 \times 5.67 \times 10^{-8}\ W/m^2 \cdot K^4\ (300\ K)^4$$

$$-2 \times 0.643 \times 5.67 \times 10^{-8}\ W/m^2 \cdot K^4\ (2000\ K)^4 - 2 \times 50\ W/m^2 \cdot K\,(2000 - 500)\,K = 0$$

$$0.8 \times G_{laser} = \left[-184.6 + 1.167 \times 10^6 + 1.500 \times 10^5\right] W/m^2$$

$$G_{laser} = 1646\ kW/m^2$$

<

(d) During the cool-down process, the total emissivity ε will decrease as the temperature decreases, $T_s(t)$. In the limit, $t \rightarrow \infty$, the sample will reach that of the enclosure, $T_s(\infty) = T_{enc}$ for which $\varepsilon = \alpha = 0.200$.

(e) Using the *IHT Lumped Capacitance Model* considering radiation exchange ($T_{enc} = 300$ K) and convection ($T_\infty = 300$ K, $h = 50\ W/m^2 \cdot K$) and evaluating the emissivity using Eq. (1) with the *Radiation Tool*, *Band Emission Factors*, the temperature-time history was determined and the time-to-cool to T(t) = 40°C was found as

$$t = 119\ s$$

<

COMMENTS: (1) From the IHT model used for part (e), the emissivity as a function of cooling time and sample temperature were computed and are plotted below. Compare these results to your sketch of part (c).

Continued...

(2) The IHT workspace model to perform the lumped capacitance analysis with variable emissivity is shown below.

```
// Lumped Capacitance Model - convection and emission/irradiation radiation processes:
/* Conservation of energy requirement on the control volume, CV.  */
Edotin - Edotout = Edotst
Edotin = As * ( + Gabs)
Edotout = As * ( + q''cv + E )
Edotst = rho * vol * cp * Der(T,t)
T_C = T - 273
// Absorbed irradiation from large surroundings on CS
Gabs = alpha * G
G =  sigma * Tsur^4
sigma = 5.67e-8                  // Stefan-Boltzmann constant, W/m^2·K^4
// Emissive power of CS
E = eps * Eb
Eb = sigma * T^4
//sigma = 5.67e-8                // Stefan-Boltzmann constant, W/m^2·K^4
//Convection heat flux for control surface CS
q''cv = h * ( T - Tinf )
/* The independent variables for this system and their assigned numerical values are */
As =     2 * 1          // surface area, m^2; unit area, top and bottom surfaces
vol =  1 * w             // vol, m^3
w = 0.001                // sample thickness, m
rho =   3900            // density, kg/m^3
cp =   760               // specific heat, J/kg·K
// Convection heat flux, CS
h =    50                // convection coefficient, W/m^2·K
Tinf =  300              // fluid temperature, K
// Emission, CS
//eps =   0.5            // emissivity; value used to test the model initially
// Irradiation from large surroundings, CS
alpha = 0.200            // absorptivity; from Part (b); remains constant during cool-down
Tsur =   300             // surroundings temperature, K

// Radiation Tool - Band emission factor:
eps = eps1 * FL1T  + eps2 * ( 1 - FL1T )
/* The blackbody band emission factor, Figure 12.14 and Table 12.1, is */
FL1T = F_lambda_T(lambda1,T)    // Eq 12.30
// where units are lambda (micrometers, mum) and T (K)
lambda1 = 3              // wavelength, mum
eps1 = 0.8               // spectral emissivity; for lambda < lambda1
eps2 = 0.2               // spectral emissivity; for lambda > lambda1
```

PROBLEM 12.97

KNOWN: Cross flow of air over a cylinder placed within a large furnace.

FIND: (a) Steady-state temperature of the cylinder when it is diffuse and gray with ε = 0.5, (b) Steady-state temperature when surface has spectral properties shown below, (c) Steady-state temperature of the diffuse, gray cylinder if air flow is parallel to the cylindrical axis, (d) Effect of air velocity on cylinder temperature for conditions of part (a).

SCHEMATIC:

Air
T_∞ = 400 K
V = 3 m/s
Cylinder, D = 30 mm, L = 150 mm
T_s
T_{sur} = 1000 K
q''_{conv}
q''_{rad}
ε_λ
0.5
0.1
0
3
λ (μm)
Part (b) surface

ASSUMPTIONS: (1) Cylinder is isothermal, (2) Furnace walls are isothermal and very large in area compared to the cylinder, (3) Steady-state conditions.

PROPERTIES: *Table A.4*, Air ($T_f \approx$ 600 K): $\nu = 52.69\times10^{-6}\ \mathrm{m^2/s}$, k = 46.9 × 10^{-3} W/m·K, Pr = 0.685.

ANALYSIS: (a) When the cylinder surface is gray and diffuse with ε = 0.5, the energy balance is of the form, $q''_{rad} - q''_{conv} = 0$. Hence,

$$\varepsilon\sigma(T_{sur}^4 - T_s^4) - \bar{h}(T_s - T_\infty) = 0\,.$$

The heat transfer coefficient, $\bar{h}$, can be estimated from the Churchill-Bernstein correlation,

$$\overline{Nu}_D = (\bar{h}\,D/k) = 0.3 + \frac{0.62\,Re_D^{1/2}\,Pr^{1/3}}{\left[1+(0.4/Pr)^{2/3}\right]^{1/4}}\left[1+\left(\frac{Re_D}{282,000}\right)^{5/8}\right]^{4/5}$$

where $Re_D = V\,D/\nu = 3\,\mathrm{m/s}\times30\times10^{-3}\,\mathrm{m}/52.69\times10^{-6}\,\mathrm{m^2/s} = 1710$. Hence,

$$\overline{Nu}_D = 20.8$$

$$\bar{h} = 20.8\times46.9\times10^{-3}\,\mathrm{W/m\cdot K}/30\times10^{-3}\,\mathrm{m} = 32.5\,\mathrm{W/m^2\cdot K}\,.$$

Using this value of $\bar{h}$ in the energy balance expression, we obtain

$$0.5\times5.67\times10^{-8}(1000^4 - T_s^4)\,\mathrm{W/m^2} - 32.5\,\mathrm{W/m^2\cdot K}(T_s - 400)\,\mathrm{K} = 0$$

which yields $T_s \approx$ 839 K. <

(b) When the cylinder has the spectrally selective behavior, the energy balance is written as

$$\alpha G - \varepsilon E_b(T_s) - q''_{conv} = 0$$

where G = E_b (T_{sur}). With $\alpha = \int_0^\infty \alpha_\lambda G_\lambda\,d\lambda / G$,

$$\alpha = 0.1\times F_{(0\to3)} + 0.5\times\left(1 - F_{(0\to3)}\right) = 0.1\times0.273 + 0.5(1-0.273) = 0.391$$

where, using Table 12.1 with λT = 3 ×1000 = 3000 μm·K, $F_{(0\to3)}$ = 0.273. Assuming T_s is such that emission in the spectral region λ < 3 μm is negligible, the energy balance becomes

Continued...

PROBLEM 12.97 (Cont.)

$$0.391\times5.67\times10^{-8}\times1000^4\ \mathrm{W/m^2} - 0.5\times5.67\times10^{-8}\times T_s^4\ \mathrm{W/m^2} - 32.5\ \mathrm{W/m^2\cdot K}(T_s - 400)\ \mathrm{K} = 0$$

which yields $T_s \approx 770$ K. <

Note that, for $\lambda T = 3 \times 770 = 2310\ \mu\mathrm{m\cdot K}$, $F_{(0\to\lambda)} \approx 0.11$; hence the assumption of $\varepsilon = 0.5$ is acceptable. Note also that the value of $\bar{h}$ based upon $T_f = 600$ K is also acceptable.

(c) When the cylinder is diffuse-gray with air flow in the longitudinal direction, the characteristic length for convection is different. Assume conditions can be modeled as flow over a flat plate of L = 150 mm. With

$$Re_L = VL/\nu = 3\ \mathrm{m/s}\times150\times10^{-3}\ \mathrm{m}/52.69\times10^{-6}\ \mathrm{m^2/s} = 8540$$

$$\overline{Nu}_L = (\bar{h}L/k) = 0.664\,Re_L^{1/2}\ Pr^{1/3} = 0.664(8540)^{1/2}0.685^{1/3} = 54.1$$

$$\bar{h} = 54.1\times0.0469\ \mathrm{W/m\cdot K}/0.150\ \mathrm{m} = 16.9\ \mathrm{W/m^2\cdot K}\ .$$

The energy balance now becomes

$$0.5\times5.667\times10^{-8}\ \mathrm{W/m^2\cdot K^4}(1000^4 - T_s^4)\mathrm{K^4} - 16.9\ \mathrm{W/m^2\cdot K}(T_s - 400)\mathrm{K} = 0$$

which yields $T_s \approx 850$ K. <

(b) Using the IHT *First Law* Model with the *Correlations* and *Properties* Toolpads, the effect of velocity may be determined and the results are as follows:

Since the convection coefficient increases with increasing V (from 18.5 to 90.6 $\mathrm{W/m^2\cdot K}$ for $1 \le V \le 20$ m/s), the cylinder temperature decreases, since a smaller value of $(T_s - T_\infty)$ is needed to dissipate the absorbed irradiation by convection.

COMMENTS: The cylinder temperature exceeds the air temperature due to absorption of the incident radiation. The cylinder temperature would approach T_∞ as $\bar{h} \to \infty$ and/or $\alpha \to 0$. If $\alpha \to 0$ and $\bar{h}$ has a small to moderate value, would T_s be larger than, equal to, or less than T_∞? Why?

PROBLEM 12.98

KNOWN: Instrumentation pod, initially at 87°C, on a conveyor system passes through a large vacuum brazing furnace. Inner surface of pod surrounded by a mass of phase-change material (PCM). Outer surface with special diffuse, opaque coating of ε_λ. Electronics in pod dissipate 50 W.

FIND: How long before all the PCM changes to the liquid state?

SCHEMATIC:

Vacuum furnace walls, $T_w = 1200K, \varepsilon_w = 0.7$

Pod, $T_s = 87°C$; $A_s = 0.040 m^2$, $M = 1.6 kg$; power dissipation, $P_e = 50W$; coating, ε_λ

Insulation

Conveyor system

ε_λ: 0.05 for $0 < \lambda < 5$, 0.9 for $\lambda > 5$; $\lambda(\mu)$

ASSUMPTIONS: (1) Surface area of furnace walls much larger than that of pod, (2) No convection, (3) No heat transfer to pod from conveyor, (4) Pod coating is diffuse, opaque, (5) Initially pod internal temperature is uniform at T_{pcm} = 87°C and remains so during time interval Δt_m, (6) Surface area provided is that exposed to walls.

PROPERTIES: Phase-change material, PCM (given): Fusion temperature, T_f = 87°C, h_{fg} = 25 kJ/kg.

ANALYSIS: Perform an energy balance on the pod for an interval of time Δt_m which corresponds to the time for which the PCM changes from solid to liquid state,

$$E_{in} - E_{out} + E_{gen} = \Delta E$$

$$\left[\left(\alpha G - \varepsilon E_b\right) A_s + P_e\right] \Delta t_m = M h_{fg}$$

εE_b, αG, T_m, A_s, M, E_{gen}, ΔE

where P_e is the electrical power dissipation rate, M is the mass of PCM, and h_{fg} is the heat of fusion of PCM.

Irradiation: $G = \sigma T_w = 5.67 \times 10^{-8}\ W/m^2 \cdot K^4 (1200\ K)^4 = 117,573\ W/m^2$

Emissive power: $E_b = \sigma T_m^4 = \sigma\left(87 + 273\right)^4 = 952\ W/m^2$

Emissivity: $\varepsilon = \varepsilon_1 F_{(0-\lambda T)} + \varepsilon_2(1 - F_{(0-\lambda T)})$ $\lambda T = 5 \times 360 = 1800\ \mu m \cdot K$

$\varepsilon = 0.05 \times 0.0393 + 0.9\ (1 - 0.0393)$ $F_{0-\lambda T} = 0.0393$ (Table 12.1)

$\varepsilon = 0.867$

Absorptivity: $\alpha = \alpha_1 F_{(0-\lambda T)} + \alpha_2(1 - F_{(0-\lambda T)})$ $\lambda T = 5 \times 1200 = 6000\ \mu m \cdot K$

$\alpha = 0.005 \times 0.7378 + 0.9\ (1 - 0.7378)$ $F_{0-\lambda T} = 0.7378$ (Table 12.1)

$\alpha = 0.273$

Substituting numerical values into the energy balance, find,

$$\left[\left(0.273 \times 117,573 - 0.867 \times 952\right) W/m^2 \times 0.040\ m^2 + 50\ W\right] \Delta t_m = 1.6 kg \times 25 \times 10^3\ J/kg$$

$$\Delta t_m = 30.7\ s = 0.51 min.$$ <

PROBLEM 12.99

KNOWN: Niobium sphere, levitated in surroundings at 300 K and initially at 300 K, is suddenly irradiated with a laser (10 W/m^2) and heated to its melting temperature.

FIND: (a) Time required to reach the melting temperature, (b) Power required from the RF heater causing uniform volumetric generation to maintain the sphere at the melting temperature, and (c) Whether the spacewise isothermal sphere assumption is realistic for these conditions.

SCHEMATIC:

ASSUMPTIONS: (1) Niobium sphere is spacewise isothermal and diffuse-gray, (2) Initially sphere is at uniform temperature T_i, (3) Constant properties, (4) Sphere is small compared to the uniform temperature surroundings.

PROPERTIES: *Table A-1*, Niobium ($\overline{T}$ = (300 + 2741)K/2 = 1520 K): T_{mp} = 2741 K, ρ = 8570 kg/m^3, c_p = 324 J/kg·K, k = 72.1 W/m·K.

ANALYSIS: (a) Following the methodology of Section 5.3 for general lumped capacitance analysis, the time required to reach the melting point T_{mp} may be determined from an energy balance on the sphere,

$$\dot{E}_{in} - \dot{E}_{out} = \dot{E}_{st} \qquad q_o'' \cdot A_c - \varepsilon\sigma A_s\left(T^4 - T_{sur}^4\right) = Mc_p\left(dT/dt\right)$$

where $A_c = \pi D^2/4$, $A_s = \pi D^2$, and $M = \rho V = \rho(\pi D^3/6)$. Hence,

$$q_o''\left(\pi D^2/4\right) - \varepsilon\sigma\left(\pi D^2\right)\left(T^4 - T_{sur}^4\right) = \rho\left(\pi D^3/6\right)c_p\left(dT/dt\right).$$

Regrouping, setting the limits of integration, and integrating, find

$$\left[\frac{q_o''}{4\varepsilon\sigma} + T_{sur}^4\right] - T^4 = \frac{\rho D c}{6\varepsilon\sigma}\frac{dT}{dt} \qquad b\int_0^t dt = \int_{T_i}^{T_{mp}} \frac{dT}{\left(a^4 - T^4\right)}$$

$$\text{where} \quad a^4 = \frac{q_o''}{4\varepsilon\sigma} + T_{sur}^4 = \frac{10\ \text{W/mm}^2\left(10^3\ \text{mm/m}\right)^2}{4\times0.6\times5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}} + \left(300\ \text{K}\right)^4 \qquad a = 2928\,\text{K}$$

$$b = \frac{6\varepsilon\sigma}{\rho D c_p} = \frac{6\times0.6\times5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4}{8570\ \text{kg/m}^3\times0.003\ \text{m}\times324\ \text{J/kg}\cdot\text{K}} = 2.4504\times10^{-11}\ \text{K}^{-1}\cdot\text{s}^{-1}$$

which from Eq. 5.18, has the solution

$$t = \frac{1}{4ba^3}\left\{\ln\left|\frac{a+T_{mp}}{a-T_{mp}}\right| - \ln\left|\frac{a+T_i}{a-T_i}\right| + 2\left[\tan^{-1}\left(\frac{T_{mp}}{a}\right) - \tan^{-1}\left(\frac{T_i}{a}\right)\right]\right\}$$

Continued …..

PROBLEM 12.99 (Cont.)

$$t = \frac{1}{4\left(2.4504\times10^{-11}\ K^{-1}\cdot s^{-1}\right)(2928\ K)^3}\left\{\ln\left|\frac{2928+2741}{2928-2741}\right| - \ln\left|\frac{2928+300}{2928-300}\right|\right.$$

$$\left. +2\left[\tan^{-1}\left(\frac{2741}{2928}\right) - \tan^{-1}\left(\frac{300}{2928}\right)\right]\right\}$$

$$t = 0.40604\left(3.4117-0.2056+2\left[0.7524-0.1021\right]\right) = 1.83s. \qquad <$$

(b) The power required of the RF heater to induce a uniform volumetric generation to sustain steady-state operation at the melting point follows from an energy balance on the sphere,

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = 0 \qquad -e s\, A_s\left(T_{mp}^4 - T_{sur}^4\right) = -\dot{E}_{gen}$$

$$\dot{E}_{gen} = \dot{q}V = 0.6\times5.67\times10^{-8}W/m^2\cdot K^4\left(p\,0.003^2\right)m^2\left(2741^4 - 300^4\right)K^4 = 54.3W. \qquad <$$

(c) The lumped capacitance method is appropriate when

$$Bi = \frac{h_r L_c}{k} = \frac{h_r\,(D/6)}{k} < 0.1$$

where h_r is the linearized radiation coefficient, which has the largest value when T = T_{mp} = 2741 K,

$$h_r = e s\,(T + T_{sur})\left(T^2 + T_{sur}^2\right)$$

$$h_r = 0.6\times5.67\times10^{-8}W/m^2\cdot K^4(2741+300)\left(2741^2+300^2\right)K^3 = 787\ W/m^2\cdot K.$$

Hence, since

$$Bi = 787\ W/m^2\cdot K\,(0.003m/3)/72.1W/m\cdot K = 1.09\times10^{-2}$$

we conclude that the transient analysis using the lumped capacitance method is satisfactory. $<$

COMMENTS: (1) Note that at steady-state conditions with internal generation, the difference in temperature between the center and surface, is

$$T_o = T_s = \frac{\dot{q}(D/2)^2}{6k}$$

and with V = $\pi D^3/6$, from the part (b) results,

$$\dot{q} = \dot{E}_{gen}/V = 54.3\ W/\left(p\times0.003^3/6\right)m^3 = 3.841\times10^9\ W/m^3.$$

Find using an approximate value for the thermal conductivity in the liquid state,

$$\Delta T = T_o - T_s = \frac{3.841\times10^9\ W/m^3(0.03m/2)^2}{6\times80\ W/m\cdot K} = 18K.$$

We conclude that the sphere is very nearly isothermal even under these conditions.

(2) The relation for ΔT in the previous comment follows from solving the heat diffusion equation written for the one-dimensional (spherical) radial coordinate system. See the deviation in Section 3.4.2 for the cylindrical case (Eq. 3.53).

PROBLEM 12.100

KNOWN: Spherical niobium droplet levitated in a vacuum chamber with cool walls. Niobium surface is diffuse with prescribed spectral emissivity distribution. Melting temperature, T_{mp} = 2741 K.

FIND: Requirements for maintaining the drop at its melting temperature by two methods of heating: (a) Uniform internal generation rate, $\dot{q}$ (W/m^3), using a radio frequency (RF) field, and (b) Irradiation, G_{laser}, (W/mm^2), using a laser beam operating at 0.632 μm; and (c) Time for the drop to cool to 400 K if the heating method were terminated.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions during the heating processes, (2) Chamber is isothermal and large relative to the drop, (3) Niobium surface is diffuse but spectral selective, (4) $\dot{q}$ is uniform, (5) Laser bean diameter is larger than the droplet, (6) Drop is spacewise isothermal during the cool down.

PROPERTIES: *Table A.1*, Niobium ($\overline{T}$ = (2741 + 400)K/2 ≈ 1600 K): ρ = 8570 kg/m^3, c_p = 336 J/kg·K, k = 75.6 W/m·K.

ANALYSIS: (a) For the RF field-method of heating, perform an energy balance on the drop considering volumetric generation, irradiation and emission,

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = 0$$

$$\left[\alpha G - \varepsilon E_b\left(T_s\right)\right]A_s + \dot{q}\forall = 0 \qquad (1)$$

where $A_s = \pi D^2$ and $V = \pi D^3/6$. The irradiation and blackbody emissive power are,

$$G = \sigma T_w^4 \qquad E_b = \sigma T_s^4 \qquad (2,3)$$

The absorptivity and emissivity are evaluated using Eqs. 12.46 and 12.48, respectively, with the band emission fractions, Eq. 12.30, and

$$\alpha = \alpha\left(\alpha_\lambda, T_w\right) = \varepsilon_1 F\left(0-\lambda_1 T_w\right) + \varepsilon_2\left[1 - F\left(0-\lambda_1 T_w\right)\right] \qquad (4)$$

$$\alpha = 0.4 \times 0.000 + 0.2\left(1 - 0.000\right) = 0.2$$

where, from Table 12.1, with $\lambda_1 T_w = 1\ \mu m \times 300\ K = 300\ \mu m \cdot K$, F(0 - λT) = 0.000.

$$\varepsilon = \varepsilon\left(\varepsilon_\lambda, T_s\right) = \varepsilon_1 F\left(0-\lambda_1 T_s\right) + \varepsilon_2\left[1 - F\left(0-\lambda_1 T_s\right)\right] \qquad (5)$$

$$\varepsilon = 0.4 \times 0.2147 + 0.2\left(1 - 0.2147\right) = 0.243$$

with $\lambda_1 T_s = 1\ \mu m \times 2741 K = 2741\ \mu m \cdot K$, F(0 - λT) = 0.2147. Substituting numerical values with T_s = T_{mp} = 2741 K and T_w = 300 K, find

Continued...

$$\left[0.2\times5.67\times10^{-8}\ \mathrm{W/m^2\cdot K^4}\,(300\,\mathrm{K})^4-0.243\times5.67\times10^{-8}\ \mathrm{W/m^2\cdot K^4}\,(2741\,\mathrm{K})^4\right]$$

$$\pi\,(0.003\,\mathrm{m})^2+\dot{q}\pi\,(0.003\,\mathrm{m})^3/6=0$$

$$\dot{q}=[-91.85+777{,}724]\,\mathrm{W/m^2}\,(6/0.003\,\mathrm{m})=1.556\times10^{9}\ \mathrm{W/m^3}$$ <

(b) For the laser-beam heating method, performing an energy balance on the drop considering absorbed laser irradiation, irradiation from the enclosure walls and emission,

$$\dot{E}_{in}-\dot{E}_{out}=0$$

$$[\alpha G-\varepsilon E_b\,(T_s)]A_s+\alpha_{las}G_{laser}A_p=0 \qquad (6)$$

where A_p represents the projected area of the droplet,

$$A_p=\pi D^2/4 \qquad (7)$$

Laser irradiation at 10.6 μm. Recognize that for the laser irradiation, G_{laser} (10.6 μm), the spectral absorptivity is

$$\alpha_{las}\,(10.6\,\mu\mathrm{m})=0.2$$

Substituting numerical values onto the energy balance, Eq. (6), find

$$\left[0.2\times\sigma\times(300\ \mathrm{K})^4-0.243\times\sigma\times(2741\ \mathrm{K})^4\right]\pi\,(0.003\ \mathrm{m})^2$$

$$+0.2\times G_{laser}\times\pi\,(0.003\ \mathrm{m})^2/4=0$$

$$G_{laser}\,(10.6\,\mu\mathrm{m})=1.56\times10^{7}\ \mathrm{W/m^2}=15.6\ \mathrm{W/mm^2}$$ <

Laser irradiation at 0.632 μm. For laser irradiation at 0.632 μm, the spectral absorptivity is

$$\alpha_{laser}\,(0.632\,\mu\mathrm{m})=0.4$$

Substituting numerical values into the energy balance, find

$$G_{laser}\,(0.632\,\mu\mathrm{m})=7.76\times10^{6}\,\mathrm{W/m^2}=7.8\ \mathrm{W/mm^2}$$ <

(c) With the method of heating terminated, the drop experiences only radiation exchange and begins cooling. Using the *IHT Lumped Capacitance Model* with irradiation and emission processes and the *Radiation Tool*, *Band Emission Factor* for estimating the emissivity as a function of drop temperature, Eq. (5), the time-to-cool to 400 K from an initial temperature, $T_s(0)$ = T_{mp} = 2741 K was found as

$T_s\,(t)=400\,\mathrm{K}$ $\qquad$ t = 772 s = 12.9 min <

COMMENTS: (1) Why doesn't the emissivity of the chamber wall, ε_w, affect the irradiation onto the drop?

(2) The validity of the lumped capacitance method can be determined by evaluating the Biot number,

Continued …..

$$\text{Bi} = \frac{\overline{h}D/6}{k} = \frac{185\,\text{W/m}^2\cdot\text{K} \times 0.003\,\text{m}/6}{75.6\,\text{W/m}\cdot\text{K}} = 0.007$$

where we estimated an average radiation coefficient as

$$\overline{h}_{rad} \approx 4\varepsilon\sigma\overline{T}^3 = 4 \times 0.2 \times 5.67 \times 10^{-8}\,\text{W/m}^2\cdot\text{K}^4\,(1600\,\text{K})^3 = 185\,\text{W/m}^2\cdot\text{K}$$

following the estimation method described in Problem 1.20. Since Bi << 0.1, the lumped capacitance method was appropriate.

(3) In the IHT model of part (c), the emissivity was calculated as a function of $T_s(t)$ varying from 0.243 at$T_s = T_{mp}$ to 0.200 at T_s = 300 K. If we had done an analysis assuming the drop were diffuse, gray with $\alpha = \varepsilon = 0.2$, the time-to-cool would be t = 773 s. How do you explain that this simpler approach predicts a time-to-cool that is in good agreement with the result of part (c)?

(4) A copy of the IHT workspace with the model of part (c) is shown below.

```
// Lumped Capacitance Model: Irradiation and Emission
/* Conservation of energy requirement on the control volume, CV.  */
Edotin - Edotout = Edotst
Edotin = As * ( + Gabs)
Edotout = As * ( + E )
Edotst = rho * vol * cp * Der(T,t)
// Absorbed irradiation from large surroundings on CS
Gabs = alpha * G
G =  sigma * Tsur^4
sigma = 5.67e-8      // Stefan-Boltzmann constant, W/m^2·K^4
// Emissive power of CS
E = eps * Eb
Eb = sigma * T^4
//sigma = 5.67e-8    // Stefan-Boltzmann constant, W/m^2·K^4
/* The independent variables for this system and their assigned numerical values are */
As =    pi * D^2        // surface area, m^2
vol =  pi * D^3 / 6      // vol, m^3
D =   0.003             // sphere diameter, m
rho =  8570             // density, kg/m^3
cp =  336               // specific heat, J/kg·K
// Emission, CS
//eps =   0.4           // emissivity
// Irradiation from large surroundings, CS
alpha =  0.2            // absorptivity
Tsur =   300            // surroundings temperature, K

// Radiation Tool - Band Emission Fractions
eps = eps1 * FL1T + eps2 * ( 1 - FL1T )
/* The blackbody band emission factor, Figure 12.14 and Table 12.1, is */
FL1T = F_lambda_T(lambda1,T)          // Eq 12.30
// where units are lambda (micrometers, mum) and T (K)
lambda1 = 1                     // wavelength, mum
eps1 = 0.4                      // spectral emissivity, lambda < lambda1
eps2 = 0.2                      // spectral emissivity, lambda > lambda1
```

PROBLEM 12.101

KNOWN: Temperatures of furnace and surroundings separated by ceramic plate. Maximum allowable temperature and spectral absorptivity of plate.

FIND: (a) Minimum value of air-side convection coefficient, h_o, (b) Effect of h_o on plate temperature.

SCHEMATIC:

ASSUMPTIONS: (1) Diffuse surface, (2) Negligible temperature gradients in plate, (3) Negligible inside convection, (4) Furnace and surroundings act as blackbodies.

ANALYSIS: (a) From a surface energy balance on the plate, $\alpha_w G_w + \alpha_{sur} G_{sur} = 2E + q''_{conv}$. Hence,

$$\alpha_w \sigma T_w^4 + \alpha_{sur} \sigma T_{sur}^4 = 2\varepsilon\sigma T_s^4 + h_o (T_s - T_\infty).$$

$$h_o = \frac{\alpha_w \sigma T_w^4 + \alpha_{sur} \sigma T_{sur}^4 - 2\varepsilon\sigma T_s^4}{(T_s - T_\infty)}$$

Evaluating the absorptivities and emissivity,

$$\alpha_w = \int_0^\infty \alpha_\lambda G_\lambda \, d\lambda / G = \int_0^\infty \alpha_\lambda E_{\lambda b}(T_w)/E_b(T_w) d\lambda = 0.3F_{(0-3\mu m)} + 0.8\left[1 - F_{(0-3\mu m)}\right]$$

With $\lambda T_w = 3\,\mu m \times 2400\text{ K} = 7200\,\mu m \cdot K$, Table 12.1 $\rightarrow F_{(0-3\mu m)} = 0.819$. Hence,

$$\alpha_w = 0.3 \times 0.819 + 0.8(1 - 0.819) = 0.391$$

Since T_{sur} = 300 K, irradiation from the surroundings is at wavelengths well above 3 μm. Hence,

$$\alpha_{sur} = \int_0^\infty \alpha_\lambda E_{\lambda b}(T_{sur})/E_b(T_{sur}) d\lambda \approx 0.800.$$

The emissivity is $\varepsilon = \int_0^\infty \varepsilon_\lambda E_{\lambda b}(T_s)/E_b(T_s) d\lambda = 0.3F_{(0-3\mu m)} + 0.8\left[1 - F_{(0-3\mu m)}\right]$. With $\lambda T_s = 5400\,\mu m \cdot K$, Table 12.1 $\rightarrow F_{(0-3\mu m)} = 0.680$. Hence, $\varepsilon = 0.3 \times 0.68 + 0.8(1 - 0.68) = 0.460$.

For the maximum allowable value of T_s = 1800 K, it follows that

$$h_o = \frac{0.391 \times 5.67 \times 10^{-8}(2400)^4 + 0.8 \times 5.67 \times 10^{-8}(300)^4 - 2 \times 0.46 \times 5.67 \times 10^{-8}(1800)^4}{(1800 - 300)}$$

$$h_o = \frac{7.335 \times 10^5 + 3.674 \times 10^2 - 5.476 \times 10^5}{1500} = 126\,W/m^2 \cdot K. \quad <$$

(b) Using the IHT *First Law* Model with the *Radiation* Toolpad, parametric calculations were performed to determine the effect of h_o.

Continued...

PROBLEM 12.101 (Cont.)

1900
1800
1700
1600
50 100 150 200 250
Temperature, T(K)
Convection coefficient, ho(W/m^2.K)

With increasing h_o, and hence enhanced convection heat transfer at the outer surface, the plate temperature is reduced.

COMMENTS: (1) The surface is not gray. (2) The required value of $h_o \geq 126\,W/m^2 \cdot K$ is well within the range of air cooling.

PROBLEM 12.102

KNOWN: Spectral radiative properties of thin coating applied to long circular copper rods of prescribed diameter and initial temperature. Wall and atmosphere conditions of furnace in which rods are inserted.

FIND: (a) Emissivity and absorptivity of the coated rods when their temperature is T_s = 300 K, (b) Initial rate of change of their temperature, dT_s/dt, (c) Emissivity and absorptivity when they reach steady-state temperature, and (d) Time required for the rods, initially at T_s = 300 K, to reach 1000 K.

SCHEMATIC:

ASSUMPTIONS: (1) Rod temperature is uniform, (2) Nitrogen is quiescent, (3) Constant properties, (4) Diffuse, opaque surface coating, (5) Furnace walls form a blackbody cavity about the cylinders, $G = E_b(T_f)$, (6) Negligible end effects.

PROPERTIES: *Table A.1*, Copper (300 K): $\rho = 8933\ kg/m^3$, $c_p = 385\ J/kg\cdot K$, $k = 401\ W/m\cdot K$; *Table A.4*, Nitrogen (T_f = 800 K, 1 atm): $\nu = 82.9 \times 10^{-6}\ m^2/s$, $k = 0.0548\ W/m\cdot K$, $\alpha = 116 \times 10^{-6}\ m^2/s$, Pr = 0.715, $\beta = (T_f)^{-1} = 1.25 \times 10^{-3}\ K^{-1}$.

ANALYSIS: (a) The total emissivity of the copper rod, ε, at T_s = 300 K follows from Eq. 12.38 which can be expressed in terms of the band emission factor, F(0 - λT), Eq. 12.30,

$$\varepsilon = \varepsilon_1 F\left(0-\lambda_1 T_s\right) + \varepsilon_2\left[1 - F\left(0-\lambda_1 T_s\right)\right] \qquad (1)$$

$$\varepsilon = 0.4 \times 0.0021 + 0.8[1 - 0.0021] = 0.799$$ <

where, from Table 12.1, with $\lambda_1 T_s = 4\ \mu m \times 300\ K = 1200\ \mu m\cdot K$, F(0 - λT) = 0.0021. The total absorptivity, α, for irradiation for the furnace walls at T_f = 1300 K, is

$$\alpha = \varepsilon_1 F\left(0-\lambda_1 T_f\right) + \varepsilon_2\left[1 - F\left(0-\lambda_1 T_f\right)\right] \qquad (2)$$

$$\alpha = 0.4 \times 0.6590 + 0.8[1 - 0.6590] = 0.536$$ <

where, from Table 12.1, with $\lambda_1 T_f = 4\ \mu m \times 1300\ K = 5200\ K$, F(0 - λT) = 0.6590.

(b) From an energy balance on a control volume about the rod,

$$\dot{E}_{st} = \rho c_p\left(\pi D^2/4\right)L\left(dT/dt\right) = \dot{E}_{in} - \dot{E}_{out} = \pi DL\left[\alpha G + \bar{h}\left(T_\infty - T_s\right) - E\right]$$

$$dT_s/dt = 4\left[\alpha G + \bar{h}\left(T_\infty - T_s\right) - \varepsilon\sigma T_s^4\right]\Big/\rho c_p D\,. \qquad (3)$$

With

$$Ra_D = \frac{g\beta\left(T_\infty - T_s\right)D^3}{\nu\alpha} = \frac{9.8\,m^2/s\left(1.25\times10^{-3}K^{-1}\right)1000\,K\left(0.01\,m\right)^3}{82.9\times10^{-6}\,m^2/s\times116\times10^{-6}\,m^2/s} = 1274 \qquad (4)$$

Eq. 9.34 gives

Continued...

$$\bar{h} = \frac{0.0548}{0.01\,m}\left\{0.60 + \frac{0.387\left(1274\right)^{1/6}}{\left[1+\left(0.559/0.715\right)^{9/16}\right]^{8/27}}\right\}^2 = 15.1\,W/m^2\cdot K \tag{5}$$

With values of ε and α from part (a), the rate of temperature change with time is

$$dT_s/dt = \frac{4\left[0.53\times5.67\times10^{-8}\,W/m^2\cdot K^4\times\left(1300\,K\right)^4 + 15.1\,W/m^2\cdot K\times1000\,K - 0.8\times5.67\times10^{-8}\,W/m^2\cdot K\times\left(300\,K\right)^4\right]}{8933\,kg/m^3\times385\,J/kg\cdot K\times0.01\,m}$$

$$dT_s/dt = 1.16\times10^{-4}\left[85{,}829+15{,}100-3767\right]K/s = 11.7\,K/s\,. \quad <$$

(c) Under steady-state conditions, $T_s = T_\infty = T_f = 1300$ K. For this situation, ε = α, hence

$$\varepsilon = \alpha = 0.536 \quad <$$

(d) The time required for the rods, initially at $T_s(0) = 300$ K, to reach 1000 K can be determined using the lumped capacitance method. Using the *IHT Lumped Capacitance Model*, considering convection, irradiation and emission processes; the *Correlations Tool*, *Free Convection*, *Horizontal Cylinder*; *Radiation Tool*, *Band Emission Fractions*; and a user-generated *Lookup Table Function* for the nitrogen thermophysical properties, find

$T_s(t_o) = 1000$ K $\qquad t_o = 81.8$ s $\quad$ <

COMMENTS: (1) To determine the validity of the lumped capacitance method to this heating process, evaluate the approximate Biot number, Bi = $\bar{h}D/k$ = 15 W/m²·K × 0.010 m/401 W/m·K = 0.0004. Since Bi << 0.1, the method is appropriate.

(2) The IHT workspace with the model used for part (c) is shown below.

```
// Lumped Capacitance Model - irradiation, emission, convection
/* Conservation of energy requirement on the control volume, CV.  */
Edotin - Edotout = Edotst
Edotin = As * ( + Gabs)
Edotout = As * ( + q''cv + E )
Edotst = rho * vol * cp * Der(Ts,t)
//Convection heat flux for control surface CS
q''cv = h * ( Ts - Tinf )
// Emissive power of CS
E = eps * Eb
Eb = sigma * Ts^4
sigma = 5.67e-8                 // Stefan-Boltzmann constant, W/m^2·K^4
// Absorbed irradiation from large surroundings on CS
Gabs = alpha * G
G = sigma * Tf^4
/* The independent variables for this system and their assigned numerical values are */
As =    pi * D * 1              // surface area, m^2
vol =   pi * D^2 / 4 * 1        // vol, m^3
rho =   8933                    // density, kg/m^3
cp =    433                     // specific heat, J/kg·K; evaluated at 800 K
// Convection heat flux, CS
//h =               // convection coefficient, W/m^2·K
Tinf =   1300       // fluid temperature, K
// Emission, CS
//eps =             // emissivity
// Irradiation from large surroundings, CS
//alpha =           // absorptivity
Tf =   1300         // surroundings temperature, K
```

Continued...

PROBLEM 12.102 (Cont.)

```
// Radiative Properties Tool - Band Emission Fraction
eps = eps1 * FL1Ts + eps2 * (1 - FL1Ts)
/* The blackbody band emission factor, Figure 12.14 and Table 12.1, is */
FL1Ts = F_lambda_T(lambda1,Ts)                  // Eq 12.30
// where units are lambda (micrometers, mum) and T (K)
alpha = eps1 * FL1Tf  + eps2 * (1- FL1Tf)
/* The blackbody band emission factor, Figure 12.14 and Table 12.1, is */
FL1Tf = F_lambda_T(lambda1,Tf)                  // Eq 12.30

// Assigned Variables:
D = 0.010          // Cylinder diameter, m
eps1 = 0.4         // Spectral emissivity for lambda < lambda1
eps2 = 0.8         // Spectral emissivity for lambda > lambda1
lambda1 = 4        // Wavelength, mum

// Correlations Tool - Free Convection, Cylinder, Horizontal:
NuDbar = NuD_bar_FC_HC(RaD,Pr)                  // Eq 9.34
NuDbar = h * D / k
RaD = g * beta * deltaT * D^3 / (nu * alphan)   //Eq 9.25
deltaT = abs(Ts - Tinf)
g = 9.8    // gravitational constant, m/s^2
// Evaluate properties at the film temperature, Tf.
Tff =Tfluid_avg(Tinf,Ts)

// Properties Tool - Nitrogen: Lookup Table Function "nitrog"
nu = lookupval (nitrog, 1, Tff, 2)
k = lookupval (nitrog, 1, Tff, 3)
alphan =  lookupval (nitrog, 1, Tff, 4)
Pr = lookupval (nitrog, 1, Tff, 5)
beta = 1 / Tff
/* Lookup table function, nitrog; from Table A.4 1 atm):
Columns: T(K), nu(m^2/s), k(W/m.K), alpha(m^2/s), Pr
300     1.586E-5        0.0259   2.21E-5   0.716
350     2.078E-5        0.0293   2.92E-5   0.711
400     2.616E-5        0.0327   3.71E-5   0.704
450     3.201E-5        0.0358   4.56E-5   0.703
500     3.824E-5        0.0389   5.47E-5   0.7
550     4.17E-5         0.0417   6.39E-5   0.702
600     5.179E-5        0.0446   7.39E-5   0.701
700     6.671E-5        0.0499   9.44E-5   0.706
800     8.29E-5         0.0548   0.000116 0.715
900     0.0001003       0.0597   0.000139 0.721
1000    0.0001187       0.0647   0.000165 0.721    */
```

PROBLEM 12.103

KNOWN: Large combination convection-radiation oven heating a cylindrical product of a prescribed spectral emissivity.

FIND: (a) Initial heat transfer rate to the product when first placed in oven at 300 K, (b) Steady-state temperature of the product, (c) Time to achieve a temperature within 50°C of the steady-state temperature.

SCHEMATIC:

ASSUMPTIONS: (1) Cylinder is opaque-diffuse, (2) Oven walls are very large compared to the product, (3) Cylinder end effects are negligible, (4) ε_λ is dependent of temperature.

PROPERTIES: *Table A-4*, Air (T_f = 525 K, 1 atm): $\nu = 42.2\times10^{-6}\ \text{m}^2/\text{s}, k = 0.0423\ \text{W/m}\cdot\text{K}$, Pr = 0.684; ($T_f$ = 850 K (assumed), 1 atm): $\nu = 93.8\times10^{-6}\ \text{m}^2/\text{s}$, k = 0.0596 W/m·K, Pr = 0.716.

ANALYSIS: (a) The net heat rate to the product is $q_{net} = A_s(q''_{conv} + \alpha G - \varepsilon E_b)$, or

$$q_{net} = \pi DL[\bar{h}(T_\infty - T) + \alpha G - \varepsilon\sigma T^4] \tag{1}$$

Evaluating properties at T_f = 525 K, $Re_D = VD/\nu = 5\,\text{m/s}\times0.025\,\text{m}/42.2\times10^{-6}\,\text{m}^2/\text{s} = 2960$, and the Churchill-Bernstein correlation yields

$$\overline{Nu}_D = \frac{\bar{h}D}{k} = 0.3 + \frac{0.62\,Re_D^{1/2}\,Pr^{1/3}}{[1+(0.4/Pr)^{2/3}]^{1/4}}\left[1+\left(\frac{Re_D}{282,000}\right)^{5/8}\right]^{4/5} = 27.5$$

Hence,

$$\bar{h} = \frac{0.0423\,\text{W/m}\cdot\text{K}}{0.025\text{m}}\times27.5 = 46.5\,\text{W/m}^2\cdot\text{K}.$$

The total, hemispherical emissivity of the diffuse, spectrally selective surface follows from Eq. 12.38,

$\varepsilon = \int_0^\infty \varepsilon_\lambda(\lambda, T_s)E_{\lambda,b}\big/\sigma T_s^4 = \varepsilon_1 F_{(0\to4\mu m)} + \varepsilon_2(1 - F_{(0\to4\mu m)})$, where $\lambda T = 4\ \mu\text{m}\times300\ \text{K} = 1200$ µm·K and $F_{(0-\lambda T)} = 0.002$ (Table 12.1). Hence, $\varepsilon = 0.8\times0.002 + 0.2\,(1 - 0.002) = 0.201$.

The absorptivity is for irradiation from the oven walls which, because they are large and isothermal, behave as a black surface at 1000 K. From Eq. 12.46, with $G_\lambda = E_{\lambda,b}$ (λ, 1000 K) and $\alpha_\lambda = \varepsilon_\lambda$,

$$\alpha = \varepsilon_1 F_{(0\to4\mu m)} + \varepsilon_2(1 - F_{(0\to4\mu m)}) = 0.8\times0.481 + 0.2(1-0.481) = 0.489$$

where, for $\lambda T = 4\times1000 = 4000$ µm·K from Table 12.1, $F_{(0-\lambda T)} = 0.481$. From Eq. (1) the net initial heat rate is $q_{net} = \pi\times0.025\text{m}\times0.2\,\text{m}[46.5\,\text{W/m}^2\cdot\text{K}(750-300)\ \text{K} + 0.489\sigma(1000)^4\text{K}^4 - 0.201\sigma(300\ \text{K})^4]$

Continued...

$q = 763$ W. <

(b) For the steady-state condition, the net heat rate will be zero, and the energy balance yields,

$$0 = \overline{h}(T_\infty - T) + \alpha G - \varepsilon\sigma T^4 \qquad (2)$$

Evaluating properties at an assumed film temperature of $T_f = 850$ K, $Re_D = VD/\nu = 5$ m/s $\times$ 0.025 m/93.8 $\times 10^{-6}$ m^2/s = 1333, and the Churchill-Bernstein correlation yields $\overline{Nu}_D = 18.6$. Hence, $\overline{h}$ = 18.6 (0.0596 W/m $\cdot$ K)/0.025 m = 44.3 W/m^2 $\cdot$ K. Since irradiation from the oven walls is fixed, the absorptivity is unchanged, in which case $\alpha = 0.489$. However, the emissivity depends on the product temperature. *Assuming* T = 950 K, we obtain

$$\varepsilon = \varepsilon_1 F_{(0\to 4\mu m)} + \varepsilon_2 (1 - F_{(0\to 4\mu m)}) = 0.8 \times 0.443 + 0.2(1 - 0.443) = 0.466$$

where for $\lambda T = 4 \times 950 = 3800$ μm·K, $F_{0-\lambda T} = 0.443$, from Table 12.1. Substituting values into Eq. (2) with $\sigma = 5.67 \times 10^{-8}$ W/m^2·K^4,

$$0 = 44.3\,(750 - T) + 0.489\,\sigma\,(1000\text{ K})^4 - 0.466\,\sigma\,T^4.$$

A trial-and-error solution yields T ≈ 930 K. <

(c) Using the IHT *Lumped Capacitance* Model with the *Correlations*, *Properties* (for copper and air) and *Radiation* Toolpads, the transient response of the cylinder was computed and the time to reach T = 880 K is

$t \approx 537$ s. <

COMMENTS: Note that $\overline{h}$ is relatively insensitive to T, while ε is not. At T = 930 K, ε = 0.456.

PROBLEM 12.104

KNOWN: Workpiece, initially at 25°C, to be annealed at a temperature above 725°C for a period of 5 minutes and then cooled; furnace wall temperature and convection conditions; cooling surroundings and convection conditions.

FIND: (a) Emissivity and absorptivity of the workpiece at 25°C when it is placed in the furnace, (b) Net heat rate per unit area into the workpiece for this initial condition; change in temperature with time, dT/dt, for the workpiece; (c) Calculate and plot the emissivity of the workpiece as a function of temperature for the range 25 to 750°C using the *Radiation* | *Band Emission* tool in *IHT*, (d) The time required for the workpiece to reach 725°C assuming the applicability of the lumped-capacitance method using the *DER(T,t)* function in *IHT* to represent the temperature-time derivative in your energy balance; (e) Calculate the time for the workpiece to cool from 750°C to a safe-to-touch temperature of 40°C if the cool surroundings and cooling air temperature are 25°C and the convection coefficient is 100 $W/m^2 \cdot K$; and (f) Assuming that the workpiece temperature increases from 725 to 750°C during the five-minute annealing period, sketch (don't plot) the temperature history of the workpiece from the start of heating to the end of cooling; identify key features of the process; determine the total time requirement; and justify the lumped-capacitance method of analysis.

SCHEMATIC:

ASSUMPTIONS: (1) Workpiece is opaque and diffuse, (2) Spectral emissivity is independent of temperature, and (3) Furnace and cooling environment are large isothermal surroundings.

ANALYSIS: (a) Using Eqs. 12.38 and 12.46, ε and α can be determined using band-emission factors, Eq. 12.30 and 12.31.

Emissivity, workpiece at 25°C

$$\varepsilon = \varepsilon_{\lambda 1} \cdot F_{(0-\lambda T)} + \varepsilon_{\lambda 2}\left(1 - F_{(0-\lambda T)}\right)$$

$$\varepsilon = 0.3 \times 1.6 \times 10^{-5} + 0.8 \times \left(1 - 1.6 \times 10^{-5}\right) = 0.8$$ <

where $F_{(0-\lambda T)}$ is determined from Table 12.1 with λT = 2.5 μm × 298 K = 745 μm·K.

Absorptivity, furnace temperature T_f = 750°C

$$\alpha = \varepsilon_{\lambda 1} \cdot F_{(0-\lambda,\ T)} + \varepsilon_2 \cdot \left(1 - F_{(0-\lambda,\ T)}\right)$$

$$\alpha = 0.3 \times 0.174 + 0.8 \times (1 - 0.174) = 0.713$$ <

where $F_{(0\ -\ \lambda T)}$ is determined with λT = 2.5 μm × 1023 K = 2557.5 μm·K.

(b) For the initial condition, $T(0) = T_i$, the energy balance shown schematically below is written in terms of the net heat rate *in*,

Continued …..

PROBLEM 12.104 (Cont.)

$$\dot{E}''_{in} - \dot{E}''_{out} = \dot{E}''_{st} \qquad \text{and} \qquad q''_{net,in} = \dot{E}''_{in} - \dot{E}''_{out}$$

$$q''_{net,in} = 2\left[q''_{cv} - \varepsilon\, E_b(T_i) + \alpha E_b(T_f)\right]$$

where $G = E_b\,(T_f)$. Substituting numerical values,

$$q''_{net,in} = 2\left[h(T_\infty - T_i) - \varepsilon\sigma T_i^4 + \alpha\sigma T_f^4\right]$$

$$q''_{net,in} = 2\Big[100\ \text{W/m}^2\cdot\text{K}\ (750-25)\text{K} - 0.8\times 5.67\times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4 (298\ \text{K})^4$$

$$+\ 0.713\times 5.67\times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4\ (1023\ \text{K})^4\Big]$$

$$q''_{net,in} = 2\times 116.4\ \text{kW/m}^2 = 233\ \text{kW/m}^2$$ <

Considering the energy storage term,

$$\dot{E}''_{st} = \rho c L \left(\frac{dT}{dt}\right)_i = q''_{net,in}$$

$$\left.\frac{dT}{dt}\right)_i = \frac{q''_{net,in}}{\rho c L} = \frac{233\ \text{kW/m}^2}{2700\ \text{kg/m}^3 \times 885\ \text{J/kg}\cdot\text{K}\times 0.010\ \text{m}} = 9.75\ \text{K/s}$$ <

(b) Energy balance, initial condition

(c) With the relation for ε of Part (a) in the *IHT* workspace, and using the *Radiation* | *Band Emission* tool, ε as a function of workpiece temperature is calculated and plotted below.

Workpiece emissivity as a function of its temperature

Continued …..

As expected, ε decreases with increasing T, and when $T = T_f = 750°C$, ε = α = 0.713. Why is that so?

(d) The energy balance of Part (b), using the lumped capacitance method with the *IHT DER (T,t)* function, has the form,

$$2\left[h\left(T_\infty - T\right) - \varepsilon\sigma T^4 + \alpha\sigma T_f^4\right] = \rho c L \ \text{DER}\ (T, t)$$

where ε = ε (T) from Part (c). From a plot of T vs. t (not shown) in the *IHT* workspace, find

$$T(t_a) = 725^\circ C \quad \text{when} \quad t_a = 186\ s$$ <

(e) The time to cool the workpiece from 750°C to the safe-to-touch temperature of 40°C can be determined using the *IHT* code from Part (d). The cooling conditions are $T_\infty = 25°C$ and h = 100 $W/m^2 \cdot K$ with $T_{sur} = 25°C$. The emissivity is still evaluated using the relation of Part (c), but the absorptivity, which depends upon the surrounding temperature, is α = 0.80. From the results in the *IHT* workspace, find

$$T(t_c) = 40^\circ C \qquad \text{when} \qquad t_c = 413\ s$$ <

(f) Assuming the workpiece temperature increases from 725°C to 750°C during a five-minute annealing period, the temperature history is as shown below.

The workpiece heats from 25°C to 725°C in t_a = 186 s, anneals for a 5-minute period during which the temperature reaches 750°C, followed by the cool-down process which takes 413 s. The total required time is

$$t = t_a + 5 \times 60\ s + t_c = (186 + 300 + 413)s = 899\ s = 15\ min$$ <

Continued …..

The Biot number based upon convection only is

$$\mathrm{Bi} = \frac{h_{cv}(L/2)}{k} = \frac{100\ \mathrm{W/m^2 \cdot K} \times 0.005\ \mathrm{m}}{165\ \mathrm{W/m \cdot K}} = 0.003 << 0.1$$

so the lumped-capacitance method of analysis is appropriate.

COMMENTS: The *IHT* code to obtain the heating time, including emissivity as a function of the workpiece temperature, Part (b), is shown below, complete except for the input variables.

```
/* Analysis.  The radiative properties and net heat flux in are calculated when the workpiece is
just inserted into the furnace.  The workpiece experiences emission, absorbed irradiation and
convection processes.  See Help | Solver | Intrinsic Functions for information on DER(T, t). */

/* Results - conditions at t = 186 s, Ts C - 725 C
FL1T      T_C       Tf        L         Tf_C      Tinf_C    eps1      eps2      h         k
          lambda1   rho       t         T
0.1607    725.1     1023      0.01      750       750       0.3       0.8       100       165
          2.5       2700      186       998.1     */

// Energy Balance
2 * ( h * (Tinf - T) + alpha * G - eps * sigma * T^4) = rho * cp * L * DER(T,t)
sigma = 5.67e-8
G = sigma * Tf^4

// Emissivity and absorptivity
eps = FL1T * eps1 + (1 - FL1T) * eps2
FL1T = F_lambda_T(lambda1, T)          // Eq 12.30
alpha = 0.713

// Temperature conversions
T_C = T - 273                    // For customary units, graphical output
Tf_C = Tf - 273
Tinf_C = Tinf - 273
```

PROBLEM 12.105

KNOWN: For the semiconductor silicon, the spectral distribution of absorptivity, α_λ, at selected temperatures. High-intensity, tungsten halogen lamps having spectral distribution approximating that of a blackbody at 2800 K.

FIND: (a) 1%-limits of the spectral band that includes 98% of the blackbody radiation corresponding to the spectral distribution of the lamps; spectral region for which you need to know the spectral absorptivity; (b) Sketch the variation of the total absorptivity as a function of silicon temperature; explain key features; (c) Calculate the total absorptivity at 400, 600 and 900°C for the lamp irradiation; explain results and the temperature dependence; (d) Calculate the total emissivity of the wafer at 600 and 900°C; explain results and the temperature dependence; and (e) Irradiation on the upper surface required to maintain the wafer at 600°C in a vacuum chamber with walls at 20°C. Use the *Look-up Table* and *Integral Functions* of *IHT* to perform the necessary integrations.

SCHEMATIC:

ASSUMPTIONS: (1) Silicon is a diffuse emitter, (2) Chamber is large, isothermal surroundings for the wafer, (3) Wafer is isothermal.

ANALYSIS: (a) From Eqs. 12.30 and 12.31, using Table 12.1 for the band emission factors, $F_{(0 - \lambda T)}$, equal to 0.01 and 0.99 are:

$$F_{(0\rightarrow\lambda 1\cdot T)} = 0.01 \text{ at } \lambda_1 \cdot T = 1437\ \mu\text{m}\cdot\text{K}$$

$$F_{(0\rightarrow\lambda 2\cdot T)} = 0.99 \text{ at } \lambda_2 \cdot T = 23{,}324\ \mu\text{m}\cdot\text{K}$$

So that we have λ_1 and λ_2 limits for several temperatures, the following values are tabulated.

T(°C)	T(K)	λ_1(μm)	λ_2(μm)
-	2800	0.51	8.33
400	673	2.14	34.7
600	873	1.65	26.7
900	1173	1.23	19.9

<

For the 2800 K blackbody lamp irradiation, we need to know the spectral absorptivity over the spectral range 0.51 to 8.33 μm in order to include 98% of the radiation.

(b) The spectral absorptivity is calculated from Eq. 12.46 in which the spectral distribution of the lamp irradiation G_λ is proportional to the blackbody spectral emissive power $E_{\lambda,b}(\lambda, T)$ at the temperature of lamps, T_ℓ 2800 K.

$$\alpha_\ell = \frac{\int_0^\infty \alpha_\lambda G_\lambda d\lambda}{\int_0^\infty G_\lambda d\lambda} = \frac{\int_0^\infty \alpha_\lambda\, E_{\lambda,b}(\lambda,\ 2800\ \text{K})}{\sigma T_\ell^4}$$

Continued …..

For 2800 K, the peak of the blackbody curve is at 1 μm; the limits of integration for 98% coverage are 0.5 to 8.3 μm according to part (a) results. Note that α_λ increases at all wavelengths with temperature, until around 900°C where the behavior is gray. Hence, we'd expect the total absorptivity of the wafer for lamp irradiation to appear as shown in the graph below.

At 900°C, since the wafer is gray, we expect $\alpha_\ell = \alpha_\lambda \approx 0.68$. Near room temperature, since $\alpha_\lambda \approx 0$ beyond the band edge, α_ℓ is dependent upon α_λ in the spectral region below and slightly beyond the peak. From the blackbody tables, the band emission fraction to the short-wavelength side of the peak is 0.25. Hence, estimate $\alpha_\ell \approx 0.68 \times 0.25 = 0.17$ at these low temperatures. The increase of α_ℓ with temperature is at first moderate, since the longer wavelength region is less significant than is the shorter region. As temperature increases, the α_λ closer to the peak begin to change more noticeably, explaining the greater dependence of α_ℓ on temperature.

(c) The integration of part (b) can be performed numerically using the *IHT INTEGRAL* function and specifying the spectral absorptivity in a *Lookup Table* file (*.lut). The code is shown in the Comments (1) and the results are:

T_w(°C)	400	600	900	
α_ℓ	0.30	0.59	0.68	<

(d) The total emissivity can be calculated from Eq. 12.38, recognizing that $\varepsilon_\lambda = \alpha_\lambda$ and that for silicon temperatures of 600 and 900°C, the 1% limits for the spectral integration are 1.65 - 26.7 μm and 1.23 - 19.9 μm, respectively. The integration is performed in the same manner as described in part (c); see Comments (2).

T(°C)	600	900	
ε	0.66	0.68	<

(e) From an energy balance on the silicon wafer with irradiation on the upper surface as shown in the schematic below, calculate the irradiation required to maintain the wafer at 600°C.

$$\dot{E}''_{in} - \dot{E}''_{out} = 0 \qquad \alpha_\ell G_\ell - 2\left[\varepsilon E_b\left(T_w\right) - \alpha_{sur} E_b\left(T_{sur}\right)\right] = 0$$

Recognize that α_{sur} corresponds to the spectral distribution of $E_{\lambda,b}(T_{sur})$; that is, upon α_λ for long wavelengths ($\lambda_{max} \approx 10$ μm). We assume $\alpha_{sur} \approx 0.1$, and with $T_{sur} = 20$°C, find

$$0.59\ G_\ell - 2\sigma\left[0.66(600+273)^4 K^4 - 0.1(20+273)^4 K^4\right] = 0$$

Continued …..

PROBLEM 12.105 (Cont.)

$$G_\ell = 73.5 \text{ kW/m}^2$$ <

where $E_b(T) = \sigma T^4$ and $\sigma = 5.67 \times 10^{-8}$ W/m$^2 \cdot$K^4.

COMMENTS: (1) The *IHT* code to obtain the total absorptivity for the lamp irradiation, α_ℓ for a wafer temperature of 400°C is shown below. Similar look-up tables were written for the spectral absorptivity for 600 and 800°C.

```
/* Results; integration for total absorptivity of lamp irradiation
T = 400 C; find abs_t = 0.30
ILb     absL    abs_t   C1       C2       T      sigma    lambda
1773    0.45    0.3012  3.742E8  1.439E4  2800   5.67E-8  10      */

// Input variables
T = 2800            // Lamp blackbody distribution

// Total absorptivity integral, Eq. 12.46
abs_t = pi * integral (ILsi, lambda) / (sigma * T^4)      // See Help | Solver
sigma = 5.67e-8

// Blackbody spectral intensity, Tools | Radiation
/* From Planck's law, the blackbody spectral intensity is */
ILsi = absL * ILb
ILb = I_lambda_b(lambda, T, C1, C2)    // Eq. 12.25
// where units are ILb(W/m^2.sr.mum), lambda (mum) and T (K) with
C1 = 3.7420e8      // First radiation constant, W·mum^4/m^2
C2 = 1.4388e4      // Second radiation constant, mum·K
// and (mum) represents (micrometers).

// Spectral absorptivity function
absL = LOOKUPVAL(abs_400, 1, lambda, 2)     // Silicon spectral data at 400 C
//absL = LOOKUPVAL(abs_600, 1, lambda, 2)   // Silicon spectral data at 600 C
//absL = LOOKUPVAL(abs_900, 1, lambda, 2)   // Silicon spectral data at 900 C

// Lookup table values for Si spectral data at 600 C
/* The table file name is abs_400.lut, with 2 columns and 10 rows
0.5     0.68
1.2     0.68
1.3     0.025
2       0.05
3       0.1
4       0.17
5       0.22
6       0.28
8       0.37
10      0.45 */
```

(2) The *IHT* code to obtain the total emissivity for a wafer temperature of 600°C has the same organization as for obtaining the total absorptivity. We perform the integration, however, with the blackbody spectral emissivity evaluated at the wafer temperature (rather than the lamp temperature). The same look-up file for the spectral absorptivity created in the part (c) code can be used.

PROBLEM 12.106

KNOWN: Solar irradiation of 1100 W/m^2 incident on a flat roof surface of prescribed solar absorptivity and emissivity; air temperature and convection heat transfer coefficient.

FIND: (a) Roof surface temperature, (b) Effect of absorptivity, emissivity and convection coefficient on temperature.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Back-side of plate is perfectly insulated, (3) Negligible irradiation to plate by atmospheric (sky) emission.

ANALYSIS: (a) Performing a surface energy balance on the exposed side of the plate,

$$\alpha_S G_S - q''_{conv} - \varepsilon E_b(T_s) = 0 \qquad \alpha_S G_S - \bar{h}(T_s - T_\infty) - \varepsilon\sigma T_s^4 = 0$$

Substituting numerical values and using absolute temperatures,

$$0.6 \times 1100 \frac{W}{m^2} - 25 \frac{W}{m^2 \cdot K}(T_s - 300)K - 0.2(5.67 \times 10^{-8}\ W/m^2 \cdot K^4)T_s^4 = 0$$

Regrouping, $8160 = 25T_s + 1.1340 \times 10^{-8} T_s^4$, and performing a trial-and-error solution,

$T_s = 321.5\ K = 48.5°C$. <

(b) Using the IHT *First Law* Model for a plane wall, the following results were obtained.

Irrespective of the value of $\bar{h}$, T decreases with increasing ε (due to increased emission) and decreasing α_S (due to reduced absorption of solar energy). For moderate to large α_S and/or small ε (net radiation transfer to the surface) T decreases with increasing $\bar{h}$ due to enhanced cooling by convection. However, for small α_S and large ε, emission exceeds absorption, dictating convection heat transfer to the surface and hence $T < T_\infty$. With increasing $\bar{h}$, $T \to T_\infty$, irrespective of the values of α_S and ε.

COMMENTS: To minimize the roof temperature, the value of ε/α_S should be maximized.

PROBLEM 12.107

KNOWN: Cavity with window whose outer surface experiences convection and radiation.

FIND: Temperature of the window and power required to maintain cavity at prescribed temperature.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Cavity behaves as a blackbody, (3) Solar spectral distribution is that of a blackbody at 5800K, (4) Window is isothermal, (5) Negligible convection on lower surface of window.

PROPERTIES: Window material: $0.2 \le \lambda \le 4$ μm, $\tau_\lambda = 0.9$, $\rho_\lambda = 0$, hence $\alpha_\lambda = 1 - \tau_\lambda = 0.1$; 4 μm < λ, $\tau_\lambda = 0$, $\alpha = \varepsilon = 0.95$, diffuse-gray, opaque

ANALYSIS: To determine the window temperature, perform an energy balance on the window,

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$\left[\alpha_{sur}G_{sur} + \alpha_S G_S - \varepsilon E_b - q''_{conv}\right]_{upper} + \left[\alpha_c G_c - \varepsilon E_b(T)\right]_{lower} = 0. \quad (1)$$

Calculate the absorptivities for various irradiation conditions using Eq. 12.46,

$$\alpha = \int_0^\infty \alpha_\lambda G_\lambda d\lambda \Big/ \int_0^\infty G_\lambda d\lambda \quad (2)$$

where $G(\lambda)$ is the spectral distribution of the irradiation.

Surroundings, α_{sur}: $G_{sur} = E_b(T_{sur}) = \sigma T_{sur}^4$

$$\alpha_{sur} = 0.1\left[F_{(0\to 4\mu m)} - F_{(0\to 0.2\mu m)}\right] + 0.95\left[1 - F_{(0\to 4\mu m)}\right]$$

where from Table 12.1, with $T = T_{sur} = (25 + 273)K = 298K$,

$$\lambda T = 0.2\mu m \times 298K = 59.6\mu m\cdot K, \quad F_{(0-\lambda T)} = 0.000$$

$$\lambda T = 4\mu m \times 298K = 1192\mu m\cdot K, \quad F_{(0-\lambda T)} = 0.002$$

$$\alpha_{sur} = 0.1[0.002 - 0.000] + 0.95[1 - 0.002] = 0.948. \quad (3)$$

Solar, α_S: $G_S \sim E_b$ (5800K)

$$\alpha_S = 0.1\left[F_{(0\to 4\mu m)} - F_{(0\to 0.2\mu m)}\right] + 0.95\left[1 - F_{(0\to 4\mu m)}\right]$$

where from Table 12.1, with T = 5800K,

$$\lambda T = 0.2\mu m \times 5800K = 1160\mu m\cdot K, \quad F_{(0-\lambda T)} = 0.002$$

$$\lambda T = 4\mu m \times 5800K = 23{,}200\mu m\cdot K, \quad F_{(0-\lambda T)} = 0.990$$

$$\alpha_S = 0.1[0.990 - 0.002] + 0.95[1 - 0.990] = 0.108. \quad (4)$$

Continued …..

PROBLEM 12.107 (Cont.)

Cavity, α_c: $G_c = E_b(T_c) = \sigma T_c^4$

$$\alpha_c = 0.1\left[F_{(0\to 4\mu m)} - F_{(0\to 0.2\mu m)}\right] + 0.95\left[1 - F_{(0\to 4\mu m)}\right]$$

where from Table 12.1 with $T_c = 250°C = 523K$,

$$\lambda T = 0.2\mu m \times 523K = 104.6\mu m \cdot K, \quad F_{0\to\lambda T} = 0.000$$

$$\lambda T = 4\mu m \times 523K = 2092\mu m \cdot K \quad F_{0\to\lambda T} = 0.082$$

$$\alpha_c = 0.1[0.082 - 0.000] + 0.95[1 - 0.082] = 0.880. \qquad (5)$$

To determine the *emissivity* of the window, we need to know its temperature. However, we know that T will be less than T_c and the long wavelength behavior will dominate. That is,

$$\varepsilon \approx \varepsilon_\lambda\,(\lambda > 4\mu m) = 0.95. \qquad (6)$$

With these radiative properties now known, the energy equation, Eq. (1) can now be evaluated using $q''_{conv} = h(T - T_\infty)$ with all temperatures in kelvin units.

$$0.948\times\sigma\,(298K)^4 + 0.108\times 800\,W/m^2 - 0.95\times\sigma T^4 - 10\,W/m^2\cdot K\,(T - 298K)$$

$$+0.880\sigma\,(523K)^4 - 0.95\times\sigma T^4 = 0$$

$$1.077\times 10^{-7}\,T^4 + 10T - 7223 = 0.$$

Using a trial-and-error approach, find the window temperature as

$$T = 413K = 139°C.$$ <

To determine the power required to maintain the cavity at $T_c = 250°C$, perform an energy balance on the cavity.

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$q_p + A_c[\rho E_b(T_c) + \tau_S G_S + \varepsilon E_b(T) - E_b(T_c)] = 0.$$

For simplicity, we have assumed the window opaque to irradiation from the surroundings. It follows that

$$\tau_S = 1 - \rho_S - \alpha_S = 1 - 0 - 0.108 = 0.892$$

$$\rho = 1 - \alpha = 1 - \varepsilon = 1 - 0.95 = 0.05.$$

Hence, the power required to maintain the cavity, when $A_c = (\pi/4)D^2$, is

$$q_p = A_c\left[\sigma T_c^4 - \rho\sigma T_c^4 - \tau_S G_S - \varepsilon\sigma T^4\right]$$

$$q_p = \frac{\pi}{4}(0.050m)^2\left[\sigma\,(523K)^4 - 0.05\sigma\,(523K)^4 - 0.892\times 800\,W/m^2 - 0.95\sigma\,(412K)^4\right]$$

$$q_p = 3.47W.$$ <

COMMENTS: Note that the assumed value of ε = 0.95 is not fully satisfied. With T = 412K, we would expect ε = 0.929. Hence, an iteration may be appropriate.

PROBLEM 12.108

KNOWN: Features of an evacuated tube solar collector.

FIND: Ideal surface spectral characteristics.

SCHEMATIC:

ANALYSIS: The outer tube should be transparent to the incident solar radiation, which is concentrated in the spectral region $\lambda \leq 3\mu m$, but it should be opaque and highly reflective to radiation emitted by the outer surface of the inner tube, which is concentrated in the spectral region above $3\mu m$. Accordingly, ideal spectral characteristics for the outer tube are

Note that large ρ_λ is desirable for the outer, as well as the inner, surface of the outer tube. If the surface is diffuse, a large value of ρ_λ yields a small value of $\varepsilon_\lambda = \alpha_\lambda = 1 - \rho_\lambda$. Hence losses due to emission from the outer surface to the surroundings would be negligible.

The opaque outer surface of the inner tube should absorb all of the incident solar radiation ($\lambda \leq 3\mu m$) and emit little or no radiation, which would be in the spectral region $\lambda > 3\mu m$. Accordingly, assuming diffuse surface behavior, ideal spectral characteristics are:

PROBLEM 12.109

KNOWN: Plate exposed to solar flux with prescribed solar absorptivity and emissivity; convection and surrounding conditions also prescribed.

FIND: Steady-state temperature of the plate.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Plate is small compared to surroundings, (3) Backside of plate is perfectly insulated, (4) Diffuse behavior.

ANALYSIS: Perform a surface energy balance on the top surface of the plate.

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$\alpha_S G_S + \alpha G_{sur} - q''_{conv} - \varepsilon E_b(T_s) = 0$$

Note that the effect of the surroundings is to provide an irradiation, G_{sur}, on the plate; since the spectral distribution of G_{sur} and $E_{\lambda,b}$ (T_s) are nearly the same, accordiing to Kirchoff's law, $\alpha = \varepsilon$. Recognizing that $G_{sur} = \sigma T_{sur}^4$ and using Newton's law of cooling, the energy balance is

$$\alpha_S G_S + \varepsilon\sigma T_{sur}^4 - \bar{h}(T_s - T_\infty) - \varepsilon \cdot \sigma T_s^4 = 0.$$

Substituting numerical values,

$$0.9 \times 900\ \text{W/m}^2 + 0.1 \times 5.67 \times 10^{-8}\text{W/m}^2 \cdot \text{K} \times (17 + 273)^4\ \text{K}^4$$
$$-20\,\text{W/m}^2 \cdot \text{K}\,(T_s - 290)\,\text{K} - 0.1\left(5.67 \times 10^{-8}\text{W/m}^2 \cdot \text{K}^4\right)T_s^4 = 0$$

$$6650\ \text{W/m}^2 = 20\,T_s + 5.67 \times 10^{-9}\,T_s^4.$$

From a trial-and-error solution, find

$$T_s = 329.2\ \text{K}.$$ <

COMMENTS: (1) When performing an analysis with both convection and radiation processes present, all temperatures must be expressed in absolute units (K).

(2) Note also that the terms $\alpha\ G_{sur}$ - $\varepsilon\ E_b$ (T_s) could be expressed as a radiation exchange term, written as

$$q''_{rad} = q/A = \varepsilon\sigma\left(T_{sur}^4 - T_s^4\right).$$

The conditions for application of this relation were met and are namely: surroundings much larger than surface, diffuse surface, and spectral distributions of irradiation and emission are similar (or the surface is gray).

PROBLEM 12.110

KNOWN: Directional distribution of α_θ for a horizontal, opaque, gray surface exposed to direct and diffuse irradiation.

FIND: (a) Absorptivity to direct radiation at 45° and to diffuse radiation, and (b) Equilibrium temperature for specified direct and diffuse irradiation components.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Opaque, gray surface behavior, (3) Negligible convection at top surface and perfectly insulated back surface.

ANALYSIS: (a) From knowledge of α_θ (θ) – see graph above – it is evident that the absorptivity of the surface to the direct radiation (45°) is

$$\alpha_{\text{dir}} = \alpha_\theta\left(45°\right) = 0.8. \qquad <$$

The absorptivity to the diffuse radiation is the hemispherical absorptivity given by Eq. 12.44. Dropping the λ subscript,

$$\alpha_{\text{dir}} = 2\int_0^{\pi/2} \alpha_\theta\left(\theta\right)\cos\theta\,\sin\theta\,d\theta \qquad (1)$$

$$\alpha_{\text{dir}} = 2\left[0.8\frac{\sin^2\theta}{2}\Bigg|_0^{\pi/3} + 0.1\frac{\sin^2\theta}{2}\Bigg|_{\pi/3}^{\pi/2}\right]$$

$$\alpha_{\text{dir}} = 0.625. \qquad <$$

(b) Performing a surface energy balance,

$$\dot{E}''_{\text{in}} - \dot{E}''_{\text{out}} = 0$$

$$\alpha_{\text{dir}}\,G_{\text{dir}} + \alpha_{\text{dif}}\,G_{\text{dif}} - \varepsilon\,\sigma\,T_s^4 = 0. \qquad (2)$$

The total, hemispherical emissivity may be obtained from Eq. 12.36 where again the subscript may be deleted. Since this equation is of precisely the same form as Eq. 12.44 – see Eq. (1) above – and since $\alpha_\theta = \varepsilon_\theta$, it follows that

$$\varepsilon = \alpha_{\text{dif}} = 0.625$$

and from Eq. (2), find

$$T_s^4 = \frac{\left(0.8\times600 + 0.625\times100\right)\text{W/m}^2}{0.625\times5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4} = 1.53\times10^{10}\,\text{K}^4, \qquad T_s = 352\text{ K}. \qquad <$$

COMMENTS: In assuming *gray* surface behavior, spectral effects are not present, and total and spectral properties are identical. However, the surface is *not diffuse* and hence hemispherical and directional properties differ.

PROBLEM 12.111

KNOWN: Plate temperature and spectral and directional dependence of its absorptivity. Direction and magnitude of solar flux.

FIND: (a) Expression for total absorptivity, (b) Expression for total emissivity, (c) Net radiant flux, (d) Effect of cut-off wavelength associated with directional dependence of the absorptivity.

SCHEMATIC:

ASSUMPTIONS: (1) Diffuse component of solar flux is negligible, (2) Spectral distribution of solar radiation may be approximated as that from a blackbody at 5800 K, (3) Properties are independent of azimuthal angle ϕ.

ANALYSIS: (a) For $\lambda < \lambda_c$ and $\theta = 45°$, $\alpha_\lambda = \alpha_1 \cos\theta = 0.707\ \alpha_1$. From Eq. (12.47) the total absorptivity is then

$$\alpha_S = 0.707\ \alpha_1 \left\{ \frac{\int_0^{\lambda_c} E_{\lambda,b}\ (\lambda, 5800\ K) d\lambda}{E_b} \right\} + \alpha_2 \left\{ \frac{\int_{\lambda_c}^{\infty} E_{\lambda,b}\ (\lambda, 5800\ K) d\lambda}{E_b} \right\}$$

$$\alpha_S = 0.707\ \alpha_1\ F_{(0\to\lambda_c)} + \alpha_2 \left[1 - F_{(0\to\lambda_c)} \right]$$ <

For the prescribed value of λ_c, $\lambda_c T$ = 11,600 μm·K and, from Table 12.1, $F_{(0\to\lambda c)} = 0.941$. Hence,

$$\alpha_S = 0.707 \times 0.93 \times 0.941 + 0.25(1 - 0.941) = 0.619 + 0.015 = 0.634$$ <

(b) With $\varepsilon_{\lambda,\theta} = \alpha_{\lambda,o}$, Eq (12.36) may be used to obtain ε_λ for $\lambda < \lambda_c$.

$$\varepsilon_\lambda(\lambda, T) = 2\alpha_1 \int_0^{\pi/2} \cos^2\theta \sin\theta\, d\theta = -2\alpha_1 \frac{\cos^3\theta}{3}\Bigg|_0^{\pi/2} = \frac{2}{3}\alpha_1$$

From Eq. (12.38),

$$\varepsilon = 0.667\,\alpha_1 \frac{\int_0^{\lambda_c} E_{\lambda,b}(\lambda, T_p) d\lambda}{E_b} + \alpha_2 \frac{\int_{\lambda_c}^{\infty} E_{\lambda,b}(\lambda, T_p) d\lambda}{E_b}$$

$$\varepsilon = 0.667\,\alpha_1\, F_{(0\to\lambda_c)} + \alpha_2 \left[1 - F_{(0-\lambda_c)} \right]$$ <

For λ_c = 2 μm and T_p = 333 K, $\lambda_c T$ = 666 μm·K and, from Table 12.1, $F_{(0\text{-}\lambda c)} = 0$. Hence,

$$\varepsilon = \alpha_2 = 0.25$$ <

Continued …..

PROBLEM 12.111 (Cont.)

(c) $$q''_{net} = \alpha_S q''_S - \varepsilon\sigma T_p^4 = 634\ W/m^2 - 0.25 \times 5.67 \times 10^{-8}\ W/m^2 \cdot K^4 (333\ K)^4$$

$$q''_{net} = 460\ W/m^2$$ <

(d) Using the foregoing model with the Radiation/Band Emission Factor option of *IHT*, the following results were obtained for α_S and ε. The absorptivity increases with increasing λ_c, as more of the incident solar radiation falls within the region of $\alpha_1 > \alpha_2$. Note, however, the limit at $\lambda \approx 3\ \mu m$, beyond which there is little change in α_S. The emissivity also increases with increasing λ_c, as more of the emitted radiation is at wavelengths for which $\varepsilon_1 = \alpha_1 > \varepsilon_2 = \alpha_2$. However, the surface temperature is low, and even for $\lambda_c = 5\ \mu m$, there is little emission at $\lambda < \lambda_c$. Hence, ε only increases from 0.25 to 0.26 as λ_c increases from 0.7 to 5.0 μm.

Surface properties

Cut-off wavelength (micrometer)

AlphaS
Epsilon

The net heat flux increases from 276 W/m^2 at $\lambda_c = 2\ \mu m$ to a maximum of 477 W/m^2 at $\lambda_c = 4.2\ \mu m$ and then decreases to 474 W/m^2 at $\lambda_c = 5\ \mu m$. The existence of a maximum is due to the upper limit on the value of α_S and the increase in ε with λ_c.

COMMENTS: Spectrally and directionally selective coatings may be used to enhance the performance of solar collectors.

PROBLEM 12.112

KNOWN: Spectral distribution of α_λ for two roof coatings.

FIND: Preferred coating for summer and winter use. Ideal spectral distribution of α_λ.

SCHEMATIC:

ASSUMPTIONS: (1) Opaque, diffuse surface behavior, (2) Negligible convection effects and heat transfer from bottom of roof, negligible atmospheric irradiation, (3) Steady-state conditions.

ANALYSIS: From an energy balance on the roof surface

$$\varepsilon \sigma T_S^4 = \alpha_S G_S.$$

Hence

$$T_S = \left(\frac{\alpha_S}{\varepsilon} \frac{G_S}{\sigma} \right)^{1/4}.$$

Solar irradiation is concentrated in the spectral region $\lambda < 4\mu m$, while surface emission is concentrated in the region $\lambda > 4\mu m$. Hence, with $\alpha_\lambda = \varepsilon_\lambda$

Coating A: $\alpha_S \approx 0.8$, $\varepsilon \approx 0.8$

Coating B: $\alpha_S \approx 0.6$, $\varepsilon \approx 0.2$.

Since $(\alpha_S/\varepsilon)_A = 1 < (\alpha_S/\varepsilon)_B = 3$, Coating A would result in the lower roof temperature and is preferred for summer use. In contrast, Coating B is preferred for winter use. The ideal coating is one which minimizes (α_S/ε) in the summer and maximizes it in the winter.

PROBLEM 12.113

KNOWN: Shallow pan of water exposed to night desert air and sky conditions.

FIND: Whether water will freeze.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Bottom of pan is well insulated, (3) Water surface is diffuse-gray, (4) Sky provides blackbody irradiation, $G_{sky} = s\,T_{sky}^4$.

PROPERTIES: *Table A-11*, Water (300 K): $\varepsilon = 0.96$.

ANALYSIS: To estimate the water surface temperature for these conditions, begin by performing an energy balance on the pan of water considering convection and radiation processes.

$$\dot{E}''_{in} - \dot{E}''_{out} = 0$$

$$a\,G_{sky} - e\,E_b - \bar{h}\left(T_s - T_\infty\right) = 0$$

$$e\,s\left(T_{sky}^4 - T_s^4\right) - \bar{h}\left(T_s - T_\infty\right) = 0.$$

Note that, from Eq. 12.64, $G_{sky} = s\,T_{sky}^4$ and from Assumption 3, $\alpha = \varepsilon$. Substituting numerical values, with all temperatures in kelvin units, the energy balance is

$$0.96 \times 5.67 \times 10^{-8} \frac{\mathrm{W}}{\mathrm{m}^2 \cdot \mathrm{K}^4}\left[(-40+273)^4 - T_s^4\right]\mathrm{K}^4 - 5\frac{\mathrm{W}}{\mathrm{m}^2 \cdot \mathrm{K}}\left[T_s - (20+273)\right]\mathrm{K} = 0$$

$$5.443 \times 10^{-8}\left[233^4 - T_s^4\right] - 5\left[T_s - 293\right] = 0.$$

Using a trial-and-error approach, find the water surface temperature,

$$T_s = 268.5\,\mathrm{K}. \qquad <$$

Since $T_s < 273$ K, it follows that the water surface will freeze under the prescribed air and sky conditions.

COMMENTS: If the heat transfer coefficient were to increase as a consequence of wind, freezing might not occur. Verify that for the given T_∞ and T_{sky}, that if $\bar{h}$ increases by more than 40%, freezing cannot occur.

PROBLEM 12.114

KNOWN: Flat plate exposed to night sky and in ambient air at T_{air} = 15°C with a relative humidity of 70%. Radiation from the atmosphere or sky estimated as a fraction of the blackbody radiation corresponding to the near-ground air temperature, $G_{sky} = \varepsilon_{sky}\, \sigma\, T_{air}$, and for a clear night, $\varepsilon_{sky} = 0.741 + 0.0062\, T_{dp}$ where T_{dp} is the dew point temperature (°C). Convection coefficient estimated by correlation, $\bar{h}\left(W/m^2 \cdot K\right) = 1.25\Delta T^{1/3}$ where ΔT is the plate-to-air temperature difference (K).

FIND: Whether dew will form on the plate if the surface is (a) clean metal with $\varepsilon_m = 0.23$ and (b) painted with $\varepsilon_p = 0.85$.

SCHEMATIC:

Ambient air
Night sky
$T_{air} = 15^oC$
RH = 70%
$h = 1.25\Delta T^{1/3}$
$G_{sky} = \varepsilon_{sky}\sigma T_{air}^4$
$\varepsilon_{sky} = 0.741 + 0.0062T_{dp}(^oC)$
T_s, Clean metallic, $\varepsilon_m = 0.23$
Painted surface, $\varepsilon_p = 0.85$
q''_{cv}
$\varepsilon E_b(T_s)$
$\alpha_{sky}G_{sky}$
T_s

ASSUMPTIONS: (1) Steady-state conditions, (2) Surfaces are diffuse, gray, and (3) Backside of plate is well insulated.

PROPERTIES: *Psychrometric charts* (Air), T_{dp} = 9.4°C for dry bulb temperature 15°C and relative humidity 70%.

ANALYSIS: From the schematic above, the energy balance on the plate is

$$\dot{E}''_{in} - \dot{E}''_{out} = 0$$

$$\alpha_{sky}\, G_{sky} + q''_{cv} - \varepsilon\, E_b\, (T_s) = 0$$

$$\varepsilon\left[\left(0.741 + 0.0062\ T_{dp}\left({}^\circ C\right)\right)\sigma T_{air}^4\right] + 1.25\left(T_{air} - T_s\right)^{4/3} W/m^2 - \varepsilon\sigma T_s^4\ W/m^2 = 0$$

where $G_{sky} = \varepsilon_{sky}\, \sigma\, T_{air}$, $\varepsilon_{sky} = 0.741 + 0.062\, T_{dp}$ (°C); T_{dp} has units (°C); and, other temperatures in kelvins. Since the surface is diffuse-gray, $\alpha_{sky} = \varepsilon$.

(a) *Clean metallic surface*, $\varepsilon_m = 0.23$

$$0.23\left[\left(0.741 + 0.0062\ T_{dp}\left({}^\circ C\right)\right)\sigma\left(15 + 273\right)^4 K^4\right]$$

$$+1.25\left(289 - T_{s,m}\right)^{4/3} W/m^2 - 0.23\,\sigma T_{s,m}^4\ W/m^2 = 0$$

$$T_{s,m} = 282.7\ K = 9.7^\circ C$$ <

(b) *Painted surface, $\varepsilon_p = 0.85$* $T_{s,p}$ = 278.5 K = 5.5°C <

COMMENTS: For the painted surface, $\varepsilon_p = 0.85$, find that $T_s < T_{dp}$, so we expect dew formation. For the clean, metallic surface, $T_s > T_{dp}$, so we do not expect dew formation.

PROBLEM 12.115

KNOWN: Glass sheet, used on greenhouse roof, is subjected to solar flux, G_S, atmospheric emission, G_{atm}, and interior surface emission, G_i, as well as to convection processes.

FIND: (a) Appropriate energy balance for a unit area of the glass, (b) Temperature of the greenhouse ambient air, $T_{\infty,i}$, for prescribed conditions.

SCHEMATIC:

G_S, G_{atm}, T_g, G_i, Outside, Inside, $T_{\infty,o}, h_o$, $T_{\infty,i}, h_i$

Conditions:
$T_g = 27°C$ $T_{\infty,o} = 24°C$
$h_i = 10 W/m^2 \cdot K$ $h_o = 55 W/m^2 \cdot K$
$G_S = 1100 W/m^2$ $G_{atm} = 250 W/m^2$
$G_i = 440 W/m^2$

ASSUMPTIONS: (1) Glass is at a uniform temperature, T_g, (2) Steady-state conditions.

PROPERTIES: Glass: $\tau_\lambda = 1$ for $\lambda \le 1 \mu m$; $\tau_\lambda = 0$ and $\alpha_\lambda = 1$ for $\lambda > 1$ μm.

ANALYSIS: (a) Performing an energy balance on the glass sheet with $\dot{E}_{in} - \dot{E}_{out} = 0$ and considering two convection processes, emission and three absorbed irradiation terms, find

$$\alpha_S G_S + \alpha_{atm} G_{atm} + h_o \left(T_{\infty,o} - T_g\right) + \alpha_i G_i + h_i \left(T_{\infty,i} - T_g\right) - 2\varepsilon\sigma T_g^4 = 0 \qquad (1)$$

where α_S = solar absorptivity for absorption of $G_{\lambda,S} \sim E_{\lambda,b}$ (λ, 5800K)
$\alpha_{atm} = \alpha_i$ = absorptivity of long wavelength irradiation ($\lambda >> 1$ μm) ≈ 1
$\varepsilon = \alpha_\lambda$ for $\lambda >> 1$ μm, emissivity for long wavelength emission ≈ 1

(b) For the prescribed conditions, $T_{\infty,i}$ can be evaluated from Eq. (1). As noted above, $\alpha_{atm} = \alpha_i = 1$ and $\varepsilon = 1$. The solar absorptivity of the glass follows from Eq. 12.47 where $G_{\lambda,S} \sim E_{\lambda,b}$ (λ, 5800K),

$$\alpha_S = \int_0^\infty \alpha_\lambda G_{\lambda,S} d\lambda / G_S = \int_0^\infty \alpha_\lambda E_{\lambda,b}(\lambda, 5800K) d\lambda / E_b(5800K)$$

$$\alpha_S = \alpha_1 F_{(0\rightarrow 1\mu m)} + \alpha_2 \left[1 - F_{(0\rightarrow 1\mu m)}\right] = 0 \times 0.720 + 1.0[1 - 0.720] = 0.28.$$

Note that from Table 12.1 for $\lambda T = 1 \mu m \times 5800K = 5800 \mu m \cdot K$, $F_{(0-\lambda)} = 0.720$. Substituting numerical values into Eq. (1),

$$0.28 \times 1100 W/m^2 + 1 \times 250 W/m^2 + 55 W/m^2 \cdot K (24 - 27) K + 1 \times 440 W/m^2 +$$
$$10 W/m^2 \cdot K \left(T_{\infty,i} - 27\right) K - 2 \times 1 \times 5.67 \times 10^{-8} W/m^2 \cdot K (27 + 273)^4 K^4 = 0$$

find that

$$T_{\infty,i} = 35.5°C.$$ <

PROBLEM 12.116

KNOWN: Plate temperature and spectral absorptivity of coating.

FIND: (a) Solar irradiation, (b) Effect of solar irradiation on plate temperature, total absorptivity, and total emissivity.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state, (2) Opaque, diffuse surface, (3) Isothermal plate, (4) Negligible radiation from surroundings.

ANALYSIS: (a) Performing an energy balance on the plate, $2\alpha_S G_S - 2E = 0$ and

$$\alpha_S G_S - \varepsilon\sigma T^4 = 0$$

For $\lambda T = 4.5\ \mu m \times 2000\ K = 9000\ \mu m\cdot K$, Table 12.1 yields $F_{(o\to\lambda)} = 0.890$. Hence,

$$\varepsilon = \varepsilon_1 F_{(0\to\lambda)} + \varepsilon_2\left(1 - F_{(o\to\lambda)}\right) = 0.95\times 0.890 + 0.03(1 - 0.890) = 0.849$$

For $\lambda T = 4.5\ \mu m \times 5800\ K = 26{,}100$, $F_{(o\to\lambda)} = 0.993$. Hence,

$$\alpha_S = \alpha_1 F_{(0\to\lambda)} + \alpha_2\left(1 - F_{(0\to\lambda)}\right) = 0.95\times 0.993 + 0.03\times 0.007 = 0.944$$

Hence,

$$G_S = (\varepsilon/\alpha_S)\sigma T^4 = (0.849/0.944)5.67\times 10^{-8}\ W/m^2\cdot K^4\,(2000\,K)^4 = 8.16\times 10^5\ W/m^2 \quad <$$

(b) Using the IHT *First Law* Model and the *Radiation* Toolpad, the following results were obtained.

The required solar irradiation increases with T to the fourth power. Since α_S is determined by the spectral distribution of solar radiation, its value is fixed. However, with increasing T, the spectral distribution of emission is shifted to lower wavelengths, thereby increasing the value of ε.

PROBLEM 12.117

KNOWN: Thermal conductivity, spectral absorptivity and inner and outer surface conditions for wall of central solar receiver.

FIND: Minimum wall thickness needed to prevent thermal failure. Collector efficiency.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Outer surface is opaque and diffuse, (3) Spectral distribution of solar radiation corresponds to blackbody emission at 5800 K.

ANALYSIS: From an energy balance at the outer surface, $\dot{E}_{in} = \dot{E}_{out}$,

$$\alpha_S q_S'' + \alpha_{sur} G_{sur} = \varepsilon \sigma T_{s,o}^4 + h_o \left(T_{s,o} - T_{\infty,o}\right) + \frac{T_{s,o} - T_{\infty,i}}{(L/k) + (1/h_i)}$$

Since radiation from the surroundings is in the far infrared, $\alpha_{sur} = 0.2$. From Table 12.1, $\lambda T = (3\ \mu m \times 5800\ K) = 17{,}400\ \mu m \cdot K$, find $F_{(0 \to 3\mu m)} = 0.979$. Hence,

$$\alpha_S = \frac{\int_0^\infty \alpha_\lambda E_{\lambda,b}(5800\ K)\, d\lambda}{E_b} = \alpha_1 F_{(0 \to 3\mu m)} + \alpha_2 F_{(3 \to \infty)} = 0.9(0.979) + 0.2(0.021) = 0.885.$$

From Table 12.1, $\lambda T = (3\ \mu m \times 1000\ K) = 3000\ \mu m \cdot K$, find $F_{(0 \to 3\mu m)} = 0.273$. Hence,

$$\varepsilon_S = \frac{\int_0^\infty \varepsilon_\lambda E_{\lambda,b}(1000\ K)\, d\lambda}{E_b} = \varepsilon_1 F_{(0 \to 3)} + \varepsilon_2 F_{(3 \to \infty)} = 0.9(0.273) + 0.2(0.727) = 0.391.$$

Substituting numerical values in the energy balance, find

$$0.885\left(80{,}000\ W/m^2\right) + 0.2 \times 5.67 \times 10^{-8}\ W/m^2 \cdot K^4 (300\ K)^4 = 0.391 \times 5.67 \times 10^{-8}\ W/m^2 \cdot K^4 (1000\ K)^4$$

$$+25\ W/m^2 \cdot K (700\ K) + (300\ K) / \left[(L/15\ W/m \cdot K) + \left(1/1000\ W/m^2 \cdot K\right)\right]$$

$$L = 0.129\ m.$$ <

The corresponding collector efficiency is

$$\eta = \frac{q_{use}''}{q_S''} = \left[\frac{T_{s,o} - T_{\infty,i}}{(L/k) + (1/h_i)}\right] / q_S''$$

$$\eta = \left[\frac{300\ K}{(0.129\ m/15\ W/m \cdot K) + \left(0.001 m^2 \cdot K/W\right)}\right] / 80{,}000\ W/m^2 = 0.391 \text{ or } 39.1\%.$$ <

COMMENTS: The collector efficiency could be increased and the outer surface temperature reduced by decreasing the value of L.

PROBLEM 12.118

KNOWN: Dimensions, spectral absorptivity, and temperature of solar receiver. Solar irradiation and ambient temperature.

FIND: (a) Rate of energy collection q and collector efficiency η, (b) Effect of receiver temperature on q and η.

SCHEMATIC:

0.9
α_λ
0.2
3
λ (μm)
D = 7 m
E
L = 12 m
q''_{conv}
T_∞ = 300 K
G_S = 80,000 W/m²
T_s

ASSUMPTIONS: (1) Steady-state, (2) Uniform irradiaton, (3) Opaque, diffuse surface.

PROPERTIES: *Table A.4*, air (T_f = 550 K): $\nu = 45.6 \times 10^{-6}$ m²/s, k = 0.0439 W/m·K, $\alpha = 66.7 \times 10^{-6}$ m²/s, Pr = 0.683.

ANALYSIS: (a) The rate of heat transfer to the receiver is $q = A_s\left(\alpha_S G_S - E - q''_{conv}\right)$, or

$$q = \pi DL\left[\alpha_S G_S - \varepsilon\sigma T_s^4 - \bar{h}\left(T_s - T_\infty\right)\right]$$

For λT = 3 μm × 5800 K = 17,400, $F_{(0\to\lambda)} = 0.979$. Hence,

$$\alpha_S = \alpha_1 F_{(0\to\lambda)} + \alpha_2\left(1 - F_{(0\to\lambda)}\right) = 0.9\times0.979 + 0.2(0.021) = 0.885$$

For λT = 3 μm × 800 K = 2400 μm·K, $F_{(0\to\lambda)} = 0.140$. Hence,

$$\varepsilon = \varepsilon_1 F_{(0\to\lambda)} + \varepsilon_2\left(1 - F_{(0\to\lambda)}\right) = 0.9\times0.140 + 0.2(0.860) = 0.298.$$

With $Ra_L = g\beta(T_s - T_\infty)L^3/\alpha\nu$ = 9.8 m/s²(1/550 K)(500 K)(12 m)³/66.7 × 10⁻⁶ m²/s × 45.6 × 10⁻⁶ m²/s = 5.06×10^{12}, Eq. 9.26 yields

$$\overline{Nu}_L = \left\{0.825 + \frac{0.387 Ra_L^{1/6}}{\left[1 + (0.492/Pr)^{9/16}\right]^{8/27}}\right\}^2 = 1867$$

$$\bar{h} = \overline{Nu}_L \frac{k}{L} = 1867\frac{0.0439\text{ W/m}\cdot\text{K}}{12\text{ m}} = 6.83\text{ W/m}^2\cdot\text{K}$$

Hence,

$$q = \pi\left(7\text{ m}\times12\text{ m}\right)\left[0.885\times80{,}000\text{ W/m}^2 - 0.298\times5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4\left(800\text{ K}\right)^4 - 6.83\text{ W/m}^2\cdot\text{K}\left(500\text{ K}\right)\right]$$

$$q = 263.9\text{ m}^2\left(70{,}800 - 6{,}920 - 3415\right)\text{W/m}^2 = 1.60\times10^7\text{ W}$$ <

The collector efficiency is $\eta = q/A_sG_S$. Hence

$$\eta = \frac{1.60\times10^7\text{ W}}{263.9\text{ m}^2\left(80{,}000\text{ W/m}^2\right)} = 0.758$$ <

Continued …..

(b) The IHT *Correlations*, *Properties* and *Radiation* Toolpads were used to obtain the following results.

Losses due to emission and convection increase with increasing T_s, thereby reducing q and η.

COMMENTS: The increase in radiation emission is due to the increase in T_s, as well as to the effect of T_s on ε, which increases from 0.228 to 0.391 as T_s increases from 600 to 1000 K.

PROBLEM 12.119

KNOWN: Dimensions and construction of truck roof. Roof interior surface temperature. Truck speed, ambient air temperature, and solar irradiation.

FIND: (a) Preferred roof coating, (b) Roof surface temperature, (c) Heat load through roof, (d) Effect of velocity on surface temperature and heat load.

SCHEMATIC:

ASSUMPTIONS: (1) Turbulent boundary layer development over entire roof, (2) Constant properties, (3) Negligible atmospheric (sky) irradiation, (4) Negligible contact resistance.

PROPERTIES: *Table A.4*, Air ($T_{s,o} \approx 300$ K, 1 atm): $\nu = 15\times10^{-6}\ \mathrm{m^2/s}$, $k = 0.026\ \mathrm{W/m\cdot K}$, $Pr = 0.71$.

ANALYSIS: (a) To minimize heat transfer through the roof, minimize solar absorption relative to surface emission. Hence, use zinc oxide white for which $\alpha_S = 0.16$ and $\varepsilon = 0.93$. <

(b) Performing an energy balance on the outer surface of the roof, $\alpha_S G_S + q''_{conv} - E - q''_{cond} = 0$, it follows that

$$\alpha_S G_S + \bar{h}(T_\infty - T_{s,o}) = \varepsilon\sigma T_{s,o}^4 + (k/t)(T_{s,o} - T_{s,i})$$

where it is assumed that convection is from the air to the roof. With

$$Re_L = \frac{VL}{\nu} = \frac{30\ \mathrm{m/s}(5\ \mathrm{m})}{15\times10^{-6}\ \mathrm{m^2/s}} = 10^7$$

$$\overline{Nu}_L = 0.037\, Re_L^{4/5}\, Pr^{1/3} = 0.037(10^7)^{4/5}(0.71)^{1/3} = 13{,}141$$

$$\bar{h} = \overline{Nu}_L (k/L) = 13{,}141(0.026\ \mathrm{W/m\cdot K}/5\ \mathrm{m} = 68.3\ \mathrm{W/m^2\cdot K}\,.$$

Substituting numerical values in the energy balance and solving by trial-and-error, we obtain

$$T_{s,o} = 295.2\ \mathrm{K}.$$ <

(c) The heat load through the roof is

$$q = (kA_s/t)(T_{s,o} - T_{s,i}) = (0.05\ \mathrm{W/m\cdot K}\times 10\ \mathrm{m^2}/0.025\ \mathrm{m})35.2\ \mathrm{K} = 704\ \mathrm{W}\,.$$ <

(d) Using the IHT *First Law* Model with the *Correlations* and *Properties* Toolpads, the following results are obtained.

Continued...

PROBLEM 12.119 (Cont.)

Temperature, Tso(K)

300
295
290
285
280

5 10 15 20 25 30

Velocity, V(m/s)

Heat load, q(W)

700
650
600
550
500

5 10 15 20 25 30

Velocity, V(m/s)

The surface temperature and heat load decrease with decreasing V due to a reduction in the convection heat transfer coefficient and hence convection heat transfer from the air.

COMMENTS: The heat load would increase with increasing α_S/ε.

PROBLEM 12.120

KNOWN: Sky, ground, and ambient air temperatures. Grape of prescribed diameter and properties.

FIND: (a) General expression for rate of change of grape temperature, (b) Whether grapes will freeze in quiescent air, (c) Whether grapes will freeze for a prescribed air speed.

SCHEMATIC:

$T_\infty = 273K$, $V = 0$ or $1 m/s$; $D = 15 mm$; G_{sky}; $T_{sky} = 235K$; dE_{st}/dt; q''_{conv}; E; G_{ea}; $T_{ea} = T_\infty = 273K$

Grape: $\varepsilon_g = 1$, $T_{fp} = -5°C$

ASSUMPTIONS: (1) Negligible temperature gradients in grape, (2) Uniform blackbody irradiation over top and bottom hemispheres, (3) Properties of grape are those of water at 273 K, (4) Properties of air are constant at values for T_∞, (5) Negligible buoyancy for V = 1 m/s.

PROPERTIES: *Table A-6*, Water (273 K): c_p = 4217 J/kg·K, ρ = 1000 kg/m^3; *Table A-4*, Air (273 K, 1 atm): ν = 13.49 × 10^{-6} m^2/s, k = 0.0241 W/m·K, α = 18.9 × 10^{-6} m^2/s, Pr = 0.714, β = 3.66 × 10^{-3} K^{-1}.

ANALYSIS: (a) Performing an energy balance for a control surface about the grape,

$$\frac{dE_{st}}{dt} = r_g \frac{p D^3}{6} c_{p \cdot g} \frac{dT_g}{dt} = \bar{h} p D^2 \left(T_\infty - T_g\right) + \frac{p D^2}{2}\left(G_{ea} + G_{sky}\right) - E p D^2.$$

Hence, the rate of temperature change with time is

$$\frac{dT_g}{dt} = \frac{6}{r_g c_{p \cdot g} D}\left[\bar{h}\left(T_\infty - T_g\right) + s\left(\left(T_{ea}^4 + T_{sky}^4\right)/2 - e_g T_g^4\right)\right].$$

<

(b) The grape freezes if $dT_g/dt < 0$ when $T_g = T_{fp}$ = 268 K. With

$$Ra_D = \frac{g b \left(T_\infty - T_g\right) D^3}{an} = \frac{9.8\, m/s^2 \left(3.66 \times 10^{-3}\ K^{-1}\right) 5K\left(0.015\ m\right)^3}{18.9 \times 10^{-6} \times 13.49 \times 10^{-6} m^4/s^2} = 2374$$

using Eq. 9.35 find

$$\overline{Nu}_D = 2 + \frac{0.589\left(2374\right)^{1/4}}{\left[1 + \left(0.469/Pr\right)^{9/16}\right]^{4/9}} = 5.17$$

$$\bar{h} = \left(k/D\right)\overline{Nu}_D = \left[\left(0.0241 W/m \cdot K\right)/\left(0.015 m\right)\right] 5.17 = 8.31 W/m^2 \cdot K.$$

Hence, the rate of temperature change is

$$\frac{dT_g}{dt} = \frac{6}{\left(1000\ kg/m^3\right) 4217\, J/kg \cdot K\left(0.015\ m\right)}\left[8.31 W/m^2 \cdot K\left(5\ K\right)\right.$$

$$\left. + 5.67 \times 10^{-8} W/m^2 \cdot K^4 \left[\left(273^4 + 235^4\right)/2 - 268^4\right] K^4\right.$$

Continued …..

$$\frac{dT_g}{dt} = 9.49 \times 10^{-5}\ \mathrm{K \cdot m^2/J}\left[41.55 - 48.56\right]\mathrm{W/m^2} = -6.66 \times 10^{-4}\ \mathrm{K/s}$$ <

and since $dT_g/dt < 0$, the grape *will freeze*.

(c) For V = 1 m/s,

$$\mathrm{Re}_D = \frac{VD}{n} = \frac{1\ \mathrm{m/s}\left(0.015\ \mathrm{m}\right)}{13.49 \times 10^{-6}\ \mathrm{m^2/s}} = 1112.$$

Hence with $(\mu/\mu_s)^{1/4} = 1$,

$$\overline{\mathrm{Nu}}_D = 2 + \left(0.4\mathrm{Re}_D^{1/2} + 0.06\mathrm{Re}_D^{2/3}\right)\mathrm{Pr}^{0.4} = 21.8$$

$$\bar{h} = \overline{\mathrm{Nu}}_D \frac{k}{D} = 21.8\frac{0.0241}{0.015} = 35\ \mathrm{W/m^2 \cdot K}.$$

Hence the rate of temperature change with time is

$$\frac{dT_g}{dt} = 9.49 \times 10^{-5}\ \mathrm{K \cdot m^2/J}\left[35\ \mathrm{W/m^2 \cdot K}\left(5\ \mathrm{K}\right) - 48.56\ \mathrm{W/m^2}\right] = 0.012\ \mathrm{K/s}$$

and since $dT_g/dt > 0$, the grape *will not freeze*. <

COMMENTS: With $Gr_D = Ra_D/Pr = 3325$ and $Gr_D/Re_D^2 = 0.0027$, the assumption of negligible buoyancy for V = 1 m/s is reasonable.

PROBLEM 12.121

KNOWN: Metal disk exposed to environmental conditions and placed in good contact with the earth.

FIND: (a) Fraction of direct solar irradiation absorbed, (b) Emissivity of the disk, (c) Average free convection coefficient of the disk upper surface, (d) Steady-state temperature of the disk (confirm the value 340 K).

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Disk is diffuse, (3) Disk is isothermal, (4) Negligible contact resistance between disk and earth, (5) Solar irradiance has spectral distribution of $E_{\lambda,b}$ (λ, 5800 K).

PROPERTIES: *Table A-4*, Air (1 atm, $T_f = (T_s + T_\infty)/2 = (340 + 300)\ K/2 = 320\ K$): $\nu = 17.90 \times 10^{-6}\ m^2/s$, k = 0.0278 W/m·K, $\alpha = 25.5 \times 10^{-6}\ m^2/s$, Pr = 0.704.

ANALYSIS: (a) The solar absorptivity follows from Eq. 12.49 with $G_{\lambda,S}\ \alpha\ E_{\lambda,b}$ (λ, 5800 K), and $\alpha_\lambda = \varepsilon_\lambda$, since the disk surface is diffuse.

$$\alpha_S = \int_0^\infty \alpha_\lambda E_{\lambda,b}(\lambda, 5800K)/E_b(5800\ K)$$

$$\alpha_S = \varepsilon_1 F_{(0\to 1\mu m)} + \varepsilon_2\left(1 - f_{(0\to 1\mu m)}\right).$$

From Table 12.1 with

$$\lambda T = 1\ \mu m \times 5800\ K = 5800\ \mu m \cdot K \quad \text{find} \quad F_{(0\to\lambda T)} = 0.720$$

giving

$$\alpha_S = 0.9 \times 0.720 + 0.2(1 - 0.720) = 0.704. \qquad <$$

Note this value is appropriate for diffuse or direct solar irradiation since the surface is diffuse.

(b) The emissivity of the disk depends upon the surface temperature T_s which we believe to be 340 K. (See part (d)). From Eq. 12.38,

$$\varepsilon = \int_0^\infty \varepsilon_\lambda E_{\lambda,b}(\lambda, T_s)/E_b(T_s)$$

$$\varepsilon = \varepsilon_1 F_{(0\to 1\mu m)} + \varepsilon_2\left(1 - F_{(0\to 1\mu m)}\right)$$

Continued …..

PROBLEM 12.121 (Cont.)

From Table 12.1 with

$$\lambda T = 1\ \mu m \times 340\ K = 340\ \mu m \cdot K \qquad \text{find} \qquad F_{(0 \to \lambda T)} = 0.000$$

giving

$$\varepsilon = 0.9 \times 0.000 + 0.2(1 - 0.000) = 0.20.$$ <

(c) The disk is a hot surface facing upwards for which the free convection correlation of Eq. 9.30 is appropriate. Evaluating properties at $T_f = (T_s + T_\infty)/2 = 320$ K,

$$Ra_L = g\beta\Delta T L^3/\nu\alpha \qquad \text{where } L = A_s/P = D/4$$

$$Ra_L = 9.8\ m/s^2 (1/320\ K)(340-300)K(0.4\ m/4)^3/17.90 \times 10^{-6}\ m^2/s \times 25.5 \times 10^{-6}\ m^2/s = 3.042 \times 10^6$$

$$\overline{Nu}_L = \bar{h}L/k = 0.54 Ra_L^{1/4} \qquad 10^4 \le Ra_L \le 10^7$$

$$\bar{h} = 0.0278\ W/m \cdot K/(0.4\ m/4) \times 0.54 \left(3.042 \times 10^6\right)^{1/4} = 6.37\ W/m^2 \cdot K.$$ <

(d) To determine the steady-state temperature, perform an energy balance on the disk.

$$\dot{E}_{in} - \dot{E}_{out} = \dot{E}_{st}$$

$$\left(\alpha_S G_{s,d} + \alpha\, G_{sky} - \varepsilon E_b - q''_{conv}\right) A_s - q_{cond} = 0.$$

Since G_{sky} is predominately long wavelength radiation, it follows that $\alpha = \varepsilon$. The conduction heat rate between the disk and the earth is

$$q_{cond} = kS(T_s - T_{ea}) = k(2D)(T_s - T_{ea})$$

where S, the conduction shape factor, is that of an isothermal disk on a semi-infinite medium, Table 4.1. Substituting numerical values, with $As = \pi D^2/4$,

$$\Big[0.704 \times 745\ W/m^2 + 0.20\sigma\ (280\ K)^4 - 0.20\sigma\ T_s^4$$

$$-6.3\ W/m^2 \cdot K(T_s - 300\ K)\Big]\pi/4(0.4\ m)^2 - 0.52\ W/m \cdot K(2 \times 0.4\ m)(T_s - 280\ K) = 0$$

$$65.908\ W + 8.759\ W - 1.425 \times 10^{-9} T_s^4 - 0.792(T_s - 300) - 0.416(T_s - 280) = 0.$$

By trial-and-error, find

$$T_s \approx 339\ K.$$ <

so indeed the assumed value of 340 K was proper.

COMMENTS: Note why it is not necessary for this situation to distinguish between direct and diffuse irradiation. Why does $\alpha_{sky} = \varepsilon$?

PROBLEM 12.122

KNOWN: Shed roof of weathered galvanized sheet metal exposed to solar insolation on a cool, clear spring day with ambient air at - 10°C and convection coefficient estimated by the empirical correlation $\bar{h} = 1.0\ \Delta T^{1/3}$ (W/m^2·K with temperature units of kelvins).

FIND: Temperature of the roof, T_s, (a) assuming the backside is well insulated, and (b) assuming the backside is exposed to ambient air with the same convection coefficient relation and experiences radiation exchange with the ground, also at the ambient air temperature. Comment on whether the roof will be a comfortable place for the neighborhood cat to snooze for these conditions.

SCHEMATIC:

Ambient air
$T_\infty = 10^oC$
$h = 1.0\ \Delta T^{1/3}$
$T_{sky} = -40^oC$
$G_S = 600\ W/m^2$
T_s, sheet metal
$\varepsilon = 0.65$
$\alpha_S = 0.8$
(a)

G_S
T_{sky}
T_∞, h
T_s, ε, α_S
T_∞, h
$T_{grd} = T_\infty$
(b)

ASSUMPTIONS: (1) Steady-state conditions, (2) The roof surface is diffuse, spectrally selective, (3) Sheet metal is thin with negligible thermal resistance, and (3) Roof is a small object compared to the large isothermal surroundings represented by the sky and the ground.

ANALYSIS: (a) For the backside-insulated condition, the energy balance, represented schematically below, is

$$\dot{E}''_{in} - \dot{E}''_{out} = 0$$

$$\alpha_{sky}\ E_b\left(T_{sky}\right) + \alpha_S G_S - q''_{cv} - \varepsilon\, E_b\left(T_s\right) = 0$$

$$\alpha_{sky}\sigma T_{sky}^4 + \alpha_S G_S - 1.0\left(T_s - T_\infty\right)^{4/3} - \varepsilon\sigma T_s^4 = 0$$

With $\alpha_{sky} = \varepsilon$ (see Comment 2) and $\sigma = 5.67\times10^{-8}\ W/m^2\cdot K^4$, find T_s.

$$0.65\ \sigma(233\ K)^4\ W/m^2\ + 0.8\times600\ W/m^2 - 1.0\left(T_s - 283\ K\right)^{4/3}\ W/m^2 - 0.65\ \sigma T_s^4 = 0$$

$$T_s = 312.5\ K = 39.5^\circ C$$ <

q''_{cv}
$\varepsilon E_b(T_s)$
$\alpha_{sky}E_b(T_{sky})$
$\alpha_S G_S$
T_s
(a)

q''_{cv}
$\varepsilon E_b(T_s)$
$\alpha_{sky}E_b(T_{sky})$
$\alpha_S G_S$
T_s
(b)
q''_{cv}
$\varepsilon E_b(T_s)$
$\alpha_{grd}E_b(T_{grd})$

Energy balances: backside condition-
(a) insulated, (b) exposed to air/ground

Continued …..

(b) With the backside exposed to convection with the ambient air and radiation exchange with the ground, the energy balance, represented schematically above, is

$$\alpha_{sky} E_b\left(T_{sky}\right) + \alpha_{grd} E_b\left(T_{grd}\right) + \alpha_S G_S - 2q''_{cv} - 2\varepsilon E_b\left(T_s\right) = 0$$

Substituting numerical values, recognizing that $T_{grd} = T_\infty$, and $\alpha_{grd} = \varepsilon$ (see Comment 2), find T_s.

$$0.65\ \sigma(233\ K)^4\ W/m^2 + 0.65\ \sigma(283\ K)^4 W/m^2 + 0.8 \times 600\ W/m^2$$
$$-2 \times 1.0\ (T_s - 283\ K)^{4/3}\ W/m^2 - 2 \times 0.65\ \sigma T_s^4 = 0$$

$$T_s = 299.5\ K = 26.5^\circ C$$ <

COMMENTS: (1) For the insulated-backside condition, the cat would find the roof quite warm remembering that 43°C represents a safe-to-touch temperature. For the exposed-backside condition, the cat would find the roof comfortable, certainly compared to an area not exposed to the solar insolation (that is, exposed only to the ambient air through convection).

(2) For this spectrally selective surface, the absorptivity for the sky irradiation is equal to the emissivity, $\alpha_{sky} = \varepsilon$, since the sky irradiation and surface emission have the same approximate spectral regions. The same reasoning applies for the absorptivity of the ground irradiation, $\alpha_{grd} = \varepsilon$.

PROBLEM 12.123

KNOWN: Amplifier operating and environmental conditions.

FIND: (a) Power generation when $T_s = 58°C$ with diffuse coating $\varepsilon = 0.5$, (b) Diffuse coating from among three (A, B, C) which will give greatest reduction in T_s, and (c) Surface temperature for the conditions with coating chosen in part (b).

SCHEMATIC:

$T_\infty = 27°C$, $h = 15\ W/m^2 \cdot K$, Amplifier, P_e, $T_{sky} = -20°C$, $G_S = 800\ W/m^2$, $T_s = 58°C$, $\varepsilon = 0.5$, $A_s = 0.13 \times 0.13\ m$

ε_λ vs $\lambda(\mu m)$: 1.0, 0, 3; curves C, A, B

ASSUMPTIONS: (1) Environmental conditions remain the same with all surface coatings, (2) Coatings A, B, C are opaque, diffuse.

ANALYSIS: (a) Performing an energy balance on the amplifier's exposed surface, $\dot{E}_{in} - \dot{E}_{out} = 0$, find

$$P_e + A_s\left[\alpha_S G_S + \alpha_{sky} G_{sky} - \varepsilon E_b - q''_{conv}\right] = 0$$

$$P_e = A_s\left[\varepsilon\sigma T_s^4 + h\left(T_s - T_\infty\right) - \alpha_S G_S - \alpha_{sky}\sigma T_{sky}^4\right]$$

$$P_e = 0.13 \times 0.13\ m^2\left[0.5 \times \sigma\ (331)^4 + 15(331 - 300) - 0.5 \times 800 - 0.5 \times \sigma\ (253)^4\right] W/m^2$$

$$P_e = 0.0169 m^2\left[0.5 \times 680.6 + 465 - 0.5 \times 800 - 0.5 \times 232.3\right] W/m^2 = 4887\ W. \quad <$$

(b) From above, recognize that we seek a coating with low α_S and high ε to decrease T_s. Further, recognize that α_S is determined by values of $\alpha_\lambda = \varepsilon_\lambda$ for $\lambda < 3\ \mu m$ and ε by values of ε_λ for $\lambda > 3\ \mu m$. Find approximate values as

Coating	A	B	C
ε	0.5	0.3	0.6
α_S	0.8	0.3	0.2
α_S/ε	1.6	1	0.333

Note also that $\alpha_{sky} \approx \varepsilon$. We conclude that coating C is likely to give the lowest T_s since its α_S/ε is substantially lower than for B and C. While α_{sky} for C is twice that of B, because G_{sky} is nearly 25% that of G_S, we expect coating C to give the lowest T_s.

(c) With the values of α_S, α_{sky} and ε for coating C from part (b), rewrite the energy balance as

$$P_e / A_s + \alpha_S G_S + \alpha_{sky}\sigma T_{sky}^4 - \varepsilon\sigma T_s^4 - h\left(T_s - T_\infty\right) = 0$$

$$4.887\ W / \left(0.13\ m\right)^2 + 0.2 \times 800\ W/m^2 + 0.6 \times 232.3\ W/m^2 - 0.6 \times \sigma T_s^4 - 15\left(T_s - 300\right) = 0$$

Using trial-and-error, find $T_s = 316.5\ K = 43.5°C$. $<$

COMMENTS: (1) Using coatings A and B, find $T_s = 71$ and $54°C$, respectively. (2) For more precise values of α_S, α_{sky} and ε, use $T_s = 43.5°C$. For example, at $\lambda T_s = 3 \times (43.5 + 273) = 950\ \mu m \cdot K$, $F_{0-\lambda T} = 0.000$ while at $\lambda T_{solar} = 3 \times 5800 = 17{,}400\ \mu m \cdot K$, $F_{0-\lambda T} \approx 0.98$; we conclude little effect will be seen.

PROBLEM 12.124

KNOWN: Opaque, spectrally-selective horizontal plate with electrical heater on backside is exposed to convection, solar irradiation and sky irradiation.

FIND: Electrical power required to maintain plate at 60°C.

SCHEMATIC:

ASSUMPTIONS: (1) Plate is opaque, diffuse and uniform, (2) No heat lost out the backside of heater.

ANALYSIS: From an energy balance on the plate-heater system, per unit area basis,

$$\dot{E}''_{in} - \dot{E}''_{out} = 0$$

$$q''_{elec} + \alpha_S G_S + \alpha G_{sky} - \varepsilon E_b(T_s) - q''_{conv} = 0$$

where $G_{sky} = \sigma T_{sky}^4$, $E_b = \sigma T_s^4$, and $q''_{conv} = h(T_s - T_\infty)$. The *solar absorptivity* is

$$\alpha_S = \int_0^\infty \alpha_\lambda G_{\lambda,S} d\lambda \Big/ \int_0^\infty G_{\lambda,S} d\lambda = \int_0^\infty \alpha_\lambda E_{\lambda,b}(\lambda, 5800\ K) d\lambda \Big/ \int_0^\infty E_{\lambda,b}(\lambda, 5800K) d\lambda$$

where $G_{\lambda,S} \sim E_{\lambda,b}$ (λ, 5800 K). Noting that $\alpha_\lambda = 1 - \rho_\lambda$,

$$\alpha_S = (1-0.2) F_{(0-2\mu m)} + (1-0.7)\left(1 - F_{(0-2\mu m)}\right)$$

where at $\lambda T = 2\ \mu m \times 5800\ K = 11{,}600\ \mu m \cdot K$, find from Table 12.1, $F_{(0-\lambda T)} = 0.941$,

$$\alpha_S = 0.80 \times 0.941 + 0.3(1 - 0.941) = 0.771.$$

The *total, hemispherical emissivity* is

$$\varepsilon = (1-0.2) F_{(0-2\mu m)} + (1-0.7)\left(1 - F_{(0-2\mu m)}\right).$$

At $\lambda T = 2\ \mu m \times 333\ K = 666\ K$, find $F_{(0-\lambda T)} \approx 0.000$; hence $\varepsilon = 0.30$. The *total, hemispherical absorptivity* for sky irradiation is $\alpha = \varepsilon = 0.30$ since the surface is gray for this emission and irradiation process. Substituting numerical values,

$$q''_{elec} = \varepsilon \sigma T_s^4 + h(T_s - T_\infty) - \alpha_S G_S - \alpha \sigma T_{sky}^4$$

$$q''_{elec} = 0.30 \times \sigma (333\ K)^4 + 10\ W/m^2 \cdot K(60-20)°C - 0.771 \times 600\ W/m^2 - 0.30 \times \sigma (233\ K)^4$$

$$q''_{elec} = 209.2\ W/m^2 + 400.0\ W/m^2 - 462.6\ W/m^2 - 50.1\ W/m^2 = 96.5\ W/m^2. \quad <$$

COMMENTS: (1) Note carefully why $\alpha_{sky} = \varepsilon$ for the sky irradiation.

PROBLEM 12.125

KNOWN: Chord length and spectral emissivity of wing. Ambient air temperature, sky temperature and solar irradiation for ground and in-flight conditions. Flight speed.

FIND: Temperature of top surface of wing for (a) ground and (b) in-flight conditions.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state, (2) Negligible heat transfer from back of wing surface, (3) Diffuse surface behavior, (4) Negligible solar radiation for $\lambda > 3\ \mu$m ($\alpha_S = \alpha_{\lambda \le 3\ \mu m} = \varepsilon_{\lambda \le 3\ \mu m} = 0.6$), (5) Negligible sky radiation and surface emission for $\lambda \le 3\ \mu$m ($\alpha_{sky} = \alpha_{\lambda > 3\ \mu m} = \varepsilon_{\lambda > 3\ \mu m} = 0.3 = \varepsilon$), (6) Quiescent air for ground condition, (7) Air foil may be approximated as a flat plate, (8) Negligible viscous heating in boundary layer for in-flight condition, (9) The wing span W is much larger than the chord length L_c, (10) In-flight transition Reynolds number is 5×10^5.

PROPERTIES: Part (a). *Table A-4*, air ($T_f \approx 325$ K): $\nu = 1.84 \times 10^{-5}$ m^2/s, $\alpha = 2.62 \times 10^{-5}$ m^2/s, k = 0.0282 W/m·K, $\beta = 0.00307$. Part (b). Given: $\rho = 0.470$ kg/m^3, $\mu = 1.50 \times 10^{-5}$ N·s/m^2, k = 0.021 W/m·K, Pr = 0.72.

ANALYSIS: For both ground and in-flight conditions, a surface energy balance yields

$$\alpha_{sky} G_{sky} + \alpha_S G_S = \varepsilon\sigma T_s^4 + \bar{h}\left(T_s - T_\infty\right) \qquad (1)$$

where $\alpha_{sky} = \varepsilon = 0.3$ and $\alpha_S = 0.6$.

(a) For the ground condition, $\bar{h}$ may be evaluated from Eq. 9.30 or 9.31, where $L = A_s/P = L_c \times W/2\ (L_c + W) \approx L_c/2 = 2$m and $Ra_L = g\beta\,(T_s - T_\infty)\,L^3/\nu\alpha$. Using the *IHT* software to solve Eq. (1) and accounting for the effect of temperature-dependent properties, the surface temperature is

$$T_S = 350.6\ \text{K} = 77.6°\text{C} \qquad <$$

where $Ra_L = 2.52 \times 10^{10}$ and $\bar{h} = 6.2$ W/m^2·K. Heat transfer from the surface by emission and convection is 257.0 and 313.6 W/m^2, respectively.

(b) For the in-flight condition, $Re_L = \rho u_\infty L_c/\mu = 0.470\ \text{kg/m}^3 \times 200\ \text{m/s} \times 4\text{m}/1.50 \times 10^{-5}\ \text{N·s/m}^2 = 2.51 \times 10^7$. For mixed, laminar/turbulent boundary layer conditions (Section 7.2.3 of text) and a transition Reynolds number of $Re_{x,c} = 5 \times 10^5$.

$$\overline{Nu}_L = \left(0.037\,Re_L^{4/5} - 871\right)Pr^{1/3} = 26{,}800$$

$$\bar{h} = \frac{k}{L}\overline{Nu}_L = \frac{0.021\,\text{W/m}\cdot\text{K} \times 26{,}800}{4\text{m}} = 141\ \text{W/m}^2\cdot\text{K}$$

Substituting into Eq. (1), a trial-and-error solution yields

$$T_s = 237.7\ \text{K} = -35.3°\text{C} \qquad <$$

Heat transfer from the surface by emission and convection is now 54.3 and 657.6 W/m^2, respectively.

COMMENTS: The temperature of the wing is strongly influenced by the convection heat transfer coefficient, and the large coefficient associated with flight yields a surface temperature that is within 5°C of the air temperature.

PROBLEM 12.126

KNOWN: Spectrally selective and gray surfaces in earth orbit are exposed to solar irradiation, G_S, in a direction 30° from the normal to the surfaces.

FIND: Equilibrium temperature of each plate.

SCHEMATIC:

$G_S = 1353\ W/m^2$ 30° T_s $T_{sur} = 0K$

Gray surface: $\alpha_\lambda = 0.95$

Coating: $\alpha_\lambda = 0.95$ for $0 \le \lambda \le 3\mu m$

$\alpha_\lambda = 0.05$ for $\lambda > 3\mu m$

ASSUMPTIONS: (1) Plates are at uniform temperature, (2) Surroundings are at 0K, (3) Steady-state conditions, (4) Solar irradiation has spectral distribution of $E_{\lambda,b}(\lambda, 5800K)$, (5) Back side of plate is insulated.

ANALYSIS: Noting that the solar irradiation is directional (at 30° from the normal), the radiation balance has the form

$$\alpha_S G_S \cos\theta - \varepsilon E_b(T_s) = 0. \qquad (1)$$

Using $E_b(T_s) = \sigma T_s^4$ and solving for T_s, find

$$T_s = \left[(\alpha_S/\varepsilon)\,(G_S \cos\theta/\sigma)\right]^{1/4}. \qquad (2)$$

For the *gray surface,* $\alpha_S = \varepsilon = \alpha_\lambda$ and the temperature is independent of the magnitude of the absorptivity.

$$T_s = \left(\frac{0.95}{0.95} \times \frac{1353\ W/m^2 \times \cos 30°}{5.67\times10^{-8}\ W/m^2\cdot K^4}\right)^{1/4} = 379\ K.$$

<

For the *selective surface,* $\alpha_S = 0.95$ since nearly all the solar spectral power is in the region $\lambda < 3\mu m$. The value of ε depends upon the surface temperature T_s and would be determined by the relation.

$$\varepsilon = 0.95\ F_{(0\to\lambda T_s)} + 0.05\left[1 - F_{(0\to\lambda T_s)}\right] \qquad (3)$$

where $\lambda = 3\mu m$ and T_s is as yet unknown. To find T_s, a trial-and-error procedure as follows will be used: (1) assume a value of T_s, (2) using Eq. (3), calculate ε with the aid of Table 12.1 evaluating $F_{(0\to\lambda T)}$ at $\lambda T_s = 3\mu m \cdot T_s$, (3) with this value of ε, calculate T_s from Eq. (2) and compare with assumed value of T_s. The results of the iterations are:

T_s(K), assumed value	633	700	666	650	655
ε, from Eq. (3)	0.098	0.125	0.110	0.104	0.106
T_s(K), from Eq. (2)	656	629	650	659	656

Hence, for the coating, $T_s \approx 656K$.

<

COMMENTS: Note the role of the ratio α_s/ε in determining the equilibrium temperature of an isolated plate exposed to solar irradiation in space. This is an important property of the surface in spacecraft thermal design and analysis.

PROBLEM 12.127

KNOWN: Spectral, hemispherical emissivity distributions for two panels subjected to solar flux in the deep space environment.

FIND: Steady-state temperatures of the panels.

SCHEMATIC:

ASSUMPTIONS: (1) Surfaces are opaque and diffuse, (2) Panels are oriented normal to solar flux with backside insulated, (3) Steady-state conditions, (4) No convection.

ANALYSIS: An energy balance on the panel is

$$q''_{in} - q''_{out} = 0$$

$$\alpha_S G_S - \varepsilon E_b(T_s) = \alpha_S G_S - \varepsilon \sigma T_s^4 = 0$$

$$T_s = \left[\left(\alpha_S/\varepsilon\right)\left(G_S/\sigma\right)\right]^{1/4}.$$

For each panel determine α_S and ε. Recognizing that $G_{\lambda,S} \sim E_{\lambda,b}$ (λ, 5800K), the solar absorptivity from Eq. 12.47 is

$$\alpha_S = \frac{\int_0^\infty \alpha_\lambda G_{\lambda,S}\, d\lambda}{\int_0^\infty G_{\lambda,S}\, d\lambda} = \frac{\int_0^\infty \varepsilon_\lambda E_{\lambda,b}(\lambda, 5800\text{K})}{E_b(5800\text{K})}.$$

Note that $\varepsilon_\lambda = \alpha_\lambda$ since the surface is diffuse. Using Eq. 12.65 and Table 12.1 find

Surface A: $\alpha_S = \varepsilon_1 F_{(0\to 2\mu m)} + \varepsilon_2\left[1 - F_{(0\to 2\mu m)}\right]\}$ $\qquad \lambda T = 2\times 5800 = 11{,}600\,\mu m\cdot K,$

$\alpha_S = 0.5\times 0.940 + 0.2[1-0.940] = 0.482\}$ $\qquad F_{(0\to 2\mu m)} = 0.940$

Surface B: $\alpha_S = 0.11\times 0.979 + 0.02[1-0.979] = 0.108\}$ $\qquad \lambda T = 3\times 5800 = 17{,}400\,\mu m\cdot K,$ $F_{(0\to 3\mu m)} = 0.979.$

To determine the total emissivity, we need to know T_s. If $T_s \le 400$K, then for $\lambda T = 3\ \mu m \times 400K = 1200K$, $F_{(0\to\lambda)} = 0.002$. That is, there is negligible power for $\lambda < 3\ \mu m$ if $T_s \le 400K$, and hence

Surface A: $\varepsilon \approx \varepsilon_2 = 0.2$ $\qquad$ *Surface B*: $\varepsilon \approx \varepsilon_2 = 0.02.$

Substituting the solar absorptivity and emissivity values, find

Surface A: $$T_s = \left(\frac{0.482}{0.20}\times\frac{1353\ W/m^2}{5.67\times10^{-8}\ W/m^2\cdot K}\right)^{1/4} = 499K$$ <

Surface B: $$T_s = \left(\frac{0.108}{0.02}\times\frac{1353\ W/m^2}{5.67\times10^{-8}\ W/m^2\cdot K}\right)^{1/4} = 599K.$$ <

COMMENTS: (1) Note the assumption that $T_s \le 400K$ used for finding ε is not satisfied; for better precision, it is necessary to perform an iterative solution. (2) Note the importance of the α_S/ε ratio which determines the surface temperature.

PROBLEM 12.128

KNOWN: Radiative properties and operating conditions of a space radiator.

FIND: Equilibrium temperature of the radiator.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible irradiation due to earth emission.

ANALYSIS: From a surface energy balance, $\dot{E}''_{in} - \dot{E}''_{out} = 0$.

$$q''_{dis} + a_S G_S - E = 0.$$

Hence

$$T_s = \left(\frac{q''_{dis} + a_S G_S}{e\ s} \right)^{1/4}$$

$$T_s = \left(\frac{1500\,W/m^2 + 0.5 \times 1000\,W/m^2}{0.95 \times 5.67 \times 10^{-8}\,W/m^2 \cdot K^4} \right)^{1/4}$$

or

$$T_s = 439K.$$ <

COMMENTS: *Passive* thermal control of spacecraft is practiced by using surface coatings with desirable values of α_S and ε.

PROBLEM 12.129

KNOWN: Spherical satellite exposed to solar irradiation of 1353 m^2; surface is to be coated with a checker pattern of evaporated aluminum film, (fraction, F) and white zinc-oxide paint (1 - F).

FIND: The fraction F for the checker pattern required to maintain the satellite at 300 K.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Satellite is isothermal, and (3) No internal power dissipation.

ANALYSIS: Perform an energy balance on the satellite, as illustrated in the schematic, identifying absorbed solar irradiation on the projected area, A_p, and emission from the spherical area A_s.

$$\dot{E}_{in} - \dot{E}_{out} = o$$

$$\left(F \cdot \alpha_{S,f} + (1-F) \cdot \alpha_{S,p}\right) G_S\, A_p - \left(F \cdot \varepsilon_f + (1-F) \cdot \varepsilon_p\right) E_b\, (T_s)\, A_s = 0$$

where $A_p = \pi D^2 / 4$, $A_s = \pi D^2$, $E_b = \sigma T^4$ and $\sigma = 5.67 \times 10^{-8}\ W/m^2 \cdot K^4$. Substituting numerical values, find F.

$$\left(F \times 0.09 + (1-F) \times 0.22\right) \times 1353\ W/m^2 \times (1/4)$$

$$-\left(F \times 0.03 + (1-F) \times 0.85\right) \sigma (300\ K)^4 \times 1 = 0$$

$$F = 0.95$$ <

COMMENTS: (1) If the thermal control engineer desired to maintain the spacecraft at 325 K, would the fraction F (aluminum film) be increased or decreased? Verify your opinion with a calculation.

(2) If the internal power dissipation per unit surface area is 150 W/m^2, what fraction F will maintain the satellite at 300 K?

PROBLEM 12.130

KNOWN: Inner and outer radii, spectral reflectivity, and thickness of an annular fin. Base temperature and solar irradiation.

FIND: (a) Rate of heat dissipation if $\eta_f = 1$, (b) Differential equation governing radial temperature distribution in fin if $\eta_f < 1$.

SCHEMATIC:

G_S = 0 or 1000 W/m^2
T_b = 400 K
E
ρ_λ
0.8
0.1
2
λ (μm)
t
G_S
E
q_r
q_{r+dr}
r_i
r_o = 0.5 m

ASSUMPTIONS: (1) Steady-state, (2) One-dimensional radial conduction, (3) Adiabatic tip and bottom surface, (4) Opaque, diffuse surface ($\alpha_\lambda = 1 - \rho_\lambda,\ \varepsilon_\lambda = \alpha_\lambda$).

ANALYSIS: (a) If $\eta_f = 1$, $T(r) = T_b = 400$ K across the entire fin and

$$q_f = \left[\varepsilon E_b(T_b) - \alpha_S G_S\right]\pi r_o^2$$

With $\lambda T = 2\ \mu m \times 5800\ K = 11{,}600\ \mu m \cdot K$, $F_{(0\to 2\mu m)} = 0.941$. Hence $\alpha_S = \alpha_1 F_{(0\to 2\mu m)} + \alpha_2\left[1 - F_{(0\to 2\mu m)}\right] = 0.2 \times 0.941 + 0.9 \times 0.059 = 0.241$. With $\lambda T = 2\ \mu m \times 400\ K = 800\ \mu m \cdot K$, $F_{(0\to 2\mu m)} = 0$ and $\varepsilon = 0.9$. Hence, for $G_S = 0$,

$$q_f = 0.9 \times 5.67 \times 10^{-8}\ W/m^2 \cdot K^4\,(400\,K)^4\,\pi\,(0.5\,m)^2 = 1026\,W \qquad <$$

and for $G_S = 1000\ W/m^2$,

$$q_f = 1026\,W - 0.241\left(1000\,W/m^2\right)\pi\,(0.5\,m)^2 = (1026 - 189)\,W = 837\,W \qquad <$$

(b) Performing an energy balance on a differential element extending from r to r+dr, we obtain

$$q_r + \alpha_S G_S (2\pi r dr) - q_{r+dr} - E(2\pi r dr) = 0$$

where

$$q_r = -k(dT/dr)2\pi r t \qquad \text{and} \qquad q_{r+dr} = q_r + (dq_r/dr)dr.$$

Hence,

$$\alpha_S G_S (2\pi r dr) - d\left[-k(dT/dr)2\pi r t\right]dr - E(2\pi r dr) = 0$$

$$2\pi r t k \frac{d^2T}{dr^2} + 2\pi t k \frac{dT}{dr} + \alpha_S G_S 2\pi r - E 2\pi r = 0$$

$$kt\left(\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr}\right) + \alpha_S G_S - \varepsilon\sigma T^4 = 0 \qquad <$$

COMMENTS: The radiator should be constructed of a light weight, high thermal conductivity material (aluminum).

PROBLEM 12.131

KNOWN: Rectangular plate, with prescribed geometry and thermal properties, for use as a radiator in a spacecraft application. Radiator exposed to solar radiation on upper surface, and to deep space on both surfaces.

FIND: Using a computer-based, finite-difference method with a space increment of 0.1 m, find the tip temperature, T_L, and rate of heat rejection, q_f, when the base temperature is maintained at 80°C for the cases: (a) when exposed to the sun, (b) on the dark side of the earth, not exposed to the sun; and (c) when the thermal conductivity is extremely large. Compare the case (c) results with those obtained from a hand calculation assuming the radiator is at a uniform temperature.

SCHEMATIC:

G_S = 1353 W/m²
T_b = 80°C
Plate, W = 6 m
ε = 0.90, α_S = 0.45
k = 300 W/m-K
T_{sur} = 4 K
t = 12 mm
L = 1 m
x

ASSUMPTIONS: (1) Steady-state conditions, (b) Plate-radiator behaves as an extended surface with one-dimensional conduction, and (c) Radiating tip condition.

ANALYSIS: The finite-difference network with 10 nodes and a space increment Δx = 0.1 m is shown in the schematic below. The finite-difference equations (FDEs) are derived for an interior node (nodes 01 - 09) and the tip node (10). The energy balances are represented also in the schematic below where q_a and q_b represent conduction heat rates, q_S represents the absorbed solar radiation, and q_{rad} represents the radiation exchange with outer space.

T00 = T_b T10 = T_L
q_f 00 01 02 09 10
A_c = Wxt
P = 2W
Δx = 0.1m t = 12 mm L = 1 m
x
q_S 03 04 05 q_a q_b q_{rad}
q_S q_f 00 01 q_b q_{rad}
q_S 09 10 q_a $q_{rad,1}$ $q_{rad,2}$

Finite-difference network, energy balances

Interior node 04

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$q_a + q_b + q_S + q_{rad} = 0$$

$$kA_c\,(T_{03} - T_{04})/\Delta x + kA_c\,(T_{05} - T_{04})/\Delta x$$

$$+\alpha_S\, G_S\,(P/2)\Delta x + \varepsilon P \Delta x \sigma\left(T_{sur}^4 - T_{04}^4\right) = 0$$

where P = 2W and A_c = W·t.

Tip node 10

$$q_a + q_S + q_{rad,1} + q_{rad,2} = 0$$

$$kA_c\,(T_{09} - T_{10})/\Delta x + \alpha_S G_S\,(P/2)\,(\Delta x/2)$$

$$+\varepsilon\, A_c \sigma\left(T_{sur}^4 - T_{10}^4\right) + \varepsilon P(\Delta x/2)\sigma\left(T_{sur}^4 - T_{04}^4\right) = 0$$

Continued …..

Heat rejection, q_f. From an energy balance on the base node 00,

$$q_f + q_{01} + q_S + q_{rad} = 0$$

$$q_f + kA_c\,(T_{01} - T_{00})/\Delta x + \alpha_S G_S (P/2)\,(\Delta x/2) + \varepsilon P(\Delta x/2)\sigma\left(T_{sur}^4 - T_{00}^4\right) = 0$$

The foregoing nodal equations and the heat rate expression were entered into the *IHT* workspace to obtain solutions for the three cases. See Comment 2 for the *IHT* code, and Comment 1 for code validation remarks.

Case	k(W/m·K)	$G_S(W/m^2)$	T_L(°C)	q_f(W)	
a	300	1353	30.5	2766	<
b	300	0	-7.6	4660	<
c	1×10^{10}	0	80.0	9557	

COMMENTS: (1) Case (c) using the *IHT* code with k = 1×10^{10} W/m·K corresponds to the condition of the plate at the uniform temperature of the base; that is T(x) = T_b. For this condition, the heat rejection from the upper and lower surfaces and the tip area can be calculated as

$$q_{f,u} = \varepsilon\sigma\left(T_b^4 - T_{sur}^4\right)\left[P \cdot L + A_c\right]$$

$$q_{f,u} = 0.65\,\sigma\left[(80+273)^4 - 4^4\right] W/m^2 [12 + 6 \times 0.012] m^2$$

$$q_{f,u} = 9565\ W/m^2$$

Note that the heat rejection rate for the uniform plate is in excellent agreement with the result of the FDE analysis when the thermal conductivity is made extremely large. We have confidence that the code is properly handling the conduction and radiation processes; but, we have not exercised the portion of the code dealing with the absorbed irradiation. What analytical solution/model could you use to validate this portion of the code?

(2) Selection portions are shown below of the *IHT* code with the 10-nodal FDEs for the temperature distribution and the heat rejection rate.

```
// Finite-difference equations
// Interior nodes, 01 to 09
k * Ac * (T00 - T01) / deltax + k * Ac * (T02 - T01) / deltax + absS * GS * P/2 * deltax + eps * P *
deltax * sigma * (Tsur^4 - T01^4) = 0
…..
…..
k * Ac * (T03 - T04) / deltax + k * Ac * (T05 - T04) / deltax + absS * GS * P/2 * deltax + eps * P *
deltax * sigma * (Tsur^4 - T04^4) = 0
…..
…..
k * Ac * (T08 - T09) / deltax + k * Ac * (T10 - T09) / deltax + absS * GS * P/2 * deltax + eps * P *
deltax * sigma * (Tsur^4 - T09^4) = 0

// Tip node 10
k* Ac * (T09 - T10) / deltax + absS * GS * P/2 * (deltax / 2) + eps * P * (deltax / 2) * sigma *
(Tsur^4 - T10^4) - eps * Ac * sigma * (Tsur^4 - T00^4) = 0

// Rejection heat rate, energy balance on base node
qf + k * Ac * (T01 - T00) / deltax + absS * GS * (P/4) * (deltax /2) + eps * (P * deltax /2) *
sigma * (Tsur^4 - T00^4) = 0
```

Continued …..

PROBLEM 12.131 (Cont.)

(3) To determine the validity of the one-dimensional, extended surface analysis, calculate the Biot number estimating the linearized radiation coefficient based upon the uniform plate condition, T_b = 80°C.

$$Bi = h_{rad}(t/2)/k$$

$$h_{rad} = \varepsilon\sigma(T_b + T_{sur})\left(T_b^2 + T_{sur}^2\right) \approx \varepsilon\sigma T_b^3 = 2.25\ W/m^2 \cdot K$$

$$Bi = 2.25\ W/m^2 \cdot K(0.012\ m/2)/300\ W/m \cdot K = 4.5 \times 10^{-5}$$

Since Bi << 0.1, the assumption of one-dimensional conduction is appropriate.

PROBLEM 12.132

KNOWN: Directional absorptivity of a plate exposed to solar radiation on one side.

FIND: (a) Ratio of normal absorptivity to hemispherical emissivity, (b) Equilibrium temperature of plate at 0° and 75° orientation relative to sun's rays.

SCHEMATIC:

ASSUMPTIONS: (1) Surface is gray, (2) Properties are independent of ϕ.

ANALYSIS: (a) From the prescribed α_θ (θ), $\alpha_n = 0.9$. Since the surface is gray, $\varepsilon_\theta = \alpha_\theta$. Hence from Eq. 12.36, which applies for total as well as spectral properties.

$$e = 2\int_0^{p/2} e_q \cos q \sin q \, dq = 2\left[0.9\frac{\sin^2 q}{2}\Bigg|_0^{p/3} + 0.1\frac{\sin^2 q}{2}\Bigg|_{p/3}^{p/2}\right]$$

$$e = 2\left[0.9(0.375) + 0.1(0.5 - 0.375)\right] = 0.70.$$

Hence

$$\frac{a_n}{e} = \frac{0.9}{0.7} = 1.286.$$

(b) Performing an energy balance on the plate,

$$a_q \, q_S'' \cos q - 2e\, s\, T_S^4 = 0$$

or

$$T_S = \left[\frac{a_q}{2e\, s} q_S'' \cos q\right]^{1/4}.$$

Hence for $\theta = 0°$, $\alpha_\theta = 0.9$ and $\cos\theta = 1$,

$$T_S = \left[\frac{0.9}{2 \times 0.7 \times 5.67 \times 10^{-8}} \times 1353\right]^{1/4} = 352\text{K}. \qquad <$$

For $\theta = 75°$, $\alpha_\theta = 0.1$ and $\cos\theta = 0.259$

$$T_S = \left[\frac{0.1}{2 \times 0.7 \times 5.67 \times 10^{-8}} \times 1353 \times 0.259\right]^{1/4} = 145\text{K}. \qquad <$$

COMMENTS: Since the surface is not diffuse, its absorptivity depends on the directional distribution of the incident radiation.

PROBLEM 12.133

KNOWN: Transmissivity of cover plate and spectral absorptivity of absorber plate for a solar collector.

FIND: Absorption rate for prescribed solar flux and preferred absorber plate coating.

SCHEMATIC:

ASSUMPTIONS: (1) Solar irradiation of absorber plate retains spectral distribution of blackbody at 5800K, (2) Coatings are diffuse.

ANALYSIS: At the absorber plate we wish to maximize solar radiation absorption and minimize losses due to emission. The solar radiation is concentrated in the spectral region $\lambda < 4\mu m$, and for a representative plate temperature of $T \leq 350K$, emission from the plate is concentrated in the spectral region $\lambda > 4\mu m$. Hence,

Coating A is vastly superior. <

With $G_{\lambda,S} \sim E_{\lambda,b}$ (5800K), it follows from Eq. 12.47

$$a_A \approx 0.85\ F_{(0-4mm)} + 0.05\ F_{(4mm-\infty)}.$$

From Table 12.1, $\lambda T = 4\mu m \times 5800K = 23{,}200\mu m \cdot K$,

$$F_{(0-4mm)} \approx 0.99.$$

Hence

$$a_A = 0.85(0.99) + 0.05(1 - 0.99) \approx 0.85.$$

With $G_S = 1000\ W/m^2$ and $\tau = 0.76$ (Ex. 12.9), the absorbed solar flux is

$$G_{S,abs} = a_A (t G_S) = 0.85\left(0.76 \times 1000\ W/m^2\right)$$

$$G_{S,abs} = 646\ W/m^2.$$ <

COMMENTS: Since the absorber plate emits in the infrared ($\lambda > 4\mu m$), its emissivity is $\varepsilon_A \approx 0.05$. Hence $(\alpha/\varepsilon)_A = 17$. A large value of α/ε is desirable for solar absorbers.

PROBLEM 12.134

KNOWN: Spectral distribution of coating on satellite surface. Irradiation from earth and sun.

FIND: (a) Steady-state temperature of satellite on dark side of earth, (b) Steady-state temperature on bright side.

SCHEMATIC:

α_λ 0.6 0.3 3 $\lambda(\mu m)$

$G_E = 340\,W/m^2$ E $G_S = 1353\,W/m^2$

ASSUMPTIONS: (1) Steady-state conditions, (2) Opaque, diffuse-gray surface behavior, (3) Spectral distributions of earth and solar emission may be approximated as those of blackbodies at 280K and 5800K, respectively, (4) Satellite temperature is less than 500K.

ANALYSIS: Performing an energy balance on the satellite,

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$a_E\, G_E \left(p\, D^2/4\right) + a_S\, G_S \left(p\, D^2/4\right) - e\, s\, T_s^4 \left(p\, D^2\right) = 0$$

$$T_s = \left(\frac{a_E\, G_E + a_S\, G_S}{4 e\, s}\right)^{1/4}.$$

From Table 12.1, with 98% of radiation below 3μm for λT = 17,400μm·K,

$$a_S \cong 0.6.$$

With 98% of radiation above 3μm for λT = 3μm × 500K = 1500μm·K,

$$e \approx 0.3 \qquad a_E \approx 0.3.$$

(a) On *dark* side,

$$T_s = \left(\frac{a_E\, G_E}{4 e\, s}\right)^{1/4} = \left(\frac{0.3 \times 340\,W/m^2}{4 \times 0.3 \times 5.67 \times 10^{-8}\,W/m^2 \cdot K^4}\right)^{1/4}$$

$$T_s = 197\ K.$$ <

(b) On *bright* side,

$$T_s = \left(\frac{a_E\, G_E + a_S\, G_S}{4 e\, s}\right)^{1/4} = \left(\frac{0.3 \times 340\,W/m^2 + 0.6 \times 1353\,W/m^2}{4 \times 0.3 \times 5.67 \times 10^{-8}\,W/m^2 \cdot K^4}\right)^{1/4}$$

$$T_s = 340K.$$ <

PROBLEM 12.135

KNOWN: Space capsule fired from earth orbit platform in direction of sun.

FIND: (a) Differential equation predicting capsule temperature as a function of time, (b) Position of capsule relative to sun when it reaches its destruction temperature.

SCHEMATIC:

ASSUMPTIONS: (1) Capsule behaves as lumped capacitance system, (2) Capsule surface is black, (3) Temperature of surroundings approximates absolute zero, (4) Capsule velocity is constant.

ANALYSIS: (a) To find the temperature as a function of time, perform an energy balance on the capsule considering absorbed solar irradiation and emission,

$$\dot{E}_{in} - \dot{E}_{out} = \dot{E}_{st} \qquad G_S \cdot p R^2 - s T^4 \cdot 4 p R^2 = r c (4/3) p R^3 (dT/dt). \tag{1}$$

Note the use of the projected capsule area (πR^2) and the surface area ($4\pi R^2$). The solar irradiation will increase with decreasing radius (distance toward the sun) as

$$G_S(r) = G_{S,e}(r_e/r)^2 = G_{S,e}\left(r_e/(r_e - Vt)\right)^2 = G_{S,e}\left(1/(1 - Vt/r_e)\right)^2 \tag{2}$$

where r_e is the distance of earth orbit from the sun and $r = r_e - Vt$. Hence, Eq. (1) becomes

$$\frac{dT}{dt} = \frac{3}{r c R}\left[\frac{G_{S,e}}{4(1 - Vt/r_e)^2} - s\, T^4\right].$$

The rate of temperature change is

$$\frac{dT}{dt} = \frac{3}{\left(4\times10^6\,J/m^3\cdot K \times 1.5m\right)}\left[\frac{1353\,W/m^2}{4\left(1 - 16\times10^3\,m/s \times t/1.5\times10^{11} m\right)^2} - s\, T^4\right]$$

$$\frac{dT}{dt} = 1.691\times10^{-4}\left(1 - 1.067\times10^{-7}\,t\right)^{-2} - 2.835\times10^{-14}\,T^4$$

where T[K] and t(s). For the initial condition, t = 0, with T = 20°C = 293K,

$$\frac{dT}{dt}(0) = -3.984\times10^{-5}\,K/s. \qquad <$$

That is, the capsule will cool for a period of time and then begin to heat.

(b) The differential equation cannot be explicitly solved for temperature as a function of time. Using a numerical method with a time increment of $\Delta t = 5\times10^5$s, find

$$T(t) = 150°C = 423\ K \qquad at \qquad t \approx 5.5\times10^6 s. \qquad <$$

Note that in this period of time the capsule traveled $(r_e - r) = Vt = 16\times10^3$ m/s $\times\ 5.5\times10^6 = 1.472\times10^{10}$ m. That is, $r = 1.353\times10^{11}$ m.

PROBLEM 12.136

KNOWN: Dimensions and spectral absorptivity of radiator used to dissipate heat to outer space. Radiator temperature. Magnitude and direction of incident solar flux.

FIND: Power dissipation within radiator.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible heat loss through sides and bottom of compartment, (3) Opaque, diffuse surface.

ANALYSIS: Applying conservation of energy to a control surface about the compartment yields

$$\dot{E}_{in} + \dot{E}_g = \dot{E}_{out}$$

$$\dot{E}_g = \left(e s T_p^4 - a G_s\right)A.$$

The emissivity can be expressed as

$$e = \int_0^\infty e_l \left(E_{l,b}/E_b\right) dl = e_1 F_{(0\rightarrow l_1)} + e_2 F_{(l_1\rightarrow\infty)}.$$

From Table 12.1: $\lambda_1 T = 1000\ \mu m \cdot K \rightarrow F_{(0\rightarrow l_1)} = 0.000321$

$$e = 0.2(0.000321) + 0.8(1 - 0.00321) = 0.8.$$

The absorptivity can be expressed as

$$a = \int_0^\infty a_l (G_l/G) dl = \int_0^\infty a_l \left[E_{l,b}(5800\ K)/E_b(5800\ K)\right] dl.$$

From Table 12.1: $l_1 T = 11{,}600\ m m \cdot K \rightarrow F_{(0\rightarrow l_1)} = 0.941,$

$$a = 0.2(0.941) + 0.8(0.059) = 0.235.$$

Hence,

$$\dot{E}_g = \left[0.8 \times 5.67 \times 10^{-8}\ W/m^2 \cdot K^4 \times (500\ K)^4 - 0.235\ \cos 30° \left(1350\ W/m^2\right)\right] 1\ m^2$$

$$\dot{E}_g = 2560\ W.$$ <

COMMENTS: Solar irradiation and plate emission are concentrated at short and long wavelength portions of the spectrum. Hence, $\alpha \neq \varepsilon$ and the surface is not gray for the prescribed conditions.

PROBLEM 12.137

KNOWN: Solar panel mounted on a spacecraft of area 1 m^2 having a solar-to-electrical power conversion efficiency of 12% with specified radiative properties.

FIND: (a) Steady-state temperature of the solar panel and electrical power produced with solar irradiation of 1500 W/m^2, (b) Steady-state temperature if the panel were a thin plate (no solar cells) with the same radiative properties and for the same prescribed conditions, and (c) Temperature of the solar panel 1500 s after the spacecraft is eclipsed by the earth; thermal capacity of the panel per unit area is 9000 $J/m^2 \cdot K$.

SCHEMATIC:

Solar panel, T, A_p = 1 m^2,
thermal capacity, 9000 J/m^2-K
G_S = 1500 W/m^2
Array, ε_a = 0.8, $\alpha_{S,a}$ = 0.8
conversion efficiency, e = 12%
Backside, ε_b = 0.7

ASSUMPTIONS: (1) Solar panel and thin plate are isothermal, (2) Solar irradiation is normal to the panel upper surface, and (3) Panel has unobstructed view of deep space at 0 K.

ANALYSIS: (a) The energy balance on the solar panel is represented in the schematic below and has the form

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$\alpha_S G_S \cdot A_p - (\varepsilon_a + \varepsilon_b)\, E_b\left(T_{sp}\right) \cdot A_p - P_{elec} = 0 \tag{1}$$

where E_b (T) = σT^4, $\sigma = 5.67 \times 10^{-8}$ $W/m^2 \cdot K^4$, and the electrical power produced is

$$P_{elec} = e \cdot G_S \cdot A_p \tag{2}$$

$$P_{elec} = 0.12 \times 1500\ W/m^2 \times 1\ m^2 = 180\ W$$ <

Substituting numerical values into Eq. (1), find

$$0.8 \times 1500\ W/m^2 \times 1\ m^2 - (0.8 + 0.7)\sigma T_{sp}^4 \times 1\ m^2 - 180\ W = 0$$

$$T_{sp} = 330.9\ K = 57.9^\circ C$$ <

EA_p $G_{S,abs}A_p$ T_{sp} P_{elec} EA_p

(a) Solar panel

EA_p $G_{S,abs}A_p$ T_p P_{elec} = 0 EA_p

(b) Thin plate

(b) The energy balance for the thin plate shown in the schematic above follows from Eq. (1) with P_{elec} = 0 yielding

$$0.8 \times 1500\ W/m^2 \times /m^2 - (0.8 + 0.7)\sigma T_p^4 \times 1\ m^2 = 0 \tag{3}$$

$$T_p = 344.7\ K = 71.7^\circ C$$ <

Continued …..

PROBLEM 12.137 (Cont.)

(c) Using the lumped capacitance method, the energy balance on the solar panel as illustrated in the schematic below has the form

$$\dot{E}_{in} - \dot{E}_{out} = \dot{E}_{st}$$

$$-(\varepsilon_a + \varepsilon_b)\sigma T_{sp}^4 \cdot A_p = TC'' \cdot A_p \frac{dT_{sp}}{dt} \qquad (4)$$

where the thermal capacity per unit area is $TC'' = (Mc / A_p) = 9000\ J/m^2 \cdot K$.

Eq. 5.18 provides the solution to this differential equation in terms of t = t (T_i, T_{sp}). Alternatively, use Eq. (4) in the *IHT* workspace (see Comment 4 below) to find

$$T_{sp}(1500\ s) = 242.6\ K = -30.4^\circ C$$ <

(c) Transient energy balance

COMMENTS: (1) For part (a), the energy balance could be written as

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = 0$$

where the energy generation term represents the *conversion process from thermal energy to electrical energy*. That is,

$$\dot{E}_g = -e \cdot G_S \cdot A_p$$

(2) The steady-state temperature for the thin plate, part (b), is higher than for the solar panel, part (a). This is to be expected since, for the solar panel, some of the absorbed solar irradiation (thermal energy) is converted to electrical power.

(3) To justify use of the lumped capacitance method for the transient analysis, we need to know the effective thermal conductivity or internal thermal resistance of the solar panel.

(4) Selected portions of the *IHT* code using the *Models Lumped | Capacitance* tool to perform the transient analysis based upon Eq. (4) are shown below.

```
// Energy balance, Model | Lumped Capacitance
/ * Conservation of energy requirement on the control volume, CV. * /
Edotin - Edotout = Edotst
Edotin = 0
Edotout = Ap * (+q"rad)
Edostat = rhovolcp * Ap * Der(T,t)
// rhovolcp = rho * vol * cp       // thermal capacitance per unit area, J/m^2·K

// Radiation exchange between Cs and large surroundings
q"rad = (eps_a + eps_b) * sigma * (T^4 - Tsur^4)
sigma = 5.67e-8    // Stefan-Boltzmann constant, W/m^2·K^4

// Initial condition
// Ti = 57.93 + 273 = 330.9                    // From part (a), steady-state condition
T_C = T - 273
```

PROBLEM 12.138

KNOWN: Effective sky temperature and convection heat transfer coefficient associated with a thin layer of water.

FIND: Lowest air temperature for which the water will not freeze (without and with evaporation).

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Bottom of water is adiabatic, (3) Heat and mass transfer analogy is applicable, (4) Air is dry.

PROPERTIES: *Table A-4*, Air (273 K, 1 atm): ρ = 1.287 kg/m^3, c_p = 1.01 kJ/kg·K, $\nu = 13.49 \times 10^{-6}$ m^2/s, Pr = 0.72; *Table A-6*, Saturated vapor (T_s = 273 K): $\rho_A = 4.8 \times 10^{-3}$ kg/m^3, h_{fg} = 2502 kJ/kg; *Table A-8*, Vapor-air (298 K): $D_{AB} \approx 0.36 \times 10^{-4}$ m^2/s, Sc = ν/D_{AB} = 0.52.

ANALYSIS: Without evaporation, the surface heat loss by radiation must be balanced by heat gain due to convection. An energy balance gives

$$q''_{conv} = q''_{rad} \qquad \text{or} \qquad h\left(T_\infty - T_s\right) = \varepsilon_s \sigma\left(T_s^4 - T_{sky}^4\right).$$

At freezing, T_s = 273 K. Hence

$$T_\infty = T_s + \frac{\varepsilon_s \sigma}{h}\left(T_s^4 - T_{sky}^4\right) = 273\ \text{K} + \frac{5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4}{25\,\text{W/m}^2\cdot\text{K}}\left[274^4 - 243^4\right]\text{K}^4 = 4.69°\text{C}.$$ <

With evaporation, the surface energy balance is now

$$q''_{conv} = q''_{evap} + q''_{rad} \ \text{ or } \ h\left(T_\infty - T_s\right) = h_m\left[\rho_{A,sat}\left(T_s\right) - \rho_{A,\infty}\right]h_{fg} + \varepsilon_s \sigma\left(T_s^4 - T_{sky}^4\right).$$

$$T_\infty = T_s + \frac{h_m}{h}\rho_{A,sat}\left(T_s\right)h_{fg} + \frac{\varepsilon_s \sigma}{h}\left(T_s^4 - T_{sky}^4\right).$$

Substituting from Eq. 6.92, with n ≈ 0.33,

$$h_m / h = \left(\rho c_p Le^{0.67}\right)^{-1} = \left[\rho c_p\left(Sc/Pr\right)^{0.67}\right]^{-1} = \left[1.287\ \text{kg/m}^3 \times 1010\,\text{J/kg}\cdot\text{K}\left(0.52/0.72\right)^{0.67}\right]^{-1} = 9.57\times10^{-4}\ \text{m}^3\cdot\text{K/J},$$

$$T_\infty = 273\text{K} + 9.57\times10^{-4}\text{m}^3\cdot\text{K/J}\times4.8\times10^{-3}\text{kg/m}^3\times2.5\times10^6\text{J/kg} + 4.69\ \text{K} = 16.2°\text{C}.$$ <

COMMENTS: The existence of clear, cold skies and dry air will allow water to freeze for ambient air temperatures well above 0°C (due to radiative and evaporative cooling effects, respectively). The lowest air temperature for which the water will not freeze increases with decreasing ϕ_∞, decreasing T_{sky} and decreasing h.

PROBLEM 12.139

KNOWN: Temperature and environmental conditions associated with a shallow layer of water.

FIND: Whether water temperature will increase or decrease with time.

SCHEMATIC:

ASSUMPTIONS: (1) Water layer is well mixed (uniform temperature), (2) All non-reflected radiation is absorbed by water, (3) Bottom is adiabatic, (4) Heat and mass transfer analogy is applicable, (5) Perfect gas behavior for water vapor.

PROPERTIES: *Table A-4*, Air (T = 300 K, 1 atm): $\rho_a = 1.161\ kg/m^3$, $c_{p,a} = 1007\ J/kg\cdot K$, Pr = 0.707; *Table A-6*, Water (T = 300 K, 1 atm): $\rho_w = 997\ kg/m^3$, $c_{p,w} = 4179\ J/kg\cdot K$; Vapor (T = 300 K, 1 atm): $\rho_{A,sat} = 0.0256\ kg/m^3$, $h_{fg} = 2.438 \times 10^6\ J/kg$; *Table A-8*, Water vapor-air (T = 300 K, 1 atm): $D_{AB} \approx 0.26 \times 10^{-4}\ m^2/s$; with $\nu_a = 15.89 \times 10^{-6}\ m^2/s$ from *Table A-4*, $Sc = \nu_a/D_{AB} = 0.61$.

ANALYSIS: Performing an energy balance on a control volume about the water,

$$\dot{E}_{st} = \left(G_{S,abs} + G_{A,abs} - E - q''_{evap}\right)A$$

$$\frac{d\left(\rho_w c_{p,w} L A T_w\right)}{dt} = \left[\left(1-\rho_S\right)G_S + \left(1-\rho_A\right)G_A - \varepsilon\sigma T_w^4 - h_m h_{fg}\left(\rho_{A,sat} - \rho_{A,\infty}\right)\right]A$$

or, with $T_\infty = T_w$, $\rho_{A,\infty} = \phi_\infty \rho_{A,sat}$ and

$$\rho_w c_{p,w} L \frac{dT_w}{dt} = \left(1-\rho_S\right)G_S + \left(1-\rho_A\right)G_A - \varepsilon\sigma T_w^4 - h_m h_{fg}\left(1-\phi_\infty\right)\rho_{A,sat}.$$

From Eq. 6.92, with a value of n = 1/3,

$$h_m = \frac{h}{\rho_a c_{p,a} Le^{1-n}} = \frac{h}{\rho_a c_{p,a}\left(Sc/Pr\right)^{1-n}} = \frac{25\,W/m^2\cdot K\left(0.707\right)^{2/3}}{1.161\,kg/m^3 \times 1007\,J/kg\cdot K\left(0.61\right)^{2/3}} = 0.0236\,m/s.$$

Hence

$$\rho_w c_{p,w} L \frac{dT_w}{dt} = \left(1-0.3\right)600 + \left(1-0\right)300 - 0.97 \times 5.67 \times 10^{-8}\left(300\right)^4$$

$$-0.0236 \times 2.438 \times 10^6\left(1-0.5\right)0.0256$$

$$\rho_w c_{p,w} L \frac{dT_w}{dt} = \left(420 + 300 - 445 - 736\right)W/m^2 = -461\,W/m^2.$$

Hence the water will *cool*. <

COMMENTS: (1) Since $T_w = T_\infty$ for the prescribed conditions, there is no convection of sensible energy. However, as the water cools, there will be convection heat transfer from the air. (2) If L = 1m, $(dT_w/dt) = -461/(997 \times 4179 \times 1) = -1.11 \times 10^{-4}$ K/s.

PROBLEM 12.140

KNOWN: Environmental conditions for a metal roof with and without a water film.

FIND: Roof surface temperature (a) without the film, (b) with the film.

SCHEMATIC:

$T_{sky}=-10°C$, $T_\infty=30°C$, $h=20 W/m^2\cdot K$, $\phi_\infty=0.65$, $G_S=700 W/m^2$, G_{sky}, E, q''_{conv}, q''_{evap} (with film), T_s

Metal roof: $\alpha_S=0.50$, $\varepsilon=0.3$

Water film: $\alpha_S=0.8$, $\varepsilon=0.9$

ASSUMPTIONS: (1) Steady-state conditions, (2) Diffuse-gray surface behavior in the infrared (for the metal, $\alpha_{sky} = \varepsilon = 0.3$; for the water, $\alpha_{sky} = \varepsilon = 0.9$), (3) Adiabatic roof bottom, (4) Perfect gas behavior for vapor.

PROPERTIES: *Table A-4*, Air ($T \approx 300$ K): $\rho = 1.16$ kg/m^3, $c_p = 1007$ J/kg·K, $\alpha = 22.5 \times 10^{-6}$ m^2/s; *Table A-6*, Water vapor ($T \approx 303$ K): $v_g = 32.4$ m^3/kg or $\rho_{A,sat} = 0.031$ kg/m^3; *Table A-8*, Water vapor-air ($T = 298$ K): $D_{AB} = 0.26 \times 10^{-4}$ m^2/s.

ANALYSIS: (a) From an energy balance on the metal roof

$$\alpha_S G_S + \alpha_{sky} G_{sky} = E + q''_{conv}$$

$$0.5\left(700 \text{ W/m}^2\right) + 0.3\times5.67\times10^{-8} \text{ W/m}^2\cdot\text{K}^4 (263 \text{ K})^4$$

$$= 0.3\times5.67\times10^{-8} \text{ W/m}^2\cdot\text{K}^4\left(T_s^4\right) + 20 \text{ W/m}^2\cdot\text{K}\left(T_s - 303 \text{ K}\right)$$

$$431 \text{ W/m}^2 = 1.70\times10^{-8} T_s^4 + 20\left(T_s - 303\right).$$ <

From a trial-and-error solution, $T_s = 316.1$ K = 43.1°C.

(b) From an energy balance on the water film,

$$\alpha_S G_S + \alpha_{sky} G_{sky} = E + q''_{conv} + q''_{evap}$$

$$0.8\left(700 \text{ W/m}^2\right) + 0.9\times5.67\times10^{-8} \text{ W/m}^2\cdot\text{K}^4 (263 \text{ K})^4 = 0.9\times5.67\times10^{-8} \text{ W/m}^2\cdot\text{K}^4\left(T_s^4\right)$$

$$+20 \text{ W/m}^2\cdot\text{K}\left(T_s - 303\right) + h_m\left(\rho_{A,sat}\left(T_s\right) - 0.65\times0.031 \text{kg/m}^3\right)h_{fg}.$$

From Eq. 6.92, assuming n = 0.33,

$$h_m = \frac{h}{\rho c_p Le^{0.67}} =$$

$$\frac{h}{\rho c_p\left(\alpha/D_{AB}\right)^{0.67}} = \frac{20 \text{ W/m}^2\cdot\text{K}}{1.16 \text{ kg/m}^3\times1007 \text{ J/kg}\cdot\text{K}\left(0.225\times10^{-4}/0.260\times10^{-4}\right)^{0.67}} = 0.019 \text{m/s}.$$

$$804 \text{ W/m}^2 = 5.10\times10^{-8} T_s^4 + 20\left(T_s - 303\right) + 0.019\left[\rho_{A,sat}\left(T_s\right) - 0.020\right]h_{fg}.$$

From a trial-and-error solution, obtaining $\rho_{A,sat}$ (T_s) and h_{fg} from Table A-6 for each assumed value of T_s, it follows that

$$T_s = 302.2 \text{ K} = 29.2°\text{C}.$$ <

COMMENTS: (1) The film is an effective coolant, reducing T_s by 13.9°C. (2) With the film $E \approx 425$ W/m^2, $q''_{conv} \approx -16$ W/m^2 and $q''_{evap} \approx 428$ W/m^2.

PROBLEM 12.141

KNOWN: Solar, sky and ground irradiation of a wet towel. Towel dimensions, emissivity and solar absorptivity. Temperature, relative humidity and convection heat transfer coefficient associated with air flow over the towel.

FIND: Temperature of towel and evaporation rate.

SCHEMATIC:

$G_S = 900\ W/m^2$; Towel: $\alpha_S = 0.65$, $\varepsilon = 0.96$, $L = 1.50\ m$, $W = 0.75\ m$

$G_{sky} = 200\ W/m^2$ (both sides); q''_{evap}; q''_{conv}; $G_g = 250\ W/m^2$ (both sides)

Air: $T_\infty = 300\ K$, $h = 20\ W/m^2\text{-}K$, $\phi_\infty = 0$

ASSUMPTIONS: (1) Steady-state, (2) Diffuse-gray surface behavior of towel in the infrared ($\alpha_{sky} = \alpha_g = \varepsilon = 0.96$), (3) Perfect gas behavior for vapor.

PROPERTIES: *Table A-4*, Air (T ≈ 300 K): $\rho = 1.16\ kg/m^3$, $c_p = 1007\ J/kg{\cdot}K$, $\alpha = 0.225 \times 10^{-4}\ m^2/s$; *Table A-6*, Water vapor ($T_\infty = 300\ K$): $\rho_{A,sat} = 0.0256\ kg/m^3$; *Table A-8*, Water vapor/air (T = 298 K): $D_{AB} = 0.26 \times 10^{-4}\ m^2/s$.

ANALYSIS: From an energy balance on the towel, it follows that

$$\alpha_S G_S + 2\alpha_{sky} G_{sky} + 2\alpha_g G_g = 2E + 2q''_{evap} + 2q''_{conv}$$

$$0.65 \times 900 W/m^2 + 2 \times 0.96 \times 200\ W/m^2 \times 2 \times 0.96 \times 250\ W/m^2$$

$$= 2 \times 0.96\ \sigma T_s^4 + 2n''_A\, h_{fg} + 2h\left(T_s - T_\infty\right) \quad (1)$$

where $n''_A = h_m\left[\rho_{A,sat}\left(T_s\right) - \phi_\infty\, \rho_{A,sat}\left(T_\infty\right)\right]$

From the heat and mass transfer analogy, Eq. 6.67, with an assumed exponent of n = 1/3,

$$h_m = \frac{h}{\rho c_p \left(\alpha / D_{AB}\right)^{2/3}} = \frac{20\ W/m^2 \cdot K}{1.16\,kg/m^3\left(1007\ J/kg \cdot K\right)\left(\dfrac{0.225}{0.260}\right)^{2/3}} = 0.0189\ m/s$$

From a trial-and-error solution, we find that for $T_s = 298\ K$, $\rho_{A,sat} = 0.0226\ kg/m^3$, $h_{fg} = 2.442 \times 10^6\ J/kg$ and $n''_A = 1.380 \times 10^{-4}\ kg/s{\cdot}m^2$. Substituting into Eq. (1),

$$\left(585 + 384 + 480\right) W/m^2 = 2 \times 0.96 \times 5.67 \times 10^{-8} W/m^2 \cdot K^4 \left(298\ K\right)^4$$

$$+2 \times 1.380 \times 10^{-4} kg/s \cdot m^2 \times 2.442 \times 10^6\ J/kg$$

$$+2 \times 20\ W/m^2 \cdot K\left(-2\ K\right)$$

$$1449\,W/m^2 = \left(859 + 674 - 80\right) W/m^2 = 1453\,W/m^2$$

The equality is satisfied to a good approximation, in which case

$$T_s \approx 298\ K = 25°C \quad <$$

and $$n_A = 2A_s\, n''_A = 2\left(1.50 \times 0.75\right) m^2 \left(1.38 \times 10^{-4}\ kg/s \cdot m^2\right) = 3.11 \times 10^{-4}\ kg/s \quad <$$

COMMENTS: Note that the temperature of the air exceeds that of the towel, in which case convection heat transfer is to the towel. Reduction of the towel's temperature below that of the air is due to the evaporative cooling effect.

PROBLEM 12.142

KNOWN: Wet paper towel experiencing forced convection heat and mass transfer and irradiation from radiant lamps. Prescribed convection parameters including wet and dry bulb temperature of the air stream, T_{wb} and T_∞, average heat and mass transfer coefficients, $\overline{h}$ and $\overline{h}_m$. Towel temperature T_s.

FIND: (a) Vapor densities, $\rho_{A,s}$ and $\rho_{A,\infty}$; the evaporation rate n_A (kg/s); and the net rate of radiation transfer to the towel q_{rad} (W); and (b) Emissive power E, the irradiation G, and the radiosity J, using the results from part (a).

SCHEMATIC:

Radiant lamp irradiation, G

Air

$T_\infty = T_{wb} = 290$ K
$\bar{h} = 28.7$ W/m^2·K
$\bar{h}_m = 0.027$ m/s

$T_s = 310$ K, $\rho_{A,s}$
$\varepsilon = 0.96$

Water-soaked towel, 92.5 x 92.5 mm

ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible heat loss from the bottom side of the towel, (3) Uniform irradiation on the towel, and (4) Water surface is diffuse, gray.

PROPERTIES: *Table A.6*, Water (T_s = 310 K): h_{fg} = 2414 kJ/kg.

ANALYSIS: (a) Since $T_{wb} = T_\infty$, the free stream contains water vapor at its saturation condition. The water vapor at the surface is saturated since it is in equilibrium with the liquid in the towel. From Table A.6,

T (K)	v_g (m^3/kg)	ρ_g (kg/m^3)
$T_\infty = 290$	69.7	$\rho_{A,\infty} = 1.435 \times 10^{-2}$
$T_s = 310$	22.93	$\rho_{A,s} = 4.361 \times 10^{-2}$

Using the mass transfer convection rate equation, the water evaporation rate from the towel is

$$n_A = \overline{h}_m A_s \left(\rho_{A,s} - \rho_{A,\infty}\right) = 0.027\,\text{m/s}\,(0.0925\,\text{m})^2\,(4.361 - 1.435)\times 10^{-2}\,\text{kg/m}^3 = 6.76\times 10^{-6}\,\text{kg/s} \quad <$$

To determine the net radiation heat rate q''_{rad}, perform an energy balance on the water film,

q_{cv} q_{rad} q_{evap} $T_s, \rho_{A,s}$

$$\dot{E}_{in} - \dot{E}_{out} = 0 \qquad q_{rad} - q_{cv} - q_{evap} = 0$$

$$q_{rad} = q_{cv} + q_{evap} = \overline{h}_s A_s \left(T_s - T_\infty\right) + n_A h_{fg}$$

and substituting numerical values find

$$q_{rad} = 28.7\,\text{W/m}^2\cdot\text{K}\,(0.0925\,\text{m})^2\,(310 - 290)\,\text{K} + 6.76\times 10^{-6}\,\text{kg/s}\times 2414\times 10^3\,\text{J/kg}$$

$$q_{rad} = (4.91 + 16.32)\,\text{W} = 21.2\,\text{W} \quad <$$

(b) The radiation parameters for the towel surface are now evaluated. The emissive power is

$$E = \varepsilon E_b\left(T_s\right) = \varepsilon\sigma T_s^4 = 0.96\times 5.67\times 10^{-8}\,\text{W/m}^2\cdot\text{K}^4\,(310\,\text{K})^4 = 502.7\,\text{W/m}^2 \quad <$$

To determine the irradiation G, recognize that the net radiation heat rate can be expressed as,

$$q_{rad} = (\alpha G - E)A_s \qquad 21.2\,\text{W} = (0.96G - 502.7)\,\text{W/m}^2 \times (0.0925\,\text{m})^2 \qquad G = 3105\ \text{W/m}^2 \quad <$$

where $\alpha = \varepsilon$ since the water surface is diffuse, gray. From the definition of the radiosity,

$$J = E + \rho G = \left[502.7 + (1 - 0.96)\times 3105\right]\text{W/m}^2 = 626.9\,\text{W/m}^2 \quad <$$

where $\rho = 1 - \alpha = 1 - \varepsilon$.

COMMENTS: An alternate method to evaluate J is to recognize that $q''_{rad} = G - J$.

PROBLEM 13.1

KNOWN: Various geometric shapes involving two areas A_1 and A_2.

FIND: Shape factors, F_{12} and F_{21}, for each configuration.

ASSUMPTIONS: Surfaces are diffuse.

ANALYSIS: The analysis is not to make use of tables or charts. The approach involves use of the reciprocity relation, Eq. 13.3, and summation rule, Eq. 13.4. Recognize that reciprocity applies to two surfaces; summation applies to an enclosure. Certain shape factors will be identified by inspection. Note L is the length normal to page.

(a) Long duct (L):

By inspection, $F_{12} = 1.0$ <

By reciprocity, $F_{21} = \frac{A_1}{A_2} F_{12} = \frac{2\,RL}{(3/4)\cdot 2\pi RL} \times 1.0 = \frac{4}{3\pi} = 0.424$ <

(b) Small sphere, A_1, under concentric hemisphere, A_2, where $A_2 = 2A$

Summation rule $F_{11} + F_{12} + F_{13} = 1$

But $F_{12} = F_{13}$ by symmetry, hence $F_{12} = 0.50$ <

By reciprocity, $F_{21} = \frac{A_1}{A_2} F_{12} = \frac{A_1}{2A_1} \times 0.5 = 0.25.$ <

(c) Long duct (L):

<

By reciprocity, $F_{21} = \frac{A_1}{A_2} F_{12} = \frac{2RL}{\pi RL} \times 1.0 = \frac{2}{\pi} = 0.637$ <

Summation rule, $F_{22} = 1 - F_{21} = 1 - 0.64 = 0.363.$ <

By inspection,

$F_{12} = 1.0$

(d) Long inclined plates (L):

Summation rule, $F_{11} + F_{12} + F_{13} = 1$

But $F_{12} = F_{13}$ by symmetry, hence $F_{12} = 0.50$ <

By reciprocity, $F_{21} = \frac{A_1}{A_2} F_{12} = \frac{20L}{10(2)^{1/2} L} \times 0.5 = 0.707.$ <

(e) Sphere lying on infinite plane

Summation rule, $F_{11} + F_{12} + F_{13} = 1$

But $F_{12} = F_{13}$ by symmetry, hence $F_{12} = 0.5$ <

By reciprocity, $F_{21} = \frac{A_1}{A_2} F_{12} \to 0$ since $A_2 \to \infty$. <

Continued …..

PROBLEM 13.1 (Cont.)

(f) Hemisphere over a disc of diameter D/2; find also F_{22} and F_{23}.

By inspection, $F_{12} = 1.0$ <

Summation rule for surface A_3 is written as

$$F_{31} + F_{32} + F_{33} = 1. \quad \text{Hence, } F_{32} = 1.0.$$

By reciprocity, $$F_{23} = \frac{A_3}{A_2} F_{32}$$

$$F_{23} = \left\{ \left[\frac{\pi D^2}{4} - \frac{\pi (D/2)^2}{4} \right] / \frac{\pi D^2}{2} \right\} 1.0 = 0.375.$$

By reciprocity, $$F_{21} = \frac{A_1}{A_2} F_{12} = \left\{ \frac{\pi}{4} \left[\frac{D}{2} \right]^2 / \frac{\pi D^2}{2} \right\} \times 1.0 = 0.125.$$ <

Summation rule for A_2, $$F_{21} + F_{22} + F_{23} = 1 \text{ or}$$

$$F_{22} = 1 - F_{21} - F_{23} = 1 - 0.125 - 0.375 = 0.5.$$ <

Note that by inspection you can deduce $F_{22} = 0.5$

(g) Long open channel (L):

Summation rule for A_1

$$F_{11} + F_{12} + F_{13} = 0$$

but $F_{12} = F_{13}$ by symmetry, hence $F_{12} = 0.50$. <

By reciprocity, $$F_{21} = \frac{A_1}{A_2} F_{12} = \frac{2 \times L}{(2\pi 1)/4 \times L} = \frac{4}{\pi} \times 0.50 = 0.637.$$

COMMENTS: (1) Note that the summation rule is applied to an enclosure. To complete the enclosure, it was necessary in several cases to define a third surface which was shown by dashed lines.

(2) Recognize that the solutions follow a systematic procedure; in many instances it is possible to deduce a shape factor by inspection.

PROBLEM 13.2

KNOWN: Geometry of semi-circular, rectangular and V grooves.

FIND: (a) View factors of grooves with respect to surroundings, (b) View factor for sides of V groove, (c) View factor for sides of rectangular groove.

SCHEMATIC:

$A_1 = A_2 = W/2 \div \sin\theta$

ASSUMPTIONS: (1) Diffuse surfaces, (2) Negligible end effects, “long grooves”.

ANALYSIS: (a) Consider a unit length of each groove and represent the surroundings by a hypothetical surface (dashed line).

Semi-Circular Groove:

$$F_{21} = 1; \qquad F_{12} = \frac{A_2}{A_1} F_{21} = \frac{W}{(\pi W/2)} \times 1$$

$$F_{12} = 2/\pi. \qquad <$$

Rectangular Groove:

$$F_{4(1,2,3)} = 1; \qquad F_{(1,2,3)4} = \frac{A_4}{A_1 + A_2 + A_3} F_{4(1,2,3)} = \frac{W}{H + W + H} \times 1$$

$$F_{(1,2,3)4} = W/(W + 2H). \qquad <$$

V Groove:

$$F_{3(1,2)} = 1; \qquad F_{(1,2)3} = \frac{A_3}{A_1 + A_2} F_{3(1,2)} = \frac{W}{\dfrac{W/2}{\sin\theta} + \dfrac{W/2}{\sin\theta}}$$

$$F_{(1,2)3} = \sin\theta.$$

(b) From Eqs. 13.3 and 13.4, $\quad F_{12} = 1 - F_{13} = 1 - \dfrac{A_3}{A_1} F_{31}.$

From Symmetry, $\quad F_{31} = 1/2.$

Hence, $F_{12} = 1 - \dfrac{W}{(W/2)/\sin\theta} \times \dfrac{1}{2}$ $\quad$ or $\quad$ $F_{12} = 1 - \sin\theta. \qquad <$

(c) From Fig. 13.4, with X/L = H/W =2 and Y/L $\rightarrow \infty$,

$$F_{12} \approx 0.62. \qquad <$$

COMMENTS: (1) Note that for the V groove, $F_{13} = F_{23} = F_{(1,2)3} = \sin\theta$, (2) In part (c), Fig. 13.4 could also be used with Y/L = 2 and X/L = ∞. However, obtaining the limit of F_{ij} as X/L $\rightarrow \infty$ from the figure is somewhat uncertain.

PROBLEM 13.3

KNOWN: Two arrangements (a) circular disk and coaxial, ring shaped disk, and (b) circular disk and coaxial, right-circular cone.

FIND: Derive expressions for the view factor F_{12} for the arrangements (a) and (b) in terms of the areas A_1 and A_2, and any appropriate hypothetical surface area, as well as the view factor for coaxial parallel disks (Table 13.2, Figure 13.5). For the disk-cone arrangement, sketch the variation of F_{12} with θ for $0 \le \theta \le \pi/2$, and explain the key features.

SCHEMATIC:

A_2, A_1, A_3, (a)

θ, A_1, A_2, A_3, (b)

ASSUMPTIONS: Diffuse surfaces with uniform radiosities.

ANALYSIS: (a) Define the hypothetical surface A_3, a co-planar disk inside the ring of A_1. Using the additive view factor relation, Eq. 13.5,

$$A_{(1,3)}\,F_{(1,3)} = A_1\,F_{12} + A_3\,F_{32}$$

$$F_{12} = \frac{1}{A_1}\left[A_{(1,3)}\,F_{(1,3)} - A_3\,F_{32}\right] \qquad <$$

where the parenthesis denote a composite surface. All the F_{ij} on the right-hand side can be evaluated using Fig. 13.5.

(b) Define the hypothetical surface A_3, the disk at the bottom of the cone. The radiant power leaving A_2 that is intercepted by A_1 can be expressed as

$$F_{21} = F_{23} \qquad (1)$$

That is, the same power also intercepts the disk at the bottom of the cone, A_3. From reciprocity,

$$A_1\,F_{12} = A_2\,F_{21} \qquad (2)$$

and using Eq. (1),

$$F_{12} = \frac{A_2}{A_1}\,F_{23} \qquad <$$

The variation of F_{12} as a function of θ is shown below for the disk-cone arrangement. In the limit when $\theta \to \pi/2$, the cone approaches a disk of area A_3. That is,

$$F_{12}\,(\theta \to \pi/2) = F_{13}$$

When $\theta \to 0$, the cone area A_2 diminishes so that

$$F_{12}\,(\theta \to 0) = 0$$

F_{13}, F_{12}, 0, 0, π/2, θ

PROBLEM 13.4

KNOWN: Right circular cone and right-circular cylinder of same diameter D and length L positioned coaxially a distance L_o from the circular disk A_1; hypothetical area corresponding to the openings identified as A_3.

FIND: (a) Show that $F_{21} = (A_1/A_2)\,F_{13}$ and $F_{22} = 1 - (A_3/A_2)$, where F_{13} is the view factor between two, coaxial parallel disks (Table 13.2), for both arrangements, (b) Calculate F_{21} and F_{22} for $L = L_o = 50$ mm and $D_1 = D_3 = 50$ mm; compare magnitudes and explain similarities and differences, and (c) Magnitudes of F_{21} and F_{22} as L increases and all other parameters remain the same; sketch and explain key features of their variation with L.

SCHEMATIC:

(*a*) Right-circular cone (*b*) Right-circular cylinder

ASSUMPTIONS: (1) Diffuse surfaces with uniform radiosities, and (2) Inner base and lateral surfaces of the cylinder treated as a single surface, A_2.

ANALYSIS: (a) For both configurations,

$$F_{13} = F_{12} \qquad (1)$$

since the radiant power leaving A_1 that is intercepted by A_3 is likewise intercepted by A_2. Applying reciprocity between A_1 and A_2,

$$A_1\,F_{12} = A_2\,F_{21} \qquad (2)$$

Substituting from Eq. (1), into Eq. (2), solving for F_{21}, find

$$F_{21} = (A_1/A_2)F_{12} = (A_1/A_2)F_{13}$$ <

Treating the cone and cylinder as two-surface enclosures, the summation rule for A_2 is

$$F_{22} + F_{23} = 1 \qquad (3)$$

Apply reciprocity between A_2 and A_3, solve Eq. (3) to find

$$F_{22} = 1 - F_{23} = 1 - (A_3/A_2)F_{32}$$

and since $F_{32} = 1$, find

$$F_{22} = 1 - A_3/A_2$$ <

Continued …..

PROBLEM 13.4 (Cont.)

(b) For the specified values of L, L_o, D_1 and D_2, the view factors are calculated and tabulated below. Relations for the areas are:

Disk-cone: $A_1 = \pi D_1^2 / 4$ $\quad A_2 = \pi D_3 / 2\left(L^2 + (D_3/2)^2\right)^{1/2}$ $\quad A_3 = \pi D_3^2 / 4$

Disk-cylinder: $A_1 = \pi D_1^2 / 4$ $\quad A_2 = \pi D_3^2 / 4 + \pi D_3 L$ $\quad A_3 = \pi D_3^2 / 4$

The view factor F_{13} is evaluated from Table 13.2, coaxial parallel disks (Fig. 13.5); find F_{13} = 0.1716.

	F_{21}	F_{22}
Disk-cone	0.0767	0.553
Disk-cylinder	0.0343	0.800

It follows that F_{21} is greater for the disk-cone (a) than for the cylinder-cone (b). That is, for (a), surface A_2 sees more of A_1 and less of itself than for (b). Notice that F_{22} is greater for (b) than (a); this is a consequence of $A_{2,b} > A_{2,a}$.

(c) Using the foregoing equations in the IHT workspace, the variation of the view factors F_{21} and F_{22} with L were calculated and are graphed below.

Note that for both configurations, when L = 0, find that $F_{21} = F_{13} = 0.1716$, the value obtained for coaxial parallel disks. As L increases, find that $F_{22} \rightarrow 1$; that is, the interior of both the cone and cylinder see mostly each other. Notice that the changes in both F_{21} and F_{22} with increasing L are greater for the disk-cylinder; F_{21} decreases while F_{22} increases.

COMMENTS: From the results of part (b), why isn't the sum of F_{21} and F_{22} equal to unity?

PROBLEM 13.5

KNOWN: Two parallel, coaxial, ring-shaped disks.

FIND: Show that the view factor F_{12} can be expressed as

$$F_{12} = \frac{1}{A_1}\left\{A_{(1,3)}\ F_{(1,3)(2,4)} - A_3\ F_{3(2,4)} - A_4\left(F_{4(1,3)} - F_{43}\right)\right\}$$

where all the F_{ig} on the right-hand side of the equation can be evaluated from Figure 13.5 (see Table 13.2) for coaxial parallel disks.

SCHEMATIC:

ASSUMPTIONS: Diffuse surfaces with uniform radiosities.

ANALYSIS: Using the additive rule, Eq. 13.5, where the parenthesis denote a composite surface,

$$F_{1(2,4)} = F_{12} + F_{14}$$

$$F_{12} = F_{1(2,4)} - F_{14} \qquad (1)$$

Relation for $F_{1(2,4)}$: Using the additive rule

$$A_{(1,3)}\ F_{(1,3)(2,4)} = A_1\ F_{1(2,4)} + A_3\ F_{3(2,4)} \qquad (2)$$

where the check mark denotes a F_{ij} that can be evaluated using Fig. 13.5 for coaxial parallel disks.

Relation for F_{14}: Apply reciprocity

$$A_1\ F_{14} = A_4\ F_{41} \qquad (3)$$

and using the additive rule involving F_{41},

$$A_1\ F_{14} = A_4\left[F_{4(1,3)} - F_{43}\right] \qquad (4)$$

Relation for F_{12}: Substituting Eqs. (2) and (4) into Eq. (1),

$$F_{12} = \frac{1}{A_1}\left\{A_{(1,3)}\ F_{(1,3)(2,4)} - A_3\ F_{3(2,4)} - A_4\left(F_{4(1,3)} - F_{43}\right)\right\}$$
<

COMMENTS: (1) The F_{ij} on the right-hand side can be evaluated using Fig. 13.5.

(2) To check the validity of the result, substitute numerical values and test the behavior at special limits. For example, as A_3, $A_4 \to 0$, the expression reduces to the identity $F_{12} \equiv F_{12}$.

PROBLEM 13.6

KNOWN: Long concentric cylinders with diameters D_1 and D_2 and surface areas A_1 and A_2.

FIND: (a) The view factor F_{12} and (b) Expressions for the view factors F_{22} and F_{21} in terms of the cylinder diameters.

SCHEMATIC:

ASSUMPTIONS: (1) Diffuse surfaces with uniform radiosities and (2) Cylinders are infinitely long such that A_1 and A_2 form an enclosure.

ANALYSIS: (a) *View factor* F_{12}. Since the infinitely long cylinders form an enclosure with surfaces A_1 and A_2, from the summation rule on A_1, Eq. 13.4,

$$F_{11} + F_{12} = 1 \qquad (1)$$

and since A_1 doesn't see itself, $F_{11} = 0$, giving

$$F_{12} = 1 \qquad (2)$$ <

That is, the inner surface views only the outer surface.

(b) *View factors* F_{22} *and* F_{21}. Applying reciprocity between A_1 and A_2, Eq. 13.3, and substituting from Eq. (2),

$$A_1 F_{12} = A_2 F_{21} \qquad (3)$$

$$F_{21} = \frac{A_1}{A_2} F_{12} = \frac{\pi D_1 L}{\pi D_2 L} \times 1 = \frac{D_1}{D_2} \qquad (4)$$ <

From the summation rule on A_2, and substituting from Eq. (4),

$$F_{21} + F_{22} = 1$$

$$F_{22} = 1 - F_{21} = 1 - \frac{D_1}{D_2}$$ <

PROBLEM 13.7

KNOWN: Right-circular cylinder of diameter D, length L and the areas A_1, A_2, and A_3 representing the base, inner lateral and top surfaces, respectively.

FIND: (a) Show that the view factor between the base of the cylinder and the inner lateral surface has the form

$$F_{12} = 2\,H\left[\left(1+H^2\right)^{1/2} - H\right]$$

where H = L/D, and (b) Show that the view factor for the inner lateral surface to itself has the form

$$F_{22} = 1 + H - \left(1+H^2\right)^{1/2}$$

SCHEMATIC:

D
A_3
A_2
A_1
L

ASSUMPTIONS: Diffuse surfaces with uniform radiosities.

ANALYSIS: (a) *Relation for F_{12}, base-to-inner lateral surface.* Apply the summation rule to A_1, noting that $F_{11} = 0$

$$F_{11} + F_{12} + F_{13} = 1$$

$$F_{12} = 1 - F_{13} \tag{1}$$

From Table 13.2, Fig. 13.5, with i = 1, *j* = 3,

$$F_{13} = \frac{1}{2}\left\{S - \left[S^2 - 4\left(D_3 / D_1\right)^2\right]^{1/2}\right\} \tag{2}$$

$$S = 1 + \frac{1+R_3^2}{R_1^2} = \frac{1}{R^2} + 2 = 4\,H^2 + 2 \tag{3}$$

where $R_1 = R_3 = R = D/2L$ and H = L/D. Combining Eqs. (2) and (3) with Eq. (1), find after some manipulation

Continued …..

PROBLEM 13.7 (Cont.)

$$F_{12} = 1 - \frac{1}{2}\left\{4\,H^2 + 2 - \left[\left(4\,H^2 + 2\right)^2 - 4\right]^{1/2}\right\}$$

$$F_{12} = 2\,H\left[\left(1 + H^2\right)^{1/2} - H\right] \quad (4)$$

(b) *Relation for F_{22}, inner lateral surface*. Apply summation rule on A_2, recognizing that $F_{23} = F_{21}$,

$$F_{21} + F_{22} + F_{23} = 1 \qquad F_{22} = 1 - 2\,F_{21} \quad (5)$$

Apply reciprocity between A_1 and A_2,

$$F_{21} = \left(A_1 / A_2\right) F_{12} \quad (6)$$

and substituting into Eq. (5), and using area expressions

$$F_{22} = 1 - 2\,\frac{A_1}{A_2}\,F_{12} = 1 - 2\,\frac{D}{4\,L}F_{12} = 1 - \frac{1}{2\,H}F_{12} \quad (7)$$

where $A_1 = \pi D^2/4$ and $A_2 = \pi DL$.

Substituting from Eq. (4) for F_{12}, find

$$F_{22} = 1 - \frac{1}{2\,H}2\,H\left[\left(1 + H^2\right)^{1/2} - H\right] = 1 + H - \left(1 + H^2\right)^{1/2}$$

<

PROBLEM 13.8

KNOWN: Arrangement of plane parallel rectangles.

FIND: Show that the view factor between A_1 and A_2 can be expressed as

$$F_{12} = \frac{1}{2\,A_1}\left[A_{(1,4)}\,F_{(1,4)(2,3)} - A_1\,F_{13} - A_4\,F_{42}\right]$$

where all F_{ij} on the right-hand side of the equation can be evaluated from Fig. 13.4 (see Table 13.2) for aligned parallel rectangles.

SCHEMATIC:

ASSUMPTIONS: Diffuse surfaces with uniform radiosity.

ANALYSIS: Using the additive rule where the parenthesis denote a composite surface,

$$A_{(1,4)}\,F^*_{(1,4)(2,3)} = A_1\,F^*_{13} + A_1\,F_{12} + A_4\,F_{43} + A_4\,F^*_{42} \qquad (1)$$

where the asterisk (*) denotes that the F_{ij} can be evaluated using the relation of Figure 13.4. Now, find suitable relation for F_{43}. By symmetry,

$$F_{43} = F_{21} \qquad (2)$$

and from reciprocity between A_1 and A_2,

$$F_{21} = \frac{A_1}{A_2}F_{12} \qquad (3)$$

Multiply Eq. (2) by A_4 and substitute Eq. (3), with $A_4 = A_2$,

$$A_4\,F_{43} = A_4\,F_{21} = A_4\,\frac{A_1}{A_2}F_{12} = A_1\,F_{12} \qquad (4)$$

Substituting for $A_4\,F_{43}$ from Eq. (4) into Eq. (1), and rearranging,

$$F_{12} = \frac{1}{2\,A_1}\left[A_{(1,4)}\,F^*_{(1,4)(2,3)} - A_1\,F^*_{13} - A_4\,F^*_{42}\right] \qquad <$$

PROBLEM 13.9

KNOWN: Two perpendicular rectangles not having a common edge.

FIND: (a) Shape factor, F_{12}, and (b) Compute and plot F_{12} as a function of Z_b for $0.05 \le Z_b \le 0.4$ m; compare results with the view factor obtained from the two-dimensional relation for perpendicular plates with a common edge, Table 13.1.

SCHEMATIC:

$Z_a = 0.4$ m, $Z_b = 0.2$ m, $Y = 0.3$ m, $X = 0.5$ m, A_1, A_2, A_3

ASSUMPTIONS: (1) All surfaces are diffuse, (2) Plane formed by $A_1 + A_3$ is perpendicular to plane of A_2.

ANALYSIS: (a) Introducing the hypothetical surface A_3, we can write

$$F_{2(3,1)} = F_{23} + F_{21}. \qquad (1)$$

Using Fig. 13.6, applicable to perpendicular rectangles with a common edge, find

$$F_{23} = 0.19: \quad \text{with } Y = 0.3, \quad X = 0.5, \quad Z = Z_a - Z_b = 0.2, \text{ and } \frac{Y}{X} = \frac{0.3}{0.5} = 0.6, \quad \frac{Z}{X} = \frac{0.2}{0.5} = 0.4$$

$$F_{2(3,1)} = 0.25: \quad \text{with } Y = 0.3, \; X = 0.5, \; Z_a = 0.4, \text{ and } \frac{Y}{X} = \frac{0.3}{0.5} = 0.6, \quad \frac{Z}{X} = \frac{0.4}{0.5} = 0.8$$

Hence from Eq. (1)

$$F_{21} = F_{2(3.1)} - F_{23} = 0.25 - 0.19 = 0.06$$

By reciprocity,

$$F_{12} = \frac{A_2}{A_1} F_{21} = \frac{0.5 \times 0.3\,\text{m}^2}{0.5 \times 0.2\,\text{m}^2} \times 0.06 = 0.09 \qquad (2) <$$

(b) Using the *IHT Tool – View Factors* for *Perpendicular Rectangles with a Common Edge* and Eqs. (1,2) above, F_{12} was computed as a function of Z_b. Also shown on the plot below is the view factor $F_{(3,1)2}$ for the limiting case $Z_b \to Z_a$.

View factors, Fij

Zb (m)

F12

limit F(3,1)2 when Zb -> Za

PROBLEM 13.10

KNOWN: Arrangement of perpendicular surfaces without a common edge.

FIND: (a) A relation for the view factor F_{14} and (b) The value of F_{14} for prescribed dimensions.

SCHEMATIC:

ASSUMPTIONS: (1) Diffuse surfaces.

ANALYSIS: (a) To determine F_{14}, it is convenient to define the hypothetical surfaces A_2 and A_3. From Eq. 13.6,

$$(A_1 + A_2) F_{(1,2)(3,4)} = A_1\, F_{1(3,4)} + A_2\, F_{2(3,4)}$$

where $F_{(1,2)(3,4)}$ and $F_{2(3,4)}$ may be obtained from Fig. 13.6. Substituting for $A_1\, F_{1(3,4)}$ from Eq. 13.5 and combining expressions, find

$$A_1\, F_{1(3,4)} = A_1\, F_{13} + A_1\, F_{14}$$

$$F_{14} = \frac{1}{A_1}\left[(A_1 + A_2) F_{(1,2)(3,4)} - A_1\, F_{13} - A_2\, F_{2(3,4)}\right].$$

Substituting for $A_1\, F_{13}$ from Eq. 13.6, which may be expressed as

$$(A_1 + A_2) F_{(1,2)3} = A_1\, F_{13} + A_2\, F_{23}.$$

The desired relation is then

$$F_{14} = \frac{1}{A_1}\left[(A_1 + A_2) F_{(1,2)(3,4)} + A_2 F_{23} - (A_1 + A_2) F_{(1,2)3} - A_2\, F_{2(3,4)}\right].$$ <

(b) For the prescribed dimensions and using Fig. 13.6, find these view factors:

Surfaces (1,2)(3,4) $\quad (Y/X) = \frac{L_1 + L_2}{W} = 1, \quad (Z/X) = \frac{L_3 + L_4}{W} = 1.45, \quad F_{(1,2)(3,4)} = 0.22$

Surfaces 23 $\quad (Y/X) = \frac{L_2}{W} = 0.5, \quad (Z/X) = \frac{L_3}{W} = 1, \quad F_{23} = 0.28$

Surfaces (1,2)3 $\quad (Y/X) = \frac{L_1 + L_2}{W} = 1, \quad (Z/X) = \frac{L_3}{W} = 1, \quad F_{(1,2)3} = 0.20$

Surfaces 2(3,4) $\quad (Y/X) = \frac{L_2}{W} = 0.5, \quad (Z/X) = \frac{L_3 + L_4}{W} = 1.5, \quad F_{2(3,4)} = 0.31$

Using the relation above, find

$$F_{14} = \frac{1}{(WL_1)}\left[(WL_1 + WL_2)0.22 + (WL_2)0.28 - (WL_1 + WL_2)0.20 - (WL_2)0.31\right]$$

$$F_{14} = \left[2(0.22) + 1(0.28) - 2(0.20) - 1(0.31)\right] = 0.01.$$ <

PROBLEM 13.11

KNOWN: Arrangements of rectangles.

FIND: The shape factors, F_{12}.

SCHEMATIC:

ASSUMPTIONS: (1) Diffuse surface behavior.

ANALYSIS: (a) Define the hypothetical surfaces shown in the sketch as A_3 and A_4. From the additive view factor rule, Eq. 13.6, we can write

$$A_{(1,3)} \overset{\surd}{F}_{(1,3)(2,4)} = A_1F_{12} + A_1 \overset{\surd}{F}_{14} + A_3 \overset{\surd}{F}_{32} + A_3F_{34} \quad (1)$$

Note carefully which factors can be evaluated from Fig. 13.6 for perpendicular rectangles with a common edge. (See √). It follows from symmetry that

$$A_1F_{12} = A_4F_{43}. \quad (2)$$

Using reciprocity,

$$A_4F_{43} = A_3F_{34}, \qquad \text{then} \qquad A_1F_{12} = A_3F_{34}. \quad (3)$$

Solving Eq. (1) for F_{12} and substituting Eq. (3) for A_3F_{34}, find that

$$F_{12} = \frac{1}{2A_1}\left[A_{(1,3)}F_{(1,3)(2,4)} - A_1F_{14} - A_3F_{32}\right]. \quad (4)$$

Evaluate the view factors from Fig. 13.6:

F_{ij}	Y/X	Z/X	F_{ij}
(1,3) (2,4)	$\frac{6}{9} = 0.67$	$\frac{6}{9} = 0.67$	0.23
14	$\frac{6}{6} = 1$	$\frac{6}{6} = 1$	0.20
32	$\frac{6}{3} = 2$	$\frac{6}{3} = 2$	0.14

Substituting numerical values into Eq. (4) yields

$$F_{12} = \frac{1}{2\times(6\times6)\text{m}^2}\left[(6\times9)\text{m}^2\times0.23 - (6\times6)\text{m}^2\times0.20 - (6\times3)\text{m}^2\times0.14\right]$$

$$F_{12} = 0.038. \qquad <$$

Continued …..

PROBLEM 13.11 (Cont.)

(b) Define the hypothetical surface A_3 and divide A_2 into two sections, A_{2A} and A_{2B}. From the additive view factor rule, Eq. 13.6, we can write

$$A_{1,3}\overset{\surd}{F}_{(1,3)2} = A_1F_{12} + A_3F_{3(2A)} + A_3\overset{\surd}{F}_{3(2B)}. \qquad (5)$$

Note that the view factors checked can be evaluated from Fig. 13.4 for aligned, parallel rectangles. To evaluate $F_{3(2A)}$, we first recognize a relationship involving $F_{(24)1}$ will eventually be required. Using the additive rule again,

$$A_{2A}F_{(2A)(1,3)} = A_{2A}\overset{\surd}{F}_{(2A)1} + A_{2A}F_{(2A)3}. \qquad (6)$$

Note that from symmetry considerations,

$$A_{2A}F_{(2A)(1,3)} = A_1F_{12} \qquad (7)$$

and using reciprocity, Eq. 13.3, note that

$$A_{2A}F_{2A3} = A_3F_{3(2A)}. \qquad (8)$$

Substituting for $A_3F_{3(2A)}$ from Eq. (8), Eq. (5) becomes

$$A_{(1,3)}\overset{\surd}{F}_{(1,3)2} = A_1F_{12} + A_{2A}F_{(2A)3} + A_3\overset{\surd}{F}_{3(2B)}.$$

Substituting for $A_{2A}\,F_{(2A)3}$ from Eq. (6) using also Eq. (7) for $A_{2A}\,F_{(2A)(1,3)}$ find that

$$A_{(1,3)}\overset{\surd}{F}_{(1,3)2} = A_1F_{12} + \left(A_1F_{12} - A_{2A}\overset{\surd}{F}_{(2A)1}\right) + A_3\overset{\surd}{F}_{3(2B)} \qquad (9)$$

and solving for F_{12}, noting that $A_1 = A_{2A}$ and $A_{(1,3)} = A_2$

$$F_{12} = \frac{1}{2A_1}\left[A_2\overset{\surd}{F}_{(1,3)2} + A_{2A}\overset{\surd}{F}_{(2A)1} - A_3\overset{\surd}{F}_{3(2B)}\right]. \qquad (10)$$

Evaluate the view factors from Fig. 13.4:

F_{ij}	X/L	Y/L	F_{ij}
(1,3) 2	$\frac{1}{1}=1$	$\frac{1.5}{1}=1.5$	0.25
(2A)1	$\frac{1}{1}=1$	$\frac{0.5}{1}=0.5$	0.11
3(2B)	$\frac{1}{1}=1$	$\frac{1}{1}=1$	0.20

Substituting numerical values into Eq. (10) yields

$$F_{12} = \frac{1}{2(0.5\times1)\,m^2}\left[(1.5\times1.0)\,m^2\times0.25 + (0.5\times1)\,m^2\times0.11 - (1\times1)\,m^2\times0.20\right]$$

$$F_{12} = 0.23.$$ <

PROBLEM 13.12

KNOWN: Two geometrical arrangements: (a) parallel plates and (b) perpendicular plates with a common edge.

FIND: View factors using "crossed-strings" method; compare with appropriate graphs and analytical expressions.

SCHEMATIC:

(a) Parallel plates (b) Perpendicular plates with common edge

ASSUMPTIONS: Plates infinite extent in direction normal to page.

ANALYSIS: The "crossed-strings" method is applicable
to surfaces of infinite extent in one direction having an obstructed view of one another.

$$F_{12} = (1/2w_1)[(ac+bd)-(ad+bc)].$$

(a) *Parallel plates:* From the schematic, the edge and diagonal distances are

$$ac = bd = \left(w_1^2 + L^2\right)^{1/2} \qquad bc = ad = L.$$

With w_1 as the width of the plate, find

$$F_{12} = \frac{1}{2w_1}\left[2\left(w_1^2+L^2\right)^{1/2} - 2(L)\right] = \frac{1}{2\times 4\ \text{m}}\left[2\left(4^2+1^2\right)^{1/2}\ \text{m} - 2(1\ \text{m})\right] = 0.781.$$ <

Using Fig. 13.4 with X/L = 4/1 = 4 and Y/L = ∞, find $F_{12} \approx 0.80$. Also, using the first relation of Table 13.1,

$$F_{ij} = \left\{\left[\left(W_i + W_j\right)^2 + 4\right]^{1/2} - \left[\left(W_i - W_j\right)^2 + 4\right]^{1/2}\right\}/2\,W_i$$

where $w_i = w_j = w_1$ and W = w/L = 4/1 = 4, find

$$F_{12} = \left\{\left[(4+4)^2+4\right]^{1/2} - \left[(4-4)^2+4\right]^{1/2}\right\}/2\times 4 = 0.781.$$

(b) *Perpendicular plates* with a common edge: From the schematic, the edge and diagonal distances are

$$ac = w_1 \qquad bd = L \qquad ad = \left(w_1^2 + L^2\right) \qquad bc = 0.$$

With w_1 as the width of the horizontal plates, find

$$F_{12} = (1/2w_1)\left[2(w_1+L) - \left(\left(w_1^2+L^2\right)^{1/2} + 0\right)\right]$$

$$F_{12} = (1/2\times 4\ \text{m})\left[(4+1)\text{m} - \left(\left(4^2+1^2\right)^{1/2}\ \text{m} + 0\right)\right] = 0.110.$$ <

From the third relation of Table 13.1, with $w_i = w_1 = 4$ m and $w_j = L = 1$ m, find

$$F_{ij} = \left\{1 + \left(w_j/w_i\right) - \left[1 + \left(w_j/w_i\right)^2\right]^{1/2}\right\}/2$$

$$F_{12} = \left\{1 + (1/4) - \left[1 + (1/4)^2\right]^{1/2}\right\}/2 = 0.110.$$

PROBLEM 13.13

KNOWN: Parallel plates of infinite extent (1,2) having aligned opposite edges.

FIND: View factor F_{12} by using (a) appropriate view factor relations and results for opposing parallel plates and (b) Hottel's string method described in Problem 13.12

SCHEMATIC:

ASSUMPTIONS: (1) Parallel planes of infinite extent normal to page and (2) Diffuse surfaces with uniform radiosity.

ANALYSIS: From symmetry consideration ($F_{12} = F_{14}$) and Eq. 13.5, it follows that

$$F_{12} = (1/2)\left[F_{1(2,3,4)} - F_{13}\right]$$

where A_3 and A_4 have been defined for convenience in the analysis. Each of these view factors can be evaluated by the first relation of Table 13.1 for parallel plates with midlines connected perpendicularly.

F_{13}: $\quad W_1 = w_1/L = 2 \qquad W_2 = w_2/L = 2$

$$F_{13} = \frac{\left[(W_1+W_2)^2+4\right]^{1/2} - \left[(W_2-W_1)^2+4\right]^{1/2}}{2W_1} = \frac{\left[(2+2)^2+4\right]^{1/2} - \left[(2-2)^2+4\right]^{1/2}}{2\times 2} = 0.618$$

$F_{1(2,3,4)}$: $\quad W_1 = w_1/L = 2 \qquad W_{(2,3,4)} = 3w_2/L = 6$

$$F_{1(2,3,4)} = \frac{\left[(2+6)^2+4\right]^{1/2} - \left[(6-2)^2+4\right]^{1/2}}{2\times 2} = 0.944.$$

Hence, find $\quad F_{12} = (1/2)\left[0.944 - 0.618\right] = 0.163.$ <

(b) Using Hottel's string method,

$$F_{12} = (1/2w_1)\left[(ac+bd) - (ad+bc)\right]$$

$$ac = \left(1+4^2\right)^{1/2} = 4.123$$

$$bd = 1$$

$$ad = \left(1^2+2^2\right)^{1/2} = 2.236$$

$$bc = ad = 2.236$$

and substituting numerical values find

$$F_{12} = (1/2\times 2)\left[(4.123+1) - (2.236+2.236)\right] = 0.163.$$ <

COMMENTS: Remember that Hottel's string method is applicable only to surfaces that are of infinite extent in one direction and have unobstructed views of one another.

PROBLEM 13.14

KNOWN: Two small diffuse surfaces, A_1 and A_2, on the inside of a spherical enclosure of radius R.

FIND: Expression for the view factor F_{12} in terms of A_2 and R by two methods: (a) Beginning with the expression $F_{ij} = q_{ij}/A_i J_i$ and (b) Using the view factor integral, Eq. 13.1.

SCHEMATIC:

ASSUMPTIONS: (1) Surfaces A_1 and A_2 are diffuse and (2) A_1 and $A_2 \ll R^2$.

ANALYSIS: (a) The view factor is defined as the fraction of radiation leaving A_i which is intercepted by surface j and, from Section 13.1.1, can be expressed as

$$F_{ij} = \frac{q_{ij}}{A_i J_i} \tag{1}$$

From Eq. 12.5, the radiation leaving intercepted by A_1 and A_2 on the spherical surface is

$$q_{1\to 2} = \left(J_1/\pi\right) \cdot A_1 \cos\theta_1 \cdot \omega_{2-1} \tag{2}$$

where the solid angle A_2 subtends with respect to A_1 is

$$\omega_{2-1} = \frac{A_{2,n}}{r^2} = \frac{A_2 \cos\theta_2}{r^2} \tag{3}$$

From the schematic above,

$$\cos\theta_1 = \cos\theta_2 \qquad\qquad r = 2R\cos\theta_1 \tag{4,5}$$

Hence, the view factor is

$$F_{ij} = \frac{\left(J_1/\pi\right) A_1 \cos\theta_1 \cdot A_2 \cos\theta_2 / 4R^2 \cos\theta_1}{A_1 J_1} = \frac{A_2}{4\pi R^2}$$

<

(b) The view factor integral, Eq. 13.1, for the small areas A_1 and A_2 is

$$F_{12} = \frac{1}{A_1}\int_{A_1}\int_{A_2} \frac{\cos\theta_1 \cos\theta_2}{\pi r^2} dA_1 dA_2 = \frac{\cos\theta_1 \cos\theta_2 A_2}{\pi r^2}$$

and from Eqs. (4,5) above,

$$F_{12} = \frac{A_2}{\pi R^2}$$

<

COMMENTS: Recognize the importance of the second assumption. We require that A_1, A_2, $\ll R^2$ so that the areas can be considered as of differential extent, $A_1 = dA_1$, and $A_2 = dA_2$.

PROBLEM 13.15

KNOWN: Disk A_1, located coaxially, but tilted 30° of the normal, from the diffuse-gray, ring-shaped disk A_2. Surroundings at 400 K.

FIND: Irradiation on A_1, G_1, due to the radiation from A_2.

SCHEMATIC:

ASSUMPTIONS: (1) A_2 is diffuse-gray surface, (2) Uniform radiosity over A_2, (3) The surroundings are large with respect to A_1 and A_2.

ANALYSIS: The irradiation on A_1 is

$$G_1 = q_{21}/A_1 = \left(F_{21} \cdot J_2 A_2\right)/A_1 \qquad (1)$$

where J_2 is the radiosity from A_2 evaluated as

$$J_2 = \varepsilon_2 E_{b,2} + \rho_2 G_2 = \varepsilon_2 \sigma T_2^4 + \left(1-\varepsilon_2\right)\sigma T_{sur}^4$$

$$J_2 = 0.7 \times 5.67 \times 10^{-8} \text{W/m}^2 \cdot \text{K}^4 \left(600\text{ K}\right)^4 + \left(1-0.7\right) 5.67 \times 10^{-8} \text{W/m}^2 \cdot \text{K}^4 \left(400\text{ K}\right)^4$$

$$J_2 = 5144 + 436 = 5580\ \text{W/m}^2. \qquad (2)$$

Using the view factor relation of Eq. 13.8, evaluate view factors between A_1', the normal projection of A_1, and A_3 as

$$F_{1'3} = \frac{D_i^2}{D_i^2 + 4L^2} = \frac{\left(0.004\text{ m}\right)^2}{\left(0.004\text{ m}\right)^2 + 4\left(1\text{ m}\right)^2} = 4.00 \times 10^{-6}$$

and between A_1' and (A_2 + A_3) as

$$F_{1'(23)} = \frac{D_o^2}{D_o^2 + 4L^2} = \frac{\left(0.012\right)^2}{\left(0.012\right)^2 + 4\left(1\text{ m}\right)^2} = 3.60 \times 10^{-5}$$

giving $$F_{1'2} = F_{1'(23)} - F_{1'3} = 3.60 \times 10^{-5} - 4.00 \times 10^{-6} = 3.20 \times 10^{-5}.$$

From the reciprocity relation it follows that

$$F_{21'} = A_1' F_{1'2}/A_2 = \left(A_1 \cos\theta_1 / A_2\right) F_{1'2} = 3.20 \times 10^{-5} \cos\theta_1 \left(A_1/A_2\right). \qquad (3)$$

By inspection we note that all the radiation striking A_1' will also intercept A_1; that is

$$F_{21} = F_{21'}. \qquad (4)$$

Hence, substituting for Eqs. (3) and (4) for F_{21} into Eq. (1), find

$$G_1 = \left(3.20 \times 10^{-5} \cos\theta_1 \left(A_1/A_2\right) \times J_2 \times A_2\right)/A_1 = 3.20 \times 10^{-5} \cos\theta_1 \cdot J_2 \qquad (5)$$

$$G_1 = 3.20 \times 10^{-5} \cos\left(30°\right) \times 5580\ \text{W/m}^2 = 27.7\ \mu\text{W/m}^2.$$ <

COMMENTS: (1) Note from Eq. (5) that $G_1 \sim \cos\theta_1$ such that G_1 is a maximum when A_1 is normal to disk A_2.

PROBLEM 13.16

KNOWN: Heat flux gauge positioned normal to a blackbody furnace. Cover of furnace is at 350 K while surroundings are at 300 K.

FIND: (a) Irradiation on gage, G_g, considering only emission from the furnace aperture and (b) Irradiation considering radiation from the cover *and* aperture.

SCHEMATIC:

Furnace, $T_f = 1000K$
$T_{sur} = 300K$
Cover, $T_c = 350K$, $\varepsilon_c = 0.2$, $D = 100mm$
Aperture, $d_a = 5mm$
Gauge, $d_g = 4mm$
$R = 1m$

ASSUMPTIONS: (1) Furnace aperture approximates blackbody, (2) Shield is opaque, diffuse and gray with uniform temperature, (3) Shield has uniform radiosity, (4) $A_g << R^2$, so that $\omega_{g-f} = A_g/R^2$, (5) Surroundings are large, uniform at 300 K.

ANALYSIS: (a) The irradiation on the gauge due *only* to aperture emission is

$$G_g = q_{f-g}/A_g = \left(I_{e,f} \cdot A_f \cos\theta_f \cdot \omega_{g-f}\right)/A_g = \frac{\sigma T_f^4}{\pi} \cdot A_f \cdot \frac{A_g}{R^2}/A_g$$

$$G_g = \frac{\sigma T_f^4}{\pi R^2} A_f = \frac{5.67\times10^{-8} W/m^2 \cdot K^4 (1000\ K)^4}{\pi (1\ m)^2} \times (\pi/4)(0.005\ m)^2 = 354.4\ mW/m^2.$$ <

(b) The irradiation on the gauge due to radiation from the *aperture* (a) and *cover (c)* is

$$G_g = G_{g,a} + \frac{F_{c-g} \cdot J_c A_c}{A_g}$$

where F_{c-g} and the cover radiosity are

$$F_{c-g} = F_{g-c}\left(A_g/A_c\right) \approx \frac{D_c^2}{4R^2 + D_c^2} \cdot \frac{A_g}{A_c} \qquad J_c = \varepsilon_c E_b(T_c) + \rho_c G_c$$

but $G_c = E_b(T_{sur})$ and $\rho_c = 1 - \alpha_c = 1 - \varepsilon_c, J_c = \varepsilon_c \sigma T_c^4 + (1-\varepsilon_c)\sigma T_{sur}^4 = (170.2 + 387.4) W/m^2$.
Hence, the irradiation is

$$G_g = G_{g,a} + \frac{1}{A_g}\left(\frac{D_c^2}{4R^2 + D_c^2} \cdot \frac{A_g}{A_c}\right)\left[\varepsilon_c \sigma T_c^4 + (1-\varepsilon_c)\sigma T_{sur}^4\right] A_c$$

$$G_g = 354.4\ mW/m^2 + \left(\frac{0.10^2}{4\times1^2 + 0.10^2}\right)\left[0.2\times\sigma(350)^4 + (1-0.2)\times\sigma(300)^4\right] W/m^2$$

$$G_g = 354.4\ mW/m^2 + 424.4\ mW/m^2 + 916.2\ mW/m^2 = 1,695\ mW/m^2.$$

COMMENTS: (1) Note we have assumed $A_f << A_c$ so that effect of the aperture is negligible. (2) In part (b), the irradiation due to radiosity from the shield can be written also as $G_{g,c} = q_{c-g}/A_g = (J_c/\pi)\cdot A_c \cdot \omega_{g-c}/A_g$ where $\omega_{g-c} = A_g/R^2$. This is an excellent approximation since $A_c << R^2$.

PROBLEM 13.17

KNOWN: Temperature and diameters of a circular ice rink and a hemispherical dome.

FIND: Net rate of heat transfer to the ice due to radiation exchange with the dome.

SCHEMATIC:

Dome, $D_2 = 35m, T_2 = 15°C$

Air, T_∞

Ice rink, $D_1 = 25m, T_1 = 0°C$

ASSUMPTIONS: (1) Blackbody behavior for dome and ice.

ANALYSIS: From Eq. 13.13 the net rate of energy exchange between the two blackbodies is

$$q_{21} = A_2 F_{21} \sigma \left(T_2^4 - T_1^4\right)$$

From reciprocity, $A_2\ F_{21} = A_1\ F_{12} = \left(\pi D_1^2 / 4\right)1$

$$A_2 F_{21} = (\pi / 4)(25\ \text{m})^2 1 = 491\ \text{m}^2.$$

Hence

$$q_{21} = 491\ \text{m}^2 \left(5.67 \times 10^{-8} \text{W/m}^2 \cdot \text{K}^4\right)\left[(288\ \text{K})^4 - (273\ \text{K})^4\right]$$

$$q_{21} = 3.69 \times 10^4\ \text{W}.$$ <

COMMENTS: If the air temperature, T_∞, exceeds T_1, there will also be heat transfer by convection to the ice. The radiation and convection transfer to the ice determine the heat load which must be handled by the cooling system.

PROBLEM 13.18

KNOWN: Surface temperature of a semi-circular drying oven.

FIND: Drying rate per unit length of oven.

SCHEMATIC:

$T_1 = 1200K,\ A_1 = \pi D/2 \cdot L$

q_{evap} q_{12}

$T_2 = 325K,\ A_2 = D \cdot L$

$D = 1m$

ASSUMPTIONS: (1) Blackbody behavior for furnace wall and water, (2) Convection effects are negligible and bottom is insulated.

PROPERTIES: *Table A-6,* Water (325 K): $h_{fg} = 2.378 \times 10^6 \text{ J/kg}$.

ANALYSIS: Applying a surface energy balance,

$$q_{12} = q_{evap} = \dot{m}\, h_{fg}$$

where it is assumed that the net radiation heat transfer to the water is balanced by the evaporative heat loss. From Eq. 13.13

$$q_{12} = A_1\, F_{12}\, \sigma \left(T_1^4 - T_2^4\right).$$

From inspection and the reciprocity relation, Eq. 13.3,

$$F_{12} = \frac{A_2}{A_1} F_{21} = \frac{D \cdot L}{(\pi D/2) \cdot L} \times 1 = 0.637.$$

Hence

$$\dot{m}' = \frac{\dot{m}}{L} = \frac{\pi D}{2} F_{12}\, \sigma \frac{\left(T_1^4 - T_2^4\right)}{h_{fg}}$$

$$\dot{m}' = \frac{\pi (1 \text{ m})}{2} \times 0.637 \times 5.67 \times 10^{-8} \frac{\text{W}}{\text{m}^2 \cdot \text{K}^4} \frac{(1200 \text{ K}) - (325 \text{ K})^4}{2.378 \times 10^6 \text{ J/kg}}$$

or

$$\dot{m}' = 0.0492 \text{ kg/s} \cdot \text{m}.$$ <

COMMENTS: Air flow through the oven is needed to remove the water vapor. The water surface temperature, T_2, is determined by a balance between radiation heat transfer to the water and the convection of latent and sensible energy from the water.

PROBLEM 13.19

KNOWN: Arrangement of three black surfaces with prescribed geometries and surface temperatures.

FIND: (a) View factor F_{13}, (b) Net radiation heat transfer from A_1 to A_3.

SCHEMATIC:

ASSUMPTIONS: (1) Interior surfaces behave as blackbodies, (2) $A_2 >> A_1$.

ANALYSIS: (a) Define the enclosure as the interior of the cylindrical form and identify A_4. Applying the view factor summation rule, Eq. 13.4,

$$F_{11} + F_{12} + F_{13} + F_{14} = 1. \tag{1}$$

Note that $F_{11} = 0$ and $F_{14} = 0$. From Eq. 13.8,

$$F_{12} = \frac{D^2}{D^2 + 4L^2} = \frac{(3m)^2}{(3m)^2 + 4(2m)^2} = 0.36. \tag{2}$$

From Eqs. (1) and (2),

$$F_{13} = 1 - F_{12} = 1 - 0.36 = 0.64.$$ <

(b) The net heat transfer rate from A_1 to A_3 follows from Eq. 13.13,

$$q_{13} = A_1\ F_{13}\ \sigma\left(T_1^4 - T_3^4\right)$$

$$q_{13} = 0.05\text{m}^2 \times 0.64 \times 5.67 \times 10^{-8}\,\text{W/m}^2 \cdot \text{K}^4 \left(1000^4 - 500^4\right)\text{K}^4 = 1700\,\text{W}.$$ <

COMMENTS: Note that the summation rule, Eq. 13.4, applies to an enclosure; that is, the total region above the surface must be considered.

PROBLEM 13.20

KNOWN: Furnace diameter and temperature. Dimensions and temperature of suspended part.

FIND: Net rate of radiation transfer per unit length to the part.

SCHEMATIC:

T_w = 1000 K, D = 2 m

T_p = 300 K

A_2, L = 1 m, A_1, A_3

ASSUMPTIONS: (1) All surfaces may be approximated as blackbodies.

ANALYSIS: From symmetry considerations, it is convenient to treat the system as a three-surface enclosure consisting of the inner surfaces of the vee (1), the outer surfaces of the vee (2) and the furnace wall (3). The net rate of radiation heat transfer to the part is then

$$q'_{1,2} = A'_3 F_{31} \sigma \left(T_w^4 - T_p^4\right) + A'_3 F_{32} \sigma \left(T_w^4 - T_p^4\right)$$

From reciprocity,

$$A'_3 F_{31} = A'_1 F_{13} = 2\,L \times 0.5 = 1\text{m}$$

where surface 3 may be represented by the dashed line and, from symmetry, F_{13} = 0.5. Also,

$$A'_3 F_{32} = A'_2 F_{23} = 2\,L \times 1 = 2\text{m}$$

Hence,

$$q'_{1,2} = (1+2)\text{m} \times 5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4 \left(1000^4 - 300^4\right)\text{K}^4 = 1.69 \times 10^5\ \text{W/m} \qquad <$$

COMMENTS: With all surfaces approximated as blackbodies, the result is independent of the tube diameter. Note that F_{11} = 0.5.

PROBLEM 13.21

KNOWN: Coaxial, parallel black plates with surroundings. Lower plate (A_2) maintained at prescribed temperature T_2 while electrical power is supplied to upper plate (A_1).

FIND: Temperature of the upper plate T_1.

SCHEMATIC:

$P_e = 17.5\,W$; Heater; $T_{sur} = 300K$; $L = 0.20m$; $A_1, D_1 = 0.20m, T_1$; $A_2, D_2 = 0.40m, T_2 = 500K$

ASSUMPTIONS: (1) Plates are black surfaces of uniform temperature, and (2) Backside of heater on A_1 insulated.

ANALYSIS: The net radiation heat rate leaving A_i is

$$P_e = \sum_{j=1}^{N} q_{ij} = A_1 F_{12}\sigma\left(T_1^4 - T_2^4\right) + A_1 F_{13}\sigma\left(T_1^4 - T_3^4\right)$$

$$P_e = A_1\sigma\left[F_{12}\left(T_1^4 - T_2^4\right) + F_{13}\left(T_1^4 - T_{sur}^4\right)\right] \qquad (1)$$

From Fig. 13.5 for coaxial disks (see Table 13.2),

$$R_1 = r_1/L = 0.10\text{ m}/0.20\text{ m} = 0.5 \qquad R_2 = r_2/L = 0.20\text{ m}/0.20\text{ m} = 1.0$$

$$S = 1 + \frac{1+R_2^2}{R_1^2} = 1 + \frac{1+1^2}{(0.5)^2} = 9.0$$

$$F_{12} = \frac{1}{2}\left\{S - \left[S^2 - 4\left(r_2/r_1\right)^2\right]^{1/2}\right\} = \frac{1}{2}\left\{9 - \left[9^2 - 4\left(0.2/0.1\right)^2\right]^{1/2}\right\} = 0.469.$$

From the summation rule for the enclosure A_1, A_2 and A_3 where the last area represents the surroundings with $T_3 = T_{sur}$,

$$F_{12} + F_{13} = 1 \qquad F_{13} = 1 - F_{12} = 1 - 0.469 = 0.531.$$

Substituting numerical values into Eq. (1), with $A_1 = \pi D_1^2/4 = 3.142\times10^{-2}\,\text{m}^2$,

$$17.5\text{ W} = 3.142\times10^{-2}\text{m}^2 \times 5.67\times10^{-8}\text{W/m}^2\cdot\text{K}^4\left[0.469\left(T_1^4 - 500^4\right)\text{K}^4 + 0.531\left(T_1^4 - 300^4\right)\text{K}^4\right]$$

$$9.823\times10^9 = 0.469\left(T_1^4 - 500^4\right) + 0.531\left(T_1^4 - 300^4\right)$$

find by trial-and-error that $\quad T_1 = 456\text{ K}.$ $\qquad <$

COMMENTS: Note that if the upper plate were adiabatic, $T_1 = 427$ K.

PROBLEM 13.22

KNOWN: Tubular heater radiates like blackbody at 1000 K.

FIND: (a) Radiant power from the heater surface, A_s, intercepted by a disc, A_1, at a prescribed location $q_{s\to1}$; irradiation on the disk, G_1; and (b) Compute and plot $q_{s\to1}$ and G_1 as a function of the separation distance L_1 for the range $0 \le L_1 \le 200$ mm for disk diameters D_1 = 25, and 50 and 100 mm.

SCHEMATIC:

ASSUMPTIONS: (1) Heater surface behaves as blackbody with uniform temperature.

ANALYSIS: (a) The radiant power leaving the inner surface of the tubular heater that is intercepted by the disk is

$$q_{2\to1} = \left(A_2 E_{b2}\right) F_{21} \qquad (1)$$

where the heater is surface 2 and the disk is surface 1. It follows from the reciprocity rule, Eq. 13.3, that

$$F_{21} = \frac{A_1}{A_2} F_{12}. \qquad (2)$$

Define now the hypothetical disks, A_3 and A_4, located at the ends of the tubular heater. By inspection, it follows that

$$F_{14} = F_{12} + F_{13} \qquad \text{or} \qquad F_{12} = F_{14} - F_{13} \qquad (3)$$

where F_{14} and F_{13} may be determined from Fig. 13.5. Substituting numerical values, with $D_3 = D_4 = D_2$,

$$F_{13} = 0.08 \quad \text{with} \quad \frac{L}{r_i} = \frac{L_1 + L_2}{D_1/2} = \frac{200}{50/2} = 8 \qquad \frac{r_j}{L} = \frac{D_3/2}{L_1 + L_2} = \frac{100/2}{200} = 0.25$$

$$F_{14} = 0.20 \quad \text{with} \quad \frac{L}{r_i} = \frac{L_1}{D_1/2} = \frac{100}{50/2} = 4 \qquad \frac{r_j}{L} = \frac{D_4/2}{L_1} = \frac{100/2}{100} = 0.5$$

Substituting Eq. (3) into Eq. (2) and then into Eq. (1), the result is

$$q_{2\to1} = A_1 \left(F_{14} - F_{13}\right) E_{b2}$$

$$q_{2\to1} = \left[\pi \left(50\times10^{-3}\right)^2 \text{m}^2/4\right] (0.20 - 0.08) \times 5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4 \,(1000\ \text{K})^4 = 13.4\,\text{W}$$ <

where $E_{b2} = \sigma T_s^4$. The irradiation G_1 originating from emission leaving the heater surface is

$$G_1 = \frac{q_{s\to1}}{A_1} = \frac{13.4\ \text{W}}{\pi\,(0.050\ \text{m})^2/4} = 6825\,\text{W/m}^2. \qquad (4)$$ <

Continued …..

PROBLEM 13.22 (Cont.)

(b) Using the foregoing equations in *IHT* along with the *Radiation Tool-View Factors* for *Coaxial Parallel Disks*, G_1 and $q_{s \to 1}$ were computed as a function of L_1 for selected values of D_1. The results are plotted below.

In the upper left-hand plot, G_1 decreases with increasing separation distance. For a given separation distance, the irradiation decreases with increasing diameter. With values of D_1 = 25 and 50 mm, the irradiation values are only slightly different, which diminishes as L_1 increases. In the upper right-hand plot, the radiant power from the heater surface reaching the disk, $q_{s \to 2}$, decreases with increasing L_1 and decreasing D_1. Note that while G_1 is nearly the same for D_1 = 25 and 50 mm, their respective $q_{s \to 2}$ values are quite different. Why is this so?

PROBLEM 13.23

KNOWN: Dimensions and temperatures of an enclosure and a circular disc at its base.

FIND: Net radiation heat transfer between the disc and portions of the enclosure.

SCHEMATIC:

ASSUMPTIONS: (1) Blackbody behavior for disc and enclosure surfaces, (2) Area of disc is much less than that of the hypothetical surfaces, $(A_1/A_2) << 1$ and $(A_1/A_3) << 1$.

ANALYSIS: From Eq. 13.13 the net radiation exchange between the disc (1) and the hemispherical dome (d) is

$$q_{1d} = A_1 F_{1d} \sigma \left(T_1^4 - T^4\right).$$

However, since all of the radiation intercepted by the dome must pass through the hypothetical area A_2, it follows from Eq. 13.8 of Example 13.1,

$$F_{1d} = F_{12} \approx \frac{D^2}{4L^2 + D^2} = \frac{1}{(2L/D)^2 + 1} = \frac{1}{1.44 + 1} = 0.410.$$

Hence

$$q_{1d} = \frac{\pi}{4}(0.02\text{ m})^2 \times 0.41 \times 5.67 \times 10^{-8}\text{ W/m}^2 \cdot \text{K}\left[(1000\text{ K})^4 - (300\text{ K})^4\right]$$

$$q_{1d} = 7.24\text{ W.}$$ <

Similarly, the net radiation exchange between the disc (1) and the cylindrical ring (r) of length L/3 is

$$q_{1r} = A_1\ F_{1r}\ \sigma\left(T_1^4 - T^4\right)$$

where

$$F_{1r} = F_{13} - F_{12} = \frac{D^2}{4(2L/3)^2 + D^2} - 0.41 = 0.61 - 0.41 = 0.20.$$

Hence

$$q_{1r} = \frac{\pi}{4}(0.02\text{ m})^2 \times 0.2 \times 5.67 \times 10^{-8}\text{W/m}^2 \cdot \text{K}^4\left[(1000\text{ K})^4 - (300\text{ K})^4\right]$$

$$q_{1r} = 3.53\text{ W.}$$ <

PROBLEM 13.24

KNOWN: Circular plate (A_1) maintained at 600 K positioned coaxially with a conical shape (A_2) whose backside is insulated. Plate and cone are black surfaces and located in large, insulated enclosure at 300 K.

FIND: (a) Temperature of the conical surface T_2 and (b) Electric power required to maintain plate at 600 K.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Plate and cone are black, (3) Cone behaves as insulated, reradiating surface, (4) Surroundings are large compared to plate and cone.

ANALYSIS: (a) Recognizing that the plate, cone, and surroundings from a three-(black) surface enclosure, perform a radiation balance on the cone.

$$q_2 = 0 = q_{23} + q_{21} = A_2 F_{23}\sigma\left(T_2^4 - T_3^4\right) + A_2 F_{21}\sigma\left(T_2^4 - T_1^4\right)$$

where the view factor F_{21} can be determined from the *coaxial parallel disks* relation (Table 13.2 or Fig. 13.5) with $R_i = r_i/L = 250/500 = 0.5$, $R_j = 0.5$, $S = 1 + \left(1+R_j^2\right)/R_i^2 = 1 + (1 + 0.5^2)/0.5^2 = 6.00$, and noting $F_{2'1} = F_{21}$,

$$F_{21} = 0.5\left\{S - \left[S^2 - 4\left(r_j / r_i\right)^2\right]^{1/2}\right\} = 0.5\left\{6 - \left[6^2 - 4\left(0.5/0.5\right)^2\right]^{1/2}\right\} = 0.172.$$

For the enclosure, the summation rule provides, $F_{2'3} = 1 - F_{2'1} = 1 - 0.172 = 0.828$. Hence,

$$0.828\left(T_2^4 - 300^4\right) = 0 + 0.172\left(T_2^4 - 600^4\right)$$

$$T_2 = 413\text{ K.}$$ <

(b) The power required to maintain the plate at T_2 follows from a radiation balance,

$$q_1 = q_{12} + q_{13} = A_1 F_{12}\sigma\left(T_1^4 - T_2^4\right) + A_1 F_{13}\sigma\left(T_1^4 - T_3^4\right)$$

where $F_{12} = A_{2'} F_{2'1} / A_1 = F_{21} = 0.172$ and $F_{13} = 1 - F_{12} = 0.828$,

$$q_1 = \left(\pi 0.5^2 / 4\right)\text{m}^2\sigma\left[0.172\left(600^4 - 413^4\right)\text{K}^4 + 0.828\left(600^4 - 300^4\right)\text{K}^4\right]$$

$$q_1 = 1312\text{ W.}$$ <

PROBLEM 13.25

KNOWN: Conical and cylindrical furnaces (A_2) as illustrated and dimensioned in Problem 13.2 (S) supplied with power of 50 W. Workpiece (A_1) with insulated backside located in large room at 300 K.

FIND: Temperature of the workpiece, T_1, and the temperature of the inner surfaces of the furnaces, T_2. Use expressions for the view factors F_{21} and F_{22} given in the statement for Problem 13.2 (S).

SCHEMATIC:

(a) Right-circular cone (b) Right-circular cylinder

$D_1 = D_3 = 50$ mm
$L = L_o = 50$ mm

$F_{21} = (A_1/A_2)F_{13}$
$F_{22} = 1 - (A_3/A_2)$

$T_{sur} = 300$ K

ASSUMPTIONS: (1) Diffuse, black surfaces with uniform radiosities, (2) Backside of workpiece is perfectly insulated, (3) Inner base and lateral surfaces of the cylindrical furnace treated as single surface, (4) Negligible convection heat transfer, (5) Room behaves as large, isothermal surroundings.

ANALYSIS: Considering the furnace surface (A_2), the workpiece (A_1) and the surroundings (A_s) as an enclosure, the net radiation transfer from A_1 and A_2 follows from Eq. 13.14,

Workpiece $\quad q_1 = 0 = A_1 F_{12}\left(E_{b1} - E_{b2}\right) + A_1 F_{1s}\left(E_{b1} - E_{bs}\right) \quad (1)$

Furnace $\quad q_2 = 50\text{ W} = A_2 F_{21}\left(E_{b2} - E_{b1}\right) + A_2 F_{2s}\left(E_{b2} - E_{bs}\right) \quad (2)$

where $E_b = \sigma T^4$ and $\sigma = 5.67 \times 10^{-8}$ W/m$^2\cdot$K^4. From summation rules on A_1 and A_2, the view factors F_{1s} and F_{2s} can be evaluated. Using reciprocity, F_{12} can be evaluated.

$$F_{1s} = 1 - F_{12} \qquad F_{2s} = 1 - F_{21} - F_{22} \qquad F_{12} = \left(A_2 / A_1\right)F_{21}$$

The expressions for F_{21} and F_{22} are provided in the schematic. With $A_1 = \pi D_1^2 / 4$ the A_2 are:

Cone: $\quad A_2 = \pi D_3 / 2\left(L^2 + \left(D_3/2\right)^2\right)^{1/2} \qquad$ *Cylinder*: $\quad A_2 = \pi D_3^2 / 4 + \pi D_3 L$

Examine Eqs (1) and (2) and recognize that there are two unknowns, T_1 and T_2, and the equations must be solved simultaneously. Using the foregoing equations in the *IHT* workspace, the results are

$$T_1 = 544\text{ K} \qquad T_2 = 828\text{ K} \qquad <$$

COMMENTS: (1) From the *IHT* analysis, the relevant view factors are: $F_{12} = 0.1716$; $F_{1s} = 0.8284$; *Cone:* $F_{21} = 0.07673$, $F_{22} = 0.5528$; *Cylinder:* $F_{21} = 0.03431$, $F_{22} = 0.80$.

(2) That both furnace configurations provided identical results may not, at first, be intuitively obvious. Since both furnaces (A_2) are black, they can be represented by the hypothetical black area A_3 (the opening of the furnaces). As such, the analysis is for an enclosure with the workpiece (A_1), the furnace represented by the disk A_3 (at T_2), and the surroundings. As an exercise, perform this analysis to confirm the above results.

PROBLEM 13.26

KNOWN: Furnace constructed in three sections: insulated circular (2) and cylindrical (3) sections, as well as, an intermediate cylindrical section (1) with imbedded electrical resistance heaters. Cylindrical sections (1,3) are of equal length.

FIND: (a) Electrical power required to maintain the heated section at T_1 = 1000 K if all the surfaces are black, (b) Temperatures of the insulated sections, T_2 and T_3, and (c) Compute and plot q_1, T_2 and T_3 as functions of the length-to-diameter ratio, with $1 \le L/D \le 5$ and D = 100 mm.

SCHEMATIC:

ASSUMPTIONS: (1) All surfaces are black, (2) Areas (1, 2, 3) are isothermal.

ANALYSIS: (a) To complete the enclosure representing the furnace, define the hypothetical surface A_4 as the opening at 0 K with unity emissivity. For each of the enclosure surfaces 1, 2, and 3, the energy balances following Eq. 13.13 are

$$q_1 = A_1F_{12}\left(E_{b1} - E_{b2}\right) + A_1F_{13}\left(E_{b1} - E_{b3}\right) + A_1F_{14} + \left(E_{b1} - E_{b4}\right) \qquad (1)$$

$$0 = A_2F_{21}\left(E_{b2} - E_{b1}\right) + A_2F_{23}\left(E_{b2} - E_{b3}\right) + A_2F_{24}\left(E_{b2} - E_{b4}\right) \qquad (2)$$

$$0 = A_3F_{31}\left(E_{b3} - E_{b1}\right) + A_3F_{32}\left(E_{b3} - E_{b2}\right) + A_3F_{34}\left(E_{b3} - E_{b4}\right) \qquad (3)$$

where the emissive powers are

$$E_{b1} = \sigma T_1^4 \qquad E_{b2} = \sigma T_2^4 \qquad E_{b3} = \sigma T_3^4 \qquad E_{b4} = 0 \qquad (4-7)$$

For this four surface enclosure, there are N^2 = 16 view factors and N (N – 1)/2 = 4 × 3/2 = 6 must be directly determined (by inspection or formulas) and the remainder can be evaluated from the summation rule and reciprocity relation. By inspection,

$$F_{22} = 0 \qquad F_{44} = 0 \qquad (8,9)$$

From the coaxial parallel disk relation, Table 13.2, find F_{24}

$$S = 1 + \frac{1 + R_4^2}{R_2^2} = 1 + \frac{1 + (0.250)^2}{(0.250)^2} = 18.00$$

$$R_2 = r_2 / L = 0.050\,\text{m} / 0.200\,\text{m} = 0.250 \qquad R_4 = r_4 / L = 0.250$$

$$F_{24} = 0.5\left\{S - \left[S^2 - 4\left(r_4 / r_2\right)^2\right]^{1/2}\right\}$$

$$F_{24} = 0.5\left\{18.00 - \left[18.00^2 - 4(1)^2\right]^{1/2}\right\} = 0.0557 \qquad (10)$$

Consider the three-surface enclosure $1 - 2 - 2'$ and find F_{11} as beginning with the summation rule,

Continued …..

$$F_{11} = 1 - F_{12} - F_{12'} \quad (11)$$

where, from symmetry, $F_{12} = F_{12'}$, and using reciprocity,

$$F_{12} = A_2 F_{21} / A_1 = \left(\pi D^2 / 4\right) F_{23} / \left(\pi DL / 2\right) = DF_{21} / 2L \quad (12)$$

and from the summation rule on A_2

$$F_{21} = 1 - F_{22'} = 1 - 0.172 = 0.828, \quad (13)$$

Using the coaxial parallel disk relation, Table 13.2, to find F_{221},

$$S = 1 + \frac{1 + R_{2'}^2}{R_2^2} = 1 + \frac{1 + 0.50^2}{0.50^2} = 6.000$$

$$R_2 = r_2 / L = 0.050\,\text{m} / (0.200/2\,\text{m}) = 0.500 \qquad R_{2'} = 0.500$$

$$F_{22'} = 0.5\left\{S - \left[S^2 - 4\left(r_{2'}/r_2\right)^2\right]^{1/2}\right\}$$

$$F_{22'} = 0.5\left\{6 - \left[6^2 - 4(1)^2\right]^{1/2}\right\} = 0.1716$$

Evaluating F_{12} from Eq. (12), find

$$F_{12} = 0.100\ \text{m} \times 0.828 / 2 \times 0.200\ \text{m} = 0.2071$$

and evaluating F_{11} from Eq. (11), find

$$F_{11} = 1 - 2 \times F_{12} = 1 - 2 \times 0.207 = 0.586$$

From symmetry, recognize that $F_{33} = F_{11}$ and $F_{43} = F_{21}$. To this point we have directly determined six view factors (underlined in the matrix below) and the remaining F_{ij} can be evaluated from the summation rules and appropriate reciprocity relations. The view factors written in matrix form, $[F_{ij}]$ are.

<u>0.5858</u>	<u>0.2071</u>	0.1781	0.02896
<u>0.8284</u>	<u>0</u>	0.1158	<u>0.05573</u>
0.1781	0.02896	0.5858	0.2071
0.1158	0.05573	0.8284	<u>0</u>

Knowing all the required view factors, the energy balances and the emissive powers, Eqs. (4-6), can be solved simultaneously to obtain:

$$q_1 = 255\,\text{W} \qquad E_{b2} = 5.02 \times 10^4\,\text{W/m}^2 \qquad E_{b3} = 2.79 \times 10^4\,\text{W/m}^2 \quad <$$

$$T_2 = 970\,\text{K} \qquad T_3 = 837.5\,\text{K} \quad <$$

Continued …..

PROBLEM 13.26 (Cont.)

(b) Using the energy balances, Eqs. (1-3), along with the *IHT Radiation Tool, View Factors, Coaxial parallel disks*, a model was developed to calculate q_1, T_2, and T_3 as a function of length L for fixed diameter D = 100 m. The results are plotted below.

For fixed diameter, as the overall length increases, the power required to maintain the heated section at T_1 = 1000 K decreases. This follows since the furnace opening area is a smaller fraction of the enclosure surface area as L increases. As L increases, the bottom surface temperature T_2 increases as L increases and, in the limit, will approach that of the heated section, T_1 = 1000 K. As L increases, the temperature of the insulated cylindrical section, T_3, increases, but only slightly. The limiting value occurs when $E_{b3} = 0.5 \times E_{b1}$ for which $T_3 \rightarrow 840$ K. Why is that so?

PROBLEM 13.27

KNOWN: Dimensions and temperature of a rectangular fin array radiating to deep space.

FIND: Expression for rate of radiation transfer per unit length from a unit section of the array.

SCHEMATIC:

Deep space

$A_3' = W, T_3 = 0$ K, $\varepsilon_3 = 1$

$A_2' = 2L$

$A_1' = W$

T

ASSUMPTIONS: (1) Surfaces may be approximated as blackbodies, (2) Surfaces are isothermal, (3) Length of array (normal to page) is much larger than W and L.

ANALYSIS: Deep space may be represented by the hypothetical surface A_3', which acts as a blackbody at absolute zero temperature. The net rate of radiation heat transfer to this surface is therefore equivalent to the rate of heat rejection by a unit section of the array.

$$q_3' = A_1' F_{13} \sigma \left(T_1^4 - T_3^4\right) + A_2' F_{23} \sigma \left(T_2^4 - T_3^4\right)$$

With $A_2' F_{23} = A_3' F_{32} = A_1' F_{12}$, $T_1 = T_2 = T$ and $T_3 = 0$,

$$q_3' = A_1' \left(F_{13} + F_{12}\right) \sigma T^4 = W \sigma T^4$$ <

Radiation from a unit section of the array corresponds to emission from the base. Hence, if blackbody behavior can, indeed, be maintained, the fins do nothing to enhance heat rejection.

COMMENTS: (1) The foregoing result should come as no surprise since the surfaces of the unit section form an isothermal blackbody cavity for which emission is proportional to the area of the opening. (2) Because surfaces 1 and 2 have the same temperature, the problem could be treated as a two-surface enclosure consisting of the combined (1, 2) and 3. It follows that $q_3' = q'_{(1,2)3} = A'_{(1,2)} F_{(1,2)3} \sigma T^4 = A_3' F_{3(1,2)} \sigma T^4 = W \sigma T^4$, (3) If blackbody behavior cannot be achieved $(\varepsilon_1, \varepsilon_2 < 1)$, enhancement would be afforded by the fins.

PROBLEM 13.28

KNOWN: Dimensions and temperatures of side and bottom walls in a cylindrical cavity.

FIND: Emissive power of the cavity.

SCHEMATIC:

ASSUMPTIONS: (1) Blackbody behavior for surfaces 1 and 2

ANALYSIS: The desired emissive power is defined as

$$E = q_3 / A_3$$

where

$$q_3 = A_1 F_{13} E_{b1} + A_2 F_{23} E_{b2}.$$

From symmetry, $F_{23} = F_{21}$, and from reciprocity, $F_{21} = (A_1/A_2) F_{12}$. With $F_{12} = 1 - F_{13}$, it follows that

$$q_3 = A_1 F_{13} E_{b1} + A_1 (1 - F_{13}) E_{b2} = A_1 E_{b2} + A_1 F_{13} (E_{b1} - E_{b2}).$$

Hence, with $A_1 = A_3$,

$$E = \frac{q_3}{A_3} = E_{b2} + F_{13} (E_{b1} - E_{b2}) = \sigma T_2^4 + F_{13} \sigma \left(T_1^4 - T_2^4 \right).$$

From Fig. 13.15, with $(L/r_i) = 4$ and $(r_j/L) = 0.25$, $F_{13} \approx 0.05$. Hence

$$E = 5.67 \times 10^{-8} \text{W/m}^2 \cdot \text{K}^4 + 0.05 \times 5.67 \times 10^{-8} \text{W/m}^2 \cdot \text{K}^4 \left(1000^4 - 700^4 \right) \text{K}^4$$

$$E = 1.36 \times 10^4 \text{W/m}^2 + 0.22 \times 10^4 \text{W/m}^2$$

$$E = 1.58 \times 10^4 \text{W/m}^2.$$ <

PROBLEM 13.29

KNOWN: Aligned, parallel discs with prescribed geometry and orientation.

FIND: Net radiative heat exchange between the discs.

SCHEMATIC:

$D_o = 0.2\,m$

$D_i = 0.15\,m$

Ring-shaped disc, $A_2, T_2 = 1000K$

$L = 1m$

Disc, $A_1, T_1 = 500K, D_1 = 30mm$

ASSUMPTIONS: (1) Surfaces behave as blackbodies, (2) $A_1 << A_2$.

ANALYSIS: The net radiation exchange between the two black surfaces follows from Eq. 13.13 written as

$$q_{12} = A_1\ F_{12}\ \sigma\left(T_1^4 - T_2^4\right).$$

The view factor can be determined from Eq. 13.8 which is appropriate for a small disc, aligned and parallel to a much larger disc.

$$F_{ij} = \frac{D_j^2}{D_j^2 + 4L^2}$$

where D_j is the diameter of the larger disk and L is the distance of separation. It follows that

$$F_{12} = F_{1o} - F_{1i} = 0.00990 - 0.00559 = 0.00431$$

where

$$F_{1o} = D_o^4 / \left(D_o^2 + 4L^2\right) = 0.2^2\,m^2 / \left(0.2^2\,m^2 + 4 \times 1\ m^2\right) = 0.00990$$

$$F_{1i} = D_i / \left(D_i^2 + 4L^2\right) = 0.15^2\,m^2 / \left(0.15^2\,m^2 + 4 \times 1\ m^2\right) = 0.00559.$$

The net radiation exchange is then

$$q_{12} = \frac{\pi\,(0.03m)^2}{4} \times 0.00431 \times 5.67 \times 10^{-8}\,\frac{W}{m^2 \cdot K^4}\left(500^4 - 1000^4\right)K^4 = -0.162\,W.$$

COMMENTS: F_{12} can be approximated using solid angle concepts if $D_o << L$. That is, the view factor for A_1 to A_o (whose diameter is D_o) is

$$F_{1o} \approx \frac{\omega_{o-1}}{\pi} = \frac{A_o / L^2}{\pi} = \frac{\pi D_o^2}{4\pi L^2} = \frac{D_o^2}{4L^2}.$$

Numerically, $F_{1o} = 0.0100$ and it follows $F_{1i} \approx D_i^2 / 4L^2 = 0.00563$. This gives $F_{12} = 0.00437$. An analytical expression can be obtained from Ex. 13.1 by replacing the lower limit of integration by $D_i/2$, giving

$$F_{12} = L^2\left[-1/\left(D_o^2/4 + L^2\right) + 1/\left(D_i^2/4 + L^2\right)\right] = 0.00431.$$

PROBLEM 13.30

KNOWN: Two black, plane discs, one being solid, the other ring-shaped.

FIND: Net radiative heat exchange between the two surfaces.

SCHEMATIC:

$D_i = 40cm$

$A_2, T_2 = 1000K$

$L = 20cm$

$A_1, T_1 = 300K$

$D_o = 80cm$

ASSUMPTIONS: (1) Discs are parallel and coaxial, (2) Discs are black, diffuse surfaces, (3) Convection effects are not being considered.

ANALYSIS: The net radiative heat exchange between the solid disc, A_1, and the ring-shaped disc, A_2, follows from Eq. 13.13.

$$q_{12} = A_1 F_{12}\,\sigma\left(T_1^4 - T_2^4\right)$$

The view factor F_{12} can be determined from Fig. 13.5 after some manipulation. Define these two hypothetical surfaces;

$$A_3 = \frac{\pi D_o^2}{4},\ \text{located co-planar with } A_2\text{, but a solid surface}$$

$$A_4 = \frac{\pi D_i^2}{4},\ \text{located co-planar with } A_2\text{, representing the missing center.}$$

From view factor relations and Fig. 13.5, it follows that

$$F_{12} = F_{13} - F_{14} = 0.62 - 0.20 = 0.42$$

F_{14}: $\quad \dfrac{r_j}{L} = \dfrac{40/2}{20} = 1, \qquad \dfrac{L}{r_i} = \dfrac{20}{80/2} = 0.5, \qquad F_{14} = 0.20$

F_{13}: $\quad \dfrac{r_j}{L} = \dfrac{80/2}{20} = 2, \qquad \dfrac{L}{r_i} = \dfrac{20}{80/2} = 0.5, \qquad F_{13} = 0.62.$

Hence

$$q_{12} = \left(\pi 0.80^2/4\right)m^2 \times 0.42 \times 5.67\times10^{-8}\,W/m^2\cdot K^4\left(300^4 - 1000^4\right)K^4$$

$$q_{12} = -11.87\ \text{kW}. \qquad <$$

Assuming negligible radiation exchange with the surroundings, the negative sign implies that q_1 = -11.87 kW and q_2 = +11.87 kW.

PROBLEM 13.31

KNOWN: Radiometer viewing a small target area (1), A_1, with a solid angle $\omega = 0.0008$ sr. Target has an area $A_1 = 0.004\ m^2$ and is diffuse, gray with emissivity $\varepsilon = 0.8$. The target is heated by a ring-shaped disc heater (2) which is black and operates at $T_2 = 1000$ K.

FIND: (a) Expression for the radiant power leaving the target which is collected by the radiometer in terms of the target radiosity, J_1, and relevant geometric parameters; (b) Expression for the target radiosity in terms of its irradiation, emissive power and appropriate radiative properties; (c) Expression for the irradiation on the target, G_1, due to emission from the heater in terms of the heater emissive power, the heater area and an appropriate view factor; numerically evaluate G_1; and (d) Determine the radiant power collected by the radiometer using the foregoing expressions and results.

SCHEMATIC:

ASSUMPTIONS: (1) Target is diffuse, gray, (2) Target area is small compared to the square of the separation distance between the sample and the radiometer, and (3) Negligible irradiation from the surroundings onto the target area.

ANALYSIS: (a) From Eq. (12.5) with $I_1 = I_{1,e+r} = J_1/\pi$, the radiant power leaving the target collected by the radiometer is

$$q_{1\rightarrow rad} = \frac{J_1}{\pi} A_1 \cos\theta_1 \omega_{rad-1} \qquad < \quad (1)$$

where $\theta_1 = 0°$ and ω_{rad-1} is the solid angle the radiometer subtends with respect to the target area.

(b) From Eq. 13.16, the radiosity is the sum of the emissive power plus the reflected irradiation.

$$J_1 = E_1 + \rho G_1 = \varepsilon E_{b,1} + (1-\varepsilon) G_1 \qquad < \quad (2)$$

where $E_{b1} = \sigma T_1^4$ and $\rho = 1 - \varepsilon$ since the target is diffuse, gray.

(c) The irradiation onto G_1 due to emission from the heater area A_2 is

$$G_1 = \frac{q_{2\rightarrow 1}}{A_1}$$

where $q_{2\rightarrow 1}$ is the radiant power leaving A_2 which is intercepted by A_1 and can be written as

$$q_{2\rightarrow 1} = A_2 F_{21} E_{b2} \qquad (3)$$

where $E_{b2} = \sigma T_2^4$. F_{21} is the fraction of radiant power leaving A_2 which is intercepted by A_1. The view factor F_{12} can be written as

Continued …..

$$F_{12} = F_{1-o} \qquad F_{1-i} = 0.5 - 0.2 = 0.3$$

where from Eq. 13.8,

$$F_{1-o} = \frac{D_o^2}{D_o^2 + 4L^2} = \frac{0.5^2}{0.5^2 + 4(0.25)^2} = 0.5 \tag{3}$$

$$F_{1-i} = \frac{D_i^2}{D_1^2 + 4L^2} = \frac{0.25^2}{0.25^2 + 4(0.25)^2} = 0.2$$

and from the reciprocity rule,

$$F_{21} = \frac{A_1 F_{12}}{A_2} = \frac{0.0004\,m^2 \times 0.3}{\pi / 4\left(0.5^2 - 0.25^2\right)m^2} = 0.000815$$

Substituting numerical values into Eq. (3), find

$$G_1 = \frac{\pi / 4\left(0.5^2 - 0.25^2\right)m^2 \times 0.000815 \times 5.67 \times 10^{-8}\,W/m^2 \cdot K^4 (1000\ K)^4}{0.0004\,m^2}$$

$$G_1 = 17,013\,W/m^2$$ <

(d) Substituting numerical values into Eq. (1), the radiant power leaving the target collected by the radiometer is

$$q_{1\to rad} = \left(6238\,W/m^2/\pi\,sr\right) \times 0.0004\,m^2 \times 1 \times 0.0008\,sr = 635\,\mu W$$ <

where the radiosity, J_1, is evaluated using Eq. (2) and G_1.

$$J_1 = 0.8 \times 5.67 \times 10^{-8}\,W/m^2 \cdot K^4 \times (500\ K)^4 + (1 - 0.8) \times 17,013\,W/m^2$$

$$J_1 = (2835 + 3403)\,W/m^2 = 6238\,W/m^2$$ <

COMMENTS: (1) Note that the emitted and reflected irradiation components of the radiosity, J_1, are of the same magnitude.

(2) Suppose the surroundings were at room temperature, T_{sur} = 300 K. Would the reflected irradiation due to the surroundings contribute significantly to the radiant power collected by the radiometer? Justify your conclusion.

PROBLEM 13.32

KNOWN: Thin-walled, black conical cavity with opening D = 10 mm and depth of L = 12 mm that is well insulated from its surroundings. Temperature of meter housing and surroundings is 25.0°C.

FIND: Optical (radiant) flux of laser beam, G_o (W/m^2), incident on the cavity when the fine-wire thermocouple indicates a temperature rise of 10.1°C.

SCHEMATIC:

ASSUMPTIONS: (1) Cavity surface is black and perfectly insulated from its mounting material in the meter, (2) Negligible convection heat transfer from the cavity surface, and (3) Surroundings are large, isothermal.

ANALYSIS: Perform an energy balance on the walls of the cavity considering absorption of the laser irradiation, absorption from the surroundings and emission.

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$A_o\, G_o + A_o\, G_{sur} - A_o\, E_b\,(T_c) = 0$$

where $A_o = \pi\, D^2/4$ represents the opening of the cavity. All of the radiation entering or leaving the cavity passes through this hypothetical surface. Hence, we can treat the cavity as a black disk at T_c. Since $G_{sur} = E_b\,(T_{sur})$, and $E_b = \sigma\, T^4$ with $\sigma = 5.67 \times 10^{-8}$ $W/m^2 \cdot K^4$, the energy balance has the form

$$G_o + \sigma(25.0 + 273)^4 K^4 - \sigma(25.0 + 10.1 + 273)^4 K^4 = 0$$

$$G_o = 63.8\ W/m^2 \qquad <$$

PROBLEM 13.33

KNOWN: Electrically heated sample maintained at T_s = 500 K with diffuse, spectrally selective coating. Sample is irradiated by a furnace located coaxial to the sample at a prescribed distance. Furnace has isothermal walls at T_f = 3000 K with ε_f = 0.7 and an aperture of 25 mm diameter. Sample experiences convection with ambient air at T_∞ = 300 K and h = 20 $W/m^2 \cdot K$. The surroundings of the sample are large with a uniform temperature T_{sur} = 300 K. A radiation detector sensitive to only power in the spectral region 3 to 5 μm is positioned at a prescribed location relative to the sample.

FIND: (a) Electrical power, P_e, required to maintain the sample at T_s = 500 K, and (b) Radiant power incident on the detector within the spectral region 3 to 5 μm considering both emission and reflected irradiation from the sample.

SCHEMATIC:

Furnace
T_f = 3000 K, ε_f = 0.7
D_f = 25 mm
Detector
T_{sur} = 300 K
A_d = 8 x 10^{-5} m^2
L_{sf} = 750 mm
L_{sd} = 1 m
T_∞ = 300 K
h = 20 $W/m^2 \cdot K$
45°
T_s = 500 K, D_s = 20 mm, ε_λ
Sample mount

ε_λ: 0.8, 0.2; 4; λ (μm)

ASSUMPTIONS: (1) Steady-state condition, (2) Furnace is large, isothermal enclosure with small aperture and radiates as a blackbody, (3) Sample coating is diffuse, spectrally selective, (4) Sample and detector areas are small compared to their separation distance squared, (5) Surroundings are large, isothermal.

ANALYSIS: (a) Perform an energy balance on the sample mount, which experiences electrical power dissipation, convection with ambient air, absorbed irradiation from the furnace, absorbed irradiation from the surroundings and emission,

$$E'_{in} - E'_{out} = 0$$

$$P_e + \left[-h\left(T_s - T_\infty\right) + \alpha_1 G_f + \alpha_{sur} G_{sur} - \varepsilon E_b\left(T_s\right)\right] A_s = 0 \quad (1)$$

where $E_b\left(T_s\right) = \sigma T_s^4$ and $A_s = \pi D_s^2/4$.

$\alpha_f G_f$ $\alpha_{sur} G_{sur}$ εE_b
q_{cv}
T_s
P_e

Irradiations on the sample: The irradiation from the furnace aperture onto the sample can be written as

$$G_f = \frac{q_{f\to s}}{A_s} = \frac{A_f F_{fs} E_{b,f}}{A_s} = \frac{A_f F_{fs} \sigma T_f^4}{A_s} \quad (2)$$

where $A_f = \pi D_f^2/4$ and $A_s = \pi D_s^2/4$. The view factor between the furnace aperture and sample follows from the relation for coaxial parallel disks, Table 13.2,

$R_f = r_f / L_{sf} = 0.0125\ m/0.750\ m = 0.01667$ $\qquad$ $R_s = r_s / L_{sf} = 0.0100\ m/0.750\ m = 0.01333$

$$S = 1 + \frac{1 + R_s^2}{R_f^2} = 1 + \frac{1 + 0.01333^2}{0.01667^2} = 3600.2$$

Continued …..

$$F_{sf} = 0.5\left\{S - \left[S^2 - 4\left(r_s / r_f\right)^2\right]^{1/2}\right\} = 0.5\left\{3600 - \left[3600^2 - 4\left(0.05/0.0625\right)^2\right]^{1/2}\right\} = 0.000178$$

Hence the irradiation from the furnace is

$$G_f = \frac{\pi\left(0.025\ m\right)^2 / 4 \times 0.000178 \times 5.67 \times 10^{-8}\,W/m^2 \cdot K^4 \left(3000\ K\right)^4}{\pi\left(0.020^2\ m^2 / 4\right)} = 1277\,W/m^2$$

The irradiation from the surroundings which are large compared to the sample is

$$G_{sur} = \sigma T_{sur}^4 = 5.67 \times 10^{-8}\,W/m^2 \cdot K\left(300K\right)^4 = 459\,W/m^2$$

Emissivity of the Sample: The total hemispherical emissivity in terms of the spectral distribution can be written following Eq. 12.38 and Eq. 12.30,

$$\varepsilon = \int_0^\infty \varepsilon_\lambda E_{\lambda,b}\left(T_s\right)d\lambda / \sigma T^4 = \varepsilon_1 F_{\left(0-\lambda_1 T_s\right)} + \varepsilon_2\left[1 - F_{\left(0-\lambda_1 T_s\right)}\right]$$

$$\varepsilon = 0.8 \times 0.066728 + 0.2\left[1 - 0.066728\right] = 0.240$$

where, from Table 12.1, with $\lambda_1 T_s = 4\ \mu m \times 500\ K = 2000\ \mu m \cdot K,\ F_{\left(0-\lambda T\right)} = 0.066728.$

Absorptivity of the Sample: The total hemispherical absorptivity due to irradiation from the furnace follows from Eq. 12.46,

$$\alpha_f = \varepsilon_1 F_{\left(0-\lambda_1 T_f\right)} + \varepsilon_2\left[1 - F_{\left(0-\lambda_1 T_f\right)}\right] = 0.8 \times 0.945098 + 0.2\left[1 - 0.945098\right] = 0.767$$

where, from Table 12.1, with $\lambda_1 T_f = 4\,\mu m \times 3000\ K = 12,000\ \mu m \cdot K,\ F_{\left(0-\lambda T\right)} = 0.945098.$ The total hemispherical absorptivity due to irradiation from the surroundings is

$$\alpha_{sur} = \varepsilon_1 F_{\left(0-\lambda_1 T_{sur}\right)} + \varepsilon_2\left[1 - F_{\left(0-\lambda_1 T_{sur}\right)}\right] = 0.8 \times 0.00234 + 0.2\left[1 - 0.002134\right] = 0.201$$

where, from Table 12.1, with $\lambda_1 T_{sur} = 4\ \mu m \times 300\ K = 1200\ \mu m \cdot K, F_{\left(0-\lambda T\right)} = 0.002134.$

Evaluating the Energy Balance: Substituting numerical values into Eq. (1),

$$P_e = \left[+20\,W/m^2 \cdot K\left(500 - 300\right)K - 0.767 \times 1277\,W/m^2\right.$$

$$\left. -0.201 \times 459\,W/m^2 + 0.240 \times 5.67 \times 10^{-8}\,W/m^2 \cdot K^4\left(500\ K\right)^4\right]\pi\left(0.020\ m\right)^2/4$$

$$P_e = 1.256\ W - 0.308\ W - 0.029\ W + 0.267\ W = 1.19\ W$$

<

(b) The radiant power leaving the sample which is incident on the detector and within the spectral region, Δλ = 3 to 5μm, follows from Eq. 12.5 with Eq. 12.30,

$$q_{s-d,\Delta\lambda} = \left[E_{s,\Delta\lambda} + G_{f,ref,\Delta\lambda} + G_{sur,ref,\Delta\lambda}\right]\left(1/\pi\right)A_s\cos\theta_s \cdot A_d\cos\theta_d / L_{sd}^2$$

where $\theta_s = 45°$ and $\theta_d = 0°$. The *emitted* component is

$$E_{s,\Delta\lambda} = \int_3^{5\mu m} \varepsilon_{\lambda,b} E_{\lambda,b}\left(T_s\right)$$

$$E_{s,\Delta\lambda} = \left\{\varepsilon_1\left[F_{\left(0-4\mu m,T_s\right)} - F_{\left(0-3\mu m,T_s\right)} + \varepsilon_2\left[F_{\left(0-5\mu m,T_s\right)} - F_{\left(0-4\mu m,T_s\right)}\right]\right]\right\}\sigma T_s^4$$

Continued …..

$$E_{s,\Delta\lambda} = \{0.8[0.066728 - 0.013754] + 0.2[0.16169 - 0.066728]\}\sigma(500K)^4 = 217.5\,W/m^2$$

where, from Table 12.1, $F_{(0-3\mu m,T_s)} = 0.013754$ at $\lambda T = 3\,\mu m \times 500\ K = 1500\ \mu m \cdot K$; $F_{(0-4\mu m,T_s)} = 0.066728$ at $\lambda = 4\,\mu m \times 500\ K = 2000\ \mu m \cdot K$; and $F_{(0-5\mu m,T_s)} = 0.16169$ at λT = 5μm × 500 K = 2500 μm·K.

The *reflected irradiation from the furnace* component is

$$G_{f,ref,\Delta\lambda} = \int_3^{5\mu m} (1-\varepsilon_\lambda) G_{f,\lambda} d\lambda$$

where $G_{f,\lambda} \approx E_{\lambda,b}(T_f)$, using band emission factors,

$$G_{f,ref,\Delta\lambda} = \left\{(1-\varepsilon)\left[F_{(0-4\mu m,T_f)} - F_{(0-3\mu m,T_f)}\right] + (1-\varepsilon_2)\left[F_{(0-5\mu m,T_f)} - F_{(0-4\mu m,T_f)}\right]\right\} G_f$$

$$G_{f,ref,\Delta\lambda} = \{0.2[0.9451 - 0.8900] + 0.8[0.9700 - 0.9451]\} 1277\,W/m^2 = 39.51\,W/m^2$$

where, from Table 12.1, $F_{(0-3\mu m,T_f)} = 0.8900$ at λT_f = 3 μm × 3000 K = 9000 μm·K; $F_{(0-4\mu m,T_f)} = 0.9451$ at λT_f = 4 μm × 3000 K = 12,000 μm·K; and, $F_{(0-5\mu m,T_f)} = 0.9700$ at λT_f = 5 μm × 3000 K = 15,000 μm·K.

The *reflected irradiation from the surroundings* component is

$$G_{sur,ref,\Delta\lambda} = \int_3^{5\mu m} (1-\varepsilon_\lambda) G_{ref,\lambda} d\lambda$$

where $G_{ref,\lambda} \approx E_\lambda(T_{sur})$, using band emission factors,

$$G_{sur,ref,\Delta\lambda} = \left\{(1-\varepsilon_1)\left[F_{(0-4\mu m,T_{sur})} - F_{(0-3\mu m,T_{sur})}\right] + (1-\varepsilon_2)\left[F_{(0-5\mu m,T_{sur})} - F_{(0-4\mu m,T_{sur})}\right]\right\} G_{sur}$$

$$G_{sur,ref,\Delta\lambda} = \{0.2[0.002134 - 0.0001685] - 0.8[0.013754 - 0.002134]\} 459\,W/m^2 = 4.44\,W/m^2$$

where, from Table 12.1, $F_{(0-3\mu m,T_{sur})} = 0.0001685$ at λT_{sur} = 3 μm ×·300 K = 900 μm·K; $F_{(0-4\mu m,T_{sur})} = 0.002134$ at λT_{sur} = 4 μm × 300 K = 1200 μm·K; and $F_{(0-5\mu m,T_{sur})} = 0.013754$ at λT_{sur} = 5 μm ×300 K=1500 μm·K. Returning to Eq. (3), find

$$q_{sd,\Delta\lambda} = [217.5 + 39.51 + 4.44]\,W/m^2 (1/\pi)\left[8\pi(0.020\ m)^2/4\right]$$
$$\cos 45° \times 8 \times 10^{-5} m^2 \times \cos 0°/(1\ m)^2 = 1.48\ \mu W$$

<

COMMENTS: (1) Note that F_{fs} is small, since A_f, $A_s << L_{sf}^2$. As such, we could have evaluated $q_{f\to s}$ using Eq. 12.5 and found

$$G_f = \frac{E_{b,f}/\pi A_f \left(A_s/L_{sf}^2\right)}{A_s} = 1276\,W/m^2$$

(2) Recognize in the analysis for part (b), Eq. (3), the role of the band emission factors in calculating the fraction of total radiant power for the emitted and reflected irradiation components.

PROBLEM 13.34

KNOWN: Water-cooled heat flux gage exposed to radiant source, convection process and surroundings.

FIND: (a) Net radiation exchange between heater and gage, (b) Net transfer of radiation to the gauge per unit area of the gage, (c) Net heat transfer to the gage per unit area of gage, (d) Heat flux indicated by gage described in Problem 3.98.

SCHEMATIC:

ASSUMPTIONS: (1) Heater and gauge are parallel, coaxial discs having blackbody behavior, (2) $A_g << A_h$, (3) Surroundings are large compared to A_h and A_g.

ANALYSIS: (a) The net radiation exchange between the heater and the gage, both with blackbody behavior, is given by Eq. 13.13 having the form

$$q_{h-g} = A_h F_{hg} \sigma \left(T_h^4 - T_g^4\right) = A_g F_{gh} \sigma \left(T_h^4 - T_g\right).$$

Note the use of reciprocity, Eq. 13.3, for the view factors. From Eq. 13.8,

$$F_{gh} = D_h^2 / \left(4L^2 + D_h^2\right) = (0.2\text{m})^2 / \left(4 \times 0.5^2 \text{m}^2 + 0.2^2 \text{m}^2\right) = 0.0385.$$

$$q_{h-g} = \left(\pi 0.01^2 \text{m}^2 / 4\right) \times 0.0385 \times 5.67 \times 10^{-8} \text{W/m}^2 \cdot \text{K}^4 \left[800^4 - 290^4\right] \text{K}^4 = 69.0 \text{ mW}. \quad <$$

(b) The net radiation *to* the gage per unit area will involve exchange with the heater and the surroundings. Using Eq. 13.14,

$$q''_{net,rad} = -q_g / A_g = q_{h-g} / A_g + q_{sur-g} / A_g.$$

The net exchange with the surroundings is

$$q_{sur-g} = A_{sur} F_{sur-g} \sigma \left(T_{sur}^4 - T_g 4\right) = A_g F_{g-sur} \sigma \left(T_{sur}^4 - T_g^4\right).$$

$$q''_{net,rad} = \frac{69.0 \times 10^{-3} \text{W}}{\pi (0.01\text{m})^2 / 4} + (1 - 0.0385) 5.67 \times 10^{-8} \text{W/m}^2 \cdot \text{K}^4 \left(300^4 - 290^4\right) \text{K}^4 = 934.5 \text{W/m}^2. \quad <$$

(c) The net heat transfer rate to the gage per unit area of the gage follows from the surface energy balance

$$q''_{net,in} = q''\big)_{net,rad} + q''_{conv}$$

$$q''_{net,in} = 934.5 \text{W/m}^2 + 15 \text{W/m}^2 \cdot \text{K} (300 - 290) \text{K}$$

$$q''_{net,in} = 1085 \text{W/m}^2. \quad <$$

(d) The heat flux gage described in Problem 3.98 would experience a net heat flux to the surface of 1085 W/m^2. The irradiation to the gage from the heater is $G_g = q_{h\to g}/A_g = F_{gh} \sigma T_h^4 = 894 \text{ W/m}^2$. Since the gage responds to net heat flux, there would be a systematic error in sensing irradiation from the heater.

PROBLEM 13.35

KNOWN: Long cylindrical heating element located a given distance above an insulated wall exposed to cool surroundings.

FIND: Maximum temperature attained by the wall and temperature at location A.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Insulated wall, (3) Negligible conduction in wall, (4) All surfaces are black.

ANALYSIS: Consider an elemental area at point x = 0; this is the location that will attain the maximum temperature. Since the wall is insulated and conduction is negligible, the net radiation leaving dA_o is zero. From Eq. 13.13,

$$q''_o = q''_{o,h} + q''_{o,sur} = F_{o,h}\,\sigma\left(T_o^2 - T_h^4\right) + F_{o,sur}\left(T_o^5 - T_{sur}^4\right) = 0 \qquad (1)$$

where $F_{o,sur} = 1 - F_{o,h}$ and $F_{o,h}$ can be found from the relation for a cylinder and parallel rectangle, Table 13.1, with s_1 = 2 mm, s_2 = 0 mm, L = 40 mm, and r = R = 10 mm.

$$F_{o,h} = \frac{r}{s_1 - s_2}\left[\tan^{-1}\frac{s_1}{L} - \tan^{-1}\frac{s_2}{L}\right] = \frac{10\,\text{mm}}{2\,\text{mm} - 0}\left[\tan^{-1}\frac{2}{40} - \tan^{-1}0\right] = 0.25 \qquad (2)$$

Rearranging Eq. (1) and substituting numerical values, find

$$T_o^4 = \left[T_h^4 + \frac{\left(1 - F_{o,h}\right)}{F_{o,h}}T_{sur}^4\right] \Big/ \left[1 + \frac{1 - F_{o,h}}{F_{o,h}}\right] \qquad (3)$$

$$T_o^4 = \left[(700\text{ K})^4 + \frac{1 - 0.25}{0.25}(300\text{ K})^4\right] \Big/ \left[1 + \frac{1 - 0.25}{0.25}\right] \qquad T_o = 507\text{ K.}$$ <

For the point A located at x = 40 mm, use the same relation of Table 13.1 to find $F_{A,h}$ (for this point, s_1 = 41 mm, s_2 = 39 mm, r = R = 10 mm, L = 40 mm),

$$F_{A,h} = \frac{10\text{ mm}}{(41 - 39)\text{mm}}\left[\tan^{-1}\frac{41}{40} - \tan^{-1}\frac{39}{40}\right] = 0.125.$$

Substituting numerical values into Eq. (3), find

$$T_A^4 = \left[(700\text{ K})^4 + \frac{1 - 0.125}{0.125}(300\text{ K})^4\right] \Big/ \left[1 + \frac{1 - 0.125}{0.125}\right] \quad T_A = 439\text{ K.}$$ <

COMMENTS: Note the importance of the assumptions that the wall is insulated and conduction is negligible. In calculating $F_{o,h}$ and $F_{A,h}$ we are finding the view factor for a small area or point. Hence, we need only specify that $s_1 - s_2$ is very small compared to L.

PROBLEM 13.36

KNOWN: Diameter and pitch of in-line tubes occupying evacuated space between parallel plates of prescribed temperature. Temperature and flowrate $\dot{m}$ of water through the tubes.

FIND: (a) Tube surface temperature T_s for $\dot{m}$ = 0.20 kg/s, (b) Effect of $\dot{m}$ on T_s.

SCHEMATIC:

T_p = 1000 K
S = 20 mm
q_{ps}
Water
T_m = 300 K, $\dot{m}$
D = 15 mm
T_s
q_{conv}
vacuum
T_p = 1000 K

ASSUMPTIONS: (1) Surfaces behave as blackbodies, (2) Negligible tube wall conduction resistance, (3) Fully-developed tube flow.

PROPERTIES: *Table A-6*, water (T_m = 300 K): $\mu = 855 \times 10^{-6}$ N·s/m^2, k = 0.613 W/m·K, Pr = 5.83.

ANALYSIS: (a) Performing an energy balance on a single tube, it follows that $q_{ps} = q_{conv}$, or

$$A_p F_{ps} \sigma \left(T_p^4 - T_s^4\right) = h A_s \left(T_s - T_m\right)$$

From Table 13.1 and D/S = 0.75, the view factor is

$$F_{ps} = 1 - \left[1 - \left(\frac{D}{S}\right)^2\right]^{1/2} + \left(\frac{D}{S}\right)\tan^{-1}\left(\frac{S^2 - D^2}{D^2}\right)^{1/2} = 0.881$$

With $Re_D = 4\dot{m}/\pi D\mu = 4(0.20\ \text{kg/s})/\pi(0.015\,\text{m})855\times10^{-6}\,\text{N}\cdot\text{s/m}^2 = 19{,}856$, fully-developed turbulent flow may be assumed, in which case Eq. 8.60 yields

$$h = \frac{k}{D}\left(0.023\,Re_D^{4/5}\,Pr^{0.4}\right) = \frac{0.613\ \text{W/m}\cdot\text{K}}{0.015\,\text{m}}(0.023)(19{,}856)^{4/5}(5.83)^{0.4} = 5220\,\text{W/m}^2\cdot\text{K}$$

Hence, with $(A_p/A_s) = 2S/\pi D = 0.849$,

$$T_s - T_m = \frac{F_{ps}\sigma}{h}\frac{A_p}{A_s}\left(T_p^4 - T_s^4\right) = \frac{0.881\times5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4}{5220\ \text{W/m}^2\cdot\text{K}}(0.849)\left(T_p^4 - T_s^4\right)$$

With T_m = 300 K and T_p = 1000 K, a trial-and-error solution yields

$$T_s = 308\ \text{K}$$ <

(b) Using the *Correlations and Radiation* Toolpads of *IHT* to evaluate the convection coefficient and view factor, respectively, the following results were obtained.

Surface temperature, Ts(K)
330
325
320
315
310
305
300
0.05
0.1
0.15
0.2
0.25
Mass flowrate, mdot(kg/s)

The decrease in T_s with increasing $\dot{m}$ is due to an increase in h and hence a reduction in the convection resistance.

COMMENTS: Due to the large value of h, $T_s \ll T_p$.

PROBLEM 13.37

KNOWN: Insulated wall exposed to a row of regularly spaced cylindrical heating elements.

FIND: Required operating temperature of the heating elements for the prescribed conditions.

SCHEMATIC:

Heating element, D=10mm
Air
T_∞=450K
h=200W/m²·K
T_2=300K
s=20mm
T_e
Insulated wall, T_1=500K

ASSUMPTIONS: (1) Upper and lower walls are isothermal and infinite, (2) Lower wall is insulated, (3) All surfaces are black, (4) Steady-state conditions.

ANALYSIS: Perform an energy balance on the insulated wall considering convection and radiation.

$$\dot{E}''_{in} - \dot{E}''_{out} = -q''_1 - q''_{conv} = 0$$

q''_{conv} q''_{12} q''_{1e}

where q''_1 is the net radiation leaving the insulated wall per unit area. From Eq. 13.13,

$$q''_1 = q''_{1e} + q''_{12} = F_{1e}\sigma\left(T_1^4 - T_e^4\right) + F_{12}\sigma\left(T_1^4 - T_2^4\right)$$

where $F_{12} = 1 - F_{1e}$. Using Newton's law of cooling for q''_{conv} solve for T_e,

$$T_e^4 = \left[T_1^4 + \frac{(1-F_{1e})}{F_{1e}}\left(T_1^4 - T_2^4\right)\right] + \frac{h}{\sigma}\frac{1}{F_{1e}}\left(T_1 - T_\infty\right).$$

The view factor between the insulated wall and the tube row follows from the relation for an infinite plane and row of cylinders, Table 13.1,

$$F_{1e} = 1 - \left[1 - \left(\frac{D}{S}\right)^2\right]^{1/2} + \left(\frac{D}{S}\right)\tan^{-1}\left(\frac{s^2 - D^2}{D^2}\right)^{1/2}$$

$$F_{1e} = 1 - \left[1 - \left(\frac{10}{20}\right)^2\right]^{1/2} + \left(\frac{10}{20}\right)\tan^{-1}\left(\frac{20^2 - 10^2}{10^2}\right)^{1/2} = 0.658.$$

Substituting numerical values, find

$$T_e^4 = \left[(500\ \text{K})^4 + \frac{1-0.658}{0.658}\left(500^4 - 300^4\right)\text{K}^4\right] + \frac{200\ \text{W/m}^2\cdot\text{K}}{5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4}\times\frac{1}{0.658}(500-450)\text{K}$$

$$T_e = 774\ \text{K}.$$ <

COMMENTS: Always express temperatures in kelvins when considering convection and radiation terms in an energy balance. Why is F_{1e} independent of the distance between the row of tubes and the wall:

PROBLEM 13.38

KNOWN: Surface radiative properties, diameter and initial temperature of a copper rod placed in an evacuated oven of prescribed surface temperature.

FIND: (a) Initial heating rate, (b) Time t_h required to heat rod to 1000 K, (c) Effect of convection on heating time.

SCHEMATIC:

q
Rod (1)
$\alpha = \varepsilon = 1$
$D = 0.01\text{m}$
Oven
$T_\infty = 1650$ K, h
Cu
$T_i = 300$ K
$T_{sur} = 1650$ K

ASSUMPTIONS: (1) Copper may be treated as a lumped capacitance, (b) Radiation exchange between rod and oven may be approximated as blackbody exchange.

PROPERTIES: *Table A-1*, Copper (300 K): $\rho = 8933$ kg/m^3, $c_p = 385$ J/kg·K, k = 401 W/m·K.

ANALYSIS: (a) Performing an energy balance on a unit length of the rod, $\dot{E}_{in} = \dot{E}_{st}$, or

$$q = Mc_p \frac{dT}{dt} = \rho\left(\frac{\pi D^2}{4} \times 1\right)c_p \frac{dT}{dt}$$

Neglecting convection, $q = q_{rad} = A_2 F_{21}\, \sigma\left(T_{sur}^4 - T^4\right) = A_1 F_{12}\, \sigma\left(T_{sur}^4 - T^4\right)$, where $A_1 = \pi D \times 1$ and $F_{12} = 1$. It follows that

$$\frac{dT}{dt} = \frac{\sigma \pi D\left(T_{sur}^4 - T^4\right)}{\rho\left(\pi D^2/4\right)c_p} = \frac{4\sigma\left(T_{sur}^4 - T^4\right)}{\rho D c_p} \qquad (1)$$

$$\left.\frac{dT}{dt}\right)_i = \frac{4\left[(1650\ \text{K})^4 - (300\ \text{K})^4\right]5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4}{8933\ \text{kg/m}^3\,(0.01\,\text{m})\,385\,\text{J/kg}\cdot\text{K}} = 48.8\ \text{K/s}.$$

<

(b) Using the *IHT Lumped Capacitance Model* to numerically integrate Eq. (2), we obtain

$$t_s = 15.0\ \text{s}$$

<

(c) With convection, $q = q_{rad} + q_{conv} = A_1 F_{12}\, \sigma\left(T_{sur}^4 - T^4\right) + hA_1 (T_\infty - T)$, and the energy balance becomes

$$\frac{dT}{dt} = \frac{4\sigma\left(T_{sur}^4 - T^4\right)}{\rho D c_p} + \frac{4h\left(T_\infty - T\right)}{\rho D c_p}$$

Performing the numerical integration for the three values of h, we obtain

h (W/m^2·K):	10	100	500
t_h (s):	14.6	12.0	6.8

COMMENTS: With an initial value of $h_{rad,i} = \sigma\left(T_{sur}^4 - T^4\right)/(T_{sur} - T) = 311$ W/m^2·K, Bi = h_{rad} (D/4)/k = 0.002 and the lumped capacitance assumption is justified for parts (a) and (b). With h = 500 W/m^2·K and $h + h_{r,i}$ = 811 W/m·K in part (c), Bi = 0.005 and the lumped capacitance approximation is also valid.

PROBLEM 13.39

KNOWN: Long, inclined black surfaces maintained at prescribed temperatures.

FIND: (a) Net radiation exchange between the two surfaces per unit length, (b) Net radiation transfer to surface A_2 with black, insulated surface positioned as shown below; determine temperature of this surface.

SCHEMATIC:

ASSUMPTIONS: (1) Surfaces behave as blackbodies, (2) Surfaces are very long in direction normal to page.

ANALYSIS: (a) The net radiation exchange between two black surfaces is given by Eq. 13.13,

$$q_{12} = A_1 F_{12} \sigma \left(T_1^4 - T_2^4 \right)$$

Noting that A_1 = width×length (ℓ) and that from symmetry, $F_{12} = 0.5$, find

$$q'_{12} = \frac{q_{12}}{\ell} = 0.1\text{ m} \times 0.5 \times 5.67 \times 10^{-8}\,\text{W/m}^2 \cdot \text{K}^4 \left(1000^4 - 800^4\right)\text{K}^4 = 1680\,\text{W/m}.$$ <

(b) With the insulated, black surface A_3 positioned as shown above, a three-surface enclosure is formed. From an energy balance on the node representing A_2, find

$$-q'_2 = q'_{32} + q'_{12}$$

$$-q_2 = A_3 F_{32} \left[E_{b3} - E_{b2}\right] + A_1 F_{12} \left[E_{b1} - E_{b2}\right].$$

To find E_{b3}, which at present is not known, perform an energy balance on the node representing A_3. Note that A_3 is adiabatic and, hence $q_3 = 0$, $q_{13} = q_{32}$.

$$A_1 F_{13} \left[E_{b1} - E_{b3}\right] = A_3 F_{32} \left[E_{b3} - E_{b2}\right]$$

Since $F_{13} = F_{23} = 0.5$ and $A_1 = A_3$, it follows that

$$E_{b3} = (1/2)\left[E_{b1} + E_{b2}\right]$$

and

$$-q'_2 = \left(A_3/\ell\right) F_{32} \left[\left(E_{b1} + E_{b2}\right)/2 - E_{b2}\right] + q'_{12}$$

$$-q'_2 = 0.1\text{ m} \times 0.5 \times 5.67 \times 10^{-8}\,\text{W/m}^2 \cdot \text{K}^4 \left[\left(1000^4 + 800^4\right)/2 - 800^4\right]\text{K}^4$$

$$+1680\text{ W/m} = 2517\text{ W/m}$$ <

Noting that $E_{b3} = \sigma T_3^4 = (1/2)\left[E_{b1} + E_{b2}\right]$, it follows that

$$T_3 = \left[\left(T_1^4 + T_2^4\right)/2\right]^{1/4} = \left[\left(1000^4 + 800^4\right)/2\right]^{1/4}\text{ K} = 916\text{ K}.$$ <

PROBLEM 13.40

KNOWN: Electrically heated tube suspended in a large vacuum chamber.

FIND: (a) Electrical power supplied to the heater, P_e, to maintain it at T_1 = 127°C, and (b) Compute and plot P_e as a function of tube length L for the range 25 ≤ L ≤ 250 mm for tube temperatures of T_1 = 127, 177 and 227°C.

SCHEMATIC:

ASSUMPTIONS: (1) All surfaces are blackbodies, (2) Tube of area A_1 is isothermal, (3) The surroundings are very large compared to the tube.

ANALYSIS: (a) Recognize that the surroundings can be represented by the surfaces A_2 and A_3, which are blackbodies at T_{sur}. This situation then permits calculation of necessary shape factors.

The net radiative heat rate from the heater, surface A_1, follows from Eq. 13.14, with $T_2 = T_3 = T_{sur}$ as

$$P_e = q_1 = A_1F_{12}\sigma\left(T_1^4 - T_2^4\right) + A_1F_{13}\sigma\left(T_1^4 - T_3^4\right). \quad (1)$$

Note that $F_{12} = F_{13}$ from symmetry considerations. Write now the summation rule for surface A_2

$$F_{21} + F_{22} + F_{23} = 1 \qquad \text{or} \qquad F_{21} = 1 - F_{23}$$

where F_{23} is determined from Fig. 13.5 using

$$\frac{r_j}{L} = \frac{0.75/2}{0.33} = 1.14 \qquad\qquad \frac{L}{r_i} = \frac{0.33}{0.75/2} = 0.88$$

giving F_{23} = 0.37. Hence F_{21} = 1 – 0.37 = 0.63. Using reciprocity, find now F_{12}

$$F_{12} = \frac{A_2}{A_1}F_{21} = \frac{\pi D^2/4}{\pi DL} \times F_{21} = \frac{\pi\left(75^2/4\right)}{\pi\left(75 \times 33\right)} \times 0.63 = 0.36.$$

Noting that $F_{12} = F_{13}$ and that $T_2 = T_3$, the electrical power using Eq. (1) with numerical values can be written as,

Continued …..

PROBLEM 13.40 (Cont.)

$$P_e = 2\left[\pi \times 0.75\text{m} \times 0.33\text{m} \times 0.36 \times 5.67 \times 10^{-8}\,\text{W/m}^2 \cdot \text{K}^4 \left[(127+273)^4 - (27+273)^4\right]\text{K}^4 = 555\,\text{W}.\right.$$ <

(b) Using the energy balance Eq. (1) of the foregoing analysis with the *IHT Radiation Tool-View Factors*, *Coaxial parallel disks*, P_e was computed as a function of L for selected tube temperatures.

As the tube length L increases, the heater power P_e required to maintain the tube at T_1 increases. Note that for small values of L, say L < 100 mm, P is linear with L. For larger values of L, P_e is not linear with L. Why is this so? What is the relationship between P_e and L for L >> 300 mm?

PROBLEM 13.41

KNOWN: Two horizontal, very large parallel plates with prescribed surface conditions and temperatures.

FIND: (a) Irradiation to the top plate, G_1, (b) Radiosity of the top plate, J_1, (c) Radiosity of the lower plate, J_2, (d) Net radiative exchange between the plates per unit area of the plates.

SCHEMATIC:

$T_1 = 1000K, \varepsilon_1 = 1$

$T_2 = 500K, \varepsilon_2 = 0.8$

ASSUMPTIONS: (1) Plates are sufficiently large to form a two surface enclosure and (2) Surfaces are diffuse-gray.

ANALYSIS: (a) The irradiation to the upper plate is defined as the radiant flux incident on that surface. The irradiation to the upper plate G_1 is comprised of flux emitted by surface 2 and reflected flux emitted by surface 1.

$\rho_2 E_{b1}$, $\varepsilon_2 E_{b2}$, T_1, T_2

$$G_1 = \varepsilon_2 E_{b2} + \rho_2 E_{b1} = \varepsilon_2 \sigma T_2^4 + (1-\varepsilon_2)\sigma T_1^4$$

$$G_1 = 0.8 \times 5.67 \times 10^{-8}\,\mathrm{W/m^2 \cdot K^4}\,(1000\ \mathrm{K})^4 + (1-0.8) \times 5.67 \times 10^{-8}\,\mathrm{W/m^2 \cdot K^4}\,(500\ \mathrm{K})^4$$

$$G_1 = 2835\,\mathrm{W/m^2} + 11{,}340\,\mathrm{W/m^2} = 14{,}175\,\mathrm{W/m^2}.$$ <

(b) The radiosity is defined as the radiant flux leaving the surface by emission and reflection. For the blackbody surface 1, it follows that

$$J_1 = E_{b1} = \sigma T_1^4 = 5.67 \times 10^{-8}\,\mathrm{W/m^2 \cdot K^4}\,(1000\ \mathrm{K})^4 = 56{,}700\,\mathrm{W/m^2}.$$ <

(c) The radiosity of surface 2 is then,

$$J_2 = \varepsilon_2 E_{b2} + \rho_2 G_2.$$

Since the upper plate is a blackbody, it follows that $G_2 = E_{b1}$ and

$$J_2 = \varepsilon_2 E_{b1} + \rho_2 E_{b1} = \varepsilon_2 \sigma T_2^4 + 1(1-\varepsilon_2)\sigma T_1^4 = 14{,}175\,\mathrm{W/m^2}.$$ <

Note that $J_2 = G_1$. That is, the radiant flux leaving surface 2 (J_2) is incident upon surface 1 (G_1).

(d) The net radiation heat exchange per unit area can be found by three relations.

$$q_1'' = J_1 - G_1 = (56{,}700 - 14{,}175)\,\mathrm{W/m^2} = 42{,}525\,\mathrm{W/m^2}$$

$$q_{12}'' = J_1 - J_2 = (56{,}700 - 14{,}175)\,\mathrm{W/m^2} = 42{,}525\,\mathrm{W/m^2}$$ <

The exchange relation, Eq. 13.24, is also appropriate with $\varepsilon_1 = 1$,

$$q_1'' = -q_2'' = q_{12}''$$

$$q_1'' = \varepsilon_2 \sigma \left(T_1^4 - T_2^4\right) = 0.8 \times 5.67 \times 10^{-8}\,\mathrm{W/m^2 \cdot K^4}\left(1000^4 - 500^4\right)\mathrm{K}^4 = 42{,}525\,\mathrm{W/m^2}.$$

PROBLEM 13.42

KNOWN: Dimensions and temperature of a flat-bottomed hole.

FIND: (a) Radiant power leaving the opening, (b) Effective emissivity of the cavity, ε_e, (c) Limit of ε_e as depth of hole increases.

SCHEMATIC:

L=24mm, D=6mm, A_1, $\varepsilon_1 = 0.8$, $T_1 = 1000K$, A_2

Thermal circuit: $q_1 \rightarrow E_{b1}$ — $\frac{1-\varepsilon_1}{\varepsilon_1 A_1}$ — J_1 — $\frac{1}{A_1 F_{12}}$ — J_2 — $\frac{1-\varepsilon_2}{\varepsilon_2 A_2}$ — $E_{b2} \rightarrow -q_2$

ASSUMPTIONS: (1) Hypothetical surface A_2 is a blackbody at 0 K, (2) Cavity surface is isothermal, opaque and diffuse-gray.

ANALYSIS: Approximating A_2 as a blackbody at 0 K implies that all of the radiation incident on A_2 from the cavity results (directly or indirectly) from emission by the walls and escapes to the surroundings. It follows that for A_2, $\varepsilon_2 = 1$ and $J_2 = E_{b2} = 0$.

(a) From the thermal circuit, the rate of radiation loss through the hole (A_2) is

$$q_1 = \left(E_{b1} - E_{b2}\right) / \left[\frac{1-\varepsilon_1}{\varepsilon_1 A_1} + \frac{1}{A_1 F_{12}} + \frac{1-\varepsilon_2}{\varepsilon_2 A_2}\right]. \quad (1)$$

Noting that $F_{21} = 1$ and $A_1 F_{12} = A_2 F_{21}$, also that

$$A_1 = \pi D^2/4 + \pi DL = \pi D\left(D/4 + L\right) = \pi\left(0.006\,\text{m}\right)\left(0.006\,\text{m}/4 + 0.024\,\text{m}\right) = 4.807\times10^{-4}\,\text{m}^2$$

$$A_2 = \pi D^2/4 = \pi\left(0.006\,\text{m}\right)^2/4 = 2.827\times10^{-5}\,\text{m}^2.$$

Substituting numerical values with $E_b = \sigma T^4$, find

$$q_1 = 5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left(1000^4 - 0\right)\text{K}^4 / \left[\frac{1-0.8}{0.8\times4.807\times10^{-4}\,\text{m}^2} + \frac{1}{2.827\times10^{-5}\,\text{m}^2} + 0\right]$$

$$q_1 = 1.580\ \text{W}. \quad <$$

(b) The effective emissivity, ε_e, of the cavity is defined as the ratio of the radiant power leaving the cavity to that from a blackbody having the same area of the cavity opening and at the temperature of the inner surfaces of the cavity. For the cavity above,

$$\varepsilon_e = \frac{q_1}{A_2 \sigma T_1^4}$$

$$\varepsilon_e = 1.580\,\text{W} / 2.827\times10^{-5}\,\text{m}^2\left(5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\right)\left(1000\ \text{K}\right)^4 = 0.986. \quad <$$

(c) As the depth of the hole increases, the term $(1 - \varepsilon_1)/\varepsilon_1 A_1$ goes to zero such that the remaining term in the denominator of Eq. (1) is $1/A_1 F_{12} = 1/A_2 F_{21}$. That is, as L increases, $q_1 \rightarrow A_2 F_{21} E_{b1}$. This implies that $\varepsilon_e \rightarrow 1$ as L increases. For L/D = 10, one would expect $\varepsilon_e = 0.999$ or better.

PROBLEM 13.43

KNOWN: Long V-groove machined in an isothermal block.

FIND: Radiant flux leaving the groove to the surroundings and effective emissivity.

SCHEMATIC:

ASSUMPTIONS: (1) Groove surface is diffuse-gray, (2) Groove is infinitely long, (3) Block is isothermal.

ANALYSIS: Define the hypothetical surface A_2 with T_2 = 0 K. The net radiation leaving A_1, q_1, will pass to the surroundings. From the two surface enclosure analysis, Eq. 13.23,

$$q_1 = -q_2 = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1-\varepsilon_1}{\varepsilon_1 A_1} + \frac{1}{A_1 F_{12}} + \frac{1-\varepsilon_2}{\varepsilon_2 A_2}}$$

Recognize that $\varepsilon_2 = 1$ and that from reciprocity, $A_1 F_{12} = A_2 F_{21}$ where $F_{21} = 1$. Hence,

$$\frac{q_1}{A_2} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1-\varepsilon_1}{\varepsilon_1}\frac{A_2}{A_1} + 1}$$

With $A_2/A_1 = 2\ell\ \tan 20° / \left(2\ell / \cos 20°\right) = \sin 20°$, find

$$q_1'' = \frac{5.67\times10^{-8}\,\mathrm{W/m^2\cdot K^4}\left(1000^4 - 0\right)\mathrm{K^4}}{\frac{(1-0.6)}{0.6}\times\sin 20° + 1} = 46.17\ \mathrm{kW/m^2}. \quad <$$

The effective emissivity of the groove follows from the definition given in Problem 13.42 as the ratio of the radiant power leaving the cavity to that from a blackbody having the area of the cavity opening and at the same temperature as the cavity surface. For the present situation,

$$\varepsilon_e = \frac{q_1''}{E_b\left(T_1\right)} = \frac{q_1''}{\sigma T_1^4} = \frac{46.17\times10^{+3}\,\mathrm{W/m^2}}{5.67\times10^{-8}\,\mathrm{W/m^2\cdot K^4}\left(1000\ \mathrm{K}\right)^4} = 0.814. \quad <$$

COMMENTS: Note the use of the hypothetical surface defined as black at 0 K. This surface does not emit and absorbs all radiation on it; hence, is the radiant power to the surroundings.

PROBLEM 13.44

KNOWN: Conical cavity formed in an isothermal, opaque, diffuse-gray material of emissivity ε and temperature T.

FIND: Radiant power leaving the opening of the cavity in terms of T, ε, r_o, and L.

SCHEMATIC:

ASSUMPTIONS: (1) Material is opaque, diffuse-gray, and isothermal, (2) Cavity opening is hypothetical black surface at 0 K.

ANALYSIS: Define A_o, the opening of the cavity, as a black surface at $T_o = 0$ K. Considering A_o and A_c as a two surface, diffuse-gray enclosure, the radiant power leaving the cavity opening is

$$q_{cavity} = -q_o = \left[E_b(T) - E_b(T_o)\right] / \left[\frac{1-\varepsilon}{\varepsilon A_c} + \frac{1}{A_c F_{co}} + \frac{1-\varepsilon_o}{\varepsilon_o A_o}\right]$$

Recognizing that $E_b(T_o) = 0$ and $\varepsilon_o = 1$ and also, using reciprocity,

$$A_c F_{co} = A_o F_{oc}$$

and from the enclosure, note $F_{oc} = 1$. Hence,

$$q_{cavity} = \frac{E_b(T)}{\frac{1-\varepsilon}{\varepsilon A_c} + \frac{1}{A_c F_{co}}} = \frac{\sigma T^4}{\frac{1-\varepsilon}{\varepsilon A_c} + \frac{1}{A_o F_{oc}}} = \frac{A_o \sigma T^4}{\frac{1-\varepsilon}{\varepsilon} \cdot \frac{A_o}{A_c} + 1}. \tag{1}$$

Noting that $A_o = \pi r_o^2$ and $A_c = \pi r_o \left(L^2 + r_o^2\right)^{1/2}$, find

$$q_{cavity} = \frac{\pi r_o^2 \cdot \sigma T^4}{\frac{1-\varepsilon}{\varepsilon} \cdot \frac{1}{\left[\left(L/r_o\right)^2 + 1\right]^{1/2}} + 1}. \qquad <$$

COMMENTS: When L increases or $A_o/A_c << 1$, the radiant power approaches that of a blackbody according to Eq. (1).

PROBLEM 13.45

KNOWN: Cavities formed by a cone, cylinder, and sphere having the same opening size (d) and major dimension (L) with prescribed wall emissivity.

FIND: (a) View factor between the inner surface of each cavity and the opening of the cavity; (b) Effective emissivity of each cavity as defined in Problem 13.42, if the walls are diffuse-gray with ε_w; and (c) Compute and plot ε_e as a function of the major dimension-to-opening size ratio, L/d, over the range from 1 to 10 for wall emissivities of ε_w = 0.5, 0.7, and 0.9.

SCHEMATIC:

Cone: d, A_2, A_1, T_1, L

Cylinder: d, L

Sphere: d, D = L

ASSUMPTIONS: (1) Diffuse-gray surfaces, (2) Uniform radiosity over the surfaces.

ANALYSIS: (a) Using the summation rule and reciprocity, determine the view factor F_{12} for each of the cavities considered as a two-surface enclosure.

Cone: $F_{21} + F_{22} = F_{21} + 0 = 1 \qquad F_{21} = 1$

$$F_{12} = A_2 F_{21}/A_1 = \left(\pi d^2/4\right)/(\pi d/2)\left[L^2 + (d/2)^2\right]^{1/2} = (1/2)\left[(L/d)^2 + 1/4\right]^{-1/2} \quad <$$

Cylinder: $F_{21} = 1$

$$F_{12} = A_2 F_{21}/A_1 = A_2/A_1 = \left(\pi d^2/4\right)/\left[\pi dL + \pi d^2/4\right] = (1 + 4L/d)^{-1} \quad <$$

Sphere: $F_{21} = 1$

$$F_{12} = A_2 F_{21}/A_1 = A_2/A_1 = \left(\pi d^2/4\right)/\left[\pi D^2 - \pi d^2/4\right] = \left(4D^2/d^2 - 1\right)^{-1}. \quad <$$

(b) The effective emissivity of the cavity is defined as

$$\varepsilon_{eff} = q_{12}/q_c$$

where $q_c = A_2 \sigma T_1^4$ which presumes the opening is a black surface at T_1 and for the two-surface enclosure,

$$q_{12} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{(1-\varepsilon_1)/\varepsilon_1 A_1 + 1/A_1 F_{12} + (1-\varepsilon_2)/\varepsilon_2 A_2} = \frac{A_1 \sigma T_1^4}{(1-\varepsilon_1)/\varepsilon_1 + 1/F_{12}}$$

since T_2 = 0K and ε_2 = 1. Hence, since $A_2/A_1 = F_{12}$ for all the cavities, with $\varepsilon_1 = \varepsilon_w$

$$\varepsilon_{eff} = \frac{1/F_{12}}{(1-\varepsilon_w)/\varepsilon_w + 1/F_{12}} = \frac{1}{F_{12}(1-\varepsilon_w)/\varepsilon_w + 1}$$

Cone: $$\varepsilon_{eff} = 1\Big/\left\{(1/2)\left[(L/d)^2 + 1/4\right]^{-1/2}(1-\varepsilon_w)/\varepsilon_w + 1\right\} \qquad (1) \quad <$$

Continued …..

PROBLEM 13.45 (Cont.)

Cylinder: $$\varepsilon_{eff} = 1 \Big/ \left\{ \left[1 + 4L/d\right]^{-1} \left(1 - \varepsilon_w\right)/\varepsilon_w + 1 \right\} \qquad (2)$$ <

Sphere: $$\varepsilon_{eff} = 1 / \left\{ \left[4D^2/d^2 - 1\right]^{-1} \left(1 - \varepsilon_w\right)/\varepsilon_w + 1 \right\} \qquad (3)$$ <

(c) Using the *IHT* Workspace with eqs. (1,2,3), the effective emissivity was computed as a function of L/d (cone, cylinder and sphere) for selected wall emissivities. The results are plotted below.

Fig. 1 Cone, cylinder, sphere cavities, eps = 0.7

Fig 2 Conical cavity

In Fig. 1, ε_{eff} is shown as a function of L/d for $\varepsilon_w = 0.7$. For larger L/d, the sphere has the highest ε_{eff} and the cone the lowest. Figures 2, 3 and 4 illustrate the ε_{eff} vs. L/d for each of the cavity types. As expected, ε_{eff} increases with increasing wall emissivity.

Fig 3 Cylindrical cavity

Fig 4 Spherical cavity

Note that for the spherical cavity, with L/d ≥ 5, $\varepsilon_{eff} > 0.98$ even with ε_w as low as 0.5. This feature makes the use of spherical cavities for high performance radiometry applications attractive since ε_{eff} is not very sensitive to ε_w.

COMMENTS: In Fig. 1, intercomparing ε_{eff} for the three cavity types, can you give a physical explanation for the results?

PROBLEM 13.46

KNOWN: Very long diffuse, gray, thin-walled tube of 1-m radius contained inside a long black duct of square cross-section, 3.2 m $\times$ 3.2 m. Top portion is open as shown schematically.

FIND: (a) Net radiant heat transfer rate per unit length of the tube from the opening, $q_1' = q_1 / L$, and the effective emissivity of the opening, ε_{eff}, for the condition when $\theta = 45°$ and (b) Compute and plot q_1' and ε_{eff} as a function of θ for the range $0 \le \theta \le 180°$.

SCHEMATIC:

θ
Long duct, 3.2 m x 3.2 m
Long tube, R = 1 m
A_3
A_1, T_1 = 1000 K, ε_1
A_3, T_3 = 1000 K, ε_3
A_2, T_2 = 300 K, ε_2 = 1

ASSUMPTIONS: (1) Tube is very long compared to its radius R and duct dimension, (2) Interior of cylinder is diffuse, gray, (3) Interior of the duct is black.

ANALYSIS: (a) Consider the two-surface enclosure formed by the inner surface of the tube, A_1, and the hypothetical surface formed by the opening, A_3. The surface A_3 behaves as a blackbody ($\varepsilon_3 = 1$) at a temperature $T_3 = T_2 = 300$ K. The net heat rate leaving the opening follows from Eq. 13.23.

$$q_1 = -q_1 = \frac{\sigma\left(T_1^4 - T_2^4\right)}{(1-\varepsilon_1)/\varepsilon_1 A_1 + 1/A_1 F_{13} + (1-\varepsilon_3)/\varepsilon_3 A_3} \qquad (1)$$

The view factor F_{13} can be determined from the reciprocity relation recognizing that $F_{31} = 1$.

$$F_{13} = A_3 F_{31} / A_1 = 1.414\text{L} \times 1/4.712\text{L} = 0.300 \qquad (2)$$

where the areas A_1 and A_3 are, with $\theta = 45°$,

$$A_1 = 2\text{R}(\pi - \theta)\text{L} = 2\times 1\,\text{m}\left[\pi - 45\times\pi/180\right]\times\text{L} = 4.712\text{L} \qquad (3)$$

$$A_3 = 2\text{R}\sin\theta\,\text{L} = 2\times 1\,\text{m}\times\sin 45°\times\text{L} = 1.414\text{L} \qquad (4)$$

Substituting numerical values, find

$$q_1 = -q_3 = \frac{5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left(1000^4 - 300^4\right)\text{K}^4}{(1-0.1)/(0.1\times4.712\text{L}) + 1/(4.712\text{L}\times0.300) + 0}$$

$$q_1/\text{L} = -q_3/\text{L} = 21{,}487\,\text{W/m}$$ <

Continued …..

PROBLEM 13.46 (Cont.)

The effective emissivity of the opening is the ratio of the radiant power leaving the opening to that of a blackbody having the area of the opening (A_3) and a temperature of the inner surface of the cavity (T_1).

$$\varepsilon_{eff} = \frac{q_1}{A_3 \sigma T_1^4} = \frac{q_1 / L}{(A_3 / L) \sigma T_1^4} \qquad (5)$$

$$\varepsilon_{eff} = \frac{21,487 \text{ W/m}}{1.414 \text{ m} \times 5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4 (1000 \text{ K})^4} = 0.268$$ <

(b) Using the foregoing equations, Eqs. (1-5), in the *IHT* workspace, $q_1' = q_1 / L$ and ε_{eff} as a function of θ were computed and are plotted below.

Note that $q_1' = 0$ when $\theta = 0°$ since the tube is closed and no power leaves the tube. At $\theta = 180°$, the area of the tube has been reduced to zero and hence, $q_1' = 0$. For small values of θ, ε_{eff} is highest and decreases as θ increases, to the limit $\varepsilon_{eff} = \varepsilon_1 = 0.1$.

PROBLEM 13.47

KNOWN: Temperature, emissivity and dimensions of a rectangular fin array radiating to deep space.

FIND: (a) Rate of radiation transfer per unit length from a unit section to space, (b) Effect of emissivity on heat rejection.

SCHEMATIC:

Deep space

$A_3' = W$, $T_3 = 0$ K, $\varepsilon_3 = 1$

$A_2' = 2L$, $T_2 = 325$ K, $\varepsilon_2 = 0.7$

L = 0.125 m

$A_1' = W$, $T_1 = 325$ K, $\varepsilon_1 = 0.7$

W = 0.025 m

ASSUMPTIONS: (1) Diffuse/gray surface behavior, (2) Length of array (normal to page) is much larger than W and L, (3) Isothermal surfaces.

ANALYSIS: (a) Since the sides and base of the U-section have the same temperature and emissivity, they can be treated as a single surface and the U-section becomes a two-surface enclosure. Deep space may be represented by the hypothetical surface A_3', which acts as a blackbody at absolute zero temperature. From Eq. (13.23), with $T_1 = T_2 = T$ and $\varepsilon_1 = \varepsilon_2 = \varepsilon$,

$$q'_{(1,2)3} = \frac{\sigma\left(T^4 - T_3^4\right)}{\dfrac{1-\varepsilon}{\varepsilon A'_{(1,2)}} + \dfrac{1}{A'_{(1,2)} F_{(1,2)3}} + \dfrac{1-\varepsilon}{\varepsilon A_3'}}$$

where $A'_{(1,2)} = 2L + W$, $A_3' = W$, $A'_{(1,2)} F_{(1,2)3} = A_3' F_{3(1,2)} = W$. Hence,

$$q'_{(1,2)3} = \frac{\sigma T^4}{\dfrac{1-\varepsilon}{\varepsilon(2L+W)} + \dfrac{1}{W} + \dfrac{1-\varepsilon}{\varepsilon W}}$$

$$q'_{(1,2)3} = \frac{5,67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\,(325\ \text{K})^4}{\dfrac{1-0.70}{0.70(0.275\text{m})} + \dfrac{1}{0.025\text{m}} + 0} = 15.2\ \text{W/m}$$ <

(b) For ε = 0.7 emission from the base of the U-section is $q_b' = \varepsilon A_1' \sigma T^4 = 0.7\times0.025\text{m} \times 5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\,(325\ \text{K})^4 = 11.1\ \text{W/m}$. The effect of ε on $q'_{(1,2)3}$ and q_b' is shown as follows.

Continued …..

PROBLEM 13.47 (Cont.)

Radiation heat rate (W/m)

16
14
12
10
8
6
4
2
0

0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

Emissivity, eps

Heat rejection with fins, q'(1,2)3 (W/m)
Emission from base, qb' (W/m)

The effect of the fins on heat transfer enhancement increases with decreasing emissivity.

COMMENTS: Note that, if the surfaces behaved as blackbodies ($\varepsilon_1 = \varepsilon_2 = 1.0$), the U-section becomes a blackbody cavity for which heat rejection is simply $A_3' \, E_b\,(T) = q_b'$. Hence, it is no surprise that the $q_b' \rightarrow q_{(1,2)3}'$ as $\varepsilon \rightarrow 1$ in the foregoing figure. For $\varepsilon = 1$, no enhancement is provided by the fins.

PROBLEM 13.48

KNOWN: Power dissipation of electronic device and thermal resistance associated with attachment to inner wall of a cubical container. Emissivity of outer surface of container and wall temperature of service bay.

FIND: Temperatures of cubical container and device.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state, (2) Device and container are isothermal, (3) Heat transfer from the container is exclusively by radiation exchange with bay (small surface in a large enclosure), (4) Container surface may be approximated as diffuse/gray.

ANALYSIS: From Eq. (13.27)

$$P_e = q = \sigma\left(6W^2\right)\varepsilon_c\left(T_{s,c}^4 - T_{sur}^4\right)$$

$$T_{s,c} = \left[\frac{q}{\sigma\left(6W^2\right)\varepsilon_c} + T_{sur}^4\right]^{1/4}$$

$$T_{s,c} = \left[\frac{50\,W}{5.67\times10^{-8}\,W/m^2\cdot K^4\times 6(0.12m)^2\times 0.8} + (150\,K)^4\right]^{1/4} = 339.4\,K = 66.4°C$$ <

With $q = \left(T_d - T_{s,c}\right)/R_t$,

$$T_d = q\,R_t + T_{s,c} = 50\,W\times 0.1\,K/W + 66.4°C = 71.4°C$$ <

COMMENTS: If the temperature of the device is too large to insure reliable operation, it may be reduced by increasing ε_c or W.

PROBLEM 13.49

KNOWN: Long, thin-walled horizontal tube with radiation shield having an air gap of 10 mm. Emissivities and temperatures of surfaces are prescribed.

FIND: Radiant heat transfer from the tube per unit length.

SCHEMATIC:

$T_2 = 35°C$
Air gap
$T_1 = 120°C$
Shield, $D_2 = 120$ mm, $\varepsilon_2 = 0.1$
Tube, $D_1 = 100$ mm, $\varepsilon_1 = 0.8$

ASSUMPTIONS: (1) Tube and shield are very long, (2) Surfaces at uniform temperatures, (3) Surfaces are diffuse-gray.

ANALYSIS: The long tube and shield form a two surface enclosure, and since the surfaces are diffuse-gray, the radiant heat transfer from the tube, according to Eq. 13.23, is

$$q_{12} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} + \dfrac{1}{A_1 F_{12}} + \dfrac{1-\varepsilon_2}{\varepsilon_2 A_2}} \tag{1}$$

By inspection, $F_{12} = 1$. Note that

$$A_1 = \pi D_1 \ell \qquad \text{and} \qquad A_2 = \pi D_2 \ell$$

where ℓ is the length of the tube and shield. Dividing Eq. (1) by ℓ, find the heat rate per unit length,

$$q'_{12} = \frac{q_{12}}{\ell} = \frac{5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}\left[(273+120)^4 - (273+35)^4\right]\text{K}^4}{\dfrac{1-0.8}{0.8\pi\left(100\times10^{-3}\,\text{m}\right)} + \dfrac{1}{\pi\left(100\times10^{-3}\,\text{m}\right)\times1} + \dfrac{1-0.1}{0.1\pi\left(120\times10^{-3}\,\text{m}\right)}}$$

$$q'_{12} = \frac{842.3\ \text{W/m}^2}{(0.7958+3.183+23.87)\,\text{m}^{-1}} = 30.2\ \text{W/m}.$$ <

COMMENTS: Recognize that convective heat transfer would be important in this annular air gap. Suitable correlations to estimate the heat transfer coefficient are given in Chapter 9.

PROBLEM 13.50

KNOWN: Long electrical conductor with known heat dissipation is cooled by a concentric tube arrangement.

FIND: Surface temperature of the conductor.

SCHEMATIC:

ASSUMPTIONS: (1) Surfaces are diffuse-gray, (2) Conductor and cooling tube are concentric and very long, (3) Space between surfaces is evacuated.

ANALYSIS: The heat transfer by radiation exchange between the conductor and the concentric, cooled cylinder is given by Eq. 13.25. For a unit length,

$$q'_{12} = \frac{q_{12}}{\ell} = \sigma \cdot 2\pi r_1 \left(T_1^4 - T_2^4\right) / \left[\frac{1}{\varepsilon_1} + \frac{1-\varepsilon_2}{\varepsilon_2}\left(\frac{r_1}{r_2}\right)\right] \qquad (1)$$

where $A_1 = 2\pi r_1 \cdot \ell$. Solving for T_1 and substituting numerical values, find

$$T_1 = \left\{T_2^4 + \frac{q'_{12}}{\sigma \cdot 2\pi r_1}\left[\frac{1}{\varepsilon_1} + \frac{1-\varepsilon_2}{\varepsilon_2}\left(\frac{r_1}{r_2}\right)\right]\right\}^{1/4}$$

$$T_1 = \left\{(27+273)^4 \text{ K}^4 + \frac{6 \text{ W/m}}{5.67\times10^{-8} \text{W/m}^2 \cdot \text{K}^4 \times 2\pi\,(0.005\text{m})}\left[\frac{1}{0.6} + \frac{1-0.9}{0.9}\left(\frac{5}{25}\right)\right]\right\}^{1/4}$$

$$T_1 = \left\{(300 \text{ K})^4 + 3.368\times10^9 \text{K}^4 \left[1.667 + 0.00222\right]\right\}^{1/4} \qquad (2)$$

$$T_1 = 342.3 \text{ K} = 69°\text{C}.$$ <

COMMENTS: (1) Note that Eq. (1) implies that $F_{12} = 1$. From Eq. (2) by comparison of the second term in the brackets involving ε_2, note that the influence of ε_2 is small. This follows since $r_1 << r_2$.

PROBLEM 13.51

KNOWN: Arrangement for direct thermophotovoitaic conversion of thermal energy to electrical power.

FIND: (a) Radiant heat transfer between the inner and outer surface per unit area of the outer surface, (b) Power generation per unit outer surface area if semiconductor has 10% conversion efficiency for radiant power in the 0.6 to 2.0 μm range.

SCHEMATIC:

ASSUMPTIONS: (1) Surfaces are diffuse-gray, (2) Surfaces approximate long, concentric cylinder, two-surface enclosure with negligible end effects.

ANALYSIS: (a) For this two-surface enclosure, the net radiation exchange per unit area of the outer surface is,

$$\frac{q_{io}}{A_o} = \frac{A_i}{A_o} \cdot \frac{\sigma\left(T_i^4 - T_o^4\right)}{\frac{1}{\varepsilon_i} + \frac{1-\varepsilon_o}{\varepsilon_o}\left(\frac{r_i}{r_o}\right)} \tag{1}$$

and since $A_i/A_o = 2\pi r_i \ell / 2\pi r_o \ell = r_1 / r_o$, the heat flux at surface A_o is

$$\frac{q_{io}}{A_o} = \left(\frac{0.0125}{0.190}\right)\frac{\left(5.67\times10^{-8}\,\mathrm{W/m^2\cdot K^4}\right)\left(1948^4 - 293^4\right)\mathrm{K}^4}{\frac{1}{0.9} + \frac{1-0.5}{0.5}\left(\frac{0.125}{0.190}\right)} = 45.62\ \mathrm{kW/m^2}. \quad (2) \quad <$$

(b) The power generation per unit area of surface A_o can be expressed as

$$P_e'' = \eta_e \cdot G_{abs}\left(0.6 \to 2.0\mu\mathrm{m}\right) \tag{3}$$

where η_e is the semiconductor conversion efficiency and G_{abs} (0.6 → 2.0μm) represents the absorbed irradiation on A_o in the prescribed wavelength interval. The *total* absorbed irradiation is $G_{abs,t}$ = q_{io}/A_o and has the spectral distribution of a blackbody at T_i since $T_o^4 << T_i^4$ and A_i is gray. Hence, we can write Eq. (3) as

$$P_e'' = \eta_e \cdot \left(q_{io}/A_o\right)\left[F_{(0\to2\mu m)} - F_{(0\to0.6\mu m)}\right]. \tag{4}$$

From Table 12.2: $\lambda T = 2 \times 1948 = 3896$ μm·K, $F_{(0-\lambda T)} = 0.461$; $\lambda T = 0.6 \times 1948 = 1169$ μm·K, $F_{(0-\lambda T)} = 0.0019$. Hence

$$P_e'' = 0.1\left(45.62\ \mathrm{kW/m^2}\right)\left[0.461 - 0.0019\right] = 2.09\ \mathrm{kW/m^2}. \quad <$$

That is, the unit produces 2.09 kW per unit area of the outer surface.

PROBLEM 13.52

KNOWN: Temperatures and emissivities of spherical surfaces which form an enclosure.

FIND: Evaporation rate of oxygen stored in inner container.

SCHEMATIC:

$D_i = 0.8m$
$T_i = 95K$
$\varepsilon_i = 0.05$
Vent and support assembly
$D_o = 1.2m$
$T_o = 280K$
$\varepsilon_o = 0.05$
Liquid oxygen
Evacuated space

PROPERTIES: Oxygen (given): $h_{fg} = 2.13 \times 10^5$ J/kg.

ASSUMPTIONS: (1) Opaque, diffuse-gray surfaces, (2) Evacuated space between surfaces, (3) Negligible heat transfer along vent and support assembly.

ANALYSIS: From an energy balance on the inner container, the net radiation heat transfer to the container may be equated to the evaporative heat loss

$$q_{oi} = \dot{m} h_{fg}.$$

Substituting from Eq. (13.26), where $q_{oi} = - q_{io}$ and $F_{io} = 1$

$$\dot{m} = \frac{-\sigma\left(\pi D_i^2\right)\left(T_i^4 - T_o^4\right)}{h_{fg}\left[\dfrac{1}{\varepsilon_i} + \dfrac{1-\varepsilon_o}{\varepsilon_o}\left(\dfrac{r_i}{r_o}\right)^2\right]}$$

$$\dot{m} = \frac{-5.67 \times 10^{-8} \text{W/m}^2 \cdot \text{K}^4 \times \pi\,(0.8\text{m})^2\left(95^4 - 280^4\right)\text{K}^4}{2.13 \times 10^5 \text{J/kg}\left[\dfrac{1}{0.05} + \dfrac{0.95}{0.05}\left(\dfrac{0.4}{0.6}\right)^2\right]}$$

$$\dot{m} = 1.14 \times 10^{-4} \text{kg/s}.$$ <

COMMENTS: This loss could be reduced by insulating the outer surface of the outer container and/or by inserting a radiation shield between the containers.

PROBLEM 13.53

KNOWN: Emissivities, diameters and temperatures of concentric spheres.

FIND: (a) Radiation transfer rate for black surfaces. (b) Radiation transfer rate for diffuse-gray surfaces, (c) Effects of increasing the diameter and assuming blackbody behavior for the outer sphere. (d) Effect of emissivities on net radiation exchange.

SCHEMATIC:

$F_{12} = 1$

$D_1 = 0.8$ m
$T_1 = 400$ K
$\varepsilon_1 = 0.1, 0.5, 1.0$

$D_2 = 1.2$ m or 20 m
$T_2 = 300$ K
$0.05 \leq \varepsilon_2 \leq 1.0$

Airspace

E_{b1}, J_1, J_2, E_{b2}; $\frac{1-\varepsilon_1}{\varepsilon_1 A_1}$, $\frac{1}{A_1 F_{12}}$, $\frac{1-\varepsilon_2}{\varepsilon_2 A_2}$

ASSUMPTIONS: (1) Blackbody or diffuse-gray surface behavior.

ANALYSIS: (a) Assuming blackbody behavior, it follows from Eq. 13.13

$$q_{12} = A_1 F_{12} \sigma \left(T_1^2 - T_2^4\right) = \pi \left(0.8\,\text{m}\right)^2 (1) 5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4 \left[\left(400\ \text{K}\right)^4 - \left(300\ \text{K}\right)^4\right] = 1995\ \text{W}. \quad <$$

(b) For diffuse-gray surface behavior, it follows from Eq. 13.26

$$q_{12} = \frac{\sigma A_1 \left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} + \frac{1-\varepsilon_2}{\varepsilon_2}\left(\frac{r_1}{r_2}\right)^2} = \frac{5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4 \pi \left(0.8\,\text{m}\right)^2 \left[400^4 - 300^4\right]\text{K}^4}{\frac{1}{0.5} + \frac{1-0.05}{0.05}\left(\frac{0.4}{0.6}\right)^2} = 191\ \text{W}. \quad <$$

(c) With $D_2 = 20$ m, it follows from Eq. 13.26

$$q_{12} = \frac{5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K} \pi \left(0.8\,\text{m}\right)^2 \left[\left(400\ \text{K}\right)^4 - \left(300\ \text{K}\right)^4\right]}{\frac{1}{0.5} + \frac{1-0.05}{0.05}\left(\frac{0.4}{10}\right)^2} = 983\ \text{W}. \quad <$$

With $\varepsilon_2 = 1$, instead of 0.05, Eq. 13.26 reduces to Eq. 13.27 and

$$q_{12} = \sigma A_1 \varepsilon_1 \left(T_1^4 - T_2^4\right) = 5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4 \pi \left(0.8\,\text{m}\right)^2 0.5 \left[\left(400\ \text{K}\right)^4 - \left(300\ \text{K}\right)^4\right] = 998\ \text{W}. \quad <$$

Continued …..

PROBLEM 13.53 (Cont.)

(d) Using the *IHT Radiation* Tool Pad, the following results were obtained

Net radiation exchange increases with ε_1 and ε_2, and the trends are due to increases in emission from and absorption by surfaces 1 and 2, respectively.

COMMENTS: From part (c) it is evident that the actual surface emissivity of a *large* enclosure has a small effect on radiation exchange with small surfaces in the enclosure. Working with $\varepsilon_2 = 1.0$ instead of $\varepsilon_2 = 0.05$, the value of q_{12} is increased by only (998 – 983)/983 = 1.5%. In contrast, from the results of (d) it is evident that the surface emissivity ε_2 of a *small* enclosure has a large effect on radiation exchange with interior objects, which increases with increasing ε_1.

PROBLEM 13.54

KNOWN: Two radiation shields positioned in the evacuated space between two infinite, parallel planes.

FIND: Steady-state temperature of the shields.

SCHEMATIC:

ASSUMPTIONS: (1) All surfaces are diffuse-gray and (2) All surfaces are parallel and of infinite extent.

ANALYSIS: The planes and shields can be represented by a thermal circuit from which it follows that

$$q_1'' = -q_2'' = \frac{\sigma\left(T_1^4 - T_2^4\right)}{R_1'' + R_2'' + R_3''} = \frac{\sigma\left(T_1^4 - T_{s1}^4\right)}{R_1''} = \frac{\sigma\left(T_{s1}^4 - T_{s2}^4\right)}{R_2''} = \frac{\sigma\left(T_{s2}^4 - T_2^4\right)}{R_3''}.$$

Since all the emissivities involved are equal, $R_1'' = \frac{A_1}{A_1 F_{12}} = 1 = R_2'' = R_3''$, so that

$$T_{s1}^4 = T_1^4 - \frac{R_1''}{R_1'' + R_2'' + R_3''}\left(T_1^4 - T_2^4\right) = T_1^4 - (1/3)\left(T_1^4 - T_2^4\right)$$

$$T_{s1}^4 = (600\text{ K})^4 - (1/3)\left(600^4 - 325^4\right)\text{K}^4 \qquad T_{1s} = 548\text{ K}$$ <

$$T_{s2}^4 = T_2^4 + \frac{R_3''}{R_1'' + R_2'' + R_3''}\left(T_1^4 - T_2^4\right) = T_2^4 + (1/3)\left(T_1^4 - T_2^4\right)$$

$$T_{s2}^4 = (325\text{ K})^4 + (1/3)\left(600^4 - 325^4\right)\text{K}^4 \qquad T_{s2} = 474\text{ K}.$$ <

PROBLEM 13.55

KNOWN: Two large, infinite parallel plates that are diffuse-gray with temperatures and emissivities of T_1 and ε_1 and T_2 and ε_2.

FIND: Show that the ratio of the radiation transfer rate with multiple shields, N, of emissivity ε_s to that with no shields, N = 0, is

$$\frac{q_{12,N}}{q_{12,0}} = \frac{\left[1/\varepsilon_1 + 1/\varepsilon_2 - 1\right]}{\left[1/\varepsilon_1 + 1/\varepsilon_2 - 1\right] + N\left[2/\varepsilon_s - 1\right]}$$

where $q_{12,N}$ and $q_{12,0}$ represent the radiation heat rate with N and N = 0 shields, respectively.

SCHEMATIC:

ASSUMPTIONS: (1) Plane infinite planes with diffuse-gray surfaces and uniform radiosities, and (2) Shield has negligible thermal conduction resistance.

ANALYSIS: Representing the parallel plates by the resistance network shown above for the "no-shield" condition, N = 0, with $F_{12} = 1$, the heat rate per unit area follows from Eq. 13.24 (see also Fig. 13.11) as

$$q''_{12,0} = \frac{E_{b1} - E_{b2}}{1/\varepsilon_1 + 1/\varepsilon_2 - 1} \tag{1}$$

With the addition of each shield as shown in the schematic above, three resistance elements are added to the network: two surface resistances, $(1 - \varepsilon_s)/\varepsilon_s$, and one space resistance, $1/F_{ij} = 1$. Hence, for the "N - shield" condition,

$$q''_{12,N} = \frac{E_{b1} - E_{b2}}{\left[1/\varepsilon_1 + 1/\varepsilon_2 - 1\right] + N\left[2(1-\varepsilon_s)/\varepsilon_s + 1\right]} \tag{2}$$

The ratio of the heat rates is obtained by dividing Eq. (2) by Eq. (1),

$$\frac{q''_{12,N}}{q_{12,0}} = \frac{\left[1/\varepsilon_1 + 1/\varepsilon_2 - 1\right]}{\left[1/\varepsilon_1 + 1/\varepsilon_2 - 1\right] + N\left[2/\varepsilon_s - 1\right]}$$ <

COMMENTS: Can you derive an expression to determine the temperature difference across pairs of the N-shields?

PROBLEM 13.56

KNOWN: Emissivities of two large, parallel surfaces.

FIND: Heat shield emissivity needed to reduce radiation transfer by a factor of 10.

SCHEMATIC:

Radiation shield
$\varepsilon_1 = 0.8$
ε_3
q_{12}
$\varepsilon_2 = 0.8$

ASSUMPTIONS: (a) Diffuse-gray surface behavior, (b) Negligible conduction resistance for shield, (c) Same emissivity on opposite sides of shield.

ANALYSIS: For this arrangement, $F_{13} = F_{32} = 1$.

Without (wo) the shield, it follows from Eq. 13.24,

$$(q_{12})_{wo} = \frac{A_1\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}.$$

With (w) the shield it follows from Eq. 13.28,

$$(q_{12})_{w} = \frac{A_1\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} + \frac{2}{\varepsilon_3} - 2}.$$

Hence, the heat rate ratio is

$$\frac{(q_{12})_w}{(q_{12})_{wo}} = 0.1 = \frac{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} + \frac{2}{\varepsilon_3} - 2} = \frac{\frac{1}{0.8} + \frac{1}{0.8} - 1}{\frac{1}{0.8} + \frac{1}{0.8} + \frac{2}{\varepsilon_3} - 2}.$$

Solving, find

$$\varepsilon_3 = 0.138. \qquad <$$

COMMENTS: The foregoing result is independent of T_1 and T_2. It is only necessary that the temperatures be maintained at fixed values, irrespective of whether or not the shield is in place.

PROBLEM 13.57

KNOWN: Surface emissivities of a radiation shield inserted between parallel plates of prescribed temperatures and emissivities.

FIND: (a) Effect of shield orientation on radiation transfer, (b) Effect of shield orientation on shield temperature.

SCHEMATIC:

ASSUMPTIONS: (1) Diffuse-gray surface behavior, (2) Shield is isothermal.

ANALYSIS: (a) On a unit area basis, the network representation of the system is

Hence the total radiation resistance,

$$R = \frac{1-\varepsilon_1}{\varepsilon_1} + 1 + \frac{1-\varepsilon_s}{\varepsilon_s} + \frac{1-2\varepsilon_s}{2\varepsilon_s} + 1 + \frac{1-\varepsilon_2}{\varepsilon_2}$$

is independent of orientation. Since $q = (E_{b1} - E_{b2})/R$, the heat transfer rate is independent of orientation.

(b) Considering that portion of the circuit between E_{b1} and E_{bs}, it follows that

$$q = \frac{E_{b1} - E_{bs}}{\frac{1-\varepsilon_1}{\varepsilon_1} + 1 + f(\varepsilon_s)}, \quad \text{where} \quad f(\varepsilon_s) = \frac{1-\varepsilon_s}{\varepsilon_s} \text{ or } \frac{1-2\varepsilon_s}{2\varepsilon_s}.$$

Hence,

$$E_{bs} = E_{b1} - \left[\frac{1-\varepsilon_1}{\varepsilon_1} + 1 + f(\varepsilon_s)\right] q.$$

It follows that, since E_{bs} increases with decreasing $f(\varepsilon_s)$ and $(1 - 2\varepsilon_s)/2\varepsilon_s < (1 - \varepsilon_s)/\varepsilon_s$, E_{bs} is larger when the high emissivity ($2\varepsilon_s$) side faces plate 1. Hence T_s is larger for case (b). $<$

PROBLEM 13.58

KNOWN: End of propellant tank with radiation shield is subjected to solar irradiation in space environment.

FIND: (a) Temperature of the shield, T_s, and (b) Heat flux to the tank, $q_1''\left(W/m^2\right)$.

SCHEMATIC:

ASSUMPTIONS: (1) All surfaces are diffuse-gray, (2) View factor between shield and tank is unity, $F_{st} = 1$, (3) Space surroundings are black at 0 K, (4) Resistance of shield for conduction is negligible.

ANALYSIS: (a) Perform a radiation balance on the shield. From the schematic,

$$\alpha_S G_S - \varepsilon_1 E_b\left(T_s\right) - q_{st}'' = 0 \qquad (1)$$

where q_{st}'' is the net heat exchange between the shield and the tank. Considering these two surfaces as large, parallel planes, from Eq. 13.24,

$$q_{st}'' = \sigma\left(T_s^4 - T_t^4\right)/\left[1/\varepsilon_2 + 1/\varepsilon_1 - 1\right]. \qquad (2)$$

Substituting q_{st}'' from Eq. (2) into Eq. (1), find

$$\alpha_s G_s - \varepsilon_1 \sigma T_s^4 - \sigma\left(T_s^4 - T_1^4\right)/\left[1/\varepsilon_2 + 1/\varepsilon_t - 1\right] = 0.$$

Solving for T_s, find

$$T_s = \left[\frac{\alpha_S G_S + \sigma T_t^4/\left[1/\varepsilon_2 + 1/\varepsilon_t - 1\right]}{\sigma\left(\varepsilon_1 + 1/\left[1/\varepsilon_2 + 1/\varepsilon_t - 1\right]\right)}\right]^{1/4}.$$

Since the shield is diffuse-gray, $\alpha_S = \varepsilon_1$ and then

$$T_s = \left[\frac{0.05 \times 1250\,W/m^2 + \sigma(100)^4\,K^4/\left[1/0.05 + 1/0.1 - 1\right]}{\sigma\left(0.05 + 1/\left[1/0.05 + 1/0.1 - 1\right]\right)}\right]^{1/4} = 338\text{ K}. \qquad <$$

(b) The heat flux to the tank can be determined from Eq. (2),

$$q_{st}'' = 5.67 \times 10^{-8}\,W/m^2 \cdot K^4\left(338^4 - 100^4\right)K^4/\left[1/0.05 + 1/0.1 - 1\right] = 25.3\,W/m^2. \qquad <$$

PROBLEM 13.59

KNOWN: Black panel at 77 K in large vacuum chamber at 300 K with radiation shield having ε = 0.05.

FIND: Net heat transfer by radiation to the panel.

SCHEMATIC:

Shield, T_s, $\varepsilon_s = 0.05$, $D_s = D$

Chamber, $T_2 = 300K$

Panel, $T_1 = 77K$, $D = 0.1m$

ASSUMPTIONS: (1) Chamber is large compared to shield, (2) Shape factor between shield and plate is unity, (3) Shield is diffuse-gray, (4) Shield is thin, negligible thermal conduction resistance.

ANALYSIS: The arrangement lends itself to a network representation following Figs. 13.10 and 13.11.

E_{b2} J_2 $J_{s,2}$ $E_{b,s}$ $J_{s,1}$ J_1 E_{b1} q_1

$\frac{1-\varepsilon_2}{\varepsilon_2 A_2}$ $\frac{1}{A_2 F_{2s}}$ $\frac{1-\varepsilon_s}{\varepsilon_s A_s}$ $\frac{1-\varepsilon_s}{\varepsilon_s A_s}$ $\frac{1}{A_s F_{s1}}$ $\frac{1-\varepsilon_1}{\varepsilon_1 A_1}$

0 since $A_2 >> A_s$ or A_1 0 since $\varepsilon_1 = 1$

Noting that $F_{2s} = F_{s1} = 1$, and that $A_2F_{2s} = A_sF_{s2}$, the heat rate is

$$q_1 = \left(E_{b2} - E_{b1}\right)/\Sigma R_i = \sigma\left(T_2^4 - T_1 4\right)/\left[\frac{1}{A_s} + 2\left(\frac{1-\varepsilon_s}{\varepsilon_s A_s}\right) + \frac{1}{A_s}\right].$$

Recognizing that $A_s = A_1$ and multiplying numerator and denominator by A_1 gives

$$q_1 = A_1\sigma\left(T_2 - T_1^4\right)\left[2 + 2\left(\frac{1-\varepsilon_s}{\varepsilon_s}\right)\right].$$

Substituting numerical values, find

$$q_1 = \frac{\pi 0.1^2\,\mathrm{m}^2}{4} \times 5.67\times10^{-8}\,\mathrm{W/m^2\cdot K^4}\left(300^4 - 77^4\right)\mathrm{K}^4/\left[2 + 2\left(\frac{1-0.05}{0.05}\right)\right]$$

$$q_1 = 89.8\,\mathrm{mW}.$$ <

COMMENTS: In using the network representation, be sure to designate direction of the net heat rate. In this situation, we have shown q_1 as the net rate *into* the surface A_1. The temperature of the shield, T_s = 253 K, follows from the relation

$$q_1 = \left(E_{bs} - E_{b1}\right)/\left[\frac{1-\varepsilon_s}{\varepsilon_s A_s} + \frac{1}{A_1 F_{s1}}\right].$$

PROBLEM 13.60

KNOWN: Dense cryogenic piping array located close to furnace wall.

FIND: Number of radiation shields, N, to be installed such that the temperature of the shield closest to the array, $T_{s,N}$, is less than 30°C.

SCHEMATIC:

Dense piping array, T_p = 0°C, ε_p = 0.6
Furnace wall T_f = 200°C, ε_f = 0.9
Installation criteria $T_{s,N} \le$ 30°C
Radiation shields, N ε_s = 0.1

ASSUMPTIONS: (1) The ice-covered dense piping array approximates a plane surface, (2) Piping array and furnace wall can be represented by infinite parallel plates, (3) Surfaces are diffuse-gray, and (4) Convection effects are negligible.

ANALYSIS: Treating the piping array and furnace wall as infinite parallel plates, the net heat rate by radiation exchange with N shields of identical emissivity, ε_s, on both sides follows from extending the network of Fig. 13.12 to account for the resistances of N shields. (See Problem 13.14 (S).) For each shield added, two surface resistances and one space resistance are added,

$$q_{fp} = \frac{\sigma\left(T_f^4 - T_p^4\right)A_f}{\left[1/\varepsilon_f + 1/\varepsilon_p - 1\right] + N\left[2/\varepsilon_s - 1\right]} \tag{1}$$

where $\sigma = 5.67 \times 10^{-8}$ W/m^2·K^4. The requirement that the N-th shield (next to the piping array) has a temperature $T_{s,N} \le 30°C$ will be satisfied when

$$q_{fp} \le \frac{\sigma\left(T_{s,N}4 - T_p4\right)A_f}{\left[1/\varepsilon_s + 1/\varepsilon_p - 1\right]} \tag{2}$$

Using the foregoing equations in the *IHT* workspace, find that $T_{s,N}$ = 30°C when N = 8.60. So that $T_{s,N}$ is less than 30°C, the number of shields required is

$$N = 9$$ <

COMMENTS: Note that when N = 0, Eq. (1) reduces to the case of two parallel plates. Show for the case with one shield, N = 1, that Eq. (1) is identical to Eq. 13.28.

PROBLEM 13.61

KNOWN: Concentric tube arrangement with diffuse-gray surfaces.

FIND: (a) Heat gain by the cryogenic fluid per unit length of the inner tube (W/m), (b) Change in heat gain if diffuse-gray shield with $\varepsilon_s = 0.02$ is inserted midway between inner and outer surfaces.

SCHEMATIC:

Cryogenic fluid in tube
$D_o = 50mm$, $T_o = 300K$, $\varepsilon_o = 0.05$
$D_i = 20mm$, $T_i = 77K$, $\varepsilon_i = 0.02$
No shield (o)
Shield, $D_s = 35mm$, $\varepsilon_s = 0.02$
With shield (s)

ASSUMPTIONS: (1) Surfaces are diffuse-gray, (2) Space between tubes is evacuated.

ANALYSIS: (a) For the *no shield* case, the thermal circuit is shown at right. It follows that the net heat gain per unit tube length is

E_{bi}, E_{bo}; $-q_i$, q_{oi}; $\frac{1-\varepsilon_i}{\varepsilon_i A_i}$, $\frac{1}{A_i F_{io}}$, $\frac{1-\varepsilon_o}{\varepsilon_o A_o}$

$$-q_1' = \frac{q_{oi}}{L} = \left(E_{bo} - E_{bi}\right) / \left[\frac{1-\varepsilon_o}{\varepsilon \pi D_o} + \frac{1}{\pi D_i F_{io}} + \frac{1-\varepsilon_i}{\varepsilon_i \pi D_i}\right]$$

where $A = \pi DL$. Note that $F_{io} = 1$ and $E_b = \sigma T^4$ giving

$$-q_1' = 5.67\times10^{-8}\,W/m^2\cdot K^4\left(300^4 - 77^4\right)K^4 / \left[\frac{1-0.05}{0.05\pi\times50\times10^{-3}} + \frac{1}{\pi 20\times10^{-3}\times1} + \frac{1-0.02}{0.02\pi\times20\times10^{-3}}\right]m^{-1}$$

$$-q_1' = 457\ W/m^2 / [121.0 + 15.9 + 779.8]m^{-1} = 0.501\ W/m.$$ <

(b) For the *with shield* case, the thermal circuit will include three additional resistances.

E_{bi}, E_{bs}, E_{bo}; $-q_i$, q_{oi}; $\frac{1-\varepsilon_i}{\varepsilon_i A}$, $\frac{1}{A_i F_{is}}$, $\frac{1-\varepsilon_s}{\varepsilon_s A_s}$, $\frac{1-\varepsilon_s}{\varepsilon_s A_s}$, $\frac{1}{A_s F_{so}}$, $\frac{1-\varepsilon_o}{\varepsilon_o A_o}$

From the network, it follows that $-q_i = \left(E_{bo} - E_{bi}\right)/\Sigma R_t$. With $F_{is} = F_{so} = 1$, find

$$-q_i' = 457\,W/m^2 / \left[121.0 + \frac{1}{\pi 35\times10^{-3}\times1} + \frac{2(1-0.02)}{0.02\pi 35\times10^{-3}} + 15.9 + 779.8\right]m^{-1}$$

$$-q_i' = 457\,W/m^2 / [121.0 + 9.1 + 891.3 + 15.9 + 779.8]m^{-1} = 0.251\,W/m.$$

The change (percentage) in heat gain per unit length of the tube as a result of inserting the radiation shield is

$$\frac{q_{i,s}' - q_{i,ns}'}{q_{i,ns}'}\times100 = \frac{(0.251 - 0.501)\,W/m}{0.501\,W/m}\times100 = -49\%.$$ <

PROBLEM 13.62

KNOWN: Heated tube with radiation shield whose exterior surface is exposed to convection and radiation processes.

FIND: Operating temperature for the tube under the prescribed conditions.

SCHEMATIC:

$\varepsilon_1 = 1$, $D_1 = 20$ mm, A_1, Heated tube, $\varepsilon_{2,i} = 0.01$, $\varepsilon_{2,o} = 0.1$, A_2, Shield, $T_2 = 42°C$, Evacuated, $D_2 = 60$ mm, $T_\infty = 27°C$, $h = 10\ W/m^2\cdot K$, $T_{sur} = 17°C$

ASSUMPTIONS: (1) Steady-state conditions, (2) No convection in space between tube and shield, (3) Surroundings are large compared to the shield and are isothermal, (4) Tube and shield are infinitely long, (5) Surfaces are diffuse-gray, (6) Shield is isothermal.

ANALYSIS: Perform an energy balance on the shield.

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$q_{12} - q_{conv} - q_{rad} = 0$$

q_{12}, q_{conv}, q_{rad}

where q_{12} is the net radiation exchange between the tube and inner surface of the shield, which from Eq. 13.25 is,

$$-q_{12} = \frac{A_1\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} + \frac{1-\varepsilon_{2,i}}{\varepsilon_{2,i}}\frac{D_1}{D_2}}$$

Using appropriate rate equations for q_{conv} and q_{rad}, the energy balance is

$$\frac{A_1\sigma\left(T_1^4 - T_2^4\right)}{1 + \frac{1-\varepsilon_{2,i}}{\varepsilon_{2,i}}\frac{D_1}{D_2}} - hA_2\left(T_2 - T_\infty\right) - \varepsilon_{2,o}A_2\sigma\left(T_2^4 - T_{sur}^4\right) = 0$$

where $\varepsilon_1 = 1$. Substituting numerical values, with $A_1/A_2 = D_1/D_2$, and solving for T_1,

$$\frac{(20/60)\times 5.67\times 10^{-8}\,W/m^2\cdot K^4\left(T_1^4 - 315^4\right)K^4}{1 + (1-0.01/0.01)(20/60)} - 10\ W/m^2\cdot K\,(315-300)K$$

$$-0.1\times 5.67\times 10^{-8}\,W/m^2\cdot K^4\left(315^4 - 290^4\right)K^4 = 0$$

$$T_1 = 745\ K = 472°C. \qquad <$$

COMMENTS: Note that all temperatures are expressed in kelvins. This is a necessary practice when dealing with radiation and convection modes.

PROBLEM 13.63

KNOWN: Cylindrical-shaped, three surface enclosure with lateral surface insulated.

FIND: Temperatures of the lower plate T_1 and insulated side surface T_3.

SCHEMATIC:

D=0.2m
L=0.2m
$A_2, T_2 = 473K, \varepsilon_2 = 0.8$
A_3, T_3, insulated
A_1, T_1, black surface
$P = 10{,}000 W = q_1$

ASSUMPTIONS: (1) Surfaces have uniform radiosity or emissive power, (2) Upper and insulated surfaces are diffuse-gray, (3) Negligible convection.

ANALYSIS: Find the temperature of the lower plate T_1 from Eq. 13.30

$$q_1 = \frac{\sigma\left(T_1^4 - T_2^4\right)}{(1-\varepsilon_1)/\varepsilon_1 A_1 + \left[A_1F_{12} + \left[(1/A_1F_{13}) + (1/A_2F_{23})\right]^{-1}\right]^{-1} + (1-\varepsilon_2)/\varepsilon_2 A_2}. \qquad (1)$$

From Table 13.2 for parallel coaxial disks,

$$R_1 = r_1/L = 0.1/0.2 = 0.5 \qquad R_2 = r_2/L = 0.1/0.2 = 0.5$$

$$S = 1 + \left(1 + R_2^2\right)/R_1^2 = 1 + \left(1 + 0.5^2\right)/0.5^2 = 6.0$$

$$F_{12} = 1/2\left\{S - \left[S^2 - 4(r_2/r_1)^2\right]^{1/2}\right\} = 1/2\left\{6 - \left[6^2 - 4(0.5/0.5)^2\right]^{1/2}\right\} = 0.172.$$

Using the summation rule for the enclosure, $F_{13} = 1 - F_{12} = 1 - 0.172 = 0.828$, and from symmetry, $F_{23} = F_{13}$. With $A_1 = A_2 = \pi D^2/4 = \pi(0.2\text{ m})^2/4 = 0.03142\text{ m}^2$ and substituting numerical values into Eq. (1), obtain

$$10{,}000\text{ W} = \frac{0.03142\text{ m}^2 \times 5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4\left(T_1^4 - 473^4\right)\text{K}^4}{0 + \left[0.172 + \left[(1/0.828) + (1/0.172)\right]^{-1}\right]^{-1} + (1-0.8)/0.8}$$

$$10{,}000 = 4.540\times10^9\left(T_1^4 - 473^4\right) \qquad T_1 = 1225\text{ K}. \qquad <$$

The temperature of the insulated side surface can be determined from the radiation balance, Eq. 13.31, with $A_1 = A_2$,

$$\frac{J_1 - J_3}{1/F_{13}} - \frac{J_3 - J_2}{1/F_{23}} = 0 \qquad (2)$$

where $J_1 = \sigma T_1^4$ and J_2 can be evaluated from Eq. 13.19,

Continued …..

PROBLEM 13.63 (Cont.)

$$q_2 = \frac{E_{b2} - J_2}{(1-\varepsilon_2)/\varepsilon_2 A_2} \qquad -10,000\ \text{W} = \frac{5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\,(473\ \text{K})^4 - J_2}{(1-0.8)/\left(0.8\times0.03142\,\text{m}^2\right)}$$

find J_2 = 82,405 W/m^2. Substituting numerical values into Eq. (2),

$$\frac{5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\,(1225\ \text{K})^4 - J_3}{1/0.172} - \frac{J_3 - 82,405\ \text{W/m}^2}{1/0.172} = 0$$

find J_3 = 105,043 W/m^2. Hence, for this insulated, re-radiating (adiabatic) surface,

$$E_{b3} = \sigma T_3^4 = 105,043\ \text{W/m}^2 \qquad T_3 = 1167\ \text{K}. \qquad <$$

PROBLEM 13.64

KNOWN: Furnace in the form of a truncated conical section, floor (1) maintained at T_1 = 1000 K by providing a heat flux $q''_{1,in} = 2200\ W/m^2$; lateral wall (3) perfectly insulated; radiative properties of all surfaces specified.

FIND: (a) Temperature of the upper surface, T_2, and of the lateral wall T_3, and (b) T_2 and T_3 if all the furnace surfaces are black instead of diffuse-gray, with all other conditions remain unchanged. Explain effect of ε_2 on your results.

SCHEMATIC:

ASSUMPTIONS: (1) Furnace is a three-surface, diffuse-gray enclosure, (2) Surfaces have uniform radiosities, (3) Lateral surface is adiabatic, and (4) Negligible convection effects.

ANALYSIS: For the three-surface enclosure, write the radiation surface energy balances, Eq. 13.21, to find the radiosities of the three surfaces.

$$\frac{E_{b,1} - J_1}{(1-\varepsilon_1)/\varepsilon_1 A_1} = \frac{J_1 - J_2}{1/A_1 F_{12}} + \frac{J_1 - J_3}{1/A_1 F_{13}} \tag{1}$$

$$\frac{E_{b,2} - J_2}{(1-\varepsilon_2)/\varepsilon_2 A_2} = \frac{J_2 - J_1}{1/A_2 F_{21}} + \frac{J_2 - J_3}{1/A_2 F_{23}} \tag{2}$$

$$\frac{E_{b,3} - J_3}{(1-\varepsilon_3)/\varepsilon_3 A_3} = \frac{J_3 - J_1}{1/A_3 F_{31}} + \frac{J_3 - J_2}{1/A_3 F_{32}} \tag{3}$$

where the blackbody emissive powers are of the form $E_b = \sigma T^4$ with $\sigma = 5.67 \times 10^{-8}\ W/m^2 \cdot K^4$. From Eq. 13.19, the net radiation leaving A_1 is

$$q_1 = \frac{E_{b,1} - J_1}{(1-\varepsilon_1)/\varepsilon_1 A_1} \tag{4}$$

$$q_1 = q''_{1,in} \cdot A_1 = 2200\ W/m^2 \times \pi (0.040\ m)^2 / 4 = 2.76\ W$$

Continued …..

Since the lateral surface is adiabatic,

$$q_3 = \frac{E_{b,3} - J_3}{(1-\varepsilon_3)/\varepsilon_3 A_3} = 0 \qquad (5)$$

from which we recognize $E_{b,3} = J_3$, but will find that as an outcome of the analysis. For the enclosure, N = 3, there are $N^2 = 9$ view factors, for which N (N - 1)/2 = 3 must be directly determined. Calculations for the F_{ij} are summarized in Comments.

With the foregoing five relations, we can determine the five unknowns: J_1, J_2, J_3, $E_{b,2}$, and $E_{b,3}$. The temperatures T_2 and T_3 will be evaluated from the relation $E_b = \sigma T^4$. Using this analysis approach with the relations in the *IHT* workspace, the results for (a) the diffuse-gray surfaces and (b) black surfaces are tabulated below.

	J_1 (kW/m^2)	J_2 (kW/m^2)	J_3 (kW/m^2)	T_2 (K)	T_3 (K)
(a) Diffuse-gray	55.76	45.30	53.48	896	986
(b) Black	56.70	46.24	54.42	950	990

COMMENTS: (1) From the tabulated results, it follows that the temperatures of the lateral and top surfaces will be higher when the surfaces are black, rather than diffuse-gray as specified.

(2) From Eq. (5) for the net heat radiation leaving the lateral surface, A_3, the rate is zero since the wall is adiabatic. The consequences are that the blackbody emissive power and the radiosity are equal, and that the emissivity of the surface has no effect in the analysis. That is, this surface emits and absorbs at the same rate; the net is zero.

(3) For the enclosure, N = 3, there are $N^2 = 9$ view factors, for which

$$N\,(N-1)/2 = 3 \times 2/2 = 3$$

must be directly determined. We used the *IHT Tools | Radiation | View Factors Relations* model that sets up the summation rules and reciprocity relations for the N surfaces. The user is required to specify the 3 F_{ij} that must be determined directly; by inspection, $F_{11} = F_{22} = 0$; and F_{12} can be evaluated using the parallel coaxial disk relation, Table 13.2 (Fig. 13.5). This model is also provided in *IHT* to simplify the calculation task. The results of the view factor analysis are:

$$F_{12} = 0.03348 \qquad F_{13} = 0.9665$$

$$F_{21} = 0.1339 \qquad F_{23} = 0.8661$$

(4) An alternative method of solution for part (a) is to treat the enclosure of part (a) as described in Section 13.3.5. For part (b), the black enclosure analysis is described in Section 13.2. We chose to use the net radiation method, Section 13.3.1, to develop a general 3-surface enclosure code in *IHT* that can also handle black surfaces (caution: use $\varepsilon = 0.999$, not 1.000).

PROBLEM 13.65

KNOWN: Two aligned, parallel square plates with prescribed temperatures.

FIND: Net radiative transfer from surface 1 for these plate conditions: (a) black, surroundings at 0 K, (b) black with connecting, re-radiating walls, (c) diffuse-gray with radiation-free surroundings at 0 K, (d) diffuse-gray with re-radiating walls.

SCHEMATIC:

0.4m; L=0.8m; $A_2, T_2 = 800K, E_{b2} = 23{,}224\ W/m^2$; $A_1, T_1 = 500K, E_{b1} = 3{,}544\ W/m^2$

ASSUMPTIONS: (1) Plates are black or diffuse-gray, (2) Surroundings are at 0 K.

ANALYSIS: (a) The view factor for the aligned, parallel plates follows from Fig. 13.4, X/L = 0.4 m/0.8 m = 0.5, Y/L = 0.4 m/0.8 m = 0.5, $F_{12} = F_{21} \approx 0.075$. When the plates are *black with surroundings at 0 K*, from Eq. 13.13,

$$q_1 = q_{12} + q_{1(sur)} = A_1F_{12}\left(E_{b1} - E_{b2}\right) + A_1F_{1(sur)}\left(E_{b1} - E_{b(sur)}\right)$$

$$q_1 = (0.4 \times 0.4)\,m^2\left[0.075(3544 - 23{,}224) + (1 - 0.075)(3544 - 0)\right]W/m^2 = 288\,W. \qquad <$$

(b) When the plates are *black with connecting re-radiating walls*, from Eq. 13.30 with $F_{1R} = R_{2R} = 1 - F_{12} = 0.925$,

$$q_1 = \frac{A_1\left[E_{b1} - E_{b2}\right]}{\left[F_{12} + \left(1/F_{1R} + 1/F_{2R}\right)^{-1}\right]^{-1}} = \frac{(0.4\,m)^2\left[3544 - 23{,}224\right]W/m^2}{\left[0.075 + (1/0.925 + 1/0.925)^{-1}\right]^{-1}} = -1{,}692\,W. \qquad <$$

(c) When the plates are *diffuse-gray ($\varepsilon_1 = 0.6$ and $\varepsilon_2 = 0.8$)* with the *surroundings at 0 K*, using Eq. 13.20 or Eq. 13.19, with $E_{b3} = J_3 = 0$,

$$q_1 = A_1F_{12}\left(J_1 - J_2\right) + A_1F_{13}\left(J_1 - J_3\right) = \left(E_{b1} - J_1\right)/\left[(1-\varepsilon_1)/\varepsilon_1 A_1\right].$$

The radiosities must be determined from energy balances, Eq. 13.21, on each of the surfaces,

$$\frac{E_{b1} - J_1}{(1-\varepsilon_1)/\varepsilon_1} = F_{12}\left(J_1 - J_2\right) + F_{13}\left(J_1 - J_3\right) \qquad \frac{E_{b2} - J_2}{(1-\varepsilon_2)/\varepsilon_2} = F_{21}\left(J_2 - J_1\right) + F_{23}\left(J_2 - J_3\right)$$

$$\frac{3{,}544 - J_1}{(1-0.6)/0.6} = 0.075\left(J_1 - J_2\right) + 0.925J_1 \qquad \frac{23{,}224 - J_2}{(1-0.8)/0.8} = 0.075\left(J_2 - J_1\right) + 0.925J_2.$$

Find $J_1 = 2682\ W/m^2$ and $J_2 = 18{,}542\ W/m^2$. Combining these results,

$$q_1 = (0.4\,m)^2(0.075)(2682 - 18{,}542)\,W/m^2 + (0.4\,m)^2(0.925)(2682 - 0)\,W/m^2 = 207\ W. \qquad <$$

(d) When the plates are *diffuse-gray with connecting re-radiating walls*, use Eq. 13.30,

$$q_1 = \frac{A_1\left[E_{b1} - E_{b2}\right]}{(1-\varepsilon_1)/\varepsilon_1 + \left[F_{12} + \left(1/F_{1R} + 1/F_{2R}\right)^{-1}\right]^{-1} + (1-\varepsilon_2)/\varepsilon_2}$$

$$q_1 = \frac{(0.4\,m)^2\left[35444 - 23{,}244\right]W/m^2}{(1-0.6)/0.6 + \left[0.075 + (1/0.925 + 1/0.925)^{-1}\right]^{-1} + (1-0.8)/0.8} = -1133\,W. \qquad <$$

PROBLEM 13.66

KNOWN: Parallel, aligned discs located in a large room; one disk is insulated, the other is at a prescribed temperature.

FIND: Temperature of the insulated disc.

SCHEMATIC:

ASSUMPTIONS: (1) Surfaces are diffuse-gray, (2) Surroundings are large, with uniform temperature, behaving as a blackbody, (3) Negligible convection.

ANALYSIS: From an energy balance on surface A_2,

$$q_2 = 0 = \frac{J_2 - J_1}{1/A_2F_{21}} + \frac{J_2 - J_3}{1/A_2F_{23}}. \qquad (1)$$

Note that $q_2 = 0$ since the surface is adiabatic. Since A_3 is a blackbody, $J_3 = E_{b3} = \sigma T_3^4$; since A_2 is adiabatic, $J_2 = E_{b2} = \sigma T_2^4$. From Fig. 13.5 and the summation rule for surface A_1, find

$$F_{12} = 0.62 \text{ with } \frac{r_j}{L} = \frac{0.2}{0.1} = 2 \text{ and } \frac{L}{r_i} = \frac{0.1}{0.2} = 0.5, \qquad F_{13} = 1 - F_{12} = 1 - 0.62 = 0.38.$$

Hence, Eq. (1) with $J_3 = 5.67 \times 10^{-8} \times 300^4$ W/m^2 becomes

$$\frac{J_2 - J_1}{1/A_2 \times 0.62} + \frac{J_2 - 459.3\,\text{W/m}^2}{1/A_2 \times 0.38} = 0 \qquad -0.62J_1 + 1.00J_2 = 174.5 \qquad (2,3)$$

The radiation balance on surface A_1 with $E_{b3} = 5.67 \times 10^{-8} \times 500^4$ W/m^2 becomes

$$\frac{E_{b1} - J_1}{(1-\varepsilon_1)/\varepsilon_1 A_1} = \frac{J_1 - J_2}{1/A_1F_{12}} + \frac{J_1 - J_3}{1/A_1F_{13}} \qquad (4)$$

$$\frac{3543.8 - J_1}{(1-0.6)/0.6A_1} = \frac{J_1 - J_2}{1/A_1 \times 0.62} + \frac{J_1 - 459.3}{1/A_1 \times 0.38} \qquad 2.50J_1 - 0.62J_2 = 5490.2 \qquad (5,6)$$

Solve Eqs. (3) and (6) to find $J_2 = 1815$ W/m^2 and since $E_{b2} = J_2$,

$$T_2 = \left(\frac{E_{b2}}{\sigma}\right)^{1/4} = \left(\frac{1815\ \text{W/m}^2}{5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4}\right)^{1/4} = 423\ \text{K}. \qquad <$$

COMMENTS: A network representation would help to visualize the exchange relations. However, it is useful to approach the problem by recognizing there are two unknowns in the problem: J_1 and J_2; hence two radiation balances must be written. Note also the significance of $J_2 = E_{b2}$ and $J_3 = E_{b3}$.

PROBLEM 13.67

KNOWN: Thermal conditions in oven used to cure strip coatings.

FIND: Electrical power requirement.

SCHEMATIC:

D=0.02m
T_1=1700K, ε_1=0.9
T_2=600K, ε_2=0.4
s_1, s_2, L
s_1=0.06m
s_2=0.02m
L=0.08m

ASSUMPTIONS: (1) Diffuse-gray surfaces, (2) Furnace wall is reradiating, (3) Negligible end effects.

ANALYSIS: The net radiant power leaving the heater surface per unit length is

$$q_1' = \frac{E_{b1} - E_{b2}}{\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1'} + \dfrac{1}{A_1'F_{12} + \left[(1/A_1F_{1R}) + (1/A_2'F_{2R})\right]^{-1}} + \dfrac{1-\varepsilon_2}{\varepsilon_2 A_2'}}$$

where $A_1' = \pi D = \pi(0.02\,\text{m}) = 0.0628\,\text{m}$ and $A_2' = 2(s_1 - s_2) = 0.08\,\text{m}$. The view factor between the heater and one of the strips is

$$F_{21} = \frac{D/2}{s_1 - s_2}\left[\tan^{-1}\frac{s_1}{L} - \tan^{-1}\frac{s_2}{L}\right] = \frac{0.01}{0.04}\left[\tan^{-1}\frac{0.06}{0.08} - \tan^{-1}\frac{0.02}{0.08}\right] = 0.10$$

and using the view factor relations find

$$A_1'F_{12} = A_2'F_{21} = 0.08\,\text{m} \times 0.10 = 0.008\,\text{m} \qquad F_{12} = (0.080/0.0628)0.10 = 0.127$$

$$F_{1R} = 1 - F_{12} = 1 - 0.127 = 0.873 \qquad F_{2R} = 1 - F_{21} = 1 - 0.10 = 0.90.$$

Hence, with $E_b = \sigma T^4$,

$$q_1' = \frac{5.67\times10^{-8}\left[(1700)^4 - (600)^4\right]}{\dfrac{1-0.9}{0.9\times0.0628} + \dfrac{1}{0.008 + \left[1/(0.0628\times0.873) + 1/(0.08\times0.90)\right]^{-1}} + \dfrac{1-0.4}{0.4\times0.08}}$$

$$q_1' = \frac{4.66\times10^5}{1.77 + 25.56 + 18.75} = 10{,}100\ \text{W/m}. \qquad <$$

COMMENTS: The radiosities for A_1 and A_2 follow from Eq. 13.19,

$$J_1 = E_{b1} - (1-\varepsilon_1)q_1'/\varepsilon_1 A_1' = 4.56\times10^5\ \text{W/m}^2$$

$$J_2 = E_{b2} + (1-\varepsilon_2)q_1'/\varepsilon_2 A_2' = 1.97\times10^5\ \text{W/m}^2.$$

From Eq. 13.31, find J_R and hence T_R as

$$0.0628\times0.873(J_1 - J_R) - 0.08\times0.90(J_R - J_2) = 0$$

$$J_R = 3.08\times10^5\ \text{W/m}^2 = \sigma T_R^4 \qquad T_R = 1527\ \text{K}.$$

PROBLEM 13.68

KNOWN: Surface temperature and emissivity of molten alloy and distance of surface from top of container. Container diameter.

FIND: Net rate of radiation heat transfer from surface of melt.

SCHEMATIC:

T_2 = 300 K
ε_2 = 1.0
R
q_1
L = 100 mm
T_1 = 900 K
ε_1 = 0.55
D = 250 mm

ASSUMPTIONS: (1) Opaque, diffuse, gray behavior for surface of melt, (2) Large surroundings may be represented by a hypothetical surface of temperature $T = T_{sur}$ and $\varepsilon = 1$, (3) Negligible convection at exposed side wall, (4) Adiatic side wall.

ANALYSIS: With negligible convection at an adiabatic side wall, the surface may be treated as reradiating. Hence, from Eq. (13.30), with $A_1 = A_2$,

$$q_1 = \frac{A_1\left(E_{b1} - E_{b2}\right)}{\frac{1-\varepsilon_1}{\varepsilon_1} + \frac{1}{F_{12} + \left[\left(1/F_{1R}\right) + \left(1/F_{2R}\right)\right]^{-1}} + \frac{1-\varepsilon_2}{\varepsilon_2}}$$

With $R_i = R_j = (D/2)/L = 1.25$ and $S = \left[1 + \left(1 + R_j^2\right)/R_i^2\right] = 2.640$, Table 13.2 yields

$$F_{12} = \frac{1}{2}\left\{S - \left[S^2 - 4\left(r_2/r_1\right)^2\right]^{1/2}\right\} = 0.458$$

Hence,

$$F_{1R} = F_{2R} = 1 - F_{12} = 0.542 \text{ and}$$

$$q_1 = \frac{\pi\,(0.25\text{m})^2 \times 5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left(900^4 - 300^4\right)\text{K}^4}{\frac{1-0.55}{0.55} + \frac{1}{0.458\,(3.69)^{-1}} + 0} = 3295\,\text{W}$$

<

PROBLEM 13.69

KNOWN: Blackbody simulator design consisting of a heated circular plate with an opening over a well insulated hemispherical cavity.

FIND: (a) Radiant power leaving the opening (aperture), $D_a = r_o/2$, (b) Effective emissivity of the cavity, ε_e, defined as the ratio of the radiant power leaving the cavity to the rate at which the circular plate would emit radiation if it were black, (c) Temperature of hemispherical surface, T_{hc}, and (d) Compute and plot ε_e and T_{hc} as a function of the opening aperture in the circular plate, D_a, for the range $r_o/8 \le D_a \le r_o/2$, for plate emissivities of $\varepsilon_p = 0.5$, 0.7 and 0.9.

SCHEMATIC:

ASSUMPTIONS: (1) Plate and hemispherical surface are diffuse-gray, (2) Uniform radiosity over these same surfaces.

ANALYSIS: (a) The simulator can be treated as a three-surface enclosure with one re-radiating surface (A_2) and the opening (A_3) as totally absorbing with no emission into the cavity (T_3 = 300 K). The radiation leaving the cavity is the net radiation leaving A_1, q_1 which is equal to $-q_3$. Using Eq. 13.30,

$$q_{cav} = q_1 = -q_3 = \frac{\sigma\left(T_1^4 - T_3^4\right)}{(1-\varepsilon_1)/\varepsilon_1 A_1 + \left[A_1 F_{13} + \left[(1/A_1 F_{12}) + (1/A_3 F_{32})\right]^{-1}\right]^{-1} + (1-\varepsilon_3)/\varepsilon_3 A_3} \qquad (1)$$

Using the summation rule and reciprocity, evaluate the required view factors:

$$F_{11} + F_{12} + F_{13} = 1 \qquad F_{13} = 0 \qquad F_{12} = 1$$

$$F_{31} + F_{32} + F_{33} = 1 \qquad F_{32} = 1.$$

Substituting numerical values with $\varepsilon_3 = 1$, T_3 = 300 K, $A_1 = \pi\left(r_o^2 - (r_o/4)^2\right) = 15\pi r_o^2/16 = 2.945 \times 10^{-2}\ m^2$, $A_3 = \pi r_a^2 = \pi (r_o/4)^2 = 1.963 \times 10^{-3}\ m^2$ and $A_1/A_3 = 15$, and multiplying numerator and denominator by A_1,

$$q_{cav} = q_1 = \frac{A_1\sigma\left(T_1 - T_3^4\right)}{(1-\varepsilon_1)/\varepsilon_1 + \left\{F_{13} + \left[(1/F_{12}) + (A_1/A_3 F_{32})\right]^{-1}\right\}^{-1} + 0} \qquad (2)$$

Continued …..

$$q_{cav} = q_1 = \frac{2.945\times10^{-2}\,m^2 \times 5.67\times10^{-8}\,W/m^2\cdot K^4\left(600^4 - 300^4\right)K^4}{(1-0.9)/0.9 + \left\{0 + \left[1+(15/1)\right]^{-1}\right\}^{-1} + 0} = 12.6\,W$$ <

(b) The effective emissivity is the ratio of the radiant power leaving the cavity to that from a blackbody having the area of the opening and temperature of the inner surface of the cavity. That is,

$$\varepsilon_e = \frac{q_{cav}}{A_3\sigma T_1^4} = \frac{12.6\,W}{1.963\times10^{-3}\,m^2\times5.67\times10^{-8}\,W/m^2\cdot K^4\times(600\,K)^4} = 0.873 \qquad (3)$$ <

(c) From a radiation balance on A_1, find J_1,

$$q_1 = 12.6\,W = \frac{E_{b1} - J_1}{(1-\varepsilon_1)/\varepsilon_1 A_1} = \frac{\sigma 600^4 - J_1}{(1-09)/0.9A_1} \qquad J_1 = 7301\,W/m^2 \qquad (4)$$

From a radiation balance on A_2 with $J_3 = E_{b3} = \sigma T_3^4 = 459.9\ W/m^2$ and $J_2 = \sigma T_2^4$, find

$$\frac{J_2 - J_1}{(1/A_1F_{12})} + \frac{J_2 - J_3}{(1/A_3F_{32})} = \frac{J_2 - 7301\,W/m^2}{\left(1/2.945\times10^{-2}\,m^2\right)} + \frac{J_2 - 459.9}{\left(1/1.963\times10^{-3}\,m^2\right)} = 0 \qquad (5)$$

$$J_2 = 6873\,W/m^2 \qquad T_2 = 590\,K.$$ <

(d) Using the foregoing equations in the *IHT* workspace, ε_e and T_2 were computed and plotted as a function of the opening, D_a, for selected plate emissivities, ε_p.

From the upper-left graph, ε_e decreases with increasing opening, D_a, as expected. In the limit as $D_a \to 0$, $\varepsilon_3 \to 1$ since the cavity becomes a complete enclosure. From the upper-right graph, T_{hc}, the temperature of the re-radiating hemispherical surface decreases as D_a increases. In the limit as $D_a \to 0$, T_2 will approach the plate temperature, $T_p = 600$ K. The effect of decreasing the plate emissivity is to decrease ε_e and decrease T_2. Why is this so?

Continued …..

COMMENTS: The *IHT Radiation, Tool, Radiation Tool, Radiation Exchange Analysis, Three-Surface Enclosure with Re-radiating Surface*, is especially convenient to perform the parametric analysis of part (c). A copy of the *IHT* workspace that can generate the above graphs is shown below.

```
// Radiation Tool – Radiation Exchange Analyses, Reradiating Surface
/* For the three-surface enclosure A1, A3 and the reradiating surface A2, the net rate of radiation transfer
from the surface A1 to surface A3 is */
q1 = (Eb1 – Eb3) / ( (1 – eps1)/(eps1 * A1) + 1/(A1 * F13 + 1/(1/(A1 * F12) + 1/(A3 * F32))) + (1 –
eps3)/(eps3 * A3))  //  Eq 13.30
/* The net rate of radiation transfer from surface A3 to surface A1 is */
q3 = q1
/* From a radiation energy balance on A2, */
(J2 – J1) / (1/(A2 * F21)) + (J2 – J3)/(1/(A2 * F23) ) = 0          // Eq 13.31
/* where the radiosities J1 and J3 are determined from the radiation rate equations expressed in terms of
the surface resistances, Eq 13.22 */
q1 = (Eb1 – J1) / ((1 – eps1) / (eps1 * A1))
q3 = (Eb3 – J3) / ((1 – eps3) / (eps3 * A3))
// The blackbody emissive powers for A1 and A3 are
Eb1 = sigma * T1^4
Eb3 = sigma * T3^4
// For the reradiating surface,
J2 = Eb2
Eb2 = sigma * T2^4              // Stefan-Boltzmann constant, W/m^2·K^4
sigma = 5.67E-8

// Effective emissivity:
epseff = q1 / (A3 * Eb1)                  // Eq (3)

// Areas:
A1 = pi * ( ro^2 · ra^2)
A2 = 0.5 * pi * (2 * ro)^2                // Hemisphere, As = 0.5 * pi * D^2
A3 = pi * ra^2

// Assigned Variables
T1 = 600                        // Plate temperature, K
eps1 = 0.9                      // Plate emissivity
T3 = 300                        // Opening temperature, K; Tsur
eps3 = 0.9999                   // Opening emissivity; not zero to avoid divide-by-zero error
ro = 0.1                        // Hemisphere radius, m
Da = 0.05                       // Opening diameter; range ro/8 to ro/2; 0.0125 to 0.050
Da_mm = Da * 1000                         // Scaling for plot
Ra = Da / 2                     // Opening radius
```

PROBLEM 13.70

KNOWN: Long hemi-cylindrical shaped furnace comprised of three zones.

FIND: (a) Heat rate per unit length of the furnace which must be supplied by the gas burners and (b) Temperature of the insulating brick.

SCHEMATIC:

Metal products, (2) $T_2 = 500K$, $\varepsilon_2 = 1$, $E_{b2} = \sigma T_2^4 = 3544 W/m^2$

(3) Insulating brick, $\varepsilon = 0.6$

(1) Ceramic plate, $T_1 = 1600K$, $\varepsilon_1 = 0.85$, $E_{b1} = \sigma T_1^4 = 371,589 W/m^2$

1m

Insulating wall separating zones

Gas burners

ASSUMPTIONS: (1) Surfaces are opaque, diffuse-gray or black, (2) Surfaces have uniform temperatures and radiosities, (3) Surface 3 is perfectly insulated, (4) Negligible convection, (5) Steady-state conditions.

ANALYSIS: (a) From an energy balance on the ceramic plate, the power required by the burner is $q'_{burners} = q'_1$, the net radiation leaving A_1; hence

$$q'_1 = A_1F_{12}(J_1 - J_2) + A_1F_{13}(J_1 - J_3) = 0 + A_1F_{13}(J_1 - J_3) \quad (1)$$

since $F_{12} = 0$. Note that $J_2 = E_{b2} = \sigma T_2^4$ and that J_1 and J_3 are unknown. Hence, we need to write two radiation balances.

A_1:
$$\frac{E_{b1} - J_1}{(1-\varepsilon_1)/\varepsilon_1 A_1} = q'_1 = 0 + A_1FF_{13}(J_1 - J_3) \quad (2)$$

A_3:
$$0 = A_3F_{31}(J_3 - J_1) + A_3F_{32}(J_3 - E_{b2})$$

$$J_3 = \frac{J_1 + E_{b2}}{2} \quad (3)$$

since $F_{31} = F_{23}$. Substituting Eq. (3) into (2), find

$$(371,589 - J_1)/(1 - 0.85)/0.85 = 1[J_1 - (J_1 + 3,544)/2]$$

$$J_1 = 341,748\ W/m^2 \qquad J_3 = 172,646\ W/m^2$$

using $E_{b1} = \sigma T_1^4 = 371,589\ W/m^2$ and $E_{b2} = \sigma T_2^4 = 3544\ W/m^2$. Substituting into Eq. (1), find

$$q'_1 = 1\ m \times 1(341,748 - 172,646)\ W/m^2 = 169\ kW/m. \quad <$$

(b) The temperature of the insulating brick, acting as a reradiating surface, is

$$J_3 = E_{b3} = \sigma T_3^4$$

$$T_3 = (J_3/\sigma)^{1/4} = \left(172,646\ W/m^2 / 5.67\times10^{-8}\ W/m^2\cdot K^4\right)^{1/4} = 1320\ K. \quad <$$

PROBLEM 13.71

KNOWN: Steam producing still heated by radiation.

FIND: (a) Factor by which the vapor production could be increased if the cylindrical side of the heater were insulated rather than open to the surroundings, and (b) Compute and plot the net heat rate of radiation transfer to the still, as a function of the separation distance L for the range $15 \le L \le 100$ mm for heater temperatures of 600, 800, 1000°C considering the cylindrical sides to be insulated.

SCHEMATIC:

ASSUMPTIONS: (1) Still and heater surfaces are black, (2) Surroundings are isothermal and large compared to still heater surfaces, (3) Insulation is diffuse-gray, (4) Negligible convection.

ANALYSIS: (a) The vapor production will be proportional to the net radiation exchange to the still. For the case when the sides are open (o) to the surroundings, the net radiation exchange leaving A_2 is from Eq. 13.13.

$$q_{2,o} = q_{21} + q_{2s} = A_2 F_{21} \sigma \left(T_2^4 - T_1^4\right) + A_2 F_{2s} \sigma \left(T_2^4 - T_{sur}^4\right)$$

where $F_{2s} = 1 - F_{21}$ and F_{21} follows from Fig. 13.5 with $L/r_i = 100/100 = 1$, $r_j/L = 100/100 = 1$.

$$F_{21} = 0.38$$

With $A_2 = \pi D^2/4$, find

$$q_{2,o} = \frac{\pi\,(0.200\text{m})^2}{4} \times 5.67 \times 10^{-8}\,\text{W/m}^2 \cdot \text{K}^4 \left\{0.38\left(373^4 - 1273^4\right)\text{K}^4 + (1 - 0.38)\left(373^4 - 300^4\right)\text{K}^4\right\}$$

$$q_{2,o} = -1752\,\text{W}. \qquad \leftarrow \text{w/o insulation}$$

With the cylindrical side insulated (i), a three-surface, re-radiating enclosure is formed. Eq. 13.30 can be used to evaluate $q_{2,i}$ and with $\varepsilon_2 = \varepsilon_1 = 1$, the relation is

$$q_{2,i} = \frac{\sigma\left(T_2^4 - T_1\right)^4}{\dfrac{1}{A_1 F_{12} + \left[(1/A_1 F_{1R}) + (1/A_2 F_{2R})\right]^{-1}}} = A_1 \left\{F_{12} + \left[1/F_{1R} + 1/F_{2R}\right]^{-1}\right\} \sigma \left(T_2^4 - T_1^4\right)$$

Recall $F_{12} = 0.38$ and $F_{1R} = 1 - F_{12} = 1 - 0.38 = 0.62$, giving

$$q_{2,i} = \frac{\pi\,(0.100\text{ m})^2}{4} \left\{0.38 + \left[1/0.62 + 1/0.62\right]^{-1}\right\} 5.67 \times 10^{-8}\,\text{W/m}^2 \cdot \text{K}\left(373^4 - 1273^4\right)\text{K}^4$$

$$q_{2,i} = -3204\text{ W}. \qquad \leftarrow \text{w insulation}$$

Continued …..

PROBLEM 13.71 (Cont.)

Hence, the vapor production rate is increased by a factor

$$\frac{q_2)_{insul}}{q_2)_{open}} = \frac{3204\,W}{1752\,W} = 1.83$$

That is, the vapor production is increased by 83%. <

(b) The *IHT Radiation Tool – Radiation Exchange Analysis* for the *Three-Surface Enclosure* with a *reradiating surface* can be used directly to compute the net heat rate to the still, $q_1 = q_2$, as a function of the separation distance L for selected heater temperatures T_1. The results are plotted below.

Note that the heat rate for all values of T_1 decreases as expected with increasing separation distances, but not markedly. For any separation distance, increasing the heater temperature greatly influences the heat rate. For example, at L = 50 mm, increasing T_1 from 600 to 800 K, causes a nearly 6 fold increase in the heat rate. But increasing T_1 from 800 to 1000 K causes only a 2 fold increase in the heat rate.

COMMENTS: When assigning the emissivity variables (ε_1, ε_2, ε_3) in the *IHT* model mentioned above, set $\varepsilon = 0.999$, rather than 1.0, to avoid a “division by zero” error message. You could also call up the *Radiation Tool, View Factor Coaxial Parallel Disk* to calculate F_{12}.

PROBLEM 13.72

KNOWN: Furnace with cylindrical heater and re-radiating, insulated walls.

FIND: (a) Power required to maintain steady-state conditions, (b) Temperature of wall area.

SCHEMATIC:

Insulated wall, $A_R, \varepsilon_R = 0.9$

Heating element, A_1 $\begin{cases} T_1 = 1500K \\ \varepsilon_1 = 1 \\ D = 10mm \end{cases}$

$h = 0.87m$

$60°$

Bottom area, A_2 $\begin{cases} T_2 = 500K \\ \varepsilon_2 = 0.6 \end{cases}$

$w = 1m$

ASSUMPTIONS: (1) Surfaces are diffuse-gray, (2) Furnace is of length ℓ where ℓ >> w, (3) Convection is negligible.

ANALYSIS: (a) Consider the furnace as a three surface enclosure with the walls, A_R, represented as a re-radiating surface. The power that must be supplied to the heater is determined by Eq. 13.30.

$$q_1 = \frac{\sigma\left(T_1^4 - T_2^4\right)}{(1-\varepsilon_1)/\varepsilon_1 A_1 + \left[A_1F_{12} + \left[(1/A_1F_{1R}) + (1/A_2F_{2R})\right]^{-1}\right]^{-1} + (1-\varepsilon_2)/\varepsilon_2 A_2}$$

Note that $A_1 = \pi d\ell$ and $A_2 = w\ell$. By inspection and the summation rule, find F_{12} = 60°/360° = 0.167, $F_{1R} = 1 - F_{12} = 1 - 0.167 = 0.833$, and $F_{2R} \approx 1$. With $q_1' = q_1/\ell$,

$$q_1' = \frac{5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left(1500^4 - 500^4\right)\text{K}^4}{0 + \left[\pi\left(10\times10^{-3}\right)\text{m}\times0.167 + \left[\left(1/\pi\left(10\times10^{-3}\text{m}\right)\times0.83\right) + (1/1\text{m}\times1)\right]^{-1}\right]^{-1} + (1-0.6)/0.6\times1\text{ m}}$$

$$q_1' = 8518\text{ W/m}.$$ <

(b) To determine the wall temperature, apply the radiation balance, Eq. 13.31,

$$\frac{J_1 - J_R}{(1/A_1F_{1R})} = \frac{J_R - J_2}{(1/A_2F_{2R})} \quad \text{or} \quad \frac{J_1 - J_R}{\left(1/\pi 10\times10^{-3}\text{m}\times0.833\right)} = \frac{J_R - J_2}{(1/1\text{m}\times1)}.$$

$$J_R = \sigma T_R^4 = (J_1 + 38.21 J_2)/39.21. \qquad (1)$$

Since A_1 is a blackbody, $J_1 = E_{b1} = \sigma T_1^4$. To determine J_2, use Eq. 13.19. Noting that $q_1' = -q_2'$, find

$$q_2 = (E_{b2} - J_2)/(1-\varepsilon_2)/\varepsilon_2 A_2 \quad \text{or} \quad J_2 = E_{b2} - q_2(1-\varepsilon_2)/\varepsilon_2 A_2$$

$$J_2 = 5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4(500\text{ K})^4 - \frac{(-8518\text{ W/m})(1-0.6)}{0.6(1\text{m})} = 9222\text{ W/m}^2.$$

Substituting this value for J_2 into Eq. (1), the wall temperature can be calculated.

$$J_R = \left(5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4(1500\text{K})^4 + 38.21\times9222\,\text{W/m}^2\right)/39.21 = 16{,}308\,\text{W/m}^2$$

$$T_R = (J_R/\sigma)^{1/4} = \left(16{,}308\,\text{W/m}^2/5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\right)^{1/4} = 732\text{ K}.$$ <

COMMENTS: Considering the entire wall as a single re-radiating surface may be a poor assumption since J_R is not likely to be uniform over this large an area. It would be appropriate to consider several isothermal zones for improved accuracy.

PROBLEM 13.73

KNOWN: Circular furnace with one end (A_1) and the lateral surface (A_2) black. Other end (A_3) is insulated.

FIND: (a) Net radiation heat transfer from each surface and (b) Temperature of A_3, and (c) Compute and plot T_3 as a function of tube length L for the range $0.1 \le L \le 0.5$ m with D = 0.3 m.

SCHEMATIC:

ASSUMPTIONS: (1) Surface A_3 is diffuse-gray, (2) Uniform radiosity over A_3.

ANALYSIS: (a) Since A_3 is insulated, the net radiation from A_3 is $q_3 = 0$. Using Eq. 13.30, find

$$q_1 = -q_2 = \frac{\sigma\left(T_1^4 - T_2^4\right)}{(1-\varepsilon_1)/\varepsilon_1 A_1 + \left[A_1F_{12} + \left[(1/A_1F_{13}) + (1/A_2F_{23})\right]^{-1}\right]^{-1} + (1-\varepsilon_2)/\varepsilon_2 A_2}$$

and since $\varepsilon_1 = \varepsilon_2 = 1$, $\quad q_1 = -q_2 = \left\{A_1F_{12} + \left[(1/A_1F_{13}) + (1/A_2F_{23})\right]^{-1}\right\}\sigma\left(T_1^4 - T_2^4\right).$

Considering A_1 and A_3 as coaxial parallel disks, from Table 13.2 (Fig. 13.5) find F_{13},

$$R_1 = r_1/L = 0.15/0.30 = 0.5 \qquad R_2 = r_2/L = 0.15/0.30 = 0.5$$

$$S = 1 + \left(1 + R_2^2\right)/R_1^2 = 1 + \left(1 + 0.5^2\right)/0.5^2 = 6$$

$$F_{13} = 0.5\left\{S - \left[S^2 - 4(r_2/r_1)^2\right]^{1/2}\right\} = 0.5\left\{6 - \left[6^2 - 4(0.5/0.5)\right]^{1/2}\right\} = 0.172.$$

From the summation rule and symmetry $\quad F_{12} = 1 - F_{13} = 1 - 0.172 = 0.828.$

From reciprocity, with $F_{32} = F_{12}$

$$F_{23} = A_3F_{32}/A_2 = \left(\pi D^2/4\right)F_{12}/(\pi DL) = DF_{12}/4L = 0.3\,\text{m} \times 0.828/4 \times 0.3\,\text{m} = 0.207.$$

With $A_1 = \pi D^2/4 = \pi(0.3\text{ m})^2/4 = 0.07069\text{ m}^2$ and $A_2 = \pi DL = \pi(0.3\text{ m})(0.3\text{ m}) = 0.2827\text{ m}^2$,

$$q_1 = -q_2 = \left\{0.07069\,\text{m}^2 \times 0.828 + \left[\left(1/0.07069\,\text{m}^2 \times 0.172\right) + \left(1/0.2827\,\text{m}^2 \times 0.207\right)\right]^{-1}\right\} 5.67 \times 10^{-8}\left(500^4 - 400^4\right)\text{K}^4 \qquad q_1 = -q_2 = 143\,\text{W}$$ <

(b) From the radiation balance on A_3, Eq. 13.31, find T_3, with $J_1 = E_{b1}$ and $J_2 = E_{b2}$,

$$\frac{E_{b1} - J_3}{(1/A_1F_{13})} - \frac{J_3 - E_{b2}}{(1/A_2F_{23})} = 0 \qquad \frac{\sigma(500\text{ K})^4 - J_3}{(1/0.07069\,\text{m} \times 0.172)} - \frac{J_3 - \sigma(400\text{ K})^4}{\left(1/0.2827\,\text{m}^2 \times 0.207\right)} = 0$$

$$J_2 = E_{b3} = \sigma T_3^4 = 1811\text{ W/m}^2 \qquad T_3 = 423\text{ K}.$$ <

(c) Using the *IHT Radiation Tools – Radiation Exchange Analysis, Three surface enclosure with a reradiating surface* and *View Factors, Three-dimensional geometries, Coaxial parallel disks – and*

Continued …..

appropriate view factor relations developed in part (a), T_3 was computed as a function of L with the diameter, D = 0.3 m, and is plotted below.

Note that T_3 decreases with increasing tube length. In the limit as $L \to \infty$, T_3 will approach T_2. In the limit as $L \to 0$, T_3 will approach T_1. Is this intuitively satisfying?

COMMENTS: The *IHT* workspace used for the part (c) analysis is copied below.

```
// Radiation Tool:
// Radiation Exchange Analysis, three-surface enclosure with a reradiating surface
/* For the three-surface enclosure A1, A2 and the reradiating surface A3, the net rate of radiation transfer
from the surface A1 to surface A2 is */
q1 = (Eb1 – Eb2) / ( (1 – eps1) / (eps1 * A1) + 1 / (A1 * F12 + 1 / (1 / (A1 * F13) + 1 / (A2 * F23))) + (1 –
eps2) / (eps2 * A2))              // Eq 13.30
/ * The net rate of radiation transfer from surface A2 to surface A1 is */
q2 = -q1
/ * From a radiation energy balance on A3, */
(J3 – J1) / (1 / (A3 * F31) ) + (J3 – J2) / (1 / (A3 * F32) ) = 0              // Eq 13.31
/* where the radiosities J1 and J2 are determined from the radiation rate equations expressed in terms of
the surface resistances, Eq 13.22 */
q1 = (Eb1 – J1) / ((1 – eps1) / (eps1 * A1))
q2 = (Eb2 – J2) / ((1 – eps2) / (eps2 * A2))
// The blackbody emissive powers for A1 and A2 are
Eb1 = sigma * T1^4
Eb2 = sigma * T2^4
// For the reradiating surface,
J3 = Eb3
Eb3 = sigma * T3^4
sigma = 5.67E-8                  // Stefan-Boltzmann constant, W/m^2·K^4

// Radiation Tool – view factor, F13:
/* The view factor, F13, for coaxial parallel disks, is * /
F13 = 0.5 * (S – sqrt(S^2 – 4 * (r3 / r1)^2))
// where
R1 = r1 / L
R3 = r3 / L
r1 = D1 / 2
r3 = r1
S = 1 + (1 + R3^2) / R1^2
// See Table 13.2 for schematic of this three-dimensional geometry.
// View factor relations:
F12 = 1 – F13                    // Summation rule, A1
F32 = F12                        // Symmetry condition
F23 = A3 * F32 / A2              // Reciprocity relation
F31 = A1 * F13 / A3              // Reciprocity relation
// Areas:
A1 = pi * D1^2/4
A2 = pi * D2 * L
A3 = A1
// Assigned variables:
T1 = 500                         // Temperature, K
D1 = 0.3                         // Diameter, m
eps1 = 0.9999                    // Emissivity; avoiding  “divide-by-zero error”
D2 = D1                          // Diameter, m
eps2 = 0.9999                    // Emissivity; avoiding “divide-by-zero error”
T2 = 400                         // Temperature, K
L = 0.3                          // Length, m
```

PROBLEM 13.74

KNOWN: Very long, triangular duct with walls that are diffuse-gray.

FIND: (a) Net radiation transfer from surface A_1 per unit length of duct, (b) The temperature of the insulated surface, (c) Influence of ε_3 on the results; comment on exactness of results.

SCHEMATIC:

Width, all sides, w = 1m

$A_1, T_1 = 1000K, \varepsilon_1 = 0.33$

$A_2, T_2 = 700K, \varepsilon_2 = 0.5$

$A_3, T_3, \varepsilon_3 = 0.8$

Insulated surface

ASSUMPTIONS: (1) Surfaces are diffuse-gray, (2) Duct is very long; end effects negligible.

ANALYSIS: (a) The duct approximates a three-surface enclosure for which the third surface (A_3) is re-radiating. Using Eq. 13.30 with $A_3 = A_R$, the net exchange is

$$q_1 = -q_2 = \frac{E_{b1} - E_{b2}}{\frac{(1-\varepsilon_1)}{\varepsilon_1 A_1} + \frac{1}{A_1 F_{12} + \left(1/A_1 F_{1T} + 1/A_2 F_{2R}\right)^{-1}} + \frac{(1-\varepsilon_2)}{\varepsilon_2 A_2}} \qquad (1)$$

From symmetry, $F_{12} = F_{1R} = F_{2R} = 0.5$. With $A_1 = A_2 = w \cdot \ell$, where ℓ is the length normal to the page and w = 1 m,

$$q_1' = q_1/\ell = (q_1/A_1)\,w$$

$$q_1' = \frac{(56,700 - 13,614)\,\text{W/m}^2 \times 1\,\text{m}}{\frac{(1-0.33)}{0.33} + \frac{1}{0.5 + (1/0.5 + 1/0.5)^{-1}} + \frac{(1-0.5)}{0.5}} = 9874\ \text{W/m.}$$ <

(b) From a radiation balance on A_R,

$$q_R = q_3 = 0 = \frac{E_{b3} - J_1}{(A_3 F_{31})^{-1}} + \frac{E_{b3} - J_2}{(A_3 F_{32})^{-1}} \qquad \text{or} \qquad E_{b3} = \frac{J_1 + J_2}{2}. \qquad (2)$$

To evaluate J_1 and J_2, use Eq. 13.19,

$$J_i = E_{b,i} - \frac{q_i}{A_i}\frac{(1-\varepsilon_i)}{\varepsilon_i} \quad \begin{cases} J_1 = 56,700 - (9874)\dfrac{1-0.33}{0.33} = 36,653\ \text{W/m}^2 \\ J_2 = 13,614 - (-9874)\dfrac{1-0.5}{0.5} = 23,488\ \text{W/m}^2 \end{cases}$$

From Eq. (2), now find

$$T_3 = (E_{b3}/\sigma)^{1/4} = \left([J_1 + J_2]/2\sigma\right)^{1/4} = \left(\frac{(36,653 + 23,488)\,\text{W/m}^2}{2\left(5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\right)}\right)^{1/4} = 853\ \text{K.}$$ <

(c) Since A_3 is adiabatic or re-radiating, $J_3 = Eb_3$. Therefore, the value of ε_3 is of no influence on the radiation exchange or on T_3. In using Eq. (1), we require uniform radiosity over the surfaces. This requirement is not met near the corners. For best results we should subdivide the areas such that they represent regions of uniform radiosity. Of course, the analysis then becomes much more complicated.

PROBLEM 13.75

KNOWN: Dimensions for aligned rectangular heater and coated plate. Temperatures of heater, plate and large surroundings.

FIND: (a) Electric power required to operate heater, (b) Heater power required if reradiating sidewalls are added, (c) Effect of coating emissivity and electric power.

SCHEMATIC:

Surroundings at 300 K or reradiating side walls; 2 m; 1 m; 0.5 m; Heater ($\varepsilon_1 = 1$) $T_1 = 700$ K; $T_2 = 400$ K; Coating ($0.1 \le \varepsilon_2 \le 1$)

ASSUMPTIONS: (1) Steady-state, (2) Blackbody behavior for surfaces and surroundings (Parts (a) and (b)).

ANALYSIS: (a) For $\varepsilon_1 = \varepsilon_2 = 1$, the net radiation leaving A_1 is

$$q_{elec} = q_1 = A_1 F_{12} \sigma \left(T_1^4 - T_2^4\right) + A_1 F_{1sur} \sigma \left(T_1^4 - T_{sur}^4\right).$$

From Fig. 13.4, with Y/L = 1/0.5 = 2 and X/L = 2/0.5 = 4, the view factors are $F_{12} \approx 0.5$ and $F_{sur} \approx 1 - 0.5 = 0.5$. Hence,

$$q_{elec} = \left(2 m^2\right) 0.5 \times 5.67 \times 10^{-8} \, W/m^2 \cdot K^4 \left[(700\ K)^4 - (400\ K)^4\right]$$

$$+ \left(2 m^2\right) 0.5 \times 5.67 \times 10^{-8} \, W/m^2 \cdot K^4 \left[(700\ K)^4 - (300\ K)^4\right] = (12,162 + 13,154)\, W = 25,316\, W. \quad <$$

(b) With the reradiating walls, the net radiation leaving A_1 is $q_{elec} = q_1 = q_{12}$. From Eq. 13.30 with $\varepsilon_1 = \varepsilon_2 = 1$ and $A_1 = A_2$,

$$q_{elec} = A_1 \sigma \left(T_1^4 - T_2^4\right) \left\{F_{12} + \left[(1/F_{1R}) + (1/F_{2R})\right]^{-1}\right\}$$

$$q_{elec} = \left(2\, m^2\right) 5.67 \times 10^{-8} \, W/m^2 \cdot K^4 \left[(700\ K)^4 - (400\ K)^4\right] \times \left\{0.5 + \left[(1/0.5) + (1/0.5)\right]^{-1}\right\}$$

$$q_{elec} = 18,243\, W. \quad <$$

(c) Separately using the *IHT Radiation* Tool Pad for a three-surface enclosure, with one surface reradiating, and to perform a radiation exchange analysis for a three-surface enclosure, with one surface corresponding to large surroundings, the following results were obtained.

Continued …..

PROBLEM 13.75 (Cont.)

In both cases, the required heater power decreases with decreasing ε_2, and the trend is attributed to a reduction in $\alpha_2 = \varepsilon_2$ and hence to a reduction in the rate at which radiant energy must be absorbed by the surface to maintain the prescribed temperature.

COMMENTS: With the reradiating walls in part (b), it follows from Eq. 13.31 that

$$J_R = E_{bR} = (J_1 + J_2)/2 = (E_{b1} + E_{b2})/2.$$

Hence, $T_R = 604$ K. The reduction in q_{elec} resulting from use of the walls is due to the enhancement of radiation to the heater, which, in turn, is due to the presence of the high temperature walls.

PROBLEM 13.76

KNOWN: Configuration and operating conditions of a furnace. Initial temperature and emissivity of steel plate to be treated.

FIND: (a) Heater temperature, (b) Sidewall temperature.

SCHEMATIC:

Radiant heaters
$\varepsilon_1 = 0.8$, $q_1 = 1.5\times10^5$ W
T_R
H=2m
W=2m
Steel plate, $\varepsilon_2 = 0.4$, $T_2 = 300$K

ASSUMPTIONS: (1) Steady-state conditions, (2) Opaque, diffuse-gray surface behavior, (3) Negligible convection, (4) Sidewalls are re-radiating.

ANALYSIS: (a) From Eq. 13.30

$$q_1 = \frac{E_{b1} - E_{b2}}{\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} + \dfrac{1}{A_1F_{12} + \left[(A_1F_{1R})^{-1} + (A_2F_{2R})^{-1}\right]^{-1}} + \dfrac{1-\varepsilon_2}{\varepsilon_2 A_2}} = 1.5\times10^5\ \text{W}$$

Note that $A_1 = A_2 = 4\ \text{m}^2$ and $E_{b2} = \sigma T_2^4 = 5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\ (300\ \text{K})^4 = 459\ \text{W/m}^2$. From Fig. 13.4, with X/L = Y/L = 1, $F_{12} = 0.2$; hence $F_{1R} = 1 - F_{12} = 0.8$, and $F_{2R} = F_{1R} = 0.8$. With $(1-\varepsilon_1)/\varepsilon_1 = 0.25$ and $(1 - \varepsilon_2)/\varepsilon_2 = 1.5$, find

$$\frac{1.5\times10^5\ \text{W}}{4\text{m}^2} = \frac{E_{b1} - 459\ \text{W/m}^2}{0.25 + \dfrac{1}{0.2 + [1.25 + 1.25]^{-1}} + 1.5} = \frac{E_{b1} - 459\ \text{W/m}^2}{3.417}$$

$$E_{b1} = 1.28\times10^5\ \text{W/m}^2 + 459\ \text{W/m}^2 = 1.29\times10^5\ \text{W/m}^2 = \sigma T_1^4$$

$$T_1 = \left(1.29\times10^5\ \text{W/m}^2 / 5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)^{1/4} = 1228\ \text{K.}$$

(b) From Eq. 13.31,it follows that, with $A_1F_{1R} = A_2F_{2R}$,

$$J_R = \sigma T_R^4 = (J_1 + J_2)/2$$

From Eq. 13.19,

$$J_1 = E_{b1} - \frac{(1-\varepsilon_1)}{\varepsilon_1 A_1} q_1 = 1.29\times10^5\ \text{W/m}^2 - \frac{0.2\times1.5\times10^5\ \text{W}}{0.8\times4\text{m}^2}$$

$$J_1 = 1.196\times10^5\ \text{W/m}^2.$$

With $q_2 = q_1 = -1.5\times10^5$ W,

$$J_2 = E_{b2} - \frac{(1-\varepsilon_2)}{\varepsilon_2 A_2} q_2 = 459\ \text{W/m}^2 + \frac{0.6}{0.4\times4\text{m}^2} 1.5\times10^5\ \text{W} = 5.67\times10^4\ \text{W/m}^2$$

$$T_R = \left(\frac{1.196\times10^5\ \text{W/m}^2 + 5.67\times10^4\ \text{W/m}^2}{2\times5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4}\right)^{1/4} = 1117\ \text{K.}$$ <

COMMENTS: (1) The above results are approximate, since the process is actually transient. (2) T_1 and T_R will increase with time as T_2 increases.

PROBLEM 13.76

KNOWN: Configuration and operating conditions of a furnace. Initial temperature and emissivity of steel plate to be treated.

FIND: (a) Heater temperature, (b) Sidewall temperature.

SCHEMATIC:

Radiant heaters
$\varepsilon_1 = 0.8,\ q_1 = 1.5\times10^5\,W$
T_R
H=2m
W=2m
Steel plate, $\varepsilon_2 = 0.4,\ T_2 = 300K$

ASSUMPTIONS: (1) Steady-state conditions, (2) Opaque, diffuse-gray surface behavior, (3) Negligible convection, (4) Sidewalls are re-radiating.

ANALYSIS: (a) From Eq. 13.30

$$q_1 = \frac{E_{b1} - E_{b2}}{\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} + \dfrac{1}{A_1F_{12} + \left[\left(A_1F_{1R}\right)^{-1} + \left(A_2F_{2R}\right)^{-1}\right]^{-1}} + \dfrac{1-\varepsilon_2}{\varepsilon_2 A_2}} = 1.5\times10^5\ \text{W}$$

Note that $A_1 = A_2 = 4\ m^2$ and $E_{b2} = \sigma T_2^4 = 5.67 \times 10^{-8}\ W/m^2 \cdot K^4\ (300\ K)^4 = 459\ W/m^2$. From Fig. 13.4, with X/L = Y/L = 1, $F_{12} = 0.2$; hence $F_{1R} = 1 - F_{12} = 0.8$, and $F_{2R} = F_{1R} = 0.8$. With $(1-\varepsilon_1)/\varepsilon_1 = 0.25$ and $(1 - \varepsilon_2)/\varepsilon_2 = 1.5$, find

$$\frac{1.5\times10^5\ \text{W}}{4\text{m}^2} = \frac{E_{b1} - 459\,\text{W/m}^2}{0.25 + \dfrac{1}{0.2 + \left[1.25 + 1.25\right]^{-1}} + 1.5} = \frac{E_{b1} - 459\,\text{W/m}^2}{3.417}$$

$$E_{b1} = 1.28\times10^5\,\text{W/m}^2 + 459\,\text{W/m}^2 = 1.29\times10^5\ \text{W/m}^2 = \sigma T_1^4$$

$$T_1 = \left(1.29\times10^5\,\text{W/m}^2 / 5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\right)^{1/4} = 1228\ \text{K.}$$

(b) From Eq. 13.31,it follows that, with $A_1F_{1R} = A_2F_{2R}$,

$$J_R = \sigma T_R^4 = \left(J_1 + J_2\right)/2$$

From Eq. 13.19,

$$J_1 = E_{b1} - \frac{\left(1-\varepsilon_1\right)}{\varepsilon_1 A_1} q_1 = 1.29\times10^5\,\text{W/m}^2 - \frac{0.2\times1.5\times10^5\,\text{W}}{0.8\times4\text{m}^2}$$

$$J_1 = 1.196\times10^5\,\text{W/m}^2.$$

With $q_2 = q_1 = -\,1.5 \times 10^5$ W,

$$J_2 = E_{b2} - \frac{\left(1-\varepsilon_2\right)}{\varepsilon_2 A_2} q_2 = 459\,\text{W/m}^2 + \frac{0.6}{0.4\times4\text{m}^2} 1.5\times10^5\ \text{W} = 5.67\times10^4\ \text{W/m}^2$$

$$T_R = \left(\frac{1.196\times10^5\ \text{W/m}^2 + 5.67\times10^4\ \text{W/m}^2}{2\times5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4}\right)^{1/4} = 1117\ \text{K.}$$ $<$

COMMENTS: (1) The above results are approximate, since the process is actually transient. (2) T_1 and T_R will increase with time as T_2 increases.

PROBLEM 13.77

KNOWN: Dimensions, surface radiative properties, and operating conditions of an electrical furnace.

FIND: (a) Equivalent radiation circuit, (b) Furnace power requirement and temperature of a heated plate.

SCHEMATIC:

$A_1, T_1 = 800K, \varepsilon_1 = 0.9$

$A_3, T_3, \varepsilon_3 = 0.6$

$A_2, T_2 = 400K, \varepsilon_2 = 0.3$

ASSUMPTIONS: (1) Steady-state conditions, (2) Opaque, diffuse-gray surfaces, (3) Negligible plate temperature gradients, (4) Back surfaces of heater are adiabatic, (5) Convection effects are negligible.

ANALYSIS: (a) Since there is symmetry about the plate, only one-half (top or bottom) of the system need be considered. Moreover, the plate must be adiabatic, thereby playing the role of a re-radiating surface.

E_{b1}, $\frac{1-\varepsilon_1}{\varepsilon_1 A_1}$, J_1, $\frac{1}{A_1F_{12}}$, J_2, $\frac{1-\varepsilon_2}{\varepsilon_2 A_2}$, E_{b2}, q, $\frac{1}{A_1F_{13}}$, $\frac{1}{A_3F_{32}}$, $J_3 = E_{b3}$

(b) Note that $A_1 = A_3 = 4\ m^2$ and $A_2 = (0.5\ m \times 2\ m)4 = 4\ m^2$. From Fig. 13.4, with X/L = Y/L = 4, $F_{13} = 0.62$. Hence

$$F_{12} = 1 - F_{13} = 0.38, \quad \text{and} \quad F_{32} = F_{12} = 0.38.$$

It follows that

$$A_1F_{12} = 4(0.38) = 1.52\,m^2$$

$$A_1F_{13} = 4(0.62) = 2.48\,m^2, \qquad (1-\varepsilon_1)/\varepsilon_1 A_1 = 0.1/3.6\,m^2 = 0.0278\,m^{-2}$$

$$A_3F_{32} = 4(0.38) = 1.52\,m^2, \qquad (1-\varepsilon_2)/\varepsilon_2 A_2 = 0.7/1.2\,m^2 = 0.583\,m^{-2}.$$

Also,

$$E_{b1} = \sigma T_1^4 = 5.67\times10^{-8}\,W/m^2\cdot K^4\,(800\ K)^4 = 23{,}224\,W/m^2,$$

$$E_{b2} = \sigma T_2^4 = 5.67\times10^{-8}\,W/m^2\cdot K^4\,(400\ K)^4 = 1452\,W/m^2.$$

The system forms a three-surface enclosure, with one surface re-radiating. Hence the net radiation transfer from a single heater is, from Eq. 13.30,

$$q_1 = \frac{E_{b1} - E_{b2}}{\frac{1-\varepsilon_1}{\varepsilon_1 A_1} + \frac{1}{A_1F_{12} + \left[1/A_1F_{13} + 1/A_3F_{32}\right]^{-1}} + \frac{1-\varepsilon_2}{\varepsilon_2 A_2}}$$

Continued …..

$$q_1 = \frac{(23,224 - 1452)\,W/m^2}{(0.0278 + 0.4061 + 0.583)\,m^{-2}} = 21.4\,kW.$$

The furnace power requirement is therefore $q_{elec} = 2q_1 = 43.8$ kW, with <

$$q_1 = \frac{E_{b1} - J_1}{(1-\varepsilon_1)/\varepsilon_1 A_1}.$$

where

$$J_1 = E_{b1} - q_1 \frac{1-\varepsilon_1}{\varepsilon_1 A_1} = 23,224\,W/m^2 - 21,400\,W \times 0.0278\,m^{-2}$$

$$J_1 = 22,679\,W/m^2.$$

Also,

$$J_2 = E_{b2} - q_2 \frac{1-\varepsilon_2}{\varepsilon_2 A_2} = 1,452\,W/m^2 - (-21,400\,W) \times 0.583\,m^{-2}$$

$$J_2 = 13,928\,W/m^2.$$

From Eq. 13.31,

$$\frac{J_1 - J_3}{1/A_1F_{13}} = \frac{J_3 - J_2}{1/A_3F_{32}}$$

$$\frac{J_1 - J_3}{J_3 - J_2} = \frac{A_3F_{32}}{A_1F_{13}} = \frac{1.52}{2.48} = 0.613$$

$$1.613J_3 = J_1 + 0.613J_2 = 22,629 + 8537 = 31,166\,W/m^2$$

$$J_3 = 19,321\,W/m^2$$

Since $J_3 = E_{b3}$,

$$T_3 = (E_{b3}/\sigma)^{1/4} = \left(19,321/5.67 \times 10^{-8}\right)^{1/4} = 764\ K.$$ <

COMMENTS: (1) To reduce q_{elec}, the sidewall temperature T_2, should be increased by insulating it from the surroundings. (2) The problem must be solved by simultaneously determining J_1, J_2 and J_3 from the radiation balances of the form

$$\frac{E_{b1} - J_1}{(1-\varepsilon_1)/\varepsilon_1 A_1} = A_1F_{12}(J_1 - J_2) + A_1F_{13}(J_1 - J_3)$$

$$\frac{E_{b2} - J_2}{(1-\varepsilon_2)/\varepsilon_2 A_2} = A_2F_{21}(J_2 - J_1) + A_2F_{23}(J_2 - J_3)$$

$$0 = A_1F_{13}(J_3 - J_1) + A_2F_{23}(J_3 - J_2).$$

PROBLEM 13.78

KNOWN: Geometry and surface temperatures and emissivities of a solar collector.

FIND: Net rate of radiation transfer to cover plate due to exchange with the absorber plates.

SCHEMATIC:

$A_1 = A_2 = A_3 = (1m \times \ell)$

$\varepsilon_2 = 1.0, T_2 = 60°C$

$\varepsilon_3 = 1.0, T_3 = 70°C$

$\varepsilon_1 = 0.9, T_1 = 25°C$

ASSUMPTIONS: (1) Isothermal surfaces with uniform radiosity, (2) Absorber plates behave as blackbodies, (3) Cover plate is diffuse-gray and opaque to thermal radiation exchange with absorber plates, (4) Duct end effects are negligible.

ANALYSIS: Applying Eq. 13.21 to the cover plate, it follows that

$$E_{b1} - J_1 = \frac{1-\varepsilon_1}{\varepsilon_1 A_1} \sum_{j=1}^{N} \frac{J_1 - J_j}{(A_i F_{ij})^{-1}} = \frac{1-\varepsilon_1}{\varepsilon_1 A_1}\left[A_1 F_{12}(J_1 - J_2) + A_1 F_{13}(J_1 - J_3)\right].$$

From symmetry, $F_{12} = F_{13} = 0.5$. Also, $J_2 = E_{b2}$ and $J_3 = E_{b3}$. Hence

$$E_{b1} - J_1 = 0.0556(2J_1 - E_{b2} - E_{b3})$$

or with $E_b = \sigma T^4$,

$$1.111 J_1 = E_{b1} + 0.0556(E_{b2} + E_{b3})$$

$$1.111 J_1 = 5.67\times10^{-8}(298)^4 \text{ W/m}^2 + 0.0556\left(5.67\times10^{-8}\right)\left[(333)^4 + (343)^4\right]\text{W/m}^2$$

$$J_1 = 476.64 \text{ W/m}^2$$

From Eq. 13.19 the net rate of radiation transfer *from* the cover plate is then

$$q_1 = \frac{E_{b1} - J_1}{(1-\varepsilon_1)/\varepsilon_1 A_1} = \frac{5.67\times10^{-8}(298)^4 - 476.64}{(1-0.9)/0.9(\ell)} = (-265.5\ell)\text{W}.$$

The net rate of radiation transfer *to* the cover plate per unit length is then

$$q_1' = (q_1/\ell) = 266 \text{ W/m}.$$ <

COMMENTS: Solar radiation effects are not relevant to the foregoing problem. All such radiation transmitted by the cover plate is completely absorbed by the absorber plate.

PROBLEM 13.79

KNOWN: Cylindrical peep-hole of diameter D through a furnace wall of thickness L. Temperatures prescribed for the furnace interior and surroundings outside the furnace.

FIND: Heat loss by radiation through the peep-hole.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Furnace interior and exterior surroundings are large, isothermal surroundings for the peep-hole openings, (3) Furnace refractory wall is adiabatic and diffuse-gray with uniform radiosity.

ANALYSIS: The open-ends of the cylindrical peep-hole (A_1 and A_2) and the cylindrical lateral surface of the refractory material (A_R) form a diffuse-gray, three-surface enclosure. The hypothetical areas A_1 and A_2 behave as black surfaces at the respective temperatures of the large surroundings to which they are exposed. Since A_r is adiabatic, it behaves as a re-radiating surface, and its emissivity has no effect on the analysis. From Eq. 13.30, the net radiation leaving A_1 passes through the enclosure into the outer surroundings.

$$q_1 = -q_2 = \frac{E_{b1} - E_{b2}}{\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} + \dfrac{1}{A_1 F_{12} + \left[(1/A_1 F_{1R}) + (1/A_2 F_{2R})\right]^{-1}} + \dfrac{1-\varepsilon_2}{\varepsilon_2 A_2}}$$

Since $\varepsilon_1 = \varepsilon_2 = 1$, and with $E_b = \sigma T^4$ where $\sigma = 5.67 \times 10^{-8}$ W/m^2·K^4,

$$q_1 = \left\{A_1 F_{12} + \left[(1/A_1 F_{1R}) + (1/A_2 F_{2R})\right]^{-1}\right\}\sigma\left(T_1^4 - T_2^4\right)$$

where $A_1 = A_2 = \pi D^2/4$. The view factor F_{12} can be determined from Table 13.2 (Fig. 13.5) for the coaxial parallel disks ($R_1 = R_2 = 125/(2 \times 250) = 0.25$ and $S = 17.063$) as

$$F_{12} = 0.05573$$

From the summation rule on A_1, with $F_{11} = 0$,

$$F_{11} + F_{12} + F_{1R} = 1$$

$$F_{1R} = 1 - F_{12} = 1 - 0.05573 = 0.9443$$

and from symmetry of the enclosure,

$$F_{2R} = F_{1R} = 0.9443$$

Substituting numerical values into the rate equation, find the heat loss by radiation through the peep-hole to the exterior surroundings as

$$q_{loss} = q_1 = 1046 \text{ W}$$ <

COMMENTS: If you held your hand 50 mm from the exterior opening of the peep-hole, how would that feel? It is standard, safe practice to use optical protection when viewing the interiors of high temperature furnaces as used in petrochemical, metals processing and power generation operations.

PROBLEM 13.80

KNOWN: Composite wall comprised of two large plates separated by sheets of refractory insulation of thermal conductivity k = 0.05 W/m·K; gaps between the sheets of width w = 10 mm, located at 1 - m spacing, allow radiation transfer between the plates.

FIND: (a) Heat loss by radiation through the gap per unit length of the composite wall (normal to the page), and (b) fraction of the total heat loss through the wall that is due to radiation transfer through the gap.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Surfaces are diffuse-gray with uniform radiosities, (3) Refractory insulation surface in the gap is adiabatic, and (4) Heat flow through the wall is one-dimensional between the plates in the direction of the gap centerline.

ANALYSIS: (a) The gap of thickness w and infinite extent normal to the page can be represented by a diffuse-gray, three-surface enclosure formed by the plates A_1 and A_2 and the refractory walls, A_R. Since A_R is adiabatic, it behaves as a re-radiating surface, and its emissivity has no effect on the analysis. From Eq. 13.30, the net radiation leaving the plate A_1 passes through the gap into plate A_2.

$$q_1 = -q_2 = \frac{E_{b1} - E_{b2}}{\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} + \dfrac{1}{A_1 F_{12} + \left[(1/A_1 F_{1R}) + (1/A_2 F_{2R})\right]^{-1}} + \dfrac{1-\varepsilon_2}{\varepsilon_2 A_2}}$$

where $E_b = \sigma T^4$ with $\sigma = 5.67 \times 10^{-8}$ W/m^2·K^4 and $A_1 = A_2 = w \cdot \ell$, but making ℓ = 1 m to obtain $q_1' \left(W/m\right)$.

The view factor F_{12} can be determined from Table 13.2 (Fig. 13.4) for aligned parallel rectangles where $\overline{X} = X/L = \infty$ since $X \to \infty$ and $\overline{Y} = Y/L = W/L = 10/50 = 0.2$ giving

$$F_{12} = 0.09902$$

From the summation rule on A_1, with $F_{11} = 0$,

$$F_{11} + F_{12} + F_{1R} = 1 \qquad\qquad F_{1R} = 1 - F_{12} = 1 - 0.09902 = 0.901$$

and from symmetry of the enclosure,

$$F_{2R} = F_{1R} = 0.901.$$

Continued …..

PROBLEM 13.80 (Cont.)

Substituting numerical values into the rate equation, find the heat loss through the gap due to radiation as

$$q'_{rad} = q'_1 = 37\ \mathrm{W/m}$$ <

(b) The conduction heat rate per unit length (normal to the page) for a 1 - m section is

$$q'_{cond} = k\,(1\ \mathrm{m})\frac{T_1 - T_2}{L} = 0.05\ \mathrm{W/m\cdot K} \times 1\ \mathrm{m}\frac{(400-35)\mathrm{K}}{0.050\ \mathrm{mm}}$$

$$q'_{cond} = 365\ \mathrm{W/m}$$

The fraction of the total heat transfer through the 1 - m section due to radiation is

$$\frac{q'_{rad}}{q'_{tot}} = \frac{q'_{rad}}{q'_{cond} + q_{rad}} = \frac{37}{365+37} = 9.2\%$$ <

We conclude that if the installation process for the sheet insulation can be accomplished with a smaller gap, there is an opportunity to reduce the cost of operating the furnace.

PROBLEM 13.81

KNOWN: Diameter, temperature and emissivity of a heated disk. Diameter and emissivity of a hemispherical radiation shield. View factor of shield with respect to a coaxial disk of prescribed diameter, emissivity and temperature.

FIND: (a) Equivalent circuit, (b) Net heat rate from the hot disk.

SCHEMATIC:

ASSUMPTIONS: (1) Surfaces may be approximated as diffuse/gray, (2) Surface 4 is reradiating, (3) Negligible convection.

ANALYSIS: (a) The equivalent circuit is shown in the schematic. Since surface 4 is treated as reradiating, the net transfer of radiation from surface 1 is equal to the net transfer of radiation to surface 3 ($q_1 = -q_3$).

(b) From the thermal circuit, the desired heat rate may be expressed as

$$q_1 = \frac{E_{b1} - E_{b3}}{\frac{1-\varepsilon_1}{\varepsilon_1 A_1} + \frac{1}{A_1 F_{12}} + \frac{2(1-\varepsilon_2)}{\varepsilon_2 A_2} + \left[A_2 F_{23} + \frac{1}{\frac{1}{A_2 F_{24}} + \frac{1}{A_3 F_{34}}}\right]^{-1} + \frac{1-\varepsilon_1}{\varepsilon_3 A_3}}$$

where $A_1 = A_3 = \pi D_1^2/4 = \pi(0.05\text{ m})^2/4 = 1.963 \times 10^3\text{ m}^2$, $A_2 = \pi D_1^2/2 = 2A_1 = 3.925 \times 10^{-3}\text{ m}^2$, $F_{12} = 1$, and $F_{24} = 1 - F_{23} = 0.7$. With $F_{34} = 1 - F_{32} = 1 - F_{23}(A_2/A_3) = 1 - 0.3(2) = 0.4$, it follows that

Continued …..

PROBLEM 13.81 (Cont.)

$$q_1 = \frac{A_1 \sigma \left(T_1^4 - T_3^4\right)}{\frac{1-\varepsilon_1}{\varepsilon_1 A_1} + \frac{1}{F_{12}} + \frac{2\left(1-\varepsilon_2\right)}{\varepsilon_2}\frac{A_1}{A_2} + \left[\frac{A_2}{A_1}F_{23} + \frac{1}{\frac{A_1}{A_2 F_{24}} + \frac{A_1}{A_3 F_{34}}}\right]^{-1} + \frac{1-\varepsilon_3}{\varepsilon_3}}$$

$$q_1 = \frac{A_1 \sigma \left(T_1^4 - T_3^4\right)}{0.667 + 1 + 49 + \left[0.6 + \frac{1}{\frac{1}{1.4} + \frac{1}{0.4}}\right]^{-1} + 1.5} = \frac{A_1 \sigma \left(T_1^4 - T_3^4\right)}{0.667 + 1 + 49 + 1.098 + 1.5}$$

$$q_1 = 0.0188\left(1.963 \times 10^{-3}\,\mathrm{m}^2\right)5.67 \times 10^{-8}\,\mathrm{W/m^2 \cdot K^4}\left(900^4 - 400^4\right)\mathrm{K}^4$$

$$q_1 = 1.32\,\mathrm{W}$$ $<$

COMMENTS: Radiation transfer from 1 to 3 is impeded and enhanced, respectively, by the radiation shield and the reradiating walls. However, the dominant contribution to the total radiative resistance is made by the shield.

PROBLEM 13.82

KNOWN: Diameter, temperature and emissivity of a cylindrical heater. Dimensions, temperature, and emissivity of a reflector. Temperature of large surroundings.

FIND: (a) Equivalent circuit and values of associated resistances and driving potentials, (b) Required electric power per unit length of heater.

SCHEMATIC:

ASSUMPTIONS: (1) Surfaces are diffuse-gray, (2) Surroundings form a large enclosure which may be represented by a hypothetical surface of temperature $T_3 = T_{sur}$ and emissivity $\varepsilon_3 = 1$.

ANALYSIS: (a) The circuit is shown in the schematic, where the hypothetical surface is part of a three-surface enclosure. On a unit length basis, the circuit resistances are

$$\frac{1-\varepsilon_1}{\varepsilon_1 A_1'} = \frac{0.2}{0.8(\pi)(0.03\,\text{m})} = 2.65\,\text{m}^{-1}$$ <

$$\frac{1}{A_1' F_{12}} = \frac{1}{\pi(0.03\,\text{m})0.625} = 16.98\,\text{m}^{-1}$$ <

$$\frac{1}{A_1' F_{13}} = \frac{1}{\pi(0.03\,\text{m})0.375} = 28.30\,\text{m}^{-1}$$ <

$$\frac{1}{A_2' F_{23}} = \frac{1}{A_3' F_{32}} = \frac{1}{(0.15\,\text{m})0.764} = 8.73\,\text{m}^{-1}$$ <

$$\frac{1-\varepsilon_2}{\varepsilon_2 A_2'} = \frac{0.9}{0.1(0.30\,\text{m})} = 30\,\text{m}^{-1}$$ <

The view factor F_{13} is obtained from the summation rule, where $F_{13} = 1 - F_{12} = 1 - 0.625 = 0.375$. Similarly, $F_{32} = 1 - F_{31}$, where $F_{31} = F_{13}\left(A_1'/A_3'\right) = 0.375(\pi D/W) = 0.375\pi(0.03\text{ m}/0.15\text{ m}) = 0.236$. Hence, $F_{32} = 0.764$. The potentials are

Continued …..

PROBLEM 13.82 (Cont.)

$$E_{b1} = \sigma T_1^4 = 5.67 \times 10^{-8}\,\mathrm{W/m^2 \cdot K^4}\,(945\ \mathrm{K})^4 = 45,220\,\mathrm{W/m^2} \qquad <$$

$$E_{b2} = \sigma T_2^4 = 5.67 \times 10^{-4}\,\mathrm{W/m^2 \cdot K^4}\,(385\ \mathrm{K})^4 = 1246\,\mathrm{W/m^2} \qquad <$$

$$E_{b3} = \sigma T_3^4 = 5.67 \times 10^{-8}\,\mathrm{W/m^2 \cdot K^4}\,(300\ \mathrm{K})^4 = 459\,\mathrm{W/m^2} \qquad <$$

(b) The required heater power may be obtained by applying Eq. (13.19) to node 1. Hence,

$$q_1' = \frac{E_{b1} - J_1}{(1-\varepsilon_1)/\varepsilon_1 A_1'}$$

The radiosity may be obtained by applying radiation balances to nodes J_1 and J_2:

$$\frac{E_{b1} - J_1}{(1-\varepsilon_1)/\varepsilon_1 A_1'} = \frac{J_1 - J_2}{1/A_1' F_{12}} + \frac{J_1 - E_{b3}}{1/A_1' F_{13}}$$

$$\frac{E_{b2} - J_2}{(1-\varepsilon_2)/\varepsilon_2 A_2'} = \frac{J_2 - J_1}{1/A_1' F_{12}} + \frac{J_2 - E_{b3}}{1/A_2' F_{23}}$$

Substituting the known potentials and solving, we obtain J_1 = 37,600 W/m^2 and J_2 = 11,160 W/m^2. Hence

$$q_1' = \frac{(45,220 - 37,600)\,\mathrm{W/m^2}}{2.65\,\mathrm{m^{-1}}} = 2875\,\mathrm{W/m} \qquad <$$

COMMENTS: Additional power would have to be supplied to compensate for heat transfer by free convection from the heater to the air.

PROBLEM 13.83

KNOWN: Cylindrical cavity with prescribed geometry, wall emissivity, and temperature.

FIND: Net radiation heat transfer from the cavity assuming the surroundings of the cavity are at 300 K.

SCHEMATIC:

Tsur = 300K; T3, A3; L = 50mm; A2; A1; D = 100mm

$$A_1 = A_3 = \pi D^2/4 = 7.85\times10^{-3}\,m^2$$

$$A_2 = \pi DL = 1.57\times10^{-2}\,m^2$$

$$F_{13} = F_{31} = 0.38 \qquad F_{32} = F_{12} = 0.62$$

$$F_{21} = F_{23} = 0.310$$

$$T_1 = T_2 = 1500\ K$$

$$\varepsilon_1 = \varepsilon_2 = 0.6$$

ASSUMPTIONS: (1) Cavity interior surfaces are diffuse-gray, (2) Surroundings are much larger than the cavity opening A_3; $T_3 = T_{sur} = 300$ K and $\varepsilon_3 = 1$.

ANALYSIS: The net radiation heat transfer from the cavity is $-q_3$, which from Eq. 13.20 is,

$$q_3 = A_3F_{31}(J_3 - J_1) + A_3F_{32}(J_3 - J_2). \qquad (1)$$

While $J_3 = E_{b3}$ since $\varepsilon_3 = 1$, J_1 and J_2 are unknown and must be obtained from the radiation balances, Eq. 13.21.

$$\frac{E_{bi} - J_i}{(1-\varepsilon_i)/\varepsilon_i A_i} = \sum_{j=1}^{N} \frac{J_i - J_i}{(A_iF_{ij})^{-1}} \qquad (2)$$

From Fig. 13.5 with $L/r_1 = 0.050/0.050 = 1$ and $r_3/L = 1$, find $F_{13} = 0.38$. From summation rule and reciprocity: $F_{32} = F_{12} = 1 - F_{12} = 0.62$ and $F_{21} = F_{23} = (A_3\,F_{32})/A_2 = 0.310$. Note also, $E_{b1} = E_{b2} = \sigma T_1^4 = \sigma(1500K)^4 = 287{,}044\ W/m^2$ and $J_3 = E_{b3} = \sigma T_3^4 = 459.3\ W/m^2$.

A_1:

$$\frac{E_{b1} - J_1}{(1-\varepsilon_1)/\varepsilon_1 A_1} = \frac{J_1 - J_2}{(A_1F_{12})^{-1}} + \frac{J_1 - J_3}{(A_1F_{13})^{-1}}$$

$$\frac{287{,}044 - J_1}{(1-0.6)/0.6} = \frac{J_1 - J_2}{(0.62)^{-1}} + \frac{J_1 - 459.3}{(0.38)^{-1}} \qquad 2.5J_1 - 0.62J_2 = 430{,}741 \qquad (3)$$

A_2:

$$\frac{E_{b2} - J_2}{(1-\varepsilon_2)/\varepsilon_2 A_2} = \frac{J_2 - J_1}{(A_2F_{21})^{-1}} + \frac{J_2 - J_3}{(A_2F_{23})^{-1}}$$

$$\frac{287{,}044 - J_2}{(1-0.6)/0.6} = \frac{J_2 - J_1}{(0.31)^{-1}} + \frac{J_2 - 459.3}{(0.31)^{-1}} \qquad -0.31J_1 + 2.140J_2 = 430{,}708 \qquad (4)$$

Solving Eqs. (3) and (4) simultaneously, find $J_1 = 230{,}491\ W/m^2$ and $J_2 = 234{,}654\ W/m^2$, and from Eq. (1), find

$$q_3 = 7.854\times10^{-3}\,m^2\left[0.38(459.3 - 230{,}491) + 0.62(459.3 - 234{,}654)\right]W/m^2 = -1827\ W.$$

Hence, 1827 W are transferred from the cavity to the surroundings. <

PROBLEM 13.84

KNOWN: Circular furnace with prescribed temperatures and emissivities of the lateral and end surfaces.

FIND: Net radiative heat transfer from each surface.

SCHEMATIC:

ASSUMPTIONS: (1) Surfaces are isothermal and diffuse-gray.

ANALYSIS: To calculate the net radiation heat transfer from each surface, we need to determine its radiosity. First, evaluate terms which will be required.

$$E_{b1} = \sigma T_1^4 = 1452\,\mathrm{W/m^2} \qquad A_1 = A_2 = \pi D^2/4 = 0.07069\,\mathrm{m^2} \qquad F_{12} = F_{21} = 0.17$$

$$E_{b2} = \sigma T_2^4 = 3544\,\mathrm{W/m^2} \qquad A_3 = \pi DL = 0.2827\,\mathrm{m^2} \qquad F_{23} = F_{13} = 0.83$$

$$E_{b3} = \sigma T_3^4 = 23,224\,\mathrm{W/m^2}$$

The view factor F_{12} results from Fig. 13.5 with $L/r_i = 2$ and $r_j/L = 0.5$. The radiation balances using Eq. 13.21, omitting units for convenience, are:

$$A_1: \quad \frac{1452 - J_1}{\dfrac{(1-0.4)}{0.4\times 0.07069}} = 0.07069\times 0.17\left(J_1 - J_2\right) + 0.07069\times 0.83\left(J_1 - J_3\right)$$

$$-2.500J_1 + 0.2550J_2 + 1.2450J_3 = -1452 \qquad (1)$$

$$A_2: \quad \frac{3544 - J_2}{\dfrac{(1-0.5)}{0.5\times 0.07069}} = 0.07069\times 0.17\left(J_2 - J_1\right) + 0.07069\times 0.83\left(J_2 - J_3\right)$$

$$-0.1700J_1 - 2.0000J_2 + 0.8300J_3 = -3544 \qquad (2)$$

$$A_3: \quad \frac{23,224 - J_3}{\dfrac{(1-0.8)}{0.8\times 0.2827}} = 0.07069\times 0.83\left(J_3 - J_1\right) + 0.07069\times 0.83\left(J_3 - J_2\right)$$

$$0.05189J_1 + 0.05189J_2 - 1.1037J_3 = -23,224 \qquad (3)$$

Solving Eqs. (1) – (3) simultaneously, find

$$J_1 = 12,877\,\mathrm{W/m^2} \qquad J_2 = 12,086\,\mathrm{W/m^2} \qquad J_3 = 22,216\,\mathrm{W/m^2}.$$

Using Eq. 13.22, the net radiation heat transfer for each surface follows:

$$q_i = \sum_{j=1}^{N} A_i F_{ij}\left(J_i - J_j\right)$$

$$A_1: \ q_1 = 0.07069\times 0.17\left(12,877 - 12,086\right)\mathrm{W} + 0.07069\times 0.83\left(12,877 - 22,216\right)\mathrm{W} = -538\,\mathrm{W} \quad <$$

$$A_2: \ q_2 = 0.07069\times 0.17\left(12,086 - 12,877\right)\mathrm{W} + 0.07069\times 0.83\left(12,086 - 22,216\right)\mathrm{W} = -603\,\mathrm{W} \quad <$$

$$A_3: \ q_3 = 0.07069\times 0.83\left(22,216 - 12,877\right)\mathrm{W} + 0.07069\times 0.83\left(22,216 - 12,086\right)\mathrm{W} = 1141\,\mathrm{W} \quad <$$

COMMENTS: Note that $\Sigma q_i = 0$. Also, note that $J_2 < J_1$ despite the fact that $T_2 > T_1$; note the role emissivity plays in explaining this.

PROBLEM 13.85

KNOWN: Four surface enclosure with all sides of equal area; temperatures of three surfaces are specified while the fourth is re-radiating.

FIND: Temperature of the re-radiating surface A_4.

SCHEMATIC:

A1, A2, A3, A4, insulated surface

A_1 $T_1 = 700K$ $\varepsilon_1 = 0.7$
A_2 $T_2 = 500K$ $\varepsilon_2 = 0.5$
A_3 $T_3 = 300K$ $\varepsilon_3 = 0.3$
A_4 Reradiating surface

ASSUMPTIONS: (1) Surfaces are diffuse-gray, (2) Surfaces have uniform radiosities.

ANALYSIS: To determine the temperature of the re-radiating surface A_4, it is necessary to recognize that $J_4 = E_{b4} = \sigma T_4^4$ and that the J_i (i = 1 to 4) values must be evaluated by simultaneously solving four radiation balances of the form, Eq. 13.21,

$$\frac{E_{bi} - J_i}{(1-\varepsilon_i)/\varepsilon_i A_i} = \sum_{j=1}^{N} \frac{J_i - j_j}{\left(A_i F_{ij}\right)^{-1}}$$

For simplicity, set $A_1 = A_2 = A_3 = A_4 = 1\ m^2$ and from symmetry, it follows that all view factors will be $F_{ij} = 1/3$. The necessary emissive powers are of the form $E_{bi} = \sigma T_1^4$.

$E_{b1} = \sigma(700\ K)^4 = 13{,}614\ W/m^2$, $E_{b2} = \sigma(500\ K)^4 = 3544\ W/m^2$, $E_{b3} = \sigma(300\ K)^4 = 459\ W/m^2$.

The radiation balances are:

A_1: $\frac{13,614 - J_1}{(1-0.7)/0.7} = \frac{1}{3}(J_1 - J_2) + \frac{1}{3}(J_1 - J_3) + \frac{1}{3}(J_1 - J_4); -1.42857J_1 + 0.14826J_2 + 0.14826J_3 + 0.14826J_4 = -13,614$

A_2: $\frac{3544 - J_2}{(1-0.5)/0.5} = \frac{1}{3}(J_2 - J_1) + \frac{1}{3}(J_2 - J_3) + \frac{1}{3}(J_2 - J_4)\quad 0.33333J_1 - 2.00000J_2 + 0.33333J_3 + 0.33333J_4 = -3544$

A_3: $\frac{459 - J_3}{(1-0.3)/0.3} = \frac{1}{3}(J_3 - J_1) + \frac{1}{3}(J_3 - J_2) + \frac{1}{3}(J_3 - J_4)\quad 0.77778J_1 + 0.77778J_2 - 3.33333J_3 + 0.77778J_4 = -459$

A_4: $0 = \frac{1}{3}(J_4 - J_1) + \frac{1}{3}(J_4 - J_2) + \frac{1}{3}(J_4 - J_3)\quad 0.33333J_1 + 0.33333J_2 + 0.33333J_3 - 1.00000J_4 = 0$

Solving this system of equations simultaneously, find

$J_1 = 11{,}572\ W/m^2$, $J_2 = 6031\ W/m^2$, $J_3 = 6088\ W/m^2$, $J_4 = 7897\ W/m^2$.

Since the radiosity and emissive power of the re-radiating surface are equal,

$$T_4^4 = J_4 / \sigma$$

$$T_4 = \left(7897\ W/m^2 / 5.67\times10^{-8}\ W/m^2\cdot K^4\right)^{1/4} = 611\ K. \quad <$$

COMMENTS: Note the values of the radiosities; are their relative values what you would have expected? Is the value of T_4 reasonable?

PROBLEM 13.86

KNOWN: A room with electrical heaters embedded in ceiling and floor; one wall is exposed to the outdoor environment while the other three walls are to be considered as insulated.

FIND: Net radiation heat transfer from each surface.

SCHEMATIC:

ASSUMPTIONS: (1) Diffuse-gray surfaces, (2) Surfaces are isothermal and irradiated uniformly, (3) Negligible convection effects, (4) $A_5 = A_{5A} + A_{5B}$.

ANALYSIS: To determine the net radiation heat transfer from each surface, find the surface radiosities using Eq. 13.20.

$$q_i = \sum_{j=1}^{5} A_i F_{ij} \left(J_i - J_j \right) \qquad (1)$$

To determine the value of J_i, energy balances must be written for each of the five surfaces. For surfaces 1, 2, and 3, the form is given by Eq. 13.21.

$$\frac{E_{bi} - J_i}{(1-\varepsilon_i)/\varepsilon_i A_i} = \sum_{j=1}^{5} \frac{J_i - J_j}{\left(A_i F_{ij}\right)^{-1}} \qquad i = 1, 2, \text{ and } 3. \qquad (2)$$

For the insulated or adiabatic surfaces, Eq. 13.22 is appropriate with $q_i = 0$; that is

$$q_i = \sum_{j=1}^{N} \frac{J_i - J_j}{\left(A_i F_{ij}\right)^{-1}} = 0 \qquad i = 4 \text{ and } 5. \qquad (3)$$

In order to write the energy balances by Eq. (2) and (3), we will need to know view factors. Using Fig. 13.4 (parallel rectangles) or Fig. 13.5 (perpendicular rectangles) find:

$F_{12} = F_{21} = 0.39$ $X/L = 10/4 = 2.5,$ $Y/L = 6/4 = 1.5$

$F_{13} = F_{14} = 0.19$ $Z/X = 4/10 = 0.4,$ $Y/X = 6/10 = 0.6$

$F_{34} = F_{43} = 0.19$ $X/L = 10/6 = 1.66,$ $Y/L = 4/6 = 0.67$

$F_{24} = F_{13} = 0.19$ $Z/X = 4/10 = 0.4,$ $Y/X = 6/10 = 0.6$

Note the use of symmetry in the above relations. Using reciprocity, find,

$$F_{32} = \frac{A_2}{A_3} F_{23} = \frac{A_2}{A_3} F_{13} = \frac{60}{40} \times 0.19 = 0.285; \qquad F_{31} = \frac{A_1}{A_3} F_{13} = \frac{60}{40} \times 0.19 = 0.285$$

$$F_{51} = \frac{A_1}{A_5} F_{15} = \frac{60}{48} \times 0.23 = 0.288; \qquad F_{53} = \frac{A_3}{A_5} F_{35} = \frac{40}{48} \times 0.25 = 0.208.$$

From the summation view factor relation,

$$F_{15} = 1 - F_{12} - F_{13} - F_{14} = 1 - 0.39 - 0.19 - 0.19 = 0.23$$

$$F_{35} = 1 - F_{31} - F_{32} - F_{34} = 1 - 0.285 - 0.285 - 0.19 = 0.24$$

Continued …..

PROBLEM 13.86 (Cont.)

Using Eq. (2), now write the energy balances for surfaces 1, 2, and 3. (Note $E_b = \sigma T^4$).

$$\frac{544.2 - J_1}{1 - 0.8/0.8 \times 60} = \frac{J_1 - J_2}{1/60 \times 0.39} + \frac{J_1 - J_3}{1/60 \times 0.19} + \frac{J_1 - J_4}{1/60 \times 0.19} + \frac{J_1 - J_5}{1/60 \times 0.23}$$

$$-1.2500J_1 + 0.0975J_2 + 0.0475J_3 + 0.570J_5 = -544.2 \qquad (4)$$

$$\frac{617.2 - J_2}{1 - 0.9/0.9 \times 60} = \frac{J_2 - J_1}{1/60 \times 0.39} + \frac{J_2 - J_3}{1/60 \times 0.19} + \frac{J_2 - J_4}{1/60 \times 0.19} + \frac{J_2 - J_5}{1/60 \times 0.23}$$

$$+0.0433J_1 - 1.111J_2 + 0.02111J_3 + 0.02111J_4 + 0.02556J_5 = -617.2 \qquad (5)$$

$$\frac{390.1 - J_3}{1 - 0.7/0.7 \times 40} = \frac{J_3 - J_1}{1/40 \times 0.285} + \frac{J_3 - J_2}{1/40 \times 0.285} + \frac{J_3 - J_4}{1/40 \times 0.19} + \frac{J_3 - J_5}{1/40 \times 0.24}$$

$$+0.1221J_1 + 0.1221J_2 - 1.4284J_3 + 0.08143J_4 + 0.1028J_5 = -390.1 \qquad (6)$$

Using Eq. (3), now write the energy balances for surfaces 4 and 5 noting $q_4 = q_5 = 0$.

$$0 = \frac{J_4 - J_1}{1/40 \times 0.285} + \frac{J_4 - J_2}{1/40 \times 0.285} + \frac{J_4 - J_3}{1/40 \times 0.19} + \frac{J_4 - J_5}{1/40 \times 0.24}$$

$$0.285J_1 + 0.285J_2 + 0.19J_3 - 1.0J_4 + 0.24J_5 = 0 \qquad (7)$$

$$0 = \frac{J_5 - J_1}{1/48 \times 0.288} + \frac{J_5 - J_2}{1/48 \times 0.288} + \frac{J_5 - J_3}{1/48 \times 0.208} + \frac{J_5 - J_4}{1/48 \times 0.208}$$

$$0.288J_1 + 0.288J_2 + 0.208J_3 + 0.208J_4 - 0.992J_5 = 0 \qquad (8)$$

Note that Eqs. (4) – (8) represent a set of simultaneous equations which can be written in matrix notation following treatment of Section 13.3.2. That is, [A] [J] = [C] with

$$A = \begin{bmatrix} -1.250 & 0.0975 & 0.0475 & 0.0475 & 0.0575 \\ 0.0433 & -1.111 & 0.02111 & 0.02111 & 0.02556 \\ 0.1221 & 0.1221 & -1.4284 & 0.08143 & 0.1028 \\ 0.285 & 0.285 & 0.190 & -1.000 & 0.240 \\ 0.288 & 0.288 & 0.208 & 0.208 & -0.992 \end{bmatrix} \qquad C = \begin{bmatrix} -544.2 \\ -617.2 \\ -390.1 \\ 0 \\ 0 \end{bmatrix} \qquad J = \begin{bmatrix} 545.1 \\ 607.9 \\ 441.5 \\ 542.3 \\ 5410 \end{bmatrix} \quad W/m^2$$

where the J_i were found using a computer routine. The net radiation heat transfer from each of the surfaces can now be evaluated using Eq. (1).

$$q_1 = A_1F_{12}(J_1 - J_2) + A_1F_{13}(J_1 - J_3) + A_1F_{14}(J_1 - J_4) + A_1F_{15}(J_1 - J_5)$$

$$q_1 = 60\ m^2[0.39(545.1 - 607.9)$$

$$+0.19(545.1 - 441.5) + 0.19(545.1 - 542.3) + 0.23(545.1 - 541.0)]\ W/m^2 = -200\ W \quad <$$

$$q_2 = 60\ m^2[0.39(607.9 - 545.1)$$

$$+0.19(607.9 - 441.5) + 0.19(607.9 - 542.3) + 0.23(607.9 - 541.0)\ W/m^2 = 5037\ W \quad <$$

$$q_3 = 40\ m^2[0.285(441.5 - 545.1) + 0.285(441.5 - 607.9)$$

$$+0.19(441.5 - 542.3) + 0.24(441.5 - 541.0)]\ W/m^2 = -4{,}799\ W \quad <$$

Since A_4 and A_5 are insulated (adiabatic), $q_4 = q_5 = 0$. $<$

COMMENTS: (1) Note that the sum of $q_1 + q_2 + q_3 = +38$ W; this indicates a precision of less than 1% resulted from the solution of the equations. (2) The net radiation for the ceiling, A_1, is into the surface. Recognize that the embedded heaters function to offset heat losses to the room air by convection.

PROBLEM 13.87

KNOWN: Cylindrical cavity closed at bottom with opening at top surface.

FIND: Rate at which radiation passes through cavity opening and effective emissivity for these conditions: (a) All interior surfaces are black and at 600 K, (b) Bottom surface of the cavity $\varepsilon = 0.6$, T = 600 K; other surfaces are re-radiating, (c) All surfaces are at 600 K with emissivity 0.6, (d) For the cavity configurations of parts (b) and (c), compute and plot ε_e as a function of the interior surface emissivity over the range 0.6 to 1.0 with all other conditions remaining the same.

SCHEMATIC:

D_o = 15 mm, A_1, A_4, L = 40 mm, A_3, A_2, D_2 = 30 mm

$$A_1 = \left(\pi D_1^2\right)/4 = 1.767 \times 10^{-4}\,\text{m}^2$$

$$A_2 = \left(\pi D_2^2\right)/4 = 7.069 \times 10^{-4}\,\text{m}^2$$

$$A_3 = \pi DL = 37.70 \times 10^{-4}\,\text{m}^2$$

$$A_4 = A_2 - A_1 = 5.302 \times 10^{-4}\,\text{m}^2$$

ASSUMPTIONS: (1) Surfaces are opaque, diffuse and gray, (2) Surfaces as subsequently defined have uniform radiosity, (3) Re-radiating surfaces are adiabatic, and (4) Surroundings are at 0 K so that $T_1 = 0$ K and $\varepsilon_1 = 1.0$.

ANALYSIS: Define the hypothetical surface A_1, the cavity opening, having $T_1 = 0$ K for which $E_{b.1} = J_1 = 0$. The radiant power passing through the cavity opening A_1 will be $-q_1$ due to exchange within the four-surface enclosure. The effective emissivity of the cavity is defined as the rate of the radiant power leaving the cavity to that from a blackbody having the same area of the cavity opening and at the temperature of the inner surface of the cavity. That is,

$$\varepsilon_e = -q_1 / A_1 \sigma T^4 \qquad (1)$$

where T is the cavity surface temperature. Recognizing that the analysis will require knowledge of view factors, begin by evaluating them now.

For this four-surface enclosure (N=4), N(N-1)/2 = 6 view factors must be directly determined. The remaining $N^2 - 6 = 10$ can be determined by the summation rules and the reciprocity relations. By inspection,

(1-4): $F_{11} = 0$ $\quad F_{14} = 0$ $\quad F_{22} = 0$ $\quad F_{44} = 0$

Using the view factor equation for coaxial parallel disks, Table 13.2, or Fig. 13.5, evaluate F_{21},

(5): $F_{21} = 0.030$ $\quad$ with $r_j/L = 7.5/40 = 0.188$, $L/r_i = 40/15 = 2.67$.

Considering the top and bottom surfaces, use the additive rule, Eq. 13.5,

(6): $F_{24} = F_{2(1.4)} - F_{21}$ $\qquad (2)$

where $F_{2(1.4)}$ can be evaluated using the coaxial parallel disk relations again

$F_{2(1.4)} = 0.111$ $\quad$ with $r_{(1.4)}/L = 15/40 = 0.375$, $L/r_2 = 40/15 = 2.67$

Substituting numerical values, find

$F_{24} = 0.111 - 0.030 = 0.081$

Continued …..

Using the summation rule for each surface, plus appropriate reciprocity relations, the remaining view factors can be determined. Written as a matrix, the F_{ij} are

$$\begin{matrix} 0^* & 0.120 & 0.880 & 0 \\ 0.0300^* & 0^* & 0.889 & 0.0811^* \\ 0.0413 & 0.167 & 0.667 & 0.125 \\ 0^* & 0.108 & 0.892 & 0^* \end{matrix}$$

The F_{ij} shown with an asterisk were independently determined.

(a) When all the internal surfaces of the cavity are black at 600 K, the cavity opening emits as a black surface and the effective emissivity is unity. Using Eq. 13.14, the heat rate leaving A_1 is

$$q_1 = \sum_{i=1}^{3} A_1 F_{ij} \sigma \left(T_1^4 - T_1^4\right) = \sigma \left(T_1^4 - T^4\right) A_1 \left[F_{12} + F_{13} + F_{14}\right] \tag{3}$$

$$q_1 = 5.67 \times 10^{-8}\, \mathrm{W/m^2 \cdot K^4} \left(0^4 - 600^4\right) \mathrm{K^4} \times 1.767 \times 10^{-4}\, \mathrm{m^2} \times [0.120 + 0.880 + 0] = -1.298\, \mathrm{W}$$ <

From Eq. (1), it follows that the effective emissivity must be unity.

$$\varepsilon_1 = 1$$ <

(b) When the bottom surface of the cavity is T_2 = 600 K with ε_2 = 0.6 and all other surfaces are re-radiating, an enclosure analysis to obtain q_1 involves use of Eqs. 13.21 and 13.22. The former will be used on A_2 and the latter on the remaining areas.

A_2:
$$\frac{E_{b2} - J_2}{(1-\varepsilon_2)/\varepsilon_2 A_2} = \frac{J_2 - J_1}{1/A_2 F_{21}} + \frac{J_2 - J_3}{1/A_2 F_{23}} + \frac{J_2 - J_4}{1/A_2 F_{24}}$$

$$\frac{7384 - J_2}{(1-0.6)/0.6 A_2} = \frac{J_2 - 0}{1/0.03 A_2} + \frac{J_2 - J_3}{1/0.889 A_2} + \frac{J_2 - J_4}{1/0.811 A_2} \tag{4}$$

where $E_{b2} = \sigma T_2^4 = \sigma(600\mathrm{K})^4 = 7348\ \mathrm{W/m^2}$ and $J_1 = E_{b1} = \sigma T_1^4 = \sigma(0)^4 = 0\ \mathrm{W/m^2}$.

A_3:
$$0 = \frac{J_3 - J_1}{1/A_3 F_{31}} + \frac{J_3 - J_2}{1/A_3 F_{32}} + \frac{J_3 - J_4}{1/A_3 F_{34}}$$

$$0 = \frac{J_3 - 0}{1/0.0413 A_3} + \frac{J_3 - J_2}{1/0.167 A_3} + \frac{J_3 - J_4}{1/0.125 A_3} \tag{5}$$

A_4:
$$0 = \frac{J_4 - J_1}{1/A_4 F_{41}} + \frac{J_4 - J_2}{1/A_4 F_{42}} + \frac{J_4 - J_3}{1/A_4 F_{43}}$$

$$0 = 0 + \frac{J_4 - J_2}{1/0.108 A_4} + \frac{J_4 - J_3}{1/0.892 A_4} \tag{6}$$

Solving Eqs. (4,5,6) simultaneously, find

J_1	J_2	J_3	J_4 ($\mathrm{W/m^2}$)
0	6450	5284	5378

and the heat rate leaving surface A_1 is

Continued …..

PROBLEM 13.87 (Cont.)

$$q_1 = \frac{J_1 - J_2}{1/A_1F_{12}} + \frac{J_1 - J_3}{1/A_1F_{13}} + \frac{J_1 - J_4}{1/A_1F_{14}} = 0.9529\ \text{W} \qquad (7)$$

From Eq. (1), the cavity effective emissivity

$$\varepsilon_e = \frac{-q_1}{A_1\sigma T_2^4} = \frac{0.9529\,\text{W}}{1.767\times10^{-4}\,\text{m}^2\sigma\left(600\ \text{K}\right)^4} = 0.734$$

<

(c) When all the interior surfaces are at 600K ($T_2 = T_3 = T_4 = 600$ K) and $\varepsilon = 0.6$ ($\varepsilon_2 = \varepsilon_3 = \varepsilon_4 = 0.6$), apply the radiation surfaces energy balances using Eq. 13.21 to A_2, A_3, and A_4

A_2:

$$\frac{E_{b2} - J_2}{\left(1-\varepsilon_2\right)/\varepsilon_2 A_2} = \frac{J_2 - J_1}{1/A_2F_{21}} + \frac{J_2 - J_3}{1/A_2F_{23}} + \frac{J_2 - J_4}{1/A_2F_{24}}$$

$$\frac{7384 - J_2}{\left(1-0.6\right)/0.6A_2} = \frac{J_2 - 0}{1/0.03A_2} + \frac{J_2 - J_3}{1/0.889A_2} + \frac{J_2 - J_4}{1/0.811A_2} \qquad (8)$$

A_3:

$$\frac{E_{b3} - J_3}{\left(1-\varepsilon_3\right)/\varepsilon_3 A_3} = \frac{J_3 - J_1}{1/A_3F_{31}} + \frac{J_3 - J_2}{1/A_3F_{32}} + \frac{J_3 - J_4}{1/A_3F_{34}} = \frac{J_3 - 0}{1/0.0413A_3} + \frac{J_3 - J_2}{1/0.167A_3} + \frac{J_3 - J_4}{1/0.125A_3} \qquad (9)$$

A4:

$$\frac{E_{b4} - J_4}{\left(1-\varepsilon_4\right)/\varepsilon_4 A_4} = \frac{J_3 - J_1}{1/A_4F_{41}} + \frac{J_4 - J_2}{1/A_4F_{42}} + \frac{J_4 - J_3}{1/A_4F_{43}} = 0 + \frac{J_4 - J_2}{1/0.108A_4} + \frac{J_4 - J_3}{1/0.892A_4} \qquad (10)$$

Solving Eqs. (8, 9, 10) simultaneously, find

J_1	J_2	J_3	J_4 (W/m^2)
0	7192	7164	7276

and the heat rate leaving surface A_1 is

$$q_1 = \frac{J_1 - J_2}{1/A_1F_{12}} + \frac{J_1 - J_3}{1/A_1F_{13}} + \frac{J_1 - J_4}{1/A_1F_{14}} = -1.267\ \text{W}$$

<

From Eq. (1), the cavity effectiveness is

$$\varepsilon_e = \frac{-q_1}{A_1\sigma T_2^4} = \frac{1.267\text{W}}{1.767\times10^{-4}\,\text{m}^2\sigma\left(600\ \text{K}\right)^4} = 0.976$$

<

(d) For the cavity configurations of parts (b) and (c) and selected cavity depths, ε_e was computed as a function of the interior surface emissivity $\varepsilon = \varepsilon_2 = \varepsilon_4$ using an *IHT* model. The model included the following tools: *Radiation-View Factors: Relations and Formulas (Coaxial parallel disks); Radiation-Radiation Surface Energy Balance Relations*. See comment 2 below.

Continued …..

PROBLEM 13.87 (Cont.)

Part (b) - Reradiating surfaces A2, A3

Effective emissivity, epseff

Bottom surface emissivity, eps2

L = 20 mm
L = 40 mm
L = 80 mm

For the cavity configurations of part (b) – re –radiating surfaces A_3 and A_4 – the ε_e vs. ε_2 plot shows that for all cavity depths, the effective emissivity increases as the emissivity of the bottom surface, ε_2, increases. Note that even when $\varepsilon_2 = 1$, the cavity effective emissivity is always less than unity. Why must that be so? The effect of increasing the cavity depth is to increase the effective emissivity.

Part (c) - Interior surfaces at 600 K with eps = 0.6

Effective emissivity, epseff

Interior surface emissivity, eps

L = 20 mm
L = 40 mm
L = 80 mm

For the cavity configuration of part (c) – all interior surfaces at 600 K and $\varepsilon = 0.6$ – the effective emissivity increases with increasing interior surface emissivity. In the limit when $\varepsilon \rightarrow 1$, $\varepsilon_e \rightarrow 1$ as expected, and the cavity performs as an isothermal enclosure. The effective emissivity increases with increasing cavity depth.

COMMENTS: (1) This arrangement of a cavity, referred to as a cylindrical cavity with a lid (A_4), is widely used for radiometric applications to calibrate radiometers, radiation thermometers, and heat flux gages. The effective emissivity can be improved by constructing the cavity with a conical bottom surface (rather than a flat bottom). Why do you think this is so?

(2) The *IHT* model used to generate the part (b) graphical results is quite extensive. It is good practice to build the code in pieces, beginning with evaluation of the view factors. To avoid divide-by-zero errors, use small values for variables which are zero, such $F_{22} = 1e^{-20}$. Also, set unity emissivity values as 0.9999 rather than 1.0. The set of equations is very stiff, especially because of the re-radiating surface where T_3 and T_4 are unknowns. You should provide *Initial Guess* minimum values for T_3 and T_4 (> 0, positive) and unknown radiosities (> 0, positive).

PROBLEM 13.88

KNOWN: Cylindrical furnace of diameter D = 90 mm and overall length L = 180 mm. Heating elements maintain the refractory liming (ε = 0.8) of section (1), L_1 = 135 mm, at T_1 = 800°C. The bottom (2) and upper (3) sections are refractory lined, but are insulated. Furnace operates in a spacecraft environment.

FIND: Power required to maintain the furnace operating conditions with the surroundings at 23°C.

SCHEMATIC:

ASSUMPTIONS: (1) All surfaces are diffuse gray, (2) Uniform radiosity over the sections 1, 2, and 3, and (3) Negligible convection effects.

ANALYSIS: By defining the furnace opening as the hypothetical area A_4, the furnace can be represented as a four-surface enclosure as illustrated above. The power required to maintain A_1 at T_1 is q_1, the net radiation leaving A_1. To obtain q_1 following the methodology of Section 13.2.2, we must determine the radiosity at all surfaces by simultaneously solving the radiation energy balance equations for each surface which will be of the form, Eqs. 13.20 or 13.21.

$$q_1 = \frac{E_{bi} - J_i}{(1-\varepsilon_i)/\varepsilon_i A_i} = \sum_{j=1}^{N} \frac{J_j - J_j}{1/A_i F_{ij}} \qquad (1,2)$$

Since ε_4 = 1, $J_4 = E_{b4}$, so we only need to perform three energy balances, for A_1, A_2, and A_3, respectively

A_1: $$\frac{E_{b1} - J_1}{(1-\varepsilon_1)/\varepsilon_1 A_1} = \frac{J_1 - J_2}{1/A_1 F_{12}} + \frac{J_1 - J_3}{1/A_1 F_{13}} + \frac{J_1 - J_4}{1/A_1 F_{14}} \qquad (3)$$

A_2: $$0 = \frac{J_2 - J_1}{1/A_2 F_{21}} + \frac{J_2 - J_3}{1/A_2 F_{23}} + \frac{J_2 - J_4}{1/A_2 F_{24}} \qquad (4)$$

A_3: $$0 = \frac{J_3 - J_1}{1/A_3 F_{31}} + \frac{J_3 - J_2}{1/A_3 F_{32}} + \frac{J_3 - J_4}{1/A_3 F_{34}} \qquad (5)$$

Note that $q_2 = q_3 = 0$ since the surfaces are insulated (adiabatic). Recognize that in the above equation set, there are three equations and three unknowns: J_1, J_2, and J_3. From knowledge of J_1, q_1 can be determined using Eq. (1). Next we need to evaluate the view factors. There are $N^2 = 4^2 = 16$ view factors and N(N – 1)/2 = 6 must be independently evaluated, while the remaining can be determined by the summation rule and appropriate reciprocity relations. The six independently determined F_{ij} are:

By inspection: (1) $F_{22} = 0$ (2) $F_{44} = 0$

Coaxial parallel disks: From Fig. 13.5 or Table 13.5,

Continued …..

PROBLEM 13.88 (Cont.)

$$F_{24} = 0.5\left\{S - \left[S^2 - 4\left(r_4 / r_2\right)^2\right]^{1/2}\right\}$$

(3) $$F_{24} = 0.5\left\{18 - \left[18^2 - 4\left(1\right)^2\right]^{1/2}\right\} = 0.05573$$

$$S = 1 + \frac{1 + R_4^2}{R_2^2} = 1 + \frac{1 + 0.250^2}{0.250^2} = 18.00 \qquad R_2 = r_2 / L = 45/180 = 0.250 \qquad R_4 = r_4 / L = 0.250$$

Enclosure 1-2-2′: from the summation rule for A_2,

(4) $F_{21} = 1 - F_{22'} = 1 - 0.09167 = 0.9083$

where $F_{22'}$ can be evaluated from the coaxial parallel disk relation, Table 13.5. For these surfaces, $R_2 = r_2/L_1 = 45/135 = 0.333$, $R_{2'} = r_2/L_1 = 0.333$, and S = 11.00. From the summation rule for A_1,

(5) $F_{11} = 1 - F_{12} - F_{12'} = 1 - 0.1514 - 0.1514 = 0.6972$

and by symmetry $F_{12} = F_{12'}$ and using reciprocity

$$F_{12} = A_2 F_{21} / A_1 = \left[\pi\left(0.090\text{m}\right)\left(2/4\right)\right] \times 0.9083 / \pi \times 0.090\text{m} \times 0.135\text{m} = 0.1514$$

Enclosure 2′-3-4: from the summation rule for A_4,

(6) $F_{43} = 1 - F_{42'} - F_{44} = 1 - 0.3820 - 0 = 0.6180$

where $F_{44} = 0$ and using the coaxial parallel disk relation from Table 13.5, with $R_4 = r_4/L_2 = 45/45 = 1$, $R_{2'} = r_2/L_2 = 1$, and S = 3.

The View Factors: Using summation rules and appropriate reciprocity relations, the remaining 10 view factors can be evaluated. Written in matrix form, the F_{ij} are

0.6972*	0.1514	0.09704	0.05438
0.9083*	0*	0.03597	0.05573*
0.2911	0.01798	0.3819	0.3090
0.3262	0.05573	0.6180*	0*

The F_{ij} shown with an asterisk were independently determined.

From knowledge of the relevant view factors, the energy balances, Eqs. (3, 4, 5), can be solved simultaneously to obtain the radiosities,

$$J_1 = 73{,}084\,\text{W/m}^2 \qquad J_2 = 67{,}723\,\text{W/m}^2 \qquad J_3 = 36{,}609\,\text{W/m}^2$$

The net heat rate leaving A_1 can be evaluated using Eq. (1) written as

$$q_1 = \frac{E_{b1} - J_1}{\left(1 - \varepsilon_1\right)/\varepsilon_1 A_1} = \frac{\left(75{,}159 - 73{,}084\right)\text{W/m}^2}{\left(1 - 0.8\right)/0.8 \times 0.03817\,\text{m}^2} = 317\,\text{W}$$ <

where $E_{b1} = \sigma T_1^4 = \sigma(800 + 273\text{K})^4 = 75{,}159\ \text{W/m}^2$ and $A_1 = \pi D L_1 = \pi \times 0.090\text{m} \times 0.135\text{m} = 0.03817\ \text{m}^2$.

COMMENTS: (1) Recognize the importance of defining the furnace opening as the hypothetical area A_4 which completes the four-surface enclosure representing the furnace. The temperature of A_4 is that of the surroundings and its emissivity is unity since it absorbs all radiation incident on it. (2) To obtain the view factor matrix, we used the *IHT Tool, Radiation, View Factor Relations*, which permits you to specify the independently determined F_{ij} and the tool will calculate the remaining ones.

PROBLEM 13.89

KNOWN: Rapid thermal processing (RTP) tool consisting of a lamp bank to heat a silicon wafer with irradiation onto its front side. The backside of the wafer (1) is the top of a cylindrical enclosure whose lateral (2) and bottom (3) surfaces are water cooled. An aperture (4) on the bottom surface provides for optical access to the wafer.

FIND: (a) Lamp irradiation, G_{lamp}, required to maintain the wafer at 1300 K; heat removal rate by the cooling coil, and (b) Compute and plot the fractional difference $(E_{b1} - J_1)/E_{b1}$ as a function of the enclosure aspect ratio, L/D, for the range $0.5 \le L/D \le 2.5$ with D = 300 mm fixed for wafer emissivities of $\varepsilon_1 = 0.75$, 0.8, and 0.85; how sensitive is this parameter to the enclosure surface emissivity, $\varepsilon_2 = \varepsilon_3$.

SCHEMATIC:

Radiant lamp bank
G_{lamp}
Wafer, A_1, D_1 = 300 mm, T_1 = 1300 K, ε_1 = 0.8
Water-cooled chamber
L = 300 mm
Enclosure, lateral surface, A_2, T_2 = 300 K, ε_2 = 0.07
Enclosure, bottom surface, A_3, $T_3 = T_2$, $\varepsilon_3 = \varepsilon_2$
Aperture, A_4, D_4 = 30 mm

ASSUMPTIONS: (1) Enclosure surfaces are diffuse, gray, (2) Uniform radiosity over the enclosure surfaces, (3) No heat losses from the top side of the wafer.

ANALYSIS: (a) The wafer-cylinder system can be represented as a four-surface enclosure. The aperture forms a hypothetical surface, A_4, at $T_4 = T_2 = T_3 = 300$ K with emissivity $\varepsilon_4 = 1$ since it absorbs all radiation incident on it. From an energy balance on the wafer, the absorbed lamp irradiation on the front side of the wafer, $\alpha_w G_{lamp}$, will be equal to the net radiation leaving the back-side (enclosure-side) of the wafer, q_1. To obtain q_1, following the methodology of Section 13.2.2, we must determine the radiosity of all the enclosure surfaces by simultaneously solving the radiation energy balance equations for each surface, which will be of the form, Eqs. 13.20 or 13.21.

$$q_i = \frac{E_{bi} - J_i}{(1-\varepsilon_i)/\varepsilon_i A_i} = \sum_{j=1}^{N} \frac{J_i - J_j}{1/A_i F_{ij}} \qquad (1,2)$$

Since $\varepsilon_4 = 1$, $J_4 = E_{b4}$, we only need to perform three energy balances, for A_1, A_2 and A_3, respectively,

A_1:
$$\frac{E_{b1} - J_1}{(1-\varepsilon_1)/A_1} = \frac{J_1 - J_2}{1/A_1 F_{12}} + \frac{J_1 - J_3}{1/A_1 F_{13}} + \frac{J_1 - J_4}{1/A_1 F_{14}} \qquad (3)$$

A_2:
$$\frac{E_{b2} - J_1}{(1-\varepsilon_2)/A_2} = \frac{J_2 - J_1}{1/A_2 F_{21}} + \frac{J_2 - J_3}{1/A_2 F_{23}} + \frac{J_2 - J_4}{1/A_2 F_{24}} \qquad (4)$$

Continued …..

A_3: $$\frac{E_{b3}-J_3}{(1-\varepsilon_3)/A_3}=\frac{J_3-J_1}{1/A_3F_{31}}+\frac{J_3-J_2}{1/A_3F_{32}}+\frac{J_3-J_4}{1/A_3F_{34}} \quad (5)$$

Recognize that in the above equation set, there are three equations and three unknowns: J_1, J_2, and J_3. From knowledge of the radiosities, the desired heat rate can be determined using Eq. (1). The required *lamp irradiation*,

$$\alpha_w\, G_{lamp} A_1 = q_1 = \frac{E_{b1}-J_1}{(1-\varepsilon_1)/\varepsilon_1 A_1} \quad (6)$$

and the heat removal rate by the cooling coil, q_{coil}, on surfaces A_2 and A_3, is

$$q_{coil} = -(q_2 + q_3) \quad (7)$$

where the net radiation leaving A_2 and A_3 are, from Eq. (1),

$$q_2 = \frac{E_{b2}-J_2}{(1-\varepsilon_2)/\varepsilon_2 A_2} \qquad q_1 = \frac{E_{b3}-J_3}{(1-\varepsilon_3)/\varepsilon_3 A_3} \quad (8,9)$$

The surface areas are expressed as

$$A_1 = \pi D_1^2/4 = 0.07069\,\text{m}^2 \qquad A_2 = \pi D_1 L = 0.2827 \quad (10,11)$$

$$A_3 = \pi\left(D_1^2 - D_4^2\right) = 0.06998\,\text{m}^2 \qquad A_2 = \pi D_4^2/4 = 0.0007069\,\text{m}^2 \quad (12,13)$$

Next evaluate the view factors. There are $N^2 = 4^2 = 16$ and $N(N-1)/2 = 6$ must be independently evaluated, and the remaining can be determined by summation rules and reciprocity relations. The six independently determined F_{ij} are:

By inspection: (1) $F_{11} = 0$ (2) $F_{33} = 0$ (3) $F_{44} = 0$ (4) $F_{34} = 0$

Coaxial parallel disks: from Fig. 13.5 or Table 13.5,

$$F_{14} = 0.5\left\{S - \left[S^2 - 4(r_4/r_1)^2\right]^{1/2}\right\}$$

(5) $$F_{14} = 0.5\left\{5.01 - \left[5.01^2 - 4(15/150)^2\right]^{1/2}\right\} = 0.001997$$

$$S = 1 + \frac{1+R_4^2}{R_1^2} = 1 + \frac{1+0.05^2}{0.5^2} = 5.010$$

$$R_1 = r_1/L = 150/300 = 0.5 \qquad R_4 = 15/300 = 0.05$$

Coaxial parallel disks: from the composite surface rule, Eq. 13.5,

(6) $F_{13} = F_{1(3,4)} - F_{14} = 0.17157 - 0.01997 = 0.1696$

where $F_{1(3,4)}$ can be evaluated from the coaxial parallel disk relation, Table 13.5. For these surfaces, $R_1 = r_1/L = 150/300 = 0.5$, $R_{(3,4)} = r_3/L = 150/300 = 0.5$, and $S = 6.000$.

The view factors: Using summation rules and reciprocity relations, the remaining 10 view factors can be evaluated. Written in matrix form, the F_{ij} are

Continued …..

0*	0.8284	0.1696	0.001997*
0.2071	0.5858	0.2051	0.002001
0.1713	0.8287	0*	0*
0.1997	0.8003	0*	0*

The F_{ij} shown with an asterisk were independently determined.

From knowledge of the relevant view factors, the energy balances, Eqs. (3, 4, 5) can be solved simultaneously to obtain the radiosities,

J_1	J_2	J_3	J_4 (W/m^2)
1.514×10^5	1.097×10^5	1.087×10^5	576.8

From Eqs. (6) and (7), the required lamp irradiation and cooling-coil heat removal rate are

$$G_{lamp} = 52,650\,\text{W/m}^2 \qquad q_{coil} = 2.89\text{ kW}$$ <

(b) If the enclosure were perfectly reflecting, the radiosity of the wafer, J_1, would be equal to its blackbody emissive power. For the conditions of part (a), $J_1 = 1.514 \times 10^5$ W/m^2 and $E_{b1} = 1.619 \times 10^5$ W/m^2. As such, the radiosity would be independent of ε_w thereby minimizing effects due to variation of that property from wafer-to-wafer. Using the foregoing analysis in the *IHT* workspace (see Comment 1 below), the fractional difference, $(E_{b1} - J_1)/E_{b1}$, was computed and plotted as a function of L/D, the aspect ratio of the enclosure.

Note that as the aspect ratio increases, the fractional difference between the wafer blackbody emissive power and the radiosity increases. As the enclosure gets larger, (L/D increases), more power supplied to the wafer is transferred to the water-cooled walls. For any L/D condition, the effect of increasing the wafer emissivity is to reduce the fractional difference. That is, as ε_w increases, the radiosity increases. The lowest curve on the above plot corresponds to the condition $\varepsilon_2 = \varepsilon_3 = 0.03$, rather than 0.07 as used in the ε_w parameter study. The effect of reducing ε_2 is substantial, nearly halving the fractional difference. We conclude that the "best" cavity is one with a low aspect ratio and low emissivity (high reflectivity) enclosure walls.

COMMENTS: The *IHT* model developed to perform the foregoing analysis is shown below. Since the model utilizes several *IHT Tools*, good practice suggests the code be built in stages. In the first stage, the view factors were evaluated; the bottom portion of the code. Note that you must set the F_{ij} which

Continued …..

are zero to a value such as 1e-20 rather than 0. In the second stage, the enclosure exchange analysis was added to the code to obtain the radiosities and required heat rate. Finally, the equations necessary to obtain the fractional difference and perform the parameter analysis were added.

```
// Enclosure Performance Parameter:
Eb1J1 = (Eb1 - J1 ) / Eb1
LoverD = L / D1
// Energy Balances - Wafer and water-cooled surfaces, Eqs (6) and (7):
alphaw * Glamp * A1 = q1   // Energy balance on wafer
alphaw = eps1              // Wafer absorptivity to lamp irradiation
qcoil = - (q2 + q3)                  // Heat rate to the cooling coil, W
// Radiation Exchange Analysis Tool - Surface Energy Balances:
/* The net heat rate leaving A1 in terms of the surface resistance is */
q1 = (Eb1 - J1) / ((1 - eps1) / (eps1 * A1))     // Eq 13.19
/* The net heat rate leaving A1 in terms of the net exchanges between enclosure surfaces is */
q1 = q12 + q13 + q14
/* where the net exchange rates expressed in terms of the space resistances are, Eq 13.20 and 13.22, */
q12 = (J1 - J2) / (1 / (A1 * F12))
q13 = (J1 - J3) / (1 / (A1 * F13))
q14 = (J1 - J4) / (1 / (A1 * F14))
/* The net heat rate leaving A2 in terms of the surface resistance is */
q2 = (Eb2 - J2) / ((1 - eps2) / (eps2 * A2))     // Eq 13.19
/* The net heat rate leaving A2 in terms of the net exchanges between enclosure surfaces is */
q2 = q21 + q23 + q24
/* where the net exchange rates expressed in terms of the space resistances are, Eq 13.20 and 13.22, */
q21 = (J2 - J1) / (1 / (A2 * F21))
q23 = (J2 - J3) / (1 / (A2 * F23))
q24 = (J2 - J4) / (1 / (A2 * F24))
/* The net heat rate leaving A3 in terms of the surface resistance is */
q3 = (Eb3 - J3) / ((1 - eps3) / (eps3 * A3))     // Eq 13.19
/* The net heat rate leaving A3 in terms of the net exchanges between enclosure surfaces is */
q3 = q31 + q32 + q34
/* where the net exchange rates expressed in terms of the space resistances are, Eq 13.20 and 13.22, */
q31 = (J3 - J1) / (1 / (A3 * F31))
q32 = (J3 - J2) / (1 / (A3 * F32))
q34 = (J3 - J4) / (1 / (A3 * F34))
/* The net heat rate leaving A4 in terms of the surface resistance is */
q4 = (Eb4 - J4) / ((1 - eps4) / (eps4 * A4))     // Eq 13.19
/* The net heat rate leaving A4 in terms of the net exchanges between enclosure surfaces is */
q4 = q41 + q42 + q43
/* where the net exchange rates expressed in terms of the space resistances are, Eq 13.20 and 13.22, */
q41 = (J4 - J1) / (1 / (A4 * F41))
q42 = (J4 - J2) / (1 / (A4 * F42))
q43 = (J4 - J3) / (1 / (A4 * F43))
// Emissive Powers:
Eb1 = sigma * T1^4          // Blackbody emissive power, W/m^2
Eb2 = sigma * T2^4
Eb3 = sigma * T3^4
Eb4 = sigma * T4^4
sigma = 5.67e-8  // Stefan-Boltzmann constant, W/m^2.K^4
// Assigned Variables - Thermal Parameters Only:
T1 = 1300                   // Wafer temperature, K
eps1 = 0.8                  // Wafer emissivity
//eps1 = 0.75
//eps1 = 0.85
T2 = 300                    // Lateral surface temperature, K
eps2 = 0.07                 // Enclosure emissivity
//eps2 = 0.03
T3 = 300                    // Bottom surface temperature, K
eps3 = 0.07                 // Enclosure emissivity
//eps3 = 0.03
T4 = 300                    // Aperture surface temperature, K
eps4 = 0.999                // Aperture emissivity; not zero to avoid divide-by-zero error
```

Continued …..

```
// Radiation Exchange Analysis Tool - View Factor Relations:
/* The summation rule for an N-surface enclosure, Eq 13.4, is */
F11 + F12 + F13 + F14 = 1
F21 + F22 + F23 + F24 = 1
F31 + F32 + F33 + F34 = 1
F41 + F42 + F43 + F44 = 1
/* Then N * (N - 1) / 2 reciprocity relations associated with an N-surface enclosure, Eq 13.3, are */
A1 * F12 = A2 * F21
A1 * F13 = A3 * F31
A1 * F14 = A4 * F41
A2 * F23 = A3 * F32
A2 * F24 = A4 * F42
A3 * F34 = A4 * F43
// Areas:
A1 =pi * D1^2 / 4              // Wafer, m^2
A2 = pi * D1 * L               // Lateral surface, m^2
A3 = pi * (D1^2 - D4^2) / 4    // Bottom surface, m^2
A4 = pi * D4^2 / 4             // Aperture, m^2
// Assigned Variables - Geometry Only:
D1 = 0.300                     // Wafer diameter, m
D4 = 0.030                     // Aperture diameter, m
L = 0.300                      // Enclosure height, m
// Independently determined Fij - by inspection:
F11 = 1e-20                    // Not zero to avoid divide-by-zero error
F33 = 1e-20
F44 = 1e-20
F34 = 1e-20
// Other independently determined Fij:
/* The view factor, F14, for coaxial parallel disks, is */
F14 = 0.5 * (Sa - sqrt(Sa^2 - 4*(r4 / r1)^2))
// where
R1 = r1 / L
R4 = r4 / L
r1 = D1 / 2
r4 = D4 / 2
Sa = 1 + (1 + R4^2) / R1^2
// Composite surface relation to find F13:
F134 = F13 + F14
/* The view factor, F1(34), for coaxial parallel disks, is */
F134 = 0.5 * (Sb - sqrt(Sb^2 - 4*(r34 / r1)^2))
// where
//R1 = r1 / L
R34 = r34 / L
r34 = r1
Sb = 1 + (1 + R34^2) / R1^2
```

PROBLEM 13.90

KNOWN: Observation cabin located in a hot-strip mill directly over the line; cabin floor (f) exposed to steel strip (ss) at T_{ss} = 920°C and to mill surroundings at T_{sur} = 80°C.

FIND: Coolant system heat removal rate required to maintain the cabin floor at T_f = 50°C for the following conditions: (a) when the floor is directly exposed to the steel strip and (b) when a radiation shield (s) ε_s = 0.10 is installed between the floor and the strip.

SCHEMATIC:

ASSUMPTIONS: (1) Cabin floor (f) or shield (s), steel strip (ss), and mill surroundings (sur) form a three-surface, diffuse-gray enclosure, (2) Surfaces with uniform radiosities, (3) Mill surroundings are isothermal, black, (4) Floor-shield configuration treated as infinite parallel planes, and (5) Negligible convection heat transfer to the cabin floor.

ANALYSIS: A gray-diffuse, three-surface enclosure is formed by the cabin floor (f) (or radiation shield, s), steel strip (ss), and the mill surroundings (sur). The heat removal rate required to maintain the cabin floor at T_f = 50°C is equal to - q_f (or, -q_s), where q_f or q_s is the net radiation leaving the floor or shield. The schematic below represents the details of the surface energy balance on the floor and shield for the conditions *without the shield* (floor exposed) and *with the shield* (floor shielded from strip).

(a) *Without the shield*. Radiation surface energy balances, Eq. 13.21, are written for the floor (f) and steel strip (ss) surfaces to determine their radiosities.

$$\frac{E_{b,f} - J_f}{(1-\varepsilon_f)/\varepsilon_f A_f} = \frac{J_f - J_{ss}}{1/A_f F_{f-ss}} + \frac{J_f - E_{b,sur}}{1/A_f F_{f-sur}} \quad (1)$$

$$\frac{E_{b,ss} - J_{ss}}{(1-\varepsilon_{ss})/\varepsilon_{ss} A_{ss}} = \frac{J_{ss} - J_f}{1/A_{ss} F_{ss-f}} + \frac{J_{ss} - E_{b,sur}}{1/A_{ss} F_{ss\text{-}sur}} \quad (2)$$

Since the surroundings (sur) are black, $J_{sur} = E_{b,sur}$. The blackbody emissive powers are expressed as $E_b = \sigma T^4$ where $\sigma = 5.67 \times 10^{-8}$ W/m^2·K^4. The net radiation leaving the floor, Eq. 13.20, is

$$q_f = A_f F_{f-ss}(J_f - J_{ss}) + A_f F_{f-sur}(J_f - E_{b,sur}) \quad (3)$$

Continued …..

PROBLEM 13.90 (Cont.)

The required view factors for the analysis are contained in the summation rule for the areas A_f and A_{ss},

$$F_{f-ss} + F_{f-sur} = 1 \qquad F_{ss-f} + F_{ss-sur} = 1 \tag{4,5}$$

F_{f-ss} can be evaluated from Fig. 13.4 (Table 13.2) for the aligned parallel rectangles geometry. By symmetry, $F_{ss-f} = F_{f-ss}$, and with the summation rule, all the view factors are determined. Using the foregoing relations in the *IHT* workspace, the following results were obtained:

$$F_{f-ss} = 0.1864 \qquad J_f = 7959\ \mathrm{W/m^2}$$

$$F_{f-sur} = 0.8136 \qquad J_{ss} = 97.96\ \mathrm{kW/m^2}$$

and the heat removal rate required of the coolant system (cs) is

$$q_{cs} = -q_f = 41.3\ \mathrm{kW} \qquad <$$

(b) *With the shield*. Radiation surface energy balances are written for the shield (s) and steel strip (ss) to determine their radiosities.

$$\frac{E_{b,s} - J_s}{(1-\varepsilon_s)/\varepsilon_s A_s} = \frac{J_s - J_{ss}}{1/A_s F_{s-ss}} + \frac{J_s - E_{b,sur}}{1/A_s F_{s-sur}} \tag{6}$$

$$\frac{E_{b,ss} - J_{ss}}{(1-\varepsilon_{ss})/\varepsilon_{ss} A_{ss}} = \frac{J_{ss} - J_s}{1/A_{ss} F_{ss-s}} + \frac{J_{ss} - E_{b,sur}}{1/A_{ss} F_{ss-sur}} \tag{7}$$

The net radiation leaving the shield is

$$q_s = A_{ss} F_{ss-s} (J_{ss} - J_s) + A_{ss} F_{ss-sur} (J_{ss} - E_{b,sur}) \tag{8}$$

Since the temperature of the shield is unknown, an additional relation is required. The heat transfer from the shield (s) to the floor (f) - the coolant heat removal rate - is

$$-q_s = \frac{\sigma\left(T_s^4 - T_f^4\right)A_s}{1 - 1/\varepsilon_s - 1/\varepsilon_f} \tag{9}$$

where the floor-shield configuration is that of infinite parallel planes, Eq. 13.24. Using the foregoing relations in the *IHT* workspace, with appropriate view factors from part (a), the following results were obtained

$$J_s = 18.13\ \mathrm{kW/m^2} \qquad J_{ss} = 98.20\ \mathrm{kW/m^2} \qquad T_s = 377^\circ\mathrm{C}$$

and the heat removal rate required of the coolant system is

$$q_{cs} = -q_s = 6.55\ \mathrm{kW} \qquad <$$

COMMENTS: The effect of the shield is to reduce the coolant system heat rate by a factor of nearly seven. Maintaining the integrity of the reflecting shield ($\varepsilon_s = 0.10$) operating at nearly 400°C in the mill environment to prevent corrosion or oxidation may be necessary.

PROBLEM 13.91

KNOWN: Opaque, diffuse-gray plate with $\varepsilon_1 = 0.8$ is at $T_1 = 400$ K at a particular instant. The bottom surface of the plate is subjected to radiative exchange with a furnace. The top surface is subjected to ambient air and large surroundings.

FIND: (a) Net radiative heat transfer to the bottom surface of the plate for $T_1 = 400$ K, (b) Change in temperature of the plate with time, dT_1/dt, and (c) Compute and plot dT_1/dt as a function of T_1 for the range $350 \le T_1 \le 900$ K; determine the steady-state temperature of the plate.

SCHEMATIC:

ASSUMPTIONS: (1) Plate is opaque, diffuse-gray and isothermal, (2) Furnace bottom behaves as a blackbody while sides are perfectly insulated, (3) Surroundings are large compared to the plate and behave as a blackbody.

ANALYSIS: (a) Recognize that the plate (A_1), furnace bottom (A_2) and furnace side walls (A_R) form a three-surface enclosure with one surface being re-radiating. The net radiative heat transfer *leaving* A_1 follows from Eq. 13.30 written as

$$q_1 = \frac{E_{b1} - E_{b2}}{\frac{1-\varepsilon_1}{\varepsilon_1 A_1} + \frac{1}{A_1 F_{12}} + \left(1/A_1 F_{1R} + 1/A_2 F_{2R}\right)^{-1}} + \frac{1-\varepsilon_2}{\varepsilon_2 A_2} \qquad (1)$$

From Fig. 13.4 with X/L = 0.2/0.2 = 1 and Y/L = 0.2/0.2 = 1, it follows that $F_{12} = 0.2$ and $F_{1R} = 1 - F_{12} = 1 - 0.2 = 0.8$. Hence, with $F_{1R} = F_{2R}$ (by symmetry) and $\varepsilon_2 = 1$.

$$q_1 = \frac{5.67\times10^{-8}\,\mathrm{W/m^2\cdot K^4}\left(400^4 - 1000^4\right)\mathrm{K}^4}{\frac{1-0.8}{0.8\times0.4\mathrm{m}^2} + \frac{1}{0.4\mathrm{m}^2\times0.20 + \left(2/0.04\mathrm{m}^2\times0.8\right)^{-1}}} = -1153\,\mathrm{W}$$ <

It follows the net radiative exchange to the plate is, $q_{rad \cdot f} = 1153$ W.

(b) Perform now an energy balance on the plate written as

$$\dot{E}_{in} - \dot{E}_{out} = \dot{E}_{st}$$

$$q_{rad.f} - q_{conv} - q_{rad,sur} = Mc_p \frac{dT_1}{dt}$$

$$q_{rad.f} - hA_s\left(T_1 - T_\infty\right) - \varepsilon_1 A_1 \sigma\left(T_1^4 - T_{sur}^4\right) = Mc_p \frac{dT_1}{dt}. \quad (2)$$

Substituting numerical values and rearranging to obtain dT/dt, find

Continued …..

PROBLEM 13.91 (Cont.)

$$\frac{dT_1}{dt} = \frac{1}{2\,kg \times 900\,J/kg\cdot K}\Big[+1153W - 25\,W/m^2\cdot K \times 0.04\,m^2\,(400-300)K$$

$$-0.8\times 0.04m^2 \times 5.67\times 10^{-8}\,W/m^2\cdot K^4\left(400^4 - 300^4\right)K^4\Big] \quad <$$

$$\frac{dT_1}{dt} = 0.57\,K/s.$$

(c) With Eqs. (1) and (2) in the *IHT* workspace, dT_1/dt was computed and plotted as a function of T_1.

When T_1 = 400 K, the condition of part (b), we found dT_1/dt = 0.57 K/s which indicates the plate temperature is increasing with time. For T_1 = 900 K, dT_1/dt is a negative value indicating the plate temperature will decrease with time. The steady-state condition corresponds to $dT_1/dt = 0$ for which

$$T_{1,ss} = 715\ K \quad <$$

COMMENTS: Using the *IHT Radiation Tools – Radiation Exchange Analysis, Three Surface Enclosure with Re-radiating Surface and View Factors, Aligned Parallel Rectangle* – the above analysis can be performed. A copy of the workspace follows:

```
// Energy Balance on the Plate, Equation 2:
M * cp * dTdt = - q1 – h * A1 * (T1 – Tinf) – eps1 * A1 * sigma * (T1^4 – Tsur^4)

/* Radiation Tool – Radiation Exchange Analysis,
Three-Surface Enclosure with Reradiating Surface: */
/* For the three-surface enclosure A1, A2 and the reradiating surface AR, the net rate of radiation transfer
from the surface A1 to surface A2 is */
q1 = (Eb1 – Eb2) / ( (1 – eps1) / (eps1 * A1) + 1 / (A1 * F12 + 1/(1/(A1 * F1R) + 1/(A2 * F2R))) + (1 –
eps2) / (eps2 * A2))  //  Eq 13.30
/* The net rate of radiation transfer from surface A2 to surface A1 is */
q2 = -q1
/* From a radiation energy balance on AR, */
(JR – J1) / (1/(AR * FR1)) + (JR – J2) / (1/(AR *FR2)) = 0      // Eq 13.31
/* where the radiosities J1 and J2 are determined from the radiation rate equations expressed in terms of
the surface resistances, Eq 13.22 */
q1 = (Eb1 – J1) / ((1 – eps1) / (eps1 * A1))
q2 = (Eb2 – J2) / ((1-eps2) / (eps2 * A2))
// The blackbody emissive powers for A1 and A2 are
Eb1 = sigma * T1^4
Eb2 = sigma * T2^4
// For the reradiating surface,
JR = EbR
```

Continued …..

```
EbR = sigma *TR^4
sigma = 5.67E-8              // Stefan-Boltzmann constant, W/m^2·K^4
```

// Radiation Tool – View Factor:

```
/* The view factor, F12, for aligned parallel rectangles, is */
F12 = Fij_APR(Xbar, Ybar)
// where
Xbar = X/L
Ybar = Y/L
// See Table 13.2 for schematic of this three-dimensional geometry.
```

// View Factors Relations:

```
F1R = 1 – F12
FR1 = F1R * A1 / AR
FR2 = FR1
A1 = X * Y
A2 = X * Y
AR = 2 * (X * Z + Y * Z)
Z = L
F2R = F1R
```

// Assigned Variables:

```
T1 = 400                     // Plate temperature, K
eps1 = 0.8                   // Plate emissivity
T2 = 1000                            // Bottom temperature, K
eps2 = 0.9999                // Bottom surface emissivity
X = 0.2                      // Plate dimension, m
Y = 0.2                      // Plate dimension, m
L = 0.2                      // Plate separation distance, m
M = 2                        // Mass, kg
cp = 900                     // Specific heat, J/kg.K,
h = 25                       // Convection coefficient, W/m^2.K
Tinf = 300                           // Ambient air temperature, K
Tsur = 300                   // Surroundings temperature, K
```

PROBLEM 13.92

KNOWN: Tool for processing silicon wafer within a vacuum chamber with cooled walls. Thin wafer is radiatively coupled on its back side to a chuck which is electrically heated. The top side is irradiated by an ion beam flux and experiences convection with the process gas and radioactive exchange with the ion-beam *grid* control surface and the chamber walls.

FIND: (a) Show control surfaces and all relevant processes on a schematic of the wafer, and (b) Perform an energy balance on the wafer and determine the chuck temperature T_c required to maintain the prescribed conditions.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Wafer is diffuse, gray, (3) Separation distance between the wafer and chuck is much smaller than the wafer and chuck diameters, (4) Negligible convection in the gap between the wafer and chuck; convection occurs on the wafer top surface with the process gas, (5) Surfaces forming the three-surface enclosure – wafer (ε_w = 0.8), grid (ε_g = 1), and chamber walls (ε_c = 1) have uniform radiosity and are diffuse, gray, and (6) the chuck surface is black.

ANALYSIS: (a) The wafer is shown schematically above in relation to the key components of the tool: the ion beam generator, the grid which is used to control the ion beam flux, q''_{ib}, the chuck which aids in controlling the wafer temperature and the process gas flowing over the wafer top surface. The schematic below shows the control surfaces on the top and back surfaces of the wafer along with the relevant thermal processes: q_{cv}, convection between the wafer and process gas; q_a, applied heat source due to absorption of the ion beam flux, q''_{ib}; $q_{1,top}$, net radiation leaving the top surface of the wafer (1) which is part of the three-surface enclosure – grid (2) and chamber walls (3), and; $q_{1,bac}$, net radiation leaving the backside of the wafer (w) which is part of a two-surface enclosure formed with the chuck (c).

<

Continued …..

(b) Referring to the schematic and the identified thermal processes, the energy balance on the wafer has the form,

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$-q_{cv} + q_a - q_{1,bac} - q_{1,top} = 0 \tag{1}$$

where each of the processes are evaluated as follows:

Convection with the process gas: with $A_w = \pi D_4^2 = \pi (0.200\text{m})^2 / 4 = 0.03142\,\text{m}^2$,

$$q_{cv} = hA_w (T_w - T_\infty) = 10\,\text{W/m}^2 \times 0.03142\text{m}^2 \times (700 - 500)\text{K} = 62.84\,\text{W} \tag{2}$$

Applied heat source – ion beam:

$$q_a = q''_{ib} A_w = 600\,\text{W/m}^2 \times 0.3142\text{m}^2 = 18.85\,\text{W} \tag{3}$$

Net radiation heat rate, back side; *enclosure (w,c):* for the two-surface enclosure comprised of the back side of the wafer (w) and the chuck, (c), Eq. 13.28, yields

$$q_{1,bac} = \frac{\sigma\left(T_w^4 - T_c^4\right)}{(1-\varepsilon_w)/\varepsilon_w A_w + 1/A_w F_{wc} + (1-\varepsilon_c)/\varepsilon_c A_c}$$

and since the wafer-chuck approximate large parallel plates, $F_{wc} = 1$, and since the chuck is black, $\varepsilon_c = 1$,

$$q_{1,bac} = \frac{\sigma\left(T_w^4 - T_c^4\right) A_w}{(1-\varepsilon_w)/\varepsilon_w + 1} \tag{4}$$

$$q_{1,bac} = \frac{0.03142\text{m}^2 \times \sigma\left(700^4 - T_c^4\right)\text{K}^4}{(1-0.6)/0.6 + 1} = 1.069 \times 10^{-9}\left(700^4 - T_c^4\right)$$

Net radiation heat rate, top surface; enclosure (1, 2, 3): from the surface energy balance on A_1, Eq. 13.20.

$$q_{1,top} = \frac{E_{b1} - J_1}{(1-\varepsilon_1)/\varepsilon_1 A_1} \tag{5}$$

where $\varepsilon_1 = \varepsilon_w$, $A_1 = A_w$, $E_{b1} = \sigma T_1^4$ and the radiosity can be evaluated by an enclosure analysis following the methodology of Section 13.2.2. From the energy balance, Eq. 13.21,

$$\frac{E_{b1} - J_1}{(1-\varepsilon_1)/\varepsilon_1 A_1} = \frac{J_1 - J_2}{1/A_1 F_{12}} + \frac{J_1 - J_3}{1/A_1 F_{13}} \tag{6}$$

where $J_2 = E_{b2} = \sigma T_g^4$ and $J_3 = E_{b3} = \sigma T_{vc}^4$ since both surfaces are black ($\varepsilon_g = \varepsilon_{vc} = 1$). The view factor F_{12} can be computed from the relation for coaxial parallel disks, Table 13.5.

$$F_{12} = 0.5\left\{S - \left[S^2 - 4(r_2/r_1)^2\right]^{1/2}\right\} = 0.5\left\{6.0 - \left[6.0^2 - 4(1)^2\right]^{1/2}\right\} = 0.1716$$

$$S = 1 + \frac{1 + R_2^2}{R_1^2} = 1 + \frac{1 + 0.5^2}{0.5^2} = 6.00$$

Continued …..

$$R_1 = r_1 / L = 100/200 = 0.5 \qquad R_4 = r_4 / L = 0.5$$

The view factor F_{13} follows from the summation rule applied to A_1,

$$F_{13} = 1 - F_{12} = 1 - 0.1716 = 0.8284$$

Substituting numerical values into Eq. (6), with $T_1 = T_w = 700$ K, $T_2 = T_g = 500$ K, and $T_3 = T_{vc} =$ 300 K, find J_1,

$$\frac{\sigma T_1^4 - J_1}{(1-\varepsilon_1)/\varepsilon_1 A_1} = \frac{J_1 - \sigma T_g^4}{1/F_{12}} + \frac{J_1 - \sigma T_{vc}^4}{1/F_{13}} \qquad (7)$$

$$J_1 = 8564 \text{ W/m}^2$$

Using Eq. (5), find $q_{1,top}$ with $E_{b2} = \sigma T_w^4 = 13,614 \text{ W/m}^2$ and $A_1 = A_w$,

$$q_{1,top} = \frac{(13,614 - 8564)\text{W/m}^2}{(1-0.6)/\left(0.6 \times 0.03142\text{m}^2\right)} = 238\,\text{W}$$

Evaluating T_c from the energy balance on the wafer, Eq. (1), and substituting appropriate expressions for each of the processes, find

$$-62.84 \text{ W/m}^2 + 18.85\text{W} - 1.069 \times 10^{-9}\left(700^4 - T_c^4\right) - 238 \text{ W} = 0$$

$$T_c = 842.5\,\text{K} \qquad <$$

From Eq. (4), with $T_c = 815$ K, the electrical power required to maintain the chuck is

$$P_c = -q_{1,bac} = 1.069 \times 10^{-9}\left(700^4 - 842.5\right) = 282\,\text{W}$$

COMMENTS: Recognize that the method of analysis is centered about an energy balance on the wafer. Identifying the processes and representing them on the energy balance schematic is a vital step in developing the strategy for a solution. This methodology introduced in Section 1.3.3 becomes important, if not essential, in analyzing complicated physical systems.

PROBLEM 13.93

KNOWN: Ice rink with prescribed ice, rink air, wall, ceiling and outdoor air conditions.

FIND: (a) Temperature of the ceiling, T_c, having an emissivity of 0.05 (highly reflective panels) or 0.94 (painted panels); determine whether condensation will occur for either or both ceiling panel types if the relative humidity of the rink air is 70%, and (b) Calculate and plot the ceiling temperature as a function of ceiling insulation thickness for $0.1 \le t \le 1$ m, identify conditions for which condensation will occur on the ceiling.

SCHEMATIC:

ASSUMPTIONS: (1) Rink comprised of the ice, walls and ceiling approximates a three-surface, diffuse-gray enclosure, (2) Surfaces have uniform radiosities, (3) Ice surface and walls are black, (4) Panels are diffuse-gray, and (5) Thermal resistance for convection on the outdoor side of the ceiling is negligible compared to the conduction thermal resistance of the ceiling insulation.

PROPERTIES: *Psychometric chart* (Atmospheric pressure; dry bulb temperature, $T_{db} = T_{\infty,i}$ = 15°C; relative humidity, RH = 70%): Dew point temperature, T_{dp} = 9.4°C.

ANALYSIS: The energy balance on the ceiling illustrated in the schematic below has the form

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$-q_o - q_{conv,c} - q_{rad,c} = 0 \qquad (1)$$

where the rate equations for each process are

$$q_o = \left(T_c - T_{\infty,o}\right)/R_{cond} \qquad R_{cond} = t/kA_c \qquad (2,3)$$

$$q_{conv,c} = h\,A_c\left(T_c - T_{\infty,i}\right) \qquad (4)$$

$$q_{rad,c} = \varepsilon E_b\left(T_c\right)A_c - \alpha A_w F_{wc} E_b\left(T_w\right) - \alpha A_i F_{ic} E_b\left(T_i\right) \qquad (5)$$

The blackbody emissive powers are $E_b = \sigma T^4$ where $\sigma = 5.67 \times 10^{-8}$ W/m²·K⁴. Since the ceiling panels are diffuse-gray, $\alpha = \varepsilon$. The view factors required of Eq. (5): determine F_{ic} (ice to ceiling) from Table 13.2 (Fig. 13.5) for parallel, coaxial disks

$$F_{ic} = 0.672$$

and F_{wc} (wall to ceiling) from the summation rule on the ice (i) and the reciprocity rule,

$$F_{ic} + F_{iw} = 1 \qquad F_{iw} = F_{cw} \text{ (symmetry)}$$

$$F_{cw} = 1 - F_{ic}$$

$$F_{wc} = \left(A_c/A_w\right)F_{cw} = \left(A_c/A_w\right)\left(1 - F_{ic}\right) = 0.410$$

Continued …..

where $A_c = \pi D^2/4$ and $A_w = \pi DL$.

Using the foregoing energy balance, Eq. (1), and the rate equations, Eqs. (2-5), the ceiling temperature is calculated using radiative properties for the two panel types,

Ceiling panel	ε	T_c (°C)		
Reflective	0.05	14.0		
Paint	0.94	8.6	$T_c < T_{dp}$	<

The dew point is 9.4°C corresponding to a relative humidity of 70% with (dry bulb) air temperature of 15°C. Condensation will occur on the painted panel since $T_c < T_{dp}$.

(b) The equations required of the analysis above were solved using *IHT*. The analysis is extended to calculate the ceiling temperatures for a range of insulation thickness and the results plotted below.

For the reflective panel (ε = 0.05), the ceiling surface temperature is considerably above the dew point. Therefore, condensation will not occur for the range of insulation thickness shown. For the painted panel (ε = 0.94), the ceiling surface temperature is always below the dew point. We expect condensation to occur for the range of insulation thickness shown.

COMMENTS: From the analysis, recognize that the radiative exchange between the ice and the ceiling is the dominant process for influencing the ceiling temperature. With the reflective panel, the rate is reduced nearly 20 times that with the painted panel. With the painted panel ceiling, for most of the conditions likely to exist in the rink, condensation will occur.

PROBLEM 13.94

KNOWN: Diameter, temperature and emissivity of boiler tube. Thermal conductivity and emissivity of ash deposit. Convection coefficient and temperature of gas flow over the tube. Temperature of surroundings.

FIND: (a) Rate of heat transfer to tube without ash deposit, (b) Rate of heat transfer with an ash deposit of diameter $D_d = 0.06$ m, (c) Effect of deposit diameter and convection coefficient on heat rate and contributions due to convection and radiation.

SCHEMATIC:

ASSUMPTIONS: (1) Diffuse/gray surface behavior, (2) Surroundings form a large enclosure about the tube and may be approximated as a blackbody, (3) One-dimensional conduction in ash, (4) Steady-state.

ANALYSIS: (a) Without an ash deposit, the heat rate per unit tube length may be calculated directly.

$$q' = \bar{h}\pi D_t \left(T_\infty - T_t\right) + \varepsilon_t \sigma \pi D_t \left(T_{sur}^4 - T_t^4\right)$$

$$q' = 100\ \text{W/m}^2\cdot\text{K}\,(\pi)\,0.05\,\text{m}\,(1800-600)\,\text{K} + 0.8\times 5.67\times 10^{-8}\,\text{W/m}^2\cdot\text{K}^4\,(\pi)(0.05\,\text{m})\left(1500^4 - 600^4\right)\text{K}$$

$$q' = (18,850 + 35,150)\,\text{W/m} = 54,000\ \text{W/m}$$ <

(b) Performing an energy balance for a control surface about the outer surface of the ash deposit, $q'_{conv} + q'_{rad} = q'_{cond}$, or

$$\bar{h}\pi D_d \left(T_\infty - T_d\right) + \varepsilon_d \sigma \pi D_d \left(T_{sur}^4 - T_d^4\right) = \frac{2\pi k \left(T_d - T_t\right)}{\ln\left(D_d / D_t\right)}$$

Hence, canceling π and considering an ash deposit for which $D_d = 0.06$ m,

$$100\,\text{W/m}^2\cdot\text{K}\,(0.06\,\text{m})(1800 - T_d)\,\text{K} + 0.9\times 5.67\times 10^{-8}\,\text{W/m}^2\cdot\text{K}^4\,(0.06\,\text{m})\left(1500^4 - T_d^4\right)\text{K}^4$$

$$= \frac{2(1\ \text{W/m}\cdot\text{K})(T_d - 600)\,\text{K}}{\ln(0.06/0.05)}$$

A trial-and-error solution yields $T_d \approx 1346$ K, from which it follows that

$$q' = \bar{h}\pi D_d \left(T_\infty - T_d\right) + \varepsilon_d \sigma \pi D_d \left(T_{sur}^4 - T_d^4\right)$$

$$q' = 100\,\text{W/m}^2\cdot\text{K}\,(\pi)\,0.06\,\text{m}\,(1800 - 1346)\,\text{K} + 0.9\times 5.67\times 10^{-8}\,\text{W/m}^2\cdot\text{K}^4\,(\pi)\,0.06\,\text{m}\left(1500^4 - 1346^4\right)\text{K}^4$$

Continued …..

PROBLEM 13.94 (Cont.)

$$q' = (8560 + 17{,}140)\,W/m = 25{,}700\,W/m$$ <

(c) The foregoing energy balance was entered into the *IHT* workspace and parametric calculations were performed to explore the effects of $\bar{h}$ and D_d on the heat rates.

For D_d = 0.06 m and $10 \le \bar{h} \le 1000\ W/m^2 \cdot K$, the heat rate to the tube, q'_{cond}, as well as the contribution due to convection, q'_{conv}, increase with increasing $\bar{h}$. However, because the outer surface temperature T_d also increases with $\bar{h}$, the contribution due to radiation decreases and becomes negative (heat transfer from the surface) when T_d exceeds 1500 K at $\bar{h} = 540\ W/m^2 \cdot K$. Both the convection and radiation heat rates, and hence the conduction heat rate, increase with decreasing D_d, as T_d decreases and approaches T_t = 600 K. However, even for D_d = 0.051 m (a deposit thickness of 0.5 mm), T_d = 773 K and the ash provides a significant resistance to heat transfer.

COMMENTS: Boiler operation in an energy efficient manner dictates that ash deposits be minimized.

PROBLEM 13.95

KNOWN: Two parallel, large, diffuse-gray surfaces; top one maintained at T_1 while lower one is insulated and experiences convection.

FIND: (a) Temperature of lower surface, T_2, when $\varepsilon_1 = \varepsilon_2 = 0.5$ and (b) Radiant flux leaving the viewing port.

SCHEMATIC:

ASSUMPTIONS: (1) Surfaces are large, diffuse-gray, (2) Lower surface experiences convection and radiation exchange, backside is perfectly insulated.

ANALYSIS: (a) Perform an energy balance on the lower surface, giving

$$q''_{conv} + q''_{rad,1} = 0 \quad (1)$$

where the latter term is equal to q''_1 or q''_{12}, the net radiant power per unit area exchanged between surfaces 1 and 2. For this two surface enclosure,

$$q''_1 = \frac{E_b(T_1) - E_b(T_2)}{(1-\varepsilon_1)/\varepsilon_1 + 1/F_{12} + (1-\varepsilon_2)/\varepsilon_2} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{(1-\varepsilon_1)/\varepsilon_1 + 1 + (1-\varepsilon_2)/\varepsilon_2} \quad (2)$$

with $F_{12} = 1$. Combining Eqs. (1) and (2),

$$h(T_\infty - T_2) + \sigma\left(T_1^4 - T_2^4\right)/\left[(1-\varepsilon_1)/\varepsilon_1 + 1 + (1-\varepsilon_2)/\varepsilon_2\right] = 0 \quad (3)$$

Substituting numerical values with $\varepsilon_1 = \varepsilon_2 = 0.5$,

$$50\ \mathrm{W/m^2 \cdot K}(300 - T_2)\mathrm{K} + 5.67\times10^{-8}\ \mathrm{W/m^2\cdot K^4}\left(400^4 - T_2^4\right)\mathrm{K}^4/[1+1+1] = 0$$

$$T_2 \approx 306\ \mathrm{K}. \quad <$$

(b) The radiant flux leaving the viewing port is $q''_{vp} = G_1$. From an energy balance on the upper plate

$$q''_1 = E_1 - \alpha_1 G_1$$

where $q''_1 = q''_{1-2}$, net exchange by radiation. But

$$q''_1 = (1/3)\sigma\left(T_1^4 - T_2^4\right)$$

$$E_1 = \varepsilon E_{b1} = 0.5\sigma T_1^4.$$

Hence, the flux is

$$G_1 = (E_1 - q_1)/\alpha = (1/0.5)\left[0.5\sigma T_1^4 - (1/3)\sigma\left(T_1^4 - T_2^4\right)\right]$$

$$G_1 = 2\sigma\left[(0.5 - 0.333)T_1^4 + 0.333T_2^4\right] = 816\ \mathrm{W/m^2}. \quad <$$

PROBLEM 13.96

KNOWN: Dimensions, emissivities and temperatures of heated and cured surfaces at opposite ends of a cylindrical cavity. External conditions.

FIND: Required heater power and outside convection coefficient.

SCHEMATIC:

$T_{sur} = 300K$; $T_\infty = 300K, h_o$; $T_2 = 400K, \varepsilon_{2,i} = 0.5, \varepsilon_{2,o} = 0.9$; $L_c = 200mm$; $T_1 = 800K, \varepsilon_1 = 0.9$; $L_b = 25mm$; $k_b = 20 W/m\cdot K$; $T_b = 300K$; $D = 120mm$; T_∞, h

ASSUMPTIONS: (1) Steady-state conditions, (2) Opaque, diffuse-gray surfaces, (3) Negligible convection within cavity, (4) Isothermal disk and heater surfaces, (5) One-dimensional conduction in base, (6) Negligible contact resistance between heater and base, (7) Sidewall is reradiating.

ANALYSIS: The equivalent circuit is

E_{b1}, $q_{1,rad}$, J_1, J_2, E_{b2}, $E_{b,sur}$, $q_{1,elec}$, $q_{1,cond}$, $q_{2,conv}$, E_{bR}

From an energy balance on the heater surface, $q_{1,elec} = q_{1,cond} + q_{1,rad}$,

$$q_{1,elec} = k_b\left(\pi D^2/4\right)\frac{T_1 - T_b}{L_b} + \frac{\sigma\left(T_1^4 - T_2^4\right)}{\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} + \dfrac{1}{A_1F_{12} + \left[\left(1/A_1F_{1R}\right) + \left(1/A_2F_{2R}\right)\right]^{-1}} + \dfrac{1-\varepsilon_{2,i}}{\varepsilon_{2,i}A_2}}$$

where $A_1 = A_2 = \pi D^2/4 = \pi(0.12\text{ m})^2/4 = 0.0113\text{ m}^2$ and from Fig. 13.5, with $L_c/r_1 = 3.33$ and $r_2/L_c = 0.3$ find $F_{12} = F_{21} = 0.077$; hence, $F_{1R} = F_{2R} = 0.923$. The required heater power is

$$q_{1,elec} = 20\text{ W/m}\cdot\text{K}\times 0.0113\text{ m}^2\frac{(800-300)\text{K}}{0.025\text{ m}}$$

$$+\frac{0.0113\text{ m}^2\times 5.67\times 10^{-8}\text{W/m}^2\cdot\text{K}^4\left(800^4 - 400^4\right)\text{K}^4}{\dfrac{1-0.9}{0.9} + \dfrac{1}{0.077 + \left[(1/0.923) + (1/0.923)\right]^{-1}} + \dfrac{1-0.5}{0.5}}$$

$$q_{1,elec} = 4521\text{ W} + 82.9\text{ W} = 4604\text{ W}. \qquad <$$

An energy balance for the disk yields, $q_{rad,2} = q_{rad,1} = h_oA_2\left(T_2 - T_\infty\right) + \varepsilon_{2,o}A_2\sigma\left(T_2^4 - T_{sur}^4\right)$,

$$h_o = \frac{82.9\text{ W} - 0.9\times 0.0113\text{ m}^2\times 5.67\times 10^{-8}\text{W/m}^2\cdot\text{K}^4\left(400^4 - 300^4\right)\text{K}^4}{0.0113\text{ m}^2\times 100\text{ K}} = 64\text{ W/m}^2\cdot\text{K}. \qquad <$$

COMMENTS: Conduction through the ceramic base represents an enormous system loss. The base should be insulated to greatly reduce this loss and hence the electric power input.

PROBLEM 13.97

KNOWN: Electrical conductors in the form of parallel plates having one edge mounted to a ceramic insulated base. Plates exposed to large, isothermal surroundings, T_{sur}. Operating temperature is T_1 = 500 K.

FIND: (a) Electrical power dissipated in a conductor plate per unit length, q_1', considering only radiative exchange with the surroundings; temperature of the ceramic insulated base T_2; and, (b) q_1' and T_2 when the surfaces experience convection with an airstream at T_∞ = 300 K and a convection coefficient of h = 24 $W/m^2 \cdot K$.

SCHEMATIC:

ASSUMPTIONS: (1) Conductor surfaces are diffuse, gray, (2) Conductor and ceramic insulated base surfaces have uniform temperatures and radiosities, (3) Surroundings are large, isothermal.

ANALYSIS: (a) Define the opening between the conductivities as the hypothetical area A_3 at the temperature of the surroundings, T_{sur}, with an emissivity ε_3 = 1 since all the radiation incident on the area will be absorbed. The conductor (1)-base (2)-opening (3) form a three surface enclosure with one surface re-radiating (2). From Section 13.3.5 and Eq. 13.30, the net radiation leaving the conductor surface A_1 is

$$q_1 = \frac{E_{b1} - E_{b2}}{\frac{1-\varepsilon_1}{\varepsilon_1 A_1} + \frac{1}{A_1 F_{13} + \left[(1/A_1 F_{12}) + (1/A_3 F_{32})\right]^{-1}} + \frac{1-\varepsilon_3}{\varepsilon_3 A_3}} \quad (1)$$

where $E_{b1} = \sigma T_1^4$ and $E_{b1} = \sigma T_3^4$. The view factors are evaluated as follows:

F_{32}: use the relation for two aligned parallel rectangles, Table 13.2 or Fig. 13.4,

$$\bar{X} = X/L = w/L = 10/40 = 0.25 \qquad \bar{Y} = Y/L = \infty$$

$$F_{32} = 0.1231$$

F_{13}: applying reciprocity between A_1 and A_3, where $A_1 = 2L\ell = 2 \times 0.040\ \text{m}\,\ell = 0.080\ \ell$ and $A_3 = w\ell = 0.010\ \ell$ and ℓ is the length of the conductors normal to the page, ℓ >> L or w,

$$F_{13} = \frac{A_3 F_{31}}{A_1} = 0.010\ell \times 0.8769 / 0.080\ell = 0.1096$$

where F_{31} can be obtained by using the summation rule on A_3,

$$F_{31} = 1 - F_{32} = 1 - 0.1231 = 0.8769$$

F_{12}: by symmetry $\qquad F_{12} = F_{13} = 0.1096$

Continued …..

Substituting numerical values into Eq. (1), the net radiation leaving the conductor is

$$q_1 = \frac{5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left(500^4 - 300^4\right)\text{K}^4}{\dfrac{1-0.8}{0.8\times0.080\ell} + \dfrac{1}{0.080\ell\times0.1096 + \left[\left(1/0.080\ell\times0.1096\right)+\left(1/0.010\ell\times0.123\right)\right]^{-1}} + 0}$$

$$q_1' = q_1/\ell = \frac{(3544-459.3)\,\text{W}}{3.1250+101.557+0} = 29.5\ \text{W/m}$$ <

(b) Consider now convection processes occurring at the conductor (1) and base (2) surfaces, and perform energy balances as illustrated in the schematic below.

Surface 1: The heat rate from the conductor includes convection and the net radiation heat rates,

$$q_{in} = q_{cv,1} + q_1 = h\,A_1\left(T_1 - T_\infty\right) + \frac{E_{b1} - J_1}{(1-\varepsilon_1)/\varepsilon_1 A_1} \quad (2)$$

and the radiosity J_1 can be determined from the radiation energy balance, Eq. 13.21,

$$\frac{E_{b1}-J_1}{(1-\varepsilon_1)/\varepsilon_1 A_1} = \frac{J_1 - J_2}{1/A_1F_{12}} + \frac{J_1 - J_3}{1/A_1F_{13}} \quad (3)$$

where $J_3 = E_{b3} = \sigma T_3^4$ since A_3 is black.

Surface 2: Since the surface is insulated (adiabatic), the energy balance has the form

$$0 = q_{cv,2} + q_2 = hA_2\left(T_2 - T_\infty\right) + \frac{E_{b2} - J_2}{1-\varepsilon_2/\varepsilon_2 A_2} \quad (4)$$

and the radiosity J_2 can be determined from the radiation energy balance, Eq. 13.21,

$$\frac{E_{b2}-J_2}{(1-\varepsilon_2)/\varepsilon_2 A_2} = \frac{J_2 - J_1}{1/A_2F_{21}} + \frac{J_2 - J_3}{1/A_2F_{23}} \quad (5)$$

There are 4 equations, Eqs. (2-5), with 4 unknowns: J_2, J_2, T_2 and q_1. Substituting numerical values, the simultaneous solution to the set yields

$$J_1 = 3417\ \text{W/m}^2 \qquad J_2 = 1745\ \text{W/m}^2 \qquad T_2 = 352\ \text{K} \qquad q_{in}' = 441\ \text{W/m}$$ <

COMMENTS: (1) The effect of convection is substantial, increasing the heat removal rate from 29.5 W to 441 W for the combined modes.

(2) With the convection process, the current carrying capacity of the conductors can be increased. Another advantage is that, with the presence of convection, the ceramic base operates at a cooler temperature: 352 K vs. 483 K.

PROBLEM 13.98

KNOWN: Surface temperature and spectral radiative properties. Temperature of ambient air. Solar irradiation or temperature of shield.

FIND: (a) Convection heat transfer coefficient when surface is exposed to solar radiation, (b) Temperature of shield needed to maintain prescribed surface temperature.

SCHEMATIC:

$G_S = 1200\, W/m^2$; $T_\infty = 300K, h$; $T_s = 320K$; E; q''_{conv}

$T_p, \varepsilon_p = 0.8$; $T_\infty = 300K, h$; q''_{ps}; q''_{conv}; $T_s = 320K$

α_λ: 1.0; $\alpha_1 = 0.9$; $\alpha_2 = 0.3$; 0; 1; λ

ASSUMPTIONS: (1) Surface is diffuse ($\alpha_\lambda = \varepsilon_\lambda$), (2) Bottom of surface is adiabatic, (3) Atmospheric irradiation is negligible,
(4) With shield, convection coefficient is unchanged and radiation losses at ends are negligible (two-surface enclosure).

ANALYSIS: (a) From a surface energy balance,

$$\alpha_S G_S = \varepsilon_s \sigma T_s^4 + h(T_s - T_\infty).$$

Emission occurs mostly at long wavelengths, hence $\varepsilon_s = \alpha_2 = 0.3$. However,

$$\alpha_S = \frac{\int_0^\infty \alpha_\lambda E_{\lambda,b}(\lambda, 5800\text{ K})d\lambda}{E_b} = \alpha_1 F_{(0-1\mu m)} + \alpha_2 F_{(1-\infty)}$$

and from Table 12.1 at $\lambda T = 5800\ \mu m\cdot K$, $F_{(0-1\mu m)} = 0.720$ and hence, $F_{(1-\infty)} = 0.280$ giving

$$\alpha = 0.9 \times 0.72 + 0.3 \times 0.280 = 0.732.$$

Hence

$$h = \frac{\alpha_S G_S - \varepsilon\sigma T_s^4}{T_s - T_\infty} = \frac{0.732\left(1200\text{ W/m}^2\right) - 0.3 \times 5.67 \times 10^{-8}\text{W/m}^2\cdot K^4 (320\text{ K})^4}{20\text{ K}}$$

$$h = 35\text{ W/m}^2\cdot K.$$ <

(b) Since the plate emits mostly at long wavelengths, $\alpha_s = \varepsilon_s = 0.3$. Hence radiation exchange is between two diffuse-gray surfaces.

$$q''_{ps} = \frac{\sigma\left(T_p^4 - T_s^4\right)}{1/\varepsilon_p + 2/\varepsilon_s - 1} = q''_{conv} = h(T_s - T_\infty)$$

$$T_p^4 = (h/\sigma)(T_s - T_\infty)\left(1/\varepsilon_p + 1/\varepsilon_s - 1\right) + T_s^4$$

$$T_p^4 = \frac{35\text{ W/m}^2\cdot K(20\text{ K})}{5.67\times10^{-8}\text{W/m}^2\cdot K^4}\left(\frac{1}{0.8} + \frac{1}{0.3} - 1\right) + (320\text{ K})^4 \qquad T_p = 484\text{ K.}$$ <

COMMENTS: For $T_p = 484$ K and $\lambda = 1\ \mu m$, $\lambda T = 484\ \mu m\cdot K$ and $F_{(0-\lambda)} = 0.000$. Hence assumption of $\alpha_s = 0.3$ is excellent.

PROBLEM 13.99

KNOWN: Long uniform rod with volumetric energy generation positioned coaxially within a larger circular tube maintained at 500°C.

FIND: (a) Center $T_1(0)$ and surface T_{1s} temperatures of the rod for evacuated space, (b) $T_1(0)$ and T_{1s} for airspace, (c) Effect of tube diameter and emissivity on $T_1(0)$ and T_{1s}.

SCHEMATIC:

D_1 = 50 mm
r_2 = 30, 50, 500 mm
Rod, $\dot{q}$ = 20 kW/m³, k = 15 W/m·K
Tube, T_2 = 773 K, $0.1 \le \varepsilon_2 \le 1$
T_{1s}, $0.1 \le \varepsilon_1 \le 1$
Vacuum or Air

ASSUMPTIONS: (1) All surfaces are diffuse-gray.

PROPERTIES: *Table A-4*, Air ($\overline{T}$ = 780 K): $\nu = 81.5 \times 10^{-6}$ m²/s, k = 0.0563 W/m·K, $\alpha = 115.6 \times 10^{-6}$ m²/s, β = 0.00128K^{-1}, Pr = 0.706.

ANALYSIS: (a) The net heat exchange by radiation between the rod and the tube is

$$q'_{12} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\left(1-\varepsilon_1\right)/\varepsilon_1 \pi D_1 + 1/\pi D_1 F_{12} + \left(1-\varepsilon_2\right)/\varepsilon_2 \pi D_2} \qquad (1)$$

and, from an energy balance on the rod, $-\dot{E}'_{out} + \dot{E}'_{gen} = 0$, or

$$q'_{12} = \dot{q}\left(\pi D_1^2/4\right). \qquad (2)$$

Combining Eqs. (1) and (2) and substituting numerical values, with F_{12} = 1, we obtain

$$\dot{q} = \frac{4}{D_1}\left[\frac{\sigma\left(T_1^4 - T_2^4\right)}{\left(1-\varepsilon_1\right)/\varepsilon_1 + 1 + \left[\left(1-\varepsilon_2\right)/\varepsilon_2\right]\left(D_1/D_2\right)}\right]$$

$$20\times10^3 \frac{W}{m^3} = \frac{4}{0.050m}\left[\frac{5.67\times10^{-8} W/m^2\cdot K^4\left(T_{1s}^4 - 773^4\right)K^4}{\left(1-0.2\right)/0.2 + 1 + \left[\left(1-0.2\right)/0.2\right]\left(0.050/0.060\right)}\right]$$

$$= 54.4\times10^{-8}\left(T_{1s}^4 - 773^4\right) W/m^3$$

$$T_{1s} = 792 \text{ K.} \qquad <$$

From Eq. 3.53, the rod center temperature is

$$T_1(0) = \frac{\dot{q}\left(D_1/2\right)^2}{4k} + T_{1s}$$

$$T_1(0) \approx \frac{20\times10^3 W/m^3\left(0.050\,m/2\right)^2}{4\times15\,W/m\cdot K} + 792 \text{ K} = 0.21 \text{ K} + 792 \text{ K} = 792.2 \text{ K.} \qquad <$$

(b) The convection heat rate is given by Eqs. 9.58 to 9.60. However, assuming a maximum possible value of (T_{s1} – T_2) = 19 K, $Ra_L = g\beta\,(T_{s,1} - T_2)L^3/\alpha\nu$ = 9.8 m/s² (0.00128 K^{-1})19 K (0.005 m)³/115.6 × 81.5 × 10^{-12} m⁴/s² = 3.16 and Ra_c^* = {[ln(D_2/D_1)]⁴/L³[(D_1)$^{-3/5}$ + (D_2)$^{-3/5}$]⁵} Ra_L = {[ln(1.2)]⁴/(0.005 m)³ [(0.05 m)$^{-3/5}$

Continued …..

$+(0.06\text{ m})^{-3/5}]^{5}\}$ 3.16 = 0.14. It follows that buoyancy driven flow is negligible and heat transfer across the airspace is by conduction. Hence, from Eq. 3.27, $q'_{cond} = 2\,\pi k\,(T_{1s} - T_2)/\ln(r_2/r_1)$.

$$q'_{cond} = \frac{2\pi k\left(T_{1s} - T_2\right)}{\ln\left(r_2/r_1\right)} = \frac{2\pi\left(0.0563\text{ W/m}\cdot\text{K}\right)\left(T_{1s} - 773\right)\text{K}}{\ln\left(30/25\right)} = 1.94\left(T_{1s} - 773\right)$$

The energy balance then becomes $\dot{q}\left(\pi D_1^2/4\right) = q'_{12} + q'_{cond}$, or

$$\dot{q} = \left(4/\pi D_1^2\right)\left(q'_{12} + q'_{cond}\right)$$

$$2\times10^4 = \left[54.4\times10^{-8}\left(T_{1s}^4 - 773^4\right) + 988\left(T_{1s} - 773\right)\right]$$

$$T_{1s} = 783\text{ K} \qquad T_1(0) = 783.2\text{ K}$$ <

(c) Entering the foregoing model and the prescribed properties of air into the *IHT* workspace, the parametric calculations were performed for $D_2 = 0.06$ m and $D_2 = 0.10$ m. For $D_2 = 1.0$ m, $Ra_c^* > 100$ and heat transfer across the airspace is by free convection, instead of conduction. In this case, convection was evaluated by entering Eqs. 9.58 – 9.60 into the workspace. The results are plotted as follows.

The first graph corresponds to the evacuated space, and the surface temperature decreases with increasing $\varepsilon_1 = \varepsilon_2$, as well as with D_2. The increased emissivities enhance the effectiveness of emission at surface 1 and absorption at surface 2, both which have the effect of reducing T_{1s}. Similarly, with increasing D_2, more of the radiation emitted from surface 1 is ultimately absorbed at 2 (less of the radiation reflected by surface 2 is intercepted by 1). The second graph reveals the expected effect of a reduction in T_{1s} with inclusion of heat transfer across the air. For small emissivities ($\varepsilon_1 = \varepsilon_2 < 0.2$), conduction across the air is significant relative to radiation, and the small conduction resistance corresponding to $D_2 = 0.06$ m yields the smallest value of T_{1s}. However, with increasing ε, conduction/convection effects diminish relative to radiation and the trend reverts to one of decreasing T_{1s} with increasing D_2.

COMMENTS: For this situation, the temperature variation *within* the rod is small and independent of surface conditions.

PROBLEM 13.100

KNOWN: Side wall and gas temperatures for adjoining semi-cylindrical ducts. Gas flow convection coefficients.

FIND: (a) Temperature of intervening wall, (b) Verification of gas temperature on one side.

SCHEMATIC:

ASSUMPTIONS: (1) All duct surfaces may be approximated as blackbodies, (2) Fully developed conditions, (3) Negligible temperature difference across intervening wall, (4) Gases are nonparticipating media.

ANALYSIS: (a) Applying an energy balance to a control surface about the wall yields

$$\dot{E}_{in} = \dot{E}_{out}.$$

Assuming $T_{g,1} > T_w > T_{g,2}$, it follows that

$$q_{rad(1\to w)} + q_{conv(g1\to w)} = q_{rad(w\to 2)} + q_{conv(w\to g2)}$$

$$A_1F_{1w}\sigma\left(T_1^4 - T_w^4\right) + hA_w\left(T_{g,1} - T_w\right) = A_wF_{w2}\sigma\left(T_w^4 - T_2^4\right) + hA_w\left(T_w - T_{g,2}\right)$$

and with

$$A_1F_{1w} = A_wF_{w1} = A_wF_{w2} = A_w$$

and substituting numerical values,

$$2\sigma T_w^4 + 2hT_w = \sigma\left(T_1^4 + T_2^4\right) + h\left(T_{g,1} + T_{g,2}\right)$$

$$11.34\times10^{-8}T_w^4 + 10T_w = 13,900.$$

Trial-and-error solution yields

$$T_w \approx 526\ \text{K}. \qquad <$$

(b) Applying an energy balance to a control surface about the hot gas (g,1) yields

$$\dot{E}_{in} = \dot{E}_{out}$$

$$hA_1\left(T_1 - T_{g,1}\right) = hA_w\left(T_{g,1} - T_w\right)$$

or

$$T_1 - T_{g,1} = \left[D/\left(\pi D/2\right)\right]\left(T_{g,1} - T_w\right)$$

$$29°\text{C} = 29°\text{C}. \qquad <$$

COMMENTS: Since there is no change in any of the temperatures in the axial direction, this scheme simply provides for energy transfer from side wall 1 to side wall 2.

PROBLEM 13.101

KNOWN: Temperature, dimensions and arrangement of heating elements between two large parallel plates, one insulated and the other of prescribed temperature. Convection coefficients associated with elements and bottom surface.

FIND: (a) Temperature of gas enclosed by plates, (b) Element electric power requirement, (c) Rate of heat transfer to 1 m × 1m section of panel.

SCHEMATIC:

D = 25 mm, s = 50 mm, $T_1 = 600\text{K}, \varepsilon_1 = 1.0, \bar{h}_1 = 10 \text{W/m}^2\cdot\text{K}$; $T_2 = 400\text{K}, \varepsilon_2 = 0.5, h_2 = 2 \text{W/m}^2\cdot\text{K}$; T_∞, T_m, $\dot{E}_{in}$, $\dot{E}_{out}$, q_1', q_2

ASSUMPTIONS: (1) Diffuse-gray surfaces, (2) Negligible end effects since the surfaces form an enclosure, (3) Gas is nonparticipating, (4) Surface 3 is reradiating with negligible conduction and convection.

ANALYSIS: (a) Performing an energy balance for a unit control surface about the gas space, $\dot{E}_{in} - \dot{E}_{out} = 0.$

$$\bar{h}_1 \pi D (T_1 - T_m) - \bar{h}_2 s (T_m - T_2) = 0$$

$$T_m = \frac{\bar{h}\pi D T_1 + \bar{h}_2 s T_2}{\bar{h}_1 \pi D + \bar{h}_2 s} = \frac{10 \text{ W/m}^2\cdot\text{K}\pi(0.025 \text{ m})600 \text{ K} + 2 \text{ W/m}^2\cdot\text{K}(0.05 \text{ m})400 \text{ K}}{10 \text{ W/m}^2\cdot\text{K}\pi(0.025 \text{ m}) + 2 \text{ W/m}^2\cdot\text{K}(0.05 \text{ m})}$$

$T_m = 577 \text{ K.}$ <

(b) The equivalent thermal circuit is

$q_{1,elec}$, $E_{b1} = J_1$, $\frac{1}{A_1F_{12}}$, J_2, $\frac{1-\varepsilon_2}{A_2\varepsilon_2}$, E_{b2}, $q_{1,conv}$, $1/A_1F_{13}$, $1/A_2F_{23}$, $q_{2,conv}$, $J_3 = E_{b3}$

The energy balance on surface 1 is

$$q'_{1,elec} = q'_{1,conv} + q'_{1,rad}$$

where $q'_{1,rad}$ can be evaluated by considering a unit cell of the form

q_1', q_2'

$$A_1' = \pi D = \pi(0.025 \text{ m}) = 0.0785 \text{ m}$$
$$A_2' = A_3' = s = 0.05 \text{ m}$$

Continued …..

The view factors are:

$$F_{21} = 1 - \left[1 - (D/s)^2\right]^{1/2} + (D/s)\tan^{-1}\left[\left(s^2 - D^2\right)/D^2\right]^{1/2}$$

$$F_{21} = 1 - [1 - 0.25]^{1/2} + 0.5\tan^{-1}(4-1)^{1/2} = 0.658 = F_{31}$$

$$F_{23} = 1 - F_{21} = 0.342 = F_{32}.$$

For the unit cell,

$$A'_2 F_{21} = sF_{21} = 0.05 \text{ m} \times 0.658 = 0.0329 \text{ m} = A'_1 F_{12} = A'_3 F_{31} = A'_1 F_{13}$$

$$A'_2 F_{23} = sF_{23} = 0.05 \text{ m} \times 0.342 = 0.0171 \text{ m} = A'_3 F_{32}.$$

Hence,

$$q'_{1,rad} = \frac{E_{b1} - E_{b2}}{R'_{equiv} + (1-\varepsilon_2)/\varepsilon_2 A'_2}$$

$$R'^{-1}_{equiv} = A'_1 F_{12} + \frac{1}{1/A'_1 F_{13} + 1/A'_2 F_{23}} = \left(0.0329 + \frac{1}{(0.0329)^{-1} + (0.0171)^{-1}}\right)\text{m}$$

$$R'_{equiv} = 22.6 \text{ m}^{-1}.$$

Hence

$$q'_{1,rad} = \frac{5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left(600^4 - 400^4\right)\text{K}^4}{\left[22.6 + (1-0.5)/0.5\times0.05\right]\text{m}^{-1}} = 138.3 \text{ W/m}$$

$$q'_{1,conv} = \bar{h}_1 \pi D (T_1 - T_m) = 10 \text{ W/m}^2\cdot\text{K}\,\pi(0.025 \text{ m})(600-577)\text{K} = 17.8 \text{ W/m}$$

$$q'_{1,elec} = (138.3 + 17.8)\,\text{W/m} = 156 \text{ W/m}. \qquad <$$

(c) Since all energy added via the heating elements must be transferred to surface 2,

$$q'_2 = q'_1.$$

Hence, since there are 20 elements in a 1 m wide strip,

$$q_{2(1\text{m}\times1\text{m})} = 20 \times q'_{1,elec} = 3120 \text{ W}. \qquad <$$

COMMENTS: The bottom panel would have to be cooled (from below) by a heat sink which could dissipate 3120 W/m^2.

PROBLEM 13.102

KNOWN: Flat plate solar collector configuration.

FIND: Relevant heat transfer processes.

SCHEMATIC:

The incident solar radiation will experience transmission, reflection and absorption at each of the cover plates. However, it is desirable to have plates for which absorption and reflection are minimized and transmission is maximized. Glass of low iron content is a suitable material. Solar radiation incident on the absorber plate may be absorbed and reflected, but it is desirable to have a coating which maximizes absorption at short wavelengths.

Energy losses from the absorber plate are associated with radiation, convection and conduction. Thermal radiation exchange occurs between the absorber and the adjoining cover plate, between the two cover plates, and between the top cover plate and the surroundings. To minimize this loss, it is desirable that the emissivity of the absorber plate be small at long wavelengths. Energy is also transferred by free convection from the absorber plate to the first cover plate and between cover plates. It is transferred by free or forced convection to the atmosphere. Energy is also transferred by conduction from the absorber through the insulation.

The foregoing processes provide for heat loss from the absorber, and it is desirable to minimize these losses. The difference between the solar radiation absorbed by the absorber and the energy loss by radiation, convection and conduction is the energy which is transferred to the working fluid. This transfer occurs by conduction through the absorber and the tube wall and by forced convection from the tube wall to the fluid.

PROBLEM 13.103

KNOWN: Operating conditions of a flat plate solar collector.

FIND: Expressions for determining the rate at which useful energy is collected per unit area.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Uniform surface heat fluxes and temperatures, (3) Opaque, diffuse-gray surface behavior for long-wave thermal radiation, (4) Complete absorption of solar radiation by absorber plate ($\alpha_{ap,S} = 1$).

ANALYSIS: From an energy balance on the absorber plate, $\dot{E}''_{in} = \dot{E}''_{out}$,

$$\alpha_{ap,S}\left(\tau_{cp,S}\right)G_S = q''_u + q''_{conv,i} + q''_{rad,ap-cp}.$$

Hence with complete absorption of solar radiation by the absorber plate,

$$q''_u = \tau_{cp,S}G_S - h_i\left(T_{ap} - T_{cp}\right) - \frac{\sigma\left(T_{ap}^4 - T_{cp}^4\right)}{1/\varepsilon_{ap} + 1/\varepsilon_{cp} - 1} \qquad (1)$$ <

where $F_{ap\text{-}cp} \approx 1$ and Eq. 13.24 is used to obtain $q''_{rad,ap-cp}$. To determine q''_u from Eq. (1), however, T_{cp} must be known. From an energy balance on the cover plate,

$$\alpha_{cp,S}G_S + q''_{conv,i} + q''_{rad,ap-cp} = q''_{conv,o} + q''_{rad,cp-sky}$$

or

$$\alpha_{cp,S}G_S + h_i\left(T_{ap} - T_{cp}\right) + \frac{\sigma\left(T_{ap}^4 - T_{cp}^4\right)}{1/\varepsilon_{ap} + 1/\varepsilon_c - 1}$$

$$= h_o\left(T_{cp} - T_\infty\right) + \varepsilon_{cp}\sigma\left(T_{cp}^4 - T_{sky}^4\right). \qquad (2)$$ <

Eq. (2) may be used to obtain T_{cp}.

COMMENTS: With T_{ap} presumed to be known, T_{cp} may be evaluated from Eq. (2) and q''_u from Eq. (1).

PROBLEM 13.104

KNOWN: Ceiling temperature of furnace. Thickness, thermal conductivity, and/or emissivities of alternative thermal insulation systems. Convection coefficient at outer surface and temperature of surroundings.

FIND: (a) Mathematical model for each system, (b) Temperature of outer surface $T_{s,o}$ and heat loss q'' for each system and prescribed conditions, (c) Effect of emissivity on $T_{s,o}$ and q''.

SCHEMATIC:

$T_{sur} = 300$ K
$T_\infty = 300$ K
$h_o = 25$ W/m²·K
$T_{s,o}$, ε_o $q''_{conv,o}$ $q''_{rad,o}$
Insulation
k = 0.09 W/m·K
q''_{cond}
$L = 25$ mm
$q''_{conv,i}$ $q''_{rad,i}$
ε_i
$T_{s,i} = 900$ K

ASSUMPTIONS: (1) Steady-state, (2) Diffuse/gray surfaces, (3) Surroundings form a large enclosure about the furnace, (4) Radiation in air space corresponds to a two-surface enclosure of large parallel plates.

PROPERTIES: *Table A-4*, air (T_f = 730 K): k = 0.055 W/m·K, $\alpha = 1.09 \times 10^{-4}$ m²/s, $\nu = 7.62 \times 10^{-5}$ m²/s, $\beta = 0.001335$ K⁻¹, Pr = 0.702.

ANALYSIS: (a) To obtain $T_{s,o}$ and q'', an energy balance must be performed at the outer surface of the shield.

Insulation:

$$q''_{cond} = q''_{conv,o} + q''_{rad,o} = q''$$

$$k = \frac{\left(T_{s,i} - T_{s,o}\right)}{L} = h_o\left(T_{s,o} - T_\infty\right) + \varepsilon_o \sigma\left(T_{s,o}^4 - T_{sur}^4\right)$$

Air Space:

$$q''_{conv,i} + q''_{rad,i} = q''_{conv,o} + q''_{rad,o} = q''$$

$$h_i\left(T_{s,i} - T_{s,o}\right) + \frac{\sigma\left(T_{s,i}^4 - T_{s,o}^4\right)}{\frac{1}{\varepsilon_i} + \frac{1}{\varepsilon_o} - 1} = h_o\left(T_{s,o} - T_\infty\right) + \varepsilon_o \sigma\left(T_{s,o}^4 - T_{sur}^4\right)$$

where Eq. 13.24 has been used to evaluate $q''_{rad,i}$ and h_i is given by Eq. 9.49

$$\overline{Nu}_L = \frac{h_i L}{k} = 0.069 Ra_L^{1/3} Pr^{0.074}$$

(b) For the prescribed conditions ($\varepsilon_i = \varepsilon_o = 0.5$), the following results were obtained.

Insulation: The energy equation becomes

$$\frac{0.09\ \text{W/m}\cdot\text{K}\left(900 - T_{s,o}\right)\text{K}}{0.025\,\text{m}} = 25\ \text{W/m}^2\cdot\text{K}\left(T_{s,o} - 300\right)\text{K} + 0.5 \times 5.67 \times 10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left(T_{s,o}^4 - 300^4\right)\text{K}^4$$

Continued …..

and a trial-and-error solution yields

$$T_{s,o} = 366\ \text{K} \qquad q'' = 1920\ \text{W/m}^2$$ <

Air-Space: The energy equation becomes

$$h_i \left(900 - T_{s,o}\right)\text{K} + \frac{5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left(900^4 - T_{s,o}^4\right)\text{K}^4}{3}$$
$$= 25\,\text{W/m}^2\cdot\text{K}\left(T_{s,o} - 300\right)\text{K} + 0.5\times5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left(T_{s,o}^4 - 300^4\right)\text{K}^4$$

where

$$h_i = \frac{0.055\ \text{W/m}\cdot\text{K}}{0.025\ \text{m}}\,0.069\,Ra_L^{1/3}\,Pr^{0.074} \qquad (1)$$

and $Ra_L = g\beta(T_{s,i} - T_{s,o})L^3/\alpha\nu$. A trial-and-error solution, which includes reevaluation of the air properties, yields

$$T_{s,o} = 598\ \text{K} \qquad q'' = 10,849\ \text{W/m}^2$$ <

The inner and outer heat fluxes are $q''_{conv,i} = 867\ \text{W/m}^2$, $q''_{rad,i} = 9982\ \text{W/m}^2$, $q''_{conv,o}$ = 7452 W/m^2, and $q''_{rad,o} = 3397\ \text{W/m}^2$.

(c) Entering the foregoing models into the *IHT* workspace, the following results were generated.

Insulation:

Continued …..

PROBLEM 13.104 (Cont.)

As expected, the outer surface temperature decreases with increasing ε_o. However, the reduction in $T_{s,o}$ is not large since heat transfer from the outer surface is dominated by convection.

In this case $T_{s,o}$ increases with increasing $\varepsilon_o = \varepsilon_i$ and the effect is significant. The effect is due to an increase in radiative transfer from the inner surface, with $q''_{rad,i} = q''_{conv,i} = 1750\ \mathrm{W/m^2}$ for $\varepsilon_o = \varepsilon_i =$ 0.1 and $q''_{rad,i} = 20{,}100\ \mathrm{W/m^2} >> q''_{conv,i} = 523\ \mathrm{W/m^2}$ for $\varepsilon_o = \varepsilon_i = 0.9$. With the increase in $T_{s,o}$, the total heat flux increases, along with the relative contribution of radiation $\left(q''_{rad,o}\right)$ to heat transfer from the outer surface.

COMMENTS: (1) With no insulation or radiation shield and $\varepsilon_i = 0.5$, radiative and convective heat fluxes from the ceiling are 18,370 and 15,000 W/m^2, respectively. Hence, a significant reduction in the heat loss results from use of the insulation or the shield, although the insulation is clearly more effective.

(2) Rayleigh numbers associated with free convection in the air space are well below the lower limit of applicability of Eq. (1). Hence, the correlation was used outside its designated range, and the error associated with evaluating h_i may be large.

(3) The *IHT* solver had difficulty achieving convergence in the first calculation performed for the radiation shield, since the energy balance involves two nonlinear terms due to radiation and one due to convection. To obtain a solution, a fixed value of Ra_L was prescribed for Eq. (1), while a second value of $Ra_{L,2} \equiv g\beta(T_{s,i} - T_{s,o})L^3/\alpha\nu$ was computed from the solution. The prescribed value of Ra_L was replaced by the value of $Ra_{L,2}$ and the calculations were repeated until $Ra_{L,2} = Ra_L$.

PROBLEM 13.105

KNOWN: Dimensions of a composite insulation consisting of honeycomb core sandwiched between solid slabs.

FIND: Total thermal resistance.

SCHEMATIC: Because of the repetitive nature of the honeycomb core, the cell sidewalls will be adiabatic. That is, there is no lateral heat transfer from cell to cell, and it suffices to consider the heat transfer across a single cell.

$T_{s,o} = -10°C$
Outer slab, k_1; $L_3 = 12.5$ mm
Air space; T_2
Side wall, k_2; $L_2 = 50$ mm
T_1
Inner slab, k_1; $L_1 = 12.5$ mm
q; $T_{s,i} = 25°C$

$W = 10$ mm; $\frac{t}{2} = 1$ mm
Cell cross section AA

ASSUMPTIONS: (1) One-dimensional, steady-state conditions, (2) Equivalent conditions for each cell, (3) Constant properties, (4) Diffuse, gray surface behavior.

PROPERTIES: *Table A-3*, Particle board (low density): $k_1 = 0.078$ W/m·K; Particle board (high density): $k_2 = 0.170$ W/m·K; For both board materials, $\varepsilon = 0.85$; *Table A-4*, Air ($\bar{T} \approx 7.5°C$, 1 atm): $\nu = 14.15 \times 10^{-6}$ m^2/s, k = 0.0247 W/m·K, $\alpha = 19.9 \times 10^{-6}$ m^2/s, Pr = 0.71, $\beta = 3.57 \times 10^{-3}$ K^{-1}.

ANALYSIS: The total resistance of the composite is determined by conduction, convection and radiation processes occurring within the honeycomb and by conduction across the inner and outer slabs. The corresponding thermal circuit is shown.

$T_{s,i}$ — $R_{cond,i}$ — T_1 — [$R_{conv,hc}$ ∥ $R_{cond,hc}$ ∥ $R_{rad,hc}$] — T_2 — $R_{cond,o}$ — $T_{s,o}$

The total resistance of the composite and equivalent resistance for the honeycomb are

$$R = R_{cond,i} + R_{eq} + R_{cond,o} \qquad R_{eq}^{-1} = \left(R_{cond}^{-1} + R_{conv}^{-1} + R_{rad}^{-1}\right)_{hc}.$$

The component resistances may be evaluated as follows. The inner and outer slabs are plane walls, for which the thermal resistance is given by Eq. 3.6. Hence, since $L_1 = L_3$ and the slabs are constructed from low-density particle board.

$$R_{cond,i} = R_{cond,o} = \frac{L_1}{k_1 W^2} = \frac{0.0125\ \text{m}}{0.078\ \text{W/m}\cdot\text{K}\left(0.01\ \text{m}\right)^2} = 1603\ \text{K/W}.$$

Continued …..

Similarly, applying Eq. 3.6 to the side walls of the cell

$$R_{cond,hc} = \frac{L_2}{k_2\left[W^2 - (W-t)^2\right]} = \frac{L_2}{k_2\left(2Wt - t^2\right)}$$

$$= \frac{0.050\ m}{0.170\ W/m\cdot K\left[2\times 0.01\ m\times 0.002\ m - (0.002\ m)^2\right]} = 8170\ K/W.$$

From Eq. 3.9 the convection resistance associated with the cellular airspace may be expressed as

$$R_{conv,hc} = 1/h(W-t)^2.$$

The cell forms an enclosure that may be classified as a horizontal cavity heated from below, and the appropriate form of the Rayleigh number is $Ra_L = g\beta(T_1 - T_2)L_2^3/\alpha\nu$. To evaluate this parameter, however, it is necessary to *assume* a value of the cell temperature difference. As a first approximation, $T_1 - T_2 = 15°C - (-5°C) = 20°C$,

$$Ra_L = \frac{9.8\ m/s^2\left(3.57\times 10^{-3}K^{-1}\right)(20\ K)(0.05\ m)^3}{19.9\times 10^{-6}m^2/s\times 14.15\times 10^{-6}m^2/s} = 3.11\times 10^5.$$

Applying Eq. 9.49 as a first approximation, it follows that

$$h = (k/L_2)\left[0.069 Ra_L^{1/3}Pr^{0.074}\right] = \frac{0.0247\ W/m\cdot K}{0.05\ m}\left[0.069\left(3.11\times 10^5\right)^{1/3}(0.71)^{0.074}\right] = 2.25\ W/m^2\cdot K.$$

The convection resistance is then

$$R_{conv,hc} = \frac{1}{2.25\ W/m^2\cdot K(0.01\ m - 0.002\ m)^2} = 6944\ K/W.$$

The resistance to heat transfer by radiation may be obtained by first noting that the cell forms a three-surface enclosure for which the sidewalls are reradiating. The net radiation heat transfer between the end surfaces of the cell is then given by Eq. 13.30. With $\varepsilon_1 = \varepsilon_2 = \varepsilon$ and $A_1 = A_2 = (W - t)^2$, the equation reduces to

$$q_{rad} = \frac{(W-t)^2\sigma\left(T_1^4 - T_2^4\right)}{2(1/\varepsilon - 1) + \left[F_{12} + \left[(F_{1R} + F_{2R})/F_{1R}F_{2R}\right]^{-1}\right.}.$$

However, with $F_{1R} = F_{2R} = (1 - F_{12})$, it follows that

$$q_{rad} = \frac{(W-t)^2\sigma\left(T_1^4 - T_2^4\right)}{2\left(\frac{1}{\varepsilon} - 1\right) + \left[F_{12} + \frac{(1-F_{12})^2}{2(1-F_{12})}\right]^{-1}} = \frac{(W-t)^2\sigma\left(T_1^4 - T_2^4\right)}{2\left(\frac{1}{\varepsilon} - 1\right) + \frac{2}{1+F_{12}}}.$$

The view factor F_{12} may be obtained from Fig. 13.4, where

$$\frac{X}{L} = \frac{Y}{L} = \frac{W-t}{L_2} = \frac{10\ mm - 2\ mm}{50\ mm} = 0.16.$$

Hence, $F_{12} \approx 0.01$. Defining the radiation resistance as

$$R_{rad,hc} = \frac{T_1 - T_2}{q_{rad}}$$

it follows that

Continued …..

$$R_{rad,hc} = \frac{2(1/\varepsilon - 1) + 2/(1 + F_{12})}{(W - t)^2 \sigma \left(T_1^2 + T_2^2\right)(T_1 + T_2)}$$

where $\left(T_1^4 - T_2^4\right) = \left(T_1^2 + T_2^2\right)(T_1 + T_2)(T_1 - T_2)$. Accordingly,

$$R_{rad,hc} = \frac{\left[2\left(\frac{1}{0.85} - 1\right) + \frac{2}{1 + 0.01}\right]}{(0.01\text{ m} - 0.002\text{ m})^2 \times 5.67 \times 10^{-8}\text{ W/m}^2 \cdot \text{K}^4 \left[(288\text{ K})^2 + (268\text{ K})^2\right](288 + 268)\text{K}}$$

where, again, it is *assumed* that T_1 = 15°C and T_2 = -5°C. From the above expression, it follows that

$$R_{rad,hc} = \frac{0.353 + 1.980}{3.123 \times 10^{-4}} = 7471\text{ K/W}.$$

In summary the component resistances are

$$R_{cond,i} = R_{cond,o} = 1603\text{ K/W}$$

$$R_{cond,hc} = 8170\text{ K/W} \qquad R_{conv,hc} = 6944\text{ K/W} \qquad R_{rad,hc} = 7471\text{ K/W}.$$

The equivalent resistance is then

$$R_{eq} = \left(\frac{1}{8170} + \frac{1}{6944} + \frac{1}{7471}\right)^{-1} = 2498\text{ K/W}$$

and the total resistance is

$$R = 1603 + 2498 + 1603 = 5704\text{ K/W}. \qquad <$$

COMMENTS: (1) The problem is iterative, since values of T_1 and T_2 were assumed to calculate $R_{conv,hc}$ and $R_{rad,hc}$. To check the validity of the assumed values, we first obtain the heat transfer rate q from the expression

$$q = \frac{T_{s,1} - T_{s,2}}{R} = \frac{25°\text{C} - (-10°\text{C})}{5704\text{ K/W}} = 6.14 \times 10^{-3}\text{W}.$$

Hence

$$T_1 = T_{s,i} - qR_{cond,i} = 25°\text{C} - 6.14 \times 10^{-3}\text{W} \times 1603\text{ K/W} = 15.2°\text{C}$$

$$T_2 = T_{s,o} + qR_{cond,o} = -10°\text{C} + 6.14 \times 10^{-3}\text{W} \times 1603\text{ K/W} = -0.2°\text{C}.$$

Using these values of T_1 and T_2, $R_{conv,hc}$ and $R_{rad,hc}$ should be recomputed and the process repeated until satisfactory agreement is obtained between the initial and computed values of T_1 and T_2.

(2) The resistance of a section of low density particle board 75 mm thick ($L_1 + L_2 + L_3$) of area W^2 is 9615 K/W, which exceeds the total resistance of the composite by approximately 70%. Accordingly, use of the honeycomb structure offers no advantages as an insulating material. Its effectiveness as an insulator could be improved (R_{eq} increased) by reducing the wall thickness t to increase R_{cond}, evacuating the cell to increase R_{conv}, and/or decreasing ε to increase R_{rad}. A significant increase in $R_{rad,hc}$ could be achieved by aluminizing the top and bottom surfaces of the cell.

PROBLEM 13.106

KNOWN: Dimensions and surface conditions of a cylindrical thermos bottle filled with hot coffee and lying horizontally.

FIND: Heat loss.

SCHEMATIC:

$D_1 = 0.07m$
$\varepsilon_1 = 0.25$, $T_1 = 75°C$
$\varepsilon_2 = 0.25$, $T_2 = 35°C$
$D_2 = 0.08m$
$L = 0.3\ m$
Airspace ($p = 1atm$), $L = 0.005m$

ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible heat loss from ends (long infinite cylinders), (3) Diffuse-gray surface behavior.

PROPERTIES: *Table A-4*, Air ($T_f = (T_1 + T_2)/2 = 328$ K, 1 atm): $k = 0.0284$ W/m·K, $\nu = 23.74 \times 10^{-6}$ m^2/s, $\alpha = 26.6 \times 10^{-6}$ m^2/s, Pr = 0.0703, $\beta = 3.05 \times 10^{-3}$ K^{-1}.

ANALYSIS: The heat transfer across the air space is

$$q = q_{rad} + q_{conv}.$$

From Eq. 13.25 for concentric cylinders

$$q_{rad} = \frac{\sigma(\pi D_1 L)\left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} + \frac{1-\varepsilon_2}{\varepsilon_2}\left(\frac{r_1}{r_2}\right)} = \frac{5.67\times10^{-8}\text{W/m}^2\cdot\text{K}^4 \pi(0.07\times0.3)\text{m}^2\left(348^4 - 308^4\right)\text{K}^4}{4 + 3(0.035/0.04)}$$

$$q_{rad} = 3.20\ \text{W}.$$

From Eq. 9.25,

$$Ra_L = \frac{g\beta(T_1 - T_2)L^3}{\alpha\nu} = \frac{9.8\ \text{m/s}^2\left(3.05\times10^{-3}\text{K}^{-1}\right)(40\ \text{K})(0.005\ \text{m})^3}{26.6\times10^{-6}\text{m}^2/\text{s}\times23.74\times10^{-6}\text{m}^2/\text{s}} = 236.7.$$

Hence from Eq. 9.60

$$Ra_c^* = \frac{[\ln(D_2/D_1)]^4 Ra_L}{L^3\left(D_1^{-0.6} + D_2^{-0.6}\right)^5} = \frac{[\ln(0.08/0.07)]^4\ 236.7}{(0.005\ \text{m})^3\left(0.07^{-0.6} + 0.08^{-0.6}\right)^5\ \text{m}^{-3}} = 7.85.$$

However, the implication of such a small value of Ra_c^* is that free convection effects are negligible. Heat transfer across the airspace is therefore by conduction ($k_{eff} = k$). From Eq. 3.27

$$q_{cond} = \frac{2\pi Lk(T_1 - T_2)}{\ln(r_2/r_1)} = \frac{2\pi\times0.3\ \text{m}\times0.0284\ \text{W/m}\cdot\text{K}(75-35)\text{K}}{\ln(0.04/0.035)} = 16.04\ \text{W}.$$

Hence the total heat loss is

$$q = q_{rad} + q_{cond} = 19.24\ \text{W}.$$ <

COMMENTS: (1) End effects could be considered in a more detailed analysis, (2) Conduction losses could be eliminated by evacuating the annulus.

PROBLEM 13.107

KNOWN: Thickness and height of a vertical air space. Emissivity and temperature of adjoining surfaces.

FIND: (a) Heat loss per unit area across the space, (b) Heat loss per unit area if space is filled with urethane foam.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Diffuse-gray surface behavior, (3) Air space is a vertical cavity, (4) Constant properties, (5) One-dimensional conduction across foam.

PROPERTIES: *Table A-4*, Air (T_f = 4°C, 1 atm): $\nu = 13.84 \times 10^{-6}$ m²/s, k = 0.0245 W/m·K, $\alpha = 19.5 \times 10^{-6}$ m²/s, Pr = 0.71, $\beta = 3.61 \times 10^{-3}$ K^{-1}; *Table A-3*, Urethane foam: k = 0.026 W/m·K.

ANALYSIS: (a) With the air space, heat loss is by radiation and free convection or conduction. From Eq. 13.24,

$$q''_{rad} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{1/\varepsilon_1 + 1/\varepsilon_2 - 1} = \frac{5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left(291^4 - 263^4\right)\text{K}^4}{1.222} = 110.7\ \text{W/m}^2.$$

With

$$Ra_L = \frac{g\beta\left(T_1 - T_2\right)L^3}{\nu\alpha} = \frac{9.8\ \text{m}^2/\text{s}\left(3.61\times10^{-3}\text{K}^{-1}\right)(18+10)\text{K}\,(0.1\ \text{m})^3}{13.84\times10^{-6}\ \text{m}^2/\text{s}\times19.5\times10^{-6}\text{m}^2/\text{s}} = 3.67\times10^6$$

and H/L = 30, Eq. 9.53 may be used as a first approximation to obtain

$$\overline{Nu}_L = 0.046 Ra_L^{1/3} = 0.046\left(3.67\times10^6\right)^{1/3} = 7.10$$

$$\bar{h} = \frac{k}{L}\overline{Nu}_L = \frac{0.0245\ \text{W/m}\cdot\text{K}}{0.1\ \text{m}} 7.10 = 1.74\ \text{W/m}^2\cdot\text{K}.$$

The convection heat flux is

$$q''_{conv} = \bar{h}\left(T_1 - T_2\right) = 1.74\ \text{W/m}^2\cdot\text{K}\,(18+10)\text{K} = 48.7\ \text{W/m}^2.$$

The heat loss is then

$$q'' = q''_{rad} + q''_{conv} = 110.7 + 48.7 = 159\ \text{W/m}^2.$$

(b) With the foam, heat loss is by conduction and

$$q'' = q''_{cond} = \frac{k}{L}\left(T_1 - T_2\right) = \frac{0.026\ \text{W/m}\cdot\text{K}}{0.1\ \text{m}}(18+10)\text{K} = 7.3\ \text{W/m}^2.$$ <

COMMENTS: Use of the foam insulation reduces the heat loss considerably. Note the significant effect of radiation.

PROBLEM 13.108

KNOWN: Temperatures and emissivity of window panes and critical Rayleigh number for onset of convection in air space.

FIND: (a) The conduction heat flux across the air gap for the optimal spacing, (b) The total heat flux for uncoated panes, (c) The total heat flux if one or both of the panes has a low-emissivity coating.

SCHEMATIC:

$\varepsilon_g = 0.90$ or $\varepsilon_c = 0.10$

Air

q''_{cond}

$T_1 = 22^oC$ $T_2 = -20^oC$

q''_{rad}

L_{op}

ASSUMPTIONS: (1) Critical Rayleigh number is $Ra_{L,c} = 2000$, (2) Constant properties, (3) Radiation exchange between large (infinite), parallel, diffuse-gray surfaces.

PROPERTIES: *Table A-4*, air [$T = (T_1 + T_2)/2 = 1°C = 274$ K]: $\nu = 13.6 \times 10^{-6}$ m^2/s, k = 0.0242 W/m·K, $\alpha = 19.1 \times 10^{-6}$ m^2/s, $\beta = 0.00365$ K^{-1}.

ANALYSIS: (a) With $Ra_{L,c} = g\beta(T_1 - T_2)L_{op}^3/\alpha\nu$

$$L_{op} = \left[\frac{\alpha\nu\,Ra_{L,c}}{g\beta(T_1 - T_2)}\right]^{1/3} = \left[\frac{19.1\times 13.6\times 10^{-12}\,m^4/s^2 \times 2000}{9.8\,m/s^2\left(0.00365\,K^{-1}\right)42°C}\right]^{1/3} = 0.0070m$$

The conduction heat flux is then

$$q''_{cond} = k(T_1 - T_2)/L_{op} = 0.0242\,W/m\cdot K(42°C)/0.0070m = 145.2\ W/m^2$$ <

(b) For conventional glass ($\varepsilon_g = 0.90$), Eq. (13.24) yields,

$$q''_{rad} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{2}{\varepsilon_g} - 1} = \frac{5.67\times 10^{-8}\,W/m^2\cdot K^4\left(295^4 - 253^4\right)K^4}{1.222} = 161.3\ W/m^2$$

and the total heat flux is

$$q''_{tot} = q''_{cond} + q''_{rad} = 306.5\ W/m^2$$ <

(c) With only one surface coated,

$$q''_{rad} = \frac{5.67\times 10^{-8}\,W/m^2\cdot K^4\left(295^4 - 253^4\right)}{\frac{1}{0.90} + \frac{1}{0.10} - 1} = 19.5\ W/m^2$$

Continued …..

PROBLEM 13.108 (Cont.)

$$q''_{tot} = 164.7\ \text{W/m}^2$$ <

With both surfaces coated,

$$q''_{rad} = \frac{5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\left(295^4 - 253^4\right)}{\frac{1}{0.10}+\frac{1}{0.10}-1} = 10.4\ \text{W/m}^2$$

$$q''_{tot} = 155.6\ \text{W/m}^2$$ <

COMMENTS: Without any coating, radiation makes a large contribution (53%) to the total heat loss. With one coated pane, there is a significant reduction (46%) in the total heat loss. However, the benefit of coating both panes is marginal, with only an additional 3% reduction in the total heat loss.

PROBLEM 13.109

KNOWN: Dimensions and emissivity of double pane window. Thickness of air gap. Temperatures of room and ambient air and the related surroundings.

FIND: (a) Temperatures of glass panes and rate of heat transfer through window, (b) Heat rate if gap is evacuated. Heat rate if special coating is applied to window.

SCHEMATIC:

$T_{sur,i} = 20°C$
$T_{sur,o} = -20°C$
$T_{s,i}$, ε_g or ε_c
$T_{s,o}$, ε_g or ε_c
H = 1 m
Room air
$T_{\infty,i} = 20°C$
q_i
q_g
q_o
Ambient air
$T_{\infty,o} = -20°C$
W = 1 m
L = 0.025 m

ASSUMPTIONS: (1) Steady-state, (2) Negligible glass pane thermal resistance, (3) Constant properties, (4) Diffuse-gray surface behavior, (5) Radiation exchange between interior window surfaces may be approximated as exchange between infinite parallel plates, (6) Interior and exterior surroundings are very large.

PROPERTIES: *Table A-4*, Air (p = 1 atm). Obtained from using *IHT* to solve for conditions of Part (a): $T_{f,i}$ = 287.4 K: $\nu_i = 14.8 \times 10^{-6}$ m²/s, k_i = 0.0253 W/m·K, $\alpha_i = 20.8 \times 10^{-6}$ m²/s, Pr_i = 0.71, β_i = 0.00348 K^{-1}. $\overline{T} = (T_{s,i} + T_{s,o})/2$ = 273.7 K: $\nu = 13.6 \times 10^{-6}$ m²/s, k = 0.0242 W/m·K, $\alpha = 19.0 \times 10^{-6}$ m²/s, Pr = 0.71, β = 0.00365 K^{-1}. $T_{f,o}$ = 259.3 K: $\nu_o = 12.3 \times 10^{-6}$ m²/s, k_o = 0.023 W/m·K, $\alpha_o = 17.1 \times 10^{-6}$ m²/s, Pr_o = 0.72, β_o = 0.00386 K^{-1}.

ANALYSIS: (a) The heat flux through the window may be expressed as

$$q'' = q''_{rad,i} + q''_{conv,i} = \varepsilon_g \sigma \left(T_{sur,i}^4 - T_{s,i}^4\right) + \overline{h}_i \left(T_{\infty,i} - T_{s,i}\right) \quad (1)$$

$$q'' = q''_{rad,gap} + q''_{conv,gap} = \frac{\sigma\left(T_{s,i}^4 - T_{s,o}^4\right)}{\frac{1}{\varepsilon_g} + \frac{1}{\varepsilon_g} - 1} + \overline{h}_{gap}\left(T_{s,i} - T_{s,o}\right) \quad (2)$$

$$q'' = q''_{rad,o} + q''_{conv,o} = \varepsilon_g \sigma \left(T_{s,o}^4 - T_{sur,o}^4\right) + \overline{h}_o \left(T_{s,o} - T_{\infty,o}\right) \quad (3)$$

where radiation exchange between the window panes is determined from Eq. (13.24) and radiation exchange with the surroundings is determined from Eq. (13.27). The inner and outer convection coefficients, $\overline{h}_i$ and $\overline{h}_o$, are determined from Eq. (9.26), and $\overline{h}_{gap}$ is obtained from Eq. (9.52).

The foregoing equations may be solved for the three unknowns $\left(q'', T_{s,i}, T_{s,o}\right)$. Using the *IHT* software to effect the solution, we obtain

$$T_{s,i} = 281.8 \text{ K} = 8.8°\text{C}$$ <

Continued …..

$T_{s,o} = 265.6\ K = -7.4°C$ <

$q = 91.3\ W$ <

(b) If the air space is evacuated $\left(\overline{h}_g = 0\right)$, we obtain

$T_{s,i} = 283.6\ K = 10.6°C$ <

$T_{s,o} = 263.8\ K = 9.2°C$ <

$q = 75.5\ W$ <

If the space is not evacuated but the coating is applied to inner surfaces of the window panes,

$T_{s,i} = 285.9\ K = 12.9°C$ <

$T_{s,o} = 261.3\ K = -11.7°C$ <

$q = 55.9\ W$ <

If the space is evacuated and the coating is applied,

$T_{s,i} = 291.7\ K = 18.7°C$ <

$T_{s,o} = 254.7\ K = -18.3°C$ <

$q = 9.0\ W$ <

COMMENTS: (1) For the conditions of part (a), the convection and radiation heat fluxes are comparable at the inner and outer surfaces of the window, but because of the comparatively small convection coefficient, the radiation flux is approximately twice the convection flux across the air gap. (2) As the resistance across the air gap is progressively increased (evacuated, coated, evacuated and coated), the temperatures of the inner and outer panes increase and decrease, respectively, and the heat loss decreases. (3) Clearly, there are significant energy savings associated with evacuation of the gap and application of the coating. (4) In all cases, solutions were obtained using the temperature-dependent properties of air provided by the software. The property values listed in the **PROPERTIES** section of this solution pertain to the conditions of part (a).

PROBLEM 13.110

KNOWN: Absorber and cover plate temperatures and spectral absorptivities for a flat plate solar collector. Collector orientation and solar flux.

FIND: (a) Rate of solar radiation absorption per unit area, (b) Heat loss per unit area.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Adiabatic sides and bottom, (3) Cover is transparent to solar radiation, (4) Sun emits as a blackbody at 5800 K, (5) Cover and absorber plates are diffuse-gray to long wave radiation, (6) Negligible end effects, (7) L << width and length.

PROPERTIES: *Table A-4*, Air ($T = (T_a + T_c)/2 = 321.5$ K, 1 atm): $\nu = 18.05 \times 10^{-6}$ m^2/s, k = 0.0279 W/m·K, $\alpha = 25.7 \times 10^{-6}$ m^2/s.

ANALYSIS: (a) The absorbed solar irradiation is

$$G_{S,abs} = \alpha_{S,a} G_S$$

where

$$G_S = q''_S \cos 30° = 900 \times 0.866 = 779.4 \text{ W/m}^2$$

$$\alpha_{S,a} = \frac{\int_0^\infty \alpha_{\lambda,a} G_{\lambda,S} d\lambda}{G_S} = \frac{\int_0^\infty \alpha_{\lambda,a} E_{\lambda,b}(5800 \text{ K}) d\lambda}{E_b(5800 \text{ K})}$$

$$\alpha_{S,a} = \alpha_{\lambda,a,1} F_{(0 \to 2\ \mu m)} + \alpha_{\lambda,a2} F_{(2 \to \infty)}.$$

For $\lambda T = 2\ \mu m \times 5800$ K = 11,600 μm·K from Table 12.1, $F_{(0 \to 2\lambda T)} = 0.941$, find

$$\alpha_{S,a} = 0.9 \times 0.941 + 0.2 \times (1 - 0.941) = 0.859.$$

Hence

$$G_{S,abs} = 0.859 \times 779.4 = 669 \text{ W/m}^2.$$ <

(b) The heat loss per unit area from the collector is

$$q''_{loss} = q''_{conv} + q''_{rad}.$$

The convection heat flux is

$$q''_{conv} = \bar{h}(T_a - T_c)$$

Continued …..

PROBLEM 13.110 (Cont.)

and with

$$Ra_L = \frac{g\beta\left(T_a - T_c\right)L^3}{\alpha\nu}$$

$$Ra_L = \frac{9.8\ m/s^2 \times \left(321.5\ K\right)^{-1}\left(343-300\right)K\left(0.02\ m\right)^3}{18.05\times10^{-6} m^2/s \times 25.7\times10^{-6} m^2/s} = 22,604$$

find from Eq. 9.54 with

$$H/L > 12,\ \tau < \tau^*,\ \cos\ \tau = 0.5,\ Ra_L\ \cos\ \tau = 11,302$$

$$\overline{Nu}_L = 1 + 1.44\left[1 - \frac{1708}{11,302}\right]\left[1 - \frac{1708\left(\sin\ 108°\right)^{1.6}}{11,302}\right] + \left[\left(\frac{11,302}{5830}\right)^{1/3} - 1\right]$$

$$\bar{h} = \overline{Nu}_L\ \frac{k}{L} = 2.30 \times \frac{0.0279\ W/m\cdot K}{0.02\ m} = 3.21\ W/m^2\cdot K.$$

Hence, the convective heat flux is

$$q''_{conv} = 3.21\ W/m^2\cdot K\left(343-300\right)K = 138.0\ W/m^2.$$

The radiative exchange can be determined from Eq. 13.24 treating the cover and absorber plates as a two-surface enclosure,

$$q''_{rad} = \frac{\sigma\left(T_a^4 - T_c^4\right)}{1/\varepsilon_a + 1/\varepsilon_c - 1} = \frac{5.67\times10^{-8} W/m^2\cdot K^4\left[\left(343\ K\right)^4 - \left(300\ K\right)^4\right]}{1/0.2 + 1/0.75 - 1}$$

$$q''_{rad} = 61.1\ W/m^2.$$

Hence, the total heat loss per unit area from the collector

$$q''_{loss} = \left(138.0 + 61.1\right) = 199\ W/m^2. \qquad <$$

COMMENTS: (1) Non-solar components of radiation transfer are concentrated at long wavelength for which $\alpha_a = \varepsilon_a = 0.2$ and $\alpha_c = \varepsilon_c = 0.75$.

(2) The collector efficiency is

$$\eta = \frac{669.3 - 199.1}{669.3} \times 100 = 70\%.$$

This value is uncharacteristically high due to specification of nearly optimum $\alpha_a(\lambda)$ for absorber.

PROBLEM 13.111

KNOWN: Diameters and temperatures of a heated tube and a radiation shield.

FIND: (a) Total heat loss per unit length of tube, (b) Effect of shield diameter on heat rate.

SCHEMATIC:

Shield
$D_o = 0.12$ m, $T_o = 35°C$, $\varepsilon_o = 0.1$

Air gap

Steam

Tube
$D_i = 0.10$ m, $T_i = 120°C$, $\varepsilon_i = 0.8$

ASSUMPTIONS: (1) Opaque, diffuse-gray surfaces, (2) Negligible end effects.

PROPERTIES: *Table A-4*, Air ($T_f = 77.5°C \approx 350$ K): k = 0.030 W/m·K, Pr = 0.70, $\nu = 20.92 \times 10^{-6}$ m²/s, $\alpha = 29.9 \times 10^{-6}$ m²/s, $\beta = 0.00286$ K^{-1}.

ANALYSIS: (a) Heat loss from the tube is by radiation and free convection

$$q' = q'_{rad} + q'_{conv}$$

From Eq. (13.25)
$$q'_{rad} = \frac{\sigma(\pi D_i)\left(T_i^4 - T_o^4\right)}{\dfrac{1}{\varepsilon_i} + \dfrac{1-\varepsilon_o}{\varepsilon_o}\left(\dfrac{r_i}{r_o}\right)}$$

or

$$q'_{rad} = \frac{5.67\times10^{-8}\dfrac{W}{m\cdot K^4}(\pi\times0.1m)\left(393^4 - 308^4\right)K^4}{\dfrac{1}{0.8} + \dfrac{0.9}{0.1}\left(\dfrac{0.05}{0.06}\right)} = 30.2\ \frac{W}{m}$$

$$Ra_L = \frac{g\beta(T_i - T_o)L^3}{\nu\alpha} = \frac{9.8\,m/s^2 \times 0.00286\ K^{-1}(85\ K)(0.01m)^3}{\left(20.92\times10^{-6}m^2/s\right)\left(29.9\times10^{-6}m^2/s\right)} = 3809$$

Hence from Eq. (9.60)

$$Ra_c^* = \frac{\left[\ell n(D_o/D_i)\right]^4 Ra_L}{L^3\left(D_i^{-3/5} + D_o^{-3/5}\right)^5} = \frac{\left[\ell n(0.12/0.10)\right]^4 3809}{(0.01m)^3\left[(0.1m)^{-0.6} + (0.12m)^{-0.6}\right]^5} = 171.6$$

and from Eq. (9.59)

$$k_{eff} = 0.386\ k\left(\frac{Pr}{0.861 + Pr}\right)^{1/4}\left(Ra_c^*\right)^{1/4}$$

Continued …..

PROBLEM 13.111 (Cont.)

$$k_{eff} = 0.386\left(0.03\frac{W}{m\cdot K}\right)\left(\frac{0.7}{0.861+0.7}\right)^{1/4}(171.6)^{1/4} = 0.0343\frac{W}{m\cdot K}$$

Hence from Eq. (9.58)

$$q'_{conv} = \frac{2\pi\, k_{eff}}{\ell n\left(D_o/D_i\right)}\left(T_i - T_o\right) = \frac{2\pi\left(0.0343\frac{W}{m\cdot K}\right)}{\ell n\left(0.12/0.10\right)}(120-35)K = 100.5\frac{W}{m}$$

$$q' = (30.2+100.5)\frac{W}{m} = 130.7\frac{W}{m}$$

(b) As shown below, both convection and radiation, and hence the total heat rate, increase with increasing shield diameter. In the limit as $D_o \to \infty$, the radiation rate approaches that corresponding to net transfer between a small surface and large surroundings at T_o. The rate is independent of ε.

COMMENTS: Designation of a shield temperature is arbitrary. The temperature depends on the nature of the environment external to the shield.

PROBLEM 13.112

KNOWN: Diameters of heated tube and radiation shield. Tube surface temperature and temperature of ambient air and surroundings.

FIND: Temperature of radiation shield and heat loss per unit length of tube.

SCHEMATIC:

$T_{sur} = 20^oC$

q'_o

q'_i

Shield
$D_o = 0.12$ m, T_o, $\varepsilon_o = 0.1$

Steam

Air gap

Ambient air

Tube
$D_i = 0.10$ m, $T_i = 120^oC$, $\varepsilon_i = 0.8$

ASSUMPTIONS: (1) Opaque, diffuse-gray surfaces, (2) Negligible end effects, (3) Large surroundings, (4) Quiescent air, (5) Steady-state.

PROPERTIES: Determined from use of *IHT* software for iterative solution. Air, $(T_i + T_o)/2 = 362.7$ K: $\nu_i = 2.23 \times 10^{-5}$ m^2/s, $k_i = 0.031$ W/m·K, $\alpha_i = 3.20 \times 10^{-5}$ m^2/s, $\beta_i = 0.00276$ K^{-1}, $Pr_i = 0.698$. Air, $T_f = 312.7$ K: $\nu_o = 1.72 \times 10^{-5}$ m^2/s, $k_o = 0.027$ W/m·K, $\alpha_o = 2.44 \times 10^{-5}$ m^2/s, $\beta_o = 0.0032$ K^{-1}, $Pr_o = 0.705$.

ANALYSIS: From an energy balance on the radiation shield, $q'_i = q'_o$ or $q'_{rad,i} + q'_{conv,i} = q'_{rad,o} + q'_{conv,o}$. Evaluating the inner and outer radiation rates from Eqs. (13.25) and (13.27), respectively, and the convection heat rate in the air gap from Eq. (9.58),

$$\frac{\sigma \pi D_i \left(T_i^4 - T_o^4\right)}{\frac{1}{\varepsilon_i} + \frac{1-\varepsilon_o}{\varepsilon_o}\left(\frac{D_i}{D_o}\right)} + \frac{2\pi k_{eff}\,(T_i - T_o)}{\ell n\left(Do/Di\right)} = \sigma \pi D_o \varepsilon_o \left(T_o^4 - T_{sur}^4\right) + \pi D_o \bar{h}_o \left(T_o - T_\infty\right)$$

From Eqs. (9.59) and (9.60)

$$k_{eff} = 0.386\ k_i \left(\frac{Pr_i}{0.861 + Pr_i}\right)^{1/4} \left(Ra_c^*\right)^{1/4}$$

$$Ra_c^* = \frac{\left[\ell n\left(D_o/D_i\right)\right]^4 Ra_L}{L^3\left(D_i^{-3/5} + D_o^{-3/5}\right)^5}$$

where $Ra_L = g\,\beta_i\,(T_i - T_o)L^3/\nu_i\,\alpha_i$ and $L = (D_o - D_i)/2$. From Eq. (9.34), the convection coefficient on the outer surface of the shield is

$$\bar{h}_o = \frac{k_o}{D_o}\left\{0.60 + \frac{0.387\ Ra_D^{1/6}}{\left[1 + \left(0.559/Pr\right)^{9/16}\right]^{8/27}}\right\}^2$$

Continued …..

PROBLEM 13.112 (Cont.)

The solution to the energy balance is obtained using the *IHT* software, and the result is

$$T_o = 332.5 \text{ K} = 59.5°\text{C}$$ <

The corresponding value of the heat loss is

$$q_i' = 88.7 \text{ W/m}$$ <

COMMENTS: (1) The radiation and convection heat rates are $q'_{rad,i} = 23.7 \text{ W/m}$, $q'_{rad,o} = 10.4 \text{ W/m}$, $q'_{conv,i} = 65.0 \text{ W/m}$, and $q'_{conv,o} = 78.3 \text{ W/m}$. Convection is clearly the dominant mode of heat transfer.

(2)With a value of T_o = 59.5°C > 35°C, the heat loss is reduced (88.7 W/m compared to 130.7 W/m if the shield is at 35°C).

PROBLEM 13.113

KNOWN: Dimensions and inclination angle of a flat-plate solar collector. Absorber and cover plate temperatures and emissivities.

FIND: (a) Rate of heat transfer by free convection and radiation, (b) Effect of the absorber plate temperature on the heat rates.

SCHEMATIC:

$L = 0.03$ m
$w = H = 2$m
$T_1 = 70$ °C
$\varepsilon_1 = 0.96$
$T_2 = 30$ °C
$\varepsilon_2 = 0.92$
$\tau = 60°$

ASSUMPTIONS: (1) Diffuse-gray, opaque surface behavior.

PROPERTIES: *Table A-4*, air $\left(\overline{T} = (T_1 + T_2)/2 = 323\ \text{K}\right)$: $\nu = 18.2 \times 10^{-6}$ m^2/s, k = 0.028 W/m·K, $\alpha = 25.9 \times 10^{-6}$ m^2/s, Pr = 704, $\beta = 0.0031$ K^{-1}.

ANALYSIS: (a) The convection heat rate is

$$q_{conv} = \overline{h}A\left(T_1 - T_2\right)$$

where A = wH=4 m^2 and, with H/L > 12 and $\tau < \tau^* = 70$ deg, $\overline{h}$ is given by Eq. 9.54. With a Rayleigh number of

$$Ra_L = \frac{g\beta\left(T_1 - T_2\right)L^3}{\alpha\nu} = \frac{9.8\,\text{m/s}^2\left(0.0031\ \text{K}^{-1}\right)\left(40°\text{C}\right)\left(0.03\,\text{m}\right)^3}{25.9\times10^{-6}\text{m}^2/\text{s}\times18.2\times10^{-6}\text{m}^2/\text{s}} = 69,600$$

$$\overline{Nu}_L = 1 + 1.44\left[1 - \frac{1708}{0.5(69,600)}\right]\left[1 - \frac{1708(0.923)}{0.5(69,600)}\right] + \left[\left(\frac{0.5\times69,600}{5830}\right)^{1/3} - 1\right]$$

$$\overline{Nu}_L = 1 + 1.44[0.951][0.955] + 0.814 = 3.12$$

$$\overline{h} = (k/L)\overline{Nu}_L = (0.028\ \text{W/m}\cdot\text{K}/0.03\,\text{m})3.12 = 2.91\ \text{W/m}^2\cdot\text{K}$$

$$q_{conv} = 2.91\ \text{W/m}^2\cdot\text{K}\left(4\ \text{m}^2\right)(70-30)°\text{C} = 466\ \text{W}$$ <

The net rate of radiation exchange is given by Eq. 13.24.

$$q = \frac{A\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} = \frac{\left(4\ \text{m}^2\right)5.67\times10^{-8}\text{W/m}^2\cdot\text{K}^4\left(343^4 - 303^4\right)\text{K}^4}{\frac{1}{0.96} + \frac{1}{0.92} - 1} = 1088\ \text{W}$$ <

(b) The effect of the absorber plate temperature was determined by entering Eq. 9.54 into the *IHT* workspace and using the *Properties* and *Radiation* Toolpads.

Continued …..

PROBLEM 13.113 (Cont.)

3200
2400
1600
800
0
Heat rate (W)
50 60 70 80 90 100
Absorber plate temperature, T1(C)
qconv + qrad
qconv
qrad

As expected, the convection and radiation losses increase with increasing T_i, with the T^4 dependence providing a more pronounced increase for the radiation.

COMMENTS: To minimize heat losses, it is obviously desirable to operate the absorber plate at the lowest possible temperature. However, requirements for the outlet temperature of the working fluid may dictate operation at a low flow rate and hence an elevated plate temperature.

PROBLEM 13.114

KNOWN: Disk heated by an electric furnace on its lower surface and exposed to an environment on its upper surface.

FIND: (a) Net heat transfer to (or from) the disk $q_{net,d}$ when T_d = 400 K and (b) Compute and plot $q_{net,d}$ as a function of disk temperature for the range $300 \le T_d \le 500$ K; determine steady-state temperature of the disk.

SCHEMATIC:

Tsur = 300 K
Quiescent air, T∞ = 300 K
Disk, Td = 400 K
εd,2 = 0.8
εd,1 = 1.0
L = 200 mm
A2, T2 = 500 K, ε2 = 1
A1, T1 = 500 K, ε1 = 0.6
D = 400 mm
qrad
qconv
qd = q3

ASSUMPTIONS: (1) Steady-state conditions, (2) Disk is isothermal; negligible thermal resistance, (3) Surroundings are isothermal and large compared to the disk, (4) Non-black surfaces are gray-diffuse, (5) Furnace-disk forms a 3-surface enclosure, (6) Negligible convection in furnace, (7) Ambient air is quiescent.

PROPERTIES: *Table A-4*, Air ($T_f = (T_d + T_\infty)/2$ = 350 K,1 atm): $\nu = 20.92 \times 10^{-6}$ m²/s, k = 0.30 W/m·K, $\alpha = 29.9 \times 10^{-6}$ m²/s.

ANALYSIS: (a) Perform an energy balance on the disk identifying: q_{rad} as the net radiation exchange between the disk and surroundings; q_{conv} as the convection heat transfer; and q_3 as the net radiation leaving the disk within the 3-surface enclosure.

$$q_{net,d} = \dot{E}_{in} - \dot{E}_{out} = -q_{rad} - q_{conv} - q_3 \quad (1)$$

Radiation exchange with surroundings: The rate equation is of the form

$$q_{rad} = \varepsilon_{d,2} A_d \sigma \left(T_d^4 - T_{sur}^4\right) \quad (2)$$

$$q_{rad} = 0.8(\pi/4)(0.400\,\text{m})^2\, 5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4 \left(400^4 - 300^4\right)\text{K}^4 = 99.8 \text{ W.}$$

Free convection: The rate equation is of the form

$$q_{conv} = \bar{h} A_d \left(T_d - T_\infty\right) \quad (3)$$

where $\bar{h}$ can be estimated by an appropriate convection correlation. Find first,

$$Ra_L = g\beta\Delta T L^3/\nu\alpha \quad (4)$$

$$Ra_L = 9.8\,\text{m/s}^2\,(1/350\text{ K})(400-300)\text{K}\,(0.400\,\text{m}/4)^3/20.92\times10^{-6}\,\text{m/s}^2 \times 29.9\times10^{-6}\,\text{m}^2/\text{s}$$

$$Ra_L = 4.476\times10^6$$

where $L = A_c/P = D/4$. For the upper surface of a heated plate for which $10^4 \le Ra_L \le 10^7$, Eq. 9.30 is the appropriate correlation,

Continued …..

PROBLEM 13.114 (Cont.)

$$\overline{Nu}_L = \bar{h}L/k = 0.54\,Ra_L^{1/4} \quad (5)$$

$$\bar{h} = 0.030\,W/m\cdot K/(0.400\,m/4)\times 0.54\left(4.476\times 10^6\right)^{1/4} = 7.45\,W/m^2\cdot K$$

Hence, from Eq. (3),

$$q_{conv} = 7.45\,W/m^2\cdot K\,(\pi/4)(0.400\,m)^2\,(400-300)K = 93.6\ W.$$

Furnace-disk enclosure: From Eq. 13.20, the net radiation leaving the disk is

$$q_3 = \frac{J_3 - J_1}{(A_3F_{31})^{-1}} + \frac{J_3 - J_2}{(A_3F_{32})^{-1}} = A_3\left[F_{31}(J_3 - J_1) + F_{32}(J_3 - J_2)\right]. \quad (6)$$

The view factor F_{32} can be evaluated from the *coaxial parallel disks* relation of Table 13.1 or from Fig. 13.5.

$$R_i = r_i/L = 200\ mm/200\ mm = 1,$$

$$R_j = r_j/L = 1,$$

$$S = 1 + \left(1 + R_j^2\right)/R_j^2 = 1 + \left(1 + 1^2\right)1^2 = 3$$

$$F_{31} = 1/2\left\{S - \left[S^2 - 4\left(r_j/r_i\right)^2\right]^{1/2}\right\} = 1/2\left\{3 - \left[3^2 - 4(1)^2\right]^{1/2}\right\} = 0.382. \quad (7)$$

From summation rule, $F_{32} = 1 - F_{33} - F_{31} = 0.618$ with $F_{33} = 0$. Since surfaces A_2 and A_3 are black,

$$J_2 = E_{b2} = \sigma T_2^4 = \sigma\,(500\ K)^4 = 3544\ W/m^2$$

$$J_3 = E_{b3} = \sigma T_3^4 = \sigma\,(400\ K)^4 = 1452\ W/m^2.$$

To determine J_1, use Eq. 13.21, the radiation balance equation for A_1, noting that $F_{12} = F_{32}$ and $F_{13} = F_{31}$,

$$\frac{E_{b1} - J_1}{(1-\varepsilon_1)/\varepsilon_1 A_1} = \frac{J_1 - J_2}{(A_1F_{12})^{-1}} + \frac{J_1 - J_3}{(A_1F_{13})^{-1}}$$

$$\frac{3544 - J_1}{(1-0.6)/0.6} = \frac{J_1 - 3544}{(0.618)^{-1}} + \frac{J_1 - 1452}{(0.382)^{-1}} \qquad J_1 = 3226\ W/m^2. \quad (8)$$

Substituting numerical values in Eq. (6), find

$$q_3 = (\pi/4)(0.400\,m)^2\left[0.382(1452 - 3226)W/m^2 + 0.618(1452 - 3544)W/m^2\right] = -248\ W.$$

Returning to the overall energy balance, Eq. (1), the net heat transfer to the disk is

$$q_{net,d} = -99.8\ W - 93.6\,W - (-248\,W) = +54.6\ W \quad <$$

That is, there is a net heat transfer rate *into* the disk.

(b) Using the energy balance, Eq. (1), and the rate equation, Eqs. (2) and (3) with the *IHT Radiation Tool, Radiation, Exchange Analysis, Radiation surface energy balances* and the *Correlation Tool, Free Convection, Horizontal Plate (Hot surface up)*, the analysis was performed to obtain $q_{net,d}$ as a function of T_d. The results are plotted below.

The steady-state condition occurs when $q_{net,d} = 0$ for which

$$T_d = 413\ K \quad <$$

Continued …..

PROBLEM 13.114 (Cont.)

Net heat rate, qnet (W) vs. Disk temperature, Td (K)

COMMENTS: The *IHT* workspace for the foregoing analysis is shown below.

```
// Radiation Tool - Three Surface Enclosure, Furnace Disk Enclosure:
/* The net heat rate leaving A1 in terms of the surface resistance is */
q1 = (Eb1 - J1) / ((1 - eps1) / (eps1 * A1))      // Eq 13.19
/* The net heat rate leaving A1 in terms of the net exchanges between enclosure surfaces is */
q1 = q12 + q13
/* where the net exchange rates expressed in terms of the space resistances are, Eq 13.20 and 13.22, */
q12 = (J1 - J2) / (1 / (A1 * F12))
q13 = (J1 - J3) / (1 / (A1 * F13))
/* The net heat rate leaving A2 in terms of the surface resistance is */
q2 = (Eb2 - J2) / ((1 - eps2) / (eps2 * A2))      // Eq 13.19
/* The net heat rate leaving A2 in terms of the net exchanges between enclosure surfaces is */
q2 = q21 + q23
/* where the net exchange rates expressed in terms of the space resistances are, Eq 13.20 and 13.22, */
q21 = (J2 - J1) / (1 / (A2 * F21))
q23 = (J2 - J3) / (1 / (A2 * F23))
/* The net heat rate leaving A3 in terms of the surface resistance is */
q3 = (Eb3 - J3) / ((1 - eps3) / (eps3 * A3))      // Eq 13.19
/* The net heat rate leaving A3 in terms of the net exchanges between enclosure surfaces is */
q3 = q31 + q32
/* where the net exchange rates expressed in terms of the space resistances are, Eq 13.20 and 13.22, */
q31 = (J3 - J1) / (1 / (A3 * F31))
q32 = (J3 - J2) / (1 / (A3 * F32))

// Emissive Powers:
Eb1 = sigma * T1^4
Eb2 = sigma * T2^4
Eb3 = sigma * T3^4
sigma = 5.67e-8              // Stefan-Boltzmann constant, W/m^2.K^4

// Radiation Tool - View Factor:
/* The view factor, F12, for coaxial parallel disks, is */
F13 = 0.5 * (S - sqrt(S^2 - 4*(r3 / r1)^2))
// where
R1 = r1 / L
R3 = r3 / L
S = 1 + (1 + R3^2) / R1^2
// See Table 13.2 for schematic of this three-dimensional geometry.

// Other View Factors and Areas Required:
F12 = 1 - F13                // Summation rule, A1
F21 = A1 * F12 / A2          // Reciprocity rule
F23 = F21                    // Symmetry condition
F31 = F13                    // Symmetry condition
F32 = F12                    // Symmetry condition
A1 = pi * r1^2               // Surface area, m^2
A2 = pi * r1 * L             // Surface area, m^2
A3 = pi * r3^2               // Surface area, m^2

// Overall plate energy balance, Eqs (1,2,3):
qnet = - qrad - qcv - q3
qrad = eps32 * A3 * sigma * (T3^4 - Tsur^4)
qcv = hLbar * A3 * (T3 - Tinf)
```

Continued …..

PROBLEM 13.114 (Cont.)

```
// Convection Tool - Free Convection, Flat Plate:
// Hot Surface Up (HSU) or Cold Surface Down (CSD)
NuLbar3 = NuL_bar_FC_HP_HSU(RaL3)    // Eq 9.30 or 31
NuLbar3 = hLbar * L3 / k3
RaL3 = g * beta3 * deltaT3 * L3^3 / (nu3 * alpha3)     //Eq 9.25
deltaT3 = abs(T3 - Tinf)
g = 9.8                          // gravitational constant, m/s^2
// Evaluate properties at the film temperature, Tf1.
Tf = Tfluid_avg(Tinf,T3)
// The characteristic length, surface area and perimeter are
L3 = As3 / P3                    // Eq 9.29
As3 =pi * r3^2
P3 = pi * r3
// Properties Tool - Air
// Air property functions : From Table A.4
// Units: T(K); 1 atm pressure
nu3 = nu_T("Air",Tf)             // Kinematic viscosity, m^2/s
k3 = k_T("Air",Tf)               // Thermal conductivity, W/m·K
alpha3 = alpha_T("Air",Tf)       // Thermal diffusivity, m^2/s
Pr3 = Pr_T("Air",Tf)             // Prandtl number
beta3 = 1/Tf                     // Volumetric coefficient of expansion, K^(-1); ideal gas

// Assigned Variables
r1 = 0.2                         // Radius, m
r3 = 0.2                         // Radius, m
L = 0.2                          // Separation distance, m
T1 = 500                         // Temperature, K
eps1 = 0.6                       // Emissivity
T2 = 500                         // Temperature, K
eps2 = 0.999                     // Emissivity; avoiding 'division by zero error'
T3 = 400                         // Temperature, K
eps32 = 0.8                      // Emissivity; upper surface
eps3 = 0.999                     // Emissivity; lower surface, enclosure side
Tinf = 300                       // Ambient air temperature, K
Tsur = 300                       // Surroundings temperature, K
```

PROBLEM 13.115

KNOWN: Radiation shield facing hot wall at T_w = 400 K is backed by an insulating material of known thermal conductivity and thickness which is exposed to ambient air and surroundings at 300 K.

FIND: (a) Heat loss per unit area from the hot wall, (b) Radiosity of the shield, and (c) Perform a parameter sensitivity analysis on the insulation system considering effects of shield reflectivity ρ_s, insulation thermal conductivity k, overall coefficient h, on the heat loss from the hot wall.

SCHEMATIC:

ASSUMPTIONS: (1) Wall is black surface of uniform temperature, (2) Shield and wall behave as parallel infinite plates, (3) Negligible convection in region between shield and wall, (4) Shield is diffuse-gray and very thin, (5) Prescribed coefficient h = 10 $W/m^2 \cdot K$ is for convection and radiation.

ANALYSIS: (a) Perform an energy balance on the shield to obtain

$$q''_{w-s} = q''_{cond}$$

But the insulating material and the convection process at the exposed surface can be represented by a thermal circuit.

In equation form, using Eq.13.24 for the wall and shield,

$$q''_{w-s} = \frac{\sigma\left(T_w^4 - T_s^4\right)}{1/\varepsilon_w + 1/\varepsilon_s - 1} = \frac{T_s - T_\infty}{L/k + 1/h} \qquad (1,2)$$

$$\frac{\sigma\left(400^4 - T_s^4\right)}{1 + 1/0.05 - 1} = \frac{\left(T_s - 300\right)K}{\left(0.025/0.016 + 1/10\right)m^2 \cdot K/W}$$

$$T_s = 350 \text{ K}.$$

where $\varepsilon_s = 1 - \rho_s$. Hence,

$$q''_{w-s} = \frac{\left(350 - 300\right)K}{\left(0.025/0.016 + 1/10\right)m^2 \cdot K/W} = 30 \text{ W/m}^2.$$ <

(b) The radiosity of the shield follows form the definition,

$$J_s = \rho_s G_s + \varepsilon_s E_b\left(T_s\right) = \rho_s\left(\sigma T_w^4\right) + \left(1 - \rho_s\right)\left(\sigma T_s^4\right). \qquad (3)$$

$$J_s = 0.95\sigma\left(400 \text{ K}\right)^4 + \left(1 - 0.95\right)\sigma\left(350 \text{ K}\right)^4 = 1421 \text{ W/m}^2.$$ <

with $\sigma = 5.67 \times 10^{-8}$ $W/m^2 \cdot K^4$.

Continued …..

PROBLEM 13.115 (Cont.)

(c) Using the Eqs. (1) and (2) in the *IHT* workspace, q''_{w-s} can be computed and plotted for selected ranges of the insulation system variables, ρ_s, k, and h. Intuitively we know that q''_{w-s} will decrease with increasing ρ_s, decreasing k and decreasing h. We chose to generate the following family of curves plotting q''_{w-s} vs. k for selected values of ρ_s and h.

Heat loss, q''w-s (W)

Thermal conductivity, k (W/m.K)

Base case: rhos = 0.95, h = 10 W/m^2.K
rhos = 0.98
rhos = 0.90
h = 5 W/m^2.K
h = 15 W/m^2.K

Considering the base condition with variable k, reducing k by a factor of 3, the heat loss is reduced by a factor of 2. The effect of changing h (4 to 24 $W/m^2\cdot K$) has little influence on the heat loss. However, the effect of shield reflectivity change is very significant. With $\rho_s = 0.98$, probably the upper limit of a practical reflector-type shield, the heat loss is reduced by a factor of two. To improve the performance of the insulation system, it is most advantageous to increase ρ_s and decrease k.

PROBLEM 13.116

KNOWN: Diameter and surface temperature of a fire tube. Gas low rate and temperature. Emissivity of tube and partition.

FIND: (a) Heat transfer per unit tube length, q', without the partition, (b) Partition temperature, T_p, and heat rate with the partition, (c) Effect of flow rate and emissivity on q' and T_p. Effect of emissivity on radiative and convective contributions to q'.

SCHEMATIC:

$T_{m,g} = 900$ K
$\dot{m}_g = 0.02, 0.05, 0.08$ kg/s
$T_s = 385$ K
$0.1 \leq \varepsilon_s \leq 1.0$
q''_{rad}
q''_{conv}
Partition, T_p, $\varepsilon_p = \varepsilon_s$
$D = 0.07$ m

ASSUMPTIONS: (1) Fully-developed flow in duct, (2) Diffuse/gray surface behavior, (3) Negligible gas radiation.

PROPERTIES: *Table A-4*, air ($T_{m,g}$ = 900 K): $\mu = 398 \times 10^{-7}$ N·s/m^2, k = 0.062 W/m·K, Pr = 0.72; air (T_s = 385 K): $\mu = 224 \times 10^{-7}$ N·s/m^2.

ANALYSIS: (a) Without the partition, heat transfer to the tube wall is only by convection. With $\dot{m}$ = 0.05 kg/s and $Re_D = 4\,\dot{m}_g/\pi D\mu = 4(0.05\ \text{kg/s})/\pi(0.07\ \text{m})398\times10^{-7}\ \text{N}\cdot\text{s/m}^2 = 22{,}850$, the flow is turbulent. From Eq. (8.61),

$$Nu_D = 0.027\,Re_D^{4/5}\,Pr^{1/3}\left(\mu/\mu_s\right)^{0.14} = 0.027(22{,}850)^{4/5}(0.72)^{1/3}(398/224)^{0.14} = 80.5$$

$$h = \frac{k}{D}Nu_D = \frac{0.062\ \text{W/m}\cdot\text{K}}{0.07\ \text{m}}80.5 = 71.3\ \text{W/m}^2\cdot\text{K}$$

$$q' = h\pi D\left(T_{m,g} - T_s\right) = 71.3\,\text{W/m}^2\cdot\text{K}(\pi)0.07\ \text{m}(900-385) = 8075\ \text{W/m}$$ <

(b) The temperature of the partition is determined from an energy balance which equates net radiation exchange with the tube wall to convection from the gas. Hence, $q''_{rad} = q''_{conv}$, where from Eq. 13.23,

$$q''_{rad} = \frac{\sigma\left(T_p^4 - T_s^4\right)}{\dfrac{1-\varepsilon_p}{\varepsilon_p} + \dfrac{1}{F_{ps}} + \dfrac{1-\varepsilon_s}{\varepsilon_s}\dfrac{A_p}{A_s}}$$

where $F_{12} = 1$ and $A_p/A_s = D/(\pi D/2) = 2/\pi = 0.637$. The flow is now in a noncircular duct for which $D_h = 4A_c/P = 4(\pi D^2/8)/(\pi D/2+D) = \pi D/(\pi + 2) = 0.611\,D = 0.0428$ m and $\dot{m}_{1/2} = \dot{m}_g/2 = 0.025$ kg/s. Hence, $Re_D = \dot{m}_{1/2}D_h/A_c\mu = \dot{m}_{1/2}D_h/(\pi D^2/8)\mu = 8(0.025\ \text{kg/s})(0.0428\ \text{m})/\pi(0.07\ \text{m})^2\,398 \times 10^{-7}\ \text{N}\cdot\text{s/m}^2 = 13{,}970$ and

$$Nu_D = 0.027(13{,}970)^{4/5}(0.72)^{1/3}(398/224)^{0.14} = 54.3$$

$$h = \frac{k}{D_h}Nu_D = \frac{0.062\ \text{W/m}\cdot\text{K}}{0.0428\ \text{m}}54.3 = 78.7\ \text{W/m}^2\cdot\text{K}$$

Continued …..

Hence, with $\varepsilon_s = \varepsilon_p = 0.5$ and $q''_{conv} = h\left(T_{m,g} - T_p\right)$,

$$\frac{5.67\times10^{-8}\,W/m^2\cdot K^4\left(T_p^4 - 385^4\right)K^4}{1+1+0.637} = 78.7\ W/m^2\cdot K\left(900 - T_p\right)K$$

$$21.5\times10^{-8}T_p^4 + 78.7T_p - 71,302 = 0$$

A trial-and-error solution yields

$$T_p = 796\ K$$ <

The heat rate to one-half of the tube is then

$$q'_{1/2} = q'_{ps} + q'_{conv} = \frac{D\sigma\left(T_p^4 - T_s^4\right)}{\frac{1-\varepsilon_p}{\varepsilon_p} + \frac{1}{F_{ps}} + \frac{1-\varepsilon_s}{\varepsilon_s}\frac{A_p}{A_s}} + h\left(\pi D/2\right)\left(T_{m,g} - T_s\right)$$

$$q'_{1/2} = \frac{0.07\ m\left(5.67\times10^{-8}\,W/m^2\cdot K^4\right)\left(796.4^4 - 385^4\right)K^4}{2.637} + 78.7\ W/m^2\cdot K\left(0.110\ m\right)\left(900-385\right)K$$

$$q'_{1/2} = 572\ W/m + 4458\ W/m = 5030\ W/m$$

The heat rate for the entire tube is

$$q' = 2q'_{1/2} = 10,060\ W/m$$ <

(c) The foregoing model was entered into the *IHT* workspace, and parametric calculations were performed to obtain the following results.

Radiation transfer from the partition increases with increasing $\varepsilon_p = \varepsilon_s$, thereby reducing T_p while increasing q'. Since h increases with increasing $\dot{m}$, T_p and q' also increase with $\dot{m}$.

Continued …..

PROBLEM 13.116 (Cont.)

Although the radiative contribution to the heat rate increases with increasing $\varepsilon_p = \varepsilon_s$, it still remains small relative to convection.

COMMENTS: Contrasting the heat rate predicted for part (b) with that for part (a), it is clear that use of the partition enhances heat transfer to the tube. However, the effect is due primarily to an increase in h and secondarily to the addition of radiation.

PROBLEM 13.117

KNOWN: Height and width of a two-dimensional cavity filled with helium. Temperatures and emissivities of opposing vertical plates.

FIND: (a) Heat rate per unit length, (b) Effect of L on heat rate.

SCHEMATIC:

Helium, p = 1 atm

H = 160 mm

T_1 = 75 °C, ε_1 = 0.8

T_2 = 15 °C, ε_2 = 0.8

L = 20 mm, $A = H/L$ = 8

ASSUMPTIONS: (1) Steady-state conditions, (2) Isothermal plates, (3) Diffuse-gray surfaces, (4) Reradiating cavity sidewalls.

PROPERTIES: *Table A-4*, Helium (T = 318 K, 1 atm): $\nu = 136 \times 10^{-6}$ m^2/s, k = 0.158 W/m·K, $\alpha = 201 \times 10^{-6}$ m^2/s, Pr = 0.679, $\beta = 0.00314$ K^{-1}.

ANALYSIS: (a) The power generated by the electronics leaving the surface A_1 is $q' = q'_{conv} + q'_{rad}$, or

$$q' = \bar{h}H(T_1 - T_2) + \frac{H\sigma\left(T_1^4 - T_2^4\right)}{\dfrac{1-\varepsilon_1}{\varepsilon_1} + \dfrac{1}{F_{12} + \left[(1/F_{1R}) + (1/F_{2R})\right]^{-1}} + \dfrac{1-\varepsilon_2}{\varepsilon_2}}$$

The free convection coefficient can be estimated using Eq. 9.50 with

$$Ra_L = \frac{g\beta(T_1 - T_2)L^3}{\alpha\nu} = \frac{9.8\,\text{m/s}^2\left(0.00314\ \text{K}^{-1}\right)60\ \text{K}\,(0.02\,\text{m})^3}{201\times10^{-6}\times136\times10^{-6}\,\text{m}^4/\text{s}^2} = 540$$

However, since $Ra_L < 1000$, free convection effects can be neglected, in which case heat transfer is by conduction and $\overline{Nu}_L = 1$. Hence,

$$\bar{h} = \overline{Nu}_L\,(k/L) = 1.0\left(0.158\ \text{W/m}\cdot\text{K}/0.02\text{m}\right) = 7.9\ \text{W/m}^2\cdot\text{K}.$$

The view factor can be found from Fig. 13.4 with X/L = 8 and Y/L = ∞. Hence, $F_{12} = 0.9$ and $F_{1R} = F_{2R} = 0.1$. It follows that

$$q' = 7.9\ \text{W/m}^2\cdot\text{K}\,(0.16\,\text{m})(60°\text{C}) + \frac{0.16\,\text{m}\times5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}\left(348^4 - 288^4\right)\text{K}^4}{0.25 + \dfrac{1}{0.9 + \left[10 + 10\right]^{-1}} + 0.25}$$

$$q' = 75.8\ \text{W/m} + 45.5\ \text{W/m} = 121\ \text{W/m}.$$ <

Continued …..

PROBLEM 13.117 (Cont.)

(b) To assess the effect of plate spacing on convection heat transfer, $q'_{conv} = \bar{h}H(T_1 - T_2)$ was computed by using both Eq. 9.50 and the conduction limit $(\overline{Nu}_L = 1)$ to determine $\bar{h}$. These expressions were entered into the *IHT* workspace, and the *Radiation* Toolpad was used to obtain the appropriate radiation rate equation and view factor.

The cross-over at L = 28 mm marks the plate spacing below and above which, respectively, the conduction limit and Eq. 9.50 are applicable. Although there is a slight increase in q'_{conv} with increasing L for L > 28 mm, the increase pales by comparison with that corresponding to a reduction in L for L < 28 mm. As the two plates are brought closer to each other in the conduction limit, the reduction in the corresponding thermal resistance significantly increases the heat rate. The total heat rate and the conduction and radiative components are also plotted for $5 \le L \le 25$ mm. There is an increase in q'_{rad} with decreasing L, due to an increase in F_{12} ($F_{12} = 0.97$ for L = 5 mm). However, the increase is small, and conduction is the dominant heat transfer mode.

COMMENTS: Even for small values of L, the total heat rate is small and the scheme is poorly suited for electronic cooling. Note that helium is preferred over air on the basis of its larger thermal conductivity.

PROBLEM 13.118

KNOWN: Diameters, temperatures, and emissivities of concentric spheres.

FIND: Rate at which nitrogen is vented from the inner sphere. Effect of radiative properties on evaporation rate.

SCHEMATIC:

D_o =1.10 m, D_i =1 m, $\dot{m}$, Helium, p = 1 atm, Liquid nitrogen, T_o = 283 K, ε_o = 0.3, T_i = 77 K, ε_i = 0.3, q

ASSUMPTIONS: Diffuse-gray surfaces.

PROPERTIES: Liquid nitrogen (given): $h_{fg} = 2 \times 10^5$ J/kg; *Table A-4*, Helium ($\bar{T} = (T_i + T_o)/2 =$ 180 K, 1 atm): $\nu = 51.3 \times 10^{-6}$ m²/s, k = 0.107 W/m·K, $\alpha = 76.2 \times 10^{-6}$ m²/s, Pr = 0.673, $\beta =$ 0.00556 K⁻¹.

ANALYSIS: (a) Performing an energy balance for a control surface about the liquid nitrogen, it follows that $q = q_{conv} + q_{rad} = \dot{m}h_{fg}$. From the Raithby-Hollands expressions for free convection between concentric spheres, $q_{conv} = k_{eff}\pi(D_i D_o/L)(T_o - T_i)$, where

$$k_{eff} = 0.74\,k\left(\frac{Pr}{0.861+Pr}\right)^{1/4}\left[\frac{L}{(D_oD_i)^4\left(D_i^{-7/5}+D_o^{-7/5}\right)^5}\right]^{1/4}$$

$$Ra_L = \frac{g\beta(T_o - T_i)L^3}{\nu\alpha} = \frac{9.8\,m/s^2\left(0.00556\ K^{-1}\right)(206\ K)(0.05m)^3}{\left(51.3\times10^{-6}m^2/s\right)\left(76.2\times10^{-6}m^2/s\right)} = 3.589\times10^5.$$

Hence,

$$k_{eff} = 0.74(0.107\,W/m\cdot K)\left(\frac{0.673}{0.861+0.673}\right)^{1/4}\left[\frac{0.05}{(1.1)^4}\frac{3.589\times10^5}{(1+0.875)^5}\right]^{1/4} = 0.309\ W/m\cdot K.$$

The heat rate by convection is

$$q_{conv} = (0.309\ W/m\cdot K)\pi\left(1.10\,m^2/0.05\,m\right)206\ K = 4399\ W.$$

From Table 13.3,

$$q_{rad} = q_{oi} = \frac{\sigma\pi D_1^2\left(T_o^4 - T_i 4\right)}{1/\varepsilon_i + ((1-\varepsilon_o)/\varepsilon_o)(D_i/D_o)^2}$$

$$= \frac{\left(5.67\times10^{-8}W/m^2\cdot K^4\right)\pi(1\ m)^2\left(283^4 - 77^4\right)K^4}{1/0.3+(0.7/0.3)(1/1.1)^2} = 216\ W.$$

Continued …..

PROBLEM 13.118 (Cont.)

Hence, $\dot{m} = q / h_{fg} = (4399 + 216)\,W / 2\times10^5\,J/kg = 0.023\,kg/s.$ <

With the cavity evacuated, *IHT* was used to compute the radiation heat rate as a function of $\varepsilon_i = \varepsilon_o$.

Clearly, significant advantage is associated with reducing the emissivities and q_{rad} = 31.8 W for $\varepsilon_i = \varepsilon_o = 0.05$.

COMMENTS: The convection heat rate is too large. It could be reduced by replacing He with a gas of smaller k, a cryogenic insulator (Table A.3), or a vacuum. Radiation effects are second order for small values of the emissivity.

PROBLEM 13.119

KNOWN: Dimensions, emissivity and upper temperature limit of coated panel. Arrangement and power dissipation of a radiant heater. Temperature of surroundings.

FIND: (a) Minimum panel-heater separation, neglecting convection, (b) Minimum panel-heater separation, including convection.

SCHEMATIC:

ASSUMPTIONS: (1) Top and bottom surfaces of heater and panel, respectively, are adiabatic, (2) Bottom and top surfaces of heater and panel, respectively are diffuse-gray, (3) Surroundings form a large enclosure about the heater-panel arrangement, (4) Steady-state conditions, (5) Heater power is dissipated entirely as radiation (negligible convection), (6) Air is quiescent and convection from panel may be approximated as free convection from a horizontal surface, (7) Air is at atmospheric pressure.

PROPERTIES: *Table A-4*, Air (T_f = (400 + 298)/2 ≈ 350 K, 1 atm): $\nu = 20.9 \times 10^{-6}$ m²/s, k = 0.03 W/m·K, Pr = 0.700, $\alpha = 29.9 \times 10^{-6}$ m²/s, $\beta = 2.86 \times 10^{-3}$ K⁻¹.

ANALYSIS: (a) Neglecting convection effects, the panel constitutes a floating potential for which the net radiative transfer must be zero. That is, the panel behaves as a re-radiating surface for which $E_{b2} = J_2$. Hence

$$q_1 = \frac{J_1 - E_{b2}}{1/A_1F_{12}} + \frac{J_1 - E_{b3}}{1/A_1F_{13}} \tag{1}$$

and evaluating terms

$$E_{b2} = \sigma T_2^4 = 5.67\times10^{-8}\,\mathrm{W/m^2\cdot K^4}\,(400\ \mathrm{K})^4 = 1452\ \mathrm{W/m^2}$$

$$E_{b3} = \sigma T_3^4 = 5.67\times10^{-8}\,\mathrm{W/m^2\cdot K^4}\,(298\ \mathrm{K})^4 = 447\ \mathrm{W/m^2}$$

$$F_{13} = 1 - F_{12} \qquad\qquad A_1 = 25\ \mathrm{m^2}$$

find that

$$\frac{75,000\ \mathrm{W}}{25\ \mathrm{m^2}} = \frac{J_1 - 1452}{1/F_{12}} + \frac{J_1 - 447}{1/(1-F_{12})}$$

$$3000\ \mathrm{W/m^2} = F_{12}(J_1 - 1452) + (J_1 - 447) - F_{12}(J_1 - 447)$$

$$J_1 = 3447 + 1005F_{12}. \tag{2}$$

Performing a radiation balance on the panel yields

$$\frac{J_1 - E_{b2}}{1/A_1F_{12}} = \frac{E_{b2} - E_{b3}}{1/A_2F_{23}}.$$

Continued …..

PROBLEM 13.119 (Cont.)

With $A_1 = A_2$ and $F_{23} = 1 - F_{12}$

$$F_1\left(J_1 - 1452\right) = \left(1 - F_{12}\right)\left(1452 - 447\right)$$

or

$$447F_{12} = F_{12}J_1 - 1005. \qquad (3)$$

Substituting for J_1 from Eq. (2), find

$$447F_{12} = F_{12}\left(3447 + 1005F_{12}\right) - 1005$$

$$1005F_{12}^2 + 3000F_{12} - 1005 = 0$$

$$F_{12} = 0.30.$$

Hence from Fig. 13.4, with X/L = Y/L and $F_{ij} = 0.3$,

$$X/L \approx 1.45$$

$$L \approx 5\text{ m}/1.45 = 3.45\text{ m}.$$

<

(b) Accounting for convection from the panel, the net radiation transfer is no longer zero at this surface and $E_{b2} \neq J_2$. It then follows that

$$q_1 = \frac{J_1 - J_2}{1/A_1F_{12}} + \frac{J_1 - E_{b3}}{1/A_1F_{13}} \qquad (4)$$

where, from an energy balance on the panel,

$$\frac{J_2 - E_{b2}}{\left(1-\varepsilon_2\right)/\varepsilon_2 A_2} = q_{conv,2} = \bar{h}A_2\left(T_2 - T_\infty\right). \qquad (5)$$

With $L \equiv A_s/P = 25\text{ m}^2/20\text{ m} = 1.25\text{ m}$,

$$Ra_L = \frac{g\beta\left(T_s - T_\infty\right)L^3}{\nu\alpha} = \frac{9.8\text{ m/s}^2\left(2.86\times10^{-3}\text{K}^{-1}\right)\left(102\text{ K}\right)\left(1.25\text{ m}\right)^3}{\left(20.9\times29.9\right)10^{-12}\text{ m}^4/\text{s}^2} = 8.94\times10^9.$$

Hence

$$\overline{Nu}_L = 0.15Ra_L^{1/3} = 0.15\left(8.94\times10^9\right)^{1/3} = 311$$

$$\bar{h} = 311\,k/L = 311\frac{0.03\text{ W/m}\cdot\text{K}}{1.25\text{ m}} = 7.46\text{ W/m}^2\cdot\text{K}$$

$$q''_{conv,2} = 7.46\text{ W/m}^2\cdot\text{K}\left(102\text{ K}\right) = 761\text{ W/m}^2.$$

From Eq. (5)

$$J_2 = E_{b2} + \frac{1-\varepsilon_2}{\varepsilon_2}q''_{conv,2} = 1452 + \frac{0.7}{0.3}761 = 3228\text{ W/m}^2.$$

Continued …..

PROBLEM 13.119 (Cont.)

From Eq. (4),

$$\frac{75,000}{25} = \frac{J_1 - 3228}{1/F_{12}} + \frac{J_1 - 447}{1/(1-F_{12})}$$

$$3000 = F_{12}(J_1 - 3228) + J_1 - 447 - F_{12}(J_1 - 447)$$

$$J_1 = 3447 + 2781F_{12}. \qquad (6)$$

From an energy balance on the panel,

$$\frac{J_1 - J_2}{1/A_1F_{12}} + \frac{E_{b3} - J_2}{1/A_2F_{23}} = \frac{J_2 - E_{b2}}{(1-\varepsilon_2)/\varepsilon_2 A_2} = q_{conv,2}$$

$$F_{12}(J_1 - 3228) + (1 - F_{12})(447 - 3228) = 761$$

$$F_{12}J_1 - 447F_{12} = 3542.$$

Substituting from Eq. (6),

$$F_{12}(3447 + 2781F_{12}) - 447F_{12} = 3542$$

$$2781F_{12}^2 + 3000F_{12} - 3542 = 0$$

$$F_{12} = 0.71.$$

Hence from Fig. 13.4, with X/L = Y/L and F_{ij} = 0.71,

$$X/L = 5.7$$

$$L \approx 5\text{ m}/5.7 = 0.88\text{ m}.$$ <

COMMENTS: (1) The results are independent of the heater surface radiative properties.

(2) Convection at the heater surface would reduce the heat rate q_1 available for radiation exchange and hence reduce the value of L.

PROBLEM 13.120

KNOWN: Diameter and emissivity of rod heater. Diameter and position of reflector. Width, emissivity, temperature and position of coated panel. Temperature of air and large surroundings.

FIND: (a) Equivalent thermal circuit, (b) System of equations for determining heater and reflector temperatures. Values of temperatures for prescribed conditions, (c) Electrical power needed to operate heater.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Diffuse-gray surfaces, (3) Large surroundings act as blackbody, (4) Surfaces are infinitely long (negligible end effects), (5) Air is quiescent, (6) Negligible convection at reflector, (7) Reflector and panel are perfectly insulated.

PROPERTIES: *Table A-4*, Air (T_f = 350 K, 1 atm): k = 0.03 W/m·K, $\nu = 20.9 \times 10^{-6}$ m^2/s, $\alpha = 29.9 \times 10^{-6}$ m^2/s, Pr = 0.70; (T_f = (1295 + 300)/2 = 800 K): k = 0.0573 W/m·K, $\nu = 84.9 \times 10^{-6}$ m^2/s, $\alpha = 120 \times 10^{-6}$ m^2/s.

ANALYSIS: (a) We have assumed blackbody behavior for A_1 and A_4; hence, $J = E_b$. Also, A_2 is insulated and has negligible convection; hence q = 0 and $J_2 = E_{b2}$. The equivalent thermal circuit is:

(b) Performing surface energy balances at 1, 2 and 3:

$$q_1 - q_{conv,1} = \frac{E_{b1} - E_{b2}}{1/A_1F_{12}} + \frac{E_{b1} - J_3}{1/A_1F_{13}} + \frac{E_{b1} - E_{b4}}{1/A_1F_{14}} \quad (1)$$

$$0 = \frac{E_{b1} - E_{b2}}{1/A_2F_{21}} + \frac{J_3 - E_{b2}}{1/A_2F_{23}} + \frac{E_{b4} - E_{b2}}{1/A_2F_{24}} \quad (2)$$

$$\frac{J_3 - E_{b3}}{(1-\varepsilon_3)/\varepsilon_3 A_3} = \frac{E_{b1} - J_3}{1/A_3F_{31}} + \frac{E_{b2} - J_3}{1/A_3F_{32}} + \frac{E_{b4} - J_3}{1/A_3F_{34}} \quad (3a)$$

where

$$\frac{J_3 - E_{b3}}{(1-\varepsilon_3)/\varepsilon_3 A_3} = q_{conv,3}. \quad (3b)$$

Continued …..

PROBLEM 13.120 (Cont.)

Solution procedure with E_{b3} and E_{b4} known: Evaluate $q_{conv,3}$ and use Eq. (3b) to obtain J_3; Solve Eqs. (2) and (3a) simultaneously for E_{b1} and E_{b2} and hence T_1 and T_2; Evaluate $q_{conv,1}$ and use Eq. (1) to obtain q_1.

For *free convection* from a heated, horizontal plate:

$$L_c = \frac{A_s}{P} = \frac{(W \times L)}{(2L + 2W)} \approx \frac{W}{2} = 0.5 \text{ m}$$

$$Ra_L = \frac{g\beta(T_3 - T_\infty)L_c^3}{\alpha\nu} = \frac{9.8 \text{ m/s}^2 (350 \text{ K})^{-1}(100 \text{ K})(0.5 \text{ m})^3}{20.9 \times 29.9 \times 10^{-12} \text{m}^4/\text{s}^2} = 5.6 \times 10^8$$

$$\overline{Nu}_L = 0.15 Ra_L^{1/3} = 0.15\left(5.6 \times 10^8\right)^{1/3} = 123.6$$

$$\bar{h}_3 = \frac{k}{L_c}\overline{Nu}_L = \frac{0.03 \text{ W/m}\cdot\text{K} \times 123.6}{0.5 \text{ m}} = 7.42 \text{ W/m}^2\cdot\text{K}.$$

$$q''_{conv,3} = \bar{h}_3(T_3 - T_\infty) = 742 \text{ W/m}^2.$$

Hence, with

$$E_{b3} = \sigma T_3^4 = 5.67 \times 10^{-8} \text{W/m}^2\cdot\text{K}^4 (400 \text{ K})^4 = 1451 \text{ W/m}^2$$

using Eq. (3b) find

$$J_3 = E_{b3} + \frac{1-\varepsilon_3}{\varepsilon_3 A_3} q_{conv,3} = (1451 + [0.3/0.7]742) = 1769 \text{ W/m}^2.$$

View Factors: From symmetry, it follows that $F_{12} = 0.5$. With $\theta = \tan^{-1}(W/2)/H = \tan^{-1}(0.5) = 26.57°$, it follows that

$$F_{13} = 2\theta/360 = 0.148.$$

From summation and reciprocity relations,

$$F_{14} = 1 - F_{12} - F_{13} = 0.352$$

$$F_{21} = (A_1/A_2)F_{12} = (2D_1/D_2)F_{12} = 0.02 \times 0.5 = 0.01$$

$$F_{23} = (A_3/A_2)F_{32} = (2/\pi)(F_{32'} - F_{31}).$$

For X/L = 1, Y/L ≈ ∞, find from Fig. 13.4 that $F_{32'} \approx 0.42$. Also find,

$$F_{31} = (A_1/A_3)F_{13} = (\pi \times 0.01/1)0.148 = 0.00465 \approx 0.005$$

$$F_{23} = (2/\pi)(0.42 - 0.005) = 0.264$$

$$F_{22} \approx 1 - F_{22'} = 1 - (A'_2/A_2)F_{2'2} = 1 - (2/\pi) = 0.363$$

$$F_{24} = 1 - F_{21} - F_{22} - F_{23} = 0.363$$

Continued …..

$F_{31} = 0.005, \qquad F_{32} = 0.415$

$F_{34} = 1 - F_{32'} = 1 - 0.42 = 0.58.$

With $E_{b4} = \sigma T_4^4 = 5.67 \times 10^{-8}\ W/m^2 \cdot K^4 (300\ K)^4 = 459\ W/m^2$,

Eq. (3a) $\rightarrow$ $0.005(E_{b1} - 1769) + 0.415(E_{b2} - 1769) + 0.58(459 - 1769) = 742$

$$0.005E_{b1} + 0.415Eb2 = 2245 \qquad (4)$$

Eq. (2) $\rightarrow$ $0.01(E_{b1} - E_{b2}) + 0.264(1769 - E_{b2}) + 0.363(459 - E_{b2}) = 0$

$$0.01E_{b1} - 0.637E_{b2} + 633.6 = 0. \qquad (5)$$

Hence, manipulating Eqs. (4) and (5), find

$$E_{b2} = 0.0157E_{b1} + 994.7$$

$$0.005E_{b1} + (0.415)(0.0157E_{b1} + 994.7) = 2245.$$

$$E_{b1} = 159,322\ W/m^2 \qquad T_1 = (E_{b1}/\sigma)^{1/4} = 1295\ K$$ <

$$E_{b2} = 0.0157(159,322) + 994.7 = 3496\ W/m^2 \qquad T_2 = (E_{b2}/\sigma)^{1/4} = 498\ K.$$ <

(c) With T_1 = 1295 K, then T_f = (1295 + 300)/2 ≈ 800 K, and using

$$Ra_D = \frac{g\beta(T_1 - T_\infty)D_1^3}{\alpha\nu} = \frac{9.8\ m/s^2 (1/800\ K)(1295 - 300)K(0.01\ m)^3}{120 \times 84.9 \times 10^{-12} m^4/s^2} = 1196$$

$$\overline{Nu}_D = 0.85Ra_D^{0.188} = 0.85(1196)^{0.188} = 3.22$$

$$\bar{h}_1 = (k/D_1)\overline{Nu}_D = (0.0573/0.01) \times 3.22 = 18.5\ W/m^2 \cdot K.$$

The convection heat flux is

$$q''_{conv,1} = \bar{h}_1(T_1 - T_\infty) = 18.5(1295 - 300) = 18,407\ W/m^2,$$

Using Eq. (1), find

$$q''_1 = q''_{conv,1} + F_{12}(E_{b1} - E_{b2}) + F_{13}(E_{b1} - J_3) + F_{14}(E_{b1} - E_{b4})$$

$$q''_1 = 18,407 + 0.5(159,322 - 3496)$$
$$+0.148(159,322 - 1769) + 0.352(159,322 - 459)$$

$$q''_1 = 18,407 + (77,913 + 23,314 + 55,920)$$

$$q''_1 = 18,407 + 236,381 = 254,788\ W/m^2$$

$$q'_1 = \pi D_1 q''_1 = \pi(0.01)254,788 = 8000\ W/m.$$ <

COMMENTS: Although convection represents less than 8% of the net radiant transfer from the heater, it is equal to the net radiant transfer to the panel. Since the reflector is a re-radiating surface, results are independent of its emissivity.

PROBLEM 13.121

KNOWN: Temperature, power dissipation and emissivity of a cylindrical heat source. Surface emissivities of a parabolic reflector. Temperature of air and surroundings.

FIND: (a) Radiation circuit, (b) Net radiation transfer from the heater, (c) Net radiation transfer from the heater to the surroundings, (d) Temperature of reflector.

SCHEMATIC:

$\varepsilon_{2i} = 0.1$, $\varepsilon_{2o} = 0.8$, $T_{sur} = 300K$, Air $T_\infty = 300K$, $\theta = 135°$, $A'_2 = A'_{2i} = A'_{2o} = 0.2 m$, $h_2 = 2 W/m^2 \cdot K$, Heater, $D_1 = 0.005 m$, $\varepsilon_1 = 0.80$, $T_1 = 1200K$, $P'_1 = 1500 W/m$

ASSUMPTIONS: (1) Steady-state conditions, (2) Heater and reflector are in quiescent and infinite air, (3) Surroundings are infinitely large, (4) Reflector is thin (isothermal), (5) Diffuse-gray surfaces.

PROPERTIES: *Table A-4*, Air (T_f = 750 K, 1 atm): $\nu = 76.37 \times 10^{-6}$ m²/s, k = 0.0549 W/m·K, $\alpha = 109 \times 10^{-6}$ m²/s, Pr = 0.702.

ANALYSIS: (a) The thermal circuit is

$\frac{1-\varepsilon_{2i}}{\varepsilon_{2i} A_2}$, E_{b2}, $\frac{1-\varepsilon_{2o}}{\varepsilon_{2o} A_2}$, J_{2o}, J_{2i}, $(A_2 F_{2i,sur})^{-1}$, $(A_2 F_{2o,sur})^{-1}$, $J_{sur} = E_{b,sur}$, $(A_1 F_{12i})^{-1}$, J_1, $(A_1 F_{1sur})^{-1}$, $\frac{1-\varepsilon_1}{\varepsilon_1 A_1}$, E_{b1}, q'_1

(b) Energy transfer from the heater is by radiation and free convection. Hence,

$$P'_1 = q'_1 + q'_{1,conv}$$

where

$$q'_{1,conv} = \bar{h}\pi D_1 (T_1 - T_\infty)$$

and

$$Ra_D = \frac{g\beta (T_1 - T_\infty) D^3}{\nu\alpha} = \frac{9.8 \text{ m/s}^2 (750 \text{ K})^{-1} (900 \text{ K})(0.005 \text{ m})^3}{76.37 \times 109 \times 10^{-12} \text{m}^4/\text{s}^2} = 176.6.$$

Using the Churchill and Chu correlation, find

$$\overline{Nu}_D = \left\{ 0.6 + \frac{0.387 Ra_D^{1/6}}{\left[1 + (0.559/Pr)^{9/16}\right]^{8/27}} \right\}^2 = \left\{ 0.6 + \frac{0.387 (176.6)^{1/6}}{\left[1 + (0.559/0.702)^{9/16}\right]^{8/27}} \right\}^2 = 1.85$$

$$\bar{h} = \overline{Nu}_D (k/D) = 1.85 (0.0549 \text{ W/m} \cdot \text{K} / 0.005 \text{ m}) = 20.3 \text{ W/m}^2 \cdot \text{K}.$$

Continued …..

Hence,

$$q'_{1,conv} = 20.3\ W/m^2 \cdot K\pi(0.005\ m)(1200-300)K = 287\ W/m$$

$$q'_1 = 1500\ W/m - 287\ W/m = 1213\ W/m.$$ <

(c) The net radiative heat transfer from the heater to the surroundings is

$$q'_{1(sur)} = A'_1 F_{1sur}(J_1 - J_{sur}).$$

The view factor is

$$F_{1sur} = (135/360) = 0.375$$

and the radiosities are

$$J_{sur} = \sigma T_{sur}^4 = 5.67\times10^{-8} W/m^2 \cdot K^4 (300\ K)^4 = 459\ W/m^2$$

$$J_1 = E_{b1} - q'_1(1-\varepsilon_1)\varepsilon_1 A'_1 = 5.67\times10^{-8} W/m^2 \cdot K^4 (1200\ K)^4 - 1213\ W/m[0.2/0.8\pi(0.005\ m)]$$

$$J_1 = 98,268\ W/m^2.$$

Hence

$$q'_{1(sur)} = \pi(0.005\ m)0.375(98,268-459)W/m^2 = 576\ W/m.$$ <

(d) Perform an energy balance on the reflector,

$$q'_{2i} = q'_{2o} + q'_{2,conv}$$

$$\frac{J_{2i} - E_{b2}}{(1-\varepsilon_{2i})/\varepsilon_{2i}A'_2} = \frac{E_{b2} - J_{sur}}{(1-\varepsilon_{2o})/\varepsilon_{2o}A'_2 + 1/A'_2 F_{2o(sur)}} + 2\bar{h}_2 A'_2 (T_2 - T_\infty).$$

The radiosity of the reflector is

$$J_{2i} = J_1 - \frac{q'_{1(2i)}}{A'_1 F_{1(2i)}} = 98,268\ W/m^2 - \frac{(1213-576)W/m}{\pi(0.005\ m)(225/360)}$$

$$J_{2i} = 33,384\ W/m^2.$$

Hence

$$\frac{33,384 - 5.67\times10^{-8}(T_2^4)}{(0.9/0.1\times0.2\ m)} = \frac{5.67\times10^{-8}(T_2^4) - 459}{(0.2/0.8\times0.2\ m) + (1/0.2m\times1)} + 2\times0.4(T_2 - 300)$$

$$741.9 - 0.126\times10^{-8}T_2^4 = 0.907\times10^{-8}T_2^4 - 73.4 + 0.8T_2 - 240$$

$$1.033\times10^{-8}T_2^4 + 0.8T_2 = 1005$$

and from a trial and error solution, find

$$T_2 = 502\ K.$$ <

COMMENTS: Choice of small ε_{2i} and large ε_{2o} insures that most of the radiation from heater is reflected to surroundings and reflector temperature remains low.

PROBLEM 13.122

KNOWN: Geometrical conditions associated with tube array. Tube wall temperature and pressure of water flowing through tubes. Gas inlet velocity and temperature when heat is transferred from products of combustion in cross-flow, or temperature of electrically heated plates when heat is transferred by radiation from the plates.

FIND: (a) Steam production rate for gas flow without heated plates, (b) Steam production rate with heated plates and no gas flow, (c) Effects of inserting unheated plates with gas flow.

SCHEMATIC:

$V = 10\ m/s$, $T_{m,i} = 1200K$

$T_w = 400K$, $p = 2.5 bar$; $D = 10mm$, $S_T = S_L = 20mm$, $N_T = 5$, $N_L = 20$, $L = 1 m$

$T_{m,o}$ (a)

$T_p = 1200K$ (b)

ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible gas radiation, (3) Tube and plate surfaces may be approximated as blackbodies, (4) Gas outlet temperature is 600 K.

PROPERTIES: *Table A-4*, Air ($\overline{T}$ = 900 K, 1 atm): ρ = 0.387 kg/m^3, c_p = 1121 J/kg·K, ν = 102.9 × 10^{-6} m^2/s, k = 0.062 W/m·K, Pr = 0.720; (T = 400 K): Pr = 0.686; (T = 1200 K): ρ = 0.29 kg/m^3; *Table A-6*, Sat. water (2.5 bars): $h_{fg} = 2.18 \times 10^6$ J/kg.

ANALYSIS: (a) With

$$V_{max} = \left[S_T / \left(S_T - D\right)\right] V = 20\ m/s$$

$$Re_D = \frac{V_{max} D}{\nu} = \frac{20\ m/s\left(0.01\ m\right)}{102.9 \times 10^{-6} m^2/s} = 1944$$

and from the Zhukauskas correlation with C = 0.27 and m = 0.63,

$$\overline{Nu}_D = 0.27\left(1944\right)^{0.63}\left(0.720\right)^{0.36}\left(0.720/0.686\right)^{1/4} = 28.7$$

$$\overline{h} = 0.062\ W/m \cdot K \times 28.7 / 0.01\ m = 178\ W/m^2 \cdot K.$$

The outlet temperature may be evaluated from

$$\frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp\left(-\frac{\overline{h}A}{\dot{m}c_p}\right) = \exp\left(-\frac{\overline{h}N\pi DL}{\rho V N_T S_T L c_p}\right)$$

$$\frac{400 - T_{m,o}}{400 - 1200} = \exp\left(-\frac{178\ W/m^2 \cdot K \times 100 \times \pi \times 0.01\ m}{0.29\ kg/m^3 \times 10\ m/s \times 5 \times 0.02\ m \times 1121\ J/kg \cdot K}\right)$$

$$T_{m,o} = 543\ K.$$

Continued …..

PROBLEM 13.122 (Cont.)

With

$$\Delta T_{\ell m} = \frac{(T_s - T_{m,i}) - (T_s - T_{m,o})}{\ln\left[(T_s - T_{m,i})/(T_s - T_{m,o})\right]} = \frac{-800 - (-143)}{\ln(800/143)} = -382\ \text{K}$$

find

$$q = \bar{h} A \Delta T_{\ell m} = 178\ \text{W/m}^2 \cdot \text{K}(100)\pi(0.01\ \text{m})1\ \text{m}(-382\ \text{K})$$

$$q = -214\ \text{kW}.$$

If the water enters and leaves as saturated liquid and vapor, respectively, it follows that $-q = \dot{m}\, h_{fg}$, hence

$$\dot{m} = \frac{214,000\ \text{W}}{2.18 \times 10^6\ \text{J/kg}} = 0.098\ \text{kg/s}. \qquad <$$

(b) The radiation exchange between the plates and tube walls is

$$q = \left[A_p F_{ps} \sigma \left(T_p^4 - T_s^4\right)\right] \cdot 2 \cdot N_T$$

where the factor of 2 is due to radiation transfer from two plates. The view factor and area are

$$F_{ps} = 1 - \left[1 - (D/S)^2\right]^{1/2} + (D/S)\tan^{-1}\left[\left(S^2 - D^2\right)/D^2\right]^{1/2}$$

$$F_{ps} = 1 - 0.866 + 0.5\tan^{-1} 1.732 = 1 - 0.866 + 0.524$$

$$F_{ps} = 0.658$$

$$A_p = N_L \cdot S_L \cdot 1\ \text{m} = 20 \times 0.02\ \text{m} \times 1\ \text{m} = 0.40\ \text{m}^2.$$

Hence,

$$q = 5 \times \left[0.80\ \text{m}^2 \times 0.658 \times 5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4 \left(1200^4 - 400^4\right)\text{K}^4\right]$$

$$q = 305,440\ \text{W}$$

and the steam production rate is

$$\dot{m} = \frac{305,440\ \text{W}}{2.18 \times 10^6\ \text{J/kg}} = 0.140\ \text{kg/s}. \qquad <$$

(c) The plate temperature is determined by an energy balance for which convection to the plate from the gas is equal to net radiation transfer from the plate to the tube. Conditions are complicated by the fact that the gas transfers energy to both the plate and the tubes, and its decay is not governed by a simple exponential. Insertion of the plates enhances heat transfer to the tubes and thereby increases the steam generation rate. However, for the prescribed conditions, the effect would be small, since in case (a), the heat transfer is already ≈ 80% of the maximum possible transfer.

PROBLEM 13.123

KNOWN: Gas-fired radiant tube located within a furnace having quiescent gas at 950 K. At a particular axial location, inner wall and gas temperature measured by thermocouples.

FIND: Temperature of the outer tube wall at the axial location where the thermocouple measurements are being made.

SCHEMATIC:

ASSUMPTIONS: (1) Silicon carbide tube walls have negligible thermal resistance and are diffuse-gray, (2) Tubes are positioned horizontally, (3) Gas is radiatively non-participating and quiescent, (4) Furnace gas behaves as ideal gas, $\beta = 1/T$.

PROPERTIES: Gas (given): $\rho = 0.32$ kg/m^3, $\nu = 130 \times 10^{-6}$ m^2/s, k = 0.070 W/m·K, Pr = 0.72, $\alpha = \nu/\Pr = 1.806 \times 10^{-4}$ m^2/s.

ANALYSIS: Consider a segment of the outer tube at the
Prescribed axial location and perform an energy balance,

$$\dot{E}'_{in} - \dot{E}'_{out} = 0$$

$$q'_{rad,i} + q'_{conv,i} - q'_{rad,o} - q'_{conv,o} = 0 \qquad (1)$$

The heat rates by *radiative* transfer are:

Inside: For long concentric cylinders, Eq. 13.25,

$$q'_{rad,i} = \frac{\sigma\pi D_i\left(T_{s,i}^4 - T_{s,o}^4\right)}{1/\varepsilon_1 + (1-\varepsilon_2)/\varepsilon_2\,(D_i/D_o)}$$

$$q'_{rad,i} = \frac{5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\pi\,(0.10\text{ m})\left(1200^4 - T_{s,o}^4\right)\text{K}^4}{1/0.6 + (1-0.6)/0.6\,(0.10/0.20)}$$

$$q'_{rad,i} = 8.906\times10^{-9}\left(1200^4 - T_{s,o}^4\right). \qquad (2)$$

Outside: For the outer tube surface to large surroundings,

$$q'_{rad,o} = \varepsilon\pi D_o\sigma\left(T_{s,o}^4 - T_{sur}^4\right) = 0.6\pi\,(0.20\text{ m})\,5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left(T_{s,o}^4 - 950^4\right)\text{K}^4$$

$$q'_{rad,o} = 2.138\times10^{-8}\left(T_{s,o}^4 - 950^4\right). \qquad (3)$$

The heat rates by *convection processes* are:

Continued …..

PROBLEM 13.123 (Cont.)

Inside: The rate equation is

$$q'_{conv,i} = h_i \pi D_o \left(T_{m,g} - T_{s,o}\right). \quad (4)$$

Find the Reynolds number with $A_c = \pi\left(D_o^2 - D_i^2\right)/4$ and $D_h = 4A_c/P$,

$$Re_D = u_m D_h/\nu \quad u_m = \dot{m}/\rho A_c = 0.13\ \text{kg/s}/\left[0.32\ \text{kg/m}^3 \times \pi/4\left(0.2^2 - 0.1^2\right)\text{m}^2\right] = 17.2\ \text{m/s}$$

$$D_h = \frac{4(\pi/4)\left(D_o^4 - D_i^2\right)}{\pi(D_o + D_i)} = \frac{\pi\left(0.2^2 - 0.1^2\right)\text{m}^2}{(0.2 + 0.1)\text{m}} = 0.100\ \text{m} \qquad Re_D = \frac{17.2\ \text{m/s} \times 0.100\ \text{m}}{130 \times 10^{-6}\ \text{m}^2/\text{s}} = 13,231.$$

The flow is turbulent and assumed to be fully developed; from the Dittus-Boelter correlation,

$$Nu_D = hD_h/k = 0.023\,Re_D^{0.8}\,Pr^{0.3}$$

$$h_i = \frac{0.070\ \text{W/m}\cdot\text{K}}{0.100\ \text{m}} \times 0.023(13,231)^{0.8}(0.720)^{0.3} = 28.9\ \text{W/m}^2\cdot\text{K}$$

Substituting into Eq. (4),

$$q'_{conv,i} = 28.9\ \text{W/m}^2\cdot\text{K} \times \pi(0.20\ \text{m})\left(1050 - T_{s,o}\right)\text{K} = 18.16\left(1050 - T_{s,o}\right). \quad (5)$$

Outside: The rate equation is

$$q'_{conv,o} = h_o \pi D_o \left(T_{s,o} - T_\infty\right).$$

Evaluate the Rayleigh number *assuming* $T_{s,o}$ = 1025 K so that T_f = 987 K,

$$Ra_D = \frac{g\beta\Delta T D_o^3}{\nu\alpha} = \frac{9.8\ \text{m}^2/\text{s}^2\,(1/987\ \text{K})(1025 - 950)\text{K}\,(0.20\ \text{m})^3}{130 \times 10^{-6}\ \text{m}^2/\text{s} \times 1.806 \times 10^{-4}\ \text{m}^2/\text{s}} = 2.537 \times 10^5.$$

For a horizontal tube, using Eq. 9.33 and Table 9.1,

$$Nu_D = h_o D_o/k = C Ra_D^n = 0.48\left(2.537 \times 10^5\right)^{1/4} = 10.77$$

$$h_o = (0.070\ \text{W/m}\cdot\text{K})/0.20\ \text{m} \times 10.77 = 3.77\ \text{W/m}^2\cdot\text{K}.$$

Substituting into Eq. (6)

$$q'_{conv,o} = 3.77\ \text{W/m}^2\cdot\text{K} \times \pi(0.20\ \text{m})\left(T_{s,o} - 950\right)\text{K} = 2.369\left(T_{s,o} - 950\right). \quad (7)$$

Returning to the energy balance relation on the outer tube, Eq. (1), substitute for the individual rates from Eqs. (2, 5, 3, 7),

$$8.906 \times 10^{-9}\left(1200^4 - T_{s,o}^4\right) + 18.16\left(1050 - T_{s,o}\right) - 2.138 \times 10^{-8}\left(T_{s,o}^4 - 950^4\right) - 2.369\left(T_{s,o} - 950\right) = 0 \quad (8)$$

By trial-and-error, find $T_{s,o}$ = 1040 K. $<$

COMMENTS: (1) Recall that in estimating h_o we assumed $T_{s,o}$ = 1025 K, such that ΔT = 75 K (vs. 92 K using $T_{s,o}$ = 1042 K) for use in evaluating the Rayleigh number. For an improved estimate of $T_{s,o}$, it would be advisable to recalculate h_o.

(2) Note from Eq. (8) that the radiation processes dominate the heat transfer rate:

	q'_{rad} (W/m)	q'_{conv} (W/m)
Inside	7948	136
Outside	7839	219

PROBLEM 13.124

KNOWN: Temperature and emissivity of ceramic plate which is separated from a glass plate of equivalent height and width by an air space. Temperature of air and surroundings on opposite side of glass. Spectral radiative properties of glass.

FIND: (a) Transmissivity of glass, (b) Glass temperature T_g and total heat rate q_h, (c) Effect of external forced convection on T_g and q_h.

SCHEMATIC:

$0 \le \lambda \le 0.4\ \mu m$: $\tau_\lambda = 0$, $\alpha_\lambda = 1$
$0.4 \le \lambda \le 1.6\ \mu m$: $\tau_\lambda = 1$, $\alpha_\lambda = 0$
$\lambda \ge 1.6\ \mu m$: $\tau_\lambda = 0$, $\alpha_\lambda = 0.9$

Air
$T_c = 1000$ K
$\varepsilon_c = 0.95$
T_g
$q''_{rad,i}$
$q''_{rad,o}$
q_h
$q''_{conv,i}$
$q''_{conv,o}$
Air
$T_\infty = 300$ K
h_o
$H = W = 2$ m
$L = 0.05$ m
$T_{sur} = 300$ K

ASSUMPTIONS: (1) Spectral distribution of emission from ceramic approximates that of a blackbody, (2) Glass surface is diffuse, (3) Atmospheric air in cavity and ambient, (4) Cavity may be approximated as a two-surface enclosure with infinite parallel plates, (5) Glass is isothermal.

PROPERTIES: *Table A-4*, air (p = 1 atm): Evaluated at $\bar{T} = (T_c + T_g)/2$ and $T_f = (T_g + T_\infty)/2$ using *IHT Properties* Toolpad.

ANALYSIS: (a) The total transmissivity of the glass is

$$\tau = \frac{\int_o^\infty \tau_\lambda E_{\lambda b} d\lambda}{E_b} = \int_{\lambda_1 = 0.4\mu m}^{\lambda_2 = 1.6\mu m} \left(E_{\lambda,b} / E_b\right) d\lambda = F_{(0\to\lambda_2)} - F_{(0\to\lambda_1)}$$

With $\lambda_2 T = 1600\ \mu m\cdot K$ and $\lambda_1 T = 400\ \mu m\cdot K$, respectively, Table 12.1 yields $F_{(0\to\lambda_2)} = 0.0197$ and $F_{(0\to\lambda_1)} = 0.0$. Hence,

$$\tau = 0.0197$$ <

With so little transmission of radiation from the ceramic, the glass plate may be assumed to be opaque to a good approximation. Since more than 98% of the incident radiation is at wavelengths exceeding 1.6 μm, for which $\alpha_\lambda = 0.9$, $\alpha_g \approx 0.9$. Also, since $T_g < T_c$, nearly 100% of emission from the glass is at $\lambda > 1.6\ \mu m$, for which $\varepsilon_\lambda = \alpha_\lambda = 0.9$, $\varepsilon_g = 0.9$ and the glass may be approximated as a gray surface.

(b) The glass temperature may be obtained from an energy balance of the form $q''_{conv,i} + q''_{rad,i} = q''_{conv,o} + q''_{rad,o}$. Using Eqs. 13.24 and 13.27 to evaluate $q''_{rad,i}$ and $q''_{rad,o}$, respectively, it follows that

$$\bar{h}_i \left(T_c - T_g\right) + \frac{\sigma\left(T_c^4 - T_g^4\right)}{\frac{1}{\varepsilon_c} + \frac{1}{\varepsilon_g} - 1} = \bar{h}_o \left(T_g - T_\infty\right) + \varepsilon_g \sigma \left(T_g^4 - T_{sur}^4\right)$$

Continued …..

where, assuming $10^4 \le Ra_L \le 10^7$, $\bar{h}_i$ and $\bar{h}_o$ are given by Eqs. 9.52 and 9.26, respectively,

$$\bar{h}_i = \frac{k_i}{L} Ra_L^{1/4} Pr_i^{0.012} (H/L)^{-0.3}$$

$$\bar{h}_o = \frac{k_o}{H} \left\{ 0.825 + \frac{0.387 Ra_H^{1/6}}{\left[1 + (0.492/Pr_o)^{9/16}\right]^{8/27}} \right\}^2$$

with $Ra_L = g\beta_i (T_c - T_g)L^3/\nu_i\alpha_i$ and $Ra_H = g\beta_o (T_g - T_\infty) H^3/\nu_o\alpha_o$. Entering the energy balance into the *IHT* workspace and using the *Correlations*, *Properties* and *Radiation* Toolpads to evaluate the convection and radiation terms, the following result is obtained.

$$T_g = 825 \text{ K}$$ <

The corresponding value of q_h is

$$q_h = 108 \text{ kW}$$ <

where $q_{conv,i}$ = 3216 W, $q_{rad,i}$ = 104.7 kW, $q_{convo,o}$ = 15,190 W and $q_{rad,o}$ = 92.8 kW. The convection coefficients are $\bar{h}_i = 4.6 \text{ W/m}^2 \cdot \text{K}$ and $\bar{h}_o = 7.2 \text{ W/m}^2 \cdot \text{K}$.

(c) For the prescribed range of $\bar{h}_o$, *IHT* was used to generate the following results.

With increasing $\bar{h}_o$, the glass is cooled more effectively and T_g must decrease. With decreasing T_g, $q_{conv,i}$, $q_{rad,i}$ and hence q_h must increase. Note that radiation makes the dominant contribution to heat transfer across the airspace. Although $q_{rad,o}$ decreases with decreasing T_g, the increase in $q_{conv,o}$ exceeds the reduction in $q_{rad,o}$.

PROBLEM 13.125

KNOWN: Conditions associated with a spherical furnace cavity.

FIND: Cooling rate needed to maintain furnace wall at a prescribed temperature.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state, (2) Blackbody behavior for furnace wall, (3) N_2 is non-radiating.

ANALYSIS: From an energy balance on a unit surface area of the furnace wall, the cooling rate per unit area must equal the absorbed irradiation from the gas (E_g) minus the portion of the wall's emissive power absorbed by the gas

$$q_c'' = E_g - \alpha_g E_b(T_s)$$

$$q_c'' = \varepsilon_g \sigma T_g^4 - \alpha_g \sigma T_s^4.$$

Hence, for the entire furnace wall,

$$q_c = A_s \sigma \left(\varepsilon_g T_g^4 - \alpha_g T_s^4\right).$$

The gas emissivity, ε_g, follows from Table 13.4 with

$$L_e = 0.65D = 0.65 \times 0.5\ \text{m} = 0.325\ \text{m} = 1.066\ \text{ft}.$$

$$p_c L_e = 0.25\ \text{atm} \times 1.066\ \text{ft} = 0.267\ \text{ft} - \text{atm}$$

and from Fig. 13.18, find $\varepsilon_g = \varepsilon_c = 0.09$. From Eq. 13.42,

$$\alpha_g = \alpha_c = C_c \left(\frac{T_g}{T_s}\right)^{0.45} \times \varepsilon_c\left(T_s, p_c L_e\left[T_s / T_g\right]\right).$$

With $C_c = 1$ from Fig. 13.19,

$$\alpha_g = 1(1400/50)^{0.45} \times \varepsilon_c\,(500\text{K},\ 0.095\ \text{ft} - \text{atm})$$

where, from Fig. 13.18,

$$\varepsilon_c\,(500\text{K},\ 0.095\ \text{ft} - \text{atm}) = 0.067.$$

Hence

$$\alpha_g = 1(1400/500)^{0.45} \times 0.067 = 0.106$$

and the heat rate is

$$q_c = \pi\,(0.5\ \text{m})^2\, 5.67 \times 10^{-8}\,\text{W/m}^2 \cdot \text{K}^4 \left[0.09(1400\ \text{K})^4 - 0.106(500\ \text{K})^4\right]$$

$$q_c = 15.1\ \text{kW}.$$ <

PROBLEM 13.126

KNOWN: Diameter and gas pressure, temperature and composition associated with a gas turbine combustion chamber.

FIND: Net radiative heat flux between the gas and the chamber surface.

SCHEMATIC:

$T_s = 500°C$; Combustion gas; $T_g = 1000°C$, $p = 1 atm$; $p_w = p_c = 0.15 atm$; $D = 0.4 m$

ASSUMPTIONS: (1) Steady-state conditions, (2) Blackbody behavior for chamber surface, (3) Remaining species are non-radiating, (4) Chamber may be approximated as an infinitely long tube.

ANALYSIS: From Eq. 13.40 the net rate of radiation transfer to the surface is

$$q_{net} = A_s\sigma\left(\varepsilon_g T_g^4 - \alpha_g T_s^4\right) \qquad \text{or} \qquad q'_{net} = \pi D\sigma\left(\varepsilon_g T_g^4 - \alpha_g T_s^4\right)$$

with $A_s = \pi DL$. From Table 13.4, $L_e = 0.95D = 0.95 \times 0.4$ m = 0.380 m = 1.25 ft. Hence, $p_w L_e = p_c L_e = 0.152$ atm × 1.25 ft = 0.187 atm-ft.

$$\text{Fig.13.16 } \left(T_g = 1273 \text{ K}\right), \rightarrow \varepsilon_w \approx 0.069.$$

$$\text{Fig.13.18 } \left(T_g = 1273 \text{ K}\right), \rightarrow \varepsilon_c \approx 0.085.$$

$$\text{Fig.13.20 } \left(p_w / \left(p_c + p_w\right) = 0.5,\ L_c\left(p_w + p_c\right) = 0.375 \text{ ft} - \text{atm},\ T_g \ge 930°C\right), \rightarrow \Delta\varepsilon \ge 0.01.$$

From Eq. 13.38,

$$\varepsilon_g = \varepsilon_w + \varepsilon_c - \Delta\varepsilon = 0.069 + 0.085 - 0.01 \approx 0.144.$$

From Eq. 13.41 for the water vapor,

$$\alpha_w = C_w\left(T_g / T_s\right)^{0.45} \times \varepsilon_w\left(T_s, p_w L_c\left[T_s / T_g\right]\right)$$

where from Fig. 13.16 (773 K, 0.114 ft-atm), → $\varepsilon_w \approx 0.083$,

$$\alpha_w = 1(1273/773)^{0.45} \times 0.083 = 0.104.$$

From Eq. 13.42, using Fig. 13.18 (773 K, 0.114 ft-atm), → $\varepsilon_c \approx 0.08$,

$$\alpha_c = 1(1273/773)^{0.45} \times 0.08 = 0.100.$$

From Fig. 13.20, the correction factor for water vapor at carbon dioxide mixture,

$$\left(p_w / \left(p_c + p_w\right) = 0.1, L_e\left(p_w + p_c\right) = 0.375,\ T_g \approx 540°C\right), \rightarrow \Delta\alpha \approx 0.004$$

and using Eq. 13.43

$$\alpha_g = \alpha_w + \alpha_c - \Delta\alpha = 0.104 + 0.100 - 0.004 \approx 0.200.$$

Hence, the heat rate is

$$q'_{net} = \pi\left(0.4 \text{ m}\right)5.67 \times 10^{-8} \text{W/m}^2 \cdot \text{K}^4\left[0.144(1273)^4 - 0.200(773)^4\right] = 21.9 \text{ kW/m.} <$$

PROBLEM 13.127

KNOWN: Pressure, temperature and composition of flue gas in a long duct of prescribed diameter.

FIND: Net radiative flux to the duct surface.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Duct surface behaves as a blackbody, (3) Other gases are non-radiating, (4) Flue may be approximated as an infinitely long tube.

ANALYSIS: With $A_s = \pi DL$, it follows from Eq. 13.40 that

$$q'_{net} = \pi D\sigma\left(\varepsilon_g T_g^4 - \alpha_g T_s^4\right)$$

From Table 13.4, $L_e = 0.95D = 0.95 \times 1\text{ m} = 0.95\text{ m} = 3.12\text{ ft}$. Hence

$$p_w L_e = 0.12\text{ atm} \times 3.12\text{ m} = 0.312\text{ atm} - \text{ft}$$

$$p_c L_e = 0.05\text{ atm} \times 3.12\text{ m} = 0.156\text{ atm} - \text{ft}.$$

With $T_g = 1400$ K, Fig. 13.16 $\rightarrow \varepsilon_w = 0.083$; Fig. 13.18 $\rightarrow \varepsilon_c = 0.072$. With $p_w/(p_c + p_w) = 0.67$, $L_e(p_w + p_c) = 0.468$ atm-ft, $T_g \geq 930°C$, Fig. 13.20 $\rightarrow \Delta\varepsilon = 0.01$. Hence from Eq. 13.38,

$$\varepsilon_g = \varepsilon_w + \varepsilon_c - \Delta\varepsilon = 0.083 + 0.072 - 0.01 = 0.145.$$

From Eq. 13.41,

$$\alpha_w = C_w\left(T_g / T_s\right)^{0.45} \times \varepsilon_w\left(T_s, p_w L_e\left[T_s / T_g\right]\right)$$

$$\alpha_w = 1(1400/400)^{0.45} \times \varepsilon_w \text{ Fig. 13.16} \rightarrow \varepsilon_w\left(400\text{ K, } 0.0891\text{ atm} - \text{ft}\right) = 0.1$$

$$\alpha_w = 0.176.$$

From Eq. 13.42,

$$\alpha_c = C_c\left(T_g / T_s\right)^{0.45} \times \varepsilon_c\left(T_s,\ p_c L_e T_s / T_g\right)$$

$$\alpha_c = 1(1400/400)^{0.45} \times \varepsilon_c \text{ Fig. 13.18} \rightarrow \varepsilon_c\left(400\text{ K, } 0.0891\text{ atm} - \text{ft}\right) = 0.053$$

$$\alpha_c = 0.093.$$

With $p_w/(p_c + p_w) = 0.67$, $L_e(p_w + p_c) = 0.468$ atm-ft, $T_g \approx 125°C$, Fig. 13.20 gives
$\Delta\alpha \approx 0.003.$

Hence from Eq. 13.43,

$$\alpha_g = \alpha_w + \alpha_c - \Delta\alpha = 0.176 + 0.093 - 0.003 = 0.266.$$

The heat rate per unit length is

$$q'_{net} = \pi(1\text{ m})5.67 \times 10^{-8}\text{ W/m}^2 \cdot \text{K}^4\left[0.145(1400\text{ K})^4 - 0.266(400\text{ K})^4\right]$$

$$q'_{net} = 98\text{ kW/m}.$$ <

PROBLEM 13.128

KNOWN: Gas mixture of prescribed temperature, pressure and composition between large parallel plates of prescribed separation.

FIND: Net radiation flux to the plates.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Furnace wall behaves as a blackbody, (3) O_2 and N_2 are non-radiating, (4) Negligible end effects.

ANALYSIS: The net radiative flux to a plate is

$$q''_{s,1} = G_s - E_s = \varepsilon_g \sigma T_g^4 - \left(1 - \tau_g\right)\sigma T_s^4$$

where $G_s = \varepsilon_g \sigma T_g^4 + \tau_g E_s$, $E_s = \sigma T_s^4$ and $\tau_g = 1 - \alpha_g\left(T_s\right)$. From Table 13.4, $L_e = 1.8L = 1.8 \times 0.75$ m = 1.35 m = 4.43 ft. Hence $p_w L_e = p_c L_e = 1.33$ atm-ft. From Figs. 3.16 and 3.18 find $\varepsilon_w \approx 0.22$ and $\varepsilon_c \approx 0.16$ for p = 1 atm. With $(p_w + p)/2 = 1.15$ atm, Fig. 13.17 yields $C_w \approx 1.40$ and from Fig. 13.19, $C_c \approx 1.08$. Hence, the gas emissivities are

$$\varepsilon_w = C_w \varepsilon_w \left(1 \text{ atm}\right) \approx 1.40 \times 0.22 = 0.31 \qquad \varepsilon_c = C_c \varepsilon_c \left(1 \text{ atm}\right) \approx 1.08 \times 0.16 = 0.17.$$

From Fig. 13.20 with $p_w/(p_c + p_w) = 0.5$, $L_e(p_c + p_w) = 2.66$ atm-ft and $T_g > 930°C$, $\Delta\varepsilon \approx 0.047$. Hence, from Eq. 13.38,

$$\varepsilon_g = \varepsilon_w + \varepsilon_c - \Delta\varepsilon \approx 0.31 + 0.17 - 0.047 \approx 0.43.$$

To evaluate α_g at T_s, use Eq. 13.43 with

$$\alpha_w = C_w \left(T_g / T_s\right)^{0.45} \varepsilon_w \left(T_s, p_w L_2 T_s / T_g\right) = C_w \left(1300/500\right)^{0.45} \varepsilon_w \left(500,\ 0.51\right)$$

$$\alpha_w \approx 1.40\left(1300/500\right)^{0.45} 0.22 = 0.47$$

$$\alpha_c = C_c \left(1300/500\right)^{0.45} \varepsilon_c \left(500,\ 0.51\right) \approx 1.08\left(1300/500\right)^{0.45} 0.11 = 0.18.$$

From Fig. 13.20, with $T_g \approx 125°C$ and $L_e(p_w + p_c) = 2.66$ atm-ft, $\Delta\alpha = \Delta\varepsilon \approx 0.024$. Hence

$$\alpha_g = \alpha_w + \alpha_c - \Delta\alpha \approx 0.47 + 0.18 - 0.024 \approx 0.63 \quad \text{and} \quad \tau_g = 1 - \alpha_g \approx 0.37.$$

Hence, the heat flux from Eq. (1) is

$$q''_{s,1} = 0.43 \times 5.67 \times 10^{-8} \text{W/m}^2 \cdot \text{K}^4 \left(1300 \text{ K}\right)^4 - 0.63 \times 5.67 \times 10^{-8} \text{W/m}^2 \cdot \text{K}^4 \left(500 \text{ K}\right)^4$$

$$q''_{s,1} \approx 67.4 \text{ kW/m}^2.$$

The net radiative flux to both plates is then $q''_{s,2} \approx 134.8 \text{ kW/m}^2$. <

PROBLEM 13.129

KNOWN: Flow rate, temperature, pressure and composition of exhaust gas in pipe of prescribed diameter. Velocity and temperature of external coolant.

FIND: Pipe wall temperature and heat flux.

SCHEMATIC:

$T_s, \varepsilon = 1.0$; $D = 0.25m$

Combustion products
$p = 1 atm$
$p_{CO_2} = p_{H_2O} = 0.1$
$T_g = T_m = 2000K$
$\dot{m} = 0.25 kg/s$

q''_r, $q''_{c,i}$, $q''_{c,o}$

Water
$V = 0.3 m/s$
$T_\infty = 300K$

ASSUMPTIONS: (1) L/D >> 1 (infinitely long pipe), (2) Negligible axial gradient for gas temperature, (3) Gas is in fully developed flow, (4) Gas thermophysical properties are those of air, (5) Negligible pipe wall thermal resistance, (6) Negligible pipe wall emission.

PROPERTIES: *Table A-4*: Air (T_m = 2000 K, 1 atm): $\rho = 0.174\ kg/m^3$, $\mu = 689 \times 10^{-7}$ kg/m·s, k = 0.137 W/m·K, Pr = 0.672; *Table A-6*: Water (T_∞ = 300 K): $\rho = 997\ kg/m^3$, $\mu = 855 \times 10^{-6}$ kg/s·m, k = 0.613 W/m·K, Pr = 5.83.

ANALYSIS: Performing an energy balance for a control surface about the pipe wall,

$$q''_r + q''_{c,i} = q''_{c,o}$$

$$\varepsilon_g \sigma T_g^4 + h_i \left(T_m - T_s\right) = \overline{h}_o \left(T_s - T_\infty\right)$$

The gas emissivity is

$$\varepsilon_g = \varepsilon_w + \varepsilon_c = \Delta\varepsilon$$

where

$$L_e = 0.95D = 0.238\ m = 0.799\ ft$$

$$p_c L_e = p_w L_e = 0.1\ atm \times 0.238\ m = 0.0238\ atm-m = 0.0779\ atm-ft$$

and from Fig. 13.16 → $\varepsilon_w \approx 0.017$; Fig. 13.18 → $\varepsilon_c \approx 0.031$; Fig. 13.20 → $\Delta\varepsilon \approx 0.001$. Hence ε_g = 0.047. Estimating the *internal flow convection coefficient*, find

$$Re_D = \frac{4\ \dot{m}}{\pi D \mu} = \frac{4 \times 0.25\ kg/s}{\pi\left(0.25\ m\right) 689 \times 10^{-7}\ kg/m \cdot s} = 18,480$$

and for turbulent flow,

$$Nu_D = 0.023 Re_D^{4/5} Pr^{0.3} = 0.023\left(18,480\right)^{4/5}\left(0.672\right)^{0.3} = 52.9$$

$$h_i = Nu_D \frac{k}{D} = 52.9 \frac{0.137\ W/m \cdot K}{0.25\ m} = 29.0\ W/m^2 \cdot K.$$

Continued …..

PROBLEM 13.129 (Cont.)

Estimating the *external convection coefficient*, find

$$Re_D = \frac{\rho VD}{\mu} = \frac{997\ kg/m^3 \times 0.3\ m/s \times 0.25\ m}{855 \times 10^{-6}\ kg/s \cdot m} = 87,456.$$

Hence

$$\overline{Nu}_D = 0.26\ Re_D^{0.6}\ Pr^{0.37}\left(Pr/Pr_s\right)^{1/4}.$$

Assuming $Pr/Pr_s \approx 1$,

$$\overline{Nu}_D = 0.26(87,456)^{0.6}(5.83)^{0.37} = 461$$

$$\overline{h}_o = \overline{Nu}_D\,(k/D) = 461(0.613\ W/m \cdot K/0.25\ m) = 1129\,W/m^2 \cdot K.$$

Substituting numerical values in the energy balance, find

$$0.047 \times 5.67 \times 10^{-8} W/m^2 \cdot K^4 (2000\ K)^4 + 29\ W/m^2 \cdot K(2000 - T_s)K$$

$$= 1129\ W/m^2 \cdot K(T_s - 300)K$$

$$T_s = 380\ K.$$ <

The heat flux due to convection is

$$q''_{c,i} = h_i(T_m - T_s) = 29\ W/m^2 \cdot K(2000 - 379.4)K = 46,997\ W/m^2$$

and the total heat flux is

$$q''_s = q''_r + q''_{c,i} = 42,638 + 46,997 = 89,640\ W/m^2.$$ <

COMMENTS: Contributions of gas radiation and convection to the wall heat flux are approximately the same. Small value of T_s justifies neglecting emission from the pipe wall to the gas. $Pr_s = 1.62$ for $T_s = 380 \rightarrow (Pr/Pr_s)1/4 = 1.38$. Hence the value of $\overline{h}_o$ should be corrected. The value would ↑, and T_s would ↓.

PROBLEM 13.130

KNOWN: Flowrate, composition and temperature of flue gas passing through inner tube of an annular waste heat boiler. Boiler dimensions. Steam pressure.

FIND: Rate at which saturated liquid can be converted to saturated vapor, $\dot{m}_s$.

SCHEMATIC:

ASSUMPTIONS: (1) Inner wall is thin and steam side convection coefficient is very large; hence $T_s = T_{sat}(2.455 \text{ bar})$, (2) For calculation of gas radiation, inner tube is assumed infinitely long and gas is approximated as isothermal at T_g.

PROPERTIES: Flue gas (given): $\mu = 530 \times 10^{-7}$ kg/s·m, k = 0.091 W/m·K, Pr = 0.70; *Table A-6*, Saturated water (2.455 bar): $T_s = 400$ K, $h_{fg} = 2183$ kJ/kg.

ANALYSIS: The steam generation rate is

$$\dot{m}_s = q / h_{fg} = \left(q_{conv} + q_{rad}\right) / h_{fg}$$

where

$$q_{rad} = A_s \sigma \left(\varepsilon_g T_g^4 - \alpha_g T_s^4\right)$$

with

$$\varepsilon_g = \varepsilon_w + \varepsilon_c - \Delta\varepsilon \qquad\qquad \alpha_g = \alpha_w + \alpha_c - \Delta\alpha.$$

From Table 13.4, find $L_e = 0.95D = 0.95$ m = 3.117 ft. Hence

$$p_w L_e = 0.2 \text{ atm} \times 3.117 \text{ ft} = 0.623 \text{ ft} - \text{atm}$$

$$p_c L_e = 0.1 \text{ atm} \times 3.117 \text{ ft} = 0.312 \text{ ft} - \text{atm}.$$

From Fig. 13.16, find $\varepsilon_w \approx 0.13$ and Fig. 13.18 find $\varepsilon_c \approx 0.095$. With $p_w/(p_c + p_w) = 0.67$ and $L_e(p_w + p_c) = 0.935$ ft-atm, from Fig. 13.20 find $\Delta\varepsilon \approx 0.036 \approx \Delta\alpha$. Hence $\varepsilon_g \approx 0.13 + 0.095 - 0.036 = 0.189$. Also, with $p_w L_e(T_s/T_g) = 0.2 \text{ atm} \times 0.95 \text{ m}(400/1400) = 0.178$ ft-atm and $T_s = 400$ K, Fig. 13.16 yields $\varepsilon_w \approx 0.14$. With $p_c L_e(T_s/T_g) = 0.1 \text{ atm} \times 0.95 \text{ m}(400/1400) = 0.089$ ft-atm and $T_s = 400$ K, Fig. 13.18 yields $\varepsilon_c \approx 0.067$. Hence

$$\alpha_w = \left(T_g / T_s\right)^{0.45} \varepsilon_w \left(T_s,\ p_w L_e T_s / T_g\right)$$

$$\alpha_w = (1400/400)^{0.45}\, 0.14 = 0.246$$

and

$$\alpha_c = \left(T_g / T_s\right)^{0.65} \varepsilon_c \left(T_s, p_c L_e T_s / T_g\right)$$

Continued …..

$$\alpha_c = (1400/400)^{0.65}\, 0.067 = 0.151$$

$$\alpha_g = 0.246 + 0.151 - 0.036 = 0.361.$$

Hence

$$q_{rad} = \pi (1\ m) 7\ m \times 5.67 \times 10^{-8} W/m^2 \cdot K^4 \left[0.189 (1400\ K)^4 - 0.361 (400\ K)^4 \right]$$

$$q_{rad} = (905.3 - 11.5) kW = 893.8\ kW.$$

For convection,

$$q_{conv} = \bar{h} \pi DL \left(T_g - T_s \right)$$

with

$$Re_D = \frac{4\dot{m}}{\pi D \mu} = \frac{4 \times 2\ kg/s}{\pi \times 1\ m \times 530 \times 10^{-7}\ kg/s \cdot m} = 48,047$$

and assuming fully developed turbulent flow throughout the tube, the Dittus-Boelter correlation gives

$$\overline{Nu}_D = 0.023 Re_D^{4/5} Pr^{0.3} = 0.023 (48,047)^{4/5} (0.70)^{0.3} = 115$$

$$\bar{h} = (k/D) \overline{Nu}_D = (0.091\ W/m \cdot K / 1\ m) 115 = 10.5\ W/m^2 \cdot K.$$

Hence

$$q_{conv} = 10.5\ W/m^2 \cdot K \pi (1\ m) 7\ m (1400 - 400) K = 230.1\ kW$$

and the vapor production rate is

$$\dot{m}_s = \frac{q}{h_{fg}} = \frac{(893.8 + 230.1) kW}{2183\ kJ/kg} = \frac{1123.9\ kW}{2183\ kJ/kg}$$

$$\dot{m}_s = 0.515\ kg/s.$$ <

COMMENTS: (1) Heat transfer is dominated by radiation, which is typical of heat recovery devices having a large gas volume.

(2) A more detailed analysis would account for radiation exchange involving the ends (upstream and downstream) of the inner tube.

(3) Using a representative specific heat of c_p = 1.2 kJ/kg·K, the temperature drop of the gas passing through the tube would be ΔT_g = 1123.9 kW/(2 kg/s × 1.2 kJ/kg·K) = 468 K.

PROBLEM 13.131

KNOWN: Wet newsprint moving through a drying oven.

FIND: Required evaporation rate, air velocity and oven temperature.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible freestream turbulence, (3) Heat and mass transfer analogy applicable, (4) Oven and newsprint surfaces are diffuse gray, (5) Oven end effects negligible.

PROPERTIES: *Table A-6*, Water vapor (300 K, 1 atm): $\rho_{sat} = 1/v_g = 0.0256\ kg/m^3$, $h_{fg} = 2438$ kJ/kg; *Table A-4*, Air (300 K, 1 atm): $\eta = 15.89 \times 10^{-6}\ m^2/s$; *Table A-8*, Water vapor-air (300 K, 1 atm): $D_{AB} = 0.26 \times 10^{-4}\ m^2/s$, $S_c = \eta/D_{AB} = 0.611$.

ANALYSIS: The evaporation rate required to completely dry the newsprint having a water content of $m''_A = 0.02\ kg/m^2$ as it enters the oven (x = L) follows from a species balance on the newsprint.

$$\dot{M}_{A,in} - \dot{M}_{A,out} = \dot{M}_{st}$$

$$\dot{M}_L - \dot{M}_0 - \dot{M}_{A,s} = 0.$$

The rate at which moisture enters in the newsprint is

$$\dot{M}_L = m''_A VW$$

hence,

$$\dot{M}_{A,s} = m''_A VW = 0.02\,kg/m^2 \times 0.2\,m/s \times 1\ m = 4\times10^{-3} kg/s.$$ <

The required velocity of the airstream through the oven, u_∞, can be determined from a convection analysis. From the rate equation,

$$\dot{M}_{A,s} = \overline{h}_m WL\left(\rho_{A,s} - \rho_{A,\infty}\right) = \overline{h}_m WL\rho_{A,sat}\left(1-\phi_\infty\right)$$

$$\overline{h}_m = \dot{M}_{A,s}/WL\rho_{A,sat}\left(1-\phi_\infty\right)$$

$$\overline{h}_m = 4\times10^{-3} kg/s/1\ m\times20\ m\times0.0256\ kg/m^3\left(1-0.2\right) = 9.77\times10^{-3} m/s.$$

Now determine what flow velocity is required to produce such a coefficient. Assume flow over a flat plate with

$$\overline{Sh}_L = \overline{h}_m L/D_{AB} = 9.77\times10^{-3} m/s\times20\ m/0.26\times10^{-4} m^2/s = 7515$$

Continued …..

and

$$Re_L = \left[\overline{Sh}_L / 0.664 Sc^{1/3}\right]^2 = \left[7515 / 0.664 (0.611)^{1/3}\right]^2 = 1.78 \times 10^8.$$

Since $Re_L > Re_{Lc} = 5 \times 10^5$, the flow must be turbulent. Using the correlation for mixed laminar and turbulent flow conditions, find

$$Re_L^{4/5} = \left[\overline{Sh}_L / Sc^{1/3} + 871\right] / 0.037$$

$$Re_L^{4/5} = \left[7515 / (0.611)^{1/3} + 871\right] / 0.037$$

$$Re_L = 5.95 \times 10^6$$

noting $Re_L > Re_{Lc}$. Recognize that u_∞^* is the velocity relative to the newsprint,

$$u_\infty^* = Re_L \nu / L = 5.95 \times 10^6 \times 15.89 \times 10^{-6} m^2/s / 20 m = 4.73 \ m/s.$$

The air velocity relative to the oven will be,

$$u_\infty = u_\infty^* - V = (4.73 - 0.2) m/s = 4.53 \ m/s. \quad <$$

The temperature required of the oven surface follows from an energy balance on the newsprint. Find

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$q_{rad} - q_{evap} = 0$$

where

$$q_{evap} = \dot{M}_{A,s} h_{fg} = 4.0 \times 10^{-3} kg/s \times 2438 \times 10^3 J/kg = 9752 \ W$$

and the radiation exchange is that for a two surface enclosure, Eq. 13.23,

$$q_{rad} = \frac{\sigma \left(T_1^4 - T_2^4\right)}{(1-\varepsilon_1)/\varepsilon_1 A_1 + 1/A_1 F_{12} + (1-\varepsilon_2)/\varepsilon_2 A_2}.$$

Evaluate,

$$A_1 = \pi/2 \ WL, \qquad A_2 = WL, \qquad F_{21} = 1, \ \text{and} \ A_1 F_{12} = A_2 F_{21} = WL$$

hence, with $\varepsilon_1 = 0.8$,

$$q_{rad} = \sigma WL \left(T_1^4 - T_2^4\right) / \left[(1/2\pi) + 1\right]$$

$$T_1^4 = T_2^4 + q_{rad} \left[(1/2\pi) + 1\right] / \sigma WL$$

$$T_1^4 = (300 \ K)^4 + 9752 \ W \left[(1/2\pi + 1)\right] / 5.67 \times 10^{-8} W/m^2 \cdot K^4 \times 1 \ m \times 20 \ m$$

$$T_1 = 367 \ K. \quad <$$

COMMENTS: Note that there is no convection heat transfer since $T_\infty = T_s = 300$ K.

PROBLEM 13.132

KNOWN: Configuration of grain dryer. Emissivities of grain bed and heater surface. Temperature of grain.

FIND: (a)Temperature of heater required for specified drying rate, (b) Convection mass transfer coefficient required to sustain evaporation, (c) Validity of assuming negligible convection heat transfer.

SCHEMATIC:

ASSUMPTIONS: (1) Diffuse/gray surfaces, (2) Oven wall is a reradiating surface, (3) Negligible convection heat transfer, (4) Applicability of heat/mass transfer analogy, (5) Air is dry.

PROPERTIES: *Table A-6*, saturated water (T = 330 K): $v_g = 8.82\ m^3/kg$, $h_{fg} = 2.366 \times 10^6$ J/kg. *Table A-4*, air (assume T ≈ 300 K): $\rho = 1.614\ kg/m^3$, $c_p = 1007$ J/kg·K, $\alpha = 22.5 \times 10^{-6}\ m^2/s$. *Table A-8*, H_2O(v) – air (T = 298 K): $D_{AB} = 0.26 \times 10^{-4}\ m^2/s$.

ANALYSIS: (a) Neglecting convection, the energy required for evaporation must be supplied by net radiation transfer from the heater plate to the grain bed. Hence,

$$q'_{rad} = \dot{m}'_{evap} h_{fg} = (2.5\ kg/h \cdot m)\left(2.366 \times 10^6\ J/kg\right)/3600\ s/h = 1643\ W/m$$

where q'_{rad} is given by Eq. 13.30. With $A'_p = A'_g \equiv A'$,

$$q'_{rad} = \frac{A'\left(E_{bp} - E_{bg}\right)}{\frac{1-\varepsilon_p}{\varepsilon_p} + \frac{1}{F_{pg} + \left[\left(1/F_{pR}\right) + \left(1/F_{gR}\right)\right]^{-1}} + \frac{1-\varepsilon_g}{\varepsilon_g}}$$

where $A' = R = 1$ m, $F_{pg} = 0$ and $F_{pR} = F_{gR} = 1$. Hence,

$$q'_{rad} = \frac{\sigma\left(T_p^4 - 320^4\right)}{0.25 + 2 + 0.111} = 2.40 \times 10^{-8}\left(T_p^4 - 320^4\right) = 1643\ W/m$$

$$2.40 \times 10^{-8} T_p^4 - 2518 = 1643$$

$$T_p = 530\ K$$ <

(b) The evaporation rate is given by Eq. 6.12, and with $A'_s = 1$ m, $n'_A = \dot{m}'_{evap}$, and $\rho_{A,\infty} = 0$,

Continued …..

PROBLEM 13.132 (Cont.)

$$h_m = \frac{n'_A}{A'_s \rho_{A,s}} = \frac{n'_A v_g}{A'_s} = \frac{2.5\,\text{kg/h}\cdot\text{m}}{1\text{m}} \times \frac{1}{3600\ \text{s}} \times 8.82\,\frac{\text{m}^3}{\text{kg}} = 6.13\times10^{-3}\,\text{m/s}$$ <

(c) From the heat and mass transfer analogy, Eq. 6.92,

$$h = h_m \rho c_p Le^{2/3}$$

where $Le = \alpha/D_{AB}$ = 22.5/26.0 = 0.865. Hence

$$h = 6.13\times10^{-3}\,\text{m/s}\left(1.161\,\text{kg/m}^3\right)1007\ \text{J/kg}\cdot\text{K}\,(0.865)^{2/3} = 6.5\ \text{W/m}^2\cdot\text{K}.$$

The corresponding convection heat transfer rate is

$$q'_{conv} = hA'\left(T_g - T_\infty\right) = 6.5\,\text{W/m}^2\cdot\text{K}\,(1\ \text{m})(330-300)\text{K} = 195\,\text{W/m}$$

Since $q'_{conv} << q'_{rad}$, the assumption of negligible convection heat transfer is reasonable.

PROBLEM 13.133

KNOWN: Diameters of coaxial cylindrical drum and heater. Heater emissivity. Temperature and emissivity of pellets covering bottom half of drum. Convection mass transfer coefficient associated with flow of dry air over the pellets.

FIND: (a) Evaporation rate per unit length of drum, (b) Surface temperatures of heater and top half of drum.

SCHEMATIC:

Drum, T_d
$D_d = 1.0$ m
Heater, T_h
$D_h = 0.10$ m, $\varepsilon_h = 0.8$
Pellets, $\rho_{A,sat}$
$T_p = 325$ K, $\varepsilon_p = 0.95$
$h_m = 0.024$ m/s
q'_p q'_{evap}

ASSUMPTIONS: (1) Steady-state, (2) Negligible heat transfer from ends of drum, (3) Diffuse-gray surface behavior, (4) Negligible heat loss from the drum to the surroundings, (5) Negligible convection heat transfer from interior surfaces of the drum, (6) Pellet surface area corresponds to that of bottom half of drum.

PROPERTIES *Table A-6*, sat. water (T = 325 K): $\rho_{A,sat} = v_g^{-1} = 0.0904$ kg/m^3, $h_{fg} = 2378$ kJ/kg.

ANALYSIS: (a) The evaporation rate is

$$n'_A = h_m(\pi D_d/2)\left[\rho_{A,sat}(T_p) - \rho_{A,\infty}\right]$$

$$n'_A = 0.024 \text{ m/s}(\pi \times 1\text{m}/2) \times 0.0904 \text{ kg/m}^3 = 0.00341 \text{ kg/s}\cdot\text{m} \qquad <$$

(b) From an energy balance on the surface of the pellets,

$$q'_p = q'_{evap} = n'_A h_{fg} = 0.00341 \text{ kg/s}\cdot\text{m} \times 2.378 \times 10^6 \text{ J/kg} = 8109 \text{ W/m} \qquad <$$

where q'_p may be determined from analysis of radiation transfer in a three surface enclosure. Since the top half of the enclosure may be treated as reradiating, net radiation transfer to the pellets may be obtained from Eq. 13.30, which takes the form

$$q'_p = \frac{E_{bh} - E_{bp}}{\frac{1-\varepsilon_h}{\varepsilon_h A'_h} + \frac{1}{A'_h F_{hp} + \left[(1/A'_h F_{hd}) + (1/A'_p F_{pd})\right]^{-1}} + \frac{1-\varepsilon_p}{\varepsilon_p A'_p}}$$

where $F_{hp} = F_{hd} = 0.5$, $A'_h = \pi D_h$ and $A'_p = \pi D_d/2$.

The view factor F_{pd} may be obtained from the summation rule,

$$F_{pd} = 1 - F_{ph} - F_{pp}$$

Continued …..

PROBLEM 13.133 (Cont.)

where $F_{ph} = A'_h F_{hp} / A'_p = (\pi D_h \times 0.5)/(\pi D_d / 2) = 0.10$ and

$$F_{pp} = 1 - (2/\pi)\left\{\left[1-(0.1)^2\right]^{1/2} + 0.1 \sin^{-1}(0.1)\right\} = 0.360$$

Hence, $F_{pd} = 1 – 0.10 – 0.360 = 0.540$, and the expression for the heat rate yields

$$8109\ \text{W/m} = \frac{E_{bh} - \sigma(325\ \text{K})^4}{\dfrac{0.25}{\pi \times 0.1\text{m}} + \dfrac{1}{\pi\left\{0.1\text{m}\times 0.5 + \left[(0.1\text{m}\times 0.5)^{-1} + (0.5\text{m}\times 0.54)^{-1}\right]^{-1}\right\}} + \dfrac{0.053}{\pi \times 0.5\text{m}}}$$

$$E_{bh} = \sigma T_h^4 = 35{,}359\ \text{W/m}^2$$

$$T_h = 889\ \text{K}$$ <

Applying Eq. (13.19) to surfaces h and p,

$$J_h = E_{bh} - q'_h (1-\varepsilon_h)/\varepsilon_h A'_h = 35{,}359\ \text{W/m}^2 - 6{,}453\ \text{W/m}^2 = 28{,}906\ \text{W/m}^2$$

$$J_p = E_{bp} + q'_p (1-\varepsilon_p)/\varepsilon_p A'_p = 633\ \text{W/m}^2 + 272\ \text{W/m}^2 = 905\ \text{W/m}^2$$

Hence, from

$$\frac{J_h - J_d}{(A'_h F_{hd})^{-1}} - \frac{J_d - J_p}{(A'_p F_{pd})^{-1}} = 0$$

$$\frac{28{,}906\ \text{W/m}^2 - J_d}{(\pi \times 0.1\text{m} \times 0.5)^{-1}} - \frac{J_d - 905\ \text{W/m}^2}{(\pi \times 0.5\text{m} \times 0.54)^{-1}} = 0$$

$$J_d = \sigma T_d^4 = 24{,}530\ \text{W/m}^2$$

$$T_d = 811\ \text{K}$$ <

COMMENTS: The required value of T_h could be reduced by increasing D_h, although care must be taken to prevent contact of the plastic with the heater.

PROBLEM 14.1

KNOWN: Mixture of O_2 and N_2 with partial pressures in the ratio 0.21 to 0.79.

FIND: Mass fraction of each species in the mixture.

SCHEMATIC:

$$\frac{p_{O_2}}{p_{N2}} = \frac{0.21}{0.79}$$

$$\mathcal{M}_{O_2} = 32 \text{ kg/kmol}$$

$$\mathcal{M}_{N_2} = 28 \text{ kg/kmol}$$

ASSUMPTIONS: (1) Perfect gas behavior.

ANALYSIS: From the definition of the mass fraction,

$$m_i = \frac{\rho_i}{\rho} = \frac{\rho_i}{\Sigma \rho_i}$$

Hence, with

$$\rho_i = \frac{p_i}{R_i T} = \frac{p_i}{(\Re / \mathcal{M}_i) T} = \frac{\mathcal{M}_i p_i}{\Re T}.$$

Hence

$$m_i = \frac{\mathcal{M}_i p_i / \Re T}{\Sigma \mathcal{M}_i p_i / \Re T}$$

or, cancelling terms and dividing numerator and denominator by the total pressure p,

$$m_i = \frac{\mathcal{M}_i x_i}{\Sigma \mathcal{M}_i x_i}.$$

With the mole fractions as

$$x_{O_2} = p_{O_2} / p = \frac{0.21}{0.21 + 0.79} = 0.21$$

$$x_{N_2} = p_{N_2} / p = 0.79,$$

find the mass fractions as

$$m_{O_2} = \frac{32 \times 0.21}{32 \times 0.21 + 28 \times 0.79} = 0.233 \qquad <$$

$$m_{N_2} = 1 - m_{O_2} = 0.767. \qquad <$$

PROBLEM 14.2

KNOWN: Partial pressures and temperature for a mixture of CO_2 and N_2.

FIND: Molar concentration, mass density, mole fraction and mass fraction of each species.

SCHEMATIC:

$p_A = p_B = 1 bar$

$T = 298K$

$A \rightarrow CO_2, M_A = 44$ kg/kmol

$B \rightarrow N_2, M_B = 28$ kg/kmol

ASSUMPTIONS: (1) Perfect gas behavior.

ANALYSIS: From the equation of state for an ideal gas,

$$C_i = \frac{p_i}{\Re T}.$$

Hence, with $p_A = p_B$,

$$C_A = C_B = \frac{1 \text{bar}}{8.314 \times 10^{-2} \text{ m}^3 \cdot \text{bar/kmol} \cdot \text{K} \times 298 \text{K}}$$

$$C_A = C_B = 0.040 \text{ kmol/m}^3.$$ <

With $r_i = M_i C_i$, it follows that

$$r_A = 44 \text{ kg/kmol} \times 0.04 \text{ kmol/m}^3 = 1.78 \text{kg/m}^3$$ <

$$r_B = 28 \text{ kg/kmol} \times 0.04 \text{ kmol/m}^3 = 1.13 \text{ kg/m}^3.$$ <

Also, with

$$x_i = C_i / \Sigma_i C_i$$

find

$$x_A = x_B = 0.04/0.08 = 0.5$$ <

and with

$$m_i = r_i / \Sigma r_i$$

find

$$m_A = 1.78/(1.78 + 1.13) = 0.61$$ <

$$m_B = 1.13/(1.78 + 1.13) = 0.39.$$ <

PROBLEM 14.3

KNOWN: Mole fraction (or mass fraction) and molecular weight of each species in a mixture of n species. Equal mole fractions (or mass fractions) of O_2, N_2 and CO_2 in a mixture.

FIND:

SCHEMATIC:

Species 1,2,... i,... n

$$x_{O_2} = x_{N_2} = x_{CO_2} = 0.333$$

or

$$m_{O_2} = m_{N_2} = m_{CO_2} = 0.333$$

$$\mathcal{M}_{CO_2} = 44$$

$$\mathcal{M}_{O_2} = 32, \ \mathcal{M}_{N_2} = 28$$

ASSUMPTIONS: (1) Perfect gas behavior.

ANALYSIS: (a) With

$$m_i = \frac{r_i}{r} = \frac{r_i}{\sum_i r_i} = \frac{p_i / R_i T}{\sum_i p_i / R_i T} = \frac{p_i \mathcal{M}_i / \Re T}{\sum_i p_i \mathcal{M}_i / \Re T}$$

and dividing numerator and denominator by the total pressure p,

$$m_i = \frac{\mathcal{M}_i x_i}{\sum_i \mathcal{M}_i x_i}.$$ <

Similarly,

$$x_i = \frac{p_i}{\sum_i p_i} = \frac{r_i R_i T}{\sum_i r_i R_i T} = \frac{(r_i / \mathcal{M}_i)\Re T}{\sum_i (r_i / \mathcal{M}_i)\Re T}$$

or, dividing numerator and denominator by the total density r

$$x_i = \frac{m_i / \mathcal{M}_i}{\sum_i m_i / \mathcal{M}_i}.$$ <

(b) With

$$\mathcal{M}_{O_2} x_{O_2} + \mathcal{M}_{N_2} x_{N_2} + \mathcal{M}_{CO_2} x_{CO_2} = 32 \times 0.333 + 28 \times 0.33 + 44 \times 0.333 = 34.6$$

$$m_{O_2} = 0.31, \qquad m_{N_2} = 0.27, \qquad m_{CO_2} = 0.42.$$ <

With

$$m_{O_2} / \mathcal{M}_{O_2} + m_{N_2} / \mathcal{M}_{N_2} + m_{CO_2} / \mathcal{M}_{CO_2} = 0.333/32 + 0.333/28 + 0.333/44$$

$$m_{O_2} = 2.987 \times 10^{-2}.$$

find

$$x_{O_2} = 0.35, \qquad x_{N_2} = 0.40, \qquad x_{CO_2} = 0.25.$$ <

PROBLEM 14.4

KNOWN: Temperature of atmospheric air and water. Percentage by volume of oxygen in the air.

FIND: (a) Mole and mass fractions of water at the air and water sides of the interface, (b) Mole and mass fractions of oxygen in the air and water.

SCHEMATIC:

$x_{O_2,air}$ Air p = 1 atm
T = 17°C

Water (liquid)
T = 17°C
$x_{O_2,liq}$

ASSUMPTIONS: (1) Perfect gas behavior for air and water vapor, (2) Thermodynamic equilibrium at liquid/vapor interface, (3) Dilute concentration of oxygen and other gases in water, (4) Molecular weight of air is independent of vapor concentration.

PROPERTIES: *Table A-6*, Saturated water (T = 290 K): $p_{vap} = 0.01917$ bars. *Table A-9*, O_2/water, H = 37,600 bars.

ANALYSIS: (a) Assuming ideal gas behavior, $p_{w,vap} = (N_{w,vap}/V) \bullet T$ and $p = (N/V) \bullet T$, in which case

$$x_{w,vap} = \left(p_{w,vap} / p_{air}\right) = (0.01917/1.0133) = 0.0194$$ <

With $m_{w,vap} = (\rho_{w,vap}/\rho_{air}) = (C_{w,vap}\mathcal{M}_w/C_{air}\mathcal{M}_{air}) = x_{w,vap}(\mathcal{M}_w/\mathcal{M}_{air})$. Hence,

$$m_{w,vap} = 0.0194\ (18/29) = 0.0120$$ <

Assuming negligible gas phase concentrations in the liquid,

$$x_{w,liq} = m_{w,liq} = 1$$ <

(b) Since the partial volume of a gaseous species is proportional to the number of moles of the species, its mole fraction is equivalent to its volume fraction. Hence on the air side of the interface

$$x_{O2,air} = 0.205$$ <

$$m_{O2,air} = x_{O2,air}\left(M_{O_2} / M_{air}\right) = 0.205(32/29) = 0.226$$ <

The mole fraction of O_2 in the water is

$$x_{O2,liq} = p_{O2,air} / H = 0.208\text{ bars} / 37{,}600\text{ bars} = 5.53\times10^{-6}$$ <

where $p_{O2,air} = x_{O2,air}\ p_{atm} = 0.205 \times 1.0133$ bars = 0.208 bars. The mass fraction of O_2 in the water is

$$m_{O2,liq} = x_{O2,liq}\left(M_{0_2} / M_w\right) = 5.53\times10^{-6}(32/18) = 9.83\times10^{-6}$$ <

COMMENTS: There is a large discontinuity in the oxygen content between the air and water sides of the interface. Despite the low concentration of oxygen in the water, it is sufficient to support the life of aquatic organisms.

PROBLEM 14.5

KNOWN: Air is enclosed at uniform pressure in a vertical, cylindrical container whose top and bottom surfaces are maintained at different temperatures.

FIND: (a) Conditions in air when bottom surface is colder than top surface, (b) Conditions when bottom surface is hotter than top surface.

SCHEMATIC:

ASSUMPTIONS: (1) Uniform pressure, (2) Perfect gas behavior.

ANALYSIS: (a) If $T_1 > T_2$, the axial temperature gradient (dT/dx) will result in an axial density gradient. However, since $d\rho/dx < 0$ there will be no buoyancy driven, convective motion of the mixture.

There will also be axial species density gradients, $d r_{O_2}/dx$ and $d r_{N_2}/dx$. However, there is no gradient associated with the mass fractions $\left(dm_{O_2}/dx = 0, d\,m_{N_2}/dx = 0\right)$. Hence, from Fick's law, Eq. 14.1, there is *no* mass transfer by diffusion.

(b) If $T_1 < T_2$, $dr/dx > 0$ and there will be a buoyancy driven, convective motion of the mixture. However, $dm_{O_2}/dx = 0$ and $dm_{N_2}/dx = 0$, and there is still *no* mass transfer. Hence, although there is motion of each species with the convective motion of the mixture, there is *no relative motion* between species.

COMMENTS: The commonly used special case of Fick's law,

$$j_A = -D_{AB}\frac{d r_A}{dx}$$

would be inappropriate for this problem since r is not uniform. If applied, this special case indicates that mass transfer would occur, thereby providing an incorrect result.

PROBLEM 14.6

KNOWN: Pressure and temperature of hydrogen stored in a spherical steel tank of prescribed diameter and thickness.

FIND: (a) Initial rate of hydrogen mass loss from the tank, (b) Initial rate of pressure drop in the tank.

SCHEMATIC:

D=100mm
Hydrogen (A)
p=10bar
T=300K
$C_{A,o}$=1.5 kmol/m³
$C_{A,L}$=0
$\dot{m}_{A,st}$
Steel (B)
D_{AB}= 0.3×10⁻¹² m²/s
$n_{A,r}$
L=2mm
$\mathcal{M}_A$=2kg/kmol

ASSUMPTIONS: (1) One-dimensional species diffusion in a stationary medium, (2) Uniform total molar concentration, C, (3) No chemical reactions.

ANALYSIS: (a) From Table 14.1

$$N_{A,r} = \frac{C_{A,o} - C_{A,L}}{R_{m,dif}} = \frac{C_{A,o}}{(1/4p D_{AB})(1/r_i - 1/r_o)}$$

$$N_{A,r} = \frac{4p\left(0.3\times10^{-12}\ m^2/s\right)1.5\ \ kmol/m^3}{(1/0.05\ m - 1/0.052\ m)} = 7.35\times10^{-12}\ kmol/s$$

or

$$n_{A,r} = M_A N_{A,r} = 2\ kg/kmol\times7.35\times10^{-12}\ kmol/s = 14.7\times10^{-12}\ kg/s. \qquad <$$

(b) Applying a species balance to a control volume about the hydrogen,

$$\dot{M}_{A,st} = -\dot{M}_{A,out} = -n_{A,r}$$

$$\dot{M}_{A,st} = \frac{d(r_A V)}{dt} = \frac{p D^3}{6}\frac{d r_A}{dt} = \frac{p D^3}{6 R_A T}\frac{dp_A}{dt} = \frac{p D^3 M_A}{6\Re T}\frac{dp_A}{dt}$$

Hence

$$\frac{dp_A}{dt} = -\frac{6\Re T}{p D^3 M_A} n_{A,r} = -\frac{6\left(0.08314\ m^3\cdot bar/kmol\cdot K\right)(300\ K)}{p\left(0.1 m^3\right)2\ kg/kmol}\times14.7\times10^{-12}\ kg/s$$

$$\frac{dp_A}{dt} = -3.50\times10^{-7}\ bar/s. \qquad <$$

COMMENTS: If the spherical shell is appoximated as a plane wall, $N_{a,x} = D_{AB}(C_{A,o})\ \pi D^2/L = 7.07 \times 10^{-12}$ kmol/s. This result is 4% lower than that associated with the spherical shell calculation.

PROBLEM 14.7

KNOWN: Molar concentrations of helium at the inner and outer surfaces of a plastic membrane. Diffusion coefficient and membrane thickness.

FIND: Molar diffusion flux.

SCHEMATIC:

$L = 0.001 m$

A (helium), B (plastic) $D_{AB} = 10^{-9} m^2/s$

$N''_{A,x}$

$C_{A,1} = 0.02 kmol/m^3$ $C_{A,2} = 0.005 kmol/m^3$

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional diffusion in a plane wall, (3) Stationary medium, (4) Uniform $C = C_A + C_B$.

ANALYSIS: The molar flux may be obtained from Eq. 14.50,

$$N''_{A,x} = \frac{D_{AB}}{L}\left(C_{A,1} - C_{A,2}\right) = \frac{10^{-9}\ m^2/s}{0.001 m}\left(0.02 - 0.005\right) kmol/m^3$$

$$N''_{A,x} = 1.5 \times 10^{-8}\ kmol/s \cdot m^2.$$

<

COMMENTS: The mass flux is

$$n''_{A,x} = \mathcal{M}_A N''_{A,x} = 4\ kg/kmol \times 1.5 \times 10^{-8}\ kmol/s \cdot m^2 = 6 \times 10^{-8}\ kg/s \cdot m^2.$$

PROBLEM 14.8

KNOWN: Mass diffusion coefficients of two binary mixtures at a given temperature, 298 K.

FIND: Mass diffusion coefficients at a different temperature, T = 350 K.

ASSUMPTIONS: (a) Ideal gas behavior, (b) Mixtures at 1 atm total pressure.

PROPERTIES: *Table A-8*, Ammonia-air binary mixture (298 K), $D_{AB} = 0.28 \times 10^{-4}\ m^2/s$; Hydrogen-air binary mixture (298 K), $D_{AB} = 0.41 \times 10^{-4}\ m^2/s$.

ANALYSIS: According to treatment of Section 14.1.5, assuming ideal gas behavior,

$$D_{AB} \sim T^{3/2}$$

where T is in kelvin units. It follows then, that for

$$NH_3 - Air: \quad D_{AB}(350\ K) = 0.28 \times 10^{-4} m^2/s (350\ K/298\ K)^{3/2}$$

$$D_{AB}(350\ K) = 0.36 \times 10^{-4} m^2/s$$

$<$

$$H_2 - Air: \quad D_{AB}(350\ K) = 0.41 \times 10^{-4} m^2/s (350/298)^{3/2}$$

$$D_{AB}(350\ K) = 0.52 \times 10^{-4} m^2/s$$

$<$

COMMENTS: Since the H_2 molecule is smaller than the NH_3 molecule, it follows that

$$D_{H_2-Air} > D_{NH_3-Air}$$

as indeed the numerical data indicate.

PROBLEM 14.9

KNOWN: The inner and outer surfaces of an iron cylinder of 100-mm length are exposed to a carburizing gas (mixtures of CO and CO_2). Observed experimental data on the variation of the carbon composition (weight carbon, %) in the iron at 1000°C as a function of radius. Carbon flow rate under steady-state conditions.

FIND: (a) Beginning with Fick's law, show that $d\rho_C / d(\ell n(r))$ is a constant if the diffusion coefficient, $D_{C\text{-}Fe}$, is a constant; sketch of the carbon mass density, $\rho_c(r)$, as function of ln(r) for such a diffusion process; (b) Create a graph for the experimental data and determine whether $D_{C\text{-}Fe}$ for this diffusion process is constant, increases or decreases with increasing mass density; and (c) Using the experimental data, calculate and tabulate $D_{C\text{-}Fe}$ for selected carbon compositions over the range of the experiment.

SCHEMATIC:

CO, CO_2

CO, CO_2

Iron cylinder, 1000°C
$\rho_c(r)$, L = 100 mm

Carbon flow rate
3.6×10^{-3} kg in 100 h

r_i = 4.30 mm
r_o = 5.70 mm

r

PROPERTIES: *Iron* (1000°C). ρ = 7730 kg/m^3.Experimental observations of carbon composition

r (mm)	4.49	4.66	4.79	4.91	5.16	5.27	5.40	5.53
Wt. C (%)	1.42	1.32	1.20	1.09	0.82	0.65	0.46	0.28

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional, radial diffusion in a stationary medium, and (3) Uniform total concentration.

ANALYSIS: (a) For the one-dimensional, radial (cylindrical) coordinate system, Fick's law is

$$j_A = -D_{AB}\, A_r \frac{d\rho_A}{dr} \qquad (1)$$

where $A_r = 2\pi rL$. For steady-state conditions, j_A is constant, and if D_{AB} is constant, the product

$$r\frac{d\rho_A}{dr} = C_1 \qquad (2)$$

must be a constant. Using the differential relation dr/r = d (ln r), it follows that

$$\frac{d\rho_A}{d\,(\ln r)} = C_1 \qquad (3)$$

so that on a ln(r) plot, ρ_A is a straight line. See the graph below for this behavior.

Continued …..

PROBLEM 14.9 (Cont.)

(b) To determine whether $D_{C\text{-}Fe}$ is a constant for the experimental diffusion process, the data are represented on a ln(r) coordinate.

Wt. carbon distribution - experimental observations

Wt Carbon (%)

Exp data

ln (r, mm)

Since the plot is not linear, $D_{C\text{-}Fe}$ is not a constant. From the treatment of part (a), if D_{AB} is not a constant, then

$$D_{AB}\frac{d\rho_A}{d(\ln r)} = C_2$$

must be constant. We conclude that $D_{C\text{-}Fe}$ will be lower at the radial position where the gradient is higher. Hence, we expect $D_{C\text{-}Fe}$ to increase with increasing carbon content.

(c) From a plot of Wt - %C vs. r (not shown), the mass fraction gradient is determined at three locations and Fick's law is used to calculate the diffusion coefficient,

$$j_C = -\rho \cdot A_r \cdot D_{C-Fe}\ \frac{\Delta\,(\text{Wt} - \%\ \text{C})}{\Delta r}$$

where the mass flow rate is

$$j_C = 3.6\times10^{-3}\ \text{kg}/100\ \text{h}\ (3600\ \text{s}/\text{h}) = 1\times10^{-8}\ \text{kg}/\text{s}$$

and ρ = 7730 kg/m^3, density of iron. The results of this analysis yield,

Wt - C (%)	r (mm)	Δ Wt-C/Δr (%/mm)	$D_{C\text{-}Fe}\times10^{11}$ (m^2/s)
1.32	4.66	-0.679	6.51
0.955	5.04	-1.08	3.79
0.37	5.47	-1.385	2.72

PROBLEM 14.10

KNOWN: Three-dimensional diffusion of species A in a stationary medium with chemical reactions.

FIND: Derive molar form of diffusion equation.

SCHEMATIC:

ASSUMPTIONS: (1) Uniform total molar concentration, (2) Stationary medium.

ANALYSIS: The derivation parallels that of Section 14.2.2, except that Eq. 14.33 is applied on a molar basis. That is,

$$N_{A,x} + N_{A,y} + N_{A,z} + \dot{N}_{A,g} - N_{A,x+dx} - N_{A,y+dy} - N_{A,z+dz} = \dot{N}_{A,st}.$$

With

$$N_{A,x+dx} = N_{A,x} + \frac{\partial N_{A,x}}{\partial x}dx, \qquad N_{A,y+dy} = ...$$

$$N_{A,x} = -D_{AB}\left(dydz\right)\frac{\partial C_A}{\partial x}, \qquad N_{A,y} = ...$$

$$\dot{N}_{A,g} = \dot{N}_A\left(dxdydz\right), \qquad \dot{N}_{A,st} = \frac{\partial C_A}{\partial t}dxdydz$$

It follows that

$$\frac{\partial}{\partial x}\left(D_{AB}\frac{\partial C_A}{\partial x}\right) + \frac{\partial}{\partial y}\left(D_{AB}\frac{\partial C_A}{\partial y}\right) + \frac{\partial}{\partial z}\left(D_{AB}\frac{\partial C_A}{\partial z}\right) + \dot{N}_A = \frac{\partial C_A}{\partial t}.$$ <

COMMENTS: If D_{AB} is constant, the foregoing result reduces to Eq. 14.38b.

PROBLEM 14.11

KNOWN: Gas (A) diffuses through a cylindrical tube wall (B) and experiences chemical reactions at a volumetric rate, $\dot{N}_A$.

FIND: Differential equation which governs molar concentration of gas in plastic.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional radial diffusion, (2) Uniform total molar concentration, (3) Stationary medium.

ANALYSIS: Dividing the species conservation requirement, Eq. 14.33, by the molecular weight, $_A$, and applying it to a differential control volume of unit length normal to the page,

$$N_{A,r} + \dot{N}_{A,g} - N_{A,r+dr} = \dot{N}_{A,st}$$

where

$$N_{A,r} = (2pr \cdot 1)\, N''_{A,r} = -2p\, r D_{AB} \frac{\partial C_A}{\partial r}$$

$$N_{A,r+dr} = N_{A,r} + \frac{\partial N_{A,r}}{\partial r} dr$$

$$\dot{N}_{A,g} = -\dot{N}_A \left(2p\, r \cdot dr \cdot 1\right) \qquad \dot{N}_{A,st} = \frac{\partial \left[C_A \left(2p\, rdr \cdot 1\right)\right]}{\partial t}.$$

Hence

$$-\dot{N}_A \left(2p\, rdr\right) + 2p\, D_{AB} \frac{\partial}{\partial r}\left(r \frac{\partial C_A}{\partial r}\right) dr = 2p\, rdr \frac{\partial C_A}{\partial t}$$

or

$$\frac{D_{AB}}{r} \frac{\partial}{\partial r}\left(r \frac{\partial C_A}{\partial r}\right) - \dot{N}_A = \frac{\partial C_A}{\partial t}. \qquad <$$

COMMENTS: (1) The minus sign in the generation term is necessitated by the fact that the reactions deplete the concentration of species A.

(2) From knowledge of $\dot{N}_A\ (r,t)$, the foregoing equation could be solved for C_A (r,t).

(3) Note the agreement between the above result and the one-dimensional form of Eq. 14.39 for uniform C.

PROBLEM 14.12

KNOWN: One-dimensional, radial diffusion of species A in a stationary, spherical medium with chemical reactions.

FIND: Derive appropriate form of diffusion equation.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional, radial diffusion, (2) Uniform total molar concentration, (3) Stationary medium.

ANALYSIS: Dividing the species conservation requirement, Eq. 14.33, by the molecular weight, $_A$, and applying it to the differential control volume, it follows that

$$N_{A,r} + \dot{N}_{A,g} - N_{A,r+dr} = \dot{N}_{A,st}$$

where

$$N_{A,r} = -D_{AB} 4\pi r^2 \frac{\partial C_A}{\partial r}$$

$$N_{A,r+dr} = N_{A,r} + \frac{\partial N_{A,r}}{\partial r} dr$$

$$\dot{N}_{A,g} = \dot{N}_A \left(4\pi r^2 dr\right), \qquad \dot{N}_{A,st} = \frac{\partial\left[C_A\left(4\pi r^2 dr\right)\right]}{\partial t}.$$

Hence

$$\dot{N}_A \left(4\pi r^2 dr\right) + 4\pi \frac{\partial}{\partial r}\left(D_{AB} r^2 \frac{\partial C_A}{\partial r}\right) dr = 4\pi r^2 \frac{\partial C_A}{\partial t} dr$$

or

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(D_{AB} r^2 \frac{\partial C_A}{\partial r}\right) + \dot{N}_A = \frac{\partial C_A}{\partial t}. \qquad <$$

COMMENTS: Equation 14.40 reduces to the foregoing result if C is independent of r and variations in ϕ and θ are negligible.

PROBLEM 14.13

KNOWN: Oxygen pressures on opposite sides of a rubber membrane.

FIND: (a) Molar diffusion flux of O_2, (b) Molar concentrations of O_2 outside the rubber.

SCHEMATIC:

Rubber (B)
Oxygen (A)
$p_{A,1} = 2\text{bar}$
$C_{A,1}$
$C_A(0)$
$N''_{A,x}$
$C_A(L)$
Oxygen (A)
$T = 25°C, C_{A,2}$
$p_{A,2} = 1\text{bar}$
$L = 0.5\text{mm}$
x

ASSUMPTIONS: (1) One-dimensional, steady-state conditions, (2) Stationary medium of uniform total molar concentration, $C = C_A + C_B$, (3) Perfect gas behavior.

PROPERTIES: *Table A-8*, Oxygen-rubber (298 K): $D_{AB} = 0.21 \times 10^{-9}\ m^2/s$; *Table A-10*, Oxygen-rubber (298 K): $S = 3.12 \times 10^{-3}\ kmol/m^3 \cdot bar$.

ANALYSIS: (a) For the assumed conditions

$$N''_{A,x} = J^*_{A,x} = -D_{AB}\frac{dC_A}{dx} = D_{AB}\frac{C_A(0) - C_A(L)}{L}.$$

From Eq. 14.33,

$$C_A(0) = Sp_{A,1} = 6.24 \times 10^{-3} \text{kmol/m}^3$$

$$C_A(L) = Sp_{A,2} = 3.12 \times 10^{-3} \text{lmol/m}^3.$$

Hence

$$N''_{A,x} = 0.21 \times 10^{-9}\ m^2/s \frac{\left(6.24 \times 10^{-3} - 3.12 \times 10^{-3}\right)\text{kmol/m}^3}{0.0005\text{ m}}$$

$$N''_{A,x} = 1.31 \times 10^{-9} \text{kmol/s} \cdot \text{m}^2. \qquad <$$

(b) From the perfect gas law

$$C_{A,1} = \frac{p_{A,1}}{\Re T} = \frac{2\text{ bar}}{\left(0.08314\text{ m}^3 \cdot \text{bar/kmol} \cdot \text{K}\right) 298\text{ K}} = 0.0807\text{ kmol/m}^3 \qquad <$$

$$C_{A,2} = 0.5 C_{A,1} = 0.0404\text{ kmol/m}^3. \qquad <$$

COMMENTS: Recognize that the molar concentrations outside the membrane differ from those within the membrane; that is, $C_{A,1} \neq C_A(0)$ and $C_{A,2} \neq C_A(L)$.

PROBLEM 14.14

KNOWN: Water vapor is transferred through dry wall by diffusion.

FIND: The mass diffusion rate through a $0.01 \times 3 \times 5$ m wall.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional species diffusion, (3) Homogeneous medium, (4) Constant properties, (5) Uniform total molar concentration, (6) Stationary medium with $x_A << 1$, (7) Negligible condensation in the dry wall.

ANALYSIS: From Eq. 14.46,

$$N''_{A,x} = -CD_{AB}\frac{dx_A}{dx} = -D_{AB}\frac{dC_A}{dx} = D_{AB}\frac{C_{A,1} - C_{A,2}}{L}.$$

From Eq. 14.33

$$C_{A,1} = Sp_{A,1} = 0.15 \times 10^{-3}\,\text{kmol/m}^3$$

$$C_{A,2} = Sp_{A,2} = 0\ \text{kmol/m}^3.$$

Hence

$$N''_A = 10^{-9}\,\text{m}^2/\text{s} \times \frac{0.15 \times 10^{-3}\,\text{kmol/m}^3}{0.01\text{m}} = 0.15 \times 10^{-10}\ \text{kmol/s} \cdot \text{m}^2.$$

Therefore

$$n_A = \mathcal{M}_A\left(A \cdot N''_A\right) = 18\text{kg/kmol} \times 15\ \text{m}^2 \times 0.15 \times 10^{-10}\,\text{kmol/s} \cdot \text{m}^2$$

or

$$n_A = 4.05 \times 10^{-9}\,\text{kg/s}.$$ <

PROBLEM 14.15

KNOWN: Pressure and temperature of CO_2 in a container of prescribed volume. Thickness and surface area of rubber plug.

FIND: (a) Mass rate of CO_2 loss from container, (b) Reduction in pressure over a 24 h period.

SCHEMATIC:

ASSUMPTIONS: (1) Loss of CO_2 is only by diffusion through the rubber plug, (2) One-dimensional diffusion through a stationary medium, (3) Diffusion rate is constant over the 24 h period, (4) Perfect gas behavior, (5) Negligible CO_2 pressure outside the plug.

PROPERTIES: *Table A-8*, CO_2-rubber (298 K, 1 atm): $D_{AB} = 0.11 \times 10^{-9}\ m^2/s$; *Table A-10*, CO_2-rubber (298 K, 1 atm): $S = 40.15 \times 10^{-3}\ kmol/m^3 \cdot bar$.

ANALYSIS: (a) For diffusion through a stationary medium,

$$N_A = AD_{AB}\frac{C_{A,1} - C_{A,2}}{L}$$

where

$$C_{A,1} = Sp_{A,1} = 40.15\times10^{-3}\ kmol/m^3 \cdot bar\times 5bar = 0.200\ kmol/m^3$$

$$C_{A,2} = Sp_{A,2} = 0.$$

Hence

$$N_A = 3\times10^{-4}m^2\left(0.11\times10^{-9}\ m^2/s\right)\frac{(0.200-0)\,kmol/m^3}{0.02\ m} = 3.30\times10^{-13}kmol/s$$

and

$$n_A = \mathcal{M}_A N_A = 44\ kg/kmol\times3.30\times10^{-13}kmol/s = 1.45\times10^{-11}\ kg/s. \quad <$$

(b) Applying conservation of mass to a control volume about the container

$$\frac{d(r_A V)}{dt} = -n_A \qquad \text{or} \qquad \frac{d(C_A V)}{dt} = -N_A.$$

Hence, with $C_A = p_A/\Re T$,

$$\frac{dp_A}{dt} = -\frac{N_A\Re T}{V} = -\frac{3.3\times10^{-13}kmol/s\times8.314\times10^{-2}m^3\cdot bar/kmol\cdot K\,(298K)}{10^{-2}\ m^3} = -8.18\times10^{-10}\ bar/s.$$

Hence

$$\Delta p_A = \left(\frac{dp_A}{dt}\right)\Delta t = -8.18\times10^{-10}\ bar/s\times24h\times3600s/h = 7.06\times10^{-5}\ bar. \quad <$$

PROBLEM 14.16

KNOWN: Pressure and temperature of helium in a glass cylinder of 100 mm inside diameter and 5 mm thickness.

FIND: Mass rate of helium loss per unit length.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional radial diffusion through cylinder wall, (3) Negligible end losses, (4) Stationary medium, (5) Uniform total molar concentration, (6) Negligible helium concentration outside cylinder.

PROPERTIES: *Table A-8*, He-SiO_2 (298 K): $D_{AB} \approx 0.4 \times 10^{-13}$ m^2/s; *Table A-10*, He-SiO_2 (298 K): $S \approx 0.45 \times 10^{-3}$ $kmol/m^3 \cdot bar$.

ANALYSIS: From Table 14.1,

$$N'_{A,r} = \frac{C_{A,S1} - C_{A,S2}}{\ln(r_2/r_1)/2\pi D_{AB}}$$

where, from Eq. 14.44, $C_{A,S} = Sp_A$. Hence

$$C_{A,S1} = Sp_{A,1} = 0.45 \times 10^{-3}\ kmol/m^3 \cdot bar \times 4\ bar = 1.8 \times 10^{-3} kmol/m^3$$

$$C_{A,S2} = SP_{A,2} = 0.$$

Hence

$$N'_{A,r} = \frac{1.8 \times 10^{-3}\ kmol/m^3}{\ln(0.055/0.050)/2\pi\left(0.4 \times 10^{-13}\ m^2/s\right)}$$

$$N'_{A,r} = 4.75 \times 10^{-15}\ kmol/s \cdot m.$$

The mass loss is then

$$n'_{A,r} = M_A N'_{A,r} = 4\ kg/kmol \times 4.75 \times 10^{-15}\ kmol/s \cdot m$$

$$n'_{A,r} = 1.90 \times 10^{-14}\ kg/s \cdot m.$$

<

PROBLEM 14.17

KNOWN: Temperature and pressure of helium stored in a spherical pyrex container of prescribed diameter and wall thickness.

FIND: Mass rate of helium loss.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Helium loss by one-dimensional diffusion in radial direction through the pyrex, (3) $C = C_A + C_B$ is independent of r, and $x_A << 1$, (4) Stationary medium.

PROPERTIES: *Table A-8*, He-SiO_2 (293 K): $D_{AB} = 0.4 \times 10^{-13}$ m^2/s; *Table A-10*, He-SiO_2 (293 K): $S = 0.45 \times 10^{-3}$ $kmol/m^3 \cdot bar$.

ANALYSIS: From Table 14.1, the molar diffusion rate may be expressed as

$$N_{A,r} = \frac{C_{A,S1} - C_{A,S2}}{R_{m,dif}}$$

where

$$R_{m,dif} = \frac{1}{4\pi D_{AB}}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = \frac{1}{4\pi\left(0.4\times10^{-13} m^2/s\right)}\left(\frac{1}{0.1m} - \frac{1}{0.11m}\right) = 1.81\times10^{12} s/m^3$$

with

$$C_{A,S1} = Sp_A = 0.45\times10^{-3} kmol/m^3 \cdot bar \times 4\ bar = 1.80\times10^{-3} kmol/m^3$$

$$C_{A,S2} = 0$$

find

$$N_{A,r} = \frac{1.80\times10^{-3} kmol/m^3}{1.81\times10^{12}\ s/m^3} = 10^{-15}\ kmol/s.$$

Hence

$$n_{A,r} = \mathcal{M}_A N_{A,r} = 4\ kg/mol \times 10^{-15} kmol/s = 4\times10^{-15} kg/s.$$ <

COMMENTS: Since $r_1 \approx r_2$, the spherical shell could have been approximated as a plane wall with L = 0.01 m and $A \approx 4\pi r_m^2 = 0.139\ m^2$. From Table 14.1,

$$R_{m,dif} = \frac{L}{D_{AB}A} = \frac{0.01m}{\left(0.4\times10^{-13} m^2/s\right)\left(0.137\ m^2\right)} = 1.8\times10^{12} s/m^3$$

and

$$N_{A,x} = \frac{C_{A,S1} - C_{A,S2}}{R_{m,dif}} = \frac{1.80\times10^{-3} kmol/m^3}{1.8\times10^{12} s/m^3} = 10^{-15} kmol/s.$$

Hence the approximation is excellent.

PROBLEM 14.18

KNOWN: Pressure and temperature of hydrogen inside and outside of a circular tube. Diffusivity and solubility of hydrogen in tube wall of prescribed thickness and diameter.

FIND: Rate of hydrogen transfer through tube per unit length.

SCHEMATIC:

$p_{A,1} = 2\,atm$; $p_{A,2} = 0.1\,atm$; $r_1 = 20\,mm$; $r_2 = 20.5\,mm$

(A) → H_2
(B) → Solid
$D_{AB} = 1.8 \times 10^{-11}\,m^2/s$
$S_{AB} = 160\,kmol/m^3 \cdot atm$

ASSUMPTIONS: (1) Steady diffusion in radial direction, (2) Uniform total molar concentration in wall, (3) No chemical reactions.

ANALYSIS: The mass transfer rate per unit tube length is

$$N'_{A,r} = \frac{C_A(r_1) - C_A(r_2)}{\ln(r_2/r_1)/2\pi D_{AB}}$$

where from Eq. 14.44, $C_{A,s} = S p_a$,

$$C_A(r_1) = S p_{A,1} = 160\ kmol/m^3 \cdot atm \times 2\ atm = 320\ kmol/m^3$$

$$C_A(r_2) = S p_{A,2} = 160\ kmol/m^3 \cdot atm \times 0.1 atm = 16\ kmol/m^3.$$

Hence,

$$N'_{A,r} = \frac{(320 - 16)\,kmol/m^3}{\ln(20.5/20)/2\pi \times 1.8 \times 10^{-11}\ m^2/s} = \frac{304\ kmol/m^3}{2.18 \times 10^8\ s/m^2}$$

$$N'_{A,r} = 1.39 \times 10^{-6}\ kmol/s \cdot m.$$ <

COMMENTS: If the wall were assumed to be plane,

$$R'_{m,dif} = \frac{L}{D_{AB}\pi D} = \frac{5 \times 10^{-4}\ m}{1.8 \times 10^{-11}\ m^2/s\,\pi\,(0.04\ m)} = 2.21 \times 10^8\ s/m^2$$

which is close to the value of $2.18 \times 10^8\ s/m^2$ for the cylindrical wall.

PROBLEM 14.19

KNOWN: Dimensions of nickel tube and pressure of hydrogen flow through the tube. Diffusion coefficient.

FIND: Mass rate of hydrogen diffusion per unit tube length.

SCHEMATIC:

Air

Nickel, $D_{AB} = 10^{-12}$ m^2/s

H_2

$p_{A,i}$ = 4 bar

D_i = 0.025 m

D_o = 0.028 m

ASSUMPTIONS: (1) Steady-state, (2) One-dimensional diffusion through tube wall, (3) Negligible pressure of H_2 in ambient air, (4) Tube wall is a stationary medium of uniform total molar concentration, (5) Constant properties.

PROPERTIES: *Table A-10* (H_2 – Ni): $S = 9.01 \times 10^{-3}$ kmol/m^3·bar.

ANALYSIS: From Table 14.1, the resistance to diffusion per unit tube length is $R_{m,dif} = \ln(D_o/D_i)/2\pi D_{AB}$, and the molar rate of hydrogen diffusion per unit length is

$$N_{A,r} = \frac{2\pi D_{AB}\left(C_{A,si} - C_{A,so}\right)}{\ln\left(D_o / D_i\right)}$$

From Eq. (14.44), the tube wall molar concentrations are

$$C_{A,si} = S\, p_{A,i} = 9.01 \times 10^{-3}\ \text{kmol/m}^3 \cdot \text{bar} \times 4\ \text{bar} = 0.036\ \text{kmol/m}^3$$

$$C_{A,so} = S\, p_{A,o} = 0$$

$$N_{A,r} = \frac{2\pi \times 10^{-12}\,\text{m}^2/\text{s} \times 0.036\ \text{kmol/m}^3}{\ln\left(0.028/0.025\right)} = 2.00 \times 10^{-12}\ \text{kmol/s}\cdot\text{m}$$

With $\mathcal{M}_A = 2\,\text{kg/kmol for } H_2$,

$$n_{A,r} = \mathcal{M}_A N_{A,r} = 2\,\text{kg/kmol} \times 2.00 \times 10^{-12}\,\text{kmol/s}\cdot\text{m} = 4.00 \times 10^{-12}\ \text{kg/s}\cdot\text{m}$$ <

COMMENTS: The hydrogen loss is miniscule.

PROBLEM 14.20

KNOWN: Conditions of the exhaust gas passing over a catalytic surface for the removal of NO.

FIND: (a) Mole fraction of NO at the catalytic surface, (b) NO removal rate.

SCHEMATIC:

Exhaust gases
T=500°C, p=1.2bar
x=L=1mm
$x_{A,L} = 0.15$
N''_{NO}
Mixture, $D_{AB}=10^{-4}\,m^2/s$
Catalytic surface,
$k_1'' = 0.05 m/s$, $A = 200 cm^2$
$x_{A,s}, N_{A,s}$

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional species diffusion through the film, (3) Effects of bulk motion on NO transfer in the film are negligible, (4) No homogeneous reactions of NO within the film, (5) Constant properties, including the total molar concentration, C, throughout the film.

ANALYSIS: Subject to the above assumptions, the transfer of species A (NO) is governed by diffusion in a stationary medium, and the desired results are obtained from Eqs. 14.60 and 14.61. Hence

$$\frac{x_{A,s}}{x_{A,L}} = \frac{1}{1+\left(Lk_1''/D_{AB}\right)} \qquad x_{A,s} = \frac{0.15}{1+0.001\text{m}\times 0.05\text{ m/s}/\ 10^{-4}\text{ m}^2/\text{s}} = 0.10.$$ <

Also

$$N''_{A,s} = -\frac{k_1'' C x_{A,L}}{1+\left(Lk_1''/D_{AB}\right)}$$

where, from the equation of state for a perfect gas,

$$C = \frac{p}{\Re T} = \frac{1.2\text{ bar}}{8.314\times 10^{-2}\text{m}^3\cdot\text{bar/kmol}\cdot\text{K}\times 773\text{ K}} = 0.0187\text{ kmol/m}^3.$$

Hence

$$N''_{A,s} = -\frac{0.05\text{ m/s}\times 0.0187\text{ kmol/m}^3\times 0.15}{1+\left(0.001\text{m}\times 0.05\text{m/s}/10^{-4}\text{ m}^2/\text{s}\right)} = -9.35\times 10^{-5}\text{ kmol/s}\cdot\text{m}^2$$

or

$$n''_{A,S} = \mathcal{M}_A N''_{A,S} = 30\text{ kg/kmol}\left(-9.35\times 10^{-5}\text{ kmol/s}\cdot\text{m}^2\right) = -2.80\times 10^{-3}\text{ kg/s}\cdot\text{m}^2.$$

The molar rate of NO removal for the entire surface is then

$$N_{A,s} = N''_{A,s}A = -9.35\times 10^{-5}\text{ kmol/s}\cdot\text{m}^2\times 0.02\text{ m}^2 = -1.87\times 10^{-6}\text{ kmol/s}$$

or

$$n_{A,S} = -5.61\times 10^{-5}\text{kg/s}.$$ <

COMMENTS: Because bulk motion is likely to contribute significantly to NO transfer within the film, the above results should be viewed as a first approximation.

PROBLEM 14.21

KNOWN: Radius of coal pellets burning in oxygen atmosphere of prescribed pressure and temperature.

FIND: Oxygen molar consumption rate.

SCHEMATIC:

$T = 1450K$, $p = 1atm$

$r_o = 0.001m$

$O_2(A)$

$CO_2(B)$

$\dot{N}_A'' = -k_1'' C_A(r_o)$

$C = C_A + C_B$

$D_{AB} = 1.71 \times 10^{-4} m^2/s$

$k_1'' = 0.1\ m/s$

ASSUMPTIONS: (1) One-dimensional diffusion in r, (2) Steady-state conditions, (3) Constant properties, (4) Perfect gas behavior, (5) Uniform C and T.

ANALYSIS: From Equation 14.53,

$$\frac{d}{dr}\left(r^2 \frac{dC_A}{dr}\right) = 0$$

$$dC_A/dr = C_1/r^2 \qquad \text{or} \qquad C_A = -C_1/r + C_2.$$

The boundary conditions at $r \to \infty$ and $r = r_o$ are, respectively,

$$C_A(\infty) = C \;\; \to \;\; C_2 = C$$

$$\dot{N}_A'' = N_A''(r_o) = -CD_{AB} \left.\frac{dx_A}{dr}\right|_{r_o} = -D_{AB} \left.\frac{dC_A}{dr}\right|_{r_o}$$

Hence

$$-k_1''(-C_1/r_o + C) = -D_{AB} C_1/r_o^2$$

$$k_1''(C_1/r_o) + D_{AB}\left(C_1/r_o^2\right) = k_1'' C \qquad \text{or} \qquad C_1 = \frac{k_1'' C}{(k_1''/r_o) + \left(D_{AB}/r_o^2\right)}.$$

The oxygen molar consumption rate is

$$N_A''(r_o) = -D_{AB} \left.\frac{dC_A}{dr}\right|_{r_o} = -D_{AB} \frac{k_1'' C}{k_1'' r_o + D_{AB}}$$

where $$C = \frac{p}{\Re T} = \frac{1\text{ atm}}{\left(8.205 \times 10^{-2} m^3 \cdot atm/kmol \cdot K\right) 1450\text{ K}} = 8.405 \times 10^{-3}\ kmol/m^3.$$

Hence,

$$N_A''(r_o) = -1.71 \times 10^{-4} m^2/s \frac{0.1\,m/s \times 8.405 \times 10^{-3} kmol/m^3}{\left(10^{-4} + 1.71 \times 10^{-4}\right) m^2/s} = -5.30 \times 10^{-4}\ kmol/s \cdot m^2$$

$$N_A(r_o) = 4p\, r_o^2 N_A''(r_o) = 4p\,(0.001\ m)^2 \times 5.30 \times 10^{-4}\ kmol/s \cdot m^2$$

$$N_A(r_o) = 6.66 \times 10^{-9}\ kmol/s.$$

<

COMMENTS: The O_2 consumption rate would increase with increasing k_1'' and approach a limiting *finite* value as k_1'' approaches infinity.

PROBLEM 14.22

KNOWN: Radius of coal particles burning in oxygen atmosphere of prescribed pressure and temperature.

FIND: (a) Radial distributions of O_2 and CO_2, (b) O_2 molar consumption rate.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional, steady-state conditions, (2) Uniform total molar concentration, (3) No homogeneous chemical reactions, (4) Coal is pure carbon, (5) Surface reaction rate is infinite (hence concentration of O_2 at surface, C_A, is zero), (6) Constant D_{AB}, (7) Perfect gas behavior.

PROPERTIES: *Table A-8*, $CO_2 \rightarrow O_2$; D_{AB} (273 K) = 0.14×10^{-4} m^2/s; D_{AB} (1450 K) = D_{AB} (273 K) $(1450/273)^{3/2} = 1.71 \times 10^{-4}$ m^2/s.

ANALYSIS: (a) For the assumed conditions, Eq. 14.53 reduces to

$$\frac{d}{dr}\left(r^2 \frac{dC_A}{dr}\right) = 0$$

$$r^2 \left(dC_A / dr\right) = C_1 \qquad \text{or} \qquad C_A = -\left(C_1 / r\right) + C_2.$$

From the boundary conditions:

$$C_A(\infty) = C \rightarrow \qquad C_2 = C$$

$$C_A(r_o) = 0 \rightarrow \qquad 0 = -C_1 / r_o + C \qquad\qquad C_1 = Cr_o.$$

Hence, recognizing that $C = C_A + C_B$,

$$C_A = C - C\left(r_o / r\right) = C\left(1 - r_o / r\right) \qquad\qquad C_B = C - C_A = C\left(r_o / r\right).$$ <

(b) The conditions correspond to equimolar, counter diffusion $\left(N''_A = -N''_B\right)$, with

$$N_{A,r} = N''_{A,r} 4\pi r^2 = -CD_{AB} 4\pi r^2 \frac{dx_A}{dr} = -D_{AB} 4\pi r^2 \frac{dC_A}{dr} = -4\pi D_{AB} r^2 \left(+\frac{Cr_o}{r^2}\right) = -4\pi D_{AB} Cr_o.$$

With

$$C = \frac{p}{\Re T} = \frac{1 \text{ atm}}{8.205 \times 10^{-2} \text{ m}^3 \cdot \text{atm/kmol} \cdot \text{K} \times 1450 \text{ K}} = 8.405 \times 10^{-3} \text{ kmol/m}^3$$

find

$$N_{A,r} = -1.71 \times 10^{-4} \text{ m}^2/\text{s} \times 4\pi \times 8.405 \times 10^{-3} \text{ kmol/m}^3 \left(10^{-3} \text{ m}\right)$$

$$N_{A,r} = 1.81 \times 10^{-8} \text{ kmol/s}.$$ <

PROBLEM 14.23

KNOWN: Pore geometry in a catalytic reactor. Concentration of reacting species at pore opening and order of catalytic reaction.

FIND: (a) Differential equation which determines concentration of reacting species, (b) Distribution of reacting species concentration along the pore.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional diffusion in x direction, (3) Stationary medium, (4) Uniform total molar concentration.

ANALYSIS: (a) Apply the species conservation requirement to the differential control volume,

$N_{A,x} - k_1'' C_A (\pi D)\, dx - N_{A,x+dx} = 0$, where

$$N_{A,x+dx} = N_{A,x} + \left(dN_{A,x}/dx\right) dx$$

and from Fick's law

$$N_{A,x} = \left(-C D_{AB} \frac{dx_A}{dx}\right)\frac{\pi D^2}{4} = -\frac{\pi D^2}{4} D_{AB} \frac{dC_A}{dx}.$$

Hence

$$-\frac{dN_A}{dx} dx - k_1'' C_A (\pi D)\, dx = \frac{\pi D^2}{4} D_{AB} \frac{d^2 C_A}{dx^2} - k_1'' C_A (\pi D)\, dx = 0$$

$$\frac{d^2 C_A}{dx^2} - \frac{4 k_1''}{D D_{AB}} C_A = 0.$$ <

(b) A solution to the above equation is readily obtained by recognizing that it is of exactly the same form as the energy equation for an extended surface of uniform cross section. Hence for boundary conditions of the form

$$C_A(0) = C_{A,0}, \qquad -D_{AB}\left(dC_A/dx\right)_{x=L} = k_1'' C_A(L)$$

the solution must be analogous to that obtained for a fin with a convection tip condition. With the analogous quantities

$$C_A \leftrightarrow \theta \equiv T - T_\infty, \qquad m \equiv \left(4 k_1''/D D_{AB}\right)^{1/2} \leftrightarrow \left(4h/Dk\right)^{1/2}$$

$$D_{AB} \leftrightarrow k, \qquad k_1'' \leftrightarrow h$$

the solution is, by analogy to Eq. 3.70

$$C_A(x) = \frac{\cosh m(L-x) + \left(k_1''/m D_{AB}\right)\sinh m(L-x)}{\cosh mL + \left(k_1''/m D_{AB}\right)\sinh mL}.$$ <

COMMENTS: The total pore reaction rate is $-D_{AB}(\pi D^2/4)\,(dC_A/dx)_{x=0}$, which can be inferred by applying the analogy to Eq. 3.72.

PROBLEM 14.24

KNOWN: Pressure, temperature and mole fraction of CO in auto exhaust. Diffusion coefficient for CO in gas mixture. Film thickness and reaction rate coefficient for catalytic surface.

FIND: (a) Mole fraction of CO at catalytic surface and CO removal rate, (b) Effect of reaction rate coefficient on removal rate.

SCHEMATIC:

Exhaust gas

$p = 1.2$ bar, $T = 500^oC$, $x_{A,L} = 0.0012$

$L = 0.01$ m

CO ↓ CO_2 ↑ O_2 ↓

Thin film
$D_{AB} = 10^{-4}\ m^2/s$

Catalytic surface
$k_1'' = 0.005$ m/s
$2CO + O_2 \rightarrow 2CO_2$

x

ASSUMPTIONS: (1) Steady-state, (2) One-dimensional species diffusion in film, (3) Negligible effect of advection in film, (4) Constant total molar concentration and diffusion coefficient in film.

ANALYSIS: From Eq. (14.60) the surface molar concentration is

$$x_A(0) = \frac{x_{A,L}}{1 + (Lk_1''/D_{AB})} = \frac{0.0012}{1 + \left(0.01m \times 0.005\,m/s / 10^{-4} m^2/s\right)} = 0.0008$$ <

With C = p/•T = 1.2 bar/(8.314 × 10^{-2} m^3·bar/kmol·K × 773 K) = 0.0187 kmol/m^3, Eq. (14.61) yields a CO molar flux, and hence a CO removal rate, of

$$N''_{A,s} = -N''_A(0) = \frac{k_1'' C x_{A,L}}{1 + (Lk_1''/D_{AB})}$$

$$N''_{A,s} = \frac{0.005\,m/s \times 0.0187\,kmol/m^3 \times 0.0012}{1 + \left(0.01m \times 0.005\,m/s / 10^{-4} m^2/s\right)} = 7.48 \times 10^{-8}\,kmol/s \cdot m^2$$ <

If the process is diffusion limited, $Lk_1''/D_{AB} >> 1$ and

$$N''_{A,s} = \frac{C\ D_{AB}\ x_{A,L}}{L} = \frac{0.0187\,kmol/m^3 \times 10^{-4} m^2/s \times 0.0012}{0.01m} = 2.24 \times 10^{-7}\,kmol/s \cdot m^2$$ <

COMMENTS: If the process is reaction limited, $N''_{A,s} \rightarrow 0$ as $k_1'' \rightarrow 0$.

PROBLEM 14.25

KNOWN: Partial pressures and temperatures of CO_2 at opposite ends of a circular tube which also contains nitrogen.

FIND: Mass transfer rate of CO_2 through the tube.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional diffusion, (3) Uniform temperature and total pressure.

PROPERTIES: *Table A-8*, $CO_2 - N_2$ ($T \approx 298$ K, 1 atm): $D_{AB} = 0.16 \times 10^{-4}\ m^2/s$.

ANALYSIS: From Eq. 14.70 the CO_2 molar transfer rate is

$$N_A = \frac{D_{AB}\left(\pi D^2/4\right)}{\Re T}\frac{p_{A,0} - p_{A,L}}{L}$$

$$N_A = \frac{0.16\times10^{-4}\ m^2/s(\pi/4)(0.05\ m)^2}{0.08205\ m^3\cdot atm/kmol\cdot K\times 298\ K}\frac{(100-50)\ mmHg}{1\ m\times 760\ mmHg/atm}$$

$$N_A = 8.45\times10^{-11}\ kmol/s.$$

The mass transfer rate is then

$$n_A = \mathcal{M}_A N_A = 44 kg/kmol\times 8.45\times10^{-11}\ kmol/s$$

$$n_A = 3.72\times10^{-9} kg/s.$$

<

COMMENTS: Although the molar transfer rate of N_2 in the opposite direction is $N_B = 8.45 \times 10^{-11}$ kmol/s, the mass transfer rate is

$$n_B = \mathcal{M}_B N_B = 28\ kg/kmol\times 8.45\times10^{-11} kmol/s = 2.37\times10^{-9} kg/s.$$

PROBLEM 14.26

KNOWN: Conditions associated with evaporation from a liquid in a column, with vapor (A) transfer occurring in a gas (B). In one case B has unlimited solubility in the liquid; in the other case it is insoluble.

FIND: Case characterized by the largest evaporation rate and ratio of evaporation rates if $p_A = 0$ at the top of the column and $p_A = p/10$ at the liquid interface.

SCHEMATIC:

$N''_{A,x}$ B

Liquid, A

$x = L,\ x_{A,L} = (p_{A,L}/p) = 0$

$p = p_A + p_B$

x

$x = 0,\ x_{A,0} = (p_{A,0}/p) = 0.1$

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional species transfer, (3) Uniform temperature and total pressure in the column, (4) Constant properties.

ANALYSIS: If gas B has unlimited solubility in the liquid, the solution corresponds to equimolar counter diffusion of A and B. From Eqs. 14.63 and 14.68, it follows that

$$N''_{A,x} = -CD_{AB}\frac{dx_A}{dx} = CD_{AB}\frac{x_{A,0} - x_{A,L}}{L}. \qquad (1)$$

If gas B is completely insoluble in the liquid, the diffusion of A is augmented by convection and from Eqs. 14.73 and 14.77

$$N''_{A,x} = -CD_{AB}\frac{dx_A}{dx} + C_A v_x^* = \frac{CD_{AB}}{L}\ln\frac{1 - x_{A,L}}{1 - x_{A,0}}. \qquad (2)$$

Comparing Eqs. (1) and (2), it is obvious that the evaporation rate for the second case exceeds that for the first case. Also

$$\frac{N''_{A,x(sol)}}{N''_{A,x(insol)}} = \frac{(CD_{AB}/L)(x_{A,0} - x_{A,L})}{(CD_{AB}/L)\ln(1 - x_{A,L})/(1 - x_{A,0})} = \frac{0.1 - 0}{\ln[(1-0)/(1-0.1)]}$$

$$\frac{N''_{A,x(sol)}}{N''_{A,x(insol)}} = 0.949.$$

<

COMMENTS: The above result suggests that, since the mole fraction of the saturated vapor is typically small, the rate of evaporation in a column is well approximated by the result corresponding to equimolar counter diffusion.

PROBLEM 14.27

KNOWN: Water in an open pan exposed to prescribed ambient conditions.

FIND: Evaporation rate considering (a) diffusion only and (b) convective effects.

SCHEMATIC:

Air, B; $\phi_\infty = 0.25$, $T_\infty = 27°C$, $p = 1atm$; $C_{A,s}$; Pan, D=0.2m; x; L=80mm; Water, A, 27°C

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional diffusion, (3) Constant properties, (4) Uniform T and p, (5) Perfect gas behavior.

PROPERTIES: *Table A-8*, Water vapor-air (T = 300 K, 1 atm), $D_{AB} = 0.26 \times 10^{-4}\ m^2/s$; *Table A-6*, Water vapor (T = 300 K, 1 atm), $p_{sat} = 0.03513$ bar, $v_g = 39.13\ m^3/kg$.

ANALYSIS: (a) The evaporation rate considering only diffusion follows from Eq. 14.63 simplified for a stationary medium. That is,

$$N_{A,x} = N''_{A,x} \cdot A = -D_{AB} A \frac{dC_A}{dx}.$$

Recognizing that $\phi \equiv p_A/p_{A,sat} = C_A/C_{A,sat}$, the rate is expressed as

$$N_{A,x} = -D_{AB} A \frac{C_{A,\infty} - C_{A,s}}{L} = \frac{D_{AB} A}{L} C_{A,sat}\left(1 - f_\infty\right)$$

$$N_{A,x} = \frac{0.26 \times 10^{-4} m^2/s\,(p/4)(0.2m)^2}{80 \times 10^{-3} m} \frac{1}{39.13\ m^3/kg \times 18\ kg/kmol}(1 - 0.25) = 1.087 \times 10^{-8} kmol/s$$

where $C_{A,s} = 1/\left(v_g M_A\right)$ with $M_A = 18$ kg/kmol.

(b)The evaporation rate considering convective effects using Eq. 14.77 is

$$N_{A,x} = N''_{A,x} \cdot A = \frac{CD_{AB} A}{L} \ln \frac{1 - x_{AL}}{1 - x_{A,0}}.$$

Using the perfect gas law, the total concentration of the mixture is

$$C = p/\Re T = 1.0133\ bar/\left(8.314 \times 10^{-2} m^3 \cdot bar/kmol \cdot K \times 300K\right) = 0.04063\ kmol/m^3$$

where p = 1 atm = 1.0133 bar. The mole fractions at x = 0 and x = L are

$$x_{A,0} = \frac{p_{A,s}}{p} = \frac{0.03531bar}{1.0133bar} = 0.0348 \qquad x_{A,L} = f_\infty x_{A,0} = 0.0087.$$

Hence

$$N_{A,x} = \frac{0.04063 kmol/m^3 \times 0.26 \times 10^{-4} m^2/s\,(p/4)(0.2\ m)^2}{80 \times 10^{-3}\ m} \ln \frac{1 - 0.0087}{1 - 0.0348} = 1.107 \times 10^{-8} kmol/s. \qquad <$$

COMMENTS: For this situation, the convective effect is very small but does tend to increase (by 1.5%) the evaporation rate as expected.

PROBLEM 14.28

KNOWN: Vapor concentrations at ends of a tube used to grow crystals. Presence of an inert gas. Ends are impermeable to the gas. Constant temperature.

FIND: Vapor molar flux and spatial distribution of vapor molar concentration. Location of maximum concentration gradient.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional, steady-state conditions, (2) Constant properties, (3) Constant pressure, hence C is constant.

ANALYSIS: Physical conditions are analogous to those of the evaporation problem considered in Section 14.4.4 with

$C_{A,0} > C_{A,L} \rightarrow$ diffusion of vapor from source to crystal,

$C_{B,L} > C_{B,0} \rightarrow$ diffusion of inert gas from crystal to source,

Impermeable ends $\rightarrow$ absolute flux of species B is zero $\left(N''_{B,x} = 0\right)$; hence $v_{B,x} = 0$.

Diffusion of B from crystal to source must be balanced by advection from source to crystal. The advective velocity is $v_x^* = N''_{A,x}/C$. The vapor molar flux is therefore determined by Eq. 14.77,

$$N''_{A,x} = \frac{CD_{AB}}{L}\ln\left(\frac{1-x_{A,L}}{1-x_{A,0}}\right)$$ <

and the vapor molar concentration is given by Eq. 14.75,

$$x_A = \frac{C_A}{C} = 1-\left(1-x_{A,0}\right)\left(\frac{1-x_{A,L}}{1-x_{A,0}}\right)^{x/L}.$$ <

From Eq. 14.72,

$$\frac{dx_A}{dx} = -N''_{A,x}\left(1-x_A\right)/CD_{AB}$$

$$\frac{dC_A}{dx} = -\frac{N''_{A,x}}{D_{AB}}\left(1-x_A\right).$$

Hence maximum concentration gradient corresponds to minimum x_A and occurs at

$$x = L.$$

COMMENTS: Vapor transfer is enhanced by the advection, which is induced by presence of the inert gas.

PROBLEM 14.29

KNOWN: Spherical droplet of liquid A and radius r_o evaporating into stagnant gas B.

FIND: Evaporation rate of species A in terms of $p_{A,sat}$, partial pressure $p_A(r)$, the total pressure p and other pertinent parameters.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional, radial, species diffusion, (3) Constant properties, including total concentration, (4) Droplet and mixter air at uniform pressure and temperature, (5) Perfect gas behavior.

ANALYSIS: From Eq. 14.31 for a radial spherical coordinate system, the evaporation rate of liquid A into a binary gas mixture A + B is

$$N_{A,r} = -D_{AB} A_r \frac{dC_A}{dr} + \frac{C_A}{C} N_{A,r}$$

where $A_r = 4\pi r^2$ and $N_{A,r} = N_A$, a constant,

$$N_A \left(1 - \frac{C_A}{C}\right) = -D_{AB} \cdot 4\pi r^2 \cdot \frac{dC_A}{dr}.$$

From perfect gas behavior, $C_A = p_A / \Re T$ and $C = p / \Re T$,

$$N_A (p - p_A) = -D_{AB} \cdot 4\pi r^2 \cdot \frac{p}{\Re T} \frac{dp_A}{dr}$$

Separating variables, setting definite limits, and integrating

$$-N_A \frac{\Re T}{p} \frac{1}{4\pi D_{AB}} \int_{r_o}^{r} \frac{dr}{r^2} = \int_{p_{A,r_o}}^{p_{A,r}} \frac{dp_A}{p - p_A}$$

find that

$$N_A = 4\pi r_o D_{AB} \frac{p}{\Re T} \frac{1}{1 - r_o / r} \ln \frac{p - p_A(r)}{p - p_{A,o}}$$ <

where $p_{A,o} = p_A(r_o) = p_{A,sat}$, the saturation pressure of liquid A at temperature T.

COMMENTS: Compare the method of solution and result with the content of Section 14.4.4, Evaporation in a Column.

PROBLEM 14.30

KNOWN: Vent pipe on a methanol distillation system condenser discharges to atmosphere at 1 bar. Cooler and vent at 21°C. Vapor volume of cooler is 0.005 m^3.

FIND: (a) Weekly loss of methanol vapor due to diffusion out the vent pipe and (b) Weekly loss due to expulsion of methanol vapor in the cooler once per hour caused by process heat rate change.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional species transport, (3) Uniform temperature and total pressure in vent pipe, (4) Constant properties, (5) Perfect gas behavior.

PROPERTIES: Methanol-air mixture (given, 273 K): $D_{AB} = 0.13 \times 10^{-4}$ m^2/s.

ANALYSIS: (a) The methanol transfer rate through the vent follows from Eq. 14.77

$$N_{A,x} = N''_{A,x} \cdot A_c = \frac{p D^2}{4} \frac{C D_{AB}}{L} \ln \frac{1 - p_{A,2}/p}{1 - p_{A,1}/p}$$

where $p_{A,2} = 0$ and $p_{A,1} = p_A = 100$ mmHg = 0.1333 bar = 13.3 kPa,

$$C = \frac{p}{\Re T} = \frac{1\text{bar}}{8.314 \times 10^{-2}\,\text{m}^3 \cdot \text{bar/kmol} \cdot \text{K}(21 + 273)\,\text{K}} = 4.093 \times 10^{-2}\,\text{kmol/m}^3$$

$$D_{AB}(294\text{K}) = D_{AB}(273)(294/273)^{3/2} = 0.13 \times 10^{-4}\,\text{m}^2/\text{s}(294/273)^{3/2} = 0.145 \times 10^{-4}\,\text{m}^2/\text{s}.$$

Substituting numerical values, find the rate on a weekly basis as

$$N_A = \frac{p\,(0.035\text{ m})^2}{4} \times 4.093 \times 10^{-2}\,\text{kmol/m}^3 \times \frac{0.145 \times 10^{-4}\,\text{m}^2/\text{s}}{0.5\text{ m}} \ln \frac{1 - 0}{1 - 0.1333/1}$$

$$\times 3600\,\text{s/h} \times 24\,\text{h/day} \times 7\,\text{day/week} = 9.883 \times 10^{-5}\,\text{kmol/week}$$

$$m_A = N_A M_A = 9.883 \times 10^{-5}\,\text{kmol/week} \times 32\text{ kg/kmol} = 0.00316\text{ kg/week.}$$ <

(b) The methanol vapor in the cooler of volume 0.005 m^3 is expelled once per hour, so that the additional mass loss is $m_A = n_A M_A$, where n_A is

$$n_A = \frac{p_A V}{\Re T} = \frac{0.1333\text{bar} \times 0.005\text{m}^3}{8.314 \times 10^{-2}\,\text{m}^3 \cdot \text{bar/kmol} \cdot \text{K} \times 294\text{ K}} = 2.728 \times 10^{-5}\,\text{kmol}$$

from which it follows that

$$m_A = 2.728 \times 10^{-5}\,\text{kmol/} \times 24 \times 7 \times 32\text{ kg/kmol} = 0.1467\text{ kg/week.}$$ <

COMMENTS: Note that the loss through the vent is approximately 2% that lost by expulsion when the process heat rate is varied.

PROBLEM 14.31

KNOWN: Clean surface with pure steam has condensate rate of 0.020 $kg/m^2 \cdot s$ for the prescribed conditions. With the presence of stagnant air in the steam, the condensate surface drops from 28°C to 24°C and the condensate rate is halved.

FIND: Partial pressure of air in the air-steam mixture as a function of distance from the condensate film.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties including pressure in air-steam mixture, (3) Perfect gas behavior.

PROPERTIES: *Table A-6,* Water vapor: p_{sat} (28°C = 301 K) = 0.03767 bar; p_{sat} (24°C = 297 K) = 0.02983 bar; *Table A-8*, Water-air (298 K, 1 bar): $D_{AB} = 0.26 \times 10^{-4}\ m^2/s$.

ANALYSIS: The partial pressure distribution of the air as a function of distance y can be found from the species (A) rate expression, Eq. 14.77,

$$N''_{A,y} = \left(CD_{AB}/y\right)\ln\left(1-x_{A,y}\right)/\left(1-x_{A,0}\right).$$

With $C = p/\Re T$, $x_{B,y} = 1 - x_{A,y}$ and $x_{B,0} = 1 - x_{A,0}$, recognizing that $x_B = p_B/p$, find

$$p_B(y) = p_{B,0}\cdot\exp\left(N''_{A,y}\frac{\Re T}{pD_{AB}}y\right)$$

$$p_{B,0} = p - p_{A,0} = p_{sat}(28°C) - p_{sat}(24°C) = (0.03767 - 0.02983)\text{bar} = 0.00784\ \text{bar}.$$

With $N''_{A,y} = -(0.020/2)\,kg/m^2\cdot s/28\,kg/kmol = 3.57\times10^{-4}\,kmol/m^2\cdot s$,

$$p_B(y) = 0.0784\ \text{bar}\times\exp\left(3.57\times10^{-4}\text{kmol}/\text{m}^2\cdot\text{s}\,\frac{8.314\times10^{-2}\text{m}^3\cdot\text{bar}/\text{kmol}\cdot\text{K}\times299\ \text{K}}{0.03767\ \text{bar}\times6.902\times10^{-4}\text{m}^2/\text{s}}\right)$$

$$p_B(y) = 784\ \text{kPa}\times\exp(-0.3415y)$$

with p_B in [kPa] and y in [mm], where T = 26°C = 299 K, the average temperature of the air-steam mixture, and $D_{AB} \approx p^{-1}\,T^{3/2} = 0.26 \times 10^{-4}\ m^2/s\ (1/0.03767)\ (299/298)^{3/2} = 6.902 \times 10^{-4}\ m^2/s$. Selected values for the pressure are shown below and the distribution is shown above:

y (mm)	0	5	10	15
$p_B(y)$ (kPa)	784	142	25.8	4.7

COMMENTS: To minimize inert gas effects, the usual practice is to pass vapor over the surfaces so that the inerts are eventually collected near the outlet region of the condenser. Our estimate shows that the effective region to be swept is approximately 10 mm thick.

PROBLEM 14.32

KNOWN: Column containing liquid phase of water (A) evaporates into the air (B) flowing over the mouth of the column.

FIND: Evaporation rate of water ($kg/h\cdot m^2$) using the known value of the binary diffusion coefficient for the water vapor - air mixture.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state, one-dimensional diffusion in the column, (2) Constant properties, (3) Uniform temperature and pressure throughout the column, (4) Water vapor exhibits ideal gas behavior, and (5) Negligible water vapor in the chamber air.

PROPERTIES: *Table A-6, water* (T = 320 K): p_{sat} = 0.1053 bar; *Table A-8, water vapor-air* (0.25 atm, 320 K): Since $D_{AB} \sim p^{-1} T^{3/2}$ find

$$D_{AB} = 0.26\times10^{-4}\ m^2/s\,(1.00/0.25)\,(320/298)^{3/2} = 1.157\times10^{-4}\ m^2/s$$

ANALYSIS: Equimolar counter diffusion occurs in the vertical column as water vapor, evaporating at the liquid-vapor interface (x = 0), diffuses up the column through air out into the chamber. From Eq. 14.7, the molar flow rate per unit area is

$$N''_{A,x} = \frac{C\,D_{AB}}{L}\ln\frac{1-x_{A,L}}{1-x_{A,0}}$$

where C is the mixture concentration determined from the ideal gas law as

$$C = \frac{p}{\mathcal{R}_u T} = \frac{0.25\ atm}{8.205\times10^{-2}\ m^3\cdot atm/kmol\cdot K\times320\ K} = 0.009397\ kmol/m^3$$

where $\mathcal{R}_u = 8.205\times10^{-2}\ m^3\cdot atm/kmol\cdot K$. The mole fractions at x = 0 and x = L are

$$x_{A,L} = 0 \qquad \text{(no water vapor in air above column)}$$

$$x_{A,0} = p_A/p = 0.1053/0.25 = 0.4212$$

where p_A is the saturation pressure for water at T = 320 K. Substituting numerical values

$$N''_{A,x} = \frac{0.009397\ kmol/m^3\times1.157\times10^{-4}\ m^2/s}{0.150\ m}\ln\frac{(1-0)}{(1-0.4212)}$$

$$N''_{A,x} = 3.964\times10^{-6}\ kmol/m^2\cdot s$$

or, on a mass basis,

$$m''_{A,x} = N''_{A,x}\mathcal{M}_A$$

$$m''_{A,x} = 3.964\times10^{-6}\ kmol/m^2\cdot s\times3600\ s/h\times18\ kg/kmol$$

$$m''_{A,x} = 0.257\ kg/m^2\cdot h$$ <

PROBLEM 14.33

KNOWN: Ground level flux of NO_2 in a stagnant urban atmosphere.

FIND: (a) Vertical distribution of NO_2 molar concentration, (b) Critical ground level flux of NO_2, $N''_{A,0,crit}$.

SCHEMATIC:

$k_1 = 0.03 s^{-1}$
$D_{AB} = 0.15 \times 10^{-4} m^2/s$
$p_A(0)_{crit} = 2 \times 10^{-6} bar$

Air (B) T=300K
$\dot{N}_A = -k_1 C_A$
x
NO_2 (A) emissions, $N''_{A,0}$

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional diffusion in a stationary medium, (3) Total molar concentration C is uniform, (4) Perfect gas behavior.

ANALYSIS: (a) For the prescribed conditions the molar concentration of NO_2 is given by Eq. 14.80, subject to the following boundary conditions.

$$C_A(\infty) = 0, \qquad \left.\frac{dC_A}{dx}\right)_{x=0} = -\frac{N''_{A,0}}{D_{AB}}.$$

From the first condition, $C_1 = 0$. From the second condition,

$$-mC_2 = -N''_{A,0}/D_{AB}.$$

Hence

$$C_A(x) = \frac{N''_{A,0}}{mD_{AB}} e^{-mx}$$ <

where $m = (k_1/D_{AB})^{1/2}$.

(b) At ground level, $C_A(0) = \frac{N''_{A,0}}{mD_{AB}}$. Hence, from the perfect gas law,

$$p_A(0) = C_A(0)\Re T = \frac{\Re T N''_{A,0}}{mD_{AB}}.$$

Hence, with $m = (0.03/0.15 \times 10^{-4})^{1/2}\ m^{-1} = 44.7\ m^{-1}$.

$$N''_{A,0,crit} = \frac{mD_{AB}p_A(0)_{crit}}{\Re T} = \frac{44.7 m^{-1} \times 0.15 \times 10^{-4} m^2/s \times 2 \times 10^{-6}\ bar}{8.314 \times 10^{-2} m^3 \cdot bar/kmol \cdot K \times 300\ K}$$

$$N''_{A,0,crit} = 5.38 \times 10^{-11}\ kmol/s \cdot m^2.$$ <

COMMENTS: Because the dispersion of pollutants in the atmosphere is governed strongly by convection effects, the above model should be viewed as a first approximation which describes a worst case condition.

PROBLEM 14.34

KNOWN: Radius of a spherical organism and molar concentration of oxygen at surface. Diffusion and reaction rate coefficients.

FIND: (a) Radial distribution of O_2 concentration, (b) Rate of O_2 consumption, (c) Molar concentration at r = 0.

SCHEMATIC:

$C_A(r_o) = C_{A,o} = 5 \times 10^{-5}$ kmol/m^3

$r_o = 10^{-4}$ m

Organism
$k_o = 1.2 \times 10^{-4}$ kmol/s-m^3
$D_{AB} = 10^{-8}$ m^2/s

ASSUMPTIONS: (1) Steady-state, one-dimensional diffusion, (2) Stationary medium, (3) Uniform total molar concentration, (4) Constant properties (k_0, D_{AB}).

ANALYSIS: (a) For the prescribed conditions and assumptions, Eq. (14.40) reduces to

$$\frac{D_{AB}}{r^2}\frac{d}{dr}\left(r^2\frac{dC_A}{dr}\right) - k_0 = 0$$

$$r^2\frac{dC_A}{dr} = \frac{k_0 r^3}{3D_{AB}} + C_1$$

$$C_A = \frac{k_0 r^2}{6D_{AB}} - \frac{C_1}{r} + C_2$$

With the requirement that $C_A(r)$ remain finite at r = 0, $C_1 = 0$. With $C_A(r_o) = C_{A,o}$

$$C_2 = C_{A,o} - \frac{k_0 r_o^2}{6D_{AB}}$$

$$C_A = C_{A,o} - \left(k_0 / 6D_{AB}\right)\left(r_o^2 - r^2\right) \qquad <$$

Because C_A cannot be less than zero at any location within the organism, the right-hand side of the foregoing equation must always exceed zero, thereby placing limits on the value of $C_{A,o}$. The smallest possible value of $C_{A,o}$ is determined from the requirement that $C_A(0) \geq 0$, in which case

$$C_{A,o} \geq \left(k_0 r_o^2 / 6D_{AB}\right) \qquad <$$

(b) Since oxygen consumption occurs at a uniform volumetric rate of k_0, the total respiration rate is $\dot{R} = \forall k_0$, or

$$\dot{R} = (4/3)\pi r_o^3 k_0 \qquad <$$

Continued …..

(c) With r = 0,

$$C_A(0) = C_{A,o} - k_0 r_o^2 / 6 D_{AB}$$

$$C_A(0) = 5\times10^{-5}\ \text{kmol/m}^3 - 1.2\times10^{-4}\ \text{kmol/s}\cdot\text{m}^3 \left(10^{-4}\ \text{m}\right)^2 / 6\times10^{-8}\ \text{m}^2/\text{s}$$

$$C_A(0) = 3\times10^{-5}\ \text{kmol/m}^3$$ <

COMMENTS: (1) The minimum value of $C_{A,o}$ for which a physically realistic solution is possible is $C_{A,o} = k_0 r_o^2 / 6 D_{AB} = 2\times10^{-5}\ \text{kmol/m}^3$.

(2) The total respiration rate may also be obtained by applying Fick's law at r = r_o, in which case $\dot{R} = -N_A(r_o) = +D_{AB}\left(4\pi r_o^2\right) dC_A/dr\big|_{r=r_o} = D_{AB}\left(4\pi R_o^2\right)\left(k_o / 6 D_{AB}\right) 2r_o = (4/3)\pi r_o^3 k_0$.
The result agrees with that of part (b).

PROBLEM 14.35

KNOWN: Radius of a spherical organism and molar concentration of oxygen at its surface. Diffusion and reaction rate coefficients.

FIND: (a) Radial distribution of O_2 concentration, (b) Expression for rate of O_2 consumption, (c) Molar concentration at r = 0 and rate of oxygen consumption for prescribed conditions.

SCHEMATIC:

$C_A(r_o) =$
$C_{A,o} = 5\text{x}10^{-5}$ kmol/m^3

$r_o = 10^{-4}$ m

Organism
$k_1 = 20\ s^{-1}$
$D_{AB} = 10^{-8}\ m^2/s$

ASSUMPTIONS: (1) Steady-state, one-dimensional diffusion, (2) Stationary medium, (3) Uniform total molar concentration, (4) Constant properties (k_1, D_{AB}).

ANALYSIS: (a) For the prescribed conditions and assumptions, Eq. (14.40) reduces to

$$\frac{1}{r^2}\frac{d}{dr}\left(D_{AB}\, r^2 \frac{dC_A}{dr}\right) - k_1 C_A = 0$$

With $y \equiv r\, C_A$, $d\, C_A/dr = (1/r)\, dy/dr - y/r^2$ and

$$\frac{1}{r^2}\frac{d}{dr}\left(D_{AB}\, r^2 \frac{dC_A}{dr}\right) = \frac{D_{AB}}{r^2}\frac{d}{dr}\left(r\frac{dy}{dr} - y\right) = \frac{D_{AB}}{r^2}\left(r\frac{d^2y}{dr^2}\right)$$

The species equation is then

$$\frac{d^2y}{dr^2} - \frac{k_1}{D_{AB}}\, y = 0$$

The general solution is of the form

$$y = C_1 \sinh\left(k_1/D_{AB}\right)^{1/2} r + C_2 \cosh\left(k_1/D_{AB}\right)^{1/2} r$$

or

$$C_A = \frac{C_1}{r}\sinh\left(k_1/D_{AB}\right)^{1/2} r + \frac{C_2}{r}\cosh\left(k_1/D_{AB}\right)^{1/2} r$$

Because C_A must remain finite at r = 0, $C_2 = 0$. Hence, with $C_A\,(r_o) = C_{A,o}$,

$$C_1 = \frac{C_{A,o}\, r_o}{\sinh\left(k_1/D_{AB}\right)^{1/2} r_o}$$

and

Continued …..

$$C_A = C_{A,o}\left(\frac{r_o}{r}\right)\frac{\sinh\left(k_1/D_{AB}\right)^{1/2} r}{\sinh\left(k_1/D_{AB}\right)^{1/2} r_o}$$ <

(b) The total O_2 consumption rate corresponds to the rate of diffusion at the surface of the organism.

$$\dot{R} = -N_A\left(r_o\right) = +D_{AB}\left(4\pi r_o^2\right) dC_A/dr\Big|_{r_o}$$

$$\dot{R} = 4\pi r_o^2 D_{AB}\, C_{A,o}\, r_o\left[-\frac{1}{r_o^2} + \frac{1}{r_o}\left(k_1/D_{AB}\right)^{1/2}\cot\left(k_1/D_{AB}\right)^{1/2} r_o\right]$$

$$\dot{R} = 4\pi r_o\, D_{AB}\, C_{A,o}\left(\alpha\coth\alpha - 1\right)$$ <

where $\alpha \equiv \left(k_1 r_o^2/D_{AB}\right)^{1/2}$.

(c) For the prescribed conditions, $(k_1/D_{AB})^{1/2} = (20\ s^{-1} \div 10^{-8}\ m^2/s)^{1/2} = 44{,}720\ m^{-1}$ and $\alpha = 4.472$.

$$C_A = \frac{5\times10^{-5}\,\text{kmol/m}^3\times10^{-4}\,\text{m}}{\sinh(4.472)}\times\frac{\sinh\left(k_1/D_{AB}\right)^{1/2} r}{r} = 1.136\times10^{-10}\,\frac{\text{kmol}}{\text{m}^3}\times\frac{\sinh\left(k_1/D_{AB}\right)^{1/2} r}{r}$$

In the limit of $r \to 0$, the foregoing expression yields

$$C_A\left(r\to 0\right) = 5.11\times10^{-6}\,\text{kmol/m}^3$$ <

$$\dot{R} = 4\pi\times10^{-4}\,\text{m}\times10^{-8}\,\text{m}^2/\text{s}\times5\times10^{-5}\,\text{kmol/m}^3\left(4.472\coth 4.472 - 1\right)$$

$$= 2.18\times10^{-15}\,\text{kmol/s}$$

COMMENTS: The total respiration rate may also be obtained by integrating the volumetric rate of consumption over the volume of the organism. That is, $\dot{R} = -\int \dot{N}_A\, d\forall = \int_0^{r_o} k_1 C_A(r) 4\pi r^2 dr$.

PROBLEM 14.36

KNOWN: Radius and catalytic reaction rate of a porous spherical pellet. Surface mole fraction of reactant and effective diffusion coefficient.

FIND: (a) Radial distribution of reactant concentration in pellet, total reactant consumption rate, and pellet effectiveness, (b) CO consumption rate and effectiveness for prescribed conditions.

SCHEMATIC:

$N_{Ar}(r_o)$; $r_o = 2.5$ mm; $x_{A,s} = 0.04$; CuO coated alumina pellet, $A_v = 10^8\ m^2/m^3$, $k_1' = 10^{-3}$ m/s; $D_{eff} = 2\times10^{-5}\ m^2/s$; $T = 550°C$, $p = 1.2$ atm

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional radial diffusion, (3) Constant properties, (4) Homogeneous chemical reactions, (5) Isothermal, constant pressure conditions within pellet, (6) Stationary medium.

ANALYSIS: (a) In spherical coordinates, the mass diffusion equation is given by

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(CD_{AB}r^2\frac{\partial x_A}{\partial r}\right)+\dot{N}_A = 0$$

where C, D_{AB} are constant and $\dot{N}_A = -k_1' A_v C_A$. Hence

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dx_A}{dr}\right)-\frac{k_1' A_v}{D_{eff}}x_A = 0.$$

The boundary conditions are $x_A(r_o) = x_{A,s}$ and $x_A(0)$ is finite. Transform the dependent variable, $y \equiv rx_A$, with

$$\frac{dx_A}{dr}=\frac{1}{r}\frac{dy}{dr}-\frac{y}{r^2} \qquad \text{or} \qquad \frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dx_A}{dr}\right)=\frac{1}{r^2}\frac{d}{dr}\left(r\frac{dy}{dr}-y\right)=\frac{1}{r^2}\left(r\frac{d^2y}{dr^2}\right).$$

Hence

$$\frac{d^2y}{dr^2}-\frac{k_1' A_v}{D_{eff}}y = 0.$$

The general solution is of the form

$$y = C_1\sinh(ar)+C_2\cosh(ar)$$

where $a \equiv \left(k_1' A_v / D_{eff}\right)^{1/2}$ giving

$$x_A = \frac{C_1}{r}\sinh(ar)+\frac{C_2}{r}\cosh(ar)$$

and using the boundary conditions,

$$x_A(0)\,\text{finite} \rightarrow C_2 = 0 \qquad\qquad x_A(r_o) = x_{A,s} \rightarrow C_1 = x_{A,s}r_o/\sinh(ar_o).$$

Continued …..

PROBLEM 14.36 (Cont.)

Hence

$$x_A(r) = x_{A,s}(r_o/r)\frac{\sinh(ar)}{\sinh(ar_o)}.$$ <

Applying conservation of species to a control volume about the pellet, $\dot{N}_{A,in} + \dot{N}_{A,g} = 0$, the total rate of consumption of A in the pellet is

$$-\dot{N}_{A,g} = \dot{N}_{A,in} = N_{A,r}(r_o) = 4\pi r_o^2 J^*_{A,r}(r_o).$$

Hence

$$N_{A,r}(r_o) = \left(4\pi r_o^2\right)\left(-CD_{eff}\frac{dx_A}{dr}\right)_{r=r_o} = \frac{4\pi r_o^3}{\sinh(ar_o)}CD_{eff}\,x_{A,s}\left[\frac{\sinh(ar)}{r^2} - \frac{\cosh(ar)}{r^2}\right]_{r=r_o}$$

$$N_{A,r}(r_o) = 4\pi r_o CD_{eff}\,x_{A,s}\left[1 - \frac{ar_o}{\tanh(ar_o)}\right].$$ <

The pellet effectiveness ε is defined as $\varepsilon \equiv N_{A,r}(r_o)/[N_{A,r}(r_o)]_{max}$ and the maximum consumption occurs if $x_A(r) = x_{A,s}$ for all $0 \le r \le r_o$. Hence

$$\left[N_{A,r}(r_o)\right]_{max} = \dot{N}_A V_p = -k_1' A_v C x_{A,s}\frac{4}{3}\pi r_o^3$$

$$\varepsilon = -\frac{3}{a^2 r_o^2}\left[1 - \frac{ar_o}{\tanh(ar_o)}\right].$$ <

(b) To evaluate the rate, first determine values for these parameters:

$$C = \frac{p}{\Re T} = \frac{1.2\,\text{atm}}{0.08205\ \text{m}^3\cdot\text{atm/kmol}\cdot\text{K}\times 823\ \text{K}} = 0.0178\ \text{kmol/m}^3$$

$$a = \left(\frac{k_1' A_v}{D_{eff}}\right)^{1/2} = \left(\frac{10^{-3}\ \text{m/s}\times 10^8\ \text{m}^2/\text{m}^3}{2\times 10^{-5}\ \text{m}^2/\text{s}}\right)^{1/2} = 7.07\times 10^4\,\text{m}^{-1}$$

$$ar_o = 176.8 \qquad \tanh(ar_o) = 1.$$

Hence the consumption rate is

$$N_{A,r}(r_o) = 4\pi(0.0025\ \text{m})\,0.0178\ \text{kmol/m}^3\times 2\times 10^{-5}\,\text{m}^2/\text{s}\times 0.04(1 - 176.8)$$

$$N_{A,r}(r_o) = -7.86\times 10^{-8}\ \text{kmol/s}$$ <

and the effectiveness is

$$\varepsilon = -\frac{3}{\left(7.07\times 10^4\ \text{m}^{-1}\right)^2(0.0025\ \text{m})^2}[1 - 176.8] = 0.0169$$ <

COMMENTS: For the range of conditions of interest, $\varepsilon \approx 3/ar_o$. Hence ε may be increased by $\downarrow r_o, \downarrow k_1', \downarrow A_v$ and $\uparrow D_{eff}$. However, $N_{A,r}(r_o)$ would decrease with $\downarrow r_o, \downarrow k_1'$ and $\downarrow A_v$.

PROBLEM 14.37

KNOWN: Molar concentrations of oxygen at inner and outer surfaces of lung tissue. Volumetric rate of oxygen consumption within the tissue.

FIND: (a) Variation of oxygen molar concentration with position in the tissue, (b) Rate of oxygen transfer to the blood per unit tissue surface area.

SCHEMATIC:

$C_A(0)$ — k_o, D_{AB} — $C_A(L)$; x from 0 to L; Lung tissue. Oxygen (A), Tissue (B)

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional species transfer by diffusion through a plane wall, (3) Homogeneous, stationary medium with uniform total molar concentration and constant diffusion coefficient.

ANALYSIS: (a) From Eq. 14.78 the appropriate form of the species diffusion equation is

$$D_{AB}\frac{d^2C_A}{dx^2} - k_o = 0.$$

Integrating,

$$dC_A/dx = (k_o/D_{AB})x + C_1 \qquad C_A = \frac{k_o}{2D_{AB}}x^2 + C_1 x + C_2.$$

With $C_A = C_A(0)$ at $x = 0$ and $C_A = C_A(L)$ at $x = L$,

$$C_2 = C_A(0) \qquad C_1 = \frac{C_A(L) - C_A(0)}{L} - \frac{k_o L}{2D_{AB}}.$$

Hence

$$C_A(x) = \frac{k_o}{2D_{AB}}x(x-L) + \left[C_A(L) - C_A(0)\right]\frac{x}{L} + C_A(0).$$ <

(b) The oxygen assimilation rate per unit area is

$$N''_{A,x}(L) = -D_{AB}(dC_A/dx)_{x=L}$$

$$N''_{A,x}(L) = -D_{AB}\left(\frac{k_o L}{D_{AB}} - \frac{k_o L}{2D_{AB}}\right) - \frac{D_{AB}}{L}\left[C_A(L) - C_A(0)\right]$$

$$N''_{A,x} = -\frac{k_o L}{2} + \frac{D_{AB}}{L}\left[C_A(0) - C_A(L)\right].$$ <

COMMENTS: The above model provides a highly approximate and simplified treatment of a complicated problem. The lung tissue is actually heterogeneous and conditions are transient.

PROBLEM 14.38

KNOWN: Combustion at constant temperature and pressure of a hydrogen-oxygen mixture adjacent to a metal wall according to the reaction $2H_2 + O_2 \rightarrow 2H_2O$. Molar concentrations of hydrogen, oxygen, and water vapor are 0.10, 0.10 and 0.20 $kmol/m^3$, respectively. Generation rate of water vapor is 0.96×10^{-2} $kmol/m^3 \cdot s$.

FIND: (a) Expression for C_{H_2} as function of distance from wall, plot qualitatively, (b) C_{H_2} at the wall, (c) Sketch also curves for $C_{O_2}(x)$ and $C_{H_2O}(x)$, and (d) Molar flux of water at x = 10mm.

SCHEMATIC:

Wall
$2H_2 + O_2 \rightarrow 2H_2O$
$\dot{N}_{H_2O} = 0.96 \times 10^{-2}$ $kmol/m^3 \cdot s$
$C_{H_2} = 0.10 kmol/m^3$
$C_{O_2} = 0.10 kmol/m^3$
$C_{H_2O} = 0.20 kmol/m^3$
0 10 x(mm)

$C_A (kmol/m^3)$
0.30
0.20
0.10
C_{H_2O}
C_{O_2}
C_{H_2}
0 10 y(mm)

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional diffusion, (3) Stationary mixture, (4) Constant properties including pressure and temperature.

PROPERTIES: Species binary diffusion coefficient (given, for H_2, O_2 and H_2O): $D_{AB} = 0.6 \times 10^{-5}$ m^2/s.

ANALYSIS: (a) The species conservation equation, Eq. 14.38b, and its general solution are

$$\frac{d^2C_A}{dx^2} + \frac{\dot{N}_A}{D_{AB}} = 0 \qquad\qquad C_A(x) = -\frac{\dot{N}_A}{2D_{AB}}x + C_1x + C_2. \qquad (1,2)$$

The boundary condition at the wall must be $dC_A(0)/dx = 0$, such that $C_1 = 0$. For the species hydrogen, evaluate C_2 from knowledge of $C_{H_2}(10\text{ mm}) = 0.10\text{ kmol/m}^3$ and $\dot{N}_{H_2} = -\dot{N}_{H_2O}$, according to the chemical reaction. Hence,

$$0.10\text{ kmol/m}^3 = -\frac{\left(-0.96\times10^{-2}\text{kmol/m}^3\cdot\text{s}\right)}{2\times0.6\times10^{-5}\text{m}^2/\text{s}}(0.010\text{ m})^2 + 0 + C_2$$

$$C_2 = 0.02\text{ kmol/m}^3.$$

Hence, the hydrogen species concentration distribution is

$$C_{H_2}(x) = -\frac{\dot{N}_{H_2}}{2D_{AB}}x^2 + 0.02 = 800x^2 + 0.02$$ <

which is parabolic with zero slope at the wall; see sketch above.

(b) The value of C_{H_2} at the wall is,

$$C_{H_2}(0) = (0 + 0.02)\text{kmol/m}^3 = 0.02\text{ kmol/m}^3.$$ <

Continued …..

PROBLEM 14.38 (Cont.)

(c) The concentration distribution for water vapor species will be of the same form,

$$C_{H_2O}(x) = -\frac{\dot{N}_{H_2O}}{2D_{AB}} x^2 + C_1 x + C_2 \qquad (3)$$

With $C_1 = 0$ for the wall condition, find C_2 from $C_{H_2O}(10\text{ mm})$,

$$0.20\ \text{kmol/m}^3 = -\frac{\left(0.96\times10^{-2}\ \text{kmol/m}^3\right)}{2\times0.6\times10^{-5}\ \text{m}^2/\text{s}}\left(0.010\ \text{m}\right)^2 + C_2 \qquad C_2 = 0.28\,\text{kmol/m}^3.$$

Hence, C_{H_2O} at the wall is,

$$C_{H_2O}(0) = 0 + 0 + C_2 = 0.28\ \text{kmol/m}^3$$

and its distribution appears as above. Recognizing that $\dot{N}_{O_2} = -0.5\dot{N}_{H_2O}$, by the same analysis, find

$$C_{O_2}(0) = 0.06\ \text{kmol/m}^3$$

and its shape, also parabolic with zero slope at the wall is shown above.

(d) The molar flux of water vapor at x = 10 mm is given by Fick's law

$$N''_{H_2O,x} = -D_{AB}\frac{dC_{H_2O}}{dx}$$

and using the concentration distribution of Eq. (3), find

$$N''_{H_2O,x} = -D_{AB}\frac{d}{dx}\left(-\frac{\dot{N}_{H_2O}}{2D_{AB}}x^2\right) = +\dot{N}_{H_2O}x$$

and evaluation at the location x = 10 mm, the species flux is

$$N''_{H_2O,x}(10\ \text{mm}) = +\left(0.96\times10^{-2}\ \text{kmol/m}^3\cdot\text{s}\right)\times0.010\ \text{m} = 9.60\times10^{-5}\text{kmol/m}^2\cdot\text{s}.$$ <

COMMENTS: Note that the generation rate of water vapor is a positive quantity. Whereas for H_2 and O_2, species are consumed and hence $\dot{N}_{H_2}$ and $\dot{N}_{O_2}$ are negative. According to the chemical reaction one mole of H_2 and 0.5 mole of O_2 are consumed to generate one mole of H_2O. Therefore, $\dot{N}_{H_2} = -\dot{N}_{H_2O}$ and $\dot{N}_{O_2} = -0.5\ \dot{N}_{H_2O}$.

PROBLEM 14.39

KNOWN: Ground level flux of NO_2 in a stagnant urban atmosphere.

FIND: (a) Governing differential equation and boundary conditions for the molar concentration of NO_2, (b) Concentration of NO_2 at ground level three hours after the beginning of emissions.

SCHEMATIC:

$x+dx$, $N''_{A,x+dx}$, $(-k_1C_A)dx$, $(\partial C_A/\partial t)dx$, x, $N''_{A,x}$

Air, x, $N''_{A,o}=3\times10^{-11}\ \frac{kmol}{s\cdot m^2}$, $D_{AB}=0.15\times10^{-4}\ m^2/s$

ASSUMPTIONS: (1) One-dimensional diffusion in a stationary medium, (2) Uniform total molar concentration, (3) Constant properties.

ANALYSIS: (a) Applying the species conservation requirement, Eq. 14.33, on a molar basis to a unit area of the control volume,

$$N''_{A,x} - (k_1 C_A)dx - N''_{A,x+dx} = \frac{\partial C_A}{\partial t}dx.$$

With $N''_{A,x+dx} = N''_{A,x} + (\partial N''_{A,x}/\partial x)dx$ and $N''_{A,x} = -D_{AB}(\partial C_A/\partial x)$, it follows that

$$D_{AB}\frac{\partial^2 C_A}{\partial x^2} - k_1 C_A = \frac{\partial C_A}{\partial t}.$$ <

Initial Condition: $C_A(x,0) = 0.$ <

Boundary Conditions: $-D_{AB}\left.\frac{\partial C_A}{\partial x}\right)_{x=0} = N''_{A,0}$, $\qquad C_A(\infty, t) = 0.$ <

(b) The present problem is analogous to Case (2) of Fig. 5.7 for heat conduction in a semi-infinite medium. Hence by analogy to Eq. 5.59, with $k \leftrightarrow D_{AB}$ and $\alpha \leftrightarrow D_{AB}$,

$$C_A(x,t) = 2N''_{A,0}\left(\frac{t}{\pi D_{AB}}\right)^{1/2}\exp\left(-\frac{x^2}{4D_{AB}t}\right) - \frac{N''_{A,0}x}{D_{AB}}\operatorname{erfc}\left(\frac{x}{2(D_{AB}t)^{1/2}}\right)$$

At ground level (x = 0) and 3h,

$$C_A(0,3h) = 2N''_{A,0}\left(\frac{t}{\pi D_{AB}}\right)^{1/2}$$

$$C_A(0,3h) = 2\left(3\times10^{-11}\ \text{kmol/s}\cdot\text{m}^2\right)\left(10{,}800\,\text{s}/\pi\times0.15\times10^{-4}\,\text{m}^2/\text{s}\right)^{1/2} = 9.08\times10^{-7}\ \text{kmol/m}^3.$$ <

COMMENTS: The concentration decays rapidly to zero with increasing x, and at x = 100 m it is, for all practical purposes, equal to zero.

PROBLEM 14.40

KNOWN: Carbon dioxide concentration at water surface and reaction rate constant.

FIND: (a) Differential equation which governs variation with position and time of CO_2 concentration in water, (b) Appropriate boundary conditions and solution for a deep body of water with negligible chemical reactions.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional diffusion in x, (2) Constant properties, including total density ρ, (3) Water is stagnant.

ANALYSIS: (a) From Eq. 14.37b, it follows that, for the prescribed conditions,

$$D_{AB}\frac{\partial^2 \rho_A}{\partial x^2} - k_1 \rho_A = \frac{\partial \rho_A}{\partial t}.$$ <

The first term on the left-hand side represents the *net* transport of CO_2 into a differential control volume by diffusion. The second term represents the rate of CO_2 consumption due to chemical reactions. The term on the right-hand side represents the rate of increase of CO_2 storage within the control volume.

(b) For a deep body of water, appropriate boundary conditions are

$$\rho_A(0,t) = \rho_{A,0}$$

$$\rho_A(\infty,t) = 0$$

and, with negligible chemical reactions, the species diffusion equation reduces to

$$\frac{\partial^2 \rho_A}{\partial x^2} = \frac{1}{D_{AB}}\frac{\partial \rho_A}{\partial t}.$$

With an initial condition, $\rho_A(x,0) \equiv \rho_{A,i} = 0$, the problem is analogous to that involving heat transfer in a semi-infinite medium with constant surface temperature. By analogy to Eq. 5.57, the species concentration is then

$$\frac{\rho_A(x,t) - \rho_{A,0}}{-\rho_{A,0}} = \operatorname{erf}\left(\frac{x}{2(D_{AB}t)^{1/2}}\right)$$

$$\rho_A(x,t) = \rho_{A,0}\operatorname{erfc}\left(\frac{x}{2(D_{AB}t)^{1/2}}\right).$$ <

PROBLEM 14.41

KNOWN: Initial concentration of hydrogen in a sheet of prescribed thickness. Surface concentrations for time t > 0.

FIND: Time required for density of hydrogen to reach prescribed value at midplane of sheet.

SCHEMATIC:

(A) → H_2
(B) → Solid
$D_{AB} = 9 \times 10^{-7} m^2/s$

L = 20 mm

x

$C_A(x,0) = 3 \text{ kmol/m}^3 = C_{A,i}$
$C_A(0,t_f) = 1.2 \text{ kg/m}^3 / 2 \text{ kg/kmol}$
$C_A(0,t_f) = 0.6 \text{ kmol/m}^3 = C_A$
$C_A(20 \text{ mm},t) = 0 = C_{A,s}$

ASSUMPTIONS: (1) One-dimensional diffusion in x, (2) Constant D_{AB}, (3) No internal chemical reactions, (4) Uniform total molar concentration.

ANALYSIS: Using Heisler chart with heat and mass transfer analogy

$$g^* = \frac{C_A - C_{A,s}}{C_{A,i} - C_{A,s}} = \frac{0.6 - 0}{3.0 - 0} = 0.2 = g_o^*$$

With $Bi_m = \infty$, Fig. D.1 may be used with

$$q_o^* = 0.2, \qquad Bi^{-1} = 0$$
$$Fo \approx 0.75.$$

Hence

$$Fo_m = \frac{D_{AB} t_f}{L^2} = 0.75$$

$$t_f = 0.75 (0.02 \text{ m})^2 / 9 \times 10^{-7} \text{m}^2/\text{s}$$

$$t_f = 333 \text{s}.$$

<

COMMENTS: If the one-term approximation to the infinite series solution

$$q^* = \sum_{n=1}^{\infty} C_n \exp\left(-V_n^2 Fo\right) \cos\left(V_n x^*\right)$$

is used, it follows that

$$g_o^* \approx C_1 \exp\left(-V_1^2 Fo_m\right) = 0.2$$

Using values of $V_1 = 1.56$ and $C_1 = 1.27$, it follows that

$$\exp\left[-(1.56)^2 Fo_m\right] = 0.157$$

$$Fo_m = 0.76$$

which is in excellent agreement with the result from the chart.

PROBLEM 14.42

KNOWN: Sheet material has high, uniform concentration of hydrogen at the end of a process, and is then subjected to an air stream with a specified, low concentration of hydrogen. Mass transfer parameters specified include: convection mass transfer coefficient, h_m, and the mass diffusivity and solubility of hydrogen (A) in the sheet material (B), D_{AB} and S_{AB}, respectively.

FIND: (a) The final mass density of hydrogen in the material if the sheet is exposed to the air stream for a very long time, $\rho_{A,f}$, (b) Identify and evaluate the parameter that can be used to determine whether the transient mass diffusion process in the sheet can be characterized by a uniform concentration at any time; *Hint*: this situation is analogous to the lumped capacitance method for a transient heat transfer process; (c) Determine the time required to reduce the hydrogen concentration to twice the limiting value calculated in part (a).

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional diffusion, (2) Material B is stationary medium, (3) Constant properties, (4) Uniform temperature in air stream and material, and (5) Ideal gas behavior.

ANALYSIS: (a) The final content of H_2 in the material will depend upon the solubility of H_2 (A) in the material (B) and its partial pressure in the free stream. From Eq. 14.44,

$$C_{A,f} = S_{AB}\, p_{A,\infty} = 160\ \text{kmol/m}^3\cdot\text{atm}\times 0.1\ \text{atm} = 16\ \text{kmol/m}^3$$

$$\rho_f = \mathcal{M}_A\, C_{A,f} = 2\ \text{kg/kmol}\times 16\ \text{kmol/m}^3 = 32\ \text{kg/m}^3$$ <

(b) The parameters associated with transient diffusion in the material follow from the analogous treatment of Section 5.2 (Fig. 5.3) and are represented in the schematic.

In the material, from Fick's law, the diffusive flux is

$$N''_{A,dif} = D_{AB}\left(C_{A,1} - C_{A,2}\right)/L \tag{1}$$

At the surface, x = L, the rate equation, Eq. 6.8, convective flux of species A is

$$N''_{A,conv} = h_m\left(C_{A,s} - C_{A,\infty}\right)$$

Continued …..

and substituting the ideal gas law, Eq. 14.9, and introducing the solubility relation, Eq. 14.44,

$$N''_{A,conv} = \frac{h_m}{S_{AB}\, \mathcal{R}_u\, T_\infty}\left(S_{AB}\, p_{A,s} - S_{AB}\, p_{A,\infty}\right)$$

$$N''_{A,conv} = \frac{h_m}{S_{AB}\, \mathcal{R}_u\, T_\infty}\left(C_{2,s} - C_{A,\infty}\right) \qquad (2)$$

where $C_{A,\infty} = C_{A,f}$, the final concentration in the material after exposure to the air stream a long time. Considering a surface species flux balance, as shown in the schematic above, with the rate equations (1) and (2),

$$\frac{D_{AB}\left(C_{A,1} - C_{A,2}\right)}{L} = \frac{h_m}{S_{AB}\, \mathcal{R}_u\, T_\infty}\left(C_{A,s} - C_{A,f}\right)$$

$$\frac{C_{A,1} - C_{A,2}}{C_{A,s} - C_{A,f}} = \frac{h_m / S_{AB}\, \mathcal{R}_u\, T_\infty}{D_{AB} / L} = \frac{\mathcal{R}''_{m,dif}}{\mathcal{R}''_{m,conv}} = Bi_m \qquad (3)$$

and introducing resistances to species transfer by diffusion, Eq. 14.51, and convection. Recognize from the analogy to heat transfer, Eq. 5.10 and Table 14.2, that when $Bi_m < 0.1$, the concentration can be characterized as uniform during the transient process. That is, the diffusion resistance is negligible compared to the convection resistance,

$$Bi_m = \frac{h_m L}{S_{AB}\, \mathcal{R}_u\, T_\infty\, D_{AB}} < 0.1 \qquad (4)$$

$$Bi_m = \frac{\left(1.5\ \text{m/h} \times 3600\ \text{s/h}\right) \times 0.003\ \text{m}}{160\ \text{kmol/m}^3 \cdot \text{atm} \times 8.205 \times 10^{-2}\ \text{m}^3 \cdot \text{atm/kmol} \cdot \text{K} \times 555\ \ \text{K} \times 2.68 \times 10^{-8}\ \ \text{m}^2/\text{s}}$$

$$Bi_m = 6.60 \times 10^{-3} \quad < \quad 0.1$$

Hence, the mass transfer process can be treated as a nearly uniform concentration situation. From conservation of species on the material with uniform concentration,

$$-N''_{A,conv} = \dot{N}''_{A,st}$$

$$-\frac{h_m}{S_{AB}\, \mathcal{R}_u\, T_\infty}\left(C_A - C_{A,f}\right) = L\ \frac{d\, C_A}{dt}$$

Integrating, with the initial condition $C_A(0) = C_{A,i}$, find

$$\frac{C_A - C_{A,f}}{C_{A,i} - C_{A,f}} = \exp\left(-\frac{h_m\, t}{L\, S_{AB}\, \mathcal{R}_u\, T_\infty}\right) \qquad (5) \quad <$$

Continued …..

PROBLEM 14.42 (Cont.)

which is similar to the analogous heat transfer relation for the lumped capacitance analysis, Eq. 5.6.

(c) The time, t_o, required for the material to reach a concentration twice that of the limiting value, $C_A\ (T_o) = 2\ C_{A,f}$, can be calculated from Eq. (5).

$$\frac{(2-1)\times 16\ \text{kmol/m}^3}{(320-16)\ \text{kmol/m}^3} = \exp\left(-\frac{1.5\ \text{m/h}\times t_o}{0.003\ \text{m}\times 160\ \text{kmol/m}^3\cdot\text{atm}\times 8.205\times 10^{-2}\ \text{m}^3\cdot\text{atm/kmol}\cdot\text{K}\times 555\ \text{K}}\right)$$

$$t_o = 42.9\ \text{hour}$$ <

PROBLEM 14.43

KNOWN: Hydrogen-removal process described in Problem 14.3 (S), but under conditions for which the mass diffusivity of hydrogen gas (A) in the sheet (B) is $D_{AB} = 1.8 \times 10^{-11}$ m^2/s (instead of 2.6×10^{-8} m^2/s). With a smaller D_{AB}, a uniform concentration condition may no longer be assumed to exist in the material during the removal process.

FIND: (a) The final mass density of hydrogen in the material if the sheet is exposed to the air stream for a very long time, $\rho_{A,f}$, (b) Identify and evaluate the parameters that describe the transient mass transfer process in the sheet; *Hint*: this situation is analogous to that of transient heat conduction in a plane wall; (c) Assuming a uniform concentration in the sheet at any time during the removal process, determine the time required to reach twice the limiting mass density calculated in part (a); (d) Using the analogy developed in part (b), determine the time required to reduce the hydrogen concentration to twice the limiting value calculated in part (a); Compare the result with that from part (c).

SCHEMATIC:

A - hydrogen
B - sheet matl

A + B

$C_A(t)$, $C_A(0) = C_{A,i} = 320$ kmol/m^3
$D_{AB} = 1.8 \times 10^{-11}$ m^2/s
$S_{AB} = 160$ kmol/m^3-atm
T = 555 K

$h_m = 1.5$ m/s
$p_{A,\infty} = 0.1$ atm
$p_\infty = 3$ atm
$T_\infty = 555$ K

A + Air

x

L = 3 mm

ASSUMPTIONS: (1) One-dimensional diffusion, (2) Material B is a stationary medium, (3) Constant properties, (4) Uniform temperature in air stream and material, and (5) Ideal gas behavior.

ANALYSIS: (a) The final content of H_2 in the material will depend upon the solubility of H_2 (A) in the material (B) at its partial pressure in the free stream. From Eq. 14.44,

$$C_{A,f} = S_{AB}\, p_{A,\infty} = 160 \text{ kmol/m}^3 \cdot \text{atm} \times 0.1 \text{ atm} = 16 \text{ kmol/m}^3$$

$$\rho_f = \mathcal{M}_A\, C_{A,f} = 2 \text{ kg/kmol} \times 16 \text{ kmol/m}^3 = 32 \text{ kg/m}^3$$ <

(b) For the plane wall shown in the schematic below, the heat and mass transfer conservation equations and their initial and boundary conditions are

Heat transfer

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$$

$$T(x,0) = T_i$$

$$\frac{\partial T}{\partial x}(0,t) = 0$$

$$-k\frac{\partial T}{\partial x}(L,t) = h\left[T(L,t) - T_\infty\right]$$

Mass (Species A) transfer

$$\frac{\partial C_A}{\partial t} = D_{AB}\frac{\partial^2 C_A}{\partial x^2}$$

$$C_A(x,0) = C_{A,i}$$

$$\frac{\partial C_A}{\partial x}(0,t) = 0$$

$$-D_{AB}\frac{\partial C_A}{\partial x}(L,t) = \frac{h_m}{S_{AB}\,\mathcal{R}_u\, T}\left[C_A(x,t) - C_f\right]$$

Continued …..

PROBLEM 14.43 (Cont.)

The derivation for the species transport surface boundary condition is developed in the solution for Problem 14.3 (S). The solution to the mass transfer problem is identical to the analogous heat transfer problem provided the transport coefficients are represented as

$$\frac{h}{k} <=> \frac{h_m / S_{AB} \mathcal{R}_u T}{D_{AB}} \tag{1}$$

(c) The uniform concentration transient diffusion process is analogous to the heat transfer lumped-capacitance process. From the solution of Problem 14.3 (S), the time to reach twice the limiting concentration, $C_A (t_o) = 2 C_{A,f,}$ can be calculated as

$$\frac{C_A(t_o) - C_{A,f}}{C_{A,i} - C_{A,f}} = \exp\left(- \frac{h_m t_o}{L S_{AB} \mathcal{R}_u T} \right) \tag{2}$$

$$t_o = 42.9 \text{ hour}$$ <

For the present situation, the mass transfer Biot number is

$$Bi_m = \frac{h_m L}{S_{AB} \mathcal{R}_u T D_{AB}}$$

$$Bi_m = \frac{(1.5 \text{ m/h} / 3600 \text{ s/h}) \times 0.003 \text{ m}}{160 \text{ kmol/m}^3 \cdot \text{atm} \times 8.205 \times 10^{-2} \text{ m}^3 \cdot \text{atm/kmol} \cdot \text{K} \times 555 \text{ K} \times 1.8 \times 10^{-11} \text{m}^2 \text{/s}}$$

$$Bi_m = 9.5 \;\; >> \;\; 0.1$$

and hence the concentration of A within B is not uniform

(d) Invoking the analogy with the heat transfer situation, we can use the one-term series solution, Eq. 5.40, with $Bi_m <=> Bi$ and

$$Fo_m <=> Fo \qquad\qquad Fo_m = \frac{D_{AB} t}{L^2} \tag{3}$$

Continued …..

PROBLEM 14.43 (Cont.)

With Bi_m = 9.5, find ζ_1 = 1.4219 rad and C_1 = 1.2609 from Table 5.1, so that Eq. 5.41 becomes

$$\frac{C_A\left(t_o\right)-C_{A,f}}{C_{A,i}-C_{A,f}}=C_1\exp\left(-\varsigma_1^2\ Fo_m\right)$$

$$\frac{(2-1)\times 16\ \text{kmol/m}^3}{(320-16)\text{kmol/m}^3}=1.2609\ \exp\left(-1.4219^2\ Fo_m\right)$$

$$Fo_m=\frac{1.8\times 10^{-11}\ \text{m}^2/\text{s}\times t_o}{(0.003\ \text{m})^2}=1.571$$

$$t_o = 218\ \text{hour}$$ <

COMMENTS: (1) Since Bi_m = 9.5, the uniform concentration assumption is not valid, and we expect the analysis to provide a longer time estimate to reach $C_A(t_o)$ = 2 $C_{A,f}$.

(2) Note that the uniform concentration analysis model of part (c) does not include D_{AB}. Why is this so?

PROBLEM 14.44

KNOWN: Radius and temperature of air bubble in water.

FIND: Time to reach 99% of saturated vapor concentration at center.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional radial diffusion of vapor in air, (2) Constant properties, (3) Air is initially dry.

PROPERTIES: *Table A-8*, Water vapor-air (300 K): $D_{AB} = 0.26 \times 10^{-4}\ m^2/s$.

ANALYSIS: Use Heisler charts with heat and mass transfer analogy,

$$g^* \equiv \frac{C_A - C_{A,s}}{C_{A,i} - C_{A,s}} = 1 - \frac{C_A}{C_{A,s}}.$$

For $g_o^* = 1 - 0.99 = 0.01$ and $Bi_m^{-1} = 0$, from Fig. D.7 find $Fo_m \approx 0.52$. Hence with

$$Fo_m = \frac{D_{AB} t}{r_o^2} = 0.52$$

$$t = 0.52\left(10^{-6}\ m^2\right)/0.26\times10^{-4}\ m^2/s = 0.02 s. \qquad <$$

COMMENTS: (1) This estimate is likely to be conservative, since shear driven motion of air within the bubble would enhance vapor transport from the surface to the center.

(2) If the one-term approximation to the infinite series solution,

$$q^* = \sum_{n=1}^{\infty} C_n \exp\left(-V_n^2 Fo\right) \frac{\sin\left(V_n r^*\right)}{V_n r^*}$$

is used, it follows that with sin 0/0 = 1,

$$g_o^* \approx C_1 \exp\left(-z_1^2 Fo_m\right) = 0.1.$$

Using values of $C_1 = 2.0$ and $V_1 = 3.11$ for $Bi_m = 100$, it follows that

$$0.01 = 2.0 \exp\left[-(3.11)^2 Fo_m\right] \qquad \text{or} \qquad Fo_m = 0.55$$

which is in reasonable agreement with the Heisler chart result.

PROBLEM 14.45

KNOWN: Initial carbon content and prescribed surface content for heated steel.

FIND: Time required for carbon mole fraction to reach 0.01 at a distance of 1 mm from the surface.

SCHEMATIC:

Carbon (A)

Steel (B), $x_A(x,0) = x_{A,i} = 0.001$

$x_{A,s} = 0.02$

$x_A(0.001m, t) = 0.01$

$D_{AB} = D_{c-s} = 2\times10^{-5} \exp[-17{,}000/T(K)]$

x

ASSUMPTIONS: (1) Steel may be approximated as a semi-infinite medium, (2) One-dimensional diffusion in x, (3) Isothermal conditions, (4) No internal chemical reactions, (5) Uniform total molar concentration.

ANALYSIS: Conditions within the steel are governed by the species diffusion equation of the form

$$\frac{\partial^2 C_A}{\partial x^2} = \frac{1}{D_{AB}} \frac{\partial C_A}{\partial t}$$

or, in molar form,

$$\frac{\partial^2 x_A}{\partial x^2} = \frac{1}{D_{AB}} \frac{\partial x_A}{\partial t}.$$

The initial and boundary conditions are of the form

$$x_A(x,0) = 0.001$$

$$x_A(0,t) = x_{A,s} = 0.02 \qquad x_A(\infty,t) = 0.001.$$

The problem is analogous to that of heat transfer in a semi-infinite medium with constant surface temperature, and by analogy to Eq. 5.57, the solution is

$$\frac{x_A(x,t) - x_{A,s}}{x_{A,i} - x_{A,s}} = \text{erf}\left(\frac{x}{2(D_{AB}t)^{1/2}}\right)$$

where

$$D_{AB} = 2\times10^{-5} \exp[-17{,}000/1273] = 3.17\times10^{-11} \text{m}^2/\text{s}.$$

Hence

$$\frac{0.01 - 0.02}{0.001 - 0.02} = 0.526 = \text{erf}\left(\frac{0.001\text{m}}{2\left(3.17\times10^{-11}t\right)^{1/2}}\right)$$

where erf w = 0.526 $\rightarrow$ w $\approx$ 0.51,

$$0.51 = 0.001/2\left(3.17\times10^{-11}t\right)^{1/2} \qquad \text{or} \qquad t = 30{,}321\text{s} = 8.42 \text{ h.}$$ <

PROBLEM 14.46

KNOWN: Thick plate of pure iron at 1000°C subjected to a carburizing process with sudden exposure to a carbon concentration $C_{C,s}$ at the surface.

FIND: (a) Consider the heat transfer analog to the carburization process; sketch the mass and heat transfer systems; explain correspondence between variables; provide analytical solutions to the mass and heat transfer situation; (b) Determine the carbon concentration ratio, $C_C(x, t)/C_{C,s}$, at a depth of 1 mm after 1 hour of carburization; and (c) From the analogy, show that the time dependence of the mass flux of carbon into the plate can be expressed as $n''_C = \rho_{C,s}\left(D_{C-Fe}/\pi t\right)^{1/2}$; also, obtain an expression for the mass of carbon per unit area entering the iron plate over the time period t.

SCHEMATIC:

Carburizing gas

Iron: $C_{C,i} = C_C(x, t \le 0) = 0$, $D_{C\text{-}Fe}$

$C_C(0, t > 0) = C_{C,s}$; $j''_{C,x}(x,t)$

Mass transfer, x

Plate: $T(x, t \le 0) = T_i$; ρ, c, k, α

$T(0, t > 0) = T_s$; $q''_x(x,t)$

Heat transfer, x

ASSUMPTIONS: (1) One-dimensional transient diffusion, (2) Thick plate approximates a semi-infinite medium for the transient mass and heat transfer processes, and (3) Constant properties.

ANALYSIS: (a) The analogy between the carburizing mass transfer process in the plate and the heat transfer process is illustrated in the schematic above. The basis for the mass - heat transfer analogy stems from the similarity of the conservation of species and energy equations, the general solution to the equations, and their initial and boundary conditions. For both processes, the plate is a semi-infinite medium with initial distributions, $C_C(x, t \le 0) = C_{C,i} = 0$ and $T(x, t \le 0) = T_i$, suddenly subjected to a surface potential, $C_C(0, t > 0) = C_{C,s}$ and $T(0, t > 0) = Ts$. The heat transfer situation corresponds to Case 1, Section 5.7, from which the following relations were obtained.

Mass transfer	***Heat transfer***
Rate equation	
$j''_C = -D_{AB}\dfrac{\partial C_c}{\partial x}$	$q''_x = -k\dfrac{\partial T}{\partial x}$
Diffusion equation	
$\dfrac{\partial}{\partial x}\left(\dfrac{\partial C_C}{\partial x}\right) = \dfrac{1}{D_{AB}}\dfrac{\partial C_C}{\partial t}$ [14.84]	$\dfrac{\partial}{\partial x}\left(\dfrac{\partial T}{\partial x}\right) = \dfrac{1}{\alpha}\dfrac{\partial T}{\partial t}$ [2.15]
Polential distribution	
$\dfrac{C_C(x, t) - C_{C,s}}{0 - C_{C,s}} =$	$\dfrac{T(x, t) - T_s}{T_i - T_s} = \text{erf}\left(\dfrac{x}{2(\alpha t)^{1/2}}\right)$ [5.58]
$\dfrac{C_C(x, t)}{C_{C,s}} = \text{erfc}\left(\dfrac{x}{2(D_{AB}\,t)^{1/2}}\right)$	

Continued …..

Flux

See Part (c) $$q''_s(t) = \frac{k\,(T_s - T_i)}{(\pi\alpha t)^{1/2}} \qquad [5.58]$$

(b) Using the concentration distribution expression above, with L = 1 mm, t = 1 h and $D_{AB} = 3 \times 10^{-11}\ m^2/s$, find the concentration ratio,

$$\frac{C_C\ (1\ mm,\ 1\ h)}{C_{C,s}} = erfc\left(\frac{0.001\ m}{2\left(3\times10^{-11}\ m^2/s\times3600\ s\right)^{1/2}}\right) = 0.0314 \qquad <$$

(c) From the heat flux expression above, the mass flux of carbon can be written as

$$n''_{C,s} = \frac{D_{C-Fe}\ (\rho_{C,s} - 0)}{(\pi\, D_{C-Fe}\ t)^{1/2}} = \rho_{c,s}\ (D_{C-Fe}/\pi\, t)^{1/2} \qquad <$$

The mass per unit area entering the plate over the time period follows from the integration of the rate expression

$$m''_C\ (t) = \int_0^t n''_{C,s}\ dt = \rho_{C,s}\ (D_{AB}/\pi)^{1/2} \int_0^t t^{-1/2}\ dt = 2\ \rho_{C,s}\ (D_{C-Fe}\ t/\pi)^{1/2}$$

PROBLEM 14.47

KNOWN: Thickness, initial condition and bottom surface condition of a water layer.

FIND: (a) Time to reach 25% of saturation at top, (b) Amount of salt transfer in that time, (c) Final concentration of salt solution at top and bottom.

SCHEMATIC:

$\rho_A(0) = 0.25\rho_{A,s}$

B- water
A- salt

$D_{AB} = 1.2\times10^{-9}\, m^2/s$

$L = 1m$

N_A

$\rho_{A,i} = 0$

$\rho_{A,s} = 380 kg/m^3 = \rho_A(x=L)$

ASSUMPTIONS: (1) One-dimensional diffusion, (2) Uniform total mass density, (3) Constant D_{AB}.

ANALYSIS: (a) With constant ρ and D_{AB} and no homogeneous chemical reactions, Eq. 14.37b reduces to

$$\frac{\partial^2 \rho_A}{\partial x^2} = \frac{1}{D_{AB}}\frac{\partial \rho_A}{\partial t}.$$

with the origin of coordinates placed at the top of the layer, the dimensionless mass density is

$$\theta^*\left(x^*, Fo_m\right) = \frac{\theta}{\theta_i} = \frac{\rho_A - \rho_{A,s}}{\rho_{A,i} - \rho_{A,s}} = 1 - \frac{\rho_A}{\rho_{A,s}}$$

Hence, $\theta^*\left(0,\ Fo_{m,1}\right) = 1 - 0.25 = 0.75.$ The initial condition is $\theta^*\left(x^*, 0\right) = 1,$ and the boundary conditions are

$$\left.\frac{\partial \theta^*}{\partial x^*}\right|_{x^*=0} = 0 \qquad \theta^*(1,\ Fo_m) = 0$$

where the condition at $x^* = 1$ corresponds to $Bi_m = \infty$. Hence, the mass transfer problem is analogous to the heat transfer problem governed by Eq. 5.34 to 5.37. Assuming applicability of a one-term approximation ($Fo_m > 0.2$), the solution is analogous to Eq. 5.40.

$$\theta^* = C_1 \exp\left(-\zeta_1^2 Fo_m\right)\cos\left(\zeta_1 x^*\right).$$

With $Bi_m = \infty$, $\zeta_1 = \pi/2 = 1.571$ rad and, from Table 5.1, $C_1 \approx 1.274$. Hence, for $x^* = 0$,

$$0.75 = 1.274 \exp\left[-(1.571)^2\, Fo_{m,1}\right]$$

$$Fo_{m,1} = -\ln\left(0.75/1.274\right)/\left(1.571\right)^2 = 0.215.$$

Hence,

$$t_1 = \frac{L^2}{D_{AB}} Fo_{m,1} = \frac{(1\ m)^2}{1.2\times10^{-9}\ m^2/s}\, 0.215 = 1.79\times10^8\, s = 2071\, days. \qquad <$$

Continued …..

PROBLEM 14.47 (Cont.)

(b) The change in the salt mass within the water is

$$\Delta M_A = M_A(t_1) - M_{A,i} = \int (r_A - r_{A,i})\, dV = A\int_0^L r_A dx$$

Hence,

$$\Delta M''_A = r_{A,s}\int_0^L (r_A / r_{A,s})\, dx$$

$$\Delta M''_A = r_{A,s} L \int_0^1 \left(1 - g^*\right) dx^*$$

$$\Delta M''_A = r_{A,s} L \int_0^1 \left[1 - C_1 \exp\left(-V_1^2 Fo_m\right)\cos\left(V_1 x^*\right)\right] dx^*$$

$$\Delta M''_A = r_{A,s} L \left[1 - C_1 \exp\left(-V_1^2 Fo_m\right)\sin V_1 / V_1\right].$$

Substituting numerical values,

$$\Delta M''_A = 380\ kg/m^3 (1\ m)\left[1 - \frac{1.274 \exp\left[-(1.571)^2\, 0.215\right]1}{1.571 rad}\right]$$

$$\Delta M''_A = 198.7\ kg/m^2. \qquad <$$

(c) Steady-state conditions correspond to a uniform mass density in the water. Hence,

$$r_A(0,\infty) = r_A(L,\infty) = \Delta M''_A / L = 198.7\ kg/m^3. \qquad <$$

COMMENTS: (1) The assumption of constant ρ is weak, since the density of salt water depends strongly on the salt composition.

(2) The requirement of $Fo_m > 0.2$ for the one-term approximation to be valid is barely satisfied.

PROBLEM 14.48

KNOWN: Temperature distribution expression for a semi-infinite medium, initially at a uniform temperature, that is suddenly exposed to an instantaneous amount of energy, $Q_o'' \left(J/m^2\right)$.

Analogous situation of a silicon (Si) wafer with a 1-μm layer of phosphorous (P) that is placed in a furnace suddenly initiating diffusion of P into Si.

FIND: (a) Explain the correspondence between the variables in the analogous temperature and concentration distribution expressions, and (b) Determine the mole fraction of P at a depth of 0.1 mm in the Si after 30 s.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional, transient diffusion, (2) Wafer approximates a semi-infinite medium, (3) Uniform properties, and (4) Diffusion process for Si and P is initiated when the wafer reaches the elevated temperature as a consequence of the large temperature dependence of the diffusion coefficient.

PROPERTIES: *Given in statement*: $D_{P\text{-}Si} = 1.2 \times 10^{-17}\ m^2/s$; Mass densities of Si and P: 2000 and 2300 kg/m^3; Molecular weights of Si and P: 30.97 and 28.09 kg/kmol.

ANALYSIS: (a) For the thermal process illustrated in the schematic, the temperature distribution is

$$T(x,\ t) - T_i = \frac{Q_o''}{\rho c(\pi \alpha t)^{1/2}} \exp\left(-x^2 / 4\alpha t\right) \qquad \text{(HT)}$$

where T_i is the initial, uniform temperature of the medium. For the mass transfer process, the P concentration has the form

$$C_P(x,\ t) = \frac{M_{P,o}''}{\left(\pi D_{P-Si}\ t\right)^{1/2}} \exp\left(-x^2 / 4\ D_{P-Si}\ t\right) \qquad \text{(MT)}$$

where $M_{P,o}''$ is the molar area density ($kmol/m^2$) of P represented by the film of concentration C_P and thickness d_o.

The correspondence between mass and heat transfer variables in the equations HT and MT involves the following conditions. The LHS represents the increase with time of the temperature or concentration above the initial uniform distribution. The initial concentration is zero, so only the C_P (x, t) appears. On the RHS note the correspondence of the terms in the exponential parenthesis and in the denominator. The thermal diffusivity and diffusion coefficient are directly analogous; this can be seen by comparing the MT and HT diffusion equations, Eq. 2.15 and 14.84. The terms $Q_o'' / \rho c$ and $M_{P,o}''$ for HT and MT represent the energy and mass instantaneously appearing at the surface. The product ρc is the thermal capacity per unit area and appears in the storage term of the HT diffusion equation. For MT, the "capacity" term is the volume itself.

Continued …..

PROBLEM 14.48 (Cont.)

(b) The molar area density (kmol/m^2) of P associated with the film of thickness d_o = 1 μm and concentration $C_{P,o}$ is

$$M''_{P,o} = C_{P,o} \cdot d_o = (\rho_P / \mathcal{M}_P) d_o$$

$$M''_{P,o} = \left(2000 \text{ kg/m}^3 / 30.97 \text{ kmol/kg}\right) \times 1 \times 10^{-6} \text{ m}$$

$$M''_{P,o} = 6.458 \times 10^{-5} \text{ kmol/m}^2$$

Substituting numerical values into the MT equation, find

$$C_p(0.1 \text{ mm}, 30 \text{ s}) = \frac{6.458 \times 10^{-5} \text{ kmol/m}^2}{\left(\pi \times 1.2 \times 10^{-17} \text{ m}^2/\text{s} \times 30 \text{ s}\right)} \exp - (0.0001 \text{ m})^2 / \left(4 \times 1.2 \times 10^{-7} \text{ m}^2/\text{s} \times 30 \text{ s}\right)\Big]$$

$$C_p = 0.08188 \text{ kmol/m}^3$$

The mole fraction of P in the Si wafer is

$$x_P = C_P / C_{Si} = C_P / (\rho_{Si} / \mathcal{M}_{Si})$$

$$x_P = 0.08188 \text{ kmol/m}^3 / \left(2300 \text{ kg/m}^3 / 28.09 \text{ kmol/kg}\right)$$

$$x_P = 2.435 \times 10^{-5}$$ <